# Lecture 12 – RLC circuit 2

Dr. Aftab M. Hussain,

Associate Professor, PATRIoT Lab, CVEST

# Parallel RLC circuit

- Fundamental equation:

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

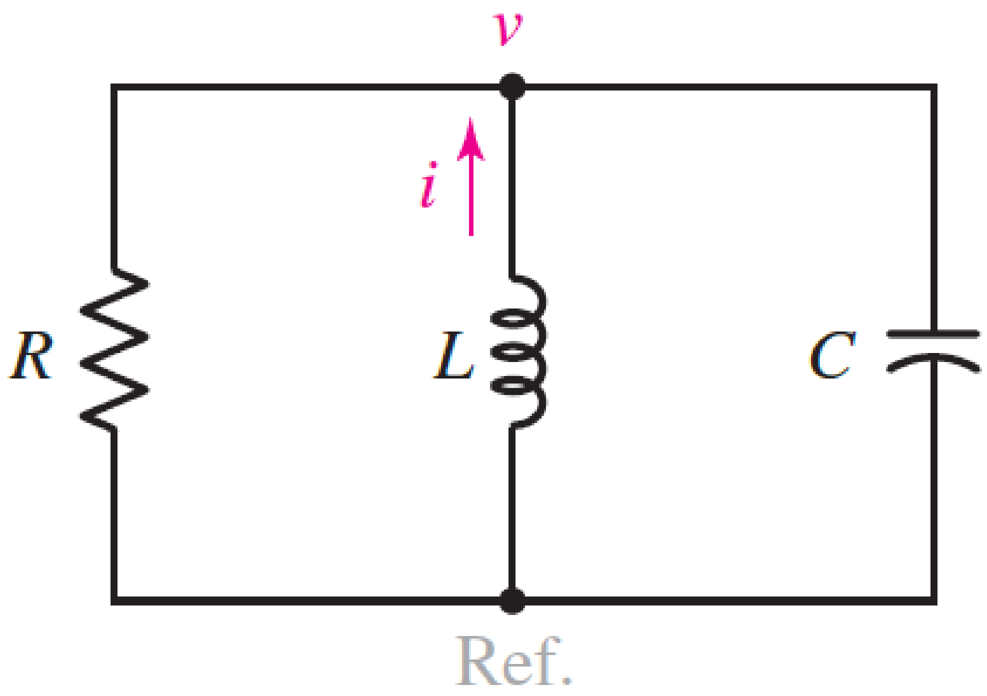
- Overdamped when:

$$\frac{1}{4R^2C^2} > \frac{1}{LC}$$

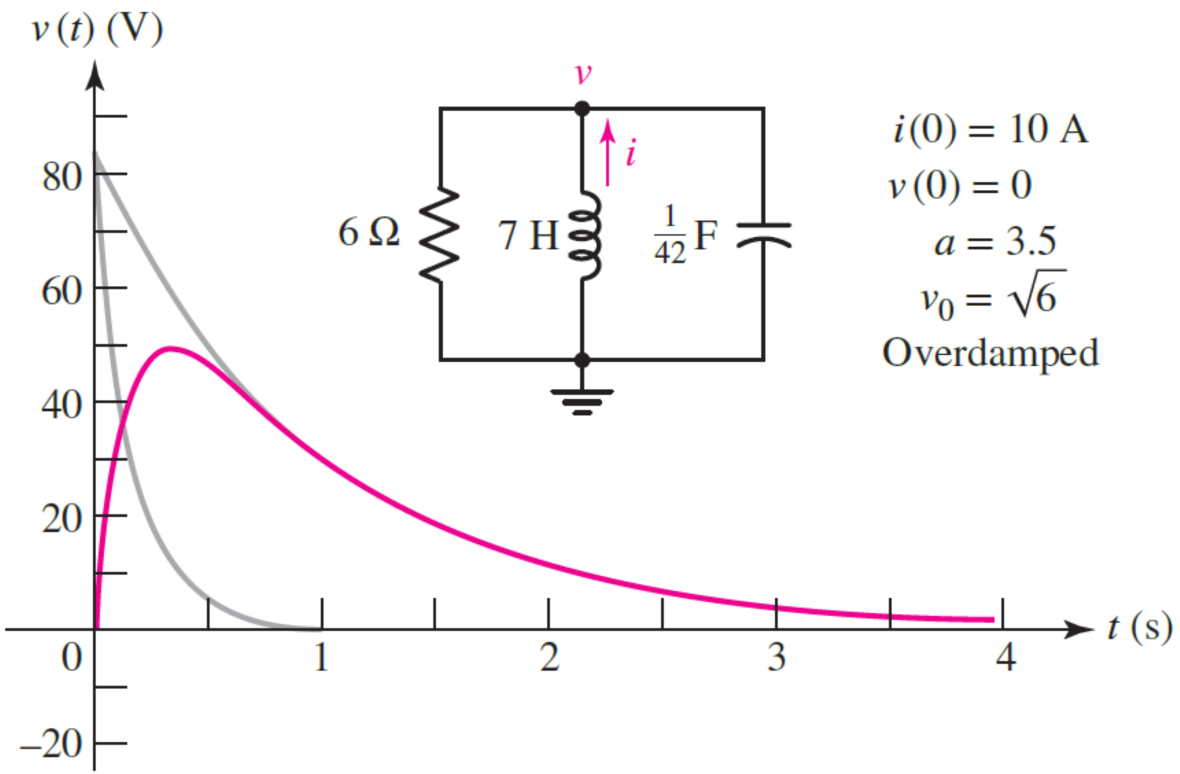
- Output: $v = A_1 e^{st} + A_2 e^{st}$
- Critically damped when:

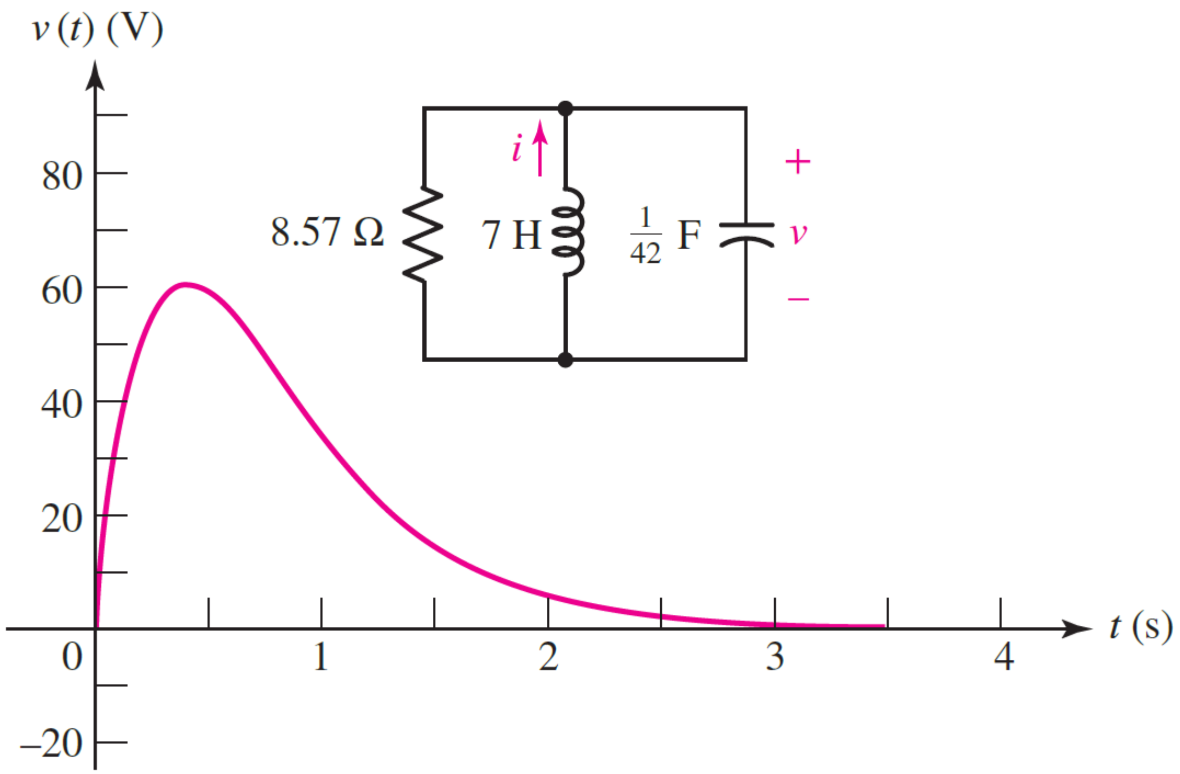
$$\frac{1}{4R^2C^2} = \frac{1}{LC}$$

- Output: $v = A_1 t e^{st} + A_2 e^{st}$



# Parallel RLC circuit



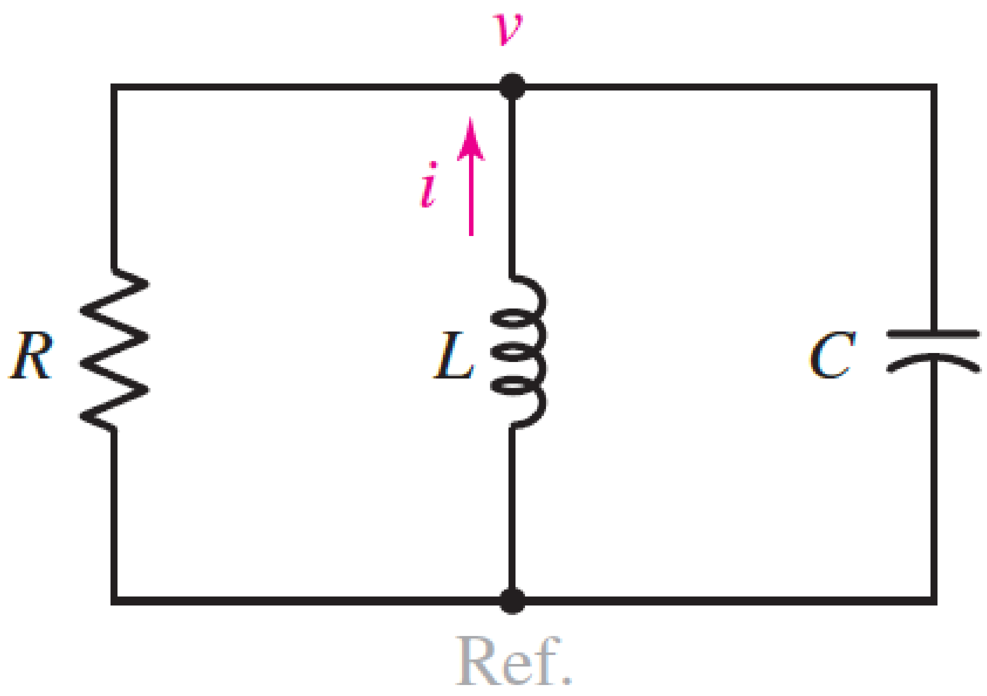
# Analysis - 3

- Finally, we look at the case when $4R^2C^2 < LC$
- In this case, both $s$ values are imaginary and complex conjugates
- We can now define some terms for better analysis of this problem
- We define:

$$\omega_0 \equiv \frac{1}{\sqrt{LC}}$$

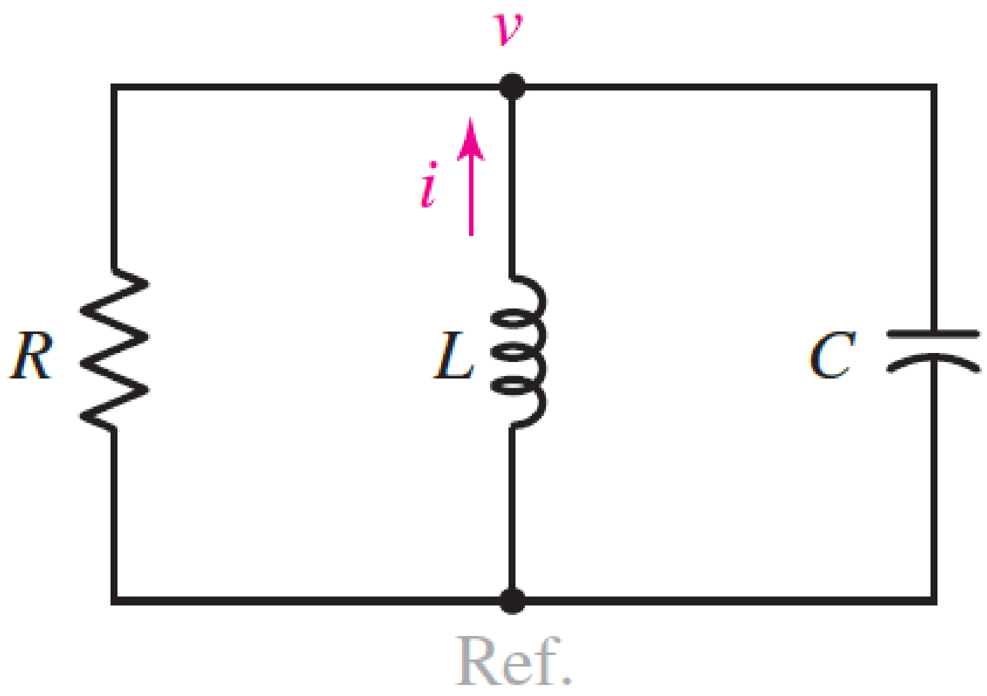
$$\alpha \equiv \frac{1}{2RC}$$

- Thus, $s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$
- We can define $\omega = \sqrt{\omega_0^2 - \alpha^2}$



# Analysis - 3

- Thus, the general solution to the differential equation becomes:
$$v = e^{-\alpha t}\left(A_1 e^{j\omega t} + A_2 e^{-j\omega t}\right)$$
- This equation can be rewritten as:
$$v = e^{-\alpha t}\left\{(A_1 + A_2)\left[\frac{\left(e^{j\omega t} + e^{-j\omega t}\right)}{2}\right] + \mathrm{j}(A_1 - A_2)\left[\frac{\left(e^{j\omega t} - e^{-j\omega t}\right)}{2j}\right]\right\}$$
$$v = e^{-\alpha t}\{B_1 \cos \omega t + B_2 \sin \omega t\}$$
- This is the general solution. To find the values of parameters, we need to take an example



# Analysis - 3

- We go back to the example with $R = 10.5\ \Omega$

$$\alpha = \frac{1}{2RC} = 2$$

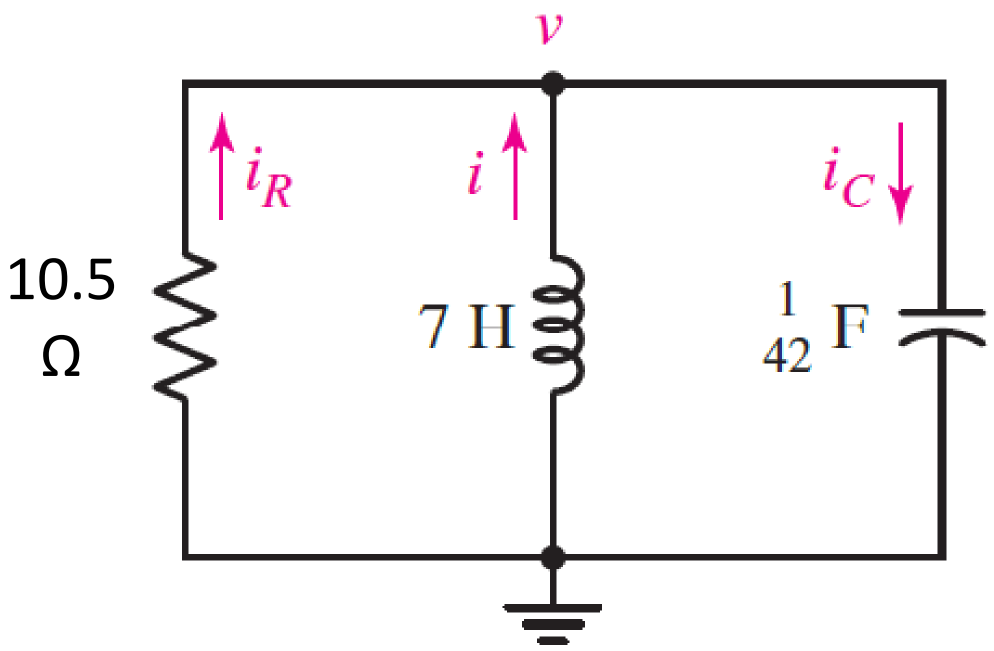
$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{6}$$

$$\omega = \sqrt{\omega_0^2 - \alpha_0^2} = \sqrt{2}$$

- Thus, the response is now:

$$v = e^{-2t}\{B_1 \cos\sqrt{2}t + B_2 \sin\sqrt{2}t\}$$

- For $B$, we take the initial condition that $i(0) = 10$ and $v(0) = 0$



# Analysis - 3

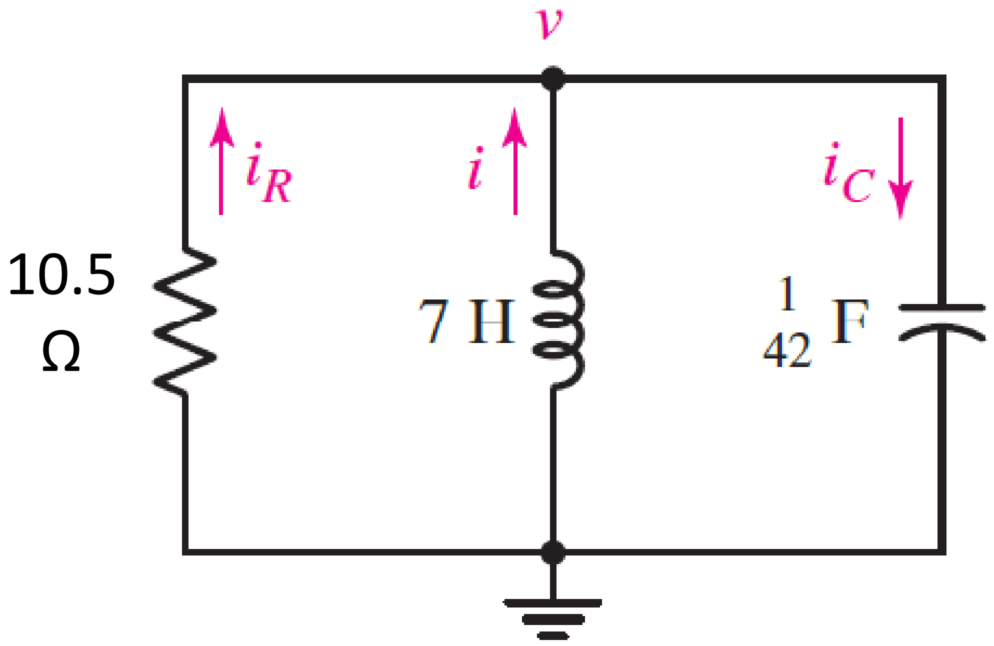
$$v = e^{-2t}\{B_1 \cos\sqrt{2}t + B_2 \sin\sqrt{2}t\}$$

- For $B$, we take the initial condition that $i(0) = 10$ and $v(0) = 0$
- If $v(0) = 0$, then $B_1$ has to be zero
- Thus,

$$v = e^{-2t} B_2 \sin\sqrt{2}t$$

Take the derivative:

$$\frac{dv}{dt} = B_2\left[\sqrt{2}e^{-2t}\cos\sqrt{2}t - 2e^{-2t}\sin\sqrt{2}t\right]$$



# Analysis - 3

$$\frac{dv}{dt} = B_2\left[\sqrt{2}e^{-2t}\cos\sqrt{2}t - 2e^{-2t}\sin\sqrt{2}t\right]$$

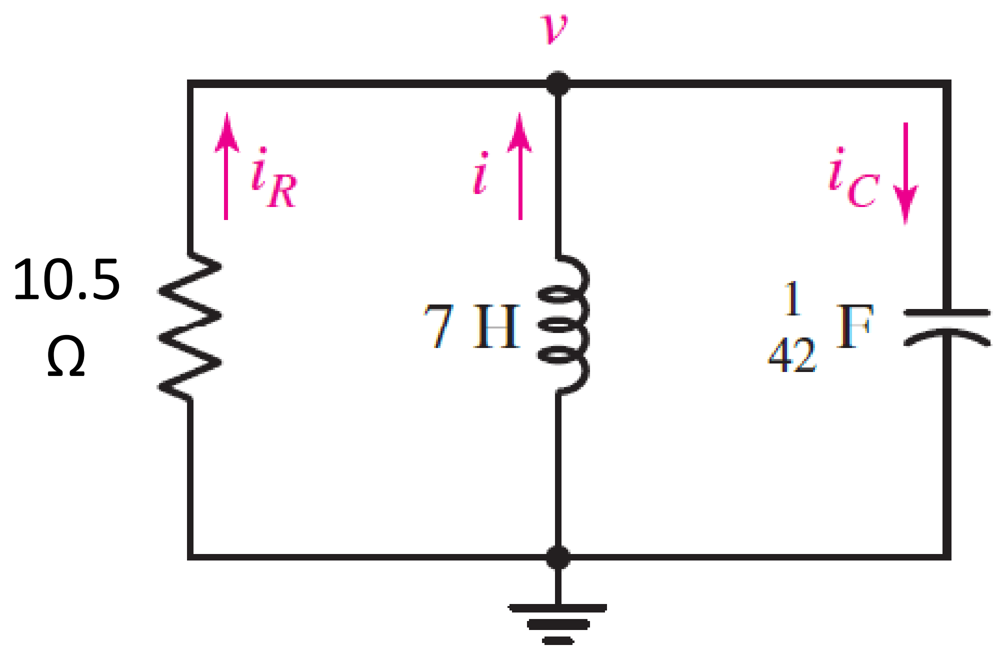
- We put:

$$\left.\frac{dv}{dt}\right|_{t=0} = \frac{i_L(0)}{C} = 420$$

$$\sqrt{2}\,B_2 = 420$$

Thus, our response is:

$$v = 210\sqrt{2}e^{-2t}\sin\sqrt{2}t$$

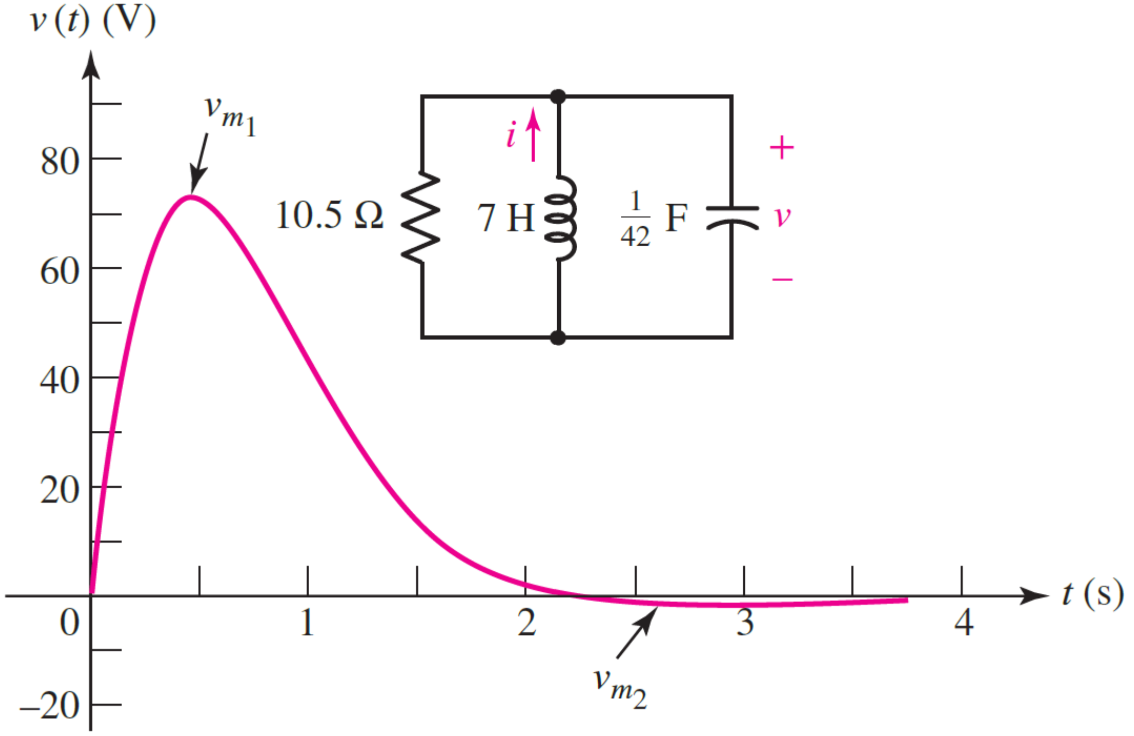


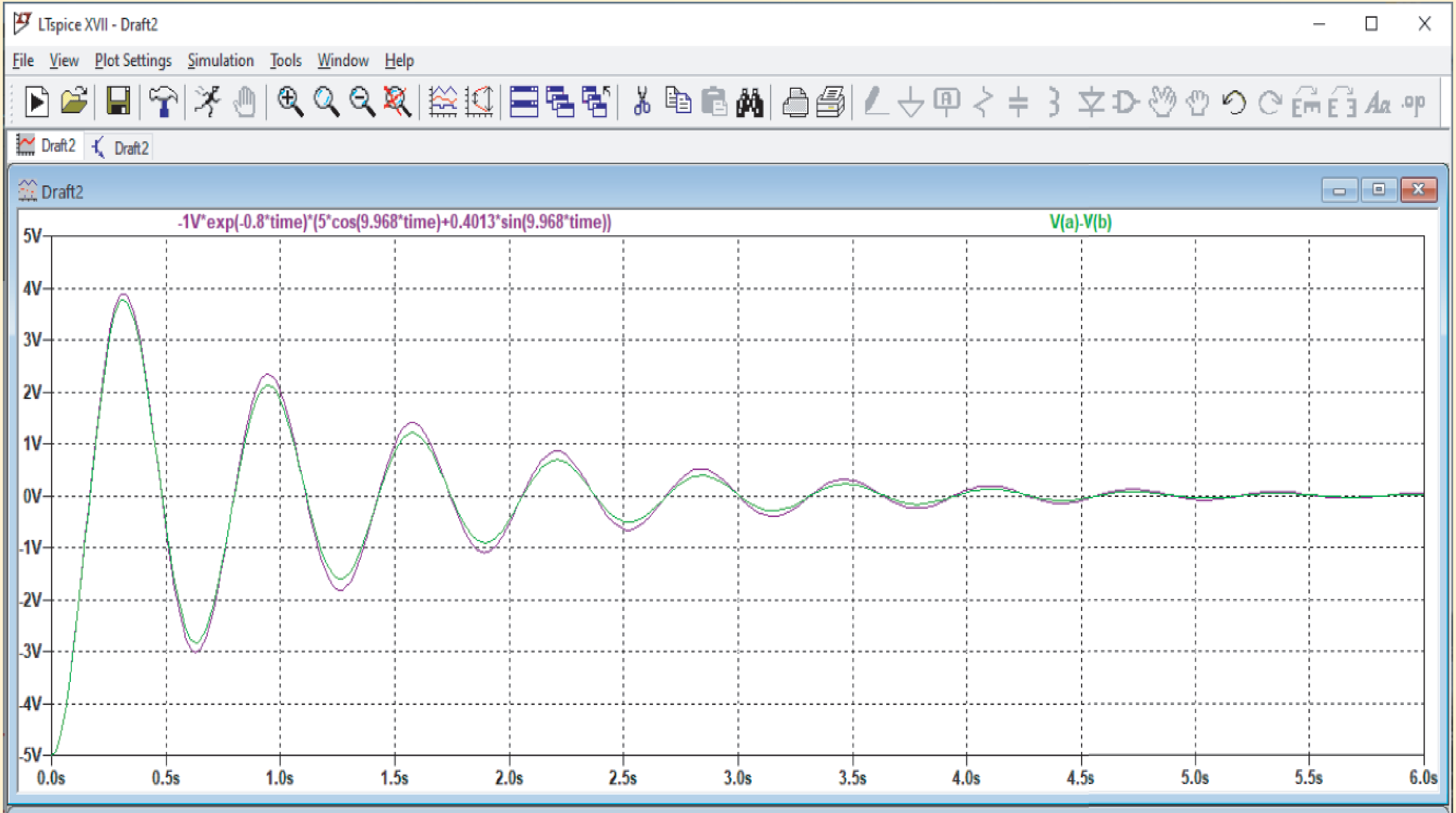
# Analysis - 3

Thus, our response is:

$$v = 210\sqrt{2}e^{-2t}\sin\sqrt{2}t$$

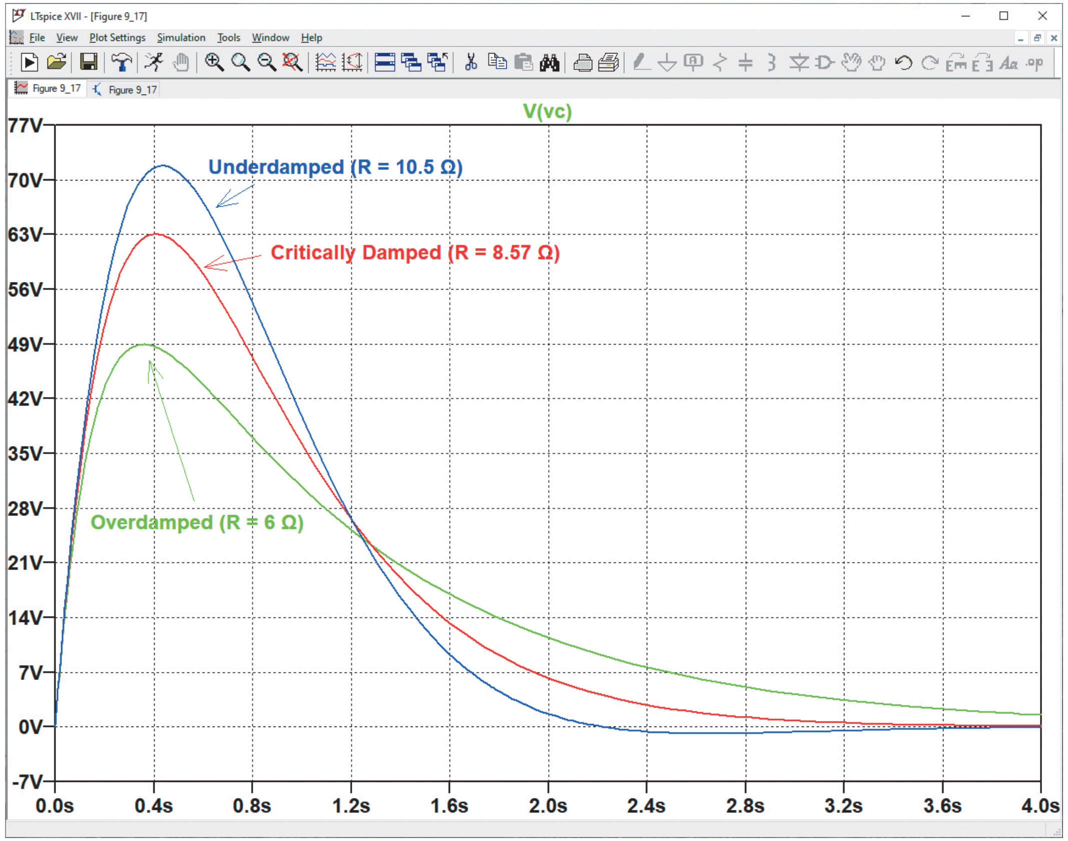
- Properties:
- The response is real!
- It has an oscillatory component that crosses zero on the x-axis for every $\sqrt{2}t = n\pi$
- It has an exponential decay component that causes the voltage to die out
- Most of the oscillations are hidden (damped) because of this decay




LTspice XVII - Draft2
File View Plot Settings Simulation Tools Window Help
Draft2
Draft2
Draft2
-1V*exp(-0.8*time)*(5*cos(9.968*time)+0.4013*sin(9.968*time))
V(a)-V(b)
5V
4V
3V
2V
1V
0V
-1V
-2V
-3V
-4V
-5V
0.0s
0.5s
1.0s
1.5s
2.0s
2.5s
3.0s
3.5s
4.0s
4.5s
5.0s
5.5s
6.0s

# The Parallel RLC circuit

- When the damping is changed by increasing the size of the parallel resistance, the maximum magnitude of the response is greater, and the amount of damping is smaller
- The response becomes oscillatory when underdamping is present
- The minimum settling time is obtained for slight underdamping



# The LC circuit

- Lets look at an extreme case
- When $R \rightarrow \infty$, i.e. R is open circuit
- This results in a "lossless LC" circuit with the characteristic equation:

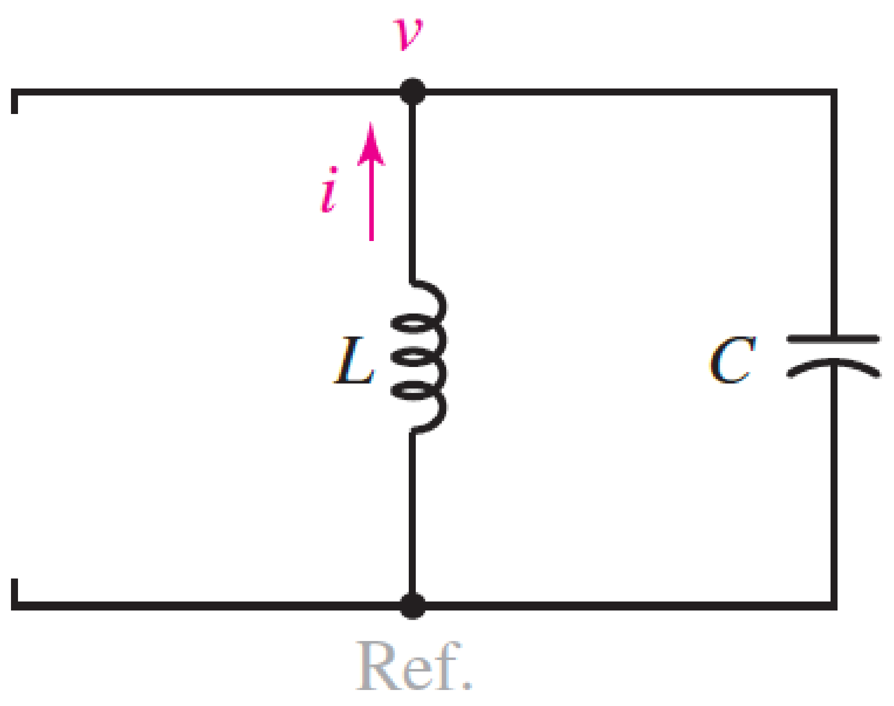
$$C\frac{d^2v}{dt^2} + \frac{1}{L}v = 0$$

- In this case as well, the general repose remains the same, with:

$$\alpha = 0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$v = B_1 \cos \omega t + B_2 \sin \omega t$$



# The LC circuit

- To calculate B we need to take some values: $v(0) = 0$ and $i(0) = 10$
- Thus,

$$v = B_2 \sin \sqrt{6}t$$

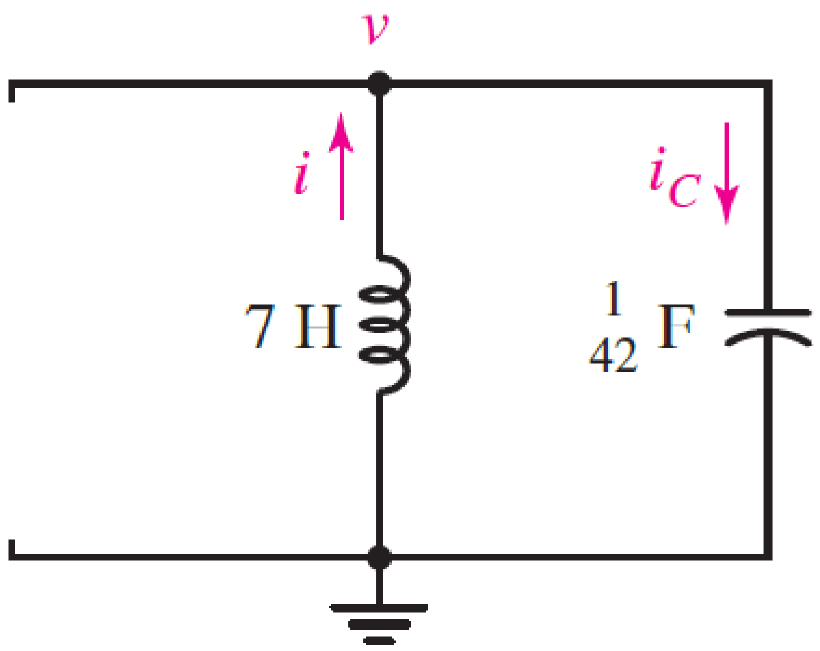
$$\frac{dv}{dt} = \sqrt{6}B_2 \cos \sqrt{6}t$$

$$\left.\frac{dv}{dt}\right|_{t=0} = \frac{i(0)}{C} = 420 = \sqrt{6}B_2$$

- Thus,

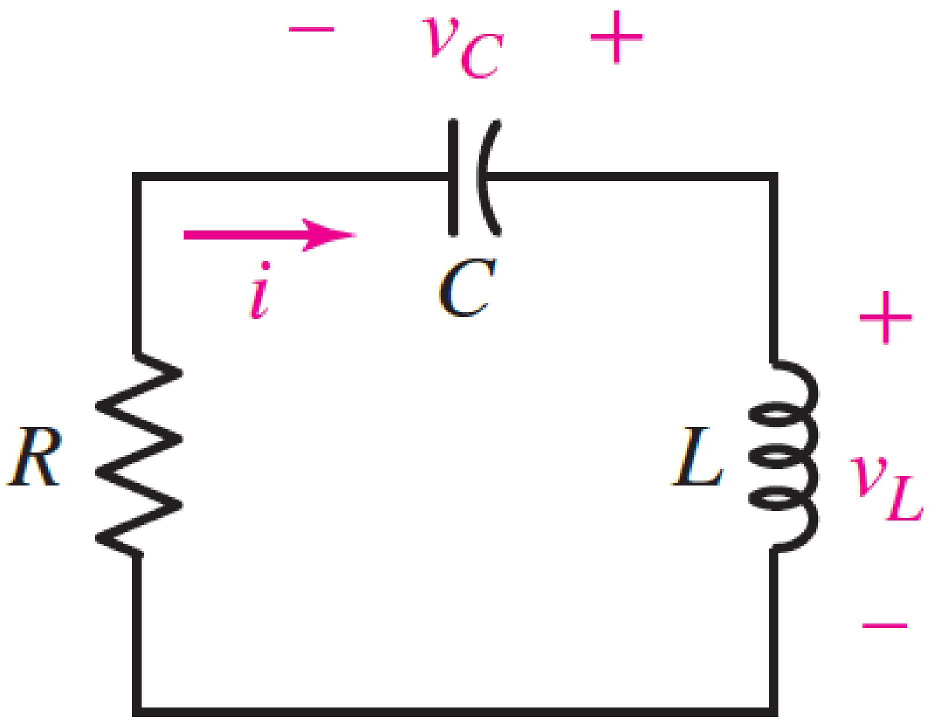
$$v = \frac{420}{\sqrt{6}} \sin \sqrt{6}t$$

- No damping at all! The oscillations will continue at the same amplitude for ever



# The series RLC circuit

- The dual of the circuit we have been analysing
- The same differential equation, now in the form of current:
$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$
- Compare this with the one for parallel:
$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$
- Our complete discussion of the parallel RLC circuit is directly applicable to the series RLC circuit, including that $i(0) = 0$ and $v(0) = 10$



# The RLC circuit

| Condition | Criteria | $\alpha$ | $\omega_0$ | Response |
|---|---|---|---|---|
| **Overdamped** | $\alpha > \omega_0$ | $\frac{1}{2RC}$ (parallel)<br>$\frac{R}{2L}$ (series) | $\frac{1}{\sqrt{LC}}$ | $A_1 e^{s_1 t} + A_2 e^{s_2 t}$, where<br>$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$ |
| **Critically damped** | $\alpha = \omega_0$ | $\frac{1}{2RC}$ (parallel)<br>$\frac{R}{2L}$ (series) | $\frac{1}{\sqrt{LC}}$ | $e^{-\alpha t}(A_1 t + A_2)$ |
| **Underdamped** | $\alpha < \omega_0$ | $\frac{1}{2RC}$ (parallel)<br>$\frac{R}{2L}$ (series) | $\frac{1}{\sqrt{LC}}$ | $e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$,<br>where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ |