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Use principle of mathematical induction to prove

① (a) Prove that $5^n - 4n - 1$ is divisible by 16 $\forall n \in \mathbb{N}$

(b) $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n} \;\; \forall \; n > 1$ (5 marks)

② For $A \subseteq \mathbb{N}$, define $f_A : \mathbb{N} \to \mathbb{R}$ by

$$f_A(n) = \begin{cases} 1 & \text{if } n \in A \\ 0 & \text{if } n \notin A \end{cases}$$

Using Cantor's theorem, show that the set $\{f_A : A \subseteq \mathbb{N}\}$ is uncountable (4 marks)

③ Show that if $f : A \to B$ and G, H are subsets of B, then (4 marks)

$f^{-1}(G \cup H) = f^{-1}(G) \cup f^{-1}(H)$ and

$f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$.

④ Prove that $|x| + |y| + |z| \leq |x+y-z| + |y+z-x| + |z+x-y| \;\; \forall \; x, y, z \in \mathbb{R}$

(3 marks)

⑤ If y is a +ve real number, show that there exists a natural number m such that

$0 < \frac{1}{2^m} < y$. (4 marks)