

# A Cascading Failure Model of Power Systems Considering Components' Multi-State Failures

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**Abstract**—It is critical to understand the cascading failure mechanisms and assess the performance of power systems under different types of natural disasters. This paper proposes a cascading failure model of power systems based on Alternating Current (AC) optimal power flow with components' multi-state failures. On the basis of the traditional cascading failure model, the proposed model considers each component's various failure states and different performance levels of each state of power systems. Meanwhile, active disconnection and passive failure of each component are considered. The proposed cascading failure model can be used as a guidance for the developing power systems' disaster recovery strategy. Then, the proposed evaluation metric for the scale of the system's cascading failure incorporates three factors, i.e., the number of working components, the amount of power flowing, and the topology of the system. Finally, taking earthquake disaster as an example, this paper provides the method to simulate natural disaster of the central type. The proposed disaster simulation method can be extended to other two types of natural disasters, such as the unilateral type (e.g., hurricane, etc.) and the global type (e.g., snowstorm, etc.). In addition, the intensity and distribution of the disaster, the attenuation of the intensity, the location of the disaster occurrence, as well as the vulnerability and physical location of each component are considered in this method. The IEEE 30-bus model is used as a testbed to demonstrate the feasibility and effectiveness of the proposed modeling methods. The proposed evaluation metric is also compared with the traditional one.

**Keywords**— power system; cascading failure; evaluation metric; optimal power flow; multi-state failure; natural disaster;

## I. INTRODUCTION

Power systems can be regarded as the most critical infrastructure of a country since the operation and management of other infrastructures are mostly dependent on the normal operation of power systems [1]. With the constant expansion of the power systems' scale, power systems have gradually transformed into a large-scale interconnected network system. Meanwhile, large-scale interconnection of grids also increases the complexity of the power systems, making the risk of the large-scale blackout continuously increase due to uncertain factors, such as hidden failures, natural disasters, operator errors, and even malicious attacks [2]. Such large-scale blackouts still occur from time to time over the world, in spite of the efforts and huge investments in mitigating the risks of power systems [3].

On the other hand, with the improvement of management systems in various countries, the number of large-scale blackouts caused by hidden failures and human factors is decreasing year by year. However, blackouts caused by natural disasters, such as hurricanes, earthquakes, snowstorms, etc., are still inevitable. Therefore, building a cascading failure model to understand the performance of power systems under different types of natural disasters is an important method to prevent cascading failures, minimize the scale of cascading failures, and ensure the security of infrastructure systems.

In general, the existing cascading failure models can be divided into two categories, including complex network-based and approximate dynamic behavior-based models.

Complex network-based cascading failure models, such as betweenness-based model [4], dynamical redistribution model [5], M-L model [6], etc., have the advantages of high-efficiency, easy solution, few input parameters, etc. However, complex network-based models usually contain a mass of approximations and assumptions of power systems. For example, the betweenness-based model assumes that electrical energy flows through the shortest path between any two nodes. Hence, it is hard to apply such type of models to the power systems due to the results' inevitable errors with the actual power systems.

In order to simulate the cascading failure process of power systems more accurately, the approximate dynamic behavior-based cascading failure models have been widely concerned and studied. This type of models can also be divided into two categories, including Direct Current (DC) power flow models and Alternating Current (AC) power flow models. DC models [7], which are the tractable relaxation models of the AC models, have such advantages of linear features, computational efficiency, and wide applications. However, DC models are sometimes unable to approximate the actual cascading process since they cannot reflect the reactive power characteristics of systems [8]. Hence, AC models are proposed. Rios et al. [9] adopted Manchester model and Monte Carlo simulation to discuss the modeling of time-dependent phenomena, such as cascade and sympathetic tripping, transient stability, and weather effect in the computation of the Value of Security in power systems. Nedic et al. [10] adopted Manchester model to investigate criticality of in a cascading failure blackout model and measure blackout size by energy unserved. However, the

regulating function of the generator and the economic dispatch have not been considered in the solution process of Manchester model. In order to address the above challenges of Manchester model, Li et al. [8] proposed an AC model considering both AC power flow and AC optimal power flow to give a reasonable consideration to economic dispatch. Their and other existing cascading failure models all share the assumption that the state of each component is binary, i.e., state is equal to 1, if working; state is equal to 0, otherwise. Nevertheless, the component's failures usually present multiple states in the actual power systems. Therefore, it is necessary to consider the multi-state failures of components in AC cascading failure modes.

The main contributions of this paper can be divided into three aspects. At first, a cascading failure model based on AC optimal power flow with components' multi-state failures is proposed. Each component's various failure states and different performance levels of each state are considered in this model. In addition, active disconnection and passive failure of each component are also considered. The actively disconnected components may be intact and no additional resources and time need to be allocated for functional repair and restoration. The recovery times and resources needed of components in different failure states are also different. The proposed cascading failure model can be used as a guidance for the developing power systems' disaster recovery strategy. In order to simulate the initial failure states of all components after a disaster, the second contribution is to first propose the simple method to simulate natural disaster of central type (e.g., earthquake, etc.). The proposed method considers the intensity and intensity's distribution of the disaster, the attenuation of the intensity, the location of the disaster occurrence, as well as the vulnerability and physical location of each component. In addition, the proposed disaster simulation method can be extended to other two types of natural disasters, including unilateral type (e.g., hurricane, etc.) and global type (e.g., snowstorm, etc.). The third contribution is to propose a metric to evaluate the scale of the system's cascading failure. The traditional evaluation metric (i.e., the giant connected component [11]), which is widely used in research, assumes that the components only belonging to the largest mutually connected component retain their functions while the smaller components fail. However, the traditional evaluation metric ignores other working components that do not belong to the largest mutually connected component and whether there is a power flow in the largest connected component. Hence, different from the traditional metric, the proposed metric, which incorporates three factors of the number of working components, the amount of power flowing, and the topology of the system, can comprehensively evaluate the scale of system's cascading failures in three perspectives. The entropy method [12] is adopted to determine the weight of each factor. At the end of the paper, the IEEE 30-bus model is used as a testbed to demonstrate the feasibility and effectiveness of the proposed modeling methods, meanwhile, the proposed evaluation metric is also compared with the traditional one.

The remaining of the paper is organized as follows. Section II proposes detailed modeling and simulation methods from the aforementioned three aspects. Section III provides an example of the proposed methods applied in IEEE 30-bus model. Section IV draws the conclusion and discusses future research.

## II. METHODOLOGIES

### A. The Cascading Failure Model based on AC Optimal Power Flow with Multi-state Failures.

The power system can be replaced by a graph  $G_p = (V_p, E_p)$ , where  $V_p$  and  $E_p$  are the nodes and branches of power systems, respectively. Each node represents a unique generating station or substation, and branches includes transmission towers, lines, and other transmission facilities. These nodes can be divided into four categories: (1) being both power supply and load ( $V_p^{SL}$ ), (2) Being only power supply ( $V_p^S$ ), (3) Being only load ( $V_p^L$ ), and (4) being neither power supply nor load ( $V_p^T$ ).

TABLE I. THE RELATIONSHIP BETWEEN FUNCTIONAL LOSS RATE AND FAILURE STATES OF COMPONENTS [13]

| Failure grade | Failure state   | The range of functional loss rates ( $F$ ) |
|---------------|-----------------|--|
| 1             | normal          | $0.00 \leq F \leq 0.10$                    |
| 2             | minor damage    | $0.10 < F \leq 0.20$                       |
| 3             | moderate damage | $0.20 < F \leq 0.45$                       |
| 4             | serious damage  | $0.45 < F \leq 0.80$                       |
| 5             | complete damage | $0.80 < F \leq 1.00$                       |

The basic assumptions of proposed cascading failure model are as follows:

- The failures of each nodes and branches present multiple states. For simplicity, all branches and nodes are assumed to have the same six failure states as shown in TABLE I.
- The capacities of each node or branch with the failure grade of 2 or 3 will be degraded, except for the nodes of type  $V_p^T$ .
- Repair is not considered in cascading failure processes.
- Initial failures are mutually independent.
- The disconnection of a node results in the active disconnection of its connected branches.

The nodes and branches with the failure grades 1 may have suffered some non-functional damage, such as appearance, etc. The capacities of these nodes and branches will not be degraded, and recovery time and repair resources are not required.

For nodes and branches with the failure grades 2, their non-critical functional parts, which have little impact on power supply, may suffer damage, such as insulation layers, circuit breakers, etc. The capacities of these nodes and branches will be degraded according to their functional loss rates. A little amount of recovery time (e.g., about one day for the substation) and repair resources are required, but these nodes and branches can be repaired with electricity.

When the failure grade of nodes and branches reaches 3, their non-critical functional parts, which have a small impact on power supply, may suffer damage, such as transformers, etc. The capacities of these nodes and branches will also be degraded according to their functional loss rates. A small amount of recovery time (e.g., about three days for the substation) and

repair resources are required. In addition, these nodes and branches need to be repaired without power.

At the failure grades 4, these damaged nodes and branches' critical functional parts, which have an impact on power supply, may suffer damage, such as chimney, etc. These nodes and branches should be immediately disconnected from the systems (i.e., active disconnection). In addition, some recovery time (e.g., more than a week for the substation) and repair resources are required.

For nodes and branches with the failure grades 5, their critical functional parts, which have a significant impact on power supply, may suffer serious damage, such as the main workshop of power plant, cooling tower, etc. In general, these nodes and branches have lost their functions so that they are not considered to be working (i.e., passive failure). In addition, some recovery time (e.g., more than a week for the substation) and repair resources are required.

When the failure grades of nodes or branches are 2 or 3, the different capacities of these nodes and branches will be degraded according to their types. Specifically, (1) for the nodes of type  $V_p^{SL}$ , the maximum reactive output ( $Q_{max}$ ), the minimum reactive output ( $Q_{min}$ ), the maximum real output ( $P_{max}$ ), the minimum real output ( $P_{min}$ ), the reactive power demand ( $Q_d$ ), and the real power demand ( $P_d$ ) will be degraded, (2) for the nodes of type  $V_p^S$ ,  $Q_{max}$ ,  $Q_{min}$ ,  $P_{max}$ , and  $P_{min}$  will be degraded, (iii) for the nodes of type  $V_p^L$ ,  $Q_d$  and  $P_d$  will be degraded, and (4) for the branches, long term rating ( $r^A$ ), short term rating ( $r^B$ ), and emergency rating ( $r^C$ ) will be degraded. The reduction in capacities of nodes and branches can be calculated as follows:

$$C'(i) = C(i) * F(i), \quad (1)$$

where  $C(i)$  represent the initial capacities of node  $i$ , including  $Q_{max}$ ,  $Q_{min}$ ,  $P_{max}$ ,  $P_{min}$ ,  $Q_d$ ,  $P_d$ ,  $r^A$ ,  $r^B$ , and  $r^C$ .  $C'(i)$  represents the nodes' capacities after degraded, and  $F(i)$  represents the functional loss rate of node  $i$ .

The steps of the proposed cascading failure model are shown in Figure 1. The iteration of the model does not stop until the active and reactive power of all islands have been calculated.

### B. The Method for Simulating Natural Disasters

According to the distribution characteristics of disaster intensity, the natural disasters can be divided into three categories, including central type, unilateral type, and global type. The disasters of central type represent the disasters with the highest intensity at the heart of the disaster area, such as earthquake, etc. Meanwhile, the intensity of such disasters attenuate as the distance increases. Different from the central type, the disasters of unilateral type represent disasters whose intensity attenuate along a straight line, and the highest intensity of the disaster occurs on one side of the disaster area, such as hurricane, etc. The disasters of global type represent disasters with the same intensity everywhere in the disaster area, such as snowstorm, etc. In this paper, taking the earthquake disasters from central type's disasters as an example, the simulation method of earthquake disasters is proposed, and the proposed method can be extended to other two types of disasters.

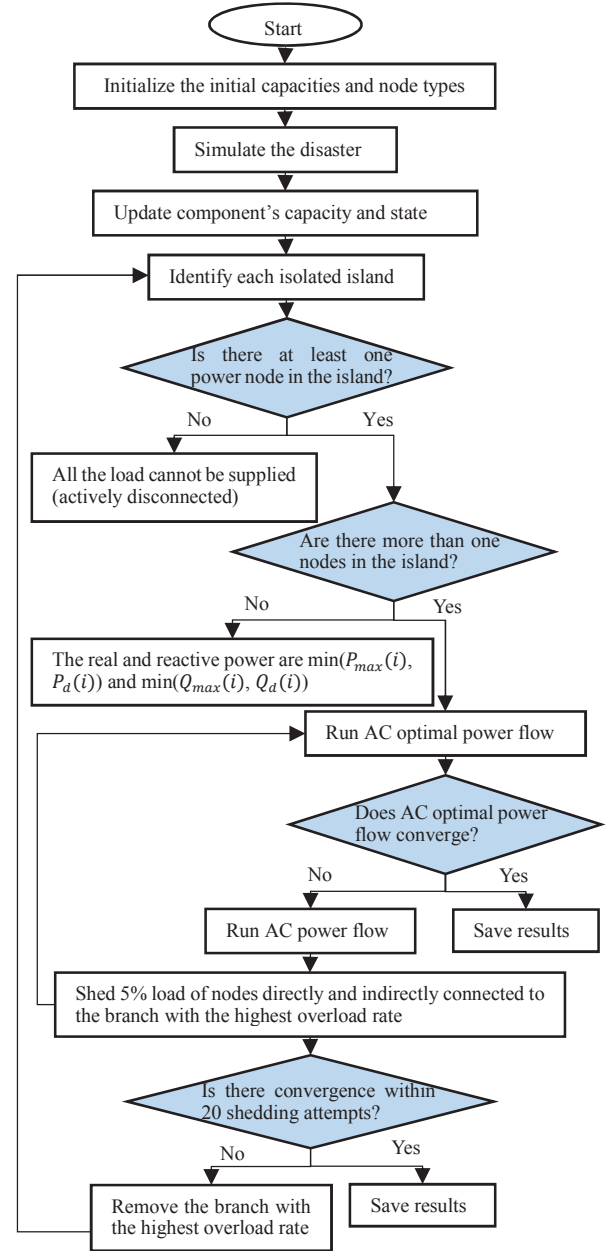


Figure 1. The steps of the proposed cascading failure model

The magnitude of an earthquake ( $M$ ) is modeled as a normal distribution with a mean of  $\mu$  and a standard deviation of  $\sigma$ ,

$$M \sim N(\mu, \sigma^2) \quad (2)$$

The values of mean  $\mu$  and variance  $\sigma$  can be obtained by fitting seismic data. The relationship between the magnitude and intensity of an earthquake and the attenuation of intensity with distance can be approximately calculated by the following equation [14]:

$$I = 0.514 + 1.500M - 0.00659R - 2.014 \log(R + 10) + \rho, \quad (3)$$

where  $I$  represents the intensity of an earthquake,  $R$  ( $R < 300km$ ) represents the distance between an node and the earthquake center, and  $\rho$  is a random variable of uncertainty,



assumed to be lognormal with a mean of 0 and a standard deviation of 0.274. This paper assumes that the intensity suffered by each branch is the higher one between its two connecting nodes.

In general, nodes are more vulnerable to earthquake damage than branches, since a node failure is the common result of building failure and electrical facility failure, either of which will lead to the failure of a node. In addition, the power plant (i.e.,  $V_p^{SL}, V_p^S$ ) is designed to withstand earthquakes of a higher intensity than other nodes and branches, when other nodes and branches can withstand earthquakes of intensity 7 or less [15]. The average functional loss rates ( $\hat{F}$ ) of nodes and branches under different intensities (i.e., component vulnerability) are shown in TABLE II. and TABLE III. [13].

TABLE II. THE AVERAGE FUNCTIONAL LOSS RATES OF NODES UNDER DIFFERENT INTENSITIES

| Fortification Intensity | Earthquake intensity |      |      |      |      |      |      |
|-------------------------|----------------------|------|------|------|------|------|------|
|                         | 6                    | 7    | 8    | 9    | 10   | 11   | 12   |
| 6 or less               | 0.20                 | 0.40 | 0.75 | 0.90 | 1.00 | 1.00 | 1.00 |
| 7                       | 0.05                 | 0.20 | 0.40 | 0.75 | 0.90 | 1.00 | 1.00 |
| 8                       | 0.00                 | 0.05 | 0.20 | 0.40 | 0.75 | 0.90 | 1.00 |
| 9                       | 0.00                 | 0.00 | 0.05 | 0.20 | 0.40 | 0.75 | 0.90 |

TABLE III. THE AVERAGE FUNCTIONAL LOSS RATES OF BRANCHES UNDER DIFFERENT INTENSITIES

| Fortification Intensity | Earthquake intensity |      |      |      |      |      |      |
|-------------------------|----------------------|------|------|------|------|------|------|
|                         | 6                    | 7    | 8    | 9    | 10   | 11   | 12   |
| 6 or less               | 0.10                 | 0.15 | 0.40 | 0.60 | 0.90 | 1.00 | 1.00 |
| 7                       | 0.00                 | 0.10 | 0.15 | 0.40 | 0.60 | 0.90 | 1.00 |
| 8                       | 0.00                 | 0.00 | 0.10 | 0.15 | 0.40 | 0.60 | 0.90 |
| 9                       | 0.00                 | 0.00 | 0.00 | 0.10 | 0.15 | 0.40 | 0.60 |

The functional loss rates of nodes and branches under different intensities can be calculated by the following equation:

$$F(i) = \hat{F}(i) + \varphi \quad (4)$$

where  $\hat{F}(i)$  represents the average functional loss rate of node  $i$  and  $\varphi$  is a random variable of uncertainty, assumed to be normal with a mean of 0 and a standard deviation of 0.033.

The steps of simulating earthquake disaster are as follows:

All nodes are mapped in a two-dimensional coordinate system, and record the maximum and minimum values of the abscissa (i.e.,  $x_{max}$  and  $x_{min}$ ) and ordinate (i.e.,  $y_{max}$  and  $y_{min}$ ), respectively (Step 1). Randomly generate a coordinate within the system area (i.e.,  $x \in [x_{min}, x_{max}]$  and  $y \in [y_{min}, y_{max}]$ ) as the earthquake location (Step 2). Randomly generate a magnitude by (2) (Step 3). Calculate the distance between all nodes and the earthquake center (Step 4). Calculate the intensity of the location of each node by (3) (Step 5). Obtain the average functional loss rate ( $\hat{F}(i)$ ) of each node and branch from TABLE II. and TABLE III. and calculate the functional loss rate ( $F(i)$ ) of each node and branch by (4) (Step 6). The simulation method of disaster can provide the initial damage data of components for the cascading failure model proposed in Section 2.A.

### C. The Metric for Evaluating the Scale of the System's Cascading Failure

The proposed metric incorporates three factors, including the number of working components, the amount of power flowing, and the topology of the system.

From the perspective of the number of working components, in general, the more working components remaining in the system after a disaster, the smaller the scale of cascading failure. The components in this paper include branches and nodes, but the importance of nodes and branches are usually different. Hence, two metrics, including the ratio of remaining working nodes ( $S_W^n$ ) and the ratio of remaining working branches ( $S_W^b$ ), are proposed to evaluate the scale of system's cascading failures. These two metrics can be calculated as follows:

$$S_W^n = N_W^n / N_{Total}^n \quad (5)$$

$$S_W^b = N_W^b / N_{Total}^b \quad (6)$$

where  $N_W^n$  represents the number of remaining working nodes after a disaster,  $N_W^b$  represents the number of remaining working branches after a disaster, and  $N_{Total}^n$  and  $N_{Total}^b$  respectively represent the number of initial working nodes and branches.

From the perspective of the amount of power flowing, this factor can be measured by the amount of load supplied in the system. The more the remaining amount of load supplied in the system after a disaster, the smaller the scale of system's cascading failure. In addition, the remaining amount of load supplied can also reflect the failure states of working nodes and branches. Hence, the metric of the ratio of remaining load supplied ( $S_W^L$ ) is proposed to evaluate the scale of system's cascading failures, as shown below:

$$S_W^L = D_W^L / D_{Total}^L \quad (7)$$

where  $D_W^L$  represents the amount of remaining load supplied and  $D_{Total}^L$  represents the amount of initial load supplied.

From the perspective of the topology of the system, in general, the fewer isolated islands the system is separated into, and the more working nodes per island, the smaller the scale of cascading failures. The traditional evaluation metric (i.e., the giant connected component) also focuses on this perspective. However, the traditional metric does not take into account whether there is a power flow in the largest island, that is, it ignores whether the nodes on this island are working. Hence, the metric of the ratio of the maximum connected working nodes ( $S_W^C$ ) is proposed to evaluate the scale of system's cascading failures, as shown below:

$$S_W^C = N_W^C / N_{Total}^C \quad (8)$$

where  $N_W^C$  represents the number of working nodes belong to the maximum working connected components after a disaster and  $N_{Total}^C$  represents the number of initial working nodes.

Note that the parameters needed to calculate the above four metrics (as in (5) to (8)) can be obtained from the proposed cascading failure model (Section II.A) and simulation method of disaster (Section II.B). Based on these four metrics from different perspectives, the comprehensive evaluation metric

( $S_{Total}$ ) for the scale of the system's cascading failure is proposed, as shown below:

$$S_{Total} = \alpha * S_W^n + \beta * S_W^b + \chi * S_W^L + \delta * S_W^C, \quad (9)$$

where  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  are constant coefficients representing the relative importance of these four metric, and satisfying the equation:  $\alpha + \beta + \gamma + \delta = 1$ . Note that due to the lack of sufficient data and experience to measure the relative importance of above four metrics, in this paper, the entropy method is adopted to determine the values of these four coefficients objectively. Specifically, based on the above proposed disaster simulation method and cascading failure model, 50 disasters are generated randomly, and the values of the four metrics after each disaster are calculated. The values of these four coefficients can be calculated through the entropy method with the 50 sets of data.

### III. AN EXAMPLE USING IEEE 30-BUS MODEL

#### A. The Cascading Failure Model and the Method for Simulating Natural Disaster

According to fit the world seismic data from January 2017 to January 2019 of China Earthquake Network Center (CENC), the distribution of earthquake magnitude in (2) can be obtained, i.e.,  $\mu = 3.88$  and  $\sigma = 1.23$ . The data of the IEEE 30-bus model used in this paper is from reference [16], and the coordinates of all nodes are estimated by the data provided in reference [17] (taking bus #2 as the reference point). There are 30 nodes and 41 branches in this model, among which #1, 2, 13, 22, 23, and 27 nodes are power generation nodes (i.e.,  $V_p^{SL}$  or  $V_p^S$ ), #5, 6, 9, 11, 25, and 28 nodes are transmission nodes (i.e.,  $V_p^T$ ), and others are load nodes (i.e.,  $V_p^L$ ). The coordinate of each node is shown in TABLE IV.

TABLE IV. THE ESTIMATED COORDINATE OF EACH NODE IN IEEE 30-BUS MODEL

| #  | Coordinate     | #  | Coordinate      | #  | Coordinate      |
|----|----------------|----|-----------------|----|-----------------|
| 1  | (0, 20)        | 11 | (102.27, 18.56) | 21 | (81.86, 35.88)  |
| 2  | (0, 0)         | 12 | (42.89, 26.6)   | 22 | (92.17, 38.14)  |
| 3  | (20.62, 11.96) | 13 | (35.06, 23.09)  | 23 | (53.82, 44.12)  |
| 4  | (42.48, 12.78) | 14 | (28.67, 38.14)  | 24 | (86.39, 47.42)  |
| 5  | (85.77, -8.04) | 15 | (43.1, 40.41)   | 25 | (96.29, 52.16)  |
| 6  | (89.07, 9.28)  | 16 | (51.5, 33.61)   | 26 | (135.67, 55.46) |
| 7  | (88.46, 2.47)  | 17 | (73.61, 27.83)  | 27 | (107.01, 71.54) |
| 8  | (128.66, 5.77) | 18 | (59.39, 39.18)  | 28 | (117.12, 71.54) |
| 9  | (93.2, 16.29)  | 19 | (66.81, 37.11)  | 29 | (102.48, 80.82) |
| 10 | (89.07, 20.82) | 20 | (67.43, 29.07)  | 30 | (83.92, 80.82)  |

In order to observe the propagation process of the system's cascading failure, earthquakes with magnitudes between 6.5 and 8 (i.e.,  $6.5 \leq M \leq 8$ ) and intensities greater than 7 at the earthquake center (i.e.,  $I \geq 7$ , when  $R = 0$ ) are focused. The power generation nodes (i.e., #1, #2, #13, #22, #23, and #27 nodes) are set to withstand an earthquake of intensity 8, and the other nodes and branches are set to 7. The randomly generated earthquake magnitude is 7.3, and the coordinate of the earthquake center is (10.13, 44.48), the results are shown in TABLE V. TABLE VI. TABLE VII.

From results, it is observed that certain components may be disconnected even if their failure grades are less than 4. For example, the #13 node is a power generation node and its failure grade is 3. However, this node is not connected to any load and cannot work properly, so this node is actively disconnected. In addition, the information of system's residual loads, the topology of the system, the amount and failure states of working nodes and branches, and the failure types (i.e., active disconnection and passive failure) of non-working nodes and branches can all be obtained. These pieces of information can be used to better understand the cascading failure mechanisms, and the developing power systems' disaster recovery strategy.

TABLE V. THE TOPOLOGY AND APPARENT POWER OF THE SYSTEM

| Amount of Nodes | # of Nodes in Isolated Island                   | Apparent Power (pu) |
|-----------------|---|---------------------|
| 14              | 1, 2, 5, 6, 7, 8, 9, 10, 11, 17, 19, 20, 21, 22 | 53.5293             |
| 7               | 23, 24, 25, 26, 27, 29, 30                      | 18.0308             |

TABLE VI. FAILURE GRADES AND WORKING CONDITION OF EACH NODE

| #  | FG <sup>a</sup> | W/A/P <sup>b</sup> | #  | FG | W/A/P | #  | FG | W/A/P |
|----|-----------------|--------------------|----|----|-------|----|----|-------|
| 1  | 3               | W                  | 11 | 3  | W     | 21 | 3  | W     |
| 2  | 3               | W                  | 12 | 4  | A     | 22 | 3  | W     |
| 3  | 4               | A                  | 13 | 3  | A     | 23 | 3  | W     |
| 4  | 4               | A                  | 14 | 4  | A     | 24 | 3  | W     |
| 5  | 3               | W                  | 15 | 4  | A     | 25 | 3  | W     |
| 6  | 3               | W                  | 16 | 4  | A     | 26 | 3  | W     |
| 7  | 3               | W                  | 17 | 3  | W     | 27 | 3  | W     |
| 8  | 3               | W                  | 18 | 4  | A     | 28 | 4  | A     |
| 9  | 3               | W                  | 19 | 3  | W     | 29 | 3  | W     |
| 10 | 3               | W                  | 20 | 3  | W     | 30 | 3  | W     |

a. Failure Grade

b. Working/ Active disconnection/ Passive failure

TABLE VII. FAILURE GRADES AND WORKING CONDITION OF EACH BRANCH

| #  | FG <sup>a</sup> | W/A/P <sup>b</sup> | #  | FG | W/A/P | #  | FG | W/A/P |
|----|-----------------|--------------------|----|----|-------|----|----|-------|
| 1  | 3               | W                  | 15 | 3  | A     | 29 | 2  | W     |
| 2  | 3               | A                  | 16 | 3  | A     | 30 | 3  | A     |
| 3  | 3               | A                  | 17 | 3  | A     | 31 | 2  | A     |
| 4  | 3               | A                  | 18 | 3  | A     | 32 | 3  | W     |
| 5  | 3               | W                  | 19 | 3  | A     | 33 | 1  | W     |
| 6  | 3               | W                  | 20 | 3  | A     | 34 | 2  | W     |
| 7  | 3               | A                  | 21 | 3  | A     | 35 | 2  | W     |
| 8  | 2               | W                  | 22 | 3  | A     | 36 | 2  | A     |
| 9  | 2               | W                  | 23 | 3  | A     | 37 | 2  | W     |
| 10 | 2               | W                  | 24 | 2  | W     | 38 | 1  | W     |
| 11 | 2               | W                  | 25 | 2  | W     | 39 | 2  | W     |
| 12 | 2               | W                  | 26 | 2  | W     | 40 | 2  | A     |
| 13 | 2               | W                  | 27 | 1  | W     | 41 | 2  | A     |
| 14 | 2               | W                  | 28 | 2  | W     |    |    |       |

a. Failure Grade

b. Working/ Active disconnection/ Passive failure

### B. The Metric for Evaluating the Scale of the System's Cascading Failure

According to the method in Section II.C, the four coefficients of the proposed evaluation metric in (9) can be calculated, i.e.,  $\alpha = 0.2087$ ,  $\beta = 0.3149$ ,  $\gamma = 0.2082$ , and  $\delta = 0.2682$ .

In order to compare the proposed metric with the traditional metric (i.e., the ratio of nodes belonging to the giant connected component), three groups of data were randomly generated, and the results of the comparison are shown in TABLE VIII.

TABLE VIII. THE TOPOLOGY AND APPARENT POWER OF THE SYSTEM

| # | $S_w^n$ | $S_w^b$ | $S_w^L$ | $S_w^C$ | Proposed metric | Traditional metric |
|---|---------|---------|---------|---------|-----------------|--------------------|
| 1 | 0.2667  | 0.1220  | 0.1859  | 0.1000  | 0.1596          | 0.1000             |
| 2 | 0.9667  | 0.9024  | 0.6514  | 0.9667  | 0.8807          | 0.9667             |
| 3 | 0.1000  | 0.0244  | 0.1159  | 0.0667  | 0.0706          | 0.1333             |

As can be seen from the results of the three groups of comparisons, the traditional metric only focuses on the number of nodes belonging to the giant connected component, and do not focus on the performance of other aspects of the system. For example, in the first group of comparison data, there are 8 working nodes (i.e.,  $30 * S_w^n = 8$ ) in the system, but only 3 nodes (i.e.,  $30 * S_w^C = 3$ ) are included in the giant connected component. Therefore, in this case, traditional metric underestimates the system's ability to resist cascading failures. In addition, in the third group of comparison data, there are 4 nodes (i.e.,  $30 * \text{Traditional metric} = 4$ ) are included in the giant connected component. However, there are only 3 nodes (i.e.,  $30 * S_w^n = 3$ ) are working in this system from the metric  $S_w^n$ . This phenomenon indicates that the nodes in the giant connected component are not working, while the 3 nodes that are working are in other islands. In this case, traditional metric overestimates the system's ability to resist cascading failures. Therefore, the proposed metric can better evaluate the scale of system's cascading failures.

### IV. CONCLUSION

This paper proposes a cascading failure model of power systems based on AC optimal power flow with components' multi-state failures. The proposed model considers each component's six failure states and different performance levels of each state. The model's results, which include system's residual loads, the topology of the system, the amount and failure states of working nodes and branches, and the failure types (i.e., active disconnection and passive failure) of non-working nodes and branches, can provide a guidance for the developing power systems' disaster recovery strategy. Second, the method of simulating natural disaster of central type is proposed to obtain the initial failure states of all components after a disaster. The information of the intensity and distribution of the disaster, the attenuation of the intensity, the location of the disaster occurrence, as well as the vulnerability and physical location of each component are all considered. In addition, the proposed disaster simulation method can be extended to other two types of natural disasters, including unilateral type and global type.

Third, an evaluation metric is proposed to comprehensively evaluate the scale of cascading failures in three perspectives, including the number of working components, the amount of power flowing, and the topology of the system. Finally, an example of IEEE 30-bus model illustrates our proposed cascading failure model and simulating method. In addition, the proposed evaluation metric is compared with the traditional metric (i.e., the giant connected component), and the superiority of the proposed metric is discussed.

In general, the operations of power systems depend on the operation of related infrastructure systems, and vice versa, such as communication systems, etc. In our future research, we will study the multi-state failures and cascading failure mechanism of power systems coupled with other interdependent systems.

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