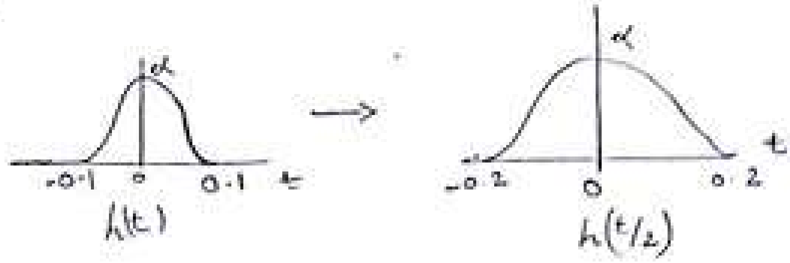
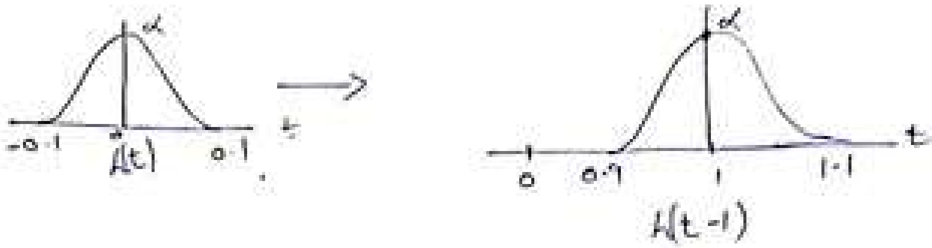
1) ⓐ Properties used :

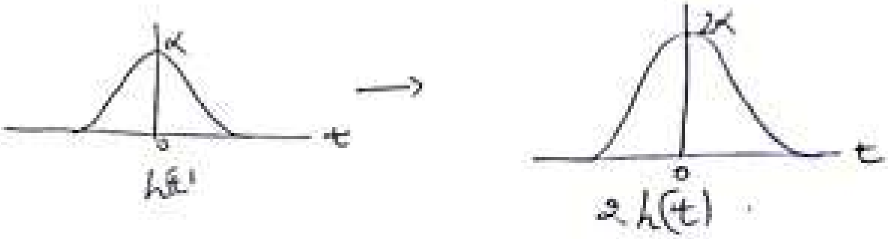
(i) Time Scaling

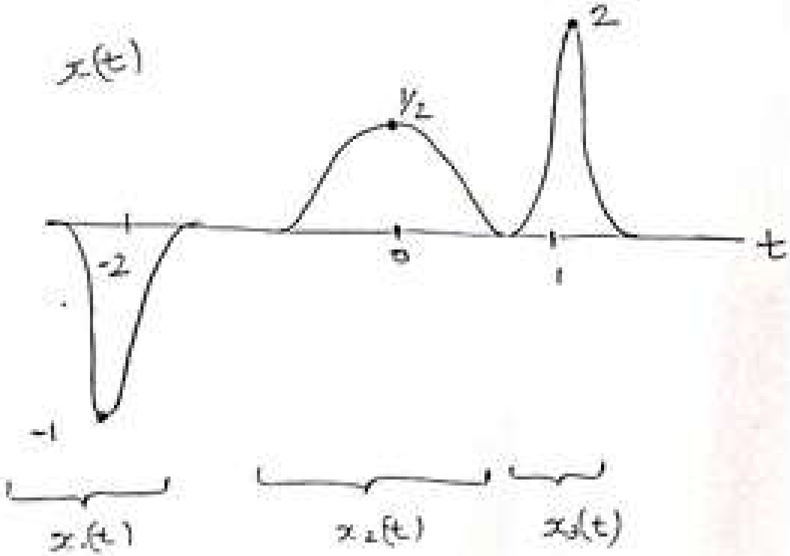


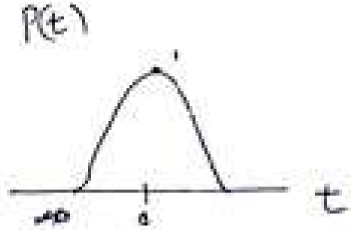
(ii) Time Shifting



(iii) Amplitude Scaling







using (ii) & (iii) : $x_1(t) = -p(t+2)$

using (i) & (iii) : $x_2(t) = \frac{1}{2} p(t/3)$

using (ii) & (iii) : $x_3(t) = 2p(t-1)$

$$x(t) = x_1(t) + x_2(t) + x_3(t)$$

$$x(t) = -p(t+2) + \frac{1}{2} p(t/3) + 2p(t-1)$$

or

$$x(t) = -p(t+2) + \frac{1}{2} p(t/2) + 2p(t-1)$$

{∵ labeling is not available in Diagram}

1 b) $p(t) \longrightarrow \boxed{} \longrightarrow \begin{cases} \cos^2(t^2-5)+2t & t\in[-2,4] \\ 0 & \text{otherwise} \end{cases}$

For an LTI system, if

$$x_1(t) \longrightarrow \boxed{h(t)} \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow \boxed{h(t)} \longrightarrow y_2(t)$$

then, $x_1(t)+x_2(t) \longrightarrow \boxed{h(t)} \longrightarrow y_1(t)+y_2(t)$

$$3p(t-2) \rightarrow \boxed{} \rightarrow \underbrace{3\left(\cos^2\left((t-2)^2-5\right)+2(t-2)\right)}_{A} \qquad -2\le t-2\le 4 \Rightarrow 0\le t\le 6$$

$$p(t+3) \rightarrow \boxed{} \rightarrow \underbrace{\cos^2\left((t+3)^2-5\right)+2(t+3)}_{B} \qquad -5\le t\le 1$$

$$\therefore\ 3p(t-2)+p(t+3) \rightarrow \boxed{} \rightarrow \begin{cases} B & -5\le t\le 0 \\ A+B & 0\le t\le 1 \\ A & 1\le t\le 6 \end{cases}$$

Since we don't have enough information regarding $p(t)$ & the system.

We can't say anything about the boundedness or causality of the system.

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Ans-2.

$x(t) = 10e^{j(240\pi t)}$

1) $\boxed{w = 240\pi}$ rad/s → (fundamental angular freq.)

$T = \frac{2\pi}{w}$

$= \frac{2\pi}{240\pi}$

$\boxed{T = \frac{1}{120} s}$

→ fundamental Time period

$2\pi f = 240\pi$

$\boxed{f = 120 Hz}$

→ fundamental freq.

2) $\boxed{k^{th} \text{ harmonic Expression} = 10e^{j(k \times 240\pi t)}}$

1^{st} harmonic expression $= 10e^{j(240\pi t)}$

2^{nd} harmonic expression $= 10e^{j(480\pi t)}$

~~harmonic~~

1^{st} harmonic freq. $= 120 Hz$

2^{nd} harmonic freq. $= 240 Hz$

(2) (c).

Total energy:

$$x(t) = 10 e^{j(240\pi t)}.$$

$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} 100 \, dt = \infty.$$

$$\boxed{\text{Energy}_{Tot} = \infty}$$

The integral diverges ⇒ energy is ∞.

∴ Signal is not energy-limited.

~~Total~~ Average Power:

$$\text{Av. Power} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt.$$

$$x(t) = 10 e^{j(240\pi t)} \Rightarrow |x(t)| = 10.$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} 100 \, dt$$

$$= 100 \times \frac{1}{T} \times (T) = 100\,W.$$

$$\boxed{\text{Av. Power} = 100\,W.}$$