# Research on physics-of-failure model for electromigration failure under multi-level stress profile based on cumulative damage theory

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***Abstract*—This paper focuses on the problem that current physics-of-failure model failed in calculating microelectronic devices' operational reliability under complex multi-level environment profile. A physics-of-failure model for electro-migration failure was analyzed as an example. In this paper, we made cumulative calculation and improved present model by using cumulative damage theory, and established new physics-of-failure model which is more applicable for electro-migration failure. This new model can simulate and predict the cycle time before failure under multi-level stress profile. At last, this paper gives an instance of calculation using the improved model under multi-level stress profile.**

***Keywords—formatting; multi-level stress; electro-migration; cumulative damage model; physics-of-failure model***

## I. INTRODUCTION

Using simulation technology to set up simulative model has become an important method and develop trend in electronic devices' operational reliability designing. However, most of the physics-of-failure models only allowed to input single stress while actual environment may provide complex multi-level stress profile. In that circumstance current physics-of-failure model would lost some accuracy in calculating devices' reliability and life under complicated environment. Because of such problem, a new physics-of-failure model with wider range of application is needed.

In this paper we improved present model by using cumulative damage theory, and established physics-of-failure model for electro-migration failure under multi-level stress profile, which can achieve simulation and prediction of cycle time before failure under multi-level thermal stress profile.

Electro-migration is transportation of metal ion in circuits or components caused by electric field force. Physics-of-failure model for electro-migration describes the influence of current density, wire material characteristics and component dimension on reliability of devices under the mechanism of electro-migration failure. Commonly used model expression is as follow [1]:

$$T_{TF} = \frac{Wd\theta^m}{Cj^n}\exp\left(\frac{E_a}{k\theta}\right) \tag{1}$$

Here, $W$ and $d$ are the width and thickness of metal wire; $\theta$ is absolute temperature; $j$ is the current density; $m$ and $n$ are failure strength index; $C$ is an constant related to the geometry size of metal, and its typical value is $8.264\times10^{-15}$; $E_a$ is active energy; $k$ is the Boltzmann constant, and its value is $1.381\times10^{-23}$ J/K.

## II. CUMULATIVE DAMAGE MODEL

Though dozens of cumulative damage models have been already set up, most of them were just qualitative model and can't give a actual reliability life. Meanwhile, some formulas are difficult to calculate, some need massive test data and some can only perform in specific situation.

In practice, we mostly use single linear cumulative damage theory(Miner model), modified Miner model [2][3], bilinear cumulative damage theory(Manson model) and nonlinear cumulative damage theory(Carten-Dolar model) [4][5].

Given that modified Miner model is more accurate than Miner model. Manson model and Carten-Dolar models' higher calculating complexity makes electro-migration model more complicate to derive the cumulative damage formula.

Hence, this paper chose modified Miner model to calculate and obtain the cumulative damage formula for electro-migration.

### *A. Single linear cumulative damage theory (Miner model)*

Whether the stress cycle is generated by heat, vibration or impact, each stress cycle produces some strain. In Miner model, it used $R$ to represent damage ratio and its mathematical expression is [6]:

$$R_{\mathrm{n}} = \sum \frac{n_i}{N_i} = 1, \qquad i = 1, 2, 3, \cdots \tag{2}$$



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Here, $n_i$ is working cycles under stress stage $i$(equals one if the stress is constant); $N_i$ is the number of cycles to failure under stress stage $i$; $R_n$ is called the Miner cumulate damage ratio.

When the sum of damage ratio equals one, a failure occurs.

### B. Modified Miner model

$$R_n=\sum\frac{n_i}{N_i}=\alpha,\qquad i=1,2,3,\cdots \tag{3}$$

In this formula, $\alpha$ is the critical damage value. It is confirmed through experiments that $\alpha$ is not a constant but a random variable with a certain distribution, and the average value is 1. The essence of the law is: the assumption is eliminated that the critical damage value is 1, and the critical damage value is determined by tests or experience.

The modified Miner model's accuracy of life estimation depends on the selection of $\alpha$. The optimum $\alpha$ is not only related to the structure and material of the product, but also to the stress type, stress level and loading order. Therefore, it would be better to select the same type of device, simulate the working environment, perform test under the load spectrum of its actual working environment, measure the value, and use it to estimate the life of similar devices. In this way, the modified Miner model's accuracy of life estimation can be greatly improved [3].

### C. The Determination of α

- Simulate the load spectrum in the actual working environment to obtain the cycle life N;
- Determine the time of operation $t_i$ or the number of cycles $n_i$;
- According to the physics-of-failure model, calculate the life $T_i$ or cycle life $N_i$ under the operation of each stress level;
- Calculate $\alpha$ based on the formula (3).

## III. Physics-of-Failure Model for Electric-Migration Failure Based on Modified Miner Model and Cumulative Damage Theory

### A. Establishment of cumulative damage failure model for electro-migration failure under segmented multi-level stress profile

In this part, we used electro-migration's physics-of-failure model and modified Miner model to establish a new model for segmented multi-level stress profile.

A typical segmented temperature stress profile is shown in Fig. 1. We separate this full stress cycle into 4 stages: 0 to $t_1$ is stage 1, $t_1$ to $t_2$ is stage 2, $t_2$ to $t_3$ is stage 3, $t_3$ to $t_4$ is stage 4.

For the stress profile shown in Fig. 1, the temperature in stage 1 and stage 3 remains unchanged, and the temperature in stage 2 and stage 4 changes slowly.

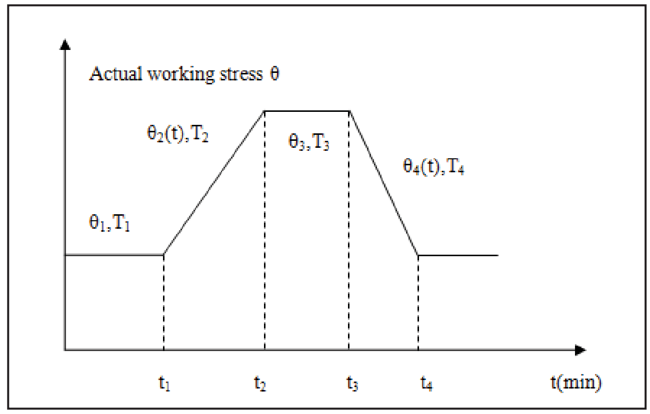


Fig. 1. Typical segmented temperature multi-level stress profile

#### 1) Damage ratio at each stage

*a)* For stage 1 and stage 3, the temperature is constant, the physics-of-failure model directly gives out the life by formula (1), and the damage ratio is calculated by formula (3).

$$T_{TF}\left(\theta_1\right)=\frac{Wd\theta_1^{\,m}}{Cj^n}\exp(\frac{Ea}{k\theta_1}) \tag{4}$$

$$R_1=\frac{T_1}{T_{TF}(\theta_1)} \tag{5}$$

$$T_{TF}\left(\theta_3\right)=\frac{Wd\theta_3^{\,m}}{Cj^n}\exp(\frac{Ea}{k\theta_3}) \tag{6}$$

$$R_3=\frac{T_3}{T_{TF}(\theta_3)} \tag{7}$$

*b)* For stage 2 and stage 4, because the temperature changes, the damage ratio is calculated by formula. (3)

$$R_2=\int_{t_1}^{t_2}\frac{1}{T_{TF}\left(\theta_2(t)\right)}dt=\int_{t_1}^{t_2}\frac{1}{\dfrac{Wd\theta_2(t)^m}{Cj^n}\exp\left(\dfrac{Ea}{k\theta_2(t)}\right)}dt \tag{8}$$

$$R_4=\int_{t_3}^{t_4}\frac{1}{T_{TF}\left(\theta_4(t)\right)}dt=\int_{t_3}^{t_4}\frac{1}{\dfrac{Wd\theta_4(t)^m}{Cj^n}\exp\left(\dfrac{Ea}{k\theta_4(t)}\right)}dt \tag{9}$$

#### 2) The total damage ratio

Based on modified Miner model, damage ratio in a cycle equals to the sum of damage ratio in each stage.

$$R_n=R_1+R_2+R_3+R_4 \tag{10}$$

*3) Cycles before failure*

$$N = \frac{\alpha}{R_n} = \alpha / (\frac{T_1}{T_{TF}(\theta_1)} + \int_{t_1}^{t_2} \frac{1}{\frac{Wd\theta_2(t)^m}{Cj^n} \exp\left(\frac{Ea}{k\theta_2(t)}\right)} dt + \tag{11}$$

$$\frac{T_3}{T_{TF}(\theta_3)} + \int_{t_3}^{t_4} \frac{1}{\frac{Wd\theta_4(t)^m}{Cj^n} \exp\left(\frac{Ea}{k\theta_4(t)}\right)} dt)$$

*B. Establishment of cumulative damage failure model for electro-migration failure under general multi-level stress profile*

In this part, we used electro-migration's physics-of-failure model and modified Miner model to establish a new model for general multi-level stress profile.

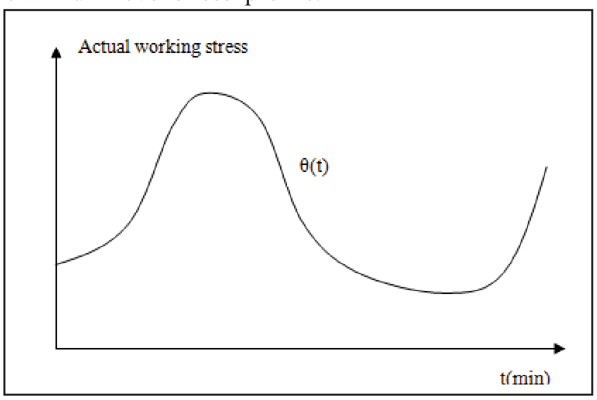


Fig. 2. General multi-level stress profile

For general multi-level stress profile shown in Fig. 2,θ(t) is the function of stress changes with time.

*1) Cycles before failure at t*

$$T_{TF}\left(\theta(t)\right) = \frac{Wd\theta(t)^m}{Cj^n} \exp\left(\frac{Ea}{k\theta(t)}\right) \tag{12}$$

*2) The total damage ratio*

$$R_n = \int_0^T \frac{1}{T_{TF}\left(\theta(t)\right)} dt = \int_0^T \frac{Cj^n}{Wd\theta(t)^m \exp\left(\frac{Ea}{k\theta(t)}\right)} dt \tag{13}$$

*3) Cycles before failure*

$$N = \frac{\alpha}{R_n} = \frac{\alpha}{\int_0^T \frac{Cj^n}{Wd\theta(t)^m \exp\left(\frac{Ea}{k\theta(t)}\right)} dt} \tag{14}$$

## IV. EXAMPLE

Fig. 1 is a typical segmented multi-level temperature stress profile, and the parameters of temperature and time are shown in TABLE I.

TABLE I. THE PARAMETER OF TEMPERATURE STRESS

| Stage | *T*(K) | *t*(min) |
|---|---|---|
| 1 | 298 | $0<t<30$ |
| 2 | $5t+148$ | $30\le t<40$ |
| 3 | 348 | $40\le t<100$ |
| 4 | $-5t+848$ | $100\le t<110$ |

A certain type of microelectronic device was taken as an example. In accordance with EM model and the formula (14), the device's internal process parameter values can be confirmed in Table 2.

TABLE II. THE PARAMETER OF DEVICE

| Parameter | Value |
|---|---|
| $W$ | 1 μm |
| $d$ | 0.8 μm |
| $j$ | 30 mA/cm$^2$ |
| $m$ | 3 |
| $n$ | 3 |
| $E_a$ | 0.59 eV |
| $C$ | $8.264\times10^{-15}$ |
| $k$ | $1.381\times10^{-23}$ J/K |

For parameters' values in Fig 1 and Table 1, and According to the formula (14), the total cycle of cumulative damage of this certain type of microelectronic device is:

$$N = \frac{\alpha}{\int_0^T \frac{Cj^n}{Wd\theta(t)^m \exp\left(\frac{E_a}{k\theta(t)}\right)} dt}$$

$$= \frac{\alpha}{\frac{30}{T_{TF}\left(298\right)} + \int_{30}^{40} \frac{dt}{T_{TF}\left(5t+148\right)} + \frac{60}{T_{TF}\left(348\right)} + \int_{100}^{110} \frac{dt}{T_{TF}\left(848-5t\right)}}$$

$$= 9399\alpha$$

$$T_{TF} = N \times 110 = 1033890\alpha$$

Therefore, the time before electro-migration failure occurs is 17232$\alpha$ hours.

## V. CONCLUSION

In this paper, we used the modified Miner model to estimate the cumulative damage of the complex multi-level stress profile for the electro-migration failure physics.

We established cumulative damage failure model for both segmentation multi-level stress profile and general functionalized multi-level stress profile for electro-migration failure respectively.

The applicability of the model is proved by a calculation case of a micro-electronic device. This model calculated cycles before failure and time before failure of electro-migration under multi-level temperature stress profile. This paper provided a new theoretical method and solution for electro-migration physics-of-failure model's application in simulation evaluation of microelectronic device reliability.

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