System Residual Useful Life Prediction with Components Following Different Exponential Life Distributions

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***Abstract*—Nowadays, as the system has been becoming more and more complex in engineering practice, the residual useful life (RUL) prediction of system is an important research content to project management. In this paper, in order to cope with the fact that the components' characteristics of useful life are diverse in the system, a method considering that the life of component follows different exponential distribution is proposed to predict the RUL of the system. This method describes the analytical relationship between RUL and reliability function. And according to different system structures, the RUL expressions of series system and parallel system are presented. With the examples, the validity and accuracy of the method are verified by comparing with the stochastic simulation.**

Keywords-exponential distribution; residual useful life; series system; parallel system

I. INTRODUCTION

With the maximization and complication of engineering system, the prediction of the system's useful life (UL) and residual useful life (RUL) plays an increasingly important role in equipment design and operational support. And in practice, the life distributions of much equipment follow exponential distribution, whereat this paper studies it.

Some researches have been made on the average useful life and residual useful life of complex systems before. Khorasgani et al. [1] develop a comprehensive methodology for system-level prognostics under different forms of uncertainty. Regarding the useful life of the unrepairable system as a fuzzy random variable, Liu et al. [2] give the specific expressions of reliability and average useful life. With the test information of equipment composed by complex systems, Li et al. [3] propose the comprehensive evaluations to the average useful life of complex systems. Yang et al. [4] establish an evaluation model for the average residual useful life of complex systems, based on the relationship between reliability and residual life. Chen [5] proposes an evaluation method adapting to various life distribution types by studying the average residual useful life of non-repairable products. And Eryilmaz et al. [6] and McLain et al. [7] also do some work on mean residual life of system.

There are also some discussions on the characteristics and properties of the average residual useful life function considering different structures of the system. Bayramoglu et al. [8] derive the average residual useful life function of series and parallel systems. Based on the useful life time of each component, Asadi et al. [9] studie the average residual useful life function of the series system and the parallel system has been reconstructed and the properties of the function. On this basis, analytic expressions and properties of the average residual useful life function have been discussed by Sadegh [10], with the components subjected to different distributions. However, these researches are not easy and effective enough to be used for engineering application.

In this case, this paper is conducted on the actual situation that the UL of component follows different exponential distributions. Given the failure rate of each component, the analytical expressions of RUL of two typical system structures will be deduced in this paper, which are series system and parallel system. This method sets a foundation for the study of UL and RUL of system.

This paper is expanded in the following order. In section 2, the expression of the relationship between RUL and reliability function is derive. Further, in section 3, the analytical expressions of the UL and RUL of system are derived respectively for systems with different reliability structures. Then, verification of the derivation is conducted in section 4 by comparing the analytical prediction method with the simulation prediction method. In section 5, conclusions are shown.

II. EXPRESSION OF RUL

Assuming that T represents the UL of product, $F(t)$, $f(t)$ and $R(t)$ are defined as the cumulative distribution function (CDF), probability density function (PDF) and reliability function of product failure process, respectively. Let

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$T_L = T - \tau$ represents the RUL of a component working until the current time τ, and the CDF and PDF of the RUL can be expressed as [6]

$$F_L(t) = P(T \le \tau + t \mid T > \tau) = \frac{F(\tau + t) - F(\tau)}{1 - F(\tau)} \tag{1}$$

$$f_L(t) = \frac{\partial F_L(t)}{\partial t} = \frac{f(\tau + t)}{1 - F(\tau)} = \frac{f(\tau + t)}{R(\tau)} \tag{2}$$

Supposing $R(t)$ as the reliability function, at the moment τ, the RUL is

$$\begin{aligned} E_\tau(T_L) &= \int_0^\infty t f_L(t)dt = \int_0^\infty \frac{tf(t+\tau)}{R(\tau)}dt \\ &= \frac{1}{R(\tau)}\int_\tau^\infty (x-\tau)f(x)dx = \frac{1}{R(\tau)}\left(\int_\tau^\infty xf(x)dx - \tau R(\tau)\right) \\ &= \frac{\int_0^\infty xf(x)dx - \int_0^\tau xf(x)dt}{R(\tau)} - \tau = \frac{A-B}{R(\tau)} - \tau \end{aligned} \tag{3}$$

where $A = \int_0^\infty xf(x)dt$, $B = \int_0^\tau xf(x)dx$. A is the estimated value of the UL. Thereby, in practical engineering, the mathematical expectation value of RUL at a specified working time can be calculated directly, as long as the UL distribution of the product is known.

In equation (3), we can get

$$\begin{aligned} \int_\tau^\infty tf(t)dt &= \int_\tau^\infty \int_0^t ds f(t)dt \\ &= \int_0^\tau \int_\tau^\infty f(t)dtds + \int_\tau^\infty \int_s^\infty f(t)dtds = \tau R(\tau) + \int_\tau^\infty R(s)ds \end{aligned} \tag{4}$$

So according to equations (3) and (4), the mathematical expectation of RUL could be presented as [11]

$$E_\tau(T_L) = \frac{\int_\tau^\infty R(t)dt}{R(\tau)} \tag{5}$$

Particularly, when it comes to a special time $\tau = 0$, the calculation expression to estimate UL can be expressed as [12]

$$E(T) = \int_0^\infty R(t)dt \tag{6}$$

Assuming that the UL of a component follow exponential distribution whose failure rate is λ, the reliability function is

$$R(t) = \mathrm{e}^{-\lambda t} \tag{7}$$

III. RUL OF TWO TYPICAL SYSTEM STRUCTURES

A. *Series System*

As we know, the reliability function of series system is

$$R_s(t) = \prod_{i=1}^{n} R_i(t) \tag{8}$$

Assuming a series system is composed by n components whose UL follow exponential distributions with the failure rates $\lambda_1, \lambda_2, \dots, \lambda_n$, UL of series system can be obtained from equations (6) and (8) as

$$\begin{aligned} E_s(T) &= \int_0^{+\infty} R_s(t)dt = \int_0^{+\infty} \prod_{i=1}^{n} R_i(t)dt \\ &= \int_0^{+\infty} e^{-\sum_{i=1}^{n}\lambda_i t}dt = \frac{1}{\sum_{i=1}^{n}\lambda_i} \end{aligned} \tag{9}$$

Particularly, when it comes to a special situation where $\lambda_1 = \lambda_2 = \dots = \lambda_n = \lambda$, the UL of series system is

$$E_s(T) = \frac{1}{n\lambda} \tag{10}$$

According to equations (5), (7) and (8), the RUL of series system is

$$\begin{aligned} E_{\tau,s}(T_L) &= \frac{\int_\tau^\infty R_s(t)dt}{R_s(\tau)} = \frac{\int_\tau^\infty e^{-\sum_{i=1}^{n}\lambda_i t}dt}{e^{-\sum_{i=1}^{n}\lambda_i \tau}} \\ &= \frac{\frac{1}{\sum_{i=1}^{n}\lambda_i} e^{-\sum_{i=1}^{n}\lambda_i \tau}}{e^{-\sum_{i=1}^{n}\lambda_i \tau}} = \frac{1}{\sum_{i=1}^{n}\lambda_i} \end{aligned} \tag{11}$$

B. *Parallel System*

For parallel system, the reliability function is

$$R_p(t) = 1 - \prod_{i=1}^{n}(1 - R_i(t)) \tag{12}$$

Then, the UL can be obtained from equations (6) and (12) as

$$
\begin{aligned}
E_p(T) &= \int_0^{+\infty} R_p(t)\,dt = \int_0^{+\infty}\left[1-\prod_{i=1}^{n}\left(1-R_i(t)\right)\right]dt \\
&= \int_0^{+\infty}\sum\nolimits_{k=1}^{n}(-1)^{k+1}\sum_{i_1 i_2 \ldots i_k} e^{-\left(\lambda_{i_1}+\lambda_{i_2}+\ldots+\lambda_{i_k}\right)t}dt \\
&= \sum\nolimits_{k=1}^{n}(-1)^{k+1}\int_0^{+\infty}\sum_{i_1 i_2 \ldots i_k} e^{-\left(\lambda_{i_1}+\lambda_{i_2}+\ldots+\lambda_{i_k}\right)t}dt \\
&= \sum\nolimits_{k=1}^{n}(-1)^{k+1}\sum_{i_1 i_2 \ldots i_k}\frac{1}{\lambda_{i_1}+\lambda_{i_2}+\ldots+\lambda_{i_k}}
\end{aligned} \tag{13}
$$

where $i_1 i_2 \ldots i_k$ represent the permutation of any k numbers of $1,2,\ldots,n$.

Particularly, when it comes to a special situation where $\lambda_1=\lambda_2=\ldots=\lambda_n=\lambda$, UL of parallel system is

$$E_p(T)=\sum\nolimits_{i=1}^{n}\frac{1}{i\lambda} \tag{14}$$

Similarly, according to equations (5), (7) and (12), the RUL of parallel system is

$$
\begin{aligned}
E_{\tau,p}(T_L) &= \frac{\int_{\tau}^{\infty}\left(1-\prod_{i=1}^{n}(1-R_i(t))\right)dt}{1-\prod_{i=1}^{n}(1-R_i(t))} \\
&= \frac{\sum_{k=1}^{n}(-1)^{k+1}\sum_{i_1 i_2 \ldots i_k}\frac{1}{\lambda_{i_1}+\lambda_{i_2}+\ldots+\lambda_{i_k}}e^{-\left(\lambda_{i_1}+\lambda_{i_2}+\ldots+\lambda_{i_k}\right)\tau}}{\sum_{k=1}^{n}(-1)^{k+1}\sum_{i_1 i_2 \ldots i_k}e^{-\left(\lambda_{i_1}+\lambda_{i_2}+\ldots+\lambda_{i_k}\right)\tau}}
\end{aligned} \tag{15}
$$

IV. VERIFICATION EXAMPLES

In order to verify the correctness of proposed method, example systems are provided.

There are two practice systems shown in Figure 1 and Figure 2. One is consisted of three components in series connection, and the other is consisted of two components in parallel connection. On the one hand, in a series system, when one component fails, the system stops working. At this point, the RUL of the first failed component is the RUL of the system. On the other hand, in a parallel system, when all the components fail, the system stops working. And the RUL of the last failed component is the RUL of the system.

In the examples, the useful life distribution of each component follows exponential distribution. In the series system, failure rates of the three components are $\lambda_1=2.7\times10^{-6}/h$, $\lambda_2=2.2\times10^{-6}/h$, $\lambda_3=1.3\times10^{-6}/h$, respectively. In the parallel system, failure rates of the two components are $\lambda_4=8.9\times10^{-6}/h$, $\lambda_5=7.1\times10^{-6}/h$, respectively. Assuming that both the systems have been working for 2 years, analytical calculation and stochastic simulation are carried out.

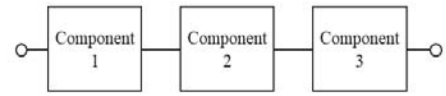

Figure 1. Reliability block diagram of the series system

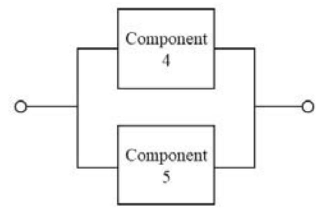

Figure 2. Reliability block diagram of the parallel system

In analytical experiment, with equations (11) and (15), the RUL of series system and parallel system are 16.129 years and 20.121 years.

In simulation experiment, the number of samples is set as 5000. And the experiment is conducted for 10 times, whose mean value is regarded as the final result. When sampling, according to the structure of system, samples of components are converted into system level and then the sample of system is obtained. Repeat the sampling process and the results will appear. As shown in Figure 3 to Figure 6, the samples and distributions of series system RUL and parallel system RUL are presented. Hence, the RUL of series system and parallel system are 16.341 years and 19.849 years.

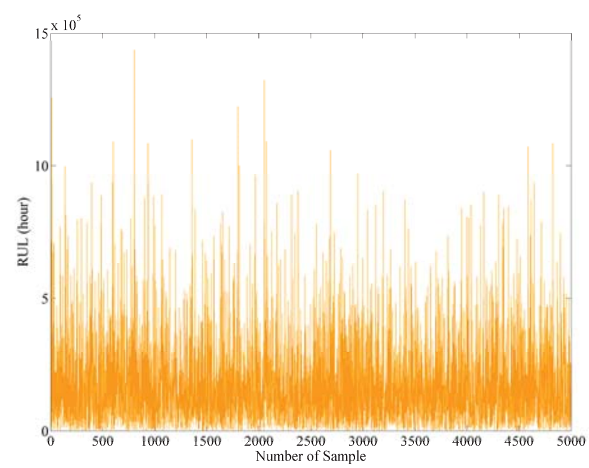

Figure 3. Samples of the series system RUL

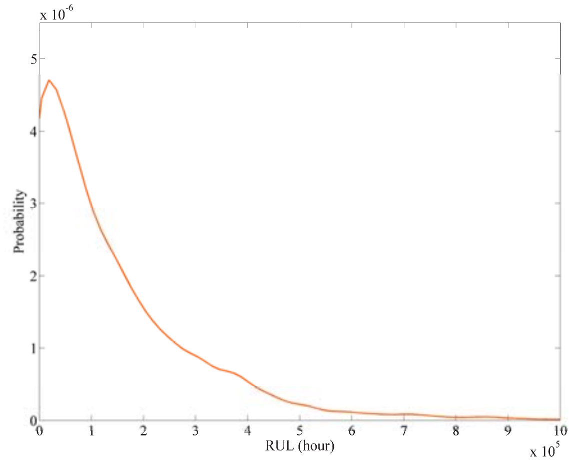

Figure 4. Distribution of the series system RUL

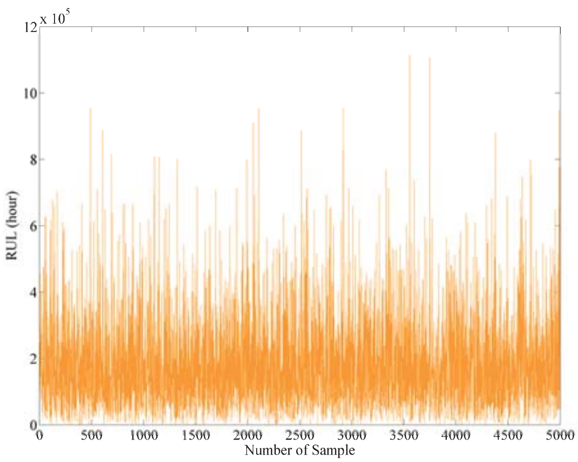

Figure 5. Samples of the parallel system RUL

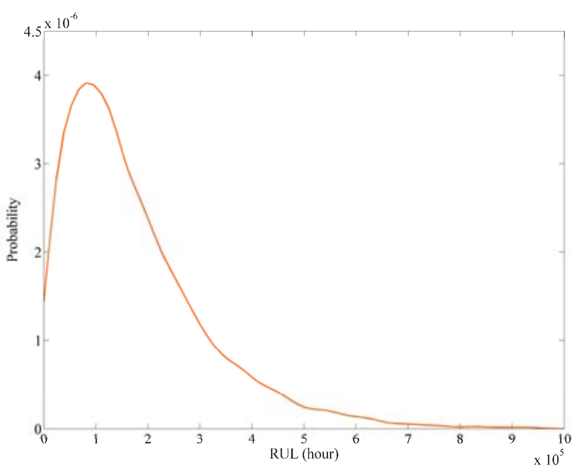

Figure 6. Distribution of the parallel system RUL

To be compared more conveniently, results of the examples are all concluded in TABLE I, where λ_i represents failure rate of each component. The deviation is defined as

$$E = \frac{|RUL_1 - RUL_2|}{RUL_2} \times 100\%$$

where RUL_1 represents the residual life from proposed method and RUL_2 represents the residual life from stochastic simulation method.

TABLE I. RESULTS OF PROPOSED METHOD AND SIMULATION METHOD

	Series System	Parallel System
λ_i (/hour)	$\lambda_1 = 2.7\times10^{-6}$, $\lambda_2 = 2.2\times10^{-6}$, $\lambda_3 = 1.3\times10^{-6}$	$\lambda_4 = 8.9\times10^{-6}$, $\lambda_5 = 7.1\times10^{-6}$
RUL_1 (years)	16.129	20.121
RUL_2 (years)	16.341	19.849
E (%)	1.30	1.37

According to TABLE I, the results of each system from proposed method and simulation method are congenial. Comparing both the methods, the deviation of series system and parallel system are only 1.30% and 1.37%, respectively, which proves the proposed analytical method processes a pretty high precision. And it can be useful in solving the residual useful life of system in practice.

V. CONCLUSIONS

Considering different exponential distributions of components' UL, an analytical method to solve the RUL of system is proposed in this paper. Given the failure rate of each component, based on two typical system structures, including series system and parallel system, the expressions of system RUL are deduced correspondingly.

Furthermore, similar analytical method would be explored to solve RUL of the other two typical system structures which are voting system and cold standby system, under the same conditions.

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