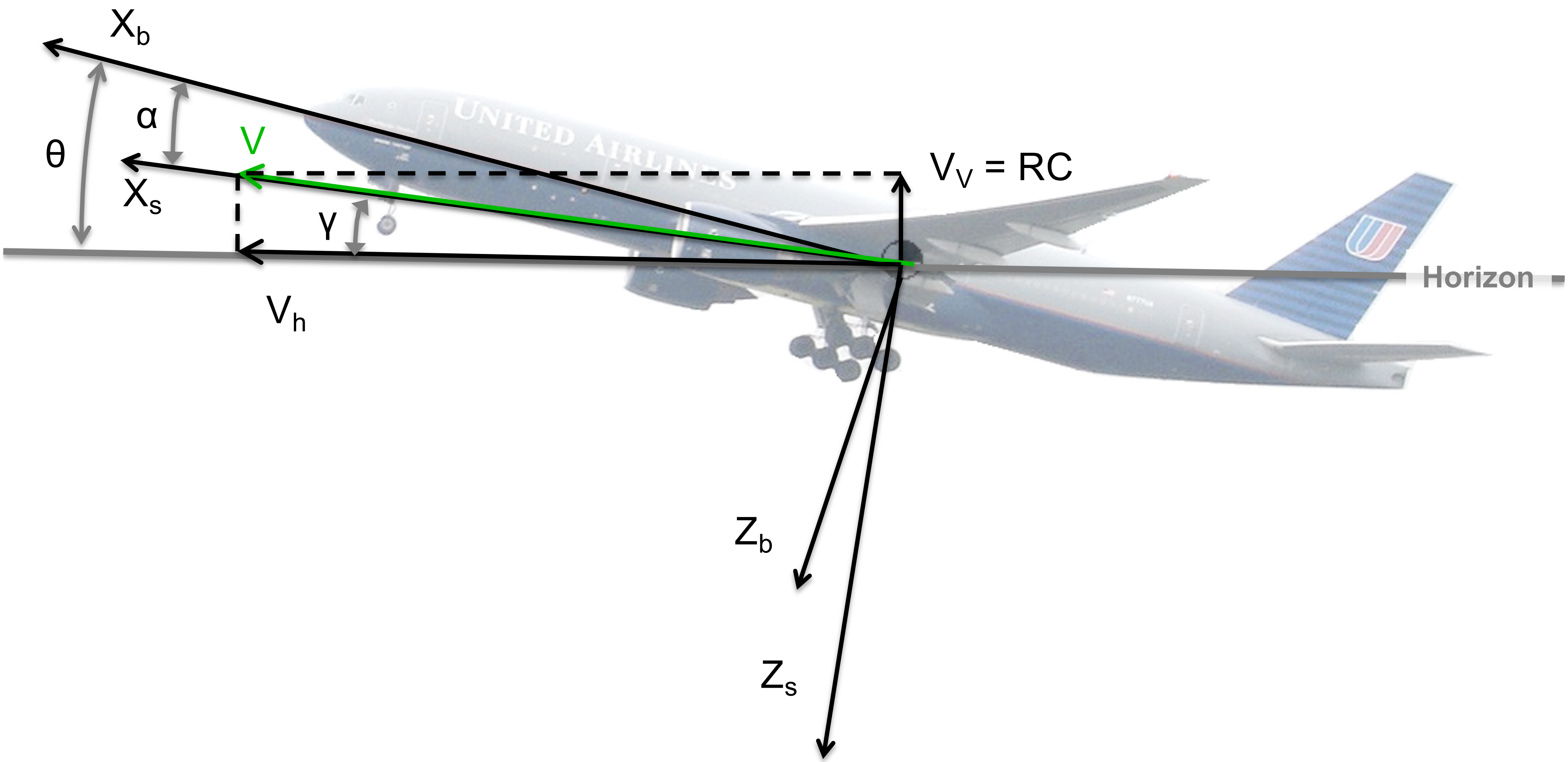
Lecture 5

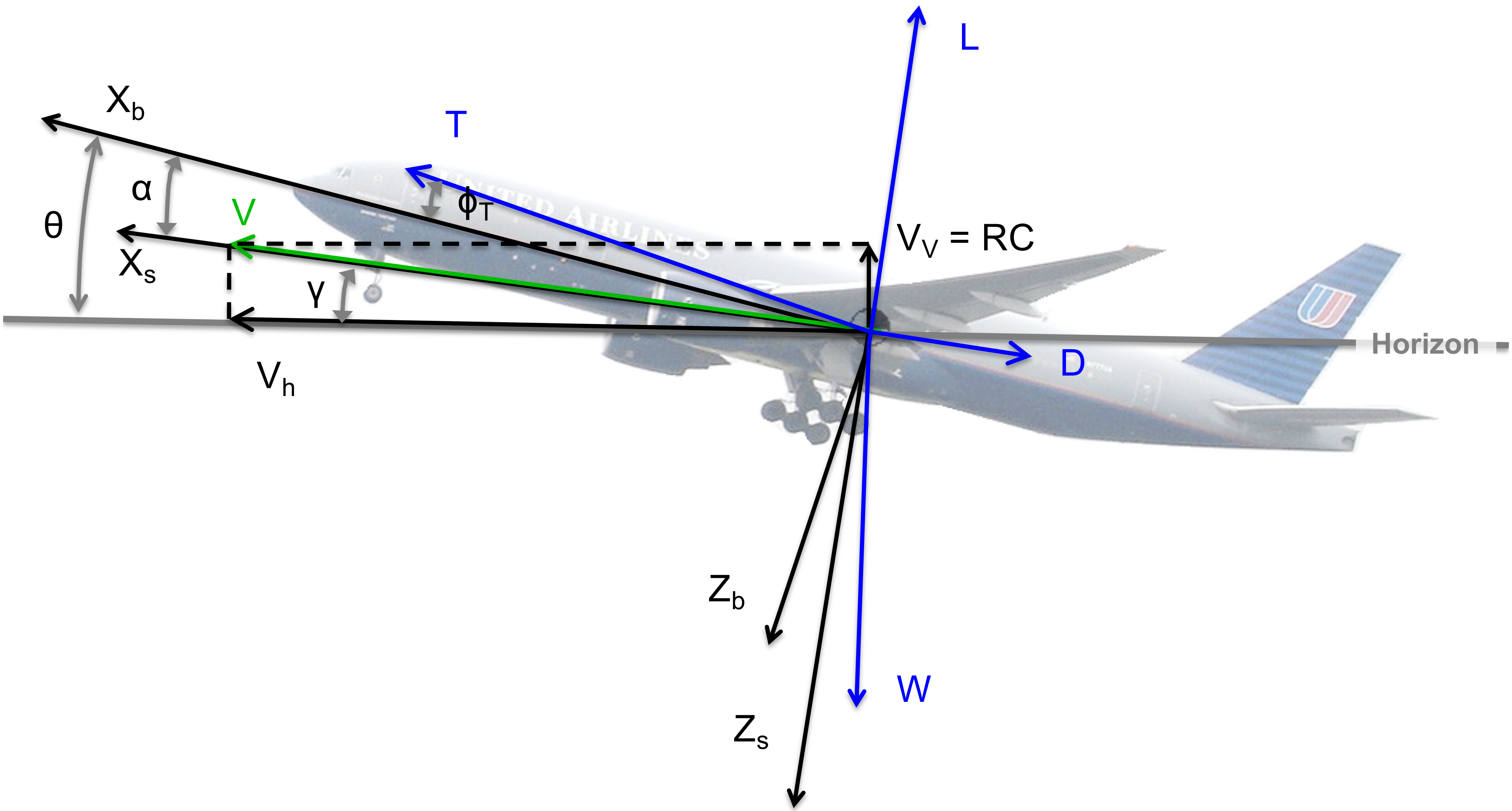
Aircraft Performance Introduction

Image Courtesy of US Navy

Stability Axis

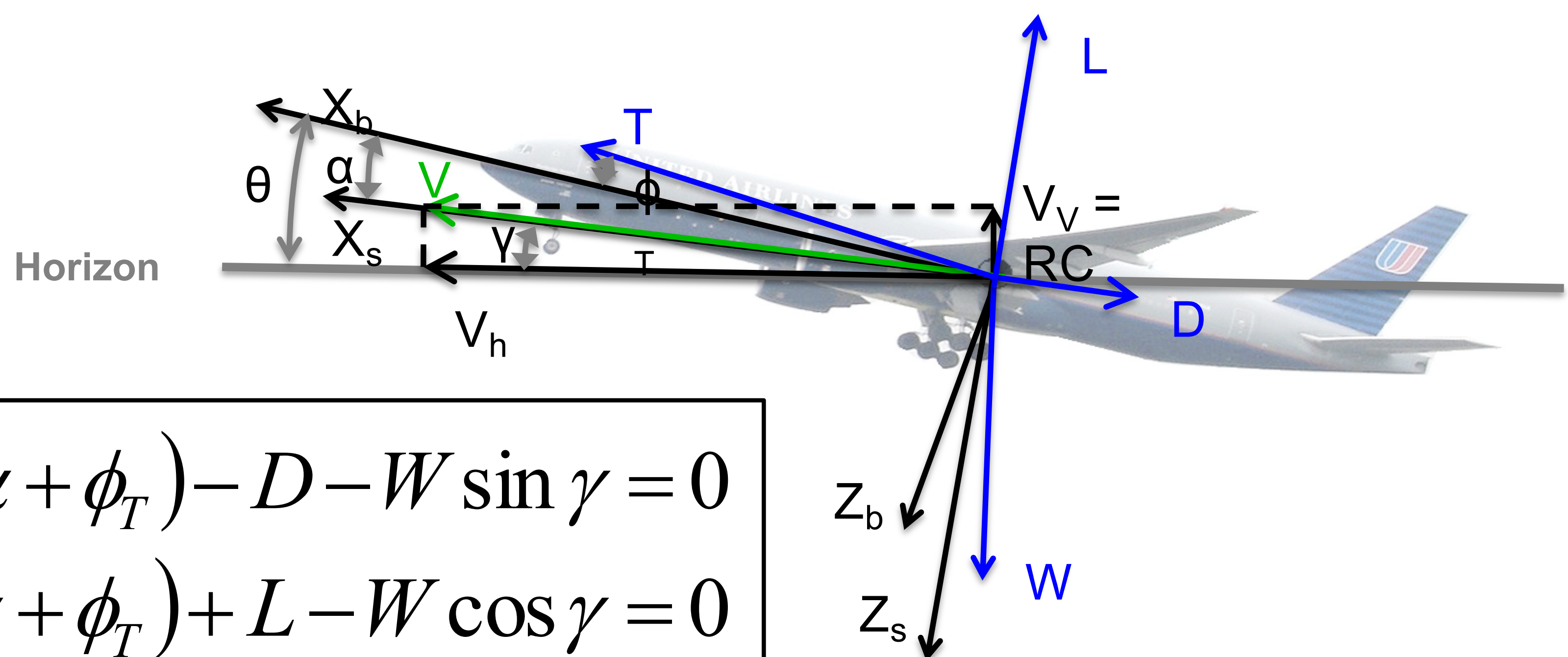


Forces



Equations of Motion for Steady Flight

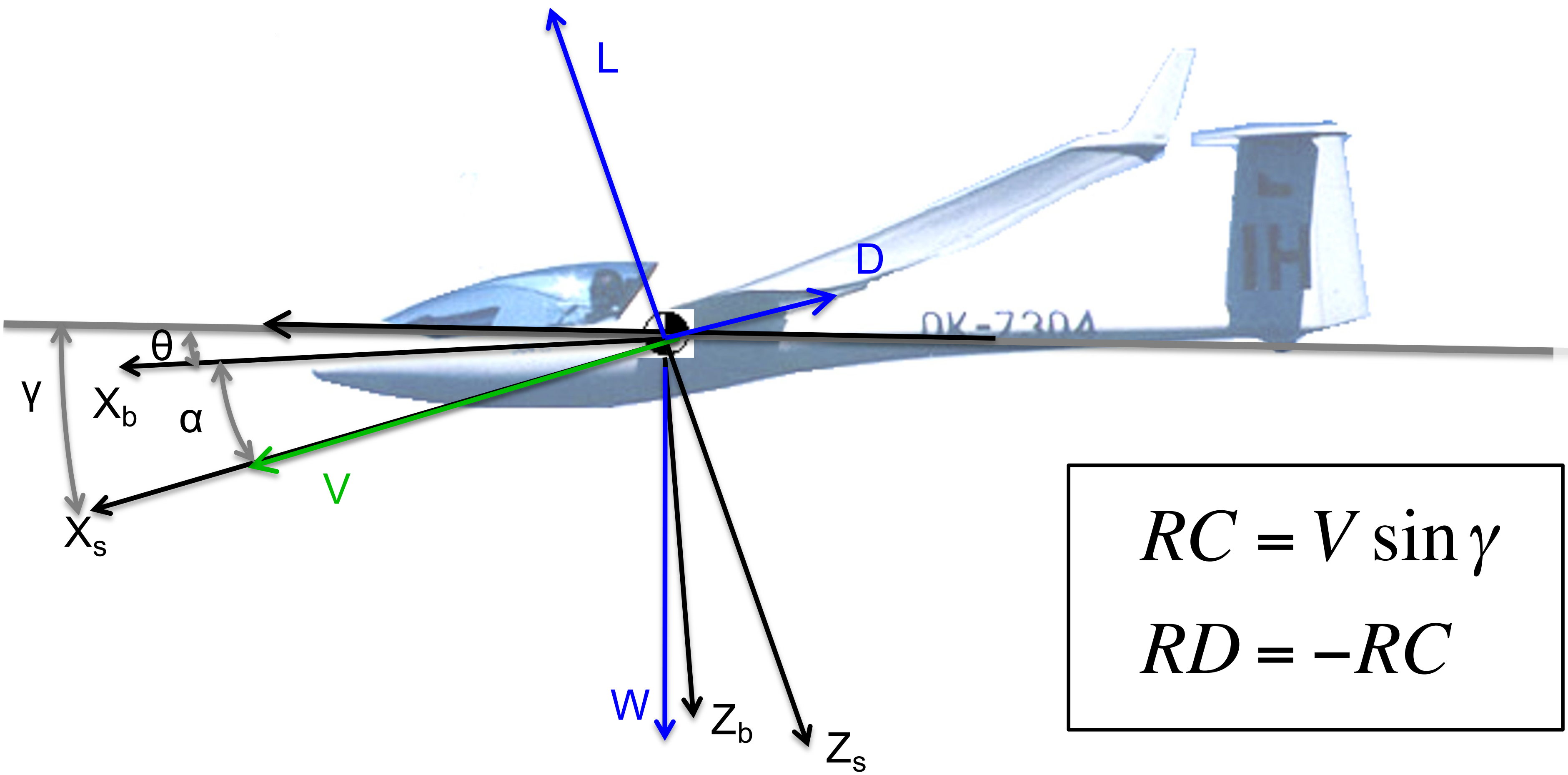
- **For level flight: γ=0**
- **In steady, level flight, we often make the approximation that L=W and T=D**
- **If thrust is known and a certain flight path angle is desired, you can compute necessary AoA and velocity**



$$T\cos(\alpha+\phi_T)-D-W\sin\gamma=0$$
$$T\sin(\alpha+\phi_T)+L-W\cos\gamma=0$$

Steady Un-powered Flight

- **For un-powered flight (T = 0), flight path angle will be negative since drag cannot be zero**



$$RC = V \sin\gamma$$

$$RD = -RC$$

Steady Un-powered Flight

Photo by DG Flugzeugbau

- **Minimizing glide slope is achieved by maximizing L/D**

$$-D - W\sin\gamma = 0$$

$$L - W\cos\gamma = 0$$

$$\sin\bar{\gamma} = \frac{D}{W} = \frac{D\cos\bar{\gamma}}{L}$$

Glide Angle $\rightarrow$ $\bar{\gamma} = -\gamma$

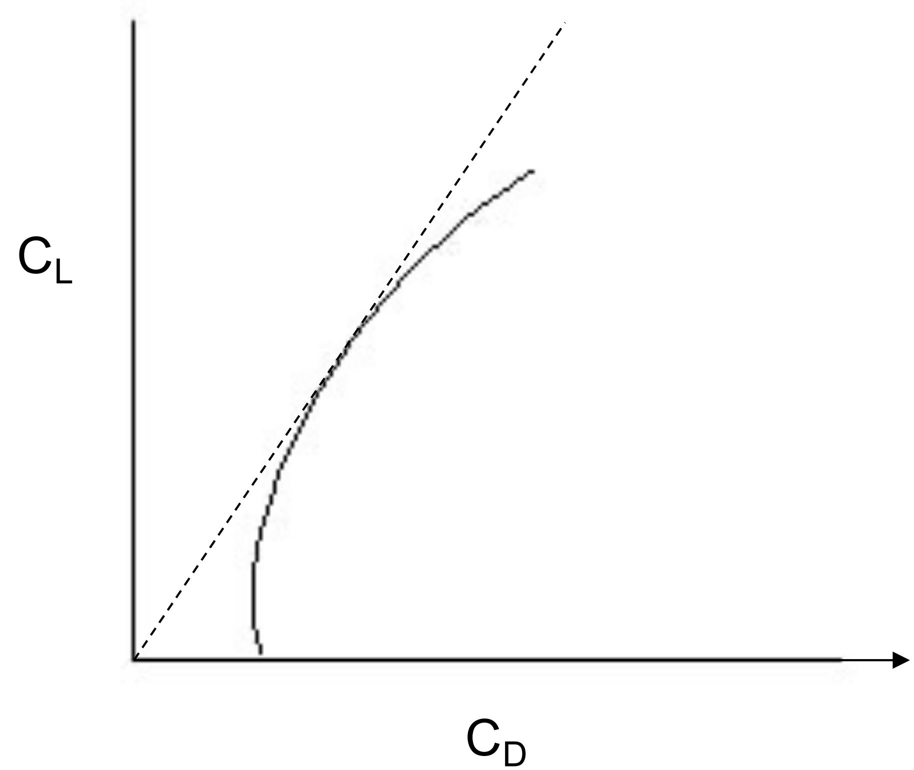
Drag Polar

- **For subsonic aircraft, the total aircraft drag can be written as $D = D_{Parasite} + D_{Induced}$**

$$C_D = C_{D,0} + \frac{C_L^2}{\pi A e}$$

- **A straight-line tangent to the drag polar indicates where the maximum L/D is located**

- **In general, the minimum C_D may be at a nonzero C_L value, but usually it is small**



Rate of Descent

- **By equating aero forces with the weight, the velocity for a steady glide can be expressed as**

$$V = \sqrt{\frac{2W}{\rho S C_L} \cos \bar{\gamma}}$$

- **The rate of descent is the negative of rate of climb: (RD = -RC)**

$$RD = V \sin \bar{\gamma} = \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{C_D^2}{C_L^3} \cos^3 \bar{\gamma}}$$

$$\sin \bar{\gamma} = \frac{D}{W} = \frac{D \cos \bar{\gamma}}{L}$$

- **Assuming a shallow glide slope, the aircraft speed and RD can be estimated as:**

$$V = \sqrt{\frac{2W}{\rho S C_L}} \qquad RD = \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{C_D^2}{C_L^3}}$$

Minimum Rate of Descent and Maximum Range

- **Maximizing gliding range is achieved by maximizing the lift to drag ratio. This minimizes the glide angle and produces the shallowest descent**

- **It can be shown that:**

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{1}{2}\sqrt{\frac{\pi A e}{C_{D,0}}}$$

- **To maximize time aloft, the aircraft should be flying at conditions that produce the minimum descent rate, which is to maximize:**

$$C_L^3 / C_D^2$$

- **These two ratios are important for both for gliding aircraft and powered flight**

Glider Example

- **Aircraft data for a hypothetical glider is given in the table below**
  - **What velocity should the glider fly at to maximize its glide distance? If its starting altitude is 1000 ft., how far can it travel before reaching the ground (assume no wind)?**

    Flying at 95 fps maximizes the L/D, which leads to the shallowest glide slope. Starting at 1000 ft, the max distance would be L/D * 1000 ft = 24,390 ft.

  - **What velocity should the glider fly at to maximize its time aloft?**

    Flying at 75 fps maximizes the $C_L^{3/2}/C_D$, which minimizes the descent rate.

V_{air} (fps)	C_L	C_D	C_L/C_D	$C_L^{3/2}/C_D$	$\bar{\gamma}$ (deg)	RD (fps)
65.00	1.59	0.09	18.53	23.37	3.09	3.51
70.00	1.37	0.07	20.26	23.74	2.83	3.45
75.00	1.20	0.05	21.73	23.76	2.63	3.45
80.00	1.05	0.05	22.89	23.46	2.50	3.50
85.00	0.93	0.04	23.71	22.87	2.41	3.58
90.00	0.83	0.03	24.21	22.05	2.37	3.72
95.00	0.74	0.03	24.39	21.05	2.35	3.89
100.00	0.67	0.03	24.31	19.93	2.36	4.11
105.00	0.61	0.03	24.00	18.74	2.39	4.38
110.00	0.56	0.02	23.50	17.52	2.44	4.68

Aircraft data generated with the following flight data:
W= 1200 lbs, S = 150 ft^2 , C_D=0.015+0.028*C_L^2, ρ=0.00238 slug/ft^3

$$\tan\bar{\gamma} = \frac{C_D}{C_L}$$

Thrust Required

- **For steady level flight, T = D and L = W**

$$L = W = q_\infty S C_L \qquad T_R = D = q_\infty S C_D = q_\infty S\left(C_{D,0} + K C_L^2\right)$$

- **Using L = W, we can solve for the lift coefficient:**

$$C_L = \frac{2W}{\rho V^2 S}$$

Note: $K = \dfrac{1}{\pi A e}$

$q_\infty = \dfrac{1}{2}\rho V^2$

- **The drag polar gives us the drag coefficient:**

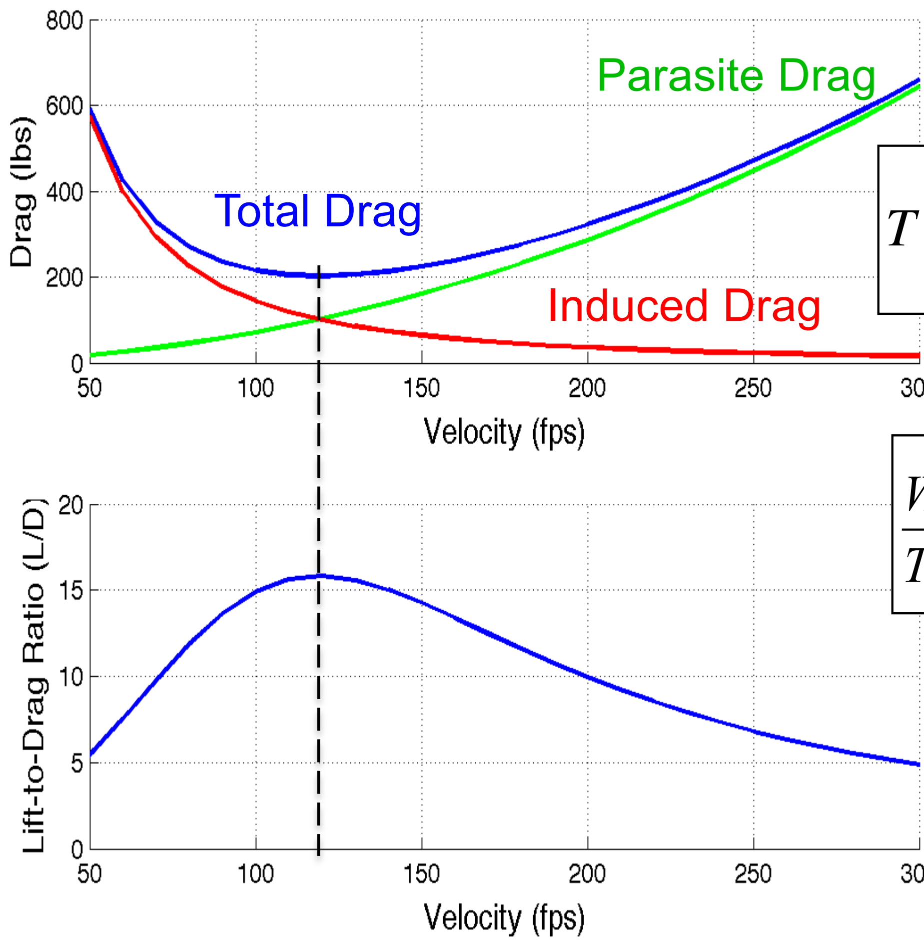
$$C_D = \left(C_{D,0} + K C_L^2\right)$$

- **Plugging in the C_L value into the drag polar, the thrust required to maintain steady level flight is:**

Parasite Drag Induced Drag

$$T_R = D = q_\infty S C_D = q_\infty S\left(C_{D,0} + K C_L^2\right) = \boxed{\frac{1}{2}\rho V^2 S C_{D,0} + \frac{2KW^2}{\rho V^2 S}}$$

Thrust Required (cont.)



$$T_R = D = \frac{1}{2}\rho V^2 S C_{D,0} + \frac{2KS}{\rho V^2}\left(\frac{W}{S}\right)^2$$

$$\frac{W}{T_R} = \frac{L}{D} = \left[\frac{\rho V^2 C_{D,0}}{2(W/S)} + \frac{2K}{\rho V^2}\left(\frac{W}{S}\right)\right]^{-1}$$

$$T_R = \frac{W}{L/D}$$

Aircraft data: W= 3200 lbs, S = 300 ft^2 , C_D=0.02+0.05*C_L^2, ρ=0.00238 slug/ft^3

Thrust Required (cont.)

- **Solving for the Velocity for a given required thrust:**

$$V^2 = \frac{(T_R/W)(W/S) \pm (W/S)\sqrt{(T_R/W)^2 - 4C_{D,0}K}}{\rho C_{D,0}}$$

- **We see that the equation for V and from the Drag vs. V plot that there are two velocities that correspond to a given required thrust, except for the minimum T_R, where there is only one value**
  - **This occurs when**

$$\left(\frac{T_R}{W}\right)^2 - 4C_{D,0}K = 0$$

  - **Which means that**

$$\left(\frac{T_R}{W}\right)_{\min} = \sqrt{4C_{D,0}K} \qquad \left(\frac{L}{D}\right)_{\max} = \frac{1}{\sqrt{4C_{D,0}K}}$$

Thrust Required (cont.)

- **We also can find minimum drag relationships by differentiating T_R with respect to the dynamic pressure**

$$\frac{dT_R}{dq_\infty} = SC_{D,0} - \frac{KW^2}{q_\infty^2 S} \longrightarrow C_{D,0} = \frac{KW^2}{q_\infty^2 S^2} = KC_L^2$$

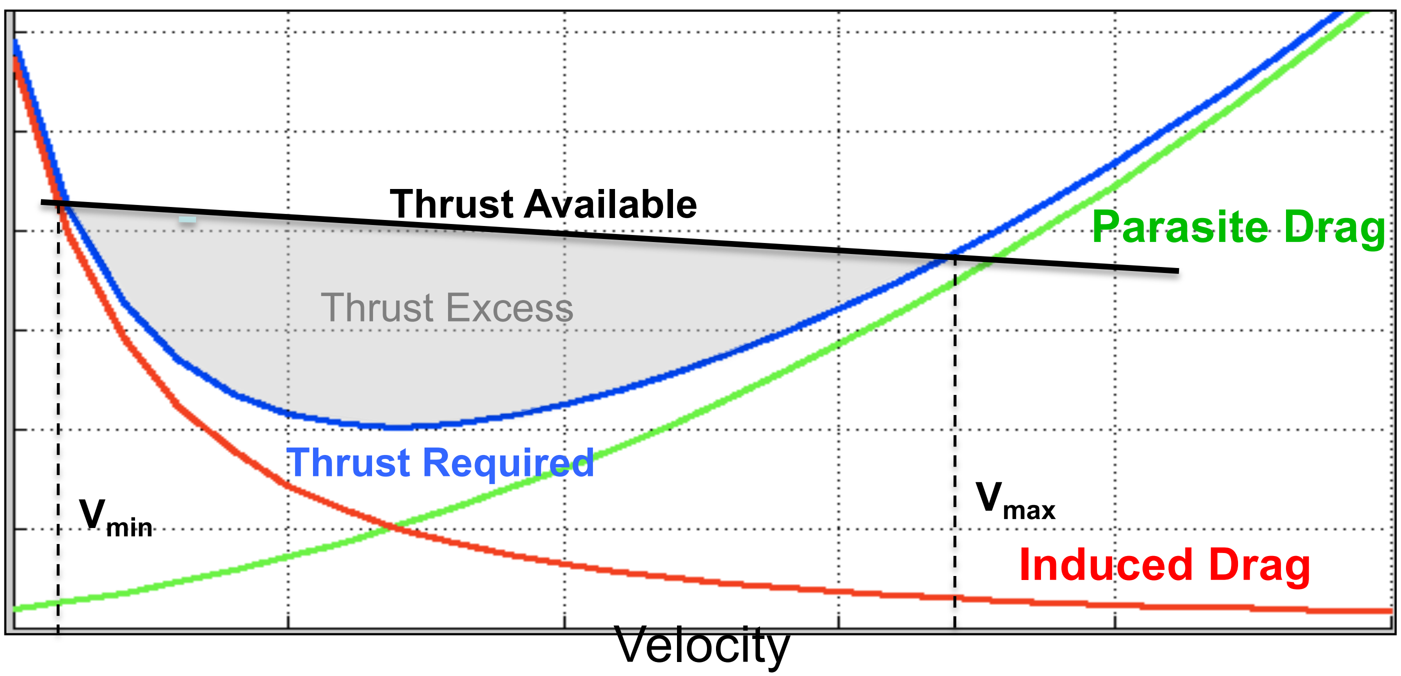
- **The minimum drag condition occurs when parasite drag and induced drag are equal**

Thrust Available

- **Thrust available represents the thrust capability of the aircraft, which is a function of power plant size and type**
  - **For piston-propeller engines, thrust decreases at higher speeds**
  - **For turbojet engines, thrust stays more constant at higher speeds and may increase slightly due to increased mass flow rate into engine. In this course, we often assume that thrust from turbojets is constant with velocity**
  - **Other engine types like turboprops and turbofans have thrust variations somewhere in between turbojets and piston-propeller engines**
- **Thrust available reduces with altitude due to the decreased air density**

Thrust Available (cont.)

- **For a given aircraft, the range of possible steady flight velocities depends upon the relative values of thrust required, T_R, and thrust available, T_A**
- **When $T_A >= T_R$, the difference between the two is the excess thrust**
- **Excess thrust is the thrust capability beyond what is required to maintain steady level flight, and it can be used for accelerating or climbing. If $T_A < T_R$, steady un-accelerated flight cannot be maintained**



Power Available and Power Required

- **When analyzing performance of propeller driven aircraft, often it is more useful to look at power instead of thrust**

- **<u>Power Available</u>:** $$P_{AV} = \eta_p\left(BHP\right) = TV$$

  – **For propeller driven aircraft, power available is roughly constant for a large range of speeds**

- **<u>Power Required</u>:** $$P_R = T_R V$$

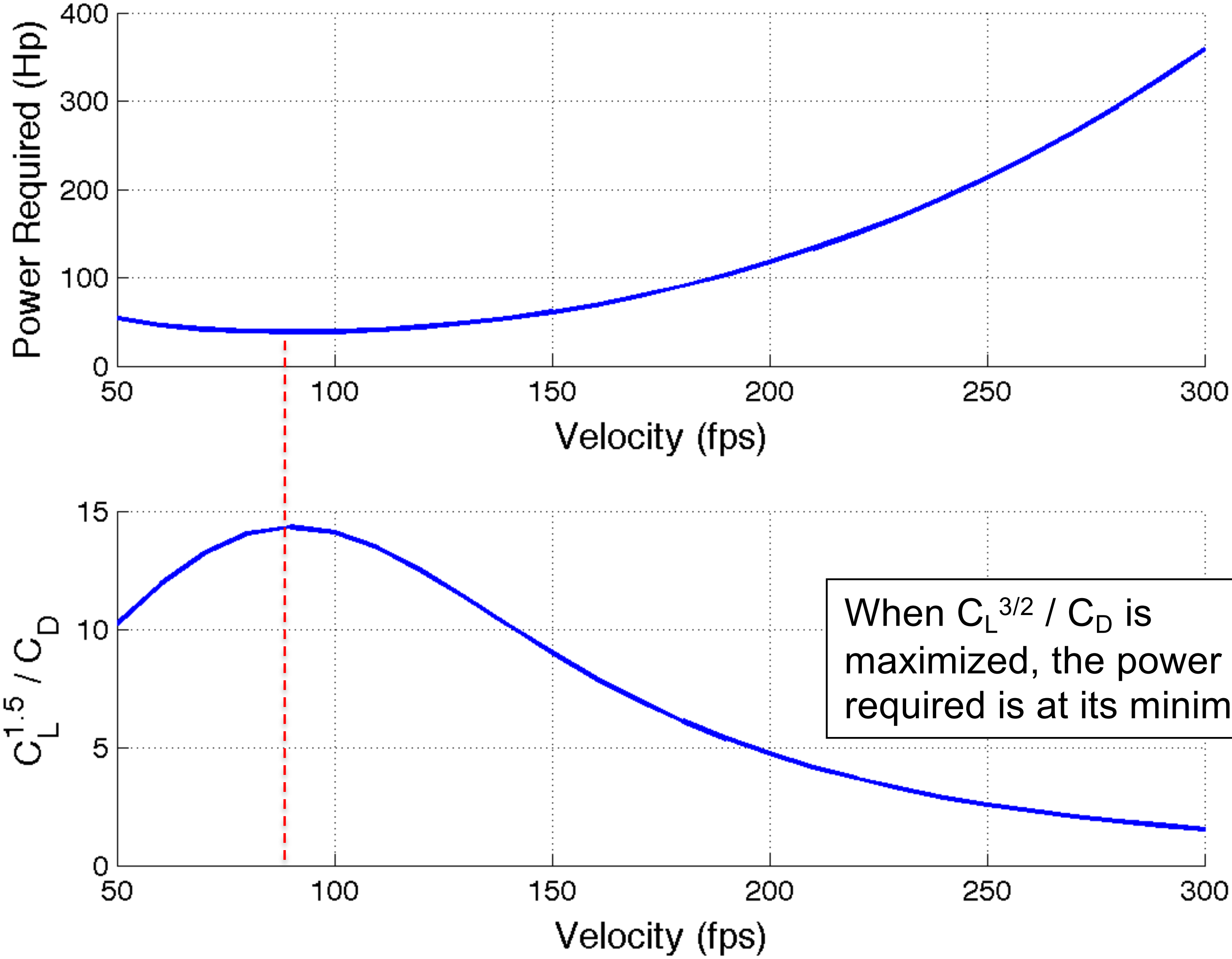
  – **P_R is inversely proportional to $C_L^{3/2} / C_D$**

  – **Power required is minimized when the ratio $C_L^{3/2} / C_D$ is maximized**

$$P_R = \frac{W}{L/D}V = \frac{W}{C_L/C_D}\sqrt{\frac{2}{\rho C_L}\left(\frac{W}{S}\right)}$$

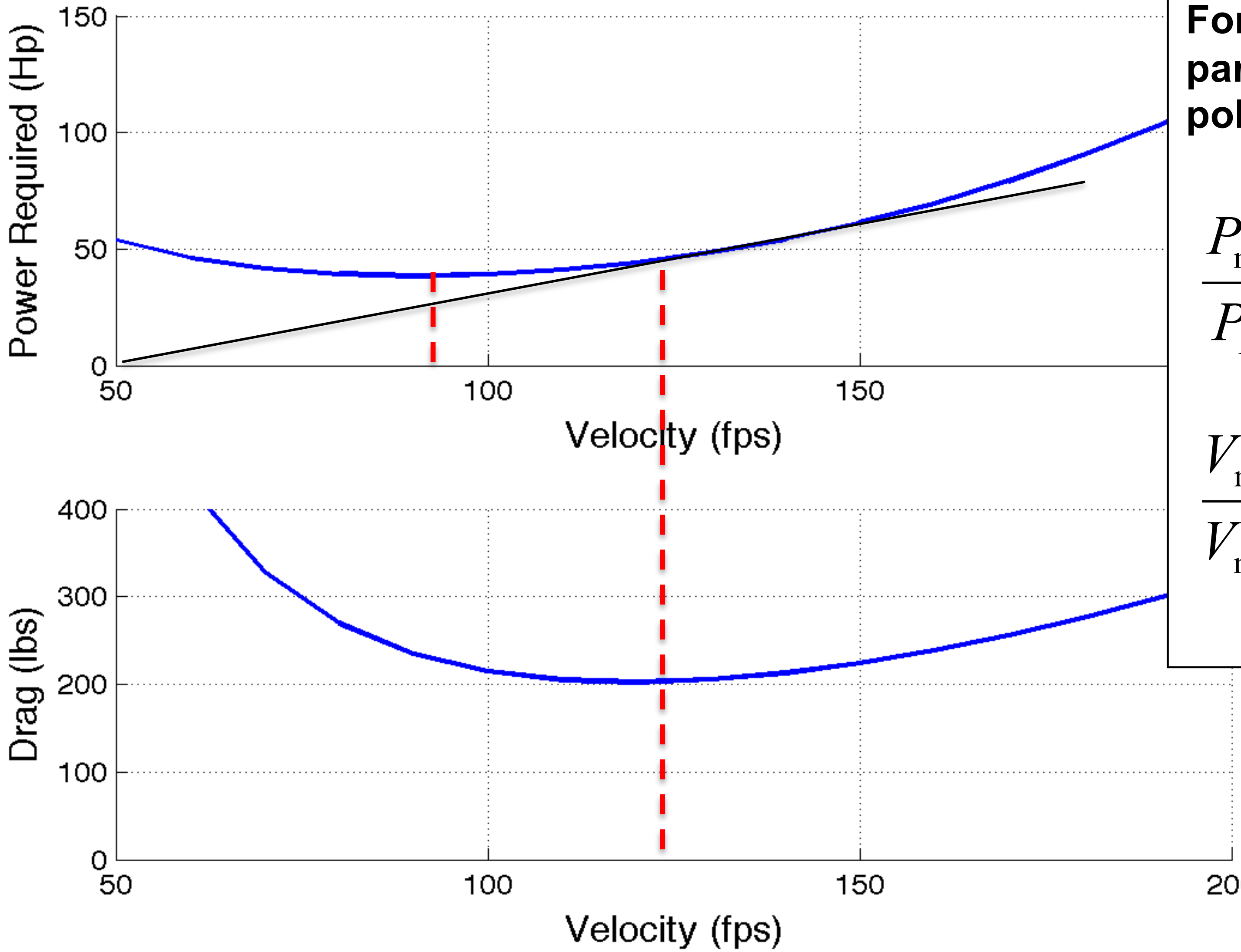
$$P_R = \sqrt{\frac{2S^2C_D^2}{\rho C_L^3}\left(\frac{W}{S}\right)^3} \propto \frac{1}{\sqrt{C_L^3/C_D^2}}$$

Power Available and Power Required



When $C_L^{3/2} / C_D$ is maximized, the power required is at its minimum

Power Available and Power Required



For airplanes with parabolic drag polars:

$$\frac{P_{\min,T_R}}{P_{R,\min}} = 1.14$$

$$\frac{V_{\min,T_R}}{V_{\min,P_R}} = 3^{1/4} = 1.32$$

References

1. **C.E.Lan, J. Roskam, *Airplane Aerodynamics and Performance*, Design Analysis Research Corporation, 1997**
2. **McCormick, B.W., *Aerodynamics, Aeronautics and Flight Mechanics*, 2nd edition, Wiley & Sons, 1995**
3. **E. Field, MAE 154S, Mechanical and Aerospace Engineering Department, UCLA, 2001**