Reliability Evaluation for Turbo Pump Component in Two-phase Development with No Failure Data

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Abstract—**Turbo pump plays a vital role in the success of a rocket launch. High reliability is thus required for turbo pump as well as its components. During the development of turbo pump component, multiple phases are usually involved with a testing after each phase. Such testing often obtains no failures, due to the high reliability of the component and the short testing period. It is not reliable to evaluate the component reliability only rely on the no failure data. To make full use the no failure data in two development phases, Bayesian method is proposed, combining with the data from similar component and experts' estimate. A case study is carried out to illustrate that the proposed reliability evaluation method yields more accurate result than the method only using no failure data.**

Keywords- turbo pump component; two-phase development; reliability evaluation; no failure data

I. INTRODUCTION

The turbo pump of a liquid rocket engine plays an important role in the operation of rocket engine, as most of the failures of rocket launches are caused by turbo pumps[1]. Therefore, turbo pump desires high reliability along with its components. Reliability is designed and realized during product development. So it is important to evaluate reliability during development, which will determine whether the reliability goal is achieved.

In the development of turbo pump components, multiple phases are usually involved, during which designs are modified after the latent defects were exposed in the testing of previous phase. Reliability will usually grow through the development phases, as defects are removed by design changes[2]. When the sample size is small, Quigley and Walls[3] proposed a reliability growth model to estimate the reliability of product in development. Awad[4] proposed a method to determine the testing time in reliability growth testing for repairable system, combining the failure information of subsystems. Jin et al.[5] considered the resources allocation in multiphase reliability growth testing, which sequentially determines and implements corrective actions. However, all the preceding models are not applicable to testing with no failure observations.

As far as no failure data is taken into account, reliability evaluation in such cases have attracted various attentions from scholars and engineers. Bailey[6] raised the question of estimating the binomial failure probability by no failure test data and compared some of the estimators. In the case of Weibull distribution with no failure data, Jiang et al.[7] proposed the use of matching distribution curve method and Bayesian method to calculate product reliability, by applying concavity or convexity and property of the distribution function. Li et al.[8] also adopted matching distribution curve method to estimate failure probability of Weibull distribution and bootstrap method is used to obtain confidence interval estimates of reliability. When additional information such as similar products or experts is available, Jiang et al.[9] presented the shrinkage estimator for Weibull distribution parameters in cases of no failure data. Most of the methods to evaluate reliability by no failure data, require that the samples should be divided into multiple groups with respective truncation times and the sample size should not be very small. Xia[10] proposed the use of grey bootstrap method for the reliability evaluation by zero-failure data under the condition of unknown probability distribution of lifetime. All the methods did not consider the multi-phase development of product and corresponding tests data. To make full use of the no failure testing data from multi-phase development, this paper propose the Bayesian method and MCMC method to estimate reliability.

This paper is organized as follows. Section 2 introduces the background of the research and the assumptions. In Section 3, we present our reliability evaluation method, combining the no failure data from the two phases in development. A case study is presented in Section 4 to illustrate our proposed method. This paper is summarized in Section 5.

II. PROBLEM FORMULATION

In the development of a key component of turbo pump for liquid rocket engine, there were two phases: In the first phase of development, n prototypes were made to validate the design, and were put into testing with censoring time t_{c1}. No failures were found in the testing. In the second phase, design changes were carried out accordingly, and the same number of prototypes were made and put into testing with censoring time t_{c2} ($t_{c1} < t_{c2}$). Also no failures were found in the testing.

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Reliability of the component is to be evaluated based on the data without failures. As Weibull distribution is widely used in engineering practices, especially in aerospace reliability engineering, we also assume that the lifetime of the component is following Weibull distribution $W(m,\eta)$, where m and η are shape parameter and scale parameter, respectively. Scale parameter η is also called the characteristic life. If variable $X \sim W(m,\eta)$, its probability density function (pdf) and survival (reliability) function can be respectively expressed by

$$f(t)=\frac{m}{\eta}\left(\frac{t}{\eta}\right)^{m-1}\exp\left[-\left(\frac{t}{\eta}\right)^{m}\right], \quad (1)$$

$$R(t)=\exp\left[-\left(\frac{t}{\eta}\right)^{m}\right]. \quad (2)$$

To formulate the problem, we make following assumptions:

(1) As shape parameter m is usually determined by the material properties, we assume that m is known and given by experts. The key to reliability evaluation is then actually to estimate the scale parameter η.

(2) We can derive the upper estimate of the scale parameter, denoted by $\underline{\eta}$, by combining the information from similar component and the estimate from the second testing.

Then we will introduce our method to estimate the scale parameter η, based on the no failure data from the tests of two phases.

III. Reliability Evaluation

A. Estimate after the first testing

In the testing at the end of the first phase, n samples were put in the testing with censoring time t_{c1} and no failures were found. Traditional reliability evaluation methods are unable to deal with this problem, as they rely on failure data. To cope with the problem, we propose the use of Modified Maximum Likelihood Estimate (MMLE) method[9].

When the shape parameter m is given, the MMLE of the scale parameter η is given by[11]

$$\underline{\eta}=\left(\frac{nt_{c1}^{m}}{-\ln\alpha}\right)^{\frac{1}{m}}, \quad (3)$$

where α is the significance level. In the case of high reliability requirements, we are suggested to choose $\alpha=0.1$ or 0.2; Otherwise, $\alpha=0.5$ for a general estimate. Therefore, in this case, as the component is important for turbo pump of liquid rocket engine, we choose $\alpha=0.1$.

Then after the first estimate of η is obtained, let the censoring time of the second testing, $t_{c2}=\underline{\eta}$. We continue the development of the component in the second phase, where design changes were made.

B. Estimate after the second testing

After the second testing, we still obtain no failures, due to the high reliability of the component and comparatively short testing period. MMLE method can be applied here, but often leads to overestimate, as the censoring time $t_{c2}=\underline{\eta}$ is usually much longer than t_{c1}. Moreover, the component design was changed in the second phase, which means the component in the second phase is not the same as in the first phase, i.e., they are not from the same population, in terms of statistics. So it is not proper to directly combine the samples in the two phases, to evaluate reliability.

To make use of the data from the first phase, Bayesian theory is proposed here. The merit as well as the debate of Bayesian theory, is the use of prior information. In our case, the prior information come from some similar component (or previous generation) and the estimate from the second testing. From the similar component, we have the estimate of η, denoted by $\overline{\eta}_1$. From the second testing, by referring to Eq.(3), when $t_{c2}=\underline{\eta}$, we can obtain the MMLE estimate of η, denoted by $\overline{\eta}_2$. Then, a weight, w, is introduced to combine the two estimates, which yields the integrated estimate of $\overline{\eta}$ (also considered as the upper estimate of η) as

$$\overline{\eta}=w\overline{\eta}_1+(1-w)\overline{\eta}_2. \quad (4)$$

As MMLE method often leads to overestimate, to obtain a better estimate $\overline{\eta}$, the information from the similar component is more important than MMLE estimate, namely, the weight w, is usually less than 0.5. Therefore, scale parameter η can be considered as a random variable following uniform distribution $U[\underline{\eta},\overline{\eta}]$. It has a prior pdf as

$$\pi(\eta)=\frac{1}{\overline{\eta}-\underline{\eta}}. \quad (5)$$

Then n samples were subjected to the second testing with censoring time t_{c2}. As no failures were observed in the testing, referring to Eq.(2), the likelihood function of η is

$$L(\mathrm{D}|\eta)=\prod_{i=1}^{n}R(t_i;\eta,m_0)=\exp\left(-\frac{nt_{c1}^{m}}{\eta^{m}}\right). \quad (6)$$

According to Bayesian theory, the posterior pdf of η can be expressed by

$$\pi(\eta\mid D)=\frac{\pi(\eta)L(D\mid\eta)}{\int\pi(\eta)L(D\mid\eta)\,\mathrm{d}\eta}=\frac{\frac{1}{\overline{\eta}-\underline{\eta}}\exp\left(-\frac{nt_{c2}^{m}}{\eta^{m}}\right)}{\int_{\underline{\eta}}^{\overline{\eta}}\frac{1}{\overline{\eta}-\underline{\eta}}\exp\left(-\frac{nt_{c2}^{m}}{\eta^{m}}\right)\mathrm{d}\eta}\propto\exp\left(-\frac{nt_{c2}^{m}}{\eta^{m}}\right) \quad (7)$$

Then the Bayes estimate of η, denoted by $\hat{\eta}$, is calculated by Monte Carlo Markov Chain(MCMC) method where Metropolis-Hastings sampling approach is applied. As m is given, and η is estimated, the reliability function $R(t)$ is determined.

IV. A CASE STUDY

A. The proposed method

To develop a key component of turbo pump for liquid rocket engine, there are two phases with respective testing at the end of each phase. 10 prototypes were made at the first development phase, and put into the testing. The first testing was censored at time t_{c1} = 800 seconds. No failures were observed during the testing.

The shape parameter m is determined to be 1.5 for the component. By referring to Eq.(3), we can obtain the lower estimate $\underline{\eta} = 2129.5$ ($\alpha = 0.1$). Then the component was modified in the second development phase. 10 prototypes were made at the end of the second development phase and put into the testing with the censoring time $t_{c2} = \underline{\eta} = 2129.5$. Also no failures were obtained due to the high reliability of the component.

From the second testing, we can obtain the MMLE estimate $\bar{\eta}_1 = 5668.5$. From a similar component, we have the estimate $\bar{\eta}_2 = 3120$. Let $w = 0.2$, by referring to Eq.(4), we can calculate the integrated estimate of $\bar{\eta} = 3629.7$. This estimate is considered to be the higher limit of scale parameter η, which means the true value of η should be no greater than $\bar{\eta}$. Then η has the prior distribution $U[2129.5, 3629.7]$.

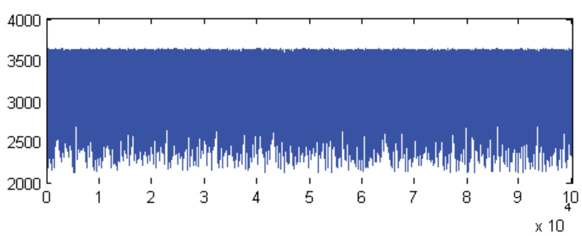

Figure 1. MCMC sampling result for parameter η

By referring to Eqs. (6) and (7), we obtain the estimate $\hat{\eta} = 3296.1$. Then the reliability of the component is

$$R(t) = \exp\left[-\left(\frac{t}{3296.1}\right)^{1.5}\right].$$

The plot of the reliability function is depicted in Figure 2. Experts have their estimate as $\hat{\eta}_e = 3500$, according to their judgments on the design changes on the components in the second phase and the similar component as well. Thus indicates our estimate is close to the experts' estimate.

It is noted here that the integrated estimate $\bar{\eta} = 3629.7$, is closer to experts' estimate $\hat{\eta}_e = 3500$, compared with our proposed estimate $\hat{\eta} = 3296.1$. This is due to the weight $w = 0.2$, and the two values $\bar{\eta}_1 = 5668.5$ and $\bar{\eta}_2 = 3120$. If any of the three values changes, the integrated estimate $\bar{\eta}$ will change dramatically.

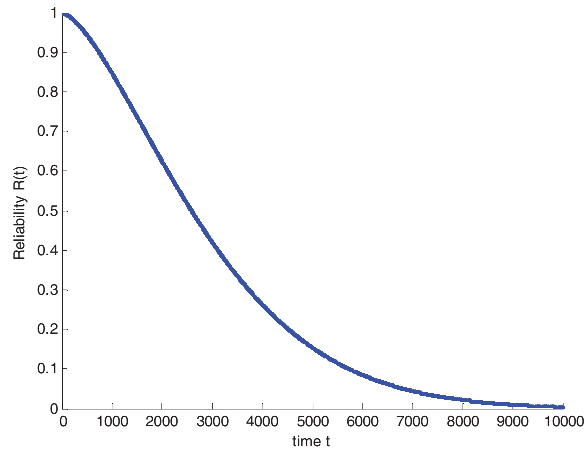

Figure 2. Plot of reliability function R(t)

B. Comparisons

As no failure data were observed in the testing, MMLE method is applied here to calculate the estimate of η.

We first calculate the estimate by combining the data from the two no failure tests: 10 samples censored at $t_{c1} = 800$, and 10 samples censored at $t_{c2} = 2129.5$. By referring to Eq.(3), we can obtain the MMLE lower estimate $\underline{\eta} = 6508.3 >> \hat{\eta} = 3296.1$ ($\alpha = 0.1$), which is overestimating the true value of η, according to experts experiences. Note that experts' estimate is $\hat{\eta}_e = 3500$, which is close to our final estimate $\hat{\eta} = 3296.1$.

As we stated above, it is not proper to directly combine the data together. Then if we only consider the data from the second testing, the estimate of η is also calculated by MMLE method. In the second testing, 10 samples censored at $t_{c2} = 2129.5$. By referring to Eq.(3), we can obtain the MMLE lower estimate $\bar{\eta}_1 = 5668.5 >> \hat{\eta} = 3296.1$ ($\alpha = 0.1$).

From above two comparisons, it is clear that either using all data in two tests or using data only from the second testing, MMLE method yields overestimate result. This can be explain by referring to Eq.(3),

$$\underline{\eta} = \left(\frac{n t_{c1}^m}{-\ln \alpha}\right)^{\frac{1}{m}} \propto t_{c1},$$

which means the MMLE estimate is proportional to the censoring time t_{c1} , when shape parameter m is given. According to the conclusion of Zhang et al.[12], as $\underline{\eta} \propto n^{\frac{1}{m}}$, the MMLE estimate is also proportional to the sample size n.

Therefore, MMLE method is not robust over censoring time and the sample size.

On the contrary, our proposed method uses the data from the two phases tests, combined with the estimate from similar component, and produces the estimate close to experts estimate $\hat{\eta}_e$, which is usually considered to be an approximate to the true value.

V. SUMMARY

During the development of components of turbo pump, as high reliability and short testing period are usually required, testing often observes no failure data. For most reliability evaluation methods dealing with no failure data, such as MMLE, the results are often overestimating. So it is not reliable to evaluate reliability only rely on the no failure data. To make full use of the testing data from multiple phases, as well as the information from similar components, Bayesian method is introduced in this paper. We assume that the lifetime of the key component is following Weibull distribution with known shape parameter *m*, and MCMC method is proposed to calculate the estimate of scale parameter η. Comparisons indicate that the information from similar components helps balance the overestimating of no failure evaluation result of the scale parameter η, which makes the final estimate dependable, as it is close to the experts estimate $\hat{\eta}_e$.

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