

Kriging - An illustration with the Meuse dataset from the gstat package

This illustration uses the Meuse dataset included in gstat which comprises of four heavy metals measured in the top soil in a flood plain along the river Meuse. Apparently polluted sediment is carried by the river and mostly deposited close to the river bank.

Packages needed for this illustration includes gstat and sp. Note that there are other packages which can handle kriging and other geostatistical tools, such as geoR, geoRglm, and fields.

Note: To use gstat, data need to be projected (i.e., not in lat-long format)

```
library(sp)
library(gstat)
data(meuse)
class(meuse)
```

```
## [1] "data.frame"
```

Let's explore the Meuse dataset a bit more.

```
names(meuse)
```

```
## [1] "x"      "y"      "cadmium" "copper" "lead"    "zinc"    "elev"
## [8] "dist"   "om"     "ffreq"   "soil"   "lime"    "landuse" "dist.m"
```

```
str(meuse)
```

```
## 'data.frame':   155 obs. of  14 variables:
## $ x      : num  181072 181025 181165 181298 181307 ...
## $ y      : num  333611 333558 333537 333484 333330 ...
## $ cadmium: num  11.7 8.6 6.5 2.6 2.8 3 3.2 2.8 2.4 1.6 ...
## $ copper  : num  85 81 68 81 48 61 31 29 37 24 ...
## $ lead   : num  299 277 199 116 117 137 132 150 133 80 ...
## $ zinc   : num  1022 1141 640 257 269 ...
## $ elev   : num  7.91 6.98 7.8 7.66 7.48 ...
## $ dist   : num  0.00136 0.01222 0.10303 0.19009 0.27709 ...
## $ om     : num  13.6 14 13 8 8.7 7.8 9.2 9.5 10.6 6.3 ...
## $ ffreq  : Factor w/ 3 levels "1","2","3": 1 1 1 1 1 1 1 1 1 1 ...
## $ soil   : Factor w/ 3 levels "1","2","3": 1 1 1 2 2 2 2 1 1 2 ...
## $ lime   : Factor w/ 2 levels "0","1": 2 2 2 1 1 1 1 1 1 1 ...
## $ landuse: Factor w/ 15 levels "Aa","Ab","Ag",...: 4 4 4 11 4 11 4 2 2 15 ...
## $ dist.m : num  50 30 150 270 380 470 240 120 240 420 ...
```

```
summary(meuse)
```

```
##          x          y          cadmium          copper
## Min.    :178605  Min.    :329714  Min.    : 0.200  Min.    : 14.00
## 1st Qu.:179371  1st Qu.:330762  1st Qu.: 0.800  1st Qu.: 23.00
## Median :179991  Median :331633  Median : 2.100  Median : 31.00
## Mean    :180005  Mean    :331635  Mean    : 3.246  Mean    : 40.32
## 3rd Qu.:180630  3rd Qu.:332463  3rd Qu.: 3.850  3rd Qu.: 49.50
## Max.    :181390  Max.    :333611  Max.    :18.100  Max.    :128.00
##
##          lead          zinc          elev          dist
## Min.    : 37.0  Min.    : 113.0  Min.    : 5.180  Min.    :0.00000
## 1st Qu.: 72.5  1st Qu.: 198.0  1st Qu.: 7.546  1st Qu.:0.07569
## Median :123.0  Median : 326.0  Median : 8.180  Median :0.21184
## Mean    :153.4  Mean    : 469.7  Mean    : 8.165  Mean    :0.24002
## 3rd Qu.:207.0  3rd Qu.: 674.5  3rd Qu.: 8.955  3rd Qu.:0.36407
## Max.    :654.0  Max.    :1839.0  Max.    :10.520  Max.    :0.88039
##
##          om          ffreq  soil  lime          landuse          dist.m
## Min.    : 1.000  1:84  1:97  0:111  W          :50  Min.    : 10.0
## 1st Qu.: 5.300  2:48  2:46  1: 44  Ah          :39  1st Qu.: 80.0
## Median : 6.900  3:23  3:12          Am          :22  Median : 270.0
## Mean    : 7.478          Fw          :10  Mean    : 290.3
## 3rd Qu.: 9.000          Ab          : 8  3rd Qu.: 450.0
## Max.    :17.000          (Other):25  Max.    :1000.0
## NA's    :2          NA's    : 1
```

Function coordinates: when the function coordinates is assigned, it will promotes the data.frame meuse into a SpatialPointsDataFrame which knows about its spatial coordinates. See the codes below. Compare the results of the function summary above and below.

```
coordinates(meuse) = ~x+y
class(meuse)
```

```
## [1] "SpatialPointsDataFrame"
## attr(,"package")
## [1] "sp"
```

```
summary(meuse)
```

```
## Object of class SpatialPointsDataFrame
## Coordinates:
##      min      max
## x 178605 181390
## y 329714 333611
## Is projected: NA
## proj4string : [NA]
## Number of points: 155
## Data attributes:
##      cadmium      copper      lead      zinc
## Min.   : 0.200   Min.   : 14.00   Min.   : 37.0   Min.   : 113.0
## 1st Qu.: 0.800   1st Qu.: 23.00   1st Qu.: 72.5   1st Qu.: 198.0
## Median : 2.100   Median : 31.00   Median :123.0   Median : 326.0
## Mean   : 3.246   Mean   : 40.32   Mean   :153.4   Mean   : 469.7
## 3rd Qu.: 3.850   3rd Qu.: 49.50   3rd Qu.:207.0   3rd Qu.: 674.5
## Max.   :18.100   Max.   :128.00   Max.   :654.0   Max.   :1839.0
##
##      elev      dist      om      ffreq soil lime
## Min.   : 5.180   Min.   :0.00000   Min.   : 1.000   1:84  1:97  0:111
## 1st Qu.: 7.546   1st Qu.:0.07569   1st Qu.: 5.300   2:48  2:46  1: 44
## Median : 8.180   Median :0.21184   Median : 6.900   3:23  3:12
## Mean   : 8.165   Mean   :0.24002   Mean   : 7.478
## 3rd Qu.: 8.955   3rd Qu.:0.36407   3rd Qu.: 9.000
## Max.   :10.520   Max.   :0.88039   Max.   :17.000
##
##      NA's      :2
##      landuse      dist.m
## W      :50   Min.   : 10.0
## Ah      :39   1st Qu.: 80.0
## Am      :22   Median : 270.0
## Fw      :10   Mean   : 290.3
## Ab      : 8   3rd Qu.: 450.0
## (Other):25   Max.   :1000.0
## NA's    : 1
```

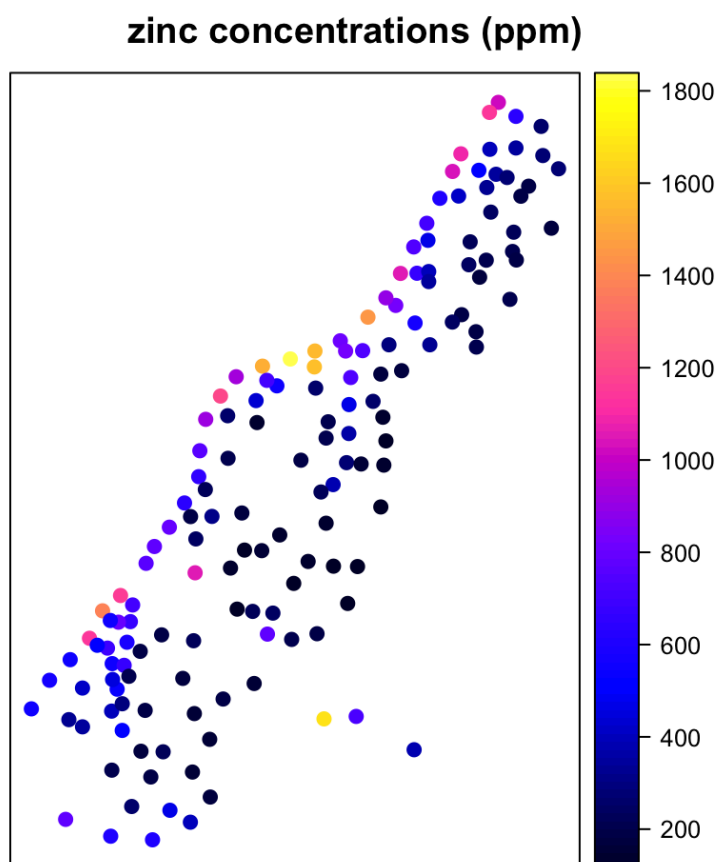
And now you can use the function `coordinates` to retrieve the spatial coordinates from a `SpatialPointsDataFrame` `meuse`.

```
coordinates(meuse)[5:15,]
```

```
##      x      y
## 5 181307 333330
## 6 181390 333260
## 7 181165 333370
## 8 181027 333363
## 9 181060 333231
## 10 181232 333168
## 11 181191 333115
## 12 181032 333031
## 13 180874 333339
## 14 180969 333252
## 15 181011 333161
```

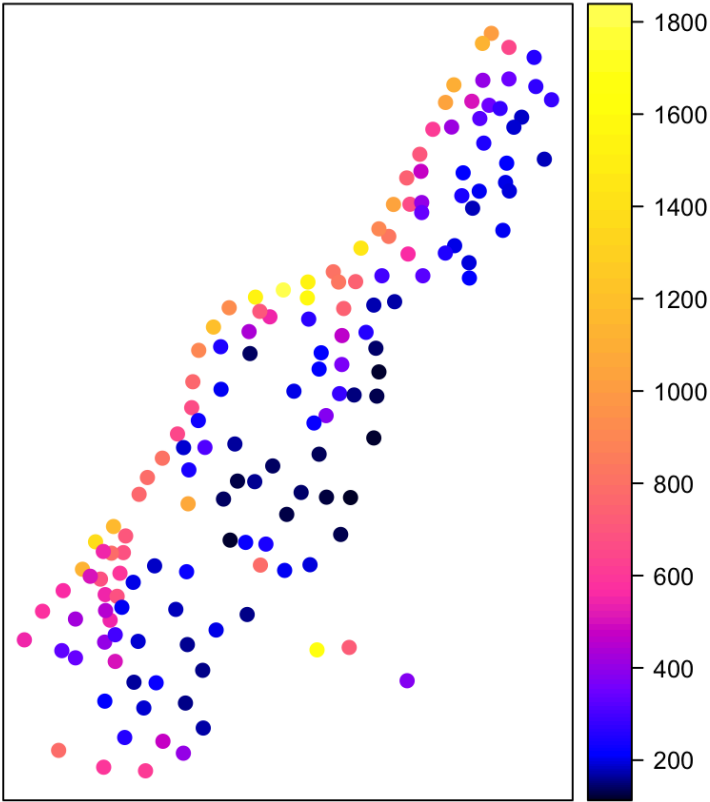
Plotting data: you can use two plotting functions `spplot` and `bubble` as illustrated below (note: the x- and y-axis are the spatial coordinates)

```
spplot(meuse, "zinc", colorkey = TRUE, main = "zinc concentrations (ppm)")
```



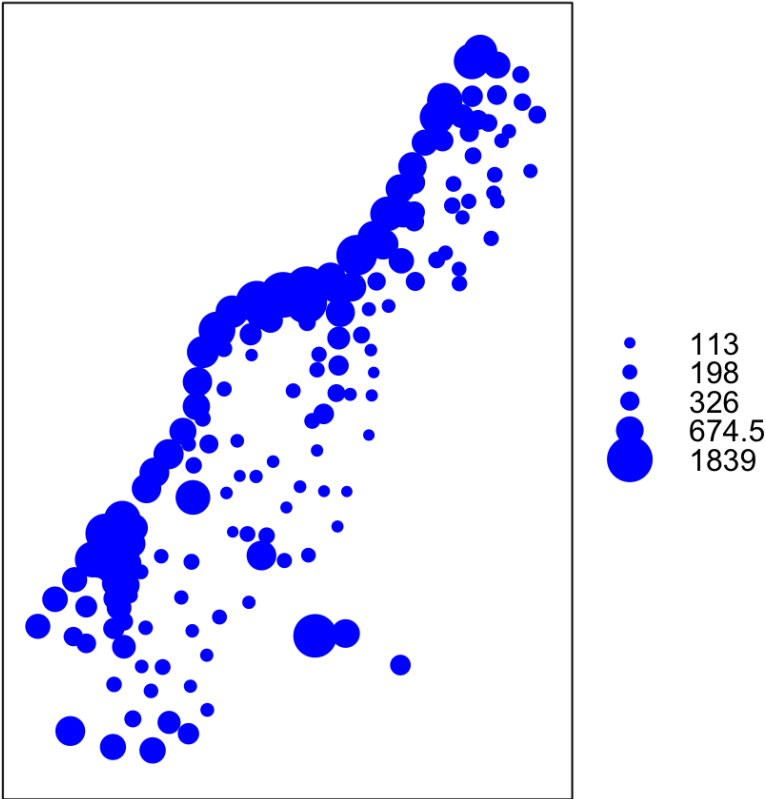
```
spplot(meuse, "zinc", do.log = T, colorkey = TRUE, main = "zinc concentrations (ppm)")
```

zinc concentrations (ppm)

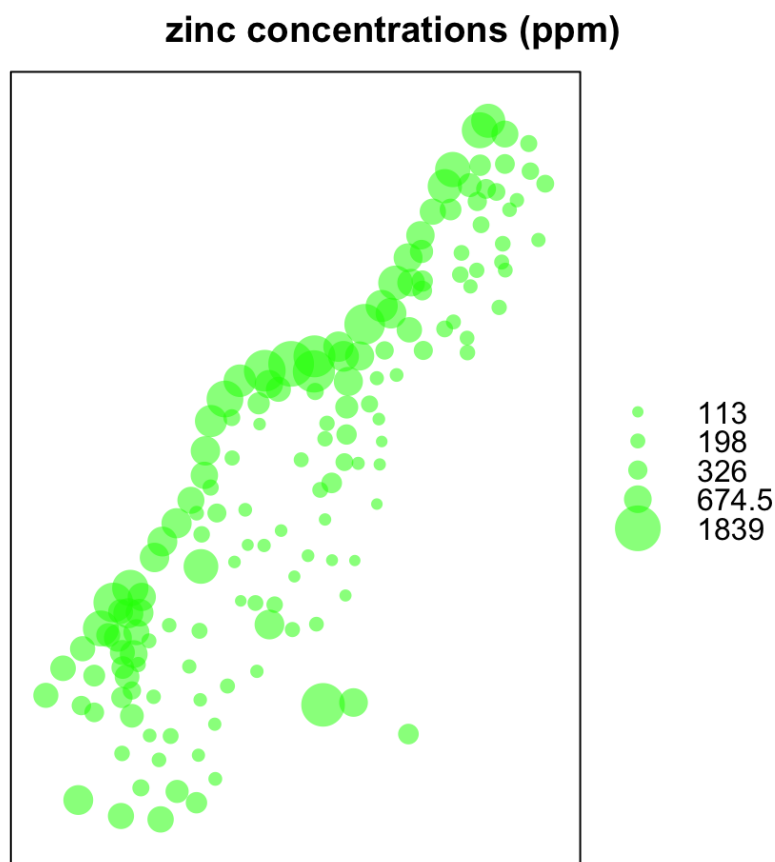


```
bubble(meuse, "zinc", col="blue", main = "zinc concentrations (ppm)")
```

zinc concentrations (ppm)



```
bubble(meuse, "zinc", col=c("#00ff0088"), main = "zinc concentrations (ppm)")
```



Now we explore the meuse.grid dataset

```
data(meuse.grid)
summary(meuse.grid)
```

```
##      x          y      part.a      part.b
## Min.   :178460  Min.   :329620  Min.   :0.0000  Min.   :0.0000
## 1st Qu.:179420  1st Qu.:330460  1st Qu.:0.0000  1st Qu.:0.0000
## Median :179980  Median :331220  Median :0.0000  Median :1.0000
## Mean   :179985  Mean   :331348  Mean   :0.3986  Mean   :0.6014
## 3rd Qu.:180580  3rd Qu.:332140  3rd Qu.:1.0000  3rd Qu.:1.0000
## Max.   :181540  Max.   :333740  Max.   :1.0000  Max.   :1.0000
##      dist      soil      ffreq
## Min.   :0.0000  1:1665  1: 779
## 1st Qu.:0.1193  2:1084  2:1335
## Median :0.2715  3: 354  3: 989
## Mean   :0.2971
## 3rd Qu.:0.4402
## Max.   :0.9926
```

```
str(meuse.grid)
```

```
## 'data.frame':   3103 obs. of  7 variables:
## $ x      : num  181180 181140 181180 181220 181100 ...
## $ y      : num  333740 333700 333700 333700 333660 ...
## $ part.a: num   1 1 1 1 1 1 1 1 1 1 ...
## $ part.b: num   0 0 0 0 0 0 0 0 0 0 ...
## $ dist   : num   0 0 0.0122 0.0435 0 ...
## $ soil   : Factor w/ 3 levels "1","2","3": 1 1 1 1 1 1 1 1 1 1 ...
## $ ffreq  : Factor w/ 3 levels "1","2","3": 1 1 1 1 1 1 1 1 1 1 ...
```

```
class(meuse.grid)
```

```
## [1] "data.frame"
```

```
coordinates(meuse.grid) = ~x+y
class(meuse.grid)
```

```
## [1] "SpatialPointsDataFrame"
## attr(,"package")
## [1] "sp"
```

```
# sSee how the class of meuse.grid changes when "gridded" is used
gridded(meuse.grid) = TRUE
class(meuse.grid)
```

```
## [1] "SpatialPixelsDataFrame"
## attr(,"package")
## [1] "sp"
```

Do some visualization

```
image(meuse.grid["dist"])
title("distance to river (red = 0)")
```

distance to river (red = 0)



```
# Now we need package gstat  
library(gstat)  
# Apply the inverse distance weighted interpolation  
zinc.idw = idw(zinc~1, meuse, meuse.grid)
```

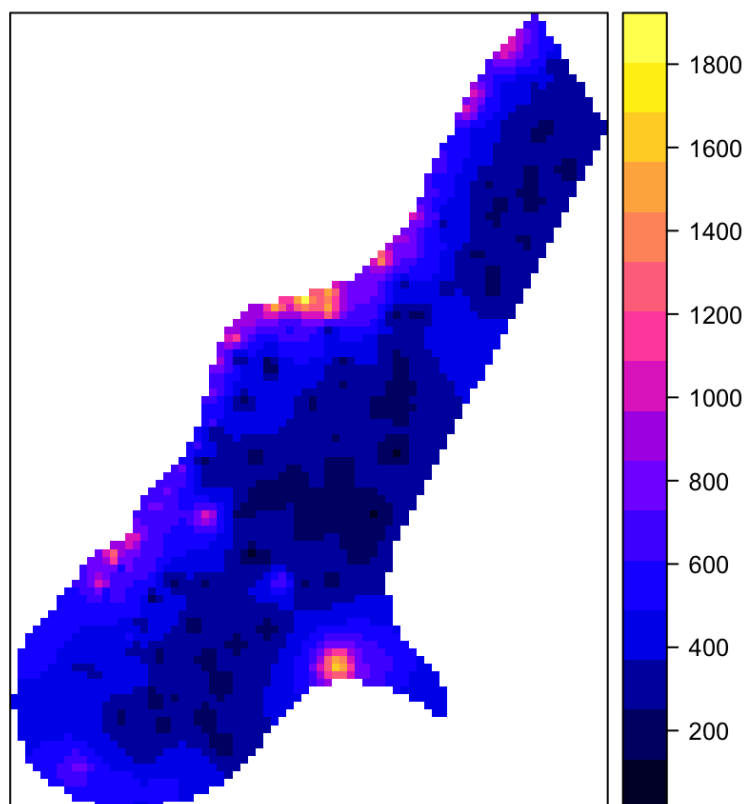
```
## [inverse distance weighted interpolation]
```

```
# class of zinc.idw  
class(zinc.idw)
```

```
## [1] "SpatialPixelsDataFrame"  
## attr(,"package")  
## [1] "sp"
```

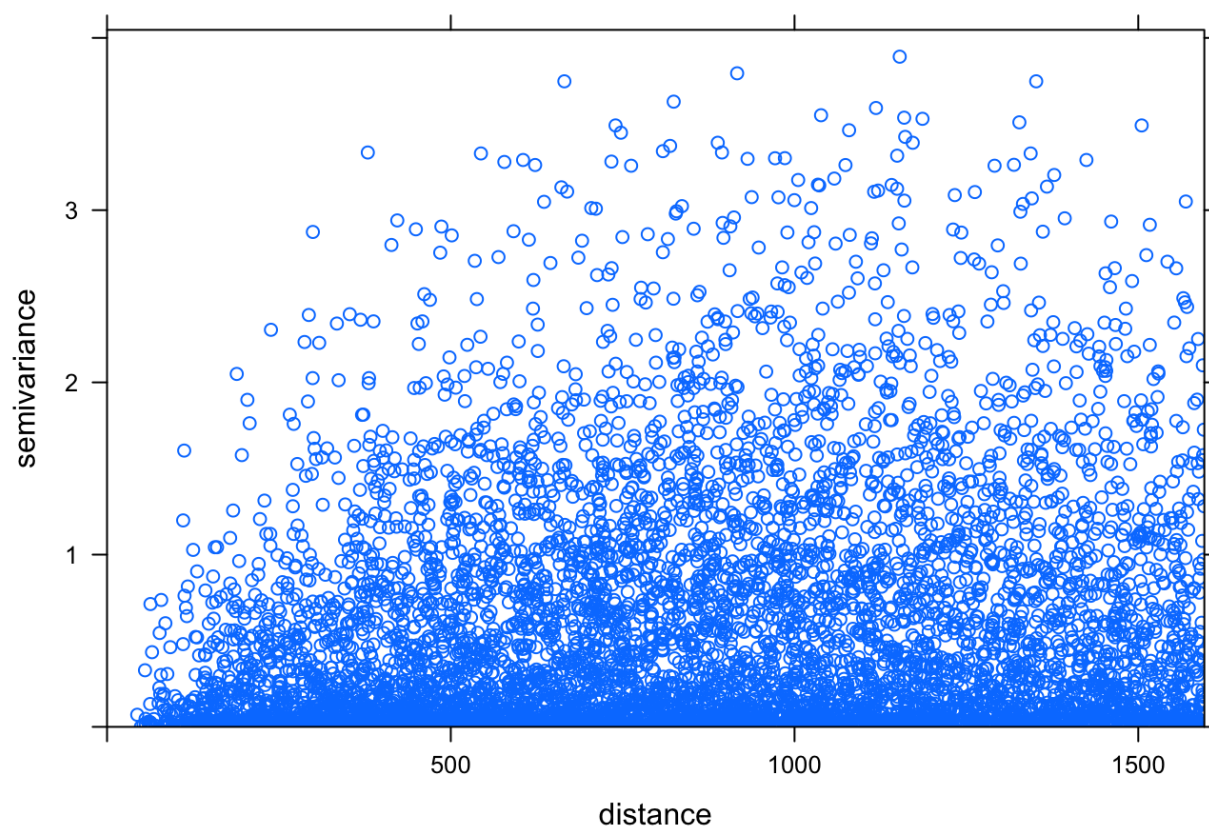
```
# plot the  
spplot(zinc.idw["var1.pred"], main = "zinc inverse distance weighted interpolations")
```


zinc inverse distance weighted interpolations



Variogram. We assume that there is a constant trend for the variable $\log(\text{zinc})$

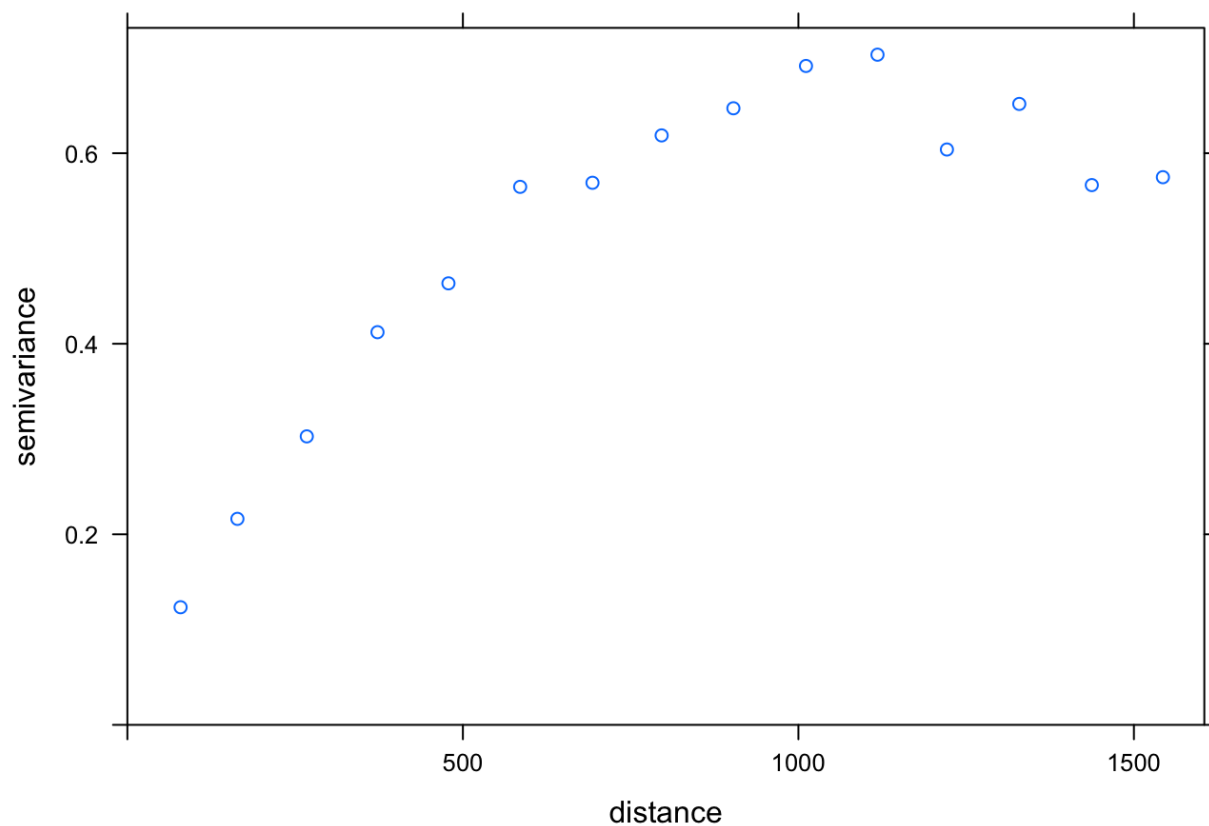
```
vgm1 = variogram(log(zinc)~1, meuse)
# plot variogram cloud
plot(variogram(log(zinc)~1, meuse, cloud=TRUE))
```



```
vgm1
```

```
##      np      dist      gamma dir.hor dir.ver  id
## 1    57   79.29244 0.1234479      0      0 var1
## 2   299  163.97367 0.2162185      0      0 var1
## 3   419  267.36483 0.3027859      0      0 var1
## 4   457  372.73542 0.4121448      0      0 var1
## 5   547  478.47670 0.4634128      0      0 var1
## 6   533  585.34058 0.5646933      0      0 var1
## 7   574  693.14526 0.5689683      0      0 var1
## 8   564  796.18365 0.6186769      0      0 var1
## 9   589  903.14650 0.6471479      0      0 var1
## 10  543 1011.29177 0.6915705      0      0 var1
## 11  500 1117.86235 0.7033984      0      0 var1
## 12  477 1221.32810 0.6038770      0      0 var1
## 13  452 1329.16407 0.6517158      0      0 var1
## 14  457 1437.25620 0.5665318      0      0 var1
## 15  415 1543.20248 0.5748227      0      0 var1
```

```
# plot vgm1
plot(vgm1)
```

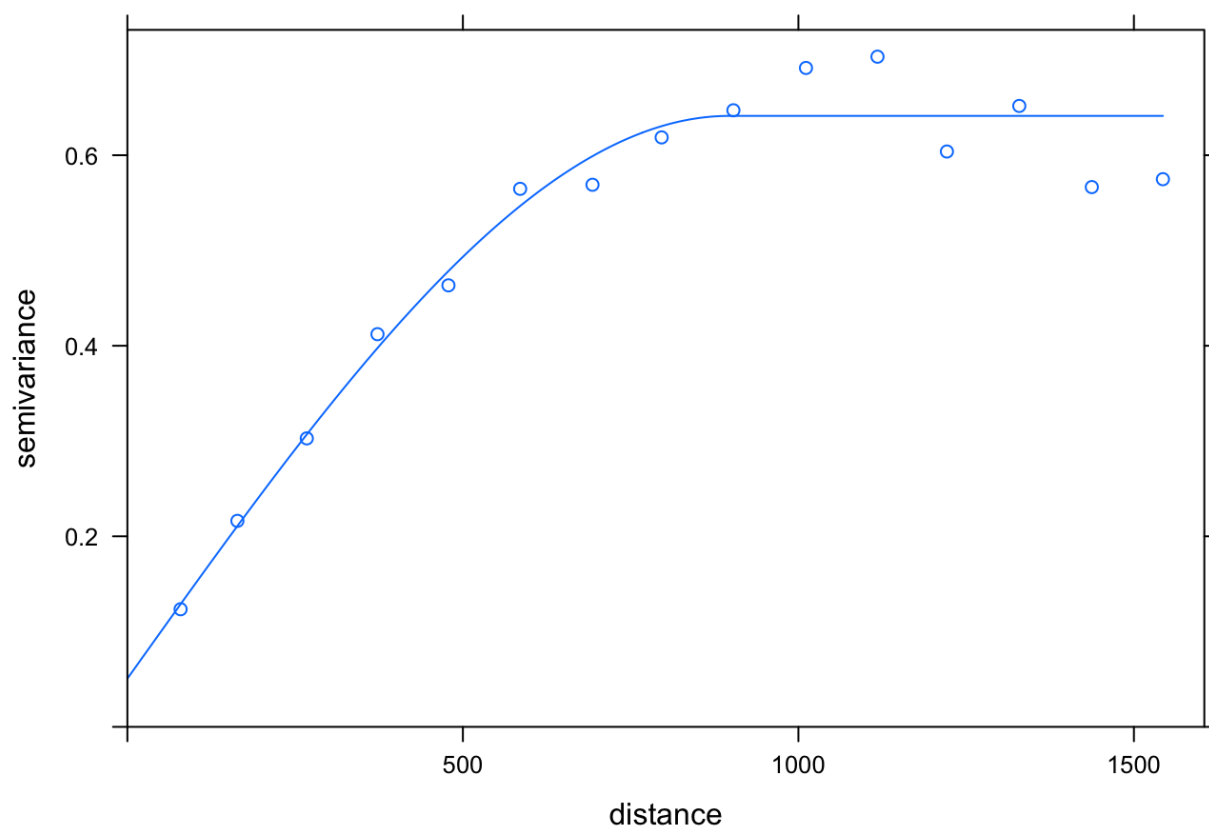


Variogram. Fit a theoretical variogram.

```
vgm1.fit = fit.variogram(vgm1, model = vgm(1, "Sph", 900, 1))  
vgm1.fit
```

```
##  model      psill  range  
## 1   Nug 0.05066243 0.0000  
## 2   Sph 0.59060780 897.0209
```

```
# plot the fitted variogram and the observed variogram on the same graph  
plot(vgm1, vgm1.fit)
```

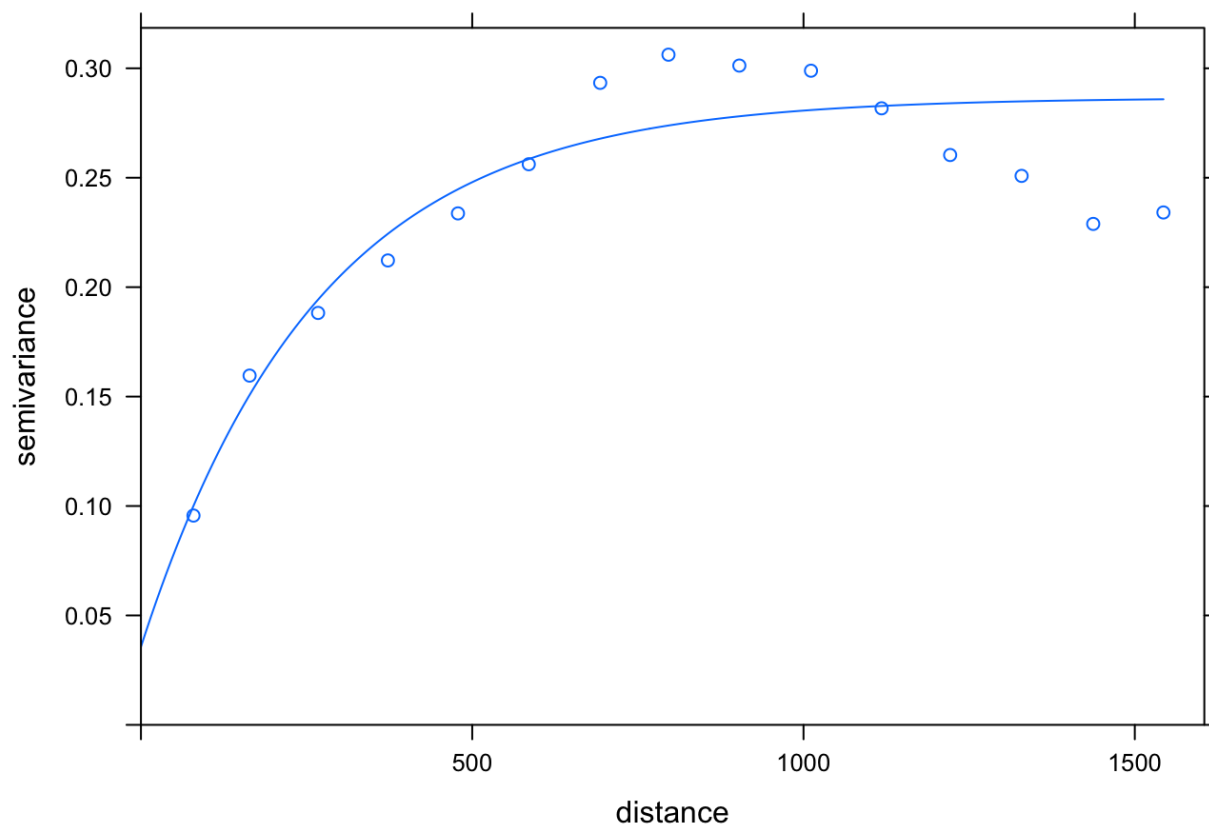


Variogram. Now we can specify a mean function for the variogram, e.g. using `~ dist` as a predictor variable

```
vgm2 = variogram(log(zinc)~dist, meuse)
vgm2.fit = fit.variogram(vgm2, model = vgm(1, "Exp", 300, 1))
vgm2.fit
```

```
##  model    psill  range
## 1  Nug 0.03563295 0.0000
## 2  Exp 0.25099411 267.2971
```

```
plot(vgm2, vgm2.fit)
```

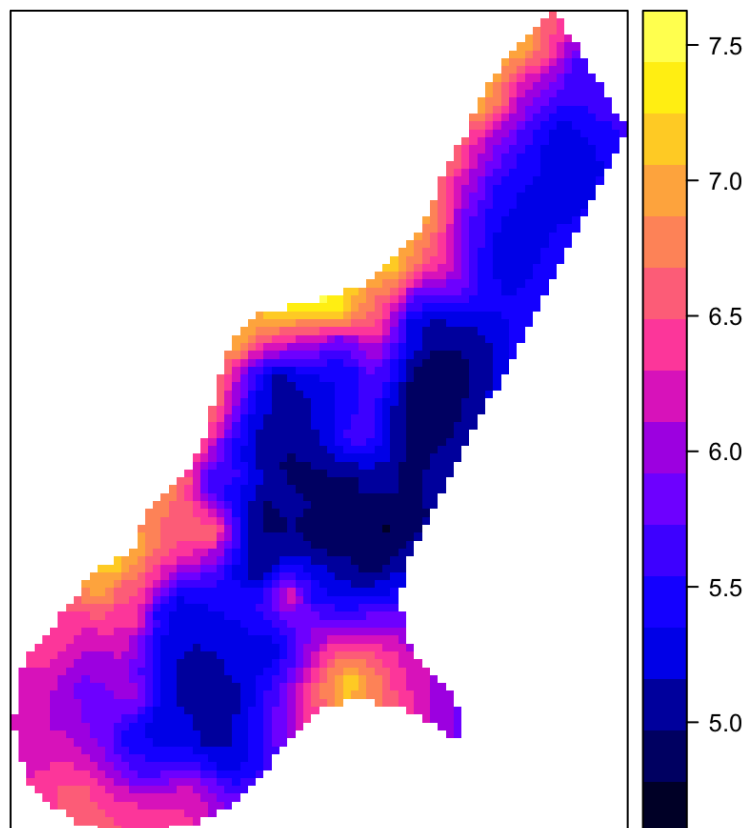


Kriging. We do an ordinary kriging.

```
vgm1.krige = krige(log(zinc)~1, meuse, meuse.grid, model = vgm1.fit)
```

```
## [using ordinary kriging]
```

```
spplot(vgm1.krige["var1.pred"])
```

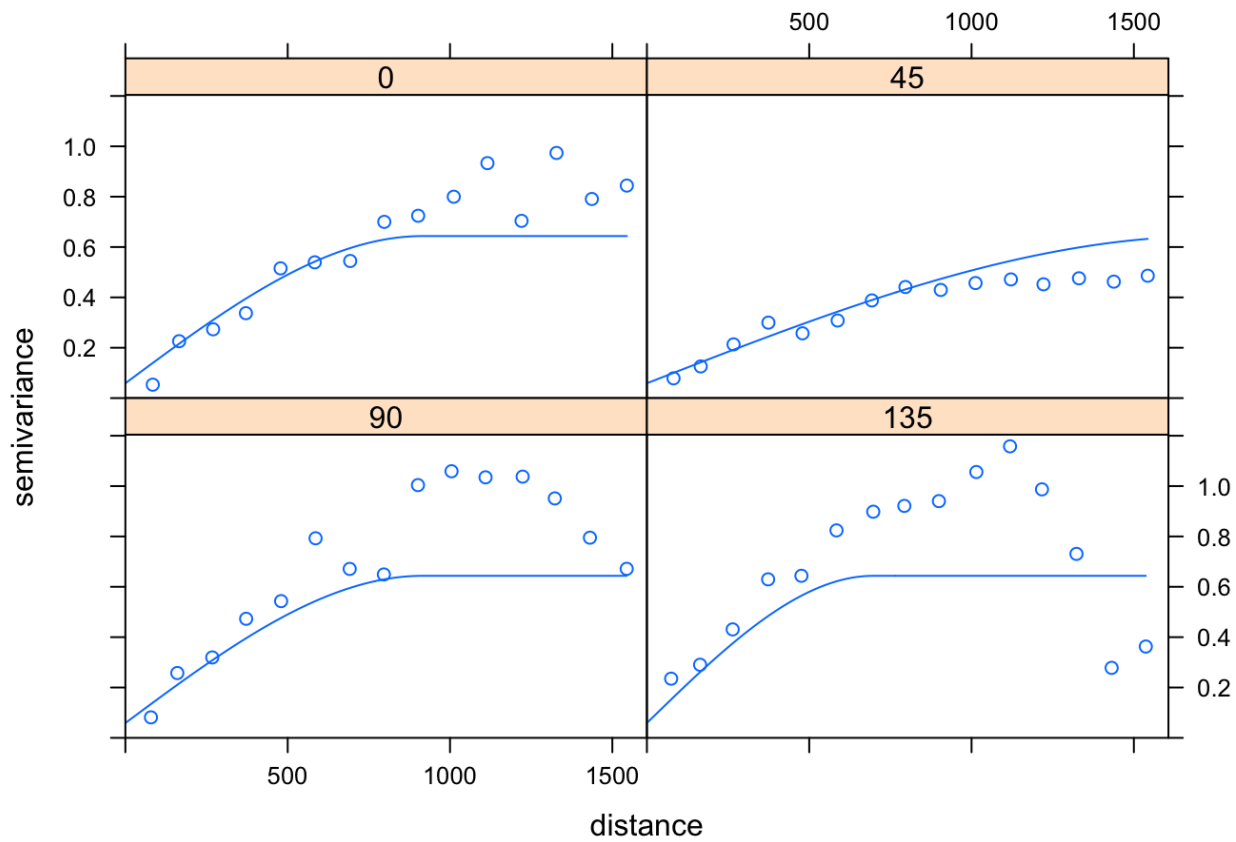


Directional variogram

```
vgm3 = variogram(log(zinc)~1, meuse, alpha = c(0, 45, 90, 135))
vgm3.fit = fit.variogram(vgm3, model = vgm(.59, "Sph", 1200, .05, anis = c(45, .4)))
vgm3.fit
```

```
##  model      psill    range ang1 anis1
## 1   Nug 0.05892218   0.000   0   1.0
## 2   Sph 0.58465000 1742.803  45   0.4
```

```
plot(vgm3, vgm3.fit, as.table = TRUE)
```

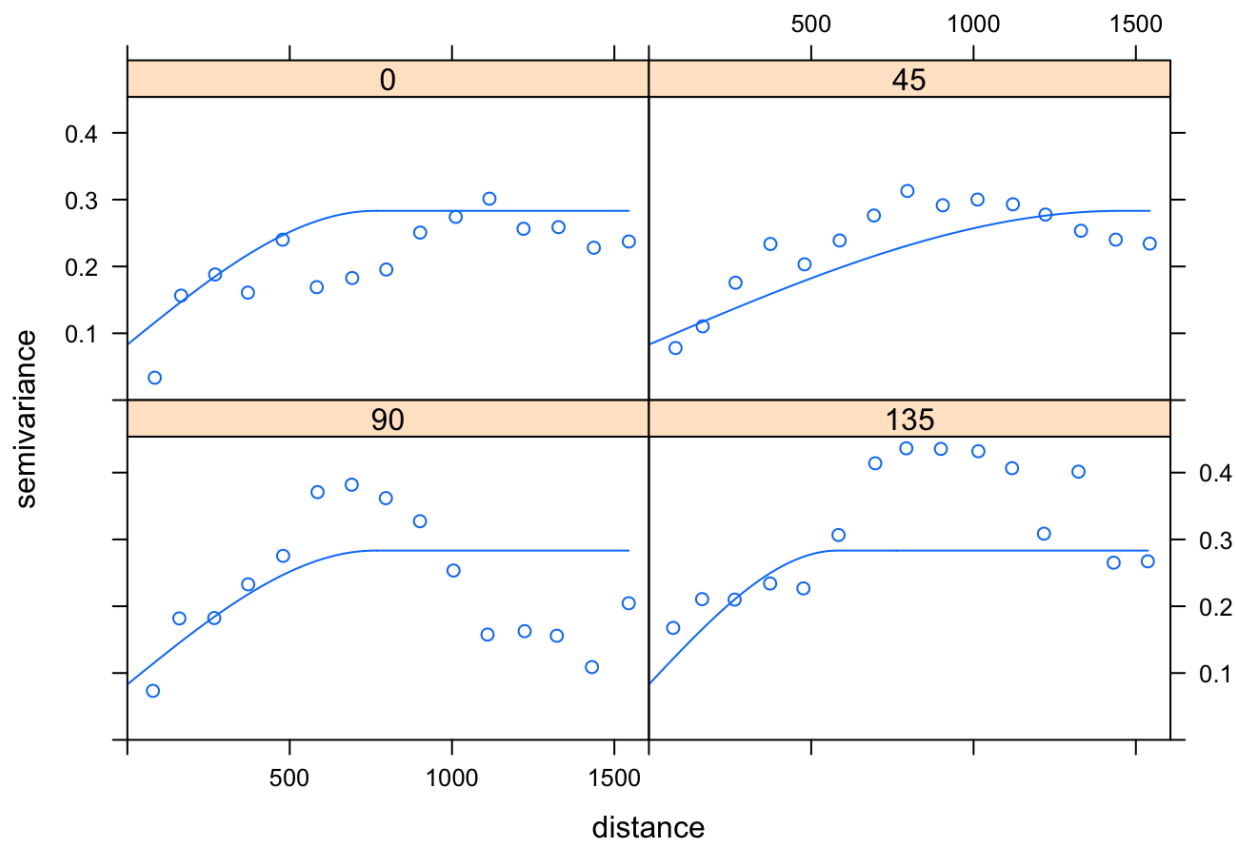


Another directional variogram

```
vgm4 = variogram(log(zinc)~dist, meuse, alpha = c(0, 45, 90, 135))
vgm4.fit = fit.variogram(vgm4, model = vgm(.59, "Sph", 1200, .05, anis = c(45, .4)))
vgm4.fit
```

```
##  model      psill    range ang1 anis1
## 1   Nug 0.08305825    0.000   0   1.0
## 2   Sph 0.20023979 1452.826  45   0.4
```

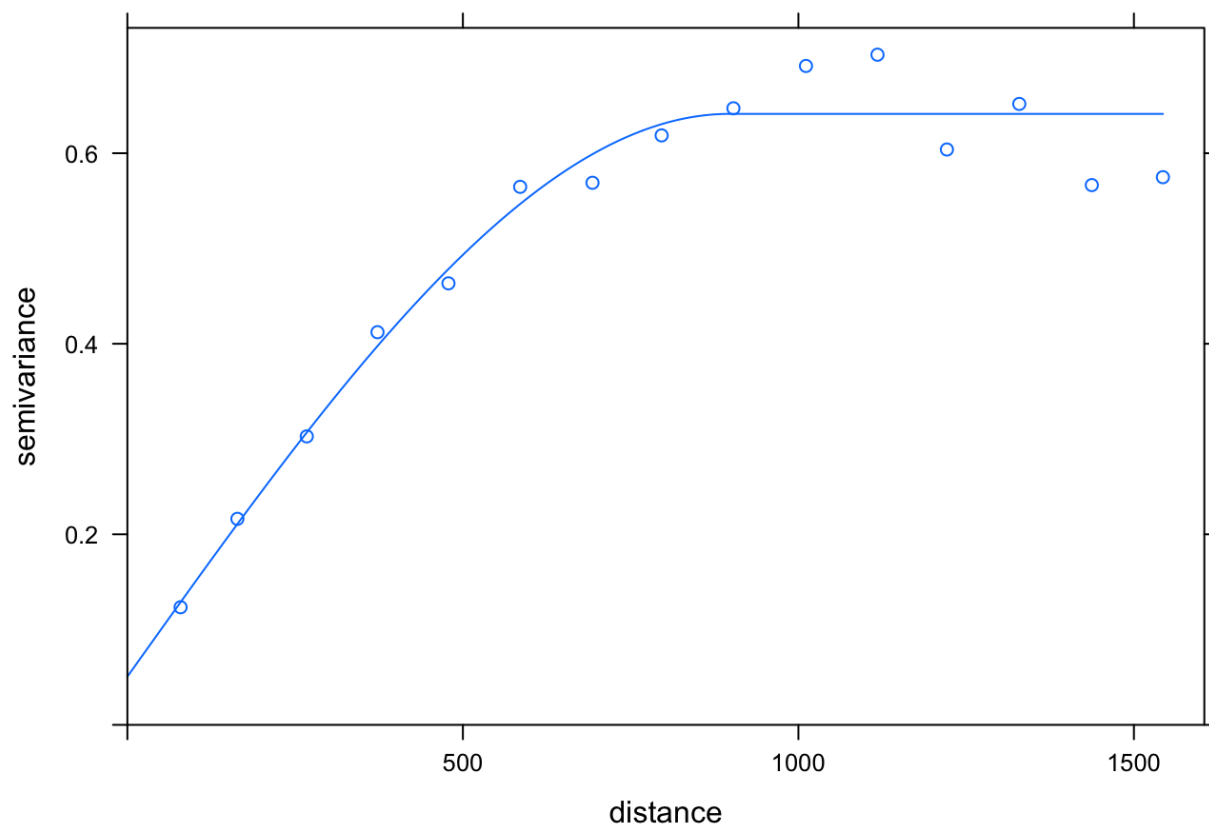
```
plot(vgm4, vgm4.fit, as.table = TRUE)
```



Blocking Kriging

```
vgm5<- variogram(log(zinc)~1, meuse)
vgm5.fit <- fit.variogram(vgm5, model=vgm(psill=1, model="Sph", range=900, nugget=1))

plot(vgm5, vgm5.fit)
```

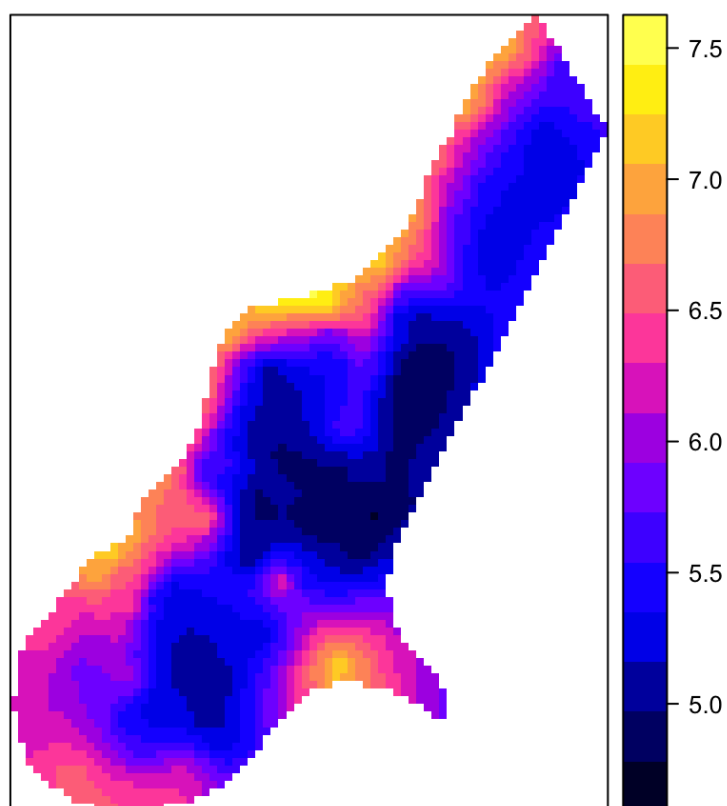



```
lzn.okriging <- krige(log(zinc)~1, meuse, meuse.grid, model = vgm5.fit )
```

```
## [using ordinary kriging]
```

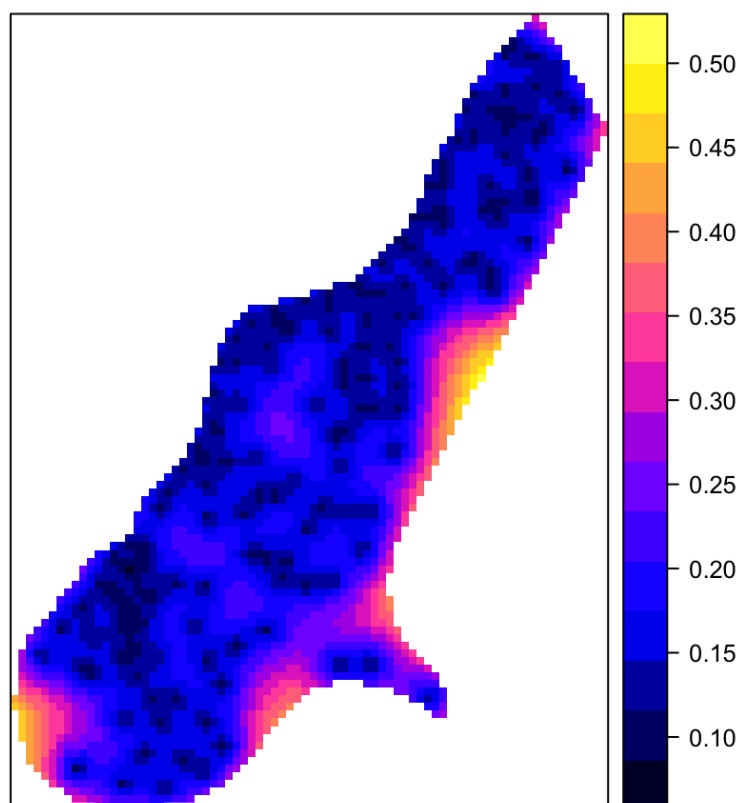
```
spplot(lzn.okriging["var1.pred"], main = "ordinary kriging predictions")
```

ordinary kriging predictions



```
spplot(lzn.okriging["var1.var"], main = "ordinary kriging variance")
```

ordinary kriging variance

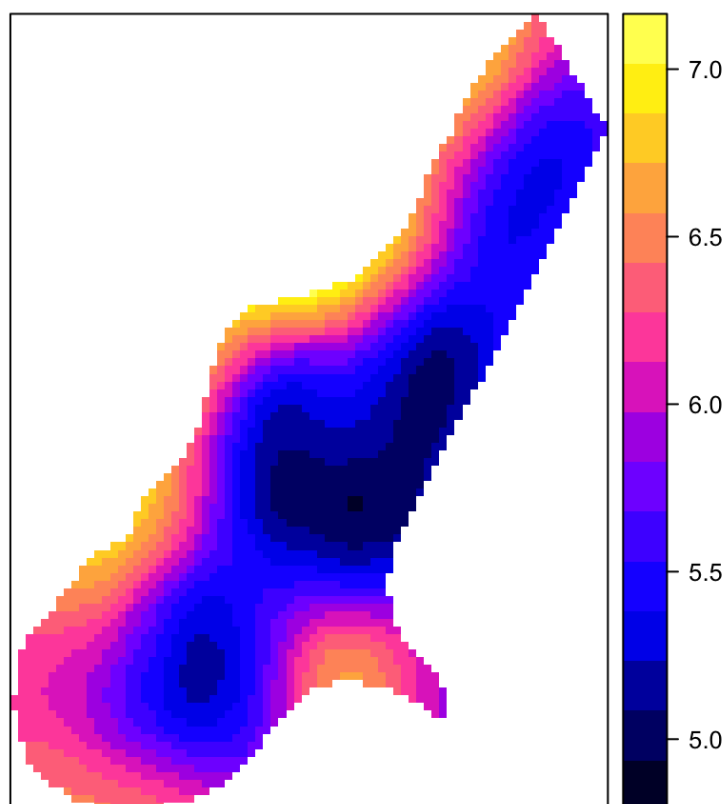


```
#Define a block size for block kriging  
lzn.bokriling <- krige(log(zinc)~1, meuse, meuse.grid, model = vgm5.fit, block=c(500,500))
```

```
## [using ordinary kriging]
```

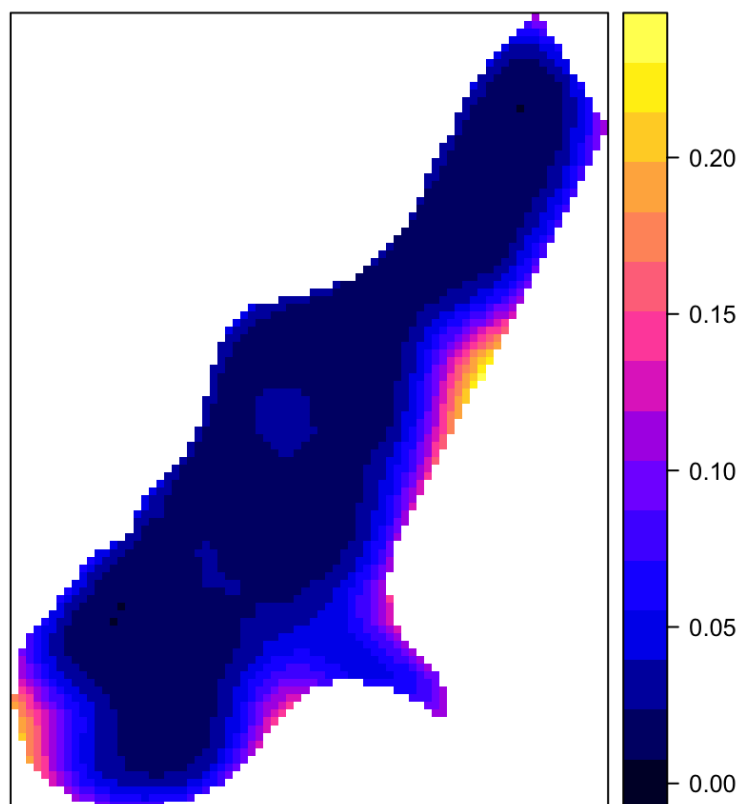
```
spplot(lzn.bokriling["var1.pred"], main = "ordinary block kriging predictions")
```

ordinary block kriging predictions



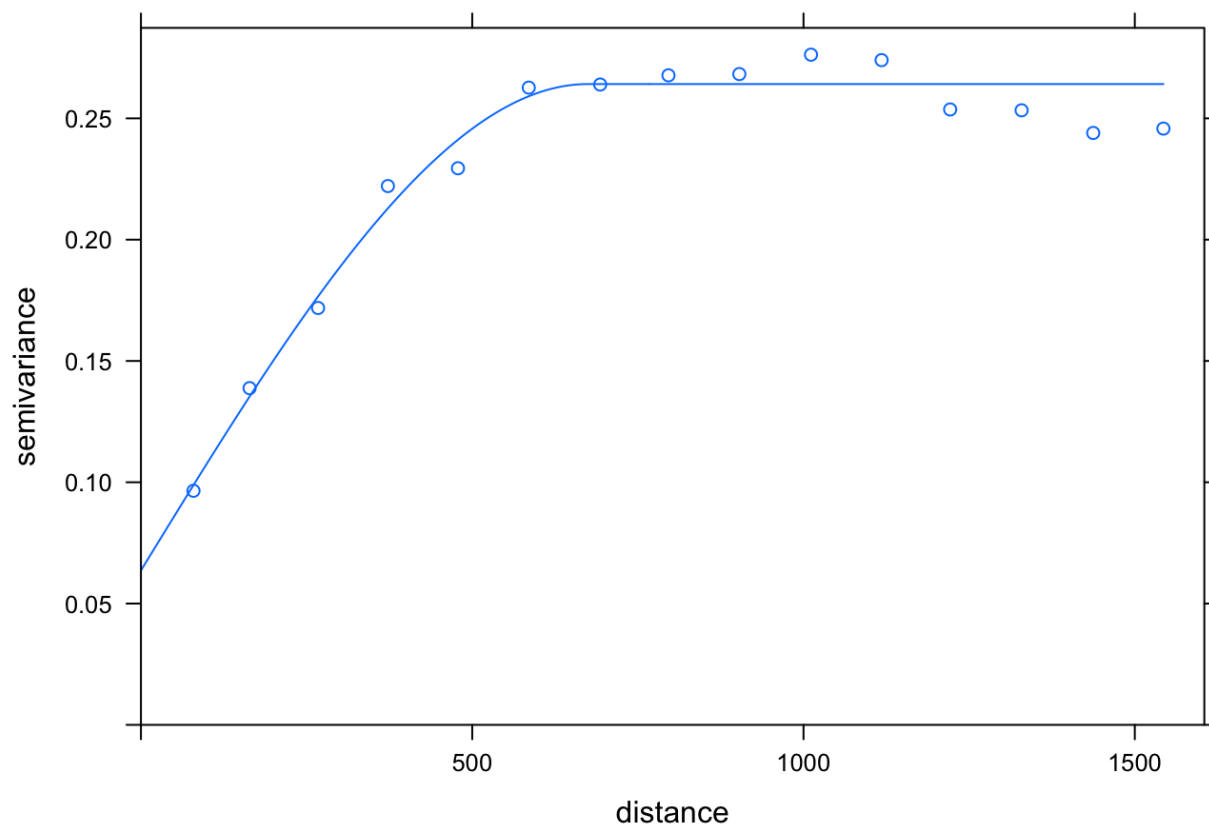
```
spplot(lzn.bokriling["var1.var"], main = "ordinary block kriging variance")
```

ordinary block kriging variance



Indicator Kriging

```
#Develop a variogram for indicator kriging  
vgm6 <- variogram(I(log(zinc)>6)~1, meuse)  
vgm6.fit <- fit.variogram(vgm6, model=vgm(psill=0.25, model="Sph", range=900, nugget=0.1))  
plot(vgm6, vgm6.fit)
```

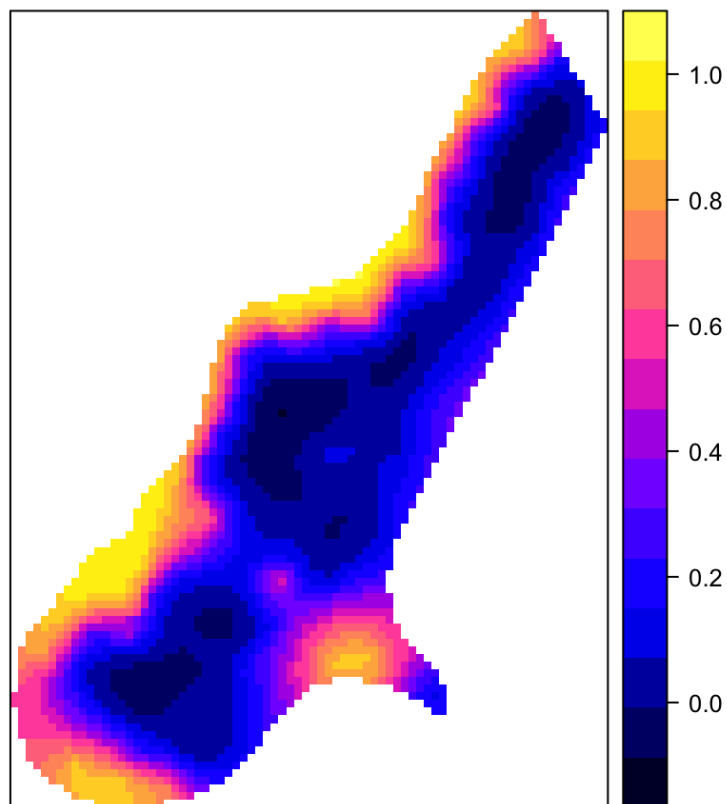


```
logzinc.ikriging <- krige(I(log(zinc)>6.21)~1, meuse, meuse.grid, vgm6.fit)
```

```
## [using ordinary kriging]
```

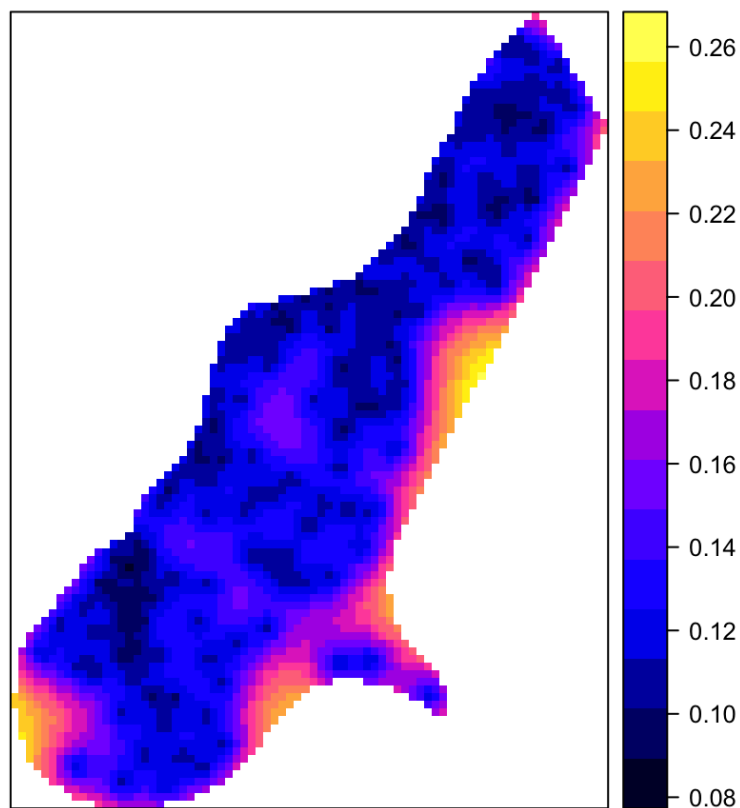
```
spplot(logzinc.ikriging["var1.pred"], main = "indicator kriging predictions")
```

indicator kriging predictions



```
spplot(logzinc.ikriging["var1.var"], main = "indicator kriging variance")
```

indicator kriging variance



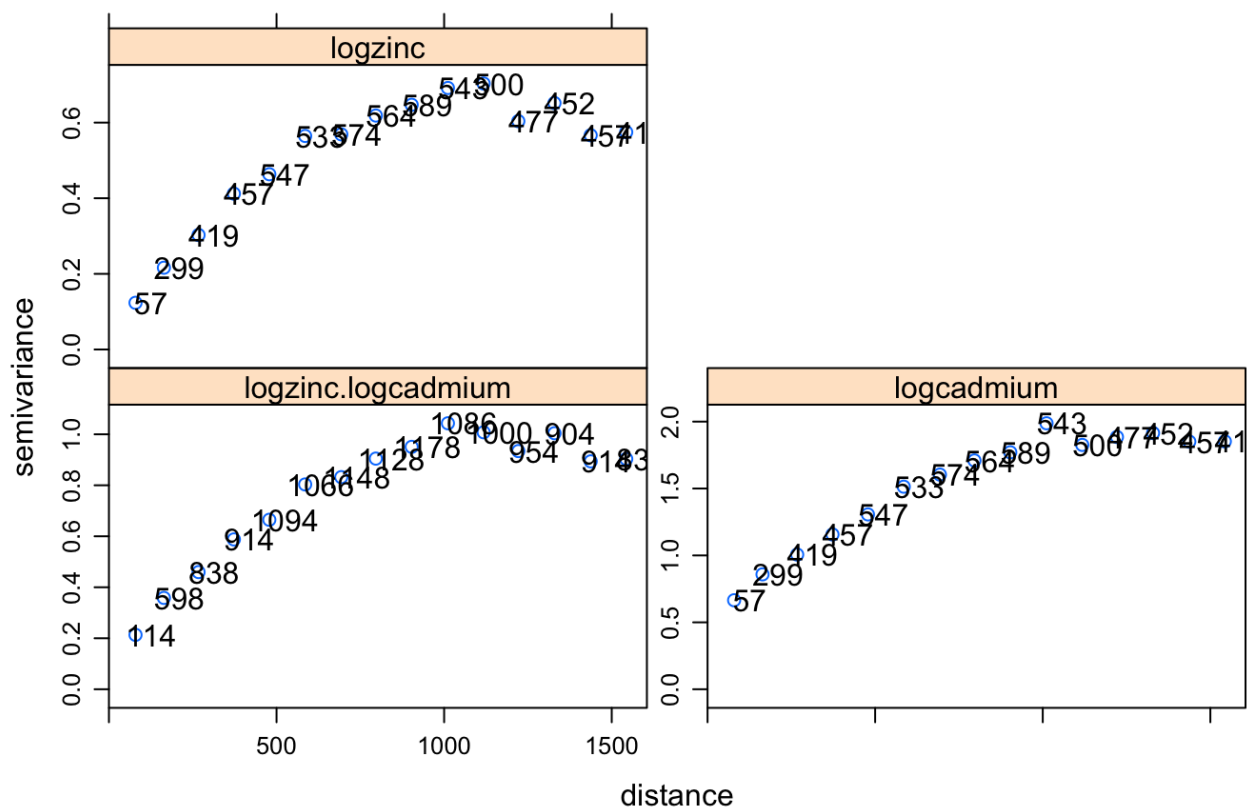
Co-Kriging

```
# Build a gstat structure containing the two sample sets
zinccadmium <- gstat(NULL, id = "logzinc", form = log(zinc) ~ 1, data=meuse)
zinccadmium <- gstat(zinccadmium, id = "logcadmium", form = log(cadmium) ~ 1, data=meuse)

# Compute and display the two direct variograms and one cross-variogram
v.cross <- variogram(zinccadmium)
str(v.cross)
```

```
## Classes 'gstatVariogram' and 'data.frame': 45 obs. of 6 variables:
## $ np : num 114 598 838 914 1094 ...
## $ dist : num 79.3 164 267.4 372.7 478.5 ...
## $ gamma : num 0.213 0.358 0.46 0.588 0.665 ...
## $ dir.hor: num 0 0 0 0 0 0 0 0 0 ...
## $ dir.ver: num 0 0 0 0 0 0 0 0 0 ...
## $ id : Factor w/ 3 levels "logzinc.logcadmium",...: 1 1 1 1 1 1 1 1 1 ...
## - attr(*, "direct")= 'data.frame': 3 obs. of 2 variables:
## ..$ id : Factor w/ 3 levels "logcadmium","logzinc",...: 3 1 2
## ..$ is.direct: logi FALSE TRUE TRUE
## - attr(*, "boundaries")= num 0 106 213 319 426 ...
## - attr(*, "pseudo")= num 0
## - attr(*, "what")= chr "semivariance"
```

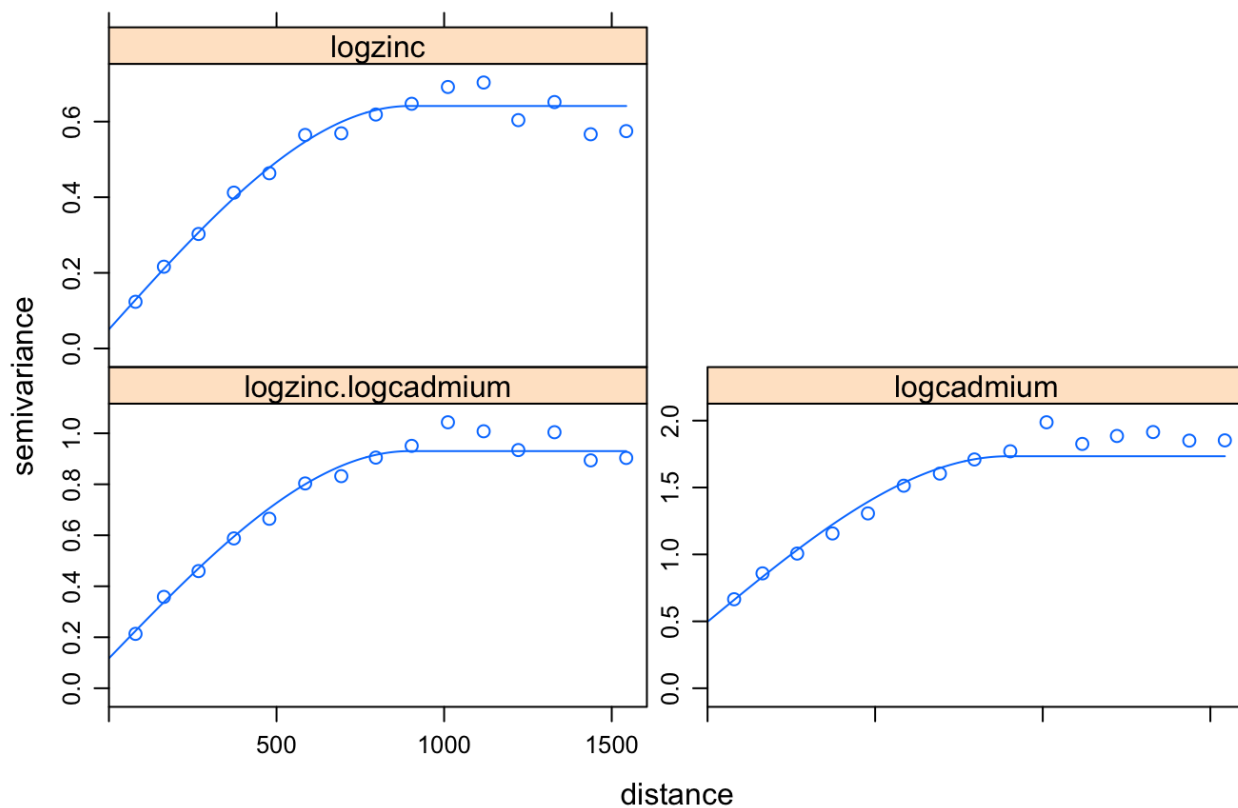
```
plot(v.cross, pl=T)
```



```
# Add variogram models to the gstat object and fit a them using the linear model of co-regionalisation
zinccadmium <- gstat(zinccadmium, id = "logzinc", model = vgm5.fit, fill.all=T)
zinccadmium
```

```
## data:
## logzinc : formula = log(zinc)~1 ; data dim = 155 x 12
## logcadmium : formula = log(cadmium)~1 ; data dim = 155 x 12
## variograms:
##           model      psill   range
## logzinc[1]      Nug 0.05066243  0.0000
## logzinc[2]      Sph 0.59060780 897.0209
## logcadmium[1]    Nug 0.05066243  0.0000
## logcadmium[2]    Sph 0.59060780 897.0209
## logzinc.logcadmium[1] Nug 0.05066243  0.0000
## logzinc.logcadmium[2] Sph 0.59060780 897.0209
```

```
# use the fit.lmc method to fit all three variograms together
zinccadmium <- fit.lmc(v.cross, zinccadmium)
plot(variogram(zinccadmium), model=zinccadmium$model)
```



```
# use the predict.gstat method for co-kriging
cokrig.model <- predict.gstat(zinccadmium, meuse.grid)
```



```
## Linear Model of Coregionalization found. Good.
## [using ordinary cokriging]
```

```
str(cokrig.model)
```

```
## Formal class 'SpatialPixelsDataFrame' [package "sp"] with 7 slots
## ..@ data      : 'data.frame':  3103 obs. of  5 variables:
## .. ..$ logzinc.pred      : num [1:3103] 6.47 6.6 6.48 6.35 6.74 ...
## .. ..$ logzinc.var       : num [1:3103] 0.318 0.25 0.271 0.294 0.175 ...
## .. ..$ logcadmium.pred   : num [1:3103] 1.62 1.81 1.66 1.5 2.01 ...
## .. ..$ logcadmium.var    : num [1:3103] 1.103 0.967 1.008 1.053 0.819 ...
## .. ..$ cov.logzinc.logcadmium: num [1:3103] 0.491 0.398 0.427 0.458 0.296 ...
## ..@ coords.nrs : int [1:2] 1 2
## ..@ grid       : Formal class 'GridTopology' [package "sp"] with 3 slots
## .. .. ..@ cellcentre.offset: Named num [1:2] 178460 329620
## .. .. .. ..- attr(*, "names")= chr [1:2] "x" "y"
## .. .. ..@ cellsize         : Named num [1:2] 40 40
## .. .. .. ..- attr(*, "names")= chr [1:2] "x" "y"
## .. .. ..@ cells.dim       : Named int [1:2] 78 104
## .. .. .. ..- attr(*, "names")= chr [1:2] "x" "y"
## ..@ grid.index : int [1:3103] 69 146 147 148 223 224 225 226 300 301 ...
## ..@ coords     : num [1:3103, 1:2] 181180 181140 181180 181220 181100 ...
## .. ..- attr(*, "dimnames")=List of 2
## .. .. ..$ : chr [1:3103] "1" "2" "3" "4" ...
## .. .. ..$ : chr [1:2] "x" "y"
## ..@ bbox       : num [1:2, 1:2] 178460 329620 181540 333740
## .. ..- attr(*, "dimnames")=List of 2
## .. .. ..$ : chr [1:2] "x" "y"
## .. .. ..$ : chr [1:2] "min" "max"
## ..@ proj4string: Formal class 'CRS' [package "sp"] with 1 slot
## .. .. ..@ projargs: chr NA
```

```
# summarize predictions and their errors
summary(cokrig.model)
```

```
## Object of class SpatialPixelsDataFrame
## Coordinates:
##      min      max
## x 178460 181540
## y 329620 333740
## Is projected: NA
## proj4string : [NA]
## Number of points: 3103
## Grid attributes:
##   cellcentre.offset cellsize cells.dim
## x           178460         40         78
## y           329620         40        104
## Data attributes:
##   logzinc.pred   logzinc.var   logcadmium.pred   logcadmium.var
## Min.    :4.752   Min.    :0.08347   Min.    :-1.0824   Min.    :0.6055
## 1st Qu.:5.238   1st Qu.:0.13556   1st Qu.: -0.4276   1st Qu.:0.7203
## Median :5.564   Median :0.16058   Median : 0.2234   Median :0.7744
## Mean    :5.713   Mean    :0.18348   Mean    : 0.3034   Mean    :0.8232
## 3rd Qu.:6.185   3rd Qu.:0.20984   3rd Qu.: 0.9662   3rd Qu.:0.8791
## Max.    :7.428   Max.    :0.49871   Max.    : 2.5737   Max.    :1.4647
## cov.logzinc.logcadmium
## Min.    :0.1674
## 1st Qu.:0.2398
## Median :0.2742
## Mean    :0.3061
## 3rd Qu.:0.3425
## Max.    :0.7376
```

```
summary(cokrig.model$logcadmium.pred)
```

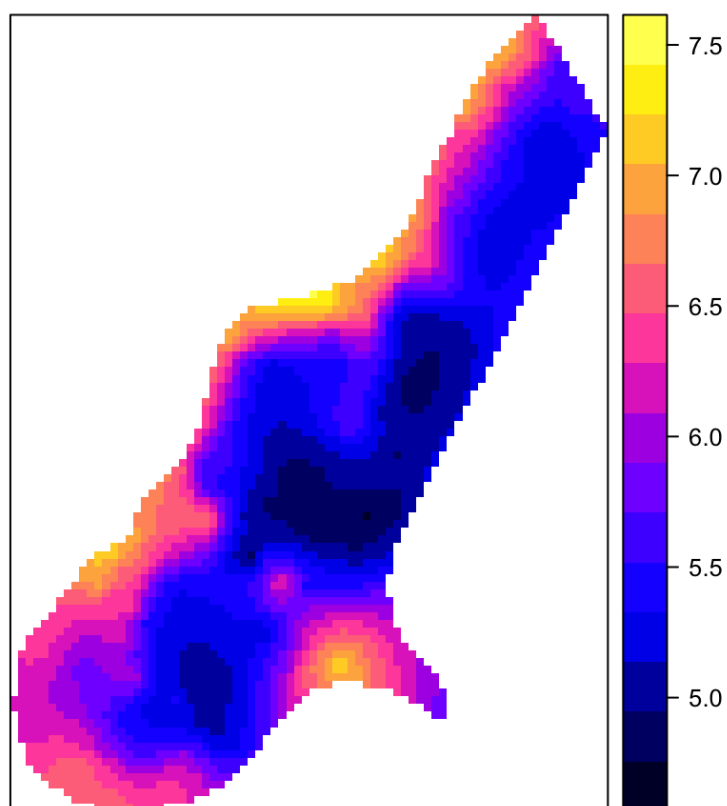
```
##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
## -1.0820 -0.4276  0.2234  0.3034  0.9662  2.5740
```

```
summary(cokrig.model$logcadmium.var)
```

```
##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
## 0.6055  0.7203  0.7744  0.8232  0.8791  1.4650
```

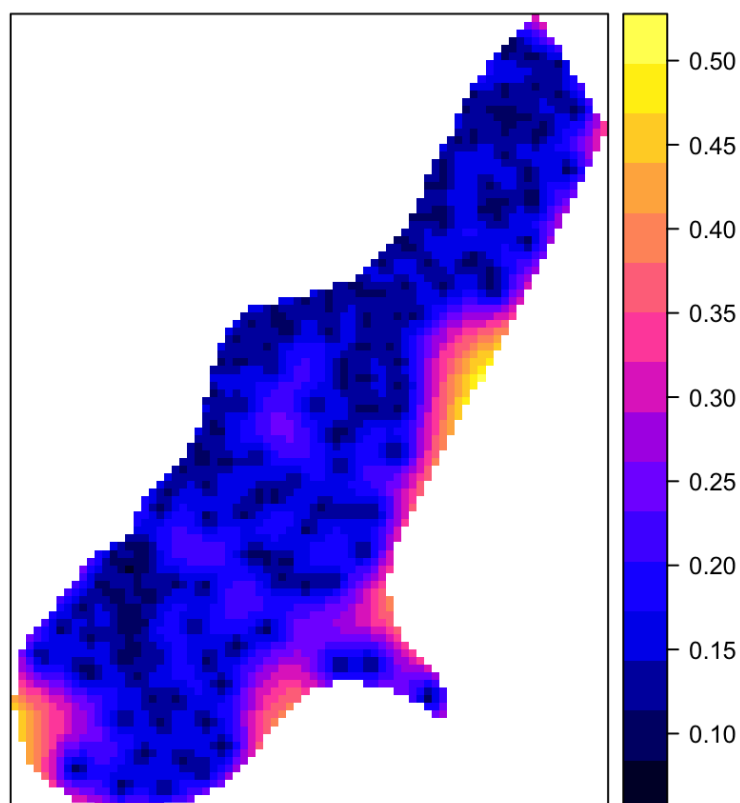
```
# Display the predictions and their errors for Log(zinc) and Log(cadmium)
spplot(cokrig.model["logzinc.pred"], main = "co kriging predictions")
```

co kriging predictions

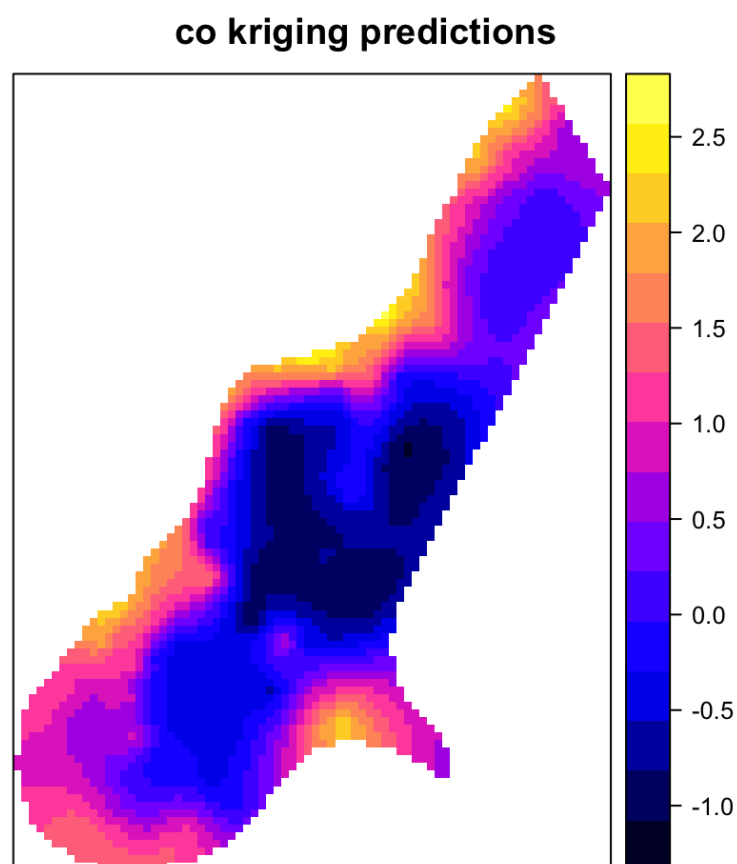


```
spplot(cokrig.model["logzinc.var"], main = "co kriging variance")
```

co kriging variance

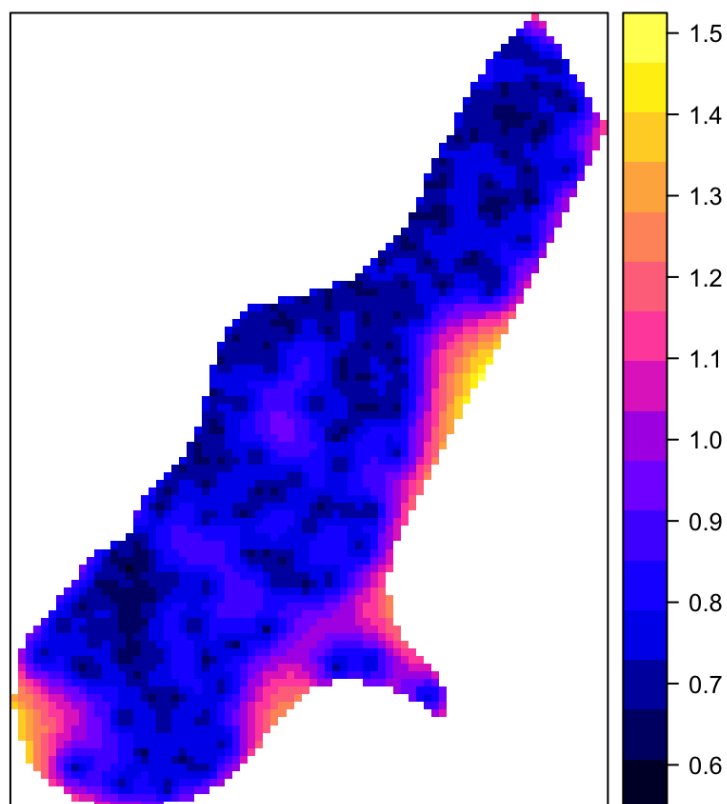


```
spplot(cokrig.model["logcadmium.pred"], main = "co kriging predictions")
```



```
spplot(cokrig.model["logcadmium.var"], main = "co kriging variance")
```

co kriging variance



Indicator Co-Kriging

```
# # Create a categorical variable of zinc (grouped by quartile)
quartzinc <- quantile(meuse$zinc)
quartzinc
```

```
##      0%    25%    50%    75%   100%
## 113.0  198.0  326.0  674.5 1839.0
```

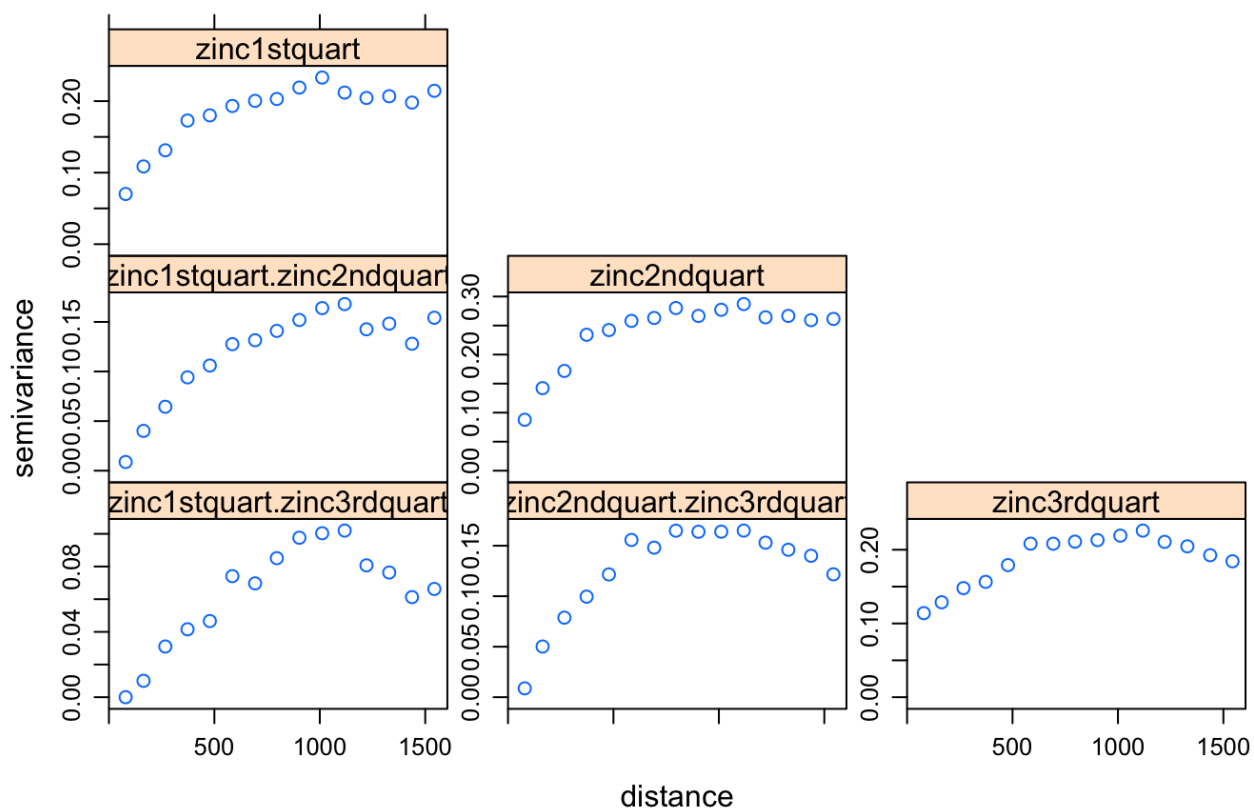
```
quartzinc[2]
```

```
## 25%
## 198
```

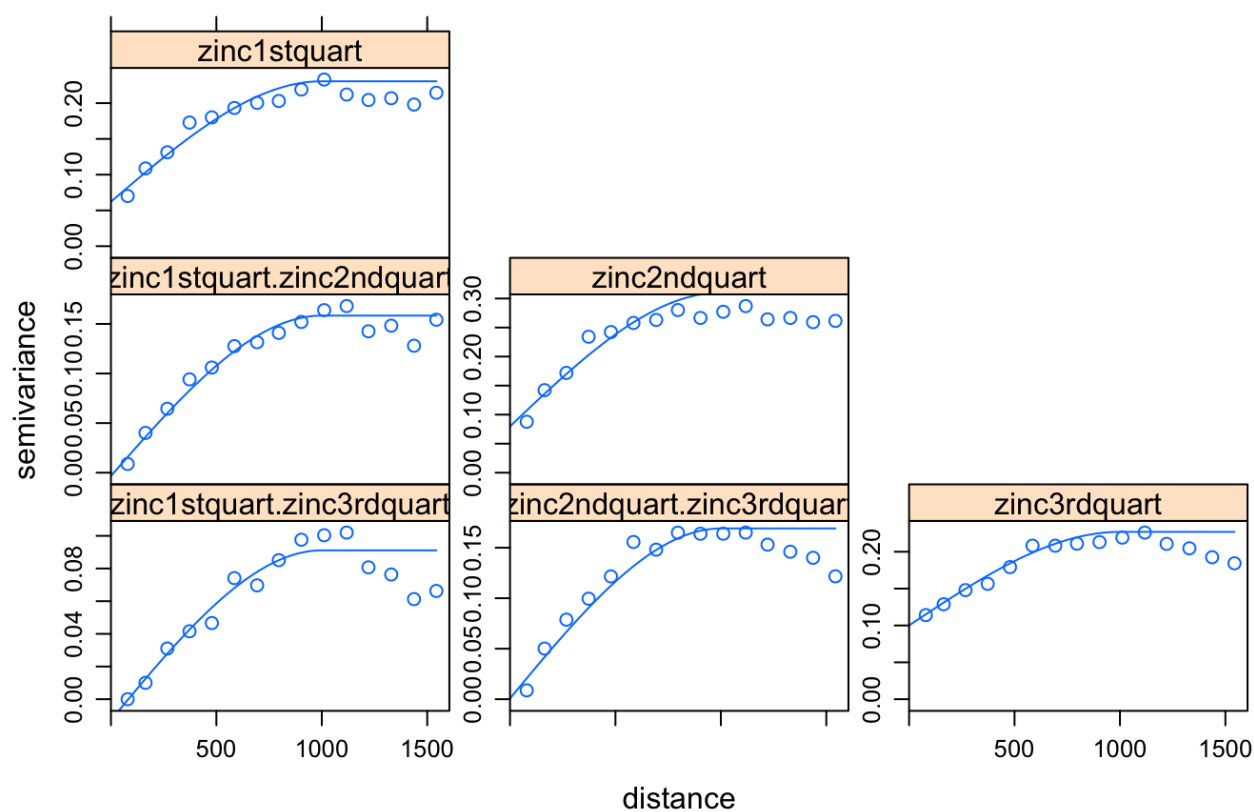
```
# create 3 separate datasets, one for each quartile. Example, one dataset will be 1-0 whether a zinc value is below the 25th percentile. Another dataset will be 1-0 whether a zinc value is below the median mark.
zinc.quart <- gstat(NULL, id = "zinc1stquart", formula = I(zinc < quantzinc[2]) ~ 1, data = meuse,
  nmax = 10, beta = 0.25, set = list(order = 4, zero = 1e-05))
zinc.quart <- gstat(zinc.quart, "zinc2ndquart", formula = I(zinc < quantzinc[3]) ~ 1, data = meuse,
  nmax = 10, beta = 0.50)
zinc.quart <- gstat(zinc.quart, "zinc3rdquart", formula = I(zinc < quantzinc[4]) ~ 1, data = meuse,
  nmax = 10, beta = 0.75)

# Create an initial semivariogram model with range equal 1200
zinc.quart <- gstat(zinc.quart, model = vgm(1, "Sph", 1000, 1), fill.all = T)

# Estimate the empirical variogram of each indicator
zincqvag <- variogram(zinc.quart)
plot(zincqvag)
```



```
# fit a single semivariogram model to all these empirical varioagrams using a linear model of coregionalization (the fit.lmc() function).
zinc.quartfit = fit.lmc(zincqvag, zinc.quart)
plot(zincqvag, model = zinc.quartfit)
```



```
# now do the co-kriging
cokrig.zincquart <- predict.gstat(zinc.quartfit, meuse.grid)
```

```
## Linear Model of Coregionalization found. Good.
## [using simple cokriging]
```

```
str(cokrig.zincquart)
```

```
## Formal class 'SpatialPixelsDataFrame' [package "sp"] with 7 slots
## ..@ data      :'data.frame':  3103 obs. of  9 variables:
## .. ..$ zinc1stquart.pred      : num [1:3103] 0.043046 0.000314 0.029345 0.042939 0 ...
## .. ..$ zinc1stquart.var       : num [1:3103] 0.14 0.123 0.128 0.134 0.106 ...
## .. ..$ zinc2ndquart.pred      : num [1:3103] 0.2503 0.1739 0.2451 0.291 0.0876 ...
## .. ..$ zinc2ndquart.var       : num [1:3103] 0.182 0.158 0.165 0.173 0.133 ...
## .. ..$ zinc3rdquart.pred      : num [1:3103] 0.576 0.517 0.574 0.613 0.451 ...
## .. ..$ zinc3rdquart.var       : num [1:3103] 0.158 0.146 0.15 0.154 0.132 ...
## .. ..$ cov.zinc1stquart.zinc2ndquart: num [1:3103] 0.064 0.0469 0.0521 0.0579 0.0285 ...
## .. ..$ cov.zinc1stquart.zinc3rdquart: num [1:3103] 0.02845 0.01716 0.02063 0.0245 0.00495 ...
## .. ..$ cov.zinc2ndquart.zinc3rdquart: num [1:3103] 0.0763 0.0593 0.0643 0.07 0.041 ...
## ..@ coords.nrs : int [1:2] 1 2
## ..@ grid       :Formal class 'GridTopology' [package "sp"] with 3 slots
## .. .. ..@ cellcentre.offset: Named num [1:2] 178460 329620
## .. .. ..- attr(*, "names")= chr [1:2] "x" "y"
## .. .. ..@ cellsize         : Named num [1:2] 40 40
## .. .. ..- attr(*, "names")= chr [1:2] "x" "y"
## .. .. ..@ cells.dim        : Named int [1:2] 78 104
## .. .. ..- attr(*, "names")= chr [1:2] "x" "y"
## ..@ grid.index : int [1:3103] 69 146 147 148 223 224 225 226 300 301 ...
## ..@ coords     : num [1:3103, 1:2] 181180 181140 181180 181220 181100 ...
## .. ..- attr(*, "dimnames")=List of 2
## .. .. ..$ : chr [1:3103] "1" "2" "3" "4" ...
## .. .. ..$ : chr [1:2] "x" "y"
## ..@ bbox       : num [1:2, 1:2] 178460 329620 181540 333740
## .. ..- attr(*, "dimnames")=List of 2
## .. .. ..$ : chr [1:2] "x" "y"
## .. .. ..$ : chr [1:2] "min" "max"
## ..@ proj4string:Formal class 'CRS' [package "sp"] with 1 slot
## .. .. ..@ projargs: chr NA
```

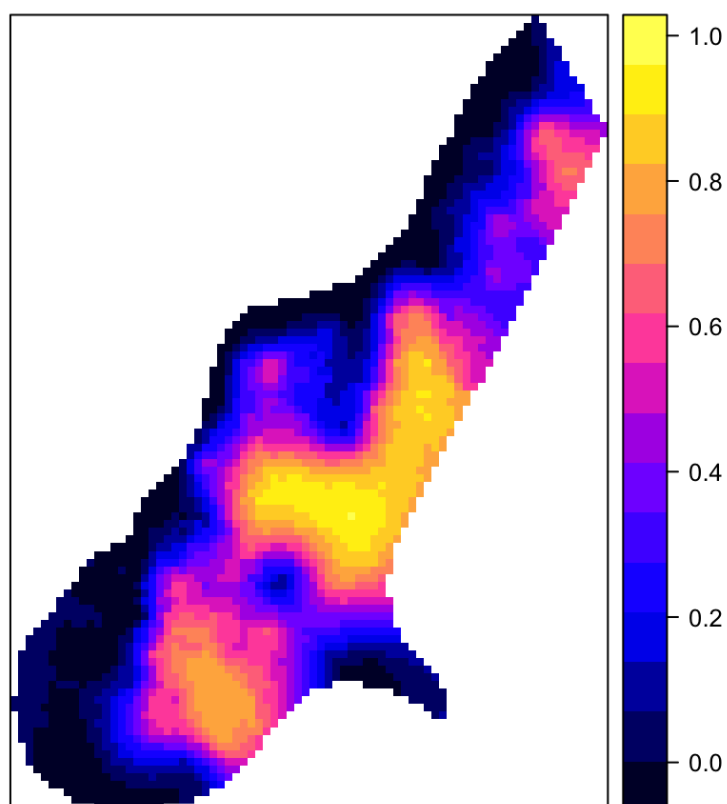
```
summary(cokrig.zincquart)
```



```
## Object of class SpatialPixelsDataFrame
## Coordinates:
##      min      max
## x 178460 181540
## y 329620 333740
## Is projected: NA
## proj4string : [NA]
## Number of points: 3103
## Grid attributes:
##   cellcentre.offset cellsize cells.dim
## x           178460         40         78
## y           329620         40        104
## Data attributes:
##   zinc1stquart.pred zinc1stquart.var zinc2ndquart.pred zinc2ndquart.var
## Min.      :0.00000   Min.      :0.07852   Min.      :0.00000   Min.      :0.0978
## 1st Qu.:0.03981   1st Qu.:0.09278   1st Qu.:0.2856   1st Qu.:0.1166
## Median :0.31988   Median :0.09936   Median :0.7084   Median :0.1257
## Mean   :0.34905   Mean   :0.10584   Mean   :0.6101   Mean   :0.1344
## 3rd Qu.:0.59063   3rd Qu.:0.11312   3rd Qu.:0.9272   3rd Qu.:0.1442
## Max.   :0.96129   Max.   :0.18936   Max.   :1.0000   Max.   :0.2516
##   zinc3rdquart.pred zinc3rdquart.var cov.zinc1stquart.zinc2ndquart
## Min.      :0.2343   Min.      :0.1121   Min.      :0.005372
## 1st Qu.:0.6127   1st Qu.:0.1228   1st Qu.:0.017969
## Median :0.9144   Median :0.1277   Median :0.024285
## Mean   :0.8023   Mean   :0.1326   Mean   :0.030428
## 3rd Qu.:1.0000   3rd Qu.:0.1381   3rd Qu.:0.036929
## Max.   :1.0000   Max.   :0.1958   Max.   :0.115197
##   cov.zinc1stquart.zinc3rdquart cov.zinc2ndquart.zinc3rdquart
## Min.      :-0.009118           Min.      :0.01487
## 1st Qu.: -0.001283           1st Qu.:0.02875
## Median : 0.002700           Median :0.03540
## Mean   : 0.006664           Mean   :0.04184
## 3rd Qu.: 0.010691           3rd Qu.:0.04890
## Max.   : 0.062379           Max.   :0.12679
```

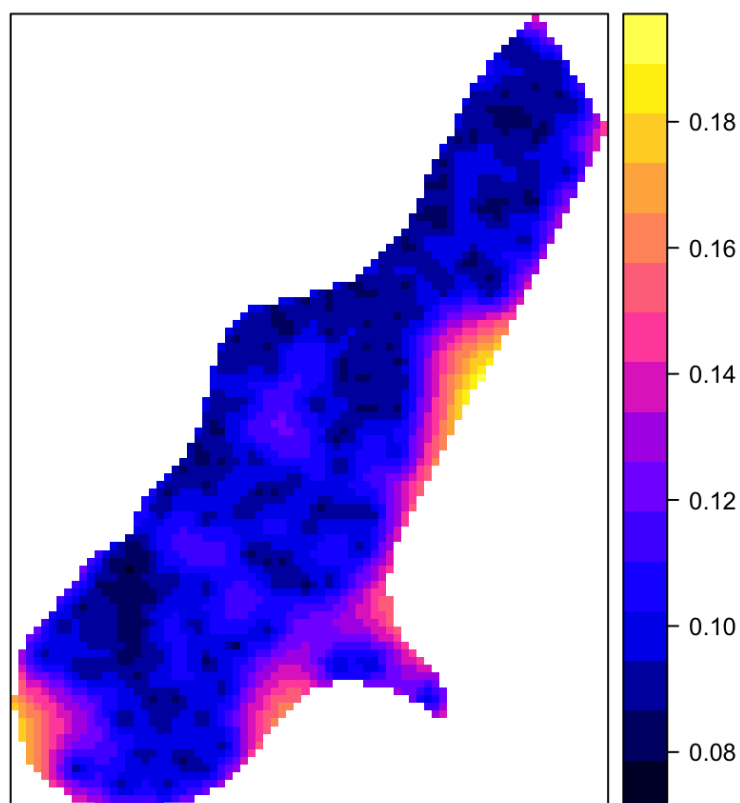
```
# plot results
spplot(cokrig.zincquart["zinc1stquart.pred"], main = "co kriging predictions")
```

co kriging predictions



```
spplot(cokrig.zincquart["zinc1stquart.var"], main = "co kriging variance")
```

co kriging variance



```
# more fancy plotting  
library(gridExtra)
```

```
## Loading required package: grid
```

```
grid.arrange(spplot(cokrig.zincquart["zinc1stquart.pred"], main = "co kriging predictions"),  
             spplot(cokrig.zincquart["zinc2ndquart.pred"], main = "co kriging predictions"),  
             ncol=2)
```

