Joint Decision Modeling of Block Inspection Maintenance and Spares Inventory Based on Three-stage Failure Process

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***Abstract*—As the maintenance event and spare parts inventory are mutually influenced, a joint decision modeling of block inspection maintenance and spare parts inventory based on three-stage failure process is proposed. A system with multiple and identical components is considered. The block inspection maintenance and spares provision policy with constant ordering interval and maximum inventory level are considered. The decision total goal is minimizing the total cost rate through optimizing maintenance relevant variables and spares relevant variables. For the sake of demonstrating the validity of the joint decision model, a numerical example is presented and the optimal decision results are gained.**

Keywords-block inspection; preventive maintenance; spare parts inventory; joint decision

I. INTRODUCTION

Maintenance management is important for equipment to improve the reliability and availability through applying effectively. Preventive replacement is a common activity in industrial applications, including age and block replacement policy. According to the two common policy in maintenance practice, Wang divided the functional inspection policy into two types: age inspection policy and block inspection policy. The former focus on single component system, which is inspected under interval *t*. The replacement is implemented because of identifying the defect or failure. However, the latter focus on multi-component system. All of the identical components are inspected under the common interval *t*, regardless of individual component is replaced within this interval. Because of the simplicity of making the inspection maintenance plan, it's very comprehensive in maintenance practice, especially for the important component. So it's meaningful to present the joint decision model of maintenance and spare inventory based on block inspection policy.

In order to optimize the maintenance variables and spares variables, there are some joint decision models of maintenance and inventory based on block inspections. Based on the two-stage delay time theory, Wang [1, 2] focus on periodical inspection system, the joint decision models are built to get the optimal inspection interval and spares parameters. In consideration of the effect of inspection and failure downtime on inspection maintenance decision, Zhang et al. [3] present the block inspection model for one-unit equipment on the basis of two-stage delay time. But the spare parts provision is not considered. Gu et al. [4] present the spare parts inventory decision model based on functional inspection, but the model don't include restrains. All of the above models are on the basis of two-stage failure process.

However, as the event that the state of equipment is expressed as three color, namely green, yellow and red, is common in industrial application, Wang creatively proposed the three-stage failure process, namely normal, minor and severe defective stage [5, 6]. On the basis of three-stage failure process, some researchers built perfect and imperfect maintenance models to optimize the relevant decision variables [7, 8]. Based on this concept, Zhao et al [9] present a joint maintenance and ordering policy. In the industrial practice, this concept is very common. Therefore, it's more innovative and practical to build the joint decision model of maintenance and spare inventory on the basis of three-stage failure process.

On the basis of this new concept, this paper focus on the identical multi-component complex system, the joint decision model of block inspection maintenance and inventory is built. Decision variables are relevant maintenance and spare inventory parameters to derive the minimum total cost rate.

Following sections are arranged as below: problem description is given in Section 2. Section 3 gives the joint decision cost model. Numerical example, results analysis and conclusion are present in Section 4 and 5 respectively.

II. MODELIING DESCRIPTION

The system includes multiple identical components and the lifetime process of every unit is followed as three stages. If the minor defect of component is distinguished at the inspection moment, the minor defect maintenance is present. If the severe defect of component is identified, the severe defect maintenance is present, which can be viewed as overhaul. The failure replacement is implemented once the component is failed. All of the components are inspected at constant interval *t*, even if the component is replaced within one interval. The spares is (T, S) policy, meaning the ordering interval is T and ordering quantity is the difference of maximum inventory quantity S and current level s. The dynamical change of maintenance and inventory is described as Figure 1. The decision objective of joint decision model is to get the minimized total cost rate. Inspection interval t, ordering interval T and S are the optimization parameters.

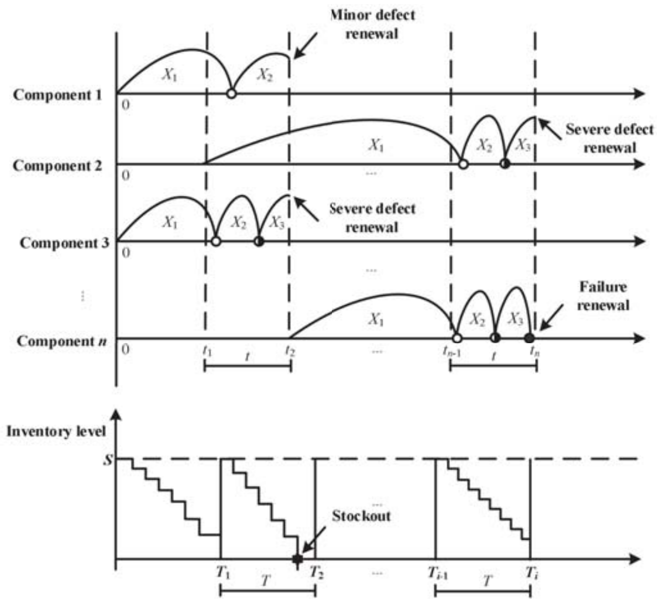

Figure 1 System maintenance and spare parts ordering

The assumption of joint decision model is given:

1) The system contains N identical components. The lifetime process is followed by normal, minor and severe defective stages.

2) Regular inspection is implemented to find system condition. All of the defects can be distinguished at the inspection point.

3) When the minor or severe defect is found, the inspection maintenance renewal is present, otherwise the component will be replaced at the failure point or wait until the next inspection point. Moreover, once the component fails, replacement will be carried out.

4) When the component needs to be replaced, if there are any residual spare parts, they will be replaced directly, otherwise they will be replaced after the next ordering spare parts arrives.

5) The spares is followed by (T, S) policy, namely the spare parts ordering interval is T and the ordering quantity is the difference of maximum inventory level and current inventory level.

6) The average inventory level is the sum of half of the expected ordering quantity and current inventory level.

7) The inspection time, replacement time and lead time of spare parts ordering are ignored.

8) The inspection maintenance and replacement of components can be regarded as component renewal.

The following notations are given:

N : quantity of identical components of system

X_n : random variable on behalf of the nth stage

$f_{X_n}(x)$: probability density function of the nth stage

$F_{X_n}(x)$: cumulative distribution function of the nth stage

S : maximum inventory level

T : inventory inspection interval

t : inspection interval

C_s : cost per inspection

C_{pm} : cost per minor defect maintenance

C_{ps} : cost per severe defect maintenance

C_f : cost per failure

C_o : cost per ordering

C_h : inventory cost per unit time for one component

C_{hs} : stockout cost for one component

$EC_{main}(t)$: relevant maintenance cost within maintenance interval

$EC_f(t)$: expected cost of failure renewal

$EC_m(t)$: expected cost of minor defect maintenance renewal

$EC_s(t)$: expected cost of severe defect maintenance renewal

$EL(t)$: expected length within renewal cycle

$EL_f(t)$: expected length of failure renewal

$EL_m(t)$: expected length of minor defect maintenance renewal

$EL_s(t)$: expected length of severe defect maintenance renewal

$E(S)$: expected inventory level at ordering point

$E(Q)$: expected ordering quantity

$EC_{main}(t,T,S)$: relevant maintenance cost rate

$EC_{inv}(t,T,S)$: relevant inventory cost rate

$EC(t,T,S)$: total cost rate

III. JOINT DECISION MODEL

In this section, joint decision model of block inspection maintenance and spare on basis of multiple-stage failure process is given. The expected total cost rate is the sum of expected maintenance relevant cost rate and expected inventory relevant cost rate. Firstly, the expected renewal cost and length of minor defect, severe defect and failure is get respectively on the basis of three cases. On the basis of these derivations, the expected maintenance relevant cost rate is given. Furthermore, the expected inventory relevant cost rate is derived. The objective function can be formulated based on these results. Therefore, derivations of expected maintenance relevant cost rate under three cases are given firstly.

A. *Expected Maintenance Relevant Cost Rate*

On the basis of the renewal theory, the expected maintenance relevant cost rate is given as

$$EC_{main}(t,T,S) = N\left(\frac{EC_{main}(t)}{EL(t)} + C_{hs}\right) \tag{1}$$

Where the $EC_{main}(t)$ and $EL(t)$ can be derived on the basis of renewal theory. As there are three cases of component renewal, namely minor defect renewal, severe defect renewal and failure renewal, the expected renewal cost and length should be derived firstly.

$$EC_{main}(t,T,S) = N\left(\frac{EC_{main}(t)}{EL(t)} + C_{hs}\right) = N\left(\frac{EC_m(t) + EC_s(t) + EC_f(t)}{EL_m(t) + EL_s(t) + EL_f(t)} + C_{hs}\right) \tag{2}$$

Let's start with the derivation of expected cost of component renewal within maintenance cycle.

The expected cost of minor defect renewal within maintenance cycle is

$$EC_m(t) = \sum_{m=1}^{\infty}(C_{pm} + mC_s)\int_{(m-1)t}^{mt} f_{X_1}(x)(1 - F_{X_2}(mt - x))dx \tag{3}$$

The expected cost of severe defect renewal within maintenance cycle is

$$EC_s(t) = \sum_{m=1}^{\infty}(C_{ps} + mC_s)\int_{(m-1)t}^{mt} f_{X_1}(x)\int_0^{mt-x} f_{X_2}(y)(1 - F_{X_3}(mt - x - y))dydx \cdot \tag{4}$$

The expected renewal cost of failure within maintenance cycle is

$$EC_f(t) = \sum_{m=1}^{\infty}(C_f + (m-1)C_s)\int_{(m-1)t}^{mt} f_{X_1}(x)\int_0^{mt-x} f_{X_2}(y)F_{X_3}(mt - x - y)dydx \tag{5}$$

Then the expected renewal length of component within maintenance round is given.

Expected length of minor defect renewal within maintenance round is

$$EL_m(t) = \sum_{m=1}^{\infty} mt\int_{(m-1)t}^{mt} f_{X_1}(x)(1 - F_{X_2}(mt - x))dx \tag{6}$$

Expected length of severe defect renewal within maintenance round is

$$EL_s(t) = \sum_{m=1}^{\infty} mt\int_{(m-1)t}^{mt}\int_0^{mt-x} f_{X_1}(x)f_{X_2}(y)(1 - F_{X_3}(mt - x - y))dydx \cdot \tag{7}$$

The expected renewal length of failure within maintenance cycle is

$$EL_f(t) = \sum_{m=1}^{\infty}\int_0^t((m-1)t + z)\int_{(m-1)t}^{(m-1)t+z} f_{X_1}(x)\int_0^{(m-1)t+z-x} f_{X_2}(y)f_{X_3}((m-1)t + z - x - y)dzdydx \tag{8}$$

Therefore, the expected maintenance relevant cost rate can be derived by formula (2).

B. *Expected Inventory Relevant Cost Rate*

The expected inventory relevant cost rate can be expressed as

$$EC_{inv}(t,T,S) = \lim_{m\to\infty} m \cdot \frac{\left(E(S) + \frac{E(Q)}{2}\right)C_h T + C_o}{mT} = \left(E(S) + \frac{E(Q)}{2}\right)C_h + \frac{C_o}{T} \tag{9}$$

Actually the maintenance relevant cost and spare parts inventory relevant cost are affected by inspection and ordering interval, maximum inventory level. For the sake of building the relationship of maintenance relevant cost and spare relevant cost, the following method is proposed. The time length is assumed as τ, the spare parts demand quantity caused by system renewal is $N\tau / EL(t)$, and the spare parts provision quantity is $\tau E(Q)/T$. Hence we assume the time length τ to be very long, spare parts demand quantity and provision quantity is same, namely $N\tau / EL(t) = \tau E(Q)/T$. This equality can be expressed as

$$E(Q) = \frac{NT}{EL(t)} \tag{10}$$

As the $E(Q) + E(S) = S$, so the equality (10) can be given as

$$EC_{inv}(t,T,S) = \left(E(S) + \frac{E(Q)}{2}\right)C_h + \frac{C_o}{T} = \left(S - \frac{E(Q)}{2}\right)C_h + \frac{C_o}{T} = \left(S - \frac{NT}{2EL(t)}\right)C_h + \frac{C_o}{T} \tag{11}$$

C. *Total Cost Rate*

The total cost rate is the sum of expected maintenance relevant cost rate and expected inventory relevant cost rate.

$$EC(t,T,S) = EC_{main}(t,T,S) + EC_{inv}(t,T,S) = N\left(\frac{EC_{main}(t)}{EL(t)} + C_{hs}\right) + \left(S - \frac{NT}{2EL(t)}\right)C_h + \frac{C_o}{T} \tag{12}$$

IV. NUMERICAL ANALYSIS

An example is present to give the effectiveness of joint decision model in this section to get the optimal decision objective and parameters.

The Weibull distribution with parameters *a* and *b* is given to show the lifetime process [15], as shown in

$$f(x;a,b) = a \cdot b(b \cdot x)^{a-1} \tag{13}$$

The lifetime distribution parameters and cost parameters are given in Tables 1 and 2 respectively.

TABLE I. PARAMETERS OF LIFETIME DISTRIBUTION

a_1	b_1	a_2	b_2	a_3	b_3
0.0565	2.14	0.107	1.78	0.145	2.65

TABLE II. PARAMETERS OF COST

C_s	C_{pm}	C_{ps}	C_f	C_o	C_h	C_{hs}	N
20	30	50	150	80	1	5	30

In the equality (12), (S-NT/2EL(t)) is the average inventory level, so we can derive the relationship $S > NT/2EL(t)$, which can be viewed as a restrain. We substitute the restrain into equality (12) for iterative solution. The solution algorithm is as follws:

Firstly, the value of T is fixed and the restrain condition is substituted to derive the range of S. Then the value of S is fixed in this range, the optimal inspection variable t to minimize the total cost rate $EC(t, T, S)$ is derived. Next the value of S is changed, relational value of $EC(t, T, S)$ and t can be get. So the minimum $EC(t, T, S)$ can be found, corresponding to the optimal inspection variable t and S. Finally, the value of T is changed and repeat the above process, the minimum total cost rate $EC^*(t, T, S)$ and optimal parameters can be derived.

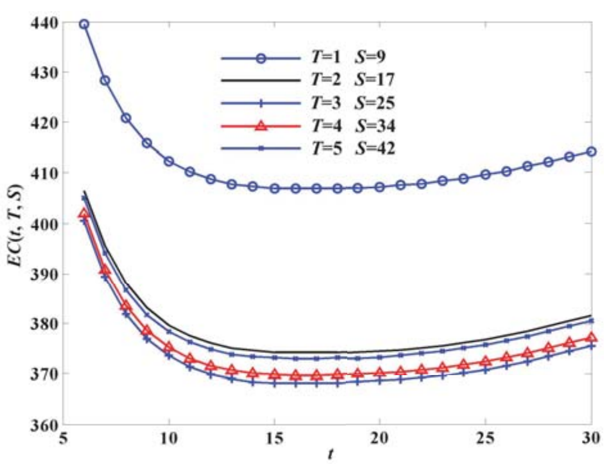

Figure 2 The corresponding relationship of $EC(t, T, S)$ and t with constant T and S

Figure 2 shows the relation of total cost rate $EC(t, T, S)$ and inspection interval t, when the ordering interval T is fixed and derived optimal maximum inventory level S. We can see that, under every group of (T, S) value, there exists an optimal t to minimize the total cost rate. The value of $EC(t, T, S)$ decreases and then increases with increasing of (T, S) value. The minimum point of $EC(t, T, S)$ is 368.098 when t*=16, T*=3, S*=25. Therefore, the optimal joint policy of inspection interval t*=16, ordering variable T*=3 and S*=25 can minimize the $EC(t, T, S)$.

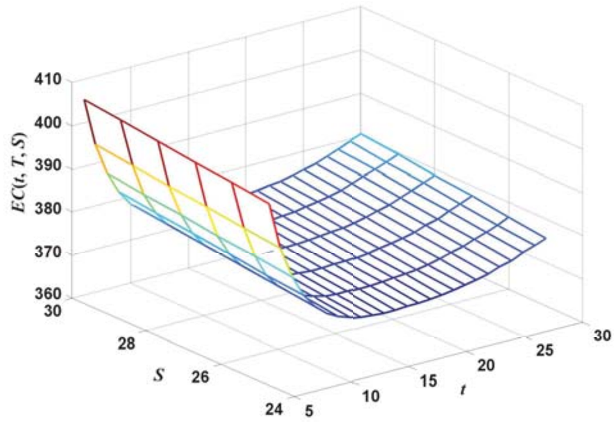

Figure 3 The corresponding relationship of $EC(t, T, S)$ with t and S

Figure 3 shows that when T*=3, the $EC(t, T, S)$ changes with the inspection variable t and S. We can see that, if the value of S is constant and t is increasing, $EC(t, T, S)$ decreases and then increases, which exists the optimal value. Moreover, if value of t is constant and S is increasing, $EC(t, T, S)$ always increases. When S*=25, the $EC(t, T, S)$ is to be minimized. Therefore, the $EC(t, T, S)$ decreases firstly and then increases with the increasing of value t, but $EC(t, T, S)$ always increases with the increasing of value S. The numerical analysis results can provide the evidence for formulating and implementing the block inspection policy and spare inventory policy.

V. CONCLUSIONS

A joint decision modeling of block inspection maintenance and spares based on three-stage failure process for identical multi-unit systems. The joint policy of system are the block inspection policy and (T, S) inventory policy. The decision goal is to derive the minimum total cost rate to derive the optimal decision variables, namely inspection interval, ordering interval and maximum inventory level. A results analysis is present to show the validity of joint decision model. Further research can be followed such as in consideration of equipment condition and monitoring information.

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