

How to Write Spells

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Abstract. Whether we want to play the isolated but dangerously intelligent wizard or the hag of {insert generic name here} Bog, one often missing feature of their characterisation is how they notate their spells. In this paper, we will discuss a potential system for notating spells with a limited set of features in their attributes (for example, the level is an attribute with set features $\{0,1,2,\dots\}$) as well as detailing the underlying maths and theory that allows us to encode and decode these shapes. The essential idea behind the system is to use binary numbers to determine which sides of a shape with n -sides we draw to get a symbol to describe an individual feature. By changing shape, while using the same vertices, we can differentiate between separate attributes in a way that allows us to stack these symbols on top of each other without overlap. By removing cyclic pairs of these binary numbers, we can ensure a totally unique, rotationally asymmetric symbol for each feature of every attribute of a spell and hence unique symbols for unique spells. Giving us a basis for how magic users may write their spells. This has certain limitations in both computational times to calculate the binary numbers and the inability to encode continuous information such as exact range. Despite the limitations, this system is flexible enough to allow a huge variety of shape and line combinations such that every individual caster could create their own symbols while following the same basic principles.

1. Introduction

Spell notation is an inherently fascinating problem, while it is vital to so much of the lore of magic. In almost every context, we rarely see a system which codifies magic in a written form such that anyone could gain literacy in it and learn the necessary skills of wizardry. In many games, we are told that wizards spend hours, even days, writing spells carefully into their Tomes, yet if I were to try and write the spell myself, I would find no internal consistency at best and more likely just nonsense. This is the problem we aim to solve here by creating a notation system that is consistent but flexible enough that it can apply to any magic system, given a few simple limitations. It is important to note that, despite my scientific language and background, this notation is entirely arbitrary and non-scientific. It is not based on any evidence that I have seen. The aim is, however, to make a more grounded attempt at creating a consistent and useful spell notation style that may prove useful in further developments of my other projects, such as the Theory of Magic. In many ways, it might be considered analogous to the rules of writing equations in real life. Why we say $E = mc^2$ and not $c^2m = E$ is entirely arbitrary, but we have agreed on the rule that the primary feature E should go on the left of the '=' while the smaller components go on the right. Similarly, we write $f(x) = mx + c$ or $y = ax^2 + bx + c$ with no "scientific" basis. This is simply a disclaimer that this system is no better than any other, and if you can find a simpler method which achieves the same goals, then you should be celebrated.

Before we begin writing a system, we must identify what we want it to achieve. Of all the possible features a spell-writing system might have, there are some that I would consider desirable:

- Distinguishing between different types of attributes (e.g. if a spell has attributes such as a school of magic and damage type, we don't want these to overlap at all and be separately decodable)
- Distinguish between different features within attribute sets (For example if spells have a 'level', we wish to distinguish between these levels (level 1 should not look like level 9). Here we use 'set' to describe the list of all unique elements in another list (e.g. the set of $[0,1,0,0,1,0,2]$ is $[0,1,2]$))
- Simple encoding and decoding systems
- Aesthetics: This is arguably the least important, but if we are going to have a system by which fantasy magic is written, it is worth making an effort to make it look as good as possible without ignoring the other requirements

The proposed solution for these issues is, in effect, quite simple but in order to fully grasp the concepts involves some more detailed maths. We will discuss these details in *Section 2* and the actual notation system without these details in *Section 3*. However, the essential proposal is that we will use 'nodes' arranged as the vertices of regular polygons (the points of squares, pentagons, hexagons etc., with all sides equal length) and the shapes we can create with the straight lines connecting these vertices. Though, as we will see later, this system is general enough that we have more flexibility still beyond the limits of a regular polygon and straight lines.

2. Maths and Algorithm

The primary idea of this theory is that we use the set of vertices on a regular polygon (the corner points) and the combinations of lines connecting them. In maths, this would fall into the purview of graph theory if you are interested in learning more. Initially, we might assume that the best system then is to assign each connecting line to a certain attribute of a feature. Say, for example, we choose to represent a spell with five levels by a pentagon, with each vertex labelled $\{1, 2, 3, 4, 5\}$, we might label level 1 as the line connecting 1 to 2, level 3, line 3 to 4 etc. Thus, every level is represented by a different line segment. Then we could represent another set of features (say they are fire, ice, and water spells). We can define fire as connecting 1 and 3, ice: 2 and 4, and water: 3 and 5. Going forward, we will use a useful factor 'k' defined such that we connect a line from point 'i' to point 'i + k' (i.e. if $k = 1$, you go to the next point along, $k = 2$, you skip to the second etc.). Therefore, in the example we just used (with a pentagon and a spell only describing level and spell type), the level is described by the $k=1$ lines and the spell type is described by $k=2$. We can then determine the value of n required (the number of vertices) by the largest length of any attribute list. So, in this case, we have five levels and three types of spell $\therefore n = 5$. However, should we add four more types of spells, we then have five levels and seven types of spells $\therefore n = 7$. I will call this the 'Single-Line Scheme' (SLS).

This scheme has the huge advantage of being exceptionally simple and results in fairly easy to read notation. If this were the system we settled on, I would be able to end this section here, yet for some reason, there are yet more words, and the section is far from over. The main issue with the SLS lies in its rotational symmetry (as pointed out by "**Beeg Yoshi**" in the Theory of Magic Discord). If you imagine yourself coming across a wizard's scroll of spells, how will you know which way it was orientated when they were writing? A line connecting vertices 1 and 2 might actually be a line connecting vertices 3 and 5 had the wizard written it upside down to how you are reading. That is unless we explicitly label each vertex which seems to defeat the purpose of a simple yet informative notation. Rather than being a useful system, I would think

of the SLS as the first step in spell notation, like how we might learn simple maths and writing before moving on to more complex prose and equations. Another solution is to add a simple, rotationally asymmetric element to the symbol as standard (for example, a T in the centre). While this might work, there is another, more elegant solution.

Instead, I suggest the ‘Unique Binary Scheme’ (UBS). In this, we will seek to find a system that is rotationally asymmetric and allows us to computationally find these combinations of symbols more easily. For this, we will have to use binary numbers:

2.1. Cyclically Unique Binary Numbers

Normally when we count, we will go 1,2,3,4,5,6,7,8,9 and then 10. We count within single digits until we reach the 10th number, and then we add a second digit which tells us how many 10s we have counted up to (breath-taking insight from me, I know). So, we know that 15 represents 1×10^1 and 5×10^0 or just 5 as $10^0 = 1$. When we count to 99, we then need to move to 100, and, the third digit tells us we have counted one 10^2 (or ten tens). Therefore, 149 represents one 10^2 s, four 10^1 s and nine 10^0 s. Similarly, the fourth digit is 10^3 , the fifth 10^4 etc. This is known as base 10 and is the most common counting system we use in day-to-day life. For our purposes here, we will instead use the second most well-known base, binary (or base 2).

In this base, when we reach the number 2, we instead move up one digit, so the first digit is 2^0 , the second is 2^1 , the third 2^2 etc., instead of 10, as we saw before. Thus the number will be comprised entirely of 1s and 0s (a surprise tool that will help us later). For example, in base-10, the twenty-first number is **21** ($2 \times 10^1 + 1 \times 10^0$), whereas, in binary, 21 would be described as **10110** ($1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$). A binary number of length ‘n’ is described as an ‘n-bit’ number. The 5-bit numbers go from 00000 \rightarrow 11111. You might be thinking this was an interesting jump into some pure maths, but why should we care as people who want to write spells? Well, one way of looking at these lines connecting our vertices is to describe them as either “on” or “off” (or alternatively 1 or 0, this is why binary is often seen as the language of computers). For example, in **Figure 2.1**, we can see how the number 10110 can be used to express different shapes within a pentagon, regardless of k, as every k-shape of an n-sided regular polygon has n-sides as well.

We can also notice that the 2nd and 3rd diagrams in **Figure 2.1** are rotationally symmetric, as are the 1st and 4th. This means that in order to maintain rotational symmetry, we must only use k values up to half of n rounded down. Thus, what we must do in order to find rotationally asymmetric shapes is to:

- (i) Calculate the number of attributes ‘ $N_{Attributes}$ ’ we have and choose $n \geq 2 \times N_{Attributes} + 1$
- (ii) Draw the vertices of an n-sides shape
- (iii) Generate all n-bit binary numbers
- (iv) Remove cyclic pairs of binary numbers (which I will discuss shortly)
- (v) For each attribute, assign each feature a number in this list of binary numbers
- (vi) Use these numbers to draw the shapes to represent these features graphically
- (vii) Combine these separate feature plots into a single graph

This is the core idea presented and will likely be paraphrased later in *Section 3* for those who chose not to read the maths (not like me and you who are secretly much cooler).

Step (iv) relies on removing so-called “cyclic pairs”. To cycle a list is to move every element one step to the left or right. When the element leaves the extent of the list (the 1st element being moved left, for example), it is moved to the other side. i.e. a list [a,b,c] can be cycled to [b,c,a], [c,a,b] and eventually back to [a,b,c] after n cycles. In our scheme, this represents rotations of our shape, and thus we must remove all but one of each group of cyclic pairs.

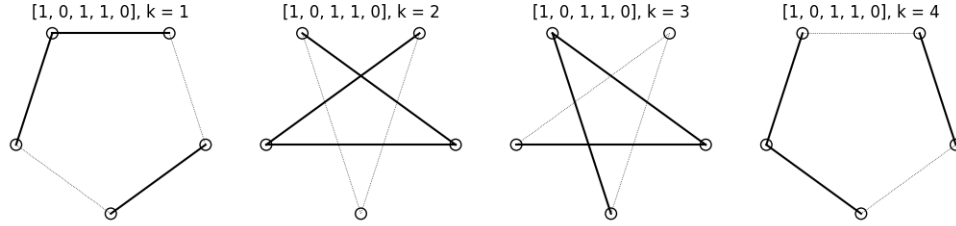


Figure 2.1. The various decoded shapes for a pentagon ($n = 5$) with input 10110 and varying k values. This shows us how we might use the same input to create different shapes. It also shows that with k values over $n/2$, the shapes become rotationally symmetric again.

Thus, we have a scheme that allows us to uniquely create shapes that can represent spell features on a set list, where a feature set is the list of all possible elements in a list ordered alphabetically for consistency.

2.2. Further Detail

One important consideration is to know how many possible shapes we have in this scheme, naively I thought this would be simple, I was wrong. The following proof **only works for prime number values of n** (only tested for up to $n = 13$). For a more exact answer you may want to look into “Pólya’s Enumeration Theorem” however, this is significantly beyond the scope of this book and a full explanation would likely take more pages than the rest of this guide. For small n ($n \leq 13$) this will be a good approximation and as we will see it likely won’t be too relevant for most casters.

First, we must calculate how many binary numbers exist in n -bits. Each bit can either be 1 or 0, or in other words, each bit has two options. To take this further: one bit has two options, two bits have two digits each with two options, thus 2^2 total numbers; for $n = 3$, we have three bits each with two options $\therefore 2^3$ combinations etc. Thus, we have in total 2^n total combinations of an n -bit binary number:

$$N_{Binary} = 2^n \quad (\text{For an } n\text{-bit number}) \quad (1)$$

The first and last numbers (0000... and 1111...) have no cyclic pairs, as every element is equal.

Thus we have:

$$N_{Cycles} = 2^n - 2 \quad (2)$$

numbers which have cyclic pairs. Each cyclic set (i.e., the number cycled until it looks the same as the initial number) has n elements thus in total we have:

$$N_{Uniques} \approx \frac{2^n - 2}{n} + 2 \quad (\text{only exact when 'n' is prime}) \quad (3)$$

total unique numbers. This is the step where the calculation stops working for non-prime 'n' as not all cyclic sets have n -elements. This lets us know how many different features of each attribute we can encode with our shape; however, it is worth noting that at higher n (which is likely for a large number of attributes), $N_{Uniques}$ is likely far larger than n (e.g. for $n = 11$, which would work for at most five different attributes $N_{Uniques} = 188$). For clarity in the Binary Dictionary (*Section 5* I have included the number of unique elements.

2.3. Variations of Notation

2.3.1. Centred Circles

Finally, as we will see in *Section 3*, the lines which connect points need not be straight as we have been using so far. The first variation I will discuss here is "Centred Circles". If we want to connect two points, $P = (x_1, y_1)$ and $Q = (x_2, y_2)$, with a circle, the simplest is with the following steps. First, we set the circle centre (a, b) as the midpoint between the two points:

$$a = \frac{x_1 + x_2}{2} \quad b = \frac{y_1 + y_2}{2} \quad (4)$$

we can then define the radius 'r' as the distance between the centre and one of the points:

$$r = \sqrt{(x_1 - a)^2 + (y_1 - b)^2} \quad (5)$$

Finally, we can limit the circle such that the arc only exists within the polygon's outermost borders (should that be something you want, this step could always be ignored by a more chaotic caster) by defining parametric equations:

$$x(\theta) = r \cos \theta + a \quad \{\theta_0 \leq \theta \leq \theta_1\} \quad (6)$$

$$y(\theta) = r \sin \theta + b \quad \{\theta_0 \leq \theta \leq \theta_1\} \quad (7)$$

where:

$$\theta_0 = \arctan \frac{y_1 - b}{x_1 - a} \quad \text{and} \quad \theta_1 = \arctan \frac{y_2 - b}{x_2 - a} \quad \text{if } y_1 \leq y_2 \quad (8)$$

or

$$\theta_1 = \arctan \frac{y_1 - b}{x_1 - a} + \pi \quad \text{and} \quad \theta_0 = \arctan \frac{y_2 - b}{x_2 - a} + \pi \quad \text{if } y_2 < y_1 \quad (9)$$

An example of this scheme is shown below:

2.3.2. Non-Centred Circles

The natural extension of the Centred Circle system is to make a similar system with circular arcs connecting points, except with the centre of these circles not being the midpoint of the two connecting points. This is simply the generalisation of the circular scheme to include a much larger variety of connecting lines (as we will see later). To derive the arcs describing these equations, we must first look at the general equation of a circle. All circles in 2D space (x,y) can be described by:

$$r^2 = (x - a)^2 + (y - b)^2 \quad (10)$$

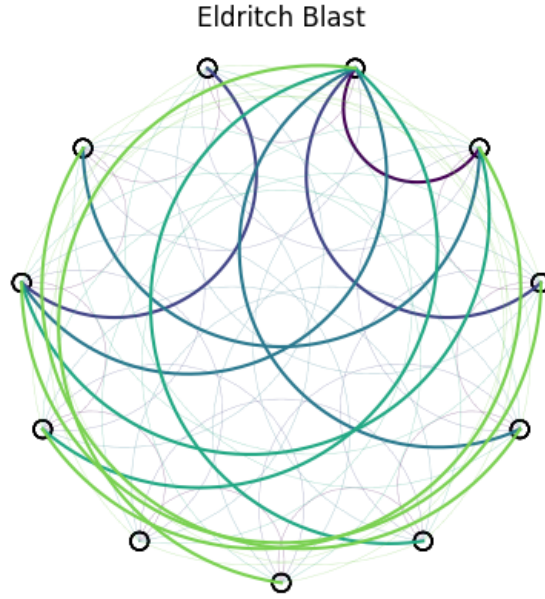


Figure 2.2. Eldritch Blast from 5e as described by a centred circular diagram

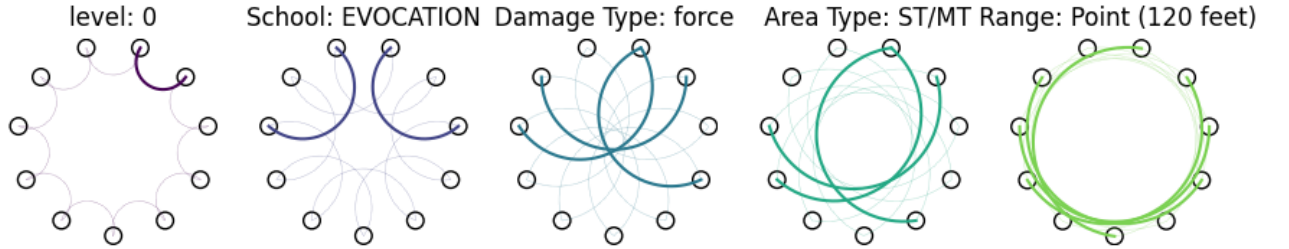


Figure 2.3. A breakdown of the centre-circular representation of 5e's Eldritch Blast diagram

Where (a, b) is the centre of the circle and r is the radius. If we are connecting points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ with the same circle, we can replace 'x' and 'y' in *Equation 10* with these points' values. Since P and Q will both lie on this mystery circle, they will have the same radius:

$$(x_1 - a)^2 + (y_1 - b)^2 = r^2 = (x_2 - a)^2 + (y_2 - b)^2 \quad (11)$$

We can then solve for the centre 'a':

$$a = \frac{x_1^2 - x_2^2 + y_1^2 - y_2^2 - 2b(y_1 - y_2)}{2(x_1 - x_2)} \quad (12)$$

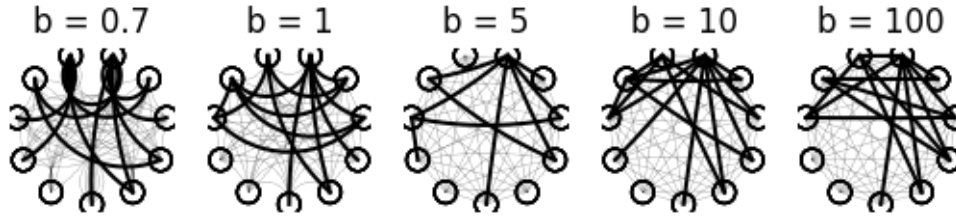


Figure 2.4. How varying values of b change how spell notation appear

This means we can then calculate the radius:

$$r = \sqrt{(x_1 - a)^2 + (x_2 - b)^2} \quad (13)$$

We can see here that ‘ b ’ is somewhat arbitrary. Since we centre our polygon on $(0,0)$, we calculate ‘ a ’ and ‘ r ’ twice, once as above and again with the negative ‘ b ’ value. We then choose the values with a smaller arc length such that our circular arcs are within the bounds of our polygon. We can then use *Equations 6 and 7* with θ_0 and θ_1 defined such that we choose the shorter arc of the connecting circle. By varying the value of ‘ b ’ we can change how our connecting lines arc as shown in **Figure 2.4**

There is also nothing stopping a caster from using even more chaotic connecting lines, but for simplicity, we will stop here. However, I think this inherent creativity is perfect for the overall goal of our system as even with varying line styles, we can still use the same base rules of connections making our system decodable and personal (similar to handwriting, though sometimes I question if mine is as decodable as I like to think).

2.3.3. Non-Polygon Bases

Another interesting way we can vary our notation and add that all-important personal spark is that we can also change our base shape such that it isn’t a regular polygon. So long as the points can be clearly connected, we can notate spells in multiple different base styles. For example, Non-centred circles with base points in a semi-circular shape **Figure 2.5**. Non-centred circle arcs connecting base points defined by the cubic function with general form $y = ax^3 + bx^2 + cx + d$ **Figure 2.6**. Non-centred circle arcs connecting base points defined by the quadratic function of general form $y = ax^2 + bx + c$ **Figure 2.7**. A quarter circle base **Figure 2.8**. And finally, quadratic (**Figure 2.9**) and quarter circles (**Figure 2.10**) with straight-line connections. The benefit of this is the flexibility of the scheme to have unique shapes and combinations for every individual caster, both in their use of shapes and linestyle.

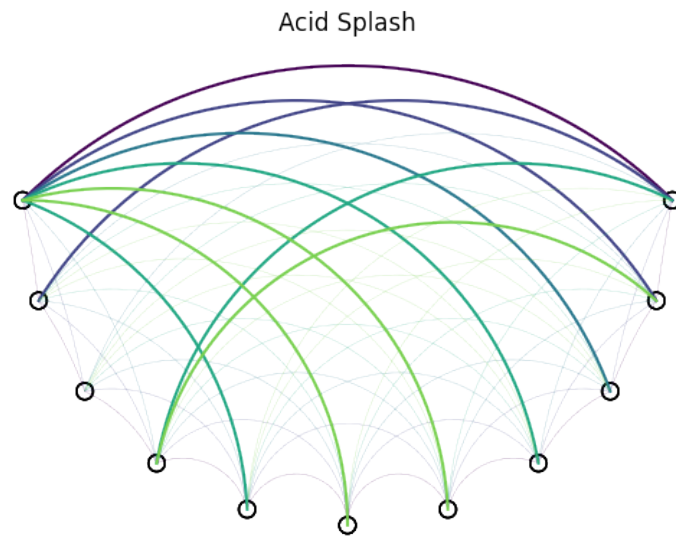


Figure 2.5. Acid Splash written with non-centred circle arc connections and a semi-circular base

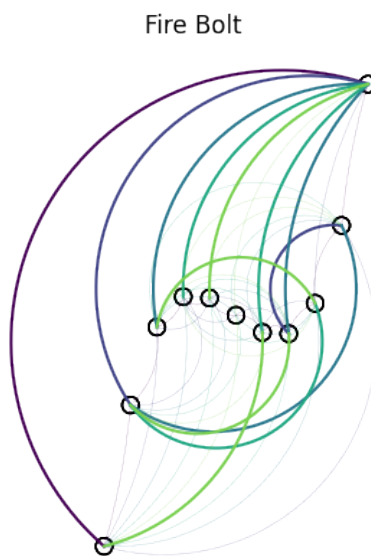


Figure 2.6. Fire Bolt written with non-centred circle arc connections and a cubic function base

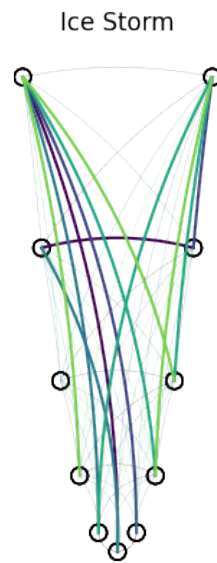


Figure 2.7. Ice Storm written with non-centred circle arc connections and a quadratic function base

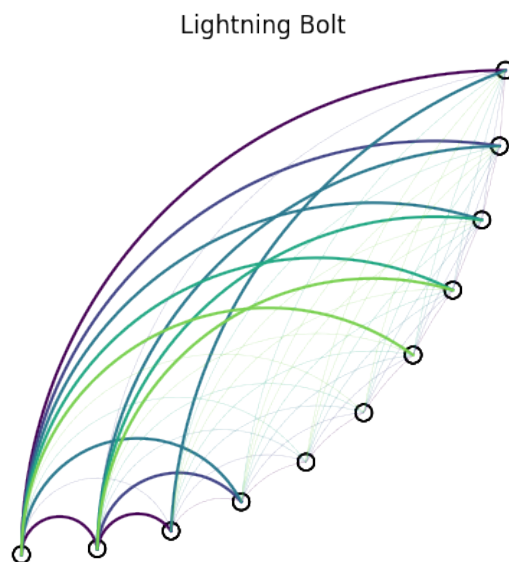


Figure 2.8. Lightning Bolt written with non-centred circle arc connections and a quarter-circle base

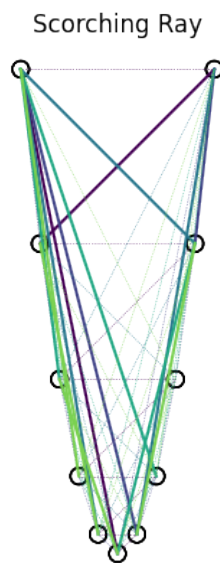


Figure 2.9. Scorching Ray written with straight-line connections and a quadratic base

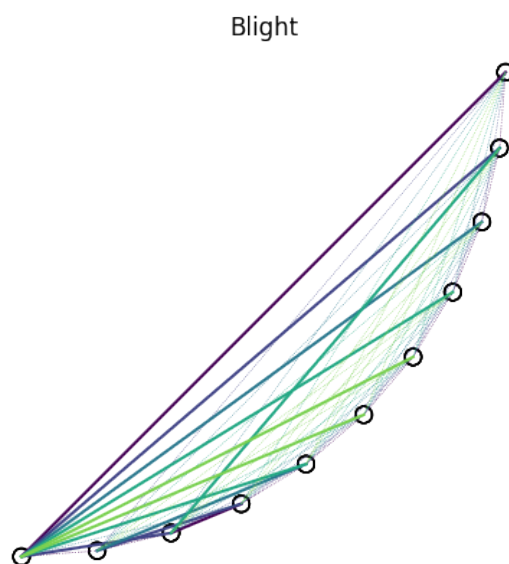


Figure 2.10. Blight written with straight-line connections and a quarter-circle base

3. Qualitative Notation System

Above was a lot of detailed maths and exploration of the concepts that underline this process. For details on why certain decisions have been made, you may wish to read that section (if you haven't already), but otherwise, this section will act as a more qualitative explanation of how to write spells. To quickly cover some of the language we are using, attributes refer to the number of different types of information we wish to encode about our spell (e.g. ['level', 'range', 'damage type' etc.], features are the individual elements within the set of attributes (e.g. levels 0,1,2,3,...,9 are features of the level attribute), with this we can describe the process as follows:

- (i) Choose the number of attributes you wish to encode, then draw a regular polygon (or another shape if you wish) with $2 \times (\text{number of attributes}) + 1$ (e.g. five attributes means we draw an 11-sided shape)
- (ii) List the features you are going to use (i.e. what level, range, and damage type the spell does)
- (iii) Either:
- (iv) Identify the relevant shapes for each feature in the dictionary below and copy these into your diagram
- (v) find a cyclically unique binary number (also in the dictionary below) and connect the points using the number as a guide (I will detail that below as well).
- (vi) Combine your individual features into one single diagram.

To read the binary numbers and no longer rely on the shapes is relatively straight forwards. First, you must pick a starting point (by design, the shapes are rotationally asymmetric, so this choice shouldn't matter, but personally, I like to start at the highest point, the rightmost if there are two). Then you read the first element of the binary number; if it is a 1, you draw the line connecting that point to the point 'k' steps along. Where different values of 'k' represents different spell attributes. The exact order you write attributes is not strictly important if you are consistent, but given my experience with notation it is probably worth noting down what attribute each 'k' represents. If the element is a 0, you do not draw the line. You continue this process until you reach the end of the binary number, et voila; you have a spell shape.

In order to create your own dictionaries, an understanding of the maths may be necessary, but below I will try to include several options. Another detail is that the dictionaries will not include variations in line connections, but you may style them as you please so long as they are distinguishable. There are, of course, some more interesting consequences this system might have within your world.

3.1. Lore Based Consequences

3.1.1. Cluster Spells

These are spells which share an edge, though difficult to find if you use a system with regular polygons (say a hexagon, which famously tile together in a very satisfying way). You can draw them in a tiled pattern so long as the shared sides are both "On" or "Off".

The discovery of these might prove difficult, but perhaps, when written, the effects might range from casting multiple spells at once to simply having a more efficient use of space in your spell book (which is obviously the far cooler option). The search for these cluster (or complimentary) spells may be a field of research of its own, trying to find spells which not only work together on paper but complement each other's effects.

3.1.2. Stellar Spells

What are stars but points in the sky? Since we have shown that the base shape of a spell does not necessarily need to be regular or sensible, could it be possible that, like ancient civilisations

in the real world, druids, wizards, and other scholars of the night sky have learned ancient spells by learning constellation shapes. Could this lead to casters who only know spells seasonally as they study the ever-moving sky? How might this be affected by elements that move in stranger ways? Would they be disturbed, or perhaps enhanced, by a different stellar body like the moon or a distant planet coming into contact with their constellations?

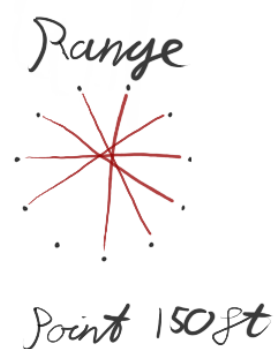
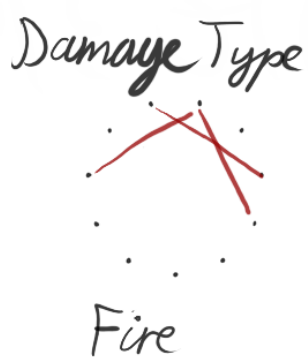
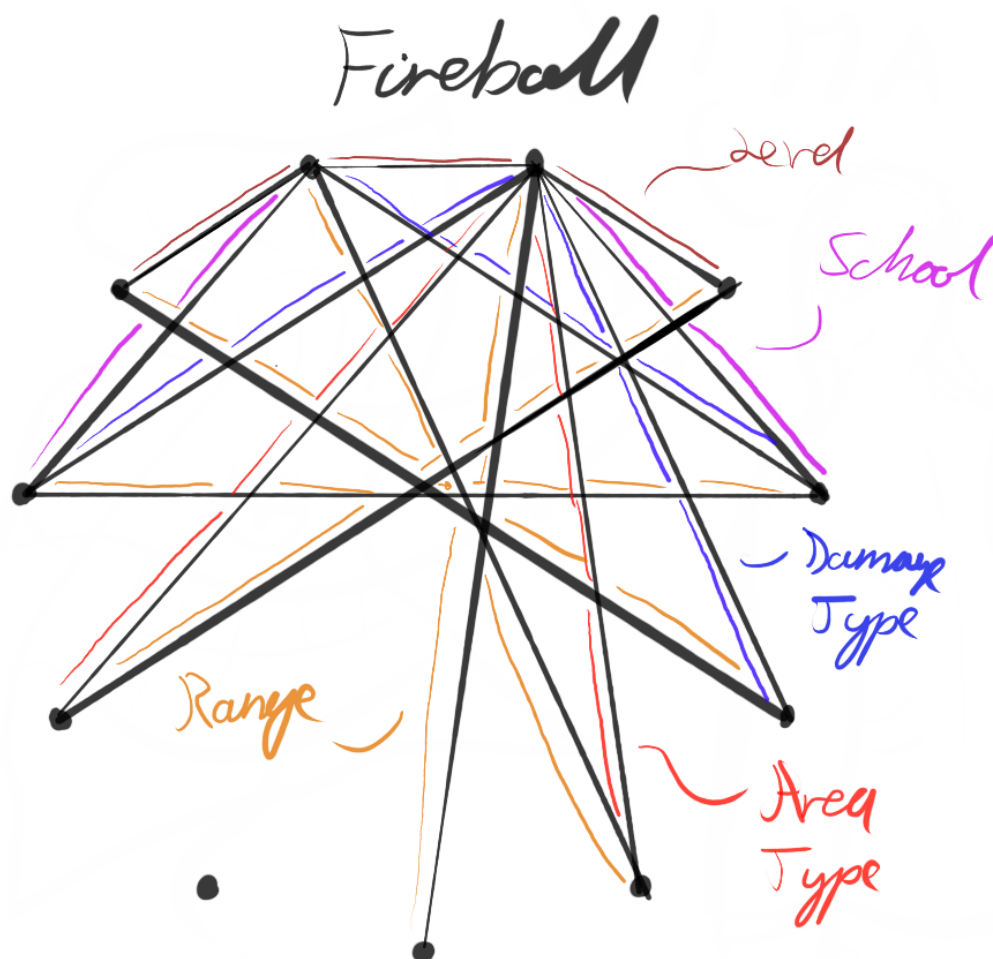
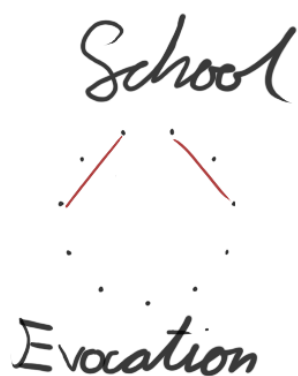
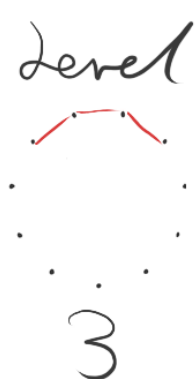
3.1.3. Enchantment

Another possible use of this system is for simple notes by those who imbue objects with magical enchantments. Perhaps the blacksmith writes the simple shape for “Fire” and the level on her latest sword so she can remember not to reveal its hidden power in a small wooden cottage. Perhaps these symbols might even work as warnings to other casters. While they might be hard to decode quickly, a cautious caster who takes the time to read them, could learn that the seemingly endless cave of ice is not as inviting as it first seemed.

3.2. Conclusion

While the implications of this system stretch as far as your own imagination, there are a few things I’d like to note. This notation system is system agnostic but does require spell attributes to have sets of features and not continuous values (for example, I can say “Range is 40 meters” in my notes, but I cannot say that in this specific instance, I am actually casting the spell to 26.451 meters). There are also aesthetic challenges where it can often become difficult to read without sitting down and decoding. I would be thrilled to see others take this work further and develop simpler systems if they can.

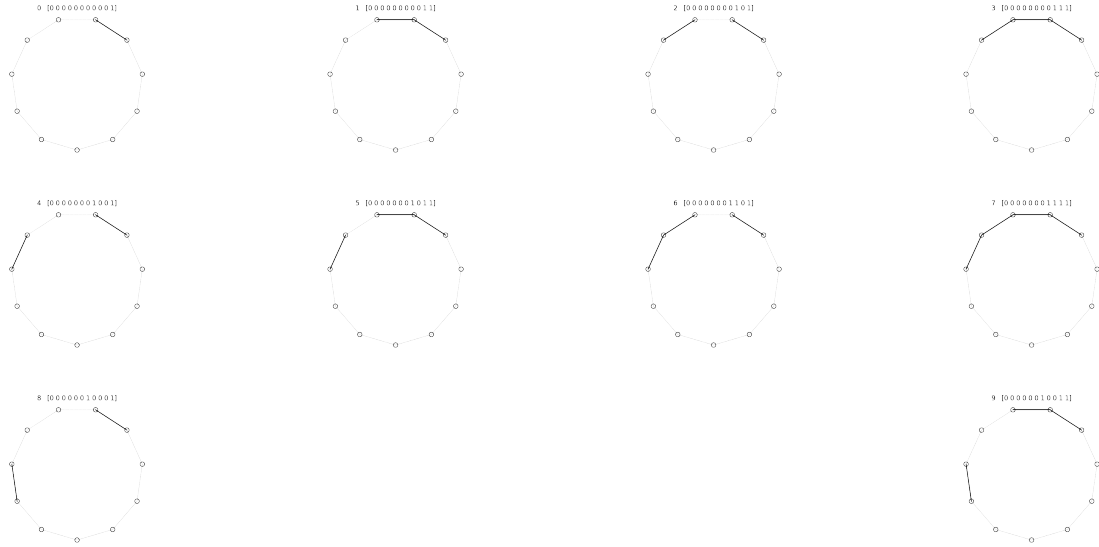
In spite of this, I believe this system to be flexible enough to work for almost any magic system with hard rules while also aesthetically flexible enough to allow casters to show their individuality through their notation without breaking the rules. Below is an example page of a Wizard’s spell book using this system, and further down is the “Dictionary” which I will split into two parts, the first being diagrams in various shapes so that if you wish, you can simply copy the diagrams for your own spell writing needs, with their corresponding binary numbers to their side. For more general purposes, I will also include the list of binary numbers for each spell shape up to $N = 13$ (the number of numbers increases exponentially, so beyond that would probably start breaking computers if we weren’t careful).



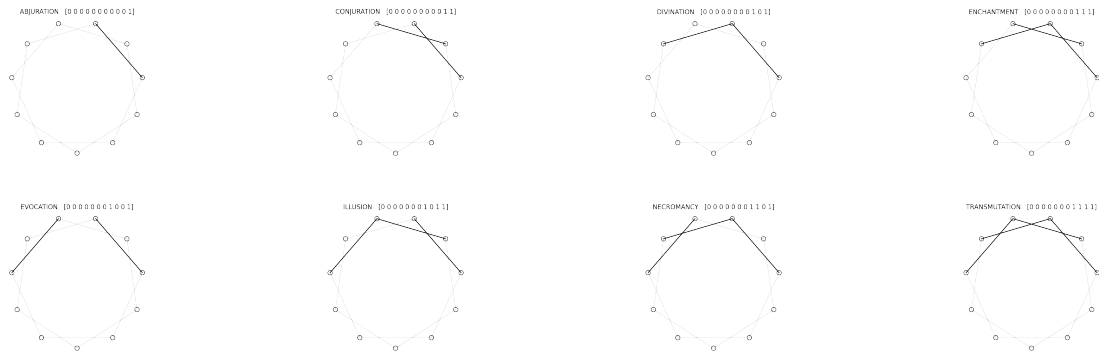
4. Symbolic Dictionary

These are simply example spell shapes for the attributes Level, School, Damage Type, Area, and Range. These are the examples that I believe would be most useful for 5e, though you may choose to use higher or lower values of n .

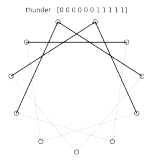
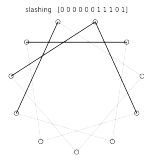
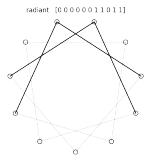
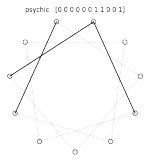
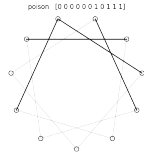
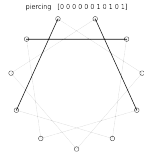
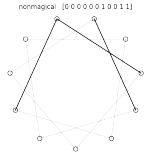
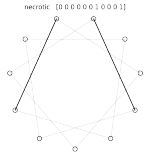
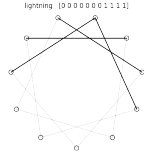
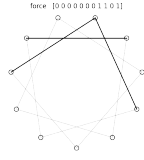
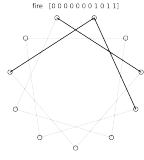
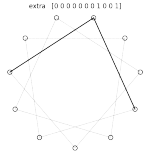
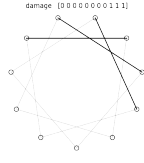
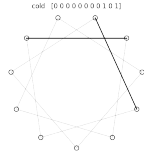
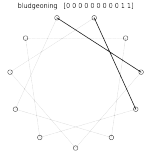
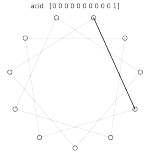
4.1. Level, $k = 1$



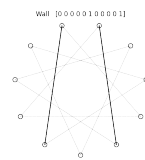
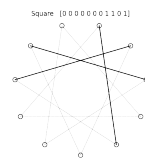
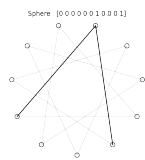
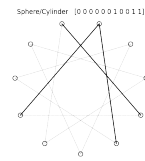
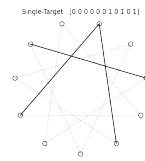
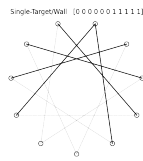
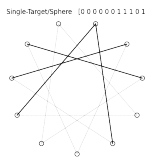
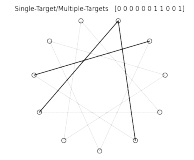
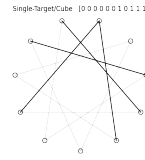
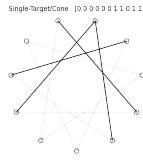
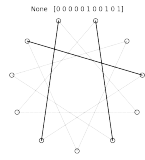
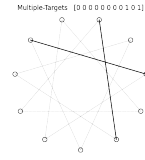
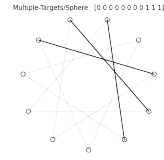
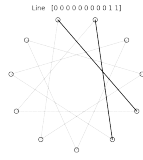
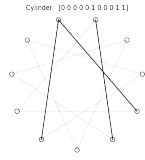
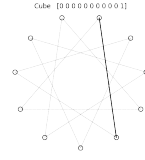
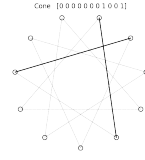
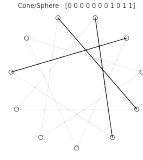
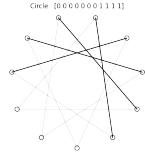
4.2. School, $k = 2$



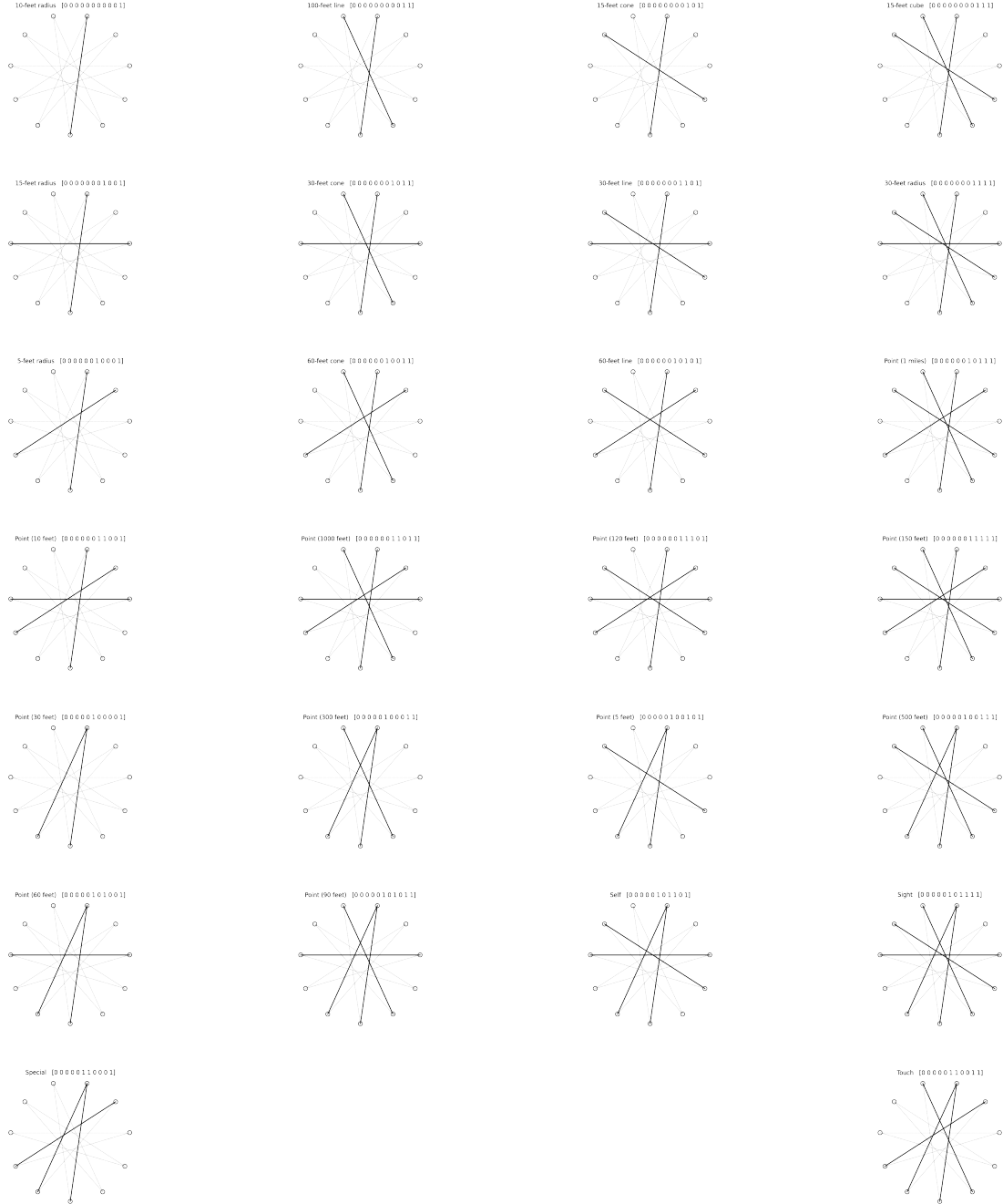
4.3. $Damage_{Type}, k = 3$



4.4. Areas, $k = 4$



4.5. Range, $k = 5$



5. Binary Dictionary

5.1. $n = 3$

$N_{Uniques} = 4$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

5.2. $n = 4$
 $N_{Uniques} = 6$

[0 0 0 0]
 [0 0 0 1]

[0 0 1 1]
 [0 1 0 1]

[0 1 1 1]
 [1 1 1 1]

5.3. $n = 5$
 $N_{Uniques} = 8$

[0 0 0 0 0]
 [0 0 0 0 1]
 [0 0 0 1 1]

[0 0 1 0 1]
 [0 0 1 1 1]
 [0 1 0 1 1]

[0 1 1 1 1]
 [1 1 1 1 1]

5.4. $n = 6$
 $N_{Uniques} = 14$

[0 0 0 0 0 0]
 [0 0 0 0 0 1]
 [0 0 0 0 1 1]
 [0 0 0 1 0 1]
 [0 0 0 1 1 1]

[0 0 1 0 0 1]
 [0 0 1 0 1 1]
 [0 0 1 1 0 1]
 [0 0 1 1 1 1]
 [0 1 0 1 0 1]

[0 1 0 1 1 1]
 [0 1 1 0 1 1]
 [0 1 1 1 1 1]
 [1 1 1 1 1 1]

5.5. $n = 7$
 $N_{Uniques} = 20$

[0 0 0 0 0 0 0]
 [0 0 0 0 0 0 1]
 [0 0 0 0 0 1 1]
 [0 0 0 0 1 0 1]
 [0 0 0 0 1 1 1]
 [0 0 0 1 0 0 1]
 [0 0 0 1 0 1 1]

[0 0 0 1 1 0 1]
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 [0 0 1 0 0 1 1]
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[0 0 1 1 1 1 1]
 [0 1 0 1 0 1 1]
 [0 1 0 1 1 1 1]
 [0 1 1 0 1 1 1]
 [0 1 1 1 1 1 1]
 [1 1 1 1 1 1 1]

5.6. $n = 8$
 $N_{Uniques} = 36$

[0 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 1]
 [0 0 0 0 0 0 1 1]
 [0 0 0 0 0 1 0 1]
 [0 0 0 0 0 1 1 1]
 [0 0 0 0 1 0 0 1]
 [0 0 0 0 1 0 1 1]
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 [0 0 0 1 0 0 1 1]
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 [0 1 1 0 1 1 1 1]
 [0 1 1 1 0 1 1 1]
 [0 1 1 1 1 1 1 1]
 [1 1 1 1 1 1 1 1]

5.7. $n = 9$
 $N_{Uniques} = 60$

[0 0 0 0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0 0 1]
 [0 0 0 0 0 0 0 0 1 1]
 [0 0 0 0 0 0 0 1 0 1]
 [0 0 0 0 0 0 0 1 1 1]
 [0 0 0 0 0 0 1 0 0 1]
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 [0 0 0 1 0 0 1 0 1]
 [0 0 0 1 0 0 1 1 1]

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 [0 0 0 1 0 1 0 1 1]
 [0 0 0 1 0 1 1 0 1]
 [0 0 0 1 0 1 1 1 1]
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[0 0 1 0 1 1 1 1 1]
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 [0 1 1 0 1 1 0 1 1]
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 [0 1 1 1 0 1 1 1 1]
 [0 1 1 1 1 1 1 1 1]
 [1 1 1 1 1 1 1 1 1]

5.8. $n = 10$
 $N_{Uniques} = 108$

[0 0 0 0 0 0 0 0 0 0]
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 [0 0 0 0 0 0 0 1 0 1]
 [0 0 0 0 0 0 0 1 1 1]
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 [0 0 0 0 1 0 0 1 1 1]
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 [0 0 1 1 0 0 1 1 0 1]

[0 0 1 1 0 0 1 1 1 1]	[0 0 1 1 1 1 0 1 0 1]	[0 1 0 1 1 0 1 1 1 1]
[0 0 1 1 0 1 0 1 0 1]	[0 0 1 1 1 1 0 1 1 1]	[0 1 0 1 1 1 0 1 1 1]
[0 0 1 1 0 1 0 1 1 1]	[0 0 1 1 1 1 1 0 1 1]	[0 1 0 1 1 1 1 0 1 1]
[0 0 1 1 0 1 1 0 1 1]	[0 0 1 1 1 1 1 1 0 1]	[0 1 0 1 1 1 1 1 1 1]
[0 0 1 1 0 1 1 1 0 1]	[0 0 1 1 1 1 1 1 1 1]	[0 1 1 0 1 1 0 1 1 1]
[0 0 1 1 0 1 1 1 1 1]	[0 1 0 1 0 1 0 1 0 1]	[0 1 1 0 1 1 1 1 1 1]
[0 0 1 1 1 0 0 1 1 1]	[0 1 0 1 0 1 0 1 1 1]	[0 1 1 1 0 1 1 1 1 1]
[0 0 1 1 1 0 1 0 1 1]	[0 1 0 1 0 1 1 0 1 1]	[0 1 1 1 1 0 1 1 1 1]
[0 0 1 1 1 0 1 1 0 1]	[0 1 0 1 0 1 1 1 1 1]	[0 1 1 1 1 1 1 1 1 1]
[0 0 1 1 1 0 1 1 1 1]	[0 1 0 1 1 0 1 0 1 1]	[1 1 1 1 1 1 1 1 1 1]

5.9. $n = 11$

$N_{Uniques} = 188$

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[0 0 0 0 0 0 0 0 0 0 1]	[0 0 0 0 1 0 0 1 0 1 1]	[0 0 0 1 0 0 1 1 1 1 1]
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[0 0 0 0 0 0 0 0 1 0 1]	[0 0 0 0 1 0 0 1 1 1 1]	[0 0 0 1 0 1 0 0 1 0 1]
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[0 0 0 0 0 0 1 0 0 0 1]	[0 0 0 0 1 0 1 1 0 1 1]	[0 0 0 1 0 1 1 0 0 1 1]
[0 0 0 0 0 0 1 0 0 1 1]	[0 0 0 0 1 0 1 1 1 0 1]	[0 0 0 1 0 1 1 0 1 0 1]
[0 0 0 0 0 0 1 0 1 0 1]	[0 0 0 0 1 0 1 1 1 1 1]	[0 0 0 1 0 1 1 0 1 1 1]
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[0 0 0 0 0 1 0 0 0 1 1]	[0 0 0 0 1 1 0 1 1 1 1]	[0 0 0 1 1 0 0 1 0 1 1]
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[0 0 0 0 0 1 0 1 1 0 1]	[0 0 0 0 1 1 1 1 0 0 1]	[0 0 0 1 1 0 1 0 1 1 1]
[0 0 0 0 0 1 0 1 1 1 1]	[0 0 0 0 1 1 1 1 0 1 1]	[0 0 0 1 1 0 1 1 0 0 1]
[0 0 0 0 0 1 1 0 0 0 1]	[0 0 0 0 1 1 1 1 1 0 1]	[0 0 0 1 1 0 1 1 0 1 1]
[0 0 0 0 0 1 1 0 0 1 1]	[0 0 0 0 1 1 1 1 1 1 1]	[0 0 0 1 1 0 1 1 1 0 1]
[0 0 0 0 0 1 1 0 1 0 1]	[0 0 0 1 0 0 0 1 0 0 1]	[0 0 0 1 1 0 1 1 1 1 1]
[0 0 0 0 0 1 1 0 1 1 1]	[0 0 0 1 0 0 0 1 0 1 1]	[0 0 0 1 1 1 0 0 1 0 1]
[0 0 0 0 0 1 1 1 0 0 1]	[0 0 0 1 0 0 0 1 1 0 1]	[0 0 0 1 1 1 0 0 1 1 1]
[0 0 0 0 0 1 1 1 0 1 1]	[0 0 0 1 0 0 0 1 1 1 1]	[0 0 0 1 1 1 0 1 0 0 1]
[0 0 0 0 0 1 1 1 1 0 1]	[0 0 0 1 0 0 1 0 0 1 1]	[0 0 0 1 1 1 0 1 0 1 1]
[0 0 0 0 0 1 1 1 1 1 1]	[0 0 0 1 0 0 1 0 1 0 1]	[0 0 0 1 1 1 0 1 1 0 1]
[0 0 0 0 1 0 0 0 0 1 1]	[0 0 0 1 0 0 1 0 1 1 1]	[0 0 0 1 1 1 0 1 1 1 1]
[0 0 0 0 1 0 0 0 1 0 1]	[0 0 0 1 0 0 1 1 0 0 1]	[0 0 0 1 1 1 1 0 0 1 1]
[0 0 0 0 1 0 0 0 1 1 1]	[0 0 0 1 0 0 1 1 0 1 1]	[0 0 0 1 1 1 1 0 1 0 1]

[0 0 0 1 1 1 1 0 1 1 1]
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5.10. $n = 12$
 $N_{Uniques} = 352$

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5.11. $n = 13$
 $N_{Uniques} = 632$

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6. Acknowledgments

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