Lecture 10

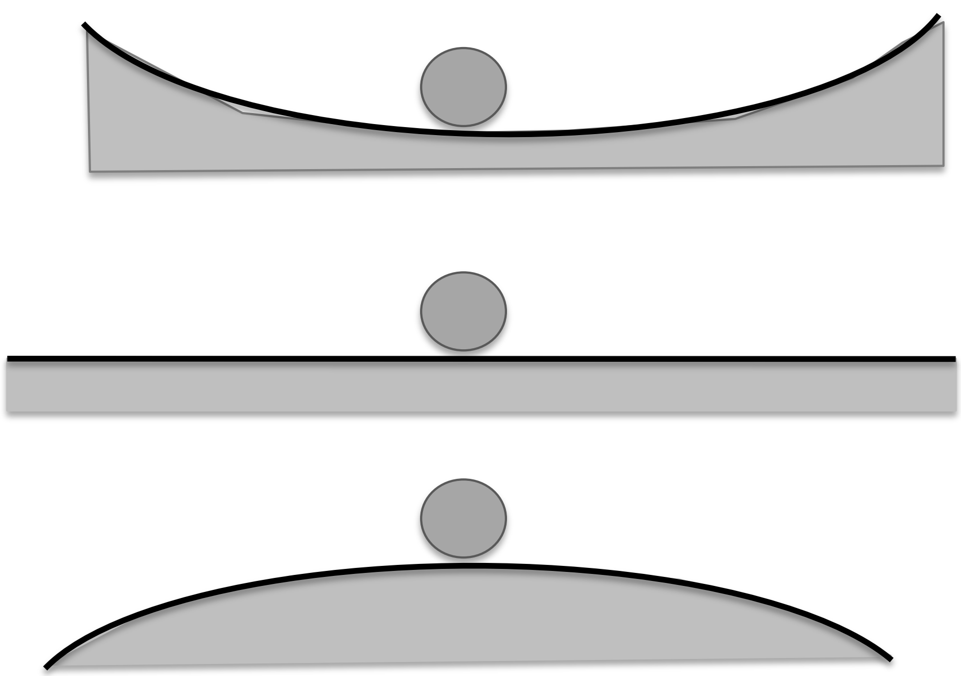
What is Stability?

- **So far this class has focused on aircraft performance, which is governed by forces that affect aircraft translational motion**

- **The 2nd half of this class examines the stability and control of aircraft, which mainly involves the moments that affect the aircraft's rotation**

Static Stability

- **Static stability exists if the forces and moments on the body caused by a disturbance tend initially to return the body toward its equilibrium position**



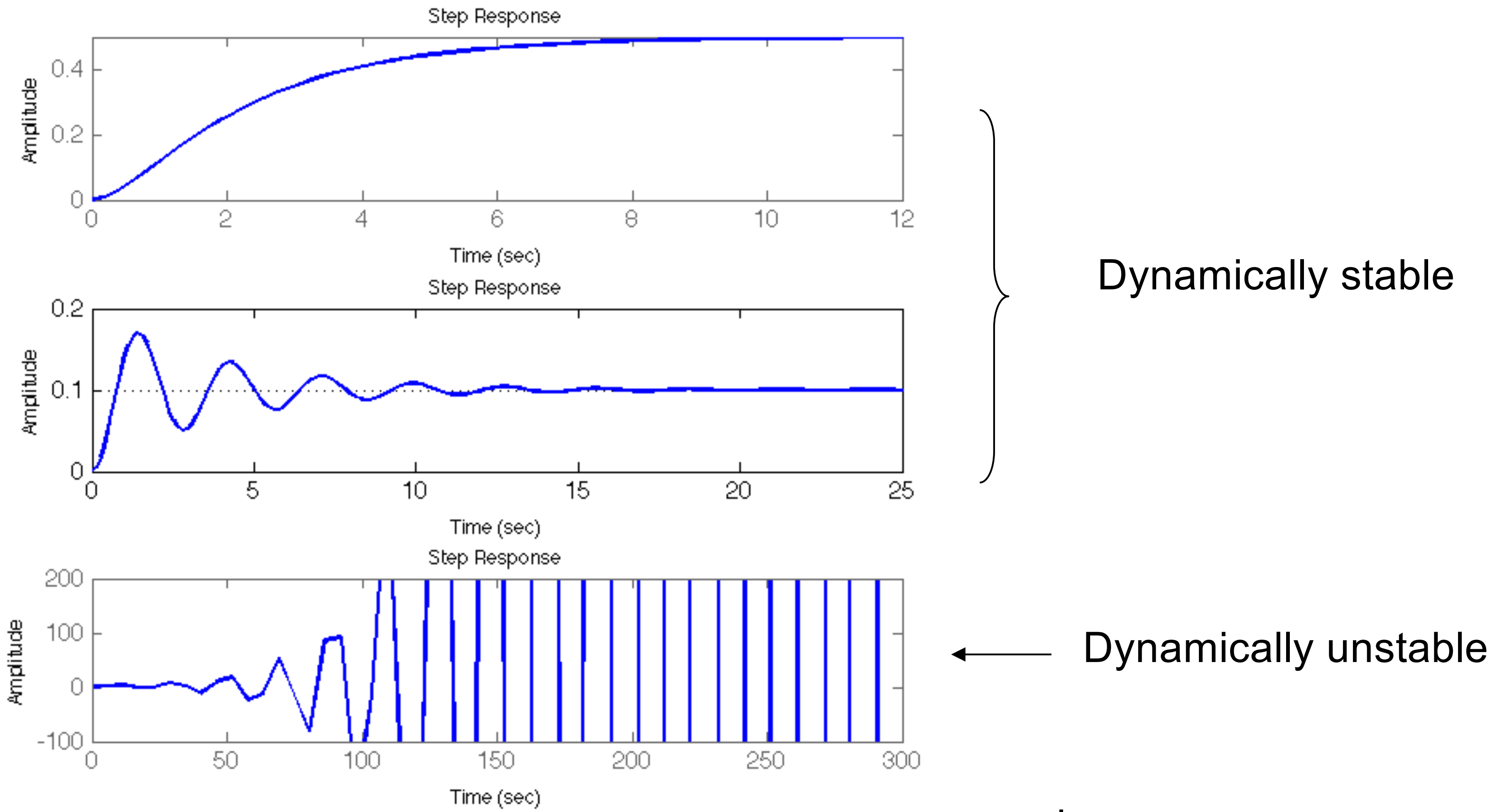
Statically stable

Neutral stability

Statically unstable

Dynamic Stability

- **Dynamic stability exists if the body eventually returns, and remains at, its equilibrium position over a period of time.**
  - **Static Stability necessary but not sufficient condition for dynamic stability**

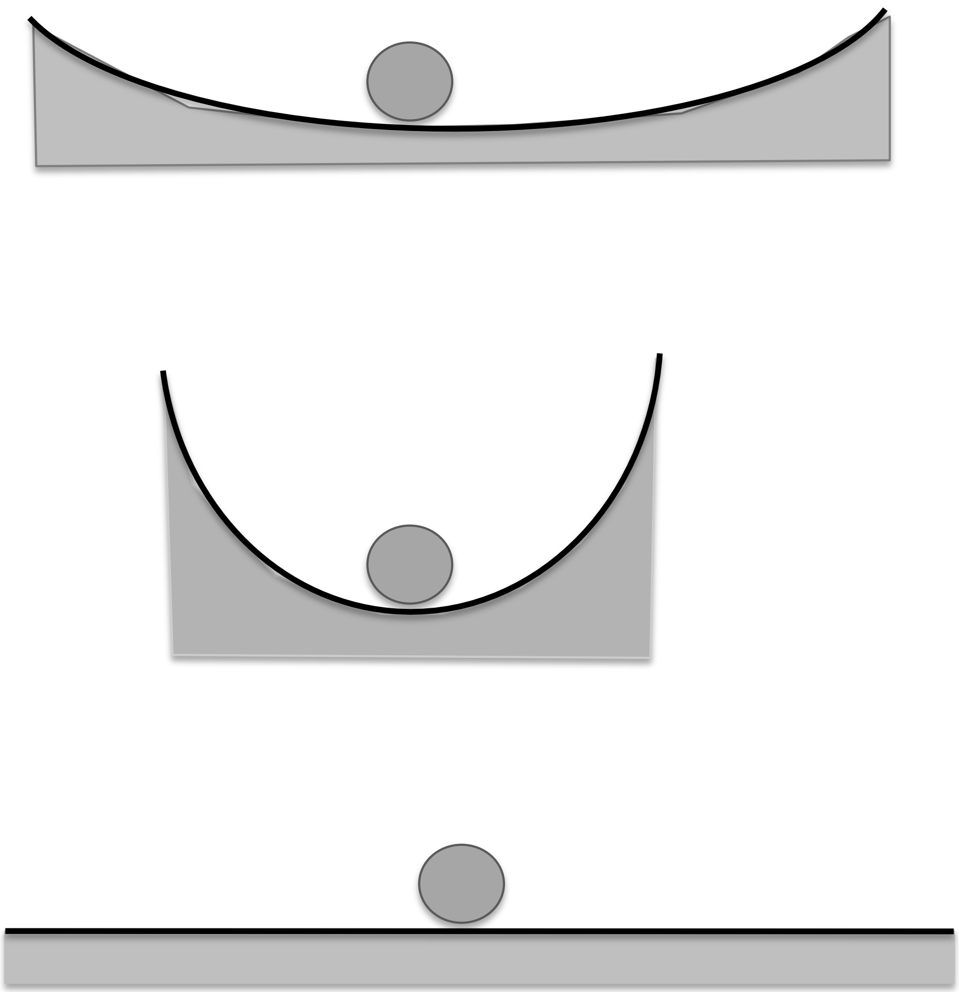


What is Control?

- **Control describes the aircraft's ability to change from one equilibrium point to another, or to produce accelerated motions and maneuvers**

- **Adequate controllability must be available for takeoff, landing and other maneuvers in flight**

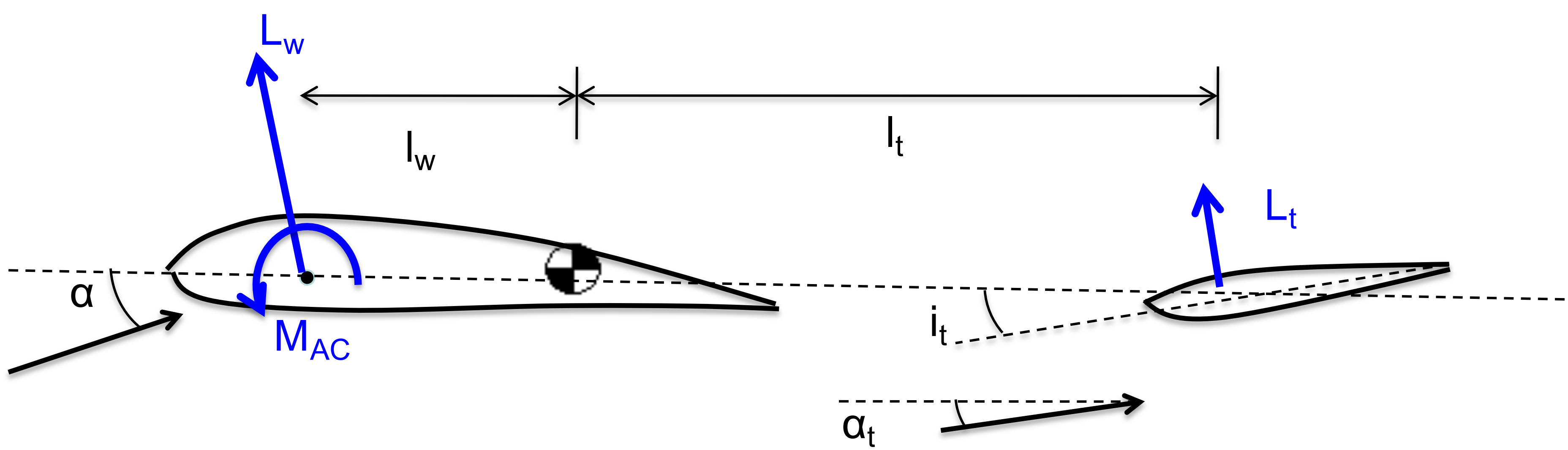
Controllability vs. Stability



- **High stability tends to reduce controllability**
- **As stability decreases, controllability increases**

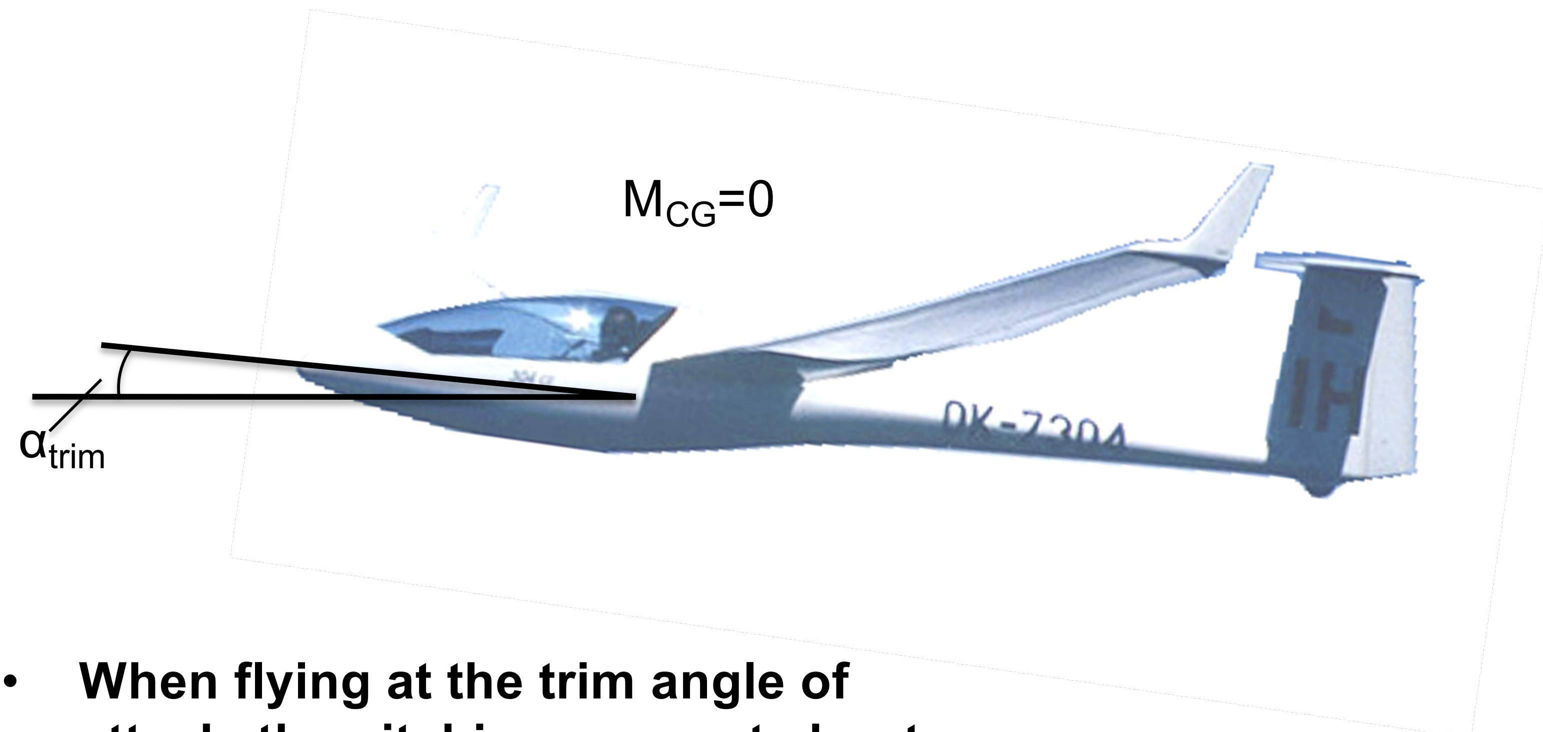
Longitudinal Static Stability

- **Longitudinal static stability is governed by the pitching moments generated by the two lifting surfaces and how they vary with angle of attack**



Longitudinal Static Stability

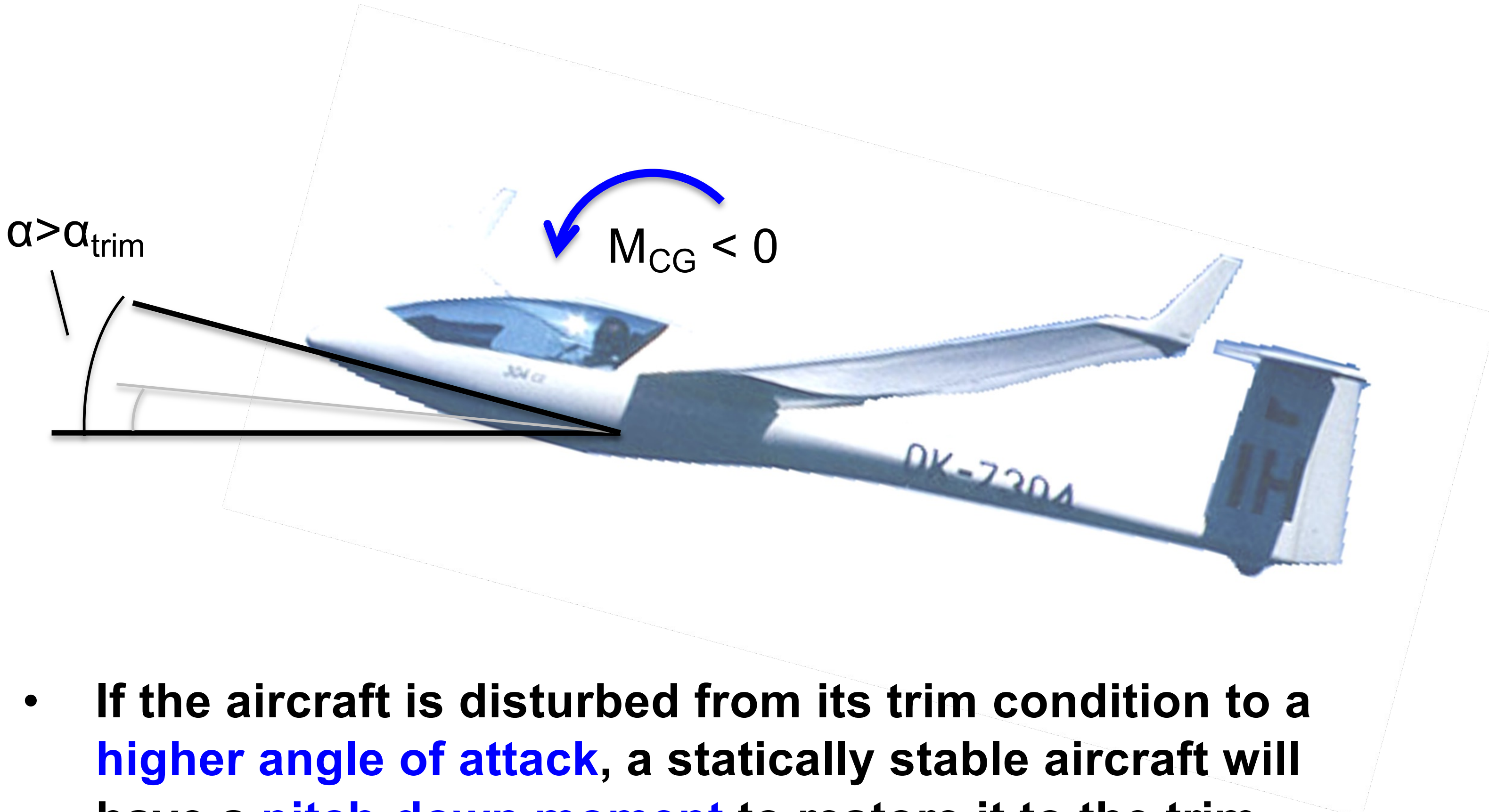
M_{CG}: pitch moment about CG



- **When flying at the trim angle of attack, the pitching moment about the CG is zero**

Longitudinal Static Stability

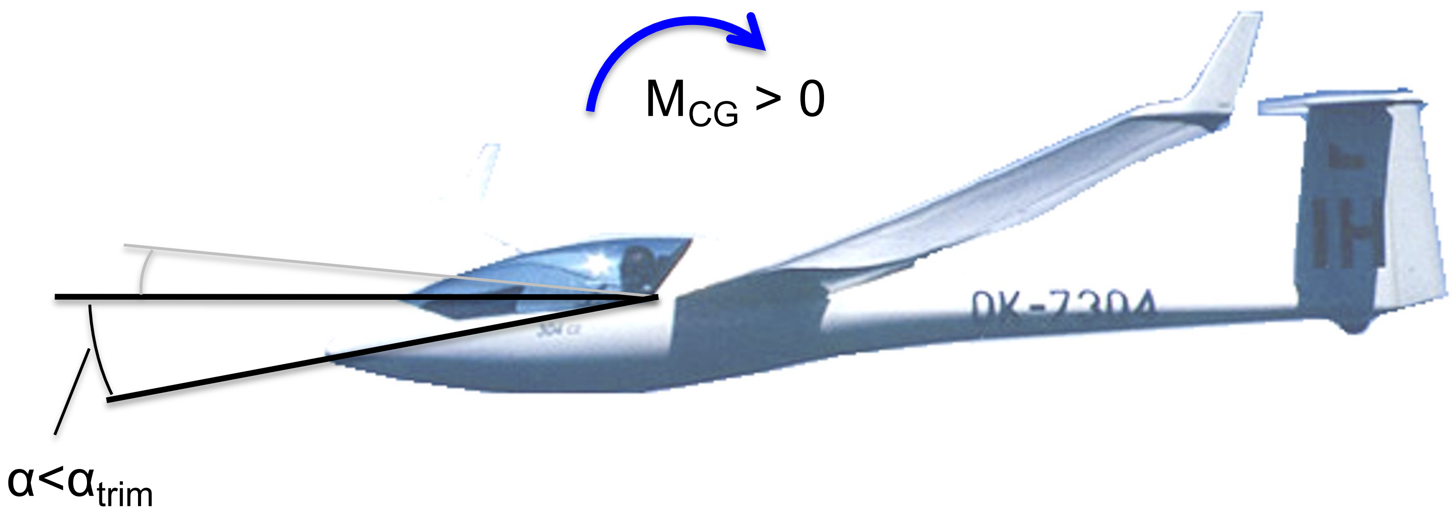
M_{CG}: pitch moment about CG



- **If the aircraft is disturbed from its trim condition to a higher angle of attack, a statically stable aircraft will have a pitch-down moment to restore it to the trim value**

Longitudinal Static Stability

M_{CG}: pitch moment about CG



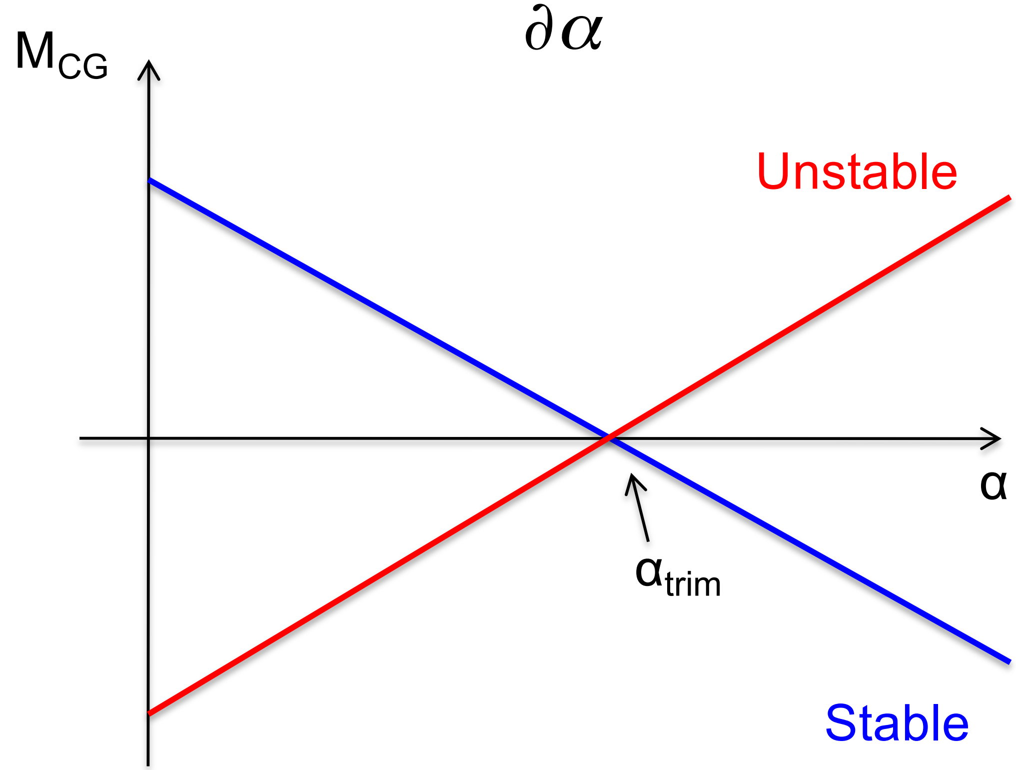
- **If the aircraft is disturbed from its trim condition to a lower angle of attack, a statically stable aircraft will have a pitch-up moment to restore it to the trim value**

Stability Criteria

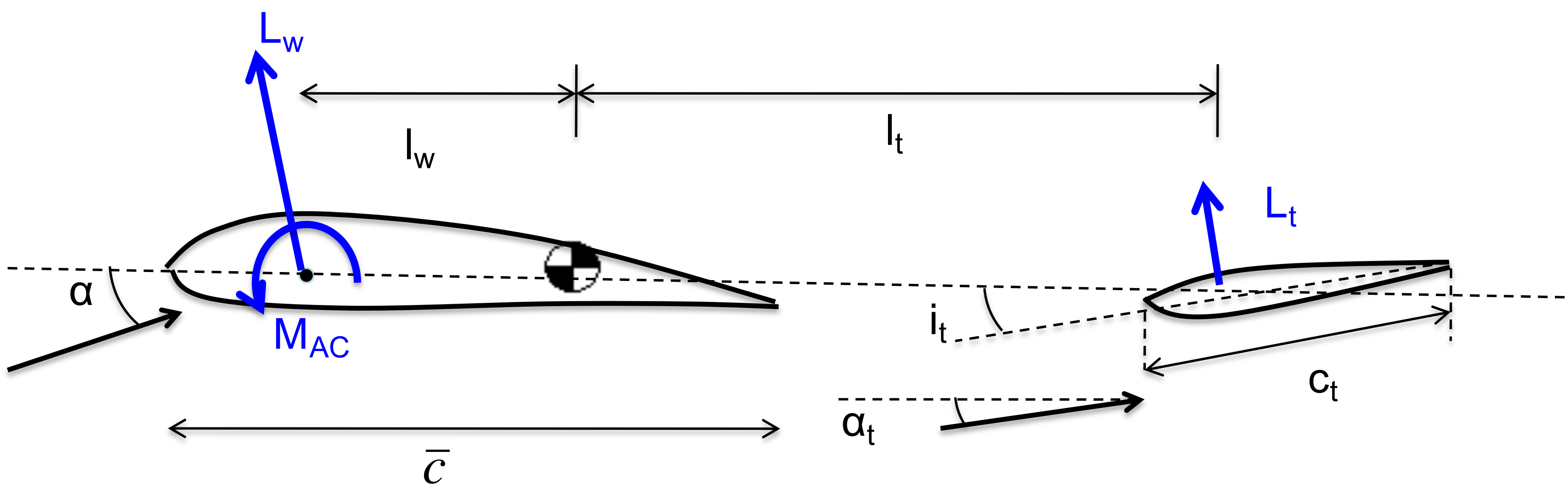
- **For longitudinal stability, the moment curve slope must be negative:**

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} < 0$$

- **And for trimmed flight (M_{CG}=0), $C_{M,0}$ must be greater than zero**



Longitudinal Static Stability

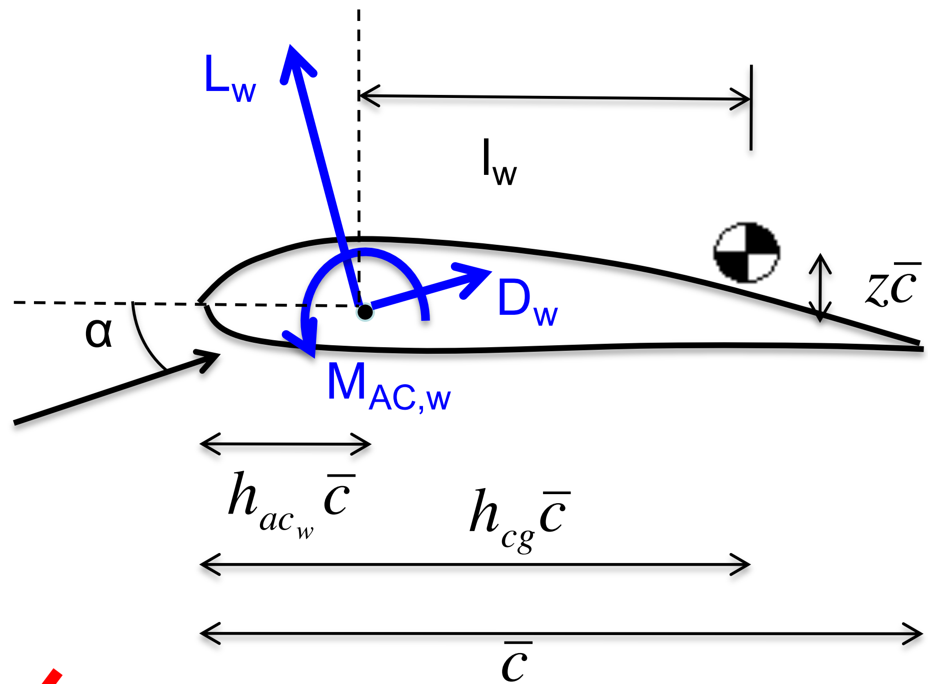


Moment due to Wing

- **When calculating the total aircraft pitching moment about the center of gravity, we first look at the wing:**
  - **Moment about wing aerodynamic center (independent of AoA)**
  - **Lift and drag forces multiplied by the moment distance between AC and CG location**

$h_{ac,w}$: distance from leading edge to A.C divided by the chord length

h_{cg}: distance from the wing leading edge to the CG divided by the chord length



Assume small angles
Assume z CG offset is small…

$$M_{cg_w} = M_{ac_w} + L_w\left(h_{cg}c - h_{ac_w}c\right)\cos\alpha + D_w\left(h_{cg}c - h_{ac_w}c\right)\sin\alpha + L_w zc\sin\alpha - D_w zc\cos\alpha$$

$$\boxed{M_{cg_w} = M_{ac_w} + L_w\left(h_{cg}c - h_{ac_w}c\right)}$$

Moment Coefficient due to Wing

- **Just as with lift and drag, it is often useful to work with non-dimensional terms**

$$C_L = \frac{L}{qS_w} \qquad C_M = \frac{M}{qS_w c}$$

- **C_M is similar to C_L and C_D except an extra term, a reference length, is needed to make it non-dimensional. The reference length is the wing chord length**

$$M_{cg_w} = M_{ac_w} + L_w\left(h_{cg}c - h_{ac_w}c\right)$$

$$C_{M_{cg,w}} = C_{M_{ac_w}} + C_{L_w}\left(h_{cg} - h_{ac_w}\right)$$

$$C_{M_{cg,w}} = C_{M_{ac_w}} + a_w\alpha\left(h_{cg} - h_{ac_w}\right)$$

$$\boxed{\begin{aligned} C_{L_w} &= a_w\alpha \\ a_w &= \frac{\partial C_{L_w}}{\partial \alpha} \end{aligned}}$$

Moment due to Tail

- **Moment on the tail is defined the same way as the wing, except usually $M_{ac,t}$ can be neglected**

$$M_{cg_t} = M_{ac_t} - L_t l_t \cos(\alpha - \varepsilon) + D_t l_t \sin(\alpha - \varepsilon) + L_t z_t c \sin(\alpha - \varepsilon) - D_t z_t c \cos(\alpha - \varepsilon)$$

- **Simplifying leads to:** $M_{cg_t} \cong -l_t L_t$

- **When putting it in non-dimensional, coefficient form, C_M is normalized by the wing area and wing chord**

$$C_{M_{cg,t}} = \frac{M_{cg_t}}{qS_w c} = -\frac{l_t L_t}{qS_w c} \qquad C_{L_t} = \frac{L_t}{qS_t}$$

- **The pitching moment due to the tail is determined by the tail lift coefficient and the tail volume ratio. The tail volume ratio captures the relative sizes and moment arms between the wing and tail**

$$C_{M_{cg,t}} = -\frac{l_t}{c}\frac{S_t}{S_w}\frac{L_t}{qS_t} = -\frac{l_t}{c}\frac{S_t}{S_w} C_{L_t}$$

Tail volume ratio

$$V_H = \frac{l_t}{c}\frac{S_t}{S_w}$$

Moment due to Tail (cont.)

- **Tail lift coefficient similar to wing, except the local angle of attack at the tail generally is not the same as the wing**
  - **Downwash induced by wing reduces tail AoA**
  - **Alignment of wing and tail may be different**
    i_t: Angle of incidence of tail

$$\varepsilon_\alpha = \frac{\partial \varepsilon}{\partial \alpha}$$

$$\alpha_t = (1 - \varepsilon_\alpha)\alpha - i_t$$

$$C_{L_t} = a_t \alpha_t$$

$$\boxed{a_t \equiv C_{L_{\alpha,t}}}$$

(tail lift curve slope)

$$C_{L_t} = a_t \alpha (1 - \varepsilon_\alpha) - a_t i_t$$

- **Plugging the lift coefficient expression into the moment equation:**

$$C_{M_{cg,t}} = V_H C_{L_t} = -\left[V_H a_t (1 - \varepsilon_\alpha)\right]\alpha + a_t V_H i_t$$

Note that $a_t V_H i_t$ term does not depend on AoA

Total Moment about CG

- **Adding up the wing and tail contributions to obtain the total pitching moment coefficient about the CG:**

$$C_{M_{cg}} = C_{M_{cg,w}} + C_{M_{cg,t}}$$

$$C_{M_{cg}} = C_{M_{ac}} + C_{L_w}\left(h_{cg} - h_{ac_w}\right) - V_H C_{L_t}$$

$$C_{M_{cg}} = a_w\left[\left(h_{cg} - h_{ac_w}\right) - V_H \frac{a_t}{a_w}\left(1 - \varepsilon_\alpha\right)\right]\alpha + C_{M_{ac}} + V_H a_t i_t$$

Total Moment about CG (cont.)

- **The terms that make up the total moment about CG can be split into those that vary with AoA and those that are independent of AoA**

$$C_{M_{cg}} = C_{M_0} + \frac{\partial C_{M_{cg}}}{\partial \alpha}\alpha$$

$$C_{M_0} = C_{M_{ac}} + V_H a_t i_t$$

- **C_{M0} must be positive, so what's that say about i_t?**

- **Partial derivative of moment with respect to angle of attack:**

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} = a_w\left[\left(h_{cg} - h_{ac_w}\right) - V_H \frac{a_t}{a_w}\left(1 - \varepsilon_\alpha\right)\right]$$

Neutral Point

- **For static stability:**

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} < 0$$

- **Similar to the aerodynamic center for an airfoil, a CG location exists for an aircraft about which the moment is constant, independent of AoA. This location is known as the aircraft's neutral point**

- **Neutral point: CG location where**

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} = 0$$

Finding the Neutral Point

- **To find the neutral point, solve for the cg location that leads to a zero change in moment due to a change in angle of attack**

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} = 0 \longrightarrow 0 = a_w\left[\left(h_{cg} - h_{ac_w}\right) - V_H \frac{a_t}{a_w}\left(1 - \varepsilon_\alpha\right)\right]$$

- **Tail volume, V_H , varies with the CG location. Rewriting in terms of tail location and CG location:**

$$V_H = \frac{l_t}{c}\frac{S_t}{S_w} \longrightarrow \frac{l_t}{c} = h_{ac_t} - h_{cg} \longrightarrow V_H = \left(h_{ac_t} - h_{cg}\right)\frac{S_t}{S_w}$$

Finding the Neutral Point

$$\frac{\partial C_{M_{cg}}}{\partial \alpha} = a_w\left[\left(h_{cg} - h_{ac_w}\right) - \left(h_{ac_t} - h_{cg}\right)\frac{S_t}{S_w}\frac{a_t}{a_w}\left(1-\varepsilon_\alpha\right)\right]$$

- **The neutral point location, ($h_n = x_n/c$), is the CG location where the derivative is zero, so:**

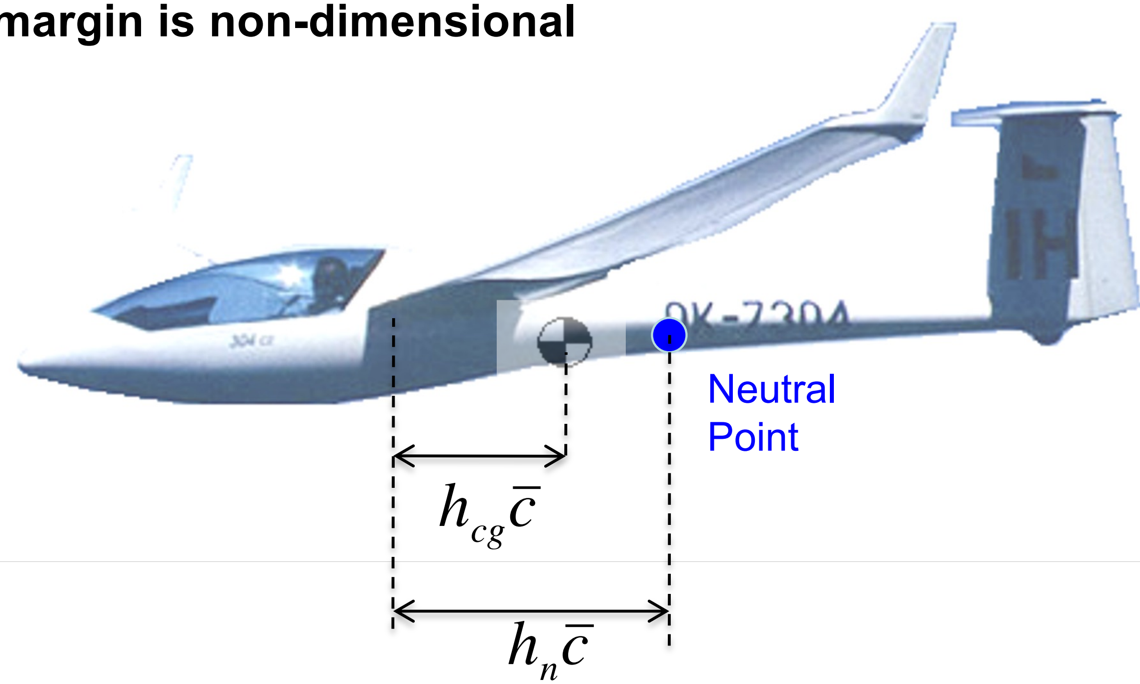
$$0 = a_w\left[\left(h_n - h_{ac_w}\right) - \left(h_{ac_t} - h_n\right)\frac{S_t}{S_w}\frac{a_t}{a_w}\left(1-\varepsilon_\alpha\right)\right]$$

- **Solving for h_n:**

$$h_n = \frac{h_{ac_w} + h_{ac_t}\dfrac{S_t}{S_w}\dfrac{a_t}{a_w}\left(1-\varepsilon_\alpha\right)}{1 + \dfrac{S_t}{S_w}\dfrac{a_t}{a_w}\left(1-\varepsilon_\alpha\right)}$$

Static Margin

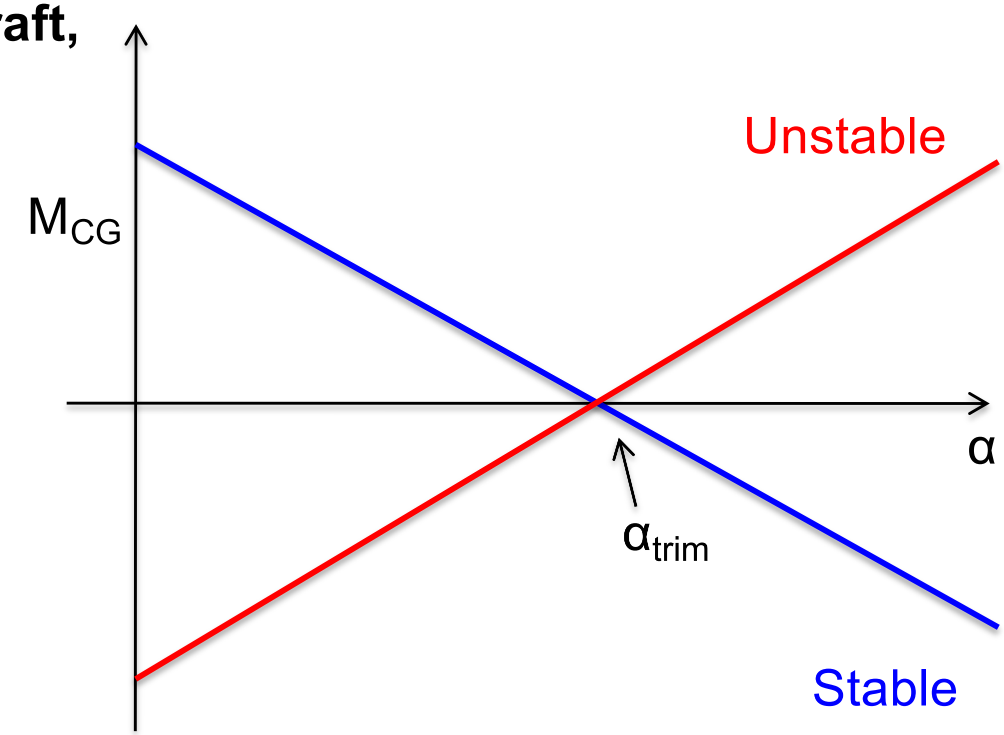
- **For static stability, the CG location must be located ahead of the neutral point**
  - **$h_n > h_{cg}$**
  - **$(h_n - h_{cg})$ is the static margin. Note that h is a fraction of the chord length, so static margin is non-dimensional**



$$C_{M_\alpha} = \frac{\partial C_{M_{cg}}}{\partial \alpha} = C_{L_\alpha}\left(h_{cg} - h_n\right) = -C_{L_\alpha}\left(h_n - h_{cg}\right)$$

Trimmed flight

- **An aircraft is in trim when L = W and M_{CG} = 0**
- **For both conditions to exists, the M_{CG} must be zero at an angle of attack where L = W is achievable**
  - **For a statically stable aircraft, the moment at zero AoA is positive so the moment line crosses zero at a positive AoA**



Longitudinal Control

- **The greater stability, the greater the control power that is required to disturb an aircraft from its state of trim**
  - **Some modern aircraft (especially fighters) are unstable and rely upon electronic flight control systems to provide stability. This makes them more maneuverable, but even these aircraft require sufficient control power to maneuver**
- **2 parts to control**
  - **Trimmed flight**
  - **Maneuvering**

Longitudinal Control

- **What is the longitudinal control required to change from one trimmed state to another?**

- **To go from a trimmed state with AoA, α_1, to another at α_2, the velocities must be different since $L_1 = L_2$**

$$L_1 = 0.5\rho V_1^2 S C_{L,1} = L_2 = 0.5\rho V_2^2 S C_{L,2}$$

$$C_{L,1} = C_{L,0} + C_{L,\alpha}\alpha_1$$

$$C_{L,2} = C_{L,0} + C_{L,\alpha}\alpha_2$$

- **For trimmed condition, moments must also be zero**

$$C_{M_{0,1}} + C_{M_\alpha}\alpha_1 = 0$$

$$C_{M_{0,2}} + C_{M_\alpha}\alpha_2 = 0$$

$$C_{M_0,1} = V_H i_{t,1} a_t + C_{M_{ac}}$$

$$C_{M_0,2} = V_H i_{t,2} a_t + C_{M_{ac}}$$

Tail incidence in terms of Lift Coefficients

- **The total lift is given as**

$$L = qS_W\left(\left[a_w + a_t \frac{S_t}{S_w}\left(1-\varepsilon_\alpha\right)\right]\alpha - a_t \frac{S_t}{S_W} i_t\right)$$

- **We can split up the lift into a constant lift term and a term that is proportional to AoA**

$$L = qSC_L \longrightarrow L = qS\left(C_{L_\alpha}\alpha + C_{L_0}\right)$$

$$C_{L_\alpha} = a_w + a_t \frac{S_t}{S_w}\left(1-\varepsilon_\alpha\right) \qquad C_{L_0} = -a_t \frac{S_t}{S_w} i_t$$

Tail incidence in terms of Lift Coefficients

- **If the tail incidence can vary, then we can express the total lift and moment coefficients as:**

Note: C_{L0} assumed to be small

$$C_L = C_{L_\alpha}\alpha + C_{L_i} i_t$$

$$C_M = C_{M_{ac}} + C_{M_\alpha}\alpha + C_{M_i} i_t$$

$$C_{L_i} = -a_t \frac{S_t}{S_w}$$

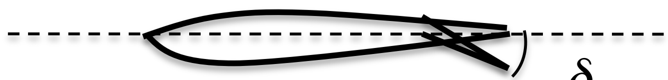
$$C_{M_i} = a_t V_H$$

- **Note that a positive incidence angle leads to a negative lift, so the C_{Li} is negative**

- **The incidence angle for trim can be written as:**

$$i_t = -\frac{C_{M_{ac}} C_{L_\alpha} + C_{M_\alpha} C_L}{C_{L_\alpha} C_{M_i} - C_{M_\alpha} C_{L_i}}$$

Elevator to Trim



Note: $C_{m,0}$ combines $C_{m,ac}$ and moment due to tail incidence

$$C_{M_0} = V_H i_t a_t + C_{M_{ac}}$$

- **For stabilizer/elevator systems, a similar equation can be developed based on elevator deflection**

$$C_m = 0 = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta e}}\delta_e \longrightarrow \delta_e = -\frac{C_{m_0} + C_{m_\alpha}\alpha}{C_{m_{\delta e}}}$$

For trim, L=W:

$$C_L = \frac{W}{qS} = C_{L_\alpha}\alpha + C_{L_{\delta e}}\delta_e \longrightarrow \alpha = \frac{C_L - C_{L_{\delta e}}\delta_e}{C_{L_\alpha}}$$

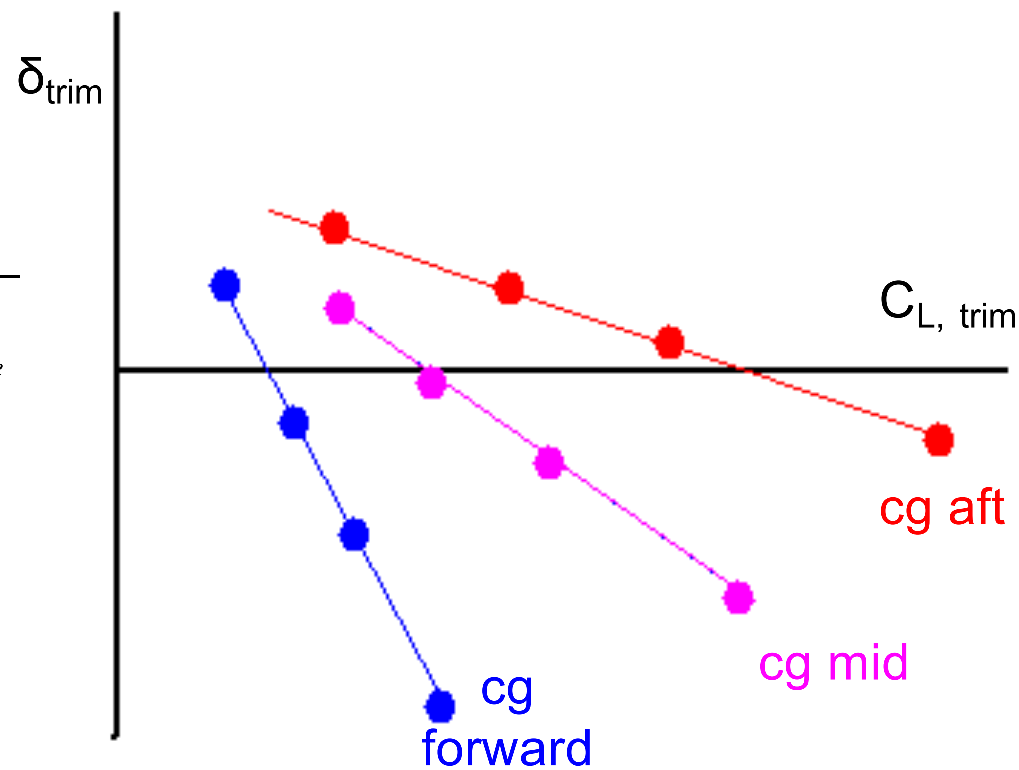
$$\delta_e = -\frac{C_{M_0} C_{L_\alpha} + C_{M_\alpha} C_L}{C_{L_\alpha} C_{M_{\delta e}} - C_{M_\alpha} C_{L_{\delta e}}}$$

($C_{L,0}$ assumed small)

Measuring Neutral Point from Flight Tests

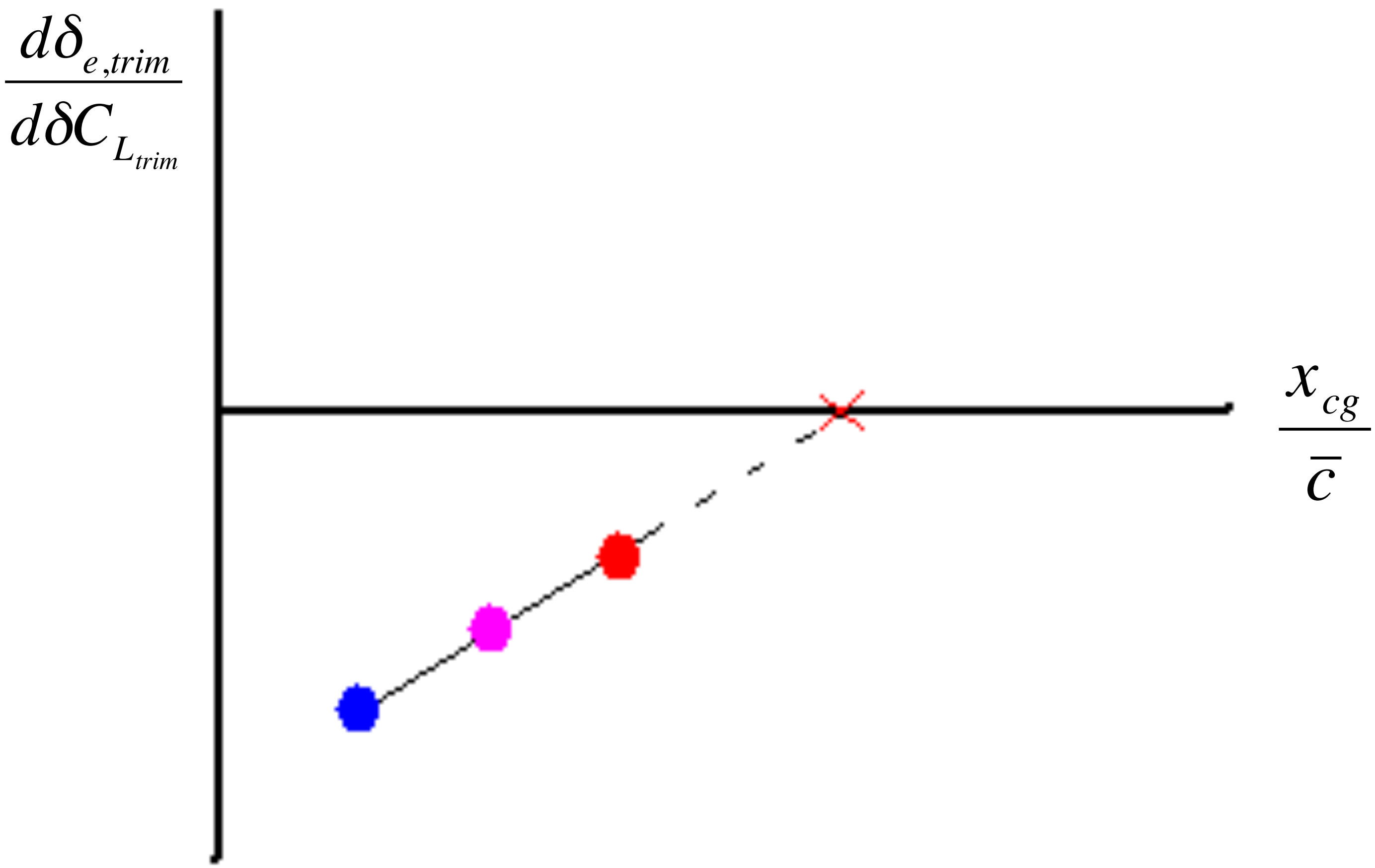
- **The neutral point location can be estimated with the trim equation**
- **Taking its derivative with respect to C_L:**

$$\frac{d\delta_e}{d\delta C_{L_{trim}}} = -\frac{C_{M_\alpha}}{C_{L_\alpha} C_{M_{\delta e}} - C_{M_\alpha} C_{L_{\delta e}}}$$



Measuring Neutral Point

- **Neutral point is located where:** $$\frac{d\delta_{e,trim}}{d\delta C_{L_{trim}}} = 0$$



Elevator Hinge Moments

- **Aero forces can produce a hinge moment on elevator**
- **Pilot exerts a force on the control stick to counter the hinge moment**
- **If pilot lets go of stick, elevator would float (deflect) to an angle that has zero hinge moment**

$$C_{h_e} = C_{h_0} + C_{h_{\alpha_t}}\alpha_t + C_{h_{\delta_e}}\delta_e + C_{h_{\delta t}}\delta_t$$

Trim tab

Stick Free Neutral Point

- **For a floating elevator, the hinge moment is zero:**

$$C_{h_e} = 0 = C_{h_{\alpha_t}} \alpha_t + C_{h_{\delta_e}} \delta_e \qquad \delta_{e\,free} = -\frac{C_{h_{\alpha_t}} \alpha_t}{C_{h_{\delta_e}}}$$

- **Assuming $i_t = 0$, the lift on the tail with a free elevator is given by:**

$$C_{L_t} = a_t \alpha_t + C_{L_{t,\delta_e}} \delta_{e\,free} \qquad C_{L_t} = a_t \alpha_t - C_{L_{t,\delta_e}} \frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}} \alpha_t$$

- **The tail lift curve slope is modified by the term in the parentheses**

$$C_{L_t} = a_t \alpha_t \left(1 - \frac{C_{L_{t,\delta_e}}}{a_t} \frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}}\right) = a_t' \alpha_t \qquad a_t' = a_t \left(1 - \frac{C_{L_{t,\delta_e}}}{a_t} \frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}}\right) = a_t f$$

Stick Free Neutral Point

- **The neutral point location can be approximated by:**

$$h_n \approx h_{ac_w} + V_H \frac{a_t}{a_w}\left(1 - \varepsilon_\alpha\right)$$

- **We can use the modified tail lift curve slope to compute a new neutral point location, the "stick free" neutral point:**

$$h_n^{'} \approx h_{ac_w} + V_H \frac{a_t^{'}}{a_w}\left(1 - \varepsilon_\alpha\right)$$

- **Difference between stick fixed and "stick free" neutral point is given by:**

$$h_n - h_n^{'} \approx \left(1 - f\right) V_H \frac{a_t}{a_w}\left(1 - \varepsilon_\alpha\right)$$

Stick Forces & Trim Tabs

- **To deflect a control surface, pilot moves the control stick (or rudder pedals for yaw control)**
- **Necessary stick force depends on hinge moment**
- **For trim conditions, it is desirable to zero out hinge moments, otherwise it might lead to pilot fatigue for long flights**
  - **Trim tabs are small control surfaces that reduce the hinge moments**

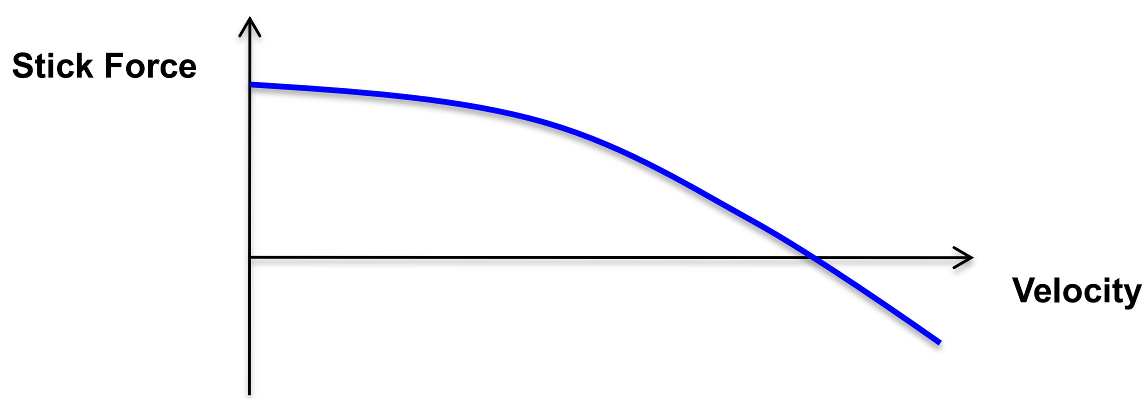
Trim tab on a Cessna 172

Stick Force Gradients

- **An important design consideration is the stick force gradient, which measures the change in stick force as the aircraft changes speed**

- **For speed stability, stick force gradient needs to be negative**

- **Assume an aircraft is in trim, and trim tabs have zeroed out the stick force**
  - **If the aircraft slows down, the stick force increases, which causes a pitch down and increases the speed**
  - **If the aircraft speeds up, the stick force decreases, causing the nose to pitch up and reduces the speed**

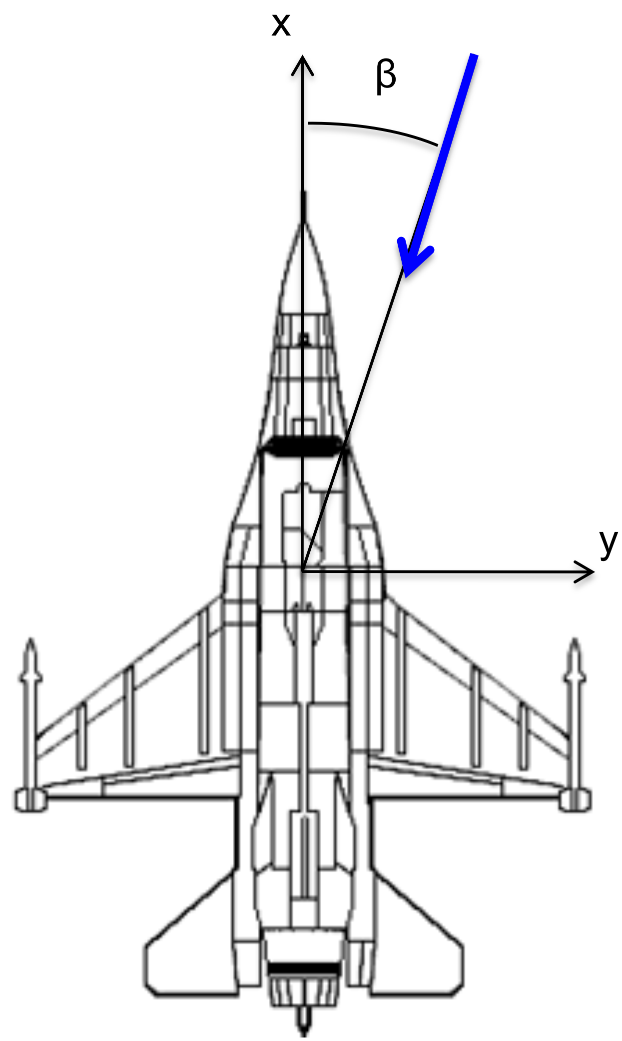


Directional Stability

- **Weathercock or directional stability is the static stability of the airplane about the z-axis**
- **For directional stability, a positive change in sideslip angle will cause a positive yaw moment**

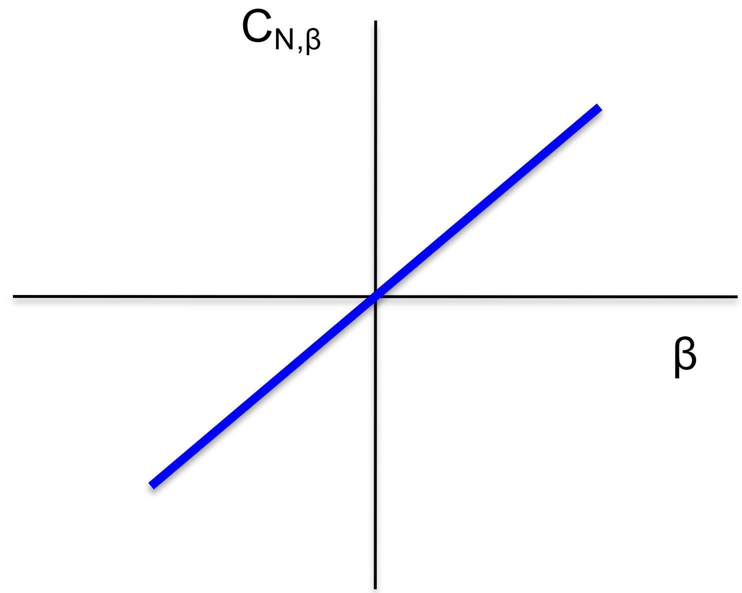
Yaw Moment $N = C_N qSb$

$$\frac{\partial N}{\partial \beta} = N_\beta > 0$$



Directional Stability

- **Yawing moment due to fuselage**
  - **Usually has a negative yawing moment when exposed to positive side slip (de-stabilizing)**
- **Yawing moment due to vertical tail surface**
  - **Just as the lift acting on the horizontal tail surface causes a pitch down moment, the side force acting on the vertical surface due to positive side slip causes a positive yawing moment (restoring moment).**



- **For directional stability** $C_{n_{\beta_v}} + C_{n_{\beta_{fw}}} > 0$

Directional Control

- **Directional control is achieved by deflecting the rudder**

- **The yawing moment depends on the lift generated by the vertical surface due to rudder deflection, as well as the moment arm between the vertical surface and center of gravity**

$$\Delta N = -l_v \Delta L_v$$

$$\Delta L_v = q S_v a_v \tau \delta_r$$

$$C_{N_\delta} = -V_v a_v \tau$$

References

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