Fault Feature Extraction of Compound Planetary Gear Based on VMD and DE

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***Abstract*—Vibration signal of compound planetary gears is complex, so it is very hard to extract fault feature and diagnosis fault. This paper proposes new fault characteristic parameters based on VMD (Variational Mode Decomposition) and DE (Dispersion Entropy). Firstly, VMD is adopted to decompose the vibration signal and obtain a set of IMF (intrinsic modal function). Second, the signals is reconstructed by some IMFs according to the mutual information criterion. Third, dispersion entropy of the reconstructed signal is calculated. Finally, DE is input as a eigenvalue to the PSO-SVM (particle swarm optimization and support vector machine) classifier to implement fault pattern recognition. The experimental results show that the features proposed in this paper can distinguish the three states of normal gear, sun gear spall and planetary gear spall with 100% accuracy.**

***Keywords*—compound planetary gears； fault feature extraction； variational mode decomposition; dispersion entropy**

I. INTRODUCTION

Compound planetary gear system is widely used in aircraft, vehicle and engineering fields. Compound planetary gearbox of military vehicle is usually working under tough operating conditions and variable load [1],which causes diverse failure forms of gears such as peeling and breaking frequently [2].Timely and accurate diagnosis of the type and location of the fault is very important in improving the safety of the vehicle.

VMD [3] could finds the optimal solution of the variational problem to determine the center frequency and bandwidth of each IMF iteratively, and divide the signal band adaptively. This method can alleviate modal aliasing effectively, and has the advantages of strong anti-noise robustness and fast calculation speed [4].

Mostafa Rostaghi and Hamed Azami [5] introduce a new method to quantify the regularity of time series, termed DE. Which is faster than sample entropy (SE) in signal process. And comparing to permutation entropy (PE), DE can detect the noise bandwidth and simultaneous frequency and amplitude change [6].

Planetary gear fault diagnosis is a typical problem of small sample pattern recognition. SVM has the characteristics of simple structure and good generalization performance. It has outstanding advantages in dealing with small sample size pattern recognition problems [7]. The pattern recognition ability of SVM is greatly affected by the penalty coefficient and kernel function parameters. PSO is a bionic intelligent optimization algorithm proposed by James Kennedy and Russell Eberhart [8]. PSO has the advantages of fast convergence speed, less parameter setting and strong anti-interference ability. This paper uses PSO to select a pair of appropriate value of SVM parameters [9].Compared with other features such as sample entropy and permutation entropy of VMD reconstructed signals, the fault feature extraction method proposed in this paper can more accurately realize the fault diagnosis of compound planetary gears.

The following chapters are organized as follows: The second part introduces the theory of VMD, DE, PSO and SVM; the third part introduces the process of fault feature extraction; the fourth part uses experimental data to verify the method; the fifth part is the conclusion.

II. THEORETICAL INTRODUCTION

A. Variational mode decomposition

VMD transforms the signal decomposition into a non-recursive variational mode decomposition mode. VMD algorithm is a plurality of adaptive Wiener filters, which have a solid theoretical foundation and strong anti-noise ability.

The variational problem is described as seeking $u_k(t)$ that minimizes the sum of the estimated bandwidths of each modality. The constrained variational problem is expressed as following:

$$\begin{gathered}\min_{\{u_k\},\{\omega_k\}}\left\{\sum_k\left\|\partial_t\left[\left(\delta(t)+\frac{j}{\pi t}\right)*u_k(t)\right]e^{-j\omega_k t}\right\|_2^2\right\}\\ s.t.\ \sum_k u_k=f\end{gathered}\tag{1}$$

Where $\{u_k\}=\{u_1,\cdots,u_K\}$ represents modal function and $\{\omega_k\}=\{\omega_1,\cdots,\omega_K\}$ represents center frequencies.

Introducing the quadratic penalty factor α and the Lagrangian multiplication operator λ, the above constraint

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problem is reduced to the unconstrained variational problem. The augmented Lagrangian multiplier expression is as follow

$$L(\{u_k\},\{\omega_k\},\lambda)=\alpha\sum_k\left\|\partial_t\left[\left(\delta(t)+\frac{j}{\pi t}\right)*u_k(t)\right]e^{-j\omega_k t}\right\|_2^2 +\left\|f(t)-\sum_k u_k(t)\right\|_2^2+\left\langle\lambda(t),f(t)-\sum_k u_k(t)\right\rangle \tag{2}$$

The alternating direction multiplier method is used to solve the augmented Lagrangian saddle point, that is, the optimal solution of the constrained variational problem is found by alternately updating u_k^{n+1}, ω_k^{n+1} and λ^{n+1}.

$$\hat{u}_k^{n+1}(\omega)=\frac{\hat{f}(\omega)-\sum_{i\neq k}\hat{u}_i(\omega)+\frac{\hat{\lambda}(\omega)}{2}}{1+2\alpha(\omega-\omega_k)^2} \tag{3}$$

$$\omega_k^{n+1}=\frac{\int_0^\infty \omega|\hat{u}_k(\omega)|^2 d\omega}{\int_0^\infty |\hat{u}_k(\omega)|^2 d\omega} \tag{4}$$

$$\hat{\lambda}^{n+1}(\omega)=\hat{\lambda}^n(\omega)+\tau\left(\hat{f}(\omega)-\sum\hat{u}_k^{n+1}(\omega)\right) \tag{5}$$

The specific process of the VMD algorithm is as follows

Step1: Initialize $\{u_k^1\},\{\omega_k^1\},\hat{\lambda}^1,n=0$,

Step2: n=n+1,

Step3: Update u_k,ω_k according to the formula (3) and (4),

Step4: k=k+1,repeat Step3,until k=K,

Step5: Update λ using (5),

Step6: if $\sum_k\left\|\hat{u}_k^{n+1}-\hat{u}_k^n\right\|_2^2\Big/\left\|\hat{u}_k^n\right\|_2^2<\varepsilon$, end. Else, repeat Step2 to Step5.

B. Dispersion Entropy

For a given signal of length $\boldsymbol{x}=\{x_1,x_2,\cdots,x_N\}$, DE calculation steps are show as follow.

Firstly, $\boldsymbol{x}=\{x_1,x_2,\cdots,x_N\}$ is mapped into $\boldsymbol{y}=\{y_1,y_2,\cdots,y_N\}$, $y_i\in(0,1)$ by NCDF (normal cumulative distribution function). NCDF can be expressed as

$$y_j=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x_j}e^{\frac{-(t-\mu)^2}{2\sigma^2}}dt \tag{6}$$

Where μ and σ^2 are the expectation and variance, respectively.

Secondly, map y to $\{1,2,\cdots,c\}$ by a linear transformation.

$$z_j^c=R(c\cdot y_j+0.5) \tag{7}$$

Where z_j^c shows the j^{th} member of the classified time series, $j=1,2,\cdots,N$.

Thirdly, embedding vector was calculated by expression

$$\boldsymbol{z}_i^{m,c}=\left\{z_i^m,z_{i+d}^m,\cdots,z_{i+(m-1)d}^m\right\} \tag{8}$$

Where m is embedding dimension, d is time delay.

Fourthly, dispersion pattern is $\pi_{v_0v_1\cdots v_{m-1}}(v=1,2,\cdots,c)$.

Next, the probability $p(\pi_{v_0v_1\cdots v_{m-1}})$ of each dispersion pattern $\pi_{v_0v_1\cdots v_{m-1}}$ is obtained as follows

$$p(\pi_{v_0v_1\cdots v_{m-1}})=\frac{Number(\pi_{v_0v_1\cdots v_{m-1}})}{N-(m-1)d} \tag{9}$$

Where $Number(\pi_{v_0v_1\cdots v_{m-1}})$ indicates the number of $\boldsymbol{z}_i^{m,c}$ maps to $\pi_{v_0v_1\cdots v_{m-1}}$.

Finally, DE is calculated as follows

$$DE(x,m,c,d)=-\sum_{\pi=1}^{c^m}p\left(\pi_{v_0v_1\cdots v_{m-1}}\right)\ln\left(p\left(\pi_{v_0v_1\cdots v_{m-1}}\right)\right) \tag{10}$$

C. SVM and PSO

1) SVM

SVM [10]can find the optimal hyper-plane to classify data accurately. Supposing that the training sample set consists of two types of samples, which can be expressed as

$$X=\left\{(\boldsymbol{x}_i,y_i)\,|\,\boldsymbol{x}_i\in\boldsymbol{R}_N,y_i\in\{-1,+1\},i=1,2,\cdots,n\right\} \tag{11}$$

Where $\boldsymbol{x}_i$ is a column vector, y_i is the category identification of the vector $\boldsymbol{x}_i$, the classification hyper-plane equation is written as

$$\boldsymbol{w}\cdot\boldsymbol{x}+b=0 \tag{12}$$

Where $\boldsymbol{w}$ is a row vector that hangs over the classified hyper-plane.

As shown in Figure.1, there are two linearly separable data on a two-dimensional plane, represented by circles and squares, respectively. H is the optimal classification hyper-plane. The point on the line H_1, H_2 is called the support vector. The distance between H_1 and H_2 is the classification interval.

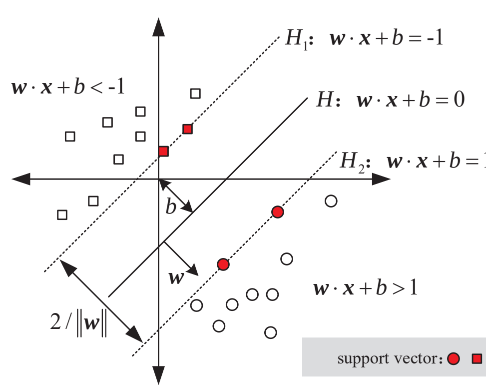

Figure.1. Schematic diagram of SVM

$$y_i\left(\boldsymbol{w}\cdot\boldsymbol{x}+b\right)-1\ge 0,\ i=1,2,\cdots,n \tag{13}$$

To obtain the optimal classification hyper-plane H, it is to find the maximum value of $1/\|\boldsymbol{w}\|$. Equivalent to the minimum $\frac{1}{2}\|\boldsymbol{w}\|^2$. Add a Lagrangian multiplier to each constraint to construct the Lagrange function

$$\mathcal{L}(\boldsymbol{w},b,\alpha)=\sum_{i=1}^{n}\alpha_i-\frac{1}{2}\sum_{i,j=1}^{n}\alpha_i\alpha_j y_i y_j \boldsymbol{x}_i^T\boldsymbol{x}_j \tag{14}$$

Where $\alpha_i\ge 0,\ i=1,2,\cdots,n$.

Find the maximum value about α, that is the optimization problem about the dual problem

$$\begin{aligned}\max_{\alpha}\quad &\sum_{i=1}^{n}\alpha_i-\frac{1}{2}\sum_{i,j=1}^{n}\alpha_i\alpha_j y_i y_j \boldsymbol{x}_i^T\boldsymbol{x}_j\\ s.t.\ &\sum_{i=1}^{n}\alpha_i y_i=0,\ \alpha_i\ge 0, i=1,\ldots,n\end{aligned} \tag{15}$$

According to equation(15), most of the solutions α_i can be found to be 0, and the corresponding ones that are not 0 are called support vectors. Set the support vector to

$$\left\{\mathbf{a}_i^*\mid \mathbf{a}_i^*>0, i=1,2,\cdots,m\right\} \tag{16}$$

Therefore, optimal weight vector $\boldsymbol{w}^*$ and the optimal offset b^* are expressed as

$$\begin{cases}\boldsymbol{w}^*=\sum\limits_{a_i>0}\boldsymbol{a}_i^* y_i\boldsymbol{x}_i\\ b^*=y_j-\sum\limits_{a_i>0}y_i\boldsymbol{a}_i^*\boldsymbol{x}_i\cdot\boldsymbol{x}_j,\ \ \forall i\in\left\{i\mid \boldsymbol{a}_i^*>0\right\}\end{cases} \tag{17}$$

The optimal classification function is

$$f(\boldsymbol{x})=\operatorname{sgn}\left(\boldsymbol{w}^*\boldsymbol{x}+b^*\right)=\operatorname{sgn}\left(\sum_{a_i>0}\boldsymbol{a}_i^* y_i\boldsymbol{x}_i\boldsymbol{x}+b^*\right) \tag{18}$$

2) PSO

The principle of PSO algorithm: assuming that the search space of population is dimension and population particle number is m, the population can be represented as $\boldsymbol{X}=\left(\boldsymbol{X}_1,\boldsymbol{X}_2,\cdots,\boldsymbol{X}_m\right)$, The spatial position of the i^{th} particle is $\boldsymbol{X}_i=\left(x_{i1},x_{i2},\cdots,x_{iD}\right)$, Velocity is $\boldsymbol{V}_i=\left(v_{i1},v_{i2},\cdots,v_{iD}\right)$ and it represents the distance the particle moves each iteration. During the search process, the speed and position of the particles are constantly updated, and the update formula is

$$v_{id}^{t+1}=wv_{id}^t+c_1r_1\left(p_{id}^t-x_{id}^t\right)+c_2r_2\left(p_{gd}^t-x_{id}^t\right) \tag{19}$$

$$x_{id}^{t+1}=x_{id}^t+v_{id}^{t+1} \tag{20}$$

Where t is the number of iterations; $d=1,2,\cdots,D$ is the dimension of the search space; $i=1,2,\cdots,m$ is the number of particles; w is the inertia weight, c_1 and c_2 are learning factors, and r_1 and r_2 are random numbers between [0, 1]; p_{id}^t is the historical optimal position of the i^{th} particle at the t^{th} iteration; p_{gd}^t is the historical optimal position of the population at the t^{th} iteration; v_{id}^t is the velocity of the i^{th} particle in the d dimensional space at the t^{th} iteration, x_{id}^t is the position of the i^{th} particle in the d dimensional space at the i^{th} iteration.

Implementation steps of PSO are as follows:

- Generating initial particles and constructing initial particle swarms X, setting initial values of parameters, initial velocity matrix $\boldsymbol{V}$, and initial optimal position p_i and p_g of each particle;
- Evaluate each particle. Calculate the particle fitness;
- Particle swarm updating. Including particle position, speed, historical optimal position and overall optimal position;
- Determine if the termination condition is met. If the iterations number reaches the initial value or the particle fitness meets the requirements, the iteration is terminated and output result.

3) SVM parameter optimization

In this paper, SVM use Gaussian kernel function to realize multi-classification. Parameters C (penalty factor) and σ (Gaussian kernel function width) need to be optimized. PSO is used to optimize C and σ. Specific steps are as follows:

- Parameter initialization. Initialize the population size, the maximum number of iterations, the position and velocity of the particles; initialize the error penalty factor of SVM and Gaussian kernel parameter;
- Calculation of fitness function. Using SVM penalty factor and Gaussian kernel parameter as the particle position, classification accuracy of SVM is taken as fitness function;
- Updating historical optimal position of the particle itself and optimal position of the population history according to fitness function;
- Updating speed and position of particles to obtain a new set of SVM parameters;
- When number of iterations reaches maximum or particle fitness function satisfies requirement, iteration is terminated and result is output. If not, return to step 2) until it is satisfied.

III. FAULT FEATURE EXTRACTION PROCESS

Based on the VMD, MI, DE and PSO-SVM, a novel fault diagnostic method for planetary gearboxes is given as follows:

- Input the vibration signal, and use the waveform method to determine the K value of the VMD;
- Performing a VMD on the signal to obtain IMFs;
- Calculating mutual information between each IMF and

the original signal;

- Selecting some IMFs, whose mutual information is greater than a threshold, to reconstruct signal;
- Calculating the dispersion entropy of the reconstructed signal;
- The dispersion entropy is putted into PSO-SVM for fault pattern recognition to realize planetary gear fault diagnosis.

IV. EXPERIMENTAL VALIDATION

A. Test bench and data introduction

The test object is a type of armored vehicle gearbox, which is a compound planetary gear train containing K1, K2 and K3 planetary rows. The K1 row structure is shown in Figure.2. It includes a sun gear with the number of teeth 29 and 31 (hereinafter referred to as “z29” and "z31"), and three planetary wheels each having a number of teeth of 15 and 18 (hereinafter referred to as "z15" and "z18"), which is a double-engaged planetary row, has a special structure and is difficult to diagnose when a fault occurs, especially a planetary gear failure. Three conditions were tested: normal, z31 spall failure, z15 spall failure. The test collected three kinds of vibration signals in three states of 1500r/min and 900N•m. The sampling frequency is 20 kHz, and each group collects 3 sets of data, and the sampling time of each set of data is 30s.

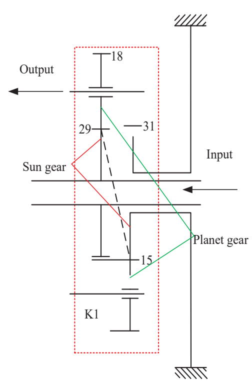

Figure.2. Schematic diagram of K1 planetary row structure

The formula for calculating the frequency of each part of the planetary gear is:

$$f_1 + i \times f_2 - (1+i) \times f_3 = 0 \tag{21}$$

$$i = \frac{z_2}{z_1} \tag{22}$$

Where f_1 is the sun rotation rate, f_2 is the ring gear rotation rate, f_3 is the planet carrier rotation rate, z_1 is the teeth number of sun gear, and z_2 is the teeth number of ring gear.

The calculation formula of the planetary gear meshing frequency is:

$$f_m = (f_1 - f_3) \times z_1 \tag{23}$$

The gear failure frequency calculation formula is

$$f_{fault} = f_m / z_{fault} \tag{24}$$

Where z_{fault} is number of teeth of faulty gear.

There are several issues to be aware of when calculating the gear fault frequency: 1) The transmission power is transmitted to the K1 sun gear through a fixed-shaft gear with a transmission ratio of 17/18. Therefore, the K1 sun gear rotation frequency is equal to the input frequency multiplied by 18/17. 2) When calculating the fault frequency of sun gear, it needs to multiply the corresponding number of planet gear; 3) When the planet gear fails, multiply the fault frequency by 2, because each time the planet gear rotates, the sun gear and the ring gear are each engaged once. According to the above formula, the fault frequencies of z15 and z31 are 52.85 Hz and 38.4 Hz, respectively. Therefore, the maximum fault period is 1/38.4=0.026s. Considering the data requirement for calculating the dispersion entropy, the length of the analysis data in this paper is chosen to be 0.1s, that is, 2000 sampling points.

B. VMD decomposition and signal reconstruction

The normal state original signal time domain waveform and spectrum are shown in Figure.3.

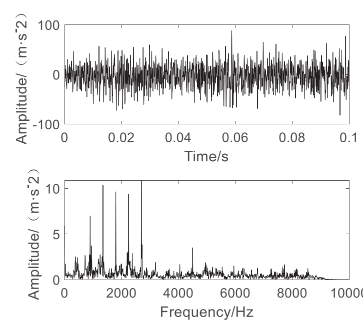

Figure.3 Time series and spectrum of the original signal

Taking normal state data as an example, the waveform method [11][12]is used to determine IMF number of VMD. The penalty factor mainly affects the bandwidth and convergence speed of each modal component, and the penalty factor is 2000. The spectrum of each modal component when different K is obtained is shown in Figure.4. When K=4, there are two frequency components with larger amplitude in u2 component, so it is necessary to further increase IMF number; when K=5, the u2 component still has a large amplitude of two frequency components; when K=6, the u2 component only has one frequency component with larger amplitude; when K=7, the u4 component is further decomposed into two components u4 and u5. The amplitudes of the two components are small relative to the amplitude of the first three components, and the frequency components do not need to be further decomposed. In combination with the original signal spectrum, there are six frequency components with large amplitudes, so K=6 is chosen. Similarly, the K corresponding to the z15 and z31 spalling fault data can be 8 and 6, respectively.

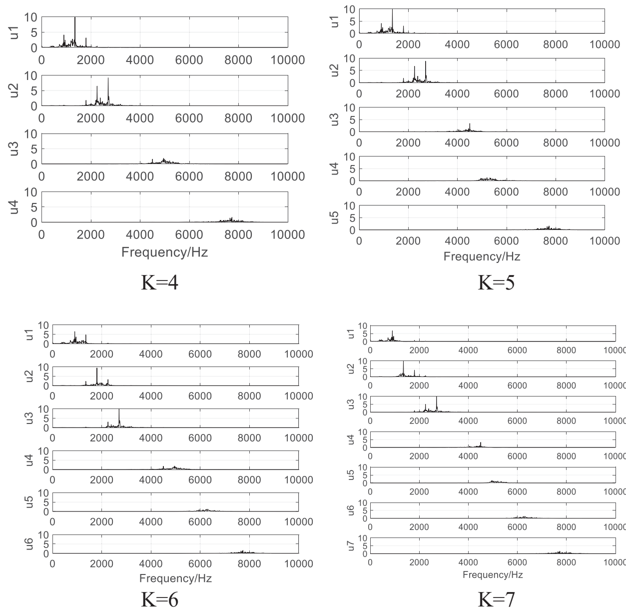

Figure.4. Spectrum of IMFs under different K value

The time series of IMFs obtained by the normal state signal VMD is shown in Figure.5. According to mutual information criterion[2], the normalized mutual information values of the respective IMFs and the original signals are as shown in Table 1. The threshold is 0.62, and the first three IMF reconstructed signals whose normalized mutual information is larger than the threshold are filtered to obtain the reconstructed signal. The time series and spectrum of reconstructed signal is shown in Figure.6.

TABLE 1 NORMALIZED MUTUAL INFORMATION VALUE

IMF	1	2	3	4	5	6	Threshold
MI	**0.98**	**1.00**	**0.94**	0.39	0.21	0.22	0.62

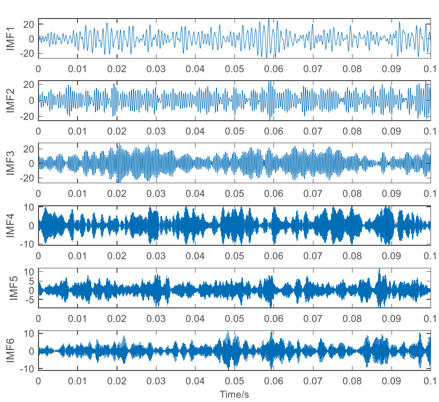

Figure.5. IMFs

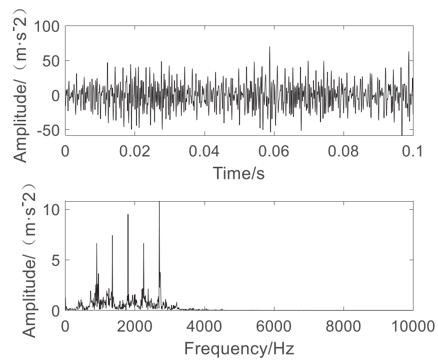

Figure.6. Time series and spectrum of reconstructed signal

C. Feature extraction and fault diagnosis

Calculate the dispersion entropy of the reconstructed signal. Each group takes 40 sets of data, and the same state data overlaps 90% before and after. Each state randomly selects 50% of the data to form a training sample, and the remaining 50% of the data is used as a test sample.

The SVM kernel function is Gaussian kernel function, and the penalty parameters C and σ need to be optimized, $C \in [0.1, 100]$, $\sigma \in [0.01, 1000]$. PSO parameter setting: the population number is 20, iterations number is 200, learning factors c_1 = 1.5 and c_2 = 1.7. Since the optimized parameters are 2, the PSO particle dimension is 2. The PSO is used to optimize (C, σ), and the fitness function is the classification accuracy rate. The K-fold cross-validation method is used to evaluate the performance of the support vector machine.

The relationship between the VMD reconstructed signal dispersion entropy and the number of sample sets is shown in Figure.7. The fitness curve is shown in Figure.8. The classification results are shown in Figure.9. The optimized penalty parameter C =0.1 and the Gaussian kernel parameter σ =70, the classification accuracy is 100%.

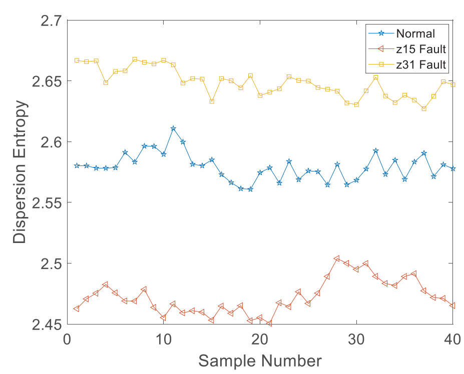

Figure.7. DE of VMD reconstructed signal

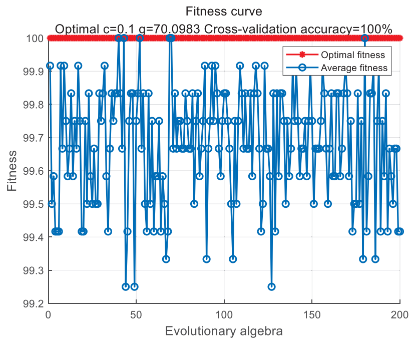

Figure.8. Fitness curve

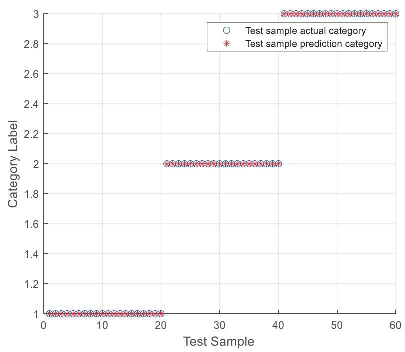

Figure.9. Classification results

V. CONCLUSIONS

The compound planetary gear fault characteristic is very weak and is difficult to diagnose directly through the time domain and frequency domain waveforms. Based on VMD and DE, this paper proposes a new fault feature index. The compound planetary gear experimental data is used to verify the feature. The results show that the feature can be input to PSO-SVM for effective identification of normal state, sun gear fault and planetary gear fault. The failure mode recognition accuracy rate is 100%.

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