Single-loop Method for Reliability-based Design Optimization of Turbine Blades with Poor Information

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***Abstract*—Turbine blade design optimization is a typical reliability-based multidisciplinary design optimization (RBMDO) problem. Due to the high experimental cost and cycle, the experimental data is limited and the design task will face the situation of poor information. In this paper, the poor information is described with interval variable and the RBMDO model based on non-probabilistic reliability is established. A single-loop method for RBMDO is proposed to improve the efficiency of turbine blade design, in which a two-stage phased construction method is presented to establish the response surface model to approximate the reliability constraint. The results of turbine blade design show that the proposed method has higher calculation efficiency with acceptable calculation accuracy and the calculation accuracy is affected by the accuracy of the response surface.**

Keywords- turbine blades; reliability-based design optimization; interval uncertainty; single-loop method

I. INTRODUCTION

Turbine blades are the key components of aero-engines, which design is directly related to the safety and reliability of the entire aero-engine and even the aircraft. The working environment and conditions of the turbine blades are extremely harsh. Surrounded by the high temperature gas, it needs to withstand the centrifugal force, thermal stress, aerodynamic drag and vibration load during high-speed rotation. Therefore, the problem of turbine blade design will involve aerodynamics, heat transfer and structural mechanics, which is a typical multidisciplinary design optimization (MDO) problem. Moreover, during the operation of turbine blades, there are uncertainties such as structural size, material parameters and working load. Small changes in these uncertain factors may cause the reliability of turbine blades to be reduced, even the aero-engine may malfunction. The traditional MDO method cannot satisfy the requirements of high reliability for the design optimization of turbine blades. It is of significance to apply the RBMDO method in the design process of turbine blades, in which the MDO method is integrated with reliability-based design optimization (RBDO).

The research on RBMDO theory has just started in recent years, and mainly focuses on the problem of random uncertainty for multidisciplinary systems. Yao [1] systematically reviewed the development, theoretical framework and key technologies of RBMDO. Chiralaksanakul [2] integrated the decoupled method of RBDO and typical MDO method and gave three kinds of methods for RBMDO model. Du [3,4] proposed a SORA-based RBMDO method, which has become the most widely used method currently. However, for complex systems such as turbine blades, due to limitations of experimental conditions, time and funding, there are few statistical data on uncertain variables in the design process, and the probability distribution of these uncertain variables cannot be accurately obtained. In this situation of poor information, the probabilistic reliability method may have low calculation accuracy [5,6]. This paper focuses on the research of RBMDO for turbine blades with poor information.

Since the interval model only needs to obtain the boundary information of uncertain variables without having to master the specific data information, and the bounds of uncertainty information are easier to obtain compared to accurate statistical data in engineering practice [7,8], the interval model was applied to represent the uncertainty of poor information in this paper. Then, the RBMDO model based on interval uncertainty was established and the single-loop method was proposed. Finally, the problem of RBDO for turbine blades was solved with the proposed method.

II. NON-PROBABILISTIC RELIABILITY MODEL BASED ON INTERVAL UNCERTAINTY

Assume that there exists a structure with interval input variables $X=(X_1, X_2, \cdots, X_n)$ and the limit-state function is expressed as:

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$$M = g(X) = g(X_1, X_2, \cdots, X_n),\ X \in \Omega \tag{1}$$

where $g(X) \geqslant 0$ indicates a safe domain while $g(X) < 0$ indicates a failure one.

According to Refs.[7,9], the non-probabilistic reliability index η is used to represent the structural reliability, which can be calculated by the following optimization problem:

$$\eta = \min(\|\delta\|_\infty)$$

$$\text{s.t. } M = g(X) = G(\delta_1, \delta_2, \cdots, \delta_n) = 0 \tag{2}$$

where the vector $\delta = \{\delta_1, \delta_2, \cdots, \delta_n\}$ is the normalized interval variables of $X = (X_1, X_2, \cdots, X_n)$; $\|\delta\|_\infty = \max(|\delta_1|, |\delta_2|, \cdots, |\delta_n|)$ denotes the infinite norm $\|\bullet\|_\infty$ in δ space. The optimal solution $\delta^* = \{\delta_1^*, \cdots \delta_n^*\}$ is called the MPP point in the non-probabilistic reliability theory.

The non-probabilistic reliability index η is defined as the shortest distance, measured under the infinite norm $\|\bullet\|_\infty$, from the origin point to the failure surface in δ space. When $\eta < 1$, the uncertainty domain is partially intersected with the failure domain, which represents the structure is unreliable with a certain probability of failure. When $\eta > 1$, the uncertainty domain is not intersected with the failure domain, which represents the structure is reliable. As the reliability index η increases, the failure surface deviates farther from the failure domain, which represents that the structure is more reliable.

III. RBMDO MODEL BASED ON INTERVAL UNCERTAINTY

A. *Multidisciplinary System with Interval Uncertainty*

A 3-discipline coupling system is illustrated in Fig. 1 to conduct the problem research.

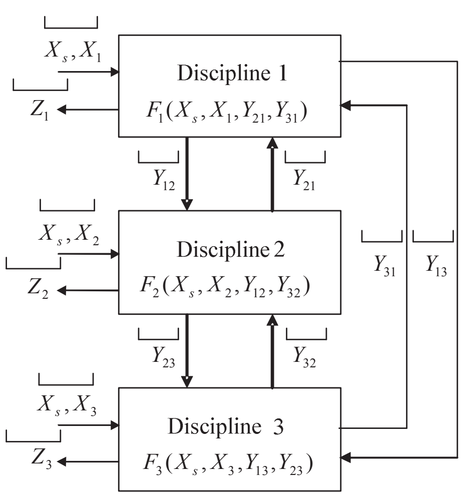

Figure 1. Multidisciplinary system with interval uncertainty

In this system, X_s denotes the vector of sharing variables involved in several disciplines as the inputs. X_i ($i = 1,2,3$) is the vector of the local input variables of discipline i . Z_i ($i = 1,2,3$) is the vector of the local output variables of discipline i . Y_{ij} ($i, j = 1,2,3$) denotes the vector of the coupling variables, which are the output variable of discipline i and the input variable of discipline j . For simplification, the coupling relationship between disciplines can be unified as:

$$Y_{ij} = F_{Yij}(X_s, X_i, Y_{\bullet i}) \tag{3}$$

where $Y_{\bullet i}$ represents the input coupling variable from other disciplines to the disciplines i . When the input variables include interval uncertainty, the coupling variables and output variables also include interval uncertainty due to the uncertainty propagation [10].

B. *RBMDO Model Based on Interval Uncertainty*

By replacing the deterministic constraints with non-probabilistic reliability constraints in the MDO model, the following RBMDO model based on interval uncertainty is established:

$$\begin{aligned} &\min\ f(D_s, D, X_s^c, X^c, Y^c) \\ &\text{s.t. } \eta_i(D_s, D_i, X_s, X_i, Y_{\bullet i}) \geqslant \eta_i^R \\ &\quad Y_{ij} = F_{Yij}(D_s, D_i, X_s, X_i, Y_{\bullet i}) \\ &\quad DV = \{D_s, D\} \end{aligned} \tag{4}$$

where $D = \{D_1, D_2, D_3\}$, $X = \{X_1, X_2, X_3\}$, $Y = \{Y_{ij}; i, j = 1,2,3\}$; X_s^c, X^c and Y^c are the means of X_s , X and Y ; η_i is the non-probabilistic reliability index of discipline i , and η_i^R is the required reliability index for discipline i in the design process. The first constraint is a non-probabilistic reliability constraint, which reflects the influence of uncertainty on the constraint condition. The second constraint is the coupling consistency constraint, which reflects the requirements of interdisciplinary coupling.

The solution of the above RBMDO model is a three-loop nested process, as shown in Fig.2. The outer loop is the system-level optimization process, which is responsible for the optimization search of the system target; the middle loop is the reliability analysis process, which is responsible for the non-probabilistic reliability analysis and judgment of the constraint; the inner loop is the multidisciplinary analysis process, which is responsible for the analysis or solution of the coupling variables. If the RBMDO model is directly solved, the computational cost will be extremely high. Therefore, it is necessary to study the efficient solution method.

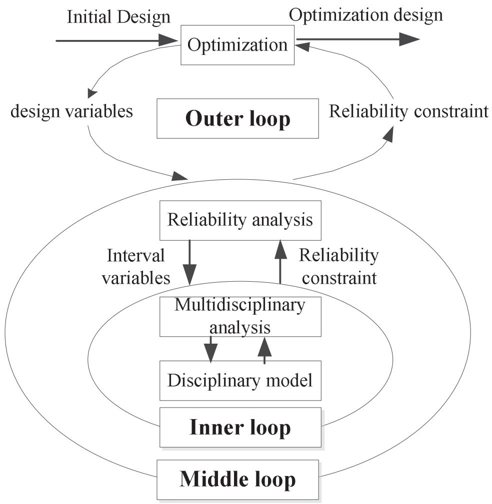

Figure 2. Three-loop nested structure of RBMDO model

IV. SINGLE-LOOP METHOD OF RBMDO

The solution framework has a great influence on the computational efficiency for the RBMDO model. Overall, from three-loop nested framework, SORA framework to single-loop framework, the computational framework becomes simpler in turn, and computational complexity and cost are reduced in turn. Therefore, the single-loop strategy is applied as the solution framework, in which the functional relationship between the design variables and the non-probabilistic reliability index is approximated with the response surface method (RSM).

For multidisciplinary systems, in order to construct the above response surface model, multidisciplinary reliability analysis for samples of design variables is inevitable. Since multidisciplinary reliability analysis is time consuming, the construction cost will be extremely high when reliability analysis is performed directly for each sample. In order to improve the construction efficiency of the response surface, a two-stage phased construction method for RSM is proposed in this paper. The calculation steps are as follows:

Phase 1：Establish a response surface model RS_1 to approximate the relationship among state variable Y, uncertain parameter X and design variable D for each constraint of the RBMDO model.

It includes the following three steps:

Step 1. Select uncertain parameter X and design variable D as experimental factors, and determine the samples based on experimental design method. Here, the uniform design (UD) method [11,12] is selected to reduce the experiment number.

Step 2. Input samples to the multidisciplinary system and perform the multidisciplinary analysis to obtain Y. Note that it is necessary to perform multiple iterations of disciplinary analysis to complete each multidisciplinary analysis.

Step 3. Establish the response surface model RS_1 based on the samples to approximate the relationship among Y, D and X. Here, the Kriging method [13,14] is selected as the approximate modelling method.

Phase 2：Perform the reliability analysis for the samples of D based on RS_1 and establish the response surface model RS_2 to approximate the relationship between reliability index η and D.

It also includes the following three steps:

Step 1. Regard D as experimental factors, and determine the samples based on the UD method.

Step 2. Perform the reliability analysis for the samples of D based on RS_1 to obtain η.

Step 3. Regard D as the input and η as the output response, and establish the response surface model RS_2 based on the above samples.

Based on the above steps, a single-loop optimization flow of the RBMDO model is established as shown in Fig.3.

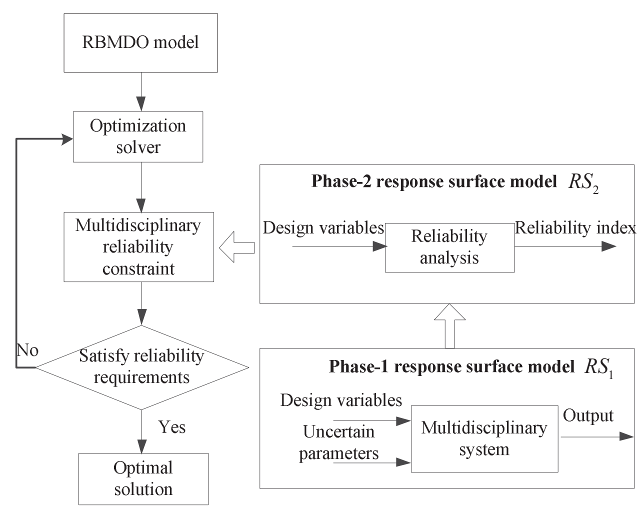

Figure 3. Single-loop optimization method of RBMDO model

V. RBDO OF TURBINE BLADES

A. Description of Problem

In the working process, turbine blades need to withstand the centrifugal force, thermal stress, aerodynamic drag and vibration load during high-speed rotation, which analysis involves multiple disciplines including aerodynamics, heat transfer, and structural mechanics. Here, the complex aerodynamic analysis is ignored and only the heat transfer and structural analysis are considered to reduce the cost of analysis.

As shown in Fig.4, the turbine blade is simplified to a rectangular thin plate. The point O is used to indicate the connection center of the turbine blade and disk, and the coordinate x is used to indicate the axial position of the blade. The given design parameters are listed in Tab.1. The design variables include blade width w and blade thickness t with design space: $0 \leqslant w \leqslant 0.03m$, $0 \leqslant t \leqslant 0.02m$.

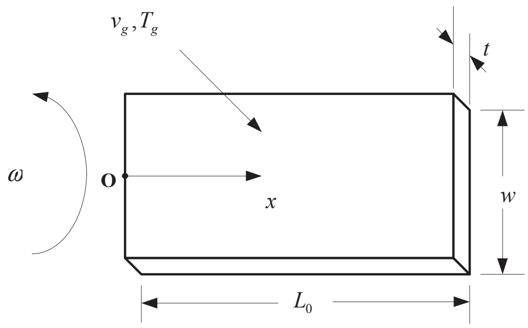

Figure 4. Geometric model of turbine blades

TABLE I. GIVEN DESIGN PARAMETERS

Blade density	8510kg/m³	**Gas density**	3.522kg/m³
Blade initial length	0.05m	**Gas speed**	100m/s
Gas temperature	900℃	**Disk radius**	0.5m
Thermal expansion coefficient	12.6×10^{-6}m/K	**Reference temperature**	300℃
Rotational speed	1150rad/s	**Drag coefficient**	2.0

Uncertain interval parameters include: convective heat transfer coefficient $\bar{h}$, thermal conductivity k, thermal expansion coefficient α, gas density ρ_g, drag coefficient C_d, blade material density ρ, elastic modulus E, and fracture strength σ_r.

Determine the maximum temperature $T_{\max}$ of blade body, maximum deformation Δ_{total} of tip and maximum stress $\sigma(x)$ of the blade body as the constraint variables, and set the required reliability index of each constraint as 0.99. The RBMDO model of turbine blade is established with the goal of minimum heat loss q and minimum blade mass m.

$$\begin{aligned} &\min_{(w,t)} \ f = \{q, m\} \\ \text{s.t.} \ &\eta\{g_1 = T_{\max} - T_{melt} \leqslant 0\} \geqslant 0.99 \\ &\eta\{g_2 = \Delta_{total} - \Delta_{allow} \leqslant 0\} \geqslant 0.99 \\ &\eta\{g_3 = \sigma(x) - \sigma_r(T(x)) \leqslant 0\} \geqslant 0.99 \end{aligned} \tag{5}$$

B. Solution and Result Analysis

Since the constraint g_1 are always satisfied and do not affect the model solution, only the response surface models of reliability index for the constraints g_2 and g_3 were established.

Then, the proposed method was used to solve the above problem, and the obtained optimization results were listed in Tab.2. Moreover, the three-loop nested method and SORA method were also applied to solve the problem and the corresponding results were listed and compared. The number of disciplinary analysis N_{dis} was used to quantity the computational cost of each method.

TABLE II. CALCULATION RESULTS OF EACH METHOD

Methods	(w^*, t^*)	f	η_2	η_3	N_{dis}
Three-loop method	(0.0173,0.0099)	0.0599	0.99	1.1851	564
SORA	(0.0173,0.0099)	0.0599	0.99	1.1851	246
Single-loop method	(0.0171,0.0097)	0.0580	0.982	1.1220	68

It can be seen that:

(1) The calculation results of the three-loop nested method and SORA method are consistent, which can better meet the reliability requirements and should be regarded as the accurate result. The calculation results of the proposed method have small deviations from the first two methods, and basically satisfy the reliability requirements. Through analyzing the reason, the three loop nested method and SORA method directly apply the true disciplinary analysis model to perform the MDO and reliability analysis, so the calculation accuracy is higher; while the proposed method applies the response surface model to approximate the reliability constraints, in which the calculation accuracy is affected by the accuracy of the response surface.

(2) The three-loop nested method has the highest computational cost, followed by SORA, and the single-loop method has the lowest computational cost. It indicates that with the simplification of the solution framework, the computational efficiency of the RBMDO model has been improved to some extent, and it also demonstrates the rationality and feasibility of the proposed method.

VI. CONCLUSION

In this paper, the interval variable is applied to describe the uncertainty with poor information and a single-loop method is proposed for the RBMDO model based on interval uncertainty, in which a two-stage phased construction method is presented to establish the response surface model to approximate the reliability constraint. Then, the proposed method, together with the three-loop nested method and the SORA, is applied to solve the problem of RBDO for turbine blades. The three-loop nested method and SORA method have higher calculation accuracy and lower efficiency. The proposed method has higher calculation efficiency with acceptable calculation accuracy, in which the accuracy of results is easily affected by the accuracy of RSM. It is the key to establish a highly accurate response surface model for this approach. Overall, the three-loop nested method and SORA method are more suitable for solving the small and medium-sized RBMDO problem. The single-loop method has lower computational cost in solving the large-scale RBMDO problem, which is suitable for the RBDO of turbine blades.

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