Vibratory Synchronization Transmission of Nonlinear Vibrating System with Frequency Capture

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***Abstract*— In the special case where frequency acquisition occurs, synchronous transmission in a vibrating system with asymmetric hysteresis is proposed. From the perspective of the hysteresis characteristics and asymmetry, a nonlinear dynamic model of a nonlinear vibration system is proposed for analyzing hysteresis characteristics. Synchronous conditions and synchronous stability conditions are also theoretically analyzed using the Hamiltonian principle. The synchronization feature of two excitation rotors and synchronous stable feature of a self-synchronizing system with asymmetric hysteresis are analyzed by the selected parameters. The synchronization feature of the double excitation rotor and the synchronization stability of the vibratory system under the action of the hysteretic force with the asymmetry have been proposed in the exceptional circumstances of frequency capture occurring. Through the differential rate of the double-excited rotor (including the initial phase difference, the initial speed difference and the rotor parameter difference), various synchronization phenomena of the vibration system under the asymmetric hysteresis force are obtained. When the differential rate of the double-excited rotor is in a limited range, the double-excited rotors can restore the synchronization by itself, the synchronization feature of the double-excited rotor and the synchronization stability of the vibratory compacting system under the action of hysteresis with asymmetry can be obtained to improve the vibrating effect.**

Keywords-vibration synchronization; self-synchronization ; frequency capture; the hysteretic characteristics

I. INTRODUCTION

In vibration systems, vibration synchronization is an important factor in multi-excited rotor drive. Vibrational synchronization is generally interpreted as the simultaneous movement of two eccentric rotors on two excitation rotors in a multi-excited rotor-driven vibration system, in the phase difference between the two excitation rotors is zero or constant and can pass a constant value The phase difference is self-synchronizing. A vibration system driven by a multi-excitation rotor can be referred to as a nonlinear vibration system. In many engineering fields, it is important to excite the synchronous nature of the rotor, especially in vibratory compression systems with multiple excitation rotors. Therefore, the research on vibration synchronization has become one of the main research directions of nonlinear vibration systems.

In daily life we can often find self-synchronization. For example, Huygens found that the two clocks on the wall were running synchronously at a normal speed. Later, he used this principle to design a clock for locating the longitude of the ship [1]. Subsequently, the clock synchronization problem [2] and the non-synchronization problem [3] also attracted the attention of many scholars. A lot of literature has research on vibration synchronization [4-5]. Scholars have studied a large number of nonlinear vibration system models, such as nonlinear stiffness models and nonlinear simplified ideal models, as well as some other nonlinear vibration model research literature [6-8]. In general, the linear model is a very practical model for analyzing vibration systems driven by multi-excited rotors, but it is not suitable for describing vibration synchronization characteristics in vibration systems. With the development of nonlinear vibration theory, scholars have developed many nonlinear models for vibration synchronization of nonlinear vibration systems, such as nonlinear models of duffing dynamic equations [9-10], nonlinear models of piecewise linear stiffness. [11-12] Etc. These models have been studied and discussed in many literatures. Many scholars have studied the relationship between the excitation frequency and the natural frequency in a vibration system driven by a multi-excitation rotor, and many related research results have been obtained in the literature. In addition, if the synchronization characteristics of the multi-excited rotor and the synchronous stability of the vibration system are realized, most of the conventional vibration machines can work in a state far beyond resonance [13]. However, the synchronization stability and vibration range obtained in this way are very limited. With the development of vibration theory, the frequency range of the vibration system is mostly driven by multi-excited rotors, such as super-resonant vibration systems [14], non-resonant vibration systems [15-17] and resonant and non-resonant vibration systems [18], there has been a long-term exhibition.

When the excitation frequency is close to the natural frequency in the vibration system driven by the double excitation rotor, that is, resonance may occur in the vibration system driven by the double excitation rotor, the synchronization characteristics of the excitation rotor and the synchronization stability of the vibration system driven by the double excitation rotor. There have been related research results [12,19]. Obviously, the relationship between the excitation frequency and the natural frequency in the vibration system affects the generation of vibration synchronization, that is, the vibration synchronization receives the constraints of both. Frequency capture is generally defined as the frequency of vibration being captured by the first natural frequency when the excitation frequency is close to the frequency range of the first natural frequency in the vibration system. Vibration systems require large amplitudes in many engineering fields, but frequency capture is an important factor in determining the magnitude of the amplitude in a vibrating system. On the basis of the nonlinear dynamic model, we should study the vibration synchronization relationship between the vibration system and the hysteretic force. However, there are few literatures on this aspect. Therefore, the study of vibration synchronization of nonlinear vibration systems with hysteretic forces has become one of the key issues in the vibration synchronization of nonlinear vibration systems driven by multi-excitation rotors.

In this paper, based on the frequency acquisition theory, the nonlinear dynamic model of the vibration and pressure system is given. The synchronization conditions and synchronization stability conditions of nonlinear vibration systems with asymmetric hysteresis are theoretically analyzed. Then, the synchronization characteristics of the two excitation rotors and the synchronization stability characteristics of the nonlinear vibration system are analyzed by substituting parameters.

II MATHEMATICAL MODEL AND THEORETICAL ANALYSIS

The dynamic model of an asymmetric lag nonlinear vibration system is shown in Figure 1. As shown in Figure 1, when the nonlinear vibration system composed of two excitation rotors is a vibration and pressure suppression system, the vibration force of the system is in the vertical direction. Oy is the coordinate system of the nonlinear vibration system with asymmetric lag, O is the coordinate center of the nonlinear vibration system (it is also the midpoint of the two excitation rotor axes), and O1, O2 are the axial center points of the two excitation rotors. Using the Lagrange equation, the differential equation of a nonlinear vibration system driven by two excited rotors under asymmetric hysteresis is defined as

$$\begin{aligned} & m\ddot{y}+c\dot{y}+ky+f(y)=m_1r_1(-\ddot{\varphi}_1\cos\varphi_1+\dot{\varphi}_1^{\,2}\sin\varphi_1) \\ & -m_2r_2(\ddot{\varphi}_2\cos\varphi_2-\dot{\varphi}_2^{\,2}\sin\varphi_2) \\ & J_{0i}\ddot{\varphi}_i=T_{mi}(\dot{\varphi}_i)-T_{fi}(\dot{\varphi}_i)-c_i\dot{\varphi}_i-m_ir_i\ddot{y}\cos\varphi_i \qquad (i=1,2) \end{aligned} \tag{1}$$

In the differential equations (1), y, $\dot{y}$ and $\ddot{y}$ are the vibration displacement, the velocity and the acceleration of nonlinear vibrating system in the vertical direction, respectively. m is the total mass of nonlinear vibrating system. m1 and m2 are the mass of the eccentric rotors on the two excited rotors, respectively. r_i(i=1,2) is the radiuses of the eccentric rotor on the excited motor around O_i (i=1,2). φ_i, $\dot{\varphi}_i$ and $\ddot{\varphi}_i$ (i=1,2) are the angular phase of the eccentric rotor i, the angular velocity of the eccentric rotors i, the angular acceleration of the eccentric block i, respectively. c is the damping of nonlinear vibrating system with the compacted soil, c_i (i=1,2) is the damping of the eccentric rotor on the excited rotors i. J_{0i} (i=1,2) is the moment of the inertia of the eccentric rotor i, $T_{mi}(\dot{\varphi}_i)$ (i=1,2) is the electromagnetic torque on the excited motor i, $T_{fi}(\dot{\varphi}_i)$ (i=1,2) is the friction torque on the excited motor i. f(y) is expressed as the hysteretic force in nonlinear vibrating system. As shown in Figure 2, the hysteretic force can be defined as

$$f(y)=\begin{cases} b_1y-b_2y^3 & (0\le y\le y_2,\ \dot{y}>0 \ \ (\text{the segment of } A \text{ to } B)) \\ b_3(y-y_1) & (y_1\le y\le y_2,\ \dot{y}<0\ (\text{the segment of } B \text{ to } C)) \\ 0 & (0\le y\le y_1,\ \dot{y}=0 \ \ (\text{the segment of } C \text{ to } A)) \end{cases} \tag{2}$$

In the differential equations (1), y is defined as the displacement in the direction of the lag force. b1, b2 and b3 are defined as coefficients of hysteresis force. y_1 and y_2 are defined as coordinate points on the axis, they should satisfy the following conditions and be expressed as $0\le y_1\le y_2$.

The angular velocity $\dot{\varphi}_i$ is generated through the rotation of the eccentric rotor on the excited motor around Oi. $\dot{\varphi}_i$ can be replaced by ω_i (i=1,2) (ω_i is the angular frequency). When $\omega_1=\omega_2$, the synchronous operation of the two excited rotors is presented in the synchronous vibration system. The two angular phases can be defined as $\varphi_1=\varphi+\frac{1}{2}\Delta\varphi$ and $\varphi_2=\varphi-\frac{1}{2}\Delta\varphi$. φ is named as the average angular phase of the two excited rotors. $\Delta\varphi$ is named as the phase difference angle of the eccentric blocks on the two excited rotors. $\dot{\varphi}$ is named as the average angular velocity of two eccentric blocks. $\dot{\varphi}$ may be expressed as $\overline{\omega}$ ($\overline{\omega}$ is the excited frequency). $\overline{\omega}$ can be defined and expressed as $\overline{\omega}=\frac{\omega_1+\omega_2}{2}$. In Eq.(1), $m_1=m_2=m_0$, $r_1=r_2=r_0$. The parameter terms of the first equation in equation (1), such as the damping term and the hysteresis term with asymmetry and excitation term, are very small quantities. When the excitation frequency is approximately equal to the first natural frequency, a nonlinear asymptotic method is used to solve the periodic solution. You can solve the following equation by calculating the periodic solution

$$y=\alpha\cos(\overline{\omega}t+\theta) \tag{3}$$

In Eq.(3), $\alpha=\dfrac{m_0r_0\overline{\omega}^2\cos\frac{1}{2}\Delta\varphi}{m\sqrt{[\omega_e(\alpha)\omega_n-\overline{\omega}^2]^2+\delta^2(\alpha)\overline{\omega}^2}}$, $\theta=\arctan\dfrac{\omega_e(\alpha)\omega_n-\overline{\omega}^2}{-\delta(\alpha)\overline{\omega}}$

$$\delta(\alpha)=\frac{c}{2m}+\frac{1}{2\pi\sqrt{km}\alpha}\left\{\frac{b_1\alpha}{2}-b_2\alpha^3+\frac{b_3\alpha}{2}\left[1-\left(\frac{y_1}{\alpha}\right)^2\right]-b_3y_1\left(1-\frac{y_1}{\alpha}\right)\right\}$$

,

$$\omega_e(\alpha)=\omega_n+\frac{1}{2\pi\sqrt{km}\alpha}\left[\begin{aligned} & b_1\alpha\left(\frac{1}{4}+\frac{\pi}{4}\right)-\frac{3}{4}b_2\alpha^3\left(\frac{1}{4}+\frac{\pi}{4}\right) \\ & +\frac{b_3\alpha}{2}\arccos\frac{y_1}{\alpha}-\frac{b_3y_1}{2}\sqrt{1-\left(\frac{y_1}{\alpha}\right)^2} \end{aligned}\right].$$

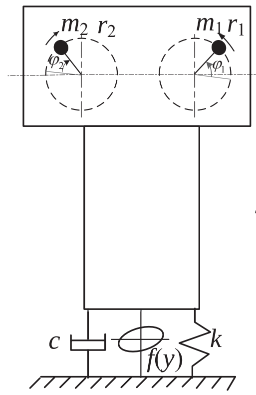

Figure 1. The model of the nonlinear vibrating system with the hysteresis force

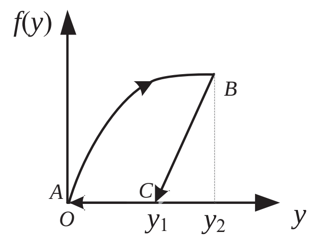

Figure 2. The hysteretic force

For the nonlinear vibration system to work safely and steadily, the two excitation rotors of the system must be synchronized. To satisfy the above conditions, the phase difference between the two exciting rotors must be within a certain range. Nonlinear vibrating system should be a complete non-conservative system, so Hamilton Principle for the nonlinear system can be expressed as

$$-\frac{(m_0 r_0 \overline{\varpi}^2)^2 \sin\theta \sin\Delta\varphi}{m\sqrt{[\omega_e(\alpha)\omega_n-\overline{\varpi}^2]^2+\delta^2(\alpha)\overline{\varpi}^2}}-(\Delta T_m-\Delta T_f)=0 \tag{4}$$

In Eq.(4), $\sin\theta=\frac{\omega_e(\alpha)\omega_n-\overline{\varpi}^2}{\sqrt{[\omega_e(\alpha)\omega_n-\overline{\varpi}^2]^2+\delta^2(\alpha)\overline{\varpi}^2}}$

$$\Delta T_m-\Delta T_f=[T_{m1}(\dot{\varphi}_1)-T_{m2}(\dot{\varphi}_2)]-[T_{f1}(\dot{\varphi}_1)-T_{f2}(\dot{\varphi}_2)].$$

In Eq.(4), D can be expressed as

$$D=\frac{(m_0 r_0 \overline{\varpi}^2)^2 W}{\Delta T_m-\Delta T_f} \tag{5}$$

In Eq.(5), $W=\frac{-[\omega_e(\alpha)\omega_n-\overline{\varpi}^2]}{m[(\omega_e(\alpha)\omega_n-\overline{\varpi}^2)^2+\delta^2(\alpha)\overline{\varpi}^2]}$.

In Eq.(5), the necessary condition for the nonlinear vibration system with hysteretic force characteristics to achieve synchronous operation is that $|D|$ is greater than or equal to 1. Therefore, the synchronization condition can be expressed by the following differential equation

$$|D|=\left|\frac{(m_0 r_0 \overline{\varpi}^2)^2 W}{\Delta T_m-\Delta T_f}\right|\geq 1 \tag{6}$$

The synchronization condition of a self-synchronizing vibration system having an asymmetric hysteresis characteristic can be expressed by the absolute value of D. When $|D|$ is larger, the vibration synchronization of the self-synchronizing vibration system with asymmetric hysteresis characteristics is easier to achieve. If $|D|\approx 1$, the synchronization condition of the self-synchronous vibrating system with the asymmetrical hysteresis is weak. When reducing $\Delta T_m-\Delta T_f$ or increasing $(m_0 r_0\overline{\varpi}^2)^2 W$, the synchronization condition of the self-synchronous vibrating system with the asymmetrical hysteresis can be improved. What's more, $\Delta T_m-\Delta T_f=0$, and it is the ideal condition. But the most unsatisfactory condition is that W=0. If the synchronization condition D is positive, namely, $\Delta T_m-\Delta T_f$ is negative, the phase different $\Delta\varphi=[0^\circ,90^\circ]$ or $\Delta\varphi=[90^\circ,180^\circ]$. If D is negative, that is $\Delta T_m-\Delta T_f$ is positive, the phase different $\Delta\varphi=[-90^\circ,0^\circ]$ or $\Delta\varphi=[180^\circ,270^\circ]$. There is a case where when there are two phase differences at each D value, there is a result that one solution is stable and the other is unstable. Therefore, it is necessary to analyze the conditions of synchronous stability in a self-synchronizing system. When the second derivative of total action to $\Delta\varphi$ is greater than zero, the synchronous stability condition can be achieved in nonlinear vibrating system. The system must satisfy the synchronous stability condition of the rotor, which can be expressed by the following differential equation:

$$\frac{(m_0 r_0\overline{\varpi}^2)^2[\overline{\varpi}^2-\omega_e(\alpha)\omega_n]\cos\Delta\varphi}{J_{01}m[(\omega_e(\alpha)\omega_n-\overline{\varpi}^2)^2+\delta^2(\alpha)\overline{\varpi}^2]}>0 \tag{7}$$

When the frequency acquisition of the system occurs, the excitation frequency of the excitation rotor is close to the first-order equivalent natural frequency in the self-synchronizing vibration pile system, that is $\overline{\varpi}=\omega_e(\alpha)$. In addition, $\omega_e(\alpha)>\omega_n$ in Eq.(3). If Eq.(7) can be satisfied, $\cos\Delta\varphi>0$ in Eq.(7), namely, the phase different $\Delta\varphi$ is at $[-90^\circ, 90^\circ]$. The phase difference of equation (7) is gradually becoming stable. Under this condition, the synchronous stability condition is satisfied in the nonlinear vibration system with asymmetric hysteresis characteristics. Therefore, it can be concluded that when the phase difference of the excitation rotor is at $[-90^\circ, 90^\circ]$, in the nonlinear vibration system, the two excitation rotors can operate stably in the system under the condition of asymmetric hysteresis. At the same time, the phase difference stability of the two excitation rotors is 0 rad.

III FREQUENCY CAPTURE

Using the nonlinear vibration system model with equations (1) - (2) with asymmetric hysteresis characteristics, select some parameters of the nonlinear vibration system with asymmetric hysteresis characteristics as follows m=78 kg, $m_1=m_2=m_0=3.5$ kg, $r_1=r_2=0.08$ m, k=1552000 N/m, $J_{01}=0.01$ kg·m^2, $J_{02}=0.01$ kg·m^2, c1=0.01Nm·s/rad, c2=0.01Nm·s/rad, b1=40, b2=20, b3=40, $y_1=0.1$, $y^2=0.2$. Simulations were performed using Matlab/Simlink. By combining equation (1-2) with the electromagnetic torque equation and the rotor torque equation, the response of the parameters in the nonlinear vibration system model with asymmetric hysteresis can be obtained. The first natural frequency is 22.5 Hz in the vertical direction, while the angular frequency of the nonlinear vibration system is 25 Hz (about 157 rad/s). From the results of the simulation analysis, when the damping = 10000 N•s / m, the response of the parameter is already available, and Figure 3 is the simulation result. According to Figure 3, it can be analyzed that the displacement of the periodic motion eventually tends to 40 mm, but the excitation frequency tends to be 24.3 Hz, that is, the speed of the double-excited rotor is finally stabilized at about 154 rad / s. The excitation frequency of the synchronous vibration system is not captured by the natural frequency, in the frequency acquisition does not occur at its own natural frequency.

According to the simulation results of Figure 4, it can be found that the damping when obtaining the response of the parameter is 100 N•s / m. When the damping varies within a certain range, such as when the excitation frequency of the system damping is close to the first natural frequency, the excitation frequency is captured by the first-order equivalent natural frequency in the self-synchronizing vibration system having electromechanical coupling characteristics. From the simulation results in Figure 4, when the nonlinear vibration system starts slowly and the operating frequency does not change after the frequency reaches the resonance point (about 22.6 Hz), the response tends to a fixed value while performing periodic motion, displacement of the periodic motion. The final trend is 0.3 meters. As shown in the simulation results of Figure 5 (motor speed diagram), the speed of the double-excited rotor eventually tends to 142 rad / s, which also confirms that the speed of the double-excited rotor is not the rated speed of the motor (about 157 rad/s). Under the condition of the natural vibration frequency of the double-excited rotor itself, the angular frequency of the double-excited rotor is captured by the first-order equivalent natural frequency, that is, the frequency capturing phenomenon occurs in the nonlinear vibration system. As shown in the simulation results in Figure 3-4, the vibration amplitude of a nonlinear vibration system with frequency acquisition characteristics is much larger than that of a nonlinear vibration system without frequency acquisition characteristics. The large amplitude of vibration of the system is beneficial to increase the speed and efficiency of the work.

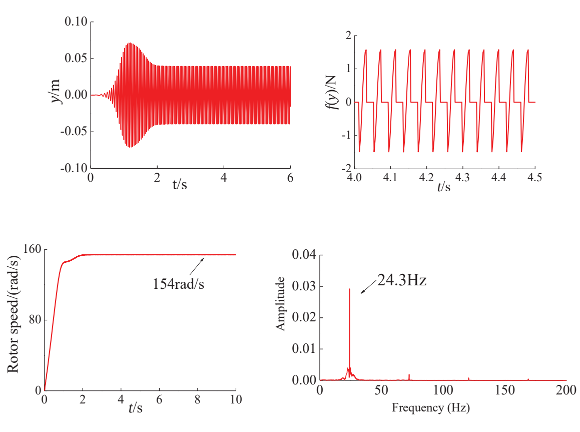

Figure 3. System simulation without frequency acquisition characteristics

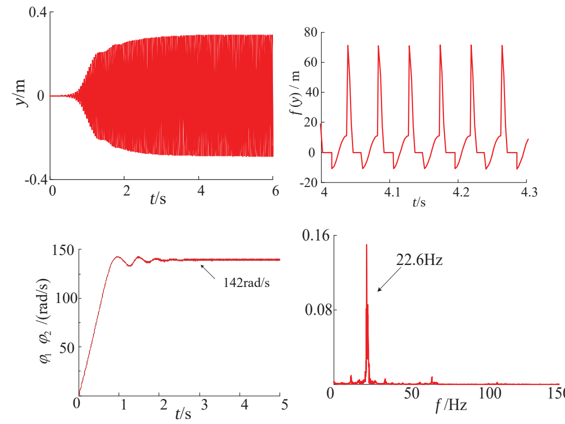

Figure 4. Simulation results of system frequency capture

IV SELF-SYNCHRONIZATION CHARACTERISTICS

In the actual engineering, the double-excited rotor of the motor is not guaranteed to be completely consistent, and the double-excited rotor is likely to have a difference. Therefore, the synchronous characteristics of the double-excited rotor must be quantitatively analyzed, and the synchronous stability of the electromechanically coupled self-synchronized vibrating pile system must also be quantitatively analyzed. In the case of a self-synchronizing vibrating pile system with electromechanical coupling characteristics, when there are differences between the two excitation rotors, it is necessary to first analyze the stability of both the phase difference and the rotational speed difference of the two rotors. Simulation of system parameters using Matlab / Simlink, simulation results of phase difference, phase plane and speed difference are shown in Figure 5-9. It can be seen from the simulation that when the initial phase, initial speed or parameters of the input are different when the double-excited rotor is simulated, the speed of the double-excited rotor is stabilized at 140 rad / s. And the rotor performs periodic motion after a certain degree of impact on the rotational speed of the double-excited rotor. Therefore, when there is a difference between the two excitation rotors, the first-order equivalent natural frequency capture excitation frequency can still be achieved, that is, the frequency acquisition phenomenon still exists in the nonlinear vibration system, and the reverse synchronization characteristics of the two are realized - the excitation rotor Synchronous stability with self-synchronizing vibrating pile systems.

As shown in the simulation results in Figure 5-9, when the angular velocity difference between the two excitation rotors is disturbed and the phase difference between the two excitation rotors is greatly disturbed, the angular velocity difference and phase difference between the two excitation rotors tend to be stable. And the phase difference is also stable at 0 rad / s. As shown in the phase plan simulation results of Figure 5-9, a limit cycle occurs in the phase plane of the angular velocity difference and the phase difference, and a stable solution of the angular velocity difference and the phase difference can be obtained. That is, the self-synchronizing vibration system can realize the synchronous stability characteristic with asymmetric hysteresis. The phase difference is stable at about 0 rad (namely 0 is at $[-90^{\circ},-90^{\circ}]$), and the simulation results are consistent with the theoretical analysis. Therefore, when the difference rate of the double-excited rotor is within a certain range, the double-excited motor has the ability to self-synchronize, the phase difference is stable at 0, the rotational speed difference is stable at 0, frequency acquisition can be realized, and the nonlinear excitation system can obtain a double-excited rotor. Synchronization characteristics and synchronization stability characteristics, and achieve increased amplitude and improved work efficiency. Therefore, it can be concluded that the nonlinear vibration system with hysteresis can obtain the synchronization characteristics and the synchronization stability characteristics of the two excitation rotors. The nonlinear vibration system with hysteresis has the ability to restore synchronization, and the nonlinear vibration system with asymmetric hysteresis has self-synchronization characteristics.

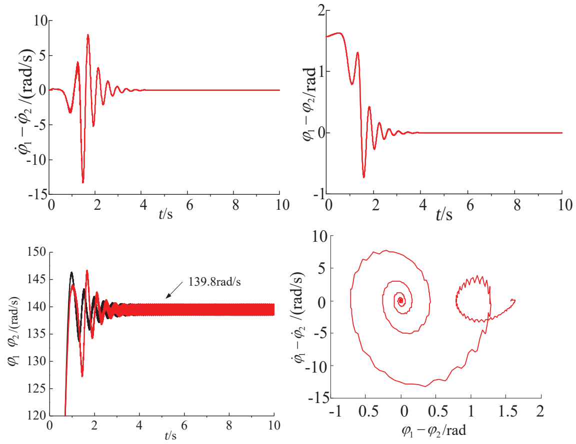

Figure 5. Simulation of systems with frequency capture characteristics under different initial phase conditions (1.57rad)

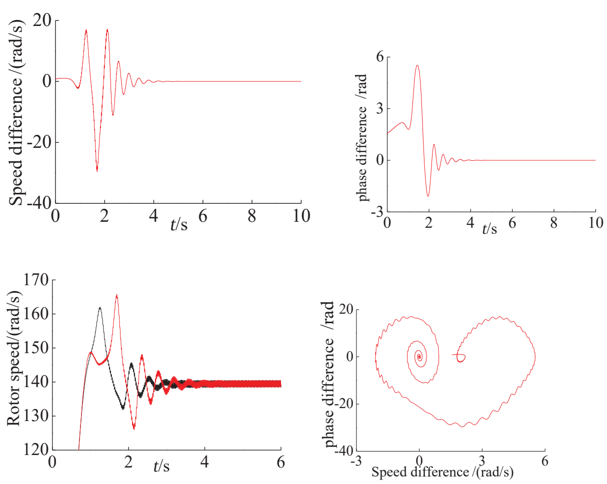

Figure 6. Simulation of the system with frequency capture in initial phase difference (1.5rad) and the initial rotational speed difference (0.8rad/s)

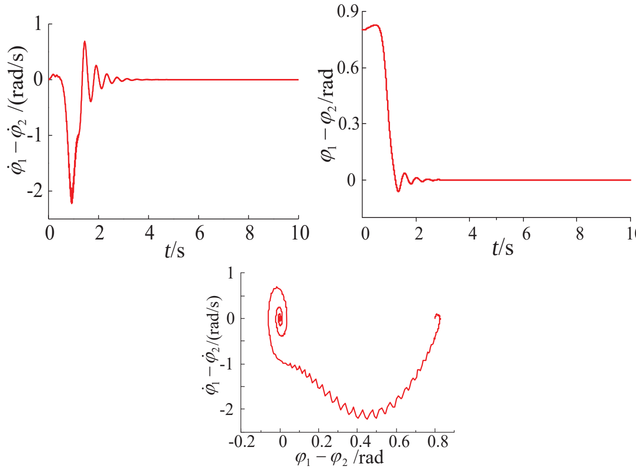

Figure 7. Initial phase difference simulation of systems with frequency capture characteristics (0.8rad)

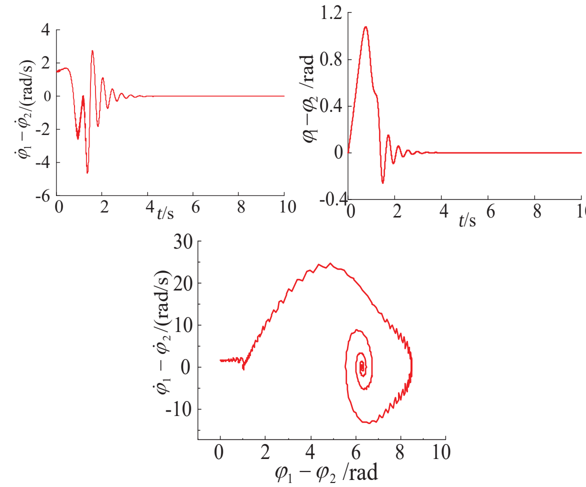

Figure 8. Initial speed difference simulation of systems with frequency capture characteristics (1.5rad)

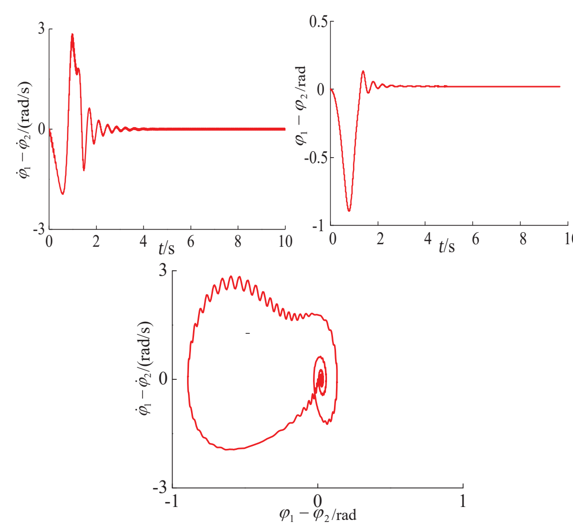

Figure 9. Simulation of rotor parameter difference for systems with frequency acquisition characteristics

Conclusions

In summary, when the excitation frequency of the excitation motor is close to the frequency range of the first-order equivalent natural frequency, a frequency trapping phenomenon occurs in the nonlinear vibration system with hysteresis force. Firstly, based on the research direction of hysteresis characteristics, a nonlinear dynamic model of nonlinear vibration system is proposed. The periodic solution of the asymmetric lag nonlinear vibration system is studied by nonlinear asymptotic method. Secondly, from the research direction of frequency acquisition (the excitation frequency of the excitation motor is captured by the first-order equivalent natural frequency), the synchronization conditions and synchronous stability conditions of the double-excitation motor are theoretically studied. When frequency capture occur, the phase difference is at $[-90^{\circ},90^{\circ}]$, namely, When the phase difference is stable at 0, the nonlinear vibration system can reach a synchronous stable state.Thirdly, this paper quantitatively analyzes the synchronization characteristics and synchronization stability of the vibratory pile system composed of dual excitation motors. When studying the difference rate between the double-excitation motors, it is found that the excitation frequency can still be captured by the first-order equivalent natural frequency, that is, the frequency synchronization and the reverse synchronization characteristics and synchronization stability of the double-excitation motor can still be realized by the excitation frequency. The vibration of the vibrating pile system is synchronized. Therefore, the nonlinear vibration system has the asymmetry and the ability to restore vibration synchronization under the hysteresis force.

ACKNOWLEDGMENT

The author gratefully acknowledges that this project was supported by the National Natural Science Foundation of China (Grant No. 51605022).

References

[1] Senator, M.. Synchronization of two coupled escapement-driven pendulum clocks. Journal of Sound & Vibration, 2006, 291(3), 566-603.

[2] Jovanovic V , Koshkin S . Synchronization of Huygens' clocks and the Poincare method. Journal of Sound & Vibration, 2012, 331(12):2887-2900.

[3] Banerjee, T., Biswas, D., & Sarkar, B. C.. Anticipatory, complete and lag synchronization of chaos and hyperchaos in a nonlinear delay-coupled time-delayed system. Nonlinear Dynamics, 2012, 72(1-2), 321-332.

[4] Blekhman, I. I. , Fradkov, A. L. , Tomchina, O. P. , & Bogdanov, D. E. . Self-synchronization and controlled synchronization: general definition and example design. Mathematics and Computers in Simulation, 2002, 58(4-6), 367-384.

[5] Blekhman, I. I., & V. B. Vasil'kov. . On some opportunities for improving vibration machines with self-synchronizing inert vibration exciters. Journal of Machinery Manufacture and Reliability, 2013, 42(3), 192-195.

[6] Potapenko, M. A.. Slow vibrations of unbalanced rotors (vibration exciters) during the disturbance of self-synchronization conditions. Journal of Machinery Manufacture and Reliability, 2008, 37(3), 228-229.

[7] Zhao, C. Y., Wen, B. C., & Zhang, X. L.. Synchronization of the four identical unbalanced rotors in a vibrating system of plane motion. Science in China Series E: Technological Sciences, 2010, 53(2), 405-422.

[8] José M. Balthazar, Felix, J. L. P. , & Brasil, R. M.. Some comments on the numerical simulation of self-synchronization of four non-ideal exciters. Applied Mathematics & Computation, 2005, 164(2), 615-625.

[9] Yamapi, R., & Woafo, P.. Dynamics and synchronization of coupled self-sustained electromechanical devices. Journal of Sound & Vibration, 2005, 285(4), 1151-1170.

[10] Zhang, N.. synchronous characteristics of a vibration piling system with electromechanical coupling. Journal of Vibroengineering, 2016, 18(5), 3305-3317.

[11] Zhang, X. , Kong, X. , Wen, B. , & Zhao, C.. Numerical and experimental study on synchronization of two exciters in a nonlinear vibrating system with multiple resonant types. Nonlinear Dynamics, 2015, 82(1-2), 987-999.

[12] Zhang, X. , Wen, B. , & Zhao, C. Vibratory synchronization transmission of a cylindrical roller in a vibrating mechanical system excited by two exciters. Mechanical Systems and Signal Processing, 2017, 96(12), 88-103.

[13] Zhang X , Wen B , Zhao C . Synchronization of three non-identical coupled exciters with the same rotating directions in a far-resonant vibrating system. Journal of Sound and Vibration, 2013, 332(9):2300-2317.

[14] Zhang X , Wen B , Zhao C. Vibratory synchronization transmission of two exciters in a super-resonant vibrating system. Journal of Mechanical Science & Technology, 2014, 28(6):2049-2058.

[15] Kandah, M. , & Meunier, J.. Theoretical and experimental study on synchronization of the two homodromy exciters in a non-resonant vibrating system. Shock & Vibration, 2015, 20(2), 327-340.

[16] Kandah, M. , & Meunier, J. . Theoretical and experimental study on synchronization of the two homodromy exciters in a non-resonant vibrating system. Shock & Vibration, 2015, 20(2), 327-340.

[17] Zhang, X. L. , Wen, B. C. , & Zhao, C. Y. .Synchronization of three homodromy coupled exciters in a non-resonant vibrating system of plane motion. Acta Mechanica Sinica (English Series), 2012, 28(5), 1424-1435.

[18] Kibirkštis E , Pauliukaitis D , Miliūnas, Valdas, et al. Synchronization of pneumatic vibroexciters operating on air cushion with feeding pulsatile pressure under autovibration regime[J]. Journal of Mechanical Science and Technology, 2018, 32(1):81-89.

[19] Kibirkštis E , Pauliukaitis D , Miliūnas, Valdas, et al. Synchronization of pneumatic vibroexciters under air cushion operating mode in a self-exciting autovibration regime[J]. Journal of Mechanical Science and Technology, 2017, 31(9):4137-4144.