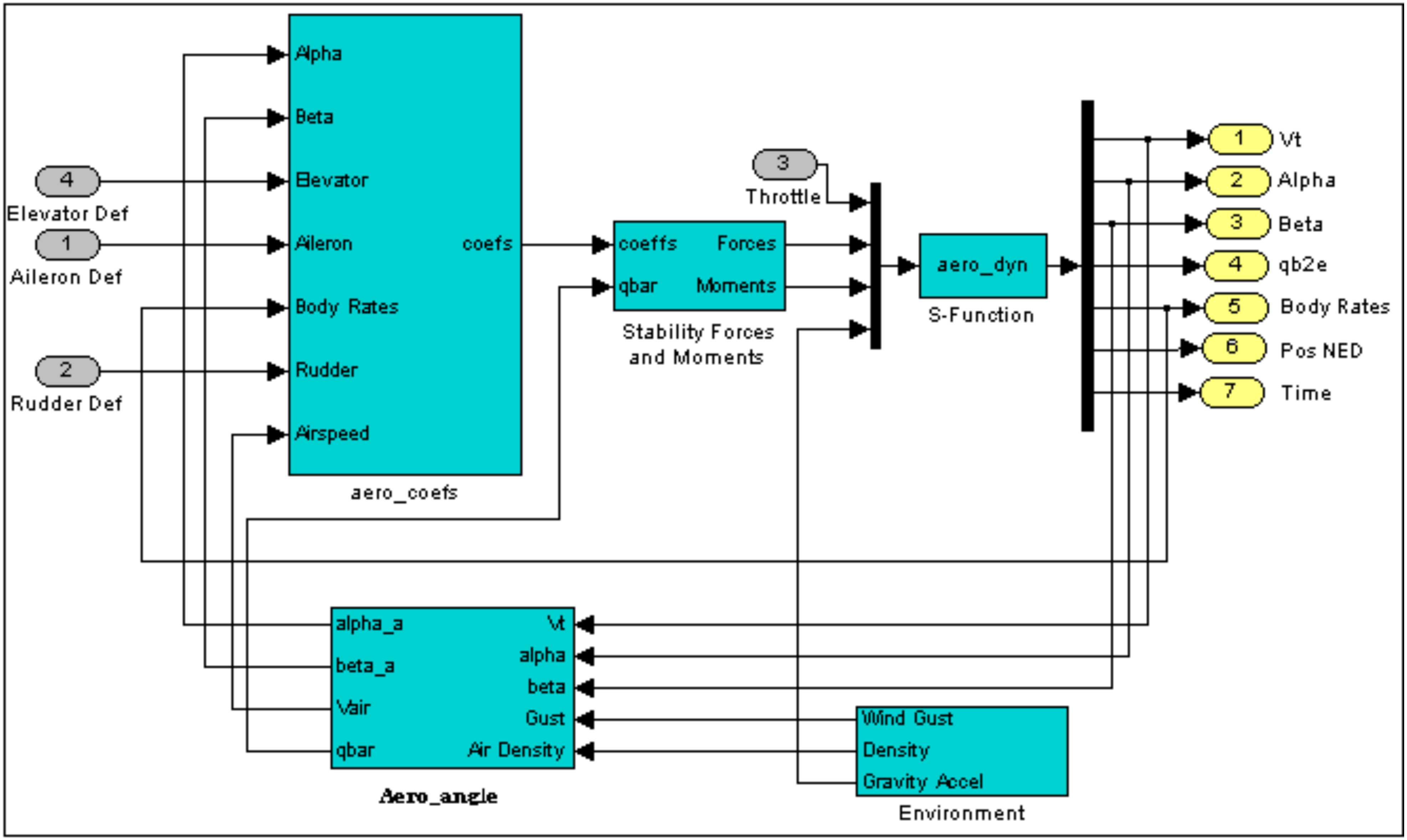
Lecture 12

Aircraft Dynamics Model in Simulink



Aircraft Dynamics Model

- **The inputs to the aircraft dynamics model are the vehicle control deflections and throttle setting, and these may be controlled either by an autopilot module, or manually controlled by the user**

- **The simulation outputs are the vehicle states:**

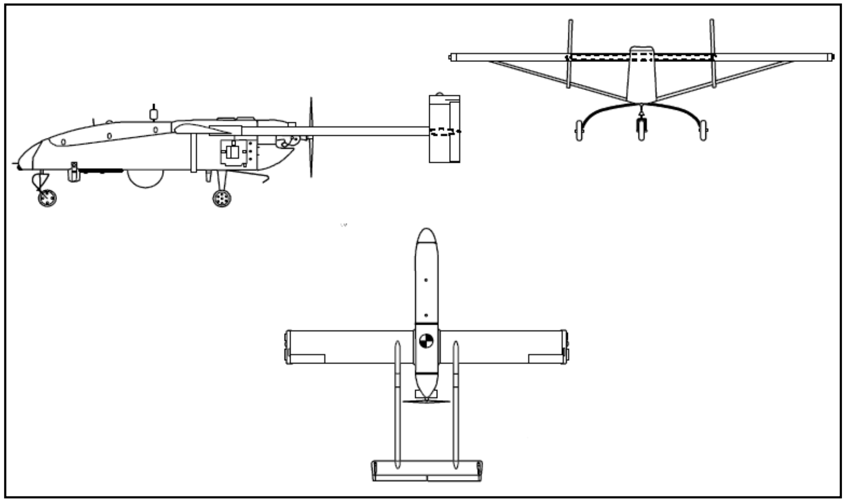
$$X^T = \begin{bmatrix} V_T & \alpha & \beta & q_0 & q_1 & q_2 & q_3 & p & q & r & p_N & p_E & p_D \end{bmatrix}$$

- **The "Aero_angle" block updates the vehicle airspeed and aero angles based on any wind effects. It also computes the dynamic pressure**

- **The "Environment" block models the atmosphere and gravity conditions. The atmosphere properties are determined from an altitude-based lookup table**

- **The "Aero_coeffs" module computes the force and moment coefficients based on the vehicle state and its stability derivatives. It calculates the contribution of each stability derivative and then sums them up to determine the total force and moment coefficients**

RQ-2 Pioneer UAV



Length	14 ft
Height	3.3 ft
Weight	452 lbs
Wingspan	16.9 ft
Speed	110 kts
Endurance	5 hrs
Ceiling	15,000 ft
Payload	75 lbs

RQ-2 Pioneer - Image Courtesy of USMC

Stability Derivatives for Pioneer UAV

CL0	0.385
CL_a	4.78
CL_adot	2.42
CL_q	8.05
CL_de	0.401
CD0	0.060
CD_a	0.430
CD_de	0.0180
Cm0	0.194
Cm_a	-2.12
Cm_adot	-11.0
Cm_q	-36.6
Cm_de	-1.76

CY_beta	-0.819
CY_dr	0.191
Cl_beta	-0.023
Cl_p	-0.450
Cl_r	0.265
Cl_da	-0.161
Cl_dr	-0.00229
Cn_beta	0.109
Cn_p	-0.110
Cn_r	-0.200
Cn_da	0.0200
Cn_dr	-0.0917

Aircraft Dynamics Model

- **The computed coefficients are passed to the "Stability Forces and Moments" block to calculate the aerodynamics forces and moments acting on the vehicle. The forces and moments are computed in the stability frame**

$$L = \bar{q} S_{ref} C_{L_{TOT}}$$

$$D = \bar{q} S_{ref} C_{D_{TOT}}$$

$$F_Y = \bar{q} S_{ref} C_{Y_{TOT}}$$

$$M = \bar{q} S_{ref} \bar{c} C_{M_{TOT}}$$

$$\bar{L} = \bar{q} S_{ref} b_w C_{l_{TOT}}$$

$$\bar{N} = \bar{q} S_{ref} b_w C_{N_{TOT}}$$

Aircraft Dynamics Model

- **At the heart of the simulation is the Matlab S-function, “aero_dyn”. It computes the state derivatives and performs the integrations. In this simulation, 13 states are simulated:**

$$X^T = \begin{bmatrix} V_T & \alpha & \beta & q_0 & q_1 & q_2 & q_3 & p & q & r & p_N & p_E & p_D \end{bmatrix}$$

- **The quaternions (q_0,q_1,q_2,q_3) describe the attitude of the vehicle. They are like Euler angles except they are more stable numerically. But they are harder to visualize**

Euler Angles

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{body} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\Phi & s\Phi \\ 0 & -s\Phi & c\Phi \end{bmatrix} \begin{bmatrix} c\theta & 0 & -s\theta \\ 0 & 1 & 0 \\ s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{NED}$$

c=cos, s=sin

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{body} = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ -c\phi s\psi + s\phi s\theta c\psi & c\phi c\psi + s\phi s\theta s\psi & s\phi c\theta \\ s\phi s\psi + c\phi s\theta c\psi & -s\phi c\psi + c\phi s\theta s\psi & c\phi c\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{NED}$$

Direction-Cosine Matrix relating Body and NED coordinate systems

State Derivative Calculation Procedure

- **External aerodynamic and thrust forces are rotated from the stability frame to the body axis**

$$F_X = F_{X,S}\cos(\alpha) - F_{Z,S}\sin(\alpha) + T_X$$

$$F_Y = F_{Y,S} + T_Y$$

$$F_Z = F_{X,S}\sin(\alpha) + F_{Z,S}\cos(\alpha) + T_Z$$

$$\bar{L} = \bar{L}_S\cos(\alpha) - N_{,S}\sin(\alpha)$$

$$M = M_{,S}$$

$$N = L_S\sin(\alpha) + N_{,S}\cos(\alpha)$$

State Derivative Calculation Procedure

- **Vehicle airspeed and aerodynamic angles are converted into body velocities: U,V,W**

$$U = V_T \cos(\alpha)\cos(\beta)$$

$$V = V_T \sin(\beta)$$

$$W = V_T \sin(\alpha)\cos(\beta)$$

State Derivative Calculation Procedure

- **Body accelerations due to thrust, gravity and aero forces are computed**

$$\dot{U} = RV - QW - g\sin(\theta) + \frac{F_X}{m}$$

$$\dot{V} = -RU + PW + g\sin(\phi)\cos(\theta) + \frac{F_Y}{m}$$

$$\dot{W} = QU - PV + g\cos(\phi)\cos(\theta) + \frac{F_Z}{m}$$

State Derivative Calculation Procedure

- **Using the body accelerations and angular velocity, the state derivatives for the airspeed and aero angles can be calculated**

$$\dot{V}_T = \frac{U\dot{U} + V\dot{V} + W\dot{W}}{V_T}$$

$$\dot{\alpha} = \frac{U\dot{W} - W\dot{U}}{U^2 + W^2}$$

$$\dot{\beta} = \frac{\dot{V}V_T - V\dot{V}_T}{V_T^{\ 2}\cos(\beta)}$$

State Derivative Calculation Procedure

- **The derivatives for the quaternion states are computed with the following kinematic equation**

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = -0.5 \begin{bmatrix} 0 & P & Q & R \\ -P & 0 & -R & Q \\ -Q & R & 0 & -P \\ -R & -Q & P & 0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

State Derivative Calculation Procedure

- **The angular accelerations are determined from the external moments (L,M,N) acting on the vehicle (c terms are inertia terms)**

$$\dot{P} = \left(c_1 R + c_2 P\right) Q + c_3 \overline{L} + c_4 N$$

$$\dot{Q} = c_5 PR - c_6\left(P^2 - R^2\right) + c_7 M$$

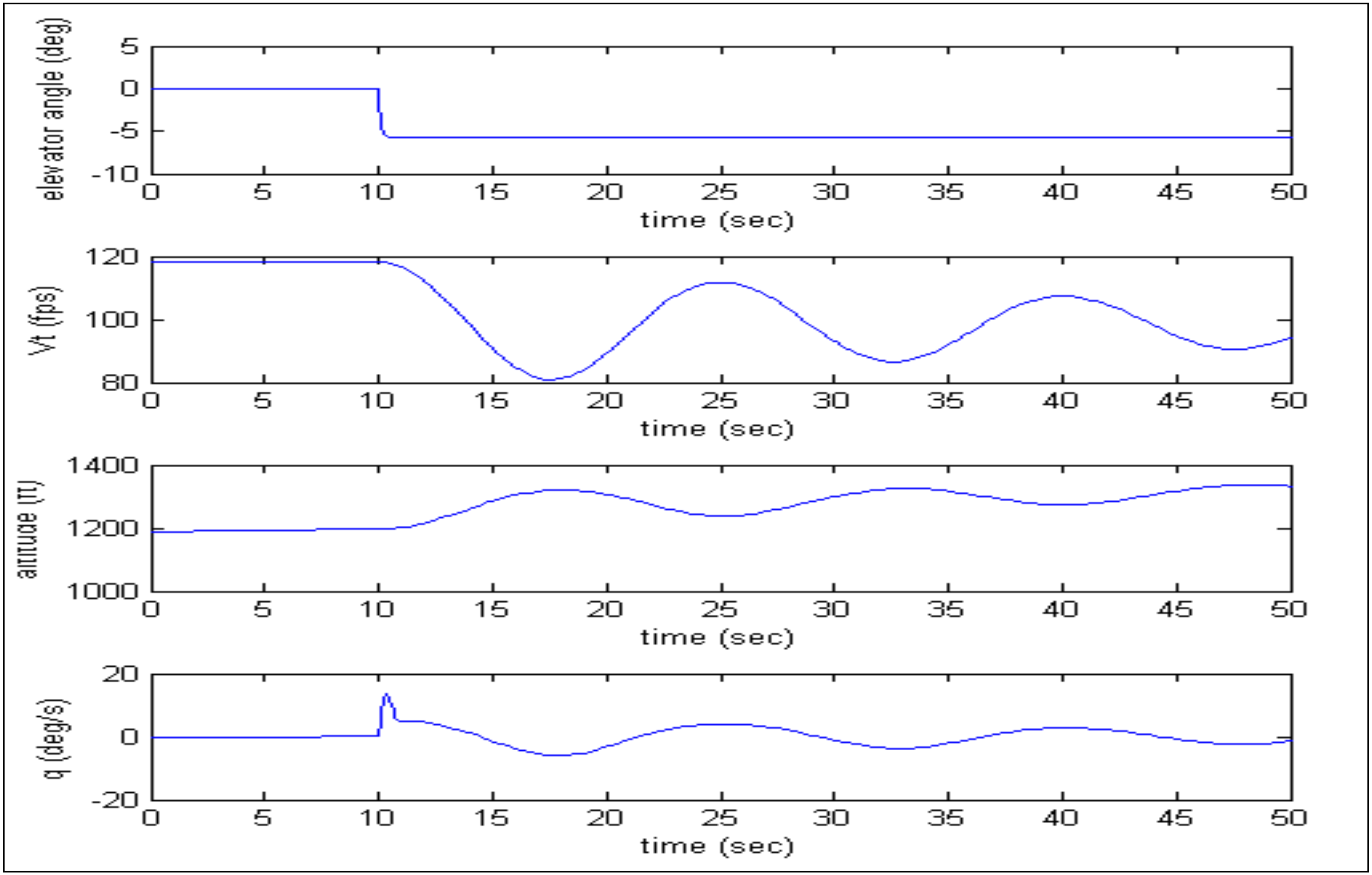
$$\dot{R} = \left(c_8 P - c_2 R\right) Q + c_4 \overline{L} + c_9 N$$

State Derivative Calculation Procedure

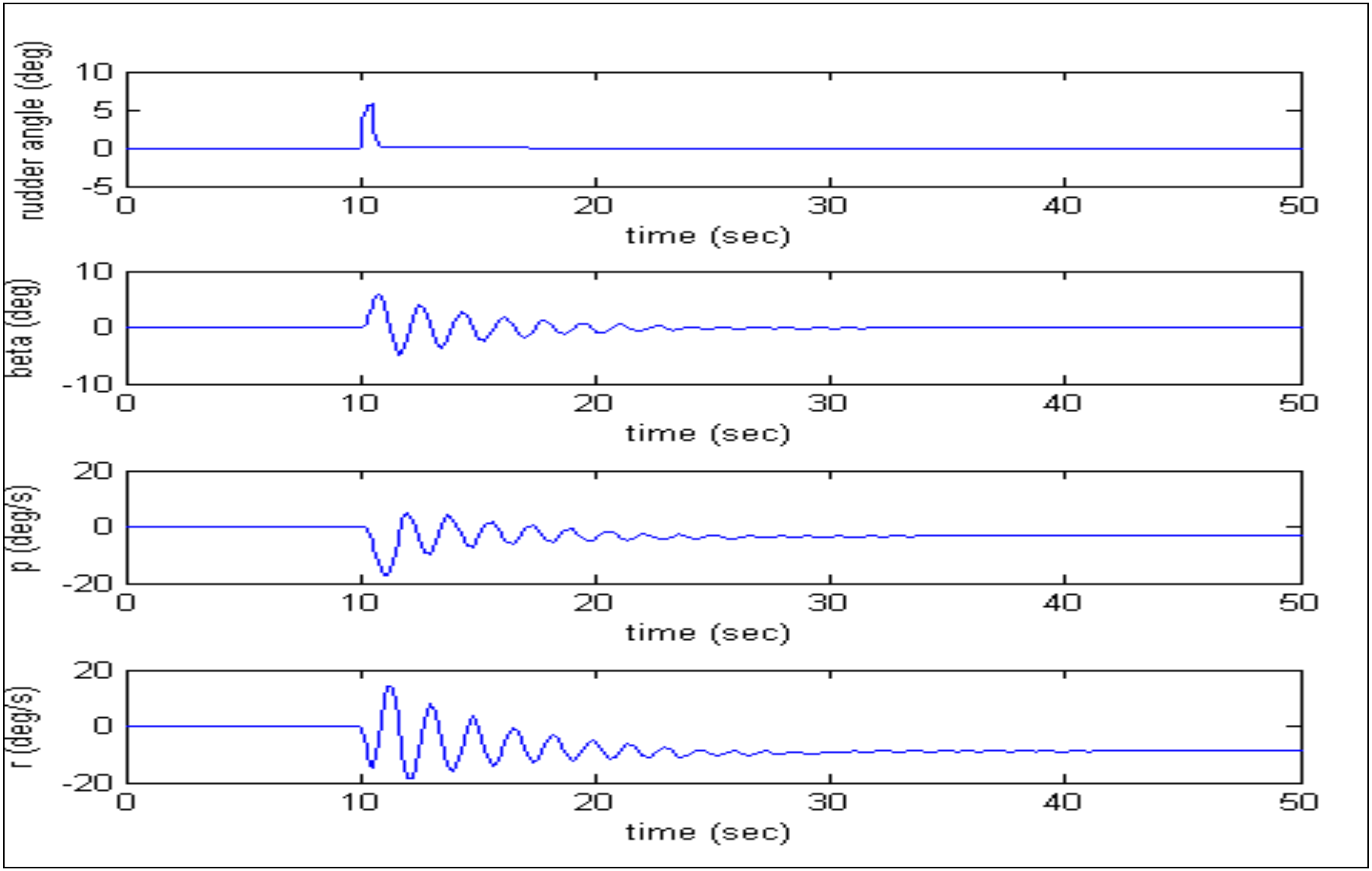
- **The derivatives of the position vector (p_N,p_E,p_D) are computed with the body velocities and Euler angles**

$$\begin{bmatrix} \dot{p}_N \\ \dot{p}_E \\ \dot{p}_D \end{bmatrix} = \begin{bmatrix} \quad DC\_B2E \quad \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

Simulation Response – elevator step input



Simulation Response – rudder kick



References

1. B.L. Stevens, F.L. Lewis, *Aircraft Control and Simulation*, John Wiley & Sons, 1992
2. Bray, Robert, M.A, *Wind Tunnel Investigation of the Pioneer Remotely Piloted Vehicle*, Monterey, CA: Naval Postgraduate School, June 1991