A Weighted Residual Useful Life Prediction Method for Weibull Distribution Model under Multiple Stress

Yuemei Zhang
College of Automation Engineering
Nanjing University of Aeronautics and Astronautics
Nanjing, China

Shaojie Zhang, Li Wang
College of Automation Engineering
Nanjing University of Aeronautics and Astronautics
Nanjing, China
zhangsj@nuaa.edu.cn

***Abstract*—This paper presents a weighted method of residual useful life (RUL) prediction based on generalized Eyring model and support vector machine (SVM) under multiple stress. In the first step, the Weibull distribution model is developed and the Weibull parameters can be obtained through maximum likelihood estimation (MLE). Secondly, this paper uses the generalized Eyring model and SVM model to establish two RUL prediction model respectively. Thirdly, a weight coefficient is introduced to allocate the two models. By minimizing the sum of error between real lifetime and estimated prediction, the value of weight coefficient is determined and the final RUL prediction model can be established. An accelerated life testing (ALT) case study of oil paper for power transformer is implemented to illustrate the performance of the proposed method under temperature-voltage stress. And the result of the ALT shows that the prediction accuracy of the weighted model is higher compared with generalized Eyring model and SVM model individually.**

Keyword—residual useful life prediction; Weibull distribution;* generalized *Eyring model; SVM; accelerated life testing

I. Introduction

With the long-term working state, the multiple stress in complex working environment has a degradation effect for component. An unexpected break down and replacement of a component may bring great loss to an enterprise. To ensure the safety and reliability of a system, the accurate RUL prediction has been playing a critical role in the reliability engineering field [1] for most working components.

Statistical distribution model of lifetime distribution law is basis of RUL prediction. Weibull distribution is a probability distribution about continuous random variable and plays an important part in the theoretical basis of reliability analysis. This paper chooses the Weibull distribution for the following reasons [2]. Weibull distribution has strong adaptability for the three failure periods of components. And because Weibull distribution is based on the weakest link model, this distribution can fully reflect the effect of environmental stress on the fatigue life of material.

There are basically two kinds of methods for RUL prediction to describe the relationship between lifetime characteristic and accelerating multiple stress: one is the model-based method [3], which uses the internal degradation theory to generate a lifetime prediction model; the other is the data-driven method [4], which establishes an automatic decision model by training and fitting the integrated and extracted information. However, the empirical model could result in a biased estimation of component's lifetime because the true relationship between lifetime characteristic and accelerating stress is usually unknown. And data-driven method makes prediction without considering the intrinsic degradation of components. Hence, the method of combining the two approaches has become a trend in the reliability estimation field [5]. The generalized Eyring model is derived from quantum mechanics, which indicates that the rate of degradation of components is related to the physical or chemical reaction rate. To increase the accuracy of RUL prediction, this paper combines the data-driven SVM method and conventional generalized Eyring relationship and proposes a weight coefficient to allocate the weight of the aforementioned two methods. Furthermore, different from most methods studying single stress, this paper discusses the RUL prediction under multiple stress, which is more consistent with the actual working environmental of components.

The paper is organized as follows. Section II provides the detailed theoretical analysis of the proposed method including developing the exact Weibull distribution model through MLE and introducing the principle of the weighted RUL prediction method. In section III, an ALT case study of oil paper for power transformer is implemented to illustrate the superiority of the proposed method. Section IV gives a conclusion of this paper.

II. The Theoretical research of RUL prediction

The RUL prediction model is based on the following assumptions [6]:

- The component lifetime follows a Weibull distribution;
- The shape parameter that represents the failure mechanism of components is invariable;
- The relationship between lifetime characteristic and accelerating stress follows the generalized Eyring model.

The detailed process of the proposed RUL prediction is shown in Fig 1.

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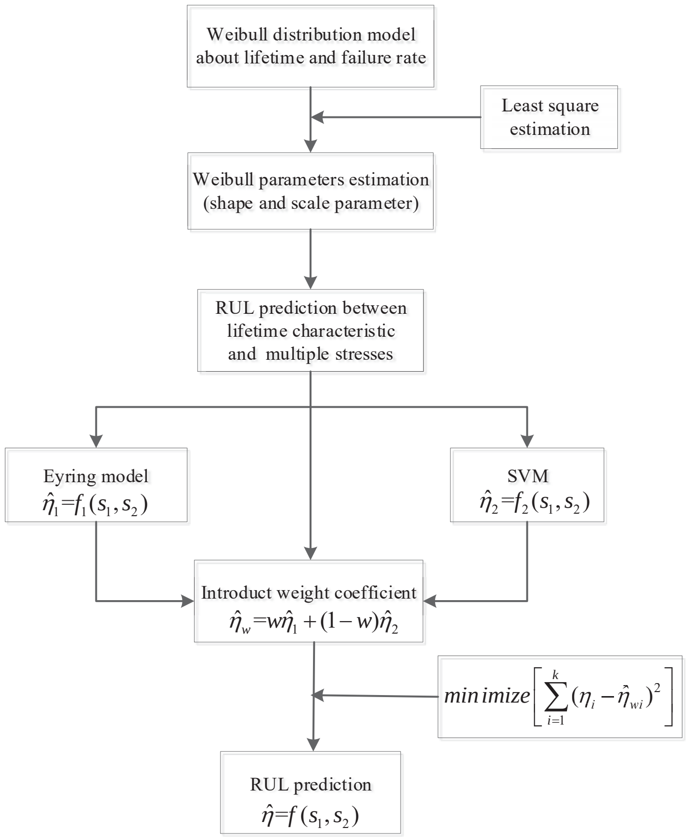

Fig 1. The flow chat of RUL prediction

A. Weibull distribution model

Weibull distribution is one of the most suitable statistic distribution models to describe the lifetime distribution law of components. The two-parameter Weibull distribution is adopted to describe the lifetime distribution of components in this paper. The probability density function of Weibull distribution is presented as [7]

$$f(t) = \frac{m}{\eta}\left(\frac{t}{\eta}\right)^{m-1} \exp\left[-\left(\frac{t}{\eta}\right)\right]^m \tag{1}$$

where m stands for shape parameter and η represents the scale parameter. The cumulative failure probability function and reliability function for Weibull distribution can be calculated from (1).

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^m\right] \tag{2}$$

$$R(t) = 1 - F(t) = \exp\left[-\left(\frac{t}{\eta}\right)^m\right] \tag{3}$$

Consider an ALT with temperature-voltage accelerating stresses, denoted as $\mathbf{s} = (T, S)^{\mathrm{T}}$; There are k accelerating stress levels, which are represented as $(\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_k)$; τ_i is the censoring time in the ith stress level $\mathbf{s}_i$. n_i and r_i is the total sample size and the number of failed components in the ith stress level $\mathbf{s}_i$.

If the jth tested component in stress level $\mathbf{s}_i$ losses efficiency at observed time t_{ij} , the log-likelihood function of (1) is

$$\ln L_{ij}{}' = \ln f(y_{ij}) = -\ln\sigma_i + \frac{y_{ij} - \mu_i}{\sigma_i} - \exp\left(\frac{y_{ij} - \mu_i}{\sigma_i}\right) \tag{4}$$

If the jth tested component in stress level $\mathbf{s}_i$ is not failed at observed time t_{ij} , the log-likelihood function of (1) is

$$\ln L_{ij}{}'' = \ln R(f_i) = \ln(1 - F(f_i)) = -\exp\left(\frac{f_i - \mu_i}{\sigma_i}\right) \tag{5}$$

where $y_{ij} = \ln t_{ij}$, $f_i = \ln\tau_i$, $\mu_i = \ln\eta_i$, $\sigma_i = \frac{1}{m_i}$.

Therefore, the log-likelihood function of the jth tested component in the ith stress level $\mathbf{s}_i$ is derived as

$$\ln L = \sum_{i=1}^{k}\sum_{j=1}^{r_i}\left(I_{ij}\ln L_{ij}{}' + (1 - I_{ij})\ln L_{ij}{}''\right) \tag{6}$$

where $I_{ij} = \begin{cases} 1\,, & y_{ij} \le f_i \\ 0\,, & y_{ij} > f_i \end{cases}$.

Taking the first partial derivative of the log-likelihood function and make it to zero, we get the transcendental equation with Weibull parameters as follows

$$\begin{cases} \dfrac{\sum_{i=1}^{r_i} t_{ij}^{m_i}\ln t_{ij} + (n_i - r_i)\tau_i^{m_i}\ln\tau_i}{\sum_{i=1}^{r_i}\ln t_{ij} + (n_i - r_i)\tau_i^{m_i}} - \dfrac{1}{m_i} = \dfrac{1}{r_i}\sum_{i=1}^{r_i}\ln t_{ij} \\ \eta_i^{m_i} = \dfrac{1}{r_i}\left[\sum_{i=1}^{r_i} t_{ij}^{m_i} + (n_i - r_i)\tau_i^{m_i}\right] \end{cases} \tag{7}$$

Solving (7) by MLE method, η_i and m_i in stress level $\mathbf{s}_i$ are obtained. Because we suppose that the failure mechanism of components is invariable, the shape parameter m is constant and considered as the weighted average of m_i . The scale parameter η_i represents the lifetime characteristic in stress level $\mathbf{s}_i$. Consequently, the lifetime characteristic $\boldsymbol{\eta} = [\eta_1, \eta_2, \ldots, \eta_k]$ of accelerating stress level is gained from the Weibull parameter estimation.

B. RUL prediction model

This section gives the introduction of the generalized Eyring model and SVM model. Based on the results of the two models, the weighted prediction model combined generalized Eyring and SVM model is developed.

1) The generalized Eyring prediction model

The actual working environment of the components is quite complex and the components are usually affected by more than one stress. Therefore, this paper chooses the generalized Eyring model to describe the relationship between RUL and multiple stress. When components are subjected to temperature and another environmental stress like vibration, voltage or humidity, generalized Eyring model can be applied to predict the RUL of components. The form of generalized Eyring model is formulated as

$$\eta = CT^{\alpha} * e^{\frac{\Delta E}{kT}+(B+\frac{A}{T})S} \tag{8}$$

where T represents absolute temperature; S represents nonthermal environmental stress; ΔE is the activation energy of failure mechanism related to component material; k is Boltzmann constant which equals to $8.617\times10^{-5}\ eV/K$; C , α , A , B is unknown constant coefficient in the model.

Irrespective of the interaction of stresses, the logarithm form of the generalized Eyring model is

$$\ln\eta_1 = \gamma_0 + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3 \tag{9}$$

where $x_1 = \ln T$, $x_2 = 1/T$, $x_3 = S$, $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ are equation coefficients that need to be solved. The estimated coefficients $\hat{\gamma}_0$, $\hat{\gamma}_1$, $\hat{\gamma}_2, \hat{\gamma}_3$ can be determined by multiple linear regression method. Based on the relationship between lifetime characteristic and multiple stress, the generalized Eyring model has a RUL prediction model of the component life as follows

$$\hat{\eta}_1 = \exp\left(\hat{\gamma}_0 + \hat{\gamma}_1 \ln T + \hat{\gamma}_2 \frac{1}{T} + \hat{\gamma}_3 S\right) \tag{10}$$

2) The SVM prediction model

SVM is developed from the idea of minimization about structural risk [8]. A test sample that its simple size is n and dimension is d can be represented as (x_i, y_i) , $x \in \mathbf{R}^d$ $i = 1, 2, \cdots, n$. There is a hyperplane $\boldsymbol{H}$: $\boldsymbol{w} \cdot x + b = 0$ that satisfies the following conditions

$$\min_{w,b,\xi} \frac{1}{2}\|\boldsymbol{w}\|^2 + \frac{C}{2}\sum_{i=1}^{m}\xi_i^2 \tag{11}$$

$$\text{s.t.}\begin{cases} y_i[\boldsymbol{w}\cdot\varphi(x_i)+b] \ge 1-\xi_i \\ \xi_i \ge 0, i = 1, 2, \cdots, m \end{cases} \tag{12}$$

where $\boldsymbol{w}$ is weight vector; $\xi_i \ge 0$ is error variable; C is penalty parameter; b is offset; $\varphi(x_i)$ is kernel function. The Lagrange multiplier α_i is introduced to develop Lagrange function as follows

$$\begin{aligned} L(\boldsymbol{w}, b, \boldsymbol{\alpha}) = {} & \frac{1}{2}\|\boldsymbol{w}\|^2 - \sum_{i=1}^{m}\alpha_i\left\{y_i[\boldsymbol{w}\cdot\varphi(x_i)+b]+\xi_i-1\right\} \\ & + \frac{C}{2}\sum_{i=1}^{m}\xi_i^2 \end{aligned} \tag{13}$$

To obtain the minimum value of (13), this paper takes the partial derivative of $\boldsymbol{w}, b, \xi_i$ and set it equal to zero

$$\begin{cases} \dfrac{\partial L}{\partial w} = 0 \Rightarrow \boldsymbol{w} = \sum_{i=1}^{m}\alpha_i y_i \varphi(x_i) \\ \dfrac{\partial L}{\partial b} = 0 \Rightarrow \sum_{i=1}^{m}\alpha_i y_i = 0 \\ \dfrac{\partial L}{\partial \xi_i} = 0 \Rightarrow C\xi_i - \alpha_i = 0 \end{cases} \tag{14}$$

The inner product $\left[\varphi(x_i) \bullet \varphi(x_j)\right]$ is represented as kernel function $\kappa(x_i, x_j)$. By duality, the minimization formula (11) can be transformed to maximization problem as

$$\max_{\alpha}\left(-\frac{1}{2}\sum_{i,j=1}^{m}\alpha_i\alpha_j y_i y_j \kappa(x_i, x_j) + \sum_{i=1}^{m}\alpha_i\right) \tag{15}$$

$$\text{s.t.}\begin{cases} 0 \le \alpha_i \le C \\ \sum_{i=1}^{m}\alpha_i y_i = 0 \end{cases} \tag{16}$$

Therefore, the SVM can be expressed as

$$f(\boldsymbol{x}) = \sum_{i=1}^{m}\alpha_i y_i \kappa\left(\boldsymbol{x}, \boldsymbol{x_i}\right) + b \tag{17}$$

The detailed SVM prediction steps are as follows:

Step1 Normalize training data and map data to $[y_{\min}, y_{\max}]$. The normalization formula is

$$x' = (y_{\max} - y_{\min})\frac{x - x_{\min}}{x_{\max} - x_{\min}} + y_{\min} \tag{18}$$

Step2 Select and apply kernel function. This paper chooses the Gaussian kernel function as

$$\kappa(\boldsymbol{x}, \boldsymbol{x_i}) = \exp\left(-\gamma\|\boldsymbol{x} - \boldsymbol{x_i}\|^2\right) \tag{19}$$

Step3 Use cross-validation and grid-search to obtain the best penalty parameter and kernel function parameter.

Step4 Train data by obtained parameter.

Step5 Test the training model.

The multiple stress level $\mathbf{s}_i = (T_i, S_i)^{\mathrm{T}}$, $i = 1, 2, \cdots, k$ is regarded as independent variable $\boldsymbol{x}_i$; the lifetime characteristic η is considered as dependent variable. With the SVM method, we get the RUL prediction model as

$$\hat{\eta}_2 = \sum_{i=1}^{k}\alpha_i y_i \kappa\left(\boldsymbol{x}, \boldsymbol{x_i}\right) + b \tag{20}$$

3) The weighted prediction model

Based on the two aforementioned prediction methods, the proposed prediction model introduces a weight coefficient w ($0 < w < 1$) to obtain a higher prediction accuracy. The combined prediction model is defined as

$$\hat{\eta}_w = w\hat{\eta}_1 + (1-w)\hat{\eta}_2\ ,\ \ 0 < w < 1 \tag{21}$$

The core problem of the weighted prediction model is the selection of weight. Appropriate weight can reasonably allocate the proportion of the two models. This paper chooses the value of w by minimizing the sum of error between real lifetime and estimated prediction. Therefore, this paper selects w as follows

$$w = \arg\min_{w}\left[\sum_{i=1}^{k}(\eta_i - \hat{\eta}_{wi})^2\right] \tag{22}$$

By selecting the proper weight coefficient, the best assignment of $\hat{\eta}_1$ and $\hat{\eta}_2$ is determined. When the accelerating stresses levels are set and corresponding failure time are recorded, the final RUL prediction model can be established.

III. CASE STUDY OF OIL PAPER

An ALT of oil paper for power transformer is conducted to illustrate the performance of the developed method. In the case study, this paper considers an ALT of oil paper that cable failure is caused under multiple accelerating stress including temperature stress and voltage stress. The proposed method is implemented to predict the RUL of oil paper for power transformer compared with the generalized Eyring model and SVM model.

A constant truncation test is chosen in this ALT [9]. The temperature stress takes values of 60, 70, 80, 90, 100, 110, and 120 (°C), and the voltage stress takes values of 6, 7, 8, 9, 10, 11, 12, and 13 (kV). From the uniform orthogonal test design [10], there are 28 sets of training data about temperature-voltage accelerating stress. The sample size of each stress level is ten. The recorded data include the temperature, voltage and failure time of the ten sample in each stress level.

For generalized Eyring model, we get the coefficient of (10) by logarithmic process and multiple linear regression and the result of coefficient is $\hat{\gamma}_0 = 66.6$, $\hat{\gamma}_1 = -9.7$, $\hat{\gamma}_2 = -2803.6$, $\hat{\gamma}_3 = -0.1$. Hence, the generalized Eyring model is

$$\hat{\eta}_1 = \exp\left(66.6 - 9.7\ln T - 2803.6\frac{1}{T} - 0.1S\right) \tag{23}$$

For SVM model, the test data are normalized to [0, 1]. The main parameters, kernel function and optimum algorithm are shown in Table I [11]. Based on the selection of these parameters, the SVM prediction model is built from training data.

Take the result of generalized Eyring model and SVM model into (22), we get the value of w equals to 0.6856. According to the value of weight, the weighted prediction model is obtained.

The relative error of the three models between training result and real lifetime is shown in Fig 2.

TABLE I. PARAMETER SELECTION OF THE SVM MODEL

parameters	selection
kernel function	Gaussian kernel function
parameter selection algorithm	cross-validation and grid-search
penalty parameter	4
kernel function paremeter	2
offset	1.4978

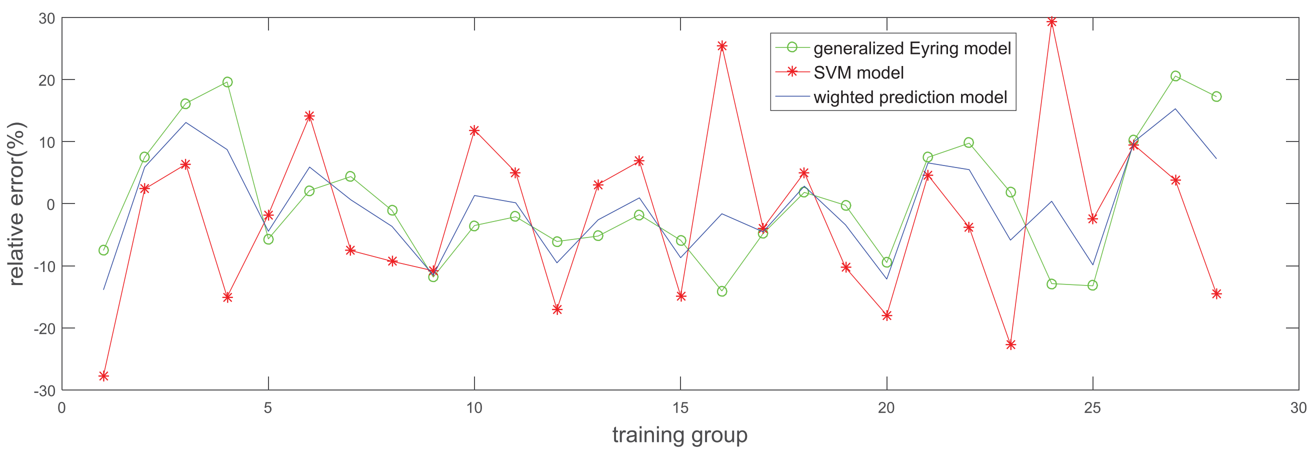

Fig 2. Relative error of training data

TABLE II. THE ERROR ANALYSIS OF THE THREE MODELS

Temperature(°C) Voltage(kV)	Test value	Prediction value			Relative error		
		Generalized Eyring model	SVM model	Weighted model	Generalized Eyring model	SVM model	Weighted model
(80, 9.5)	3841.6	3493.9	3807.9	3745.1	-9.0509	-0.8772	-6.4811
(80, 10.5)	1652.2	1680.0	1715.5	1708.4	1.6826	3.8313	2.3581
(70, 11.5)	1224.3	1313.8	1170.2	1198.9	7.3103	-4.4189	3.6227
(90, 11.5)	460.3	472.9	413.4	425.3	2.7373	-10.1890	-1.3267

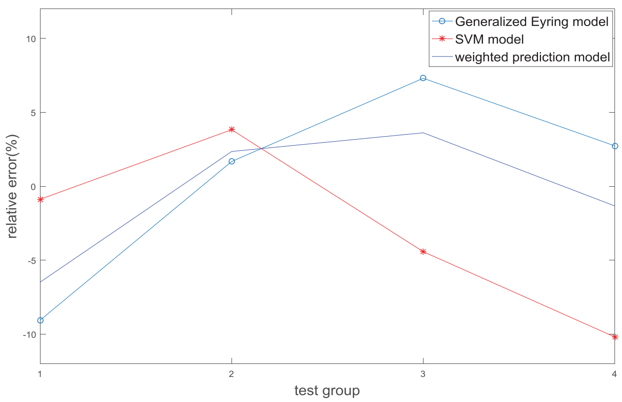

Fig 3. Relative error of test data

To verify the superiority of the proposed method, we uses four additional set data to predict the RUL of oil paper for power transformer. The error analysis of the three model is shown in Table II and the relative error of training data is shown in Fig 3.

From the prediction results and error analysis, the RUL prediction of the weighted method is generally closer to actual lifetime than the other two methods. Due to the different prediction effects of generalized Eyring model and SVM model of different components, the weighted method increases the weight of the model with good fitting effect and thus obtains a better model.

IV. CONCLUSION

The weighted prediction method proposed in this paper is based on the generalized Eyring model and SVM model. By minimizing the sum error of training data, the weight is determined and the weighted prediction model is established. Then four sets of test data are used to verify the feasibility of the proposed method. Compared to generalized model and SVM model, the weighted prediction model has higher accuracy and stronger universality for different components and stress types. In engineering practice, this method can help test personnel estimate RUL of products effectively and obtain a referable lifetime prediction of components.

In the following research, we intend to obtain a more meaningful weight value by further precision of the generalized Eyring model and the SVM model. Furthermore, the selection of weight can be determined in a more efficient and accurate method. These research contents deserve further improved.

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