Lecture 10 – Duality

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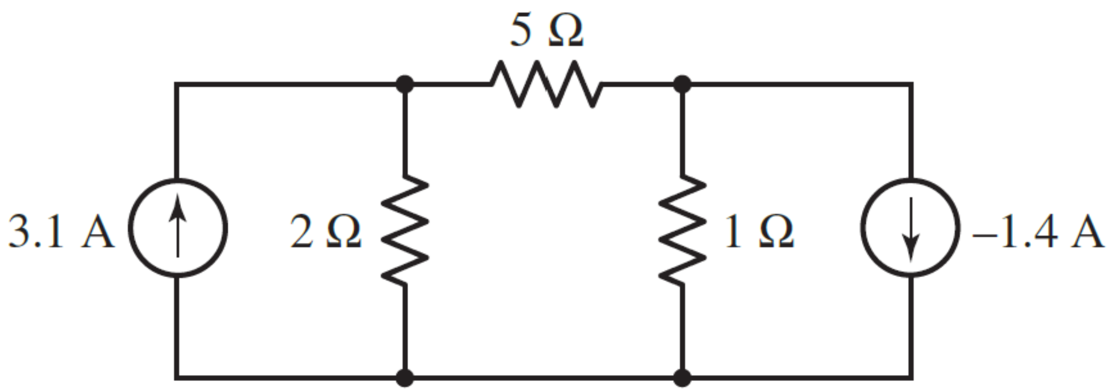
Nodal analysis

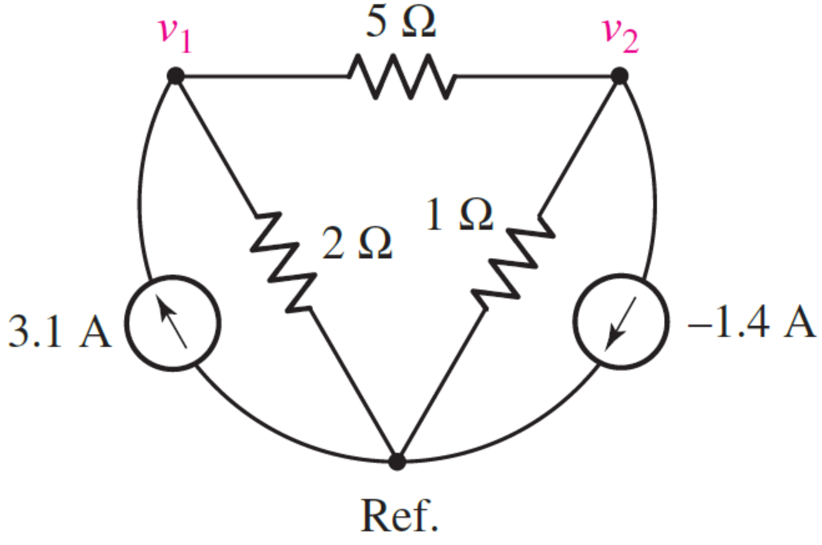
- Nodal analysis is based on KCL
- We take count the number of unique nodes in the circuit
- Designate one of them as reference, and assign voltage values to the remaining
- Now, lets apply KCL to the nodes:

$$3.1 = \frac{v_1}{2} + \frac{v_1 - v_2}{5}$$

$$-(-1.4) = \frac{v_2}{1} + \frac{v_2 - v_1}{5}$$

- Solving these two equations in two unknowns provides the all the answers to the circuit



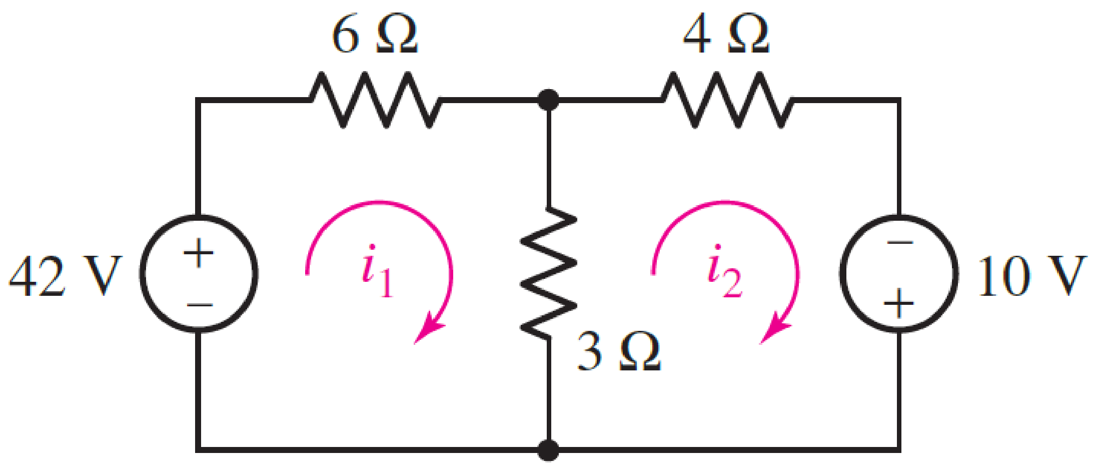


Mesh analysis

- We define a mesh as a loop that does not contain any other loops within it
- We can define currents through each mesh and write KVL equations for each mesh
- For the given circuit:

$$42 = 6i_1 + 3(i_1 - i_2)$$
$$10 = -3(i_1 - i_2) + 4i_2$$

- Solving these gives the complete analysis of the circuit



Duality

- In electronics, a dual of a relationship is formed by interchanging voltage and current in an expression
- For instance:

$$v = Ri \leftrightarrow i = Gv$$

- The interchange in current and voltage causes some circuit elements to change into another:

$$v = L\frac{di}{dt} \leftrightarrow i = C\frac{dv}{dt}$$

Duality

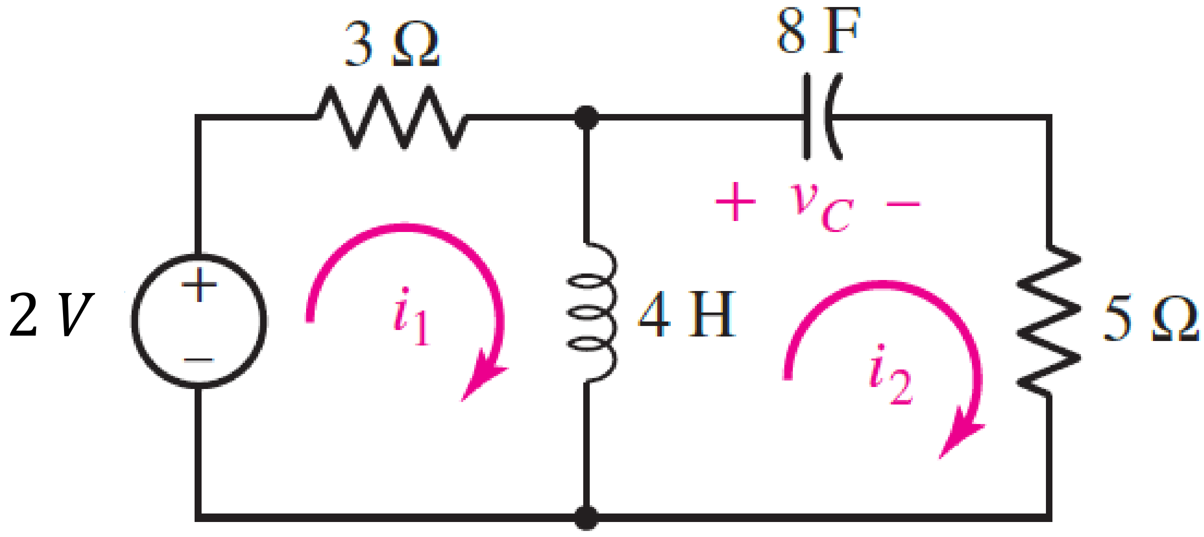
- Two circuits are “duals” if the mesh equations that characterize one of them have the same mathematical form as the nodal equations that characterize the other
- They are said to be exact duals if each mesh equation of one circuit is numerically identical with the corresponding nodal equation of the other
- Duality itself merely refers to any of the properties exhibited by dual circuits

Duality

- Let us use the definition to construct an exact dual circuit by writing the two mesh equations for the circuit shown
- Two mesh currents i_1 and i_2 are assigned, and the mesh equations are:

$$-2 + 3i_1 + 4\frac{d}{dt}(i_1 - i_2) = 0$$

$$-4\frac{d}{dt}(i_1 - i_2) + \frac{1}{8}\int i_2 dt + 5i_2 = 0$$



Duality

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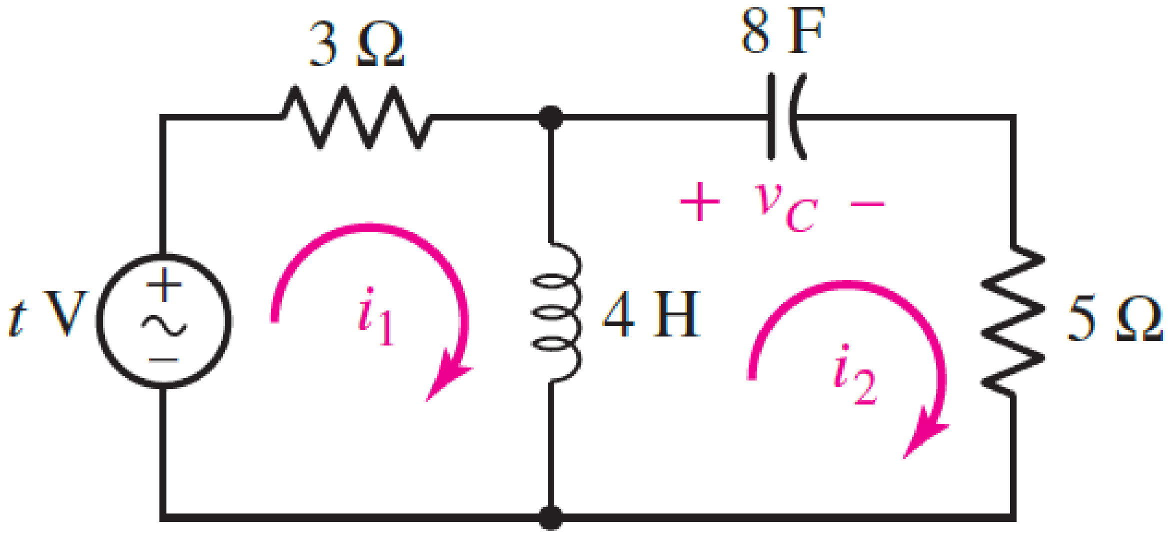
- Now, let us construct the duals of these equations
- We do this by replacing current terms with voltage terms:

$$-2 + 3v_1 + 4\frac{d}{dt}(v_1 - v_2) = 0$$

$$-4\frac{d}{dt}(v_1 - v_2) + \frac{1}{8}\int v_2 dt\ + 5v_2 = 0$$



Duality

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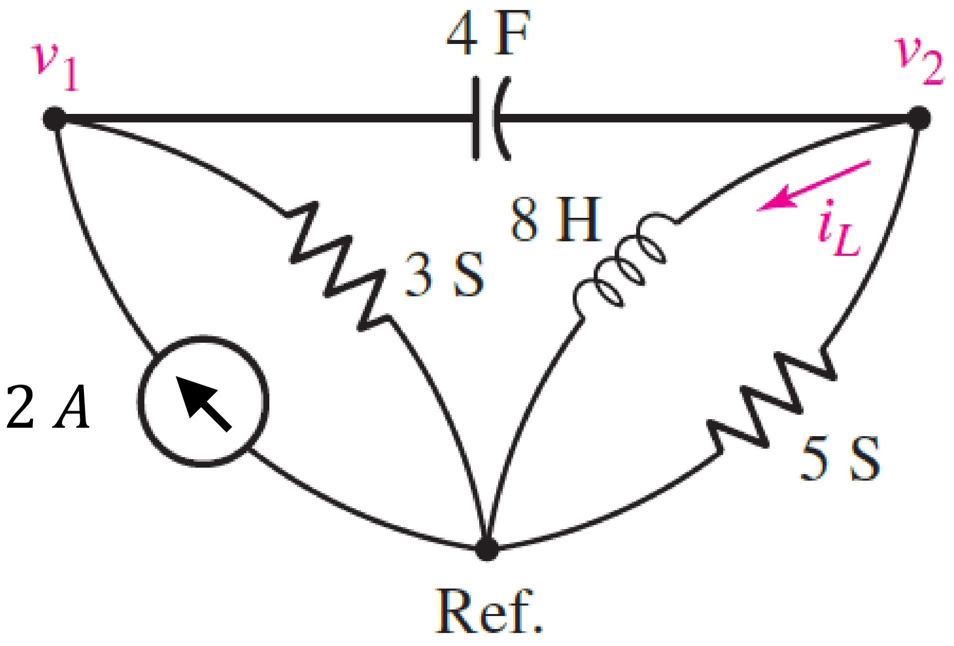
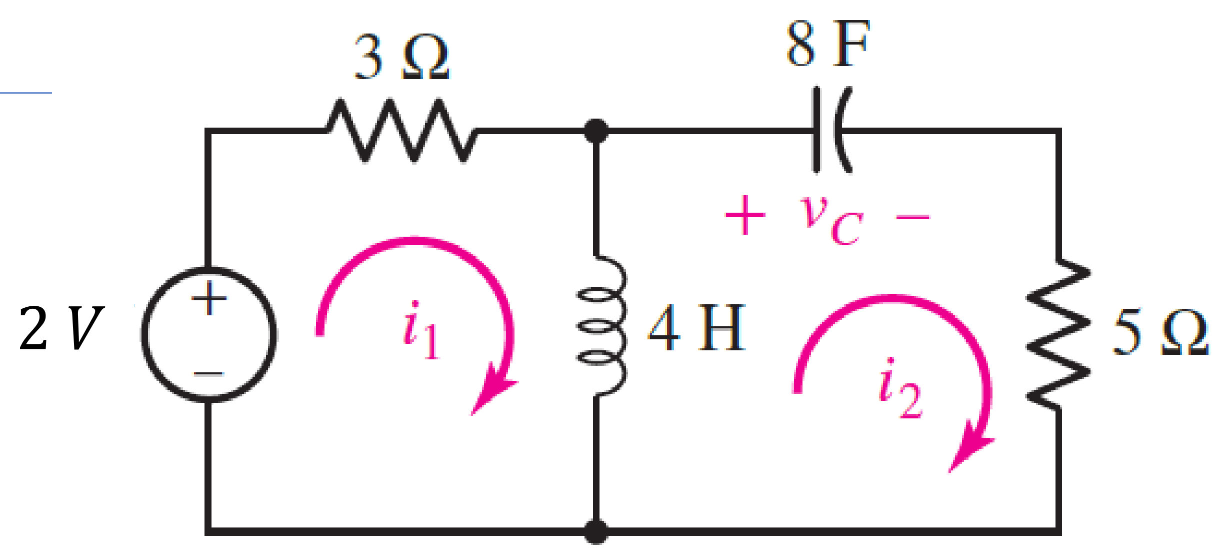
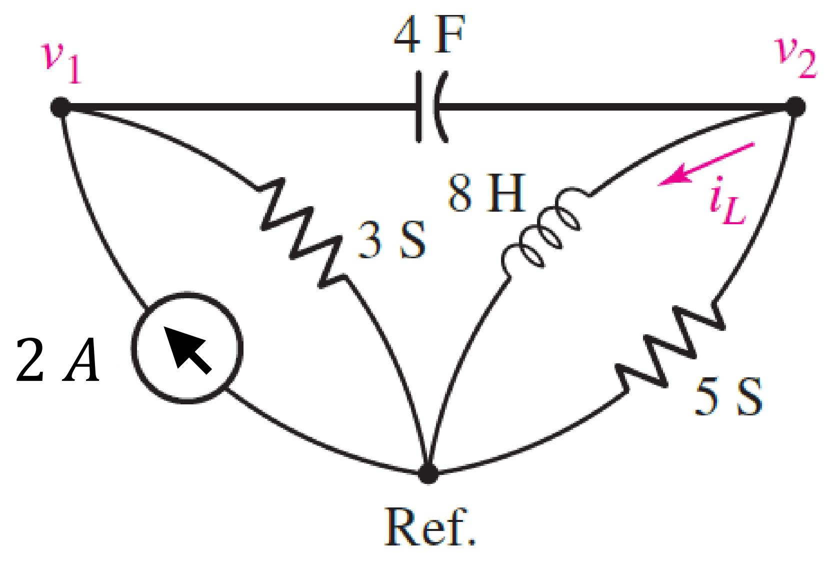
- These are equations for a new circuit
- With two nodes with voltages v_1 and v_2
- The elements in the circuit are defined such that the equations resemble a nodal analysis



Duality

- Dual circuits may be obtained more readily than by this method, for the equations need not be written
- To construct the dual of a given circuit, we think of the circuit in terms of its mesh equations
- With each mesh we must associate a nonreference node, and, in addition, we must supply the reference node
- Elements that appear only in one mesh must have duals that appear between the corresponding node and the reference node





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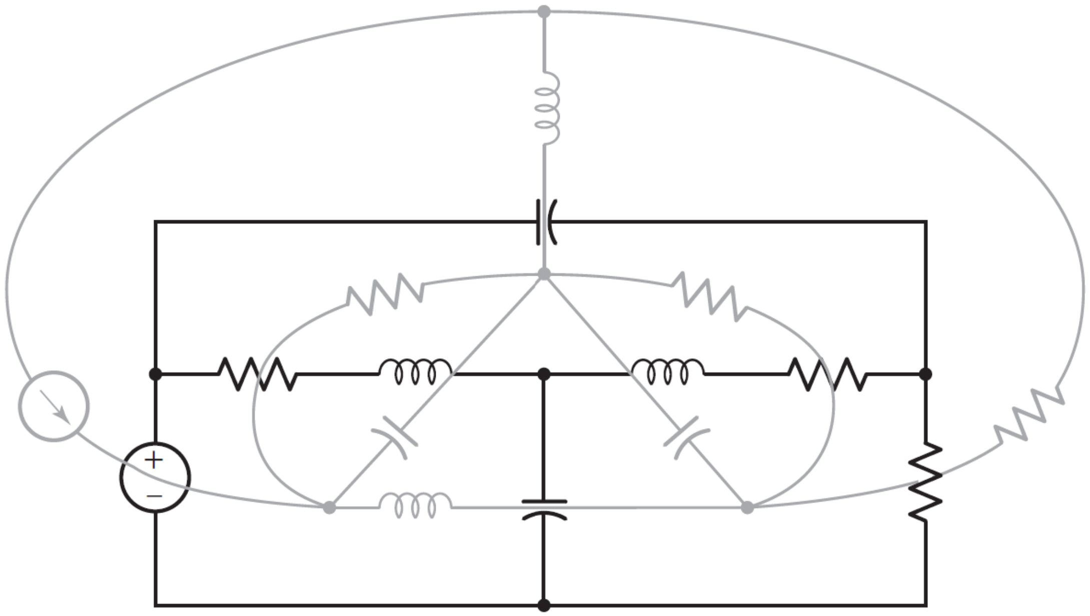
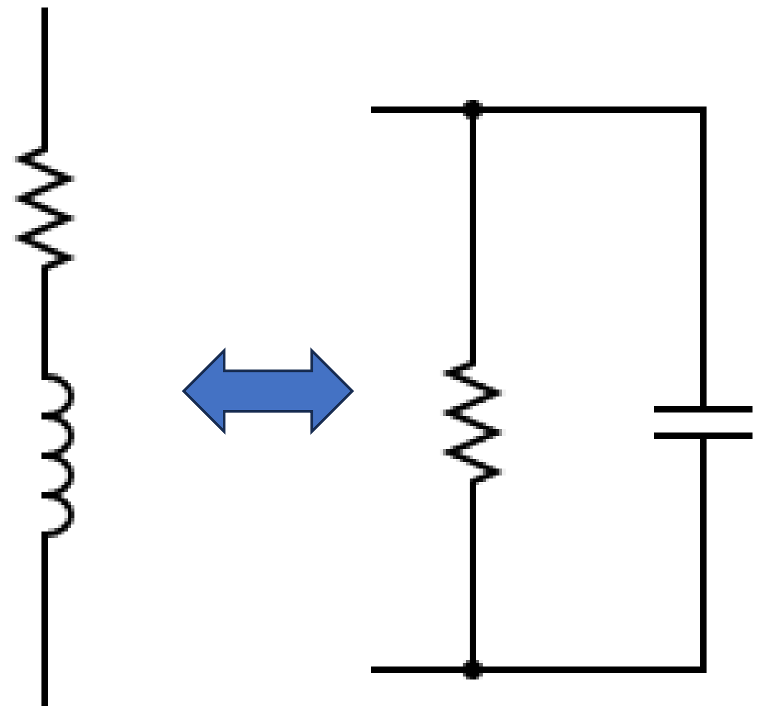


Duality

- Just like circuits, we can have duals of networks
- For example, a simple RL network with two terminals
- If we assume an arbitrary voltage source across this, we can convert that circuit into a dual by replacing the source with a current source, the resistor with the equal conductance, and the inductor with a capacitor
- In some cases, this makes the analysis of the network/circuit simpler



Duality

KVL	KCL
Voltage source	Current source
Capacitor (store of voltage)	Inductor (store of current)
Capacitor opposes change in voltage	Inductor opposes change in current
No voltage in inductor at steady state	No current in capacitor at steady state
Mesh	Node
Open circuit	Short circuit
Parallel	Series

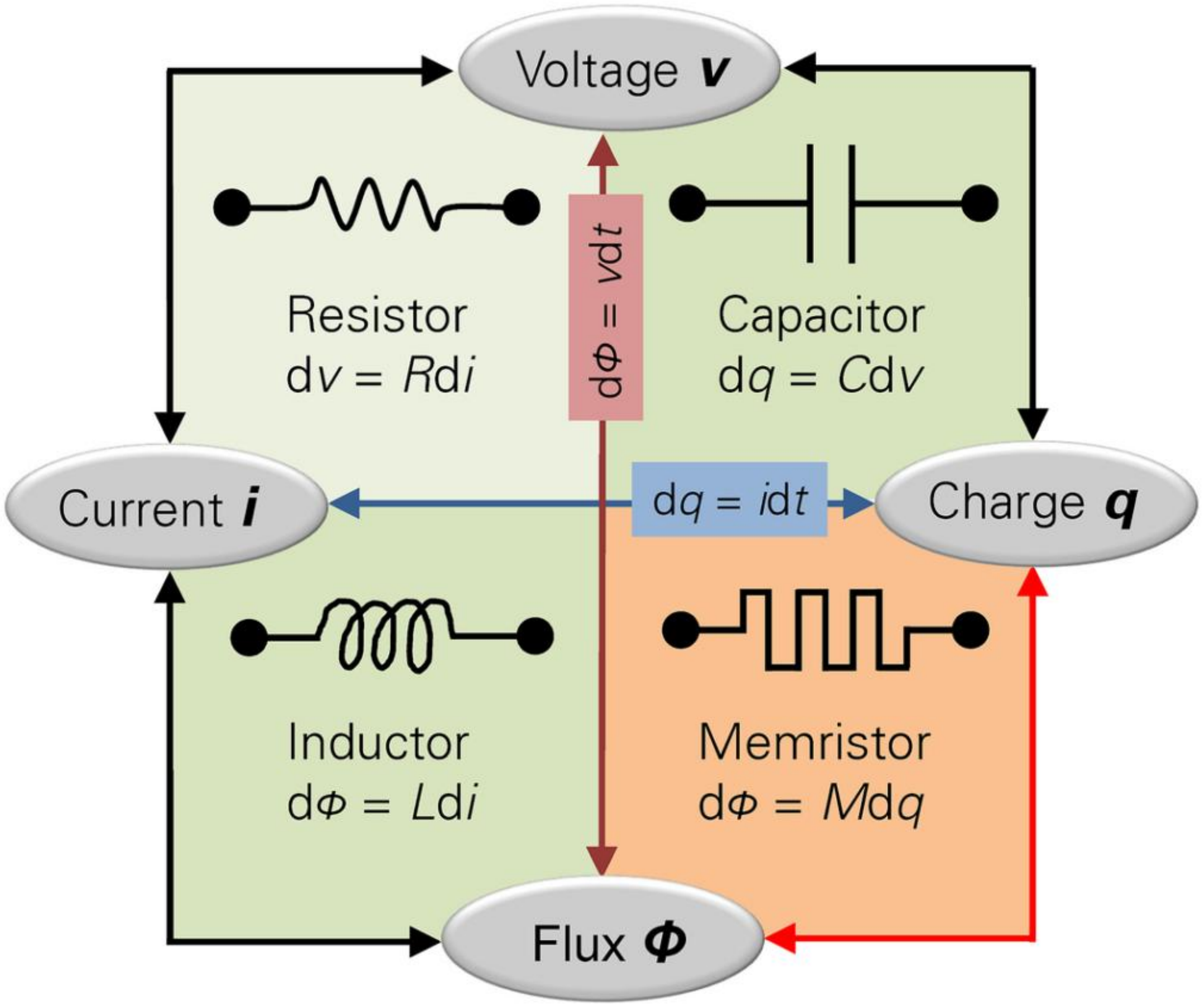
Dual of a dual is the original circuit

The missing element

- We have been introduced to three different two-terminal passive elements: the resistor, the capacitor, and the inductor
- Each has been defined in terms of its current–voltage relationship:

$$v = Ri \qquad i = C\frac{dv}{dt} \qquad v = L\frac{di}{dt}$$

- From a more fundamental perspective, however, we can view these three elements as part of a larger picture relating four basic quantities, namely, charge q, current i, voltage v, and flux linkage φ
- The three circuit elements provide relations between current i, voltage v, current i, flux linkage φ, and voltage v, charge q
- However, there should be a fourth element defining a relationship between flux linkage φ and charge q



The missing element

- In 1971, Leon Chua published a paper titled, “Memristor-The Missing Circuit Element”
- He wanted it to conform to the characteristic

$$d\varphi = Mdq$$

- Based on this, he theorize that this new element should have characteristics based on its history, i.e., it should have a “memory”
- In 2008, three separate papers simultaneously announced that they found the missing memristor, however, many counterclaims have been published and *real memristors* are still elusive!