Reliability Assessment Method By Integrating Accelerated Degradation and Life Data Based on Nonlinear Wiener Process

Xuesong Yang, Shunong Zhang, Chenqi Lv
School of Reliability and Systems Engineering
Beihang University
Beijing, China
yangxuesong@buaa.edu.cn & zsn@buaa.edu.cn & lvchenqibuaa@163.com

***Abstract*—A performance degradation process is often considered as a stochastic process that is affected by environment, stresses, internal materials, and many other random factors. The Wiener process represents a continuous incremental random process and has the favorable nature of computing and analysis, and many scholars use the linear Wiener processes to describe the performance degradation process. However, not all degradation processes are linear time functions, which means that the nonlinear degradation process needs to be emphasized. At the same time, for some products, accelerated performance degradation data and accelerated life data can be available, and both of them contain reliability information. Therefore, this paper proposes a reliability modeling method for nonlinear Wiener degradation failure process by integrating accelerated performance degradation data and life data. Firstly, the nonlinear process is transformed into a linear process by time scale function, and then two independent modeling processes based on accelerated degradation data and accelerated life data are given respectively. Based on the joint modeling of two independent processes, a one-dimensional nonlinear model is established. Then as a example, the Maximum Likelihood Estimation (MLE) method is used to obtain the point estimate of the model parameters. The simulation data of a kind of electrical connector are used to prepare for the estimation results. The results show that the proposed joint modeling method can accurately describe the nonlinear Wiener process of product performance degradation.**

Keywords-Wiener Process; Nonlinear; Accelerated Degradation; Accelerated Life; Joint Modeling;

I. INTRODUCTION

A reliability modeling and analysis based on accelerated data is a common practice for designing, testing, and evaluating high-reliability and long-life products. Most traditional statistical analysis methods are based on pseudo-life data or degraded lifetime data to fit the life distribution form of the product. However, the product performance degradation process is a stochastic process that is affected by environment, stresses, internal materials, and many other random factors[1]. Compared with traditional statistical analysis methods, modeling based on stochastic process seems to reveal the probabilistic nature of degradation. The Wiener process represents a continuous incremental random process and has the favorable nature of computing and analysis , and many scholars use the linear Wiener processes to describe the performance degradation process. Therefore, the accelerated test combined with the Wiener process in recent years has received considerable attention in the field of reliability. The Wiener process was first applied to engineering by Doksom and Hoyland[2]. Park and Padgett describe the carbon film resistors performance degradation by modeling the Wiener process based on the resistance[3]. Further, the optimization design of the accelerated stress acceptance test under the Wiener process is proposed in literature [4].

It is known from the mathematical properties of the Wiener process that it has good linear characteristics. A case of characterizing performance degradation through a linear Wiener process is discussed in literature [5]. However, not all degradation processes are linear time functions, which means that the nonlinear degradation process needs to be emphasized. Turning a nonlinear process into a linear process is a hot issue. Whitmore studied the conversion of time parameters and transformed nonlinear degradation into a linear drift Wiener process [6]. Wang and Nair conducted a similar study in which nonparametric transformations were considered [7].

At present, most reliability modeling studies based on Wiener processes are only one application of degradation data or lifetime data. However, for some products, accelerated performance degradation data and accelerated life data can be available, and both of them contain reliability information. Making full use of all reliability data is of great significance for modeling. Wilson et al. used censored data and degradation data to establish a likelihood function in the timing censor modeling [8]. More researches on joint modeling at home and abroad are discussed in literature [9]. The current research is still in the exploratory stage, and there is no effective comprehensive method for joint modeling of degradation data and lifetime data. The main difficulty is reflected in the process of establishing and solving the likelihood function of life data. Since the Wiener process can clearly represent the form of life distribution, joint modeling and estimation of degradation data and lifetime data can be achieved. In order to improve the accuracy of life prediction

978-1-7281-0860-5/19/$31.00 ©2019 IEEE

based on Wiener process, this paper establishes a mathematical model that can fully utilize accelerated degradation data and accelerated life data to avoid the problem of ignoring some of the available reliability data.

II. WIENER STOCHASTIC PROCESS

A. Definition and Transformation

Let $X(t)$ represents the performance degradation at time t, the stochastic process $X(t)$ is defined as Wiener process if it meets the following three properties:

1) $\Delta X(t)$ is the degradation increment between the time interval of $[t, t+\Delta t]$

$$\Delta X(t) = X(t+\Delta t) - X(t) \sim N\left(\mu\Delta t, \sigma^2 \Delta t\right) \tag{1}$$

2) During any disjoint time periods $[t_1, t_2]$ and $[t_3, t_4]$, the degradation increments $\left(X(t_2) - X(t_1)\right)$ is independent of $\left(X(t_4) - X(t_3)\right)$

3) $X(0) = 0$, and $X(t)$ is continued at the time $t = 0$. The Wiener process with parameters is denoted by:

$$X(t) = \mu t + \sigma W(t) \tag{2}$$

where μ is drift parameter, σ is diffusion coefficient, $W(t)$ is a standard Brownian motion.

From the viewpoint of failure physics, the degradation ΔX between the time interval of $[t, t+\Delta t]$ is the sum of a great deal tiny independent identically distributed random performance loss. If the amount of these tiny loss is proportional to Δt, then ΔX obeys normal distribution [10]. By the definition of the Wiener process, one-dimensional Wiener process can be applied to the product performance degradation process modeling.

B. Time-Scale Transformation for Nonlinear Wiener Process

From equation (1) and (2), it is not hard to find that:

$$E\left[X(t)\right] = \mu t \tag{3}$$

$$Var\left[X(t)\right] = \sigma^2 t \tag{4}$$

From equation (3) and (4), the expectancy and variance of wiener process is a linear function of time which means it is only fit for linear degradation process. In many cases, degradation process is nonlinear. There are three main types of nonlinear Wiener process for the degradation modeling, that is, log transformation [11], time-scale transformation [12], direct modeling [13]. The time-transformed Wiener process is most commonly used to model the accelerated data. In this paper, therefore, we use the time-transformed Wiener process to model the data. The transformation can be expressed as:

$$\tau = \Lambda(t) = t^{\beta} \tag{5}$$

where τ is the calculating time which is a positive non-decreasing function and t denotes the real time. What's more, the transformation of time scale can be denoted by any function, which fulfills the following conditions that continuous, strictly increase monotonically, and starts from original point. It is derived from equation (2) as:

$$X(t) = \mu\Lambda(t) + \sigma W\left(\Lambda(t)\right) \tag{6}$$

Or

$$Y(\tau) = \mu\tau + \sigma W(\tau) \tag{7}$$

equation (7) is regarded as linear one-dimensional Wiener process by time-scale transformation.

C. Life Distribution

If $Y(\tau)$ reaches a pre-specified failure threshold l for the first time, a product is announced to be failed. The product's failure time (or life), denoted T, is then defined as the first-passage time of $W(\tau)$, i.e.,

$$T = \inf\left\{\tau \mid Y(\tau) > l\right\} \tag{8}$$

It is well known that the Wiener process crossing a constant threshold l obeys an inverse Gaussian distribution whose mean value is $\frac{l}{\mu}$ and shape parameter is $\frac{l^2}{\sigma^2}$ [14]. For the degradation process as shown in equation (7), the cumulative distribution function (CDF) and density probability function (PDF) of the life can be obtained as follows:

$$F(\tau \mid \mu, \sigma, l) = \Phi\left(\frac{\mu\tau - l}{\sigma\sqrt{\tau}}\right) + \exp\left(\frac{\mu l}{2\sigma}\right) \times \Phi\left(\frac{-l-\mu\tau}{\sigma\sqrt{\tau}}\right) \tag{9}$$

$$f(\tau \mid \mu, \sigma, l) = \frac{l}{\sqrt{2\pi\sigma^2\tau^3}} \exp\left[-\frac{(l-\mu\tau)^2}{2\sigma^2\tau}\right] \tag{10}$$

Then, the functions for reliability is as follows:

$$R(\tau \mid \mu, \sigma, l) = \Phi\left(\frac{l-\mu\tau}{\sigma\sqrt{\tau}}\right) - \exp\left(\frac{\mu l}{2\sigma}\right) \times \Phi\left(\frac{-l-\mu\tau}{\sigma\sqrt{\tau}}\right) \tag{11}$$

where $\Phi(\bullet)$ is standard normal distribution function.

III. RELIABILITY MODELING

A. Accelerated Degradation Data Based Modeling

Assume that Y_{ijk} is the kth measurement value of the jth product under the stress, by this means, we can define that t_{ijk} is time point of the kth measurement of the jth product under the ith stress, τ_{ijk} is the conversion value corresponding to t_{ijk}. ΔY_{ijk} and $\Delta\tau_{ijk}$ are degradation increments and time increments respectively.

$$\Delta Y_{ijk} = Y_{ijk} - Y_{ij(k-1)} \tag{12}$$

$$\Delta\tau_{ijk}=\tau_{ijk}-\tau_{ij(k-1)} \tag{13}$$

(1) can change into

$$\Delta Y_{ijk}\sim N\left(\mu\Delta\tau_{ijk},\sigma^2\Delta\tau_{ijk}\right) \tag{14}$$

Likelihood function derived from performance degradation data can obtained as:

$$L\left(\mu_i,\sigma_i\right)=\prod_{i=1}^{N_1}\prod_{j=1}^{N_2}\prod_{k=1}^{N_3}\frac{1}{\sqrt{2\pi\sigma_i^2\Delta\tau_{ijk}}}\exp\left[-\frac{\left(\Delta Y_{ijk}-\mu_i\Delta\tau_{ijk}\right)^2}{2\sigma_i^2\Delta\tau_{ijk}}\right] \tag{15}$$

where N_1, N_2, N_3 are the number of stress, test samples, measuring respectively. μ_i and σ_i are drift parameter and diffusion coefficient under the ith stress.

B. Accelerated Life Data Based Modeling

As is mentioned before, the life distribution of the products whose degradation failure obeying Wiener process is the Inverse Gaussian Distribution. So the life data based method can be used for modeling [15].

Suppose that there are N_4 different stress levels which exist life data, S_i is the number of life data at every single level, including P_i failure time ($\gamma_{i1},\gamma_{i2},......,\gamma_{iS_i}$) and Q_i censored life time ($\eta_{i1},\eta_{i2},......,\eta_{iS_i}$). $\tau_{\gamma_{ij}}$ and $\tau_{\eta_{ij}}$ are the conversion value corresponding to γ_{ij} and η_{ij} respectively. Likelihood function derived from life data can obtained as:

$$\begin{aligned}L\left(\mu_i,\sigma_i\right)=\prod_{i=1}^{N_4}\Bigg\{&\prod_{j=1}^{P_i}\frac{l}{\sqrt{2\pi\sigma_i^2\tau_{\gamma_{ij}}^{3}}}\exp\left[-\frac{\left(l-\mu_i\tau_{\gamma_{ij}}\right)^2}{2\sigma_i^2\tau_{\gamma_{ij}}}\right]\\ \bullet&\prod_{j=1}^{Q_i}\left[\Phi\left(\frac{l-\mu_i\tau_{\eta_{ij}}}{\sigma_i\sqrt{\tau_{\eta_{ij}}}}\right)-\exp\left(\frac{2\mu_i l}{\sigma_i^2}\right)\Phi\left(\frac{-l-\mu_i\tau_{\eta_{ij}}}{\sigma_i\sqrt{\tau_{\eta_{ij}}}}\right)\right]\Bigg\}\end{aligned} \tag{16}$$

$$S_i=P_i+Q_i \tag{17}$$

Where μ_i and σ_i are drift parameter and diffusion coefficient under the ith stress. Particularly, if all life data is failure time data without any censored life data, the likelihood function is

$$L\left(\mu_i,\sigma_i\right)=\prod_{i=1}^{N}\left[\prod_{j=1}^{S_i}\frac{l}{\sqrt{2\pi\sigma_i^2\tau_{\gamma_{ij}}^{3}}}\exp\left[-\frac{\left(l-\mu_i\tau_{\gamma_{ij}}\right)^2}{2\sigma_i^2\tau_{\gamma_{ij}}}\right]\right] \tag{18}$$

C. Joint Modeling Based on Accelerated Degradation Data and Accelerated Life Data

Suppose that the two parameters of one-dimensional Wiener process are the function of stress. Based on physics of failure, we would build the models of the performance degradation parameters to depict the relation between the degradation rate or time to failure and stresses, including temperature, humidity, vibration, electricity, etc. The conventional acceleration model includes the Arrhenius model, Inverse power law model and Eyring model. And the Arrhenius model is commonly used for the thermal stress. For more details of the acceleration model, see [16]. Based on the previous paper [17], if using Arrhenius model, the general assumption is the diffusion coefficient is not changed with varying stress, see equation (19) and the relation between drift parameter and stress meets equation (20).

$$\sigma\left(T\right)=\sigma \tag{19}$$

$$\mu\left(T\right)=a\exp\left(-\frac{b}{T}\right) \tag{20}$$

Where T is the temperature of the test.

Suppose that there are various stress ($T_1,T_2......,T_N$), under every single level, the degradation data is obtained. It is not essential that life data will be acquired under every single level, which means it is vital to find life data under some stress T_i though. Combining with equation (15), (16), (17), (18), (19), (20), the joint modeling is denoted by:

$$\begin{aligned}L\left(\mu\left(T_i\right),\sigma\left(T_i\right)\right)=&\prod_{i=1}^{N_1}\prod_{j=1}^{N_2}\prod_{k=1}^{N_3}\frac{1}{\sqrt{2\pi\sigma^2\Delta\tau_{ijk}}}\\ &\bullet\exp\left[-\frac{\left(\Delta Y_{ijk}-a\exp\left(-\frac{b}{T_i}\right)\Delta\tau_{ijk}\right)^2}{2\sigma^2\Delta\tau_{ijk}}\right]\\ &\bullet\prod_{i=1}^{N_4}\Bigg\{\prod_{j=1}^{P_i}\frac{l}{\sqrt{2\pi\sigma^2\tau_{\gamma_{ij}}^{3}}}\exp\left[-\frac{\left(l-a\exp\left(-\frac{b}{T_i}\right)\tau_{\gamma_{ij}}\right)^2}{2\sigma^2\tau_{\gamma_{ij}}}\right]\\ &\bullet\prod_{j=1}^{S_i-P_i}\left[\Phi\left(\frac{l-a\exp\left(-\frac{b}{T_i}\right)\tau_{\eta_{ij}}}{\sigma\sqrt{\tau_{\eta_{ij}}}}\right)\right.\\ &\left.-\exp\left(\frac{2a\exp\left(-\frac{b}{T_i}\right)l}{\sigma^2}\right)\Phi\left(\frac{-l-a\exp\left(-\frac{b}{T_i}\right)\tau_{\eta_{ij}}}{\sigma\sqrt{\tau_{\eta_{ij}}}}\right)\right]\Bigg\}\end{aligned} \tag{21}$$

Theoretically, according to conventional solving process which is partial derivative, we can figure out unknown parameters a, b, σ, θ by substituting ΔY_{ijk}, $\Delta\tau_{ijk}$, $\tau_{\gamma_{ij}}$, $\tau_{\eta_{ij}}$, T_i into equation (20). It is not easy to achieve direct solution owing to these sophisticated equations, though. Considering a special case, that is, the life data are all failure data without any censored life data. Under this circumstance, the joint modeling is denoted by

$$L\left(\mu(T_i),\sigma(T_i)\right)=\prod_{i=1}^{N_1}\prod_{j=1}^{N_2}\prod_{k=1}^{N_3}\frac{1}{\sqrt{2\pi\sigma^2\Delta\tau_{ijk}}}$$
$$\bullet\exp\left[-\frac{\left(\Delta Y_{ijk}-a\exp\left(-\frac{b}{T_i}\right)\Delta\tau_{ijk}\right)^2}{2\sigma^2\Delta\tau_{ijk}}\right] \tag{22}$$
$$\bullet\prod_{i=1}^{N_4}\left\{\prod_{j=1}^{S_i}\frac{l}{\sqrt{2\pi\sigma^2\tau_{\gamma_{ij}}{}^3}}\exp\left[-\frac{\left(l-a\exp\left(-\frac{b}{T_i}\right)\tau_{\gamma_{ij}}\right)^2}{2\sigma^2\tau_{\gamma_{ij}}}\right]\right.$$

Taking its log, the function can be changed as:

$$L\left(\mu(T_i),\sigma(T_i)\right)=\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\sum_{k=1}^{N_3}\left[-\frac{1}{2}\ln(2\pi\sigma^2\Delta\tau_{ijk})-\frac{\left(\Delta Y_{ijk}-a\exp\left(-\frac{b}{T_i}\right)\Delta\tau_{ijk}\right)^2}{2\sigma^2\Delta\tau_{ijk}}\right]$$
$$+\sum_{i}^{N_4}\sum_{j}^{S_i}\left[\ln(l)-\frac{1}{2}\ln(2\pi\sigma^2\tau_{\gamma_{ij}}{}^3)-\frac{\left(l-a\exp\left(-\frac{b}{T_i}\right)\tau_{\gamma_{ij}}\right)^2}{2\sigma^2\tau_{\gamma_{ij}}}\right] \tag{23}$$

Now, maximum likelihood estimation (MLE) can be obtained directly by the partial derivatives of the parameters in the likelihood function equation (23):

$$\frac{\partial L}{\partial a}=\frac{\partial L}{\partial b}=\frac{\partial L}{\partial\sigma^2}=0 \tag{24}$$

$$\frac{\partial L}{\partial\sigma^2}=\frac{1}{2\sigma^2}\left\{\frac{1}{\sigma^2}\left[\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\sum_{k=1}^{N_3}\left(\frac{\left(\Delta Y_{ijk}\right)^2}{\Delta\tau_{ijk}}\right)+a^2\sum_{i=1}^{N_1}\left[\exp\left(-\frac{b}{T_i}\right)\right]^2\tau_i-\right.\right.$$
$$\left.2a\sum_{i=1}^{N_1}Y_i\exp\left(-\frac{b}{T_i}\right)-\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\sum_{k=1}^{N_3}1\right]+ \tag{25}$$
$$\frac{1}{\sigma^2}\left[\sum_{i=1}^{N_4}\sum_{j=1}^{S_i}\frac{l^2}{\tau_{\gamma_{ij}}}+a^2\sum_{i=1}^{N_1}\sum_{j=1}^{S_i}\left[\exp\left(-\frac{b}{T_i}\right)\right]^2\tau_{\gamma_{ij}}-\right.$$
$$\left.\left.2al\sum_{i=1}^{N_4}\sum_{j=1}^{S_i}\exp\left(-\frac{b}{T_i}\right)-\sum_{i=1}^{N_4}\sum_{j=1}^{S_i}1\right]\right\}$$

$$\frac{\partial L}{\partial a}=\frac{1}{\sigma^2}\left\{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\sum_{k=1}^{N_3}\left[-\Delta Y_{ijk}\exp\left(-\frac{b}{T_i}\right)+a\exp\left(-\frac{2b}{T_i}\right)\Delta\tau_{ijk}\right]+\right.$$
$$\left.\sum_{i=1}^{N_4}\sum_{j=1}^{S_i}\left[-l\exp\left(-\frac{b}{T_i}\right)+a\exp\left(-\frac{2b}{T_i}\right)\tau_{\gamma jk}\right]\right\} \tag{26}$$

$$\frac{\partial L}{\partial b}=\frac{1}{\sigma^2}\left\{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\sum_{k=1}^{N_3}\left[\frac{a\Delta Y_{ijk}\exp\left(-\frac{b}{T_i}\right)-a^2\exp\left(-\frac{2b}{T_i}\right)\Delta\tau_{ijk}}{T_i\Delta\tau_{ijk}}\right]+\right.$$
$$\left.\sum_{i=1}^{N_4}\sum_{j=1}^{S_i}\left[\frac{2al\exp\left(-\frac{b}{T_i}\right)-a^2\exp\left(-\frac{2b}{T_i}\right)\tau_{\gamma jk}}{T_i}\right]\right\} \tag{27}$$

Note that Y_i is the total degradation data for all products at every single stress $(i=1,2,3....N_1)$, not degradation increment data ΔY_{ijk}. τ_i is the total measurement time data that has been converted for all products at every single stress $(i=1,2,3....N_1)$, not time increment data $\Delta\tau_{ijk}$.

IV. A CASE STUDY

A. Data Simulation

As a basic electromechanical component, the electrical connector is equipped with wide application, transmitting signal and energy by mechanical means. The failure of any one of the electrical connectors may cause the whole system to be abnormal. Taking temperature as accelerating factor, the research on ALT and ADT of electric connector has been conducted for years. It is suggested that increased contact resistance is a common failure mode for electrical connectors. Reference [18] shows that wiener process can be applied to assessment for performance degradation of electrical connector. In this section, we take the accelerated degradation simulation data of electrical connector as an example to implement the proposed models to illustrate the correctness of joint modeling. The performance characteristic is contact resistance.

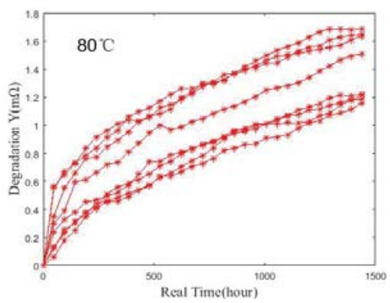
- The given estimated parameters are $\hat{a}=370.124$, $\hat{b}=2026.633$, $\hat{\sigma}=0.14$ respectively.
- 24 samples are selected and 3 stress levels are determined. 8 samples are tested at per stress level which is $T_1=80^{\circ}\mathrm{C}$, $T_2=100^{\circ}\mathrm{C}$, $T_3=125^{\circ}\mathrm{C}$ respectively.
- At the T_1 stress level, we carry out 30 data sampling and simulation and the time interval is 48 hours *i.e.*, $\Delta t=48$ h. At the T_2 stress level, we carry out 25 data sampling and simulation and the time interval is

36 hours. At the T_3 stress level, we carry out 20 data sampling and simulation and the time interval is 24 hours.

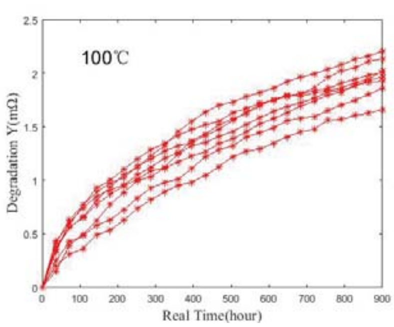
- According to the technical documentation of *a* certain type electrical connector, the threshold of contact resistance failure is $70\,\mathrm{m}\Omega$, *i.e.,* $l = 70\mathrm{m}\Omega$.
- According to [18], the performance degradation of electrical connector and the 1/2th power of time meet certain function relation, that is, time-scale transformation for nonlinear wiener process can be described as:

$$\tau = \Lambda(t) = \sqrt{t} \tag{26}$$

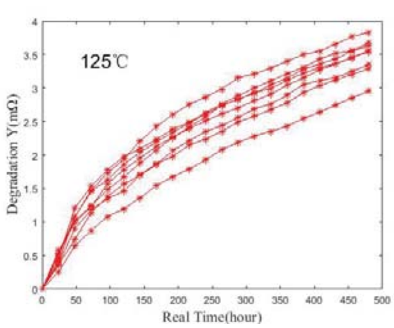
- Suppose that normal operating temperature is $T_0 = 40\,^{\circ}\mathrm{C}$. Owing to the randomness of sampling, after various attempts we decide to, obtain 10 accelerated life data at $80\,^{\circ}\mathrm{C}$ and $100\,^{\circ}\mathrm{C}$ respectively via simulation.



(a) temperature stress level is 80℃



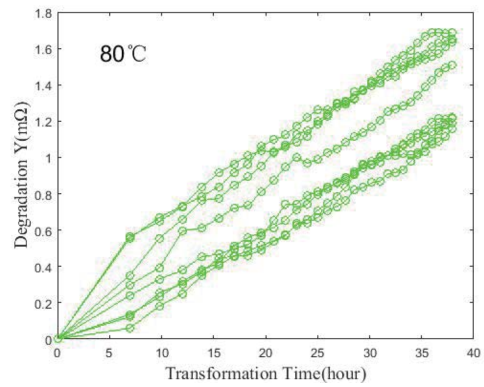
(b) temperature stress level is 100℃



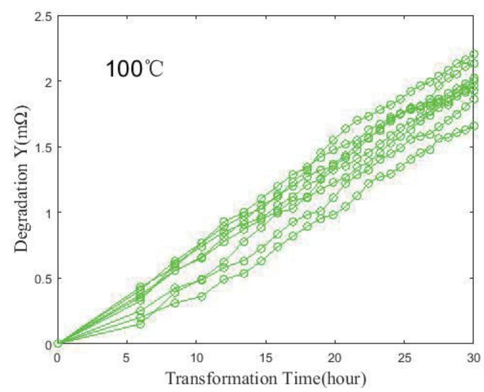
(c) temperature stress level is 125℃

Figure 1. Degradation data at 3 temperature stress levels

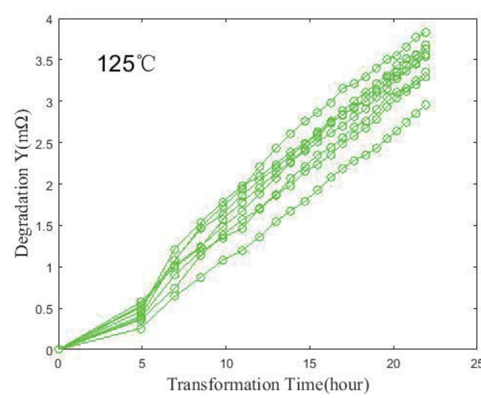
Thus, the degradation data of samples at 3 stress levels are shown in Fig. 1 and the degradation data after time scale transformations are shown in Fig. 2. The life data are shown in table I.



(a) temperature stress level is 80℃



(b) temperature stress level is 100℃



(c) temperature stress level is 125℃

Figure 2. Degradation data after time scale transformations

TABLE I. THE ACCELERATED LIFE DATA OF ELECTRICAL CONNECTOR

	80℃	**100℃**
1	7853.4	3846.6

2	6927.7	3702.7
3	7008.3	3704.9
4	6252.0	3677.1
5	6827.1	3795.2
6	6421.1	3825.4
7	6908.9	3688.0
8	6293.7	3688.9
9	5867.8	3797.4
10	6780.5	3648.7

B. Reliability Assessment and Life Prediction

Taking tim scale transformation of accelerated degradation and life data into equation (23), (24), we can obtain the maximum likelihood estimation (MLE).

$$\left(\hat{a},\hat{b},\hat{\sigma}\right)=\left(370.458,2031.818,0.1375\right)$$

When normal operating temperature is $T_0 = 40\,^{\circ}\mathrm{C}$, according to equation (19) and (20), $\hat{\mu}_0 = 0.5635$, $\hat{\sigma}_0 = 0.1375$, therefore probability density function (PDF) of the life base on equation (10) can be derived:

$$f\left(t \mid \mu,\sigma,l\right)=\frac{290.1}{t^{3/4}}\exp\left[-\frac{\left(100-0.5635\times t^{1/2}\right)^2}{3.781\times 10^{-2}\times t^{1/2}}\right] \quad (27)$$

And then from equation (11), reliability function is derived:

$$R\left(t \mid \mu,\sigma,l\right)=\Phi\left(\frac{100-0.5635\times t^{1/2}}{0.1375\times t^{1/4}}\right) -2.816\times 10^{4}\times\Phi\left(\frac{-100-0.5635\times t^{1/2}}{0.1375\times t^{1/4}}\right) \quad (28)$$

The property of reverse Gaussian distribution indicates that the life expectancy is $\xi = l/\mu$. Since time scale transformation τ, the point estimate of life expectancy is as:

$$\hat{\xi}_0=\left(\frac{l}{\mu_0}\right)^2=31493 \text{ h} \quad (29)$$

In order to show the accuracy of solution further, point estimates and given values are compared in table II.

TABLE II. THE COMPARISON BETWEEN POINT ESTIMATE AND TARGET VALUE

	Point Estimate	**Target Value**	**Deviation**
a	370.458	370.124	0.090%
b	2031.818	2026.633	0.256%
μ	0.5635	0.5724	1.555%
σ	0.1375	0.140	1.786%
ξ	31493	30522	3.181%

As seen from Table II, the parameter's maximum likelihood point estimate has a small error, which can be within 2%, and the MTTF error is only 3%.

It can be obtained that the joint modeling scheme based on the Wiener process can accurately describe the life distribution of the product.As The probability density function (PDF) and reliability function(R) are shown as Fig. 3 and Fig. 4

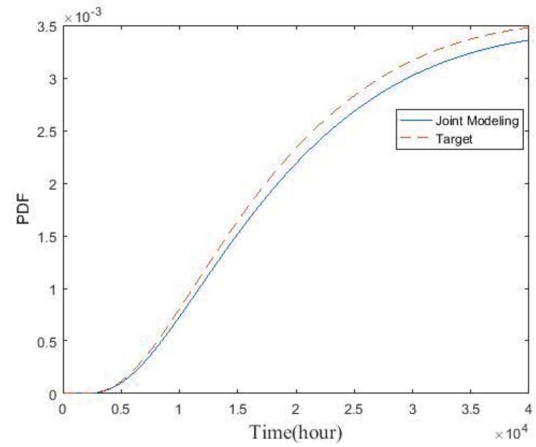


Figure 3. Probability density curve at T_0

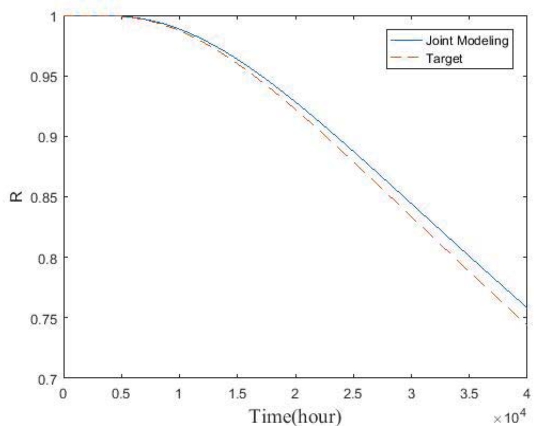


Figure 4. Reliability curve at T_0

Similarly, the simulation test scheme is used to solve the maximum likelihood function based on single degradation data or single lifetime data, and obtain the point estimation of the product life distribution parameter. In view of the space limitations, the solution process will not be described, the results are shown in the table below.

TABLE III. THE COMPARISON BETWEEN POINT ESTIMATE AND TARGET VALUE BASED ON DEGRADATION DATA

	Point Estimate	**Target Value**	**Deviation**
a	333.810	370.124	9.811%
b	1979.865	2026.633	2.308%
μ	0.5994	0.5724	4.717%
σ	0.1563	0.140	11.643%

ξ	27835	30522	8.803%

TABLE IV. THE COMPARISON BETWEEN POINT ESTIMATE AND TARGET VALUE BASED ON LIFE DATA

	Point Estimate	Target Value	Deviation
a	379.380	370.124	2.501%
b	2053.245	2026.633	1.313%
μ	0.5389	0.5724	5.853%
σ	0.1263	0.140	9.786%
ξ	34432	30522	12.810%

Therefore, the same set of product parameters are assumed to be simulated for the scenario, and the required degradation amount data and life data are obtained. The result data is substituted into three different models, and the maximum likelihood function is solved in turn, After analysis, three sets of point estimates for product life distribution are obtained.

The obtained parameter estimation value is compared with the actual assumed product parameter, so that the estimation result based on the joint modeling of the degradation data and the life data can be obtained closest to the true value. In summary, the effect of joint modeling is better than the effect based on single data modeling.

V. CONCLUSION

This paper first analyzes the applicability of the Wiener process in describing the process of product performance degradation, and transforms the nonlinear degradation process into a linear process through time-scale function transformation. For the case where both accelerated degradation data and lifetime data can be available, the effective modeling of all reliability information can more accurately predict lifetime of the product. Finally, the simulation results of the electrical connector are used to prepare for the numerical analysis example. The results show that the proposed joint modeling method which contains degradation data and lifetime data can accurately describe the nonlinear Wiener process of product performance degradation.

VI. FUTURE WORK

In the future work, the real test verification should be conducted. A point estimate of the maximum likelihood function is solved based on the measured test data to determine the life distribution parameter of the product. At the same time, the experimental measured data was fitted to the product life distribution curve as a reference for the standard results.

Therefore, the fit of real-life product life cycle data based on degraded data modeling, life-based data modeling, and joint modeling based on both can be compared, which can further highlight joint degradation data and life data modeling`s accuracy.

REFERENCE

[1] Z. Wang, W. Ren, and G. Zhai, "Review of Accelerated Degradation Testing and Accelerated Life Testing". Low Voltage Apparatus.London ,vol. 17, pp. 506-517, April 2010.

[2] K. A. Doksum and A. Hóyland, "Models for Variable-Stress Accelerated Life Testing Experiments Based on Wener Processes and the Inverse Gaussian Distribution". Technometrics, 1992, 37(1):170–172.

[3] C. Park, and W. J. Padgett, "Accelerated degradation models for failure based on geometric Brownian motion and gamma processes." Lifetime Data Analysis, vol. 25, no. 1, pp. 511-527, Jun. 2005.

[4] C. C. Tsai, C. T. Lin, and N. Balakrishnan. "Optimal Design for Accelerated-Stress Acceptance Test Based on Wiener Process." IEEE Transactions on Reliabilit ,vol. 64, no. 2, pp. 603-612, Jun. 2015.

[5] J. Cai and Y. M. Wang. "Reliability Analysis of Production Based on Wiener Process Degradation". Journal of Hunan Institute of Science & Technology, vol. 25, no. 2, pp. 603-612, Jun. 2012.

[6] G. A. Whitmore, "Estimating degradation by a Wiener diffusion process subject to measurement error.". Lifetime Data Analysis, vol. 1, no. 3, pp. 307-319, July. 1995.

[7] X. Wang and V. A. Nair class of degradation model based on nonhomogeneous Gaussian process. University of Michigan, 2005.

[8] A. G. Wilson, Reese C S. "Advances in Data Combination, Analysis and Collection for System Reliability Assessment." Statistical Science, vol. 21, no. 4, pp. 514-531, April. 2007.

[9] V Couallier, "Some recent results on joint degradation and failure time modeling. Probability," Statistics and Modelling in Public Health. vol. 17, no. 2, pp. 73-89, April. 2006.

[10] B. Dong, Wang S, Meng Y and G.Zhai, "Reliability Assessment Method for Aerospace Relay Based on Wiener Process." Electrical & Energy Management Technology, vol. 11, no. 3, pp. 77-87, Sept. 2015.2015, 11: 003.

[11] K. A. Kaiser and N. Z. Gebraeel, "Predictive maintenance management using sensor-based degradation models." IEEE Transactions on Systems, vol. 39, no. 4, pp. 840849, April. 2009.

[12] Z. S. Ye, Y. Wang, K. L. Tsui and M. Pecht, "Degradation data analysis using Wiener processes with measurement errors." IEEE Transactions on Reliability, vol. 62, no. 4, pp. 772-780, April. 2013.

[13] X. S. Si, W. Wang, C. H. Hu and D. H. Zhou, "Remaining useful life estimation based on a nonlinear diffusion degradation process." IEEE Transactions on Reliability, vol. 61, no. 1, pp. 50-67 May. 2012.

[14] R. Li, L. X. Wang, G. Liu and Z. J. Zhou, "Nonlinear characteristic analysis and storage life forecast for accelerometer based on accelerate degradation model."Zhongguo Guanxing Jishu Xuebao/journal of Chinese Inertial Technology, vol. 22, no. 1, pp. 125-130, April. 2014.

[15] B. H. Peng and J. L. Zhou, "Reliability assessment for product with Wiener process degradation based on marker data. "IEEE International Conference on Industrial Engineering and Engineering Management. IEEE, 2010:2394-2397.

[16] G. V. Mark, "System Reliability Theory: Models and Statistical Method. "Statistical methods in reliability theory and practice. E. Horwood, 1996,pp.79-80.

[17] H. W. WANG, T. X. XU and Y. Z. HE, "Application of Wiener Processes on Modeling Nonlinear Degradation. "Science & Technology Review, in press.

[18] R. J. Lin, W. H. Chen and J. Liu, "Research on feasibility of accelerated degradation test for aerospace electrical connector."Journal of Engineering Design, vol. 17, no. 4, pp.317-320, April. 2010.