Lecture 13 – RLC circuit 3

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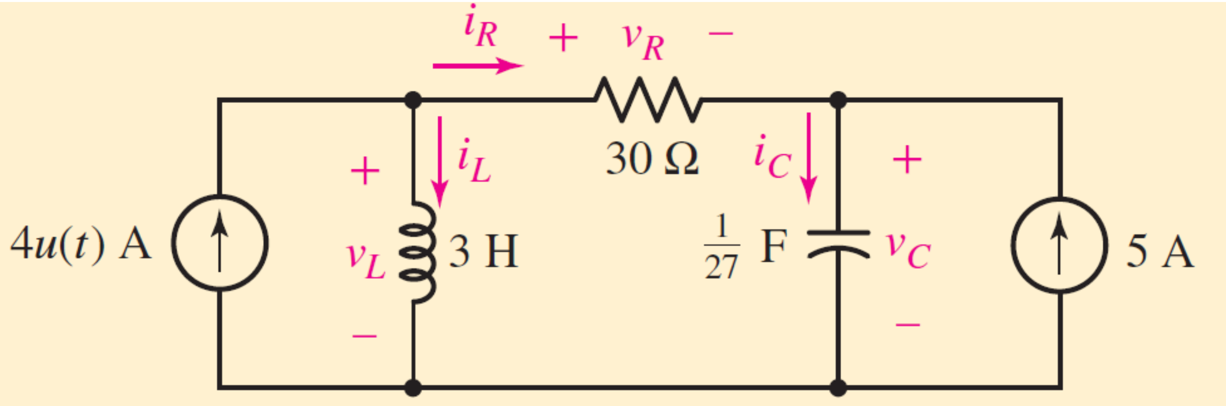
RLC circuit with sources

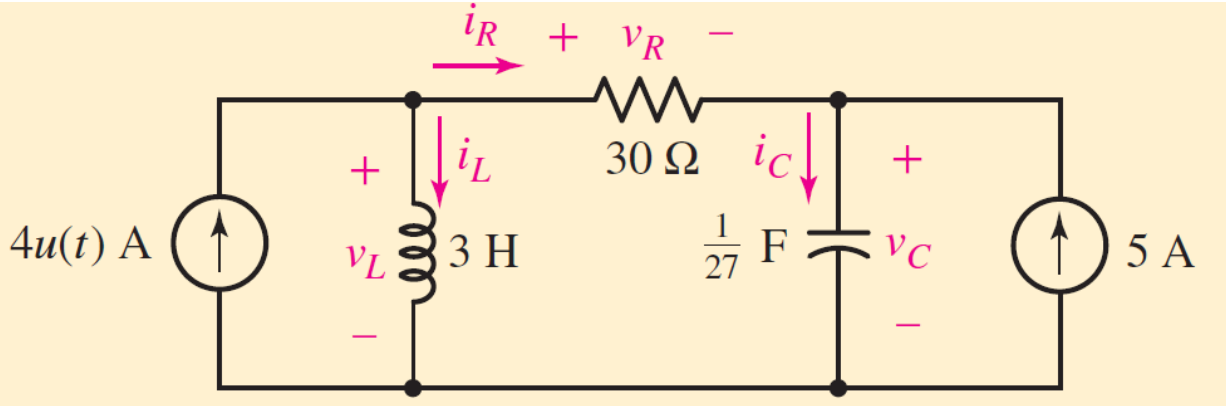
- We use the concept of superposition
- Because we know the source-free, or “natural” response of a circuit, we can inactivate the sources and find that response for the initial conditions
- Then, we can obtain the response because of the addition of the sources
- Arithmetic addition of these responses provides the complete solution



RLC circuit with sources

- Initial values of the circuit at $t = 0^-$:
- It is assumed that the circuit is in this state for a long time
- Any oscillatory response is long gone
- Thus,

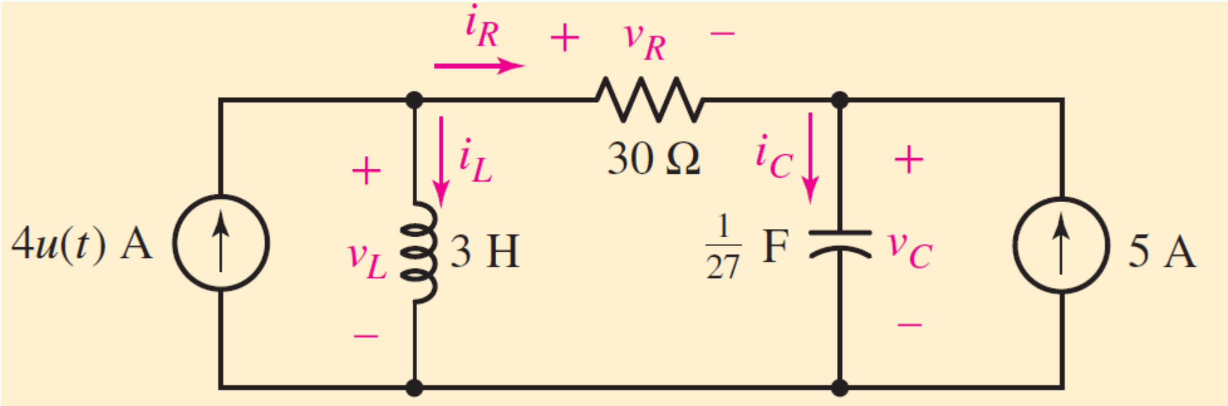
	$v(0^-)$	$i(0^-)$
Resistance	-150	-5
Inductance	0	5
Capacitance	150	0



RLC circuit with sources

- As soon as the unit voltage comes in, the currents (and voltages) change
- However, the current in the inductor and the voltage across the capacitor do not change
- Thus,

	$v(0^+)$	$i(0^+)$
Resistance	-30	-1
Inductance	120	**5**
Capacitance	**150**	4



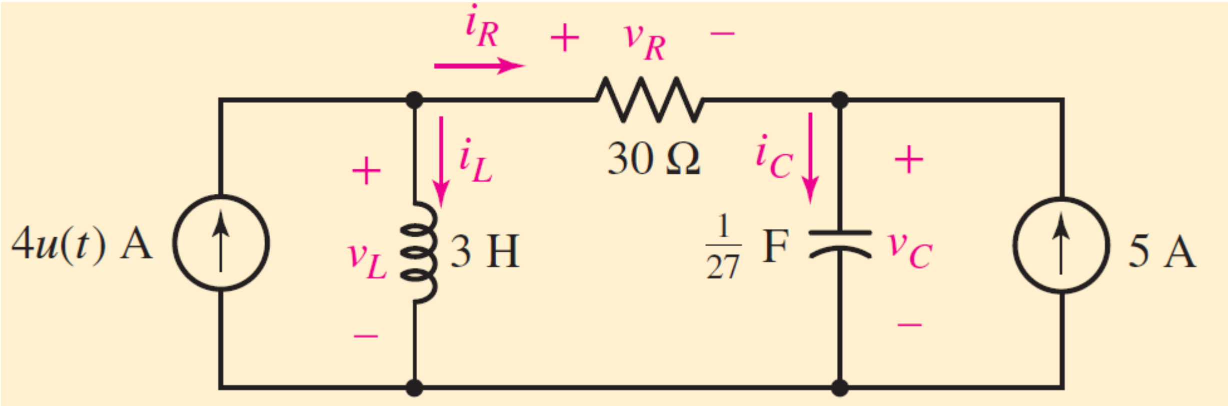
RLC circuit with sources

- To get the initial conditions, we also need to know the derivatives of the currents and voltages across the elements

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{120}{3} = 40$$

$$\left.\frac{dv_C}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = \frac{4}{1/27} = 108$$

	$\left.\frac{dv}{dt}\right\|_{t=0^+}$	$\left.\frac{di}{dt}\right\|_{t=0^+}$
Resistance	-1200	-40
Inductance	-1092	40
Capacitance	108	-40



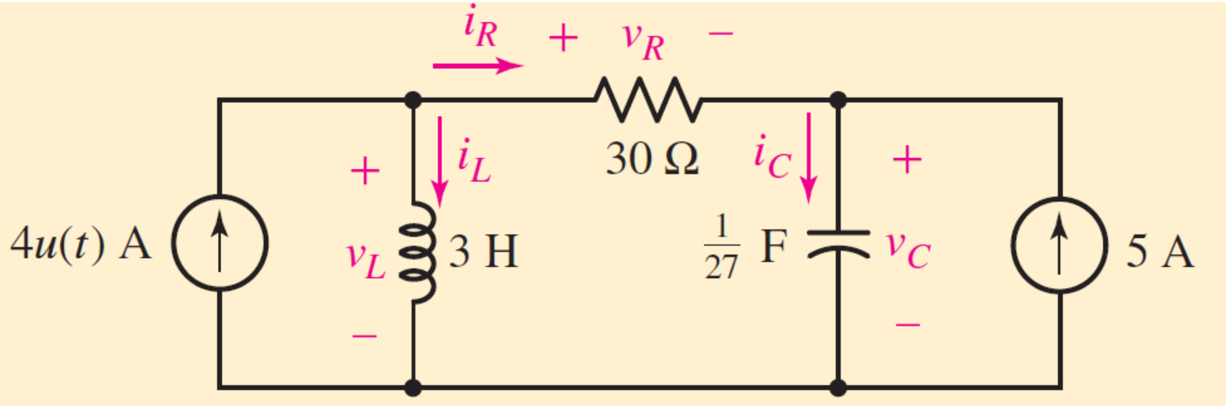
RLC circuit with sources

- With all these initial conditions, we can find any response
- For example, lets calculate $v_C(t)$
- With the sources inactive, it is a series RLC circuit

$$\alpha = 5 \text{ and } \omega_0 = 3$$

- Thus, it is an overdamped circuit with $s_1 = -1$ and $s_2 = -9$
- Thus, the natural response is:

$$f_n = A_1 e^{-t} + A_2 e^{-9t}$$



RLC circuit with sources

- For the forced response, we look at the circuit as $t \to \infty$
- We have: $v_C(\infty) = -v_R(\infty) = 150$
- Thus,

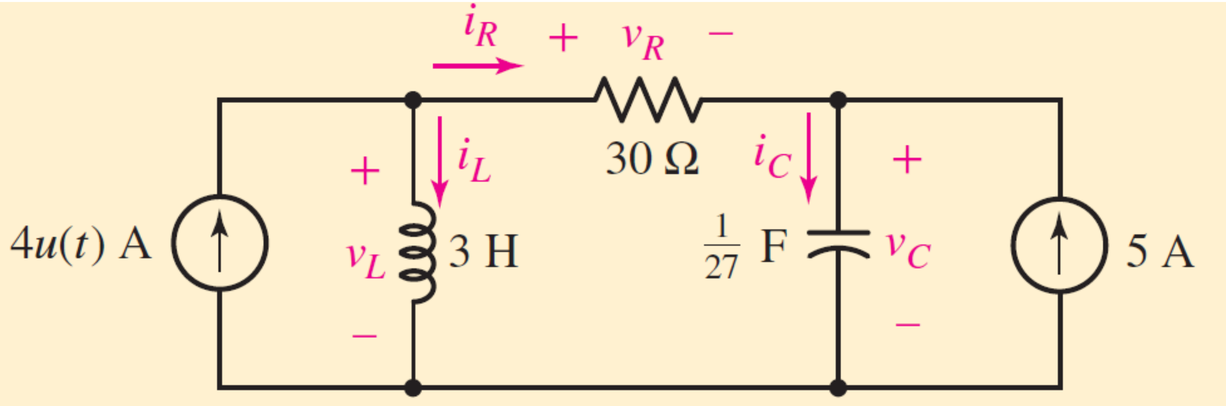
$$f_f = 150$$

- Hence, the total response is:

$$v_C = f_n + f_f$$

$$v_C = 150 + A_1 e^{-t} + A_2 e^{-9t}$$

- We have, $v_C(0^+) = 150$
- Thus, $A_1 + A_2 = 0$



RLC circuit with sources

$$\frac{dv_C}{dt} = -A_1 e^{-t} - 9A_2 e^{-9t}$$

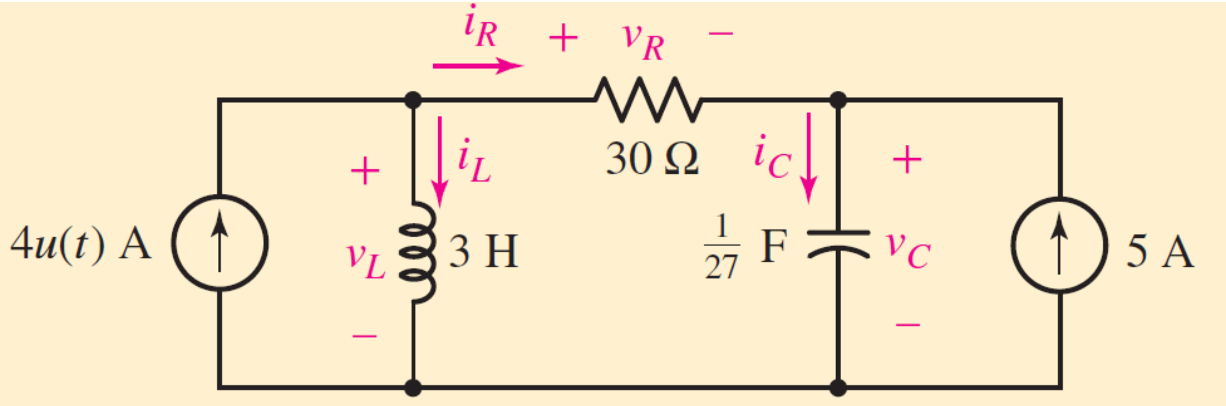
$$\left.\frac{dv_C}{dt}\right|_{t=0^+} = -A_1 - 9A_2 = 108$$

- Thus,

$$A_1 = 13.5 \text{ and } A_2 = -13.5$$

- The complete response for the capacitor voltage is:

$$v_C = 150 + 13.5\left(e^{-t} - e^{-9t}\right)$$



Complex frequency

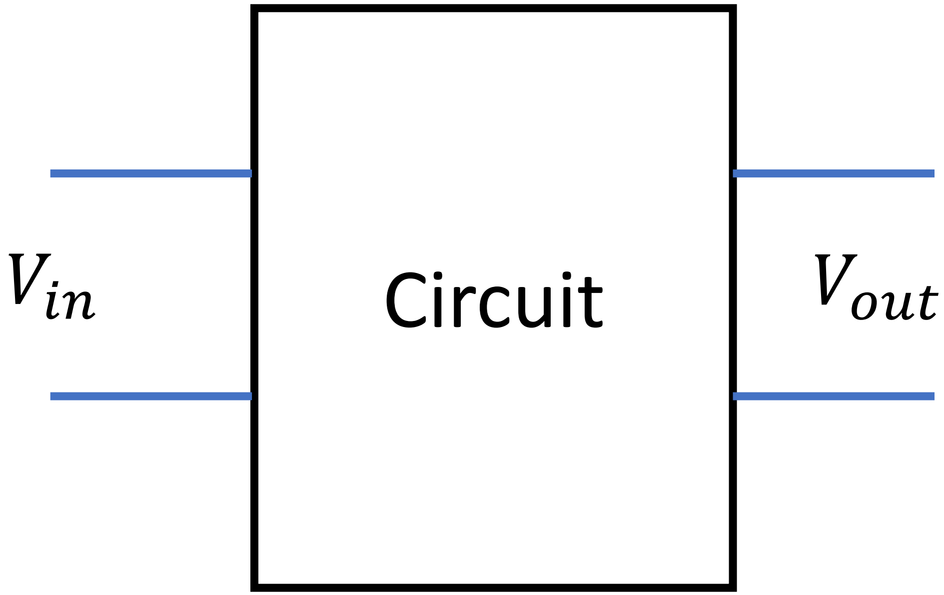
- In the study of a circuit analysis, we are often presented with a circuit with a given input, and we are interested in the output response
- We realize that we can represent the voltages using single standard function of the form:

$$v(t) = \sum K\, e^{st}$$

- Here, we define a term "complex frequency", s, as having the form:

$$s = \sigma + j\omega$$

- Using this, we realize that we can obtain any arbitrary voltage function



Complex frequency

- For DC: $s = 0$
- For Exponential: $\omega = 0$
- For Sinusoidal: $v(t) = V_m \cos(\omega t + \theta)$

$$v(t) = \frac{1}{2} V_m \left[e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}\right]$$

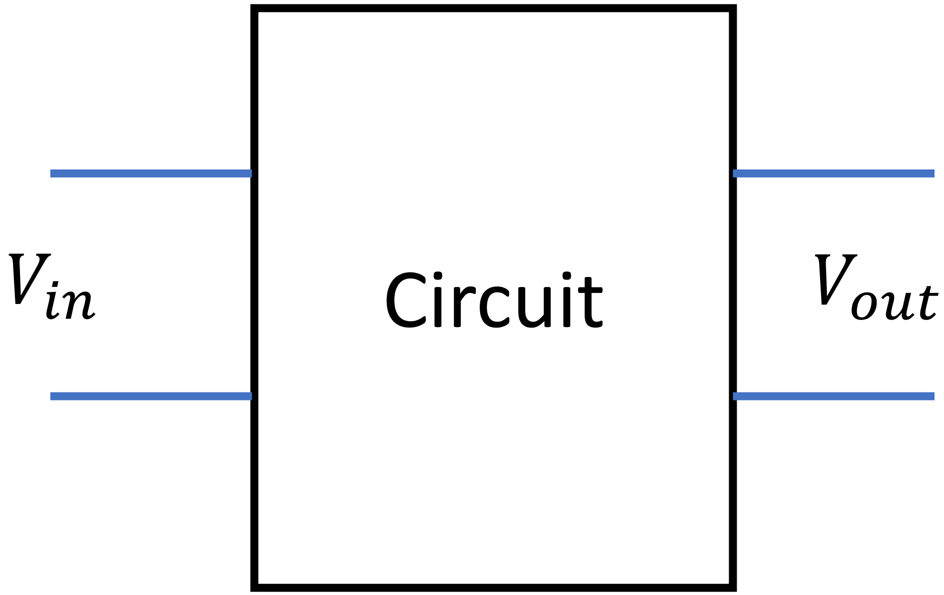
$$v(t) = \left(\frac{1}{2} V_m e^{j\theta}\right) e^{j\omega t} + \left(\frac{1}{2} V_m e^{-j\theta}\right) e^{-j\omega t}$$

- Thus, we have, $s_1 = j\omega$, $s_2 = -j\omega$ (complex conjugates), and $K_1 = \frac{1}{2} V_m e^{j\theta}$, $K_2 = \frac{1}{2} V_m e^{-j\theta}$ (complex conjugates)

- Exponentially damped: $v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$

$$v(t) = \left(\frac{1}{2} V_m e^{j\theta}\right) e^{(\sigma+j\omega)t} + \left(\frac{1}{2} V_m e^{-j\theta}\right) e^{(\sigma-j\omega)t}$$

- Thus, we have, $s_1 = \sigma + j\omega$, $s_2 = \sigma - j\omega$ (complex conjugates)



Frequency domain

- To go from a given input voltage function to the output, we need to solve the integrodifferential equations for the circuit, which is a tedious task even for a simple RLC circuit
- Any more complexity, and things will get out of hand
- Hence, we look at one of the most popular techniques to simplify our circuit analysis – The Laplace Transform
- Introduced by Pierre Simon Laplace, in 1785, to solve some of the problems being studied by the likes of Euler and Lagrange

The Laplace transform

- We transform the voltage function into its frequency domain form using the Laplace transform

$$F(s) = \int_{-\infty}^{\infty} e^{-st} f(t)dt$$

- For circuit analysis, we typically use the one-sided Laplace (assuming the circuit was "switched on" at $t = 0$)
- Thus,

$$F(s) = \int_{0^-}^{\infty} e^{-st} f(t)dt$$

- Also, Laplace transform follows linearity, ie, the transform of the sum of two time-domain functions is the sum of their transforms

$$\mathcal{L}\big(af_1(t) + bf_2(t)\big)$$
$$= a\int_{0^-}^{\infty} e^{-st} f_1(t)dt + b\int_{0^-}^{\infty} e^{-st} f_2(t)dt$$
$$= aF_1(s) + bF_2(s)$$

- Also applies to the inverse Laplace:

$$\mathcal{L}^{-1}\big(aF_1(s) + bF_2(s)\big) = \mathcal{L}^{-1}\big(aF_1(s)\big) + \mathcal{L}^{-1}\big(bF_2(s)\big)$$
$$= af_1(t) + bf_2(t)$$