Lecture 9 – The Inductor

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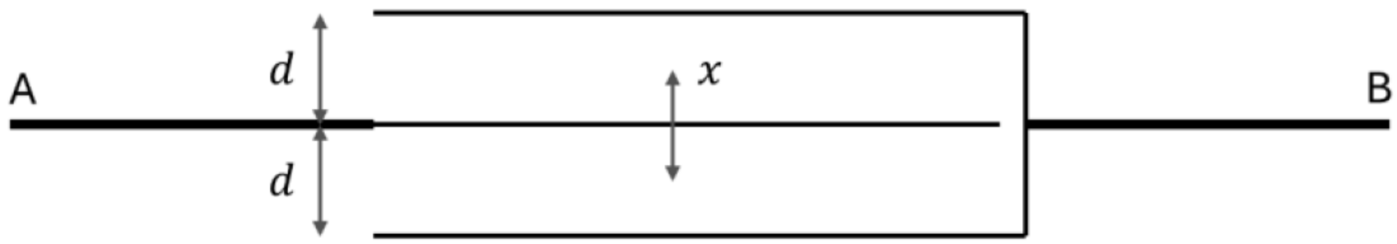
Q1

Q1. Say we are tasked with designing a 5V, 2A regulated power supply. We chose the Zener diode circuit to do this, with an input power supply that can go from 6 to 9 V. Assume the Zener chosen has a 10 mA minimum current for the Zener effect. What should be the specifications for the Zener and the series resistor?

[10 marks]

Q2

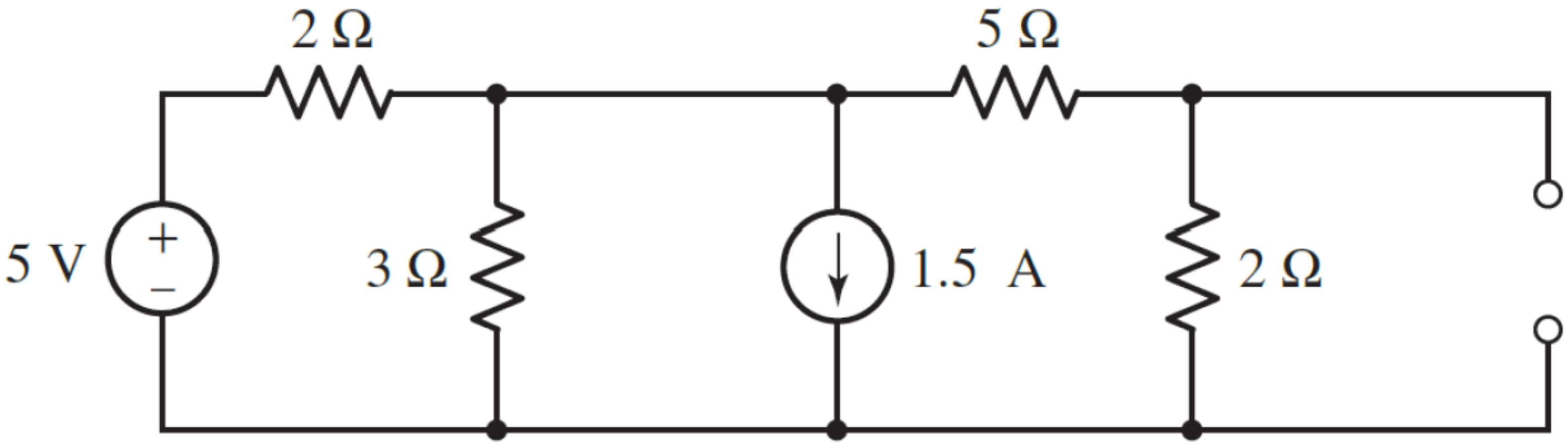
Q2. Accelerometers are made using a capacitive structure with a movable plate. Refer to the structure shown in the figure. Initially the plate is in the middle ($x = 0$). If the middle plate moves vertically by x, what is the equivalent capacitance of the system with respect to the initial capacitance of the system (C_0)? Make a rough plot of C/C_0 with x. Assume plate area is the same in all cases. Ignore fringe capacitance effects. Assume $x < d$ <<linear dimensions.



[10 marks]

Q3

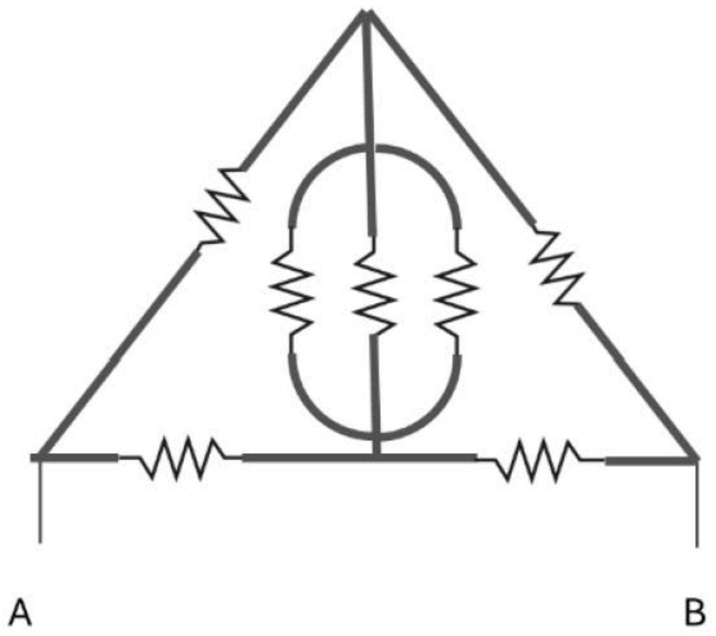
Q3. Create the Thevenin equivalent circuit for the circuit given below:



[10 marks]

Q4

Q4. Find the equivalent resistance of the Deathly Hallows symbol, between A and B. All resistors are R.



[10 marks]

Current and magnetic field

- Experiments involving currents and magnetic field are very old
- Danish scientist Hans Christian Ørsted showed that a current-carrying conductor produced a magnetic field (i.e. compass needles were affected in the presence of a wire when current was flowing)
- Shortly thereafter, Ampère (in competition with Biot-Savart) made some careful measurements which demonstrated that this magnetic field was *linearly* related to the current which produced it

$$B \propto i$$

Hans Christian Ørsted

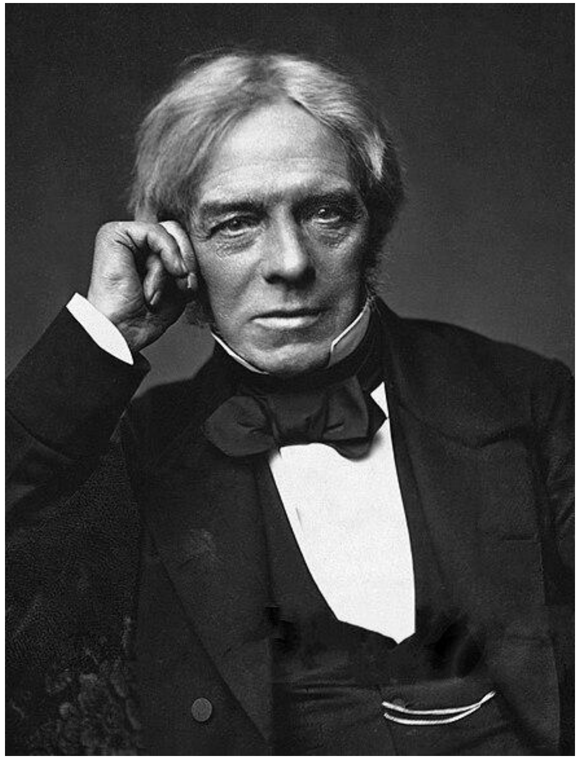
André-Marie Ampère

Electromagnetic induction

- Some years later, the inverse effect was discovered – that magnetic fields can also produce currents
- In this case, the magnetic fields had to be changing, and the voltage induced because of them was proportional to the rate of change of the field

$$v \propto \frac{dB}{dt}$$

- Discovered by Micheal Faraday (in competition with Joseph Henry)



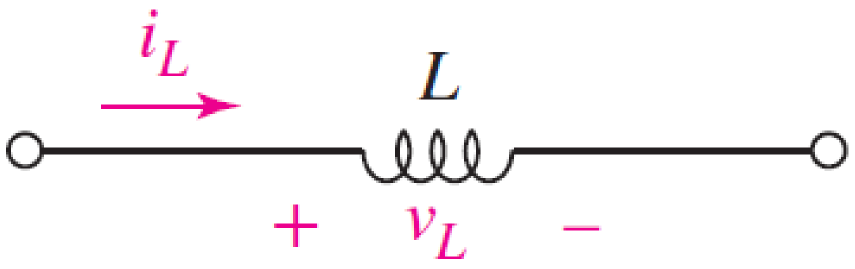
Michael Faraday

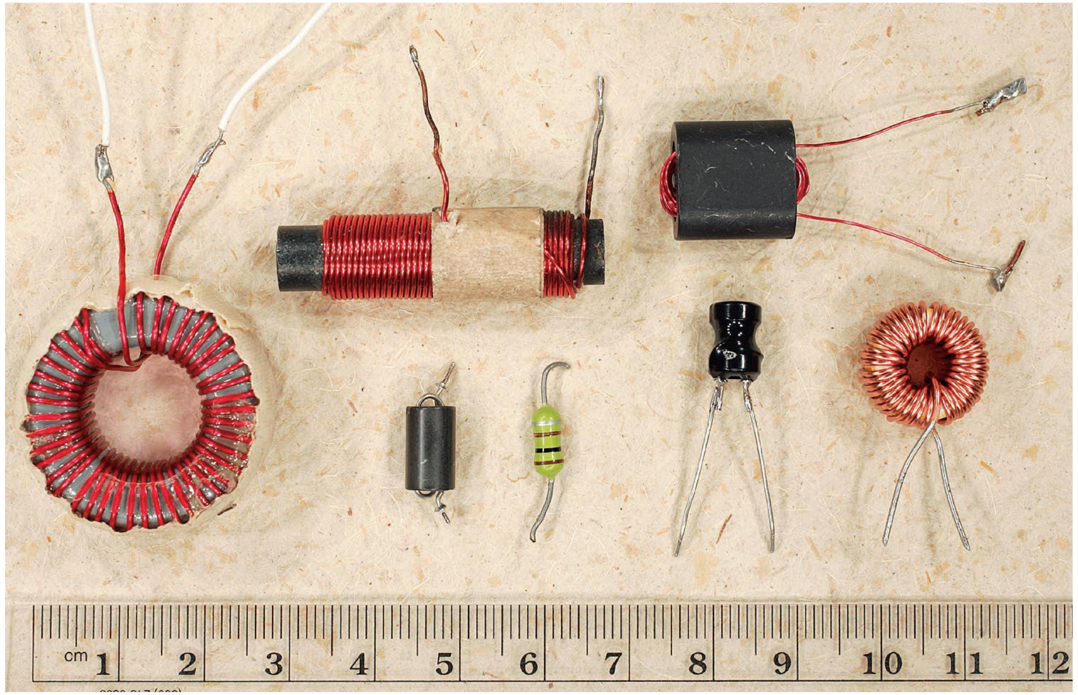
Joseph Henry

Self induction

- As it turns out, a voltage is induced in the wire carrying the current itelf, ie, a change in the current of a wire causes a change in the magnetic field around it
- This change in the magnetic field causes a voltage to be induced in the wire itself
- This is self inductance, and the coefficient of proportionality is the inductance of a wire

$$v = L\frac{di}{dt}$$





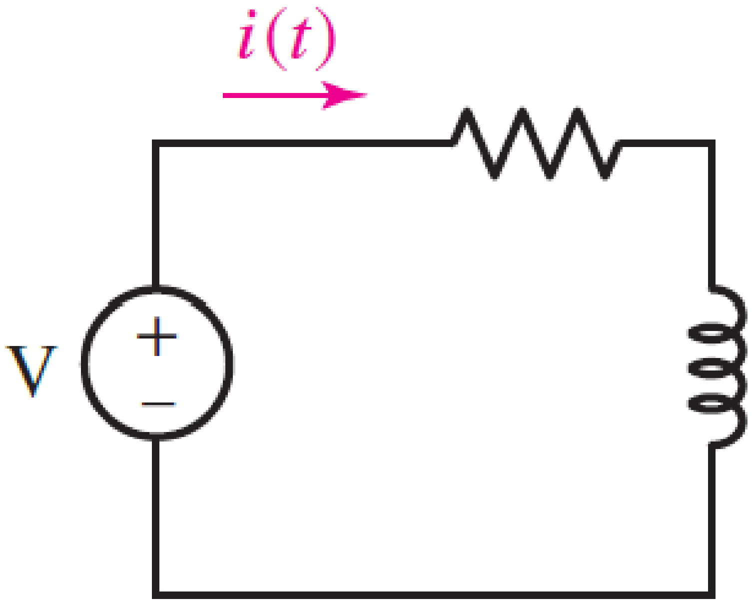
Energy storage

- For an increasing current $i(t)$, there is a voltage induced $v(t)$ that is opposing the change
- Thus, the current needs to exert “work” to increase
- This work is stored as potential energy in the coil

$$p(t) = i(t)v(t)$$

$$p = iL\frac{di}{dt}$$

$$E = \int p\,dt = L\int_0^I i di = \frac{1}{2}LI^2$$



Inductors in series

- From KVL:

$$v_s = v_1 + v_2 + \cdots + v_n$$

$$v_s = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \cdots + L_n \frac{di}{dt}$$

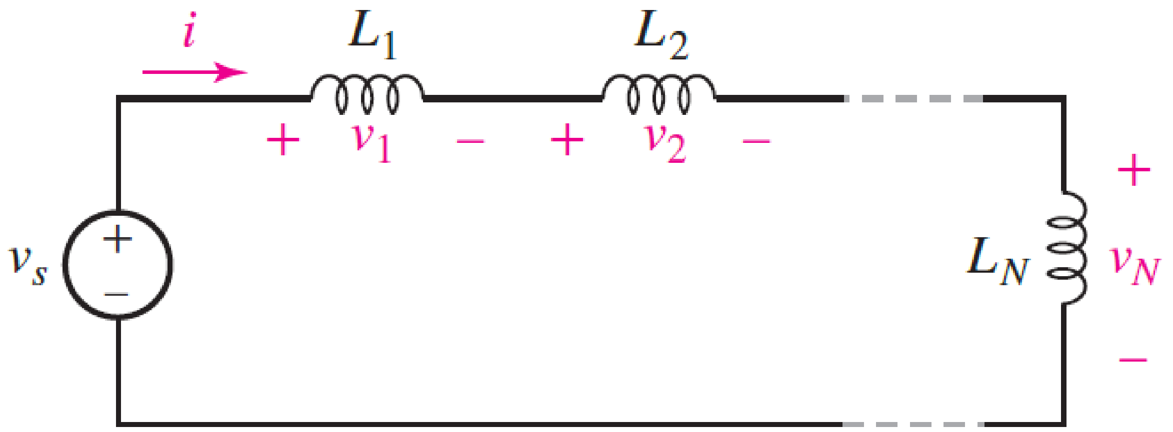
$$v_s = (L_1 + L_2 + \cdots + L_n) \frac{di}{dt}$$

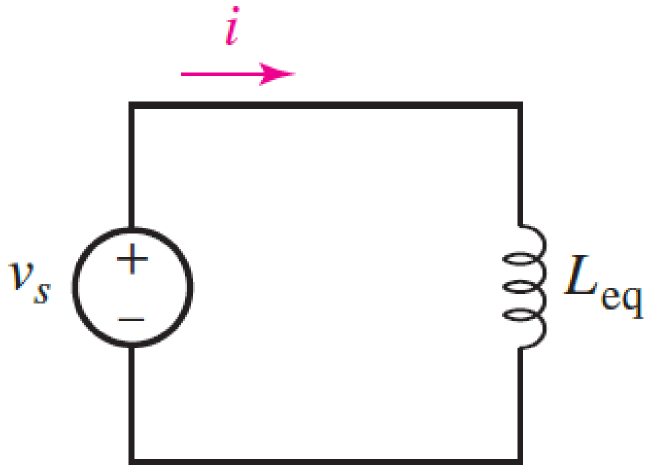
- For the equivalent circuit:

$$v_s = L_{eq} \frac{di}{dt}$$

- Thus,

$$L_{eq} = L_1 + L_2 + \cdots + L_n$$





Inductors in parallel

- From KCL: $i_s = i_1 + i_2 + \cdots + i_n$

$$v = L_i \frac{di_i}{dt}$$

$$i_i = \frac{1}{L_i} \int v\, dt$$

$$i_s = \frac{1}{L_1} \int v\, dt + \frac{1}{L_2} \int v\, dt + \cdots + \frac{1}{L_n} \int v\, dt$$

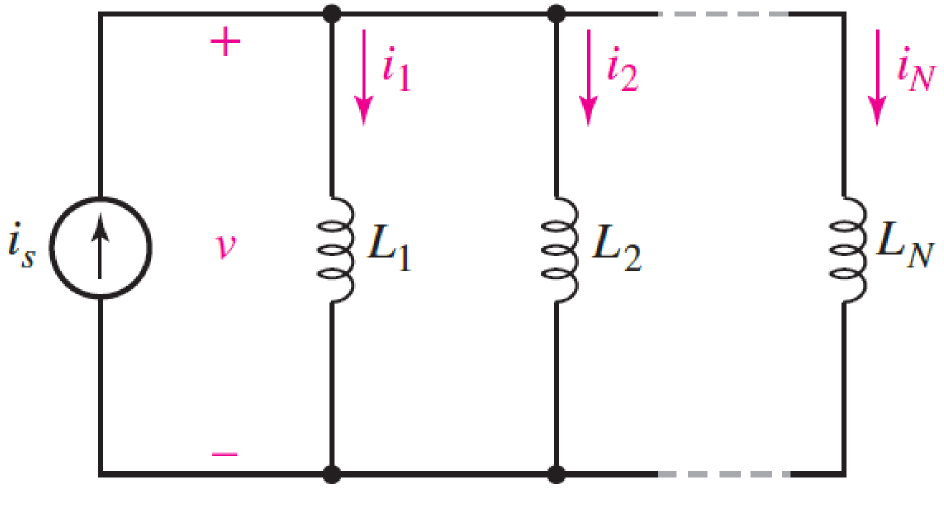
$$i_s = \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n} \right) \int v\, dt$$

- For the equivalent:

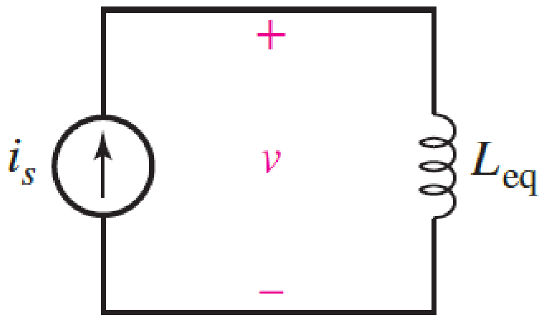
$$i_s = \frac{1}{L_{eq}} \int v\, dt$$

- Thus,

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$$



(a)



(b)

RL circuit

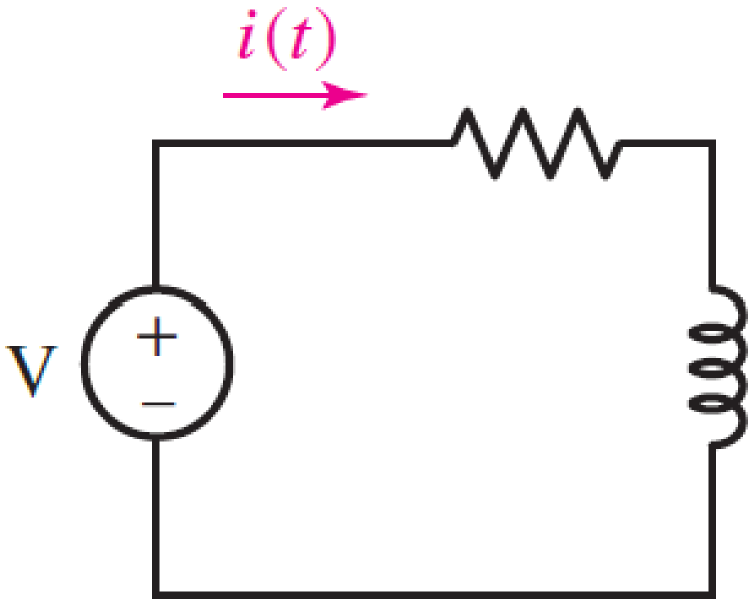
- When we connect an inductor to the a voltage source through a resistor, the current in the circuit wants to rise immediately, ie, from zero to V/R
- However, the inductor does not allow that to happen, creating a back EMF of di/dt
- At any given time, the KVL can be written:

$$V = iR + v_L$$

$$V = iR + L\frac{di}{dt}$$

$$\frac{di}{(V - iR)} = \frac{dt}{L}$$

$$-\ln(V - iR) = \frac{Rt}{L} + A$$



RL circuit

$$-\ln(V - iR) = \frac{Rt}{L} + A$$

At $t = 0, i = 0$

Thus,

$$\ln\left(\frac{V}{V - iR}\right) = \frac{Rt}{L}$$

$$V - iR = Ve^{-\frac{Rt}{L}}$$

$$i = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

Thus, at $t = 0$, the current is zero. It gradually increases to V/R as $t \to \infty$

$$v_L = L\frac{di}{dt} = Ve^{-\frac{Rt}{L}}$$