Lecture 1 – Charge, potential and current flow

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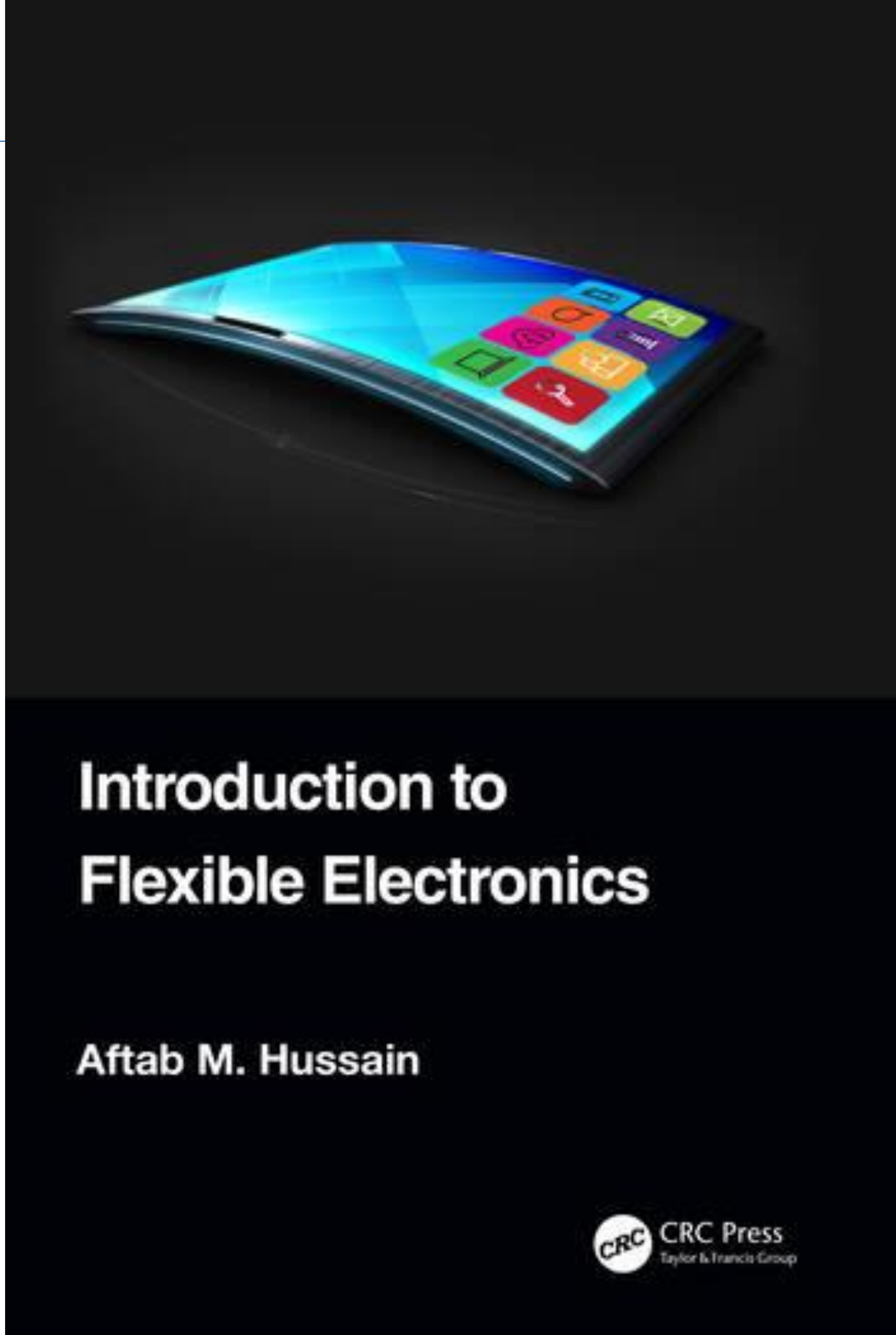
Introductions

- B. Tech in IIT Roorkee (2009):
- After B. Tech.:
  - Design Engineer, Analog Devices India (2011)
- Joined KAUST as M.S. in 2011
- Continued as Ph.D. from Jan 2013
- Postdoc in Harvard University up to Jan 2018
- Assistant Prof., IIITH, 2018 - 2023
- Associate Prof., IIIT H, 2023 - present

- Total of 110+ research papers and 14 patents in the last 10 years

Courses

- Networks, Signals and Systems [ECE UG1 core]
  - "Engineering Circuit Analysis", Hayt, William Hart.
- Communications and Controls in IoT [ECE UG2 elective]
- Principles of Semiconductor Devices [Open Elective, Monsoon]
  - "Advanced Semiconductor Fundamentals", Sze
- Flexible Electronics [Open Elective, Spring]
  - "Introduction to Flexible Electronics", A. M. Hussain



Here. We. Go.

Charge

- Origins of charges are not well known
- However, we know this:
  - Charge on an electron is constant
  - Charge is conserved in a closed system
- Represented by the unit Coulomb
  - 1 Coulomb is a fairly large charge
- Electron charge: $e = 1.6 \times 10^{-19}$ C

Charles-Augustin de Coulomb

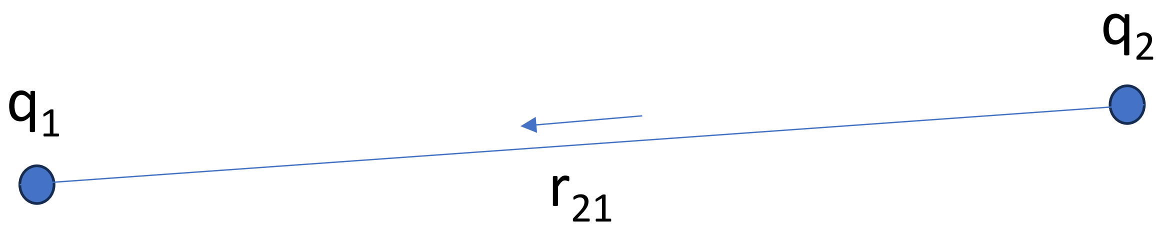
1779, force of a charge is inversely proportional to square of distance

Electrostatic Force

- Forces are experienced by two charged particles because of each other
- Force on q_1:

$$F_1 = k\frac{q_1 q_2}{r^2}$$

$$\boldsymbol{F_1} = k\frac{q_1 q_2}{r^3}\boldsymbol{r_{21}}$$



Charles-Augustin de Coulomb

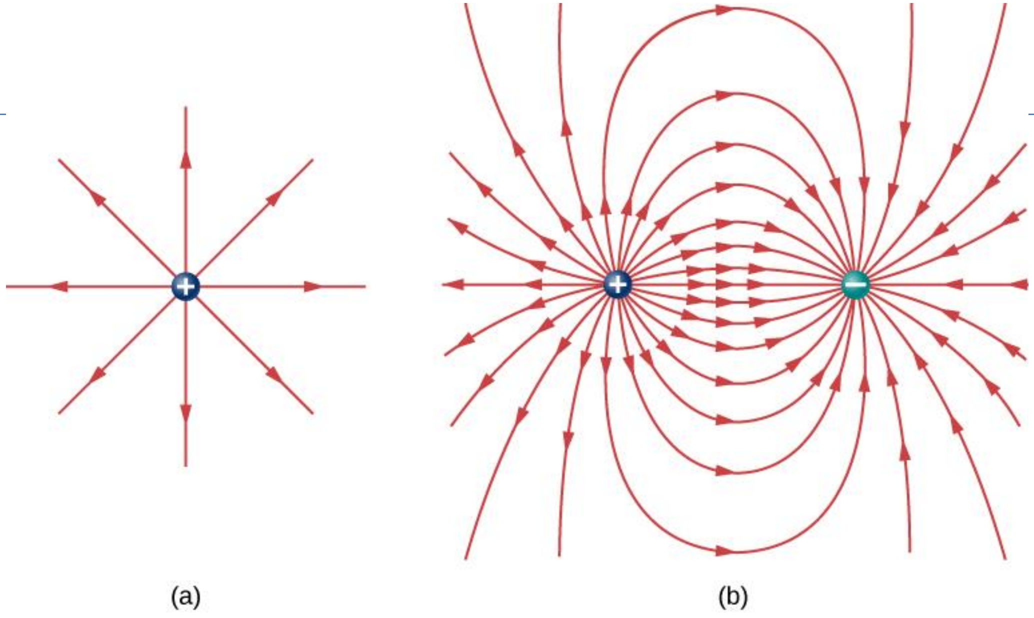
1779, force of a charge is inversely proportional to square of distance

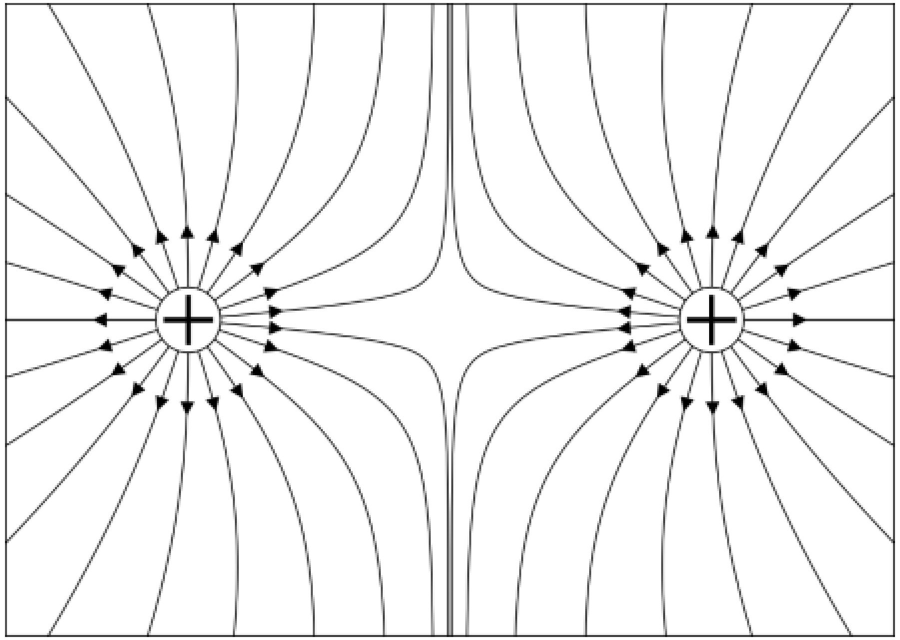
Electric Field

- For a given charge distribution, at any point, we can define the "electric field", as the force experienced by a unit charge at that point
- Units are N/C and it is a vector

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{q}$$

- The direction of the resultant force defines the direction of the field
- This is indicated by field lines



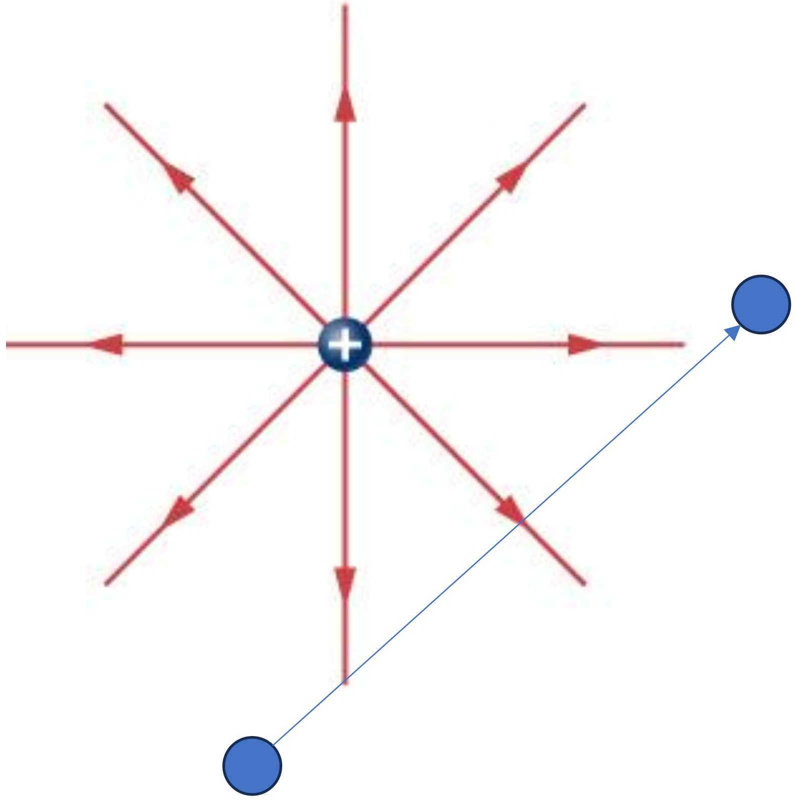


Potential Difference

- Amount of energy per unit charge required to move a unit charge from a specific point (A) to another point (B), given a charge distribution
- If we know the field, then we know the force experienced by the particle of a given charge
- Work done is the integration of this force

$$W = \int_A^B \boldsymbol{F} \cdot \boldsymbol{dl}$$

$$V = \frac{W}{q} = \int_A^B \boldsymbol{E} \cdot \boldsymbol{dl}$$



Potential Difference

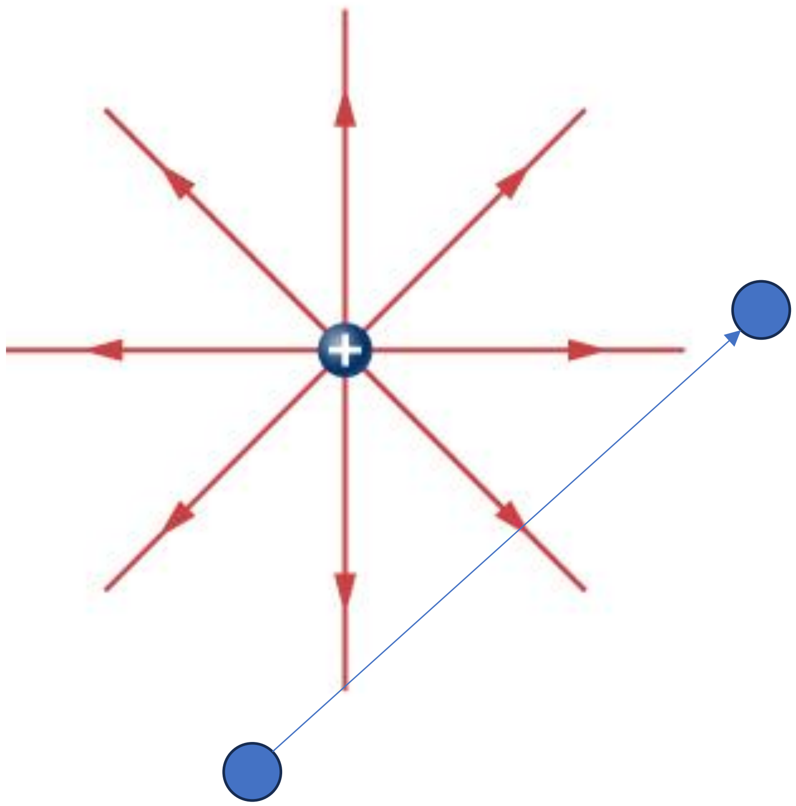
- We can write that:

$$\boldsymbol{E} = -\nabla V$$

- Thus, the electric field points towards decreasing potential
- We can say that given a potential difference between two points A and B, separated by a distance d, the field between the two points as:

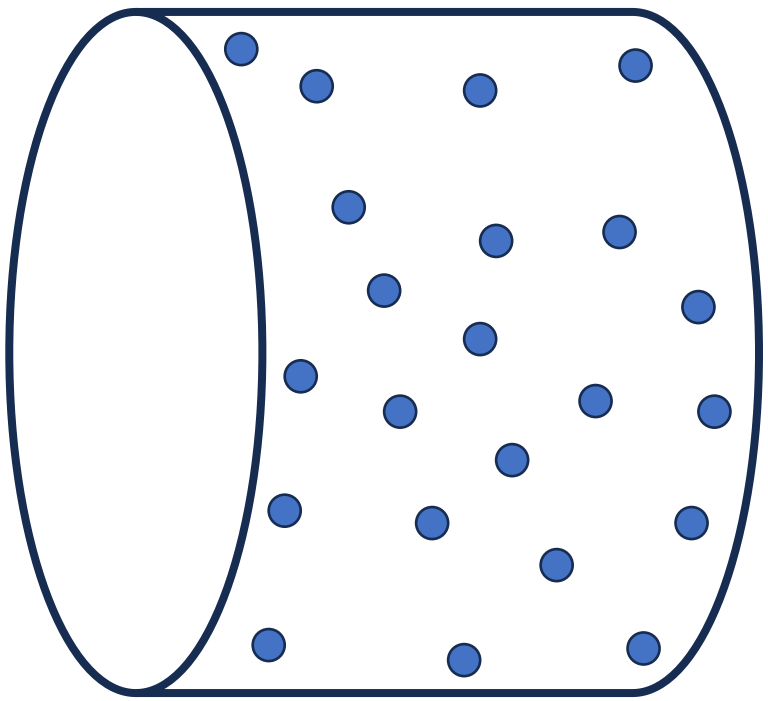
$$E_{AB} = \frac{V}{d}$$

- Assuming that the field is constant throughout the distance



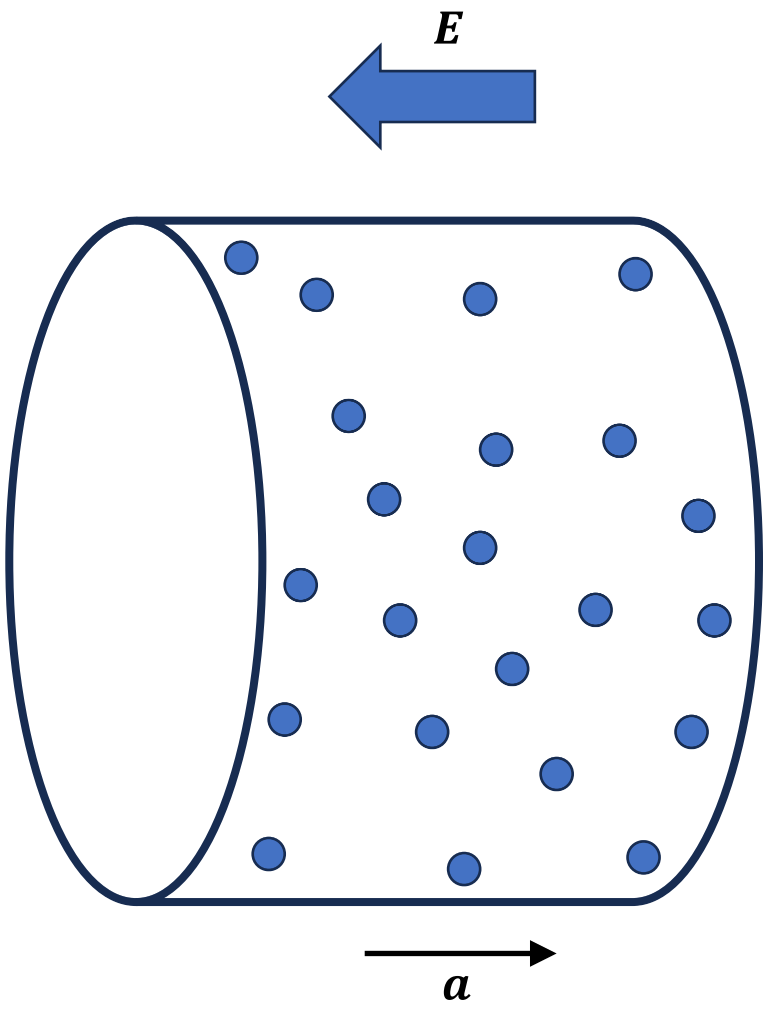
Inside a wire

- Now, lets jump inside a wire
- Metals typically have a large number of “free” electrons
- A wire made of a metal, say copper, will have these free electrons such that they can respond to a specific applied field and experience a force
- Force -> Acceleration -> Motion
- This motion is called “current”
- Let us start with a potential difference, and analyze the situation



EMF

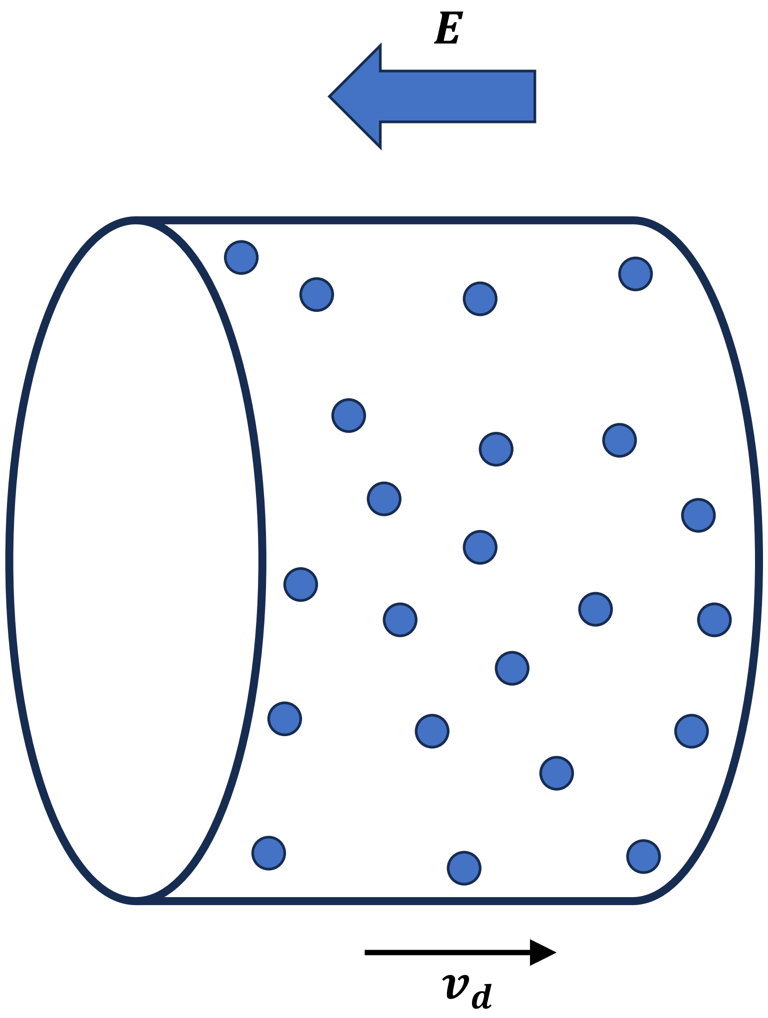
- Applying a potential difference to a wire refers to the application of a difference in potential energy at two points in a wire (say end points)
- This potential energy difference is experienced by electrons as a force (EMF)
- The field produced because of this potential causes the electrons to accelerate (in the opposite direction)
- Leading to a flow if electrons



Terminal velocity

- Electrons experience a "drag" because of the scattering they experience within the wire
- Scattering is caused by the atoms of the metal lattice
- Because of this, electrons reach a "terminal" velocity $\boldsymbol{v_d}$
- This is proportional to the applied field

$$\boldsymbol{v_d} = -\mu \boldsymbol{E}$$



Mobility

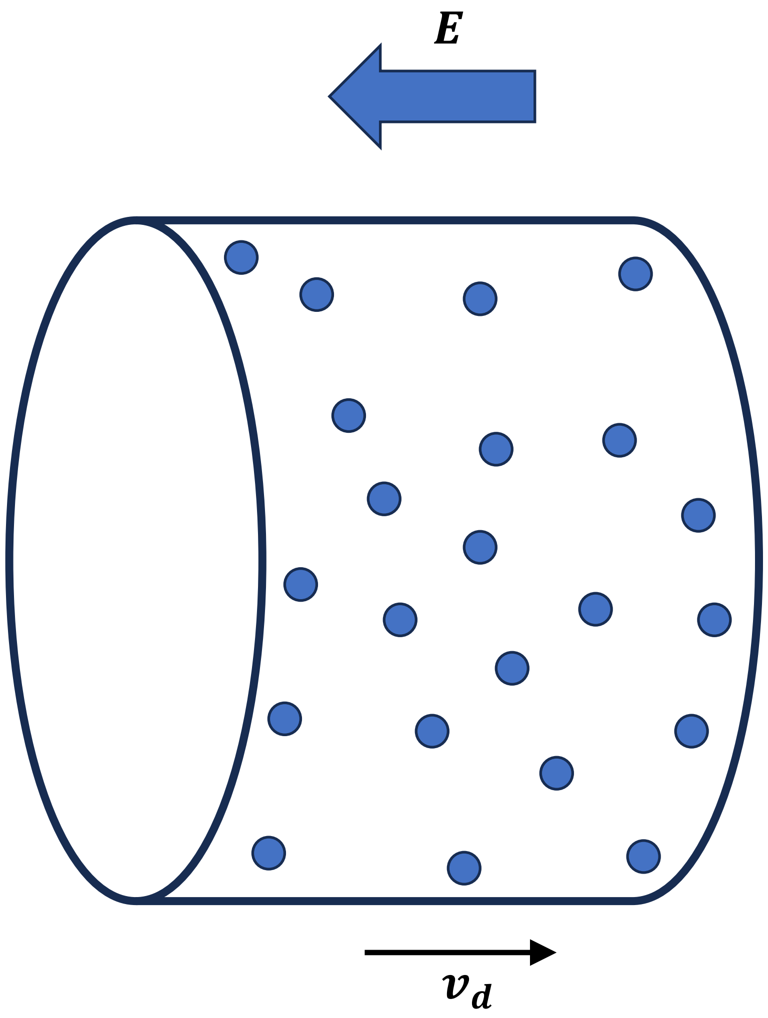
- Assume that average time between collisions is t
- The average velocity is:

$$v_d = \frac{at}{2}$$

$$v_d = \frac{eEt}{2m^*}$$

$$\mu = \frac{e\tau_c}{m^*}$$

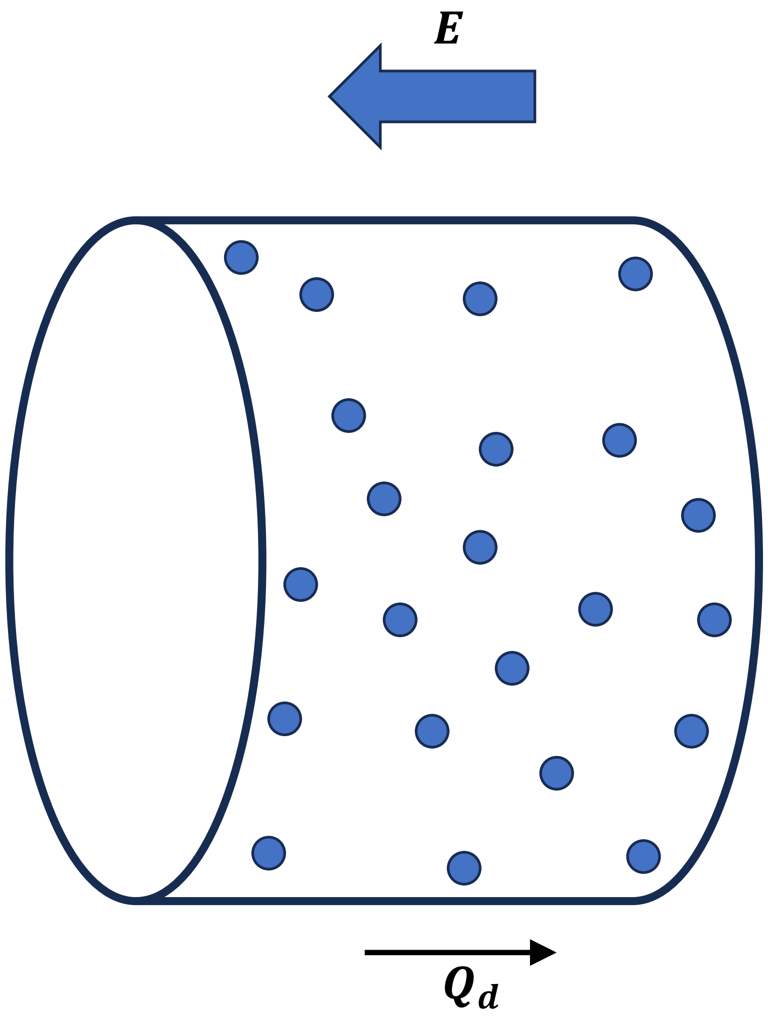
- τ_c is the mean free time



Current

- Flow of charge
  - Charge is conserved in total, but goes from one point to another in a circuit
- Hence, it is charge per unit time (typically across a specific cross-section)
- Units are Ampere (Coulomb/time)
- Consider a cross section of the wire, where electrons are flowing with a velocity $\boldsymbol{v_d}$, across a cross-section area A, the charge crossing in time dt is given by:

$$dQ_d = v_d dt \times A \times n \times e$$



Current

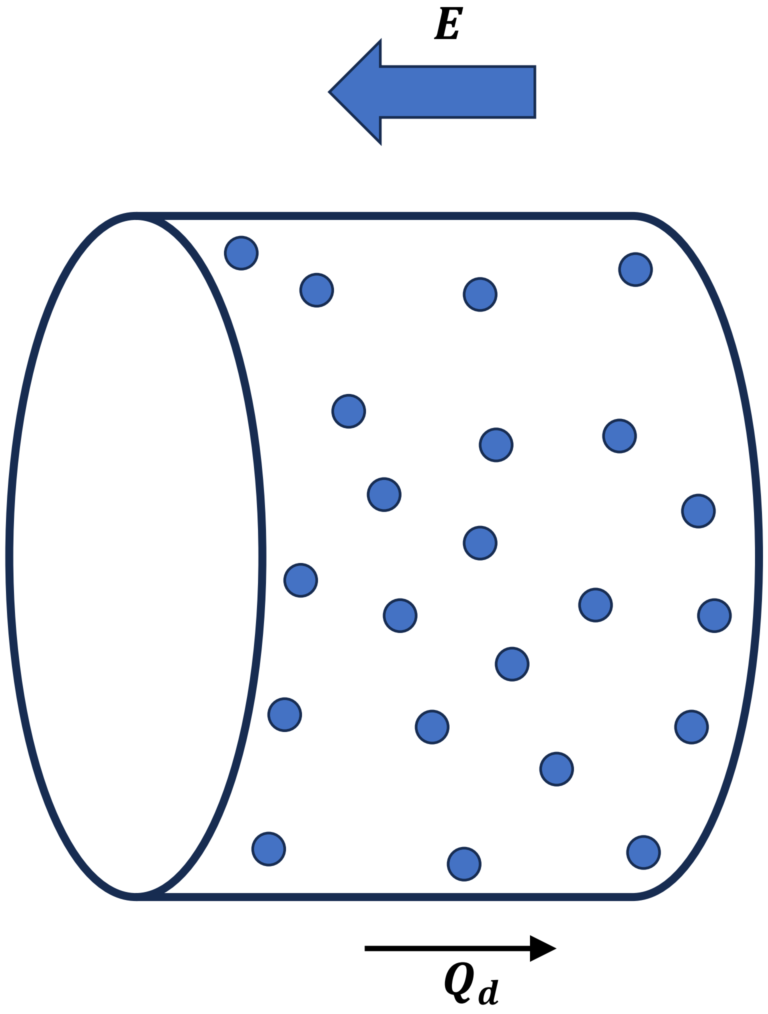
$$dQ_d = v_d dt \times A \times n \times e$$

$$\frac{dQ_d}{dt} = v_d Ane$$

$$I = v_d Ane$$

$$I = \mu E Ane$$

$$I = \mu \frac{V}{L} Ane$$



Current

$$I = \left(\frac{\mu A n e}{L}\right) V$$

- The Ohm's Law!
- The corner stone for every electronic circuit ever!
    - Not to mention, applicable to heat flow, fluid flow, magnetic circuits, etc.

$$R = \frac{L}{\mu A n e}$$

$$R = \frac{\rho L}{A}$$

$$\rho = \frac{1}{\mu n e}$$

George Ohm

1827, *The Galvanic Circuit Investigated Mathematically,* provides a model for current versus length of conductor

Summary

- Discussed charge, potential, and current
- Derived Ohm’s law for a resistor (using the Drude model)
- The modern quantum mechanical view can also be approximated to this model:
  - Using effective mass of free electrons in a particular energy band (Schrödinger)
  - The number of electrons available for conduction (Fermi-Dirac statistics)