Degradation Modeling of Digital Multimeter with Multiple-performance Indicators in Multi-stress Dynamic Marine Environment Based on Vine Copula

Zixuan Yu
School of Reliability & Systems Engineering
Beihang University
Beijing, China
yuzixx@buaa.edu.cn

Tingting Huang
School of Reliability & Systems Engineering
Beihang University
Beijing, China
htt@buaa.edu.cn

Xin Wu
School of Reliability & Systems Engineering
Beihang University
Beijing, China
wuxin123@buaa.edu.cn

Kun Zhou
Southwest Technical Engineering Institute
& Environmental Test Center
Chongqing, China
zkdudu@163.com

***Abstract*—A digital multimeter working in the marine environment suffers from complex environmental stresses of time-varying temperature, relative humidity and salinity. It is used to measure five indicators of Resistance (R), Direct Current Voltage (DCV), Alternating Current Voltage (ACV), Direct Current (DC) and Alternating Current (AC), and there is an interactive relationship between the five indicators due to the complex structure among the components of multimeter.**

Considering the measurement errors data of five indicators of digital multimeter as the degradation signal, this paper establishes a degradation model to predict the reliability of each single indicator considering the time-varying environmental stresses based on Brownian motion. The typical D-vine copula is utilized to describe the correlations of multiple performance indicators, the parameters of the optimal D-vine model can be estimated by maximum likelihood estimation. In this paper, lifetime of the multimeter working in marine environment with multiple-performance indicators can be predicted accurately. A case study is presented as an application of this method.

Keywords-reliability; vine copula; degradation modeling; dynamic environments; multi-performance indicators

I. Introduction

The conventional degradation modeling methods are based on the assumption that a product is under static environmental stresses during the process of degradation, and focused on a single indicator of the product. Thus, conventional degradation modeling methods are not suitable for some practical conditions. Recently, degradation modeling methods have been improved to deal with the problems of products with multiple performance indicators operating in dynamic environment.

Considering the impact of dynamic environmental stresses, linear and nonlinear degradation models have been established based on stochastic processes, thus, the effect of environment or load on degradation process can be established directly. For example, Doksum and Hoyland [1] obtained the failure time distribution by applying Brownian motion to the accelerated life test, and converted the non-stationary Brownian motion into a stationary wiener process by time scale transformation to obtain the expression of residual life. Cinlar [2] used Markov process to express environmental effects and described the degradation process as an increasing Lévy process. The above degradation methods are more suitable for the actual situation by considering dynamic environmental factors, but they only focus on a single performance indicator.

In order to analyze the degradation process of products with multiple-performance indicators in practical production, many researches have carried out on the degradation modeling considering the influence of multiple performance indicators on the degradation process of a product. For example, Crk [3] assumed that the system fault is controlled by several independent parts, different performance indicators reflect the characteristics of different aspects of the product, a degradation modeling method was proposed, the reliability of the system with multiple-performance indicators can be predicted by monitoring each performance indicator. However, this method ignored the interaction between multiple performance indicators, it is not appropriate for a product containing multiple interacting performance indicators.

Data fusion method can be applied to model the degradation modeling of products with multiple performance indicators. At present, data level fusion, feature level fusion and decision level fusion are three categories of data fusion

This work was supported by NSFC71601009 and JSZL2015601B010.

method [4]. Volponi, Tom and Robert [5] applied feature level fusion method to maximize useful information by fusing collected information, and comprehensively diagnose the health condition of aviation gas turbines. Sun [6] proposed a decision level fusion method based on Bayesian method to monitor the vehicle health condition, the degradation process of vehicle is characterized by fusing the degradation data of two indicators, vibration signal and oil sample are the two indicators to measure the degradation of vehicle. Liu, Gebraeel and Shi [7] proposed a new method to construct a health indicator which presents the health condition of the system, data-level fusion is used to fuse single indicators into a health indicator, and the reliability of system is predicted according to the health indicator. It is verified that the comprehensive health indicator is more accurate than single performance indicator in predicting the residual life of the system by a case study.

Copula can handle the relationship between random variables and connect them flexibly, so it has been introduced into reliability field to better solve the problems of the system with complex dependencies [8]. There are two main approaches in the current study based on copula function. The first approach directly extends pair copula function to multidimensional copula function, the multidimensional copula function can connect multiple marginal distribution functions of reliability into a joint distribution function of reliability. Ellipsoidal copula and Archimedes copula have the property of directly dealing with multidimensional marginal distribution functions [9]. Sun, Liu, Li and Liao [10] proposed an accelerated degradation model with multiple dependent indicators based on Wiener process, a multidimensional copula function was applied to connect the reliability marginal distribution of each degradation indicator. Navarro and Durante[11] applied the high-dimensional copula to predict the lifetime of the associated system in which components are correlated. However, the high-dimensional copula function has only one parameter to describe the correlation between multiple indicators in the system, it cannot accurately describe the relationship between any two degradation indicators of the system with complex dependences.

To overcome the limitation that multidimensional copulas cannot flexibly describe the correlations between multiple variables, recently, vine copula with pair-copula constructions were introduced by Aas, Czado, Frigessi and Bakken [12] as the second approach. A lot of new studies have introduced vine copula into the field of reliability prediction. Xu et al. [13][14] utilized the vine copula to fuse multiple dependent performance characteristics of the product, so as to predict the reliability accurately. The degradation process of each parameter indicator was established based on Brownian motion with drift, and the reliability marginal distribution of each performance characteristic were connected by a vine copula.

In order to accurately predict the reliability of the digital multimeter with multiple-performance indicators, this paper make some improvements based on the method proposed in [15]. In this paper, the degradation model of each performance indicator is established based on Brownian motion with drift considering three dynamic stresses of marine environment. The vine copula is used to describe the correlation among multiple performance indicators, and the optimal vine copula is constructed by maximum likelihood estimation method. Finally, the joint reliability can be obtained based on the optimal vine copula structure by connecting each performance indicator.

This paper is organized as follows, the degradation model of each performance indicator of the digital multimeter considering dynamic environments is proposed in section II. Section III introduces the fusion method of multi-performance indicators based on vine copula. Section IV illustrates the proposed method by a case study. Section V draws some conclusions.

II. Degradation Model of each performance indicator

A. Establishment of Degradation Model

The digital multimeter working in marine environments suffers from three dynamic stresses, they are temperature, relative humidity and salinity respectively. Measurement errors of R, DCV, ACV, DC and AC can be considered as the degradation signal of the five indicators.

The degradation process of each indicator considering three marine dynamic stresses is established based on Wiener process, the expression is shown as (1),

$$X(t) = X(0) + \int_0^t r\left[w_1(v), w_2(v), w_3(v)\right] \mathrm{d}v + \sigma B(t) \quad (1)$$

where, $X(0)$ represents the initial value of degradation signal, $w_1(t)$, $w_2(t)$ and $w_3(t)$ respectively represents temperature, relative humidity and salinity, $r[w_1(t),w_2(t),w_3(t)]$ represents the degradation rate, $B(t)$ represents a standard Wiener process, σ is the diffusion parameter.

B. Degradation Rate Model

In reference to [15], the expression of degradation rate is defined to be a linear form of independent terms of three environmental stresses, the expression is shown as (2),

$$r\left[w_1(t), w_2(t), w_3(t)\right] = b_0 + b_1 \cdot \frac{1}{w_1(t)} + b_2 \cdot w_2(t) + b_3 \cdot w_3(t) \quad (2)$$

where, b_0, b_1, b_2, and b_3 are parameters to be estimated.

C. Parameter Estimation

By approximating the integral form in degradation model to the sum form, the expression is shown as (3),

$$X(t) \approx X(0) + \sum_{i=1}^{m} r\left[w_1(t_i), w_2(t_i), w_3(t_i)\right] \Delta t_i + \sigma B(t) \quad (3)$$

where, m is the cumulative number of observations of the degradation signal, $\Delta t_i = t_i - t_{i-1}$ is the time interval between the i-1*th* and i*th* degradation signals, $w_k(t_i)$ is the environmental stress in the time interval $[t_{i-1}, t_i]$.

By eliminating the cumulative effect of degradation rate from the degradation process, it can be modified to $H(t)$,

$$\begin{aligned} H(t) &= X(t) - X(0) - \sum_{i=1}^{m} r\left[w_1(t_i), w_2(t_i), w_3(t_i)\right]\Delta t_i \\ &= \sigma B(t) \end{aligned} \tag{4}$$

Based on the independent increment property of the Wiener process, the likelihood function can be denoted as (5),

$$L(\sigma) = \prod_{i=1}^{m} \frac{1}{\sigma\sqrt{2\pi\Delta t_i}} \cdot \exp\left[-\frac{\Delta H(t_i)^2}{2\sigma^2\Delta t_i}\right] \tag{5}$$

The logarithmic likelihood function is,

$$l(\sigma) = -\frac{1}{2}\sum_{i=1}^{m}\ln(2\pi\Delta t_i) - \frac{1}{2}m\ln\sigma^2 - \sum_{i=1}^{m}\left[\frac{\Delta H(t_i)^2}{2\sigma^2\Delta t_i}\right] \tag{6}$$

To maximize the logarithmic likelihood function (6), the estimation result of diffusion parameter σ can be obtained,

$$\hat{\sigma}^2 = \frac{1}{m}\sum_{i=1}^{m}\left[\frac{\Delta \mathrm{H}(t_i)^2}{\Delta \mathrm{t}_i}\right] \tag{7}$$

D. Lifetime Prediction

After establishing the degradation model of each indicator and estimating the parameters of the model, the reliability of each performance indicator can be predicted according to the degradation threshold and the future environment. The reliability of each indicator can be obtained by (8) and (9),

$$R(t) = 1 - \int_0^t f(v)\,\mathrm{d}v \tag{8}$$

Assuming the threshold is expressed as D, $f(t)$ is the pdf of the first passage time, according to [16] and [17], $f(t)$ is approximately expressed as (9),

$$\begin{aligned} f(t) = \frac{1}{\sqrt{2\pi t}} &\left[\frac{D - X(0) - \int_0^t r(w(v))\,\mathrm{d}v + r(w(t))\cdot t}{t\sigma}\right] \\ &\cdot \exp\left(-\frac{\left[D - X(0) - \int_0^t r(w(v))\,\mathrm{d}v\right]^2}{2t\sigma^2}\right) \end{aligned} \tag{9}$$

III. FUSION OF MULTPLE INDICATORS BY COPULA

A. Construct Vine Copula Structure

Considering the degradation process is affected by multiple-performance indicators simultaneously, connect multiple indicators by vine copula. Based on the correlation among the five performance indicators, the typical D-vine copula is selected to fuse the degradation processes of multiple indicators of the digital multimeter. For a n-dimensional D-vine structure, it contains n-1 layers tree, and variable nodes in each layer are arranged in a straight line, each edge between adjacent nodes are connected by a suitable pair copula [18]. A D-vine with five variables is shown as Fig. 1.

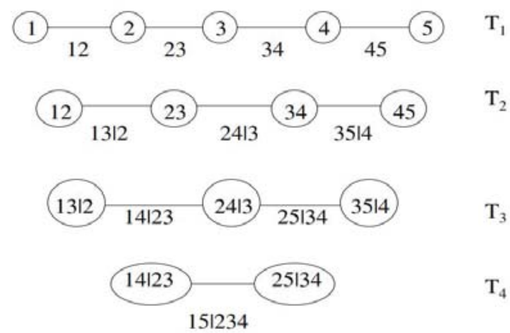


Figure 1. A D-vine model with five variables

The nodes in the lowest layer tree T_1 of D-vine is set as DCV, DC, ACV, AC and R of digital multimeter respectively. Firstly, the optimal form of each pair copula in D-vine need to be determined. In this paper, three classical Archimedes pair copulas are the optional forms, the expressions of the cumulative density function of them are shown in TABLE I.

TABLE I. THE CUMULATIVE DENSITY FUNCTIONS OF CLASSICAL PAIR COPULAS

Copula	**Cumulative density functions**
Gumbel	$C_\alpha^{Gumbel}(u,v) = \exp\left(-\left[(-\ln u)^\alpha + (-\ln v)^\alpha\right]^{\alpha-1}\right)$ $\alpha \ge 1$
Frank	$C_\alpha^{Frank}(u,v) = -\frac{1}{\alpha}\ln\left(1 + \frac{(\exp(-\alpha u)-1)(\exp(-\alpha v)-1)}{\exp(-\alpha)-1}\right)$ $\alpha \in (-\infty, +\infty)\setminus\{0\}$
Clayton	$C_\alpha^{Clayton}(u.v) = \max\left[\left(u^{-\alpha} + v^{-\alpha} - 1\right)^{\frac{-1}{\alpha}}, 0\right]$ $\alpha > 0$

Because the higher trees in D-vine model contains the conditional distributions of nodes in lower trees, so the optimal forms of pair copulas in D-vine between adjacent nodes need to be determined layer by layer, from lower trees to higher trees.

For any two random variables X_1 and X_2, $F_1(X_1)$ and $F_2(X_2)$ are the cdf of variables, the corresponding pdf are $f_1(X_1)$ and $f_2(X_2)$. Suppose a sample set $\{X_{1i}, X_{2i}\}$, i=1,2,3...m, m is the sample size. The log-likelihood function of each pair copula can be established, the expression is shown as (10),

$$\ln L = \sum_{i=1}^{m} \ln\left[c\left(F_1(X_{1i}), F_2(X_{2i}) | \alpha\right) \cdot f_1(X_{1i}) \cdot f_2(X_{2i})\right] \quad (10)$$

Where, $c(.)$ is the pdf of pair copula, α is the parameter of copula.

The pair copula with the minimum result of Aikaike information criterion (AIC) is selected to be the optimal form of the three alternative pair copulas. AIC [19] formula is written as (11),

$$AIC = -2\ln L + 2k \quad (11)$$

where $\ln L$ is the log-likelihood function, k is the number of parameters.

The higher trees of the D-vine except T_1 involves the conditional distributions of the nodes in lower trees, the conditional distributions in the higher trees can be calculated according to Joe's result [20],

$$F_{x|v} = \frac{\partial C_{x,v_j|v_{-j}}\left(F(x|v_{-j}), F(v_j|v_{-j})\right)}{\partial F(v_j|v_{-j})} \quad (12)$$

where $C_{xv_j|v_{-j}}(.)$ presents the cdf of the pair copula, v is a vector, $v = (v_1, v_2, \ldots, v_k)$, v_j is a random vector selected from v and v_{-j} is the vector without it .

For the classical Archimedes pair copulas in TABLE I, the conditional distributions of them are expressed in TABLE II.

TABLE II. THE FIRST DERIVATIVES FUNCTIONS OF CLASSICAL PAIR COPULAS

Copula	$F_{u\|v} = \frac{dC(u,v)}{dv}$
Gumbel	$C(u,v\|\alpha)\frac{1}{v}(-\log v)^{\alpha-1}\left((-\log u)^{\alpha}+(-\log v)^{\alpha}\right)^{1/\alpha-1}$
Frank	$\frac{\exp(-\alpha v)\left[\exp(-\alpha u)-1\right]}{(\exp(-\alpha)-1)+(\exp(-\alpha u)-1)(\exp(-\alpha v)-1)}$
Clayton	$v^{-\alpha-1}\left(u^{-\alpha}+u^{-v}-1\right)^{\frac{-\alpha-1}{\alpha}}$

When determining the optimal forms of pair copulas, the results of parameter estimation of pair copulas are local optimal.

B. *Parameter estimation of pair copulas*

After determining the optimal form of each pair copula in D-vine, the global optimal parameter estimations of each pair copula need to be obtained in one step by maximum likelihood estimation.

The joint density function $f(x_1, x_2, x_3, x_4, x_5)$ of the digital multimeter based on D-vine can be expressed as follows,

$$\begin{aligned} f(x_1,x_2,x_3,x_4,x_5) = {} & c_{12}\cdot c_{23}\cdot c_{34}\cdot c_{45}\cdot c_{13|2}\cdot c_{24|3}\cdot c_{35|4} \\ & \cdot c_{14|23}\cdot c_{25|34}\cdot c_{15|234}\cdot \prod_{i=1}^{5} f_i(x_i) \end{aligned} \quad (13)$$

where $c_{i,i+j|i+1,\ldots,i+j+1}$ is the simplified form of the pdf of pair copula.

The log-likelihood function can be expressed as (14),

$$\begin{aligned} \ln L = {} & \ln\left(c_{12}\cdot c_{23}\cdot c_{34}\cdot c_{45}\cdot c_{13|2}\cdot c_{24|3}\cdot c_{35|4}\cdot c_{14|23}\cdot c_{25|34}\cdot c_{15|234}\right) \\ & + \sum_{i=1}^{5}\ln\left(f_i(x_i)\right) \end{aligned} \quad (14)$$

By maximizing the log-likelihood function (14), the parameters of pair copula in D-vine can be obtained simultaneously, the results of parameter estimation are global optimal. Based on the optimal form of pair copula and the results of global optimal parameter estimation, the optimal D-vine model is determined.

C. *Reliability Estimation*

Based on the determined optimal D-vine model, the failure probability density function of the digital multimeter with five indicators is denoted as $f(x_1, x_2, x_3, x_4, x_5)$. It can be decomposed into the multiplication form of pair copulas as (13), and its cumulative failure probability function F_{12345} can be obtained by integral calculation as follows,

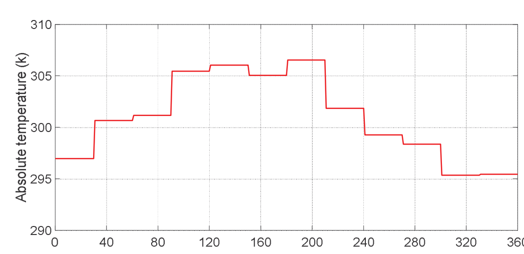
$$F_{12345} = \int \ldots \int f(x_1, x_2, x_3, x_4, x_5)\, dx_1, \ldots, dx_5 \quad (15)$$

Based on the reliability formula R_{12345}=1-F_{12345}, the joint reliability R_{12345} of the digital multimeter can be calculated, then the lifetime can be predicted according to it.

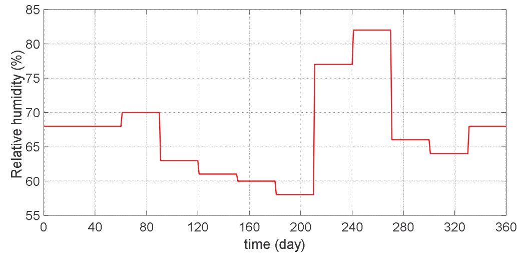
IV. CASE STUDY

A. *Degradation Test*

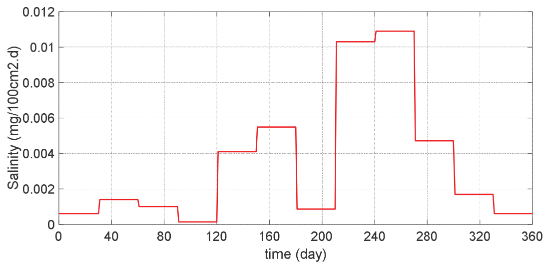
The environment profile of temperature, relative humidity and salinity in 12 months are setted as Fig. 2. The measurement error results of R, DCV, ACV, DC and AC in the first 9 months are recorded as degradation signals of the digital multimeter.



(a) Temperature



(b) Relative humidity



(c) Salinity

Figure 2. Environmental profile

B. *Parameter Estimation of Degradation Rate Model*

The parameters of the degradation rate model, b_0, b_1, b_2 and b_3 are estimated by linear regression analysis method based on the environment profile and the measurement errors of five indicators in the first 9 months. The diffusion parameter σ is obtained by maximum likelihood method, it is calculated as (7). The results are shown in TABLE III.

TABLE III. PARAMETERS ESTIMATIONS OF DEGRADATION RATE MODEL

	b_0	b_1	b_2	b_3	σ
DCV	7.28 e-03	-24.645	1.48 e-04	-0.232	5.37 e-03
DC	-0.471	154.116	5.42 e-04	0.158	7.56 e-03
ACV	-0.139	43.491	9.14 e-06	-0.086	0.021
AC	-0.139	41.982	1.05 e-04	-0.057	2.13 e-04
R	-0.805	295.662	2.43 e-03	-1.497	0.0171

C. *Reliability Prediction of each indicator and Multi-performance Indicators Fusion based on D-vine*

1) *Reliability Prediction of each indicator*

The thresholds of R, DCV, ACV, DC and AC are shown in TABLE IV.

TABLE IV. THRESHOLD

R	DCV	ACV	DC	AV
1.125	0.36	1.6875	1.275	1.9875

Suppose the future environmental profile is the same as the profile in the degradation test. After determining the estimation results of parameters and the thresholds of each indicator, the reliability of each indicator in the future 48 months can be predicted according to (8) and (9).

2) *Construct the optimal D-vine model*

The nodes in T_1 is set as DCV, DC, ACV, AC and R of digital multimeter respectively. The optimal form of each pair copula in D-vine need to be determined layer by layer. The results of the local optimal parameter estimation of the three alternative pair copulas are shown in TABLE V.The optimal form of each pair copula in D-vine can be determined according to the value of AIC.

TABLE V. PARAMETERS ESTIMATIONS OF PAIR COPULAS

copula	Gumbel copula		Frank copula		Clayton copula	
	α	*AIC*	α	*AIC*	α	*AIC*
c_{12}	1.42	-156.19	23.13	-201.41	0.72	-594.56
c_{23}	1.87	-169.11	8.02	-95.43	0.17	-146.27
c_{34}	3.04	-165.70	16.80	-131.45	1.80	-2.02
c_{45}	1.57	-89.19	8.38	-96.68	0.19	-127.23
$c_{13\|2}$	1.00	2.00	0.82	0.11	0.01	1.95
$c_{24\|3}$	1.08	-0.08	1.18	-2.98	0.02	1.02
$c_{35\|4}$	1.45	-94.66	4.36	-46.42	0.10	-23.21
$c_{14\|23}$	1.00	2.00	-1.65	-4.65	0.00	2.00
$c_{25\|34}$	3.36	-328.37	20.07	-175.86	0.47	-336.87
$c_{15\|234}$	1.03	-7.91	3.09	-26.12	0.25	-5.84

After determining the optimal form of each pair copula, the results of the global optimal parameter estimation of each pair copula are obtained in one step based on the maximum likelihood estimation method. By maximizing log-likelihood function of D-vine, the optimal form of each pair copula and the results of parameter estimation are shown in TABLE VI.

TABLE VI. RESULTS OF PARAMETER ESTIMATION

	Optimal pair copula	Local parameter estimation	Global parameter estimation
c_{12}	Clayton	0.72	0.64
c_{23}	Gumbel	1.87	1.77
c_{34}	Gumbel	3.04	2.89
c_{45}	Clayton	0.19	0.079
$c_{13\|2}$	Frank	0.82	1.35
$c_{24\|3}$	Frank	1.18	1.43
$c_{35\|4}$	Gumbel	1.45	1.52
$c_{14\|23}$	Frank	-1.65	-1.29
$c_{25\|34}$	Clayton	0.47	0.46
$c_{15\|234}$	Frank	3.09	7.41
ln*L*		-779.94	-794.09

3) *Reliability Prediction of each indicator*

The optimal D-vine model of digital multimeter is determined by the above two steps. Then utilize it to fuse the indicators of digital multimeter, the reliability prediction of the digital multimeter in the future 48 months can be obtained. Plot the joint reliability R_{12345} obtained based on optimal D-vine as shown in Fig. 3.

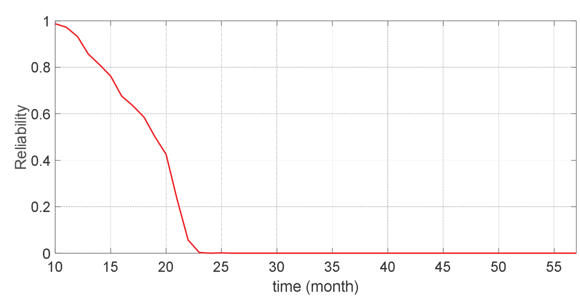


Figure 3. Joint reliability of digital multimeter

V. CONCLUSION

The digital multimeter working in marine environments suffers three time-varying environmental stresses, and has its complex structure with five performance indicators which are R, DCV, ACV, DC and AC, there are complex relationships among the five indicators. On the one hand, the degradation model in this paper considers the effect of dynamic environments, on the other hand it can connect multiple-performance indicators, so the reliability of digital multimeter can be accurately predicted.

The degradation model of each indicator considering three marine dynamic stresses is established based on Wiener process, according to the future environment profile and the threshold, the reliability marginal distribution function of each indicator can be obtained. The joint reliability of digital multimeter is obtained by fusing the marginal distribution function of each indicator, based on the optimal D-vine. In the optimal D-vine, the alternative pair copula with the minimum AIC value is selected as the optimal form of each pair copula and all parameters are estimated in one step by the maximum likelihood estimation method. A case study is presented as an application of this method.

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