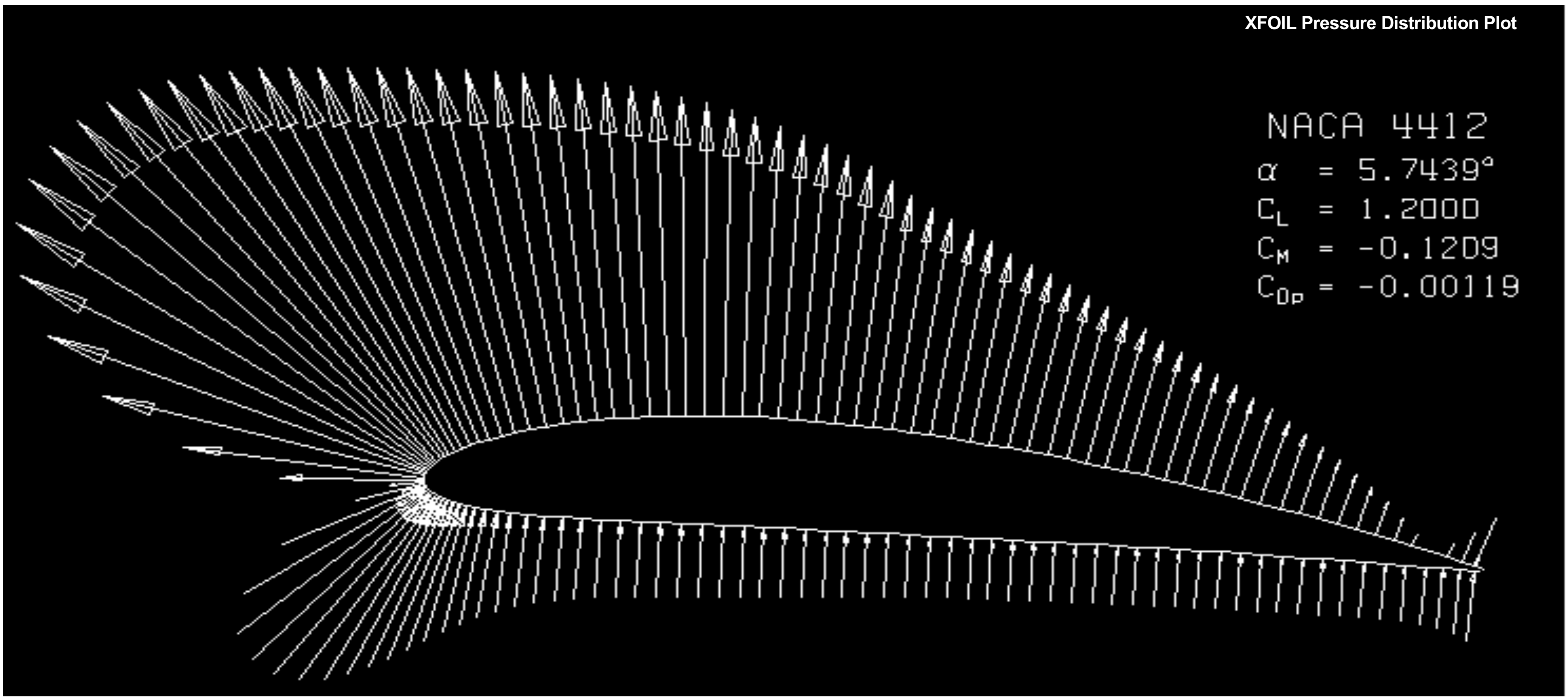
Lecture 2

Lift
2D Airfoils & 3D Wings



Lift and Drag

- **Lift: The aerodynamic force produced by a surface in the direction normal to the velocity vector**

- **Drag: The aerodynamic force parallel to the velocity vector**

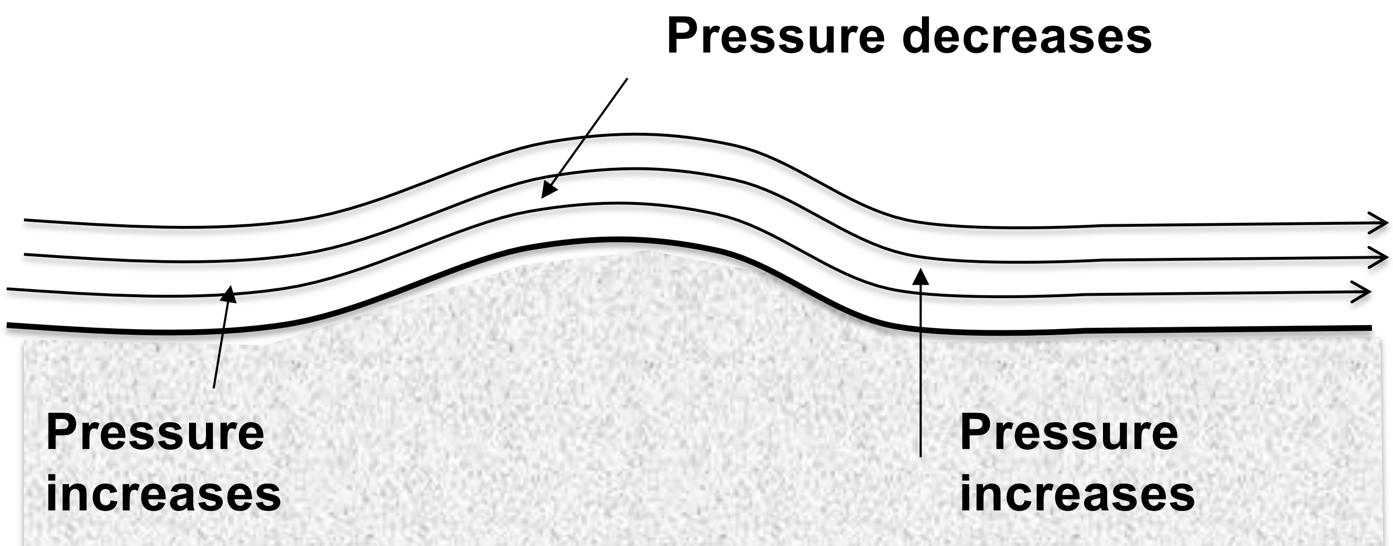
$$L = \frac{1}{2}\rho V^2 S C_L$$

$$D = \frac{1}{2}\rho V^2 S C_D$$

S = reference area – wing area
ρ = density of air
C_L= lift coefficient
C_D= drag coefficient
q = ½ ρV^2, dynamic pressure

Where does lift come from?

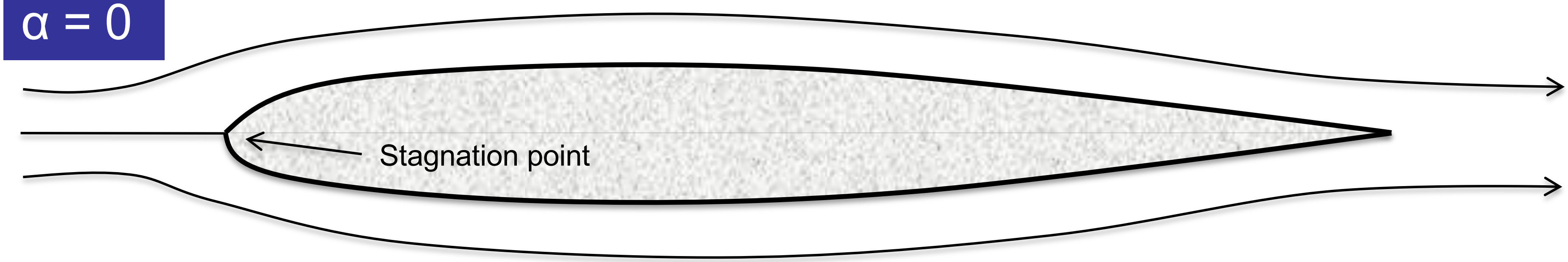
- **Transverse pressure gradient forces fluid to follow prescribed path**
  - **Pressure increases when turned by a concave surface**
  - **Pressure decreases when turned by a convex surface**



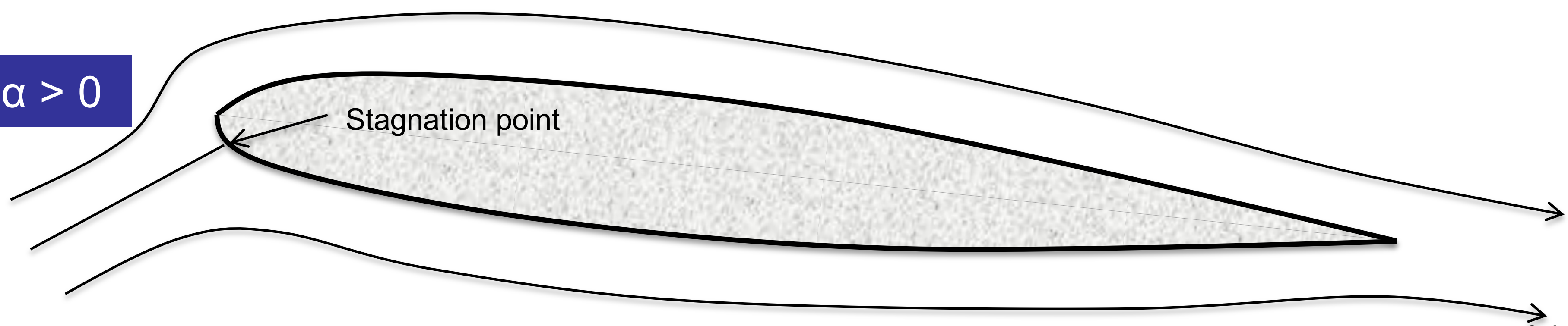
Where does lift come from?

- **Pressures acting on airfoil come from the airfoil shape and the angle of attack of the oncoming air**
  - **When the angle of attack = 0, a symmetric airfoil will see 0 net lift since the pressure distribution is symmetric**
  - **At a positive angle of attack, the pressure gradients are no longer balanced. The flow on the upper whips around the leading edge, and this leads to a more severe pressure change**

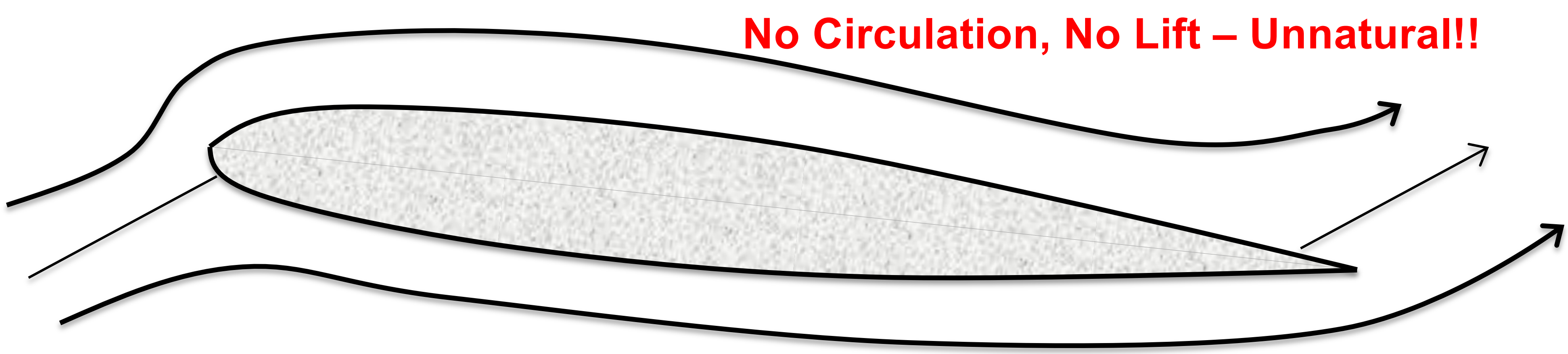
$\alpha = 0$



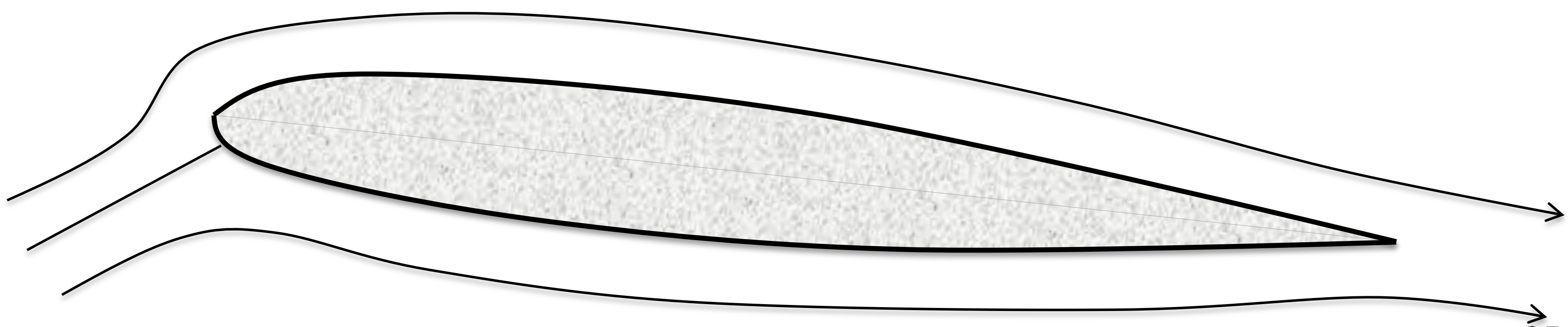
$\alpha > 0$



Where does lift come from?



- **Kutta Condition: Air does not want to turn the corner at a sharp trailing edge. Rear stagnation point will be located at trailing edge so air can flow smoothly**
- **Circulation is generated to enforce Kutta condition. As angle of attack increases, the amount of circulation needed to satisfy Kutta condition will also increase, and more lift is generated**

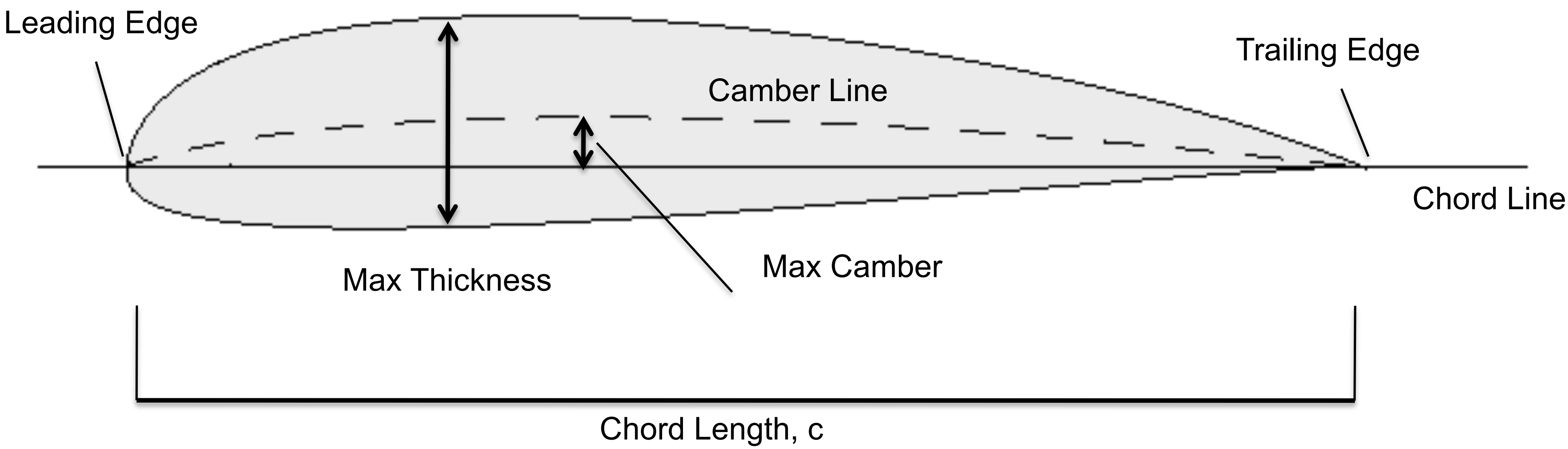


Where does lift come from?

- ***Equal transit theory of lift*** ***(INCORRECT)***
  - ***The upper surface is shaped to provide a longer path than the bottom surface***
  - ***Air molecules on the top must move faster to meet up with the molecules that go underneath***
  - ***According to Bernoulli's principle, the faster air on top is at a lower pressure than the air on the bottom, and this pressure differential creates lift***

- ***More correct definition of lift***
  - ***Lift is created when a moving fluid is turned by a solid object. Flow is turned in one direction, and lift is generated in the opposite direction***
  - ***Both upper and lower surfaces contribute to the turning flow***
  - ***The turning of flow is also known as circulation. Amount of lift generated is proportional to the amount of circulation created to satisfy the Kutta Condition***
  - ***Bernoulli's equation holds - lower pressure does correspond to an increased velocity - but turns out*** **that the velocity on the upper surface is much faster than what equal transit theory predicts**

Airfoil Definitions



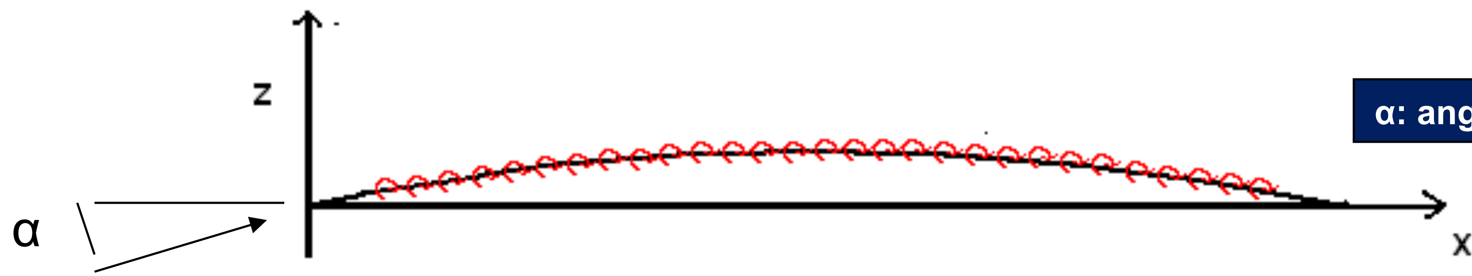
NACA 4-digit airfoils

- **1st digit: max camber in % chord**
- **2nd digit: position of max camber in 1/10 of c**
- **Last 2 digits: max thickness in % chord**
- **Example: NACA 4412**
  - **4% camber, at 40% chord, 12% thickness**

Thin Airfoil Theory - Quick Derivation

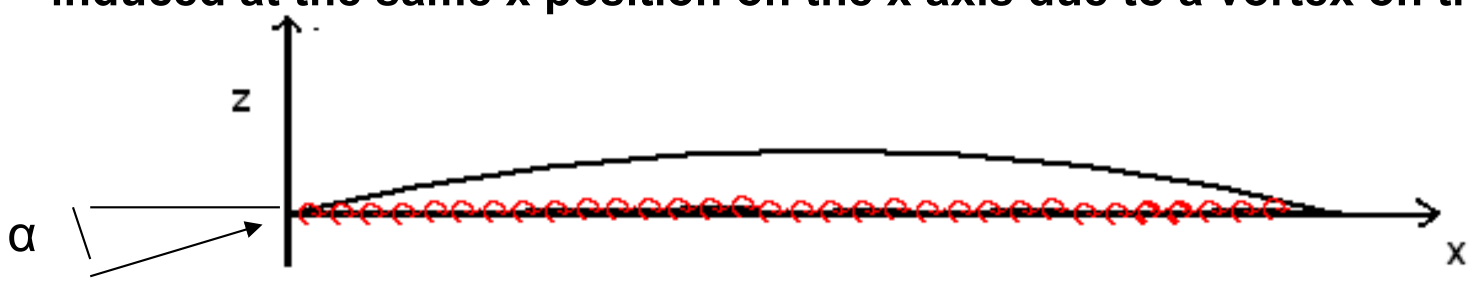
- **Potential Flow theory can be used to estimate the aerodynamic forces on a thin airfioil**

- **A distribution of vorticity on the airfoil will be a solution to Laplace's equation. It will satisfy the boundary conditions if the velocity induced by the vortices cancels the component of the freestream normal to the plate:**

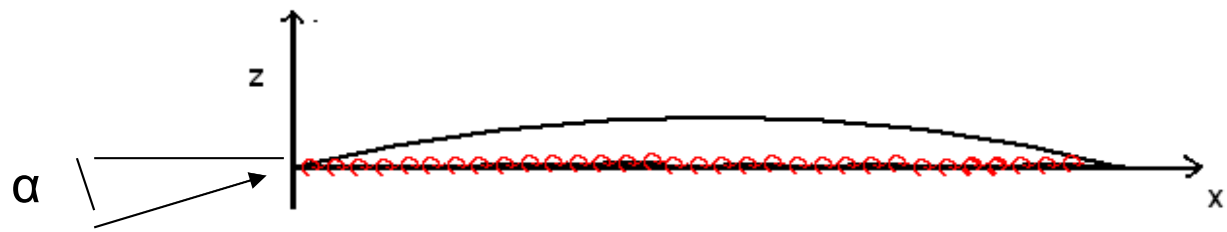


α: angle of attack

- **The basic approximation of thin airfoil theory is that the velocity induced at some point x due to the vorticity at x' may be approximated by the velocity induced at the same x position on the x axis due to a vortex on the x axis:**



Thin Airfoil Theory – Derivation cont.



The downward velocity induced at x due to the elemental vortex at x':

$$dw(x) = \frac{\gamma(x')dx'}{2\pi(x - x')}$$

Total induced velocity at x is the integral:

$$w(x) = \frac{1}{2\pi}\int_0^1 \frac{\gamma(x')dx'}{x - x'}$$

Combine the vortex sheet relation with the tangent flow condition:

$$\frac{1}{2\pi U_\infty}\int_0^1 \frac{\gamma(x')dx'}{x - x'} = \left(\alpha - \frac{dz(x)}{dx}\right)$$

- **α: angle of attack**
- **$\alpha_{L=0}$: angle of attack at zero lift**
- $\gamma(x)$ **: vorticity distribution**
- $w(x)$ **: induced velocity**
- **U_∞: Free stream velocity**
- **C_l: 2D Lift coefficient**
- **$C_{l\alpha}$: 2D lift curve slope**
- **$C_{m,LE}$: Moment coefficient about leading edge**
- **$C_{m,1/4}$: Moment coefficient about ¼ chord.**
- **C_p: Pressure coefficient**
- **x_{AC} : location of aerodynamic center.**
- **x_{CP} : location of center of pressure**

Thin Airfoil Theory – Derivation cont.

To solve, make a change in variables:

$$\cos\theta = 1 - 2x$$

Theta varies from 0 to pi

$$\frac{1}{2\pi U_\infty}\int_0^\pi \frac{\gamma(\theta')\sin\theta' \, d\theta}{\cos\theta' - \cos\theta} = \left(\alpha - \frac{dz(x)}{dx}\right)$$

And it turns out that the relation for gamma that solves this equation can be written as a Fourier series

$$\gamma(\theta) = 2U_\infty\left[A_0 \cot\frac{\theta}{2} + \sum_1^\infty A_n \sin(n\theta)\right]$$

Thin Airfoil Theory – Derivation cont.

Doing some substitution and using a few trig relations leads to a workable equation,

$$A_0 - \sum_1^{\infty} A_n \cos(n\theta) = \alpha - \frac{dz}{dx}$$

where

$$\boxed{\begin{aligned} A_0 &= \alpha - \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}\, d\theta \\ A_n &= \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx} \cos(n\theta)\, d\theta \end{aligned}}$$

Thin Airfoil Theory – Derivation cont.

From Kutta-Joukowski relationship:

$$l = \int_0^1 \rho U_\infty \gamma(x)\, dx \qquad m_{LE} = -\int_0^1 \rho U_\infty x \gamma(x)\, dx$$

Plugging in A_0 and A_n into the solution for gamma leads to:

$$\boxed{C_l = 2\pi A_0 + \pi A_1}$$

$$\boxed{C_{m,LE} = -\frac{\pi}{2}\left(A_0 + A_1 - {}^{A_2}\!/_{2}\right)}$$

$C_{m,1/4}$ does not depend on A_0, therefore does not vary with AoA

Moment about ¼ chord…

$$C_{m,1/4} = C_{m,LE} + C_l \times \frac{1}{4}$$

$$= -\frac{\pi}{2}\left(A_0 + A_1 - {}^{A_2}\!/_{2}\right) + \frac{1}{4}\left(2\pi A_0 + \pi A_1\right) = \boxed{-\frac{\pi}{4}\left(A_1 - A_2\right)}$$

Aerodynamic Center and Center of Pressure

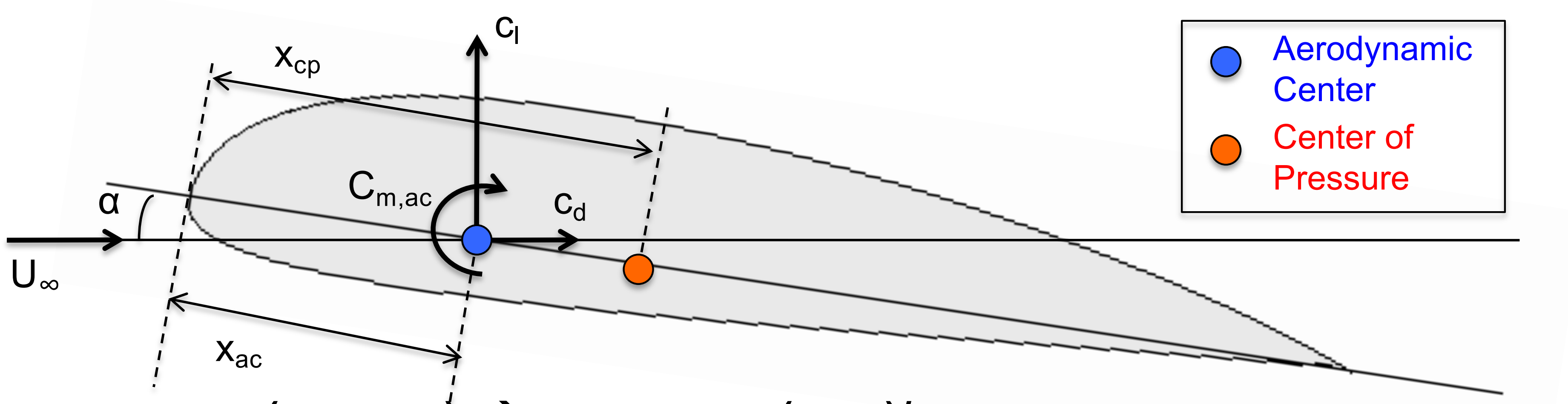
- **Definition: The aerodynamic center of an airfoil is defined as that point on its chord which the pitching moment coefficient is invariant with angle of attack**

- **Definition: The center of pressure of an airfoil is that point on its chord where the resultant of the pressure distribution acts – can be thought of as the centroid of the total lift distribution**

- **The lift distribution of an airfoil can be broken into two parts**
  - **Lift that depends on airfoil shape (camber)**
  - **Additional lift that depends linearly on angle of attack**
  - **The aerodynamic center can be thought of as the centroid of the additional lift distribution**

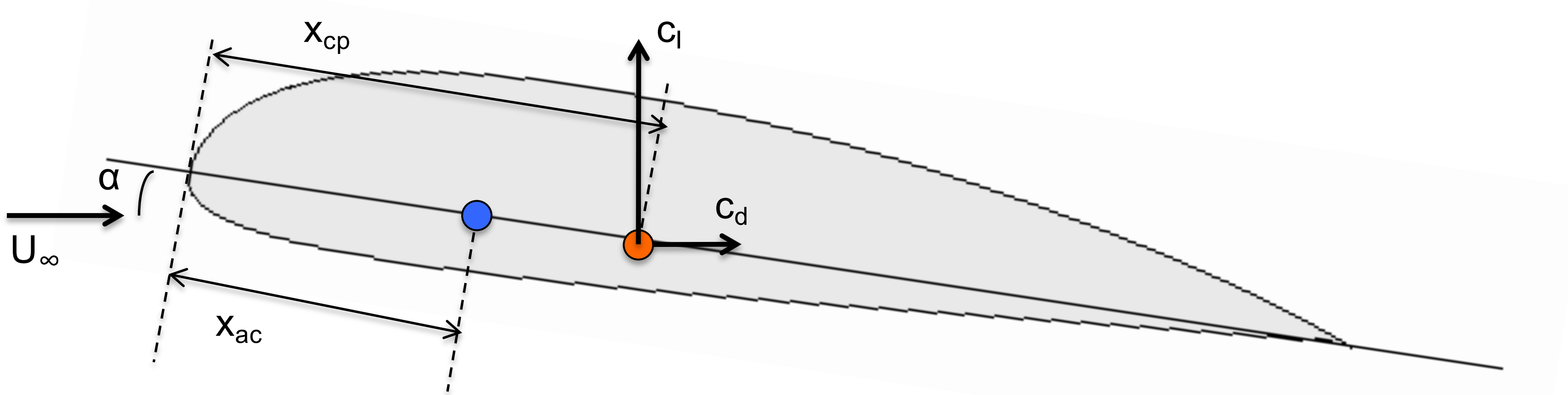
- **For a symmetric airfoil, the center of pressure and the aerodynamic center coincide**

Location of Center of Pressure



- $c_{m,ac} = -c_l(x_{cp} - x_{ac}) \rightarrow x_{cp} = x_{ac} - (c_{m,ac})/c_l$

- **For positive camber airfoils, $c_{m,ac}$ is negative, so center of pressure normally is behind aerodynamic center**



Thin Airfoil Summary

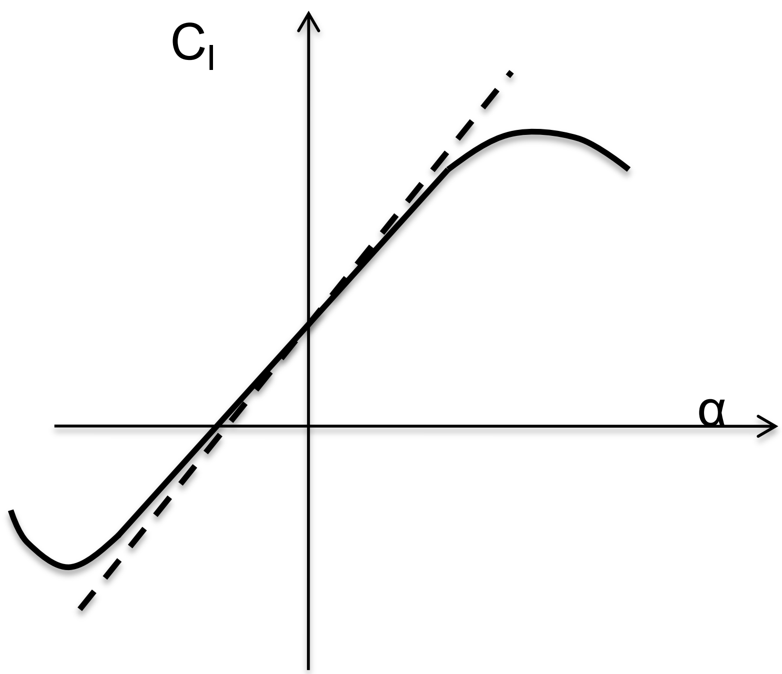
- **Thin arifoil theory assumes**
  - **2D flow, inviscid, incompressible, irrotational flow**
  - **Small AoA, small thickness, small camber**
- $C_l = 2\pi\left(\alpha - \alpha_{L=0}\right)$

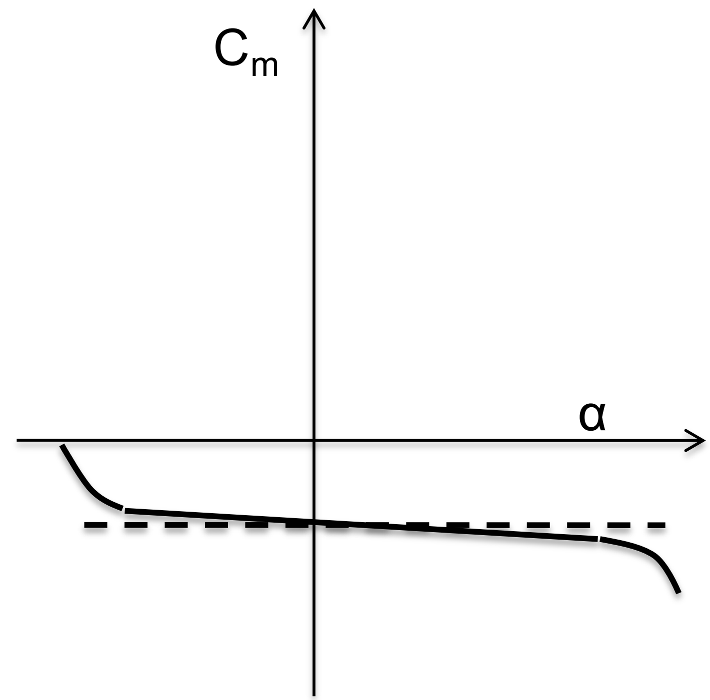
$$\alpha_{L=0} = \frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}\left(1-\cos\theta\right)d\theta$$

$$x = \frac{c}{2}\left(1-\cos\theta\right)$$

- **Moment at quarter chord is independent of angle of attack**
  - **c/4 = aerodynamic center for all airfoils**
  - **$M_{c/4}$ only dependent on camber**
  - **$M_{c/4}$ for symmetric airfoils is zero**

How accurate is potential flow theory?





- **In general, potential flow theory can be accurate through the linear region of the lift-curve slope, but since it does not model flow separation, it does not capture the effects of stall**

- **Potential flow theory predicts a constant Cm value at the ¼ chord. This is a good approximation for many airfoils, but it doesn't hold at high angles of attack**

Principal Effects of Airfoil Properties

- **Airfoil Thickness**
  - **Max lift coefficient, $c_{l,max}$**
  - **Aerodynamic Center**
- **Shape of the mean line (camber)**
  - **Zero lift angle of attack**
  - **Max lift coefficient, $c_{l,max}$**
  - **Pitching moment coefficient at zero lift**
- **Leading edge radius, leading edge shape**
  - **Max lift coefficient, $c_{l,max}$**
  - **End of linear AoA range**
- **Trailing edge angle**
  - **Aerodynamic center**

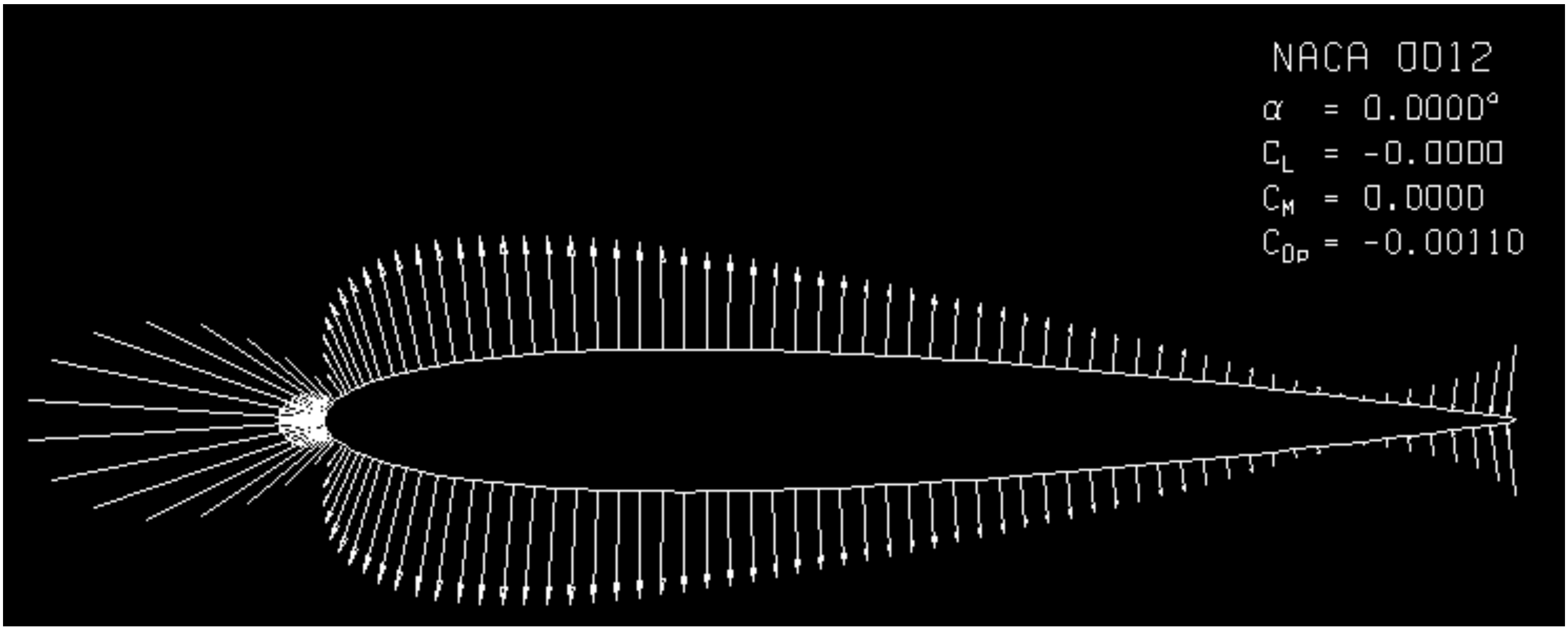
Photo by Dave_S. from Witney, England

XFOIL

- **XFOIL is an interactive program for designing and analyzing subsonic airfoils developed by Prof. Marc Drela at MIT**
- **Free download at http://web.mit.edu/drela/Public/web/xfoil/**

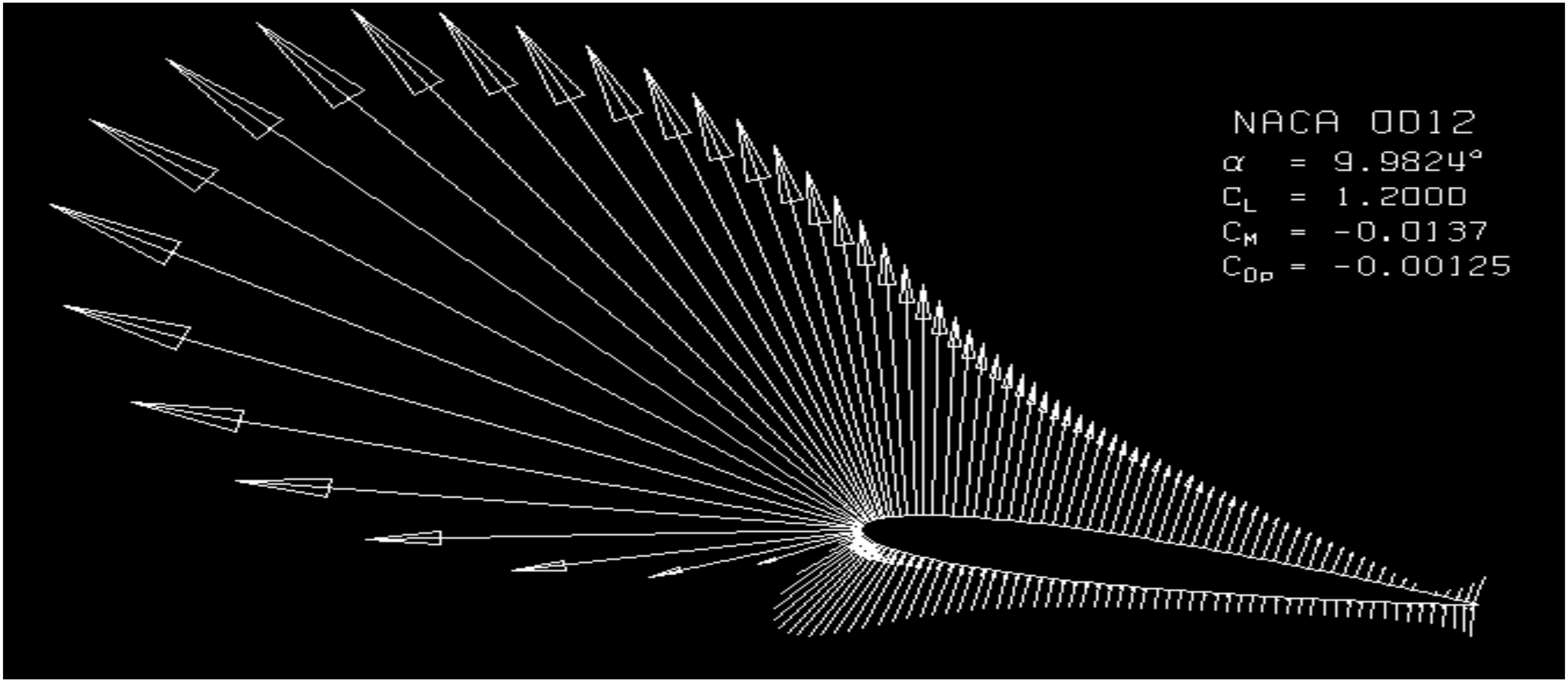
NACA 0012

- **Pressure distribution around NACA 0012 airfoil (symmetric) at zero AoA**



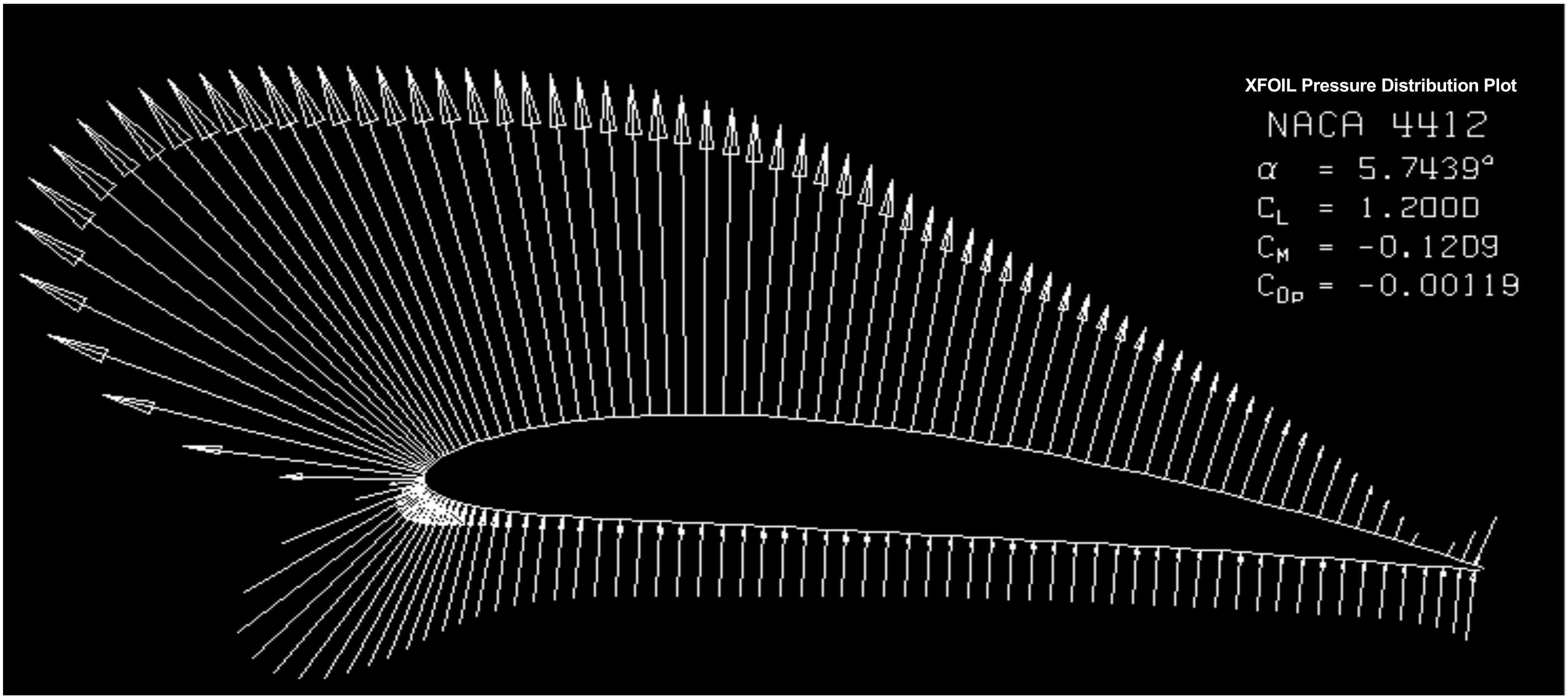
NACA 0012

- **NACA 0012 at AoA ~ 10 deg shows strong leading edge spike**



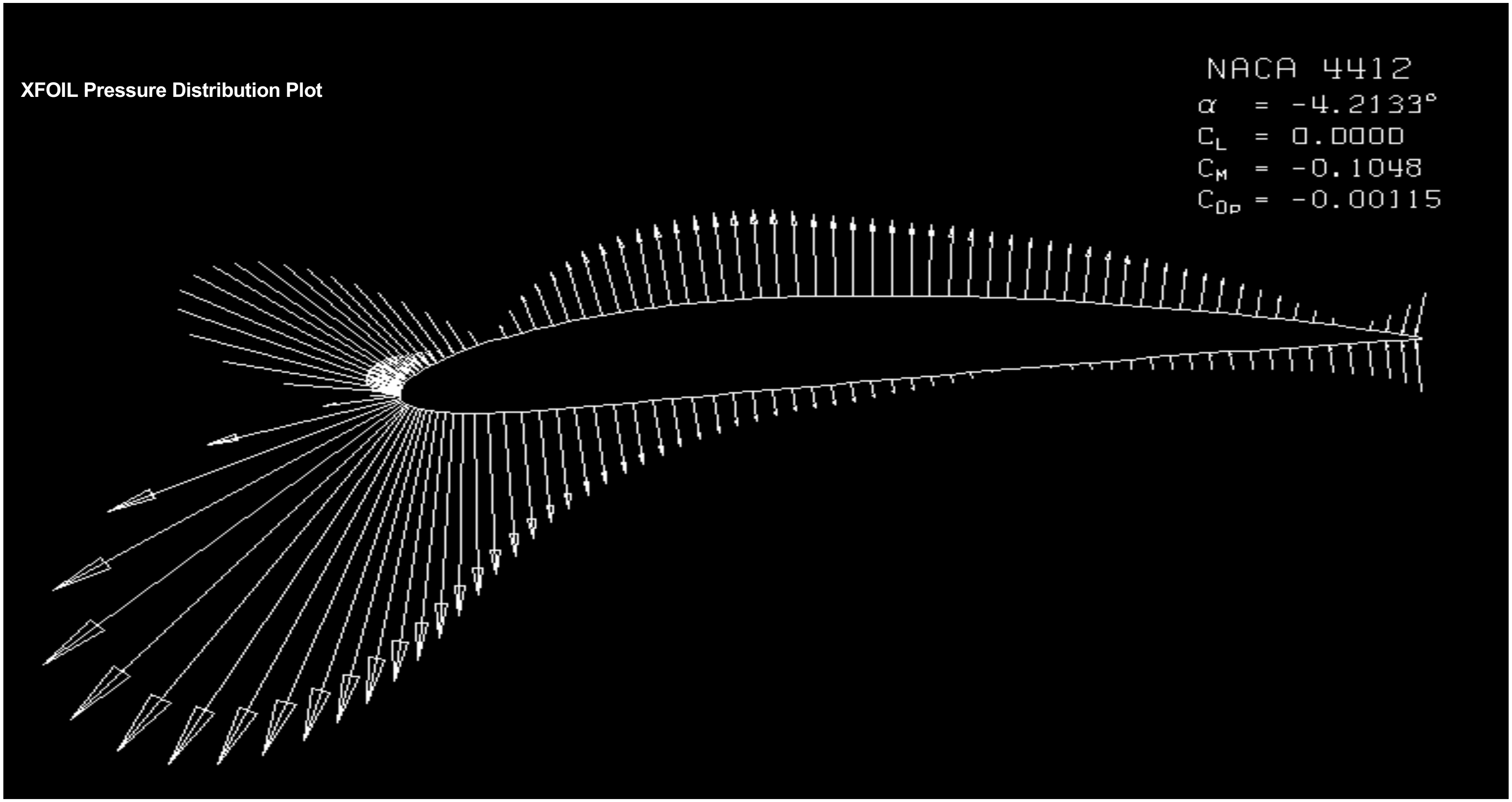
NACA 4412

- **Adding camber to the airfoil reduces the leading edge spike.**



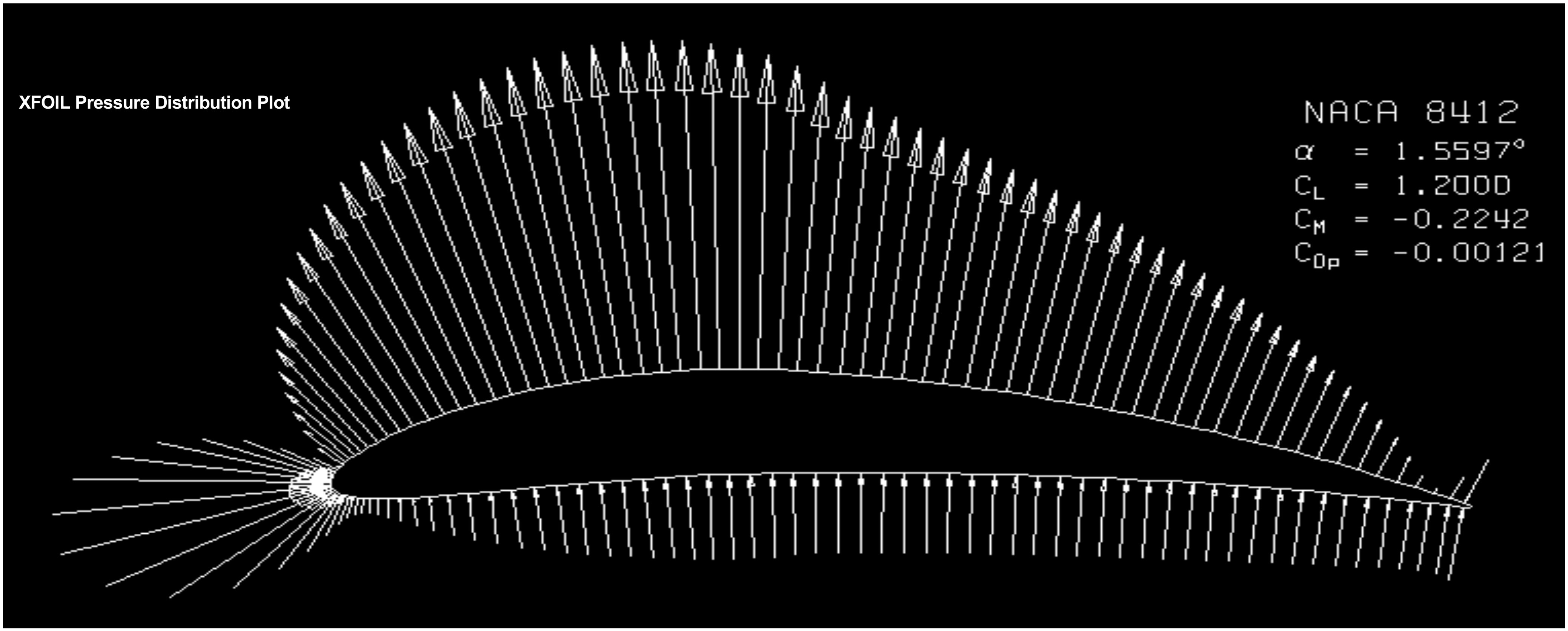
NACA 4412

- **At C_l = 0, spike is seen on underside of airfoil, but it is not large**



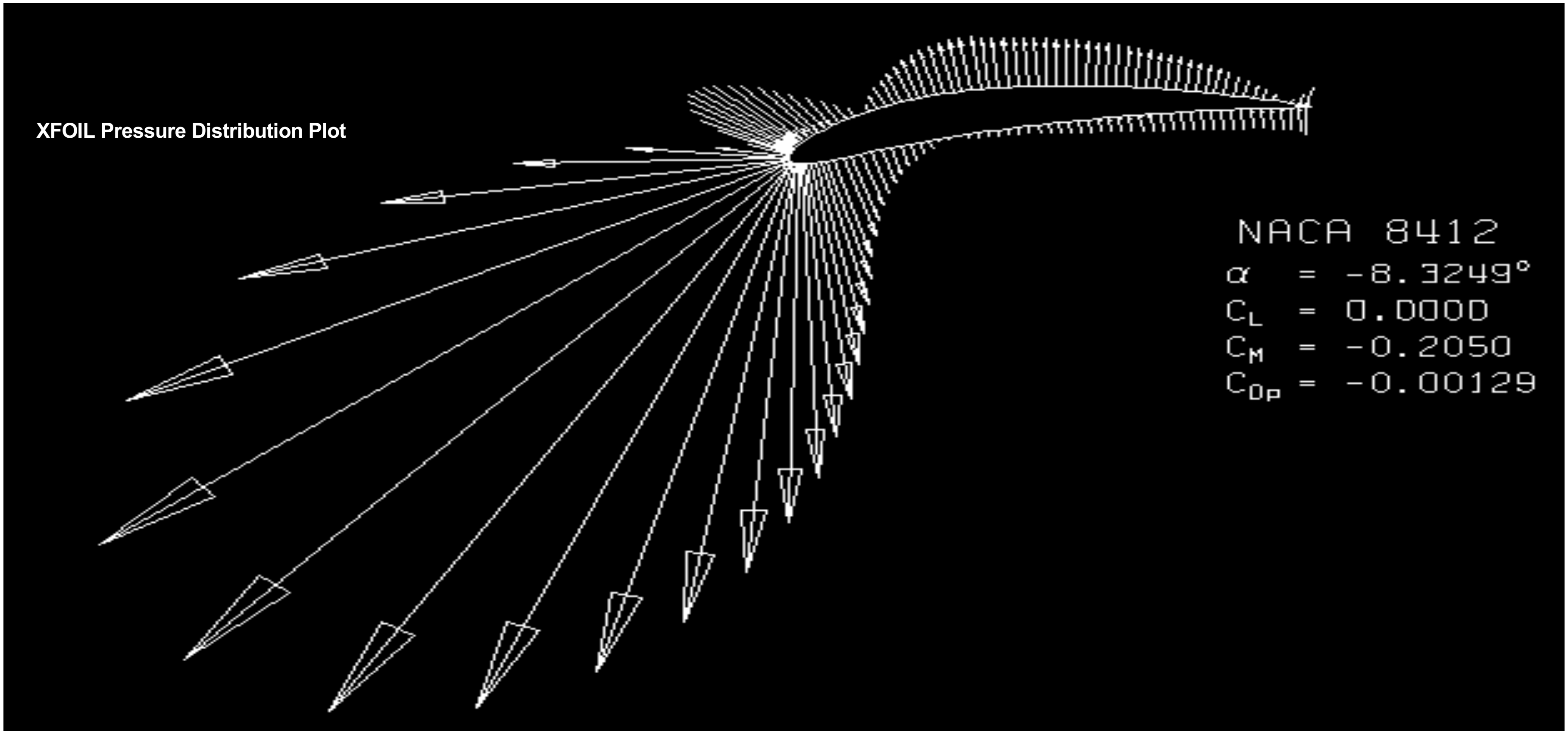
NACA 8412

- **Increasing camber to 8% further reduces leading edge spike, which will help delay separation**

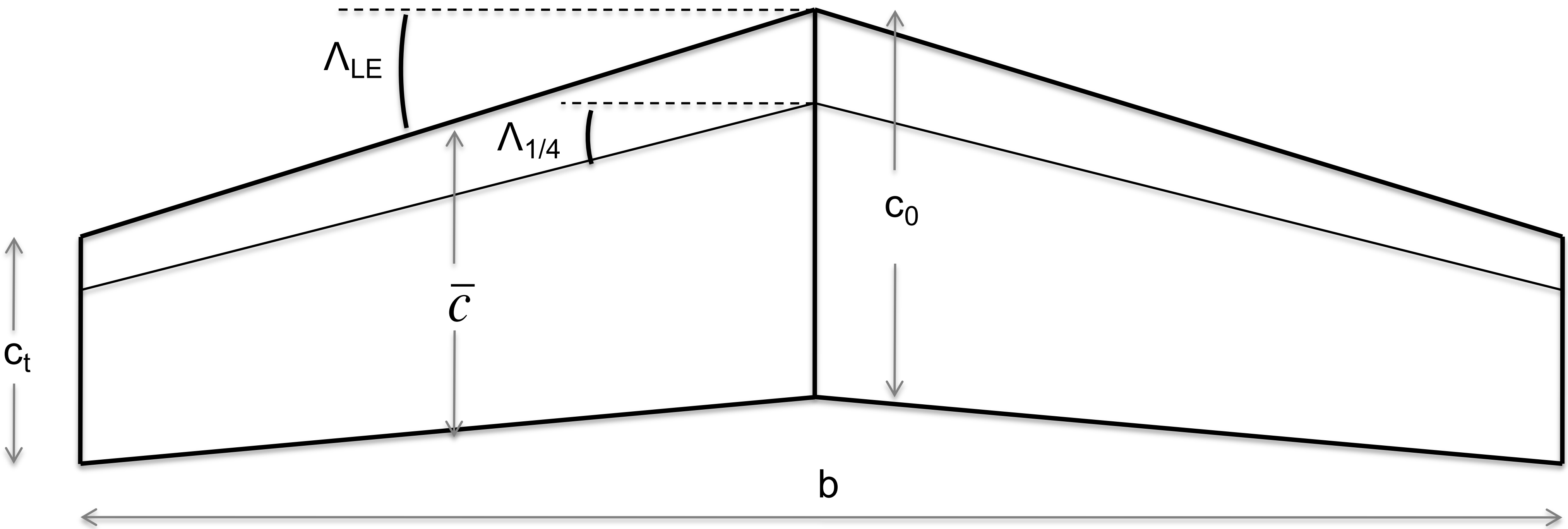


NACA 8412

- **However, C_l =0 (at high speeds) has a very large spike on the bottom – significant drag penalty**



Wing Definitions

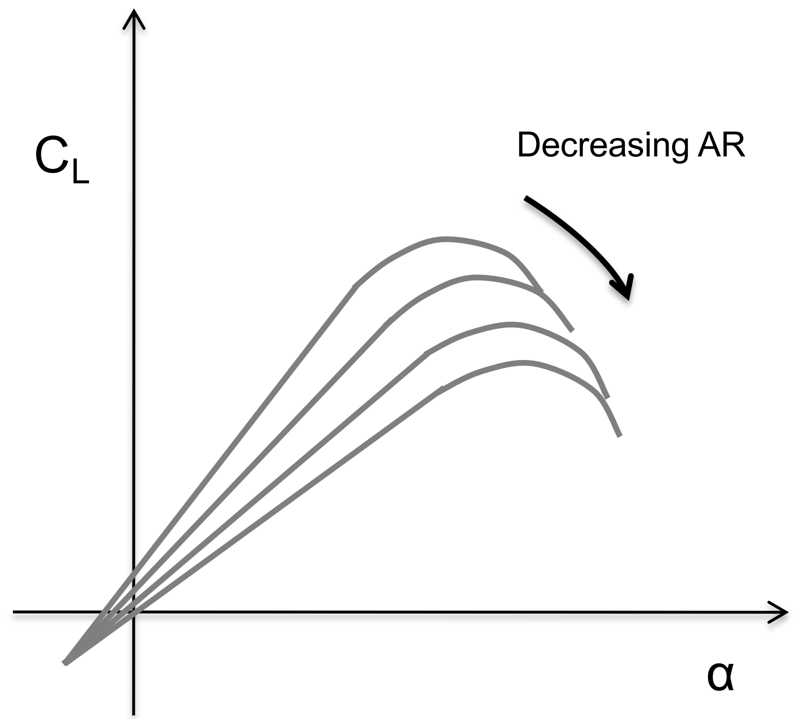


- b: Span
- c: Chord
  - c_0: chord at mid-span (root chord)
  - c_t: chord at tip of wing
  - $\bar{c}$: mean aerodynamic chord (MAC)

- λ: taper ratio | $\lambda = c_t/c_o$
- A: Aspect Ratio | b^2/S
- S: Wing Area
- Λ=Wing Sweep

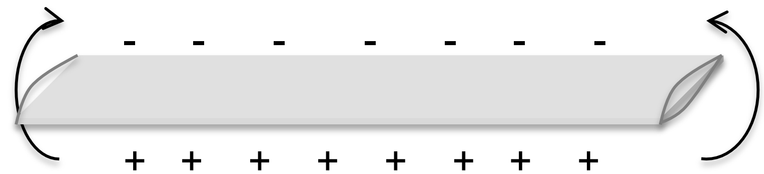
3D Lift Effects

- $C_{L\alpha}$ (3D lift curve slope) depends the airfoil's 2D lift curve slope, planform geometry and Mach No
- For subsonic speeds, in general:
  - $C_{L\alpha}$ increases with increasing aspect ratio
  - $C_{L\alpha}$ decreases with increasing sweep angle
  - $C_{L\alpha}$ increases with increasing Mach number
- For wings with low wing sweep, the aerodynamic center, X_{AC} is usually close to the 25% chord point on the mean aerodynamic chord
  - As sweep angles increase, X_{AC} moves aft
  - As Mach No. approaches transonic range, X_{AC} moves aft



Wing Downwash

- **Pressure differences due to lift production lead to vortices about the wing tip, as higher pressure under the wing forces air outward and around the wing tip to the upper surface**



- **The trailing vortex system causes upwash outside the wing's span, and downwash inside the wing span**

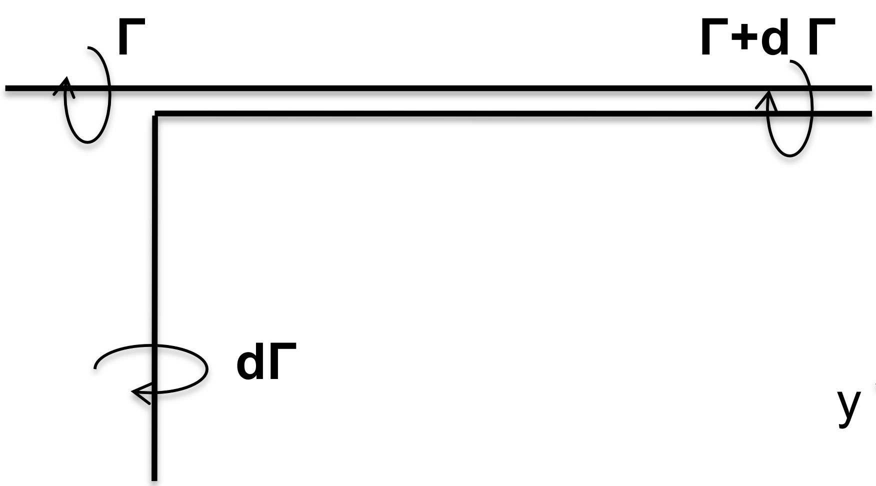
- **The downwash reduces the effective angle of attack on the wing, which degrades the lift-curve slope**

- **The downwash effect also rotates the lift vector, creating a component of lift in the direction of the free-stream air. This is the induced drag, and is major source of drag on the vehicle, especially at low speeds**

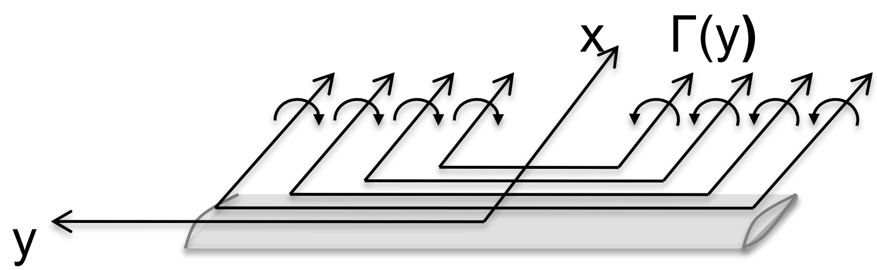
Lifting Line Model

- **Assumptions for Prandtl's lifting line theory**
  - **Potential flow (inviscid, irrotational)**
  - **High aspect ratio**
  - **Low sweep angle**
- **Kutta-Joukowski relationship: lift per unit span is proportional to circulation**

$$\frac{dL(y)}{dy} = \rho V \Gamma(y)$$



- **α_i : induced angle of attack**
- **α_{eff}: effective angle of attack**
- **α_{geo}: geometric angle of attack (same as α)**
- **A : Aspect ratio (sometimes written "AR")**
- **Γ : Circulation**
- **U_∞: Free stream velocity**
- **C_L: 3D Lift coefficient**
- **$C_{L\alpha}$: 3D lift curve slope**
- **$C_{M,AC}$: Moment coefficient about aerodynamic center**
- **$C_{M\alpha}$: Moment curve slope**
- **D_i : induced drag**
- **e : Oswald's efficiency factor**



Lifting Line Theory

-- Effective angle of attack accounts for induced AoA

-- Lift coefficient is influenced by induced AoA

-- Downwash velocity at y_o due to elemental trailing vortex at y is inversely proportional to distance between y_0 and y

-- Integrate trailing vortex system to solve for induced AoA

-- Combining equations yields the integral relation for the circulation distribution across the wing

$$\Gamma = \pi c V_\infty \alpha_{eff}$$

$$\alpha_{eff}(y) = \alpha_{GEO}(y) + \frac{w(y)}{V_\infty}$$

$$dw(y_0) = \frac{d\Gamma(y)}{4\pi(y_0 - y)}$$

$$\alpha_i(y_0) = \frac{1}{4\pi V_\infty} \int_{-b/2}^{b/2} \frac{d\Gamma(y)}{(y_0 - y)}$$

$$\alpha_{eff}(y_0) = \alpha_{geo} + \frac{1}{4\pi V_\infty} \int_{-b/2}^{b/2} \frac{d\Gamma(y)}{(y_0 - y)} = \frac{\Gamma}{\pi c V_\infty}$$

Elliptical Lift Distribution

Just like with thin airfoil theory, making a coordinate transformation helps to make the equation easier to work with

$$y = \frac{b}{2}\cos\theta$$

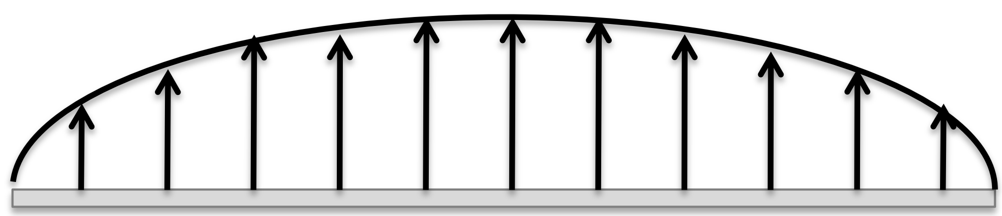
Relation for induced AoA becomes:

$$\alpha_i(\theta_0) = \frac{1}{2\pi V_\infty b}\int_0^\pi \frac{d\Gamma(\theta)}{(\cos\theta - \cos\theta_0)}$$

If lift distribution is elliptical, then the downwash is constant across the span of the wing

$$\Gamma = \Gamma_0\sqrt{1-\left(\frac{2y}{b}\right)^2} \rightarrow \Gamma_0 \sin\theta$$

$$\boxed{\alpha_i = \frac{\Gamma_0}{2bV_\infty}}$$



Elliptical Wings

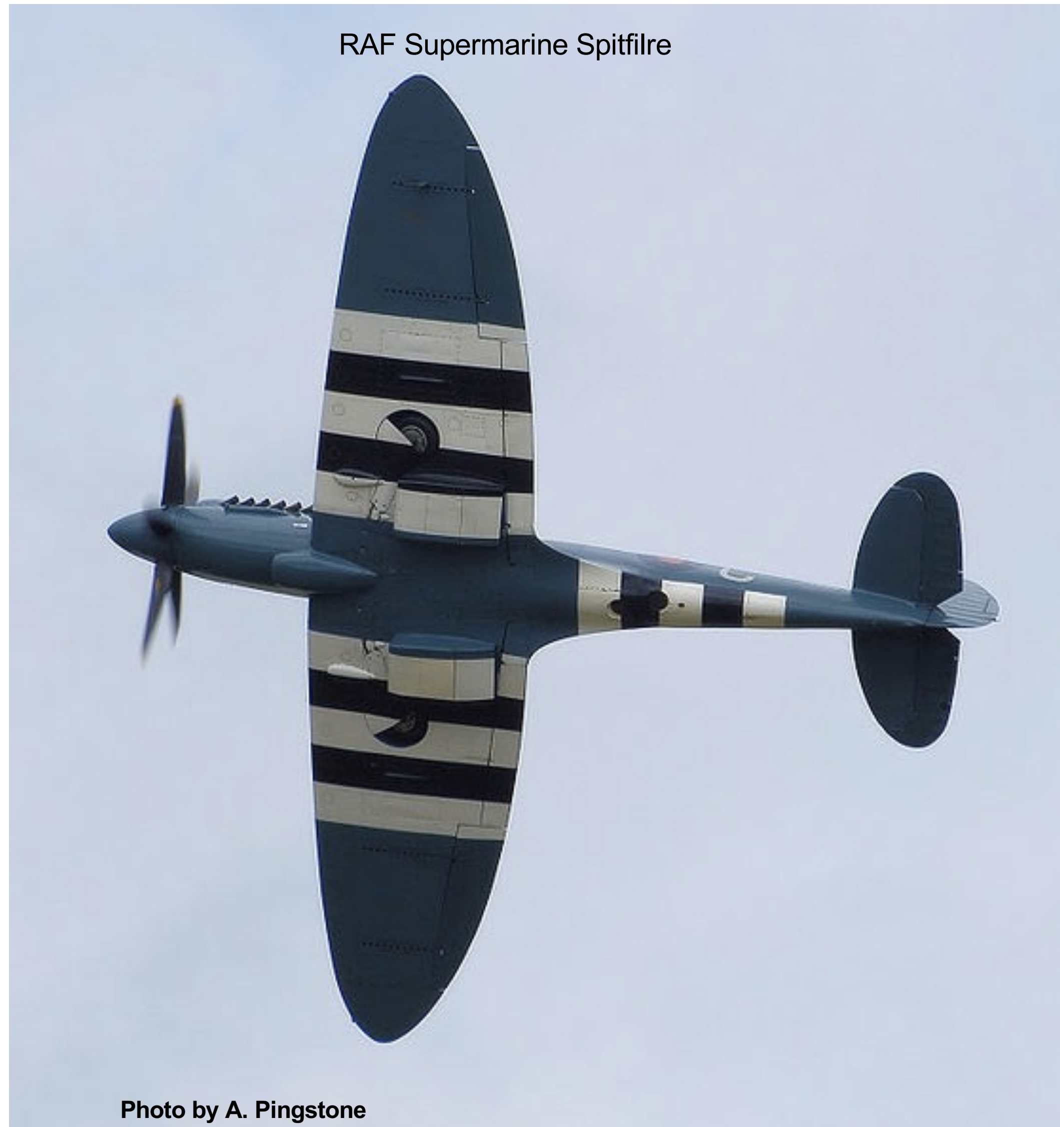
- **It turns out that the planform shape that produces an elliptical lift distribution is an elliptical wing**

- **Important results for an elliptical lift distribution:**

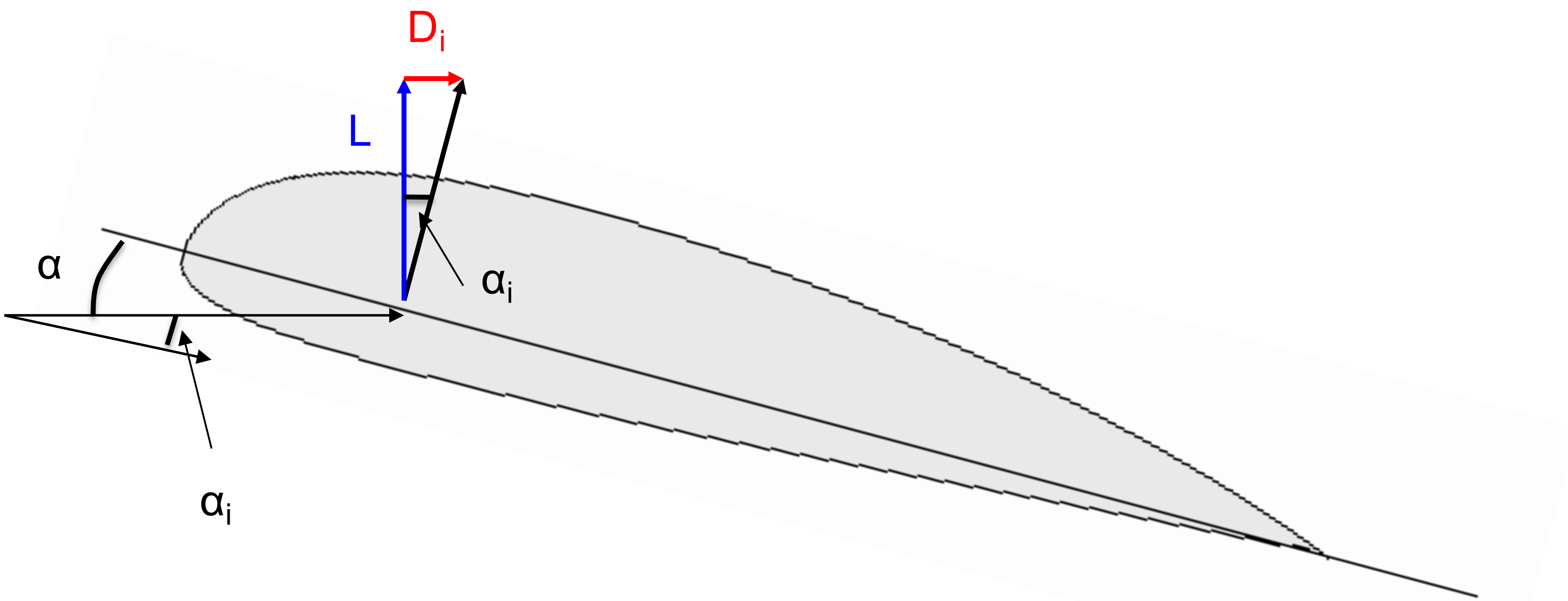
$$\alpha_i = \frac{C_L}{\pi A}$$

$$C_L = C_{l,\alpha}\left(\alpha - \frac{C_L}{\pi A}\right)$$

$$C_{L\alpha} = \frac{C_{l,\alpha}}{1 + \frac{C_{l,\alpha}}{\pi A}}$$



Induced Drag



- **Induced angle of attack tilts lift vector backward**
- **With respect to free stream AoA, a component of the lift vector is in drag direction**

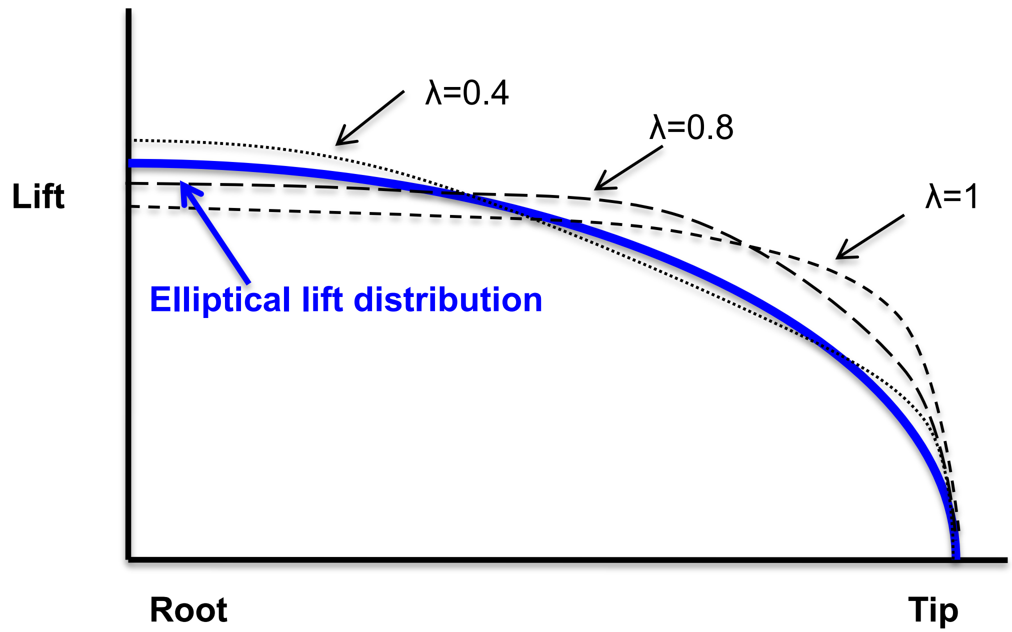
$$D_i = L\alpha_i \qquad C_{D_i} = C_L\alpha_i = C_L\left(\frac{C_L}{\pi A}\right) = \left(\frac{C_L^2}{\pi A}\right)$$

Oswald's Efficiency

- **Actual wings aren't perfectly elliptical. Even the Spitfire has a fuselage and other appendages that affect the downwash and reduce the wing's efficiency**
- **Oswald's efficiency factor, e, added to the induced drag equation to account for realistic wing efficiency**
- **Taper ratios can approximate the lift distribution of an elliptical planform**
  - **Taper ratios between 0.3 and 0.4 come close to an elliptical lift distribution and are more efficient than a rectangular wing**

$$C_{D_i} = \left(\frac{C_L^2}{\pi A e} \right)$$

$$C_{L\alpha} = \frac{C_{l\alpha}}{1 + \frac{C_{l\alpha}}{\pi A e}}$$

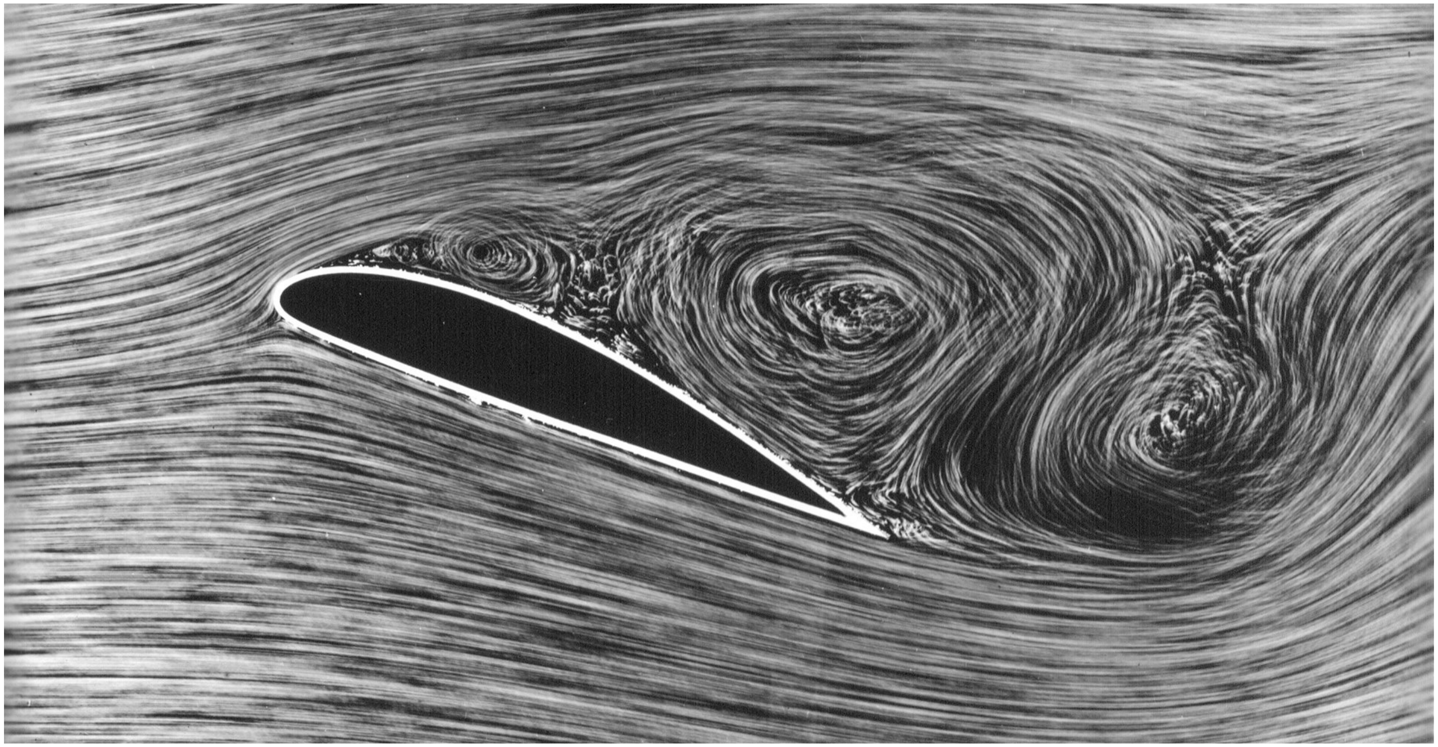


Stall

There are limits to how much lift a wing can produce

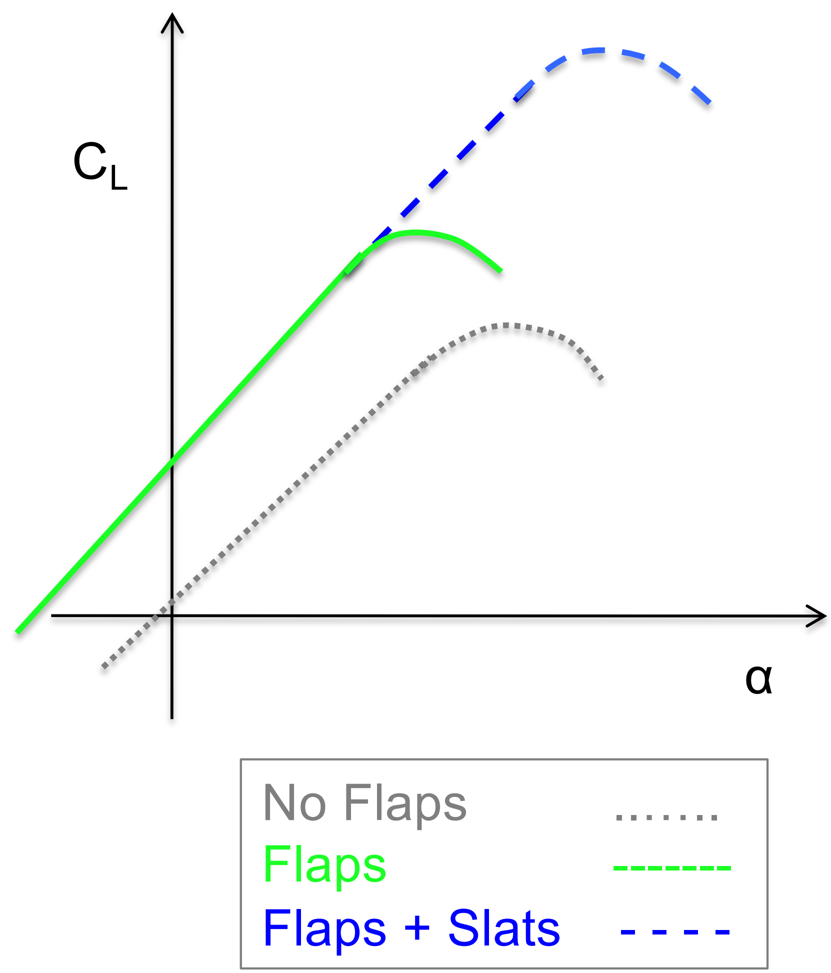
Lift increases with increasing angle of attack, but if the angle of attack exceeds a critical value, the lift will decrease due to flow separation

Max values for angle of attack varies with airfoil and wing aspect ratio, but typical stall angles are about 15 degrees



High Lift Devices

- **Flaps**
  - **Increase Max C_L by deflecting the airfoil trailing edge, which increases the camber and shifts the lift curve slope upward without significantly altering the lift-curve slope**
  - **The deflected trailing edge effectively rotates the airfoil and often results in an earlier stall AoA**
- **Leading edge flap (slats, Krueger flaps)**
  - **Helps avoid leading edge separation**
  - **Normally used to supplement trailing edge flaps by increasing max C_l**



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