Q1. frame {B} is obtained ~~by~~ from {A} by (1) rotation about the ~~across~~ A-frame x-axis by θ, then (2) a translation by $t = [2\,3\,4]^T$ expressed in the {A} frame.

$$^{A}p = {}^{A}T_{B}\,{}^{B}p,$$

so $^{A}T_{B}$ maps coordinates in {B} to coordinates in {A}.

(a) $^{A}T_{B}$

Rotation about the x-axis by θ:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

The Homogeneous transformation (rotation then translation t expressed in A):

$$^{A}T_{B} = \begin{bmatrix} R_x(\theta) & t \\ 000 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & \cos\theta & -\sin\theta & 3 \\ 0 & \sin\theta & \cos\theta & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) $^{B}T_{A}$

The inverse of a rigid body homogeneous transform is:

$$^{B}T_{A} = \begin{bmatrix} R_x(\theta)^T & -R_x(\theta)^T t \\ 000 & 1 \end{bmatrix}$$

Since $R_x(\theta)^T = R_x(-\theta)$

$$^{B}T_{A} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & \cos\theta & \sin\theta & -(0\cdot 2 + \cos\theta\cdot 3 + \sin\theta\cdot 4) \\ 0 & -\sin\theta & \cos\theta & -(0\cdot 2 + \sin\theta\cdot 3 + \cos\theta\cdot 4) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
{}^{B}T_{A} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & \cos\theta & \sin\theta & -(3\cos\theta + 4\sin\theta) \\ 0 & -\sin\theta & \cos\theta & 3\sin\theta - 4\cos\theta \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

(c) $\theta = \pi/3$ (60°) ${}^{A}p = [1\ 2\ 3]^T$ Compute ${}^{B}p$

With $\theta = \pi/3$

$\cos\theta = 0.5$, $\sin\theta = \frac{\sqrt{3}}{2} \approx 0.8660$

from relation ${}^{A}p = {}^{A}T_{B}\,{}^{B}p$ we get ${}^{B}p = {}^{B}T_{A}\,{}^{A}p$. A convinient form is
${}^{B}p = R_x(-\theta)({}^{A}p - t)$

Compute ${}^{A}p - t = [1-2,\ 2-3,\ 3-4]^T$
$= [-1, -1, -1]^T$

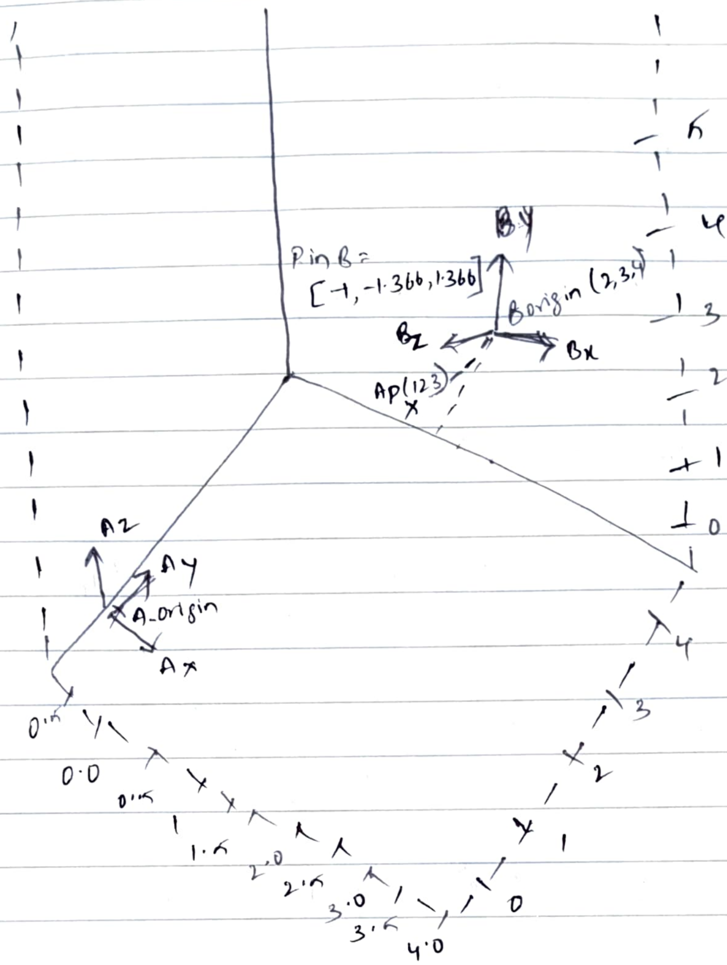
Multiply by $R_x(-\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$

$$
{}^{B}p = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.8660 \\ 0 & -0.8660 & 0.5 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -0.5 - 0.8660 \\ 0.8660 - 0.5 \end{bmatrix} \approx \begin{bmatrix} -1.0 \\ -1.3660 \\ 0.3660 \end{bmatrix}
$$

So ${}^{B}p \approx [-1, -1.366, 0.366]^T$

(d)

(d)



Q2. Frame {A} → (rotate by about A_y by θ) → {B} → (rotate about B_z by ϕ) → {C}. Find 3×3 rotation matrix ${}^{A}R_{C}$ that gives:

$$ {}^{A}p = {}^{A}R_{C}\,{}^{C}p. $$

Because rotation compose ; ${}^{A}R_{C} = {}^{A}R_{B}\,{}^{B}R_{C}$

Here ${}^{A}R_{B} = R_y(\theta)$; ${}^{B}R_{C} = R_z(\theta)$

$$ R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \qquad R_z(\theta) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

$$A_{R_C} = \begin{bmatrix} \cos\theta\cos\phi & -\cos\theta\sin\phi & \sin\theta \\ \sin\phi & \cos\phi & 0 \\ -\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{bmatrix}$$

This 3×3 matrix maps coordinates from frame {C} to {A} as requested.

Q3. Assumptions & coordinate conventions

1. Base frame {0}: origin at the robot.
   Axes: x_0 points towards the table, y_0 points to the robot's right, z_0 points upward.
2. Table: the near corner {1} has origin at top front of the table, with axes parallel to {0}. Its origin position in {0} is $^0O_1 = [1.0, 0.0, 1.0]^T$
3. The table top is 1m above ground and is 1m × 1m.
   The cube is placed centered on the table.
   The cube's frame {2} is located on the center of the cube's bottom face.
   So $^0O_2 = [1.5, 0.5, 1.0]^T$ m.
4. The camera frame {3} is located directly above the cube center. 2.0m above the table top, so $^0O_3 = [1.5, 0.5, 3.0]^T$ m.
   The camera points downwards to match camera measurement convention in part (c), we take the camera z_3 access pointing down (positive down). The camera x_3, y_3 axes are aligned with base x_0, y_0.

(a) Find $^2T_3$

$$^2p = {}^2T_3 \, {}^3p$$

under the conventions :

→ the camera origin expressed in cube coordinates is at $[0, 0, 2]^T$

→ the rotation from camera frame to cube frame is a flip of Z: $R_{2 \to 3} = \text{diag}(1, 1, -1)$.

$$ {}^{2}T_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(b) ${}^{0}T_{1}$, ${}^{0}T_{2}$, ${}^{0}T_{3}$

1. Frame 1 (table at corner $[1.0, 0.0, 1.0]$)

$$ {}^{0}T_{1} = \begin{bmatrix} 1 & 0 & 0 & 1.0 \\ 0 & 1 & 0 & 0.0 \\ 0 & 0 & 1 & 1.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

2. Frame 2 (cube bottom face center at $[1.5, 0.5, 1.0]$

$$ {}^{0}T_{2} = \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 1 & 1.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

/ /

3. Frame (3) (Camera a [1.5, 0.5, 3.0] with Camera z flipped

$$ {}^{0}T_{3} = \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & -1 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}p = {}^{0}T_{i}\, {}^{i}p . $$

(c) $$ {}^{3}p_{2} = \begin{bmatrix} -0.4 \\ 0.2 \\ 2.0 \end{bmatrix} $$

$$ {}^{0}p_{2} = {}^{0}T_{3}\, {}^{3}p_{2} $$

$$ {}^{0}p_{2} = \begin{bmatrix} 1 & 0 & 0 & 1.5 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & -1 & 3.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.4 \\ 0.2 \\ 2.0 \\ 1 \end{bmatrix} $$

$$ = \begin{bmatrix} 1.5 + (-0.4) \\ 0.5 + 0.2 \\ 3.0 + (-1) \times 2.0 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 0.7 \\ 1.0 \end{bmatrix} m $$