Reliability Evaluation of Rolling Bearing Based on Wavelet Packet Energy Entropy

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***Abstract*—Based on the time-domain analysis of bearing vibration data, the root-mean-square is selected from the 14 time domain indicators as the prediction index and the key threshold is set according to the image. According to the wavelet packet decomposition and reconstruction theory, the vibration signal analysis of bearing is carried out. In this way, the characteristic signals of each frequency band are extracted, and the change of bearing band energy in outer ring wear is analyzed by combining with the wavelet packet energy spectrum. Finally, from the perspective of information entropy and integrated with the state parameter method, the running state of rolling bearing is evaluated and predicted. Compared to the traditional threshold method, the method used in this paper can judge the running state of bearing more effectively. And through the wavelet packet information entropy, the bearing running state is divided into four levels, which can make a more detailed and reasonable evaluation of the bearing's health and reliability. This provides a basis for reliability assessment in areas such as machining.**

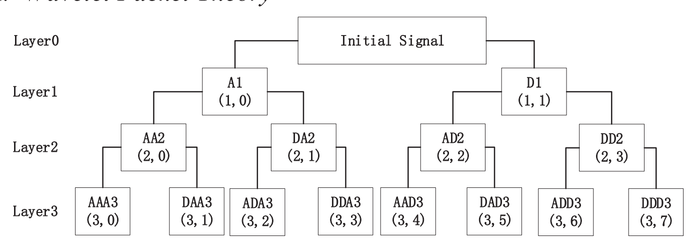
Keywords-rolling bearing; vibration signal; wavelet packet decomposition; energy entropy; operational reliability

I. INTRODUCTION

In the field of industrial machinery, rolling bearing is both one of the key components and the most vulnerable parts. For this reason, the bearing performance directly concerns the equipment's key indicators such as load capacity, temperature resistance, machining accuracy, limit speed, operating life, and reliability. The quality requirements for bearings, as the continuous development of industrial technology and improvement of equipment performance requirements, have also increased. Aerospace industry urgently demands for high-quality and high-reliability bearings. Meanwhile, the bearing technology, in the design of aero-engine structures, accounts for more than 90-95%, which also represents the reliability level of an engine [1]. Due to the reasons of processing, installation and overload, the bearing will suffer from severe wear, breakage, overheating, etc. [2-3] Once the bearing failure occurs, it will directly affect the normal operation of equipment, and even endanger personal safety. Considering the large dispersion of the service life of rolling bearing, the service life, fault type and failure degree of the same batch of processed bearings are greatly different. Therefore, it is necessary to grasp the real-time status information of bearing operation, analyze and predict the state operation, and give reasonable guidance so as to improve the operational reliability as well as safety of the entire equipment.

II. BASIC THEORY OF WAVELET PACKET

A. Wavelet Packet Theory



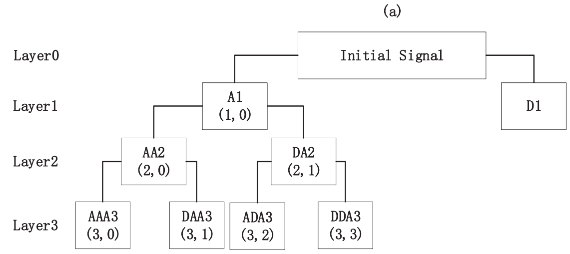


Figure 1. Tree diagram.

(a) Wavelet packet decomposition; (b) Wavelet decomposition

Proposed by Mcyer, Wickerhauser et al. in 1989, the concept of wavelet packet is a more sophisticated method of signal analysis, which improved the time domain resolution of signal. With the purpose of overcoming the poor frequency resolution in the high frequency band and poor time resolution in the low frequency band of wavelet decomposition, wavelet packet can decompose both the low-frequency signal and the high-frequency part without any redundancy or omission. On this

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basis, wavelet packet is able to conduct a better time-frequency local analysis on the signal containing a large amount of high-frequency information. Figure 1 is the tree diagram of wavelet packet decomposition (a) and wavelet decomposition (b).

In order to define wavelet packet, it is necessary to give priority to the spatial decomposition of multiresolution analysis:

$$L^2(R)=\bigoplus_{j\in z} W_j \tag{1}$$

This equation shows that the multiresolution analysis equation decomposes the Hilbert space $L^2(R)$ into subspaces W_j of multiple wavelet functions $\psi(t)$ based on different scale factors j. In order to improve the frequency resolution, a frequency subdivision on W_j shall be performed according to binary fraction:

$$\begin{cases} U_j^0 = V_j, \\ U_J^1 = W_j, \end{cases} j \in Z \tag{2}$$

wherein, U_j^n is the new subspace, V_j is the scale subspace. Then the orthogonal decomposition $V_{j+1} = V_j \oplus W_j$ can be uniformly decomposed into:

$$U_j^n = U_j^0 \oplus U_j^1, j \in Z \tag{3}$$

Define the subspace U_j^n is the closure space of function $U_n(t)$, while $U_n(t)$ is the closure space of function $U_{2n}(t)$. And let $U_n(t)$ satisfy the following two-scale equation:

$$\begin{cases} u_{2n}(t) = \sqrt{2}\sum_{k\in Z} h_k u_n(2t-k) \\ u_{2n+1}(t) = \sqrt{2}\sum_{k\in Z} g_k u_n(2t-k) \end{cases} \tag{4}$$

wherein $g_k = (-1)^k h_{1-k}$, that is, the two coefficients are in orthogonal relation. When $n=0$, then:

$$\begin{cases} u_0(t) = \sqrt{2}\sum_{k\in Z} h_k u_0(2t-k) \\ u_1(t) = \sqrt{2}\sum_{k\in Z} g_k u_0(2t-k) \end{cases} \tag{5}$$

The scaling function $u_0(t)$ and the wavelet function $u_1(t)$ can be separately reduced to the $\varphi(t)$ and $\phi(t)$ in the two-scale equations of the multiresolution analysis.

$$\begin{cases} \varphi_0(t) = \sqrt{2}\sum_{k\in Z} h_k \varphi(2t-k), \{h_k\} \in l^2 \\ \phi_1(t) = \sqrt{2}\sum_{k\in Z} g_k \phi(2t-k), \{g_k\} \in l^2 \end{cases} \tag{6}$$

wherein, $\{h_k\}$ and $\{g_k\}$ are the conjugate filter defined in the multiresolution analysis, $\{h_k\}$ is a low-pass filter bank and $\{g_k\}$ is a high-pass filter bank. The signal obtained from the original signal $u(t)$, after the action of $\{h_k\}$, is a low-frequency component, which is also called the approximate part; the signal obtained after the action of $\{g_k\}$ is a high-frequency component, which is also called the detail part.

According to equations (5) and (6), the spatial decomposition is obtained as follows:

$$U_{j+1}^n = U_j^{2n} \oplus U_j^{2n+1}, j \in Z, n \in Z_+ \tag{7}$$

Then the wavelet packet is defined as a sequence constructed by (5) and (6), $\{u_n(t)\}_{n\in Z}$ is called as orthogonal wavelet base determined by the basis function $u_0(t)=\varphi(t)$. Since $\varphi(t)$ is uniquely determined by h_k, $\{u_n(t)\}_{n\in Z}$ is the orthogonal wavelet packet of sequence $\{h_k\}$.

B. Wavelet Packet Algorithm

Wavelet packet algorithm is divided into decomposition algorithm and reconstruction algorithm. If let function sequence $g_j^n(t) \in U_j^n$, then $g_j^n(t)$ can be expressed as:

$$g_j^n(t)=\sum_l d_l^{j,n} u_n(2^j t - l) \tag{8}$$

The decomposition and reconstruction algorithms can be defined separately as follows:

1) Decomposition Algorithms:

Obtaining $\{d_l^{j,2n}\}$ and $\{d_l^{j,2n+1}\}$ through $\{d_l^{j+1,n}\}$,the equation is as follows:

$$\begin{cases} d_l^{j,2n}=\sum_k a_{k-2l} d_k^{j+1,n} \\ d_l^{j,2n+1} = \sum_k b_{k-2l} d_k^{j+1,n} \end{cases} \tag{9}$$

where j and n stand for the node number of wavelet packet; d indicates the decomposition coefficient of wavelet packet; l and k represent the number of decomposition layers.

For the original signal $u(t)$ with the sampling frequency of f_s, the 2^i frequency bands are obtained after being decomposed by layer i, wherein the frequency range of the j th frequency band is $\frac{f_s}{2^{i+1}} j \sim \frac{f_s}{2^{i+1}}(j+1)$, of which $j = 1,2,\ldots,2i+1$.

2) Reconstruction algorithms:

Obtaining $\{d_l^{j+1,n}\}$ through $\{d_l^{j,2n}\}$ and $\{d_l^{j,2n+1}\}$,the equation is as follows:

$$d_l^{j+1,n}=\sum_k \left[h_{l-2k} d_k^{j,2n} + g_{l-2k} d_k^{j,2n}\right] \tag{10}$$

C. Wavelet packet energy spectrum and energy entropy

By conducting i th layer of wavelet packet decomposition on original signal $u(t)$, a wavelet packet decomposition sequence $S_{i,j}\,(j=1,2,\ldots,2^i)$ can be obtained. The secondary energy type is used to indicate the reconstructed signal corresponding to each frequency band; then the energy spectrum [6] of the j th frequency band of i th layer of wavelet packet decomposition is:

$$E_{i,j}(k)=\left|x_{i,j}(k)\right|^2 \tag{11}$$

wherein, $x_{i,j}(k)$ is the discrete point amplitude of the reconstructed signal $S_{i,j}$, j is the frequency band serial number of the i th layer after decomposition, k is the sampling point serial number $(k=1,2,\ldots,n)$, n is the total number of signal sampling points. Then after decomposition, the energy of each band in the last layer is:

$$E_{i,j}=\int \left|S_{i,j}(t)\right|^2 dt=\sum_{k=1}^{n}\left|x_{jk}\right|^2 \tag{12}$$

In this way, the wavelet packet energy spectrum of each frequency band can be obtained:

$$E_i=\left[E_{i,1}, E_{i,2}, \cdots, E_{i,2^i}\right]^T \tag{13}$$

According to the characteristics of orthogonal wavelet transform, the total signal power E at certain time window is equal to the sum of the power $E_{i,j}$ of each component, so that an orthogonal feature division is formed. Let $p_j=E_{i,j}/E$ and $\sum p_j=1$, then the corresponding wavelet energy spectrum entropy can be given according to the measurement of information entropy, that is:

$$s=-\sum_j p_j \log p_j \tag{14}$$

III. Analysis process and case

This paper selects the bearing data provided by the Institute of Design Science and Basic Components at Xi'an Jiaotong University [7] as input. This data is obtained from several accelerated degradation tests of the normal operation to the severe failure of rolling bearing. During the acceleration, it is considered as severe failure once the amplitude of vibration signal reaches 10 times the maximum amplitude ($A_{\max}$) at the normal operation phase of the bearing. The bearing model is LDK UER204, and its bearing test bench is shown in Figure 2. The test conditions are changed by changing the rotational speed and radial force.

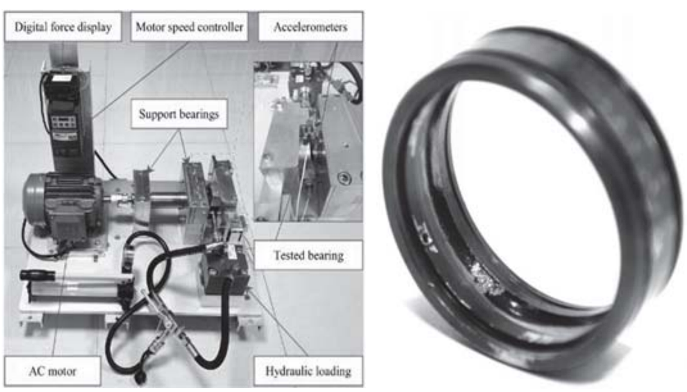

Figure 2. Bearing testbed (a) and failure bearing with outer race wear (b)[7]

The bearing with machining condition of 35 Hz/12 kN and the fault type of outer ring wear (Figure 2) is selected as the analysis object. The specific parameters are shown in Table I, and the original signal is shown in Figure 3.

TABLE I. Parameters of the tested bearings [7]

Parameter	Value	Parameter	Value
Outer race diameter	39.80mm	Inner race diameter	29.30mm
Bearing mean diameter	34.55mm	Ball diameter	7.92 mm
Number of balls	8	Contact angle	0°
Load rating(static)	6.65kN	Load rating(dynamic)	12.82 kN
Bearing lifetime	2 h 3 min	Operating condition	35 Hz(2100 rpm)/12 kN
Sampling frequency	25.6 kHz	Sampling period	1 min

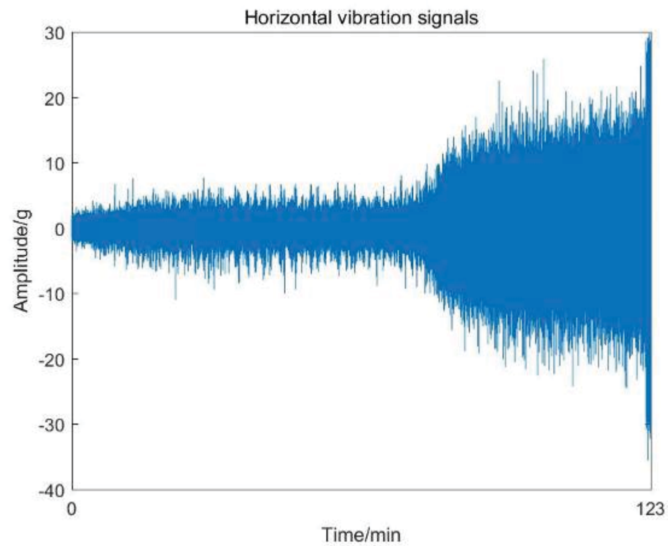

Figure 3. Horizontal vibration original signals of the tested bearing.

A. *Threshold analysis*

1) *Time domain analysis.*

Time domain analysis is the analysis of data directly in the time domain. The time domain signal indicatores are mainly used to judge the hidden fault, failure degree and its development trend of equipment, which is featured by intuitiveness and accuracy. In this paper, 14 commonly used time domain indicatores are adopted for data analysis, including mean value, absolute mean value, root mean square value (RMS), amplitude of RMS, peak-to-peak value, peak factor, standard deviation, kurtosis, kurtosis factor, form factor, pulse factor, margin factor, skewness and skewness factor. According to the relevant formulas of various indicators, a time domain indicator map (Figure 4) is made.

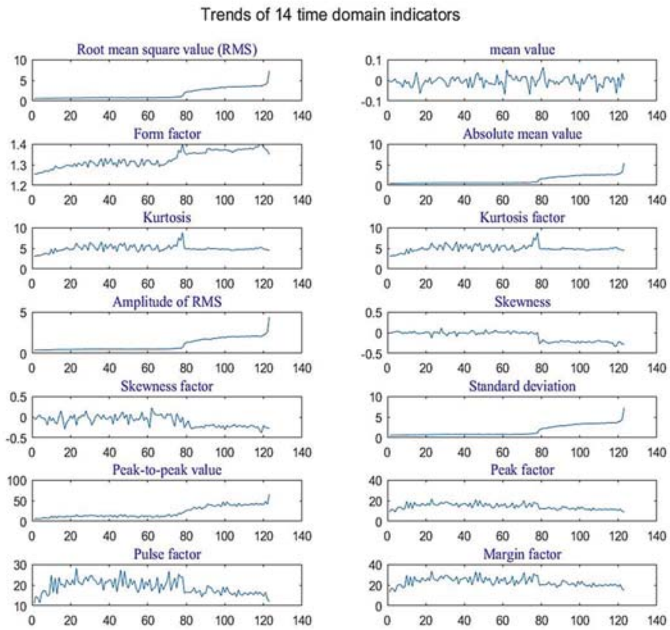

Figure 4. Trends of 14 time domain indicators

Finally, from the perspective of image trend and computational simplicity, the root-mean-square is selected as the subsequent analysis indicator. According to the image trend, the bearing operation can be divided into three stages:

(I) Normal operation stage;

(II) Fault growth stage;

(III) Failure stage;

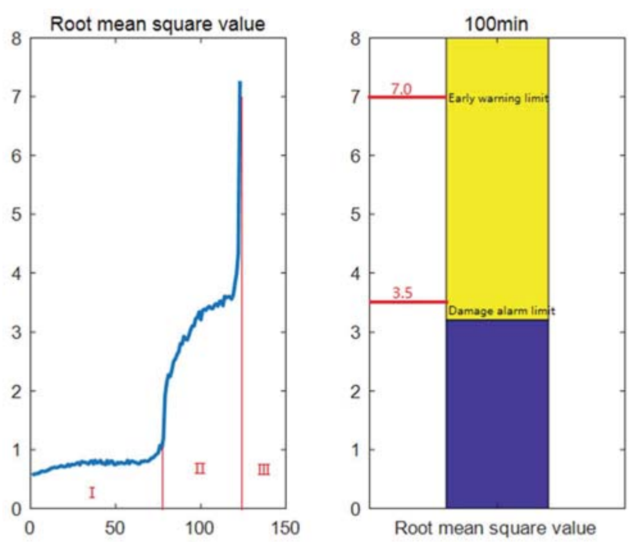

Figure 5. Three stages of bearing operation

One point is separately selected from the three stages for time-frequency analysis, and the respective images are shown in Figure 5. It can be seen from the images that as the increase of failure degree, the amplitude of bearing time domain signal increases continuously; and the fault frequency is more concentrated in the low frequency with increased amplitude.

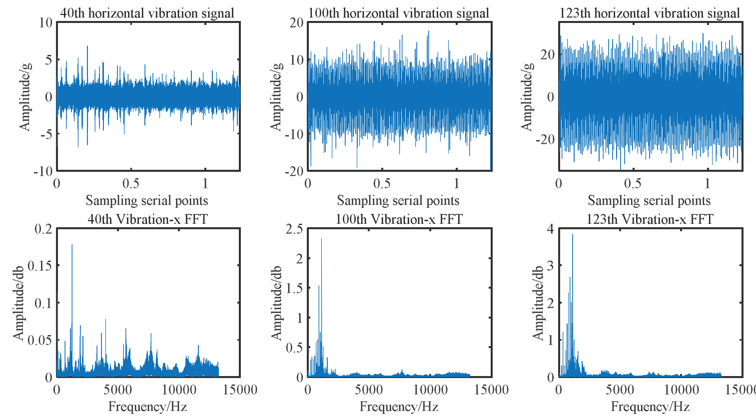

Figure. 6. Three-stage time domain and frequency domain figure

2) Traditional Thresholding

The thresholding evaluates the state of running equipment by setting the fault limit of an indicator. In the industrial application, the statistical indicator bar chart is commonly used for analysis. In this data, the root-mean-square is selected as the statistical indicator, and two thresholds are set according to the image:

1) Limit of failure warning limit (limit of alarm level I, lim1);

2) Limit of damage alarm limit (limit of alarm level II, lim2);

When the color bar breaks through the failure warning limit lim1, it indicates fault in the equipment but it can still run. When the color bar breaks through the damage alarm limit lim2, it indicates that the fault is so dangerous that the equipment needs to be shut down and repaired instantly. This paper takes lim1=3.5 (according to the value of the warning line below), lim2=7.0 (according to the maximum amplitude of the data). It can be seen that when t = 79min, the fault reaches the warning limit lim1, When t=122min, the fault reaches the failure warning limit lim2.

B. Wavelet packet analysis

1) Selection of layers of wavelet packet decomposition.

The layers of wavelet packet decomposition can represent the scale of its decomposition. According to engineering experience, the formula of layer numbers is determined as:

$$J \le \log_2 \frac{f_s}{100} \tag{15}$$

In this data set, the sampling frequency of the signal is $f_s = 25.6kHz$, and a total of 32,768 data points (1.28 s) are recorded each time, that is, the sampling period is 1 min. According to formula (15), $J \le 8$ can be obtained. Since the excessive number of decomposition layers becomes too large, while small number of decomposition layers is insufficient. For this reason, 4 layers are selected as the decomposition layer for test.

2) Wavelet packet decomposition and reconstruction

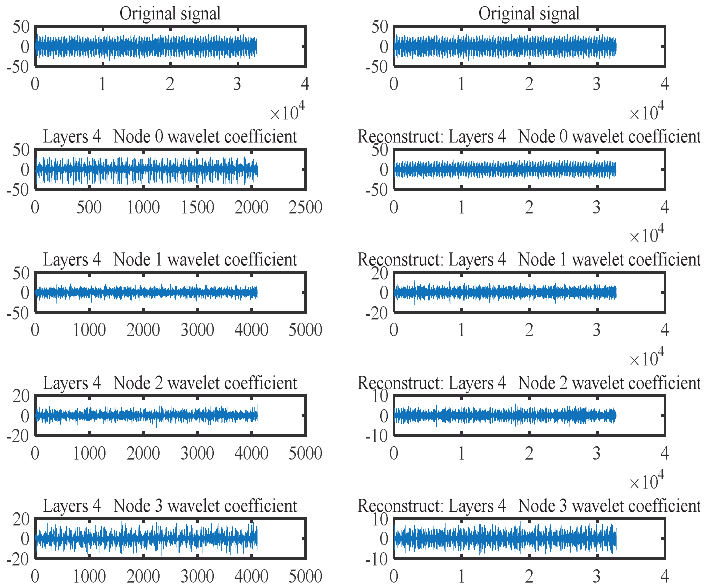

Figure 7. The 123min data wavelet packet decomposition and reconstruction figure (partial)

The "db5" wavelet is selected to decompose the vibration signal. In this way, the characteristic signal $S_{4j}(j=0,1,\ldots,15)$ of 16 frequency bands at the fourth layer from low frequency to high frequency is obtained. The wavelet packet coefficients are reconstructed so that the signal length of each node at the fourth layer is the same as the original signal, so that the reconstructed signal SS_{4j} is obtained. The corresponding decomposition and reconstruction are shown in Figure 7, which is the decomposition and reconstruction map of the first 8 nodes of at the 4th layer of 123min data.

3) *Wavelet packet energy and energy entropy*

Firstly, the total band energy at each node in the 4th layer is calculated, so that the energy at the j th node can be obtained according to (12). Then the total energy $E=\sum_{j=0}^{15} E_{4j}$ of the 4th layer of wavelet packet decomposition is calculated, so that the energy ratio $p_{4j}=E_{4j}/E$ of different frequency bands is obtained. The energy and energy specific gravity of the three stages are shown in Figure 8.

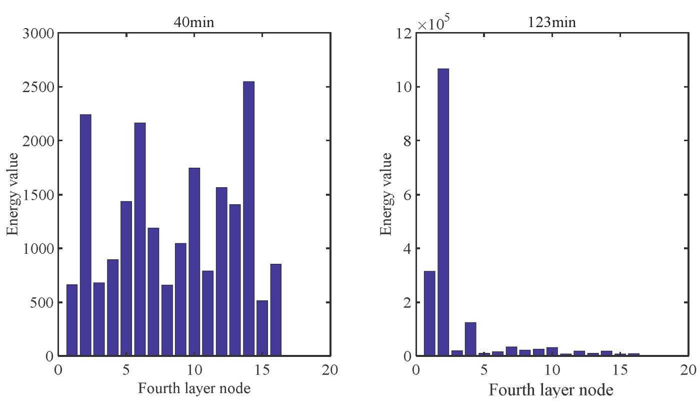

Figure 8. Fourth layer node energy

According to (14), the energy entropy of the wavelet packet decomposition of this signal can be obtained:

$$s=-\sum_{j=0}^{15} p_j \log p_j \tag{16}$$

According to literature [8], the more the energy approaches to average distribution, the higher the energy entropy value; the more the energy approaches to concentrated distribution, the lower the energy entropy value. When the rolling bearing is in normal and stable operation, its main frequency components include the rotating fundamental frequencies such as the inner ring, outer ring and holder, and their frequency multiplication. The energy is higher at the system inherent frequency. When one or more faults occur, the energy is concentrated nearby the fault frequency. Based on the characteristics of energy entropy, the operational reliability of bearings and systems can be judged through the change of energy entropy.

The energy entropy of the three stages is: $s_{40}=2.41$, $s_{100}=1.517$, $s_{123}=1.181$, and its overall trend is consistent with the above inference.

C. *Operational reliability calculation*

Reliability degree is an important indicator of reliability analysis. Operational reliability $R(t)$ can be defined as the normalized measurement indicator of equipment when completing specified function under specific conditions and during the service time, especially under the operational status of the components. $R(t)$ is the function of time t, the normalized measurement interval is in the range [0,1].

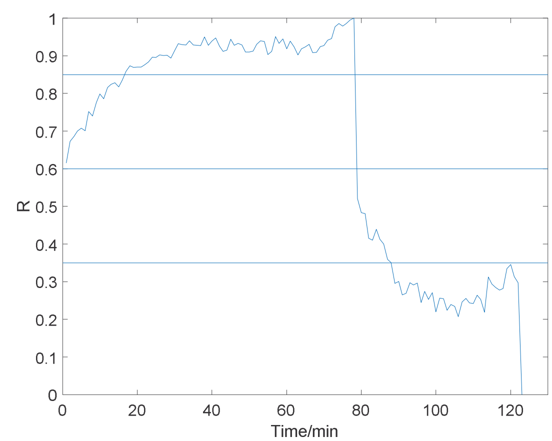

Figure 9. Operational reliability

In this paper, the state parameter method is adopted to calculate the running reliability of rolling bearings. As the equipment components are damaged during operation, the increase of certain characteristic value v of its signal means the decrease of equipment running reliability. The relevant calculation formula is as follows:

$$R(v)=\begin{cases}1 & v\le m_1\\ 1-\dfrac{v-m_1}{m_2-m_1} & m_1<v\le m_2\\ 0 & v>m_2\end{cases} \tag{17}$$

where m_2 is the upper limit of the bearing operating characteristics, m_1 is the lower limit, and the operational reliability $R(v)\in[0,1]$

Different wavelet packet energy entropies at the same scale can reflect different operational and fault states. In this paper, the wavelet packet energy entropy is selected as the characteristic parameter. The lower limit m_1=1.818 is taken as the final entropy value when the bearing is faulty, and the upper limit m_2=2.490 is the maximum entropy value during normal operation. The operational reliability $R(v)$ of the rolling bearing can be obtained by the formula (17).

Since there is no clear definition of the healthy running state of rolling bearing, in order to better describe the health state of the rolling bearing, the running state of rolling bearing can be divided into four classes combined with the classification criteria of the operating state of rotating equipment in literature [9], as shown in Table II.

TABLE II. HEALTH LEVEL OF BEARING OPERATING CONDITIONS

No.	State Class	Operating Situation	Risk Degree	$R(v)$ Interval
Ⅰ	Healthy	The monitoring data and indicators are within the allowable range and away from the warning value r_1. The bearing is in good operation.	Small possibility of bearing failure, no maintenance work is required, and the maintenance cycle can be extended.	[1-0.85]
Ⅱ	General	The monitoring data and indicators are within the allowable range, which do not reach or are approaching the warning value r_1. The bearing can still operate normally.	Possibility of bearing failure is within acceptable limits. The monitoring and maintenance can be performed according to maintenance schedules.	[0.85-0.60]
□	Obviously degraded	The monitoring data and indicators fluctuate grammatically nearby the warning value r_1, or are lower the warning value r_1.	Bearings are likely to fail, and failures may occur in the short term. It shall be shut down for maintenance in advance.	[0.60-0.35]
Ⅳ	Failure	The monitoring data reaches or exceeds the warning value r_2. There is a bad condition in the operation log and the bearing cannot operate normally.	The bearing is about to fail or has failed. It is recommended to shut down for reparation or replacement.	[0.35-0]

Combined with Figure 9, it can be seen that the reliability of the bearing is lower than the warning value r_1 at t=79min; the bearing reliability is lower than the warning value r_2 when t=88min.

The comparison of these two methods discovers that the state analysis based on wavelet packet energy entropy can early predict the possible failure time of the bearing. On the other hand, compared with the traditional thresholding, this method can evaluate the health state of bearing's operation. By classifying the processed normalized wavelet entropy into four categories, the health state of bearing can be refined, and corresponding maintenance instructions can be made for each different level. The state of bearing can indirectly reflect the operation state of the equipment and system in which bearing locates or the precision of machined parts.

IV. CONCLUSIONS AND OUTLOOK

In this paper, the algorithms of wavelet packet energy spectrum and energy entropy are introduced. Combined with the energy entropy, the reliability assessment of the bearing's operating state is performed by using the state parameter method. The root-mean-square is selected as the time-domain indicator to judge the bearing reliability via traditional thresholding, so that the methods are compared to analyze the cases. The experimental results show that the method based on wavelet packet information entropy and state parameters can efficiently evaluate and judge the running state of equipment. The classification of four states helps to specifically and reasonably judge the running state of bearing. In combination of various eigenvalues such as information entropy and time domain parameters, the failure of rolling bearing can be classified based on operational reliability, so that the failure information of bearing operation can be further improved.

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