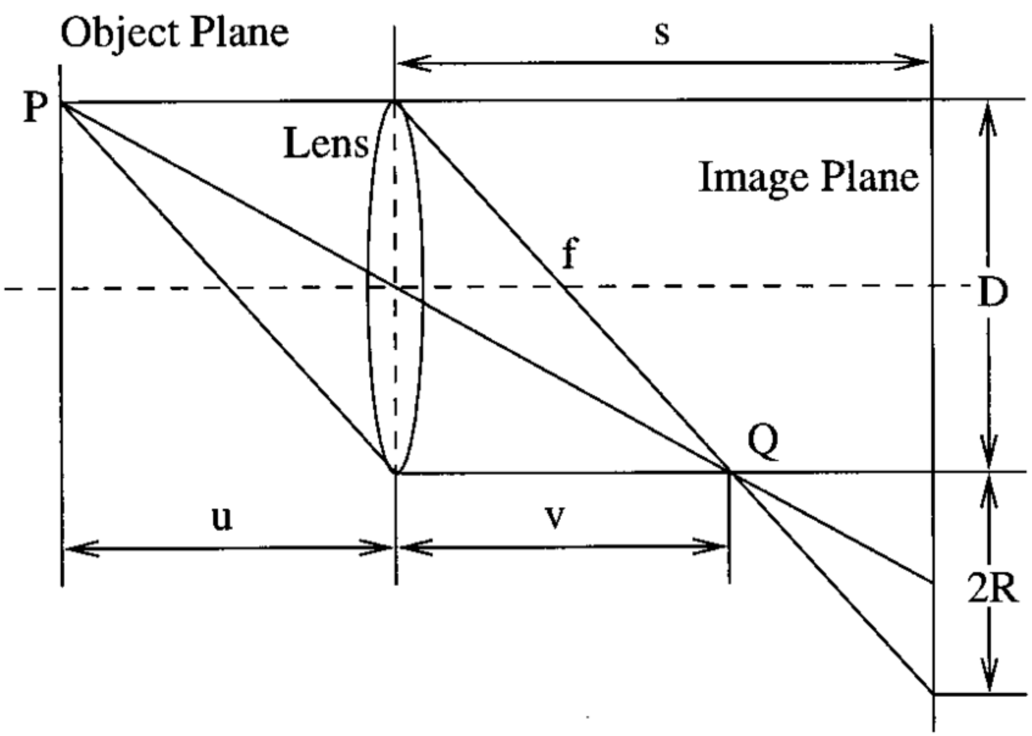
Lecture 12
Camera Calibration

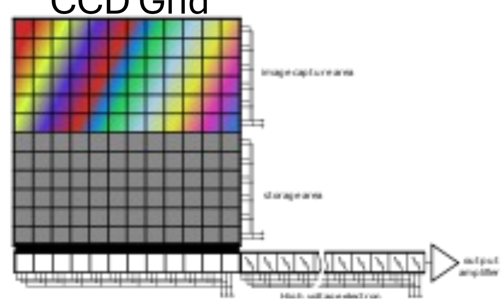
ECE 1390/2390

Syllabus

Week	Date	Topic	In class work	Due date
8	10/14/2024	FALL Break [No Class]		
	10/16/2024	Camera Calibration	Python problems (HW5)	Due 10/23
9	10/21/2024	Depth estimation	Group Project work	
	10/23/2024	3D Reconstruction and Pose estimation	Python problems (HW6)	Due 10/30
10	10/28/2024	Haar Cascade Classifiers	Python problems (HW7)	Due 11/4
	10/30/2024	HOG and Custom Detectors	Project updates	
11	11/4/2024	Object Tracking	Project updates	
	11/6/2024	OCR Text Detection	Python problems (HW8)	Due 11/13
12	11/11/2024	MediaPipe/SLAM	Group Project work	
	11/13/2024	OpenCV DNN	Python problems (HW9)	Due 11/20
13	11/18/2024	Super Resolution	Python problems (HW10)	Due 12/4
	11/20/2024	Image compression	Python problems	
14	11/25/2024	Thanksgiving recess		
	11/27/2024	Thanksgiving recess		
15	12/2/2024	Group Project work	Group Project work	
	12/4/2024	Group Project work	Group Project work	Final project commit
16	12/9/2024	Final Projects	Project presentations	Final group ratings

Camera Calibration





Ideal camera

$$K = \begin{bmatrix} f_x & 0 & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

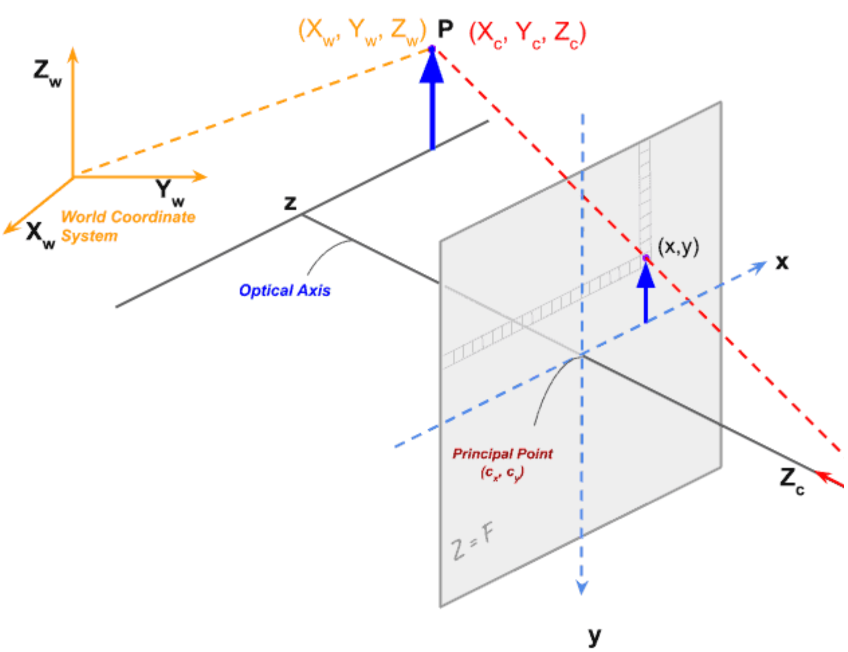
Real camera

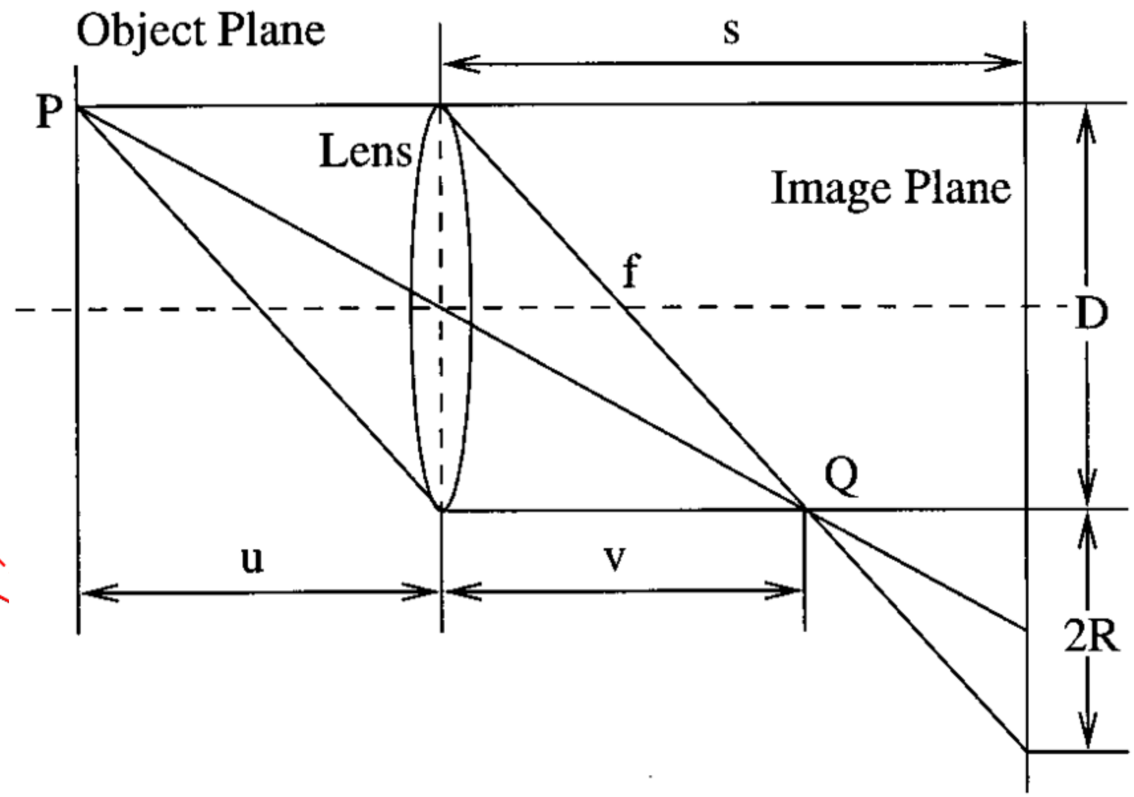
X/Y CCD grid skew

$$K = \begin{bmatrix} f_x & \gamma & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

Offset of CCD center from optical axis

Camera Calibration





Fourier Optics

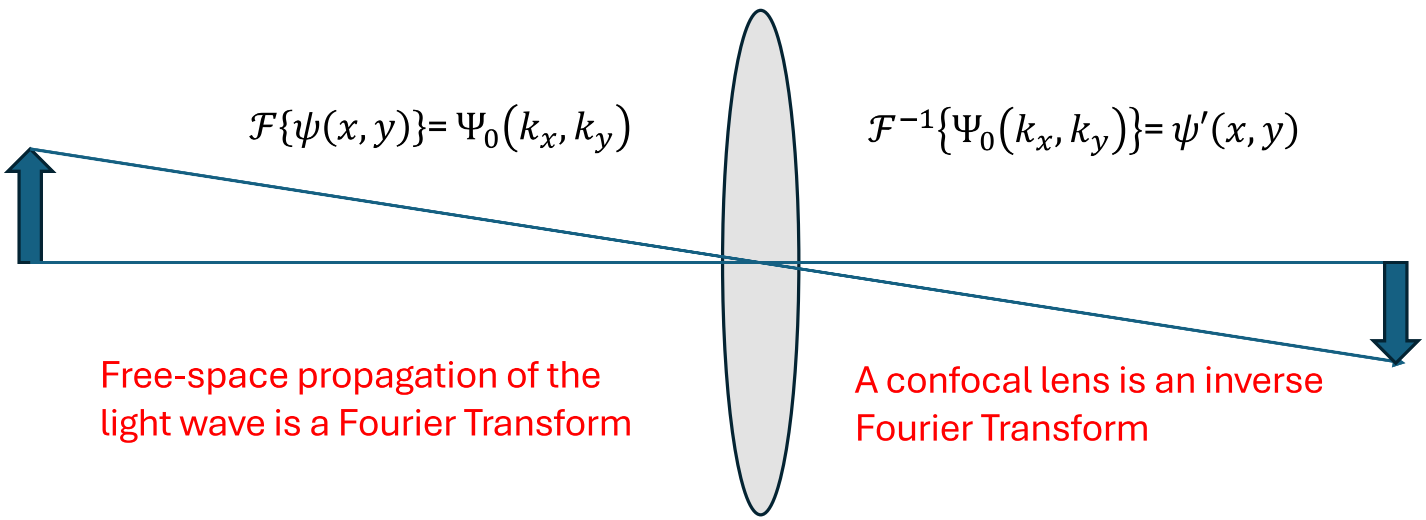
$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)u(r,t) = 0$$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial t^2}$$

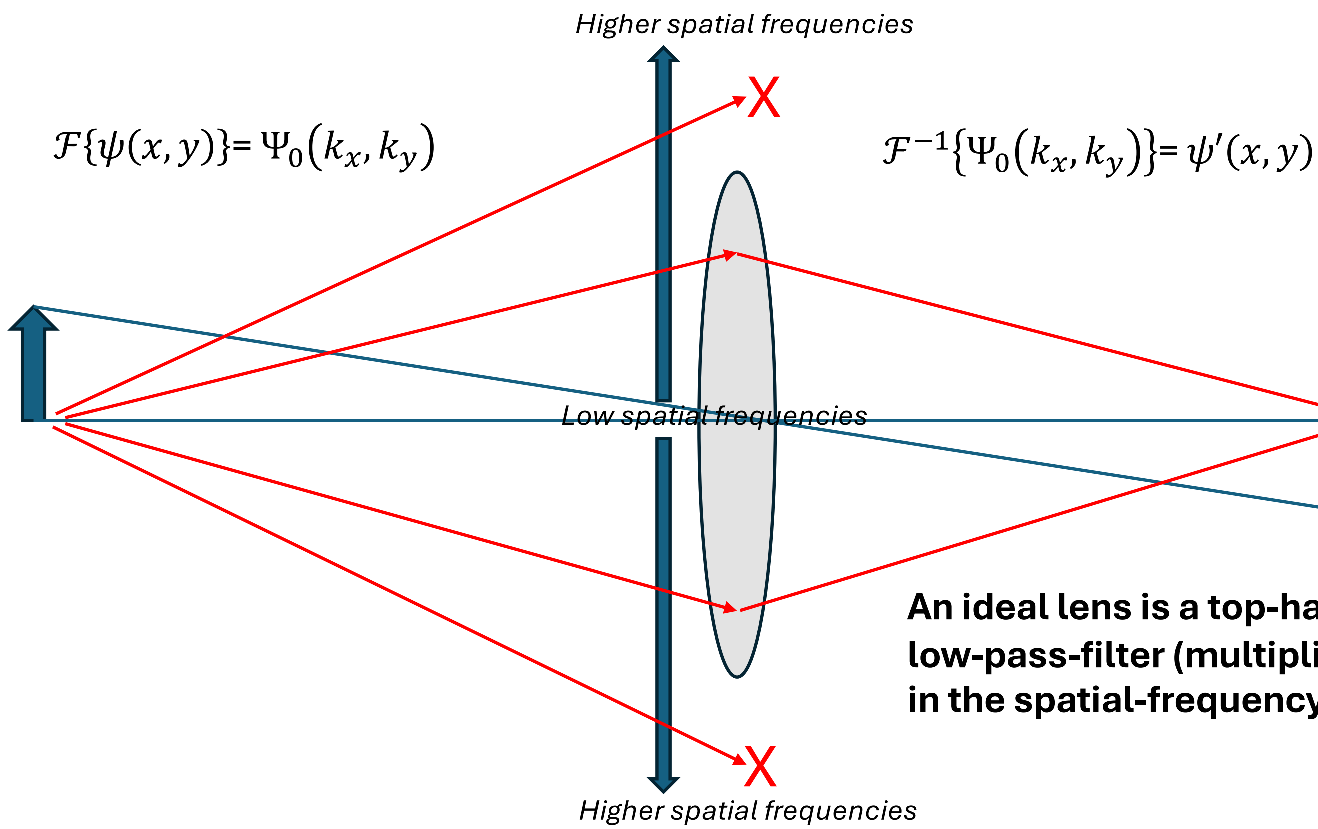
$$k_x = \frac{2\pi f_x}{c}$$

$$\psi(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk_x \cdot dk_y \cdot \Psi_0(k_x,k_y)e^{i\cdot(k_x x + k_y y)}$$

Solving for the spherical wave propagation at point (x,y)



Fourier Optics

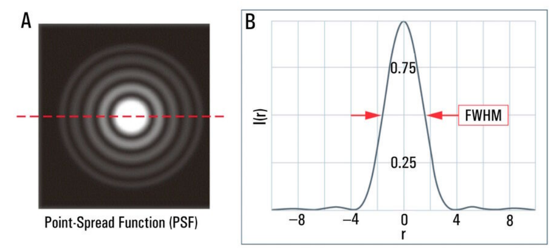


An ideal lens is a top-hat low-pass-filter (multiplication) in the spatial-frequency domain

Fourier Optics

$$\Delta x_{min} = \frac{\lambda}{2 \cdot NA}$$

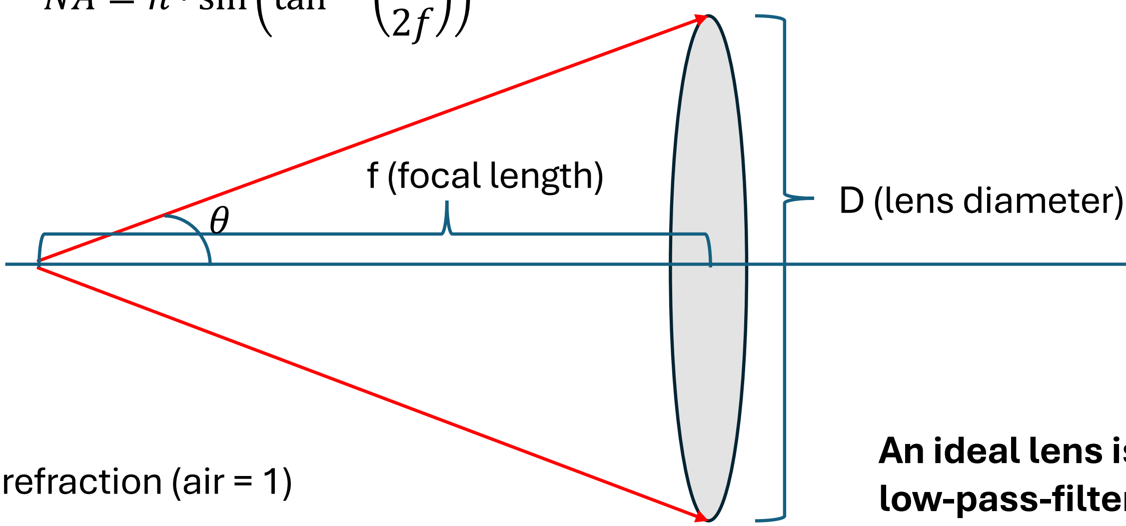
Diffraction resolution limit of a microscope



Numerical Aperture

$$NA = n \cdot \sin(\theta)$$

$$NA = n \cdot \sin\left(\tan^{-1}\left(\frac{D}{2f}\right)\right)$$



n = Index of refraction (air = 1)

An ideal lens is a top-hat low-pass-filter (multiplication) in the spatial-frequency domain

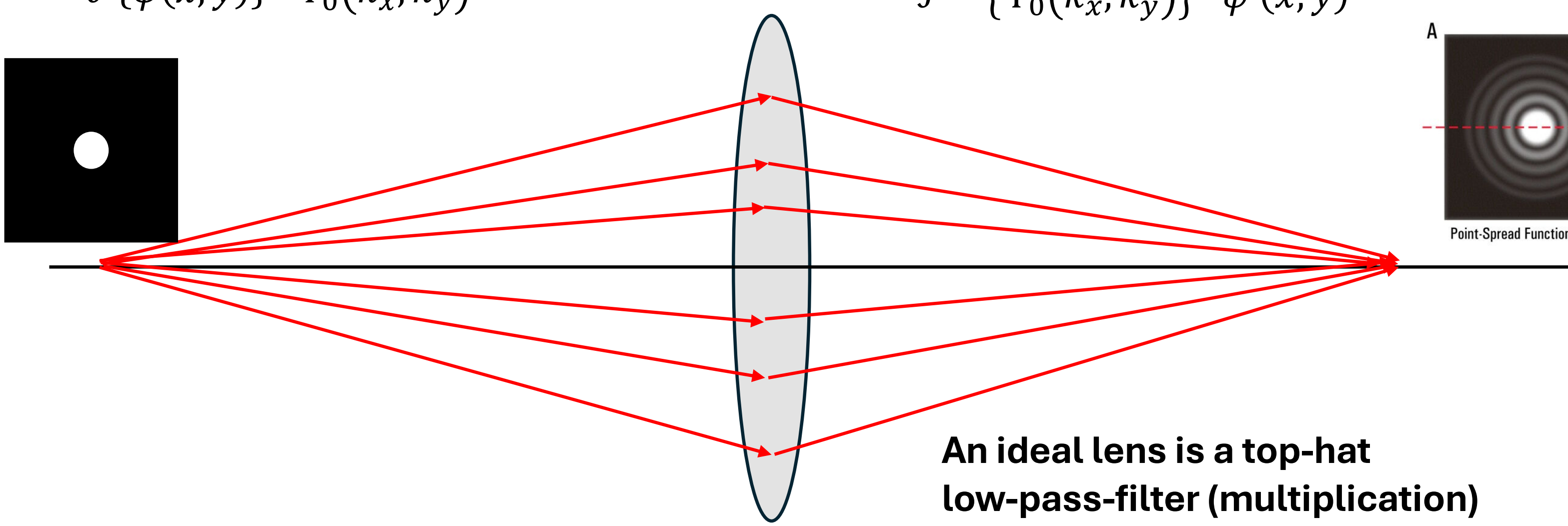
Fourier Optics

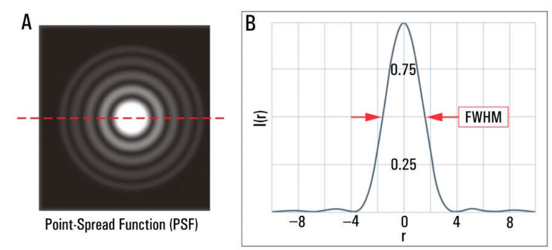
$$\mathrm{I}_{\mathrm{photo}}(u, v\,) = K * \mathcal{F}^{-1}\{\mathcal{F}\{I_{object}(x, y\,)\} \cdot l(k_x, ky, \lambda)\}$$

$$\mathrm{I}_{\mathrm{photo}}(u, v\,) = K * (I_{object}(x, y\,) \otimes L(x, y, \lambda))$$

$\mathcal{F}\{\psi(x, y)\} = \Psi_0\left(k_x, k_y\right)$

$\mathcal{F}^{-1}\{\Psi_0\left(k_x, k_y\right)\} = \psi'(x, y)$





An ideal lens is a top-hat low-pass-filter (multiplication) in the spatial-frequency domain

Fourier Optics

Spherical aberration

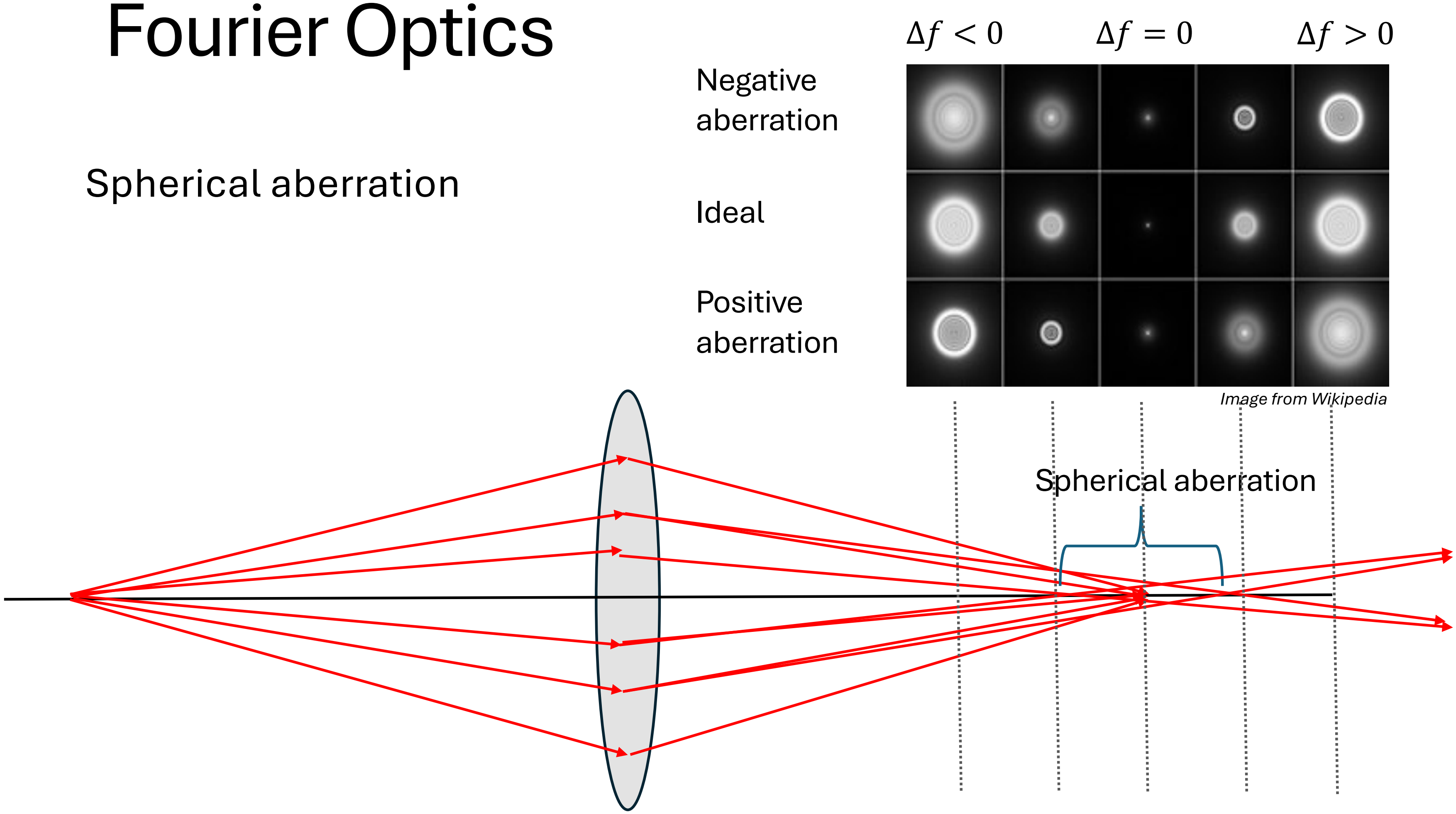
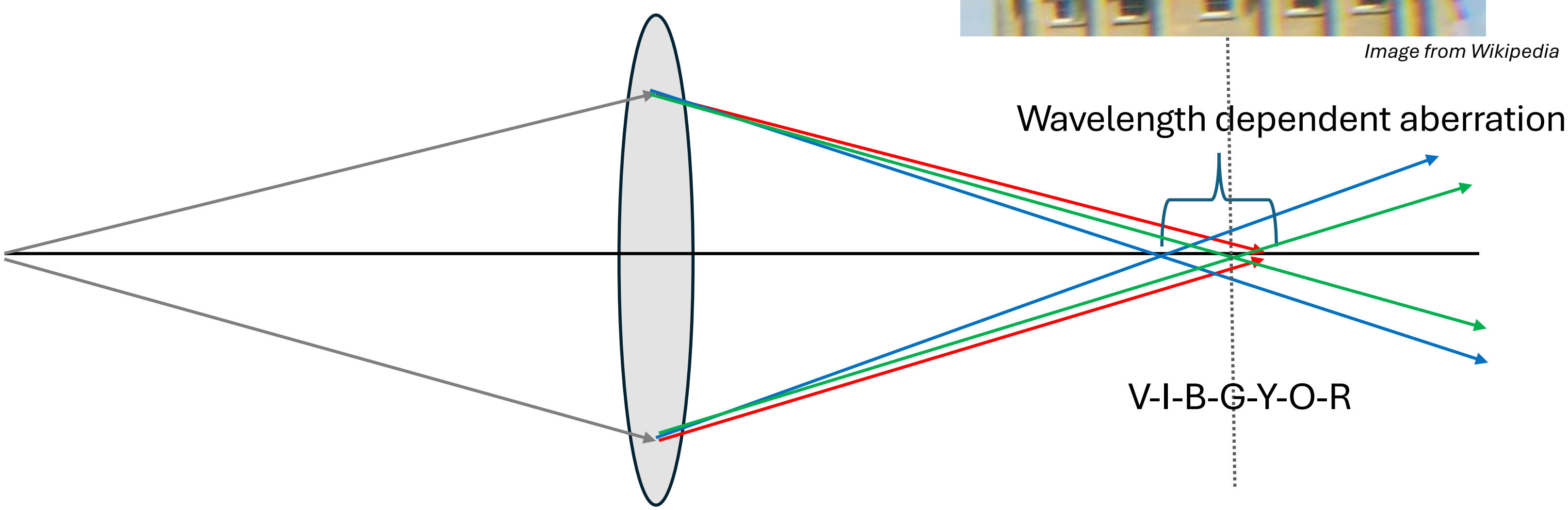


Image from Wikipedia

Fourier Optics

Chromatic aberration

Image from Wikipedia

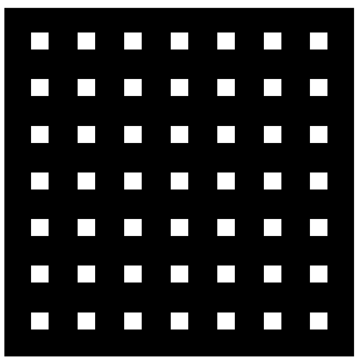


Deconvolution (Wiener Filter)

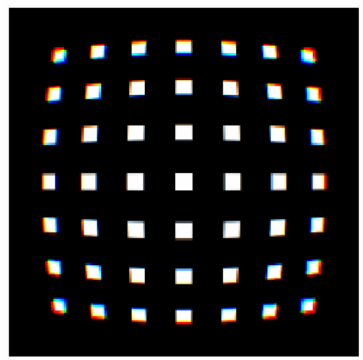
$$l(kx, ky, c) \ = \mathcal{F}\{\mathrm{I}_{\mathrm{Raw}}(u, v, c\ )\}/\mathcal{F}\{\mathrm{I}_{Target}(u, v, c\ )\}$$

$$I_{Corrected}(x, y, c\ ) = \mathcal{F}^{-1}\{\mathcal{F}\{\mathrm{I}_{\mathrm{Raw}}(u, v, c\ )\}/l(k_x, ky, c)\}$$

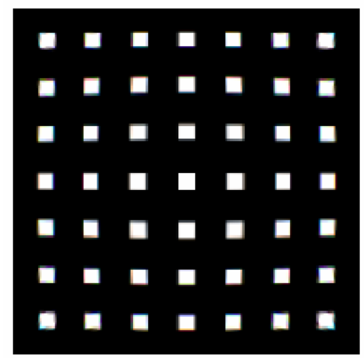
Calibration Target



Photograph Raw



Corrected



Imperfect

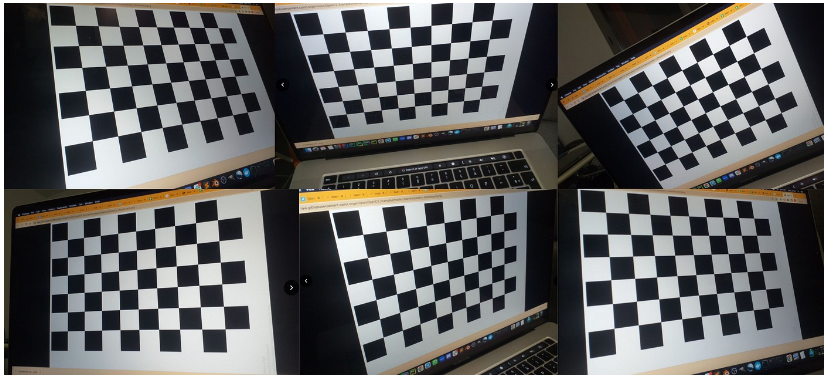
Ng, Yiu-Ming Harry. “Correcting the Chromatic Aberration in Barrel Distortion of Endoscopic Images.” (2003).

OpenCV Camera Calibration

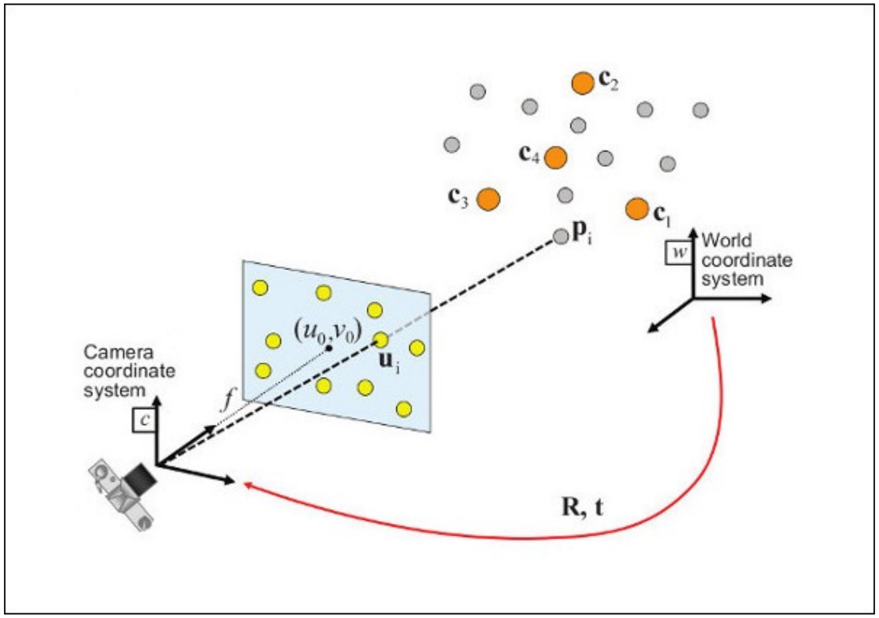
- Internal
  - Focal
  - Optical center
  - Radial Distortion

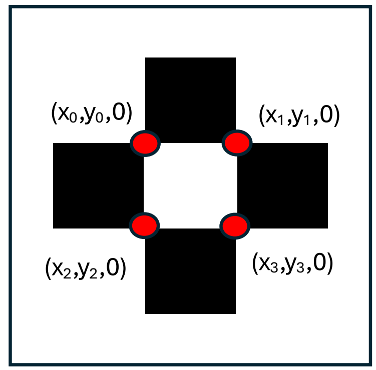
$$K = \begin{bmatrix} f_x & \gamma & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

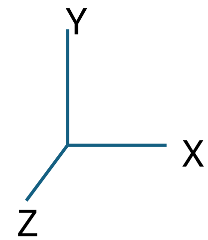
- External
  - Orientation of target relative to camera



Prospective-n-Point (PnP)







$\boldsymbol{\theta}_x$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix} A$$

$\boldsymbol{\theta}_y$

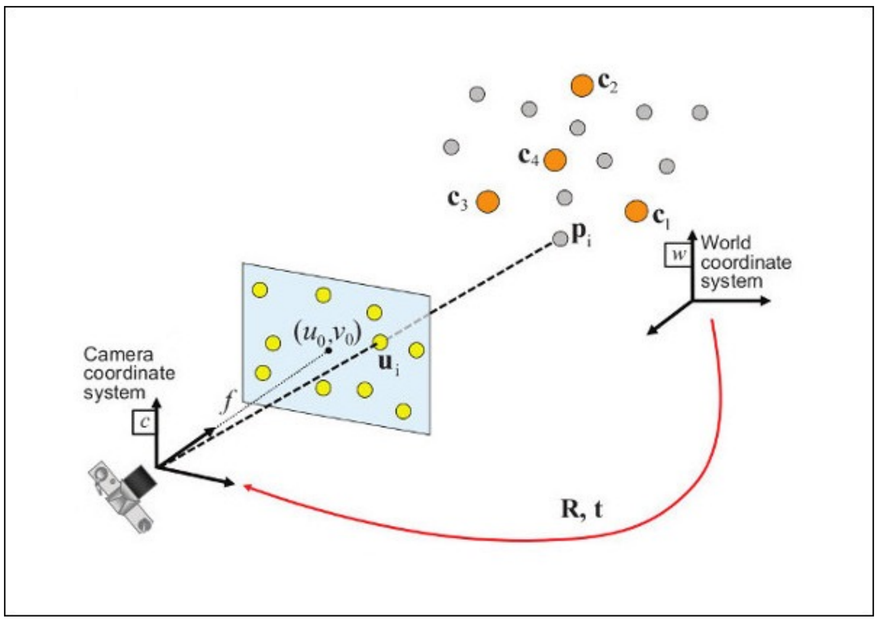
$$B = \begin{bmatrix} \cos\theta_Y & 0 & \sin\theta_Y \\ 0 & 1 & 0 \\ -\sin\theta_Y & 0 & \cos\theta_Y \end{bmatrix} A$$

$\boldsymbol{\theta}_z$

$$B = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 & 0 \\ \sin\theta_z & \cos\theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_Y & 0 & \sin\theta_Y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_Y & 0 & \cos\theta_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x & 0 \\ 0 & \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \Delta X \\ 0 & 1 & 0 & \Delta Y \\ 0 & 0 & 1 & \Delta Z \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} A_x \\ A_y \\ A_z \\ 1 \end{bmatrix}$$

Prospective-n-Point (PnP)



$$\begin{bmatrix} B_x \\ B_y \\ B_z \\ 1 \end{bmatrix} = \begin{bmatrix} r_{0,0} & r_{0,1} & r_{0,2} & \Delta X \\ r_{1,0} & r_{1,1} & r_{1,2} & \Delta Y \\ r_{2,0} & r_{2,1} & r_{2,2} & \Delta Z \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} A_x \\ A_y \\ A_z \\ 1 \end{bmatrix}$$

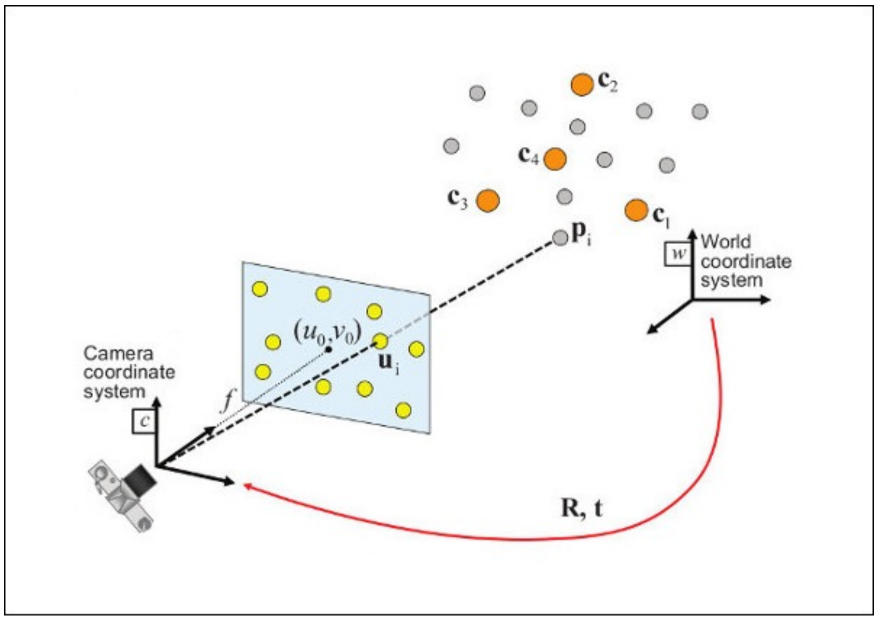
3D to 2D projection

$$\begin{bmatrix} u_{ideal} \\ v_{ideal} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \\ 1 \end{bmatrix}$$

Intrinsic Camera distortion

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & \gamma & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{ideal} \\ v_{ideal} \\ 1 \end{bmatrix}$$

Prospective-n-Point (PnP)



$$\begin{bmatrix} B_x \\ B_y \\ B_z \\ 1 \end{bmatrix} = \begin{bmatrix} r_{0,0} & r_{0,1} & r_{0,2} & \Delta X \\ r_{1,0} & r_{1,1} & r_{1,2} & \Delta Y \\ r_{2,0} & r_{2,1} & r_{2,2} & \Delta Z \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} A_x \\ A_y \\ A_z \\ 1 \end{bmatrix}$$

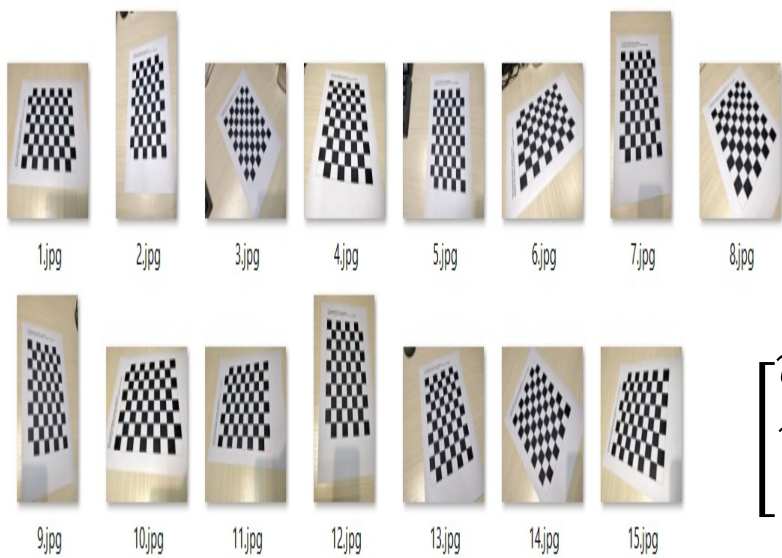
3D to 2D projection

$$\begin{bmatrix} u_{ideal} \\ v_{ideal} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \\ 1 \end{bmatrix}$$

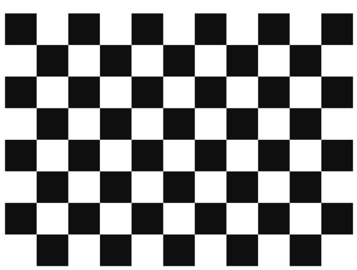
Intrinsic Camera distortion

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & \gamma & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{ideal} \\ v_{ideal} \\ 1 \end{bmatrix}$$

Prospective-n-Point (PnP)



$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} A_x \\ A_y \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} B_x \\ B_y \\ B_z \\ 1 \end{bmatrix} = \begin{bmatrix} r_{0,0} & r_{0,1} & r_{0,2} & \Delta X \\ r_{1,0} & r_{1,1} & r_{1,2} & \Delta Y \\ r_{2,0} & r_{2,1} & r_{2,2} & \Delta Z \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} A_x \\ A_y \\ A_z \\ 1 \end{bmatrix}$$

Differs per image

3D to 2D projection

$$\begin{bmatrix} u_{ideal} \\ v_{ideal} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \\ 1 \end{bmatrix}$$

Same across images

Intrinsic Camera distortion

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & \gamma & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{ideal} \\ v_{ideal} \\ 1 \end{bmatrix}$$

OpenCV Camera Calibration

```
img_points =[]
obj_points = []

for gray in images:
        Step 1.
                retval,corners = cv2.findChessBoardCorners(image=gray, patternSize=(10,8))

        (optional)
                corners = cv2.cornerSubPix(gray,corners,(11,11),(-1,-1),criteria)
        (optional)
                cv2.drawChessboardCorners
        Step 2.
                img_points.append(corners)
                obj_points .append(real_coors)

Step 3.
        ret, cameramtx, distcorr, rot, trans = cv2.calibrateCamera(obj_points,img_points,gray.shape[::-1],None,None)
```

Finds corners of checkerboard

Estimates sub-pixel position of corners

These two arrays need to have corresponding points

Camera (K) matrix

Distortion parameters

Estimated rotation/translation of each image

OpenCV Camera Calibration

From previous steps

```
# shape of image needed to design undistortion filter
h,w = img.shape[:2]

# make camera matrix and valid ROI
newcameramtx, roi = cv2.getOptimalNewCameraMatrix(cameramtx, distcorr, (w,h), 1, (w,h))

# undistort image
dst = cv.undistort(img, mtx, dist, None, newcameramtx)

# crop the image
x,y,w,h = roi
dst = dst[y:y+h, x:x+w]
```