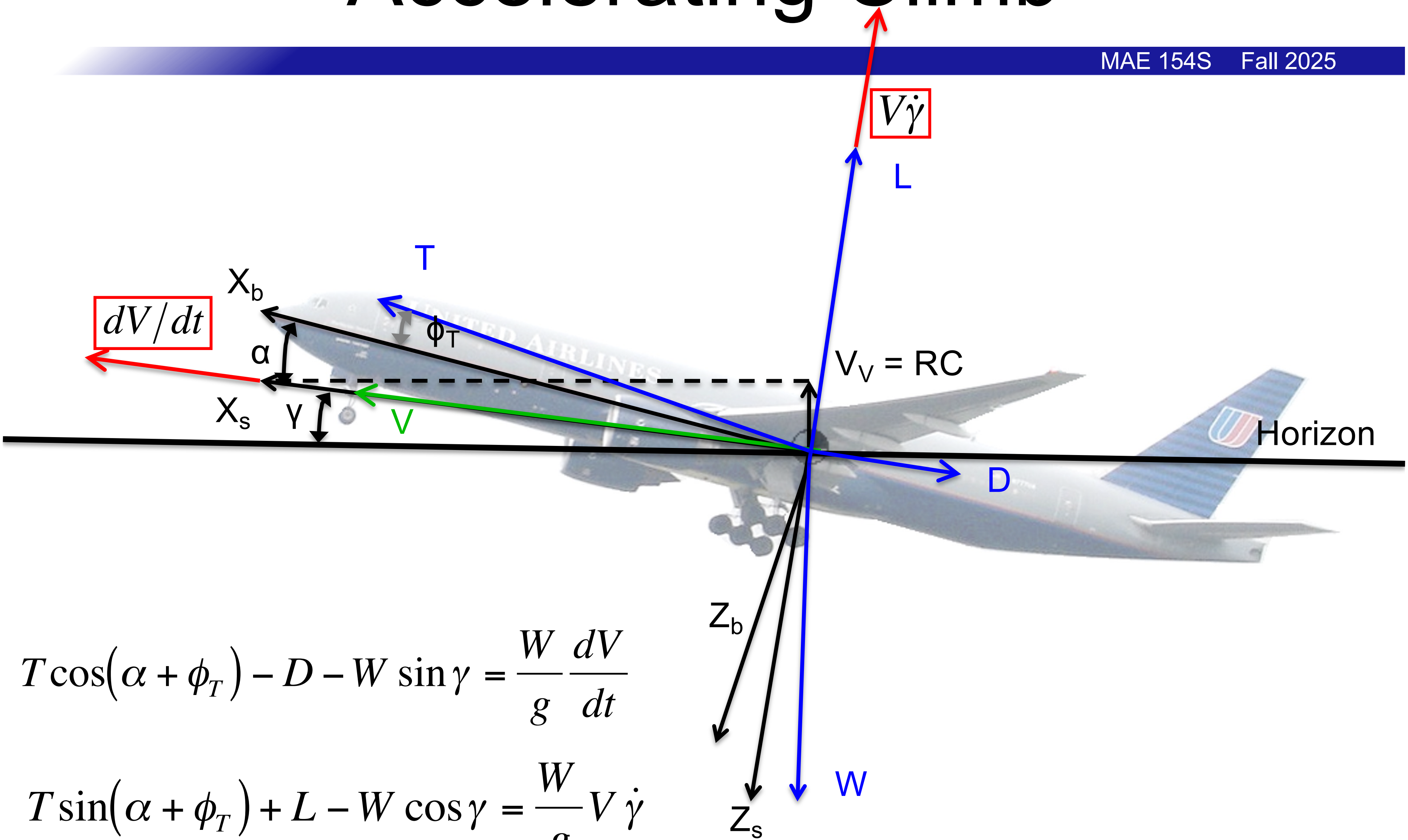
Lecture 6 – Climbing Performance

Accelerating Climb



$$T\cos(\alpha+\phi_T)-D-W\sin\gamma=\frac{W}{g}\frac{dV}{dt}$$

$$T\sin(\alpha+\phi_T)+L-W\cos\gamma=\frac{W}{g}V\dot{\gamma}$$

Equations of Motion

$$T\cos\left(\alpha+\phi_T\right)-D-W\sin\gamma=\frac{W}{g}\frac{dV}{dt}$$

$$T\sin\left(\alpha+\phi_T\right)+L-W\cos\gamma=\frac{W}{g}V\dot{\gamma}$$

- **Assuming straight line flight path, the centrifugal acceleration term drops away**

$$T\sin\left(\alpha+\phi_T\right)+L-W\cos\gamma=0$$

- **With small angle assumptions, the equations of motion can be rewritten as:**

$$\boxed{\begin{aligned} T-D-W\sin\gamma&=\frac{W}{g}\frac{dV}{dt}\\ L&=W\end{aligned}}$$

Rate of Climb Derivation

$$T - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt}$$

- **Given that RC = V * sin(γ), sin(γ) can be rewritten as: RC/V:**

$$RC = \frac{dh}{dt} = V\sin\gamma \rightarrow \sin\gamma = \frac{RC}{V}$$

$$T - D - W\left(\frac{dh/dt}{V}\right) = \frac{W}{g}\frac{dV}{dt} \qquad \text{OR} \qquad \frac{dh}{dt} = \frac{(T-D)V}{W} - \frac{V}{g}\frac{dV}{dt}$$

Rate of Climb Derivation

- **Excess power can be used to increase the aircraft's energy**

RC: Rate of Climb

T_{AV}*V: Available Power

D*V: Required Power

(T-D)*V: Excess Power

$$V\frac{dV}{dt} = \frac{d}{dt}\left(\frac{1}{2}V^2\right)$$

$$(T-D)V = W\left(\frac{dh}{dt}\right) + V\frac{W}{g}\frac{dV}{dt}$$

$$\underbrace{(T-D)V}_{\text{Excess Power}} = \frac{d}{dt}\left(\underbrace{Wh}_{\text{P.E}} + \underbrace{\frac{1}{2}\frac{W}{g}V^2}_{\text{K.E}}\right)$$

Rate of Climb Derivation

- **The rate of climb equations can be expressed differently depending on whether we are dealing with a jet aircraft of propeller driven aircraft**

- **For jet aircraft, thrust is nearly constant with speed, so it's convenient to write the rate of climb equation and climb angle equations as:**

$$RC = \frac{\left(T_{AV} - T_{reqd}\right)V}{W} \qquad \gamma = \frac{RC}{V} = \frac{\left(T_{AV} - T_{reqd}\right)}{W}$$

- **For propeller aircraft, thrust decreases with increasing speed, but power remains nearly constant for much of the aircraft's velocity range. It can be useful to write the RC equation in terms of power available and power required:**

$$RC = \frac{\left(P_{AV} - P_{reqd}\right)}{W} \qquad \gamma = \frac{\left(P_{AV} - P_{reqd}\right)}{WV}$$

Maximizing Climb rate

- **To maximize climb rate, need to maximize excess power:**

$$P_{excess} = P_{AV} - P_{reqd}$$

- **For propeller driven aircraft, if we assume that P_{AV} is constant, then the maximum power excess occurs when P_{reqd} is minimized**
  - **Minimum P_{reqd} occurs at velocity for max $C_L^{3/2}/C_D$**

- **For jet aircraft, available power increases significantly with velocity, so the velocity for excess power is a little harder to find**

Max Climbing Rate for Propeller Aircraft

- **For propeller aircraft, thrust is not constant with speed, but P_{AV} is approximately constant with speed**

$$RC = \frac{\left(P_{AV} - P_{reqd}\right)}{W}$$

- **From the RC equation, maximum climb rate will occur when power is minimized**
  - **Power is minimized with $C_L^{3/2}/C_D$ is maximized:**

$$\frac{\partial}{\partial C_L}\left(\frac{C_L^{3/2}}{C_D}\right) = \frac{\partial}{\partial C_L}\left(\frac{C_L^{3/2}}{C_{D,0} + C_L^2 / \pi Ae}\right) = 0$$

$$\frac{3}{2} C_L^{1/2}\left(C_{D,0} + \frac{C_L^2}{\pi Ae}\right) - C_L^{3/2}\frac{2C_L}{\pi Ae} = 0$$

$$C_{D,0} = \frac{C_L^2}{3\pi Ae} \longrightarrow C_{L,\max RC} = \sqrt{3C_{D,0}\pi Ae}$$

Max Climb Rate for Turbojet Aircraft

- **Turbojets can be approximated as having constant thrust with speed**

$$W = L = \frac{1}{2}\rho V^2 S C_L \longrightarrow V = \sqrt{\frac{2W}{\rho S C_L}}$$

- **With T_{AV} constant, we can examine how RC varies with C_L :**

T_{reqd} =D

L = W

$$\longrightarrow T_{reqd} = \frac{W}{C_L / C_D}$$

$$RC = \frac{\left(T_{AV} - T_{reqd}\right)V}{W} = \sqrt{\frac{2W}{\rho S C_L}}\left(\frac{T_{AV}}{W} - \frac{C_D}{C_L}\right)$$

$$RC = \sqrt{\frac{2W}{\rho S}}\left(\frac{T_{AV}}{W} C_L^{-1/2} - \frac{C_{D,0} + \frac{C_L^2}{\pi A e}}{C_L^{3/2}}\right) \longrightarrow$$

For maximum rate of climb differentiate with respect to C_L and find

$$\frac{\partial}{\partial C_L} = 0$$

Max Climb Gradient for Turbojet Aircraft

- **Flight path angle is found from**

$$\gamma = \frac{RC}{V} = \frac{\left(T_{AV} - T_{reqd}\right)}{W}$$

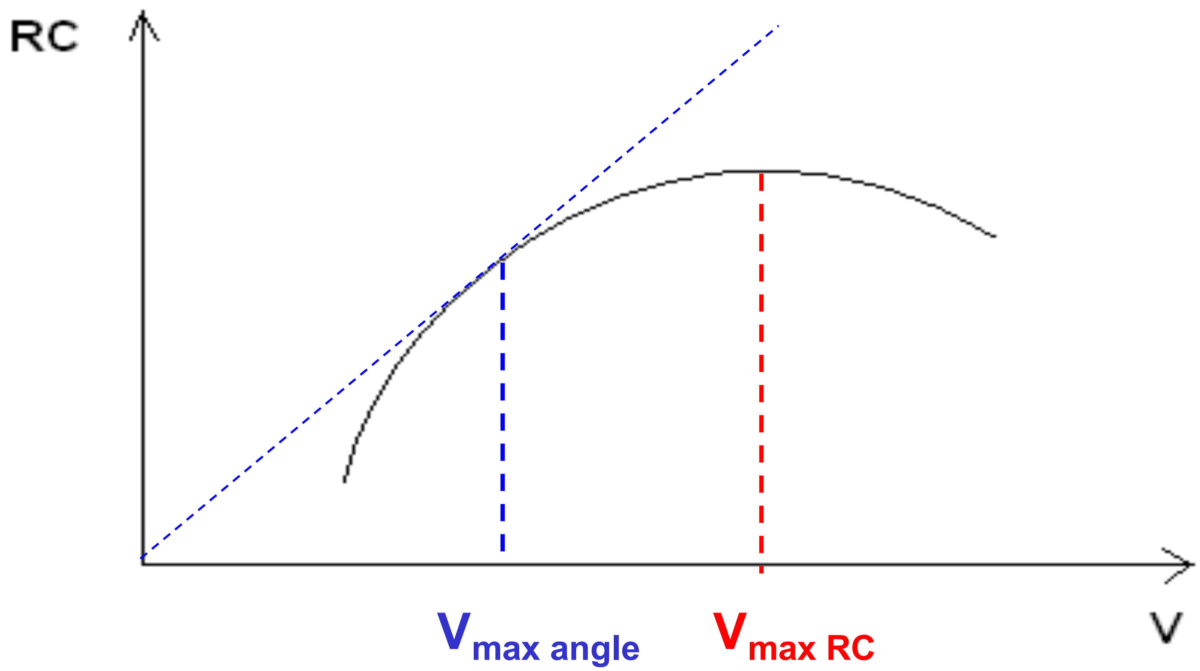
- **For turbojets, since T_{AV} is constant with speed, maximum climbing angle occurs when at a speed where T_{reqd} is minimized**
  - **T_{reqd} is minimized when L/D is maximized**
  - **Max L/D occurs with $C_{D,i}$=$C_{D,0}$**
- **Therefore, max climbing gradient occurs at max L/D, which corresponds to a C_L equal to**

$$C_{L,\max\angle} = \sqrt{C_{D,0}\pi Ae}$$

TURBOJETS ONLY

Maximum Climb Gradient for Propeller Aircraft

- **Climb gradient equation for props:** $$\gamma = \frac{\left(P_{AV} - P_{reqd}\right)}{WV}$$
- **The additional velocity term in equation makes solving for maximum gradient more challenging, but it can be found by differentiating equation with respect to CL, or by graphing RC vs. V**



Steep Angle Climbs

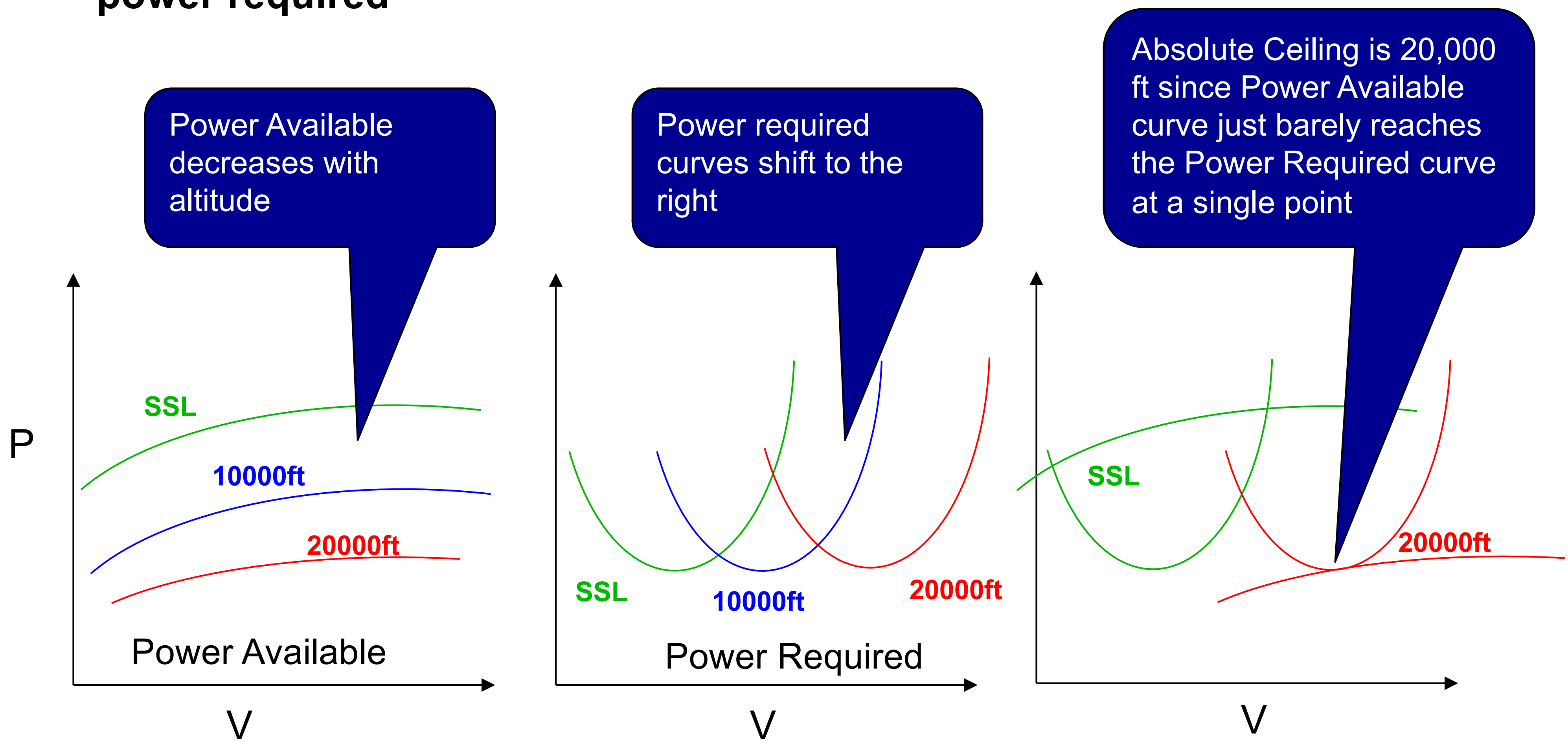
- So far, our derivations for climbing performance have assumed small flight path angles
  - At small angles, we can assume that L = W
- Aircraft with higher thrust to weight ratios may have a high climb angle
  - At large angles, the L=W assumption no longer is a very good approximation
  - And since drag depends on lift (induced drag), T_{reqd} is no longer independent of γ
- To find flight path angle, the two equations can be solved iteratively:
  - Find CL corresponding to zero flight path angle
  - Use computed CL to find drag due to lift
  - Compute flight path angle from (T-D)/W
  - Use new flight path angle to obtain new value for lift (CL)
  - Repeat process until flight path angle converges

$$L = W\cos\gamma$$

$$T - D = W\sin\gamma$$

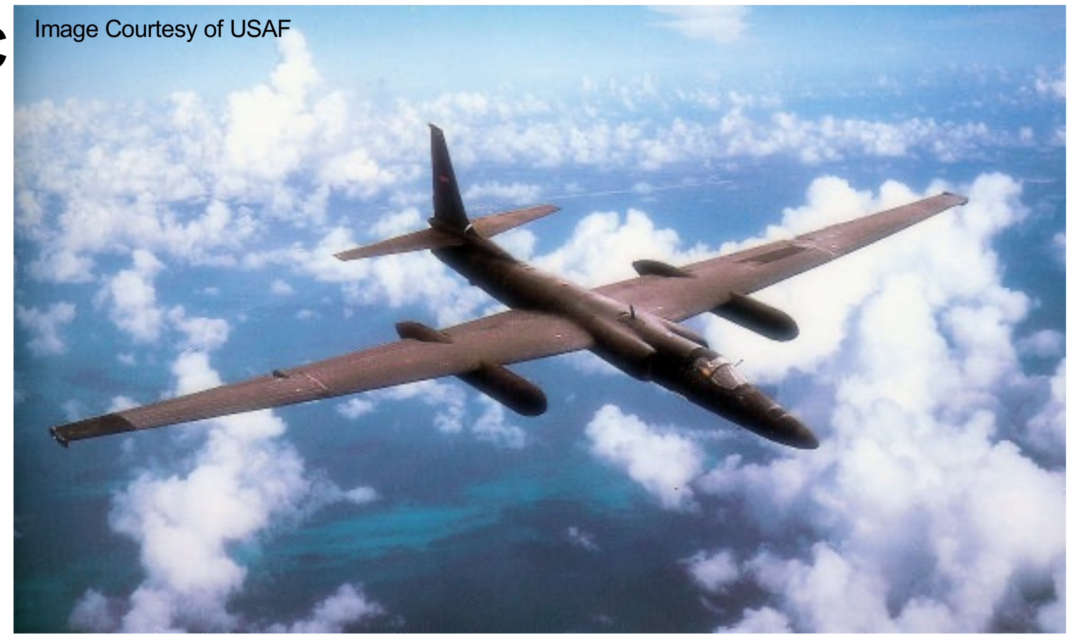
Absolute Ceiling

- **Absolute ceiling is where the power available is tangent with the power required**



Ceiling Definitions

- **Service Ceiling:**
  - **The altitude of which the RC equals**
    - **100 fpm for commercial piston-propeller aircraft**
    - **500 fpm for commercial jet aircraft**
    - **100 fpm for military aircraft**



- **Absolute Ceiling:**
  - **The altitude where RC equals 0**
  - **Not achievable**
  - **Only obtained experimentally by extrapolating measurements**

Lockheed U-2 has a service ceiling of 80,000 ft

Engine Out Conditions

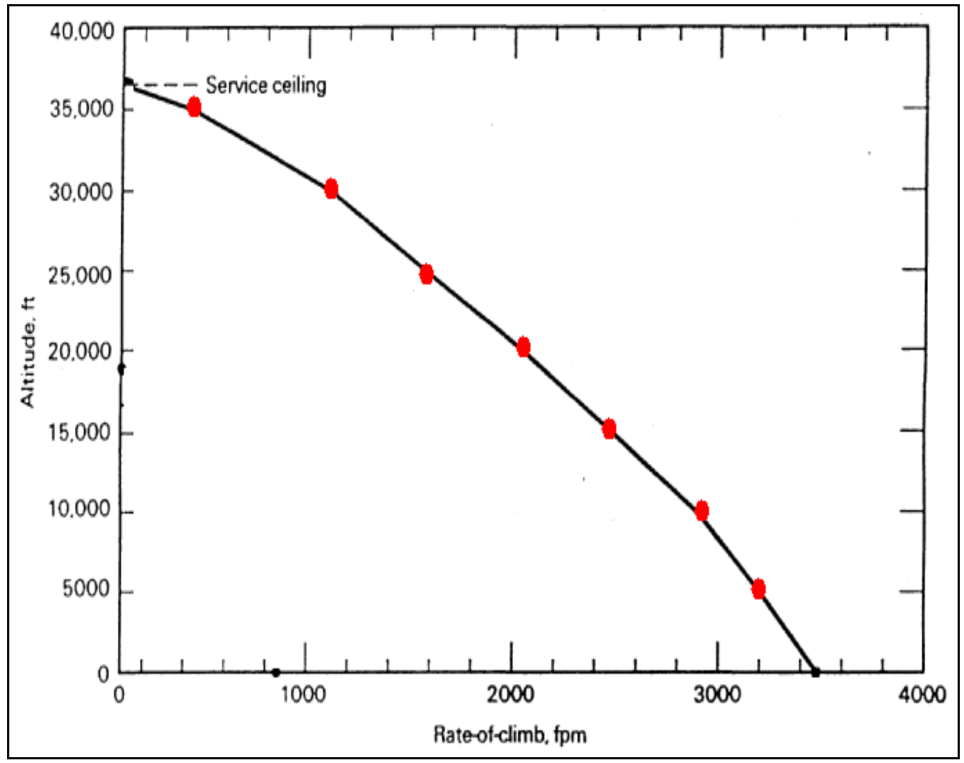
- **If a multi-engine aircraft loses an engine, its thrust may drop below the thrust required to maintain its current altitude**
- **Drag will also increase due to the inoperative engine, as well as from additional trim drag**
- **Ceiling will drop**
  - **Can the plane sustain flight on one engine?**
  - **Will the ceiling be higher than the terrain the plane is flying over?**

Time to Climb

- **To find the time to climb, we need to integrate the rate of climb with respect to altitude:**

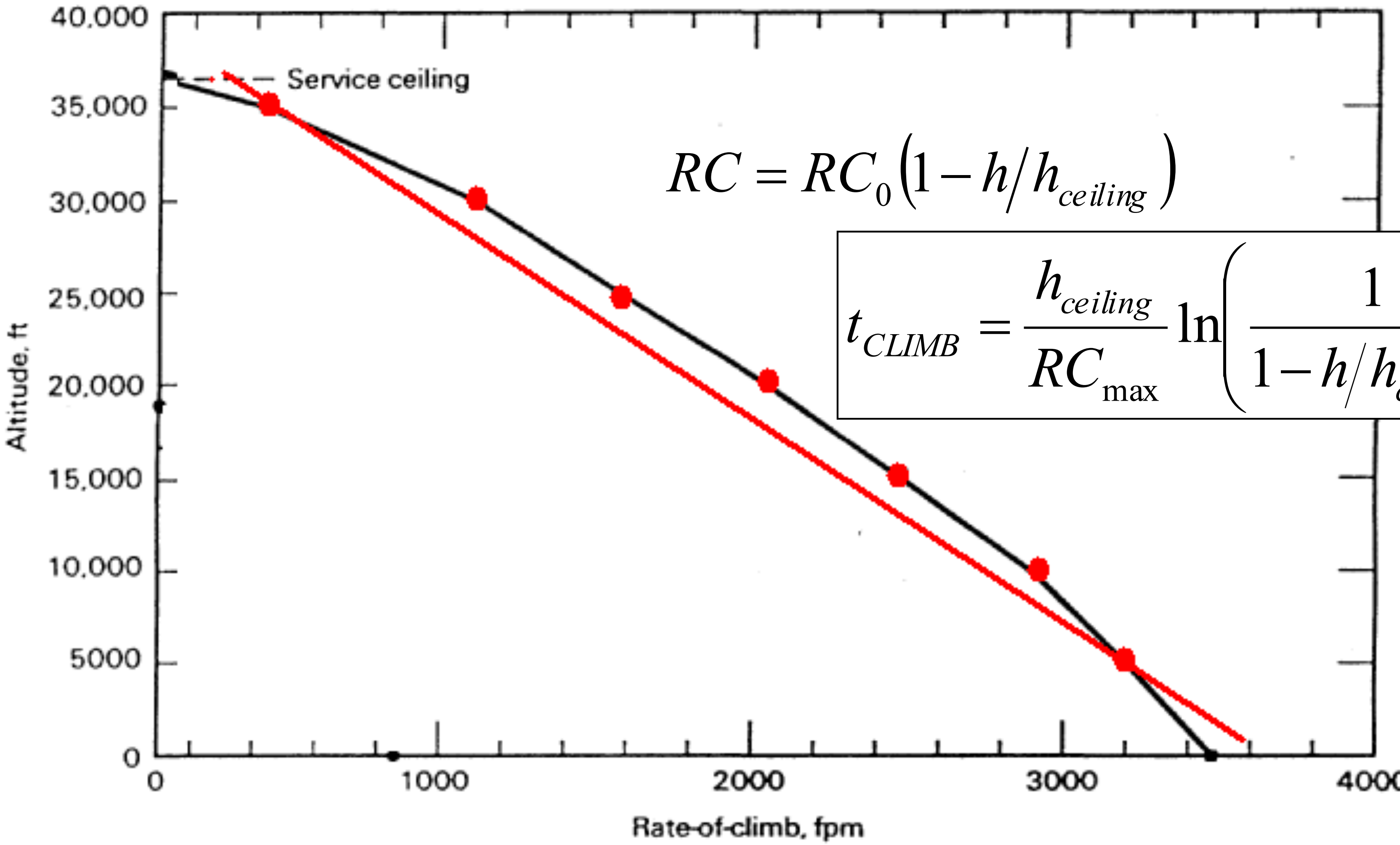
$$RC = \frac{dh}{dt} \longrightarrow t_2 - t_1 = \int_{h_1}^{h_2} \frac{dh}{RC}$$

- **Need to first determine how RC varies with altitude**
  - **Available thrust or power decreases with altitude, which significantly complicates the problem**
  - **One way is to compute RC values at various altitude increments and evaluate the integral numerically**



Time to Climb (cont.)

- **Another method for obtaining a time to climb estimate is to assume a linear profile for RC with respect to altitude**



$$RC = RC_0\left(1 - h/h_{ceiling}\right)$$

$$\boxed{t_{CLIMB} = \frac{h_{ceiling}}{RC_{\max}} \ln\left(\frac{1}{1 - h/h_{ceiling}}\right)}$$

References

1. **C.E.Lan, J. Roskam, *Airplane Aerodynamics and Performance*, Design Analysis Research Corporation, 1997**
2. **McCormick, B.W., *Aerodynamics, Aeronautics and Flight Mechanics*, 2nd edition, Wiley & Sons, 1995**
3. **E. Field, MAE 154S, Mechanical and Aerospace Engineering Department, UCLA, 2001**