Improved GM(1,N) Model for Equipment Development Cost Prediction

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***Abstract*—Aiming at the large error of traditional grey GM (1,N) model in cost prediction, this paper analyses the defects of the model itself, and puts forward a model improvement scheme. Firstly, on the basis of grey relational analysis, weighting factor is introduced to optimize the background value; secondly, linear correction term and grey action are introduced to make the model structure more reasonable, and an improved grey GM (1,N) model is proposed. Finally, an example is given to simulate and compare the model. The results show that the improved model can effectively improve the prediction accuracy.**

Keywords-weapon equipment; cost prediction; GM (1,N) model; background value optimization

I. INTRODUCTION

With the increasing complexity of weapon equipment and the increasing cost of weapon equipment, it is particularly necessary to explore the factors affecting the cost of weapon equipment and its estimation model, and accurately estimate the cost of the development process of weapon equipment. At the same time, the cost estimation has important guiding significance to strengthen the scientific management of equipment cost, the comprehensive development plan and the comprehensive economic benefits, to further improve the management of equipment cost plan, to rationally use military expenditure and to enhance the operational efficiency of equipment [1-2].

As one of the main applications of grey system theory, grey Prediction is suitable for weapon equipment cost Prediction with small sample and poor information because of its simple principle, simple calculation and less data information. In the [3], although grey GM (1,1) model is applied to weapon system cost prediction, and the prediction accuracy is improved, the model can only reflect the changing law of weapon equipment cost itself, and cannot reflect the influence of external factors on the changing law of equipment cost synthetically. Obviously, its Prediction results are not convincing. Reference [4-5] are based on grey relational analysis, and put forward multi-factor grey model and multi-variable grey model respectively, which take into account the influence of many factors, but do not involve data processing and background value optimization, and its prediction accuracy is difficult to meet the requirements. Therefore, it is necessary to establish an improved multi-factor grey prediction model considering the factors affecting the cost comprehensively.

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In this paper, aiming at the large error of traditional GM (1, N) model, a cost prediction method based on improved GM (1, N) model is proposed, taking aircraft development cost as the research object. Firstly, on the basis of grey relational analysis, weighting factor is introduced to optimize the background value; secondly, linear correction term and grey action are introduced to overcome the shortcomings of the model itself, so that the model structure is more reasonable, and an improved GM (1,N) model is established; finally, some aircraft development costs are predicted and analyzed and compared with the models.

II. TRADITIONAL GM(1,N) MODEL AND ITS PROBLEMS

A. Traditional GM(1,N) Model

Grey theory was first put forward by Professor Deng in 1982, and has been widely used in various fields. Grey GM(1,N) model is one of the main contents of grey system theory. It is a first-order differential equation composed of multi-variables. It mainly fits and predicts the dominant factors and related variables in some complex systems under the condition of "small sample and poor information", reveals the changing rules of dominant factors, and predicts the future development and change situation [6-7].

The traditional GM (1,N) model prediction steps are as follows.

Step 1 conduct cumulative generation operations

Assume $\mathbf{X}_i^{(0)}(i=1,2,\cdots,n)$ is primitive nonnegative sequence. If there are m data corresponding to n variables, n sequence $\mathbf{X}_i^{(0)}$ can be formed.

$$\mathbf{X}_i^{(0)}=\left\{x_i^{(0)}(1),x_i^{(0)}(2),\cdots,x_i^{(0)}(m)\right\}$$

Accumulate the original sequence once to generate n sequences.

$$\mathbf{X}_i^{(1)}=\left\{x_i^{(1)}(1),x_i^{(1)}(2),\cdots,x_i^{(1)}(m)\right\}$$

In the formula, $x_i^{(1)}(k)=\sum_{j=1}^{k}x_i^{(0)}(j),\quad(k=1,2,\cdots,m)$.

Step 2 generate the neighboring mean sequence

In order to make the cumulative generated sequence smoother, the nearest mean value of $\mathbf{X}_i^{(0)}$ is generated below.

$$\mathbf{Z}_1^{(1)} = (z_1^{(1)}(2), z_1^{(1)}(3), \cdots, z_1^{(1)}(m))$$

In the formula, $z_1^{(1)}(k) = 0.5[x_1^{(1)}(k) + x_1^{(1)}(k-1)]$ (1)

Then GM (1, N) model is

$$x_1^{(0)}(k+1) + az_1^{(1)}(k+1) = \sum_{i=2}^{n} b_i x_i^{(1)}(k+1) \qquad (2)$$

The corresponding whitening equation is

$$\frac{dx_1^{(1)}(k)}{dt} + ax_i^{(1)}(k) = \sum_{i=2}^{n} b_i x_i^{(1)}(k) \qquad (3)$$

In the formula, $\mathbf{Z}_1^{(1)}$ is the background value, a is the system development coefficient, and b_i is the system driving coefficient.

Step 3 establish approximate time response formula

The estimation of parameter column $\hat{a}$ can be obtained by least square method.

$$\hat{a} = (\mathbf{B}^{\mathbf{T}}\mathbf{B})^{\mathbf{-1}}\mathbf{B}^{\mathbf{T}}\mathbf{Y} \qquad (4)$$

$$\hat{a} = \left[a, b_2, b_3 \cdots b_n\right]^T \qquad (5)$$

In the formula,

$$\mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & x_2^{(1)}(2) & \cdots & x_n^{(1)}(2) \\ -z^{(1)}(3) & x_2^{(1)}(3) & \cdots & x_n^{(1)}(3) \\ \vdots & \vdots & \cdots & \vdots \\ -z^{(1)}(m) & x_2^{(1)}(m) & \cdots & x_n^{(1)}(m) \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} x_1^{(0)}(2) & x_1^{(0)}(3) & \cdots & x_1^{(0)}(m) \end{bmatrix}^{\mathrm{T}}$$

The approximate time response formula of GM(1,N) model can be obtained.

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{1}{a}\sum_{i=2}^{n} b_i x_i^{(1)}(k+1)\right]e^{-ak}$$

$$+\frac{1}{a}\sum_{i=2}^{n} b_i x_i^{(1)}(k+1), \; k = 1,2,3,\cdots,n-1 \qquad (6)$$

Step 4 cumulative reduction and error analysis

The prediction model of GM (1,N) can be obtained by using the cumulative reduction operation for $\hat{x}^{(1)}(k+1)$.

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \qquad (7)$$

B. Problems in Traditional Model

On the basis of in-depth study of traditional model definition and modeling process, it is found that the model has defects in modeling mechanism, parameter usage and model structure, which leads to the instability of the traditional model [8].

- The mechanism defect. From the traditional GM (1,N) model modeling process, it can be seen that there is a highly idealized simplified process from the solution of model whitening equation to the derivation of approximate time response function. That is to say, assuming that the change range of accumulation sequence $\mathbf{X}_i^{(1)}$ is very small, $\sum_{i=2}^{n} b_i x_i^{(1)}(k)$ is regarded as a grey constant. In fact, different variables have different characteristics, dynamic laws and development trends. It is difficult to ensure that the change range of $\mathbf{X}_i^{(1)}$ is very small, which is not in line with the actual situation, leading to the instability of traditional models in cost Prediction.
- Parameter defect. The parameter sequence of the traditional model is based on (2) and estimated by least square method, which can minimize the deviation of the system characteristic data sequence. However, the time response formula of the model is not derived from (2), but from (3). That is to say, the estimates of parameter series are derived from (2), but in the traditional model, the parameter series are taken as the parameters of (6). This "misalignment" leads to the instability of model performance.
- Structural defects. Equation (2) shows that the structure of the traditional model is relatively simple, the grey action is not excavated from the model itself, and the influence of the term k linear relationship on the performance of the model is not considered. At the same time, when $N = 1$, the model cannot be equivalent to GM (1,1) model. Therefore, the defect of the model structure is also the objective reason for the large prediction error of the model.

III. Improved GM(1,N) Model Based on Background Value Optimization

A. Grey Relational Analysis

When using GM (1,N) model to predict costs, it is necessary to clarify the relationship between costs and various factors. The selection of significant factors in the model will directly affect the final prediction accuracy. The selection of significant factors in the prediction model is too few to effectively reflect the impact of external factors on equipment costs. Excessive selection of significant factors will result in excessive gray level of prediction results[9]. Therefore, the grey relational analysis of the original data can effectively improve the prediction accuracy of the model.

Assume the original data matrix collected is $\mathbf{S}^{(0)}$.

The parent sequence is

$$\mathbf{S}_1^{(0)} = \{s_1^{(0)}(1), s_1^{(0)}(2) \cdots s_1^{(0)}(m)\}$$

The subsequence is

$$\mathbf{S}_j^{(0)} = \{s_j^{(0)}(1), s_j^{(0)}(2) \cdots s_j^{(0)}(m)\},$$

$j = 2,3,\cdots,n$

Dimensionless processing of raw data.

$$s_1'^{(0)}(k) = s_1^{(0)}(k) / s_1^{(0)}(1)$$

$$s_j'^{(0)}(k) = s_j^{(0)}(k) / s_j^{(0)}(1)$$

In the formula, $k = 1, 2, \cdots, m$.

Thus the grey correlation coefficient of sequence $\mathbf{S}_1^{(0)}$ and $\mathbf{S}_j^{(0)}$ at point k can be obtained.

$$\xi_j(k) = \frac{\min \Delta_j(k) + \rho \max \Delta_j(k)}{\Delta_j(k) + \rho \max \Delta_j(k)} \tag{8}$$

In the formula, $\Delta_j(k) = | s_1'^{(0)}(k) - s_j'^{(0)}(k) |$; $\rho (0 < \rho < 1)$ is the resolution coefficient. The theoretical study shows that when $\rho \le 0.5463$, the resolution of grey relational degree analysis of the model is better [10].

The correlation degree between the parent sequence $\mathbf{S}_1^{(0)}$ and the j^{th} subsequence $\mathbf{S}_j^{(0)}$ can be calculated as follows.

$$\gamma_j = \frac{1}{m} \sum_{k=1}^{m} \xi_j(k) \tag{9}$$

If $\gamma_j \ge 0.5$, the parent sequence can be considered to be associated with the corresponding subsequence.

B. Background Value Optimization

The construction form of background value is one of the main factors affecting the prediction accuracy of the model. The improvement of background value can effectively reduce the prediction error of the model and enhance the applicability of the model [11]. On the basis of studying the relationship between the background value of the model and the actual background value, the optimization formulas of the background value are respectively given in [12-14], which can effectively reduce the error caused by the background value of the model and improve the prediction accuracy. On the basis of studying the construction of background value, this paper puts forward an optimization method of GM (1,N) model background value, so as to improve the prediction accuracy of the model.

Reconstructing (1) by introducing weighting factor λ.

$$z^{(1)}(k) = \lambda x^{(1)}(k) + (1-\lambda) x^{(1)}(k-1),$$

$$k = 2, 3, \cdots, n \tag{10}$$

$\lambda \ (0 < \lambda < 1)$ is called the weighted coefficient, the (1) is the special case when $\lambda = 0.5$.

When the average relative error of the difference between the original value and the prediction value of the model reaches the minimum,

$$\overline{\Delta} = \frac{1}{n} \sum_{k=1}^{n} \frac{\left| x^{(0)}(k) - \hat{x}^{(0)}(k) \right|}{x^{(0)}(k)} = \min \tag{11}$$

At this time, the value of λ which satisfies the above conditions can calculate the minimum error of the prediction result, that is the optimal solution.

C. Improved GM(1,N) Model

Aiming at the shortcomings of the traditional grey GM (1, N) model itself, based on the background value optimization, the traditional model is modified and optimized, and an improved GM (1,N) model is proposed. In the improved GM (1,N) model, there is no hypothesis that the change range of cumulative sequence $\mathbf{X}_i^{(1)}$ is very small and $\sum_{i=2}^{n} b_i x_i^{(1)}(k)$ is regarded as a grey constant; there is no "dislocation" between the parameter series of the model and the whitening equation; at the same time, linear correction term and grey action are introduced in the improved model to make the structure of the model more consistent [15].

By introducing the linear correction term kc and the grey action d, the (2) can be transformed.

$$x_1^{(0)}(k+1) + a z_1^{(1)}(k+1) = \sum_{i=2}^{n} b_i x_i^{(1)}(k+1) + kc + d \tag{12}$$

The corresponding whitening equation is as follows.

$$\frac{dx_1^{(1)}(k)}{dt} + a x_i^{(1)}(k) = \sum_{i=2}^{n} b_i x_i^{(1)}(k) + (k-1)c + d \tag{13}$$

The parameters of the improved model are listed as follows.

$$\hat{a}' = \left[a, b_2, b_3 \cdots b_n, c, d \right]^T \tag{14}$$

And the least squares estimate satisfies

$$\hat{a}' = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{Y} \tag{15}$$

In the formula,

$$\mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & x_2^{(1)}(2) & \cdots & x_n^{(1)}(2) & 1 & 1 \\ -z^{(1)}(3) & x_2^{(1)}(3) & \cdots & x_n^{(1)}(3) & 2 & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ -z^{(1)}(m) & x_2^{(1)}(m) & \cdots & x_n^{(1)}(m) & m-1 & 1 \end{bmatrix}$$

$$\mathbf{Y} = \left[x_1^{(0)}(2) \quad x_1^{(0)}(3) \quad \cdots \quad x_1^{(0)}(m) \right]^{\mathrm{T}}$$

The following time response formula of the improved GM (1,N) model is as follows.

$$\hat{x}_1^{(1)}(k+1) = \sum_{t=1}^{k} [\mu_1 \sum_{i=2}^{n} \mu_2^{t-1} b_i x_i^{(1)}(k-t+2)] +$$

$$\mu_2^k \hat{x}_1^{(1)}(1) + \sum_{j=0}^{k-1} \mu_2^j [(k-j+1)\mu_3 + \mu_4] \tag{16}$$

In the formula, $\mu_1 = \dfrac{1}{1+0.5a}$, $\mu_2 = \dfrac{1-0.5a}{1+0.5a}$, $\mu_3 = \dfrac{kc}{1+0.5a}$, $\mu_4 = \dfrac{d-kc}{1+0.5a}$, $k = 1, 2, \cdots, n-1$.

Finally, the prediction value $\hat{x}^{(0)}(k+1)$ of GM (1,N) model can be obtained by using the cumulative reduction operation of (7).

D. Calculation Steps of Improved Model

Based on the Prediction model improvement countermeasures, the improved model is applied to weapon equipment cost Prediction, and the calculation steps are summarized as follows.

Step 1 calculate grey correlation degree

The original data matrix $\mathbf{S}^{(0)}$ is established by collecting the index of weapon equipment cost and related performance parameters, and the grey relational degree analysis is carried out to find the correlation degree γ_j between cost and influencing factors. Then the data sequence satisfying the minimum correlation degree ($\gamma_j \geq 0.5$) between dependent variable and independent variable is selected as the original data sequence $\mathbf{X}^{(0)}$.

Step 2 calculate the optimal background value

The original modeling data are accumulated once to generate an accumulation sequence. The background value function $z^{(1)}(k)$ is constructed by (10), and the parameters are estimated. The matrix **B** and **Y** are constructed. Then we judge whether matrix **B** is non-singular. If matrix **B** is singular, repeat Step1; otherwise the improved GM (1,N) model is established by calculating the parameter value, and the weighting factor $\overline{\Delta}$ satisfying the minimum λ is obtained by combining with the simulation of MATLAB.

Step 3 establish an improved GM(1,N) model based on optimal background value

The improved GM (1,N) model is reconstructed by the optimized background value, and the value of **B** is obtained. According to the least squares method, the value of $a, b_2, b_3 \cdots b_n, c, d$ is calculated by (14) and (15). The value of $\hat{x}^{(1)}(k+1)$ is obtained by substituting them into (16).

Step 4 calculate prediction value and error

Equation (7) is used for cumulative reduction to obtain the final prediction value $\hat{x}^{(0)}(k+1)$ of the model and calculate the relative error; Step 2~4 is repeated until the prediction task is completed . Finally, the prediction value and error are output.

The calculation flow chart of the improved GM (1,N) model is shown in Fig. 1.

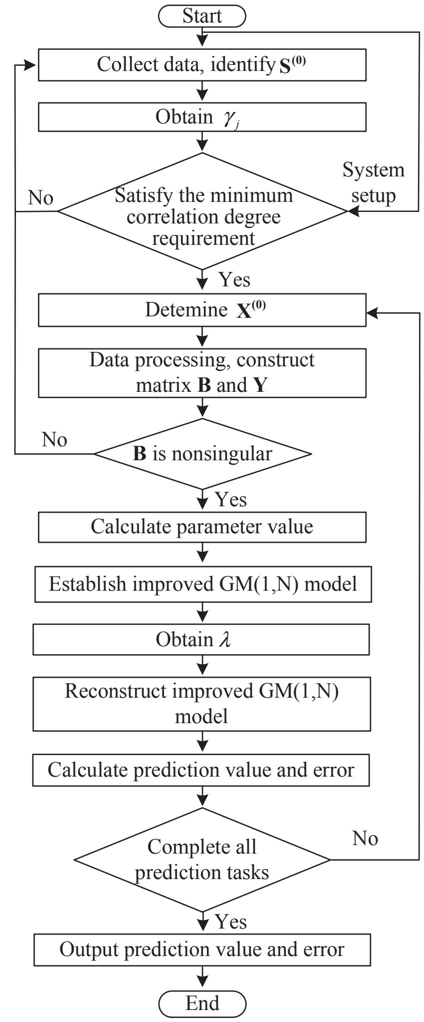

Figure 1. Calculation flow chart of improved GM (1,N) model

IV. EXAMPLE CALCULATION

In order to verify the superiority of the improved GM (1,N) model, the data in [16] are used for calculation and analysis, as shown in TABLE I. Among them, X_0 is the cost of aircraft development (1 million dollar), X_1 is the wingspan (m), X_2 is the wingspan area (m^2), X_3 is the maximum take-off mass (kg), X_4 is the maximum level Mach number, and X_5 is the range (km).

TABLE I. PRIMARY DATA OF AIRCRAFT DEVELOPMENT COST AND CHARACTERISTIC PARAMETERS

Aircraft model	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
X_0	62.5	51	68	67	92.6	29	57.7	31.5	45	37.1
X_1	14.7	19.54	15.16	14.7	13.56	13.956	13.05	9.45	9.13	10.8
X_2	62.04	52.49	62.04	62	78	37.3	56.49	27.87	41	45.7
X_3	33000	33724	38800	44350	27216	17800	36741	19187	17000	24500
X_4	2.35	2.34	2.35	1.8	2	2.35	2.5	2	2.2	1.8
X_5	3680	3220	3400	4000	4050	2550	4445	3890	3333	3600

Step 1 calculate grey correlation degree

In Table I, the original data matrix $\mathbf{S}^{(0)}$ of grey relational analysis model was established by analyzing the development cost and the five cost influencing factors of the first six aircraft. The last four types of aircraft are to be predicted. The resolution coefficient ρ is 0.5, and the grey correlation degree γ_j=(0.6910, 0.8187, 0.6844, 0.6120, 0.6531), the original data sequence $\mathbf{X}^{(0)}$ is established.

Step 2 calculate the optimal background value

The original modeling data are accumulated once to generate an accumulation sequence. The background value function $z^{(1)}(k)$ is constructed by (10), and the parameters are estimated. The matrix **B** and **Y** are constructed. Then we judge whether matrix **B** is non-singular. If matrix **B** is singular, repeat Step1; otherwise the improved GM (1,6) model is established by calculating the parameter value. The weighting factor λ =0.086, satisfying the minimum $\bar{\Delta}$ by combining with the simulation of MATLAB.

Step 3 establish an improved GM (1,6) model based on optimal background value

The improved GM (1,6) model is reconstructed by using the optimized background value, and the value of **B** is obtained. According to the least square method, the value of $a, b_2, b_3 \cdots b_n, c, d$ is obtained by using (14), (15), and the value of $\hat{x}^{(1)}(k+1)$ is obtained by substituting them into (16).

Step 4 calculate prediction value and error

The final prediction value $\hat{x}^{(0)}(k+1)$ of the model is obtained by using (7) and the relative error is calculated. Step 2-4 is repeated until the prediction task is completed. Finally, the prediction value and error are output. The results are shown in TABLE II.

V. Comparative Analysis

In order to highlight the superiority of the improved GM (1,6) model in cost prediction, the prediction results are compared with the linear regression model and the traditional GM (1,6) model. The prediction results and relative errors are shown in TABLE II. The average relative errors of the three models are 10.44%, 14.85% and 2.29% respectively. From the prediction results in TABLE II, it can be seen that although the above three methods can predict the development cost of aircraft, from the average relative error, the improved GM (1,6) model proposed in this paper has the smallest error and the traditional GM (1,6) model has the greatest error when the original data information satisfies the prediction requirements of the model through the analysis and comparison of the results. Combining with the prediction results in TABLE II, this paper further presents the comparison curves of actual and prediction values and relative errors of the three methods, as shown in Fig. 2 and Fig. 3.

TABLE II. Comparison of Prediction Results of Different Models

Aircraft model	Development cost	Linear regression model		Traditional model		Proposed model	
		Prediction value (1 million dollar)	Relative error (%)	Prediction value (1 million dollar)	Relative error (%)	Prediction value (1 million dollar)	Relative error (%)
7	57.70	61.99	7.43%	64.43	11.66%	59.34	2.85%
8	31.50	28.20	10.46%	27.58	12.44%	29.93	4.99%
9	45.00	41.33	8.15%	40.44	10.13%	45.58	1.30%
10	37.10	42.93	15.71%	46.44	25.16%	37.10	0.00%

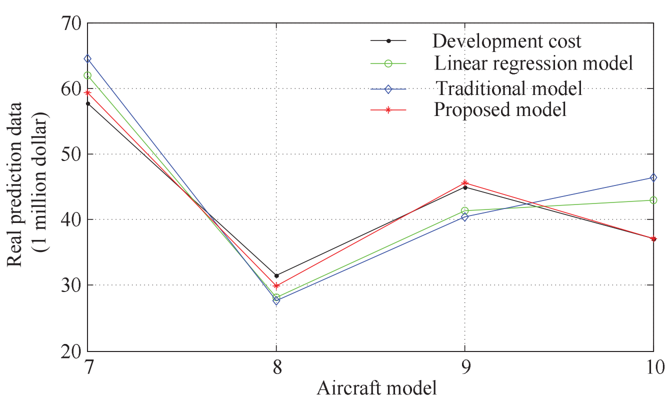

Figure 2. Comparison curve of actual value and prediction value

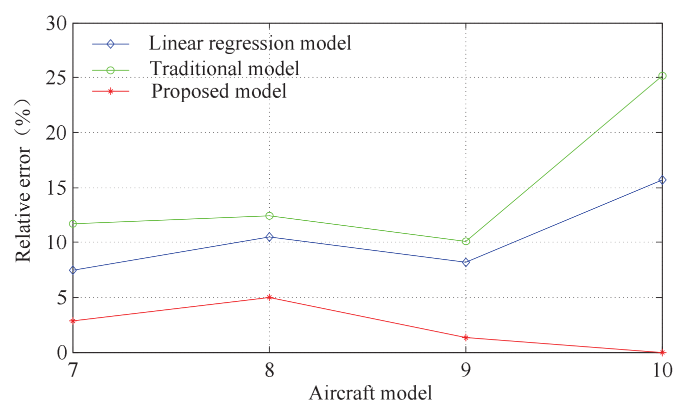

Figure 3. Comparison curve of relative error

As can be seen from Fig. 2, the aircraft development cost is volatile and unstable. Multivariate linear regression model and traditional GM (1,6) model have poor fitting effect with the actual data curve. In contrast, the prediction accuracy is lower. It cannot meet the actual demand well when the prediction accuracy is high. The prediction data of the improved model proposed in this paper fit the actual data better, is closer to the actual purchase cost curve, and the results are more in line with the actual situation. It shows that the prediction accuracy of this method is higher.

From the comparative analysis of Fig. 3, it can be concluded that the linear regression model and the traditional GM (1,6) model have larger prediction errors, among them the traditional GM (1,6) model has the greatest error, while the prediction method proposed in this paper has the smallest error. This shows that the improved model is better than the linear regression model in the case of small sample and poor information, and the linear regression model is superior to the traditional GM (1,N) model.

In summary, the improved GM (1,N) model proposed in this paper improves the fitting effect between the model and the actual data by optimizing the background value and reconstructing the model on the basis of grey relational analysis. The prediction results are more in line with the actual situation, giving full play to the advantages of less data and high prediction accuracy of grey system prediction. The requirement of original data is not high, and there is no need to have a typical distribution law. In contrast, it is more practical. Therefore, the improved GM (1,N) model proposed in this paper is superior to the linear regression model and the traditional GM (1,N) model in terms of the accuracy of aircraft development cost prediction, and has a strong practicability in aircraft cost prediction.

VI. Conclusions

In the Prediction of weapon equipment cost, aiming at the problems of large error and poor fitting effect in traditional GM (1,N) model prediction, the traditional GM (1,N) model is improved on the basis of studying the traditional GM (1,N) model. Through grey relational analysis, the relativity between data and prediction objects are improved. Meanwhile, weighting factors are introduced to reconstruct background values, which can dynamically adjust the size of background values according to the changes of original modeling data. Then linear correction term and grey action are introduced to overcome the defects of the model itself, so as to make the structure of the model more reasonable. It effectively solves the problem of large prediction error, improves the prediction accuracy of the model, and has certain applicability in the actual cost prediction. Finally, the prediction results of linear regression model, traditional GM (1,N) model and the improved model proposed in this paper are obtained through an example. And the comparisons and analyses are made with the actual development costs. The results show that the optimized GM (1,N) model can significantly improve the prediction accuracy, and is suitable for weapon equipment cost prediction. It has important application value for strengthening equipment cost management and reasonably formulating equipment development planning.

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