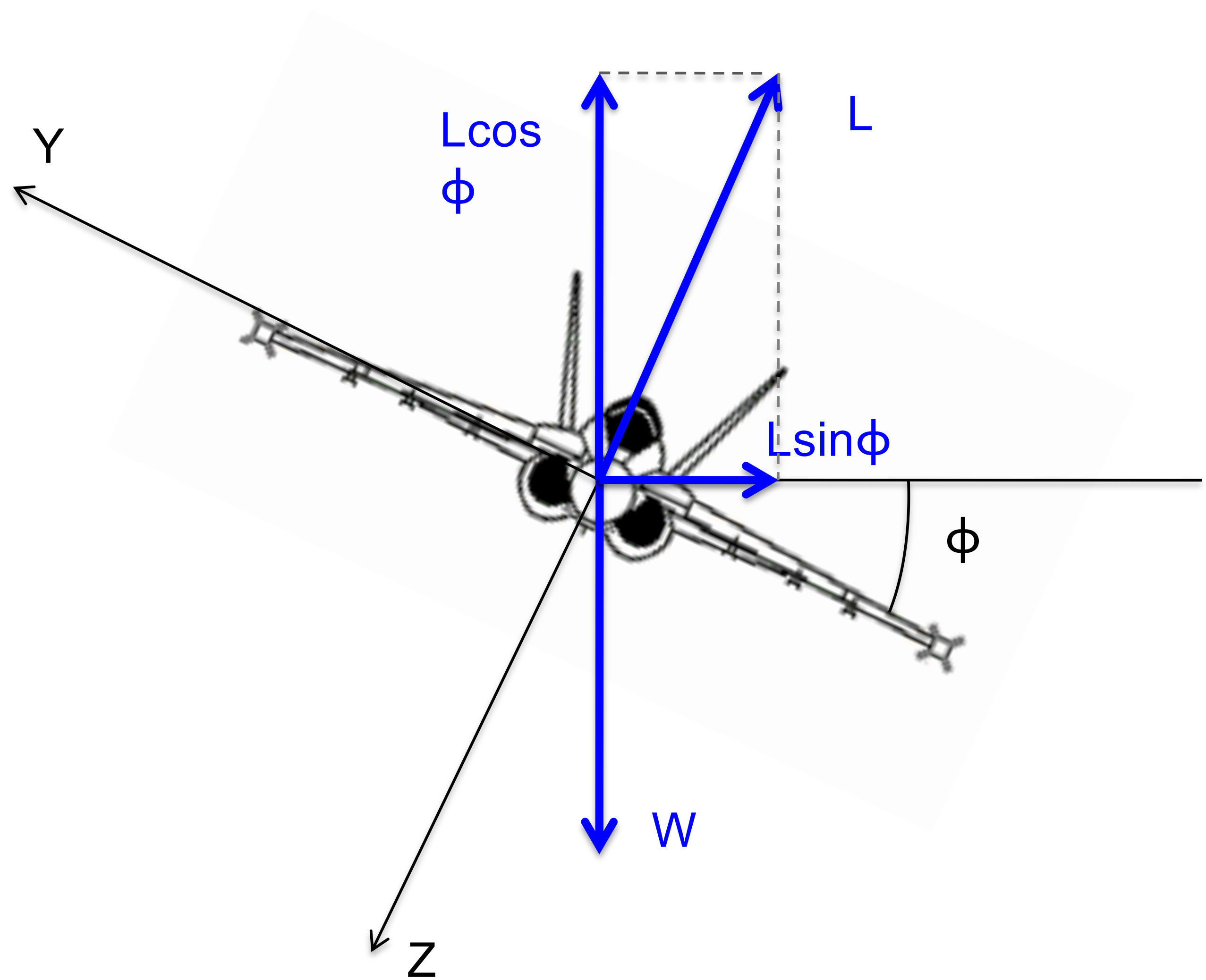
Lecture 9 – Aircraft Maneuvering

Extra 300 - Image Courtesy of A. Pingstone

Turning Flight



Turn Rate & Turn Radius

- **During a coordinated turn, the lift vector is tilted by the roll angle, Φ**
- **In order to maintain level flight, the vertical component of the lift vector must equal the weight**

$$L\cos\Phi = W$$

- **Horizontal component of tilted lift vector produces a centripetal force**

$$F_r = L\sin\Phi = m\frac{V^2}{R} \qquad R = \frac{mV^2}{L\sin\Phi}$$

- **Turn rate, ω, can be found from:**

$$\omega = \frac{d\Psi}{dt} = \frac{V}{R}$$

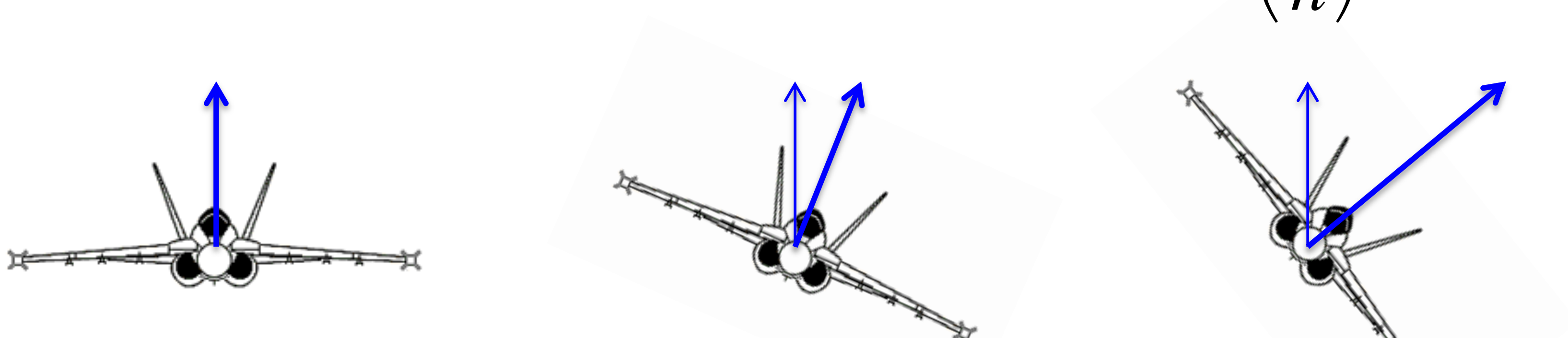
Load Factor

- **Load factor is defined as**

$$n \equiv \frac{L}{W}$$

- **In terms of load factor, the relationship between lift and roll angle becomes**

$$L\cos\Phi = W \longrightarrow \Phi = \cos^{-1}\left(\frac{1}{n}\right)$$



ϕ = 0 deg
n = 1

ϕ = 30 deg
n = 1.16

ϕ = 60 deg
n = 2

Load Factor

- **Solving for turning radius in terms of load factor:**

$$R = \frac{mV^2}{L\sin\Phi} = \frac{W}{L}\frac{V^2}{g\sin\Phi} = \frac{V^2}{gn\sin\Phi}$$

$$R = \frac{V^2}{g\sqrt{n^2-1}}$$

- **Turn rate becomes..**

$$\omega = \frac{d\Psi}{dt} = \frac{V}{R} = \frac{g\sqrt{n^2-1}}{V}$$

- **For best turning, fly at highest load factor at lowest possible velocity**

Lift Constrained Load Factor

- **At low speeds, high load factors require high lift coefficients**

- **Load factor is constrained by maximum lift coefficient**

$$n \equiv \frac{L}{W} = \frac{1}{2}\rho V^2 C_L \frac{S}{W}$$

$$n_{\max LC} = \frac{1}{2}\rho V^2 \frac{C_{L_{\max}}}{\left(W/S\right)}$$

Banking Stall

- **Solving for the velocity:**

$$V = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\left(\frac{n_{\max_{LC}}}{C_{L_{\max}}}\right)} = V_{stall}$$

- **If n = 1 (bank angle =0), V = V_S for straight and level flight**

- **As bank angle increases, V_S increases**

Sustained Turning

- **High load factors require a lot of lift, but as lift increases, so does drag**

$$T_{reqd} = D = \frac{1}{2}\rho V^2 S\left(C_{D_0} + KC_L^2\right) \qquad L = nW = \frac{1}{2}\rho V^2 S C_L$$

- **In order to maintain altitude and speed, the thrust must increase over straight and level flight**

$$T_{reqd} = D = \frac{1}{2}\rho V^2 S\left[C_{D_0} + K\left(\frac{2nW}{\rho V^2 S}\right)^2\right]$$

- **Solving for n, the load factor for sustained turning is given as,**

$$n = \left[\frac{\frac{1}{2}\rho V^2}{K(W/S)}\left(\frac{T_{reqd}}{W} - \frac{1}{2}\rho V^2 \frac{C_{D_0}}{W/S}\right)\right]^{1/2}$$

Thrust Constrained Load Factor

- **The maximum possible load factor depends on the maximum available thrust:**

$$n_{\max TC} = \left[\frac{\frac{1}{2}\rho V^2}{K\left(W/S\right)} \left(\left(\frac{T_A}{W}\right)_{\max} - \frac{1}{2}\rho V^2 \frac{C_{D_0}}{W/S} \right) \right]^{1/2}$$

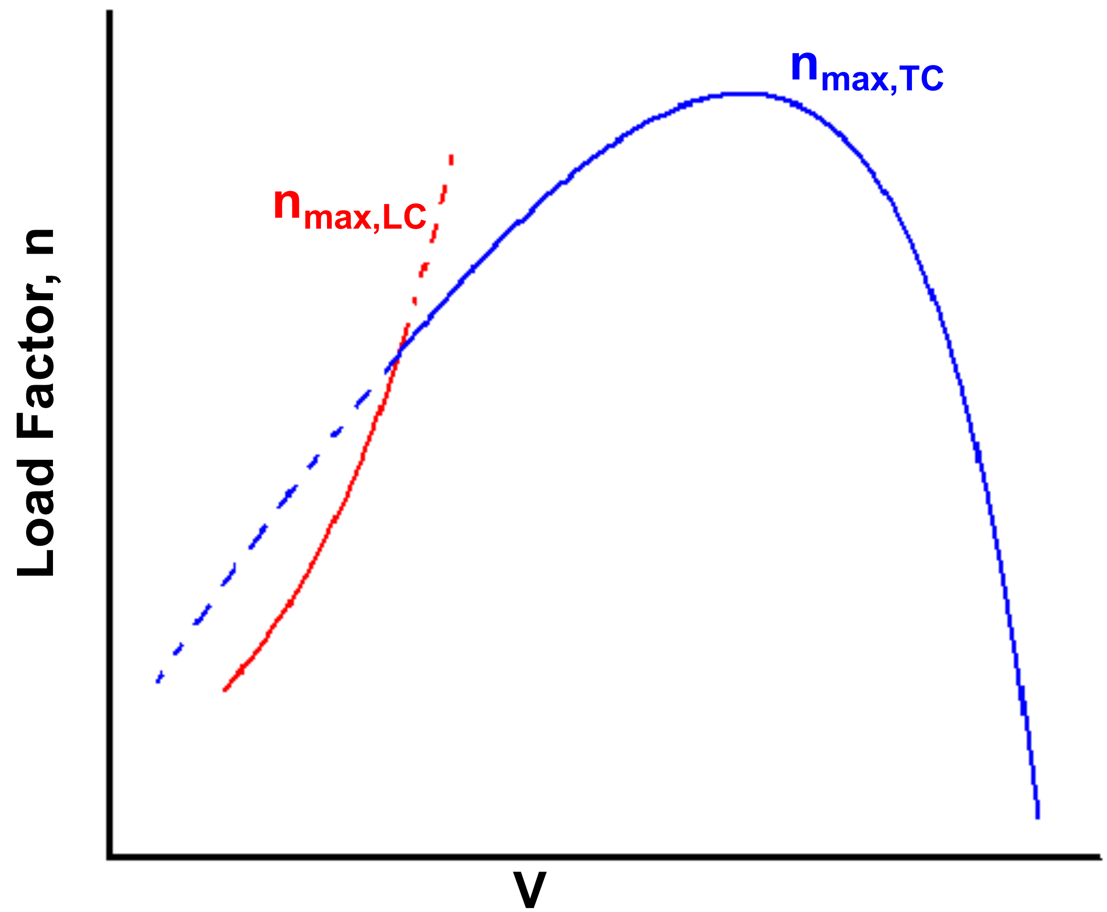
- **From the definition of load factor, for a given available thrust, the load factor increases as lift-to-drag ratio increases**

$$n \equiv \frac{L}{W} = \left(\frac{L}{D}\right)\left(\frac{D}{W}\right) = \left(\frac{L}{D}\right)\left(\frac{T_{reqd}}{W}\right) \longrightarrow n_{\max TC} = \left(\frac{L}{D}\right)\left(\frac{T_A}{W}\right)_{\max}$$

Maximum Load Factor

- **At high speeds, maximum sustainable load factor is limited by the thrust available**

- **At low speeds, maximum load factor constrained by stall line**



Minimum Turn Radius (Thrust Constrained)

- **Previous equations for turning radius and load factor:**

$$R = \frac{2q_\infty}{g\rho\sqrt{n^2 - 1}} \qquad n = \left[\frac{q_\infty}{K(W/S)}\left(\frac{T}{W} - q_\infty \frac{C_{D_0}}{W/S}\right)\right]^{1/2}$$

- **Differentiating with respect to dynamic pressure and setting equal to zero:**

$$\left(q_\infty\right)_{R_{\min}} = \frac{2K(W/S)}{(T/W)} \qquad V_{R_{\min}} = \sqrt{\frac{4K(W/S)}{\rho(T/W)}}$$

- **Solving for the load factor and minimum turning radius at that velocity:**

$$n_{R_{\min}} = \sqrt{2 - \frac{4KC_{D_0}}{(T/W)^2}} \qquad R_{\min} = \frac{4K(W/S)}{g\rho(T/W)\sqrt{1 - \frac{4KC_{D_0}}{(T/W)^2}}}$$

Max Load Factor

- **To minimize turn radius and maximize turn rate, decrease wing loading, and decrease K**
- **Easiest way to decrease K is to increase aspect ratio, but a longer, more slender wing has structural issues**

Bi-Planes

Pitts Special - Image Courtesy of G.Bird

- **Biplanes split their lift between two wings. For a given wing loading and wingspan, the aspect ratio is higher than a monoplane**

Symmetric Pull-up

- **In a pull-up maneuver, the aircraft lift exceeds its weight, generating a centripetal force**

$$F = m\frac{V^2}{R} = L - W\cos\theta$$

- **If Θ =0, the turn rate and loop radius are:**

$$R = m\frac{V^2}{F} = \frac{W}{g}\frac{V^2}{W(n-1)} = \frac{V^2}{g(n-1)} \qquad \omega = \frac{V}{R} = \frac{g(n-1)}{V}$$

References

1. **C.E.Lan, J. Roskam, *Airplane Aerodynamics and Performance*, Design Analysis Research Corporation, 1997**

2. **McCormick, B.W., *Aerodynamics, Aeronautics and Flight Mechanics*, 2nd edition, Wiley & Sons, 1995**