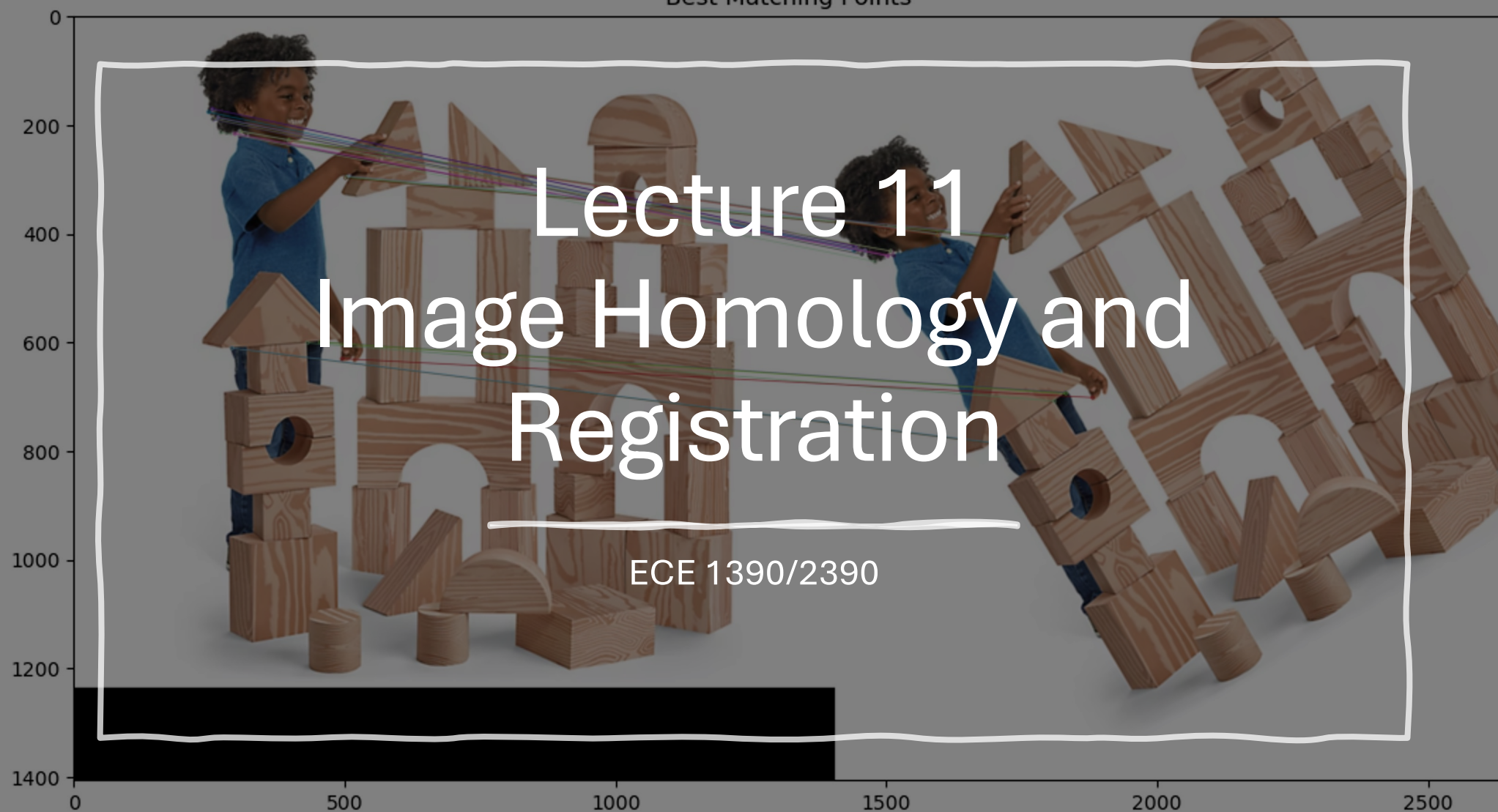


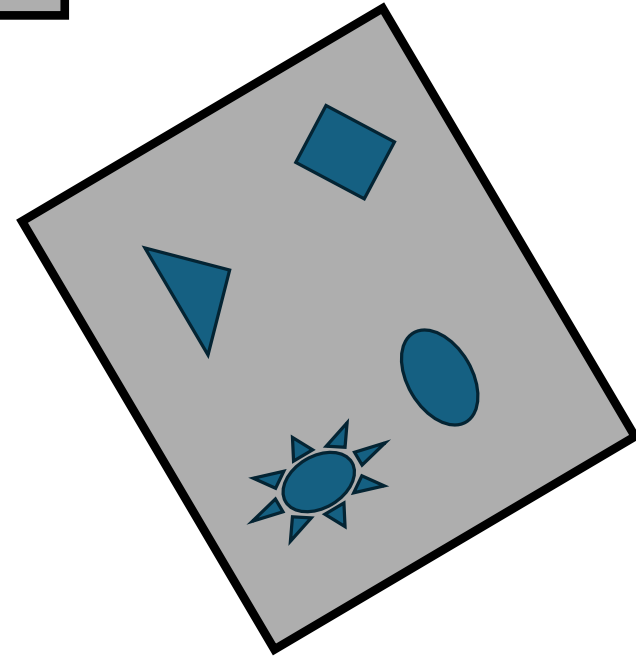
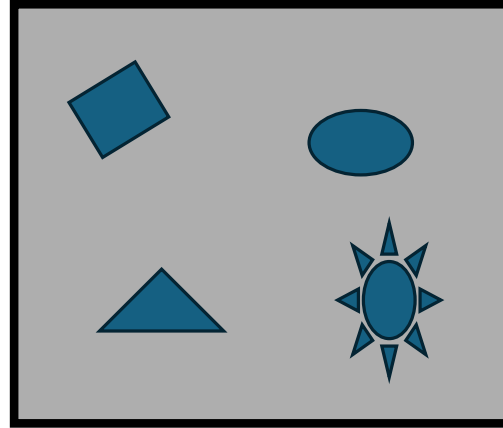
Best Matching Points



Affine transform

$$\begin{bmatrix} x_0 & y_0 \\ x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_n & y_n \end{bmatrix} = \begin{bmatrix} x'_0 & y'_0 \\ x'_1 & y'_1 \\ x'_2 & y'_2 \\ \vdots & \vdots \\ x'_n & y'_n \end{bmatrix} * R + T$$

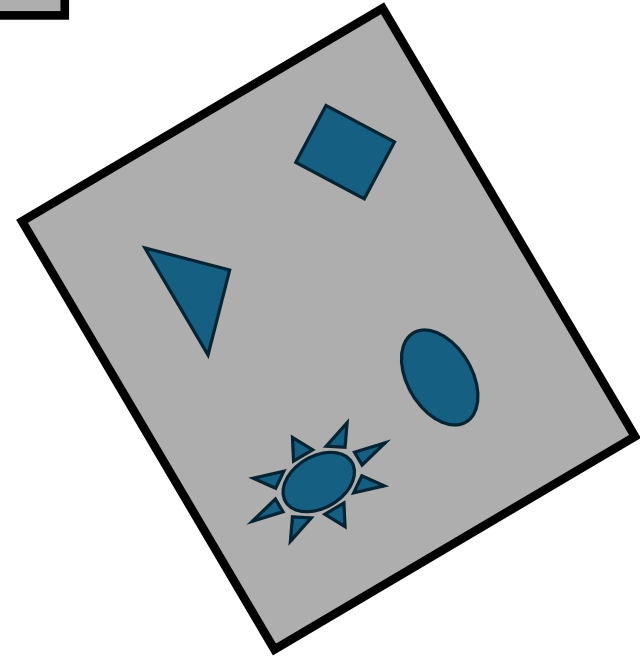
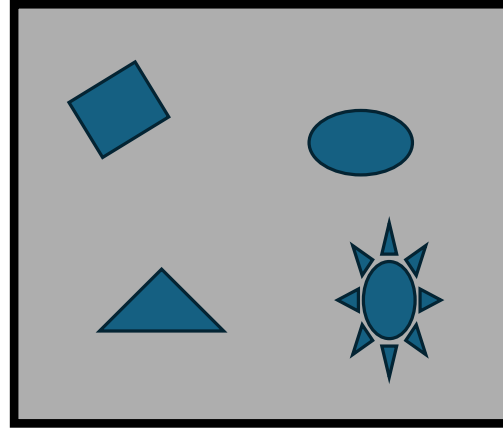
Corresponding pairs



Affine transform

$$\begin{bmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix} = \begin{bmatrix} x'_0 & y'_0 & 1 \\ x'_1 & y'_1 & 1 \\ x'_2 & y'_2 & 1 \\ \vdots & \vdots & \vdots \\ x'_n & y'_n & 1 \end{bmatrix} * R'$$

$$R' = \begin{bmatrix} a & c & 0 \\ b & d & 0 \\ \Delta x & \Delta y & 1 \end{bmatrix}$$



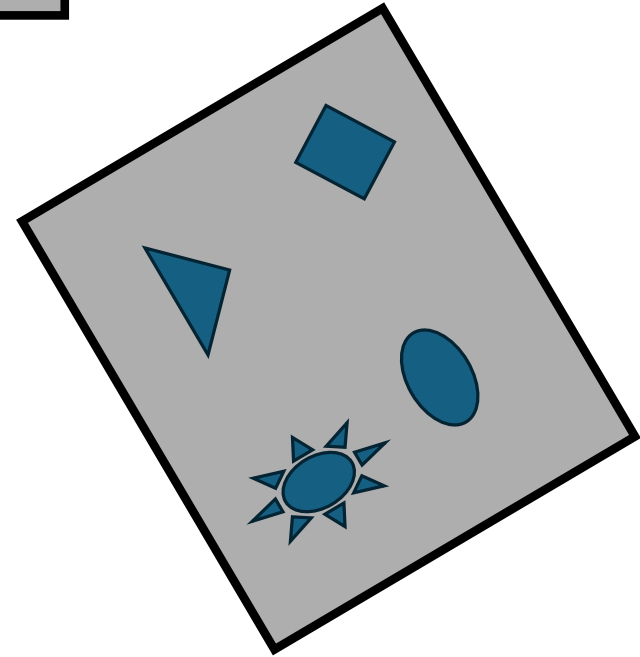
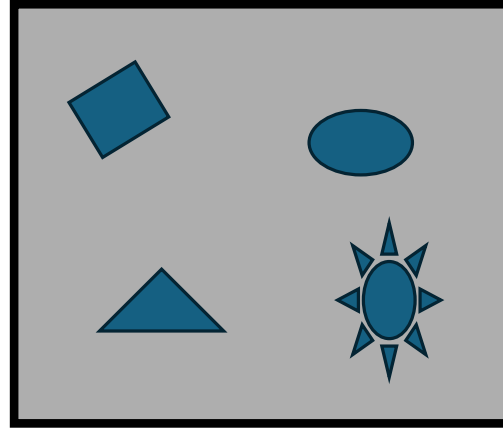
Affine transform

$$A = B * R'$$

$$B^{-1} * A = R'$$

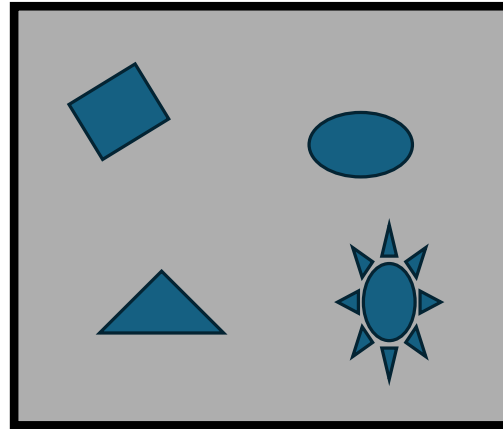
$$(B^T * B)^{-1} * B^T * A = R'$$

$$R' = \begin{bmatrix} a & c & 0 \\ b & d & 0 \\ \Delta x & \Delta y & 1 \end{bmatrix}$$



Rigid-body transform

$$A = B * R'$$

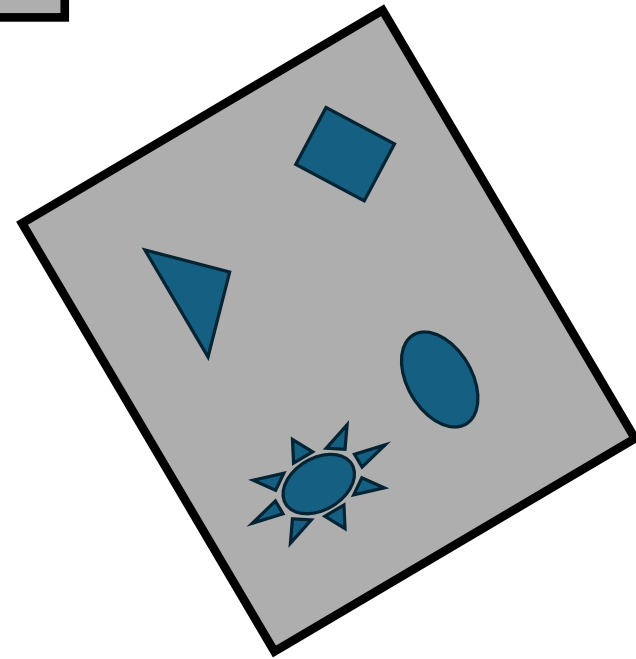


$$R_{rigid}' = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ \Delta x & \Delta y & 1 \end{bmatrix}$$

$$R_{stretch}' = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ \Delta x & \Delta y & 1 \end{bmatrix}$$

$$R_{sheer}' = \begin{bmatrix} k_x & k_{xy} & 0 \\ k_{yx} & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ \Delta x & \Delta y & 1 \end{bmatrix}$$

Same as affine



ICP (iterative closest point)

$$\begin{bmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix} = \begin{bmatrix} x'_0 & y'_0 & 1 \\ x'_1 & y'_1 & 1 \\ x'_2 & y'_2 & 1 \\ \vdots & \vdots & \vdots \\ x'_n & y'_n & 1 \end{bmatrix} * R'$$

Initial Guess of correspondence (lst1, lst2)

While():

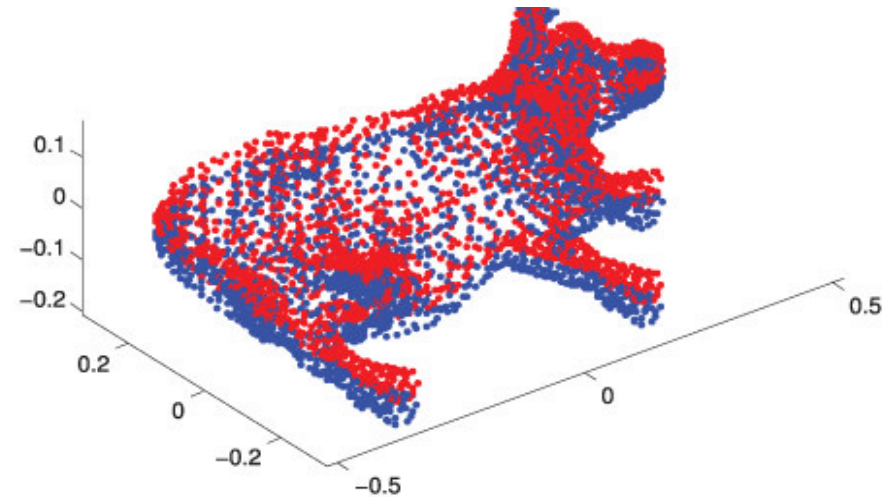
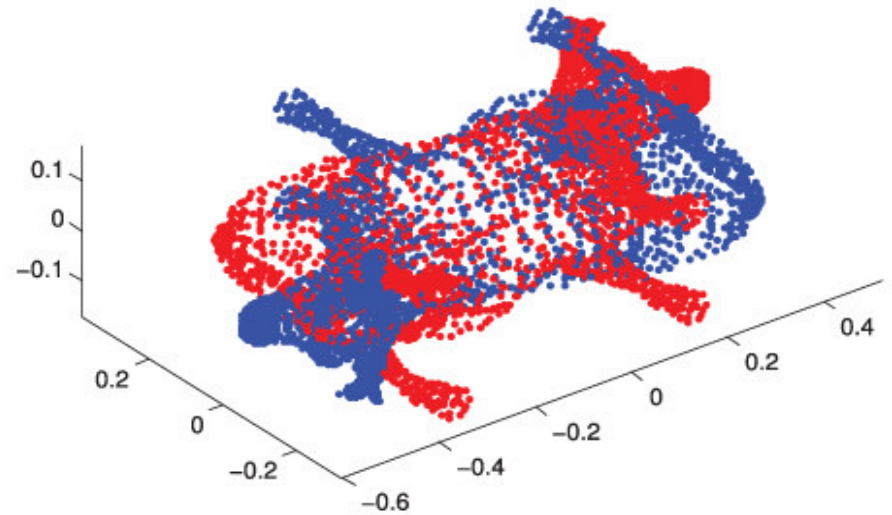
 A(lst1) = R * B(lst2)

 Calculate transform

 Apply transform

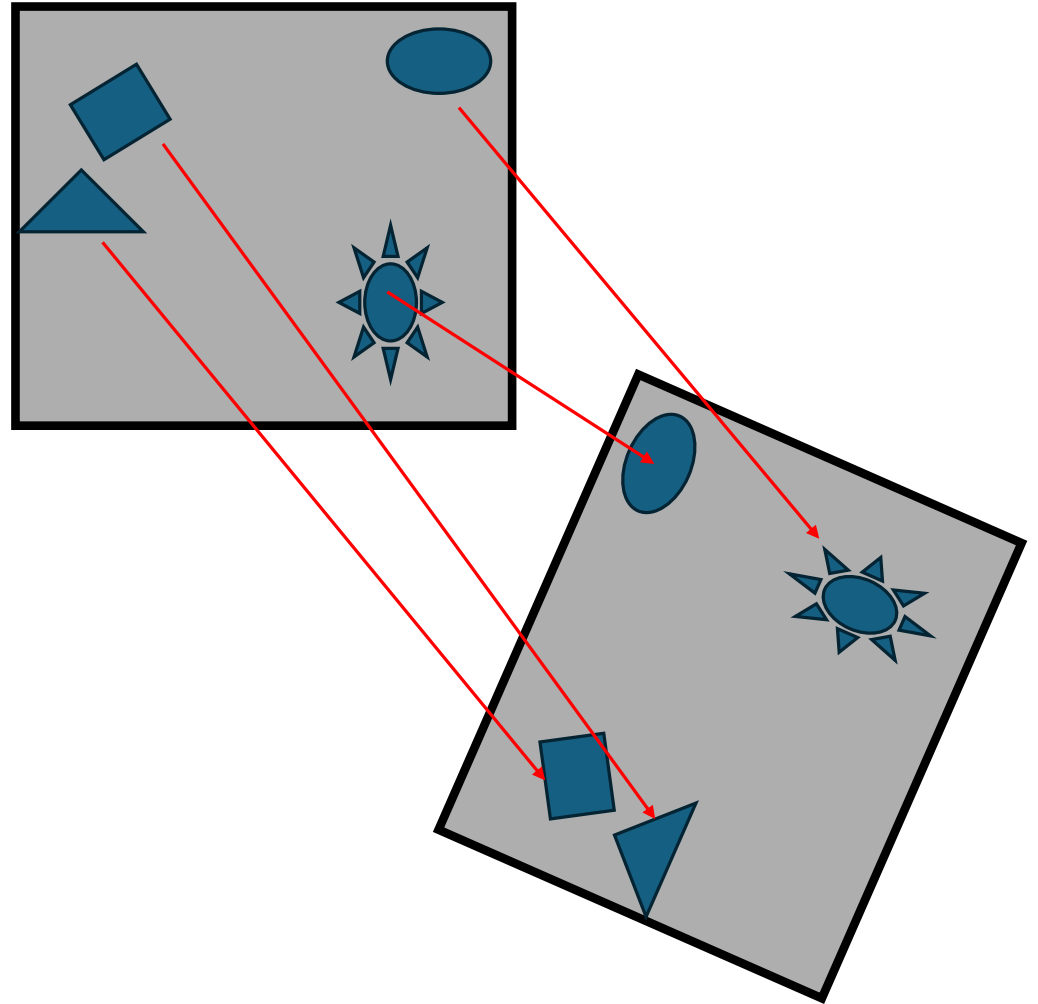
 Remove bad points

 update correspondence (lst1, lst2)

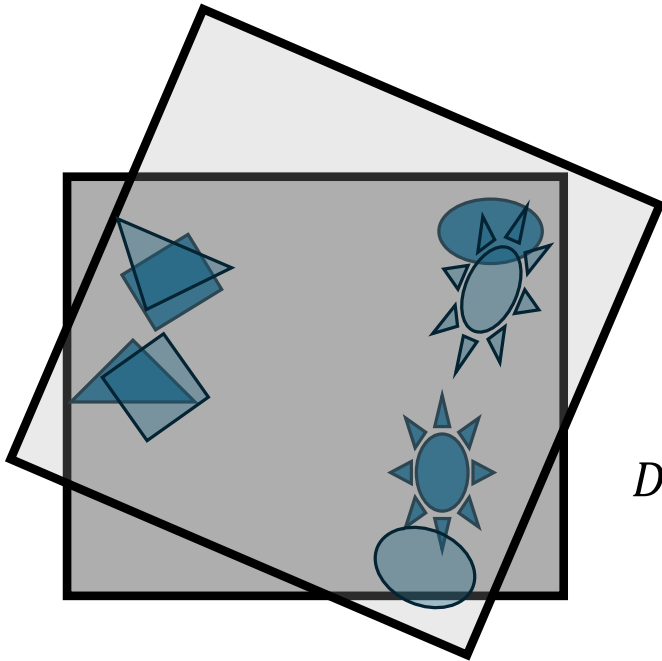


ICP (iterative closest point)

$$\begin{bmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_n & y_n & 1 \end{bmatrix} = \begin{bmatrix} x'_0 & y'_0 & 1 \\ x'_1 & y'_1 & 1 \\ x'_2 & y'_2 & 1 \\ \vdots & \vdots & \vdots \\ x'_n & y'_n & 1 \end{bmatrix} * R'$$



ICP (iterative closest point)



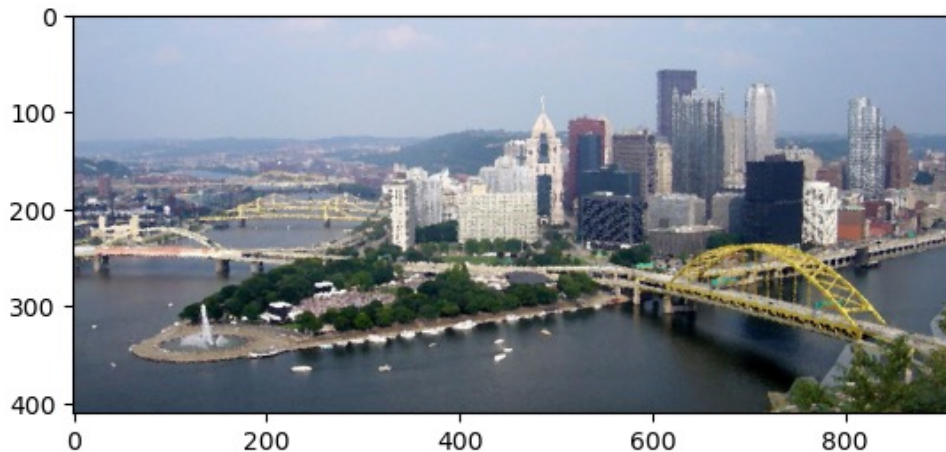
$$Distance = \sqrt{(x - x')^2 + (y - y')^2}$$

$$Distance = \sqrt{(x - x')^2 + (y - y')^2 + (shape - shape')^2}$$

Feature Descriptors

Color information

- Dominant color
 - Most prominent color in a region around point



Grab pixels in kernel around point

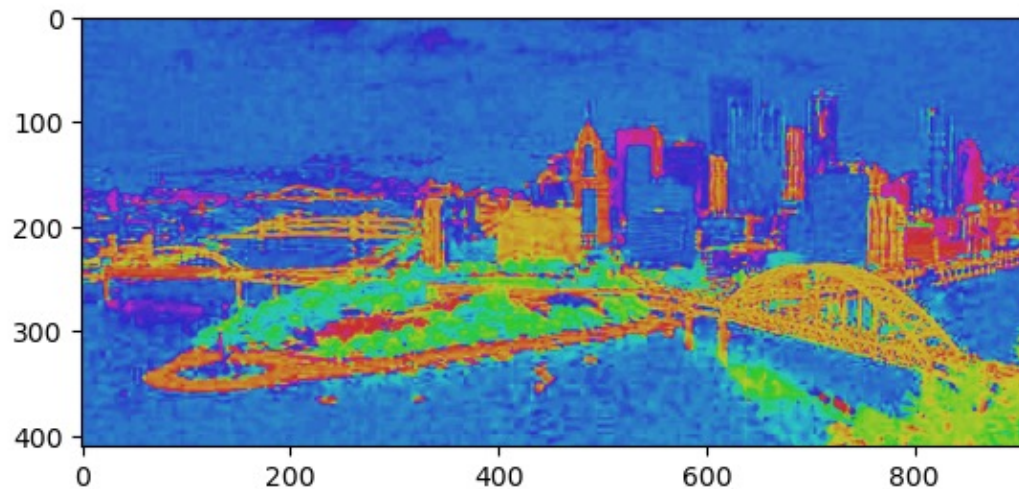
Bin to histogram or k-means cluster

Report largest bin

Feature Descriptors

Color information

- Scalable color
 - Convert to HSV → hue



Convert to HSV

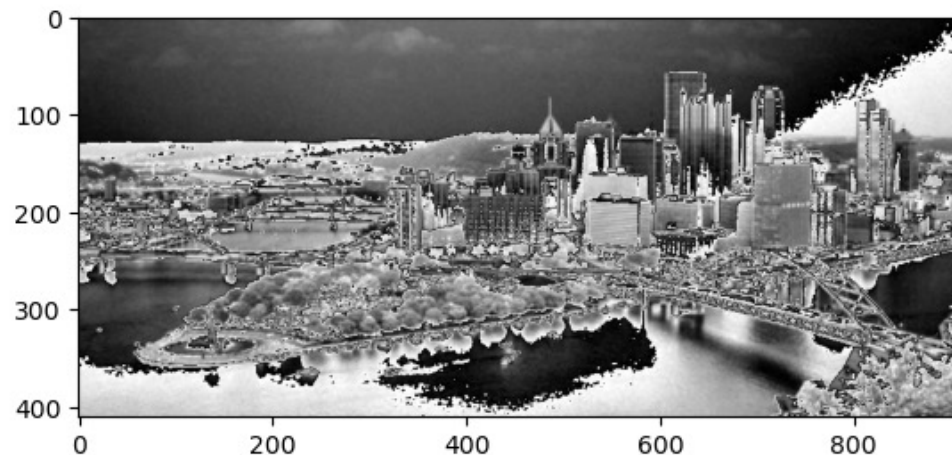
Discard S and V channels

Report Hue

Feature Descriptors

Color layout description

- Spatial color information around point
 - Convert to YCrCb format
 - Discrete Cosine Transform



Convert to YCrCr (brightness (luma),
Blue-luma, Red-luma)

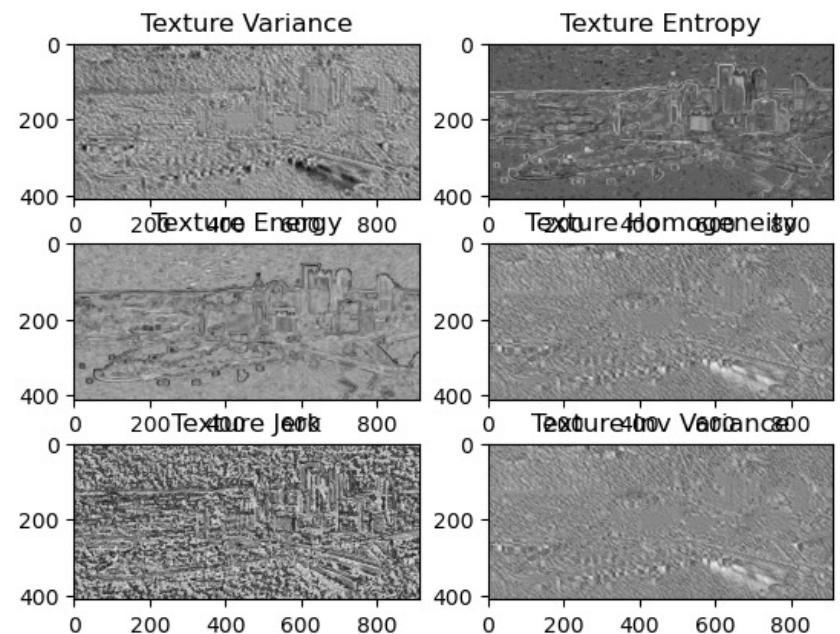
Run a DCT on brightness

Report DCT coefficients

Feature Descriptors

Statistical Texture Descriptors

- Compute the statistical properties in a region
 - Variance ($\langle x^2 \rangle$)
 - Jerk ($\langle x^3 \rangle$)
 - Homogeneity $\langle 1/(1+|x|) \rangle$
 - Inverse Var $\langle 1/(x^2) \rangle$
 - Entropy $\langle \text{prob} * \log(\text{prob}) \rangle$
 - Energy $\langle \text{prob}^2 \rangle$



Feature Descriptors

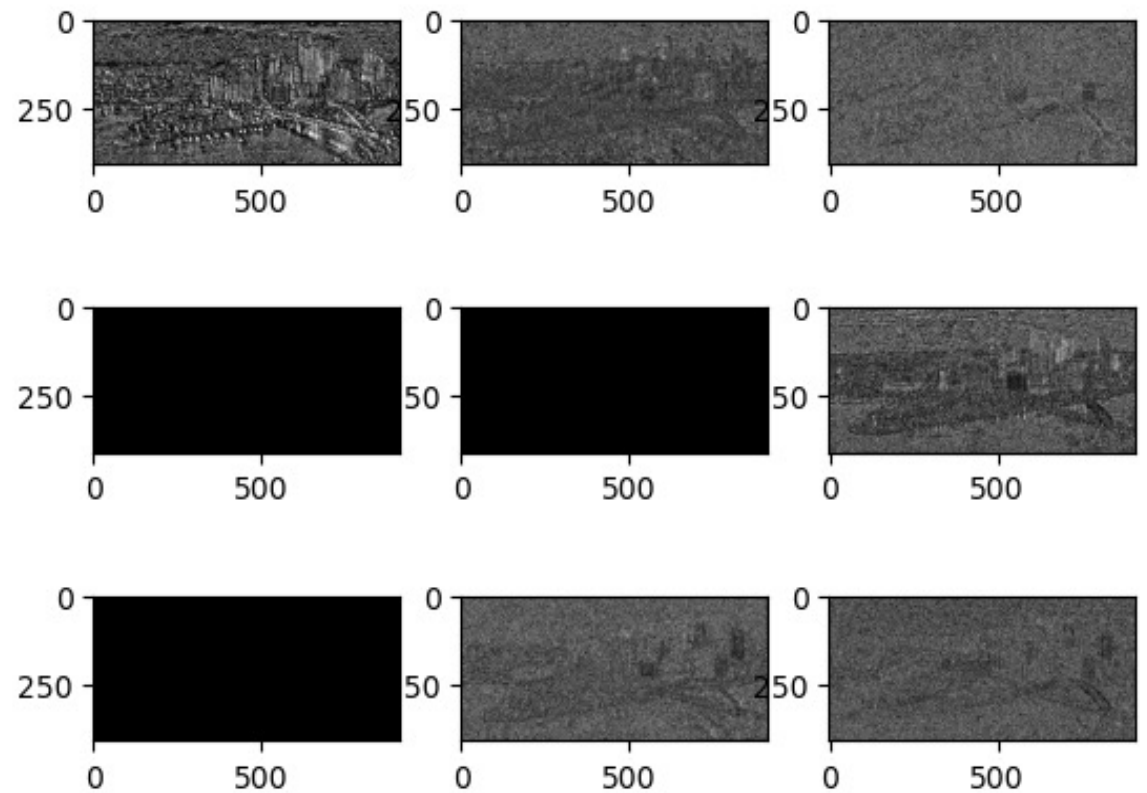
Texture Energy Descriptors

- Convolution kernels
 - [1 4 6 4 1] (ridge)
 - [-1 -2 0 +2 +1] (edge)
 - [-1 0 2 0 -1] (spot)
 - [+1 -4 +6 -4 +1] (ripple)
 - [-1 +2 0 -2 +1] (wave)

Feature Descriptors

Texture Energy Descriptors

- Convolution kernels
 - 4 x 4
 - discard symmetric
 - = 9 terms



Feature Descriptors

Structured Texture Descriptors

- Convolution kernel is texture sample
 - Regional correlation with texture sample

