Periodic Inspection Policies of a System Subject to Shocks with Random Lead-time

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***Abstract*—This paper studies a system exposed to shocks, and the effect of the corresponding shocks may be fatal or accumulated. Assume the spare unit for the system has a random lead-time, we study the periodic inspection policy of this system. We formulate a model for the sake of minimizing the average cost per unit time. We give a numerical example to calculate the optimal inspection time.**

Keywords- periodic inspection; random lead-time; replacement; shock model

I. INTRODUCTION

In reliability engineering, we often hope to design a system which can operate reliably and safely. However, in the running process of the system, its function inevitably deteriorates because of its wear and age, and other damage factors. Maintenance costs of system failures are often high. Therefore, maintenance policies need to be developed to reduce unexpected system hazards and enhance the system function [1]. Assume that maintenance costs is determined by the maintenance or replacement of the system, a maintenance model with minimal repair was developed in terms of a repair-cost limit policy [2]. A Degradation-Threshold-Shock model, having dependent failure modes, is established by Huynh et al. [3], and they presented the time-based maintenance policy and condition-based minimal repair policy.

These above maintenance policies have a common assumption that the spare units can be supplied at any time. In practice, in some industries, it is reasonable to assume that only one spare unit can be used for replacement, due to the problems such as storage costs and wear during the storage. Therefore, when the spare unit is replaced, it is necessary to reorder the spare unit. We assume that the lead-time of ordering the spare unit is random. For this situation, we cannot ignore the impact of the random lead-time of delivery of spare units on the system. Many literature have developed some replacement policies for the system having a random lead-time for delivering the spare unit. For example, based on random lead-time, Chang [4] presented a generalized age replacement policy and gave the optimal replacement time. Assume that the spare unit has a random lead-time, Wang [5] presented a replacement policy of a deteriorating system based upon the systems real-time health condition, and obtained the ordering policy for the spare unit. Under the condition that the spare component has a random lead time, Zhang and Zeng [6] built an imperfect maintenance model for determining the jointly optimal policy. Chien and Chen [7] studied an age-replacement model for the system having minimal repair. Sheu and Griffith [8] considered the cost incurred by the storage of a spare and the downtime of a system, and they established an age replacement model. Cheng and Li [9] presented the deteriorating system's replacement policies.

In some real industrial applications, periodic inspection maintenance policies are commonly carried out. For instance, periodic inspection, including cleaning and servicing the system in total productive maintenance, and we replaced the deteriorated units in order to prevent serious shutdown problems of the system. Klutke and Yang [10] calculated the availability of maintenance systems and presented a periodic inspection policy. Kharoufeh et al. [11] considered a single-unit system under a periodical inspection, and they obtained the lifetime distribution by using the Laplace–Stieltjes transform, and they also calculated the limiting availability for this system with hidden failure. This paper studies the periodic inspection policy of a maintained deterioration system where failure can be detected only by inspection. In addition, we assume that the system works in a random environment. We use a random shock model to describe the influence of the random environment on the system.

The purpose of this paper is to determining an optimal periodic inspection policy for a system with a spare unit having a random lead-time. Section II gives the proposed model. In Section III, we give a detailed analysis of the model. Section IV gives a numerical example.

II. MODEL ASSUMPTIONS

In this section, a maintenance model is introduced for a deteriorating unit which has two types of failures, and the spare unit has a random lead time. We give the following assumptions for this model.

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1. A new unit starts running at time zero. It is inspected at deterministic periodic intervals $\tau, 2\tau, \ldots$. We assume that the failure of components can not be detected in time, and it can be detected only when it is inspected. We replace the unit by an available s-identical spare unit.

2. When an operating unit reaches age T_p, we give the unit preventive replacement if the spare unit has arrived. Within the preventive replacement interval $(0, T_p)$, periodic inspection is performed every $\tau\ (0 < \tau < T_p)$ time with a cost c_{in}. Hence, there are at most M periodic inspections in $(0, T_p)$, where

$$M = \begin{cases} [T_p/\tau] - 1, & \text{if} \quad T_p/\tau = [T_p/\tau], \\ [T_p/\tau], & \text{if} \quad T_p/\tau > [T_p/\tau]. \end{cases}$$

3. Let c_p be the preventive replacement cost of the unit. When the unit failure occurs between two successive inspections, the failure state will be continue until the next inspection. Let c_s be the cost of per unit time for failure state of the unit. In addition, denote the cost of failure replacement by c_f.

4. We use a non-homogeneous Poisson process $\{N(t), t \geq 0\}$ to describe the arrival process of shocks. Let the intensity function and mean value function of the non-homogeneous Poisson process be denoted by $v(t)$ and $\Lambda(t) = \int_0^t v(u)\mathrm{d}u$, respectively. We assume that the shock process will restart after the unit is replaced.

5. We consider an extreme shock and accumulated shock. The extreme shock can cause the unit to fail immediately. The accumulated shock can increase the wear of unit, and the unit is failure if the accumulated wear exceeds a random threshold R. Let $W(t)$ be an accumulated wear at time t. It is reasonable to suppose that it is an increasing stochastic process. Assume that $W(t) = t$ if there is no shocks occur. In addition, external random shocks can also have an impact on the failure process of the unit. Let $\{N(t), t \geq 0\}$ denote times of shocks, and shocks arrival times be denoted by $T_i, i = 1, 2, \ldots$. Suppose that the ith shock occurs at time t_i, and it leads to immediate failure of the unit with probability $p(t_i)$, and it increases the wear of a unit by a random increment W_i, with probability $q(t_i) = 1 - p(t_i)$. Let T_f be the random lifetime of the unit subjected to shocks, and its distribution function be denoted by $F(t)$. According to Cha and Finkelstein [12], at time t, the accumulated wear of the unit is presented by $W(t) = t + \sum_{i=0}^{N(t)} W_i$, where random variables $W_1, W_2, \ldots$ are independent and identically distributed.

6. The lead-time of delivering a spare is uncertain, and we denote it by a random variable Y. Assume that $G(t)$ is the distribution function of Y. Let the unit operation and the random lead-time both start at time zero. If the spare unit has arrived before the preventive replacement or before the revealed failure of the unit, we put the spare unit into stock. Let c_h be the cost of per unit time of stocking a unit. If the failure unit has been found at inspection or at preventive replacement time and no spare unit is available, then the unit is in down state. The replacement is delayed with a down-time cost c_d per unit time. The delayed replacement will be carried out until the spare unit arrives.

7. Assume that $c_p < c_f$ and $c_d < c_s$. The assumption $c_p < c_f$ represents that the preventive replacement cost is smaller than corrective replacement cost. The assumption $c_d < c_s$ is due to c_s includes the cost of producing defective products because of the unit failure during the production process.

III. ANALYTICAL MODEL

As an extreme shock or the accumulated wear can cause unit failure, the conditional reliability function of the unit can be expressed as

$$\begin{aligned} & P\left(T_f > t \mid N(s), 0 \leq s \leq t; W_1, W_2, \ldots, W_{N(t)}; R\right) \\ & = \prod_{i=0}^{N(t)} q(T_i) \mathrm{I}\left(W(t) < R\right). \end{aligned} \tag{1}$$

Assume that R has the exponential distribution with failure rate λ. Therefore, by assumptions 4 and 5 in the second section, the reliability function of the unit can be expressed as ([12])

$$\begin{aligned} & \bar{F}(t) = P(T_f > t) = \exp\{-\lambda t - \int_0^t v(x)\mathrm{d}x \\ & + M_W(-\lambda) \int_0^t q(x) v(x) \mathrm{d}x\}, t \geq 0, \end{aligned} \tag{2}$$

where $M_W(t)$ is the moments generating function of W_i.

In this model, let T_i denote the length of the ith replacement, and let C_i denote the operational cost over the renewal interval T_i, $i = 1, 2, \ldots$. Therefore, the long-run average cost per unit time of a replacement cycle can be expressed as (see, e.q., Ross [13])

$$\lim_{t \to \infty} \frac{D(t)}{t} = \frac{E(C_1)}{E(T_1)}. \tag{3}$$

According to the assumptions given in Section II, we give six states between successive replacements as follows:

- State 1. $T_f \in ((k-1)\tau, k\tau),\quad k \in \{1,2,\ldots,M\}$. If the spare unit arrives before time $k\tau$, then the spare unit is put into stock. The unit is replaced at time $k\tau$.
- State 2. $T_f \in ((k-1)\tau, k\tau),\ k \in \{1,2,\ldots,M\}$. The unit is shut down if the spare unit arrives after time $k\tau$. We replace the unit until the spare unit arrives.
- State 3. $T_f \in (M\tau, T_p)$. If the spare unit arrives before time T_p, then the spare unit is put into stock. We replace the unit by the spare unit at time T_p.
- State 4. $T_f \in (M\tau, T_p)$. The unit is shut down if the spare unit arrives after time T_p. We replace the unit until the spare unit arrives.
- State 5. $T_f \in (T_p, \infty)$. When the spare unit arrives before time T_p, then the spare unit is stored. We replace the unit by the spare unit at time T_p.
- State 6. $T_f \in (T_p, \infty)$. The unit is shut down if the spare unit arrives after time T_p. We replace the unit by the spare unit until the spare unit arrives.

According to the six cases described above, the length of a replacement cycle can be expressed as

$$T_1 = \begin{cases} k\tau, & if \quad (k-1)\tau < T_f \le k\tau, Y \le k\tau, k \in \{1,2,\ldots,M\}, \\ Y, & if \quad (k-1)\tau < T_f \le k\tau, Y > k\tau, k \in \{1,2,\ldots,M\}, \\ T_p, & if \quad M\tau < T_f \le T_p, Y \le T_p, \\ Y, & if \quad M\tau < T_f \le T_p, Y > T_p, \\ T_p, & if \quad T_f > T_p, Y \le T_p, \\ Y, & if \quad T_f > T_p, Y > T_p. \end{cases} \tag{4}$$

From equation (4), the expected time of a replacement cycle can be given as

$$\begin{aligned} E[T_1] &= \sum_{k=1}^{M}\left[\left(F(k\tau) - F((k-1)\tau)\right)\cdot G(k\tau)\cdot k\tau\right] \\ &\quad + \sum_{k=1}^{M}\left[\left(F(k\tau) - F((k-1)\tau)\right)\cdot \int_{k\tau}^{\infty} t\mathrm{d}G(t)\right] + \\ &\quad \left(F(T_p) - F(M\tau)\right)\cdot G(T_p)\cdot T_p + \left(F(T_p) - F(M\tau)\right) \\ &\quad \cdot \int_{T_p}^{\infty} t\mathrm{d}G(t) + \bar{F}(T_p)\cdot G(T_p)\cdot T_p + \bar{F}(T_p)\cdot \int_{T_p}^{\infty} t\mathrm{d}G(t) \\ &= \sum_{k=1}^{M}\left[\left(F(k\tau) - F((k-1)\tau)\right)\left(G(k\tau)\cdot k\tau + \int_{k\tau}^{\infty} t\mathrm{d}G(t)\right)\right] \\ &\quad + \left(1 - F(M\tau)\right)\left(G(T_p)\cdot T_p + \int_{T_p}^{\infty} t\mathrm{d}G(t)\right) \end{aligned} \tag{5}$$

In addition, the cost for a replacement cycle is given by

$$C_1 = \begin{cases} c_f + c_h\left(k\tau - Y\right) + c_s\left(k\tau - T_f\right) + kc_{in} \\ if \quad (k-1)\tau < T_f \le k\tau, Y \le k\tau, k \in \{1,2,\ldots,M\}, \\ c_f + c_d\left(Y - k\tau\right) + c_s\left(k\tau - T_f\right) + kc_{in} \\ if \quad (k-1)\tau < T_f \le k\tau, Y > k\tau, k \in \{1,2,\ldots,M\}, \\ c_f + c_h\left(T_p - Y\right) + c_s\left(T_p - T_f\right) + Mc_{in} \\ if \quad M\tau < T_f \le T_p, Y \le T_p, \\ c_f + c_d\left(Y - T_p\right) + c_s\left(T_p - T_f\right) + Mc_{in} \\ if \quad M\tau < T_f \le T_p, Y > T_p, \\ c_p + c_h\left(T_p - Y\right) + Mc_{in} \quad if \quad T_f > T_p, Y \le T_p, \\ c_p + c_d\left(Y - T_p\right) + Mc_{in} \quad if \quad T_f > T_p, Y > T_p. \end{cases} \tag{6}$$

From equation (5), the expected cost of a replacement cycle is given by

$$\begin{aligned} E[C_1] = &\sum_{k=1}^{M}\Big[c_f\cdot\left(F(k\tau) - F((k-1)\tau)\right)\cdot G(k\tau) + \\ &c_h\cdot\left(F(k\tau) - F((k-1)\tau)\right)\cdot\int_0^{k\tau}(k\tau - t)\mathrm{d}G(t) \\ &+ c_s\cdot G(k\tau)\cdot\int_{(k-1)\tau}^{k\tau}(k\tau - t)\mathrm{d}F(t) + \\ &k\cdot c_{in}\cdot\left(F(k\tau) - F((k-1)\tau)\right)\cdot G(k\tau)\Big] + \\ &\sum_{k=1}^{M}\Big[c_f\cdot\left(F(k\tau) - F((k-1)\tau)\right)\cdot\bar{G}(k\tau) + \\ &c_d\cdot\left(F(k\tau) - F((k-1)\tau)\right)\cdot\int_{k\tau}^{\infty}(t - k\tau)\mathrm{d}G(t) \\ &+ c_s\cdot\bar{G}(k\tau)\cdot\int_{(k-1)\tau}^{k\tau}(k\tau - t)\mathrm{d}F(t) \\ &+ k\cdot c_{in}\cdot\left(F(k\tau) - F((k-1)\tau)\right)\cdot\bar{G}(k\tau)\Big] \\ &+ c_f\cdot\left(F(T_p) - F(M\tau)\right)\cdot G(T_p) \\ &+ c_h\cdot\left(F(T_p) - F(M\tau)\right)\cdot\int_0^{T_p}(T_p - t)\mathrm{d}G(t) \\ &+ c_s\cdot G(T_p)\cdot\int_{M\tau}^{T_p}(T_p - t)\mathrm{d}F(t) \\ &+ M\cdot c_{in}\cdot\left(F(T_p) - F(M\tau)\right)\cdot G(T_p) \\ &+ c_f\cdot\left(F(T_p) - F(M\tau)\right)\cdot\bar{G}(T_p) \\ &+ c_d\cdot\left(F(T_p) - F(M\tau)\right)\cdot\int_{T_p}^{\infty}(t - T_p)\mathrm{d}G(t) \\ &+ c_s\cdot\bar{G}(T_p)\cdot\int_{M\tau}^{T_p}(T_p - t)\mathrm{d}F(t) \\ &+ M\cdot c_{in}\cdot\left(F(T_p) - F(M\tau)\right)\cdot\bar{G}(T_p) \\ &+ c_p\cdot\bar{F}(T_p)\cdot G(T_p) + c_h\cdot\bar{F}(T_p)\cdot\int_0^{T_p}(T_p - t)\mathrm{d}G(t) \\ &+ M\cdot c_{in}\cdot\bar{F}(T_p)\cdot G(T_p) + c_p\cdot\bar{F}(T_p)\cdot\bar{G}(T_p) \end{aligned} \tag{7}$$

$$
\begin{aligned}
&+c_d \cdot \bar{F}(T_p) \cdot \int_{T_p}^{\infty} (t-T_p) \mathrm{d}G(t) + M \cdot c_{in} \cdot \bar{F}(T_p) \cdot \bar{G}(T_p) \\
= \; & \sum_{k=1}^{M} \Big[\big(F(k\tau) - F((k-1)\tau) \big) \Big(c_f + c_h \int_0^{k\tau} (k\tau - t) \mathrm{d}G(t) \\
&+ c_d \int_{k\tau}^{\infty} (t - k\tau) \mathrm{d}G(t) + k c_{in} \Big) + c_s \int_{(k-1)\tau}^{k\tau} (k\tau - t) \mathrm{d}F(t) \Big] \\
&+ \big(F(T_p) - F(M\tau) \big) \Big(c_f + c_h \int_0^{T_p} (T_p - t) \mathrm{d}G(t) + c_d \int_{T_p}^{\infty} (t \\
&- T_p) \mathrm{d}G(t) + M c_{in} \Big) + \bar{F}(T_p) \Big(c_p + c_h \int_0^{T_p} (T_p - t) \mathrm{d}G(t) \\
&+ c_d \int_{T_p}^{\infty} (t - T_p) \mathrm{d}G(t) + M c_{in} \Big) + c_s \int_{M\tau}^{T_p} (T_p - t) \mathrm{d}F(t).
\end{aligned}
$$

Hence, from equation (5) and equation (7), the long-run expected cost per unit time of a replacement cycle is

$$B(\tau) = \frac{E(C_1)}{E(T_1)} . \tag{8}$$

By equation (8), given preventive replacement time T_p, the optimal τ^* minimizing $B(\tau)$ can be obtained.

IV. NUMERICAL EXAMPLES

The following numerical example assumes that W_i has an exponential distribution with failure rate $\frac{1}{\mu_W}$, i.e.,

$$F_W(x) = 1 - \exp\{-x/\mu_W\} . \tag{9}$$

Then,

$$M_W(-\lambda) = \frac{1}{1 + \mu_W \cdot \lambda} . \tag{10}$$

Further, suppose that the lead-time Y has an exponential distribution $G(x) = 1 - \exp\{-\delta x\}$, $q(x) \equiv q$, $v(x) \equiv v$. The detail values for parameters of the unit are given by Table Ⅰ. The cost parameters of the unit maintenance are provided in Table Ⅱ.

TABLE I. DETAILED VALUES FOR DISTRIBUTION PARAMETERS.

Parameter	λ	μ_W	v	δ	q
value	0.02	0.015	0.8	0.25	0.9

TABLE II. DETAILED VALUES FOR COST PARAMETERS.

Parameter	c_f	c_p	c_h	c_d	c_s	c_{in}
value	20000	6000	1000	2000	4000	200

As shown in Table Ⅲ, we give the optimal periodic detection time τ^* and minimum unit cost $B(\tau^*)$ of the unit under different preventive replacement times T_p.

TABLE III. EFFECT OF LEAD-TIME

T_p	τ^*	$B(\tau^*)$
5	1.6	3181.69
10	1.1	3033.09
20	1.1	2963.12
30	1.3	2927.71
40	1.2	2927.14
60	1.3	2915.77

In addition, we consider the unit without performing periodic inspection policy, i.e., $M = 0$. Hence, the expected time of a replacement cycle is given as

$$E[T_1] = G(T_p) \cdot T_p + \int_{T_p}^{\infty} t \mathrm{d}G(t) . \tag{11}$$

The expected cost of a replacement cycle can be given by

$$
\begin{aligned}
E[C_1] = F(T_p)c_f + \bar{F}(T_p)c_p + c_h \int_0^{T_p} (T_p - t) \mathrm{d}G(t) \\
+ c_d \int_{T_p}^{\infty} (t - T_p) \mathrm{d}G(t) + c_s \int_0^{T_p} (T_p - t) \mathrm{d}F(t)
\end{aligned} . \tag{12}
$$

Hence, from equation (11) and equation (12), the long-run expected cost per unit time of a replacement cycle is

$$B(T_p) = \frac{\begin{aligned} F(T_p)c_f + \bar{F}(T_p)c_p + c_h \int_0^{T_p} (T_p - t) \mathrm{d}G(t) \\ + c_d \int_{T_p}^{\infty} (t - T_p) \mathrm{d}G(t) + c_s \int_0^{T_p} (T_p - t) \mathrm{d}F(t) \end{aligned}}{G(T_p) \cdot T_p + \int_{T_p}^{\infty} t \mathrm{d}G(t)} . \tag{13}$$

The detail values for parameters of the unit are given by Table Ⅰ. The cost parameters of the unit maintenance are provided in Table Ⅱ. Table IV gives the unit cost $B(T_p)$ under different preventive replacement times T_p.

TABLE IV. UNIT COST OF THE SYSTEM UNDER DIFFERENT PREVENTIVE REPLACEMENT TIMES

T_p	$B(T_p)$
5	3290.72
10	3542.08
20	3977.59
30	4245.51
40	4413.91
60	4602.49

Table IV presents the expected cost per unit time about T_p without performing periodic inspection policy. From Table III and Table IV, we can see that the unit cost without performing periodic inspection policy are higher than that with performing periodic inspection policy.

V. CONCLUDING

This paper studies the periodic inspection policy of maintained deterioration unit in a random environment. We consider the spare unit has a random lead time. Various costs are considered. We give the long-run expected cost per unit time of a replacement cycle. Finally, a numerical example is given to show the advantage of periodic inspection in this model.

ACKNOWLEDGMENT

This work was supported by the Foundation of Hebei Education Department (QN2018129), the Science Foundation of Hebei Normal University (L2017B24), and the Young Talent Support Plan of Hebei Province.

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