Remaining Useful Lifetime Prediction for the Equipment with the Random Failure Threshold

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***Abstract*—Prognostics and health management (PHM) technology is widely used in industrial production, and its core is to predict the remaining useful life (RUL) of the equipment. For the existing research of RUL prediction, the impact of random failure threshold (RFT) has not been analyzed. To solve this problem, an RUL prediction method based on the Kalman filter is proposed. Firstly, a nonlinear Wiener degradation model is built in this paper. Then, the parameters of the degradation model are estimated by the maximum likelihood estimation (MLE) method and the distribution coefficients of RFT are calculated by the expected maximum (EM) algorithm. In addition, the Kalman filtering technique is applied to renewal the degradation states by obtaining condition monitoring (CM) data. Finally, the analytical expression of probability density function (PDF) for the RUL is derived by considering the RFT. The simulation example shows that the method in this paper has advantages of RUL prediction, and thus can have potentially engineering application value.**

Keywords—RUL prediction; Kalman Filter; random failure threshold; EM algorithm; MLE method

I. INTRODUCTION

The RUL prediction of equipment plays a key function in PHM technology and is an important basis for obtaining preventive maintenance and protection strategy selection. Therefore, the prediction method of equipment's RUL has become a hot spot in reliability research in the past two decades [1]-[3].

In general, the RUL of equipment is determined by two key factors which are the degradation process and the failure threshold (FT). Due to differences in system composition and operating environment, the FT and degradation process of different equipment is always different, which means that the FT and degradation process are stochastic and can't be described by a determination process or value. Today, the Wiener degradation process is widely used to construct degradation models of equipment due to its non-monotonic nature and favorable mathematical properties. Under normal conditions, the randomness of the linear Wiener process is determined by its drift parameters. For the nonlinear Wiener process that is prevalent in reality, how to establish the accuracy degradation model is the hotspot and difficulty of current research. The traditional modeling theory of nonlinear model assumes that the nonlinear model can be transformed into a linear model by using the time scale transformation theory [4] or the logarithmic transformation theory [5]. However, many degradation processes cannot be converted to linearity in practice. To solve this problem, Si introduces a mild hypothesis that can transform a nonlinear Wiener process into a linear process without data conversion [6]. Based on the study of Si, many scholars have studied the effects of random drift parameters of nonlinear Wiener processes on equipment RUL prediction [6]-[8]. However, the measurement error (ME) was not considered in the above study, and the degradation state of the degradation model was not updated under the observation of the CM data. Tang et al. [9] introduced the nonlinear function, ME and random drift parameters into the degradation model, and the Bayesian method is used to update the drift parameters. However, practice shows that the Bayesian method does not achieve good accuracy. In order to improve the accuracy of RUL prediction, based on the establishment of the state-space model, Zheng et al [10] use Kalman filtering to update the degradation state and drift parameters, which achieve good results.

The FTs in the above studies are all fixed values. However, due to individual differences between each other, the fault thresholds of different equipment are not always the same. For example, the vibration level of the machine [11], the fatigue degradation of the metal material [12] and the degree of wear of the lathe [13] are random values, but not fixed. Based on the assumption that the equipment failure threshold is random, many scholars use the proportional hazard model [14] or the Gamma process [15] to describe the randomness of the FT. Usynin et al [16] derived the PDF integral expression based on the linear Wiener process. However, this paper cannot give a distribution of RTFs. Wang et al [17] introduced a method that applies an exponential distribution to describe the randomness of the FT and derives the integral expression of PDF to the RUL. However, the estimation method of the RTF distribution coefficient is not given in the paper.

With the analysis of the above paper, it is clear that the RUL prediction based on RTF has not been thoroughly studied. Aiming at this problem, the impact of RFT in RUL prediction is analysis in this paper. First, random degradation and MEs are introduced into the traditional nonlinear Wiener process. Then, the parameters are estimated by the MLE and EM methods. At the same time, the degradation state and drift parameters are updated based on the Kalman filtering. After

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that, the PDF of the RUL is derived by considering the RTF. Finally, a simulation example is used to verify the effectiveness and rationality of the proposed method.

II. NONLINEAR DEGRADATION MODELING

The nonlinear Wiener process consists of three parts, which can be written as (1).

$$X(t) = X(0) + \alpha\Theta(t,\xi) + \sigma_B B(t) \tag{1}$$

where, $X(t)$ is the state variables of the equipment at t, $B(t)$ is the standard Brownian motion, σ_B and α are diffusion and drift coefficient respectively. $\Theta(t,\eta)$ is a monotonic continuous function for time t with parameter η. There are mainly two types of $\Theta(t,\eta)$, one is $\Theta(t,\eta) = t\wedge\eta$ and another is $\Theta(t,\eta) = \exp(\eta t) - 1$. Under normal circumstances, $X(0)$ can be equal to 0 and α is independent of $B(t)$. In addition, α is often assumed to be subject to a normal distribution, $\alpha \sim (\mu_\alpha, \sigma_\alpha^2)$, which can reflect the individual differences between the different equipment. While σ_B is treated as a fixed value which can extract the common features of the equipment.

$Y(t)$ is used to represent the measurements of performance degradation $X(t)$ with measurement error ε. And we also called $Y(t)$ as CM value.

$$Y(t) = X(t) + \varepsilon \tag{2}$$

where ε represents the MEs and is assumed to be subject to a normal distribution $N(0,\sigma_\varepsilon^2)$.

III. RUL PREDICTION WITH RTF

Deriving the PDF of the equipment's RUL is the key technology of RUL prediction. In this section, we use the MLE method in literature [6] to get the parameters estimation of the degradation model, i.e. $\hat{\mu}_\alpha, \hat{\sigma}_\alpha^2, \hat{\eta}, \hat{\sigma}_B^2, \hat{\sigma}_\varepsilon^2$. And the distribution coefficients ($\mu_\omega, \sigma_\omega$) of RTF are calculated by the EM algorithm. And the approximate analytic expression of PDF to RUL is derived from the concept of first arrival time (FAT). What's more, a state-space model is built in this paper to update the degradation states and drift parameters based on the Kalman filtering theory.

A. Distribution coefficients of RTF estimation

It can be seen from the above analysis that the RTF ω should be greater than zero. And We assume that $\boldsymbol{\omega} = [\omega_1', \omega_2', \cdots, \omega_R', \omega_1'', \omega_2'', \cdots, \omega_S'']$ obey normal distribution $N(\mu_\omega, \sigma_\omega^2)$, where $\boldsymbol{\omega}' = [\omega_1', \omega_2', \cdots, \omega_R']$ are less than zero, and $\boldsymbol{\omega}'' = [\omega_1'', \omega_2'', \cdots, \omega_S'']$ are greater than zero. Based on the non-negative assumption of the RTF, $\boldsymbol{\omega}''$ can be used to represent the failure threshold data of S samples, and $\boldsymbol{\omega}'$ represents the unobserved virtual failure threshold data. Based on the above analysis, the profile likelihood function can be written as (3).

$$\ln LN(\omega|\boldsymbol{\omega}) = -\frac{R+S}{2}\ln(2\pi\sigma_\omega^2) - \sum_{i=1}^{R}\frac{(\omega_i' - \mu_\omega)^2}{2\sigma_\omega^2} - \sum_{i=1}^{S}\frac{(\omega_i'' - \mu_\omega)^2}{2\sigma_\omega^2} \tag{3}$$

Let $\mu_{\omega,j}$ and $\sigma_{\omega,j}^2$ represent the estimation results of the j-th iteration of the EM algorithm, then the j+1-th iterative process can be decomposed into E steps and M steps.

E step: The expectation of $\boldsymbol{\omega}'$ for equation (3) can be written as:

$$\begin{aligned} W(\mu_\omega, \sigma_\omega^2 | \mu_{\omega,j}, \sigma_{\omega,j}^2) &= E_{\omega'}\left(\ln L(\omega|\boldsymbol{\omega})\right) = \\ -\frac{R+S}{2}\ln(2\pi\sigma_\omega^2) &- \sum_{i=1}^{S}\frac{(\omega_i'' - \mu_\omega)^2}{2\sigma_\omega^2} - \\ \sum_{i=1}^{R}\frac{\left(E(\omega_i'|\mu_{\omega,j},\sigma_{\omega,j}^2) - \mu_\omega\right)^2}{2\sigma_\omega^2} &- \sum_{i=1}^{R}\frac{D(\omega_i'|\mu_{\omega,j},\sigma_{\omega,j}^2)}{2\sigma_\omega^2} \end{aligned} \tag{4}$$

M step: Find the maximum value of $W(\mu_\omega, \sigma_\omega^2 | \mu_{\omega,j}, \sigma_{\omega,j}^2)$, then

$$\left(\mu_{\omega,j}, \sigma_{\omega,j}^2\right) = \underset{\mu_\omega,\sigma_\omega^2}{\arg\max} W(\mu_\omega, \sigma_\omega^2 | \mu_{\omega,j}, \sigma_{\omega,j}^2) \tag{5}$$

Based on the above analysis, the iterative formula of μ_ω and σ_ω can be further obtained.

$$\mu_{\omega,j+1} = \frac{\sum_{i=1}^{S}\omega_i'' + R_j E(\omega'|\mu_{\omega,j},\sigma_{\omega,j}^2)}{S + R_j} \tag{6}$$

$$\begin{aligned} \sigma_{\omega,j+1}^2 &= \frac{R_j}{R_j+S}\left(E(\omega'|\mu_{\omega,j},\sigma_{\omega,j}^2) - \mu_{\omega,j+1}\right)^2 + \\ &\frac{1}{R_j+S}\sum_{i=1}^{S}(\omega_i'' - \mu_{\omega,j+1})^2 + \frac{R_j}{R_j+S}D(\omega'|\mu_{\omega,j},\sigma_{\omega,j}^2) \end{aligned} \tag{7}$$

where,

$$R_j = \frac{S\left(1 - \Phi(\mu_{\omega,j}/\sigma_{\omega,j})\right)}{\Phi(\mu_{\omega,j}/\sigma_{\omega,j})} \tag{8}$$

B. Degradation states updating

For online update degradation status of equipment, the essence is to use $Y(t)$ to update $X(t)$ and λ. In this paper, the parameter estimation value obtained by the MLE method is used as the initial distribution parameter of the drift parameter, i.e. $\lambda_0 \sim N(\mu_{\lambda_0}, \sigma_{\lambda_0}^2)$. For the degradation process represented as (2), the degradation states updating mechanism is concluded as follows:

$$\begin{cases} x_k = x_{k-1} + \alpha_{k-1}(\Theta(t_k,\xi) - \Theta(t_{k-1},\xi)) + \Gamma_k \\ \alpha_k = \alpha_{k-1} \\ y_k = x_k + \varepsilon \end{cases} \tag{9}$$

where, $\Gamma_k = \sigma_B B(t_k - t_{k-1})$, $Y(t_k) = y_k$, $X(t_k) = x_k$. Due to the property of the standard Brownian motion $B(t)$, we have: $\Gamma_k \sim N(0, \sigma_B^2 \Delta t_k)$, and $\Delta t_k = t_k - t_{k-1}$, $t_0 = 0$.

To facilitate the inference, (3) can be written as:

$$\begin{cases} \boldsymbol{Z}_k = \boldsymbol{A}_k \boldsymbol{Z}_{k\text{-}1} + \boldsymbol{B}_k \\ y_k = \boldsymbol{C} \boldsymbol{Z}_{k\text{-}1} + \varepsilon \end{cases} \tag{10}$$

where, $\boldsymbol{Z}_k = (x_k, \alpha_k)^{\mathrm{T}}$, $\boldsymbol{B}_k = (\Gamma_k, 0)^{\mathrm{T}}$, $\boldsymbol{C} = (1, 0)$, $\boldsymbol{A}_k = \begin{bmatrix} 1 & \Theta(t_k, \eta) - \Theta(t_{k-1}, \eta) \\ 0 & 1 \end{bmatrix}$.

In this paper, $\boldsymbol{Y}_{1:k} = (y_1, y_2, \cdots, y_k)^{\mathrm{T}}$ is represented as the measured value of state variables at the time t_1, t_2, $\cdots$, t_k. Correspondingly, $\boldsymbol{X}_{1:k} = (x_1, x_2, \cdots, x_k)^{\mathrm{T}}$ is the actual state variables of the equipment.

Firstly, the variance and expectation of state variable $\boldsymbol{Z}_k$ are defined as follows.

$$E(\boldsymbol{Z}_k | \boldsymbol{Y}_{1:k}) = \hat{\boldsymbol{Z}}_{k|k} = (\hat{x}_{k|k}, \hat{\alpha}_{k|k}) \tag{11}$$

$$Var(\boldsymbol{Z}_k | \boldsymbol{Y}_{1:k}) = \hat{\boldsymbol{P}}_{k|k} = \begin{bmatrix} \delta_{x,k}^2 & \delta_{\theta,k}^2 \\ \delta_{\theta,k}^2 & \delta_{\alpha,k}^2 \end{bmatrix} \tag{12}$$

where, $\delta_{x,k}^2 = Var(x_k | \boldsymbol{Y}_{1:k})$, $\delta_{\lambda,k}^2 = Var(\alpha_k | \boldsymbol{Y}_{1:k})$, $\delta_{\theta,k}^2 = \mathrm{cov}(x_k, \alpha_k | \boldsymbol{Y}_{1:k})$, $\hat{x}_{k|k} = E(x_k | \boldsymbol{Y}_{1:k})$, $\hat{\alpha}_{k|k} = E(\alpha_k | \boldsymbol{Y}_{1:k})$.

Then, the one-step predicted variance and expectation of $\boldsymbol{Z}_k$ are defined as follows.

$$E(\boldsymbol{Z}_k | \boldsymbol{Y}_{1:k\text{-}1}) = \hat{\boldsymbol{Z}}_{k|k\text{-}1} = (\hat{x}_{k|k\text{-}1}, \hat{\alpha}_{k|k\text{-}1}) \tag{13}$$

$$Var(\boldsymbol{Z}_k | \boldsymbol{Y}_{1:k\text{-}1}) = \hat{\boldsymbol{P}}_{k|k\text{-}1} = \begin{bmatrix} \delta_{x,k\text{-}1}^2 & \delta_{\theta,k\text{-}1}^2 \\ \delta_{\theta,k\text{-}1}^2 & \delta_{\alpha,k\text{-}1}^2 \end{bmatrix} \tag{14}$$

Finally, applying the Kalman filtering to update x_k and α, filtering formulations are as follows.

$$\hat{\boldsymbol{Z}}_{k|k} = \hat{\boldsymbol{Z}}_{k|k\text{-}1} + \boldsymbol{\Psi}_k (y_k - \boldsymbol{C} \hat{\boldsymbol{Z}}_{k|k\text{-}1}) \tag{15}$$

$$\hat{\boldsymbol{Z}}_{k|k\text{-}1} = \boldsymbol{A}_k \hat{\boldsymbol{Z}}_{k-1|k\text{-}1} \tag{16}$$

$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} - \boldsymbol{\Psi}_k \boldsymbol{C} \boldsymbol{P}_{k|k-1} \tag{17}$$

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{D}_k + \boldsymbol{A}_k \boldsymbol{P}_{k-1|k-1} \boldsymbol{A}_k^{\mathrm{T}} \tag{18}$$

$$\boldsymbol{\Psi}_k = \boldsymbol{P}_{k|k\text{-}1} \boldsymbol{C}^{\mathrm{T}} (\boldsymbol{C} \boldsymbol{P}_{k|k\text{-}1} \boldsymbol{C}^{\mathrm{T}} + \sigma_\varepsilon^2)^{-1} \tag{19}$$

$$\boldsymbol{D}_k = \begin{bmatrix} \sigma_B^2 \Delta t_k & 0 \\ 0 & 0 \end{bmatrix} \tag{20}$$

$$\hat{\boldsymbol{Z}}_{0|0} = \begin{bmatrix} 0 \\ \mu_{\alpha_0} \end{bmatrix} \tag{21}$$

$$\boldsymbol{P}_{0|0} = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_{\alpha_0}^2 \end{bmatrix} \tag{22}$$

Due to the Gaussian properties of Kalman filtering, it can be obtained that $\boldsymbol{Z}_k | \boldsymbol{Y}_{1:k} \sim N(\hat{\boldsymbol{Z}}_{k|k}, \boldsymbol{P}_{k|k})$, and we can obtain that

$$\alpha_k | \boldsymbol{Y}_{1:k} \sim N(\mu_{\alpha_k}, \sigma_{\alpha_k}^2) \tag{23}$$

where, $\mu_{\alpha_k} = \hat{\alpha}_{k|k}$, $\sigma_{\alpha_k}^2 = \delta_{\alpha,k}^2$.

C. RUL distribution estimation

The PDF of RUL is the key point of the RUL prediction analysis. In this section, we derived the PDF expression of RUL with considering the RFT. In traditional research, the equipment's RUL is often defined as the runtime from the current moment to the failure moment, i.e. $l_t = T - t | T > t$, where T is the lifetime and t represents the current moment. According to the literature [2], the definition of T is:

$$T = \inf\{t : X(t) \geq \omega | X(0) < \omega\} \tag{24}$$

where ω is the FT of the equipment.

Given the failure threshold ω, time t_k and performance degradation $\boldsymbol{X}_{1:k}$, the definition of l_{t_k} can be written as follows.

$$L_{t_k} = \inf\{l_{t_k} : X(t_k + l_{t_k}) \geq \omega | \boldsymbol{X}_{1:k}\} \tag{25}$$

So as to derive the PDF of the RUL under nonlinear degradation model, this paper introduces Lemma1.

Lemma 1 [9] Aiming at the degradation process as (2), the CM data $\boldsymbol{Y}_{1:k}$ and fixed failure threshold ω are obtained, then the PDF $f_{L_k|\omega,\boldsymbol{Y}_{1:k}}(l_k | \omega, \boldsymbol{Y}_{1:k})$ of the equipment's RUL l_{t_k} can be formulated as

$$\begin{aligned} f_{L_{t_k}|\boldsymbol{Y}_{1:k},\omega}(l_{t_k} | \boldsymbol{Y}_{1:k}, \omega) \approx & \sqrt{\frac{1}{2\pi l_{t_k}^2 \left(\psi(l_{t_k})^2 \sigma_{\alpha_k}^2 + \sigma_B^2 l_{t_k} + \sigma_\varepsilon^2\right)}} \cdot \\ & \exp\left(-\frac{\left(\omega - y_k - \mu_{\alpha_k} \psi(l_{t_k})\right)^2}{2\left(\psi(l_{t_k})^2 \sigma_{\alpha_k}^2 + \sigma_B^2 l_{t_k} + \sigma_\varepsilon^2\right)}\right) \cdot \\ & \left[\omega - y_k - \mu_{\alpha_k} \beta(l_{t_k}) - \frac{\omega - y_k - \mu_{\alpha_k} \psi(l_{t_k})}{\psi(l_{t_k})^2 \sigma_{\alpha_k}^2 + \sigma_B^2 l_{t_k} + \sigma_\varepsilon^2} \left(\psi(l_{t_k})^2 \sigma_{\alpha_k}^2 + \sigma_\varepsilon^2\right)\right] \end{aligned} \tag{26}$$

where

$$\psi(l_{t_k}) = \Theta(t_k + l_{t_k}, \eta) - \Theta(t_k, \eta) \tag{27}$$

$$\beta(l_{t_k}) = \psi(l_{t_k}) - l_{t_k} \, \mathrm{d}\psi(l_{t_k}) / \mathrm{d}l_{t_k} \tag{28}$$

In the actual applications, the equipment's failure threshold is not always a fixed value but a random variable. In reality, the failure threshold generally satisfies the assumption of a normal distribution, i.e. $\omega \sim N(\mu_\omega, \sigma_\omega^2)$. It's easy to observe that the failure threshold ω may sometimes be negative. But it does not match the actual degradation process. In order to make the RTF always non-negative, i.e. $\omega > X(0) = 0$, the truncated normal distribution (TND) is introduced into this paper to describe the RTF. Assuming that ω obeys normal distribution and is always positive, i.e., $\omega > 0$, and the ω obeys TND which can be written as $\omega \sim TN(\mu_\omega, \sigma_\omega^2)$. The distribution function of RFT can be written as (29).

$$f(\omega) = \frac{1}{\sqrt{2\pi\sigma_\omega^2}\Phi(\mu_\omega/\sigma_\omega)} \exp\left(-\frac{(\omega-\mu_\omega)^2}{2\sigma_\omega^2}\right) \tag{29}$$

According to the nonlinear degradation model with RTF, the PDF of RUL can be derived by the lemma2 and Theorem1, which are presented as follows.

Lemma 2 [18] If $D \sim TN(\mu, \sigma^2)$, $E, F \in R$ and $G \in R^+$, then

$$\begin{aligned} & E_D\left[(D-E)\exp\left(-\frac{(D-F)^2}{2G}\right)\right] = \frac{1}{\sqrt{2\pi}\Phi(\mu/\sigma)} \cdot \\ & \exp\left(-\frac{(\mu-F)^2}{2(G+\sigma^2)}\right)\left[\frac{G\sigma}{G+\sigma^2}\exp\left(-\frac{(F\sigma^2+G\mu)^2}{2(G+\sigma^2)G\sigma^2}\right) + \right. \\ & \left.\left(\frac{F^2\sigma^2+G\mu}{G+\sigma^2} - E\right)\sqrt{\frac{2\pi G}{G+\sigma^2}}\Phi\left(\frac{F\sigma^2+G\mu}{\sqrt{(G+\sigma^2)G\sigma^2}}\right)\right] \end{aligned} \tag{30}$$

Theorem 1: If the nonlinear Wiener degradation process is written as (2), and the CM data $\boldsymbol{Y}_{1:k}$ are obtained, then the PDF expression of RUL can be derived.

$$\begin{aligned} & f_{L_{t_k}|\boldsymbol{Y}_{1:k}}(l_{t_k}|\boldsymbol{Y}_{1:k}) \approx \\ & \frac{\sigma_B^2}{2\pi\Phi(\mu_\omega/\sigma_\omega)H_1}\exp\left(-\frac{(\mu_\omega - H_2)^2}{2(H_1+\sigma_\omega^2)}\right) \cdot \\ & \left[\frac{\sqrt{H_1\sigma_\omega^2}}{H_1+\sigma_\omega^2}\exp\left(-\frac{(H_2\sigma_\omega^2 + H_1\mu_\omega)^2}{2(H_1+\sigma_\omega^2)H_1\sigma_\omega^2}\right) + \sqrt{\frac{2\pi}{H_1+\sigma_\omega^2}} \cdot \right. \\ & \left.\left(\frac{H_2^2\sigma_\omega^2 + H_1\mu_\omega}{H_1+\sigma_\omega^2} - H_3\right)\Phi\left(\frac{H_2\sigma_\omega^2+H_1\mu_\omega}{\sqrt{(H_1+\sigma_\omega^2)H_1\sigma_\omega^2}}\right)\right] \end{aligned} \tag{31}$$

where

$$H_1 = \psi(l_{t_k})^2\sigma_{\lambda_k}^2 + \sigma_B^2 l_{t_k} + \sigma_\varepsilon^2 \tag{32}$$

$$H_2 = y_k + \mu_{\lambda_k}\psi(l_{t_k}) \tag{33}$$

$$\begin{aligned} H_3 = & \frac{\left(y_k + \mu_{\lambda_k}\beta(l_{t_k})\right)H_1}{\sigma_B^2 l_{t_k}} - \\ & \left(\psi(l_{t_k})^2\sigma_{\lambda_k}^2 + \sigma_\varepsilon^2\right)\frac{y_k + \mu_{\lambda_k}\psi(l_{t_k})}{\sigma_B^2 l_{t_k}} \end{aligned} \tag{34}$$

The proof of Theorem1:

According to the law of total probability, $f_{L_{t_k}|\boldsymbol{Y}_{1:k}}(l_{t_k}|\boldsymbol{Y}_{1:k})$ can be derived:

$$\begin{aligned} f_{L_{t_k}|\boldsymbol{Y}_{1:k}}(l_{t_k}|\boldsymbol{Y}_{1:k}) = & \int_0^{+\infty} f_{L_{t_k}|\omega,\boldsymbol{Y}_{1:k}}(l_{t_k}|\omega,\boldsymbol{Y}_{1:k})P(\omega)\mathrm{d}\omega = \\ & E_\omega\left[f_{L_{t_k}|\omega,\boldsymbol{Y}_{1:k}}(l_{t_k}|\omega,\boldsymbol{Y}_{1:k})\right] \end{aligned} \tag{35}$$

The Theorem1 can be derived straightforwardly based on the (29), (20) and Lemma2.

IV. EXAMPLES

A simulation example is built to illustrate the effectiveness and rationality of the proposed RUL prediction method. By comparing with the present methods [9][18], it can be easily concluded that our method has higher prediction accuracy. For the sake of simplicity, our method is called M0, and the RUL prediction method that does not consider the RTF in [9] is called M1. The method without considering the nonlinear degradation process in [18] is expressed as M2.

In this paper, the Monte Carlo method is used to simulate the nonlinear degradation model with MEs and random drift parameters. Then, we set $\Theta(t,\eta) = t^\wedge\eta$ and $\mu_\alpha = 3.5$, $\sigma_\alpha^2 = 0.02^2$, $\sigma_B^2 = 0.1$, $\eta = 1.8$, $\sigma_\varepsilon^2 = 0.05^2$, $\Delta t = 0.1\mathrm{h}$. Based on the above simulation parameters, six items' CM data are simulated in Tab. I.

TABLE I CM DATA

CM/h	Item1	Item2	Item3	Item4	Item5	Item6
0.1	0.16	0.02	0.10	0.06	0.14	0.01
0.2	0.35	0.18	0.04	0.35	0.23	0.19
0.3	0.57	0.53	0.26	0.50	0.48	0.40
0.4	0.74	0.82	0.69	0.72	0.61	0.66
0.5	1.03	0.97	1.25	1.24	0.92	1.26
0.6	1.57	1.29	1.47	1.71	1.44	1.69
0.7	1.79	1.68	2.03	2.15	1.82	2.04
0.8	2.18	2.25	2.41	2.64	2.39	2.52
0.9	2.66	2.81	3.06	3.20	2.82	3.10
1.0	3.24	3.31	3.92	3.71	3.62	3.64

In this paper, we use the fourth item as the target equipment. Assuming that the CM data at time $t_k = 1h$ for each item are their own failure threshold. Fitting the failure threshold of each item and we can obtain the distribution coefficients by EM algorithm as $\omega \sim N(3.7189, 0.0398)$. Then, the parameters estimation by using MLE under M0, M1, and M2 are listed in Tab. II.

TABLE II THE PARAMETER ESTIMATION RESULTS

	μ_α	σ_α^2	σ_B^2	σ_ε^2	η
Real value	3.5000	0.0004	0.1000	0.0025	1.8000
M0	3.5551	0.0319	0.0415	0.0044	0.0044
M1	3.5551	0.0319	0.0415	0.0044	0.0044
M2	3.5339	0.0668	0.3439	0.0059	1.0000

In order to further compare the difference between the above methods, we apply the mean square error (MSE) to describe the model fitting of different methods. The definition of MSE can be written as:

$$\mathrm{MSE} = \int_0^\infty f_{L_{t_k}|\mathbf{Y}_{1:k}}(l_{t_k}|\mathbf{Y}_{1:k})(t_k + l_{t_k} - T)^2 \,\mathrm{d}\, l_{t_k}$$

In this paper, we calculated the MSE of M0, M1, and M2 from CM time 0.4h to 1h. The MSEs in different CM times are shown in Figure 1.

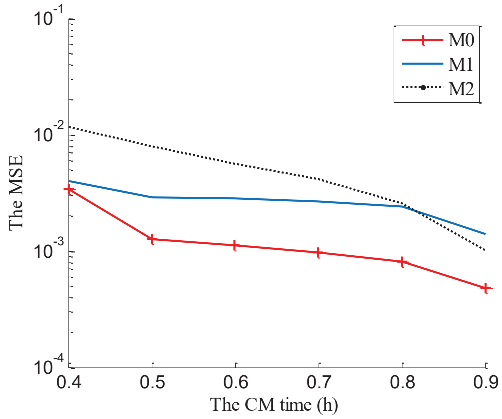

Figure1 MSEs of M0, M1, and M2 in different CM times

From Figure 1, it can be found that the M0's MSE is smaller than M1 and M0. It illustrates that the M0 has a better model fitting than M1 or M0, and M0 is more realistic. Furthermore, the PDF of equipment's RUL at time $t_k = 0.7$h in different approaches are shown in Figure 2.

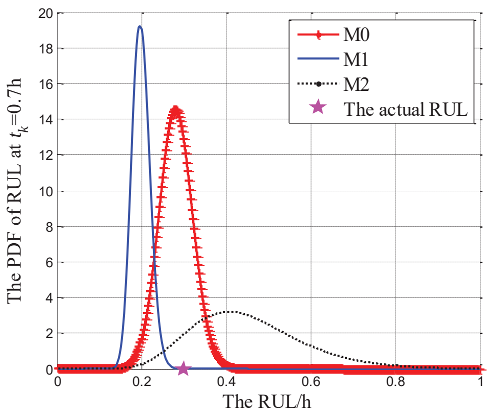

Figure2 PDF of RUL at t_k=0.7h in different methods

In Figure2, the actual RUL of the target equipment is in the PDF under M0 and M2, while the PDF of M1 cannot cover the actual RUL. It shows that M0 and M2 have higher prediction accuracy than M1. Since the PDF under M0 is sharper than M2, M0 has a higher prediction accuracy than M2 in consideration of the nonlinear degradation process.

To further compare the different methods, this paper calculates the 95% confidence interval of the equipment RUL, as shown in Figure3. Obviously, M0 has the clearest 95% confidence interval on the premise of covering the actual RUL of the target equipment, which also indicates that M0 has higher RUL prediction accuracy.

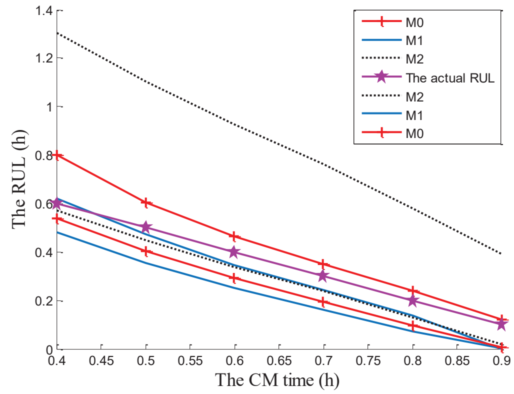

Figure3 95% confidence intervals of RUL in different methods

In summary, the simulation example shows that the proposed RUL prediction method has advantages in prediction accuracy. In other words, the RFT and nonlinear degradation process have a significant impact on the RUL prediction.

V. Conclusion

This paper analyzes the impact of RTF on the RUL prediction of nonlinear degradation equipment. The simulation results show that the method has higher prediction precision and accuracy.

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