Lecture 8 – The Capacitor 2

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The Superposition theorem

The response in a linear circuit having more than one independent source can be obtained by the algebraic addition of the responses caused by the sources acting alone

- In essence, if there are many power sources, we can take one of them at a time, and calculate the current through a branch or voltage at a node because of this source
- Then algebraic addition of this response, gives the response because of all the combined sources

- We define a circuit as linear if it is composed of linear components
- We a component as linear if the multiplication of the current through the element by a constant K results in the multiplication of the voltage across the element by the same constant K

Demonstration

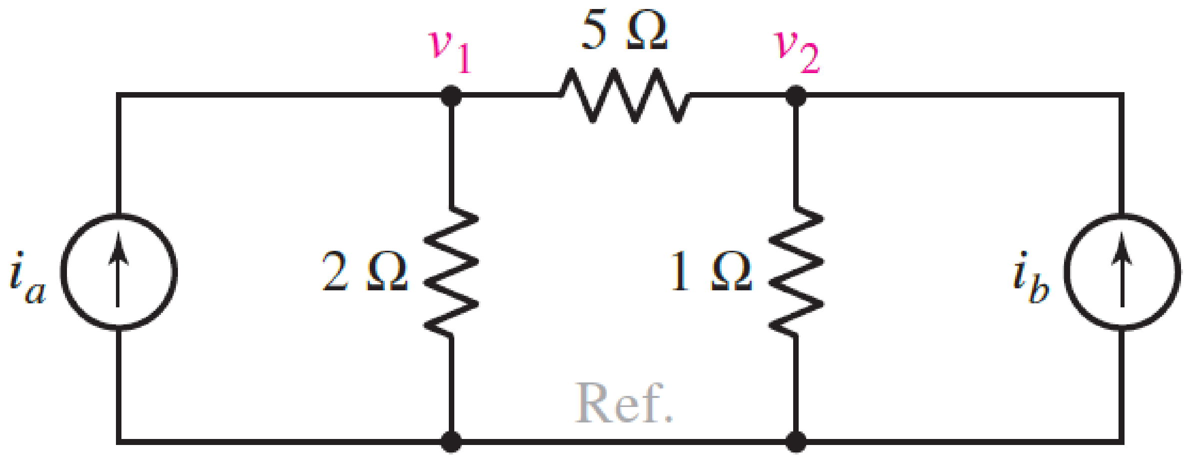
- Consider this circuit
- We can write the following equations:

$$i_a = \frac{v_1}{2} + \frac{v_1 - v_2}{5}$$

$$i_b = v_2 - \frac{v_1 - v_2}{5}$$

Thus,

$$i_a = 0.7v_1 - 0.2v_2$$
$$i_b = -0.2v_1 + 1.2v_2$$



Demonstration

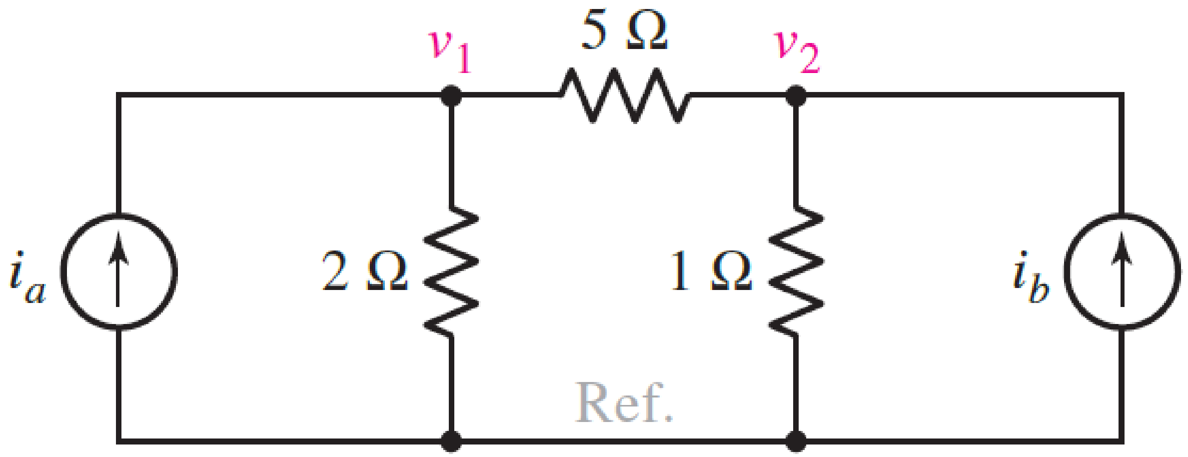
- Now, we can change i_a and i_b to any value say i_{ax} and i_{bx}, or i_{ay} and i_{by}

$$i_{ax} = 0.7v_{1x} - 0.2v_{2x}$$
$$i_{bx} = -0.2v_{1x} + 1.2v_{2x}$$

- Or,

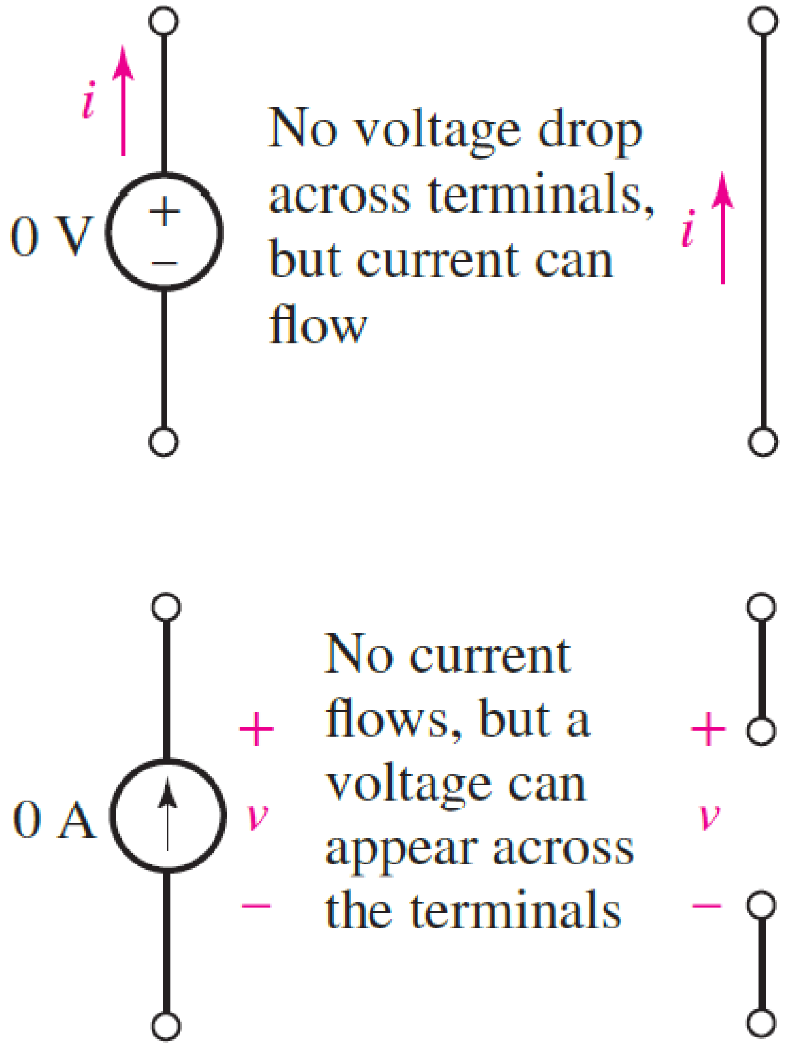
$$i_{ay} = 0.7v_{1y} - 0.2v_{2y}$$
$$i_{by} = -0.2v_{1y} + 1.2v_{2y}$$

- We realize that if we chose $i_a = i_{ax} + i_{ay}$ and $i_b = i_{bx} + i_{by}$, then the node voltages will be given by $v_1 = v_{1x} + v_{1y}$ and $v_2 = v_{2x} + v_{2y}$
- Without loss of generality, we can choose $i_{ax} = i_a$, $i_{ay} = 0$, and $i_{bx} = 0$, $i_{by} = i_b$
- Thus, we end up with the superposition theorem

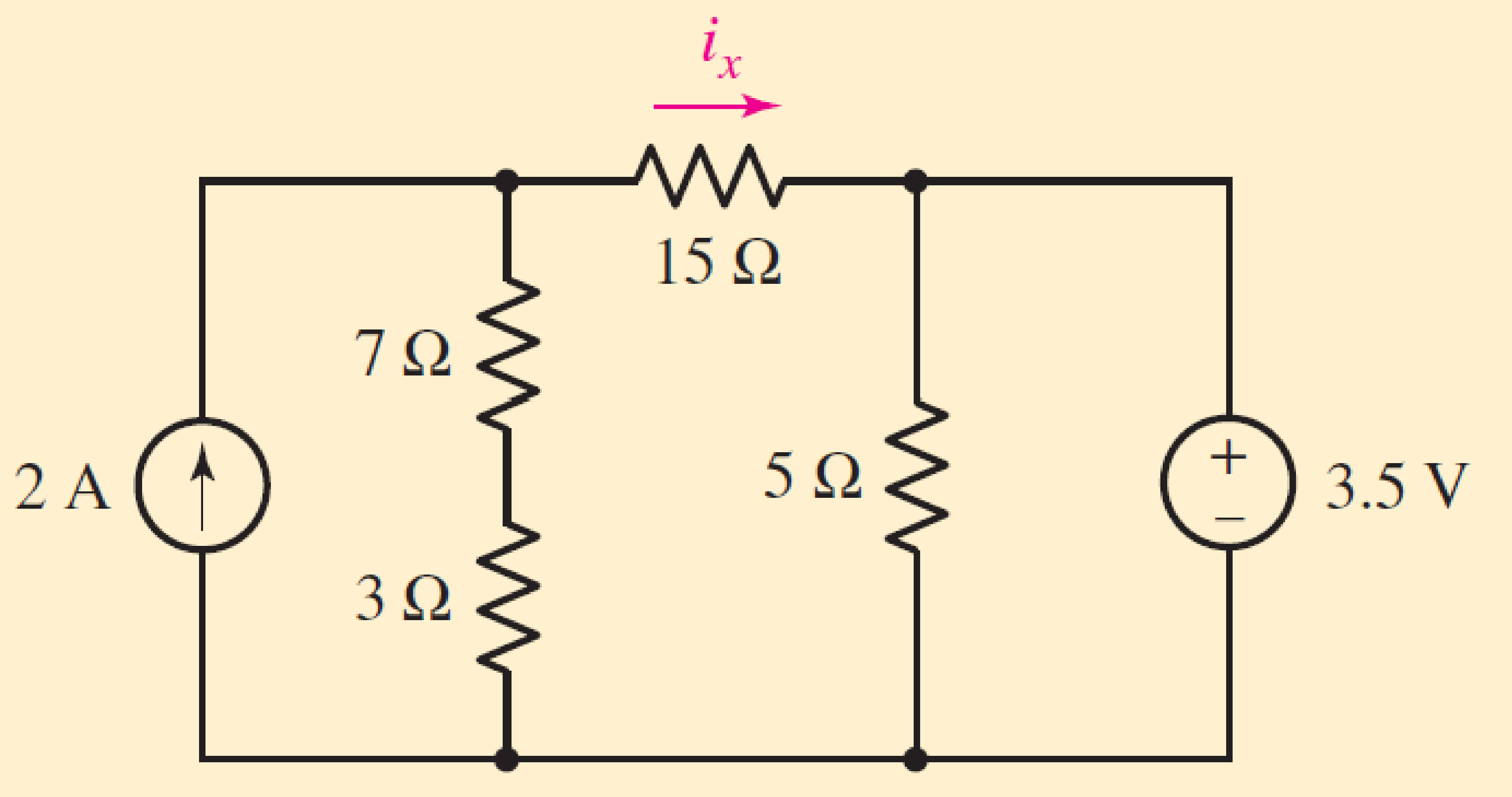


Zeroing the sources

- Thus, if there are N independent sources, we must perform N experiments, each having only one of the independent sources active and the others inactive/turned off/zeroed out
- If we reduce a voltage source to zero volts, we have effectively made it into a short circuit
- If we reduce a current source to zero amperes, we have effectively created an open circuit
- There is also no reason that an independent source must assume only its given value or a zero value in the several experiments; it is necessary only for the sum of the several values to be equal to the original value



Example



The non-ideal capacitor

- In the real world, the dielectric of a capacitor does not have an infinite resistance
- The resistivity of the dielectric, although high, provides a path for the charges to dissipate over time
- If the resistivity of the capacitor dielectric is ρ, $R = \rho d/A$
- For an isolated charged capacitor, this process can be modeled as a capacitor in series with a resistance (its internal resistance)
- Say a non-ideal capacitor with voltage V_0 is left alone
- The current through the internal resistance is given by: $i = \frac{V_0 A}{\rho d}\left(1 - e^{-\frac{t}{\tau}}\right)$
- Thus, the time constant is: $\tau = RC = \left(\frac{\rho d}{A}\right)\left(\frac{\epsilon A}{d}\right) = \rho\epsilon$
- Charge is given by: $Q = Q_0 e^{-\frac{t}{\tau}}$

The non-ideal capacitor

- In a circuit, the non-ideal capacitor is modeled as a resistance parallel to the capacitor
- The voltage across the capacitor causes a current through the leakage resistor

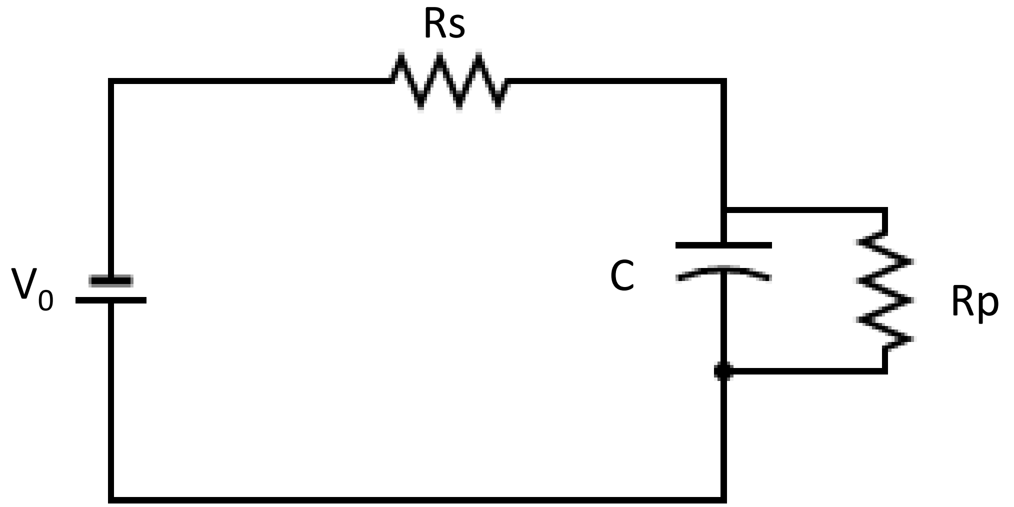
$$V_0 = IR_s + \frac{q}{C}$$

$$V_0 = (i + i_1)R_s + \frac{q}{C}$$

$$V_0 = iR_s + \frac{q}{C}\frac{R_s}{R_p} + \frac{q}{C}$$

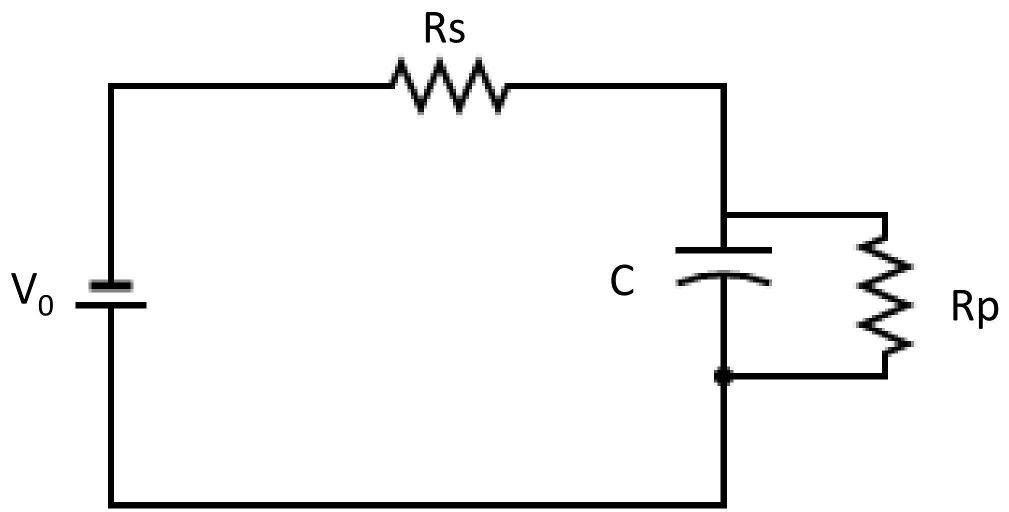
$$V_0 = iR_s + \frac{q}{C}\left(1 + \frac{R_s}{R_p}\right)$$

So, if we replace $C_{eq} = C\left(\frac{R_p}{R_s+R_p}\right)$, we can continue the remaining analysis



The non-ideal capacitor

- The step response for a non-ideal capacitor can be used to obtain information about the values of R_s, C and R_p
- The peak of the current provides information about R_s
- The settling value of current provides R_p
- And, the time constant of decay provides C



"Realizing the potential of dielectric elastomer artificial muscles", January 24, 2019, 116 (7) 2476-2481

Capacitors in series

- From KVL, we have:

$$v_s = v_1 + v_2 + \cdots + v_N = \sum v_i$$

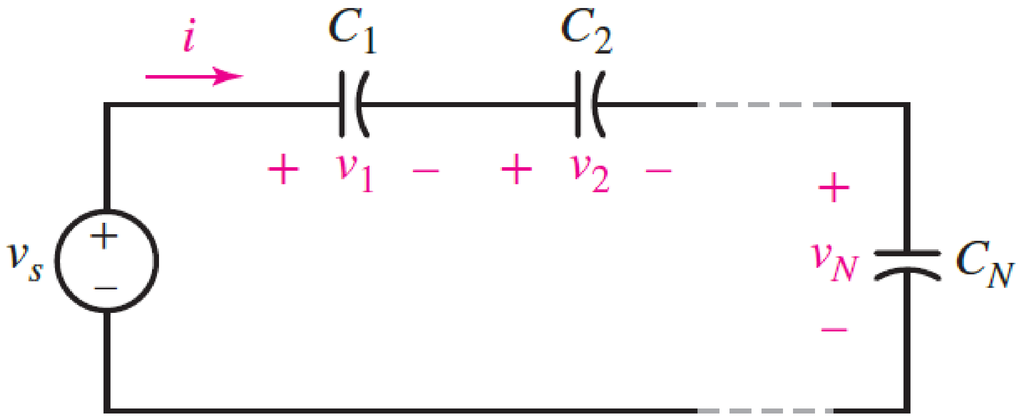
- For a capacitor,

$$v_i(t) = \frac{q_i(t)}{C_i}$$

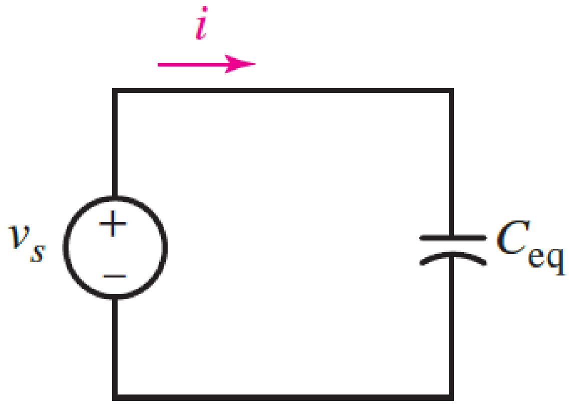
- For the circuit in (a):

$$v_i(t) = \frac{1}{C_i} \int i(t) + v_i(t_0)$$

$$v_s = \sum \left(\frac{1}{C_i} \int i(t) + v_i(t_0) \right)$$



(a)



(b)

Capacitors in series

$$v_s = \left(\sum \frac{1}{C_i}\right) \int i(t) + \sum v_i(t_0)$$

- In the equivalent circuit:

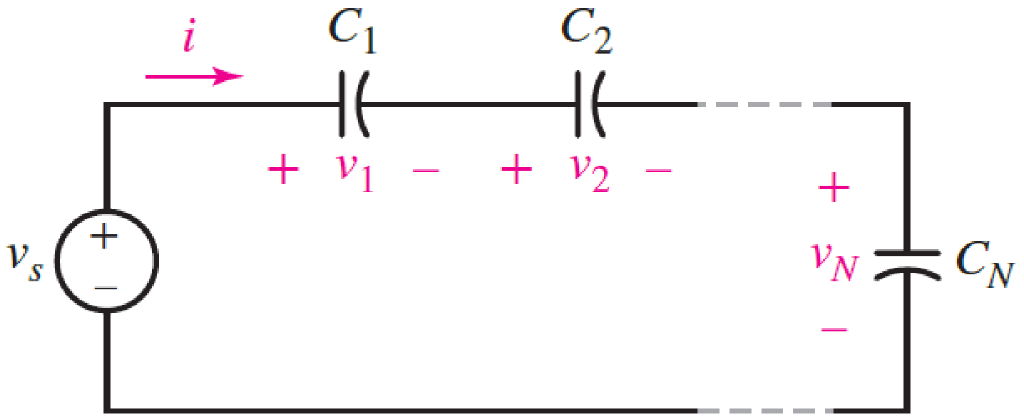
$$v_s = \frac{1}{C_{eq}} \int i(t) + v_c(t_0)$$

- From KVL at t_0:

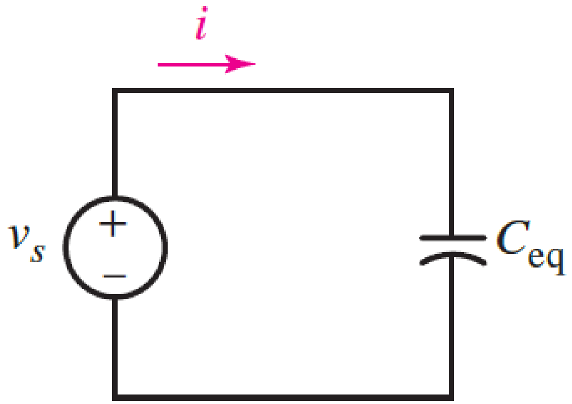
$$v_c(t_0) = \sum v_i(t_0)$$

- Thus,

$$\frac{1}{C_{eq}} = \left(\sum \frac{1}{C_i}\right)$$



(a)



(b)

Capacitors in parallel

- For parallel case, we consider a current source
- From KCL:

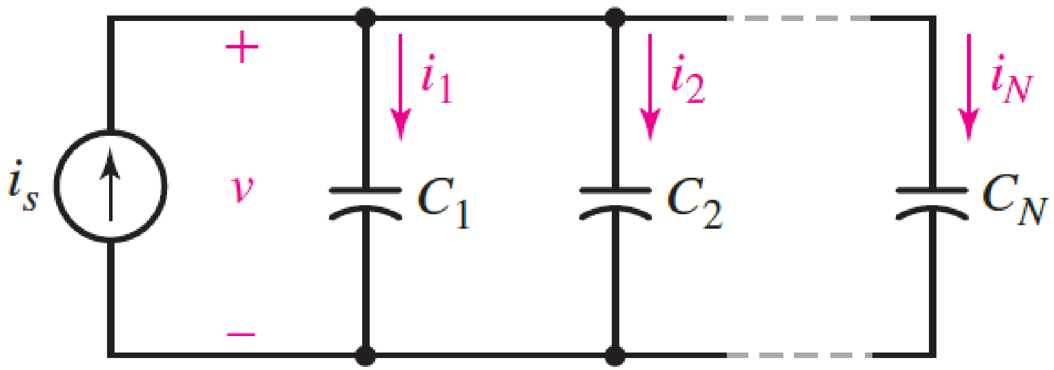
$$i_s = i_1 + i_2 + \cdots + i_N = \sum i_i$$

- For one capacitor:

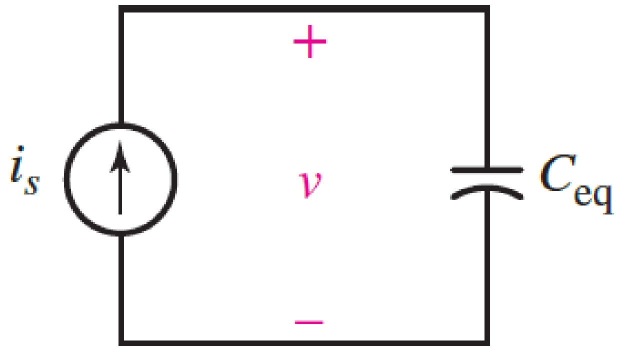
$$q_i(t) = C_i v(t)$$
$$i_i(t) = C_i \frac{dv(t)}{dt}$$

- Thus,

$$i_s = \sum \left(C_i \frac{dv(t)}{dt} \right)$$



(a)



(b)

Capacitors in parallel

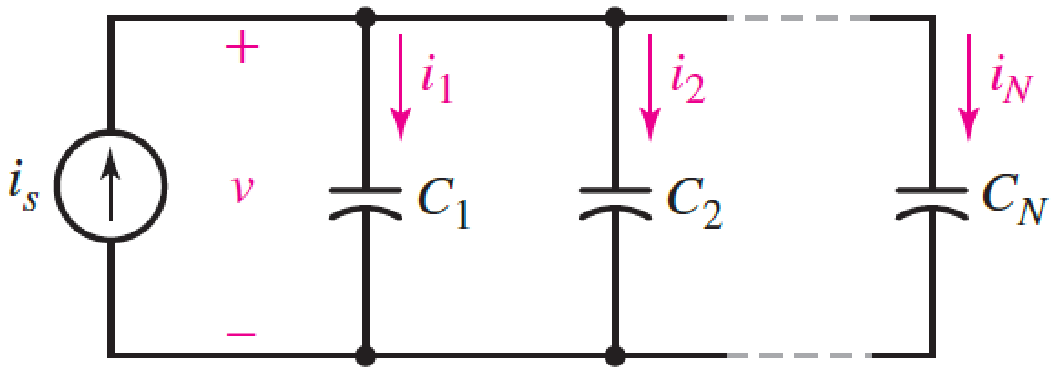
$$i_s = \left(\sum C_i\right)\frac{dv(t)}{dt}$$

- For the equivalent circuit:

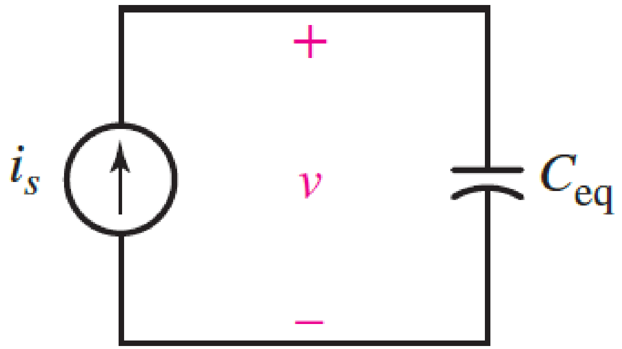
$$i_s = C_{eq}\frac{dv(t)}{dt}$$

- Thus,

$$C_{eq} = \sum C_i$$

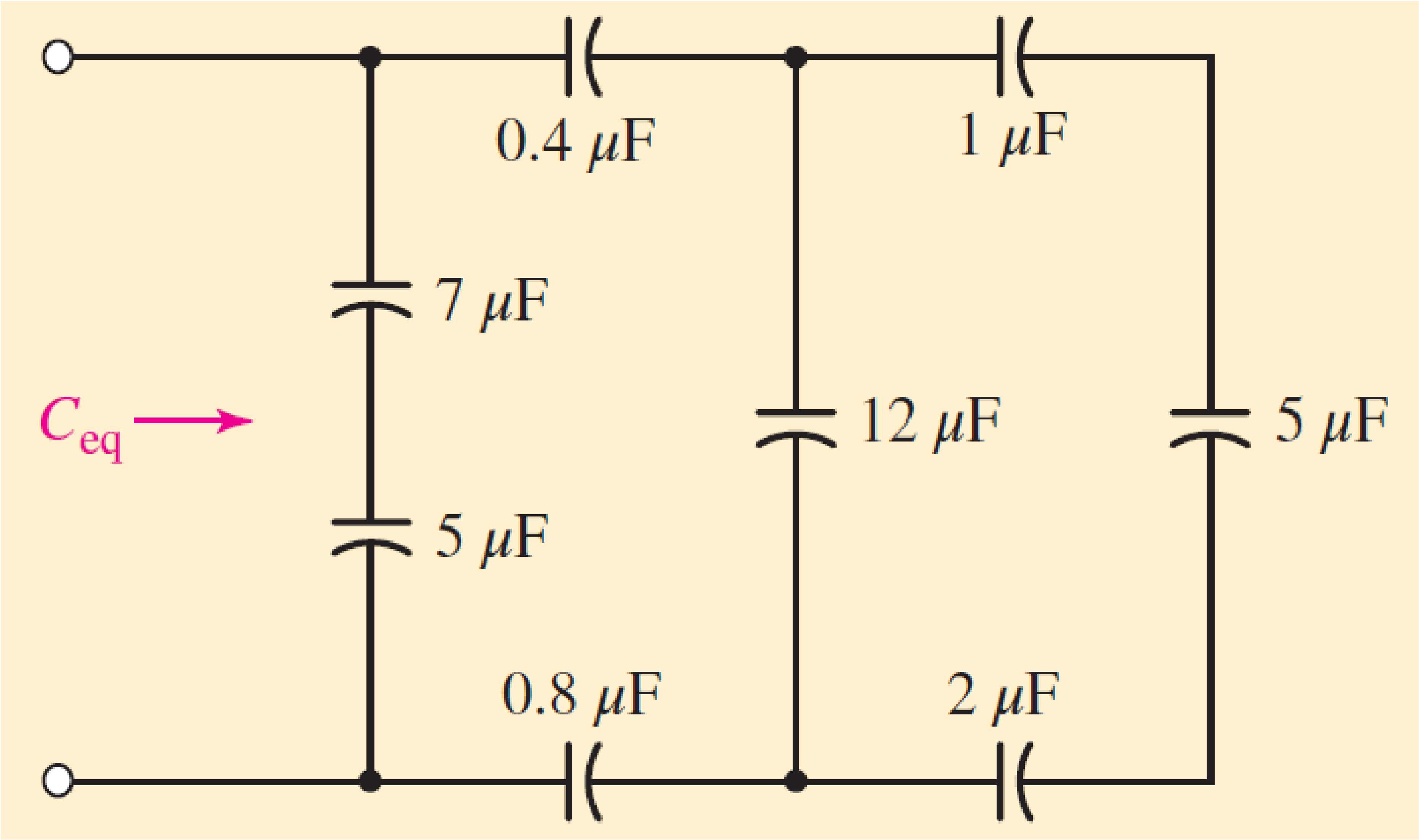


(a)



(b)

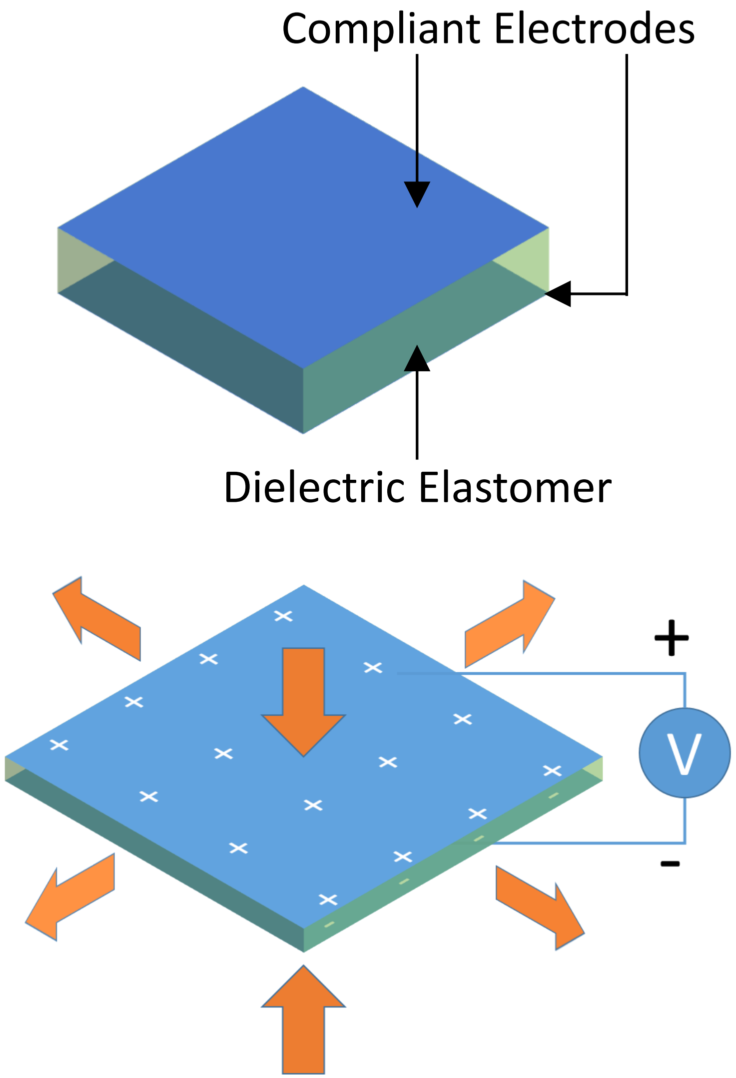
Example



Research – Dielectric Elastomer Actuators

- Motivation:
  - Polymer-based actuators promise a new paradigm for soft robotics applications
  - Creation of mechanical force using completely soft and flexible materials
- Concept:
  - Dielectric Elastomer Actuators (DEAs) are complaint capacitors that produce motion on application of voltage
  - When an electric field is applied, the oppositely charged plates create an attraction force causing the soft polymer to compress normal to the field and expand laterally
  - Maxwell stress:

$$\sigma = \epsilon \left(\frac{V}{d}\right)^2$$



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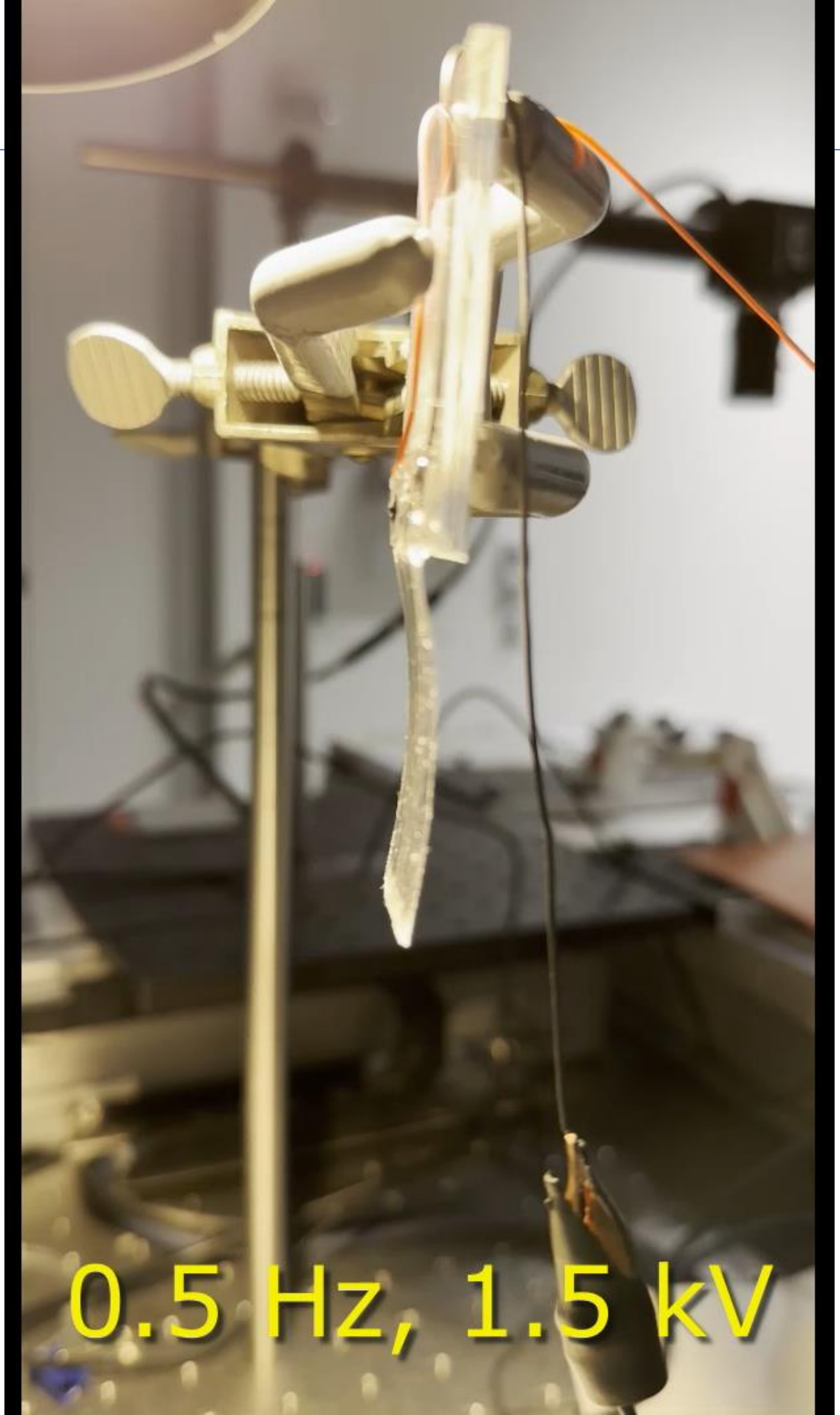

Pi-Star transformation

- Homework!