

IIIT-H

Networks, Signals, and Systems

Monsoon-2024

Prasad Krishnan, Aftab Hussain

Assignment 1

Submission Date & Time: 25-Aug-2024, 8PM (Hard Deadline : 30th Aug, 8PM)

Instructions:

- **You are free to discuss the problems with co-students, TAs, and instructor if needed. However, you should write down all answers yourself.**
 - All steps should be justified in detail.
 - **Any attempt at plagiarism will result in ZERO for the assignment, apart from other academic consequences.**
 - TAs will likely (but not guaranteed) also upload the solutions (some solutions may not be elaborate but gives essential ideas) after the submission. This will be treated as the master document for evaluation.
 - **Evaluation done by TAs/Instructor will generally be final. Appeals on the evaluation will generally not be permitted for assignments, unless the solution is wrong for some question.**
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1. (12 marks : each part carries 2 marks, last question carries 2 marks) Let $x(t)$ be an arbitrary (real or complex) signal. Identify which of the following signals are even or odd (no matter what $x(t)$ is).

- (a) $|x^2(t)|$.
- (b) $x(t^2)$.
- (c) $x(t) + x(-t)$.
- (d) $e^{x(t)}$.
- (e) $x(t) - x(-t)$.

Can you show from parts (c) and (e) in the above that, for any $x(t)$, there exists two signals, one being even denoted by $x_e(t)$ and another odd denoted $x_o(t)$, such that $x(t) = x_e(t) + x_o(t)$?

2. (5 marks) Let $x(t) = 0.5 \cos(2\pi f_1 t + \phi_1) + e^{j(2\omega_2 t + \phi_2)}$ denote a sum of sinusoidal signals. Is this a periodic signal always? Are there relationships between f_1 and f_2 such that this sum is always periodic? Are there examples of f_1 and f_2 such that this sum is not periodic? Do the values of ϕ_1 and ϕ_2 influence the answer? Give your analysis in a few statements. What happens if more sinusoids are added?

3. (18+6=24=each part carries 3 marks, with the last question carrying $2 \times 3 = 6$ marks)
Consider a signal defined as follows.

$$x(t) = \begin{cases} 2 - t & \text{if } t \in (0, 2], \\ 2 + t & \text{if } t \in [-2, 0], \\ 0 & \text{otherwise} \end{cases}$$

(Exactly) Sketch the below signals $y(t)$ (with proper markings on the graph) given by the following equations.

- (a) $y(t) = x(t)$.
- (b) $y(t) = x(3t + 5)$
- (c) $y(t) = x(2 - 0.5t) + 0.5x(2(t + 3))$ (describe this in words first and then you might find it easier to sketch).
- (d) $y(t) = x(t)x(t + 1)$ (describe this in words first and then you might find it easier to sketch) .
- (e) $y(t) = \sum_{k \in \mathbb{Z}} x(t - 6k)$.
- (f) $y(t) = \lim_{t \rightarrow 0} x(t)$.

For the signals in parts (a) (b) and (e), calculate the (average) power and the energy.

4. (15 marks = each part carries 3 marks) Consider the systems given by the following relationships, where $x(t)$ is the input and $y(t)$ is the output, as follows. (assume $x(t)$ is a real signal throughout)

- (a) $y(t) = e^t x^2(t)$.
- (b) $y(t) = t - x(t)$.
- (c) $y(t) = \frac{e^{2x(t)}}{t}$
- (d) $y(t) = \int_{-\infty}^{t/2} h(t - \tau)x(\tau)d\tau$, where $h(\tau)$ is some other real function of finite energy.
- (e) $y(t) = \frac{d}{dt}x(t + t_0)$, for some t_0 being constant.

Verify, using proper arguments, whether the above systems are (1) LTI, (2) causal, and (3) bounded.