Lecture 5 – Equivalent circuits

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Equivalent circuits

- Many times, we replace a part of an electronic circuit with an equivalent

- The idea is to simply the analysis of the behaviour of the rest of the system

- To qualify as an “equivalent”, a circuit should have the following properties:
  - Same number of terminals as the original
  - Same current at all nodes and same voltage across all terminals, as the original circuit
  - Same behaviour when an external condition is changed

Resistance networks

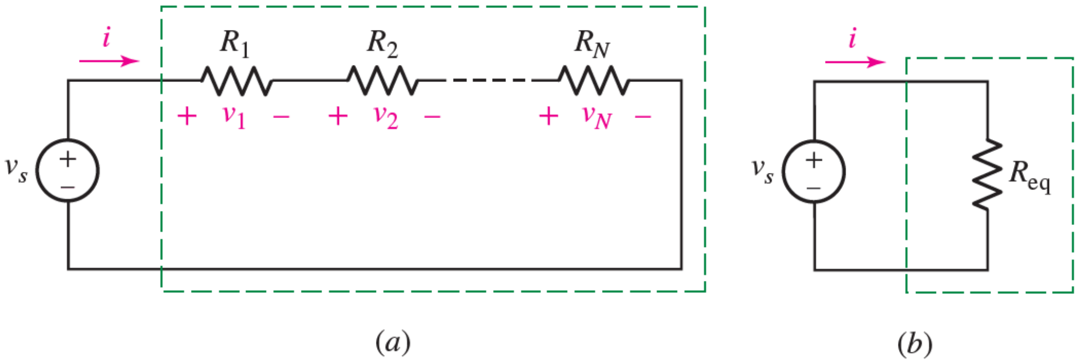
- The simplest example of equivalent circuits is resistive networks
- We know that, for resistances in series:

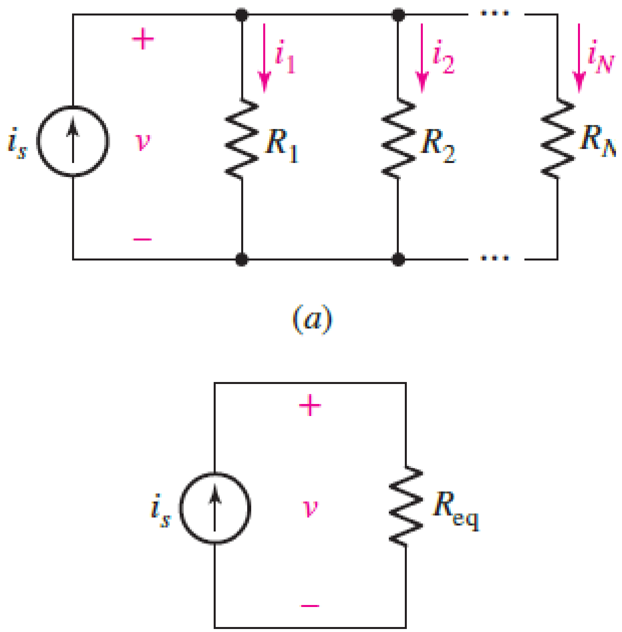
$$R_{eq} = \sum R_i$$

- For parallel resistors:

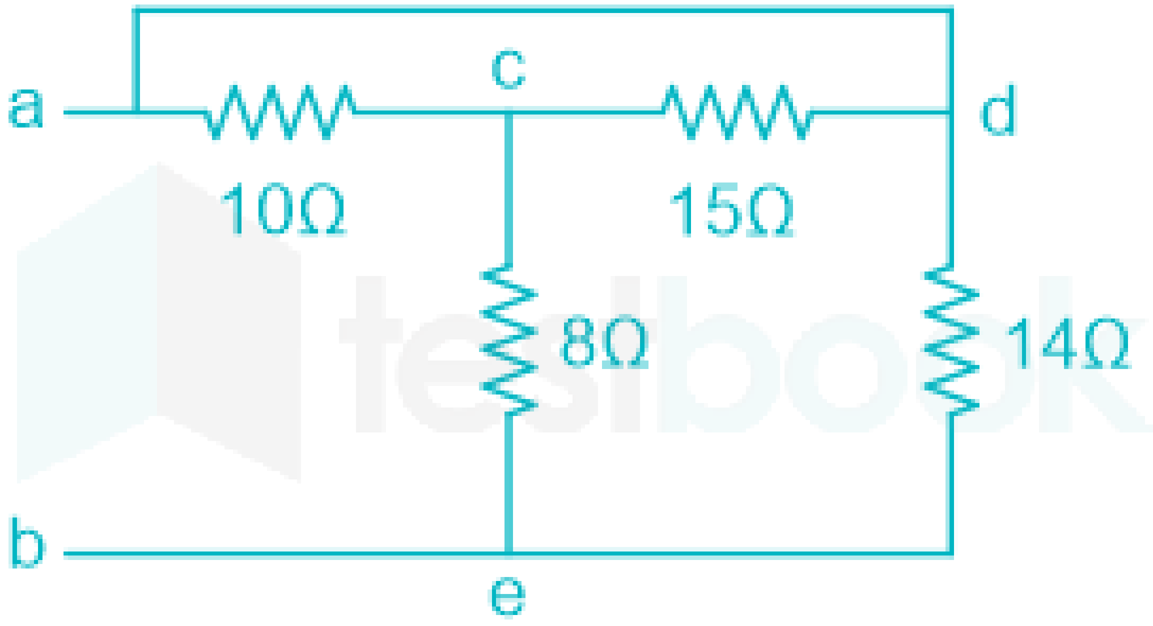
$$\frac{1}{R_{eq}} = \sum \frac{1}{R_i}$$

- In the case when the resistive network is more complex, we can apply this technique multiple times to get an equivalent

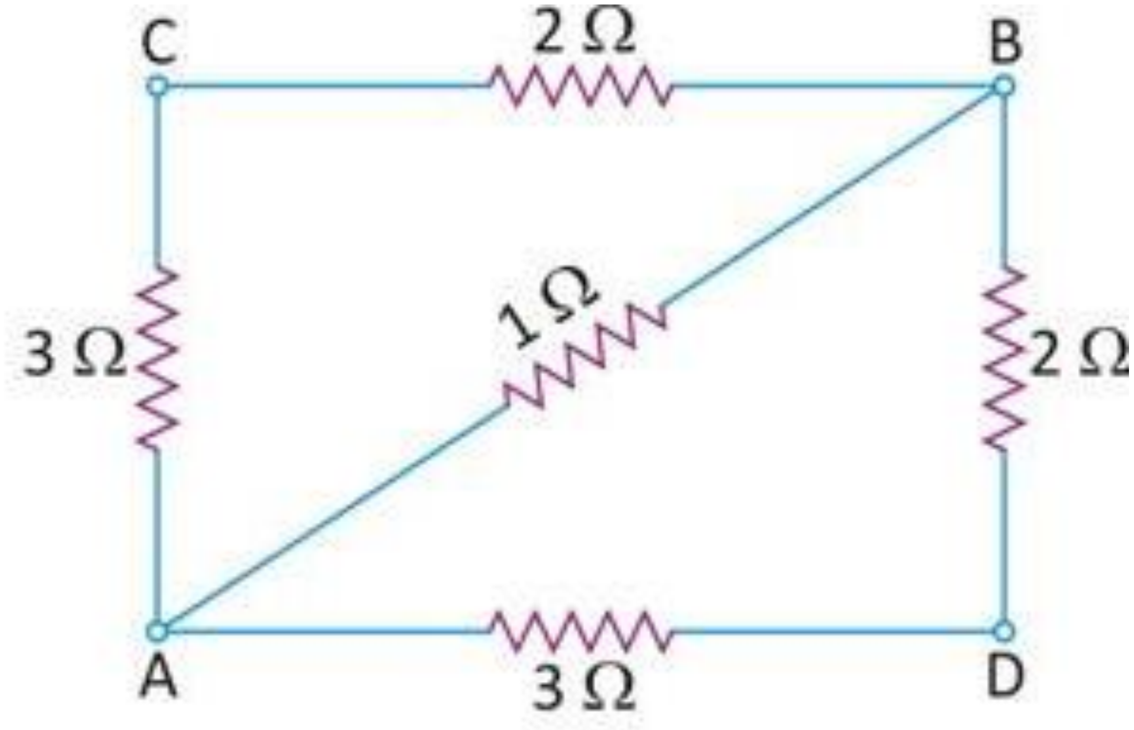




Example 1



Example 2



$R_{AC} = 51/35\,\Omega$

$R_{AB} = 25/35\,\Omega$

$R_{BD} = 46/35\,\Omega$

$R_{BC} = 46/35\,\Omega$

$R_{AD} = 51/35\,\Omega$

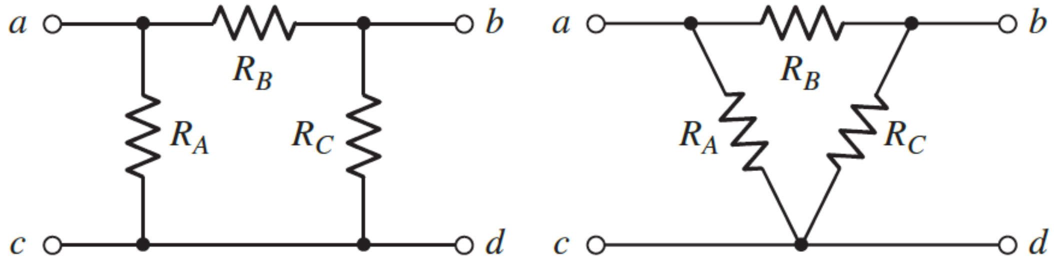
$R_{CD} = ?$

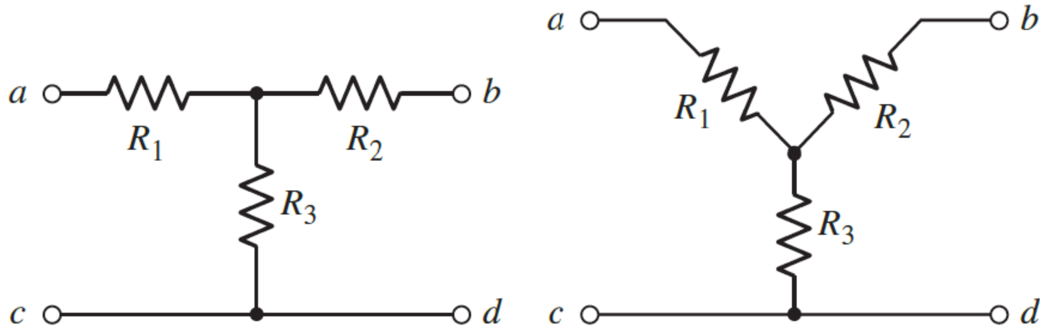
Pi-Star transformation

- In some cases, the combination of series-parallel can be difficult to obtain

- In these cases, we need to perform some special operations and circuit equivalences

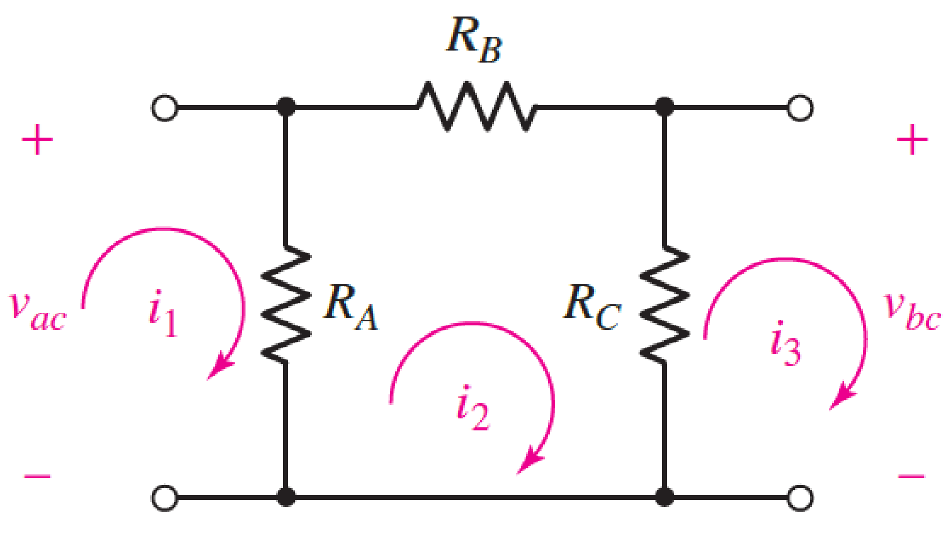
- One such equivalence is the Pi-Star equivalence, also known as Delta-Star or Delta-Wye transformation



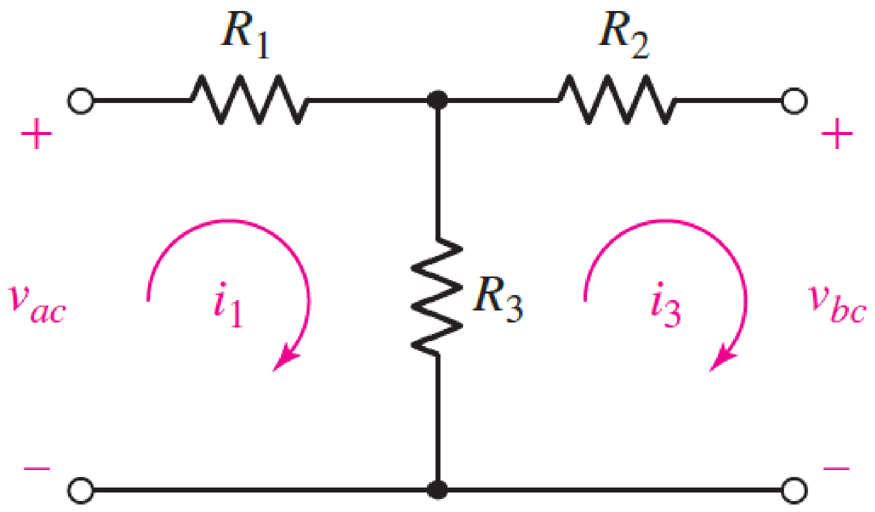


Pi-Star transformation

- Say we want these two circuits to be "equivalent"
- Then, we need to make sure that voltages at the terminals and the currents entering/leaving the terminals are the same
- Thus, v_{ac}, v_{bc}, i_1 and i_3 should be same. i_2 is an internal current for the first circuit, and does not have a constraint



(a)



(b)

Pi-Star transformation

- For network (a):

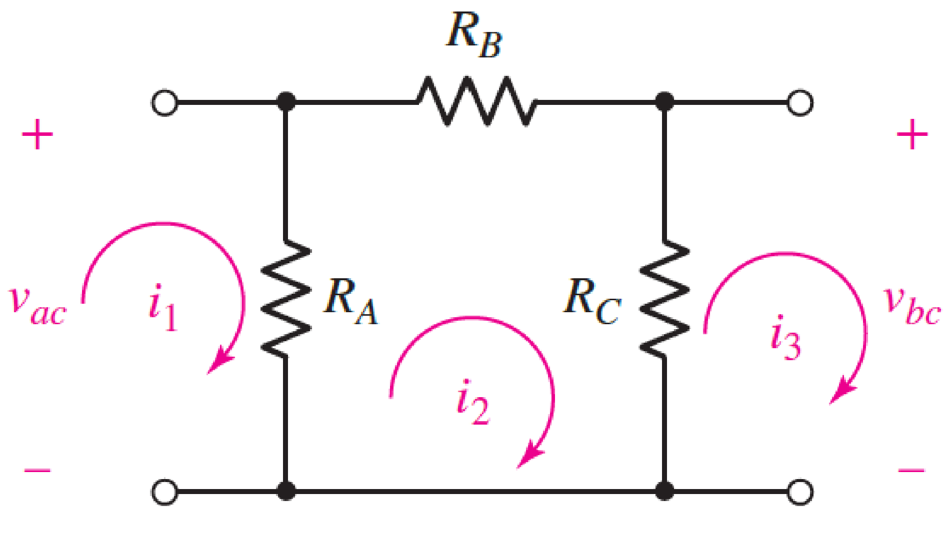
$$R_A(i_1 - i_2) = v_{ac}$$
$$-R_A(i_1 - i_2) + R_B i_2 + R_C(i_2 - i_3) = 0$$
$$R_C(i_2 - i_3) = v_{bc}$$

- For network (b):

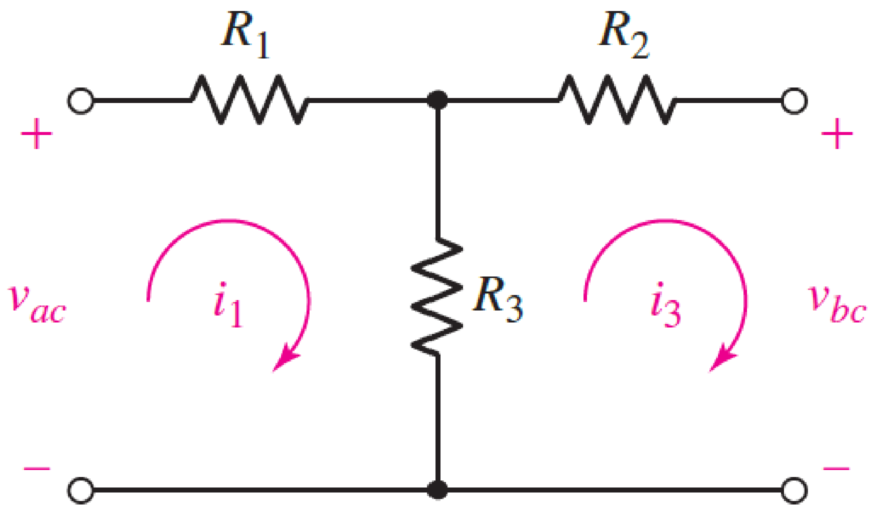
$$R_1 i_1 + R_3(i_1 - i_3) = v_{ac}$$
$$R_3(i_1 - i_3) - R_2 i_3 = v_{bc}$$

We can get i_2 as:

$$i_2 = \frac{R_A i_1 + R_C i_3}{R_A + R_B + R_C}$$



(a)



(b)

Pi-Star transformation

- For network (a):

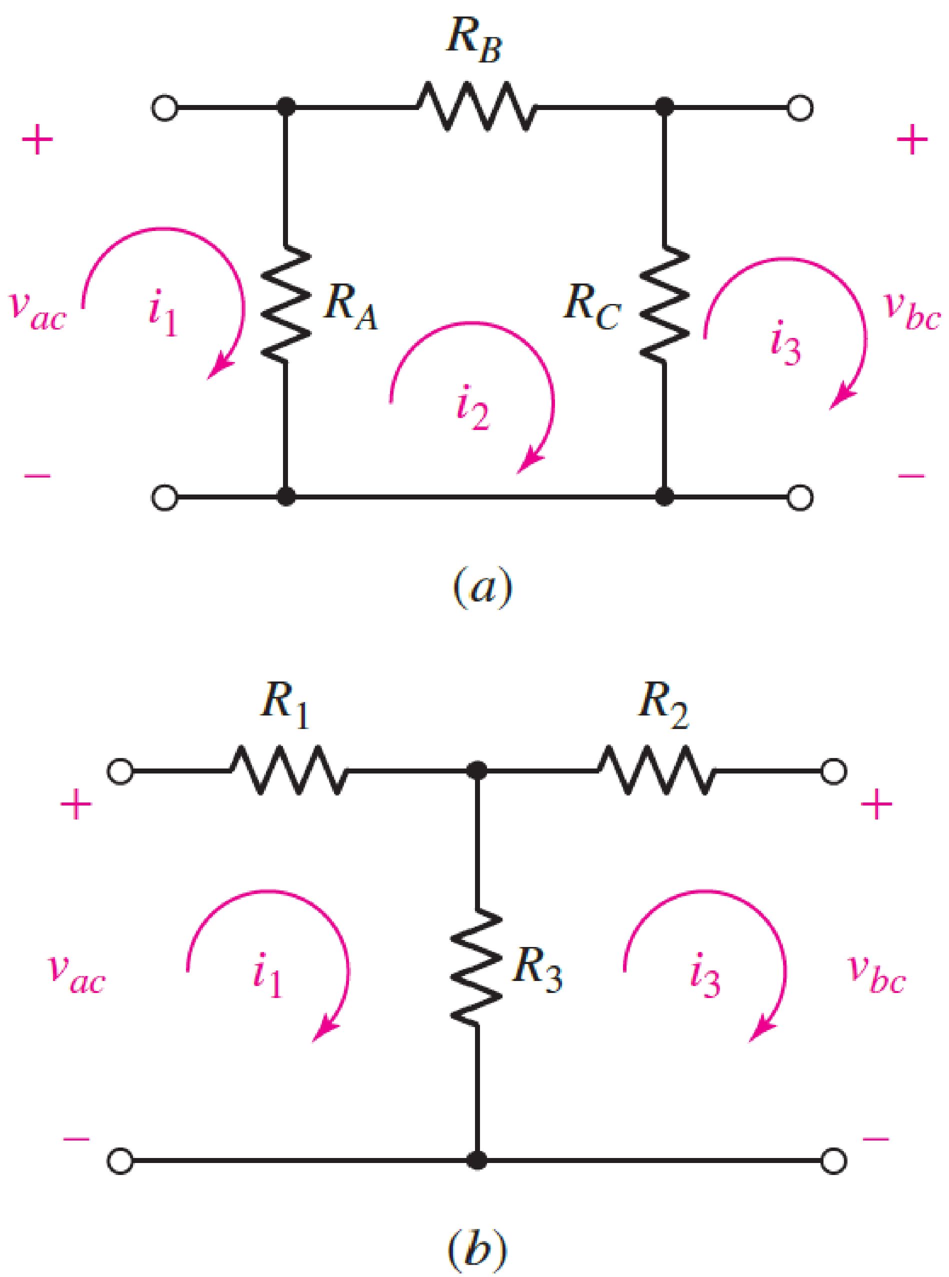
$$R_A\left(i_1 - \frac{R_A i_1 + R_C i_3}{R_A + R_B + R_C}\right) = v_{ac}$$

$$\frac{R_A(R_B+R_C)i_1}{R_A + R_B + R_C} - \frac{R_A R_C i_3}{R_A + R_B + R_C} = v_{ac}$$

$$\frac{R_A R_C i_1}{R_A + R_B + R_C} - \frac{R_C(R_A+R_B)i_3}{R_A + R_B + R_C} = v_{bc}$$

- For network (b):

$$(R_1+R_3)i_1 - R_3 i_3 = v_{ac}$$
$$R_3 i_1 - (R_2+R_3)i_3 = v_{bc}$$



Pi-Star transformation

- For complete equivalence, the two sets of equations should have the same coefficients
- Thus,

$$R_3 = \frac{R_A R_C}{R_A + R_B + R_C}$$

- For the complete conversion:

$$R_1 = \frac{R_A R_B}{R_A + R_B + R_C}$$

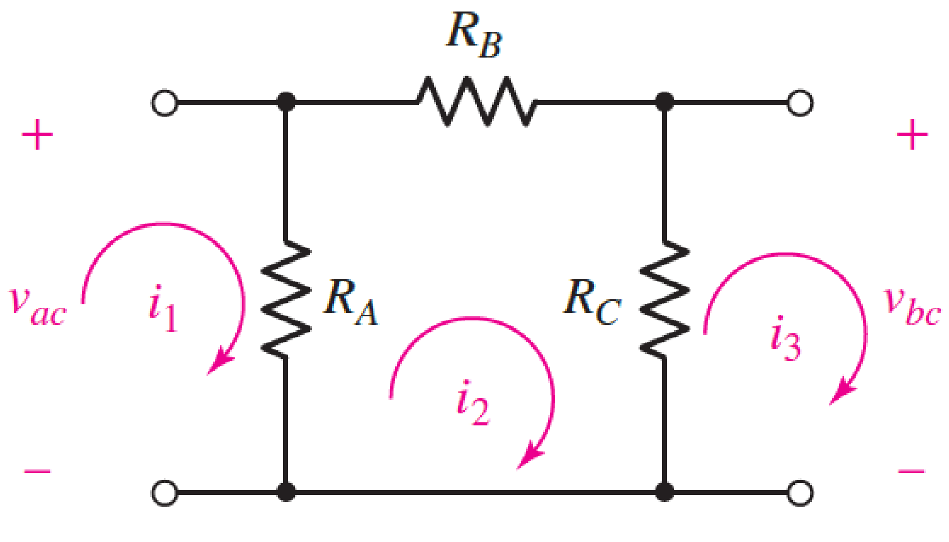
$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_C R_A}{R_A + R_B + R_C}$$

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$



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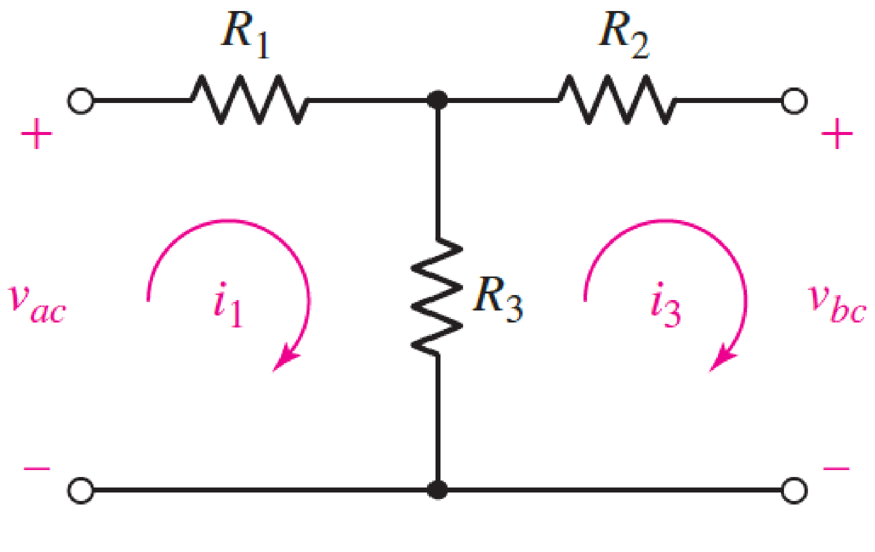
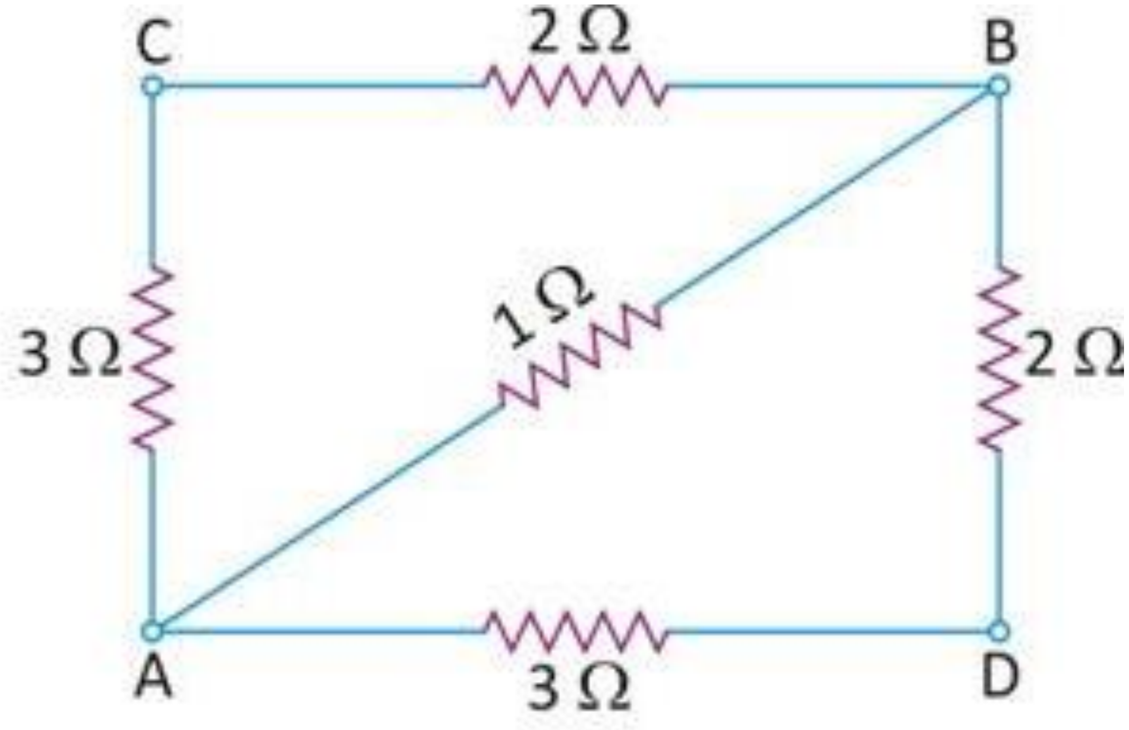


(b)

Example 2



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