

Recurrence Plot Quantitative Analysis-Based Fault Recognition Method of Rolling Bearing*

Wanlu Jiang

Hebei Provincial Key Laboratory of Heavy Machinery
Fluid Power Transmission and Control
Yanshan University
Qinhuangdao, China
wljiang@ysu.edu.cn

Zhenbao Li*, Anqi Jiang, Yafei Lei, Haonan Wang

Key Laboratory of Advanced Forging & Stamping
Technology and Science (Yanshan University)
Ministry of Education of China
Qinhuangdao, China

*Corresponding author: 1053097147@qq.com

Abstract—Aiming at the problem that rolling bearings are widely used but have high failure rate. A fault diagnosis method combined with local projective noise reduction method and recurrence plots quantification analysis for rolling bearings is presented. The fault vibration signals of rolling bearings are taken as the analysis objects. Firstly, the vibration signal is denoised by local projective noise reduction method. Then, the recurrence plots of denoised vibration signals are drawn. The system dynamic behavior reflected in the recurrence plots is extracted by recurrence quantification analysis. The determinism (DET) and entropy (ENTR) are selected to form the characteristic vectors. Finally, the characteristic vectors of the training samples are clustered by kernel fuzzy C-means (KFCM) clustering method. The minimum Euclidean distance principle is used to identify the test samples. Comparing the recurrence plot quantification analysis method with the quantitative feature method of phase space complex network. The results show that the recurrence plots quantification analysis-based fault diagnosis method of rolling bearings has a higher diagnosis rate.

Keywords- complex network; recurrence quantification analysis; fault diagnosis; rolling bearing

I. INTRODUCTION

Fault diagnosis technology is to process the information collected by the sensors, judge the type of fault occurrence, locate the fault and predict the development tendency of the fault. In the production process, the fault diagnosis technology can be used to detect the damaged and failed components in time, so as to avoid the occurrence of major accidents [1]. The rotating machinery is usually under the operating conditions of long time and high load, so the rotating machinery has a higher rate of failure [2]. The rolling bearings sustain the load in the rotating machinery. Once it malfunctions, it can cause equipment to be shut down and even cause casualties. According to statistics, about 30% of rotating machinery failures are caused by rolling bearing failures [3]. Therefore, fault diagnosis of rolling bearings is of great significance.

Recurrence complex network analysis method is an effective method to analyze nonlinear time series [4,5]. Eckmann et al. proposed a method to represent the recurrence behavior on two-dimensional plotting, so that the recurrence behavior of attractor can be expressed through two-dimensional

lattice. Such two-dimensional graphs are called recurrence plots. The recurrence plots is established based on the reconstruction of phase space. It can directly reflect the space manifold of the dynamic system and reveal the dynamic characteristics of the system. At the same time, it is also suitable for the characteristic analysis of non-stationary vibration signals. However, recurrence plots are qualitative analysis, and its judgment results are highly dependent on the subjective experience of researchers. Therefore, Zbilut J P and Webber C L proposed the method of recurrence quantification analysis (RQA) on the basis of recurrence plots [6], which provides the basis for quantitative analysis of recurrence plots.

In this paper, the vibration signal of rolling bearing is taken as the research object. Firstly, the structure and the statistical characteristics of complex network are studied. The local projective noise reduction method based on phase space reconstruction is used to signal denoising. Secondly, the denoised signals are plotted as recurrence plots. The RQA method is used to describe the recurrence plots quantitatively. Finally, the kernel fuzzy C-means(KFCM) clustering method is used to diagnosis the fault. The result of fault recognition proves that the fault diagnosis method based on RQA can effectively solve the problem of fault diagnosis of rolling bearings and has high engineering application value.

II. CONSTRUCTION OF COMPLEX NETWORK AND LOCAL PROJECTIVE NOISE REDUCTION METHOD

Complex networks can be used to analyze the information hidden in discrete points. Digging out the information hidden in nonlinear time series is the key to describe the dynamic behavior of the system. The first step in building phase space complex networks is phase space reconstruction [7]. Phase space reconstruction can reconstruct attractor according to one-dimensional data to study the dynamic behavior of the system. In the high-dimensional phase space, the phase space can be decomposed into orthogonal subspaces by local projective noise reduction method. Then the time series are separated into different components [8].

A. Statistical Characteristics of Complex Networks

In recent years, many indexes have been proposed to describe the statistical characteristics of complex network

structures. Among them, Average Path Length, Clustering Coefficient and Average Degree are three important basic indexes [9,10].

B. Phase Space Complex Network and Their Reconstruction Parameters

Packard et al. proposed the construction method of phase space complex network in 1980. The data points of one-dimensional time series are reconstructed to high-dimensional space through delay time τ to describe attractors of the dynamic system. The basic method of phase space reconstruction is as follows:

Set the one-dimensional time series as $\{x_1, x_2, \dots, x_n\}$. Select the appropriate delay time τ and embedding dimension m to reconstruct the phase space and obtain N m -dimension vectors \mathbf{x}_i :

$$\mathbf{x}_i = [x_i \quad x_{i+\tau} \quad x_{i+2\tau} \quad \dots \quad x_{i+(m-1)\tau}] \quad (1)$$

where, $i = 1, 2, \dots, N$, $N = n - (m-1)\tau$.

The quality of the reconstructed phase space depends on the selection of delay time τ and embedding dimension m . The quality of the reconstruction parameters determines whether the phase space can truly restore the dynamics law of the system or not.

The mutual information function method [11] is an effective method to estimate the delay time τ of the reconstructed phase space. The mutual information $I_N(\tau)$ of \mathbf{x}_i is the function of variable τ :

$$I_N(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) = \sum_{i=1}^N [H(\mathbf{x}_i) - H(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)] \quad (2)$$

where, $H(\mathbf{x}_i)$ is the information entropy of \mathbf{x}_i . $H(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)$ is the joint entropy of $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N$.

Shaw proposed that the time delay can be used as the time delay of phase space reconstruction when mutual information $I_N(\tau)$ reaches the minimum value for the first time.

Cao method [12] is an effective method to determine the optimal embedding dimension m . The Cao method is an improved method for calculating the embedding dimension by false nearest neighbors. The algorithm is as follows. Define:

$$E(m) = \frac{1}{n - m\tau} \sum_{i=1}^{n-m\tau} \frac{\|\mathbf{x}_{m+1,i} - \mathbf{x}_{m+1,i}^{NP}\|_2}{\|\mathbf{x}_{m,i} - \mathbf{x}_{m,i}^{NP}\|_2} \quad (3)$$

$$E_1(m) = \frac{E(m+1)}{E(m)} \quad (4)$$

where, $\mathbf{x}_{m,i}^{NP}$ is the nearest neighbor point of the phase point $\mathbf{x}_{m,i}$ in the m -dimensional phase space. $\mathbf{x}_{m+1,i}^{NP}$ is the nearest

neighbor point of the phase point $\mathbf{x}_{m+1,i}$ in the $m+1$ -dimensional phase space.

For the determined time series, $E_1(m)$ will tend to a constant value as the embedding dimension m increasing. For the random time series, $E_1(m)$ will increasing as the embedding dimension m increases. In practical application, a judgment criterion is usually added to determine whether the value of $E_1(m)$ has been stable or not.

$$E^*(m) = \frac{\sum_{i=1}^{n-m\tau} [\|\mathbf{x}_{m+1,i} - \mathbf{x}_{m+1,i}^{NP}\|_2^2 - \|\mathbf{x}_{m,i} - \mathbf{x}_{m,i}^{NP}\|_2^2]^2}{n - m\tau} \quad (5)$$

$$E_2(m) = \frac{E^*(m+1)}{E^*(m)} \quad (6)$$

If the time series is random, $E_2(m)$ will always be 1. This indicates that the time series has no correlation. If time series have deterministic components, the correlation of time series is related to m . There are always some m values that make $E_2(m)$ not be equal to 1.

C. Local Projective Noise Reduction Method

Based on the phase space reconstruction, local projective noise reduction method selects the appropriate neighborhood points in the track space of attractor. Then, in each local neighborhood, orthogonal projection of chaotic data with noise is performed on the linear hyperplane. So that the off track manifolds can approach the real dynamic trajectory gradually.

According to formula (1), set \mathbf{x}_k as the time series that reflects the dynamic behavior of the system under test. If it is polluted by noise δ_k , the actual observed time series \mathbf{s}_k can be expressed as:

$$\mathbf{s}_k = \mathbf{x}_k + \delta_k \quad (7)$$

In the case of time series with noise, the linear expansion of the system in the neighborhood of point \mathbf{s}_k is as follows:

$$\mathbf{A}^k \cdot \mathbf{R} \cdot (\mathbf{s}_k - \bar{\mathbf{s}}_k) = \boldsymbol{\eta}_k \quad (8)$$

where, $\boldsymbol{\eta}_k$ is the sum of noise. \mathbf{R} is the diagonal weight matrix of $m \times m$. \mathbf{A}^k is the direction matrix of $m \times m$. $\bar{\mathbf{s}}_k$ is the center of mass of the phase point of \mathbf{s}_k in the neighborhood μ_k .

For a deterministic system, attractors usually exist only in phase space with lower dimensions when one dimensional time series are reconstructed into m dimensional phase space. Assuming that the dimension of this low dimensional subspace is m_0 . According to formula (8), the components between m -dimensional space and m_0 -dimensional subspace is caused by noise. The fundamental principle of the local projective noise reduction method is to find out the zero subspace. Then the

noise components are removed from the time series to reduce the noise.

Assuming that the dimension of the zero subspace is $Q = m - m_0$. Then Q orthogonal vectors \mathbf{a}_{m_0+q} ($q = 1, 2, \dots, Q$) can be found and the local projections of $\mathbf{R}(s_k - \bar{s}_k)$ on these vectors are minimums. Assuming that $\mathbf{z}_k = \mathbf{R}(s_k - \bar{s}_k)$. The projection of \mathbf{z}_k on the zero subspace is $\sum_{q=1}^Q \mathbf{a}_{m_0+q} \cdot (\mathbf{a}_{m_0+q} \cdot \mathbf{z}_k)$.

Where \mathbf{a}_{m_0+q} is the normalized vector. The projection is limited to the neighborhood μ_k of phase point s_k . The appropriate \mathbf{a}_{m_0+q} is selected to minimize the projection of

$$\sum_{k' \in \mu_k} \left[\sum_{q=1}^Q \mathbf{a}_{m_0+q} \cdot (\mathbf{a}_{m_0+q} \cdot \mathbf{z}_{k'}) \right]^2 \text{ in the neighborhood } \mu_k. \text{ If}$$

introducing Lagrange multipliers λ_q and $\|\mathbf{a}_{m_0+q}\|^2 = 1$, then the minimized Lagrange operator is:

$$L = \sum_{k' \in \mu_k} \left[\sum_{q=1}^Q \mathbf{a}_{m_0+q} (\mathbf{a}_{m_0+q} \cdot \mathbf{z}_{k'}) \right]^2 - \sum_{q=1}^Q \lambda_q (\mathbf{a}_{m_0+q} \cdot \mathbf{a}_{m_0+q} - 1) \quad (9)$$

for each independent q , there is:

$$\mathbf{C} \mathbf{a}_{m_0+q} - \lambda_q \mathbf{a}_{m_0+q} = 0 \quad (10)$$

where, $q = 1, 2, \dots, Q$, \mathbf{C} is the $m \times m$ covariance matrix of vector $\mathbf{z}_{k'}$ in the neighborhood μ_k . Then:

$$c_{ij} = \sum_{k' \in \mu_k} (\mathbf{z}_{k'})_i \cdot (\mathbf{z}_{k'})_j \quad (11)$$

The eigenvector \mathbf{a} and eigenvalue λ of matrix \mathbf{C} can be obtained from formula (10). The eigenvectors corresponding to the Q minimum eigenvalues are caused by noise. These components should be subtracted from the time series. Therefore, the final form of the local projective noise reduction method is:

$$\hat{s}_k = s_k - \mathbf{R}^{-1} \sum_{q=1}^Q \mathbf{a}_{m_0+q} \cdot [\mathbf{a}_{m_0+q} \cdot \mathbf{R} \cdot (s_k - \bar{s}_k)] \quad (12)$$

III. RECURRENCE QUANTITATIVE ANALYSIS AND KERNEL FUZZY C-MEANS CLUSTERING PRINCIPLE

A. Recurrence Plots Algorithm

In 1987, Eckmann et al. proposed a method to analyze short-term non-linear time series, namely, Recurrence Plot (RP) analysis method [13,14]. Its basic algorithm is:

Based on the method of space reconstruction, the phase points x_i can be obtained by reconstructing the phase space of one-dimensional time series, as shown in formula (1). The distance between phase point x_i and x_j in phase space is calculated:

$$d_{i,j} = \|x_i - x_j\| \quad (13)$$

where, $i, j = 1, 2, 3, \dots, N$, $\|\cdot\|$ is the distance of Euclidean. N is the number of phase points x_i .

Calculate the recurrence value:

$$R_{i,j} = \Theta(d_{i,j} - r) \quad (14)$$

where, r is a pre-specified threshold constant. $\Theta(x)$ is the function of Heaviside.

The distance between the all phase points $d_{i,j}$ are calculated. The value of $R_{i,j}$ is got through the distance between the phase point and the size of the given threshold r . $R_{i,j} = 0$ and $R_{i,j} = 1$ represent the white points and black points of corresponding positions in the two-dimensional diagram respectively. Then all the values of $R_{i,j}$ are plotted on a $i-j$ two-dimensional graph to get a recurrence plot. Finally, the dynamic law of nonlinear time series is described by analyzing the recurrence plot.

B. Quantitative Analysis of Recurrence Plot

The recurrence plot can only analyze the signals of different characteristics qualitatively, but cannot describe the dynamic characteristics of the recurrence points in the recurrence plot quantitatively. Zbilut et al. proposed a method to quantitatively describe the structure of the small scales of the recurrence plot by using statistics method. Namely Recurrence Quantification Analysis [15] (RQA). The recurrence plot features extracted by RQA mainly include:

1) *Recurrence Ratio (RR)*: the percentage of recurrence points in a recurrence plot.

$$RR = \frac{1}{N^2} \sum_{i,j=1}^N R_{i,j} \quad (15)$$

2) *Determinism (DET)*: the percentage of recurrence points that constitute a line segment paralleling to the main diagonal in the total recurrence points.

$$DET = \frac{\sum_{l=l_{\min}}^N lP(l)}{\sum_{i,j=1}^N R_{i,j}} \quad (16)$$

Where $P(l)$ is the distribution probability that a line segment paralleling to the main diagonal and having a length of l . l_{\min} is the initial value of the length taken in the diagonal structure, generally taking $l_{\min} = 2$.

3) *Laminarity (LAM)*: the percentage of recurrence points that constitute the vertical/horizontal line segments of a recurrence plot in the total recurrence points.

$$LAM = \frac{\sum_{v=l_{\min}}^N vP(v)}{\sum_{i,j}^N R_{i,j}} \quad (17)$$

where $P(v)$ is the distribution probability of the line segment with length v in the vertical/horizontal line segment, generally taking $v_{\min}=2$.

4) *Maximum Diagonal Length (L_{\max})*: The length of the longest diagonal except the main diagonal.

$$L_{\max} = \max(l_i) \quad (18)$$

where $i=1,2,\dots,N-1$ and $1 \leq l_i \leq N-1$.

5) *Entropy (ENTR)*: The Shannon entropy of the length distribution of the line segments paralleling to the main diagonal in a recurrence plot.

$$ENTR = - \sum_{l=l_{\min}}^N P(l) \ln P(l) \quad (19)$$

C. Kernel Fuzzy C-Means Clustering Algorithm

Kernel fuzzy C-means (KFCM) clustering algorithm maps the data samples to be classified from original space to high-dimensional space through Kernel function [16]. Nonlinear mapping can extract and amplify useful feature information effectively. Therefore, the clustering recognition of features is realized, and the convergence rate is faster [17].

The KFCM algorithm adopts nonlinear mapping $\Phi: X \rightarrow F$. The original space X is mapped to high-dimensional feature space F . If the input space samples are $\{x_1, x_2, \dots, x_n\}$, then the sample x_i ($i=1,2,\dots,n$) is mapped to $\Phi(x_i)$ for clustering analysis. The expression of the function is:

$$J_m(U, v) = \sum_{i=1}^S \sum_{k=1}^n u_{ik}^\alpha \|\Phi(x_k) - \Phi(v_i)\|^2 \quad (20)$$

where, $U=[u_{ik}]$ is the membership matrix. u_{ik} is the membership degree value of the k -th sample to the i -th class set.

u_{ik} meets the condition $0 \leq u_{ik} \leq 1$ and $\sum_{i=1}^S u_{ik} = 1$. v_i is the i -th cluster center. Where, $i = \{1, 2, \dots, S\}$, S is the number of clusters. α is the weighted index or the smoothing index, and $\alpha > 1$. In this paper takes $\alpha = 2$.

The kernel function $K(x, y)$ is defined as $K(x, y) = \Phi(x)^T \Phi(y)$, so the Euclidean distance is:

$$\|\Phi(x_k) - \Phi(v_i)\|^2 = K(x_k, x_k) + K(v_i, v_i) - 2K(x_k, v_i) \quad (21)$$

According to the Lagrange multiplier optimization method, the minimum value of the objective function given in Formula

(20) can be obtained by iterations of Formula (22) and Formula (23).

$$u_{ik} = \frac{\left[\frac{1}{\|\Phi(x_k) - \Phi(v_i)\|^2} \right]^{\frac{1}{\alpha-1}}}{\sum_{i=1}^S \left[\frac{1}{\|\Phi(x_k) - \Phi(v_i)\|^2} \right]^{\frac{1}{\alpha-1}}} \quad (22)$$

$$v_i = \frac{\sum_{k=1}^n u_{ik}^\alpha K(x_k, v_i) x_k}{\sum_{k=1}^n u_{ik}^\alpha K(x_k, v_i)} \quad (23)$$

The calculation steps of KFCM clustering algorithm are as follows:

1. Set the number of clusters as S and the weighted index as α .
2. Initialize each cluster center v_i .
3. Repeat the following operations and stop the iteration until $\|U^{t-1} - U^t\| \leq \varepsilon$.

(1) The membership degree is updated according to the current cluster center and Formula (22).

(2) Each cluster center is updated according to the current cluster center and Formula (23).

The kernel function selected in this paper is Gaussian kernel function.

IV. FAULT FEATURE EXTRACTION METHOD BASED ON RECURRENCE QUANTITATIVE ANALYSIS

A. Test Equipment and Data Acquisition

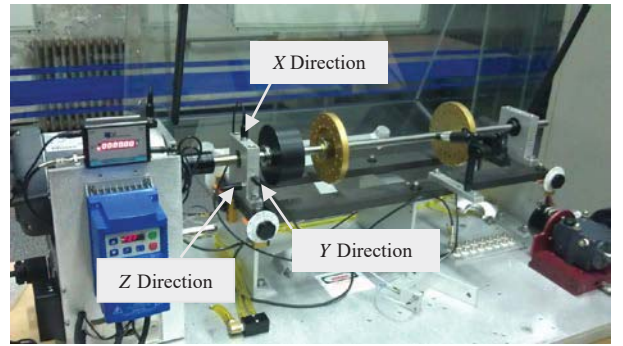


Figure 1. Mechanical failure simulation test bench

In this test, the Machinery Fault Simulator Magnum (MFS-MG) designed by SQI was adopted. The type of bearing selected is ER-12K and its parameters are shown in TABLE I. Set the motor rotation frequency to 40Hz during the test. The type of acceleration sensor is 608A11 with frequency range of 5Hz~10kHz and sensitivity of 100mV/g. The installation of

acceleration sensor is BNC. As shown in Fig.1, a 5-kilogram load is placed on the working shaft of the test stand.

TABLE I. MAIN PARAMETERS OF BEARING (ER-12K)

Ball	Ball Diameter	Pitch Diameter	BPFI	BSF	BPFO
8	0.3125in	1.318in	4.95	1.99	3.05

In this test, a NI9234 data acquisition card of NI company was used. The bearing vibration signals of directions x , y and z (Fig.1) and rotational speed signal of the shaft are collected through four channels. Set the sampling frequency as 50kHz. Set the sampling numbers of each sample as 10k.

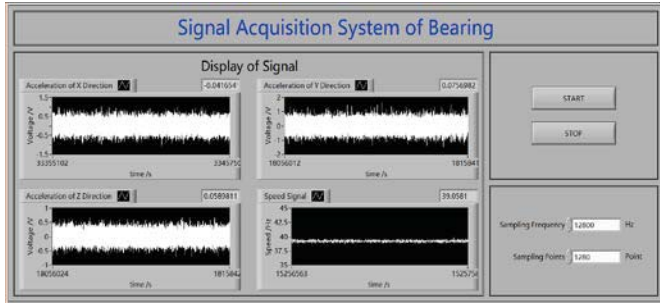


Figure 2. LabVIEW data acquisition system

As shown in Fig.2, a data acquisition system was developed under the LabVIEW software platform. The system can display and store the collected data signals in real time. The parameters can be set on the front panel to control the signal sampling frequency and sampling points, to start and stop of the collection state.

B. The Denoising of Rolling Bearing Vibration Signal

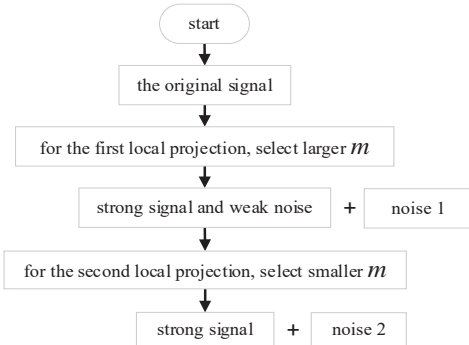


Figure 3. The process of local projective noise reduction

The frequency spectrum caused by noise is complex. In order to extract the effective characteristic signal, the vibration signal needs to be de-noised. During the first local projective noise reduction, a larger embedding dimension m is selected to eliminate the strong noise signal in the original signal. At this time, the obtained signal contains only the useful signal and the weak noise signal. During the second local projective noise reduction, a smaller embedding dimension m is selected to eliminate the weak noise signal in the signal that has been denoised by a large embedding dimension above mentioned. Then only useful signal is remained in the signal. The process of denoising is shown in Fig.3.

The specific denoising steps are as follows:

- 1) Calculate the embedding dimension m and delay time τ . A larger m is selected to reconstruct the phase space of original vibration signal.
- 2) Calculate the mass center of the phase points in the neighborhood of x_i . Take the diagonal weight matrix, $R_{11} = R_{mm} = 10^3$ and the rest $R_{ii} = 1$.
- 3) Calculate the covariance matrix Cov of each phase point.
- 4) Calculate the eigenvector a_i and eigenvalue λ_i of the matrix.
- 5) The eigenvalues λ_i of the covariance matrix Cov are sorted from large to small. The components corresponding to several smaller eigenvalues are noise.
- 6) Remove the eigenvectors caused by noise according to Formula (12) and eliminate the part of noise. The de-noising phase points are reconstructed according to their corresponding positions.
- 7) Return to step 1) and select a smaller m to denoise the signal that got in step 6) with local projective noise reduction again.

Taking the rolling ball fault signal of rolling bearing as examples, the phase space reconstruction parameters of vibration signal are calculated by mutual information function method and Cao method. The time delay when mutual information $I_N(\tau)$ reaches the minimum value for the first time is taken as the time delay τ of phase space reconstruction. The calculation results of τ are shown in Fig.4. For the deterministic system, there is an optimal embedding dimension m which makes the $E_1(m)$ tend to be stable. $E_2(m)$ is the supplement judgment criterion to judge whether the value of $E_1(m)$ has been stabled. The calculation results of the embedding dimension m are shown in Fig.5.

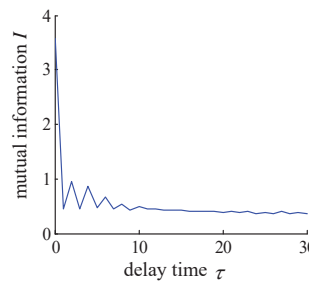


Figure 4. The delay time τ of rolling ball fault signal calculated by mutual information method

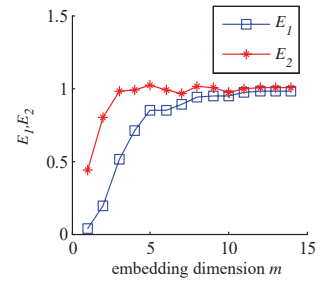


Figure 5. The embedding dimension m of rolling ball fault signal calculated by Cao method

The mutual information method and the Cao method are respectively applied to the vibration signals of the normal state, inner race fault, rolling ball fault and outer race fault of the rolling bearing. The parameters of phase space reconstruction of original bearing vibration signals of each state are obtained. That is, the optimal embedding dimension m and the optimal delay time τ . The calculation results are shown in TABLE II.

TABLE II. THE PHASE SPACE RECONSTRUCTION PARAMETERS OF BEARING VIBRATION SIGNALS IN DIFFERENT STATES

state type	delay time τ	the smaller m	the larger m
normal condition	1	9	11
inner race fault	2	6	8
rolling ball fault	1	6	8
outer race fault	1	7	9

Based on the optimal parameters determined by mutual information method and Cao method, the bearing vibration signal of normal state is denoised by local projective noise reduction method. Hilbert envelope demodulation is carried out for bearing vibration signals after denoising. Since the fault frequency of bearing is within the 0-1000Hz frequency band, the envelope signal is down-sampled and reduced its sampling frequency to 2kHz. Then the power spectrum of envelope signal is analyzed.

Before denoising, the vibration signal of rolling bearing contains the frequency component of fault characteristic frequency. Fig.6 shows the envelope power spectrum before and after denoising of outer race fault vibration signal of rolling bearing. Although the fault characteristic frequency can be recognized before de-noising, as shown in Fig.6a), there is strong noise interference. After denoising, as shown in Fig.6b), the frequency of noise is filtered and the failure frequency is more obvious.

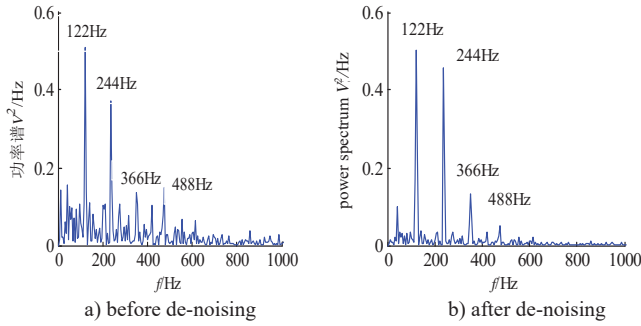


Figure 6. Envelope power spectrum before and after denoising of outer race fault vibration signal

C. Recurrence Quantification Analysis and KFCM Clustering Method

The optimal embedding dimension m and the optimal delay time τ of the vibration signal after denoising are calculated by mutual information method and Cao method. The calculation results are shown in TABLE III. Select $r=1.2\sigma$ (σ is the standard deviation) as the threshold of bearing vibration signal. 1000 points of denoised bearing vibration signal in each state are intercepted and recurrence plots are drawn, as shown in Fig.8.

TABLE III. RECURRENCE PLOT PARAMETERS OF BEARING VIBRATION SIGNALS IN DIFFERENT STATE

state type	delay time τ	embedding dimension m
normal state	3	3
inner race fault	3	4
rolling ball fault	2	3
outer race fault	3	3

As shown in Fig.7, although the recurrence plots drawn by bearing vibration signals in different states are obviously different. It can only be diagnosed qualitatively. The recurrence quantification analysis can quantify the points and line segments in the recurrence plot according to the statistical concept. The system dynamics information reflected by the recurrence plot is analyzed quantitatively. The bearing vibration signals in different states are analyzed by recurrence quantification analysis. A group of data is selected from the recurrence plot of each state for analysis. The recurrence quantification eigenvalues are shown in TABLE IV.

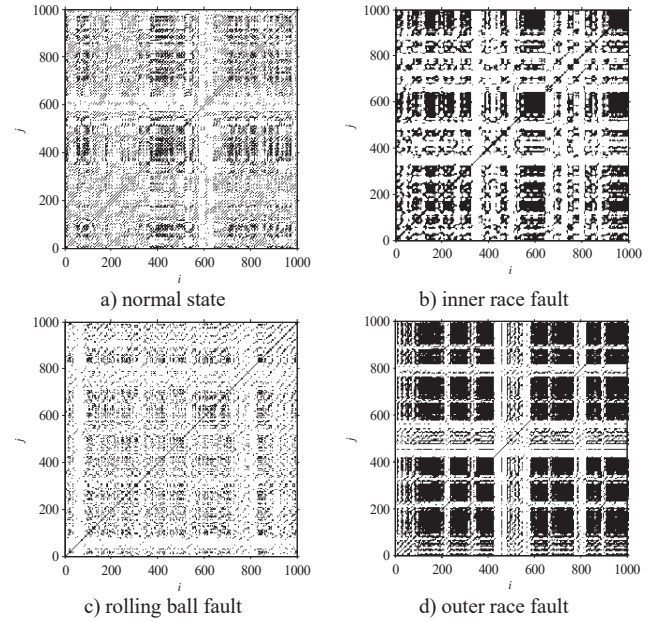


Figure 7. Recurrence plot of bearing vibration signal in different state

TABLE IV. RECURRENCE QUANTITATIVE EIGENVALUES OF BEARING VIBRATION SIGNALS IN DIFFERENT STATES

state type	RR (%)	DET (%)	LAM (%)	Lmax	ENTR
normal state	0.1089	0.5803	0.7520	54	1.4480
inner race fault	0.2489	0.7320	0.7535	113	0.6738
rolling ball fault	0.1106	0.2602	0.3130	21	2.0347
outer race fault	0.5204	0.9103	0.9439	196	0.8835

It can be seen from TABLE IV that the recurrence quantification eigenvalues of vibration signals of rolling bearings in different states have great differences. They can be used to identify different state types of rolling bearings. It provides a new idea for the feature extraction of rolling bearing vibration signal.

By calculating, the eigenvector is constructed by DET and ENTR. Then the eigenvector sets of training samples and test samples can be obtained. The eigenvectors of training samples can be analyzed by KFCM clustering algorithm. Finally, the clustering of training samples feature set can be completed.

For each of the 20 training samples of 4 state types, DET and ENTR of recurrence quantification eigenvalues are selected to construct an 80×2 -dimensional feature set of training samples for KFCM clustering. Set the number of

clusters $S=4$, the weighted index $\alpha =2$, Gaussian kernel parameter $\varepsilon =10$. The calculation is stopped when the absolute value of the membership values between two iterations is less than 0.001. The clustering result of training sample set is shown in Fig.8. The clustering centers of 4 bearing states are shown in TABLE V.

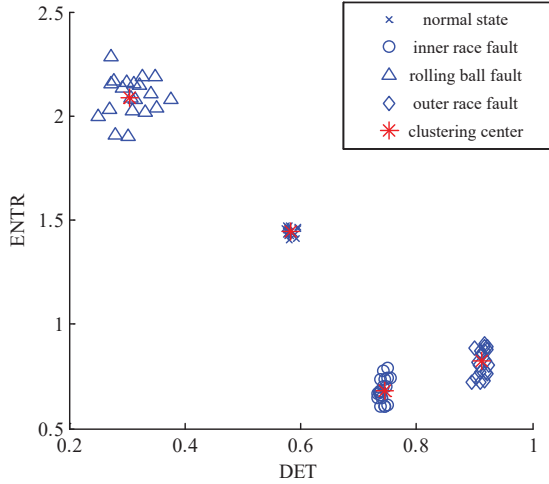


Figure 8. KFCM clustering result of recurrence quantification analysis

TABLE V. CLUSTERING CENTERS OF BEARING VIBRATION SIGNALS OF FOUR STATES

state type	clustering center	state type	clustering center
normal state	(0.5819, 1.4443)	rolling ball fault	(0.3057, 2.0882)
inner race fault	(0.7451, 0.6833)	outer race fault	(0.9123, 0.8259)

It can be seen from Fig.8 that four types of training samples with 80 groups of data are clustered into four clusters. The clustering results are effective.

V. COMPARISON ANALYSIS OF FAULT DIAGNOSIS RESULTS

Fault identification is carried out for the eigenvectors of 20 groups of test samples in normal state, inner race fault, rolling ball fault and outer race fault. i is the cluster center label of training sample and is assigned as $i=1,2,3,4$ according to four states. j is the eigenvector label of test samples. In the identification process, the Euclidean distance $d(i, j)$ between the i -th cluster center and the j -th eigenvector of test samples is used as the standard:

$$d(i, j) = \sqrt{(X_i - x_j)^2 + (Y_i - y_j)^2} \quad (24)$$

where, X_i is the DET value of the i -th cluster center. Y_i is the ENTR value of the i -th cluster center. x_j is the DET value of the j -th test sample. y_j is the ENTR value of the j -th test sample and $1 \leq j \leq 80$. If $d(i, j)$ meets the minimum Euclidean distance:

$$d(i, j) = \min \left\{ \sqrt{(X_i - x_j)^2 + (Y_i - y_j)^2} \right\} \quad (25)$$

Then the j -th eigenvector of test sample belongs to the i -th cluster type. The fault recognition results of rolling bearings are shown in TABLE VI:

TABLE VI. FAULT RECOGNITION RESULTS OF ROLLING BEARING

state type	correct	error	accuracy
normal state	20	0	100%
inner race fault	20	0	100%
rolling ball fault	20	0	100%
outer race fault	20	0	100%

In order to compare the fault accuracy recognition rate of bearings, 40 sets of vibration signals in each state are selected for feature extraction under the same conditions. The basic statistical characteristic clustering coefficient C and average degree $\langle k \rangle$ of the phase-space complex network are selected to constitute the eigenvector. The first 20 groups of each state are served as training samples and the last 20 groups were served as test samples. KFCM clustering algorithm is performed on the eigenvectors. The clustering result of training sample set is shown in Fig.9. The cluster centers of the training sample are shown in TABLE VII.

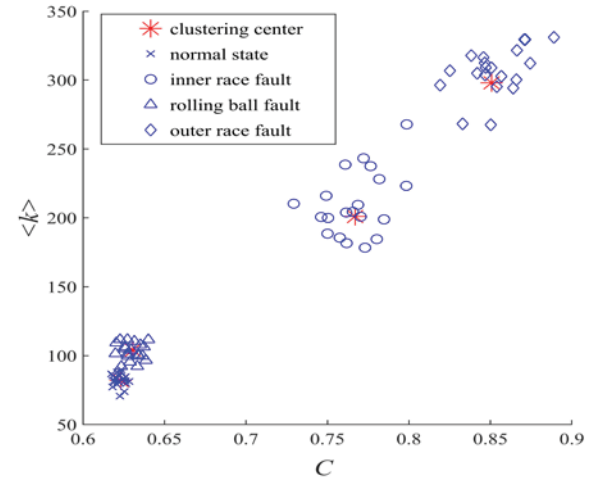


Figure 9. KFCM clustering result of complex network

TABLE VII. CLUSTER CENTERS OF TRAINING SAMPLES IN FOUR STATES BY COMPLEX NETWORK AND KFCM

state type	clustering center	state type	clustering center
normal state	(0.6231, 82)	rolling ball fault	(0.6306, 104)
inner race fault	(0.7670, 201)	outer race fault	(0.8507, 298)

The clustering results show that the distribution of the same type of eigenvector points is large. The fault recognition results are shown in TABLE VIII.

TABLE VIII. FAULT RECOGNITION RESULTS OF ROLLING BEARING BY COMPLEX NETWORK AND KFCM

state type	correct	error	accuracy
normal state	18	2	90%
inner race fault	16	4	80%
rolling ball fault	17	3	85%
outer race fault	17	3	85%

By comparing TABLE VI and TABLE VIII, it can be found that the fault recognition accuracy of recurrence quantitation analysis method is significantly higher than that of phase space complex network method under the same situation.

VI. CONCLUSIONS

In this paper, the rolling bearing is taken as the research object. The local projective noise reduction method and the recurrence quantification analysis method based on phase space reconstruction are deeply studied. The mutual information method and Cao method are introduced to calculate the optimal delay time τ and the optimal embedding dimension m respectively. DET and ENTR in the recurrence quantification feature are selected to form the eigenvectors. Fault recognition of bearing vibration signals in different states is carried out by combining KFCM clustering algorithm and principle of minimum Euclidean distance judgment. The main research conclusions are as follows:

1) The optimal delay time τ and the embedded dimension m of phase space reconstruction are calculated by mutual information method and Cao method. The optimal parameters make the complex network of phase space to retain many properties of attractors. It lays the foundation for denoising and fault recognition of rolling bearing vibration signals.

2) Combined with the best delay time and the best embedding dimension calculated by mutual information method and Cao method, the local projective noise reduction method is introduced into the de-noising of rolling bearing vibration signal. This method has got effective results in bearing vibration signals de-noising.

3) The recurrence quantitation analysis method is applied to the fault feature extraction of rolling bearings. The features of recurrence plots are fully revealed. Combined with KFCM clustering algorithm and the judgment criterion of minimum Euclidean distance, the fault types of rolling bearings can be identified accurately.

4) Comparing with the features of phase space complex network, the features of recurrence quantitation analysis method can reveal the substantive characteristics of the rolling bearing vibration signals more comprehensively. The fault recognition accuracy of the rolling bearing vibration signals in different states is higher.

ACKNOWLEDGEMENTS

This work is supported by National Natural Science Foundation of China (Grant No. 51875498, 51475405) and Key Project of Natural Science Foundation of Hebei Province, China (Grant No. E2018203339). The support is gratefully

acknowledged. The authors would also like to thank the reviewers for their valuable suggestions and comments.

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