Lecture 5
Frequency and Spatial Domain Filters

ECE 1390/2390

Learning Objectives:

- Frequency domain filtering
- Window-method for FIR design

High spatial frequencies

Low spatial frequencies

DFT

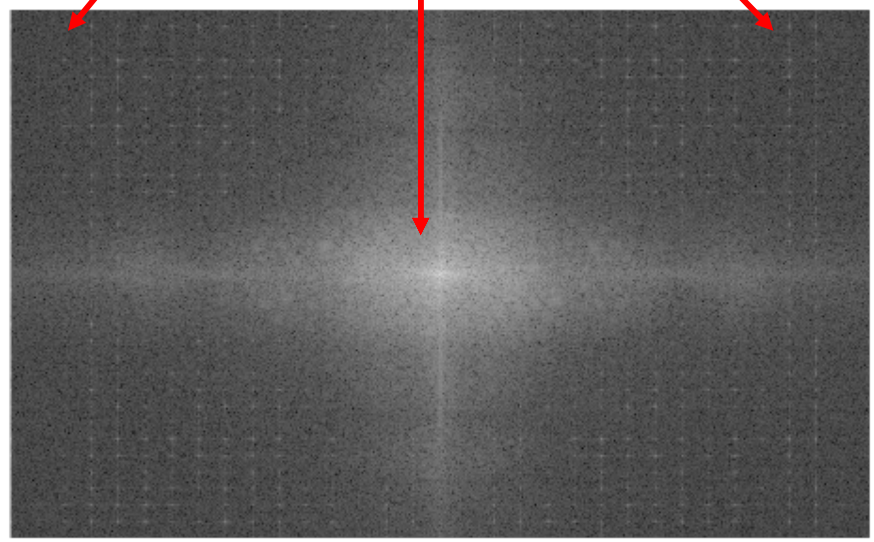


Image Resolution

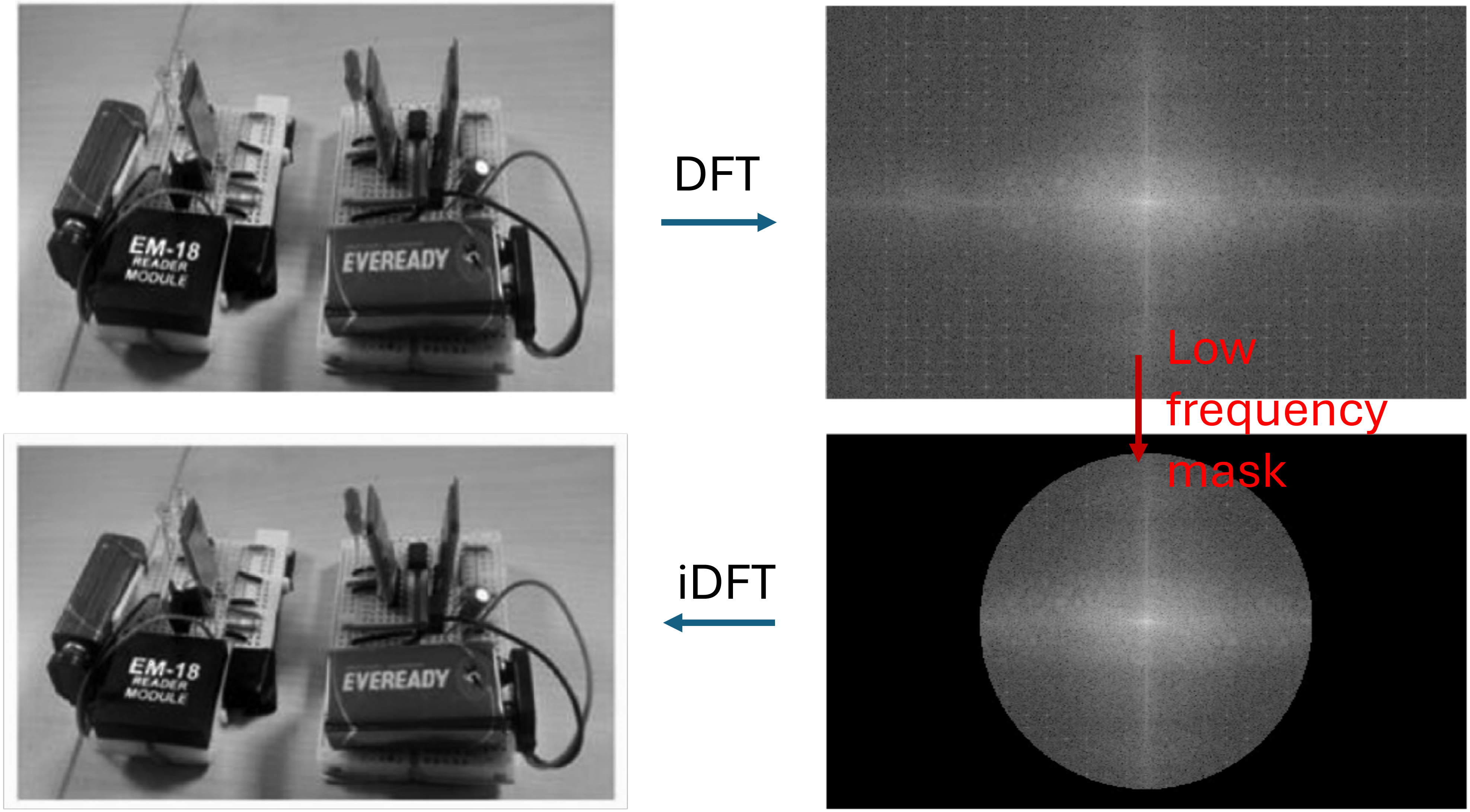
1000 x 1000 [15cm x 10cm]
= 0.15 mm/pixel in X-direction
= 0.10 mm/pixel in Y-direction

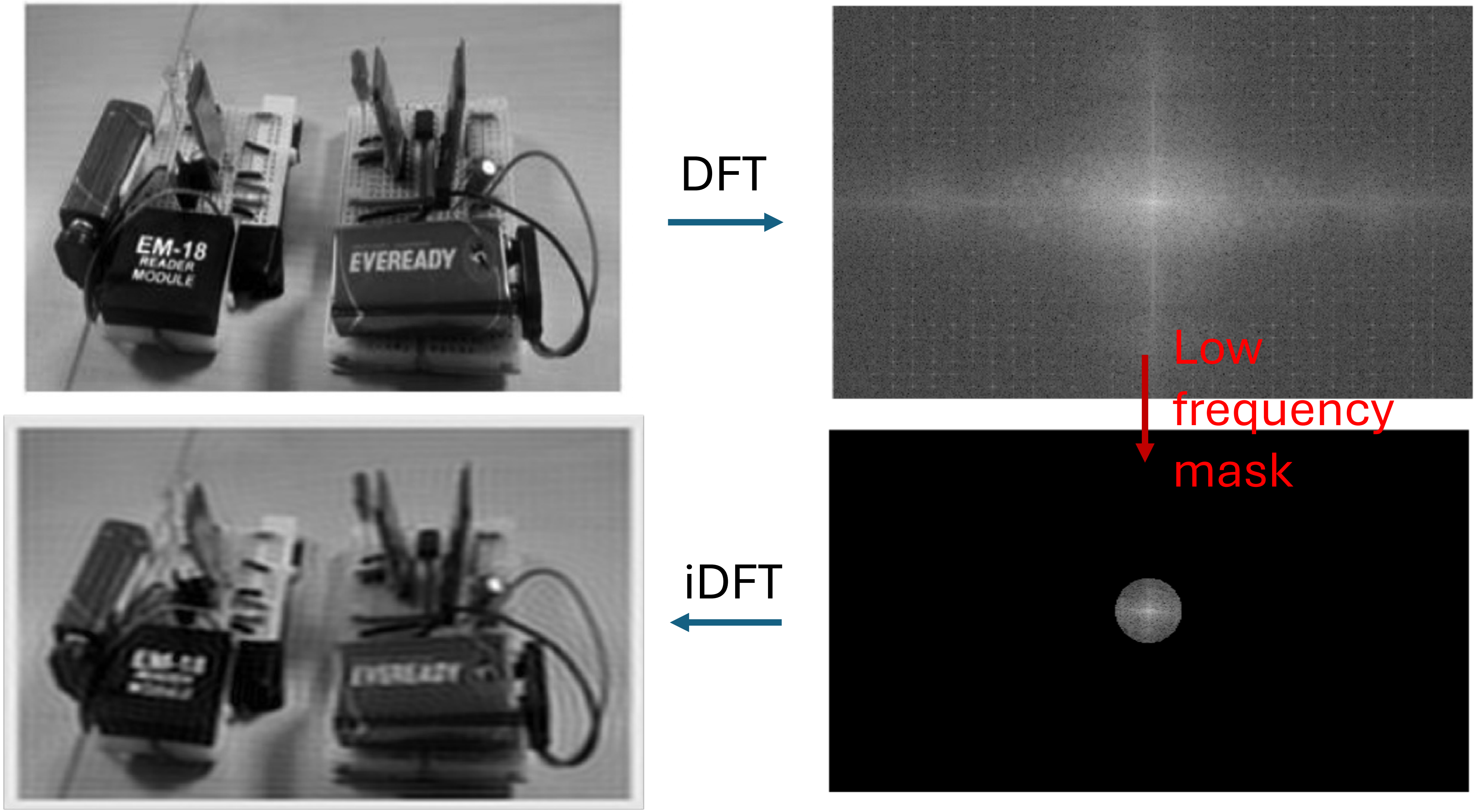
Sample Rate

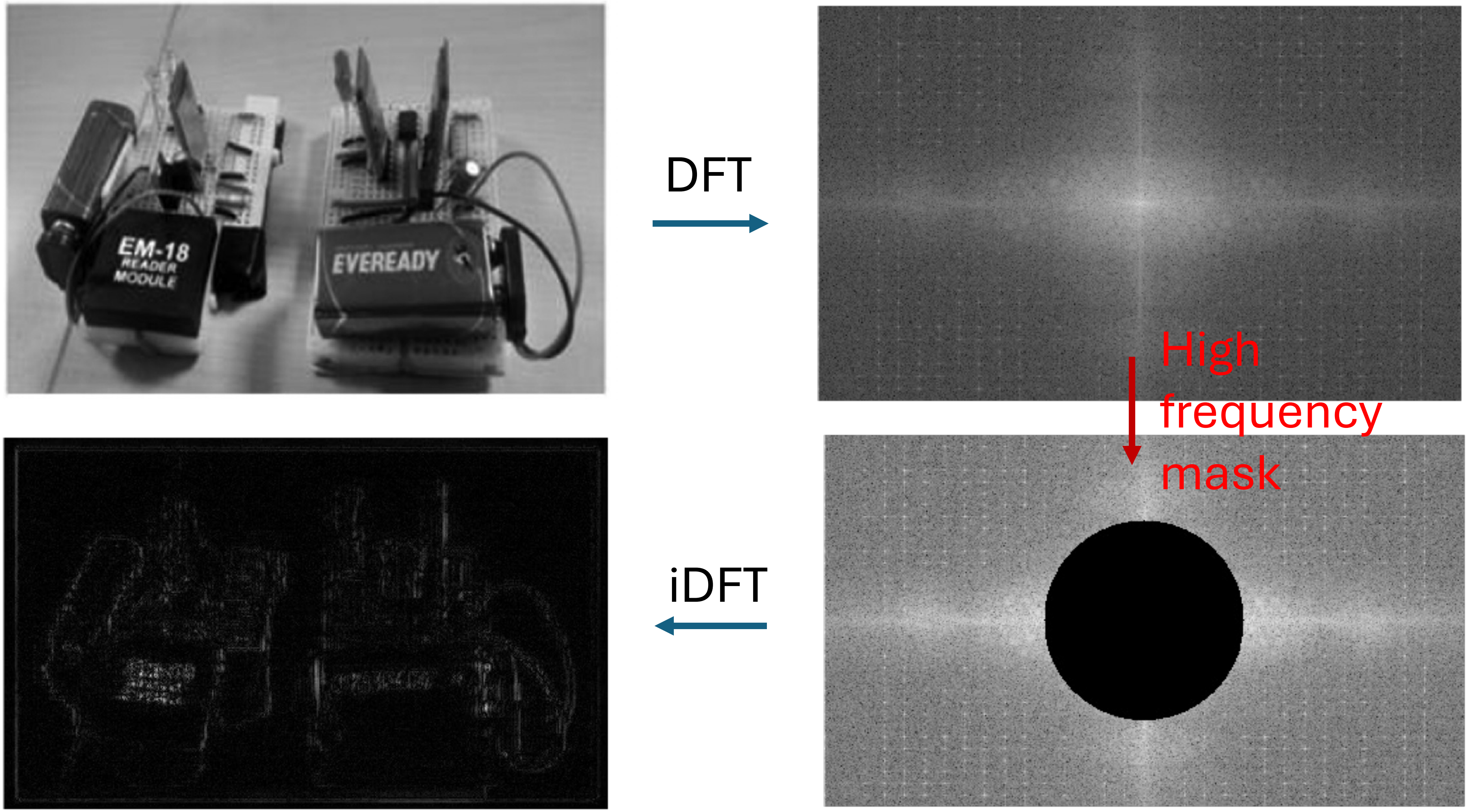
1/0.15 pixel/mm = 6.7 pixel/mm
1/0.10 pixel/mm = 10.0 pixel/mm

Nyquist Spatial Frequency

3.3 pixel/mm = 3.3 mm^{-1}
5.0 pixel/mm = 5.0 mm^{-1}


DFT
Low
frequency
mask
iDFT
EM-18
READER
MODULE
EVEREADY

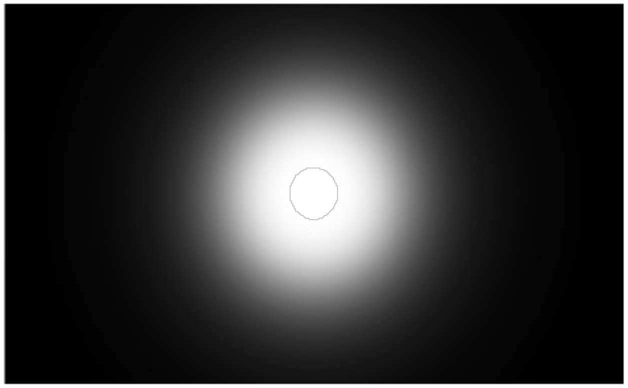

DFT
EM-18
READER
MODULE
EVEREADY
Low frequency mask
iDFT

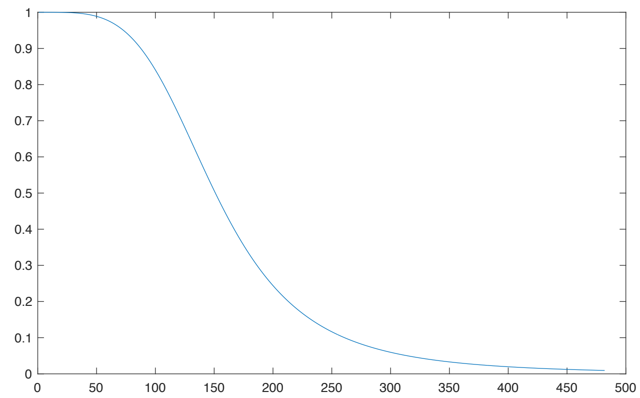

DFT
High frequency mask
iDFT
EM-18
READER
MODULE
EVEREADY

Butterworth

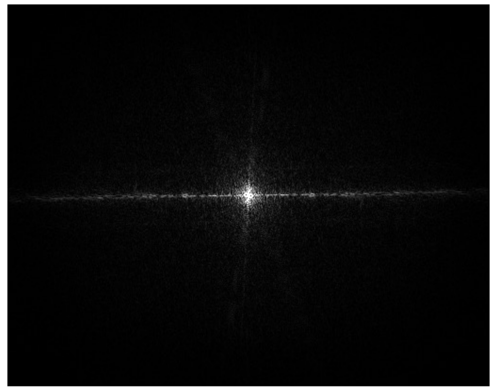
$$H(u,v)=\frac{1}{1+\left[D(u,v)/D_0\right]^{2n}}$$

Corner frequency

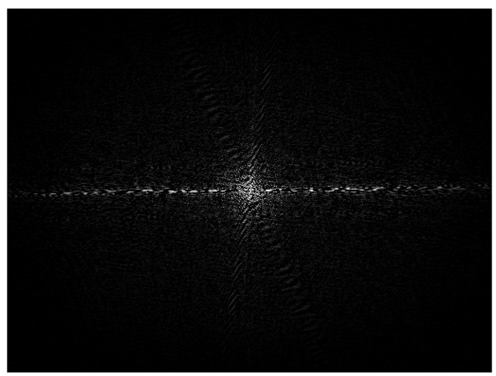




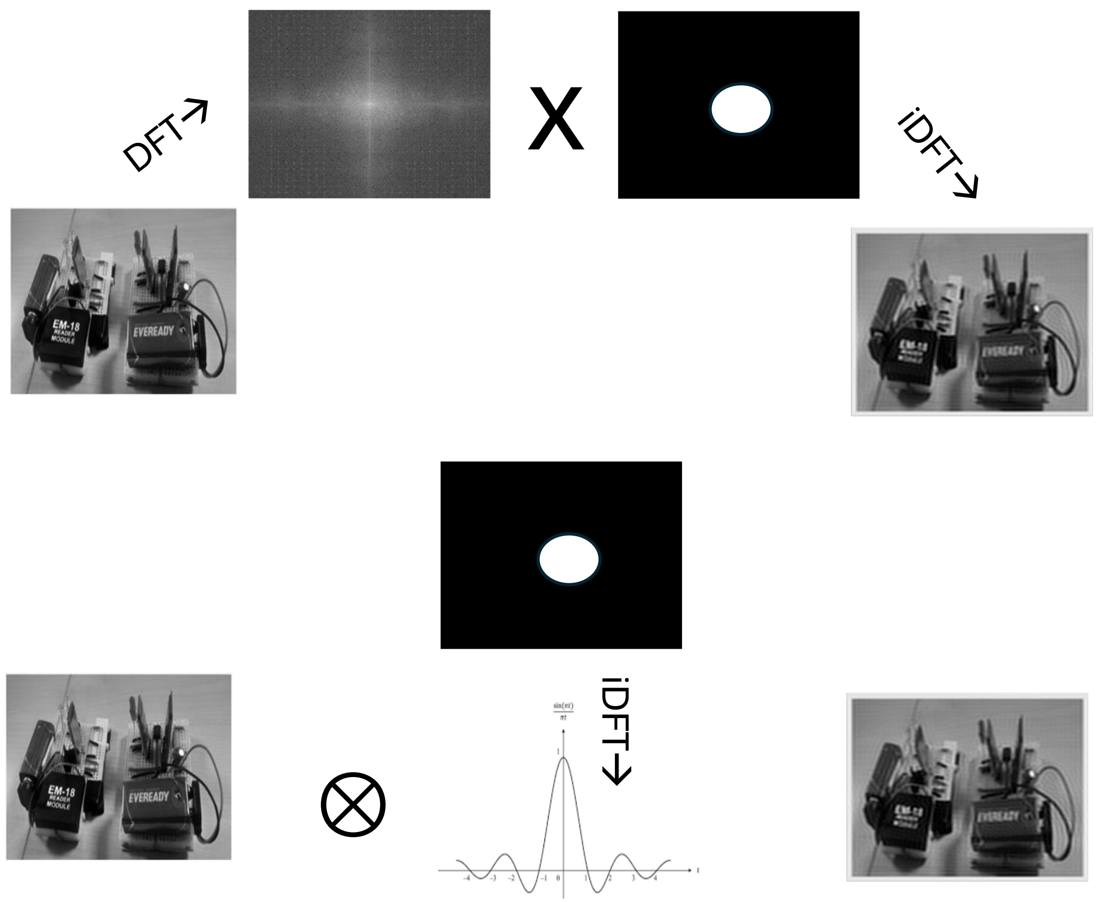
$$\left|F_{real}(i,j) + F_{imaginary}(i,j)\right|$$



$$\left|F_{real}(i,j) + \cancel{F_{imaginary}(i,j)}\right|$$



Frequency → Spatial domain

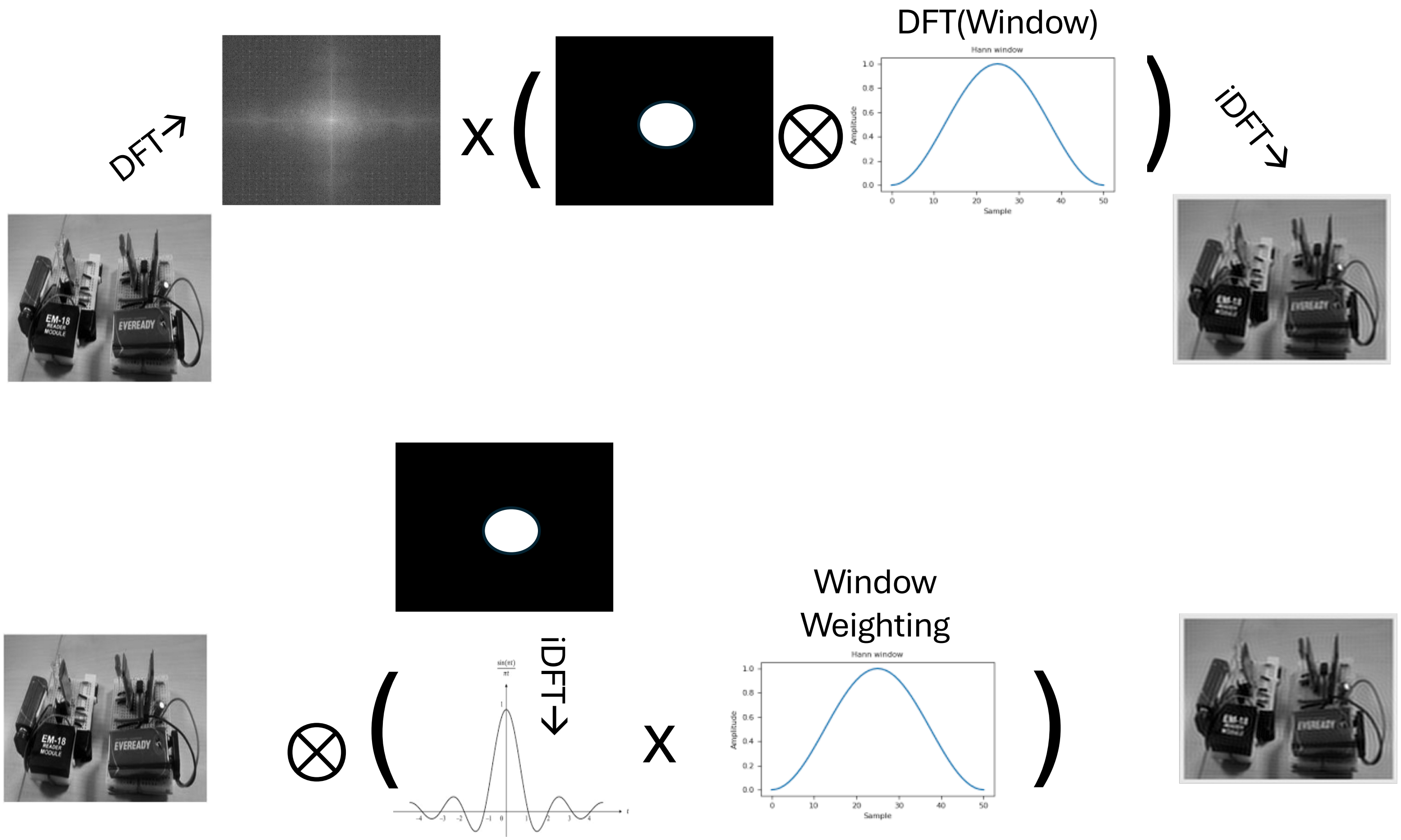


An ideal <u>top-hat function</u> in the frequency-domain would be a <u>sinc function</u> in the spatial domain.

But, the sinc function in the spatial domain would have infinite size.

Need to truncate the sinc kernel. Where?

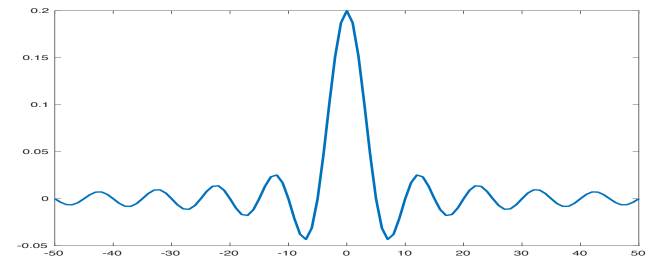
Window method for FIR design



Low-pass

$$\frac{\sin(kw_c)}{\pi k}$$

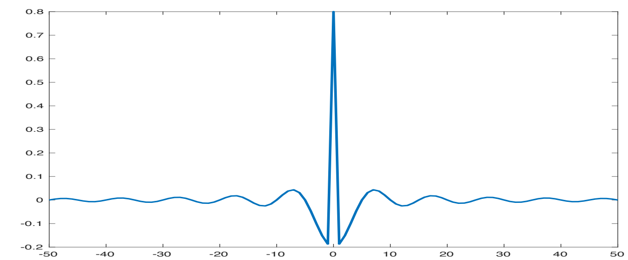
$$\lim_{n\to 0} = \frac{w_c}{\pi}$$



High-pass

$$\frac{-\sin(kw_c)}{\pi k}$$

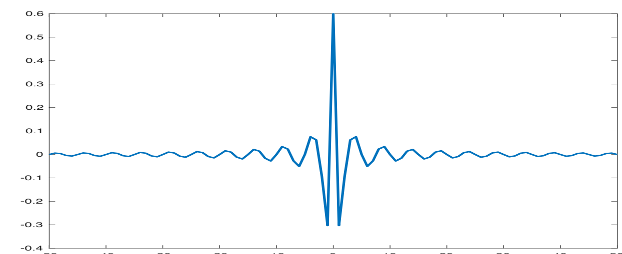
$$\lim_{n\to 0} = 1 - \frac{w_c}{\pi}$$



Band-pass

$$\frac{\sin(kw_{up})}{\pi k} - \frac{\sin(kw_{low})}{\pi k}$$

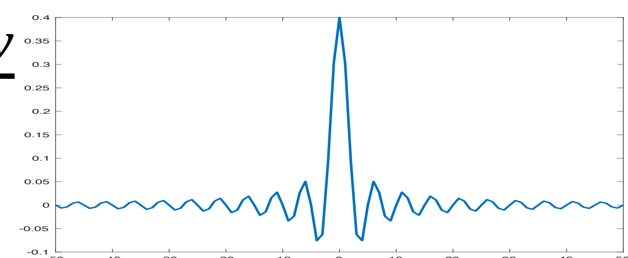
$$\lim_{n\to 0} = \frac{w_{up} - w_{low}}{\pi}$$



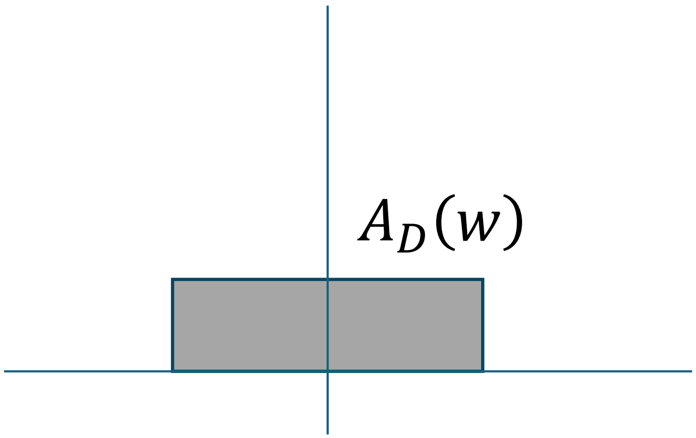
Notch

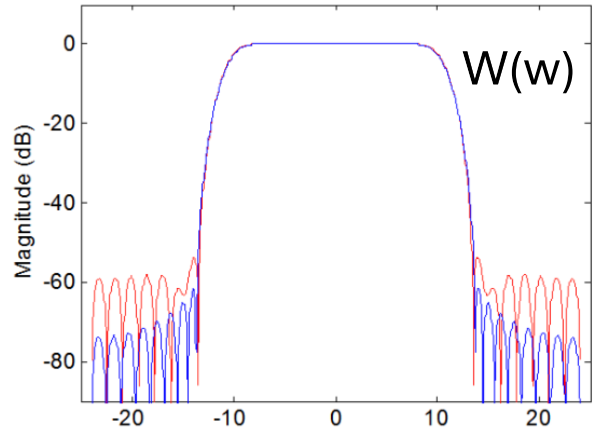
$$\frac{\sin(kw_{low})}{\pi k} - \frac{\sin(kw_{up})}{\pi k}$$

$$\lim_{n\to 0} = 1 - \frac{w_{up} - w_{low}}{\pi}$$

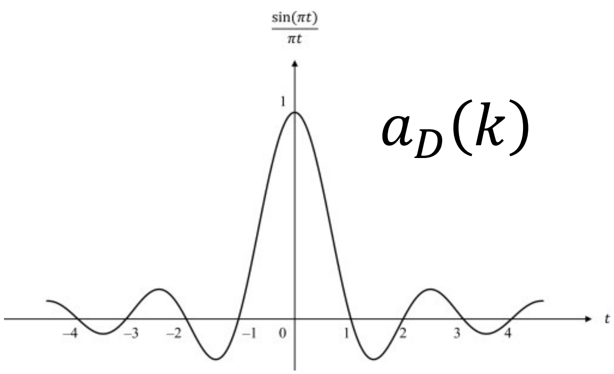


Window method for FIR design

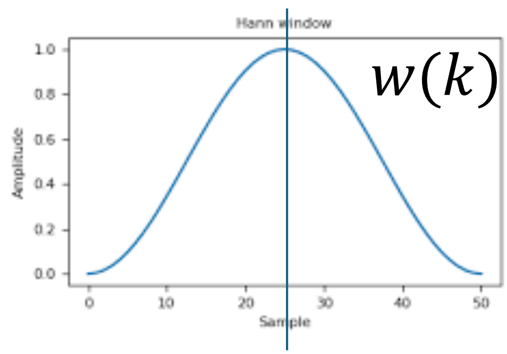




$$A(w) = A_D(w) * W(w)$$

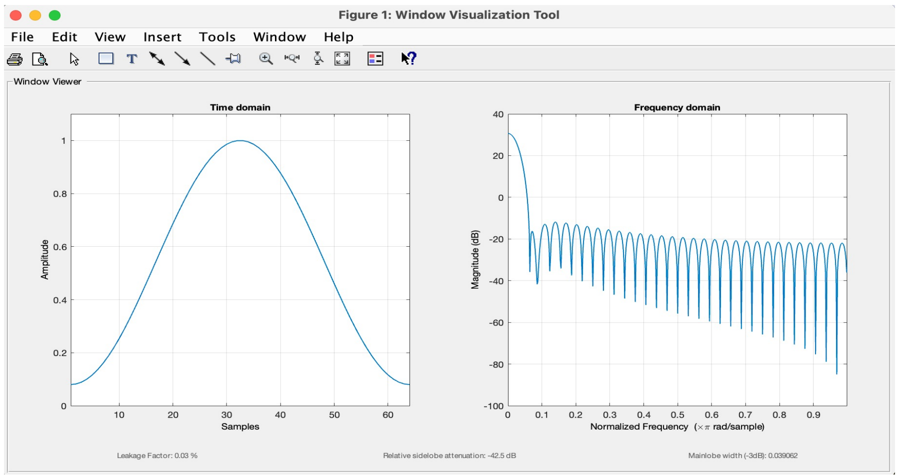


$H(e^{jw})$



$$a(k) = a_d(k) \cdot w(k)$$

Hamming

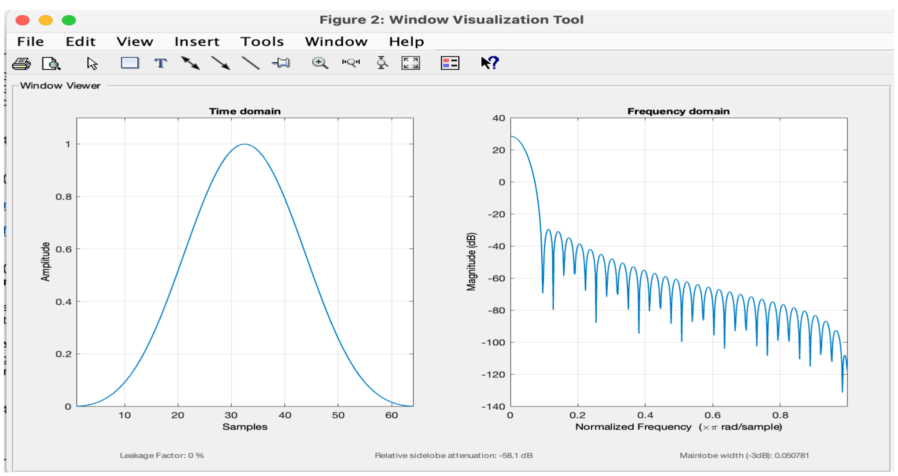


Larger main-lobe width

↓

Sharper transition bandwidth

Blackman



More side lobes

↓

Less ripple in pass/stop band

Window	Passband ripple (db)	Stopband attenuation (dB) A_m	First side-lobe (dB)	Transition width Δf (norm. Hz)	δ_m
Rectangular	0.7416	21	-13	$0.9/N$	0.0891
Kaiser, A=30, β=2.12	0.270	30	-19	$1.5/N$	0.0316
Hanning	0.0546	44	-31	$3.1/N$	0.00632
Kaiser, A=50, β=4.55	0.0274	50	-34	$2.9/N$	0.00316
Hamming	0.0194	53	-41	$3.3/N$	0.00224
Kaiser, A=70, β=6.76	0.00275	70	-49	$4.3/N$	0.000316
Blackman	0.0017	74	-57	$5.5/N$	0.000196
Kaiser, A=90, β=8.96	0.000275	90	-66	$5.7/N$	0.0000316

Sharpest transition/
Largest ripple

Kaiser- Adjustable
trade-offs

2D FIR filtering

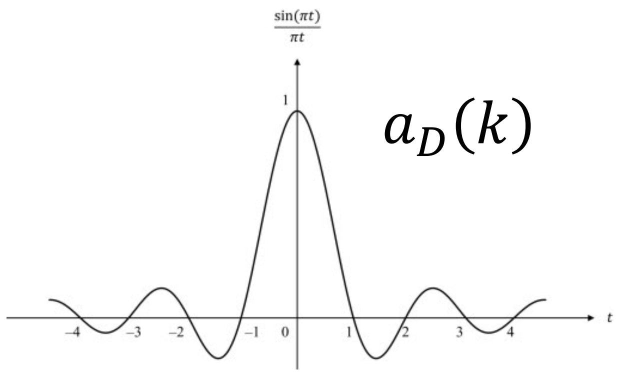
homogenous sampling

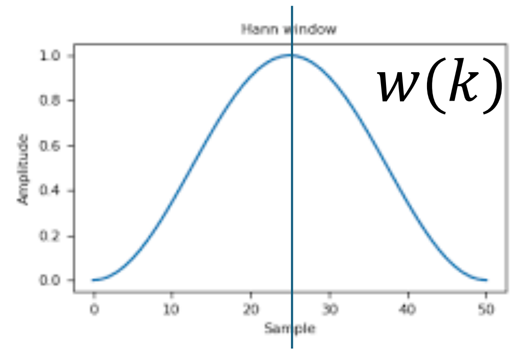
$$yf[n] = x[n] * h[n]$$

$$yf[i,j] = x[i,j] * h[i,j]$$

$$yf[i,j] = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h[n,m] \cdot x[i-n, j-m]$$

Window method for FIR design





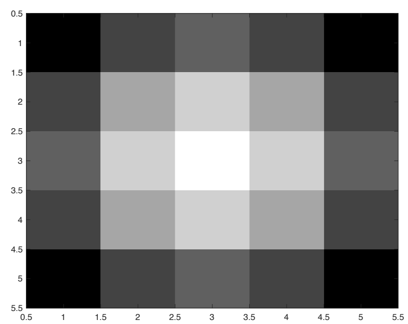
$a(k) = a_d(k) \cdot w(k)$

"T" – temporal sampling interval → "voxel size"
"F" – temporal frequency (s^{-1}) → spatial frequency (cm^{-1})
"w" – frequency in radians/s → same definition

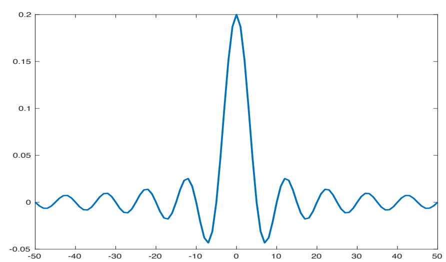
- Images are typically a lot fewer samples than time (e.g. 1024x1024 vs #time-points)
- Larger filter kernels will have more edge effects (edges on all 4 sides of an image)
- Zero-padding needed

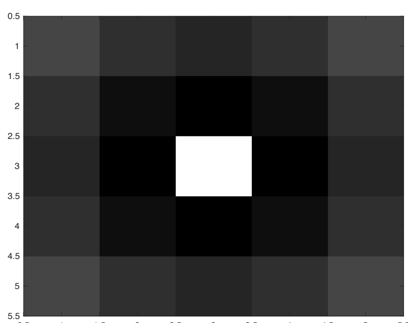
$$yf[i,j] = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h[n,m] \cdot x[i-n, j-m]$$

Gaussian filter

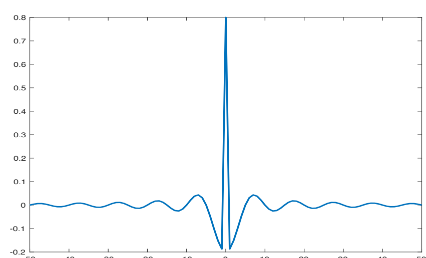


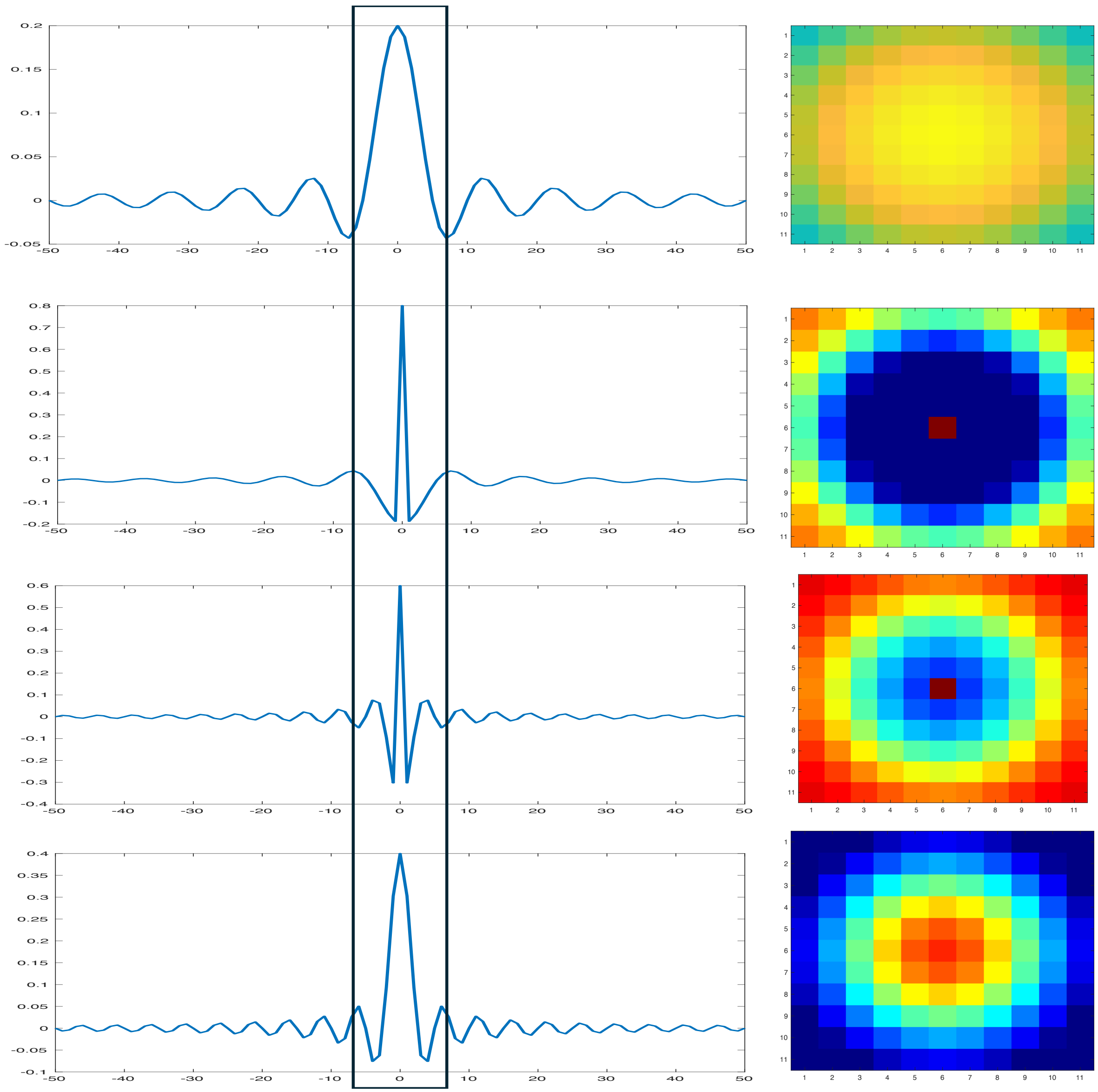
FIR LPF





FIR HPF





$$\frac{\sin(rw_c)}{\pi r} \qquad \lim_{r\to 0} = \frac{w_c}{\pi}$$

$$\frac{-\sin(rw_c)}{\pi r} \qquad \lim_{r\to 0} = 1 - \frac{w_c}{\pi}$$

$$\frac{\sin(rw_{up})}{\pi r} - \frac{\sin(rw_{low})}{\pi r} \qquad \lim_{r\to 0} = \frac{w_{up} - w_{low}}{\pi}$$

$$\frac{\sin(rw_{low})}{\pi r} - \frac{\sin(rw_{up})}{\pi r} \qquad \lim_{r\to 0} = 1 - \frac{w_{up} - w_{low}}{\pi}$$