

Condition-Based Maintenance Optimization with Safety Constraints under Imperfect Inspection

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Abstract—The maintenance decision needs to consider various aspects such as cost, safety, system condition etc. This paper presents a condition-based maintenance optimization model with a risk acceptance criterion under imperfect inspection. The inspection interval can adaptively change with the system condition. In order to make the maintenance decision more reasonable, the optimization model, which minimizes the expected long-term cost rate with safety constraints, also considers the influence of imperfect inspection. The method is illustrated through the case of a feeder pipe of a nuclear power plant. The results show that the safety constraint and measurement error cannot be ignored.

Keywords—Gamma process; risk acceptance criterion; risk-based maintenance; measurement error

I. INTRODUCTION

Many systems in the oil industries, chemical industries and nuclear power plants must be operated under strict safety regulations, as they suffer from gradual deterioration due to environmental factors, operational stresses and aging effects. The random failures caused by this degradation reaching a critical state can result in great financial losses and serious safety/environmental consequences, e.g. the Fukushima radiation contamination in 2011. Maintenance is an important dimension of the safety management of critical systems [1]. The study by Khan and Abbasi [2] showed a strong relationship between maintenance practices and the occurrence of serious accidents. The major challenge lies in how to implement an appropriate preventive maintenance (PM) strategy in order to keep the system operating safely and effectively.

In recent years, there has been an increasing number of risk-based maintenance (RBM) approaches to incorporate the risk aspect with maintenance optimization. Risk is defined as the combination of the probability of a failure and the consequence (related to safety, economy, and environment) of a failure [3]. Through analyzing the probability and consequence of failure, risk-based approaches use the risk as a criterion on which to prioritize and plan inspection and maintenance activities. The main aim of the risk-based approach is to reduce the overall risk whilst maintaining acceptable costs.

Khan and Haddara [4] proposed an integrated RBM framework based on the comprehensive understanding of the previous maintenance strategies from a risk perspective. In order to make the RBM more practical, Hu et al. [5] considered the imperfect maintenance for the RBM application in a petrochemical reforming reaction system. Sepeda [6] proposed an approach based on the proper categorization of risks and the appropriate levels of maintenance attention, to help keep the facility operating safely. However, most of the RBM strategies are time-based maintenance (TBM) strategies that do not consider the system state. This may result in the risk of a system not being reduced effectively.

As Vatn and Aven [7] mentioned, the traditional RBM optimization models only refer to the computed optimization result without properly reflecting on the risks and uncertainties in a practical setting. Condition-based maintenance (CBM) is a maintenance program that recommends maintenances based on the system condition[8,9]. Through a comparison between TBM and CBM, Arunraj and Maiti [10] and Ahmad and Kamaruddin [11] found that CBM is preferable to TBM, as CBM has the better risk reduction capability and is more realistic than TBM.

In this paper, a CBM optimization model that considers safety constraints under imperfect inspection is developed to aid the manager's decision-making. There exist two novel developments: (i) the risk acceptance criteria and system condition are incorporated into the development of the maintenance strategy, and (ii) in order to make the maintenance strategy more practical, the measurement error for the non-periodic inspection is considered. This maintenance model not only reduces the maintenance cost but also satisfies the safety requirement.

This paper is organized as follows. A degradation model that considers measurement error is given in Section II. Section III presents the maintenance policy with the safety constraint and the non-periodic inspection plan. The evaluation of the maintenance policy with measurement error and safety constraint consideration is given in Section IV. A feeder pipe case study and sensitivity analysis are presented in Section V. Finally, conclusions are drawn in Section 7.

II. DEGRADATION MODEL WITH MEASUREMENT ERROR

A. Degradation model

Assume that a system deteriorate monotonically over time. The degradation state of system at time t can be described by a random variable $X(t)$, and the process $\{X(t): t \geq 0\}$ is a gamma process with parameters α and β . The degradation process has the following properties:

- $X(0)=0$,
- $\{X(t): t \geq 0\}$ has stationary and independent increments,
- For $t > 0, s > 0$, the increment $X(t+s)-X(s)$ follows a Gamma distribution with shape parameter αt and scale parameter β , and the increment probability density function can be represented as follows:

$$f_{\alpha, \beta}(x) = \frac{\beta^{\alpha t}}{\Gamma(\alpha t)} x^{\alpha t - 1} e^{-\beta x}, \text{ for } x \geq 0 \quad (1)$$

where $\Gamma(\alpha(t-s)) = \int_{t=0}^{\infty} t^{\alpha(t-s)-1} e^{-t} dt$, $\alpha(t-s) > 0$, $\beta > 0$.

For the accumulated degradation process, let T_L denote the failure time of the system, and $T_L = \inf\{t \mid X(t) \geq L\}$, where L is the predetermined system failure threshold. The failure time distribution of the system can be described by

$$F(t) = P\{T_L \leq t\} = P\{X(t) \geq L\}. \quad (2)$$

B. Measurement error

The system is monitored discretely and the inspection result may contain measurement errors [9, 12]. Because the detection device shows performance degradation with the usage time. Assuming that the measurement error follows a Normal distribution, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. At inspection time t , the measured degradation is $Z(t)$ and $Z(0)=0$, i.e. the initial system state is new without degradation. The inspection result includes the true value $X(t)$ and the measurement error, then we have

$$Z(t) = X(t) + \varepsilon. \quad (3)$$

The pdf of $Z(t)$ can be determined by the convolution

$$f_{Z(t)}(z) = \int_{-\infty}^{\infty} f_X(z - \varepsilon \mid \alpha t, \beta) f_E(\varepsilon) d\varepsilon \quad (4)$$

where $f_X(\bullet)$ is the pdf of $X(t)$ and $f_E(\bullet)$ is the pdf of ε .

III. MAINTENANCE STRATEGY WITH SAFETY CONSTRAINTS

The inspection scheme of the proposed maintenance strategy is an adaptive process, since inspection intervals change with the system state. The inspection time T_{i+1} is optimally determined at the inspection time T_i by an inspection scheduling function, which is used to determine the next

inspection interval based on the current inspection result. The system state can only be determined by inspection (i.e. determine the value of $Z(T_i)$). Without considering the risk factor, we can obtain an alarm threshold L_a ($L_a < L$) to trigger PM and achieve the minimum maintenance cost [13, 14]. The maintenance decision maker not only pays attention to the maintenance cost, but also to the system failure risk. Risk acceptance criterion [15] is a rational criterion for deciding whether the system safety is acceptable, and can be involved in the maintenance optimization model to control the system risk. The risk acceptance criterion, L_R , (the upper limit of acceptable risk) has a great influence on the maintenance plan. In order to obtain a maintenance plan that satisfies the risk acceptance criterion with a lower maintenance cost, the final PM threshold L_a should satisfy $L_a \leq L_R$.

A. Maintenance strategy

When $L_a \leq L_R$, the adopted maintenance strategy is as follows:

- At $T = T_i \mid i \geq 1$, if $Z(T_i) < L_a$, then do nothing and perform the next inspection at T_{i+1} . The inspection cost is C_i .
- If $L_a \leq Z(T_i) < L$, perform a perfect PM and the current cycle ends (we call this the ‘replacement zone’). The system returns to its original state and the cost per PM is C_p .
- If $Z(T_i) \geq L$, perform a perfect corrective maintenance (CM) or replacement and the current cycle ends. The failure replacement incurs cost C_f .
- If $Z(T_{i-1}) < L_a$ and $Z(T_i) \geq L$, there is a period $d(t)$ during which the system stays in the failed state. Note that the system is considered as failed as soon as the system state exceeds the critical level L . C_d is the cost of the system staying in the failed state per unit time.

B. Inspection scheduling

Assuming the system state is $X(T_i)$ at time T_i , the next inspection time is given by Grall et al. [16] as

$$T_{i+1} = T_i + \Delta T = T_i + m(X(T_i)), \quad (5)$$

where $m(x)$ is the inspection scheduling function based on the system state x .

Note that the scheduling function is a useful tool for condition-based inspection planning. Here we will make a comparison between the linear function, convex function and concave function, to choose a suitable scheduling function for keeping the system running more efficiently and effectively. The inspection scheduling functions we are interested in are given as follows.

$$m_1(x) = \max\{1, A - \frac{A-1}{B}x\} \quad (6)$$

$$m_2(x) = \begin{cases} 1 + \frac{(x-B)^2}{B^2}(A-1), & 0 \leq x \leq B \\ 1, & x > B \end{cases} \quad (7)$$

and

$$m_3(x) = \begin{cases} A - \left(\frac{\sqrt{A-1}}{B}x\right)^2, & 0 \leq x \leq B \\ 1, & x > B \end{cases} \quad (8)$$

where $A > 1$, $B > 0$ for all these three inspection scheduling functions. If $A=1$, the inspection policy becomes a periodic inspection policy with $\Delta T=1$. The evolution trends of these three inspection-scheduling functions are illustrated in Fig. 1.

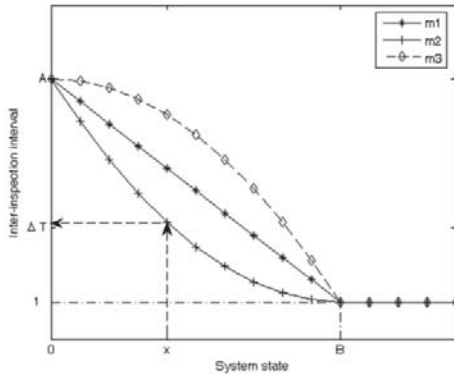


Figure 1. Three inspection scheduling functions vs. system state

Fig.1 presents the corresponding relationship between the system state x and the inter-inspection interval by m_i , $i=1, 2, 3$. The parameter A corresponds to the first inter-inspection interval with $m_i(0)=A$. The parameter B controls the change for frequency of inspections with $m_i(x_t)=1$, if $x_t \geq B$.

IV. MAINTENANCE OPTIMIZATION

For the system evolution after each inspection is independent of the past process. According to the property of a semi-regenerative process, we can use the first inspection interval $[0, T_1]$ to express the expected cost rate (ECR) [16]:

$$C = \lim_{t \rightarrow \infty} E\left[\frac{C(t)}{t}\right] = \frac{E_\pi[C(T_1)]}{E_\pi[T_1]} \\ = \frac{C_i E_\pi[N_i(T_1)]}{E_\pi[T_1]} + \frac{C_p E_\pi[N_p(T_1)]}{E_\pi[T_1]} + \frac{C_f E_\pi[N_c(T_1)]}{E_\pi[T_1]} + \frac{C_d E_\pi[d(T_1)]}{E_\pi[T_1]} \quad (9)$$

where $E_\pi[\bullet]$ denotes the expectation with respect to the stationary law π ; $N_i(t)$, $N_p(t)$ and $N_c(t)$ are the random number of inspections, PM and CM within time t , respectively; and $d(t)$ is the elapsed time of the system in the failed state.

According to Grall et al.[16], the stationary law satisfies

$$\pi(dy) = a\delta_0(dy) + (1-a)b(y)dy, \quad (10)$$

$$\text{with } a = \frac{1}{1 + \int_0^{L_a} B(y)dy}, b(y) = \frac{a}{1-a} B(y), \text{ and}$$

$$B(y) = f_Z(y | \alpha m(0), \beta) + \int_0^y B(z) \tilde{f}_Z(y-z) dz \text{ almost}$$

$$\text{everywhere.} \quad (11)$$

Then Eq.(9) can be computed in the following way:

The expected number of inspections on $[0, T_1]$ is

$$E_\pi[N_i(T_1)] = 1. \quad (12)$$

The expected number of preventive replacements on $[0, T_1]$ is

$$E_\pi[N_p(T_1)] = P_\pi(L_a \leq Z(T_1^-) < L) \\ = \int_{[0, L_a)} (\bar{F}_{\alpha \cdot m(z), \beta}(L_a - z) - \bar{F}_{\alpha \cdot m(z), \beta}(L - z)) \pi(dz) \\ = a(\bar{F}_{\alpha \cdot m(0), \beta}(L_a) - \bar{F}_{\alpha \cdot m(0), \beta}(L)) + \\ a \int_{(0, L_a)} (\bar{F}_{\alpha \cdot m(z), \beta}(L_a - z) - \bar{F}_{\alpha \cdot m(z), \beta}(L - z)) B(z) dz \quad (13)$$

The expected number of corrective replacements on $[0, T_1]$ is

$$E_\pi[N_c(T_1)] = P_\pi(Z(T_1^-) \geq L) = \int_{[0, L_a)} \bar{F}_{\alpha \cdot m(z), \beta}(L - z) \pi(dz) \\ = a\bar{F}_{\alpha \cdot m(0), \beta}(L) + a \int_{(0, L_a)} \bar{F}_{\alpha \cdot m(z), \beta}(L - z) B(z) dz \quad (14)$$

The expected elapsed time after system failure on $[0, T_1]$ is

$$E_\pi[d(T_1)] = \int_{[0, L_a)} E_z\left(\int_0^{T_1} I_{\{Z(t) \geq L\}} dt\right) \pi(dz) \\ = \int_{[0, L_a)} \left(\int_0^{m(z)} \bar{F}_{\alpha \cdot t, \beta}(L - z) dt\right) \pi(dz) \\ = a \int_0^{m(0)} \bar{F}_{\alpha \cdot t, \beta}(L) dt + a \int_{(0, L_a)} \left(\int_0^{m(z)} \bar{F}_{\alpha \cdot t, \beta}(L - z) dt\right) B(z) dz \quad (15)$$

Fig. 2 shows the possible phenomena of early CM and late CM with measurement errors. The measurement error will result in the following events on the first inspection interval $[0, T_1]$ of early CM or late CM.

- Early CM means that the real system state is below the critical threshold L at T_1 , but the inspection result $Z(T_1)$

has crossed L and an undesired CM is triggered. Early CM will increase the cycle cost due to the unnecessary CM.

- Late CM means that the inspection result $Z(T_1)$ indicates that the system state is still under the PM threshold L_a but the real system state has crossed L , and the desired CM is missed due to measurement error. The late CM will introduce additional unavailability time and increase the renewal cycle loss.

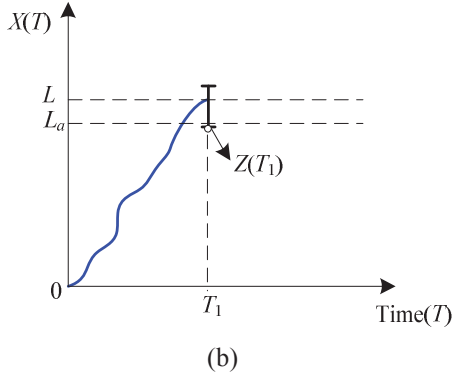
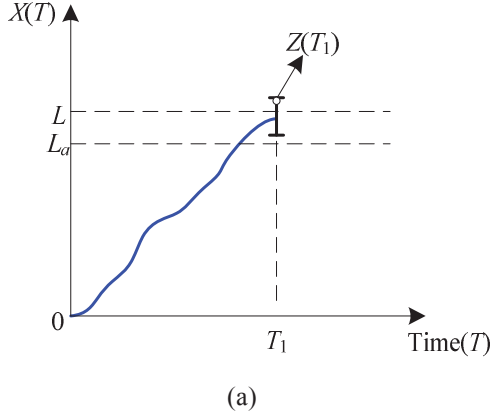


Figure 2. Influences of measurement error to CM: (a) early CM (b) late CM

Considering the influences of early CM and late CM, we need to adjust the result of Eq.(15). The unavailability time $d1(T_1)$ caused by the early CM needs to be removed from Eq.(15), and the neglected unavailability time $d2(T_1)$ caused by the late CM should be added to Eq.(15), that is,

$$\begin{aligned}
 E_\pi[d1(T_1)] &= \int_{[0, L_a)} E_z \left(\int_0^{T_1} I_{\{Z(t) \geq L \cap X(t) < L\}} dt \right) \pi(dz) \\
 &= \int_{[0, L_a)} E_z \left(\int_0^{T_1} I_{\{Z(t) \geq L \cap w > Z(t) - L\}} dt \right) \pi(dz) \\
 &= \int_{[0, L_a)} E_z \left(\int_0^{T_1} \left(\int_{L-z}^{+\infty} \int_{s-L}^{+\infty} f_{\alpha, \beta}(s-w) f_w(w) dw ds \right) dt \right) \pi(dz) \\
 &= \int_{[0, L_a)} \left(\int_0^{m(z)} \left(\int_{L-z}^{+\infty} \int_{s-L}^{+\infty} f_{\alpha, \beta}(s-w) f_w(w) dw ds \right) dt \right) \pi(dz)
 \end{aligned} \quad (16)$$

and

$$\begin{aligned}
 E_\pi[d2(T_1)] &= \int_{[0, L_a)} E_z \left(\int_0^{T_1} I_{\{Z(t) < L_a \cap X(t) \geq L\}} dt \right) \pi(dz) \\
 &= \int_{[0, L_a)} E_z \left(\int_0^{T_1} I_{\{Z(t) < L_a \cap w \leq Z(t) - L\}} dt \right) \pi(dz) \\
 &= \int_{[0, L_a)} E_z \left(\int_0^{T_1} \left(\int_0^{L_a-z} \int_{-\infty}^{s-L} f_{\alpha, \beta}(s-w) f_w(w) dw ds \right) dt \right) \pi(dz) \\
 &= \int_{[0, L_a)} \left(\int_0^{m(z)} \left(\int_0^{L_a-z} \int_{-\infty}^{s-L} f_{\alpha, \beta}(s-w) f_w(w) dw ds \right) dt \right) \pi(dz)
 \end{aligned} \quad (17)$$

respectively.

The adjusted expected unavailability time $E_\pi[d^*(T_1)]$ is

$$\begin{aligned}
 E_\pi[d^*(T_1)] &= E_\pi[d(T_1)] - E_\pi[d1(T_1)] + E_\pi[d2(T_1)] \\
 &= \int_{[0, L_a)} \left(\int_0^{m(z)} \left(\int_{L-z}^{+\infty} \int_{s-L}^{+\infty} f_{\alpha, \beta}(s-w) f_w(w) dw ds \right) dt \right) \pi(dz) \\
 &\quad + \int_{[0, L_a)} \left(\int_0^{m(z)} \left(\int_0^{L_a-z} \int_{-\infty}^{s-L} f_{\alpha, \beta}(s-w) f_w(w) dw ds \right) dt \right) \pi(dz)
 \end{aligned} \quad (18)$$

The expected first inspection interval length is

$$E_\pi[T_1] = \int_{[0, L_a)} m(z) \pi(dz) = am(0) + a \int_{[0, L_a)} m(z) B(z) dz \quad (19)$$

Through numerical integration, we can compute the expected cost rate (ECR) based on Eq. (9)–(19).

For achieving the minimal maintenance costs under the acceptance risk criterion, the optimization problem is formed as

$$\begin{aligned}
 (L_a, A, B) &= \arg \min \lim_{t \rightarrow \infty} E \left[\frac{C(t)}{t} \right] \\
 s.t. \quad &\begin{cases} 0 < L_a \leq L_R \\ A \geq 1 \\ B > 0 \end{cases}
 \end{aligned} \quad (20)$$

A searching algorithm can be used to determine the optimal results of ECR, L_a and the scheduling function parameters (A, B).

V. NUMERICAL EXAMPLE

Feeder pipes are important components of the heat transport system in a nuclear power plant. They are susceptible to flow-accelerated corrosion (FAC) [17]. This corrosion will cause the feeder pipe performance to deteriorate and threaten the safety of the nuclear power plant, such as with the leakage of radioactive materials. In order to keep the nuclear power plant running safely, we need to consider the feeder pipe deterioration in the maintenance optimization. The gamma

process has been proved to be a suitable stochastic process to model the corrosion damage mechanism [17,18]. Therefore, the gamma process is applied to model the FAC process. Here we choose the FAC of feeder pipes case from Lu [19] to illustrate the proposed method. The wall thickness of the feeder pipe was measured from 5.16–14.3 effective full power years (EFPY). The data set is taken from 50 feeder pipes, with 26 pipes measured twice, 13 measured three times, and the remaining 11 measured four or five times. Fig.3 shows the variation of the measured wall thickness of some typical feeder pipes over time.

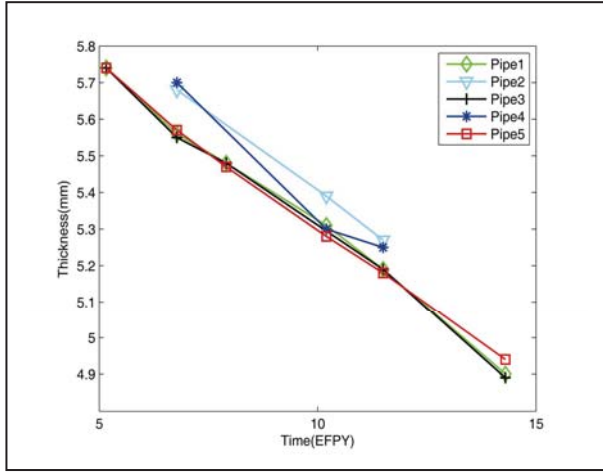


Figure 3. Sample data of some feeder pipes

The inspection is not perfect, and the measurement error follows a normal distribution with mean 0 and $\sigma_e=0.05$. The maximum allowable wall thickness loss of feeder pipes is given as $L=3\text{mm}$.

Considering the measurement error, $\hat{\alpha}=4.89$ and $\hat{\beta}=66.67$ can be obtained by the maximum likelihood method proposed by Lu [19]. Suppose $C_i=25$, $C_p=50$, $C_f=200$, $C_d=400$, and the acceptable failure risk is $P_0=0.005$. The constraint $L_a \leq L_R$ can be achieved by $E_\pi[N_c(T_1)] \leq P_0$.

Considering measurement errors, the expected failure time is $L/\hat{\mu} = L\hat{\beta}/\hat{\alpha} \approx 41$ EFPY. In order not to miss the optimal results, we choose the upper limit of scheduling function parameter A is 41. According to the value of L , we choose the upper limit of scheduling function parameter B as 3. Here, two cases with and without safety constraints are considered under the imperfect inspection.

A. With safety constraints

Utilizing the searching algorithm with safety constraints, we can obtain the optimal numerical results for the three scheduling functions considering the measurement error, as shown in Case I in Table I. P^* represents the system failure probability related to the optimal results.

TABLE I. OPTIMAL RESULTS CONSIDERING MEASUREMENT ERRORS

With safety constraints Case I	Without safety constraints Case II
$(L_a^*, C^*, A^*, B^*, P^*)$	$(L_a^{**}, C^{**}, A^{**}, B^{**}, P^{**})$
$m_1: (1.4, 3.0494, 33, 2, 0.0025)$	$m_1: (1.4, 3.0083, 34, 1, 0.0073)$
$m_2: (1.2, 3.0499, 33, 1, 0.0027)$	$m_2: (1.2, 3.0065, 34, 2, 0.0074)$
$m_3: (1.2, 3.0491, 33, 2, 0.0025)$	$m_3: (1.4, 3.0095, 34, 2, 0.0073)$

From Table I, we notice that the minimum cost rates for m_1 , m_2 and m_3 are similar, but the minimum value $C^*=3.0491$ is achieved by m_3 among the three scheduling functions. In order to minimize the system maintenance cost, $L_a^*=1.2\text{mm}$ and m_3 function with $A^*=33$, $B^*=2$ are chosen as the optimal maintenance decision variables. Meanwhile, Table I shows that C^* has a varying trend that, $C^* < C^* < C^*$ when considering the risk constraint. The optimal A^* of the three scheduling functions are the same in Table I. This means that the first inspection intervals for the three scheduling functions are the same. Because P^* for m_3 is smaller than P_0 , the optimal CBM strategy is a risk-averse strategy.

B. Without safety constraints

In order to prove the necessity of considering the safety constraint, we make a comparison with the optimal results without considering the safety constraint under imperfect inspection (case II). The optimal results of Case II are listed in Table I. Results in Table I show that the minimum cost rate $C^{**}=3.0065$ is achieved by scheduling function m_2 with $A^{**}=34$, $B^{**}=2$, and PM threshold $L_a^{**}=1.2$. We found that $C^{**} < C^{**} < C^{**}$ and $A^{**}=34$ is the same for different scheduling functions. Because $P^{**}=0.0074 > 0.005$, the optimal maintenance strategy without considering safety constraint with measurement errors is a risk-prone strategy.

Influenced by the safety constraints, the optimal results for the three scheduling functions in Table I have $L_a^* \leq L_a^{**}$, $A^* < A^{**}$, which means the optimal policy in Case I will have earlier PM actions and more frequent inspections than the optimal policy in Case II. Also, the system failure probabilities of case I and case II have $P^{**} > P^* > P_0$. The above differences make $C^* < C^{**}$ in Table I for different scheduling functions. The final results prove that the safety constraint has importance influence on maintenance optimization with better safety, but more cost.

VI. CONCLUSIONS

To make the maintenance strategy more applicable, this paper presents a CBM optimization model that considers the risk acceptance criterion and measurement errors. The determined maintenance strategy not only satisfies the risk acceptance criterion, but also achieves the minimum maintenance cost. By analyzing the influences of measurement errors, we have developed an analytical optimization model that considers measurement errors. In the feeder pipe case, we prove that the influence on the optimal results caused by the safety constraint and the measurement error should be

concerned, particularly. We find that the scheduling function m_3 is more suitable than the other two functions for considering the measurement error and safety constraints in the example. Through a comparison between the optimal result with and without safety constraints, the proposed model is shown to be able to provide an effective and realistic maintenance plan for the decision maker. It is worth to point out that the searching algorithm for the optimal result is time-consuming, thus it needs to explore more efficient method in the future research.

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