Machine Learning Based Dynamic Failure Criteria for Reliability Analysis of Bearings Via Kriging Model

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***Abstract*. Accurate estimation of the reliability for different mechanical components plays an important role in the design and maintenance of mechanical systems. In this regard, a new method is proposed for increasing the accuracy of reliability prediction of bearings by introducing a new approach to determine dynamic failure criteria. To be specific, a Bayesian network classifier is applied to establish a machine learning approach for the determination of failure criteria at each time step with varying working and physical condition. The resulted failure criteria at each time are utilized together with a Kriging estimator to express an updated limit state function. Consequently, the second order reliability method is used for the calculation of time-varying reliability. Finally, the presented method is applied for reliability analysis of rolling element bearings and the resulted reliability curve for both accelerated and normal working conditions are presented. The outcome of this work can result in a pertinent approach for further calculation of the reliability of complex mechanical systems.**

Keywords: Bayesian Network Classifier; Surrogate Modelling; Kriging Model; SORM; Mechanical Reliability; Rolling Element Bearing, Time-varying Reliability

I. INTRODUCTION

In many situations, reliability analysis can be considered as a multi-dimensional integral of system variables joint probability density over the specified range of a limit state function. Usually, an analytical solution of this integral or even finding the limit state function is not possible, therefore, simulation approaches like Monte-Carlo simulation (MCS) or first/second order reliability methods are utilized in the literature to calculate system reliability [1]. Even though simulation methods are helpful and powerful in many cases, they face deficiencies in some aspects, e.g., when the joint probability density function (PDF) of system parameters is time-varying, they cannot provide an accurate estimation of the system response. In such cases, the probability density evolution method (PDEM) [2], [3] and methods based on mathematical degradation models [4] were proposed to calculate a time-varying PDF for stochastic systems.

Moreover, methods based on surrogate modeling are also suggested to overcome computational difficulties of the mentioned reliability analysis methods. However, the surrogate models can also be further improved to increase their accuracy of reliability estimation [5].

Kriging method is a widely used surrogate model for structural reliability analysis [1]. The Kriging method is well known for its numerical precision and its potential to be upgraded via training methods [6]. Applying the Kriging model, an efficient global reliability analysis of structures can be established using a set of training samples [6]. Although this method showed acceptable performance for reliability analysis, it still requires high computational effort for high dimensional systems, hence, Zhang et al. suggested an expected feasibility function to gain new training set [7]. The latter method is a powerful approach, which can be used for many reliability analysis problems. However, it uses assumptions, like a predefined failure criterion, which makes it difficult to find a proper reliability estimation for some applications, especially when dealing with highly random physical and environmental variables or in accelerated testing conditions.

Aiming towards the reliability assessment of rolling element bearings, here a Bayesian classifier is used together with the Kriging estimator in order to construct an accurate reliability prediction method using the Second Order Reliability Method (SORM). Thus far, the reliability analysis of rolling element bearings have been reported in several publications [8]–[10] most of which using numerical models to predict the reliability. Ying et al. also suggested a method to calculate the reliability of bearings based on vibration data. Their method uses online monitoring data to predict the reliability using a knowledge-based model. Here, a model for finding the bearing failure criteria is developed using accelerated tests. Then the estimated failure criteria is used to predict the lifetime reliability of the bearing. The method will be introduced here and its application for reliability analysis of a commercial bearing under accelerated tests will be evaluated using experimental data.

II. DYNAMIC FAILURE CRITERIA

Many reliability estimation methods have been developed and utilized for different structural and mechanical systems. Understanding the failure criteria and the reliability analysis are always tied together and no one can maintain a reliability analysis without specifying proper failure criteria. Acquiring performance-based failure criteria is not always easy. In other

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words, there is a slight difference between failure, fault, and error. A failure is an event that happens at a specific time but a fault is a state and can last for an interval of time. Besides, error can be considered as the difference between an estimated value and the correct value.

There can be different faults in a system and different failure criteria can be defined for each fault. Hence, there may be different measurements that cannot represent a system fault individually, or at least they cannot represent the system fault in an optimal manner. In such cases, dynamic failure criteria can be developed as a function of different measurements and working conditions using several systematic tests. Here, the application of a Bayesian Network Classifier (BNC) is represented for the specification of time-varying dynamic failure criteria. It should be noted that one may assume a conservative failure criterion to represent all fault elements, but using such a static failure criteria will result in a lower reliability at each point of time, which leads to a pretty large error for the final (end-of-life) calculated reliability as compared to the real reliability value.

A. BNC for Dynamic Failure Criteria

Bayes' theorem is used to find the posterior probability of desired parameters using given data. Here we are going to use the Bayesian method in order to distinguish failed and safe states based on monitoring data. In this respect, several time responses of the mechanical component of interest are assumed with prior knowledge about their failure state (this knowledge may be concluded from inspection or using expert's supervision). The gathered data of all samples will establish a dataset $X=[X_1,X_2,\dots,X_n]$ which is known. Each X_i has a corresponding property vector (physical and environmental) and category vector (faille or safe), A and Y respectively, $X_i=<A, Y>$. When there is a new set of data, $C=[C_1,C_2,\dots,C_n]$, the Bayesian classification method assigns the proper category Y (the category with the largest probability) to the new dataset, C. In Bayesian classification method, the Bayesian rule is utilized in order to calculate $P(Y_i|C)$, as presented in (1):

$$P(Y_i|C) = \frac{P(C|Y_i)P(Y_i)}{P(C)} \qquad (1)$$

Since each sampling process can be considered as an independent process, the P(C) value can be considered as a constant. Therefore, maximizing the value of $P(C|Y_i)P(Y_i)$ can result in a classification approach, which yields the healthy or failed condition of each new sample. Considering the mentioned characteristics of a BNC machine, the BNC-based Dynamic Failure Criteria (BDFC) can be established at each time step, using the measurable properties of the performing system together with the known fault elements and their associated failure criteria.

B. Establishing the BDFC using pre-evaluation tests

In order to establish an accurate BDFC, a proper knowledge based on sampling, testing and inspection is required to train the BNC machine. The property vector should be consist of all the random variables, loading parameters, and all possible measurements. The gathered data at each time can form the property vector for each sampling time. For example, if we have the data of vertical and horizontal accelerations as well as the strain and loading parameters vs time, these data in addition to the physical parameters and all other random variables can form the property vector at each time. The category vector can also be assumed as {fail, safe} vector, or even more complicated cases like semi-fail condition. The probability of the category vector is easily concluded from the inspection or simulation. In addition, $P(C|Y_i)$ can be calculated as a joint probability density using multivariate probability density calculation schemes. Now, the BNC machine can be formed and get used for finding the safe margin for all parameter within the property vector. Besides, any subset of the property vector can be utilize as a dynamic failure criteria based on our desired or measurable limit state function.

III. Reliability Analysis Using Kriging Estimation and SORM

As stated before, intended for numerical prediction of mechanical reliability through the MCS, one should continually calculate the performance functions through simulation methods. This method is not always computationally feasible or at least requires too much processing time. In such cases, usually the first/second order reliability methods (FORM/SORM) are utilized in the literature. In order to estimate the reliability value using SORM, firstly, a limit state function has to be specified. In this sense, consider the property (or random variables) vector as $\Theta = (\theta_1, \theta_2, \dots, \theta_n)$, a limit state function, which ensures the reliability to remain in safe domain can be defined as $g(\Theta) < 0$, where g is a function of system variables and the failure criteria. Defining the limit state function, one can calculate the failure probability and hence the Reliability as follows:

$$P_f = P\{g(\theta) < 0\} = \int_{g(\theta)<0} f_\theta(\theta)\, dx \qquad (2)$$

$$R = 1 - P_f = P\{g(\theta) > 0\} = \int_{g(\theta)>0} f_\theta(\theta)\, dx \qquad (3)$$

where, $f_\theta(\theta)$ represents the joint probability density function (*pdf*) of random variables.

Generally, the reliability analysis results using FORM/SORM and MCS should be close to each other. However, sometimes due to the computational limitations tied with the MCS or the simplifications associated with FORM and SORM, the results are not as close as expected. In the current study, as our procedure requires different computational efforts, we preferred to apply the SORM to avoid the potential computational costs of the MCS.

A. Second Order Reliability Method

In most cases, calculation of the integral of (3) is not easy, therefore, the idea of using FORM and SORM for simplifying the limit state function is introduced [11]. FORM and SORM use the first and second order Taylor expansion respectively to approximate the answer of equation $g(\theta) = 0$ for further calculation of (3). In this paper, we use SORM to estimate $g(\theta)$ using following equation:

$$g(\boldsymbol{U}) \approx q(\boldsymbol{U}) = g(\boldsymbol{u}^*) + \boldsymbol{\nabla}(\boldsymbol{u}^*)(\boldsymbol{U} - \boldsymbol{u}^*)^T + \frac{1}{2}(\boldsymbol{U} - \boldsymbol{u}^*)\mathbf{H}(\boldsymbol{u}^*)(\boldsymbol{u}^*)(\boldsymbol{U} - \boldsymbol{u}^*)^T \quad (4)$$

Here, $\mathbf{H}(\boldsymbol{u}^*)$ is the Hessian matrix at the most probable point, $\boldsymbol{u}^*$. Further details about the SORM are presented in [11]. Until now, it is designated that how we can calculate the reliability using SORM with predefined failure criteria. However, a major step is still missing, which is calculation of the limit state function itself. This calculation can be carried out using several approaches like structural analysis, system identification, etc. Here, we use the Kriging model with experimental data to estimate the system response.

B. The Kriging Method

In order to find a proper mathematical model for system response using limited number of sampling conditions/points, the following model is suggested to find a deterministic response Y(x) as:

$$\mathbf{Y}(\mathbf{x}) = \mathbf{f}^{\mathbf{T}}(\mathbf{x})\boldsymbol{\beta} + \mathbf{Z}(\mathbf{x}) \quad (5)$$

The $\mathbf{Y}(\mathbf{x})$ in (5), is an unknown function which we are going to estimate and use it as the limit state function, where $\mathbf{f}(\mathbf{x})$ is the known (or measured) function of $\mathbf{x}$ and $\boldsymbol{\beta}$ is a vector of regression coefficients. In addition, $\mathbf{Z}(\mathbf{x})$ is assumed as a Gaussian stationary process to compensate the deviation from $\mathbf{f}^{\mathbf{T}}(\mathbf{x})\boldsymbol{\beta}$ and make the model as accurate as possible. Further details about the method for calculating $\boldsymbol{\beta}$ and $\mathbf{Z}(\mathbf{x})$ are provided in [1].

C. Proposed BDFC-based reliability analysis method

The proposed algorithm for finding the reliability of a mechanical system with dynamic failure criteria using limited number of tests is shown in Fig. 2 and explained as follows:

1) vibration signals are gathered and used together with the loading specification and inspection results to train a BNC machine which yields the dynamic failure criteria.

2) Loading and physical parameters are used as an input to the Kriging estimation algorithm for finding the estimated performance curve. The estimation curve may be one dimensional or can be presented in higher dimensions based on the selected failure criteria.

3) Using the performance curve and the dynamic failure criteria, the limit state function can be estimated.

4) Assuming proper probability density functions (PDF) for all random variables, and using the SORM approach, the reliability curve is concluded (the selected PDFs for our case study are described in section IV)

5) In the validation stage, we can change the pdf of the variables in order to minimize the error of the calculated reliability.

Here, the error is defined based on the discrepancy between different reliability curves calculated for different loading conditions. To be more clear, consider a specific mechanical system or component which may fail in different points of time during accelerated tests. Regarding the definition of reliability, the system/component reliability should be a constant value for different samples of a same product. Hence, the calculated reliability curves using different accelerated tests should converge to each other. Accordingly, using different tests, and trying to converge different reliability curves to each other, we can minimize the error and hence maximize the accuracy of the reliability method. Then, the BDFC and uncertainty setting resulted from minimizing this error can be used to predict a more accurate life-time reliability curve.

IV. Reliability Analysis of Bearings

In order to evaluate the efficiency of the presented method, data of the accelerated degradation tests of several rolling element bearings are utilized. The data are firstly used to find the reliability under accelerated testing condition, and then using the developed model, the estimated reliability for a normal working condition is presented.

A. Data Description

Here, the data for the evaluation objective is the same as the data used in [12]. The bearing test experimental setup is shown in Fig. 1. Accelerated degradation tests of fifteen bearings (type: LDK UER204) are conducted using this platform and data gathered as presented in Table I. Three different loading conditions have been applied for the bearings and different failure modes occurred as demonstrated in Fig. 3. Further details about the experiments are provided in [12]. Additionally, regarding the data description provided in [12], a normal distribution with an standard deviation of 10% of the mean is considered for all the physical and loading parameters to account for the possible uncertainties in the measurement and identification process. It should be noted that the selection of PDFs is absolutely depend on the characteristics of the system and the testing facilities as well as environmental factors.

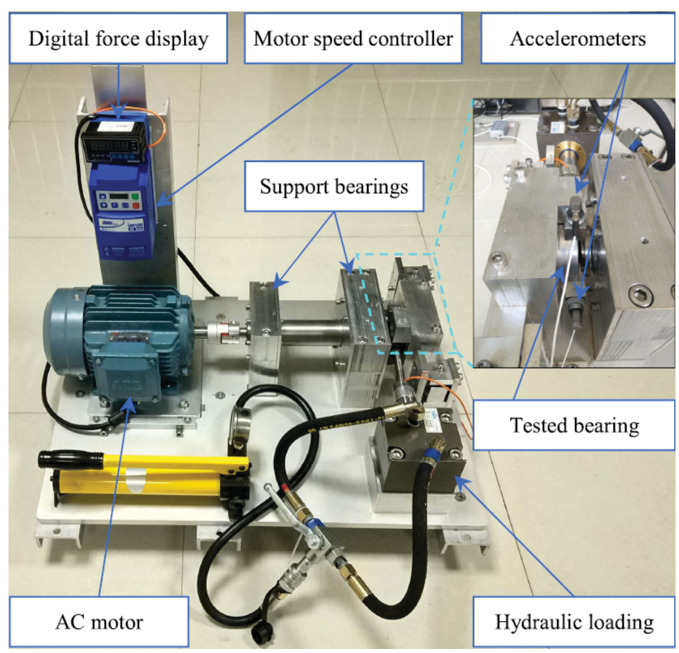

Figure 1. Bearing testing experimental setup [12]

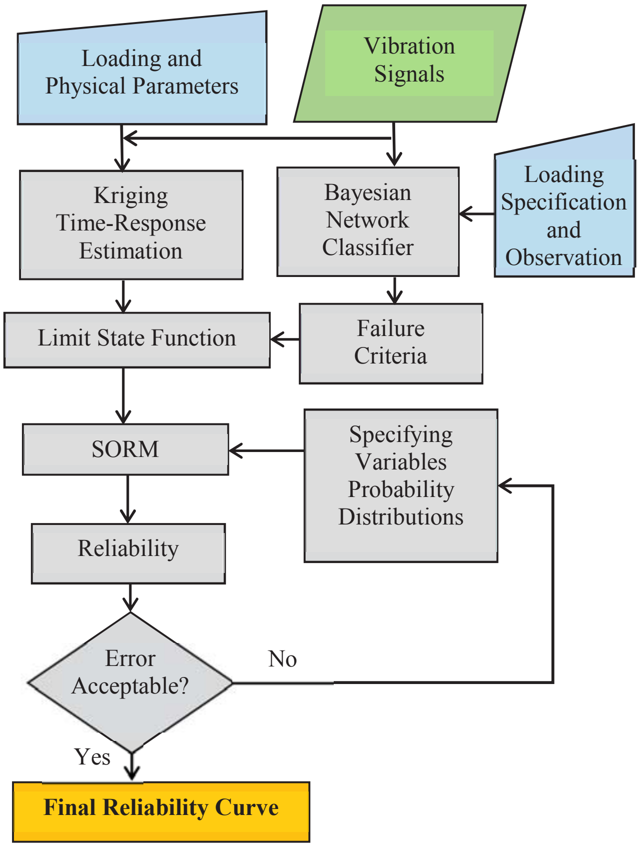

Figure 2. Flowchart of BDFC-based reliability analysis method

Figure 3. Failed Bearings: (a) Wear of inner race (b) Fracture of the cage (c) Wear of the outer race (d) Fracture of outer race [12]

TABLE I. TESTED BEARINGS DATASETS

Loading Condition	Bearing Number	Bearing Lifetime (min)	Fault Element
35 HZ 12 kN	1-1	123	Outer race
	1-2	161	Outer race
	1-3	158	Outer race
	1-4	122	Cage
	1-5	52	Inner& Outer race
37.5 Hz 11 kN	2-1	491	Inner race
	2-2	161	Outer race
	2-3	533	Cage
	2-4	42	Outer race
	2-5	339	Outer race
40 Hz 10 kN	3-1	2538	Outer race
	3-2	2496	Inner & Outer race, ball, cage
	3-3	371	Inner race
	3-4	1515	Inner race
	3-5	114	Outer race

As exposed in Fig. 4, in the degradation phase, the amplitudes of vibration increase versus time. This means that the vibration signals in this phase can be considered as one of the several possible failure criteria. Wang et al., have assumed that the time when the amplitude of the vibration signal exceeds 20g is the failure time of the bearings under operating condition [12]. Here, based on the described BDFC method, and the data gathered from all fifteen samples, time-varying failure criteria for the horizontal vibration amplitude is calculated. The BDFC invokes different failure criterion based on maximum horizontal vibration amplitude at each time, considering different loading conditions. For instance, Fig. 5 represents the maximum safe horizontal vibration amplitude after 2500 minutes of operation for different loading conditions.

Now, using the output of BDFC together with the Kriging model, the SORM can be utilized to calculate the reliability curve for each sample and each working condition. All the samples were from a same bearing type and so the calculated curve based on each sample should be the same. The latter fact helps to complete the validation loop in Fig. 1 for fine-tuning the estimation parameters. Three reliability curves for three different working conditions (as in table I) are represented in Fig. 6.

The estimation parameters, which have been tuned using test samples, are now used to find a reliability curve for real working condition. The real working condition is defined as (30 Hz,6 kN) and its corresponding reliability curve after 15000 hours is presented in Fig. 7. As it can be seen from Fig. 7, the reliability suddenly decreases after 300000 minutes of operation, which means that this time may be the best operational lifetime for this product. It should be noted that if we increase the number of testing samples, a more reliable reliability curve could be concluded. However, this method is able to perform the analysis even with a few number of testing trials or even using some computational analysis.

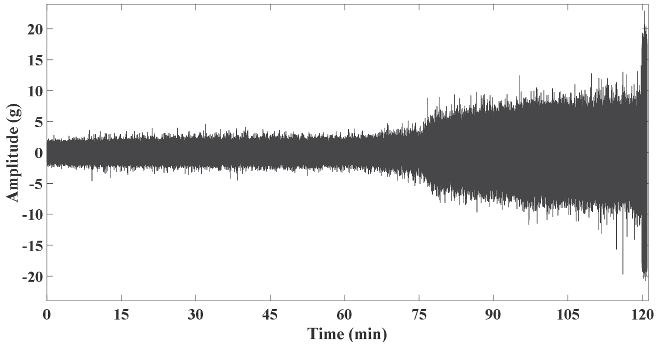

Figure 4. Horizontal vibration signal for bearing 1-1

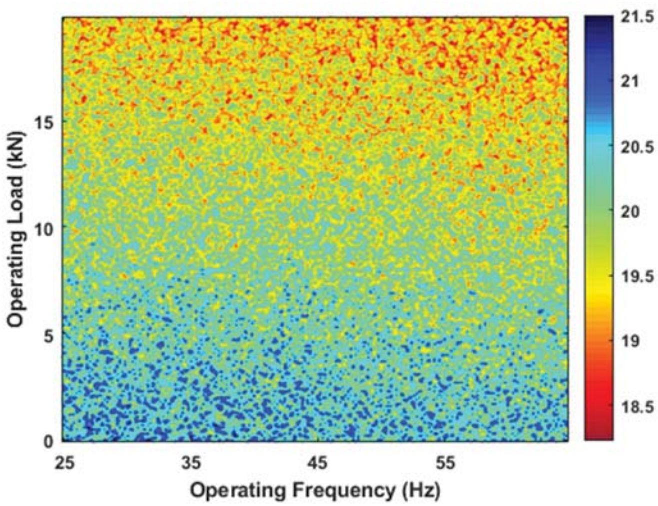

Figure 5. Failure criteria at t=2500 min for different loading condition

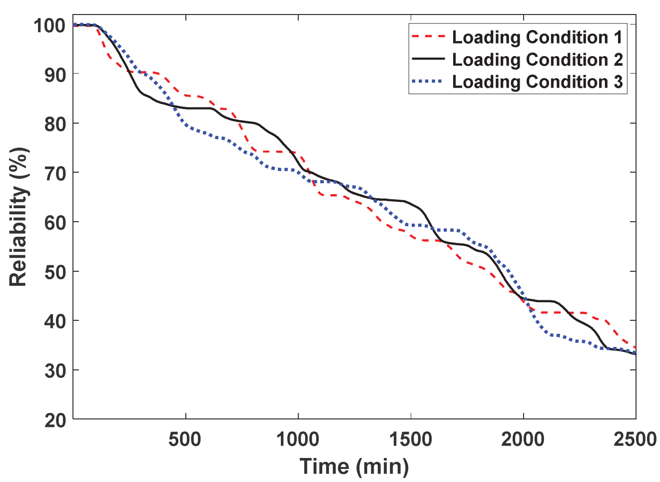

Figure 6. Reliability curves for different accelerated tests

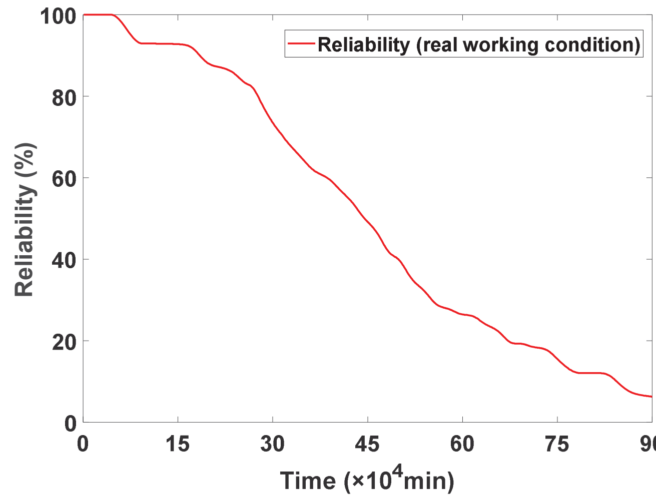

Figure 7. Reliability curve for a real working condition after 15×10^3 hours

V. Conclusions

A time-varying reliability calculation method based on dynamic failure criteria and using a Kriging model is presented in this paper. The suggested approach uses a Bayesian network classifier to find accurate time-varying failure criteria considering different working conditions and uncertain physical and environmental properties. A Kriging surrogate model is also utilized in order to discover the best system performance function using existing data. The Kriging model together with the dynamic failure criteria are coming together to shape a dynamic limit state function. Using the calculated limit state function and the second order reliability method, a time-varying reliability curve can be concluded. The method is finally applied to calculate the reliability of a rolling element bearing using a set of accelerated test data. Minimizing the difference between different reliability curves from different tests, we fine-tune the parameters of reliability estimation to use them for reliability prediction of a normal working condition. The resulting curves for both accelerated testing condition and a normal testing condition are plotted. It is shown that the presented method is capable of calculating a lifetime reliability curve using a few accelerated degradation tests. Consequently, the outcome of this research may pave the way for further reliability analysis during design, manufacturing and operational phases with a minimal set of accelerated performance tests.

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