Optimal Trading Stops and Algorithmic Trading

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Abstract

Trading stops are often used by traders to risk manage their positions. In this note, we show how to derive optimal trading stops for generic algorithmic trading strategies when the P&L of the position is modelled by a Markov modulated diffusion. Optimal stop levels are derived by maximising the expected discounted utility of the P&L. The approach is independent of the signal used to enter the position. We analyse in details the case of trading signals with a limited (random) life. We show how to calibrate the model to market data and present a series of numerical examples to illustrate the main features of the approach.

Keywords: Optimal trading stops, Algorithmic trading, Stop loss, Target profit, Utility functions, Markov chains

1 Introduction

Traders often remark that the risk management of existing position is as important, if not more important than having accurate entry signals for their trading strategies. Stop loss and target profit thresholds are commonly used by market practitioners to help manage the risk of their portfolios. Building upon the seminal work of Imkeller and Rogers [11], we try to give a plausible answer to the following questions: once a trade has been entered, when is the right time to exit it? What level of losses is reasonable to tolerate? When should a trader exit a profitable position? Are trading stops related to the strength of the signal of the underlying strategy? Do they depend on the risk aversion of individual traders? How can they be efficiently employed in an algorithmic trading context?

Assume that a position on an asset or portfolio of assets has been entered based on a given trading signal and that trading stops are placed with reference to the running P&L of the position. It is natural to expect that strategies or traders with a strong track record will be allowed run the position for a longer time, i.e. stops will be relatively far from zero. Likewise, traders with low risk aversion will tend to tolerate higher losses or let profit cumulate for longer before closing their position.

Rogers and Imkeller [11] propose to choose the stopping strategy which maximise the expected discounted utility of the P&L of a give position. They model the P&L as a arithmetic Brownian motion with constant (either known or unknown) drift and analyse a variety of stopping strategies, notably fixed stops and trailing stops. The authors above prove that, for reasonable model parameters, it is never optimal to place trading stops when the drift of the P&L is known. In order to overcome this difficulty, they suggest to model the P&L with a constant but unknown drift. In other words, when entering the position, the trader does not know for sure whether the strategy is a good or a bad one (positive or negative drift respectively) but estimates the drift in a Bayesian fashion.

In this paper, we adopt the utility approach of [11] but allow the P&L to have a stochastic drift and volatility. As as example, imagine to enter a position based on a given signal with a positive initial drift. However as time (e.g. seconds) goes by, other market participants spot the same trading opportunity, enter the same position and move the market. It is conceivable after enough time has elapsed the P&L of the position will dominated by noise rather than the initial signal. It makes sense in these circumstances to exit the trade before the pure noisy state takes over. The example above, as well as more complex ones can be modelled by letting the P&L be a Markov modulated diffusion of the type

$$dX_t = \mu(y_t)dt + \sigma(y_t)dW_t \tag{1.1}$$

where y_t is a continuous time Markov chain. In the previous example, y_t would have two possible values 1 and 2 with $\mu(1) > 0$ and $\mu(2) = 0$ and 2 would be an absorbing state (no way out).

One the advantages of this approach when used together with relatively high frequency strategies is that it allows the strength as well as the average life of the signal to be calibrate to market data. Of course, more general behaviours can be captured by the P&L dynamics in (1.1), e.g. a decreasing drift which may go negative, random initial drift, etc, by increasing the number of chain states. However, calibration becomes more complex due to the higher number of parameters in those cases.

The paper is organised as follows: in section 2 we briefly review the the approach of Imkeller and Rogers [11]. In section 3 we solve the optimal stops problem when the P&L is modelled by a Markov Modulated diffusion. Section 4 shows how to calibrate the model parameters in closed form in a simplified yet relevant specification of the model. Section 5 deals with the calibration of the more general n-state chain model. Finally section **??** illustrates the main features of the approach via a series of numerical examples.

2 Optimal Stops with Constant P&L Drift

We being by briefly summarising the basic idea of the approach introduced by [11]. The authors above start by modelling the P&L of a position as a Brownian motion with constant drift

$$X_t = \sigma W_t + \mu t \tag{2.1}$$

and assume that the cost of exiting the position at the random (stopping) time T is equal to c. They consider several stopping strategies, including the simple rule

$$T \equiv \inf\{t : X_t = -a \quad or \quad X_t = b\} \tag{2.2}$$

One way to formalise the trading stop problem is to consider an agent wishing to maximise the expected utility of a position's P&L, i.e.

$$\phi(a, b) = E[e^{-\rho T} U(X_T - c)] \tag{2.3}$$

for some increasing and concave utility function $U(x)$.

Consider the expected utility of the P&L as a function of the initial value of X_0

$$f(x) \equiv E[e^{-\rho T} U(X_T - c) \mid X_0 = x]. \tag{2.4}$$

A simple application of Ito's formula shows that $f(x)$ must satisfy the following ODE

$$\frac{1}{2}\sigma^2 f^{''}(x) + \mu f^{'}(x) - \rho f(x) = 0 \tag{2.5}$$

It is thus sufficient to solve equation (2.5) with boundary conditions

$$\begin{aligned} f(b) &= U(b-c) && (2.6)\\ f(a) &= U(-a-c) && (2.7) \end{aligned}$$

to obtain a solution for the expected utility of the P&L by setting $\phi(a,b) = f(0;a,b)$. The optimal stop loss and target profit thresholds can then be easily obtained by maximising $\phi(a,b)$ as a function of the parameters a and b.

For some specific choice of utility function, e.g. CARA utility,

$$U(x) = 1 - \exp(-\gamma x), \tag{2.8}$$

ODE (2.5) can be solved in closed form and one can obtain explicit solution for the objective function $\phi(a,b)$

$$\begin{aligned} \phi(a,b) &= E[e^{-\rho T}] - e^{-\gamma c} E[e^{-\rho T - \gamma X_T}] && (2.9)\\ &= L(\rho, 0) - e^{\gamma c} L(\rho, \gamma) && (2.10) \end{aligned}$$

where

$$L(\rho,\gamma) = E[e^{-\rho T - \gamma X_T}] \tag{2.11}$$

Solving the ODE, we obtain

$$L(\rho,\gamma) = \frac{e^{\gamma a}(e^{\beta b} - e^{\alpha b}) + e^{-\gamma b}(e^{\alpha a} - e^{-\beta a})}{e^{\alpha a + \beta b} - e^{-\alpha b - \beta a}} \tag{2.12}$$

where α and β are equal to

$$\alpha = -\frac{\mu}{\sigma^2} + \frac{1}{\sigma}\sqrt{\frac{\mu^2}{\sigma^2} - 2\rho} \tag{2.13}$$

and

$$\beta = -\frac{\mu}{\sigma^2} - \frac{1}{\sigma}\sqrt{\frac{\mu^2}{\sigma^2} - 2\rho}. \tag{2.14}$$

Note however that in the simple model we are considering, it is never optimal to place stops for a reasonable choice of paramters. Assume that the drift is deterministic and positive; if the P&L at time t is negative a trader will wait for the positive drift to prevail over the gaussian noise bringing X back into positive territory. If the $P\&L$ is positive, the trader will expect to achieve an ever greater gain by keeping the position. On the other hand, if the drift is deterministic and negative, the trader will not enter the position in the first place.

In order to overcome this contradiction (in real markets, trader make common use of trading stops), Imkeller and Rogers [11] suggest to let μ be a random variable with known distribution. In order to find the optimal a and b, one has to solve a number of ODEs of the form of (2.5) for different levels of μ (both positive and negative). The expected utility of the P&L will be then a weighted average of the solutions

$$\phi(a,b) = \int E^{(\mu)}[e^{-\rho T}U(X_T - c)]\psi(\mu)d\mu. \tag{2.15}$$

By introducing uncertainty about the drift, non trivial solution to the optimisation exist and placing finite stops can improve a trader's expected utility.

3 Optimal Stops with Stochastic Drift

In this section we propose a generalisation of the approach of [11] by considering a P&L process with stochastic drift and volatility. It is in fact reasonable to expect that the drift of a strategy as well as its volatility may change over time. For example, the drift may be high and positive when the trade is entered and weaken over time (or even become negative) as other market participants spot the same opportunity or new information and other exogenous factors start affecting the price of the asset(s).

In order to capture such a behaviour one can model the P&L as a Markov-modulated diffusion

$$dX_t = \mu(y_t)dt + \sigma(y_t)dW_t, \tag{3.1}$$

where y_t is a continuous time Markov chain, independent from W_t, with infinitesimal generator Q.

The example above, could be modelled by choosing $y_t \in \{1,2\}$, $\mu(1) = \bar{\mu} > 0$ and $\mu(2) = 0$. In this set up, the P&L has an initial positive drift which dies out at the random time when the chain changes state. If $y_t = 2$ is an absorbing state, then the trader is left with pure noise after a certain (random) time elapses. It is then conceivable that the trader may want to exit his position after the positive effect of his good signal has elapsed.

The optimisation problem is similar to the one summarised in the previous section

$$\phi(a,b) = E[e^{-\rho T} U(X_T - c) \mid X_0 = 0], \tag{3.2}$$

where X_t is now the Markov modulated diffusion with dynamics (3.1).

In order to find a solution to expectation (3.2), consider a function $f(x,y) \in \mathcal{C}^{2,0}$. A simple application of Ito's formula and Dynkin's formula to the function $\tilde{f} \equiv e^{-\rho t} f(X_t, y_t)$ shows that

$$\begin{aligned} d(e^{-\rho t} f(X_t, y_t)) &= e^{-\rho t}(\mu(y_t) f_x(X_t, y_t) + \frac{1}{2}\sigma^2(y_t) f_{xx}(X_t, y_t) \\ &+ (Qf)(X_t, y_t) - \rho f(X_t, y_t))dt + dM_t^f, \end{aligned}$$

where M_t^f is a local Martingale. Since y_t can only take a finite number of values, with a slight abuse of notation we can think of $f(X_t)$ as a vector valued function with element i equal to $f_i(X_t) \equiv f(X_t, i)$. For $\tilde{f}$ to be a local Martingale we require that

$$\frac{1}{2}\Sigma f''(x) + M f'(x) + (Q - R) f(x) = 0. \tag{3.3}$$

Here Σ, M and R are diagonal matrices with ith diagonal entries equal to $\sigma(i)$, $\mu(i)$ and $\rho(i)$ respectively. Since $f(x)$ is bounded in the interval $[-a, b]$, it follows from the optional stopping theorem that

$$f_i(x) = E^{x,i}[e^{-\rho T} U(X_T - c)],$$

where $y_0 = i$ is the initial state of the chain. The boundary conditions are independent from the state of the chain and are equal to

$$\begin{cases} f_i(-a) = U(-a-c) & i \in \{1, \ldots, n\} \\ f_i(b) = U(b-c) & i \in \{1, \ldots, n\} \end{cases} \tag{3.4}$$

The system of ODEs above admits solutions of the form

$$f(x) = v e^{-\lambda x}, \tag{3.5}$$

where v is a vector of dimension n equal to the number of states of the chain and λ is a scalar. Substituting (3.5) into the system (3.3) and re-arranging we obtain

$$\lambda^2 v - 2\lambda\Sigma^{-1}Mv + 2\Sigma^{-1}(Q-R)v = 0. \tag{3.6}$$

The system above is a quadratic eigenvalue problem which can be reduced to a canonical eigenvalue problem by way of some standard transformation,

$$\begin{pmatrix} 2\Sigma^{-1}M & -2\Sigma^{-1}(Q-R) \\ & \\ I & 0 \end{pmatrix} \begin{pmatrix} h \\ v \end{pmatrix} = \lambda \begin{pmatrix} h \\ v \end{pmatrix}$$

where

$$h = \lambda v.$$

This is a standard eigenvalue problem which admits n solutions in the positive half plane and n in the negative half plane. The solution to our ODE system will thus take the form

$$f(x) = \sum_{i=1}^{2n} w_i v_i e^{-\lambda_i x}.$$

The $2n$ coefficients w_i can be derived by solving the system

$$\begin{cases} \sum_{i=1}^{2n} w_i v_i e^{-\lambda_i b} = \bar{U}(b-c) \\ \sum_{i=1}^{2n} w_i v_i e^{\lambda_i a} = \bar{U}(-a-c) \end{cases}$$

Here $\bar{U}(z)$ is an n dimensional vector with ith entry equal to $U(z)$.

It is thus possible to derive the function $f(x)$ for a utility function of choice and a n-states chain via a series of simple linear algebra operations. As before the expected utility function (3.2) is obtained by setting the initial level of the P&L to zero in the function $f(x)$.

Remark 3.1. *In practice it is not realistic to expect that the whole position will be unwound at once. It is more common to observe a gradual unwind of the assets. It is possible to explain such a behaviour in the framework of the paper by imagining that the position is entered by a series of agents, each characterised by a different level of risk aversion. Each agent will be allocated an arbitrary portion of the total capital. In general each agent have different optimal stops. The more risk averse agents will be the first to unwind their position in case of loss or gain, while the more risk neutral agent will keep their position, everything else being equal, for longer.*

4 Calibration

Markov modulated models allow a large degree of flexibility in modelling the dynamics of the P&L. Model users have the ability to set the number of chain states (which in principle can be very large), the vector of drifts μ and volatilities σ corresponding to each state and the infinitesimal generator Q. In particular, Q governs the distribution of the Markov chain and the probabilities of jumping from any state i to another state j in any given time frame. The model flexibility of the Markov chain approach however has to be used with caution as may lead to over-fitting and instability problems at the calibration stage.

We consider first a very simple special case of the Markov modulated model introduced in the previous section. In particular we assume that there exist a strategy (long, short, long/short) which has a positive drift when an entry signal is detected. However the life of the signal is limited and after a while the drift goes to zero and the P&L of the position is governed by pure noise. In other words, we assume that $n = 2$ (two states chain) and whenever a trade is entered, the Markov chain is in state $y_0 = 1$ with $\mu(1) > 0$. The chain jumps to state $y_t = 2$ at some future time t with intensity q. State 2 is absorbing, i.e. the probability of jumping from state 2 back to state 1 is zero. Volatility is assumed to be constant. More precisely, we need to set

$$\mu = \begin{pmatrix} \mu_1 \\ 0 \end{pmatrix}$$

,

$$\sigma = \begin{pmatrix} \bar{\sigma} \\ \bar{\sigma} \end{pmatrix}$$

,

and

$$Q = \begin{pmatrix} -q & q \\ 0 & 0 \end{pmatrix}.$$

The calibration routine analysed below is better suited for strategies where intra day (e.g. tick, second, minute) data are available, entry signals are detected relatively often and a large history of data is available. Note that in this paper we are not concerned with finding the best entry strategy, which we assume given, but how to best deal with trading stops for generic entry strategies.

The data used for the calibration are collected following the steps below:

1. Run entry strategy on single underlying asset or basket.
2. Once an entry signal is detected, start recording the P&L and keep recording it for a fixed (but sufficiently) long period of time
3. Go back to step 1.

In the simplified model under consideration, there are only three parameters to calibrate: μ_1, $\bar{\sigma}$ and q. Volatility σ can be estimated using standard techniques (e.g. standard deviation of returns). Parameters μ_1 and q can be calibrated in closed form, i.e. without resorting to any minimisation routine, by calculation the integral transform

$$A_1 \equiv \int_0^\infty E[X_t]\lambda e^{-\lambda t} dt, \tag{4.1}$$

$$A_2 \equiv \int_0^\infty tE[X_t]e^{-\lambda t} dt, \tag{4.2}$$

both empirically as well as analytically. Here λ is an arbitrary decay parameter which depends on the length of the available P& L time series. In order to calculate A_1 and A_2 analytically, we need an explicit expression for $E[X_t]$. Note that the expected P&L can be easily calculated using the fact that the jump times of the chain are exponentially distributed. In particular,

$$\begin{aligned} E[X_t] &= \int_0^t E[\mu(y_s)]ds = \int_0^t \mu_1 P(T > s)ds \\ &= \int_0^t \mu_1 e^{-qs} ds = \frac{\mu_1}{q}(1 - e^{-qt}). \end{aligned}$$

Substituting the result above into (4.1) and (4.2), we obtain A_1 and A_2 as a function of μ_1 and q

$$A_1 = \frac{\mu_1}{q + \lambda}, \tag{4.3}$$

$$A_2 = \frac{\mu_1}{\lambda^2} \frac{q + 2\lambda}{q + \lambda}. \tag{4.4}$$

The next step is to estimate A_1 and A_2 empirically, which can be done from the backtested sample path

$$
\begin{aligned}
A_1 &\approx \int_0^{\bar{t}} \frac{1}{n} \sum_{j=1}^{n} \tilde{X}_t^j \lambda e^{-\lambda t} \\
A_2 &\approx \int_0^{\bar{t}} \frac{1}{n} \sum_{j=1}^{n} \tilde{X}_t^j t e^{-\lambda t}
\end{aligned}
$$

where $\bar{t}$ is the cut off time for each individual backtesting. Using equations (4.3) and (4.4) in conjunction with the estimates above and solving for mu_1 and q, we obtain

$$
q = \frac{\lambda(2A_1 - A_2\lambda^2)}{A_2\lambda^2 - A_1} \tag{4.5}
$$

$$
\mu_1 = \frac{\lambda A_1^2}{A_2\lambda^2 - A_1} \tag{4.6}
$$

We tested the accuracy of the calibration algorithm on simulated data and its dependence on the level of the calibrated parameter. In particular, we consider high and low levels of volatility, jump intensity and drift. The reference time horizon for the variable under consideration is one day, e.g. $\sigma = 0.025$ corresponds to a daily volatility of 2.5%. A jump intensity parameter q corresponds to an expected time in state $1/q$. For example $q = 2$ corresponds to an average life in state 1 equal to half a day.

The first three columns represent the true parameters of the simulation, whereas the last three column are the calibrated parameters. Note that the results are fairly accurate even for high levels of q, i.e. for very short lived signals. Also a relatively high level of noise does not materially worsen the calibration output.

Table 1: Calibration Test Results

q	μ_1	σ	$\tilde{q}$	$\tilde{\mu}_1$	$\tilde{\sigma}$
2	0.05	0.015	2.0841	0.0502	0.0152
2	0.05	0.05	1.9526	0.0505	0.0505
10	0.025	0.015	9.625	0.0261	0.0150
10	0.025	0.05	11.87	0.0260	0.0493

5 Calibration: n-States Model

The calibration problem can be generalised to accommodate more complex modelling requirements. As mentioned in the previous section, it is advisable to introduce some structure in the input parameters and in particular in the infinitesimal generator of the chain to avoid over-fitting problems. Consider a n-states Markov chain with generator Q and initial state distribution π. One possible approach is to minimise the integral of the square distance between the theoretical and empirical expected P&L,

$$
\begin{aligned}
F &\equiv \int_0^{\bar{t}} \left(E[X_t] - \hat{X}_t \right)^2 dt \\
&\approx \sum_{j=1}^{m(\bar{t})} \left(E[X_{t_j}] - \hat{X}_{t_j} \right)^2 \Delta_j,
\end{aligned}
$$

where

$$
\hat{X}_t \equiv \frac{1}{N} \sum_{j=1}^{n} \tilde{X}_t^j
$$

and N is the number of backtesting. The expected P&L conditional of the initial state of the chain, can be calculated analytically

$$
\begin{aligned}
E[X_t \mid y_0 = i] &= E^i \left[\int_0^t \mu(y_s) ds \right] \\
&= \int_0^t (e^{Qs}\mu)_i ds \\
&= \left(Q^{-1} \left(e^{Q} t - I \right) \mu \right)_i .
\end{aligned}
$$

The unconditional expected P&L can be derived easily calculating the weighted sum of the conditional P&L with respect to the initial distribution of the chain π,

$$
E[X_t] = \sum_{i=1}^{n} \left(Q^{-1} \left(e^{Qt} - I \right) \mu \right)_i \pi_i.
$$

If no information on the initial state on the chain is available, then one can simply set $\pi = 1/n$.

Note that if the number of state of the chain is equal to two, the infinitesimal generator Q is not invertible. The expected P&L can be derived by substituting the Taylor expansion of the matrix Q exponential in (5.1) and integrating term by term, i.e.

$$\int_0^t (e^{Qs}\mu)_i ds = \sum_{j=0}^{\infty} \int_0^t \frac{((Qs)^j \mu)_i}{j!}.$$

6 Numerical Examples

In this section we shall analyse the dependency of the optimal stops on the choice of parameters. The graphs below shows the expected utility of P&L $\phi(a, b)$ for different levels of a (stop loss) and b (target profit). The values of a and b for which ϕ reaches its maximum are the optimal stopping levels. Both a and b are expressed in percentage terms. The axes go from 0 to 1 (i.e. 100%).

In figure 1, we consider a strategy characterised by a constant and positive drift. The cost of exiting the position is equal to 1%. As one would expect, $\phi(a, b)$ is increasing in both arguments. This implies that stopping is never optimal whenever μ is known and positive; the positive drift will eventually prevail on the random noise and the trader can increasing its utility by keeping the position.

In figure 2, the strategy starts off with a positive drift $\mu_1 = 0.15$ per unit of time, however the drift dies off with intensity $q = 0.5$, i.e. on average it take two units of time (e.g. two days, two hours, etc) for the chain to jump from state 1 to state 2. Once the chain is in state two, the strategy has completely lost its drift component and the P&L is driven by pure random noise. From the shape of the expected utility function it is clear that now a maximum exists. In figure 2, the signal of the strategy is quite resilient and as consequence a and b are relatively high. In figure 3 we consider a strategy with a higher jump intensity $q = 2$. The expected life of state one is now one-half time units. As a consequence, the optimal a and b are smaller in this case as the signal dies off faster and the strategy is overall less profitable .

Optimal stops are also dependent on the exit cost c. For highly illiquid securities for example, it may not be profitable to enter the trade in the first place. In figure 4 we consider the optimal expected utility surface when the exit cost $c = 15\%$. While such a high exit cost is not very common in functioning markets, it helps us highlight the features of the optimal solution. The maximum expected utility is reached for $a = b = 0$, i.e. it is optimal not to enter the trade.

In picture 5 we consider the dependency of the optimal strategy on the level of risk aversion of the agent. For a very low levels of γ (i.e. low risk aversion), the optimal a and b will be very high. For γ approaching zero, i.e. for risk neutral agents, it is not optimal to place stop losses or target profits.

Finally in table 6 we summarise the dependence of a and b with respect to μ_1 and q. Note first that the optimal target profit level b is greater than a in all cases considered. As σ increases, everything else being equal, b increases significantly, while a decreases moderately. A greater b is consequence of the fact that a stronger initial drift enhances the expected profits and thus the expected utility of the agent. A lower a is due to the fact that a significantly negative P&L when the drift initial drift is relatively strong is likely to be due to a chain jump to state two (the pure noise state) and hence losses should be cut. Note also how an increase in q will move a and b closer to zero. Again, this is to be expected as the chain is more likely to jump early on from state one (positive drift) to state two (pure noise).

Table 2: Optimal stops for different parameters

q	μ_1	σ	a	b
2	0.025	0.05	0.07	0.09
2	0.05	0.05	0.06	0.14
2	0.1	0.05	0.05	0.21
10	0.025	0.05	0.0325	0.04
10	0.2	0.05	0.0275	0.1

Constant drift - $\mu = 0.15$, $\sigma = 0.25$, $c = 0.01$**.**

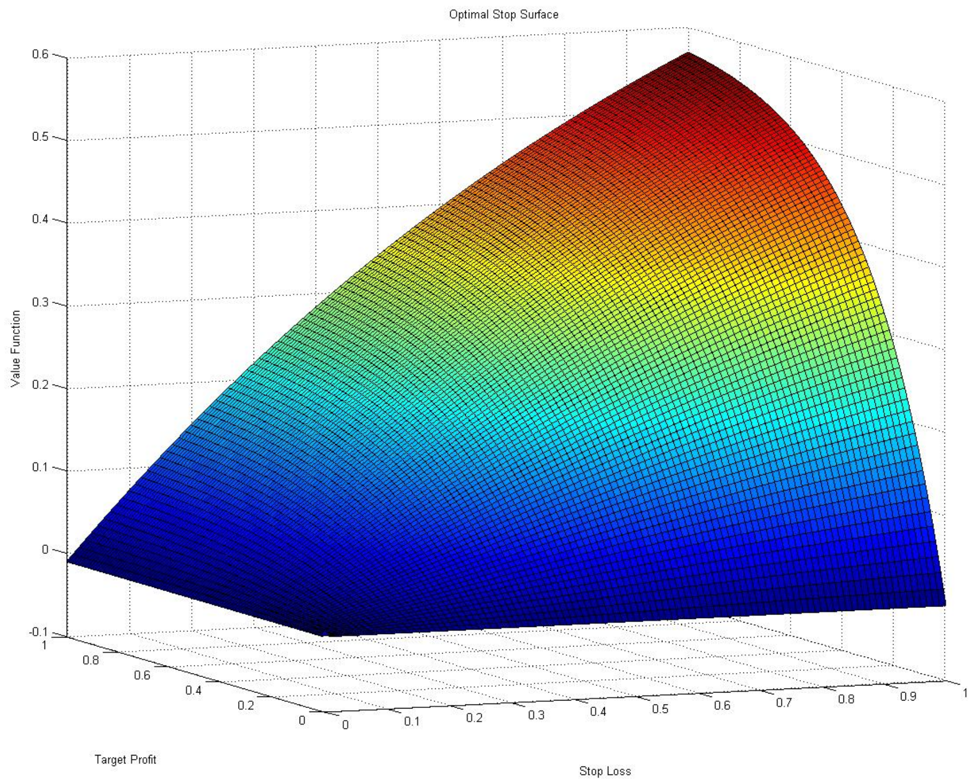

Figure 1: Optimal stop surface with constant and known μ. It is not optimal to stop when the drift is constant and positive. and positive.

Signal with slow decay - $\mu_1 = 0.15$**,** $\mu(2) = 0$**,** $q = 0.5$**,** $c = 0.01$**.**

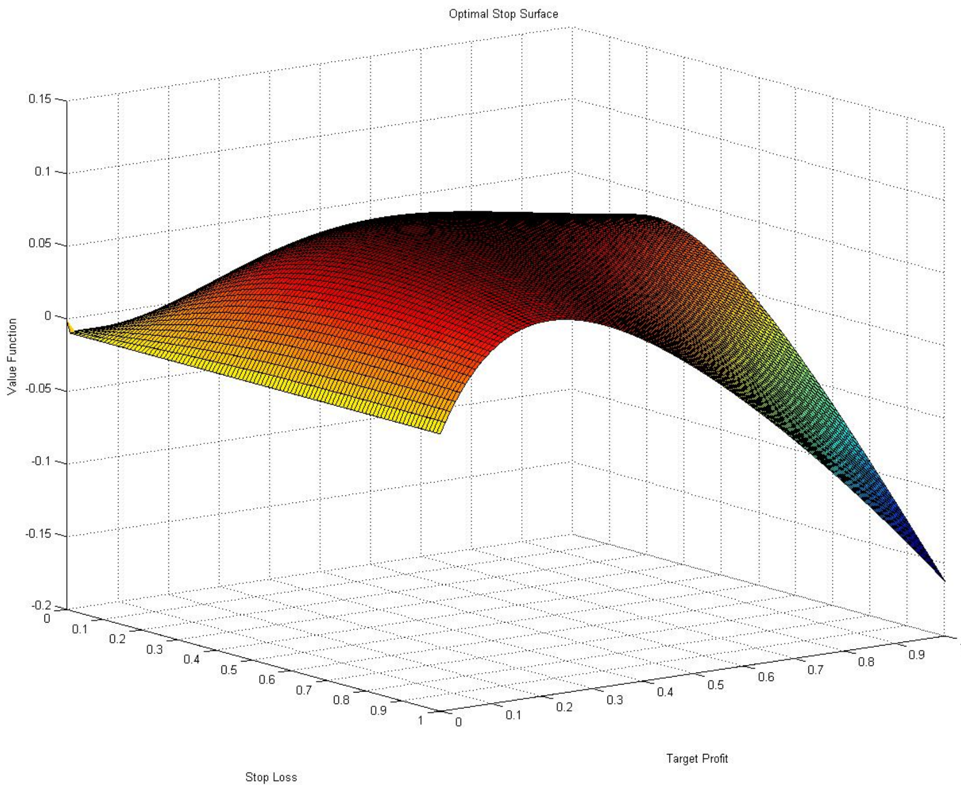

Figure 2: The P&L starts off with a drift of $\mu_1 = 0.15$ and takes on average two units of time to jump to the state with zero drift. Non trivial optimal stops exist in this case

Signal with fast decay - $\mu(1) = 0.15$**,** $\mu(2) = 0$**,** $\sigma = 0.25$**,** $q = 2$**,** $c = 0.01$

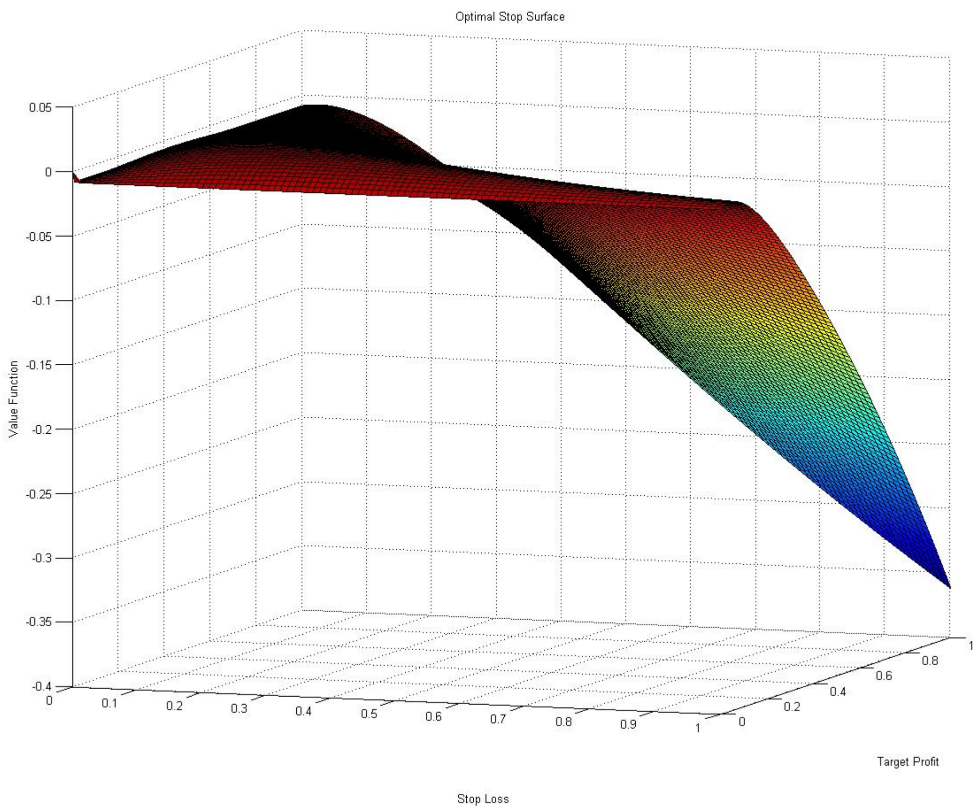

Figure 3: The P&L starts off with a drift of $\mu_1 = 0.15$ and takes on average one half unit of time to jump to the state with zero drift. The chain spends less time on average in the state with positive drift and thus the optimal stops are closer to zero.

Illiquid Security - $\mu(1) = 0.15$, $\mu(2) = 0$, $\sigma = 0.25$, $q = 0.5$, $c = 0.15$

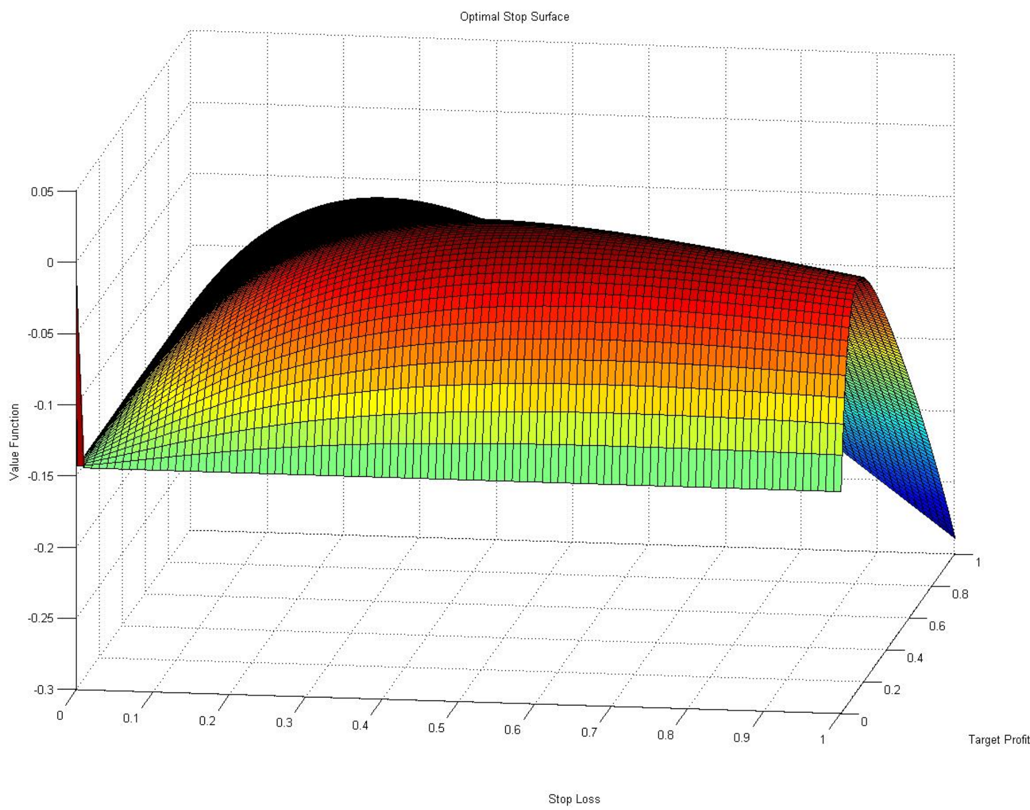

Figure 4: When transaction costs are relatively high, it may be optimal not to enter the position in the first place.

Low risk aversion - $\mu(1) = 0.15,\ \mu(2) = 0,\ \sigma = 0.25,\ q = 0.5,\ c = 0.15,\ \gamma = 0.05$

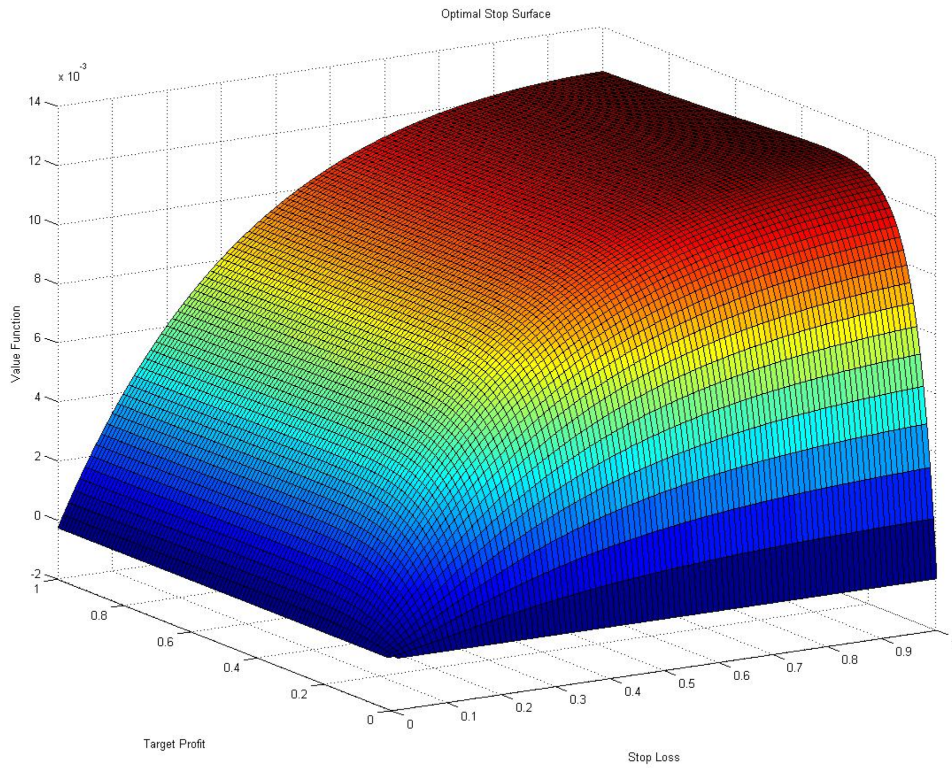

Figure 5: The existence of optimal stops is closely linked to the risk aversion of the agents. It is optimal for the risk neutral agent not to place any trading stop.

7 Conclusions and further research

We have shown how to derive optimal fixed stops when the P&L of a position is modelled by a continuous Markov modulated diffusion. Practitioners make wide use of other types of stops and notably trailing stops, i.e. a position is exited when the drop from the running maximum P& L reaches a pre-specified levels. Optimal trailing stops were derived by Imkeller and Rogers [**?**]for P&L processes with constant drift. Trailing stops for Markov modulated diffusions would be a natural extension of the work presented in this paper and will be the subject of a future research paper. For specific applications, for example spread trading, mean reverting dynamics for the P& L may be appropriate and may be worthwhile investigating.

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