

Failure Analysis of Coupler Knuckle Considering Truncated and Censored Lifetime Data

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Abstract—Increased axle load, heavy load, and increased speed of railway wagons need high reliability of coupler knuckle. However, the failure analysis of coupler knuckle is troublesome under incomplete lifetime data due to extensive management. In this paper, maximum likelihood estimation method is employed to with a mixture data of left truncation, interval censoring, and right censoring. The parameter estimations are done by Newton-Raphson method and bootstrap resampling. The results of case study shows that the proposed method is reasonable and could reflect the reality. These results have important application value in reality for further refinement of the maintenance strategy according to the remaining life of components, thereby reducing the maintenance cost and improving the reliability of components.

Keywords- truncation; censoring; Maximum likelihood estimation; parametric bootstrap

I. INTRODUCTION

Railway wagon transport is an effective way of transporting bulk cargo. Heavy load is an important development of railway wagon technology. Under heavy load conditions, the reliability of the railway wagon parts, especially the coupler and draft gear is particularly prominent. Coupler knuckle, which can transfer traction and impact, is the key connecting part between railway wagon vehicles. Under heavy load conditions, coupler knuckle is more frequently subjected to pulling force, compressive force and impact force, resulting in more cracks, wear and other failures, directly affecting the operational safety of the vehicle [1]. Thus, failure analysis of coupler knuckle is very necessary.

Unlike high-speed rail trains, the management of railway wagon is relatively rough, so there is a great difficulty in data acquisition and processing of component failure. Limitations in observation capabilities and measuring instrument capabilities can cause data to be censored or truncated [2]. In other words, it is difficult to observe the complete process of components from the beginning of use to the failure, and it is not convenient to analyze the life distribution of components. At present, scholars' research on the lifetime analysis of coupler knuckle mainly includes statistical analysis of field data, mechanism analysis of crack formation, and reliability analysis based on data simulation. Liu [3] and Li [4] predicted the service life of coupler knuckle by calculating the critical fracture dimension. Li [5] and Song [6] calculated the reliability life of coupler knuckle based on finite element analysis. Xue [7] made a statistical analysis of

the fatigue life of coupler knuckle based on the censoring specimen using the maximum likelihood method.

This study focuses on failure analysis of coupler knuckle considering truncated and censored lifetime data. Dealing with truncated and censored data is an important part of failure analysis (survival analysis), especially in the biomedical field. Lin et al. [8] proposed a method of molding medical expenses, and used the Kaplan-Meier to obtain the estimations of the overall cost average. Sun et al [9] used the Cox proportional hazard model to perform regression analysis on AIDS latency data containing both right censored and interval censored.

The maximum likelihood estimation (MLE) method, which is based on the idea of “finding the parameter estimations that maximizes the probability of occurrence of the set of data”, and has strong applicability of modeling truncated and censored data. Balakrishnan and Mitra [10] established a likelihood function for the left truncated and right censored data, and then used the expectation maximization (EM) method to estimate the parameters of several common life distributions. Hong et al. [11] obtained the prediction intervals of the remaining life of transformers based on the Weibull distribution likelihood function using the left truncated and right censored lifetime data.

In the case of relatively small samples, the theory of the asymptotic normality of the maximum likelihood estimators as a parameter estimation method is not working well. The bootstrap method, which does not need to assume the overall distribution of observations and easy to implement, has been a hot method for interval estimation in recent years. Li [12] studied the application of bootstrap method in small samples and found that the parameter bootstrap method is more efficient than the nonparametric method when the distribution is known. Wang [13] proposed a parameter estimation method of unbalanced panel data, and found that the parameter bootstrap method can better control the probability of making the first type of error. In summary, bootstrap method is suitable for complex and unclear data distribution.

In summary, lifetime data analysis that considers truncated and censored data has not been studied comprehensive in literature. In practice, to the best of the authors' knowledge, the comprehensive analysis of coupler knuckle has not been reported yet. In addition, since the consistency of the casting process could not be effectively guaranteed, the failure time of

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coupler knuckle has great dispersions [14]. These gaps motived this study. In this paper, we consider failure analysis of coupler knuckle considering truncated and censored lifetime data. The main methodology is based on maximum likelihood method and bootstrap resampling technique. Actual engineering data is used to illustrate and evaluate the proposed method. This study will help to estimate the remaining useful life of coupler knuckle, reasonably establish the maintenance cycle, and realize the control of the life cycle cost. At the same time, it will help to provide reference for the development of the safety standard assessment system for heavy load coupler knuckle under the new operating conditions.

The rest of the paper is organized as follows: In Section II, the coupler knuckle data structure is introduced and the main methods used in this study is summarized. In Section III, the results are presented with detailed discussion. In Section IV, the paper is concluded with a general summary and suggestions for further research directions.

II. METHOD AND MATERIAL

A. The coupler knuckle data

The data used in this study comes from a maintenance factory, which was established in 2015 and began careful archival record keeping at that time. In other words, there is no information on units installed and failed before 2015. Thus, coupler knuckle that were installed before 2015 but were not yet failed must be viewed as transformers sampled from truncated distribution(s).

Fig. 1 shows different truncated data scenarios based on observation time. Subject *a*, *b*, and *c* are unobservable since both the beginning time of service and the time of failure occur before the start of the observation. The beginning time of service of subject *d*, *e*, and *f* are before the start of the observation, while their time of failure are after the start of the observation. These three subjects are observable, and they are from the left truncated distribution. For subject *g*, both the beginning time of service and the time of failure occur within the observation window, making it a complete observation data point.

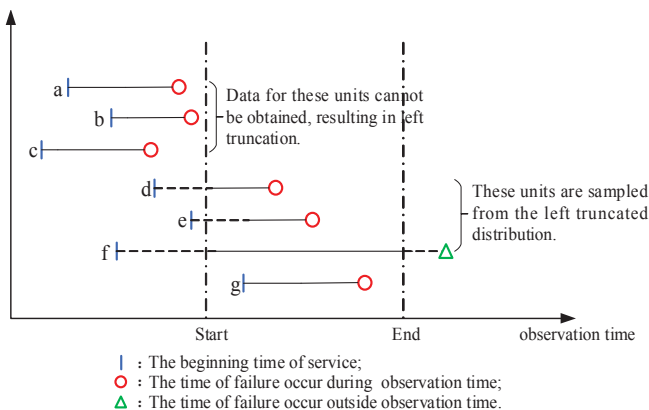


Figure 1. Schematic diagram of truncated data (based on observation time)

On the other hand, since the maintenance factory still used the interval maintenance policy (the inspection frequency is

about 1.5 to 2 years), there are right censored and interval censored data in the raw data. Fig. 2 shows different censored data scenarios based on length of service. All units were pull back to the same starting point for the analysis of the length of service. It is known that coupler knuckle is inspected every 2 years and three inspection records are shown in Fig. 2.

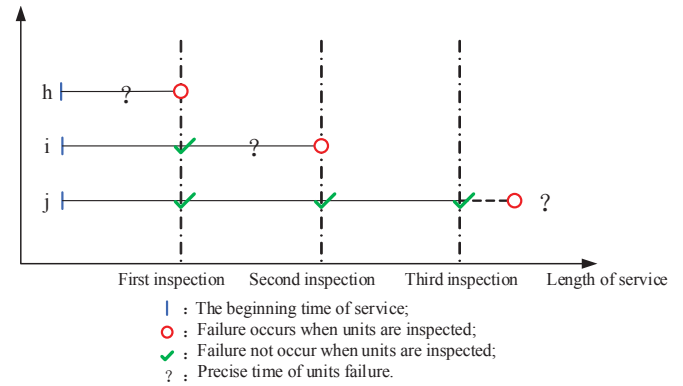


Figure 2. Schematic diagram of censored data (based on usage time)

During the first inspection, subject *h* is found to have already failed, its life data are left censored data, meaning its failure occurred before the first inspection. Subject *i* passed the first inspection and failed in the second inspection, its life data are interval censored data, meaning its precise failure time is between the first inspection and the second one. Subject *j* passed all three inspections and its life data are right censored data, meaning its precise failure time is larger than six years. In summary, the truncation is that the data entry information is incomplete, while the censoring is that the information of the start and/or the end of the data is incomplete, and these phenomena can happen simultaneously. In this study, coupler knuckle (type 16H) data are a mixed data set of left truncated, failed (interval censored) data, and right censored data.

TABLE I. SUMMARY OF THE NUMBER OF FAILED, CENSORED, AND TRUNCATED UNITS FOR THE DIFFERENT MANUFACTURERS

MFG ^a	F&T ^b	F&T ^c	C&T ^d	C&T ^e	Total
MA	211	332	90	208	841
MB	544	100	67	7	718
MC	472	19	176	16	683
MD	315	216	62	46	639
ME	0	65	0	318	383
MF	183	0	75	0	258
MG	160	0	34	0	194
Other	140	17	53	2	212

a. Manufacturers; b. Failed and Truncated; c. Failed but not Truncated; d. Censored and Truncated; e. Censored but not Truncated.

A total of 3,928 data were collected through on-site measurement, including 2,774 failure data. Table I gives a summary of the number of failed, censored, and truncated units for the different manufacturers.

B. Maximum likelihood estimation

Maximum likelihood methods could be used for parameter estimation in these mixed data set of left truncated, interval censored data and right censored data. The basic idea of maximum likelihood methods is finding (estimating) the parameters that make the probability (likelihood) of sample data is maximum. These probabilities are reflected by the likelihood function. For censored data, the probabilities are represented by the difference of cumulative probability density function. For truncated data, the probabilities are represented by conditional probability.

Let the index i be the serial number of coupler knuckle, in this case $i \in \{1, \dots, 4425\}$. Let the index j be the serial number of inspections. Then, the length of service of the i^{th} coupler knuckle in the j^{th} ($j \geq 1$) inspection is represented as t_i^j and t_i^{j-1} represents the length of service of the i^{th} coupler knuckle in the $(j-1)^{th}$ inspection. Let τ_i^L represents the left-truncated time of the i^{th} coupler knuckle. In other words, the difference between its beginning time of service and 2015, if its beginning time of service is before 2015. The indicator of truncation of the i^{th} coupler knuckle is represented by binary indicator α_i . When coupler knuckle started its service before 2015, then $\alpha_i = 0$, meaning there is truncation. When coupler knuckle started its service after 2015, then $\alpha_i = 1$, meaning there is no truncation. The indicator of censoring β_i is defined in a similar way. If the data is right censored, then $\beta_i = 1$, and $\beta_i = 0$ means that there is interval censored. Thus, the likelihood function of coupler knuckle lifetime data is

$$L(\theta | \text{DATA}) = \prod_{i=1}^n [1 - F(t_i^{j-1}; \theta)]^{\alpha_i \beta_i} \times \left[\frac{1 - F(t_i^{j-1}; \theta)}{1 - F(\tau_i^L; \theta)} \right]^{(1-\alpha_i)\beta_i} \times [F(t_i^{j-1}; \theta) - F(t_i^j; \theta)]^{\alpha_i(1-\beta_i)} \times \left[\frac{F(t_i^{j-1}; \theta) - F(t_i^j; \theta)}{1 - F(\tau_i^L; \theta)} \right]^{(1-\alpha_i)(1-\beta_i)} \quad (1)$$

The log-likelihood function is

$$\log L = \sum_{i=1}^n \alpha_i \beta_i * \log[1 - F(t_i^{j-1}; \theta)] + \sum_{i=1}^n (1 - \alpha_i) \beta_i * \{\log[1 - F(t_i^{j-1}; \theta)] - \log[1 - F(\tau_i^L; \theta)]\} + \sum_{i=1}^n \alpha_i (1 - \beta_i) * \log[F(t_i^{j-1}; \theta) - F(t_i^j; \theta)] + \sum_{i=1}^n (1 - \alpha_i)(1 - \beta_i) * \{\log[F(t_i^{j-1}; \theta) - F(t_i^j; \theta)] - \log[1 - F(\tau_i^L; \theta)]\} \quad (2)$$

Here θ is a vector that gives the parameters for each coupler knuckle. The exact structure of θ depends on the context of the model. In this study, coupler knuckle has many potential crack locations, and the failure of coupler knuckle is determined by the occurrence of its first crack. In other words, a coupler knuckle's lifetime is determined by a minimum distribution. Since Weibull distribution is one of the most widely used minimum distribution, this study selects Weibull distribution as the distribution for lifetime of coupler knuckle. In addition, Weibull distribution has good adaptability in the case of incomplete data and can provide reasonable life analysis and prediction [15]. The cumulative density function (cdf) and probability density

function (pdf) of Weibull distribution could be represented as the following equations.

$$F(x) = 1 - e^{-\left(\frac{x}{\lambda}\right)^k} \quad (3)$$

$$f(x) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \quad (4)$$

For Weibull distribution, the log-likelihood function can be written as:

$$\begin{aligned} \log L(\lambda, k | \text{DATA}) &= \sum_{i=1}^n \alpha_i \beta_i * \left(-\left(\frac{t_i^{j-1}}{\lambda}\right)^k\right) + \\ &\sum_{i=1}^n (1 - \alpha_i) \beta_i * \left[\left(-\left(\frac{t_i^{j-1}}{\lambda}\right)^k\right) - \left(-\left(\frac{\tau_i^L}{\lambda}\right)^k\right)\right] + \\ &\sum_{i=1}^n \alpha_i (1 - \beta_i) * \log\left[\exp\left(-\left(\frac{t_i^{j-1}}{\lambda}\right)^k\right) - \exp\left(-\left(\frac{t_i^j}{\lambda}\right)^k\right)\right] + \\ &\sum_{i=1}^n (1 - \alpha_i)(1 - \beta_i) * \left\{\log\left[\exp\left(-\left(\frac{t_i^{j-1}}{\lambda}\right)^k\right) - \exp\left(-\left(\frac{t_i^j}{\lambda}\right)^k\right)\right] - \left(-\left(\frac{\tau_i^L}{\lambda}\right)^k\right)\right\} \\ &= \sum_{i=1}^n \beta_i * \left(-\left(\frac{t_i^{j-1}}{\lambda}\right)^k\right) - \sum_{i=1}^n (1 - \alpha_i) * \left(-\left(\frac{\tau_i^L}{\lambda}\right)^k\right) + \\ &\sum_{i=1}^n (1 - \beta_i) * \log\left[\exp\left(-\left(\frac{t_i^{j-1}}{\lambda}\right)^k\right) - \exp\left(-\left(\frac{t_i^j}{\lambda}\right)^k\right)\right] \end{aligned} \quad (5)$$

$$(6)$$

Obviously, this likelihood function is very complicated and it is difficult to get an analytical solution. It is necessary to apply a numerical approximation method for parameter estimation. Newton-Raphson method is an approach for approximate solving equations in real and complex domains by using Taylor expansion to estimate nonlinear equations in linear form.

C. Basic bootstrap confidence intervals

Bootstrap is a resampling process based on simulation. The basic idea is that a function of the true parameter θ and the actual estimate $\hat{\theta}_n$, $h(\theta, \hat{\theta}_n)$, can be approximated the empirical distribution of that same function of the actual estimate $\hat{\theta}_n$ and the bootstrap estimates $\theta_{n,m}^*$. In other words, the empirical distribution of bootstrap is used to estimate $h(\theta, \hat{\theta}_n)$. Since the parameter true value θ is not available, its estimated value $\hat{\theta}_n$ is used as the true value θ in simulation. The comparison function $h(\theta, \hat{\theta}_n)$ could help reduce impact of the true value on the parameter estimation of sampling distribution. The basic steps for parameter bootstrap are summarized in Figure 3.

As shown in Figure 3, the estimated value $\hat{\theta}_n$ is based on the observed data, and the simulation data $\mathbf{y}^* = (y_1^*, \dots, y_n^*)$ is simulated based on $\hat{\theta}_n$. Bootstrap estimated value $\theta_{n,m}^*$ could be calculated based on the simulation data \mathbf{y}^* . This resampling process is repeated by M times and the set of bootstrap estimates is $\{\hat{\theta}(\mathbf{y}_{n,m}^*); m = 1, \dots, M\}$. Bootstrap confidence interval could be obtained by this set. Davison and Hinkley proposed the basic bootstrap confidence interval in 1997, using the comparison function $h(\theta, \hat{\theta}_n) = n^{\frac{1}{2}}(\hat{\theta}_n - \theta)$. Then the $(1 - \alpha)100\%$ bootstrap confidence interval can be represented as:

$$L_\alpha \approx \hat{\theta}_n - (\theta_n^* - \hat{\theta}_n)_{[(M+1)(1-\frac{\alpha}{2})]} = 2\hat{\theta}_n - \theta_{n,[(M+1)(1-\frac{\alpha}{2})]}^* \quad (7)$$

$$U_\alpha \approx \hat{\theta}_n - (\theta_n^* - \hat{\theta}_n)_{[(M+1)\frac{\alpha}{2}]} = 2\hat{\theta}_n - \theta_{n,[(M+1)\frac{\alpha}{2}]}^* \quad (8)$$

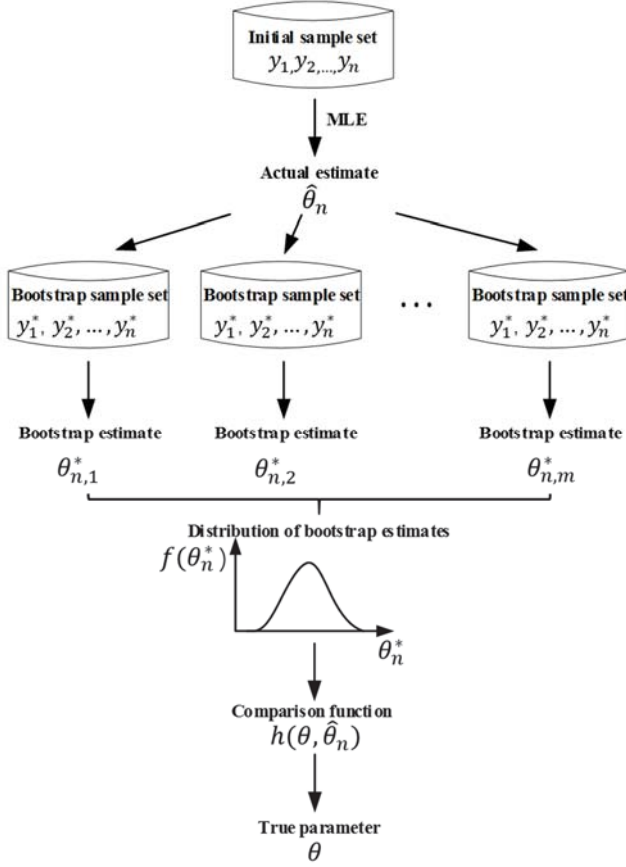


Figure 3. Basic process of parameter bootstrap

$$(2\hat{\theta}_n - \theta_{n,[(M+1)(1-\frac{\alpha}{2})]}^*, 2\hat{\theta}_n - \theta_{n,[(M+1)\frac{\alpha}{2}]}^*) \quad (9)$$

III. RESULTS AND DISCUSSION

A. Basic results

Apply the coupler knuckle data in the Newton-Raphson method, the MLE of λ and k are $\hat{\lambda}=1697.05$, $\hat{k}=2.24$. Based on a significant level of $\alpha=0.05$ and bootstrap resampling time of $M=999$, the basic bootstrap interval estimation of λ and k are summarized in Table II. Based on these results, the average service life is 1503 days, about 4 years, and the expression for the risk rate is

$$h(x) = \frac{2.24}{1697.05} \left(\frac{x}{1697.05} \right)^{1.24} \quad (10)$$

TABLE II. PARAMETER INTERVAL ESTIMATES FOR THE BASIC BOOTSTRAP

Statistics	Scale parameter λ	Shape parameter k
$\theta_{n,[(M+1)(1-\frac{\alpha}{2})]}^*$	1674.003347	2.183281

Statistics	Scale parameter λ	Shape parameter k
$\theta_{n,[(M+1)\frac{\alpha}{2}]}^*$	1721.259075	2.286439
L_α	1672.836719	2.184061
U_α	1720.092447	2.287219

B. Discussion on the effect of truncated and censored data on failure analysis

Three methods are compared in this section. The first model (Model 1) is Kaplan-Meier method considering only the right censoring. The second model (Model 2) is MLE method that only considers interval censoring and right censoring. The third model (Model 3) is the proposed model that established in Section 2.

Kaplan-Meier method is a commonly used method for calculating empirical reliability functions. Let $t_{(i)}$ be the ordered failure time, and r_i be the number of units that are not failure at the i^{th} failure. Then the Kaplan-Meier estimators of reliability function is as follows.

$$\hat{R}(t) = \prod_{\{i: t_i \leq t\}} \left(1 - \frac{1}{r_i}\right) \quad (11)$$

The likelihood function of Model 2 is

$$L(\theta | \text{DATA}) = \prod_{i=1}^n [1 - F(t_i^{j-1}; \theta)]^{\beta_i} \times [F(t_i^j; \theta) - F(t_i^{j-1}; \theta)]^{1-\beta_i} \quad (12)$$

Fig. 4 shows the reliability function of these three models, and it is clear that model selection has a great impact on reliability.

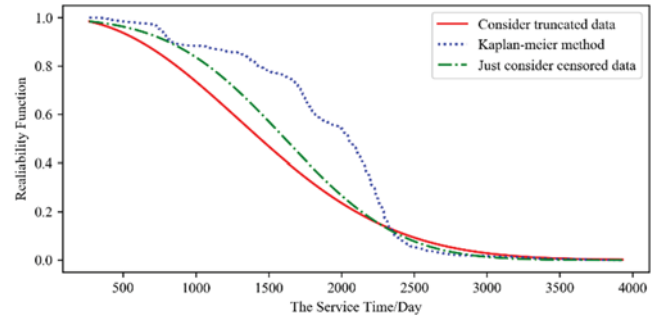


Figure 4. Comparison of the reliability function of three data scenarios

Since the Kaplan-Meier method calculates the failure probability with the endpoint as the failure time, the actual cumulative failure probability is underestimated. In a similar way, Method 2 underestimated the failure rate of early age since part of these information is omitted by ignoring the truncation. Based on these results, the proposed model fixed the underestimation issues by considering the truncated data.

TABLE III. COMPARISON OF THE ESTIMATED VALUES OF THE RELIABILITY FUNCTION FOR THREE DATA SCENARIOS

Year	Estimation of the cumulative distribution function		
	Model 3	Model 2	Model 1
1	0.9683	0.977	0.9977
2	0.8592	0.921	0.9642
3	0.6869	0.796	0.8752
4	0.4895	0.5961	0.7924
5	0.3084	0.3665	0.5977
6	0.1706	0.1774	0.3505
7	0.0824	0.0656	0.0461
8	0.0346	0.0181	0.0176
9	0.0126	0.0037	0.011
10	0.0039	0.0006	0.0025
11	0.0011	1E-04	0

Based on Model 3, half of coupler knuckle will fail at the fourth year, and nearly 90% of coupler knuckle will fail within seven years. In view of the current maintenance cycle of two years, it is suggested to adjust the period of mandatory retirement of coupler knuckle.

IV. FUTURE WORK

In this model, a framework for failure analysis of coupler knuckle considering truncated and censored lifetime data is presented. Maximum likelihood estimation method is employed to model the truncated and censored data. Newton-Raphson method is used to get numerical parameter estimations. The confidence interval of parameter is estimated by parametric bootstrap method. The numerical results show that the proposed method is more reasonable than the Kaplan-Meier method and considering truncation is very important in this case.

The authors are working on extending this work for several directions. First of all, the improved numerical methods for estimating MLE are needed. The standard Newton-Raphson method is subject to the quality of initial value. When the portion of truncated data is relatively large, the overall accuracy of the parameter estimation method needs to be further improved for flat likelihood function. An improved algorithm based on the idea of finite element is under development by the authors and the preliminary test results are encouraging. Second, the effects of comparison function in bootstrap interval estimations need further attention. Comparative analysis of other parameter interval estimation methods is necessary. Last but not the least, predictions of the cumulative number of failed components are needed. Random weighted bootstrap method is a promising way to solve this issue. In summary, the failure analysis is essential for further prognostic and health management, and further studies are great needed.

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