# Reliability Analysis Method of Nuclear Cleaning System Considering Performance Degradation

Lujie Liu, Guangyan Zhao, Yufeng Sun\*, Weiwei Hu School of Reliability and Systems Engineering Beihang University Beijing, China e-mail: syf@buaa.edu.cn

Abstract—As a degenerate system, nuclear cleaning system contains performance degradation components, which will lead to performance degradation of the whole system. In this paper, continuous degradation process of the component was discretized into several discrete states. The process of degradation and maintenance of the component was described by a Markov state transition diagram. A new extended multi-state voting gate, which can be combined with traditional multi-state logical gate to build a multi-state fault tree, was established to describe the relationship between the performance of the component and the subsystem. Monte Carlo method was used to simulate the multi-state fault tree to obtain the reliability parameters of the system. Finally, the application verifies the method presented in this paper, which provides a theoretical basis for reliability analysis of nuclear cleaning system considering performance degradation.

Keywords-Performance degradation; Multi-state Fault Tree; Monte Carlo simulation; Nuclear cleaning system

#### I. INTRODUCTION

The action of particles embrittles the material of nuclear cleaning equipment, which is an important factor leading to the degradation of equipment performance [1]. Research on the degradation and maintenance of nuclear equipment is of great importance for its safe and reliable operation.

A component with degraded characteristics will experience a performance degradation process from normal to failure. The study of performance degradation began in the 1970s. Gertsbackh and Kordonskiy first proposed reliability analysis methods based on performance degradation data [2]. Soro I W gave a reliability evaluation method for multi-state degradation systems with preventive maintenance [4]. The effect of different maintenance strategies on the availability of nuclear power systems with performance degradation was discussed [5]. Based on the general generation function, Linjie Yan analysed the reliability of spatial mechanism systems with degenerate characteristics [6]. Eryilmaz gave a calculation method for the performance of a system composed of multistate components [7]. Liu discretized the degradation process of the machine tool holder and analyzed the reliability of the tool holder with performance degradation through the multistate fault tree model [8]. In the literature [9], an analysis method for multi-state fault trees considering component degradation was given.

In the past, for the reliability study of degraded systems, most scholars studied the degenerate component and the two-state component separately. Less research has been done on the simultaneous existence of degenerate components and two-state components. Traditional logic gates cannot handle system performance problems but fault logic relationships between components.

In this paper, the continuous degradation process of a component was discretized into multiple states, and the performance degradation of the component and subsystem was shown by Markov state transition process. Extending the voting gate, a new logic gate was created that describes the performance relationship between the component and the subsystem. Combined with traditional multi-state logic gates, a new fault tree model of the degraded system was established. After that the method based on Monte Carlo simulation was given to calculate the reliability parameters of the system.

The remainder of the paper is organized as follows. In section II, the modeling of degradation system is given. The simulation method to analyse the model is described in section III. In section IV, an example of nuclear cleaning system with performance degradation is analysed using the model and method above. Finally, we conclude paper and suggest future work.

#### II. DEGRADED SYSTEM MODELING

A performance degradation system exhibits multiple performances at work and is a typical multi-state system.

- A. Multi-state model of performance degradation component
  - 1) Basic assumption of the model
  - The state of the degraded component can only be transferred from the current state to the next degraded state and would not directly fail.
  - The state variables of different components are independent of each other, and the state transition is a random event.

#### 2) Establishment of model

During the operation of the system, the component degrades due to factors such as radiation, wear and thermal fatigue. As the working time increases, the component degrades from the state of good performance to fault. The state

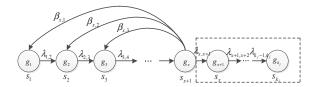


Figure 1. Multi-state transition diagram of degenerate component

transition process is shown in Fig. 1. When the component moves to state  $s_k$ , the component performance does not meet the requirements and fails. The component needs to be repaired.  $\lambda_{a,b}^{(j)}, \beta_{a,b}^{(j)}$  represent the failure rate and repair rate of the component j from state a to b, respectively.

The performance value of component j is discretized into  $k_j$  kinds. The performance at time t is  $g_j(t)$ .

$$g_{j}(t) \in \{g_{j_{1}}, \dots, g_{j_{i}}, \dots, g_{j_{k_{j}}}\}, j = 1, \dots, n, i = 0, 1, \dots, k_{j}.$$
 (1)

 $g_{j}$  is the performance of component j in state i; n is the total number of components in system.

The state probability of the component is expressed as  $q_i(t)$ .

$$q_{j}(t) \in \left\{ q_{j_{1}}(t), \dots, q_{j_{k}}(t), \dots, q_{j_{k_{j}}}(t) \right\},$$

$$q_{j_{i}}(t) = \Pr \left\{ g_{j}(t) = g_{j_{j}} \right\}, \sum_{i=1}^{k_{j}} q_{j_{i}}(t) = 1.$$
(2)

 $q_{i}$  is the probability of component j in state i.

### B. Multi-state model of performance degradation subsystem

Similarly, as time increases, the states of the subsystem change. According to the structure function between the system and the component, the state of the system and corresponding probability can be expressed as follows:

$$\delta(t) = f(g_1(t), \dots, g_n(t)),$$

$$q_s(t) = \prod_{i=1}^n q_i(t), \sum_{k=1}^{k_1} \sum_{i=1}^{k_2} \dots \sum_{i=1}^{k_n} \prod_{j=1}^n q_{j_{i_j}}(t) = 1.$$
(3)

 $f(\cdot)$  represents the relationship between subsystem performance and component performance.

It is known from (1) that there is a total combination of  $L = \prod_{j=1}^{n} k_j$  kinds system performances. When the system performance is degraded to  $G_x$ , the maintenance will be performed because the performance does not meet the working requirements. In order to improve the calculation efficiency, the states and probabilities that do not meet the performance requirements are divided into one group. After merging and

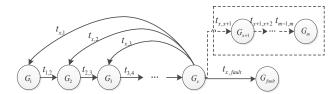


Figure 2. Multi-state transition diagram of degenerate subsystem

grouping, the performance and state probability of the subsystem are obtained:

$$G(t) = \{G_1, \dots, G_x, G_{fault}\},\$$

$$p(t) = \{p_1(t), \dots, p_x(t), p_{fault}(t)\}.$$
(4)

Similarly, the relationship between components and subsystems can be expressed as follows:

$$G(t) = \phi(\delta(t)) = \phi(f(g_1(t), \dots, g_n(t)))$$
(5)

 $\phi(\cdot)$  represents the mapping function of the system performance combination and grouping. G(t) represents the state of degenerate subsystem.

In this case, the state transition process of the subsystem is shown in Fig. 2.  $t_{i,i+1}$  is the degeneration time from state i to state i+1.  $t_{x,i}$  is the repair time from state x to state i. G(t) represents the state of degenerate subsystem. Degraded subsystems deteriorate over time, while the state of components will be refined due to maintenance.

# C. System multi-state fault tree

Existing multi-state logic gates can only describe states of the component, but cannot describe the performance relationship between components and subsystems. In this section, the multi-state voting gate is extended to enable calculation of subsystem states based on performance of the input component, first. The output of the logic gate is a value between 0 and 1. Second, the multi-state fault tree of the degraded system is constructed by multi-state logic gates.

# 1) Establishment of extended multi-state voting gate

The extended multi-state voting gate contains z input events. The component state corresponding to event i is  $s_i(t)$ , and the output event Y is the state of the subsystem. The graphical representation is as follows:



Figure 3. Extended multi-state voting gate

The calculation process of the output event Y in the logic gate can be divided into the following two steps:

### Step 1: Subsystem performance calculation

According to the actual system, the calculation method of the performance relationship between the component and the subsystem in the multi-state voting gate can be divided into the following three types:

- (1) The relationship between component performance and subsystem performance can be given by (5), when the physical structure relationship between the component and the subsystem is clear.
- (2) The system structure is complex and cannot be expressed in an explicit form and requires high precision in the performance relationship between the component and the subsystem. For such cases, a virtual prototype can be built in the computer to get the performance relationship between the component and the subsystem.
- (3) When the physical structure between the component and the subsystem is unknown, since the performance of the subsystem depends on the comprehensive performance level of the component, the relationship between the performance of the component and the performance of the subsystem can be given by (6). The coefficients of the component are obtained by actual measurement.

$$G(t) = \sum_{i=1}^{k} w_i g_i(t)$$
 (6)

G(t) is the performance of the subsystem;  $w_j$  is the coefficient of element j, indicating the degree of performance contributes to subsystem performance of component.

#### Step 2: Subsystem state calculation

During the operation of the subsystem with degraded characteristics, the performance of the subsystem is deteriorating. When the performance of the subsystem does not meet the requirements of the work, it is determined to be a fault. Due to the different ranges of the subsystem performance, the system performance needs to be componentized to facilitate the establishment of the fault tree.

For production systems and energy transmission systems, the greater the system output value, the better the subsystem performance. For such subsystems, the state is calculated as follows:

$$Y = \begin{cases} \frac{G_1 - G(t)}{G_1}, & G(t) \ge w \\ 1, & G(t) < w \end{cases}, Y \in [0, 1]$$
 (7)

 $G_1$  is subsystem in good performance; w is the minimum performance requirement of the subsystem, when the performance below this value ,the system will be fault, Y=1.

However, for systems such as control systems and mechanical actuators, the smaller the subsystem deviation, the

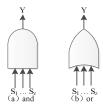


Figure 4. Multi-state and gate & Multi-state or gate

better the subsystem performance. For such subsystems, the state is calculated as follows:

$$Y = \begin{cases} \frac{G(t)}{w}, & G(t) \le w \\ 1, & G(t) > w \end{cases}, Y \in [0,1]$$
 (8)

w is the maximum allowable error value of the subsystem and fault will occur when the subsystem error is higher than this value.

# 2) Establishment of system multi-state fault tree

The multi-state AND gate, OR gate contains z input events. The component state corresponding to event i is  $s_i(t)$ , and the output event Y is the state of the subsystem. (Fig. 4)

The calculation rules for multi-state logic gates are as follows:

Multi-state AND gate: Output Y is the smallest of the z input events.

$$Y = \min(s_1(t), \dots, s_n(t)) \tag{9}$$

Multi-state OR gate: Output Y is the largest of the z input events.

$$Y = \max(s_1(t), \dots, s_z(t)) \tag{10}$$

Performance degradation of actual systems may only be related to performance degradation of certain components. The state of the other two-state components will only determine if the system will be failed. Therefore, the number of states of the system is equal to the number of states of the degraded subsystem. The state of the degenerate system is expressed as (11). For sub-fault tree, which contains components related to system performance degradation, can be established by extending the multi-state voting gate to calculate the state of the subsystem. Then through the multi-state fault tree of the system, the system state can be solved.

System state:

$$S(t) \in \left\{ S_1, \dots, S_x, S_{fault} \right\} \tag{11}$$

 $P_{S}$  is state probability of system:

$$P_{S} = q_{1}(t) \cdot q_{2}(t) \cdots q_{n}(t) \tag{12}$$

#### III. MONTE CARLO ANALYSIS OF RELIABILITY OF SYSTEM

Monte Carlo simulation method is one of the most commonly used methods to solve multi-state fault tree in engineering. In this paper, it is used to solve the multi-state fault tree established in section 2.

# A. Specific simulation process

The specific simulation process is as follows and the corresponding simplified Monte Carlo simulation flow chart is shown in Fig. 5.

- Step 1: Input the information needed for simulation. It includes failure distribution function, maintenance distribution function, total simulation time and total simulation times of all bottom events.
- Step 2: Multiple degenerate processes of the component are discretized into multiple states. According to the component failure state rate and maintenance rate, the component failure time and maintenance time are sampled by using the sampling formula. The sample failure time and maintenance time are put into the event queue and the event queue is sorted to get the first event.
- Step 3: According to the proposed modeling method of extended multi-state voting gate, the relationship between the performance of the component and the degenerate subsystem is given. Then, the state of subsystem can be calculated by (7), (8). The output of the logic gate is a value between 0 and 1. Use (9), (10) to establish the multi-state fault tree of the system. Based on the logical relationship between the bottom events, the state of the top event is solved by sweeping fault tree. Calculate the cumulative failure time and repair time of the system. When the simulation time steps to the event occurrence, the event is removed from the event queue.
- Step 4: Determine whether the single simulation time reaches the predetermined value, if yes, record the cumulative simulation times and add one to the system simulations times, then, jump to Step5; otherwise, jump to Step2.
- Step 5: To determine whether the predetermined number of simulations N is reached. The number of simulation times in this paper is set to 10<sup>5</sup> times. If yes, jump to Step6; If not, jump to Step1.
- Step 6: Calculating the reliability simulation parameters of the system based on the simulation results. The specific of statistical parameters are shown in part B.

#### B. Parameters statistics

### 1) System availability

A system in a state of 1 indicates that the system is failed and need to be repaired. Therefore, the availability of the system A(t) can be defined as the probability that the multistate fault tree output event is not 1 and is given by:

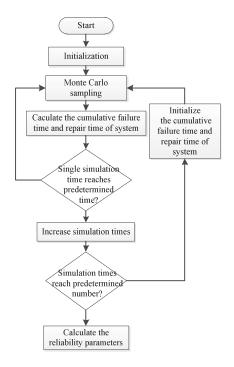


Figure 5. Flow chart of reliability simulation of system

$$A(1,t) = \Pr\{S < 1\} = \sum_{i=1}^{fault} \left[ P_{S_i}(t) \cdot I_{S < 1}(S_i) \right]$$
 (13)

 $I(\cdot)$  represents indicative function. When  $S_i < 1, I_{S<1}(S_i) = 1$ . When  $S_i \ge 1, I_{S<1}(S_i) = 0$ .

### 2) System state

The state of the multi-state system at time t, which can be represented by the expected state of the system  $\xi(t)$  and is given by:

$$\xi(t) = \sum_{i=1}^{finult} S_i P_{S_i}(t), \quad i = 1, \dots, fault, \sum_{i=1}^{finult} P_{S_i}(t) = 1, 0 \le t \le T.$$
 (14)

 $P_{S_i}(t)$  is state probability of the system in state i at time t.

# IV. RELIABILITY ANALYSIS OF NUCLEAR CLEANING SYSTEM

Nuclear radioactive material cleaning system mainly consists of laser generator, optical fiber, mechanical arm, drive motor and control system [10]. The function of the laser subsystem is to generate and transmit lasers. Motion subsystem implements movement of the mechanism. The constituent materials of the laser generator and the optical fiber are brittle under the action of radiation stress, resulting in degradation of performance. For the mechanical arm and the drive motor, this paper assumes that it has only two states during operation and assumes that the reliability of the control system is 1. Functional diagram of the cleaning system for nuclear power plants is shown in Fig. 6. Required output power is not less than 300w.

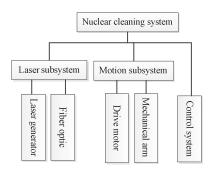


Figure 6. Functional diagram of nuclear cleaning system

TABLE I. STATE DIVISION AND PERFORMANCE OF LASER GENERATOR

Number	State	Power/w
1	normal	500
2	light degradation	350
3	moderate degradation	150
4	failure	0

TABLE II. STATE DIVISION AND PERFORMANCE OF OPTICAL FIBERS

Number	State	Attenuation coefficient /(db/km)
1	perfect functioning	-25
2	degraded	-100
3	failure	-200

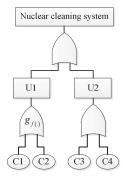


Figure 7. Multiple fault tree of nuclear cleaning system

# A. Multi-state model of the component

In this paper, the failure rate and maintenance rate of components are obtained from literature [1, 6, 11].

## 1) Laser generator

According to the literature [11], the degradation phenomenon of the laser generator during its operation shows that the output power will continue to decrease when the current is constant. The degradation process of the laser generator is divided into four discrete states according to the output power, and the performance corresponding to each discrete state is shown in Table I:

Assume that the time of each state transition of the laser generator obeys the exponential distribution, and the failure rates are  $\lambda_{12}^{(1)} = 1.3 \times 10^{-3}$ ,  $\lambda_{23}^{(1)} = 2 \times 10^{3}$ ,  $\lambda_{34}^{(1)} = 3 \times 10^{3}$ . Repair rate is  $\beta_{0}^{(1)} = 0.05 \cdot h^{-1}$ .

# 2) Optical fiber

According to the literature [1], radiation damage leads to an increase in the attenuation coefficient of the optical fiber, which reduces the transmission power of the optical fiber. According to the optical fiber attenuation coefficient, the degradation process of the fiber is divided into three states: perfect functioning, degraded and failed. The performance level is shown in the Table II. Let the time of each state transition of the fiber obey the exponential distribution, and the failure rates are  $\lambda_{12}^{(2)} = 5.71 \times 10^3 h^{-1}$ ,  $\lambda_{23}^{(2)} = 4.35 \times 10^3 h^{-1}$ . Repair rate is  $\beta_{31}^{(2)} = 0.05 \cdot h^{-1}$ .

### 3) Mechanical arm and drive motor

According to the literature [6], this paper assumes that the drive motor and the mechanical arm have only fault and normal states, and their fault and repair time are subject to exponential distribution.

Drive motor failure rate and repair rate are  $\lambda_{12}^{(3)} = 2.5 \times 10^{-4} h^{-1}$ ,  $\beta_{21}^{(3)} = 0.05 \cdot h^{-1}$ .

Mechanical arm failure rate and repair rate are  $\lambda_{12}^{(4)} = 2 \times 10^{-4} h^{-1}$ ,  $\beta_{21}^{(4)} = 0.05 \cdot h^{-1}$ .

#### B. System multi-state fault tree

The multi-state fault tree of the nuclear cleaning system is established according to the method proposed in this paper, as shown in Fig. 7. The input events C1, C2 of the extended multi-state voting gate are the state of the laser generator and the fiber. Any failure of the mechanical arm or the drive motor will lead the system to failed, so the OR gate is used to combine C3 and C4. Finally, OR gate combines subtrees of each subsystem together.

Since the physical laws of laser conduction in the optical fiber are clear, the performance relationship between the component and the subsystem can be directly given in the extended multi-state voting gate. According to the conduction energy attenuation equation (15) and (5), the expression of the output performance of the subsystem composed of the laser generator and the optical fiber can be given by (16).

$$a = \frac{1}{L} 10 \lg \frac{P_1}{P_0} \tag{15}$$

$$G(t) = P(t) / 10^{0.1La(t)}$$
(16)

 $P_0$  Input Power (w); L Fiber length (km), the length of the fiber in this paper is 10m.

Then, by (7), the state of the subsystem is calculated as the output of the extended multi-state voting gate.

#### C. Results of Monte Carlo simulation

In this paper, Monte Carlo simulation time is set to 2000 hours, and the total number of simulation time is 10<sup>5</sup>. The availability of the nuclear cleaning system can be obtained and

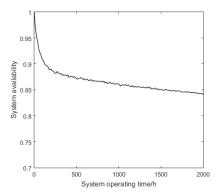


Figure 8. System availability

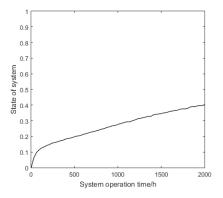


Figure 9. State of the system

is shown in Fig. 8. The system is in good condition at the beginning of operation. Due to the degradation or sudden failure of the system component, the availability of the system decreases with time. Since the system is repaired immediately after a failure, the availability of the system does not continue to decrease. System availability presents a monotonic convergence state and reaches a stable level of 0.85 at around 2000 hours.

The system is in good condition when it starts running. (Fig. 9) The state of the system continues to degrade as the laser generator and fiber are degraded. Repairs are performed when the system performance is lower than the required performance. At first, the system is in good condition and deteriorates quickly. Then, the degradation rate becomes slower because of the maintenance. Under the simultaneous action of system degradation and maintenance, the system state does not change. The final system average state tends to stabilize at 0.4, which means that the system works in the degradation state of 0.4 under the current maintenance strategy.

#### V. CONCLUSION AND FUTURE WORKS

This paper studied the repairable system including both performance degradation components and two-state components for system availability and the state of system. An analysis of the performance state over time was given. The proposed method provided a theoretical basis for reliability analysis of systems considering degradation.

Fault tree is a very common method to analyze system reliability in engineering. In view of strict assumptions, traditional fault trees cannot take into account performance degradation characteristics of components. This paper provides an improved method to describe the degradation characteristics in the system. So it is more accurate than traditional fault tree. At the same time it can help engineers to reduce the difficulty of actual modeling than any other models.

In the future, the extended multi-state voting gates will be combined with dynamic logic gates to analyze more complex dynamic systems considering degradation.

#### ACKNOWLEDGMENT

I would like to express my gratitude to Jie Xuan for her selfless help. The authors are thankful to the editor and anonymous reviewers for their valuable comments that helps us improve the quality of the paper.

#### REFERENCES

- [1] Hongchen Zhang, "The performance degradation mechanism and mechanism of quartz polarization-maintaining fiber under irradiation environment," Dissertation, Harbin Institute of Technology, 2013.
- [2] Gertsbakh I B, Kordonsky K B, Models of Failure, English translation from the Russian version, New York: Springer, 1969.
- [3] Crk V, "Component and system reliability assessment from degradation data," Dissertation, University of Arizona, 1998.
- [4] Soro I W, Nourelfath M, Daoud Ait-Kadi, "Performance evaluation of multi-state degraded systems with minimal repairs and imperfect preventive maintenance," Reliability Engineering & System Safety, vol. 95, no. 2, pp. 65-69, Feb, 2010.
- [5] Yanlong Shang, Qi Cai, Xinwen Zhao, "Analysis of availability of nuclear power systems considering performance degradation and multilevel guarantee," Atomic Energy Science and Technology, vol. 48, no. 4, pp. 703-708, Mar, 2014.
- [6] Linjie Geng, Jianguo Zhang, Yidong Wang, "Multi-state reliability analysis of on-orbit space system based on performance degradation and generalized generation function," Journal of Mechanical Engineering, vol. 11, no. 6, pp. 123-129, Jun, 2017.
- [7] Eryilmaz, Serkan, "Reliability analysis of multi-state system with three-state components and its application to wind energy," Reliability Engineering & System Safety, vol. 172, no. 4, pp. 58-63, Apr, 2018.
- [8] Chenxi Liu, "Reliability Evaluation and Structure Optimization of Servo Turret Tool Holder," Journal of Mechanical Engineering, vol. 44, no. 3, pp. 538-543, May, 2014.
- [9] Olivier Pourret, Jerome Collet, JL Bo, "Boolean modeling and evalutation of a multistate-component system," International Journal of Reliability, Quality and Safety Engineering, vol. 09, no. 02, pp. 183-192, Jau, 2002.
- [10] Gao Z, "Experimental simulation of radioactive decontamination with excimer laser," The International Society for Optical Engineering, vol. 8786, no. 22, pp. 291-296, Jul, 2013.
- [11] Lingling Li, Xunhua Gu, Fengqiang Li, "Reliability measurement method based on performance degradation of GaAs lasers," Journal of Engineering Design, vol. 19, no. 3, pp. 166-169, Apr, 2012.
- [12] Yuedong Shi, Yuqiao Chen, Jiashan Jin, "Method for solving general generation function of ship equipment multi-state repairable system reliability," Systems Engineering and Electronics, vol. 38, no. 9, pp. 2215-2220, Sep, 2016.