Lecture 7 – The Capacitor

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The capacitor

- A “passive” element that can store and delivery energy
- The current-voltage relationship is time-dependent, which, if used well, can be very useful
- Typically, active elements are defined as those that can provide electrical energy, like power supplies
- Now, we need to be specific in saying that active elements are those that can provide a finite average power over an infinite amount of time

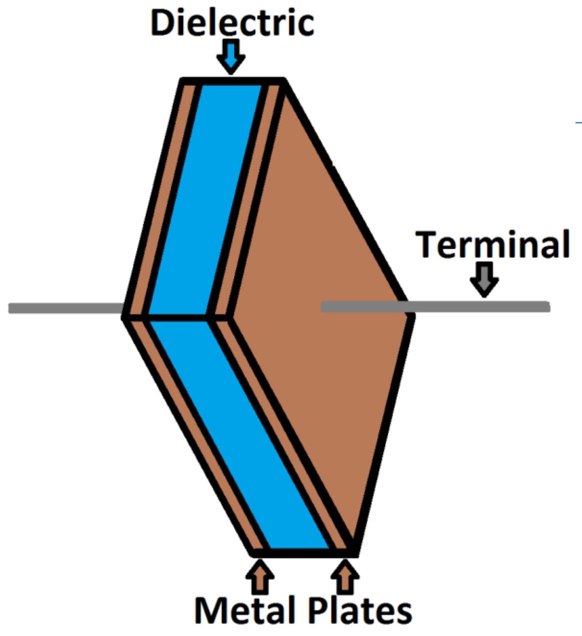
The capacitor

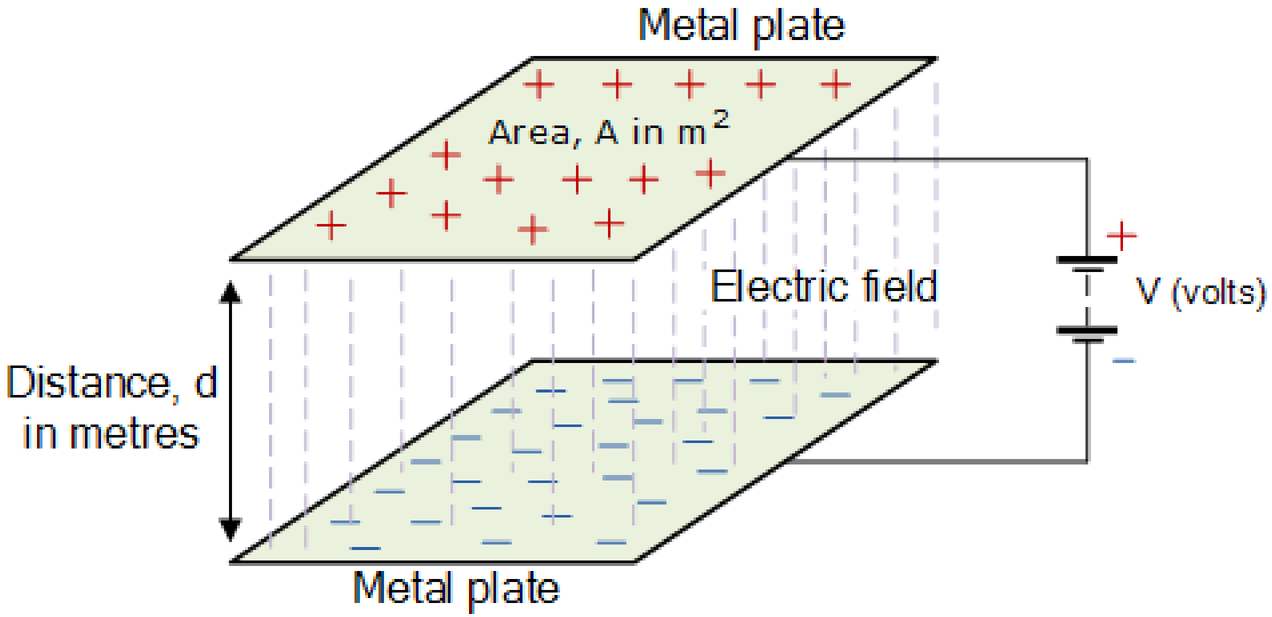
- A capacitor is a parallel plate structure separated by a dielectric
- When subjected to a potential difference across the plates, there is an electric field that is present in the dielectric
- This electric field requires the build up of charges on the metal plates
- The total charge on the metal plate is proportional to the applied potential difference

$$Q \propto V$$

- The constant of proportionality is the “capacitance” of the structure

$$Q = CV$$





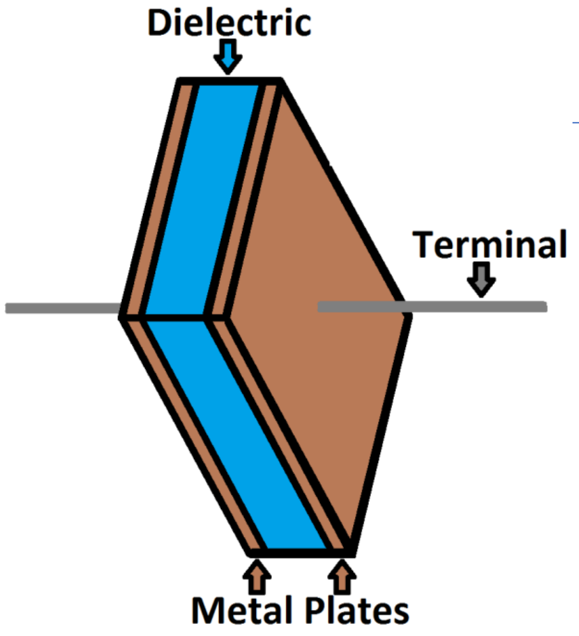
The capacitor

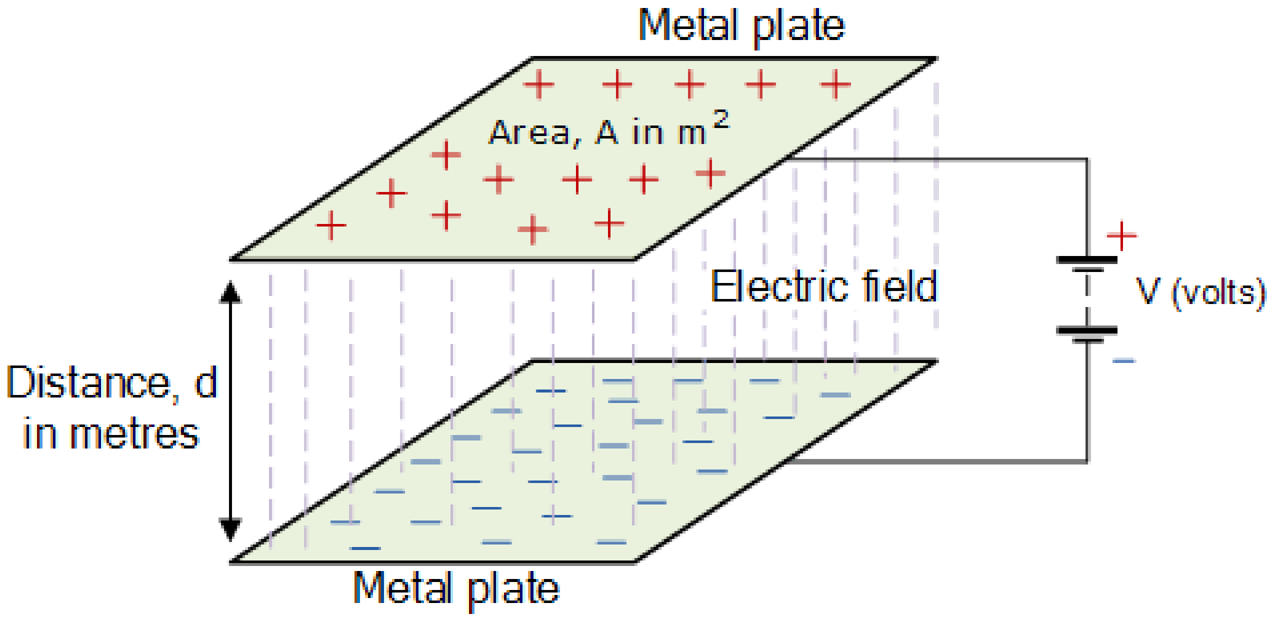
- Capacitance is measured in Farad
- In a 1 Farad capacitor, you will get 1 C charge for 1 V potential difference
- Now, a small change in charge leads to a small change in the voltage across the capacitor

$$\frac{dQ}{dt} = C\frac{dV}{dt}$$

- If connected in a circuit, this dQ/dt is the current flowing through the capacitor branch
- Thus, the current voltage relationship is given by

$$i = C\frac{dV}{dt}$$





Capacitance of a parallel plate

- The electric field because of an infinite sheet is

$$E = \frac{\sigma}{2\epsilon}$$

- Because of the two plates (one positive and one negative), the two fields add:

$$E_c = \frac{\sigma}{\epsilon}$$

- For a constant electric field:

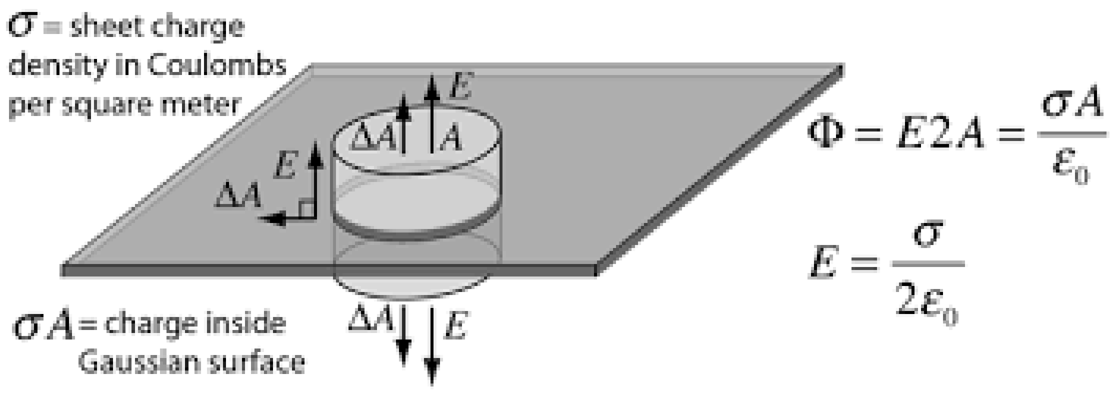
$$V = \int E dx = Ed$$

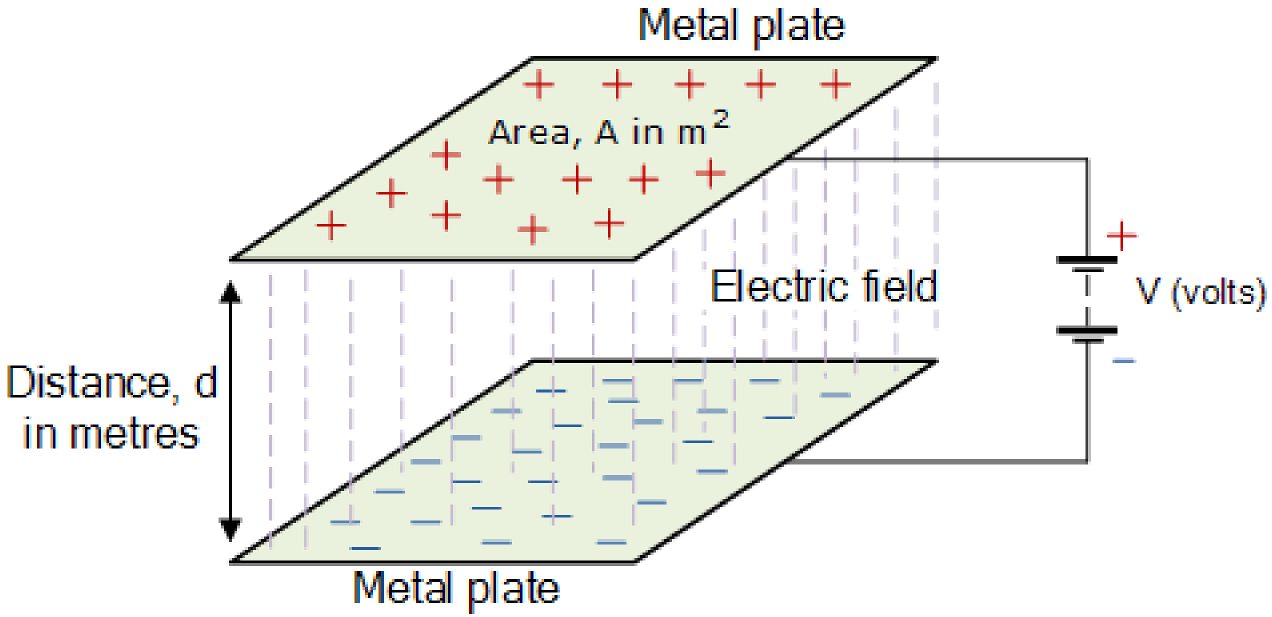
- By definition:

$$C = \frac{Q}{V} = \frac{A\sigma}{E_c d}$$

$$\boldsymbol{C = \frac{\epsilon A}{d}}$$

**Key assumption: d << linear dimensions* → **Key assumption: d << linear dimensions*



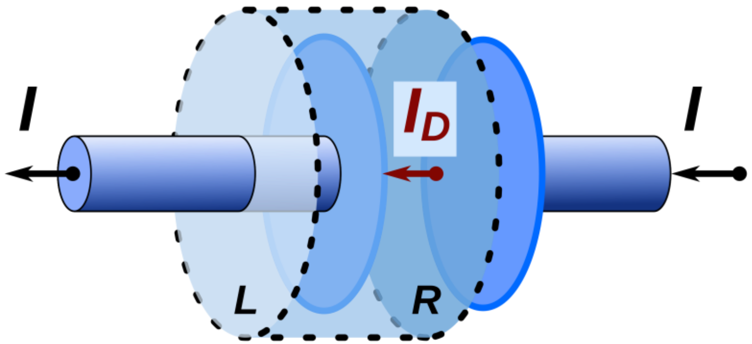


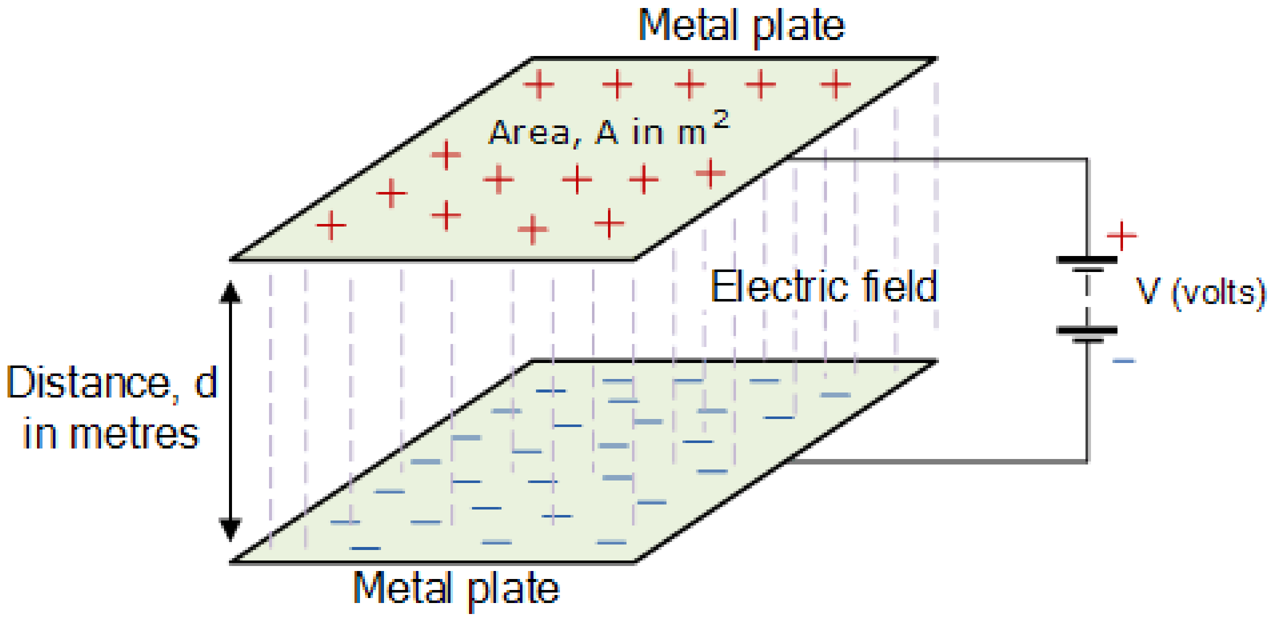
The KCL problem

- A capacitor, when connected in a circuit cause current to flow into (and away from) the plates
- However, in an ideal capacitor, the current does not flow through the dielectric
- If one of the plates of the capacitor is considered, then the KCL gets violated because current is entering that plate, but not leaving
- This can be resolved by defining an electric "displacement current" as:

$$J_d = \frac{dD}{dt} = \epsilon \frac{dE}{dt}$$

$$I_d = \epsilon A \frac{dE}{dt} = \frac{\epsilon A}{d} \frac{dV}{dt} = C \frac{dV}{dt}$$





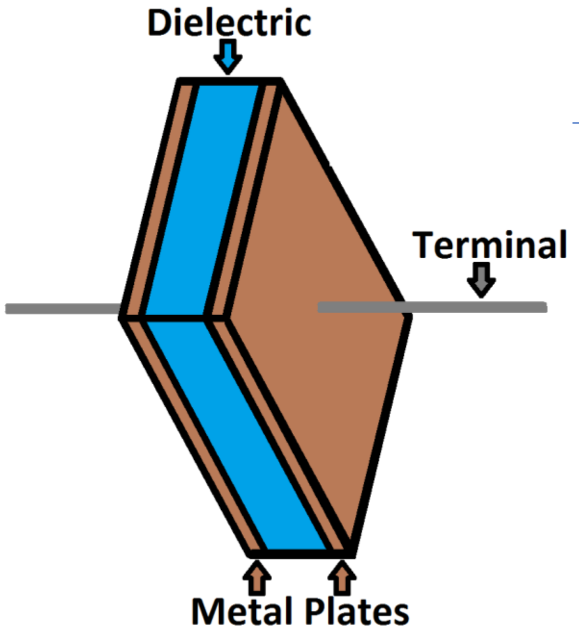
Energy storage

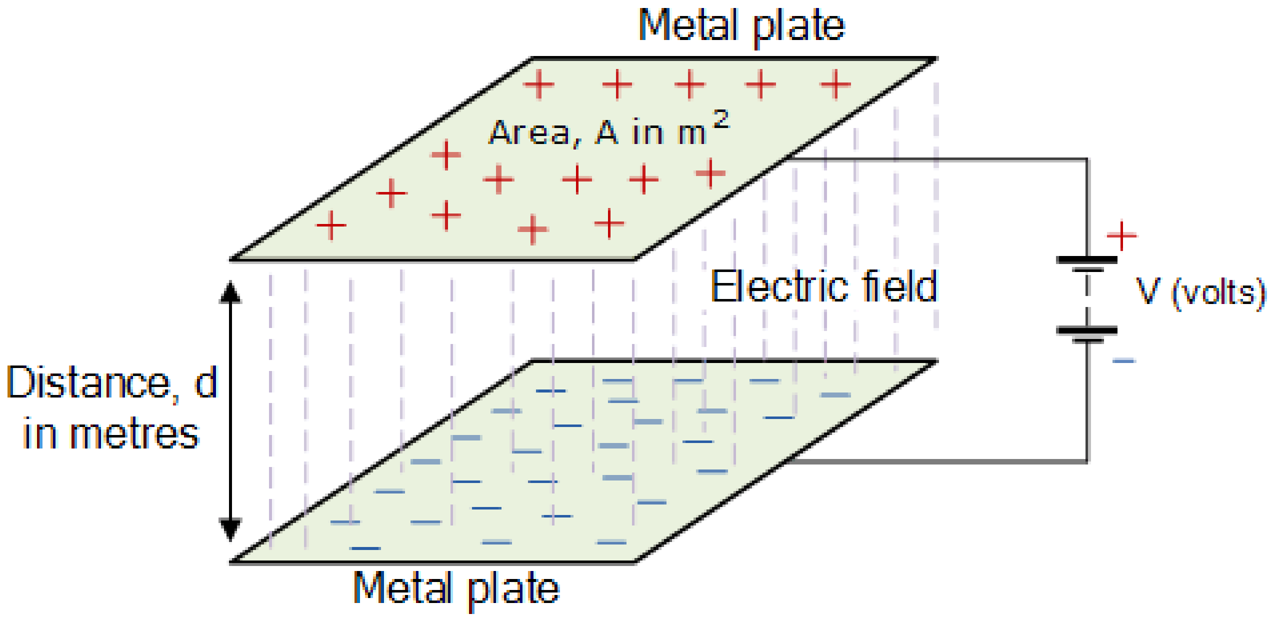
- Now, let us consider the energy stored in a capacitor
- For every new charge dQ introduced, work is done against the already present at the plate
- This is the potential energy stored in the capacitor

$$dW = VdQ = \frac{Q}{C}dQ$$

Total energy stored:

$$W = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

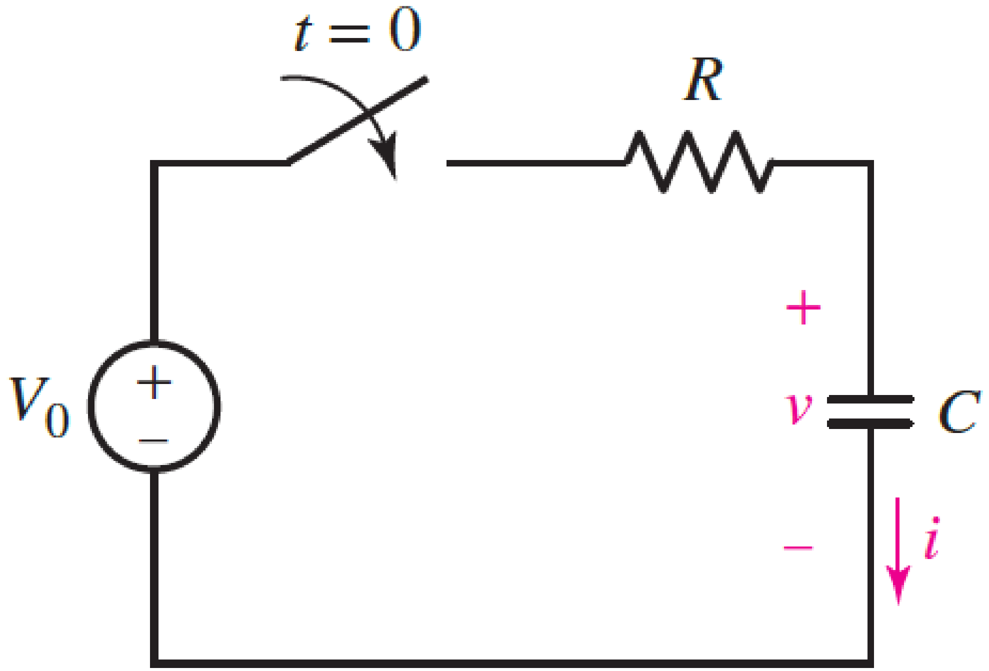




Capacitor in a circuit

- Let us connect our capacitor to a voltage source
- At first ($t = 0$), there is no charge across the capacitor and hence no voltage
- Because of KVL and OL, there should be a current $i = V_0/R$
- Because of this current, there is now a charge, and a voltage across the capacitor
- This is given by $v = q/C$
- The KVL now becomes:

$$V_0 = iR + v_c$$



Capacitor in a circuit

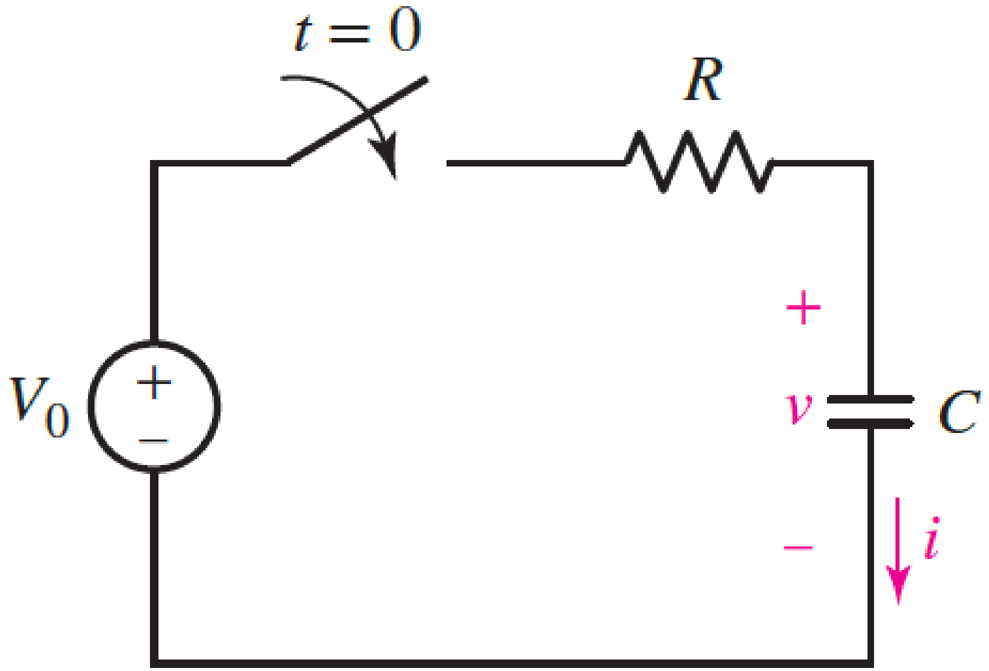
$$V_0 = iR + \frac{q}{C}$$

$$V_0 = \frac{dq}{dt}R + \frac{q}{C}$$

$$\frac{CV_0 - q}{C} = \frac{dq}{dt}R$$

$$\frac{dt}{RC} = \frac{dq}{CV_0 - q}$$

$$\frac{t}{RC} = -\ln(CV_0 - q) + A$$



Capacitor in a circuit

$$\frac{t}{RC} = -\ln(CV_0 - q) + A$$

At $t = 0$, $q = 0$. Thus, $A = \ln(CV)$

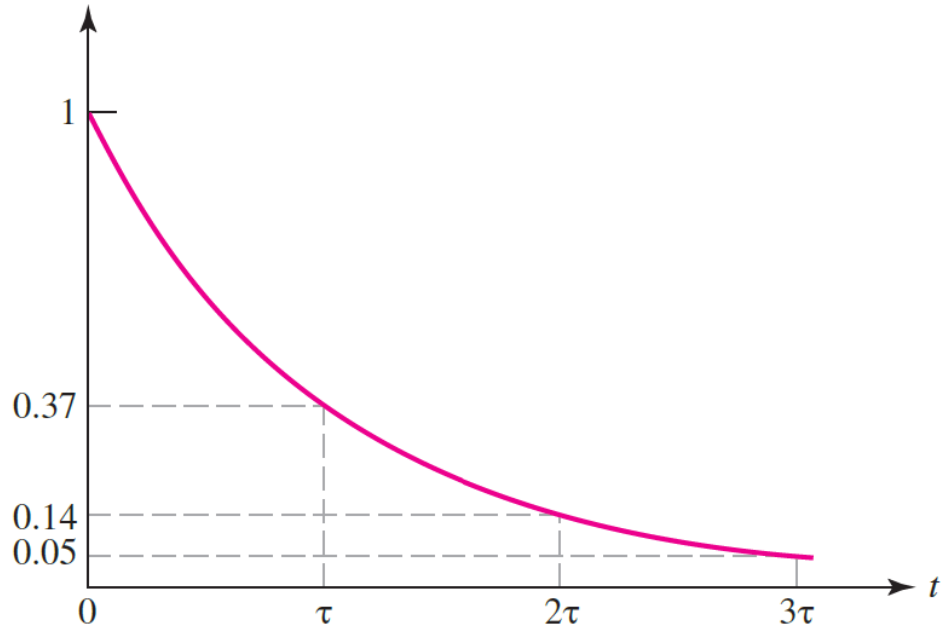
$$\frac{t}{RC} = \ln\left(\frac{CV_0}{CV_0 - q}\right)$$

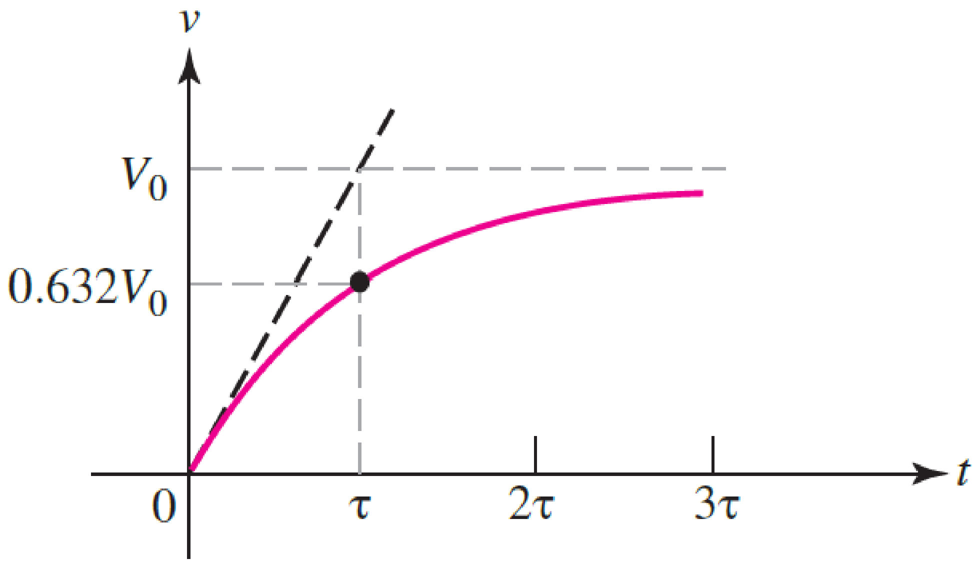
$$\frac{CV_0}{CV_0 - q} = e^{\frac{t}{RC}}$$

$$\frac{q}{CV_0} = 1 - e^{-\frac{t}{RC}}$$

$$\boldsymbol{v = V_0\left(1 - e^{-\frac{t}{RC}}\right)}$$

$$\boldsymbol{i = \frac{dq}{dt} = C\frac{dv}{dt} = \frac{V_0}{R}e^{-\frac{t}{RC}}}$$





Step response

- Say we have a step up (or down) in applied voltage. At a specific time $t = t_1$, voltage applied goes from $V = V_0$ to $V = V_1$

- What is the response of the capacitive circuit?

- In this case, we can consider the voltage source as two separate voltage sources, with one switching on at t_0 and the other at t_1

- The response of the circuit is a superposition of the two responses