Order Based Modal Analysis Using Vold-Kalman Filter

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***Abstract*—Different from the operational modal analysis (OMA), the order based modal analysis (OBMA) is based on periodic sweep to obtain the dynamic behaviors of machinery. Therefore, its applicable condition is that the mechanical system generates periodic excitation force during the operational process. Due to the imbalance and misalignment of the rotating mechanical, it generates periodic excitation force whose frequency is proportional to the rotational speed during revolution. This paper utilizes OBMA to identify the resonances from the simulated signal with crossing order and white noise. First, the Vold-Kalman filter based order tracking (VK) method is utilized to extract harmonic response known as engine orders. Finally, the least-squares complex frequency-domain estimation method (PolyMAX) is applied to identify the resonance frequency, damping and modal shapes. Especially, two modes whose natural frequencies are close are successfully separated.**

Keywords-Order based modal analysis; Vold-Kalman filter based order tracking; Rotating machine;

I. INTRODUCTION

Operational Modal Analysis (OMA) and Experimental Modal Analysis (EMA) are two main methods to identify modal parameters. Compared with EMA, OMA has two representative advantages. First, OMA obtains the vibration characteristics in operating state, which is different from the characteristics in static state obtained by EMA. Besides, the input signal is not required. But the input signal is necessary to EMA, and it is hard to excite artificially large structures by hammer or shaker. Therefore, OMA is widely used in identifying modal parameters of rotating machinery and large structures.

Operational Modal Analysis is based on the white noise assumption, that is, the environmental excitation can be regarded as Gaussian noise. Then OMA constructs pseudo-Frequency Response Functions (pseudo-FRFs) using the power spectrum density function (PSD) to identify modal parameters. The excitation of some large structures which exposed to natural environment usually is regarded as white noise. Therefore, the PSD-based OMA method is widely used in modal analysis of large structures such as football stadium and bridge [1] [2].

However, for rotating machinery in operational state, due to the residual imbalance and misalignment, the external excitation contains the harmonics with frequency multiplies of its rotating speed. The assumption of the broad-band white noise excitation is no longer valid. Even if the external excitation is broad-band excitation, the highest frequency of each engine order may be identified as a physical pole of the system as a result of “end-of-order” effect [3]. Based on the above reasons, OMA cannot be applied directly in rotating machinery.

The order based modal analysis (OBMA) which is first proposed by Janssens is based on periodic sweep excitation, which requires the rotating machinery to generate periodic excitation force whose frequency is proportional to the rotating speed during the process of rotation [4][5]. When there is residual imbalance in the system, the excitation force generated by the imbalance satisfies the condition. Therefore, it is of great significance for modal parameter identification in the operational state of rotating machinery.

The OBMA method combines Order Tracking (OT) techniques and OMA method. First, the OT method is used to extract the amplitude and phase response of forced vibration under different speed. The response curves of different engine orders after being processed can be taken as the pseudo-FRF curve for resonance identification. The OBMA method has been validated by some industrial examples, such as a gear test rig and wind turbine gearboxes [6][7][8].

The Vold-Kalman filter based order tracking (VK)is based on the classical Kalman filter. Generally, the amplitude and phase response curves extracted by the VK method under different speed have higher frequency resolution [9].

The aim of the present paper is to validate that the VK method has a good effect on extracting high quality response curves to identify modal parameters. The theoretical background about OBMA and VK method is introduced in section 2. Section 3 illustrates these methods via a simulated forced vibration signal under sweeping harmonic excitation with crossing order. The VK method and the least-squares complex frequency-domain estimation method, which is known as PolyMAX, are applied to analysis response data.

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II. THEORETICAL BACKGROUND

A. Order based modal analysis

Due to the imbalance, misalignment and impact of the rolling bearing in the rotor system, the rotor generates excitation in multiples of the rotational frequency, which is known as Engine Orders (EO). Taking the imbalance as an example, as shown in (1), the rotor generates excitations of similar magnitude on the X and Y axes simultaneously during the revolution, with a phase difference of 90 degrees. Where ω_0 is the rotationat speed, l is the engine order of harmonics, A_{lx}, A_{ly} and θ_l are the amplitude and phase of the orthogonal excitation of engine order l respectively.

$$\begin{Bmatrix} f_X(t) \\ f_Y(t) \end{Bmatrix} = \begin{Bmatrix} \sum_{l=1}^{P} A_{l,x}\cos(l\omega_0 t + \theta_l) \\ \sum_{l=1}^{P} A_{l,y}\sin(l\omega_0 t + \theta_l) \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} F_X(\omega) \\ F_Y(\omega) \end{Bmatrix} = \begin{Bmatrix} \sum_{l=1}^{P} A_{l,x} e^{j\theta_l}\delta(\omega - l\omega_0) \\ \sum_{l=1}^{P} A_{l,y} e^{j(\theta_l \pm 90^\circ)}\delta(\omega - l\omega_0) \end{Bmatrix} \tag{2}$$

In the frequency domain, $f_X(t)$ and $f_Y(t)$ can be written as $F_X(\omega)$ and $F_Y(\omega)$ in (2). $\delta(\bullet)$ represents unit pulse function.

Then, the measured response $X(\omega)$ of forced vibration can be represented in (3) [7]. $H(\omega)$ is the corresponding Frequency Response Functions (FRF) matrix.

$$\{X(\omega)\} = \left[H_{(:,X)}(\omega) \quad H_{(:,Y)}(\omega) \right] \begin{Bmatrix} F_X(\omega) \\ F_Y(\omega) \end{Bmatrix} \tag{3}$$

It is assumed that the amplitude of external excitation $F(\omega)$ is proportional to the squared rotational speed ω_0^2.Combing the (2) and (3), the response of rotor machinery can be written as (4).

$$X(\omega) \propto \omega_0^2 (H_{(:,X)}(\omega) - jH_{(:,Y)}(\omega))\delta(\omega - l\omega_0) \tag{4}$$

The FRF matrix can be expressed in the form of modal poles, shapes and participation factors as (5).

$$H_{(:,\bullet)}(\omega) = V(j\omega I - \Lambda)^{-1} L_\bullet + \frac{1}{\omega^2} LR_\bullet + UR_\bullet \tag{5}$$

Where, Λ is the diagonal matrix of modal poles, which can be written in the form of modal frequencies and dampings. V is the matrix of modal shapes and L is the modal participation factors. The above parameters are complex valued. LR and UR represent the real-valued lower and upper modal residuals, respectively.

As shown in (1), $F(\omega)$ is made up of multiple orders of rotational speed ω_0, Combing the (4) and (5) we obtain:

$$\begin{aligned} X(\omega) \propto \omega_0^2 (V(j\omega I - \Lambda)^{-1}(L_x - jL_y) + \cdots \\ \frac{1}{\omega^2}(LR_x - jLR_y) + (UR_x - jUR_y)) \end{aligned} \tag{6}$$

It is clear that (6) will have the similar form as FRF if divided by ω_0^2. Equation (6) also contains the information of poles and modal shapes. It means that the response curves during a run-up or run-down process can be used for modal parameter identification as pseudo-FRFs. Furthermore, if the response measured is the acceleration response, the amplitude response needs to be divided by the fourth rotational speed ω_0^4.

In the present paper, the VK method is used to extract the amplitude and phase response under different rotational speed in rotating machinery. Then response curves are fitted by the operational PolyMAX [10].

B. Vold-Kalman filter based order tracking

Analyzing frequency components related to rotational speed is the aim of order tracking. The orders could be estimated both in terms of amplitude and phase. In 1993, Vold proposed the first generation of the VK method, but only one order component was extracted [11]. In 1997, Vold proposed the second generation VK method, which can extract multiple orders at the same time and decouple close and cross orders [12]. Compared with other order tracking methods, such as Angle Domain techniques (AD) and Time Variant Discrete Fourier Transform (TVDFT), VK has no leakage problem due to time-frequency transform, and the high time-frequency resolution problem cannot be simultaneously satisfied owing to the Heisenberg uncertainty principle. Therefore, The VK method achieves higher resolution.

The VK method is based on the structural and data equations. During the operation of the rotor system, the sum of the various order components to be extracted $x(n)$ can be expressed as (7).

$$x(n) = \sum_{k=-\infty}^{+\infty} x_k(n) = \sum_{k=-\infty}^{+\infty} a_k(n)\theta_k(n) \tag{7}$$

$$\theta_k(n) = \exp(kj\sum\nolimits_{m=0}^{n} \omega(m)\Delta T) \tag{8}$$

Where, the integral k denotes the order be tracked. $x_k(n)$ denotes the kth order signal. $a_k(n)$ denotes the kth order complex envelope and $\theta_k(n)$ is a carrier wave which can be written as (8). $\omega(m)$ and ΔT represent respectively the speed of the reference shaft and sampling interval.

Generally, $a_k(n)$ can be regarded as a low-order polynomial in the local range. Using low-order polynomials to fit the amplitude change of the order signal, the so-called structural equation can be written as (9).

$$\nabla^{s} a_k(n) = \varepsilon_k(n) \tag{9}$$

Where, ∇ represents the difference operator, the index s is the given differentiation order. Generally, we choose s=2, then (9) can be shown as (10). $\varepsilon_k(n)$ means the unknown non-homogeneity term.

$$\nabla^2 a_k(n) = a_k(n) - 2a_k(n+1) + a_k(n+2) = \varepsilon_k(n) \tag{10}$$

When simultaneously track K order components, especially in the case of multi-axial reference rotating speeds, the data equation can be written as (11).

$$y(n) = \sum_{k=i} a_k(n)\theta_k(n) + \eta(n), i = \pm 1, \pm 2, \cdots, \pm K \tag{11}$$

Where, $y(n)$ is the measured signal, and $\eta(n)$ comprises non-extracted order components and measurement errors.

The unknown quantity $a_k(n)$ cannot be solved just using the structural and data equation. In order to obtain $a_k(n)$, the sum of squares of $\eta(n)$ and $\varepsilon_k(n)$ is minimized by least-squares method. In order to balance the proportion of the unknown non-homogeneity term $\varepsilon_k(n)$ and non-extracted order components and measurement errors $\eta(n)$, the Harmonic Confidence Factor (HCF) r is introduced as shown in (12).

$$J = \sum_{k=i} r^2 \varepsilon_k^T(n)\varepsilon_k(n) + \eta^T(n)\eta(n) \ . \tag{12}$$

In order to minimize J, calculate derivative of function J with respect to a_k and make it equal to 0 as shown in (13).

$$\frac{\partial J}{\partial a_k} = 0 \tag{13}$$

Then the complex envelope of the extracted multiple orders $a_k(n)$ can be obtained.

III. Numerical Simulation

To validate the proposed methodologies, a rotor was mounted on a plate-shape foundation of size 300mm*800mm as shown in Fig. 1. Six modes of plate were defined. The modal frequency and dampings of the plate were listed in the first column of Table. I. There were three bending modes and three torsional modes shapes.

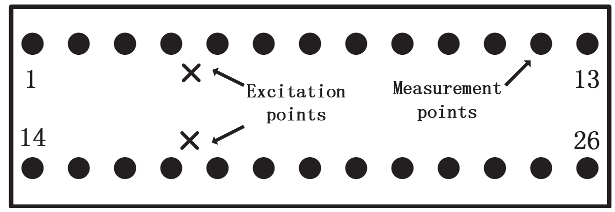


Figure 1. Excitation points and measurement points of a plate

There were two sets of reference speed with crossing order in the rotor system on two excitation points. Speed 1 was assumed to run up from 360 rpm to 1800 rpm in 120 seconds with a linear sweep rate of 720 rpm/min. Speed 2 was assumed to run up from 720 rpm to 1440 rpm in 120 seconds with a linear sweep rate of 360 rpm/min. The simulated signal contained two sets of order components based on the corresponding reference speed, one with orders 1 to 20, and the other with orders 1 to 6.

The acceleration responses on 26 homogeneously distributed measurement points was calculated after determining the excitation of system. First, the acceleration response of single degree of freedom system under each order mode force was obtained by applying Duhamel integral method. Then the acceleration responses at 26 points was obtained by applying modal superposition method. Finally, the white Gaussian noise whose S/R is 10 dB is added to the acceleration signal. The Fig. 2 shows the simulated time domain vibration signal polluted by noise.

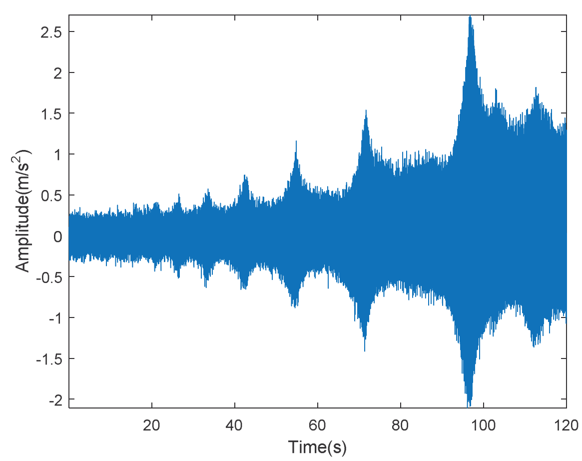
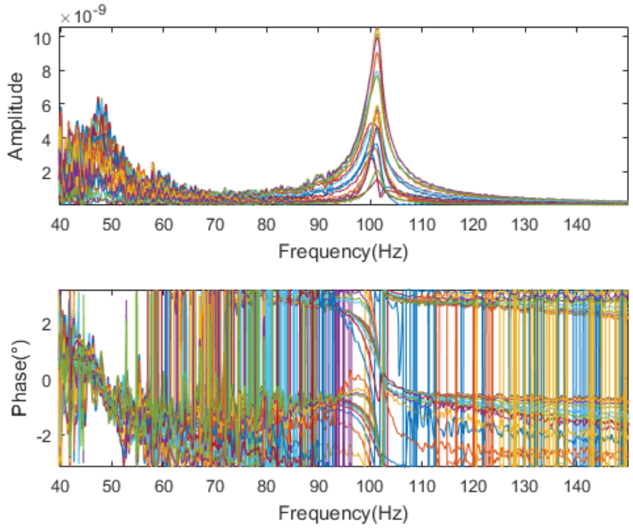


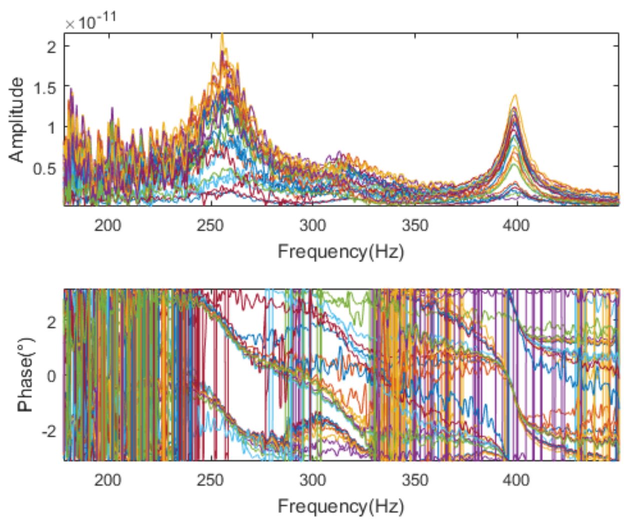
Figure 2. The simulated time domain vibration signal

The bandwidth of the filter needs to be set before extracting orders. We choose a bandwidth value equal to 2 Hz to balance the computational time and the quality of orders tracked. The response curves extracted from engine order 5 and 15 of Speed1 are shown in Fig.3, which has been divided by the fourth power of rotational frequency ω_0^4 as introduced in previous section. This means the response curves can be applied to estimates the modal parameters as pseudo-FRFs.

Then these pseudo-FRFs could be regarded as input to the operational PolyMAX method to identify the modal parameters. The stabilization diagrams are shown in Fig.4. The vertical lines of ‘s’ represents the physical poles, ‘f’ represents modal frequency unstable, ‘d’ represents damping unstable, ‘v’ represents modal vector unstable.

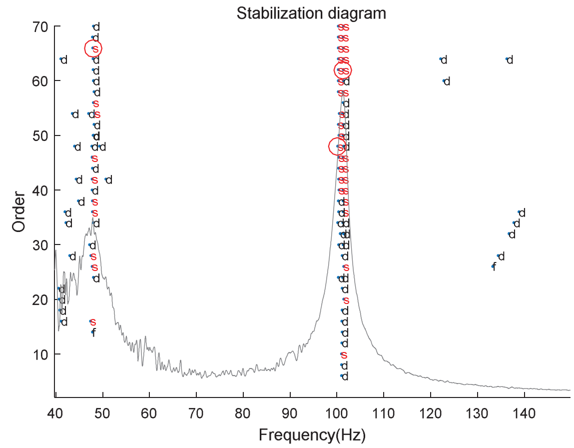


(a) *Order* 5 of the 26th meaturement points

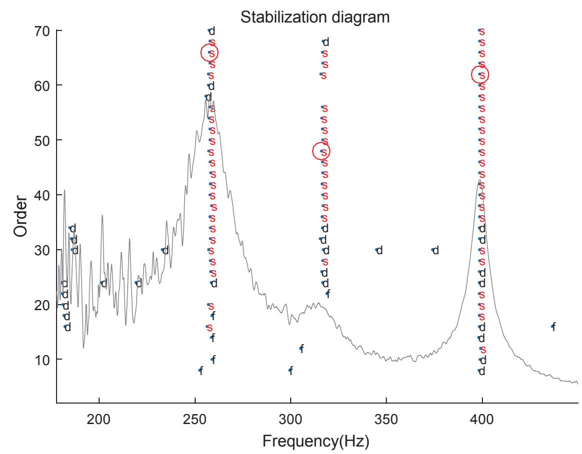


(b) *Order* 15 of the 26th measurement points

Figure 3. Amplitude and phase of order 5 and order 15 pseudo-FRFs extracted from the simulate signal



(a) *Order* 5



(b) *Order* 15

Figure 4. Stabilization plot of order 5 and and order 15

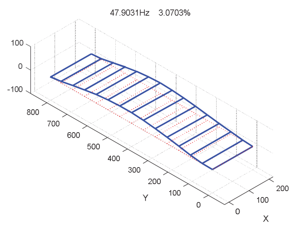
The modal parameters identified using pseudo-FRFs of Order 5 and Order 15 are shown in Table. I . 6 modes were superposed in the simulated signal and 6 modes are all identified using the VK method and the PolyMAX method. Especially MODE 2 and MODE 3 whose natural frequencies are close are successfully separated.

TABLE I. COMPARISON OF IDENTIFIED MODAL PARAMETERS AND SIMULATED MODAL PARAMETERS

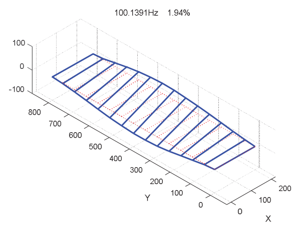
True value		**Order 5**		**Order 15**	
Frequency (Hz)	***Damping (%)***	***Frequency (Hz)***	***Damping (%)***	***Frequency (Hz)***	***Damping (%)***
47.71	3.00	47.90	3.07	-	-
100.18	2.00	100.14	1.94	-	-
101.12	1.00	101.35	0.92	-	-
257.48	4.00	-	-	257.49	3.94

True value		Order 5		Order 15	
315.62	4.00	-	-	316.02	3.20
398.45	1.00	-	-	398.57	1.00

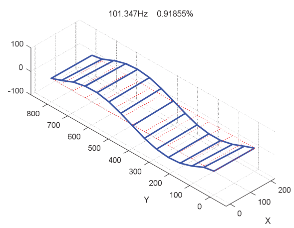
The modal shapes identified from MODE 1 to MODE 6 are shown in Fig.5. Three bending modes and three torsional modes are exhibited.



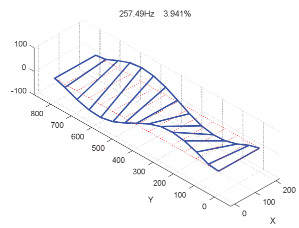
(a) *MODE* 1



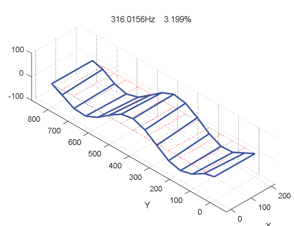
(b) *MODE* 2



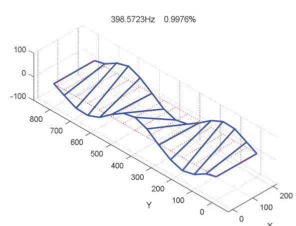
(c) *MODE* 3



(d) *MODE* 4



(e) *MODE* 5



(f) *MODE* 6

Figure 5. Shapes of modes identified

IV. CONCLUSIONS

The method combing the VK method and PolyMAX method has a great development prospect for modal parameters identification of rotating machinery. The response curves was extracted with high quality in the case of crossing orders. Besides, due to accuracy of pseudo-FRFs extracted by the VK method, two modes whose natural frequencies are close are successfully separated in white noise environment. In future work, the experiment data can be used to analysis. And in this paper, the bandwidth in the VK method is fixed for all the frequencies. The better results may be obtained if the bandwidth changes with frequency.

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