Investigation on the Influence of Dynamic Tooth Wear on Gear Dynamic Characteristics

Lisha ZHU
School of Mechanical and Automotive Engineering
Zhaoqing University
Zhaoqing, Guangdong Province 526021, P.R. China
neulisachu@163.com

Haonan CHEN
School of Mechanical and Automotive Engineering
Zhaoqing University
Zhaoqing, Guangdong Province 526021, P.R. China

Zunling DU
School of Mechanical and Automotive Engineering
Zhaoqing University
Zhaoqing, Guangdong Province 526021, P.R. China

Zisheng LIN
School of Mechanical and Automotive Engineering
Zhaoqing University
Zhaoqing, Guangdong Province 526021, P.R. China

Yonghui HE
Shanghai Marine Diesel Engine Research Institute
No. 3111 Huaning Road Shanghai, P.R. China
13262821690@163.com

Hairong HAN
China Ship Development and Design Center
Zhangzhidong Road, Wuchang District, Wuhan, Hubei, 430064, P.R. China
haironghan@163.com

***Abstract*—In order to study the effect of tooth surface wear on gear dynamics, based on the Archard wear model, considering the dynamic load distribution between teeth under the geometrical normal clearance and influence of the contact point on the tooth profile points in the surrounding area, an accurate wear model of tooth surface is established and the dynamic characteristics of gear are analyzed by coupling the wear of gear surface into the gear dynamics model, and a dynamic wear calculation model of gear surface is established. The results indicate that early wear has little effect on gear dynamics, with the increase of wear cycle, the vibration of the non-resonant region is increased, while in the resonance region first invariant and then increased.**

Keywords-gear; dynamic wear; dynamics; coupling

I. INTRODUCTION

Under high contact pressure, relative rolling and sliding are taken place on the surface of the meshing teeth. Although most gears are lubricated with grease, the lubrication state is boundary lubrication or mixed lubrication. The meshing surfaces cannot be completely separated by the lubricant, and the friction surface of metals will generate direct friction. In this case, the surface of meshing gear teeth under the action of low speed and heavy load will cause the transfer of the material and the spalling of the surface metal. This phenomenon is called gear wear. The wear of gear will cause the gap between tooth surfaces and thus affect the dynamic characteristics of the system. This paper mainly studies the calculation method of tooth surface wear and the impact of wear on gear dynamics.

Research on gear wear, scholars at home and abroad generally based on the Archard wear model. However, the

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meshing load and sliding distance of tooth surface are not the same when determining the gear. Anddersson[1] obtained the relationship between the sliding distance and meshing position in the meshing process through the gear geometry theory, and the load of double-tooth meshing was evenly distributed. Flodin[2,3] determined the sliding speed and meshing pressure through the single point observation method, and the time-varying meshing stiffness was not taken into account when the load was distributed. Scholar at home and abroad have paid more and more attention to the interaction between gear wear and dynamics in recent years. Kuang et al. [4] established a single-stage gear torsion dynamic model after considering the friction on the tooth surface, studied the impact of tooth clearance caused by wear on dynamic meshing force, and then proposed the wear model. Liu et al. [5] combined a quasi-static model with a nonlinear dynamic model of the system to obtain a dynamic wear prediction model. In these models, the dynamic load distribution in the double-tooth meshing area is not considered, and the impact of contact points on the tooth profile points in the surrounding area is not considered. In this paper, the total transfer load in the calculation of the wear amount of the tooth surface is derived from the results of the dynamic analysis, and the wear amount distribution of the tooth surface is calculated after considering the dynamic load distribution in the double-tooth meshing area, and the cumulative wear model is coupled to the dynamic model, and finally the influence of the dynamic wear model on the dynamics is discussed.

II. NONLINEAR DYNAMIC MODEL OF SINGLE-STAGE GEAR WITH SIDE CLEARANCE

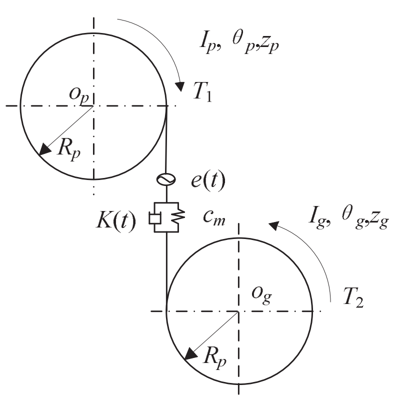


FIGURE 1. DYNAMIC MODEL OF SINGLE GEAR PAIR

Fig.1 shows a typical single-stage torsion gear dynamic model, assuming that the stiffness of the supporting shaft and bearing is very large. The displacement motion of the gears and the influence of the supporting bearing and the friction force on the tooth surface during rotation are not considered, only considering the torsion motion of the respective centers of the two gears.According to Newton's laws of mechanics, the dynamic equation of the system can be obtained:

$$\begin{aligned}
&I_p\ddot{\theta}_p + c_m(R_p\dot{\theta}_p - R_g\dot{\theta}_g - \dot{e}(t))R_p \\
&+k(t)f(R_p\theta_p - R_g\theta_g - e(t))R_p = T_1 \\
&I_g\ddot{\theta}_g - c_m(R_p\dot{\theta}_p - R_g\dot{\theta}_g - \dot{e}(t))R_g \\
&-k(t)f(R_p\theta_p - R_g\theta_g - e(t))R_g = -T_2
\end{aligned} \tag{1}$$

where R_p, R_g are the radius of the base circle of the master and slave driving wheels, I_p, I_g are the moment of inertia of two gears, θ_p, θ_g is the torsional displacement of two gears, T_1, T_2 is the torque applied to the master and slave gears, c_m is the static transfer error, $k(t)$ is the time-varying stiffness of gears, $e(t)$ is the static transfer error. The function f in the formula is the nonlinear function of the meshing force of gears with side clearance. $\delta(t)=R_p\theta_p$ - $R_p\theta_p$ - $e(t)$ is the dynamic transfer error.

Here, taken the side clearance of the gear pair as fixed side clearance $2b$, the gap nonlinear f function is expressed as:

$$f(x)=\begin{cases} x-b & x>b \\ 0 & -b \le x \le b \\ -(-x-b) & x<-b \end{cases} \tag{2}$$

III. QUASI-STATIC WEAR MODEL OF TOOTH SURFACE

Wear of spur gears is simulated based on the generalized Archard wear equation.

$$\frac{V}{s} = K\frac{W}{H} \tag{3}$$

where V is the volume wear of the material, s is the sliding distance of the meshing tooth surface, W (W_1 or W_2) is the applied normal load between the teeth, H is the material hardness of the observed surface, and K is the dimensionless wear coefficient, which is related to the lubrication state and wear mechanism.

Flodin[2,3] proposed a slight wear prediction model for spur gears based on the generalized Archard wear equation, assuming that the surface pressure p_i and slip velocity v_i at point i on the tooth profile remain unchanged in a very short time. Therefore, the wear amount at point i can be expressed as:

$$h_{i,n} = h_{i,n-1} + \Delta tkN \sum_{j=1}^{s} p_{i,j} v_{i,j} \tag{4}$$

$h_{i,n}$ is the wear depth after n times of wear at meshing point i, $h_{i,n-1}$ is the wear depth after n-1 times of wear at meshing point i, n is the current wear times, Δt is the time step, k is the coefficient of wear, N is the number of revolutions per wear interval, the gear profile will not be updated during this period, s is the point within the Hertz contact radius. $p_{i,j}$ is the surface pressure at point i when point j contacts, and $v_{i,j}$ is the slip velocity at point i.

In a meshing cycle, the double tooth meshing and single tooth meshing appear alternately, in double teeth mesh areas, two pairs of meshing gear pair share method to load, if only considering the elastic deformation under the condition of no account of error, two pairs of meshing gears can be regarded as a pair of parallel springs to carry on the load distribution, the need to consider along the meshing line the time-varying stiffness of gear pair. However, low-speed and heavy-duty gear inevitably forms a gap between the teeth due to wear, which causes the meshing point to deviate from the ideal involute position. Therefore, it is necessary to establish a more practical load distribution formula between teeth under geometrical side gaps [6].

In the double-tooth meshing area, if the first pair of gears contact first, then the load borne by the 2 pairs of gears respectively is:

$$\left.\begin{aligned} W_1 &= \frac{k_1(t)(W+|\Delta|\cdot k_2(t))}{k_1(t)+k_2(t)} \\ W_2 &= \frac{k_2(t)(W-|\Delta|\cdot k_1(t))}{k_1(t)+k_2(t)} \end{aligned}\right\} \tag{5}$$

If the second pair contacts first, the load borne by the two pairs of gears is:

$$\left.\begin{aligned} W_1 &= \frac{k_1(t)(W-|\Delta|\cdot k_2(t))}{k_1(t)+k_2(t)} \\ W_2 &= \frac{k_2(t)(W+|\Delta|\cdot k_1(t))}{k_1(t)+k_2(t)} \end{aligned}\right\} \tag{6}$$

IV. Dynamic wear calculation model

The quasi-static wear model of gear is coupled to the nonlinear dynamic model of gear clearance, and the nonlinear dynamic model of gear with tooth surface wear can be obtained. According to the quasi-static wear model, the wear depth $h(t)$ of each position on the tooth surface can be calculated, and the wear depth will increase the tooth side clearance, so the new tooth side clearance becomes: $b+h(t)$ and $h(t)$ which is half of the sum of the wear of the master and slave gears at the meshing point, and the clearance function becomes:

$$f(x) = \begin{cases} x-(b+h(t)) & x > b+h(t) \\ 0 & -(b+h(t)) \le x \le b+h(t) \\ x+(b+h(t)) & x < -(b+h(t)) \end{cases} \tag{7}$$

The dynamic transfer error $\delta(t)$ can be obtained by solving the nonlinear dynamic equation of wear on the tooth surface, then the total transfer dynamic load after considering the wear of the tooth surface is:

$$W = \begin{cases} k(t)(\delta(t)-(b+h(t)))+c_m\dot{\delta}(t) & \delta(t) > b+h(t) \\ 0 & -(b+h(t)) < \delta(t) \le b+h(t) \\ k(t)(\delta(t)+(b+h(t)))+c_m\dot{\delta}(t) & \delta(t) \le -(b+h(t)) \end{cases} \tag{8}$$

By substituting the dynamic load into formula (5) or (6), the wear amount of the tooth surface can be calculated.

V. Example

This example is used to solve the dynamic problem of Gears Considering the dynamic wear of tooth surfaces. It is the same as the dynamic model established by Tamminana. V. K. [7] when studying the relationship between dynamic excitation factors and dynamic transmission errors of gears. The specific parameters are shown in Table I.

TABLE I. PARAMETERS OF THE EXAMPLE MODEL

Parameter Name	Gear	Pinion
Number of teeth	50	50
Pressure angle(°)	20	20
Addendum circle diameter(mm)	157	157
meterial	42CrMo	42CrMo
gear surface roughness (μm)	0.3	0.3
Mass (kg)	1.788	1.788
Moment of inertia (Kg·mm2)	4980	4980
Poisson's ratio	0.3	0.3
rotate speed (rpm)	500~4000	500~4000
Backlash (mm)	0.13	
Center distance (mm)	151	
Module (mm)	3	3
tooth width (mm)	15	15
Base circle diameter (mm)	140.95	140.95
Density (kg·m-3)	7830	7830
Quality grade	6	6
Elasticity modulus (Gpa)	206	206
Meshing damping (N·s/m)	0.3	0.3
Accuracy grade	5	5
Addendum coefficient	0.1708	0.1708
Transfer torque (N·m)	300	

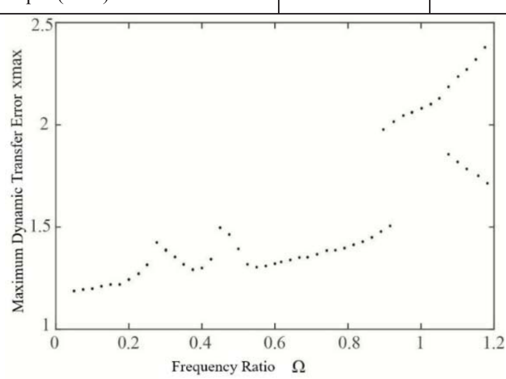


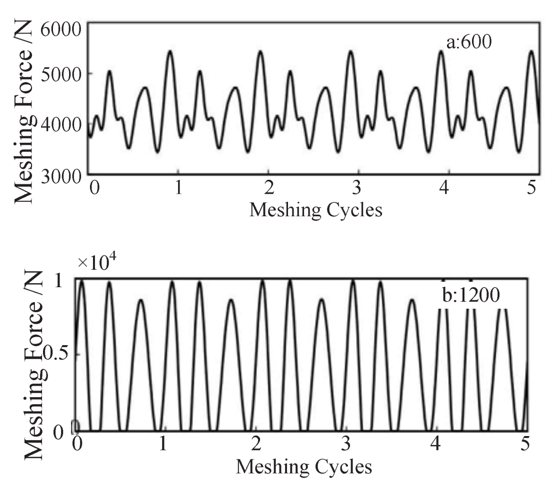
FIGURE 2. AMPLITUDE FREQUENCY RESPONSE（THE ABSCISSA IS THE FREQUENCY AND THE ORDINATE IS THE MAXIMUM DYNAMIC TRANSFER ERROR）

The dynamic equation is obtained by calculating the parameters. The dynamic equation is solved by variable-step Runge-Kutta. The frequency response graph ($\Omega=\omega/\omega_n$) of the maximum dynamic transfer error is obtained by solving the initial value of multiple transformations. The graph shows the amplitude-frequency curve near the primary resonance (Ω=1) and the amplitude-frequency curve of the second harmonic resonance of 1/2 order (Ω=1/2) and 1/3 order (Ω=1/3), which shows that the system has strong sub-harmonic resonance.

The operating speed of gear system is 500 to 4000rpm (meshing frequency is 415 to 3330 Hz). The calculated primary resonance frequencies are about 3050 Hz, and the frequencies of first, second and third order subharmonic resonances are 1525, 1017 and 763 Hz, respectively.

Five typical rotational speeds are selected: (1) n = 600 rpm, far from all levels of resonance peaks of the system, in the non-resonance region; (2) n = 1200 rpm, near the second-order sub-harmonic resonance peak of the system; (3) n = 1800 rpm, near the upper branch of the first-order sub-harmonic resonance peak of the system; (4) n = 2200 rpm is far from the main resonance peak and the first-order sub-harmonic resonance peak, in the non-resonance region; (5) n = 3600 rpm, near the lower main resonance peak of the system. Branch.

Under five rotational speeds (a:800 rpm, b:1200 rpm, c:1800 rpm, d:2200 rpm and e:3600 rpm), the dynamic results of the system can be obtained without considering wear and wear, so as to study the effect of wear on the dynamic response of the system.



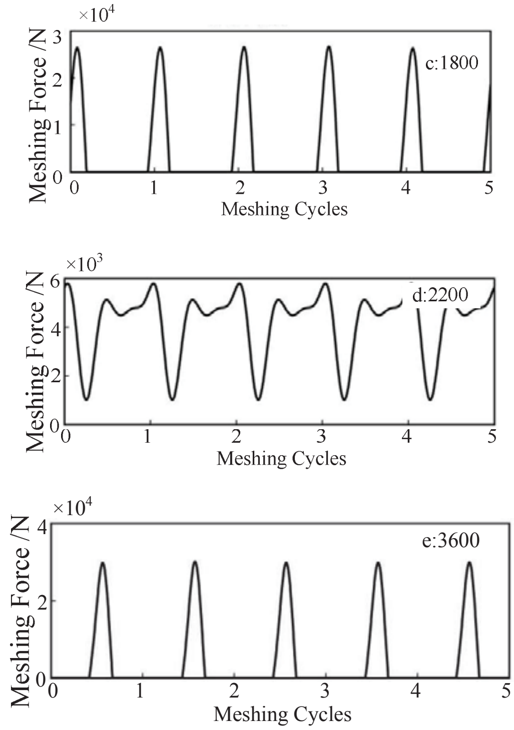
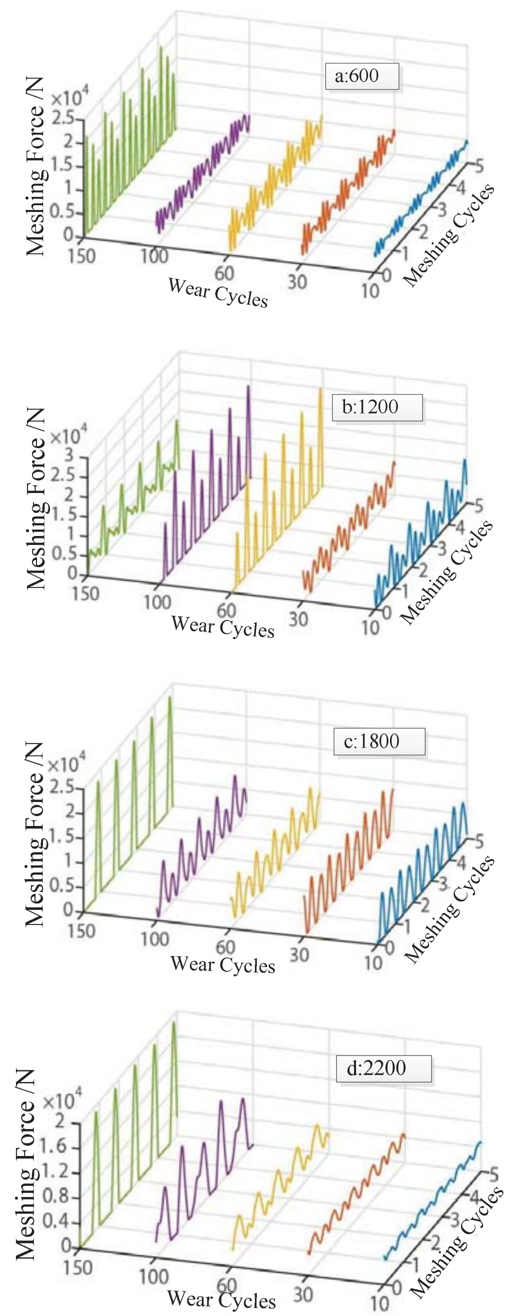


FIGURE 3. MESH FORCE WITHOUT WEAR AT FIVE SPEED（A:600, B:1200, C:1800, D:2200, E:3600）

Without considering the transmission error and clearance excitation caused by tooth surface wear, the dynamic meshing force transmitted by tooth surface is obtained. As shown in Figure 3, no impact occurs on both a and d, while unilateral impact occurs on b, C and D conditions. The meshing tooth surface is separated in 28% of the time in B conditions, 68% in C conditions and 70% in D conditions. Without considering the transmission error and clearance excitation caused by tooth surface wear, the dynamic meshing force transmitted by tooth surface is obtained. As shown in Figure 3, no impact occurs on both a and d, while unilateral impact occurs on b, C and D conditions. The meshing tooth surface is separated in 28% of the time in B conditions, 68% in C conditions and 70% in D conditions.

From Fig. 4, it can be seen that the meshing force is greater than zero in the initial wear period for both working conditions a and d, which indicates that there is no impact phenomenon in the system at the beginning of operation, but with the accumulation of wear, the meshing force appears to be equal to zero, which indicates that the gear system has the situation of unilateral impact. For b, C and e working conditions, unilateral impact occurs in the wear-free stage. With the increase of cycle, impact decreases, and then with the increase of wear, unilateral impact occurs and vibration increases.



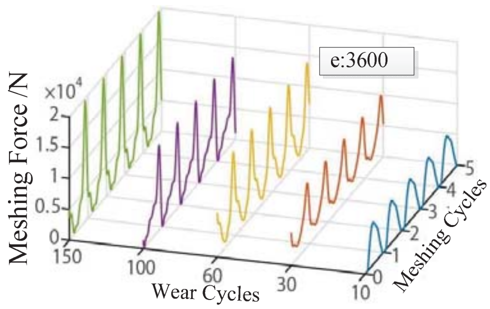


FIGURE 4. MESH FORCE WITH WEAR AT FIVE SPEED（A:600, B:1200, C:1800, D:2200, E:3600）

VI. CONCLUSION

In this paper, the effects of dynamic wear on the dynamic characteristics of gear system under five typical working conditions are studied. the following conclusions can be drawn:

For early wear, wear has little effect on the dynamic response of gears in non-resonant region.

In the gear system operating in resonance zone, the early increase of wear will reduce the vibration of gear system, avoid unilateral impact, and make the system stable. However, with the further increase of wear, the vibration of gear system will intensify, and the impact of gear will appear again.

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