

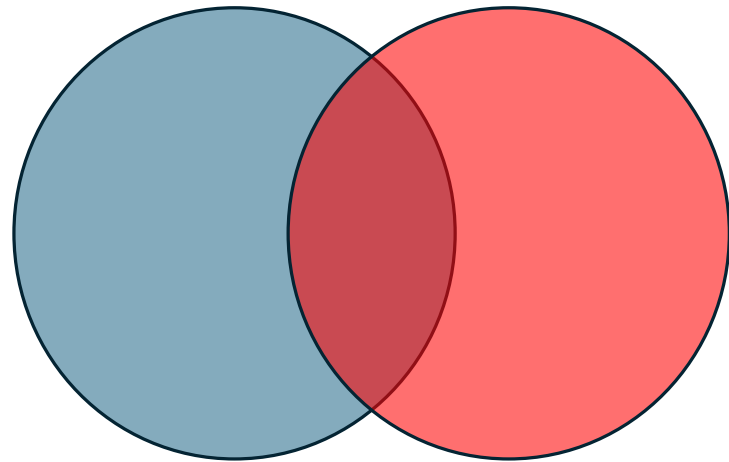
Lecture 2

Operations on Images

ECE 1390/2390

Learning Objectives:

- Mathematical operations
- Bitwise/Logical operations
- Dealing with overflow
- Masking and thresholding



An image is a matrix of information
(tensor for color images)

Math based operations

$$Y = a * X$$

$$Y = fn(X)$$

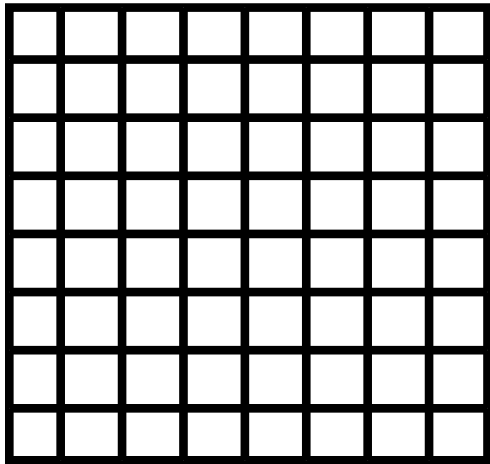
$$Y = H * X * H'$$

Morphological operations

E.g. Dilate/Erode

```
For (i=0; i<n; i++)  
    do something at {i}  
end
```

X



Brightness

$$Y = X + a$$

Changing the brightness of an image means adding/subtracting a scalar from the image

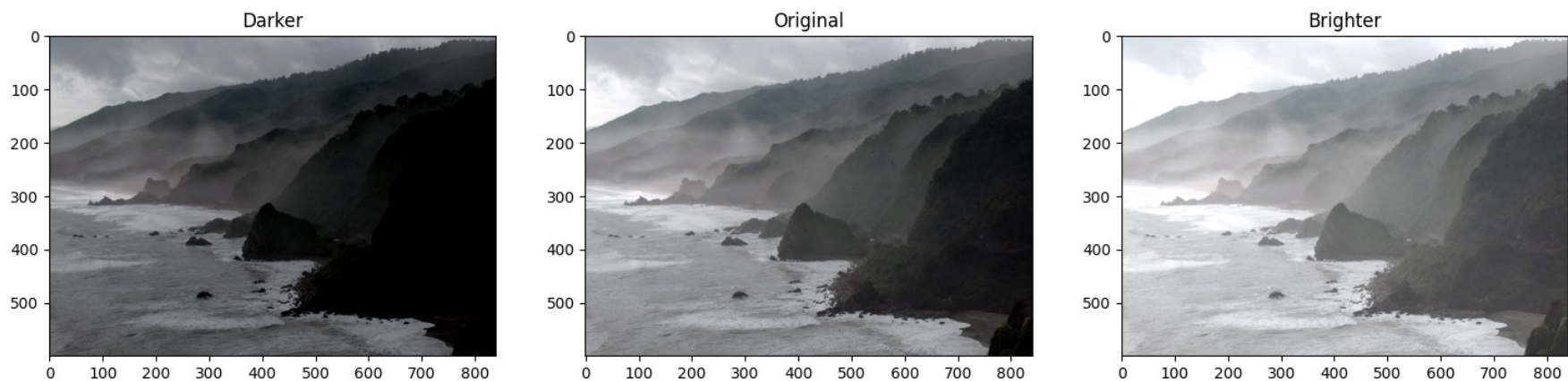


From Lecture1/04_Image_Enhancement.ipnb

```
matrix = np.ones(img_rgb.shape, dtype="uint8") * 50
```

```
img_rgb_brighter = cv2.add(img_rgb, matrix)
```

```
img_rgb_darker = cv2.subtract(img_rgb, matrix)
```



Contrast

$$Y = a * X$$

Contrast is the range of values in the image. Multiplying by a scalar changes the contrast

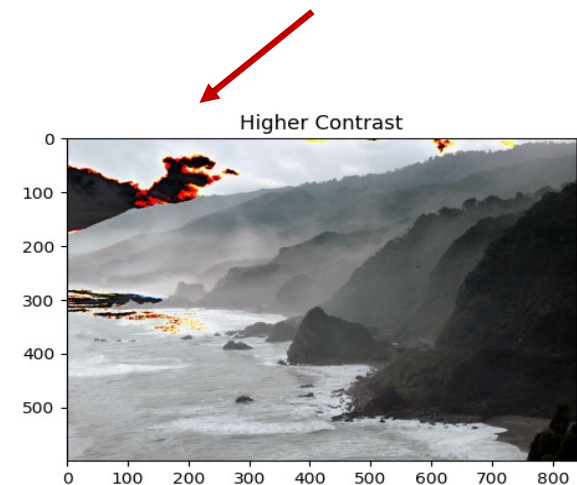
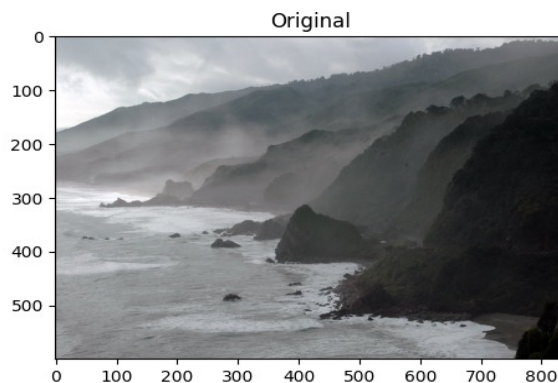
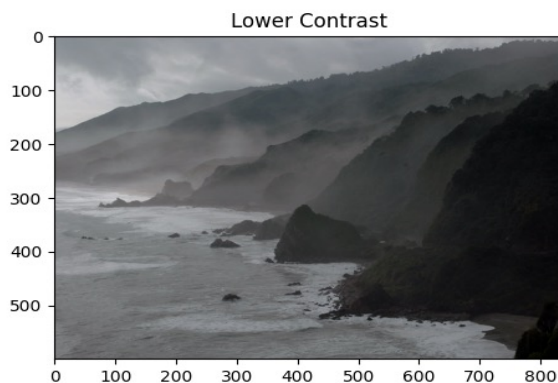


From Lecture1/04_Image_Enhancement.ipnb

```
matrix_low_contrast = np.ones(img_rgb.shape) * 0.8  
matrix_high_contrast = np.ones(img_rgb.shape) * 1.2
```

```
img_rgb_darker = np.uint8(cv2.multiply(np.float64(img_rgb),  
matrix_low_contrast))
```

```
img_rgb_brighter = np.uint8(cv2.multiply(np.float64(img_rgb),  
matrix_high_contrast))
```



Clipping

Python:

```
[1]: import numpy as np  
import cv2
```

```
[2]: a=np.array(250,dtype=np.uint8)  
b=np.array(10,dtype=np.uint8)
```

```
[3]: # In openCV  
cv2.add(a,b)
```

```
[3]: array([[255]], dtype=uint8)
```

```
[4]: # Directly as NP array  
a+b
```

```
[4]: 4
```

```
[5]: a+10
```

```
[5]: 260
```

OpenCV clips at 255

numpy rolls over (260 -> 255, 0, 1, 2, 3, 4)

np + scalar => converts type

Clipping

Python:

`a = [100 150 200 250] → convert to uint16`

`b = [10 10. 10. 10] → convert to uint16`

`a + b = [110 160 210 260] (uint16)`

Normalize

`= [110 160 210 260] * 255 / 260`

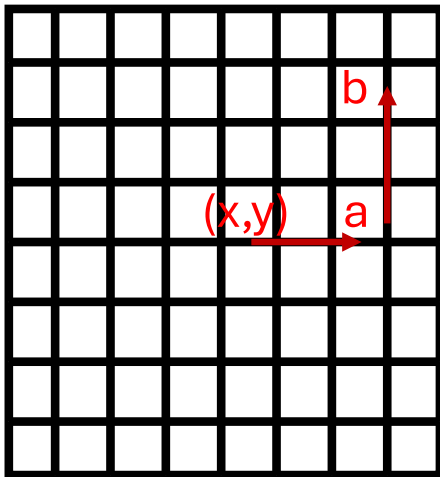
`= [107 157 206 255]`

Point Spread Function

General

$$h(x, a, y, b)$$

Operation varies depending on which position (x,y) in the image

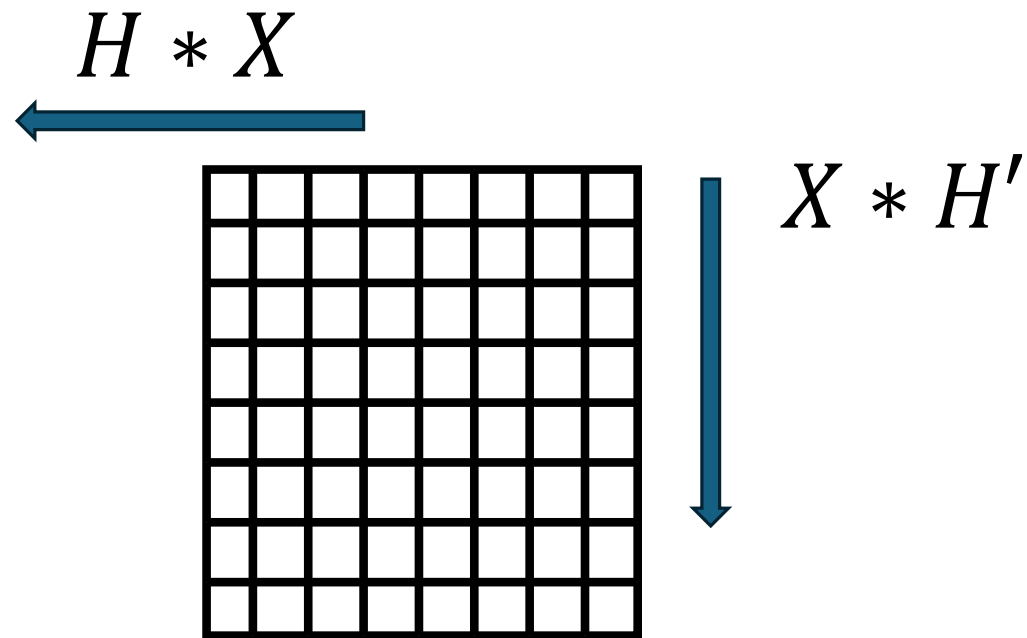


Shift invariant

$$h(a - x, b - y)$$

Operation is independent of the position (x,y) in the image

$$Y = H * X * H'$$



Out = kron(kron(H,X), H')

Out = np.convolution(np.convolution(X,H,axis=0), H,axis=1)

2D convolution as a matrix operation

a b c d e f
 g h ...
 \vdots

z

$\langle 6 \times 6 \rangle$

1 2 3
 4 5 6
 7 8 9

$\langle 3 \times 3 \rangle$

6

<div style="border-top: 1px solid black; width: 100%; margin: 0 auto;"></div>					
1	2	3	0	0	0
0	1	2	3	0	0
0	0	1	2	3	0
0	0	0	1	2	3

4	5	6	0	0	0
0	4	5	6	0	0
0	0	4	5	6	0
0	0	0	5	6	7

7	8	9	0	0	0
0	7	8	9	0	0
0	0	7	8	9	0
0	0	0	7	8	9

$\left. \begin{array}{c} \text{[Red Box]} \\ \text{[Blue Box]} \\ \text{[Green Box]} \end{array} \right\} n = (6 - 3) + 1$

Construction of “H” matrix

2D convolution as a matrix operation

1	2	3	0	0	0
0	1	2	3	0	0
0	0	1	2	3	0
0	0	0	1	2	3

4	5	6	0	0	0
0	4	5	6	0	0
0	0	4	5	6	0
0	0	0	5	6	7

7	8	9	0	0	0
0	7	8	9	0	0
0	0	7	8	9	0
0	0	0	7	8	9

= H

<16 x 36>

1	2	3	0	0	0
0	1	2	3	0	0
0	0	1	2	3	0
0	0	0	1	2	3

4	5	6	0	0	0
0	4	5	6	0	0
0	0	4	5	6	0
0	0	0	5	6	7

7	8	9	0	0	0
0	7	8	9	0	0
0	0	7	8	9	0
0	0	0	7	8	9

1	2	3	0	0	0
0	1	2	3	0	0
0	0	1	2	3	0
0	0	0	1	2	3

4	5	6	0	0	0
0	4	5	6	0	0
0	0	4	5	6	0
0	0	0	5	6	7

7	8	9	0	0	0
0	7	8	9	0	0
0	0	7	8	9	0
0	0	0	7	8	9

$$n = (6 - 3) + 1$$

1	2	3	0	0	0
0	1	2	3	0	0
0	0	1	2	3	0
0	0	0	1	2	3

4	5	6	0	0	0
0	4	5	6	0	0
0	0	4	5	6	0
0	0	0	5	6	7

7	8	9	0	0	0
0	7	8	9	0	0
0	0	7	8	9	0
0	0	0	7	8	9

2D convolution as a matrix operation

a b c d e f
 g h ...

$\langle 16 \times 36 \rangle$

$\langle 36 \times 1 \rangle$

$\langle 6 \times 6 \rangle$

$$H * g' = x$$

$reshape(x) \rightarrow \langle 4 \times 4 \rangle$

a
 b
 c
 d
 e
 f
 g
 \vdots

$= g'$

2D convolution as a matrix operation

In MATLAB: `conv2(B,A,'valid')`

$H = \text{kron}(\text{eye}(N), \text{reshape}(A', 1, []))$

$g = \text{reshape}(B', 1, [])$

$x = \text{reshape}(H * g', N, N)$

$\langle 6 \times 6 \rangle * \langle 3 \times 3 \rangle = \langle 4 \times 4 \rangle$

$N = (6 - 3) + 1$

If you want the same size output as input image:

$N=6$

$= (8 - 3)$

need to pad the image from $\langle 6 \times 6 \rangle$ to $\langle 8 \times 8 \rangle$

<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>f</i>	0	0	0	0	0	0
<i>a</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>f</i>	0	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>
<i>g</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>l</i>	0	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>
\vdots							\vdots	\vdots					

z *z*

2D convolution as a matrix operation

```
kernel= np.ones((3,3))/9
```

```
img_smooth = cv2.filter2D(img,-1,kernel)
```

Depth

-1 : gives output image data type as input

cv2.CV_8U = 0 uint8

cv2.CV_8S = 1 int8

cv2.CV_16U = 2 uint16

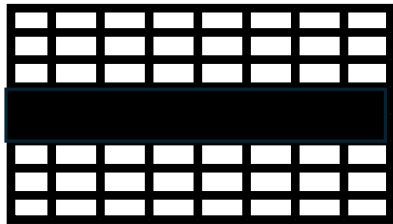
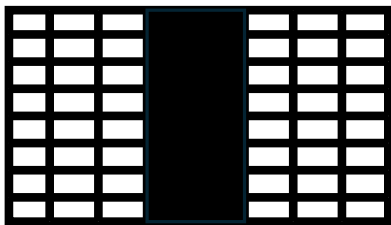
cv2.CV_16S = 3 int16

cv2.CV_32S = 4 int32

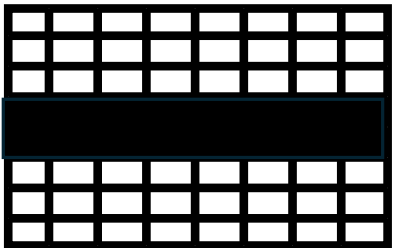
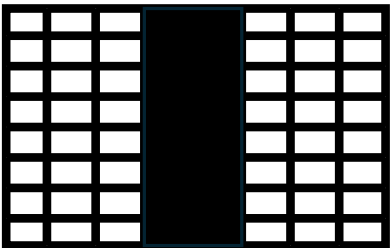
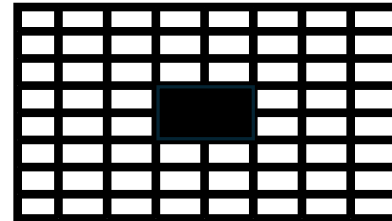
cv2.CV_32F = 5 float32

cv2.CV_64F = 6 float64

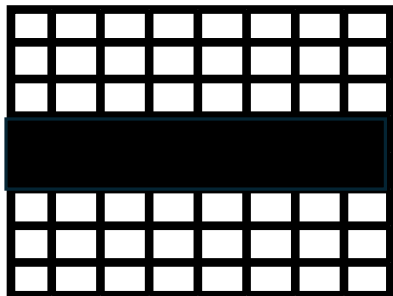
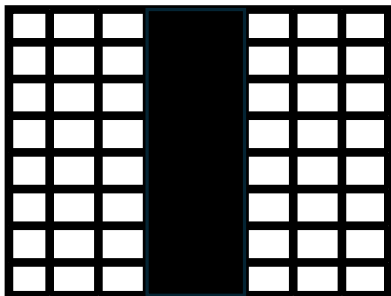
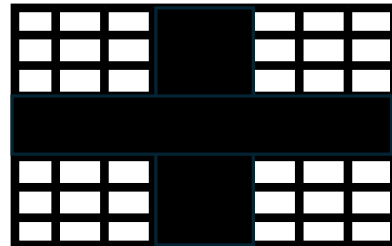
Boolean (Bit-wise) operations



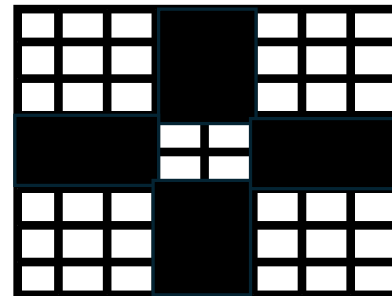
AND



OR



XOR

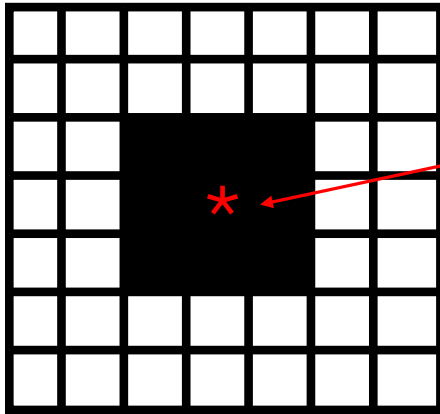


Morphological Operations

Erosion/Dilation

Kernel is Boolean (mask)

Structure/Kernel



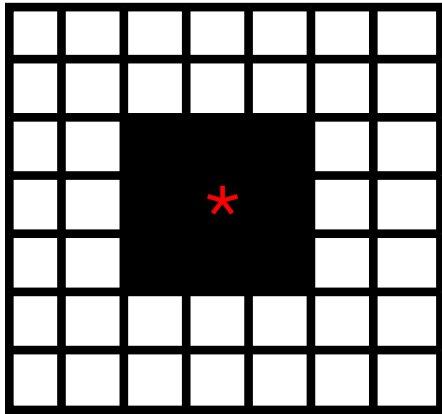
Anchor point



Erosion/Dilation

Anchor point

Structure/Kernel



For $\{i,j\}$ in image:

move kernel anchor to $\{i,j\}$

list = find (kernel == true)

newVal $\{i,j\}$ = fcn (image[list])

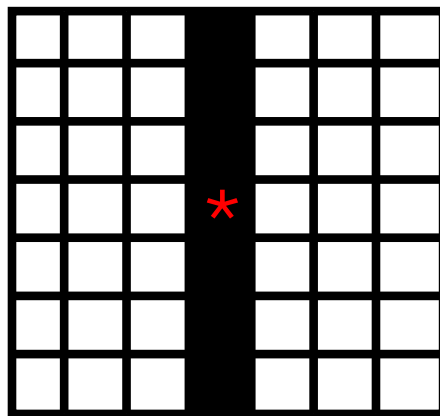
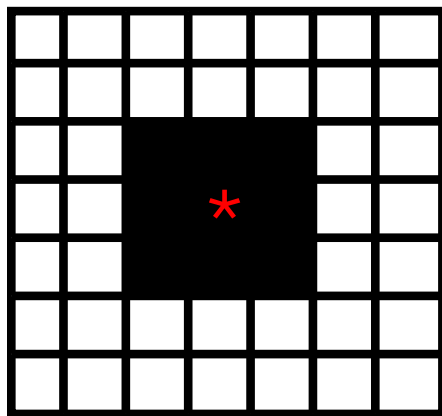
Dilation - MAX (image[list])

Erosion - MIN (image[list])

Denoising - MEDIAN (image[list])

Smoothing - MEAN (image[list])

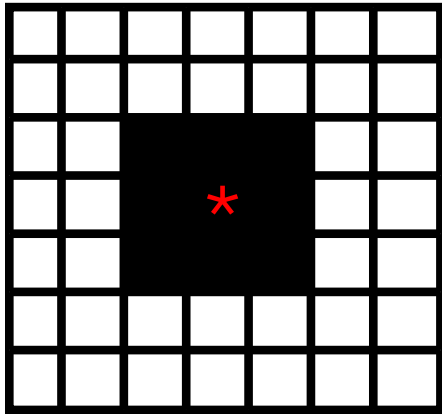
Erosion/Dilation



Convolutional (Filtering) Operations

Anchor point

Structure/Kernel



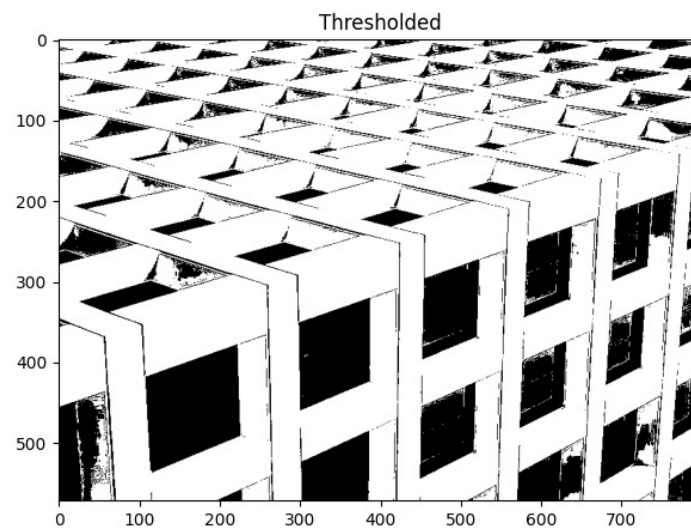
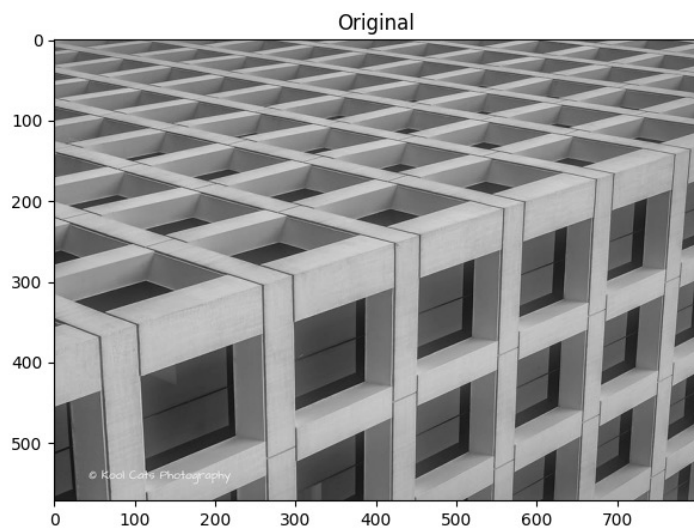
For $\{i,j\}$ in image:

move kernel anchor to $\{i,j\}$

$\text{newVal}\{i,j\} = \text{sum}(\text{image} .* \text{kernel})$

Thresholding

`thres, img_out = cv2.threshold(img, thres, replacement value, type)`



Thresholding

`thres, img_out = cv2.threshold(img, thres, replacement value, type)`

Types:

- THRESH_BINARY
- THRESH_BINARY_INV
- THRESH_TRUNC
- THRESH_TOZERO
- THRESH_TOZERO_INV

If $x > \text{thresh}$; $x = \text{rep_value}$; else $x = 0$

If $x > \text{thresh}$; $x = 0$; else $x = \text{rep_value}$

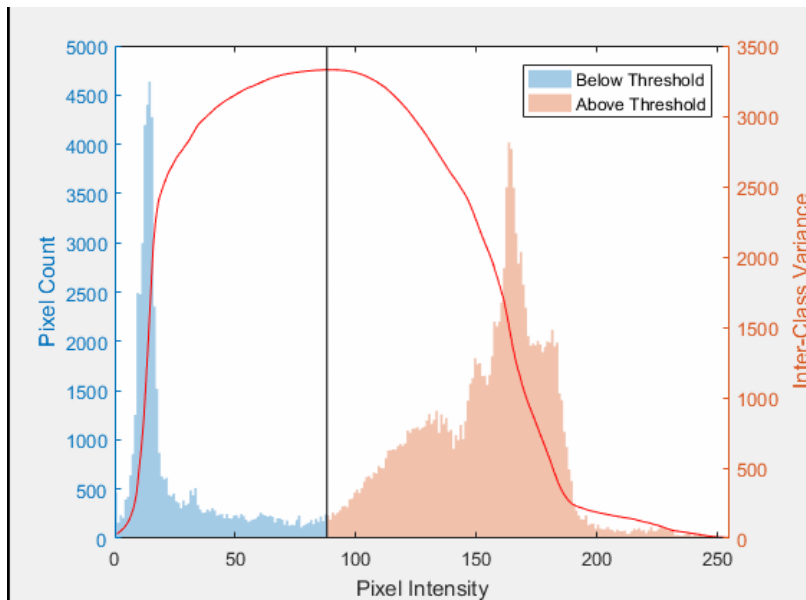
If $x > \text{thresh}$; $x = x$; else $x = \text{thresh}$

If $x > \text{thresh}$; $x = x$; else $x = 0$

If $x > \text{thresh}$; $x = 0$; else $x = x$

Otsu's thresholding

- Nobuyuki Otsu (1979). "A threshold selection method from gray-level histograms". *IEEE Transactions on Systems, Man, and Cybernetics*. **9** (1): 62–66.



Variance values below thresh

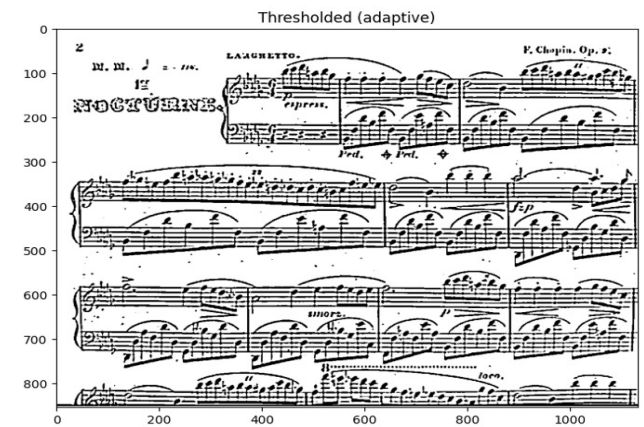
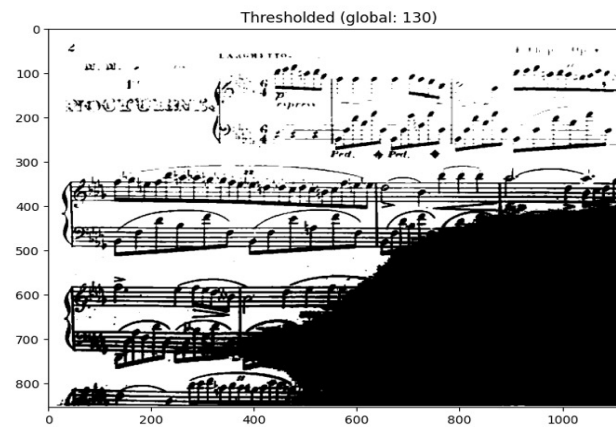
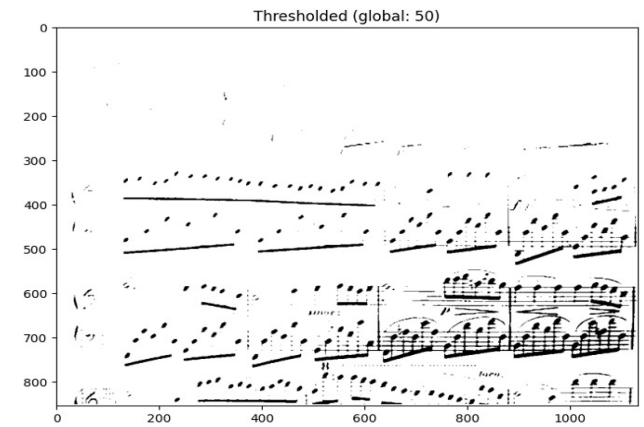
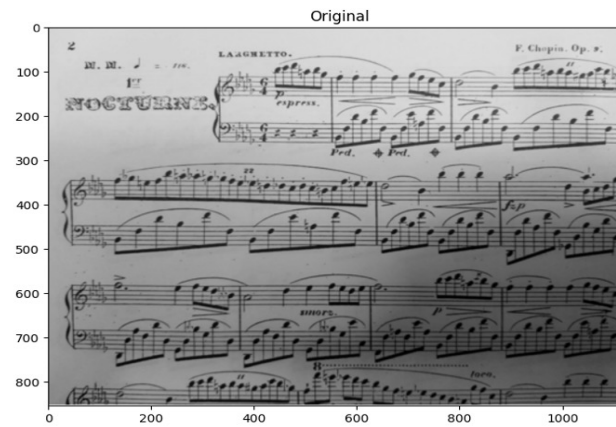
Variance values above thresh

$$\sigma^2 = w_0 \cdot \sigma_0^2 + w_1 \cdot \sigma_1^2$$

Probability of values below thresh

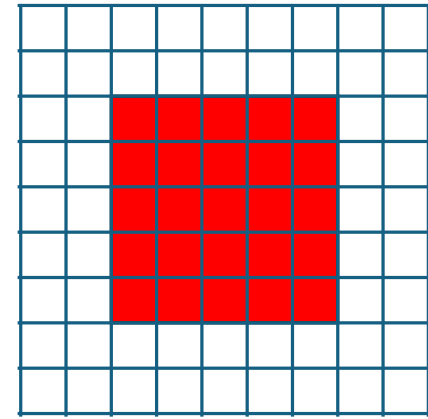
Probability of values above thresh

Adaptive thresholding



Adaptive thresholding

$$thres\{i,j\} = \left(\frac{1}{n * m} \sum_{i=1}^n \sum_{j=1}^m x[i,j] \right) - C$$



$$thes\{i,j\} = \left(\frac{1}{\sum_{i=1}^n \sum_{j=1}^m G[i,j]} \sum_{i=1}^n \sum_{j=1}^m G[i,j] * x[i,j] \right) - C$$

