

# Singular Spectrum Decomposition Based Nonlinear Energy Operator for Rolling Bearing Faulty Diagnosis

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**Abstract**—Fault signals of rolling bearings are non-stationary and the fault characteristics are often submerged in the background noise. Unfortunately, the fault characteristic frequency detection from composite fault signals of rolling bearings is difficult. To address this problem, during this paper, an improved method is proposed, which combines the singular spectrum decomposition (SSD) with the nonlinear energy operator (NEO). SSD can separate a hybrid signal into several sub-signals in accordance with their different frequencies adaptively and greatly alleviate mode mixing, etc. The decomposed components name singular spectrum components (SSCs) whose optimal component comprises fault features. Meanwhile, NEO is an excellent tool to demodulate modulating signals with enhancing amplitudes advantage. Subsequently, the selected SSC is demodulated by NEO. Toward upon testifying the effectiveness of the enhanced method, comparison investigation with EMD is also given in the paper. The comparison result validly testifies the improved method is more effective to detect fault characteristic frequency.

**Keywords**—rolling bearings fault; singular spectrum decomposition; nonlinear energy operator; fault detection

## I. INTRODUCTION

Bearings are widely applied a variety of engineering situations [1]. Four main parts, which are inner ring, outer ring, ball and cage, besides little parts, they consist of rolling bearings, when the essential parts of bearings are failed, these will bring about low efficiency of mechanical equipment, and moreover, any loss of property or injury caused during the period of mechanical operation. Therefore, it is necessary to detect faults position and category in rolling bearing in time. Vibration signals analysis method is regarded as a key tool for detecting faults by extracting the fault period and frequency. All kinds of techniques have been proposed in recent years. One of the most popular methods is Fast Fourier Transform (FFT). However, the signal is analyzed, it is nonlinear and non-stationary, which is generated from working conditions and so on. Result is represented in frequency domain; ‘aliasing’ phenomenon cannot detect features directly. Based on that, there is a need to come up with an effective method of processing.

In recent years, time-frequency analysis (TFA) is a commonly used analysis method. It has been widely accepted because it can display both time and frequency information [2]. The typical approaches are proposed [3-5]. Nevertheless, these methods have intrinsic shortcomings for non-stationary signal processing and always sink into low time or frequency resolution. Thus, inspired by the above methods, vibration signals are decomposed by other methods adaptively. For instance, Adaptive and improved ideas are proposed [6-8]. And other researchers proposed local mean decomposition (LMD) [9] and variational mode decomposition (VMD) [10]. Vibration signals are divided into several intrinsic mode functions (IMFs) by empirical mode decomposition (EMD). Nevertheless, both decomposed components will be at mode mixing and end effects. Wu and Huang proposed ensemble empirical mode decomposition (EEMD) that is based on EMD method, the uniform distribution of Gaussian white noise is used for mix white noise into the input signal so that mode mixing phenomenon could be decreased. In order to eliminate redundant noise components, in 2010, N.E. Huang proposed the complementary ensemble empirical mode decomposition (CEEMD) method, by the means of adding pairs of complementary white noise to the original signal, which validly removes residual noise. However, it is difficult to take the ensemble mean. LMD also faced mode mixing issue. VMD is better than EMD during the section of mode mixing restraint, signal separation and noise robustness.

Singular spectrum decomposition (SSD) was presented by Bonizzi [11], in 2014. The vibration signals are divided into a series of singular spectrum components (SSCs) by SSD. The SSCs are arranged from high value frequency to low value frequency. It is similar to EMD, The principle of SSD, on the basis of on the collection of energy connected with a variety of intrinsic time scales and effectively alleviates mode mixing, and furthermore, it could achieve accurate intermittent parts separation at the cross-over points. However, under the strong noise, the fault information is overwhelmed and cannot easily detect from frequency domain.

Nonlinear Energy Operator is proved a great tool since it is easily to differentiate between the conceivable impulse and the interrupted noise according to own energies are differential.

The advantage of this method is that the output signal is proportional to the product of the amplitude and frequency of the input signal.

This paper is arranged as follows. In section II, we introduce detailed of SSD and NEO. In section III, the proposed method is applied in experiment rolling bearing data and compared with EMD method. In section IV, the comparison result conclusion is given.

## II. METHODOLOGY

### A. Singular spectrum decomposition

SSD is a signal processing technology, which can divide hybrid signal into meaningful components through the extraction of the components one by one. The principle of SSD is as follows:

Step one: collected a time series  $y(m)$ , and length of  $M$ . Then given an embedding dimension  $N$ , the matrix  $(N \times M)$  is presented as  $Y = [y_1^T, y_2^T, \dots, y_N^T]^T$ , for example, the signal can be described  $y(m) = \{c_1, c_2, c_3, c_4, c_5\}$ , set an embedding dimension  $N = 3$ , the relevant trajectory matrix can be

$$Y_{or} = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 & c_5 \\ c_2 & c_3 & c_4 & c_5 & c_1 \\ c_3 & c_4 & c_5 & c_1 & c_2 \end{pmatrix}$$

The left-hand sub-matrix corresponds to the trajectory matrix  $Y$  was applied in singular spectrum analysis (SSA) [12]. It is a powerful tool to strengthen the oscillation of original signal and make decrease of the residual component energy. The new trajectory matrix is as

$$Y = \begin{pmatrix} & & c_1 & & \\ & c_1 & & & \\ c_1 & c_2 & c_3 & c_4 & c_5 \\ c_2 & c_3 & c_4 & c_5 & * \\ c_3 & c_4 & c_5 & * & * \end{pmatrix}$$

Step two: the embedding dimension  $N$  is adaptively at iteration  $p$ , the power spectral density (PSD) is computed, where corresponds to the residual term  $x_p$  at iteration  $p$ :

$$x_p(m) = y(m) - \sum_{k=1}^{p-1} x_k(m), (x_0(m) = y(m))$$

The next step is that the most dominant peak of PSD corresponds to the frequency  $f_{\max}$  is estimated. Given two circumstances: one is the normalized frequency  $f_{\max}/f_s$  (the

$f_s$  presents sampling frequency) which is less than the set threshold (is given 0.01 in the experiment) at iteration 1, given

$N$  is  $M/3$ . The other is for iterations  $p > 1$ , the embedding

$$\text{dimension is } N = 1.2 \frac{f_s}{f_{\max}}.$$

Step three: the  $p$ -th part series is reconstructed at first iteration, if only a major trend is caught, the first left and right eigenvectors are applied to get  $e^{(1)}(m)$ , thus,  $y_1 = \sigma_1 u_1 x_1^T$  and  $e^{(1)}(m)$  is obtained from diagonal averaging of  $Y_1$ . If iterations  $p > 1$ ,  $e^{(1)}(m)$  is focused on the frequency band  $[f_{\max} - df, f_{\max} + df]$  ( $df$  means the half band width of the dominant peak in the PSD of the residual term). The subset  $R_p(R_p = \{r_1, \dots, r_q\})$  is built by one eigentriple which contains the most energy of the dominant peak. Then, by the diagonal averaging of the matrix  $Y_{R_p} = Y_{r_1} + \dots + Y_{r_q}$ , the corresponding component series is reconstructed possibly.

Step four: the stopping criterion is established and a new residual is calculated when  $e^{(p)}(m)$  is estimated:

$$x^{(p+1)}(m) = x^{(p)}(m) - e^{(p)}(m)$$

The above letters can be described as the input to the next iteration, a new component respectively. The normalized mean squared error between input signal and the residual:

$$NMSE^{(p)} = \frac{\sum_{r=1}^M (x^{(p+1)}(r))^2}{\sum_{r=1}^M (y(r))^2}$$

Given the threshold is 0.01, once  $NMSE$  is more than it, the process is stopped. The final result is devoted:

$$y(m) = \sum_{k=1}^n e^{(k)}(m) + x^{(n+1)}(m)$$

The above formula which is the  $n$  presents the number of component series.

### B. Nonlinear operator energy

NEO is proposed by Kaiser for the nonlinear speech modeling [13]. The prominent of the technique which is the output signal is in direct proportion to the product of the input signal in amplitude and frequency [13-14]. For discrete series the principle of NEO is as follow:

$$\psi_N \left[ \tilde{y}(m) \right] = \tilde{y}^2(m) - \tilde{y}(m+N) \tilde{y}(m-N)$$

where  $\tilde{y}(m)$  is the original signal,  $N$  is an integer referred to as the lag parameter.

## III. VALIDATION

With the purpose of illustrating the proposed algorithm in this paper, a flow chart of algorithm is as follow Figure 1:

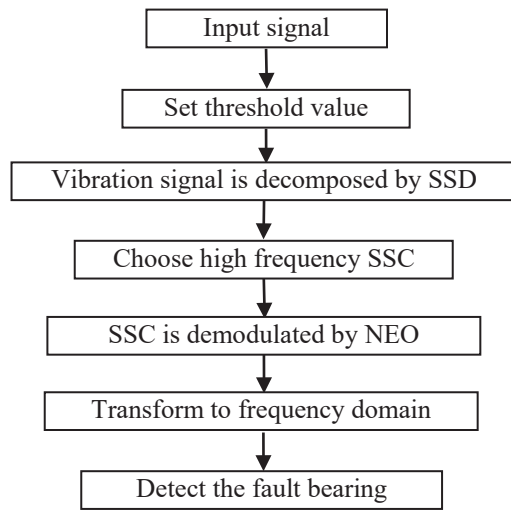


Figure 1. Flow chart of the proposed algorithm

In this paper, the experiment data were acquired by Machinery Fault Simulator (MFS) test-rig of SpectraQuest Co. Type ER-12K inner fault rolling bearing is applied to prove to the proposed algorithm. The parameters about ball diameter, pitch diameter, number of elements and the contact angle are  $d = 7.9375$  mm,  $D = 33.4772$  mm,  $z = 8$  and  $\theta = 0^\circ$  respectively. And sampling frequency  $f_s$  is 25600 Hz. the rotating speed is 1792.2 r/min. Thus, according to the parameters, the theoretical fault frequency is 147.8565 Hz. The experimental configuration is shown in Figure 2. Figure 3 is represented the experiment signal. The signal is smeared by heavy noise, cannot detect fault period directly. The threshold is set 0.01.

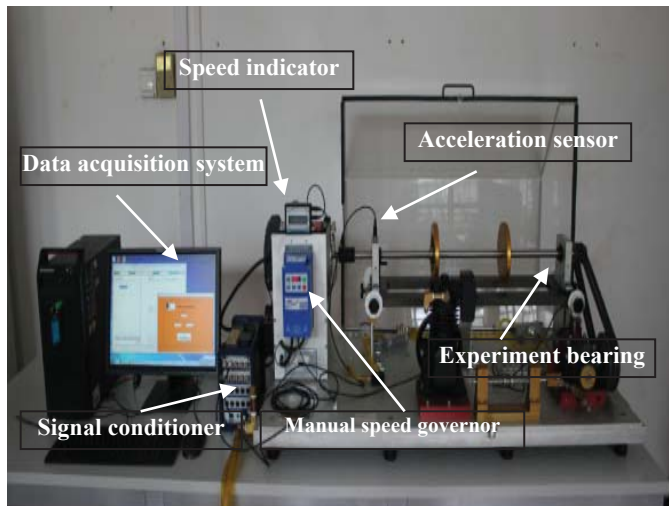


Figure 2. The configuration of machinery fault simulator (MFS)

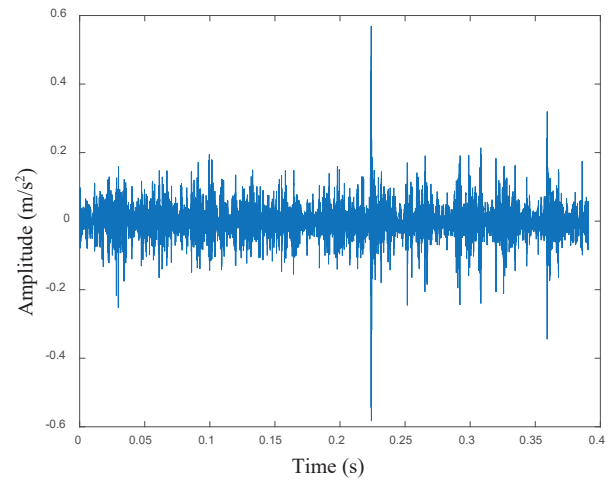


Figure 3. The vibration signal in this paper

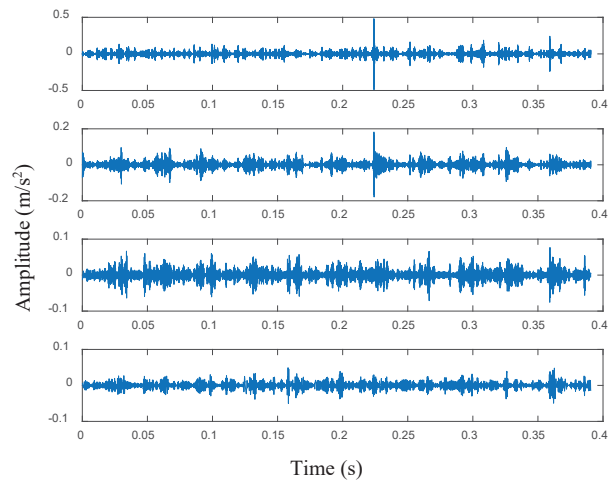


Figure 4. The signal is decomposed by EMD

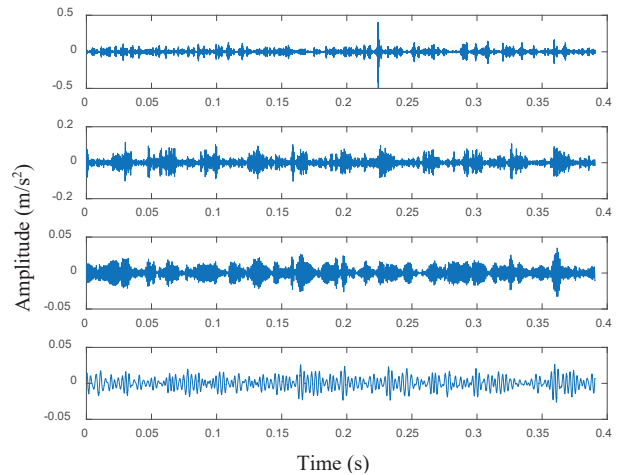


Figure 5. The signal is decomposed by SSD

It is essential to verify the proposed algorithm. Compare SSD with EMD methods in fault data. The EMD result is displayed in Figure 4. Figure 5 is SSD result. The decomposition configuration shows the fault impulse is not detect obviously. For the decomposition result of SSD and

EMD, the SSC and IMF are chosen which are based on the principle of kurtosis respectively. Nevertheless, the component that the maximum kurtosis component always polluted by strongly noise. Therefore, we choose the SSC or IMF, which is belonged to No.2 kurtosis value. Both SSC2 and IMF2 are demodulated by NEO. The envelope spectrum result of SSC2 and IMF2 are shown in Figure 6, and Figure 7. It is obviously that the fault frequency and its harmonic components can be extracted 148.5 Hz, 297 Hz, and 445.4 Hz in Figure 7. However, IMF2 cannot detect fault frequency and its harmonic components. The Hilbert envelope spectrum result of SS2 is shown in Figure 8. The amplitude is weakened, which result in fault feature frequency couldn't detect.

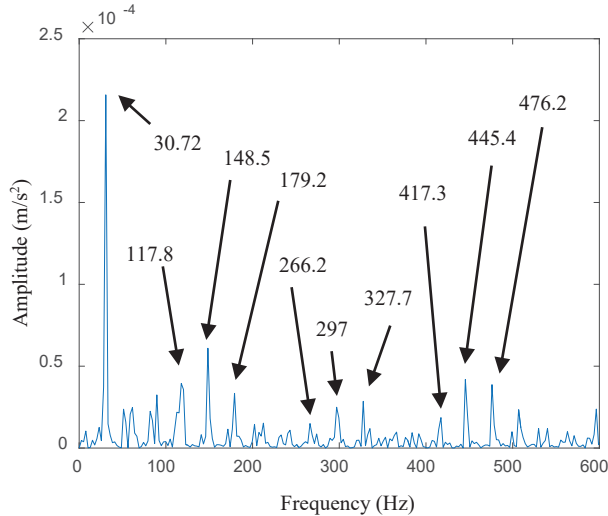


Figure 6. The spectrum of SSC2

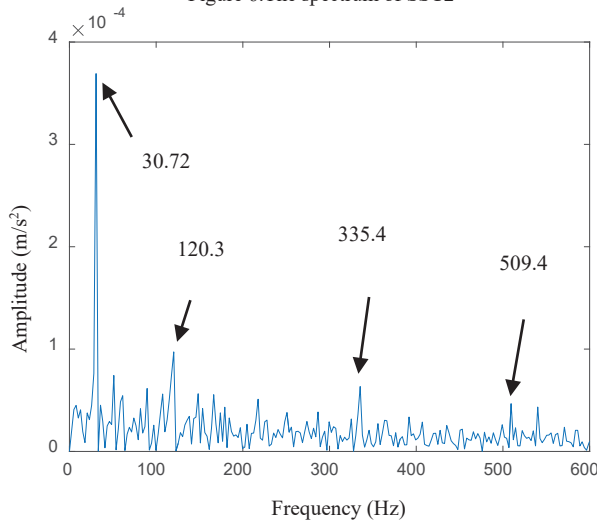


Figure 7. The spectrum of IMF2

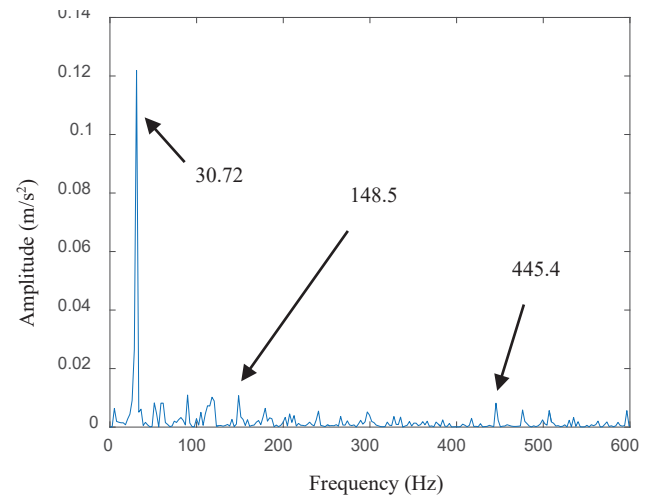


Figure 8. The spectrum of SSC2 which is demodulated by Hilbert

#### IV. CONCLUSION

At the end of this paper, an improved method is to removal both irrelevant and relevant interference of the fault frequencies is put forward. Based on the SSD and NED algorithms, it is clear that the enhanced method effectively detect the rolling bearing fault frequency. By comparing SSD with EMD, the result represents that the former which is a better choice for component decomposition. Furthermore, NEO method is better than Hilbert envelope to enhance the prominent of amplitude.

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