Joint Optimization of Spare Ordering and Preventive Replacement Policy Based on RUL

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***Abstract*—This paper presents a joint spare ordering and preventive replacement policy for critical systems based on the remaining useful life (RUL). The spare ordering depends on a threshold, which is related to the difference between the estimated RUL at some condition monitoring point and the lead-time. Considering the availability of the spare, replacement could be carried out immediately at the time of failure immediately, or is delayed to the arrival time of the ordered spare. Moreover, the preventive replacement is required if the degradation level exceeds a pre-determined threshold. Then, the optimization model is established to minimize the expected cost rate using the renewal award theory, and a discrete event-driven simulation algorithm is designed to solve it. Finally, a case study is given to illustrate the applicability of the proposed model and algorithm.**

Keywords-remaining useful life; preventive replacement; spare ordering; control limit policy

I. Introduction

An assumption that a spare used for replacement is always available is shared by most previous works related to preventive maintenance (PM), which ignores the lead time, i.e., the duration from a spare's ordering to the arrival time. Obviously, if a spare has been ordered and it is needed before the arrival time, replacement can not been performed immediately, resulting in a delayed replacement at the arrival time of the ordered spare or an emergency order with a higher cost. Therefore, the availability of a spare should be merged into the modeling of the PM policy, rather than make separated decisions for them. Previously, some papers focused on optimizing the age-replacement policy and the spare's availability jointly [1]. The models in these studies assumed that the spare is ordered at the beginning time of the system's operation and arrives after a fixed or random lead time. According to the age-replacement policy, the replacement arises either at the specified age or at the failure time, thereby generating the demand for the spare parts. It is noted that however the demand is determinate at the time of age-replacement, but random at the failure time. Although the cost associated with inspection, replacement and spare are taken into consideration in the objective function, actually only the age-replacement interval is optimized.

Wang et al proposed firstly a condition-based order-replacement policy for single-unit systems with continuous state, in which a regular inspection scheme is undertaken to identify the degradation level and then make decisions to place order, preventive and corrective replacement [2]. For a finite multi-state deteriorating system, the concept of the two-stage delay time is used to build the relationship between the inspection interval and the number of required spare for replacement in the works of Wang [3-5], in which the system is composed of with a number of identical components. Furthermore, Zhao et al presented a joint inspection-based PM and spare ordering policy for single-unit systems based on a three-stage degradation process, and different decisions are made in terms of the identification state at inspection, and the corresponding model is established to find the optimal inspection scheme [6]. However, the above studies considering the degradation condition ignored the utilization of RUL at the condition monitoring time for making decisions for spare ordering.

Recently, RUL is integrated into condition-based prognostics and spare inventory strategies. To the best of our knowledge, Elwany and Gebraeel studied a sensor-driven prognostic model for making component replacement and inventory decisions [7]. Followed by it, the costs related to condition monitoring and spare ordering are added into the optimization model by Wang et al [8]; the random lead time is further considered in [9]. Louit et al presented a prognostic-information-based spare ordering policy for critical systems, in which the ordering decision relies on the conditional reliability function of the item and only the ordering time is optimized [10].

It is investigated in the industry that decision makers commonly place an order for replacement at the time of condition monitoring when the lead time is much longer and RUL is relatively shorter. Otherwise, the spare has not been ordered or delivered once it is required; if the spare is ordered when RUL is much larger, it is possible to increase the holding cost. Motivated by it, this paper presents a joint preventive replacement and spare ordering policy based on RUL, in which

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a threshold is introduced to determine whether it is necessary to place an order.

The remaining part of this paper is outlined as follows: Section Ⅱ gives the problem description, and in Section Ⅲ RUL is estimated and a discrete event-driven simulation algorithm is designed in Section Ⅳ; a case study is given to illustrate the proposed model and algorithm in Section Ⅴ,and this paper is concluded in Section Ⅵ.

II. PROBLEM DESCRIPTION

A. Joint Preventive Replacement and Spare Ordering Policy Based on RUL

A single-unit system that deteriorates subject to Wiener process is considered, and an inspection scheme is carried out with the interval ΔT to check the degradation level. The value of ΔT is often various depending on different systems in the industry, and it could be 1 day, 1 hour, 1 minute, or 1 second. So the degradation level can be regarded as continuous. A control limit policy is considered, by which the maintenance actions are implemented in terms of the degradation level. That is, if the degradation level reaches to a failure threshold L_c, the corrective replacement (CR) is needed; the preventive replacement (PR) is done when the degradation level reaches to a PR threshold L_p ($L_p < L_c$).

At each condition monitoring time, the remaining useful time is predicted to determine whether to place an order or not. If the estimated RUL at the kth condition monitoring point (t_k) is L_k and the difference between RUL L_k and the lead time L equals to or is smaller than a ordering threshold h, i.e, the constraint L_k-$L \leq h$ holds, place an order at t_k and the ordering time is $t_o=t_k$. The ordered spare arrives after the delivery time L and is stored for replacement. If L_k-$L>h$ is met, no ordering at t_k until the next monitoring time. As shown in Figure 1, if the ordering threshold $h=h_1$, then place an order at t_k; however, when h equals to h_2, there is no need to place an order. Therefore, we treat the ordering threshold h as a decision variable.

At the time of preventive or corrective replacement requirement, the spare provisioning is considered. There are three situations as follows: no order before the time of required replacement, a spare has been ordered but not delivered, and the ordered spare is available. If no spare is ordered, then an emergency replenishment is made with a higher cost and negligible lead time than that of a regular order. Replacement has to be delayed until the arrival time of the regular ordered spare. It is natural that an immediate PR/CR occurs once the spare is available and the system after replacement is as good as new.

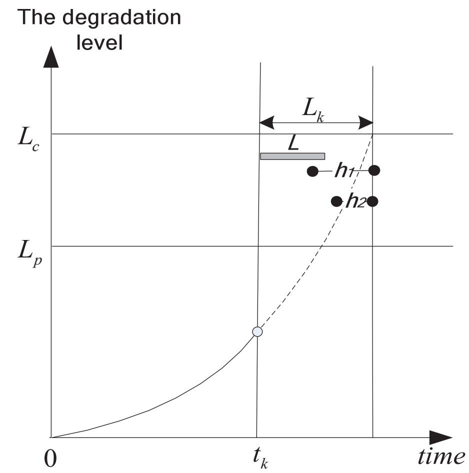

Figure 1. Degradation process

B. Renewal Scenarios

Based on the proposed joint policy, there are six mutually independent and exclusive renewal scenarios, which are sketched as follows.

Scenario 1: if the degradation level $X(t_k)$ satisfies $L_p \leq X(t_k) < L_c$ before the ordering time of a spare, an emergency order is placed to replace the system immediately at t_k. It leads to an emergency PR.

Scenario 2: if the degradation level $X(t_k)$ satisfies $L_p \leq X(t_k) < L_c$ after the ordering time of a spare, but before the arrival of the ordered spare, a delayed PR is incurred at t_o+L.

Scenario 3: When a PR is required for the degradation system with $L_p \leq X(t_k) < L_c$ after the arrival of the ordered spare, an immediate PR is incurred at t_k.

Scenario 4: The system deteriorates and reaches to L_c, before which no spare is ordered. This generates an emergency CR.

Scenario 5: The system fails between the ordering time and the arrival of the ordered spare, so wait for the spare and replace at t_o+L It is termed as a delayed CR.

Scenario 6: When the system exceeds L_c after the arrival of the ordered spare, an immediate PR is incurred at the failure time.

C. Notations

Some notations are given and used to the subsequent modeling.

c_i : average cost per condition monitoring

c_r : average cost of the regular order

c_e : average cost of the emergency order ($c_e > c_r$)

c_{PR} : average cost of preventive replacement

c_{CR} : average cost of corrective replacement

c_h : average holding cost per time unit

c_s : average shortage cost per time unit

L : the lead time of the regular ordered spare

$X(t_k)$: the degradation level of the system at time t_k ($t_k = k \cdot \Delta T$)

III. REMAINING USEFUL LIFT PREDICTION

A. Degradation Modeling

The degradation of the single-unit system is modeled by the Wiener process [11], which is represented as:

$$X(t_k) = \lambda t_k + \sigma B(t_k) , \tag{1}$$

where λ is the drift coefficient, σ is the diffusion coefficient (σ >0), and $B(t_k)$ is the standard Brownian motion with $B(t_k) \sim N(0, t_k)$. Moreover, the degradation level $X(t_k)$ has the following properties:

(1) $X(0) = 0$ holds at time 0.

(2) The expectation of $X(t_k)$ is $E(X(t_k)) = \lambda t_k$, and corresponding variance is $\mathrm{var}(X(t_k)) = \sigma^2 t_k$ representing the uncertainty of the degradation at time t_k .

(3) The random increment $X(t_k + \Delta t) - X(t_k) \sim N(0, \sigma^2 \Delta t)$.

B. RUL Prediction

The commonly used definition of the lifetime T is the time to reach the pre-set failure threshold L_c for the first time, namely the first hitting time (FHT). Then, we have the lifetime T in (2), and T has an inverse Gaussian distribution with probability density function (pdf) in (3).

$$T = \inf\{t : X(t) \geq L_c \,|\, X(0) < L_c\} . \tag{2}$$

$$f_T(t) = \frac{L_c}{\sqrt{2\pi\sigma^2 t^3}} \exp\left(-\frac{(L_c - \lambda t)^2}{2\sigma^2 t} \right). \tag{3}$$

Similarly, using the FHT, the RUL L_k at the current condition monitoring t_k is defined as the duration from t_k to the point that the degradation first exceeds to L_c, and denoted by

$$L_k = \inf\{l_k : X(t_k + l_k) \geq L_c \,|\, X(t) < L_c, \forall t \leq t_k\} . \tag{4}$$

The degradation level at $t_k + l_k$ is described as $X(t_k + l_k) = X(t_k) + \lambda l_k + S(l_k)$, in which $l_k \geq 0$, $S(l_k) = B(t_k + l_k) - B(t_k)$. The pdf of L_k conditional on x_k ($X(t_k)= x_k$) is derived as

$$f_{L_k}(l_k | x_k) = \frac{L_c - x_k}{\sqrt{2\pi\sigma^2 l_k^3}} \exp\left(-\frac{(L_c - x_k - \lambda l_k)^2}{2\sigma^2 l_k} \right), \tag{5}$$

where the mean and variance of RUL L_k are $\frac{L_c - x_k}{\lambda}$ and $\frac{(L_c - x_k)\sigma^2}{\lambda^3}$, respectively.

IV. DISCRETE EVENT-DRIVEN SIMULATION ALGORITHM

A. Obejctive Function of the Proposed Joint Policy

Based on all possible renewal scenarios as described in Section Ⅱ, the objective function to minimize the long-run expected cost per unit time is formulated as

$$\begin{aligned} &\min\ E_C(L_p, h) = \sum_{i=1}^{6} C_i(L_p, h) \Big/ \sum_{i=1}^{6} L_i(L_p, h) \\ &s.t.\ L_k - L \leq h \end{aligned}, \tag{6}$$

where $C_i(L_p, h)$ and $L_i(L_p, h)$ are the expected cost and length of the ith renewal scenario, respectively; the constraint in (6) is set to place an order at time t_k. The decision variables are the PR threshold and the ordering threshold, (L_p, h) . The explicit expressions of $C_i(L_p, h)$ and $L_i(L_p, h)$ in (6) can be derived, but complicated. Hence, we design a discrete event-driven simulation algorithm to solve (6).

B. Simulation Algorithm

Figure 2 shows the proposed simulation algorithm and the simulation procedure is as follows.

Step 1: Set the initial cost parameters, the lead time L, the drift coefficient, the diffusion coefficient, ΔT , L_c , and we plan to run $N_{\max}$ simulations. Let T_C and T_L denote the total cost and length as the algorithm runs, respectively.

For $L_p = 1, 2, ...$

For $h = 1, 2, ...$

let $T_C = 0$, $T_L = 0$, i=0;

Step 2: $i=i+1$, $x_0 = 0$, $t=1$, and the ordering time is assumed to be zero t_o =0.

Step 3: Generate random values from $N(0,t)$, and $N(0,t-\Delta T)$, respectively; then, calculate the difference $S(\Delta T)$ between them and obtain the degradation level at time $x(t)=x(t-\Delta T)+\lambda\Delta T+S(\Delta T)$.

Step 4: Calculate the RUL as $L(t)=L_c-x(t)/\lambda$ $+(L_c-x(t))\sigma^2/\lambda^3$.

Step 5: Decide whether to place an order or not according to $L(t)-L\le h$. If it is met, a regular order occurs at time t with the ordering time $t_o=t$ and the arrival time $t_a=t+L$; and, go to Step 6. If the constraint is rejected, go to Step 6.

Step 6:

If $x(t)<L_p$ then, $\begin{matrix} t=t+\Delta T \\ T_C=T_C+c_i \end{matrix}$ and return to Step 3

Elseif $x(t)\ge L_c$ % A CR is required;

if $t_o=0$ % Scenario 4 occurs;

$T_C=T_C+c_i+c_e+c_{CR}, T_L=T_L+t$; go to Step 7

Elseif $t<t_a$ % Scenario 5 occurs;

$\begin{matrix} T_C=T_C+c_i+c_r+c_{CR}+c_s(t_a-t) \\ T_L=T_L+t_a \end{matrix}$; go to Step 7

Else % Scenario 6 occurs;

$\begin{matrix} T_C=T_C+c_i+c_r+c_{CR}+c_h(t-t_a) \\ T_L=T_L+t \end{matrix}$; go to Step 7

End;

Else % A PR is required;

if $t_o=0$ % Scenario 1 occurs;

$T_C=T_C+c_i+c_e+c_{PR}, T_L=T_L+t$; go to Step 7

Elseif $t<t_a$ % Scenario 2 occurs;

$\begin{matrix} T_C=T_C+c_i+c_r+c_{PR}+c_s(t_a-t) \\ T_L=T_L+t_a \end{matrix}$; go to Step 7

Else % Scenario 3 occurs;

$\begin{matrix} T_C=T_C+c_i+c_r+c_{PR}+c_h(t-t_a) \\ T_L=T_L+t \end{matrix}$; go to Step 7

End;

End;

Step 7: if $i=N_{\max}$

Output the mean cost rate $E_C(L_p,h)=T_C/T_L$;

Else Return to Step 2;

End;

End; %Find the optimal h^* that minimizes $E_C(L_p,h)$ under the given L_p .

End; %Find the optimal (L_p^*,h^*) that minimizes $E_C(L_p,h)$.

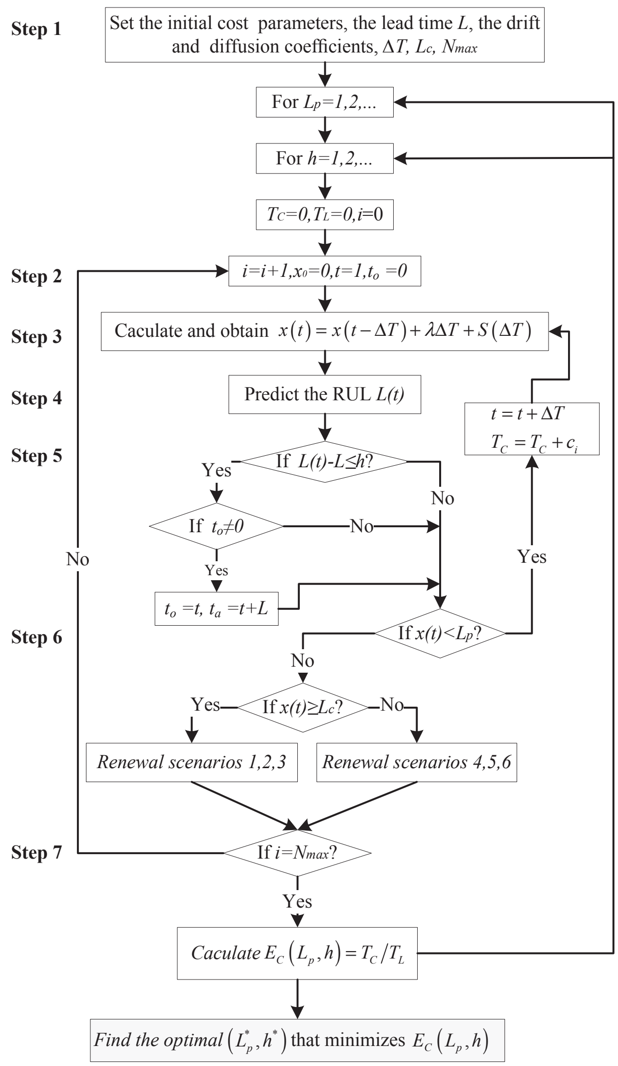

Figure 2. The proposed simulation algorithm

V. CASE STUDY

The health condition of engine is commonly indicated by take-off exhaust gas temperature margin (EGTM) and the EGTM value reduces gradually as it is used. In this Section, the engine is selected as our case study from the literature [12]. The degradation process is estimated using historical EGTM information of 18 engines, and is represented by $X(t)=75-0.01478t+0.39997B(t)$, in which $X(0)$=75 , λ=-0.01478 and σ=0.39997 . It is clear that the degradation level is a decreasing function of time t, so the PM threshold is larger than the failure threshold in this case. $\Delta T = 100, L_c = 0$ and the cost parameters are set as c_i=500 , c_r=100 , c_e=2000 , c_{CR}=50000 , c_h=50 , c_s=250 , L=300 . With these parameters, the discrete event-driven simulation algorithm shown in Figure 3 is run using the Matlab Software.

Figure 3 illustrates the change of the cost rate as the preventive replacement threshold L_p under different ordering thresholds. It is clear that the optimal solutions are $\left(L_p^*, h^*\right)=\left(4{,}300\right)$ with the minimal cost rate 7.4694. Obviously, the cost rate decreases firstly and then increase with the increase of L_p for the given ordering threshold. This can be explained that more frequent preventive replacement with a smaller L_p decreases the availability due to earlier renewal, but it has to cost much more due to a random failure if PR is required with a higher L_p. On other hand, it is also noted that the cost rate shows the same trend as the ordering threshold increases, which is as we expected. Since when the optimal ordering time is too small, it is more likely to increase the holding cost due to the earlier ordering; otherwise, the shortage cost is incurred and then the cost rate increases.

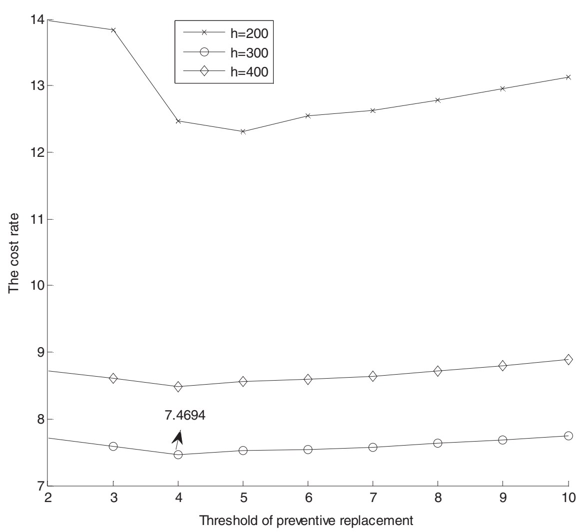

Figure 3. The change of the cost rate as the increase of L_p

VI. CONCLUSIONS

A joint preventive replacement and spare ordering policy using the remaining useful lifetime is proposed for a single-unit system. The system degrades following the Wiener process and a control limit policy is considered. Preventive and corrective replacement are performed by a new and identical spare once the degradation level exceeds the PR/CR thresholds, respectively. The spare is ordered only and if only the difference between the estimated RUL at the condition monitoring time and the lead time is not larger than the ordering threshold. The objective function of the proposed policy is established and the corresponding discrete event-driven simulation algorithm is designed to solve it. The results in the case study shows the proposed joint policy can decrease the average cost rate. In the future, the analytical optimization model will be formulated.

ACKNOWLEDGMENT

This research report is partially supported by the NSFC (71701038 and 71601019), China Ministry of Education Humanities and Social Sciences Research Youth Fund Project (16YJC630174), the Fundamental Research Funds for the Central Universities of China (N172304017).

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