Networks, Signals, and Systems: Mid Exam

Prasad Krishnan, Aftab Hussain

Monsoon-2025 | Marks: 40 | Time : 1.5 hrs

- Clearly state the assumptions (*if any*) made that are not specified in the questions.
- Throughout the paper, $\delta(t)$ and $u(t)$ refer to the unit impulse and unit step functions respectively.
- Throughout each question, we follow the same notation and terminology, in all parts.

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1. (17 marks)

   (a) (2+2+1 marks) Consider a system with a rational system function $H(s) = \frac{A(s)}{B(s)}$, where $A(s)$ and $B(s)$ are polynomials that do not share any common factor. Define the notion of ROC of this system. Using this, explain why the ROC of this system cannot include any poles of $H(s)$. Must they surely include all zeroes of $H(s)$? What do you think (with reasons)?

   (b) (7 marks) Consider a system with system function $H(s) = \frac{5000(s-1)}{(s+1)(s-2)}$. Suppose the system is known to be stable. Find the impulse response of this system. (**Hint:** *Perform the partial fraction simplification, then use known signal-transform pairs, and the property that the Laplace transform must satisfy in a stable system.*)

   (c) (5 marks) In the context of signal processing or communication systems, a *low-pass filter* is one which attenuates (i.e., reduces the amplitudes of) high frequency signals, while letting through low frequency signals (with higher, or same amplitudes). A *high-pass filter* has the opposite effect, i.e., to attenuate low-frequency signals while letting through high-frequency signals. Is the system in part (b) a low-pass filter or a high-pass one? Argue with reasons (***Hint:*** *Use the input-output relationship in terms of the Fourier-Transforms, and the magnitudes of this relationship, to argue*).

2. (13 marks)

   (a) (2 marks) First, obtain the Laplace Transform of $u(t)$ from known Laplace Transform pairs (done in the class). Don't forget the ROC. Then, do the following.

   (b) (5 marks) Let $x(t)$ and $y(t)$ be two time-signals, with respective Laplace transforms $X(s)$ (ROC R_1) and $Y(s)$ (ROC R_2). Derive then, the Laplace transform of the convolved signal $x(t) * y(t)$. Remark on the ROC of the same.

   (c) (3 marks) Show that the signal $z(t) \triangleq \int_{-\infty}^{t} x(\tau)d\tau$ is the same as the signal $x(t) * u(t)$.

   (d) (3 marks) Using all parts above, find the relationship between the transforms $X(s)$ and $Z(s)$, along with the ROC conditions.

3. (10 marks) A square wave with period T, as shown in Fig. 1 (next page), is sent through a LTI system with impulse response $e^{-2t}u(t) + e^{5t}u(-t)$. Find the output signal. (**Hint:** You may need to use Fourier Series and recall what happens if you send sinusoids through an LTI system. Remember that the ROC matters, always. You need the precise answer, but absolutely simplified version of the answer is not expected.)

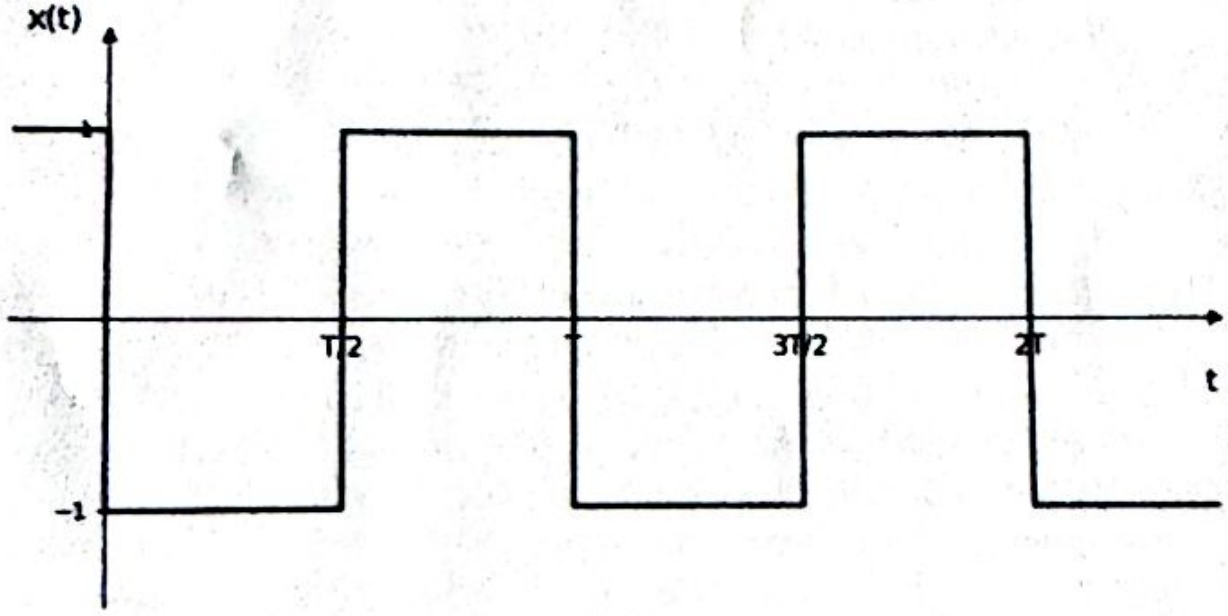

Figure 1: Square Wave