

# Life Extrapolation Model for Lithium-ion Battery with Accelerated Degradation Test

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**Abstract**—Battery life estimation is a key part for lithium cells for a long cycle life. The purpose of this paper is to develop a life extrapolation model for evaluating the lithium-ion battery cycle life with accelerated degradation test (ADT) data. An ADT is carried out including lithium-ion battery cells discharged with different current. The ADT data are used for parameterization with the accelerated model and distribution model. The lifetime of normal working condition is obtained by the fusion of accelerated model and accelerated data. To improve ability for life extrapolation, the proposed method is modeled with uncertainty expression by confidence lower limits and confidence lower limits for the reliability for the extrapolated life. Finally, extrapolation trajectory with uncertainty expression is obtained and the extrapolation result indicates that the proposed model can provide more accurate estimates with life extrapolation. In addition, the remaining useful life corresponding to any discharge current can be also calculated.

**Keywords**- accelerated degradation test; life extrapolation model; lithium-ion battery

## I. INTRODUCTION

Lithium-ion batteries play a crucial role for the energy storage systems. As the high requirement for varieties of applications, accurate life prediction of lithium-ion battery is a key part for the product reliability [1,2]. However, the long service life of lithium-ion batteries up to 15~20 years limits the practical application of the life prediction. Therefore, it is significant for finding solutions to estimate the battery life in the limit time. The solid electrolyte interface (SEI) is the major factor for capacity fade of lithium-ion batteries [3,4]. As the battery continues to circulate, the chemical reactions between the negative electrode and the electrolyte are intensified [5-9]. That leads to that the SEI film continues to accumulate and thicken. As a solution of time limit, accelerated testing is applied through increasing stress on the working condition of lithium-ion batteries. The degradation features are recorded to describe the capacity fade during the process of accelerated testing. The method of this accelerated testing is called accelerated degradation test (ADT) [10].

Due to that the ADT is able to accelerate the capacity fade of lithium-ion battery, the degradation process can be modeled with the accelerated degradation data. Furthermore, the lifetime prediction can also be obtained. Approaches for predicting battery life based on the accelerated degradation test (ADT) model have been developed and used. Gang Ning et al.

conducted the cycle test for the Sony US 18650 Li-ion batteries with 3 different discharge rates (1C, 2C, and 3C) [11]. Through the accelerated experiment, the authors got conclusion that the capacity degraded more rapidly and the resistance increase was more with the higher discharge rate. This study well interpreted the mechanism of battery degradation with high discharge rate. Thomas et al. analyzed the effect of cylindrical lithium-ion battery on the different temperature for the fixed charging rate and cycle interval [12]. The results showed that the battery had faster life degradation and shorter cycle life when working at high temperatures. Zheng et al. used the temperature and state of charge (SoC) of lithium-ion battery as the accelerated factors to study the lifetime with different work condition [13]. The experiment was carried out by single stress loading. Accelerated life test results showed that for every 15 °C increase, the lifetime was reduced by approximately 17.35%. However, in these studies, the battery life of normal level, which is a key part of product reliability, has not been incorporated in the ADT model.

For the battery life extrapolation of normal level, Nelson compared the mixed effect model with mathematical fitting model [14]. The result indicated that the extrapolation performance of mathematical fitting model was better. Unfortunately, the extrapolation performance was also unsatisfied with mathematical fitting model when the degradation data were not sufficient. The stochastic process was applied to describe the random effects of degradation test. However, the calculation process is too complicated and limits the scope of application [15-17]. Worse more, there is still a problem of weak ability for the stochastic process supporting uncertainty presentation. Shiau and Lin proposed the nonparametric degradation trajectory model [18]. However, there were also problems with unsatisfactory extrapolation effects.

For the studies stated above, it can be seen that there are two problems to be solved for life extrapolation model with accelerated degradation test. The first is to simplify the calculation model for reducing the amount of calculation. The second is to improve the ability for uncertainty presentation. To solve the problems stated above, this paper proposes a new method for life extrapolation with uncertainty bounds mixed the accelerated model and life distribution. For the first issue, to improve the robustness, the life extrapolation is modeled fused the accelerated model and accelerated working condition. For the second issue, the uncertainty of life extrapolation is

presented by the two factors. One of them is confidence lower limits for the extrapolated life with the fixed number of samples. The another one is confidence lower limits for the reliability with a given lifetime. The two factors are obtained by the life distribution and accelerated model. Finally, to validate the proposed model, a case study is carried out with accelerated data of lithium-ion battery.

The remainder of this paper is classified into four parts as follows. The accelerated model and life distribution are introduced in Section 2. The framework of the proposed model is developed in Section 3. In Section 4, we provide the case study and analysis for the results. In Section 5, the conclusions are drawn and the future work is introduced.

## II. METHODOLOGIES

### A. Accelerated Model

The accelerated model is used for presenting the relationship between accelerated stress and lifetime. The three models are used most commonly, Arrhenius model (AM), Eyring Model (EM) and Inverse Power Law model (IPLM). The three models are shown as follows.

Arrhenius model:

$$\xi = Ae^{E/K \cdot T} \quad (1)$$

where  $\xi$  is the lifetime of specified material,  $E$  reflects activation energy,  $K$  represents Boltzmann coefficient,  $T$  stands for the temperature, and  $A$  is the constant coefficient.

Eyring Model:

$$\xi = A \exp\left(\frac{B}{KT}\right) \quad (2)$$

where  $\xi$  is the lifetime of specified material,  $A$  and  $B$  are the constant coefficient,  $K$  represents Boltzmann coefficient, and  $T$  stands for the temperature.

Inverse Power Law model:

$$\xi = Av^{-C} \quad (3)$$

where  $\xi$  is the lifetime of specified material,  $A$  is the constant coefficient,  $C$  represents coefficient of activation energy, and  $v$  stands for the electrical stress.

Since the AM is a physical model derived from temperature changes, it is often applied to temperature variables in thermodynamic phenomena. The EM is developed based on quantum mechanics, it is used for temperature variables or non-temperature variables such as humidity. The IPLM is applied for non-thermodynamic model. In this paper, the discharged rate is selected as the stress to accelerate lithium-ion battery degradation. Thus, the IPLM is chosen for the accelerated model describing the relationship between accelerated stress and lifetime.

In the following, the lifetime distribution model will be introduced.

### B. Lifetime Distribution Model

The lifetime distribution mode is the failure probability distribution function of the battery. The reliability and property of failure can be obtained through the lifetime distribution. Exponential distribution (ED), normal distribution (ND) and Weibull distribution (WD) are commonly used for life distribution in reliability. If the stress level and lifetime are set to be  $S$  and  $T$ , the three lifetime distributions are shown as follows.

Exponential distribution:

$$F(t | S) = 1 - \exp[-\lambda(S)t], \quad t > 0 \quad (4)$$

where  $\lambda(S)$  is the failure rate function under the stress level  $S$ . Under the condition of ED, average life defined by  $\theta(S) = 1 / \lambda(S)$  acts as the accelerate model parameter.

Normal distribution:

$$F(t | S) = \frac{1}{\sqrt{2\pi}\sigma(S)} \int_{-\infty}^t \exp\left[-\frac{(t - \mu(S))^2}{2\sigma^2(S)}\right] dt, \quad t > 0 \quad (5)$$

where  $\mu(S)$  is the mean life, and  $\sigma(S)$  reflects the standard deviation of lifetime. If the degradation obeys normal distribution, it is assumed that standard deviation is independent of stress level. Meanwhile, the mean life works as the accelerate model parameter.

Weibull distribution:

$$F(t | S) = 1 - \exp\left[1 - \left(\frac{t}{\eta(S)}\right)^{m(S)}\right], \quad t > 0 \quad (6)$$

where  $m(S)$  is the shape parameter, and  $\eta(S)$  reflects the Scale parameter which is also defined as characteristic life. Under the condition of WD, it is assumed that the shape parameters are independent of stress levels. Besides, the characteristic life is defined as the accelerate model parameter.

ED is widely used in reliability fields such as electronic components, complex systems and complete machines. It is suitable for the case where the failure rate is constant. ND is a basic distribution in reliability used for describing the life of a material. WD has a wide range of applications in reliability theory. It can be transformed the formation of ED, ND. In addition to these three distributions, the lognormal distribution, Poisson distribution and Gamma distribution are also widely used in reliability theory. WD has a good performance on fitting the failure time of lithium-ion battery [18]. Hence, in this paper, the WD is chosen for fitting the lifetime of lithium-ion battery.

## III. DEVELOPMENT OF LIFE EXTRAPOLATION MODEL

### A. The Framework of Proposed Model

This paper focuses on a new method for life extrapolation with uncertainty bounds fused the accelerated model and life distribution. To improve the robustness, the life extrapolation is modeled fused the accelerated model and accelerated

working condition. Meanwhile, the uncertainty of life extrapolation is presented by the two factors obtained with lifetime distribution. Depending on the station above, the flowchart for the proposed hybrid method is shown in Figure 1.

According to Figure 1, the remaining useful life (RUL) of different working condition is obtained by the accelerated

degradation data. Then, the life distribution can be modeled with the prediction of RUL for the accelerated condition. At the same time, the accelerated model can be obtained with the RUL for the accelerated condition. Then, the life extrapolation can be modeled combining accelerated model and life distribution. The extrapolated life is appraised by the evaluation criteria.

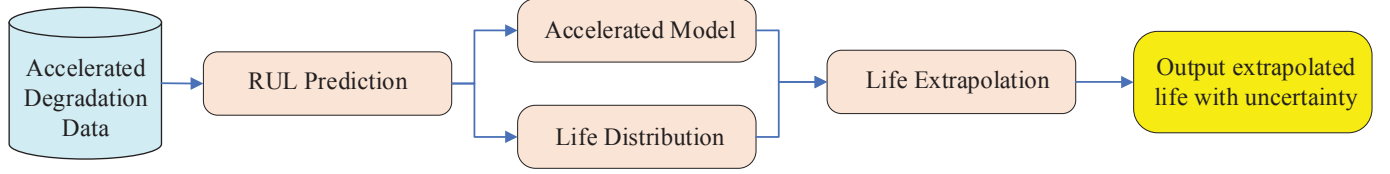


Figure 1. The framework of proposed hybrid method.

### B. Life Extrapolation Model

As the IPLM is chosen for the accelerated model, (1) can be linearized as follow:

$$\ln \xi_i = \alpha + \gamma \varphi(v_i) \quad (7)$$

where,  $\alpha = \ln A$ ,  $\varphi(v_i) = \ln v_i$ ,  $\gamma = -C$ .  $\xi_i$  is the characteristic life on the condition of stress  $v_i$ .

The mean life  $t_i$  is chosen for characteristic life and the relationship between lifetime and stress is shown in Figure 2.

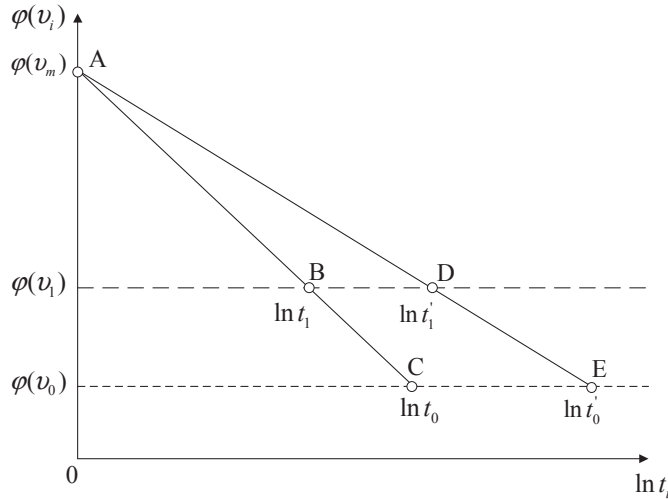


Figure 2. The relationship between lifetime and stress of linearized IPLM.

In Figure 2,  $v_m$  represents the maximum stress level with lifetime  $t_m$ ,  $t_1$  is the characteristic life with accelerated stress level  $v_1$ , and  $t_0$  stands for the characteristic life with used stress level  $v_0$  [19]. Then, Line  $ADE$  reflects the relationship between the true life and the accelerated stress. D reflects the true life of the product under accelerated life testing defined as  $t_1'$ . However, the D is also unknown, so this line is not available. B is the accelerated life test stop point whose true

life is defined as  $t_1$ . Compared with line  $ADE$ , the result of  $ABC$  is more conservative.

Through mathematical derivation,  $\alpha$  and  $\gamma$  can be obtained as follows.

$$\alpha = \frac{\varphi(v_1) \ln t_m - \varphi(v_m) \ln t_1}{\varphi(v_1) - \varphi(v_m)} \quad (8)$$

$$\gamma = \frac{\ln t_1 - \ln t_m}{\varphi(v_1) - \varphi(v_m)}$$

lifetime  $t_0$  with normal stress level  $v_0$  can be extrapolated by the  $t_m$  and  $t_1$ , shown as (9).

$$t_0 = \exp \left\{ \frac{\ln t_1 [\varphi(v_0) - \varphi(v_m)] + \ln t_m [\varphi(v_1) - \varphi(v_0)]}{\varphi(v_1) - \varphi(v_m)} \right\} \quad (9)$$

$$= t_1^{\frac{\varphi(v_m) - \varphi(v_0)}{\varphi(v_m) - \varphi(v_1)}} \cdot t_m^{\frac{\varphi(v_1) - \varphi(v_0)}{\varphi(v_1) - \varphi(v_m)}}$$

As the WD is chosen for fitting the lifetime of lithium-ion battery, confidence lower limits  $t_{R,low}$  for the extrapolated life and confidence lower limits  $R_{low}$  for the reliability can be derived based on the known shape parameter  $\beta$  [20], shown as follows.

$$t_{R,low} = t_0 \left[ (N+1) \frac{\ln R}{\ln(1-\gamma)} \right]^{\frac{1}{\beta}} \quad (10)$$

$$R_{low} = \exp \left\{ \frac{\ln(1-\gamma)}{N+1} \left( \frac{t}{t_0} \right)^{\beta} \right\} \quad (11)$$

where  $N$  reflects the number of samples,  $R$  is the reliability, and  $\gamma$  stands for the confidence level.

Equation (10) and equation (11) can be expressed as (12) and (13) with known  $t_0$ .

$$t_{R,low} = t_1^{\frac{\varphi(v_m) - \varphi(v_0)}{\varphi(v_m) - \varphi(v_1)}} \cdot t_m^{\frac{\varphi(v_1) - \varphi(v_0)}{\varphi(v_1) - \varphi(v_m)}} \left[ (N+1) \frac{\ln R}{\ln(1-\gamma)} \right]^{\frac{1}{\beta}} \quad (12)$$

$$R_{low} = \exp \left\{ \frac{\ln(1-\gamma)}{N+1} \left( \frac{t}{t_1 \frac{\varphi(v_m)-\varphi(v_0)}{\varphi(v_1)-\varphi(v_0)} \cdot t_m \frac{\varphi(v_1)-\varphi(v_0)}{\varphi(v_1)-\varphi(v_m)}} \right)^\beta \right\} \quad (13)$$

In this paper, the uncertainty of life extrapolation is presented by (12) and (13).

#### IV. EXPERIMENTS AND DISCUSSION

In this paper, the accelerated degradation test was carried out with 3 different discharge current (8.5A, 18.75A, and 20A), while retaining the other stress factors constant. The experiment procedure contained the charge cut-off voltage 4.15V and fixed depth of discharge. The time consuming of these 3 working condition is 4550 minutes, 2663.1 minutes and 2347.5 minutes. The cells charged with 8.5A work on normal condition, and other two discharged current are used for accelerating degradation tests. Specification on the discharge current condition, the number of samples and test intervals are provided in Table I.

TABLE I. CELLS ARRANGEMENT AND SAMPLE DISTRIBUTIONS

Discharge Current (A)	Numbers of cells	Test Interval (Cycles)
8.5	5	500
17.5	2	500
20	3	500

The discharge cut-off voltage is chosen for representing the degradation of lithium-ion battery in this paper. Figure 3 shows the normalized discharge cut-off voltage profile for the cells.

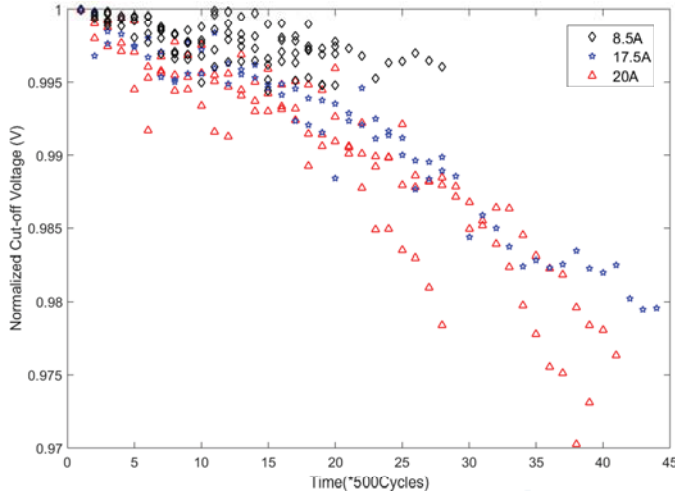


Figure 3. The normalized discharge cut-off voltage profile for the cells.

In this paper, the batteries are failure when the normalized cut-off voltage reaches about 80% of the rated capacity (from 4.15V to about 3.3V). The trend lines were fitted for the ADT data, and then the RUL of batteries can be obtained, shown as Table II.

TABLE II. RUL ESTIMATION OF CELLS

Discharge Current (A)	Cell's Number	RUL (years)
8.5	1	25.47
	2	21.06
	3	16.79
	4	51.05
	5	16.28
17.5	1	8.32
	2	16.01
20	1	6.63
	2	11.26
	3	8.57

Different life distributions can express the RUL of Table II, shown as Figure 4. The performance of life distributions is estimated by the Anderson-Darling (AD) and hypothesis-testing factor P. The smaller the AD value, the better the fit of the distribution. On the condition of 95% confidence interval, if P is less than 0.05, the hypothesis test cannot be established.

The performance for the 4 distribution models is shown in Table III. It is clear that, the performance of WD is better than other three distribution model. Therefore, choosing the Weibull distribution to describe battery life is reasonable.

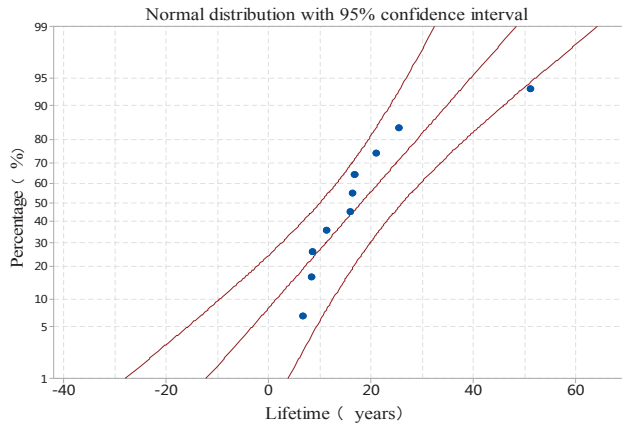
TABLE III. COMPARISON WITH DIFFERENT DISTRIBUTION MODEL

Distribution Model	AD	P
Normal ditribution	0.836	0.020
Exponential distribution	0.375	0.250
Gamma ditribution	1.018	0.096
Weibull ditribution	0.364	0.232

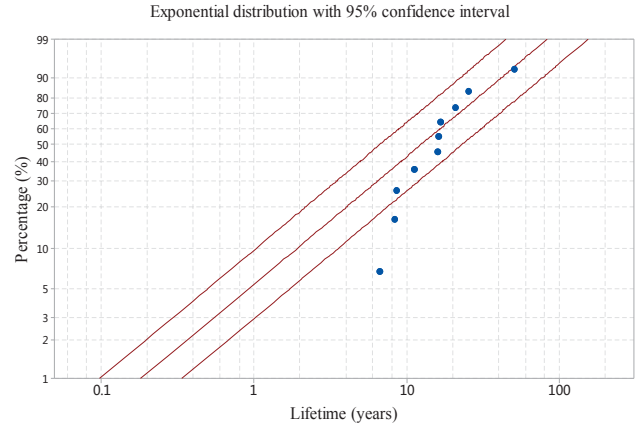
According to the RUL estimation, the parameters for (7) can be obtained by the maximum likelihood estimate (MLE). That is  $\gamma=-1.10$  and  $\alpha=6.02$ . The relationship between accelerated stress and lifetime is presented as follow.

$$\ln \xi_i = 6.02 - 1.10 \ln v_i \quad (14)$$

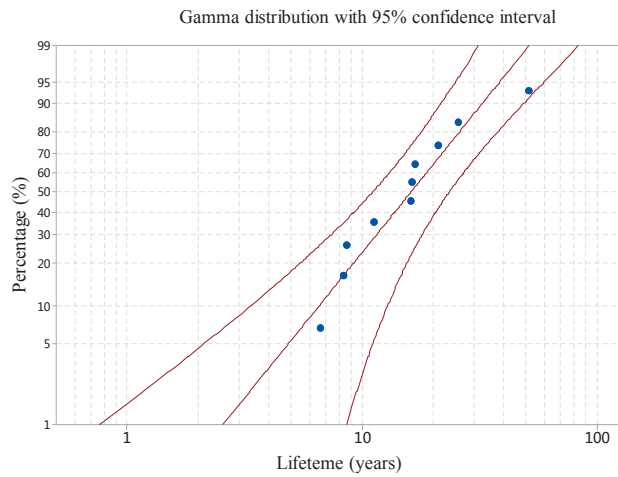
According to the traditional method, the lifetime for normal working condition can be obtained, which is 39.25 years. The mean life is selected as the characteristic life in this paper. The lifetime for normal working condition can be extrapolated by (9), which is 19.98 years. The mean life of normal waking condition in Table I is 26.13 years. Compared the accelerated model, the performance for extrapolation of proposed model is better. The uncertainty presentation trajectory can be fitted by (12) and (13), shown as Figure 5.



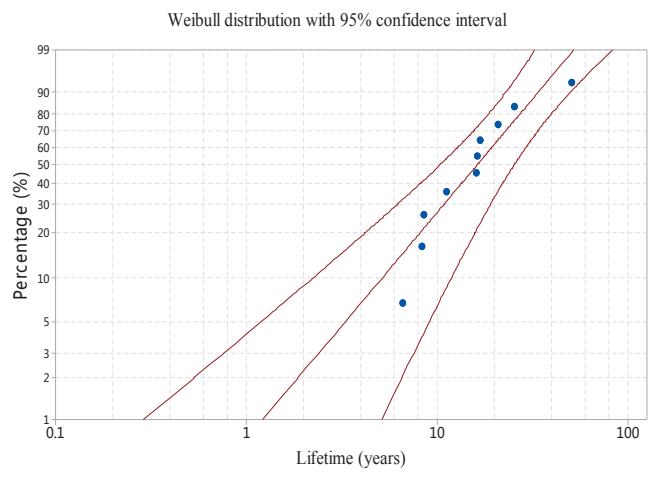
(a)



(b)



(c)



(d)

Figure 4. The lifetime distribution with different models. (a) is normal distribution with 95% confidence interval. (b) is exponential distribution with 95% confidence interval. (c) is normal distribution with 95% confidence interval. (d) is Weibull distribution with 95% confidence interval.

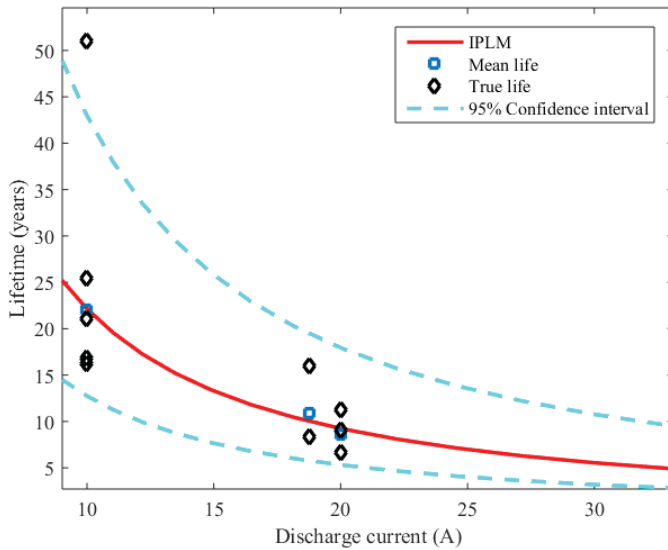


Figure 5. The extrapolation trajectory by proposed model.

## V. CONCLUSION

The research orients general method for extrapolating lifetime with ADT data. A life extrapolation model fused the accelerated model and accelerated working condition is proposed and compared with traditional life extrapolation method. The advantage of the proposed model is that an adequate data is used for modeling the extrapolation model. Besides, for the ADT data, different tests for life distribution model are carried out to help analyze the model matching. The extrapolation result indicates that the proposed model can provide more accurate estimates with life extrapolation. Meanwhile, the proposed model is also able to provide the ability for uncertainty presentation.

In the future, the life extrapolation model is considered for the wider application and higher accuracy. The proposed method shows the satisfied performance with life estimation for ADT data. However, the life extrapolation model proposed in this paper did not take consideration with differences between samples. Hence, it is meaningful to discover a method that



modeled considering random effects. This will make sense of the wider applications and higher accuracy of the model.

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