DS Tutorial 1: Logic, Predicates, and Quantifiers

Truth Tables

Problems:
For each pair of propositions below, construct the necessary truth tables to determine if the propositions in the pair are logically equivalent.

(i) $p \to (\neg q \lor r)$ and $(p \to q) \lor (\neg p \to r)$

(ii) $p \to q$ and $\neg p \lor q$

Predicates and Quantifiers

Problems:

- Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny," and the domain consists of all people.
  (a) $\forall x\big(C(x) \to F(x)\big)$
  (b) $\exists x\big(C(x) \land F(x)\big)$

- Let $P(x)$ be the statement "$x = x^2$." If the domain consists of all integers, what are these truth values?
  (a) $P(2)$
  (b) $\exists x\, P(x)$
  (c) $\forall x\, P(x)$

Negation of Quantified Statements

Problems:
Express the negations of these propositions using quantifiers and in English.

- There is a student in this class who has never seen a computer.
- There is a student in this class who has been in at least one room of every building on campus.

Bonus Problem

"For every prime number p, there exists some other prime number q such that q is greater than p but less than $2p$." (Bertrand's Postulate)

Let the domain be the set of all integers. Define the following predicate:

- $P(x)$: "x is a prime number."

Tasks:

1. Translate the statement into a formal logical expression using quantifiers $(\forall, \exists)$ and the predicate $P(x)$.
2. Write the negation of your formal expression and simplify.
3. Translate your final negated expression back into a clear and natural English sentence.