A Virtual Model To Predict The Influence Of Indexing Errors On The Transmission Error Of Spur Gears

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***Abstract*—The transmission error of a gear pair is one of the main excitations that cause the vibration and noise problems of gearboxes. The root causes of the transmission error include the gear manufacturing errors, the installing errors and the elastic deformation of meshing gear teeth. Although the transmission error have a significant influence on the dynamics of gear pairs, most of the previous studies just employ simplified mathematical functions to qualitatively represent its periodicity. Only recently, the experimental study was conducted to investigate the detailed properties of the transmission errors in quasi-static conditions, which requires strong expertise and costly precision equipment. Therefore, to give a quick evaluation of the properties of transmission error, this paper proposes a virtual model to numerically predict the transmission error of a spur gear pair in the static condition. The model is capable to simulate the transmission error that is caused by typical gear manufacturing and installing errors like the indexing errors and run-out errors. The simulated transmission errors agree with the experimental phenomenon observed in previous published works. The proposed virtual model has the potential to assist in the in-depth analysis and modeling of dynamic behavior of gear transmissions.**

Keywords-virtual model; indexing error; transmission error; run out error;

I. INTRODUCTION

The gearboxes, one of the most important components in drive-trains, are often prone to the vibration and noise problem. Previous research [1-6] works have identified that the transmission error and time-varying mesh stiffness are the two main internal excitations on the vibration of gearboxes. The detailed properties of time-varying mesh stiffness [7-9] has already been well studied and modeled by many researchers. However, there are just few work quantified the transmission error that is usually caused by the gear manufacturing errors, the installing errors and the elastic deformation of meshing gear teeth. Most of the researchers just employ the sinusoidal harmonics with arbitrary amplitude to represent the influence of transmission error on the dynamics of gear pairs. Only in the recent, a systematic experimental study was conducted to measure the quasi-static transmission errors by typical manufacturing and installing errors [10]. It was shown that the transmission errors are very sensitive with the manufacturing and installing errors and the different transmission errors have strong impact on the properties of gear dynamic response. Thus, it is important to investigate the detailed properties of the transmission errors. However, the experiment based approach requires manufacturing, installation and commissioning of many sets of gears with different specifications, which is time-consuming and expensive. Moreover, such kind of experiment requires strong expertise and costly precision equipment.

Therefore, this paper addresses the lack of tools that could provide an assessment of the static transmission error that is caused by typical gear manufacturing and installing errors like the indexing errors and run-out errors. The rest of the paper

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begins with a brief review of the definition of transmission error, indexing error and run-out error. Afterwards, the details of the proposed virtual model, that is capable to numerically assess the transmission error, is introduced. Finally, the effectiveness of the proposed virtual model is validated by comparing the simulated results with the previously published experimental results. The approach presented in this paper can provide a quick prediction for the static transmission error and quantitatively assess the influence of typical gear manufacturing and installing errors like the indexing errors and run-out errors on the transmission errors.

II. TRANSMISSION ERROR, INDEXING ERROR AND RUN-OUT ERROR

For the ease of readers, this section gives a brief review on the transmission error, indexing error and run-out error.

A. *Static Transmission Error*

In a gear transmission system, it can define the number of teeth of the driving wheel as N_w, the number of teeth of the driven pinion as N_p. When the driving wheel rotates an angle θ_w, the theoretical rotational angle of the driven pinion is θ_p = $\theta_w \times \frac{N_w}{N_p}$. However, during the practical operation of gear pairs, the actual rotational angle of the driven pinion θ_p may not be equal to the theoretical rotational angle $\theta_w \times \frac{N_w}{N_p}$. This phenomenon is called as the transmission error. The root causes of the transmission error [11] include the gear manufacturing errors, the installing errors, the elastic deformation of meshing gear teeth and even backlashes. The quantitative value of the transmission error is usually defined as a linear displacement along the line of action:

$$e = \theta_w N_w - \theta_p N_p \tag{1}$$

This paper restricts to predict and simulate the static transmission error that is mainly contributed by the typical gear manufacturing and installing errors: indexing error [12] and run-out error [13].

B. *Indexing Error*

The indexing error, also known as pitch error or tooth pitch error, is one of the most common gear manufacturing errors in practical gear transmissions. The indexing error is defined as the displacement deviation of any tooth flank from its theoretical position relative to a reference tooth flank. Fig.1 illustrates the definition of the indexing error. The green colored tooth flank is chosen as the reference tooth flank. For the ease of the demonstration, the dedendum circle of the gear is projected to a horizontal axis in Fig. 1. The blue curves represent the theoretical position of the i^{th} gear tooth and the red curves represent the actual position of the i^{th} gear tooth. It can be calculated that the distance between the theoretical flank of the i^{th} gear tooth and the reference tooth flank equals to $d = (i-1)\frac{2\pi r}{N}$, where r is the radius of the reference circle and N is the total number of the gear teeth. Define as the distance between the actual flank of the i^{th} gear tooth and the reference tooth flank. If deviates 10μm from d as shown in Fig. 1, then it is called the i^{th} gear tooth has an indexing error of 10μm. In the existing of the indexing error, the gear tooth flank will have a delayed or advanced mesh contact compared with the ideal gear mesh, which significantly increase the transmission error [14].

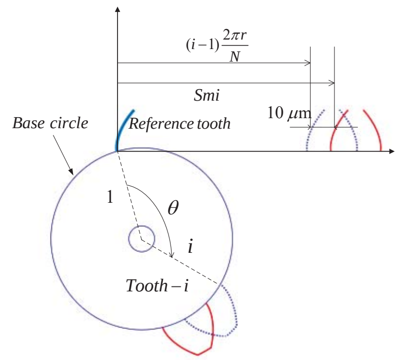

Figure 1. Indexing error

C. *Run-out Error*

As illustrated in Fig. 2, the run-out error is the deviation of the geometric center of the gear with respect to the center of the rotational shaft on which the gear is mounted [15]. The run-out error can either originate from the manufacturing errors, such as the excessive tolerances of gear holes, shaft and bearing housing, or installing error like the incorrect assembly. The run-out error has similar effects as the indexing error to result in cumulative variations of the transmission error, which lags or leads the contact instant during gear meshing.

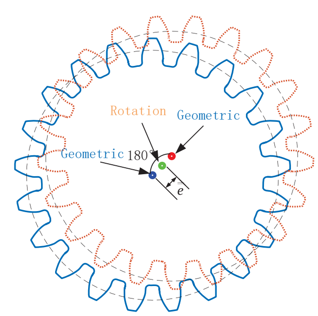

Figure 2. Run-out error diagram

III. THE VIRTUAL GEAR MODEL

A. *Mathematical Model of Involute Tooth Profile*

The main problem of the virtual model of a spur gear is to accurately establish the involute tooth flank. As illustrated in Fig. 3(a), take an arbitrary point B' on the involute tooth flank for example, the position of B' can be easily described in the polar coordinate as (r_k, β). Based on the definition of the involute curve, r_k and β can be further represented as Eq. (2) and Eq. (3), respectively:

$$r_k = \frac{r_b}{\cos\alpha} \tag{2}$$

$$\beta = inv\alpha = \tan\alpha - \alpha \tag{3}$$

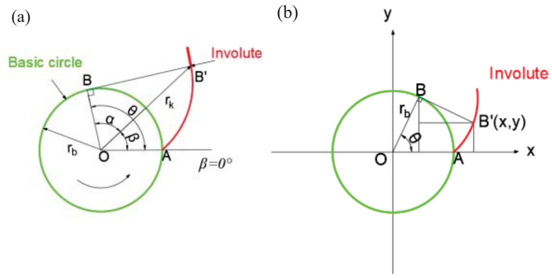

Figure 3. (a) Involute curve in the polar coordinate; (b) Involute curve in the Cartesian coordinate.

where r_b is the radius of base circle and α is the pressure angle for the arbitrary point B'. As shown in Fig. 3(b), the arbitrary point B' on the involute curve can also be written in the Cartesian coordinate by transformed from the polar coordinate:

$$x = r_b\cos\theta + \theta r_b\sin\theta \tag{4}$$

$$y = r_b\sin\theta - \theta r_b\cos\theta \tag{5}$$

where θ is the roll angle and $\theta = \alpha + \beta$. Then, Eq. (4) and Eq. (5) can be further written as:

$$x = \frac{Nm\cos a_0}{2}\left(\cos(\tan\alpha) + \tan\alpha\sin(\tan\alpha)\right) \tag{6}$$

$$y = \frac{Nm\cos a_0}{2}\left(\sin(\tan\alpha) - \tan\alpha\cos(\tan\alpha)\right) \tag{7}$$

where m is the modulus, α_0 is the pressure angle at the pitch circle that usually equals to 20° , N is the number of teeth. It can be seen from Eq. (6) and Eq. (7), the only variable of an involute gear flank is the pressure angle α. For a standard non-modified spur gear, the range of pressure angle α is between:

$$a_d \le a \le a_a \tag{8}$$

where α_d and α_a is the pressure angle at the dedendum circle and addendum circle, respectively [14]. The pressure angle at the dedendum circle and addendum circle can be given by:

$$a_d = \arccos\frac{r_b}{r_d} = \arccos\frac{N\cos a_0}{N - 2\left(h^* + c^*\right)} \tag{9}$$

$$a_a = \arccos\frac{r_b}{r_a} = \arccos\frac{N\cos a_0}{N + 2h^*} \tag{10}$$

where r_d is the radius of dedendum circle, r_a is the radius of addendum circle, h^* is the addendum factor that usually equals to 1, c^* is the dedendum factor that usually equals to 0.25. Figure. 4 shows the developed virtual gear profile based on the mathematical equations presented in this section.

B. The Virtual Indexing and Run-out Error

As illustrate in section II, indexing error is the deviation of tooth flank. Therefore, the tooth profile (x', y') having the indexing error can be generated based on the following equations:

$$x' = x\cos(\varphi) - y\sin(\varphi) \tag{11}$$

$$y' = x\sin(\varphi) + y\cos(\varphi) \tag{12}$$

where φ is the angle of the indexing error on the pitch circle, r is the radius of pitch circle. For example, if there is a 10μm indexing error on a tooth. Here φ is calculated by :

$$\varphi = \frac{10^{-6}}{2\pi r} \tag{13}$$

As shown in Fig. 5, the green curve is the normal tooth profile, the red curve is the tooth profile that has a positive 10μm indexing error. The indexing error tooth profile is got by normal tooth profile rotate an angle.

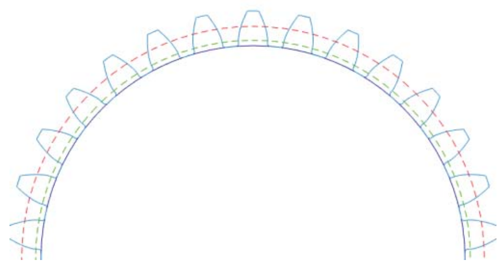

Figure 4. The developed virtual gear profile.

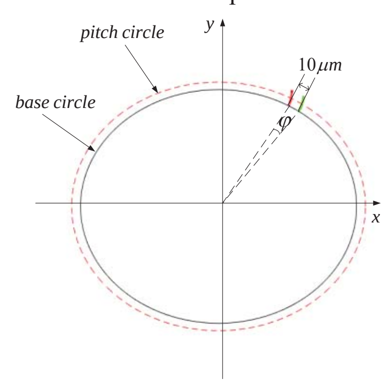

Figure 5. The virtual indexing error.

For the run-out error case, when the gear rotating, the rotating center is not the geometric center of its circle. If choosing the origin of the coordinate as the rotating center, the teeth profiles (x'', y'') can be generated using the following equations to seed the run-out error in the model:

$$x'' = (x - e\cos(\theta_e))\cos(\omega) + (y - e\sin(\theta_e))\sin(\omega) + e\cos(\theta_e) \tag{14}$$

$$y'' = (y - e\sin(\theta_e))\cos(\omega) - (x - e\cos(\theta_e))\sin(\omega) + e\sin(\theta_e) \tag{15}$$

where e is the value of run out error, θ_e is the angle of run out

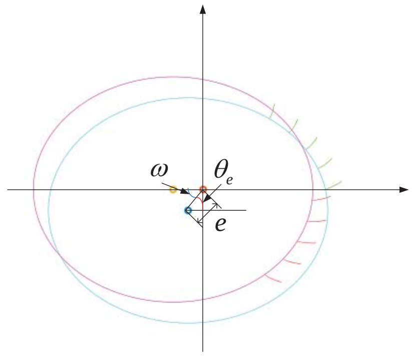

Figure 6. The virtual run out error.

error. In the Fig. 6 the blue dot is geometric center, the brown dot is rotation center , the yellow dot is the geometric center rotated an angle ω .

C. *Program Framework of the Virtual Model*

Based on the formulation of the tooth profile described above, the tested gear pair can be numerically generated and virtually displayed in the computer. The various indexing and run-out errors can be seeded in this virtual model, which is realized by numerically changing the virtual tooth profile or the position of the rotational center of the virtual gear body. The algorithm of the assessment of the static transmission error with this virtual model is given in Fig. 5. In the presence of the indexing and run-out error, the paired tooth flank may have a delayed or advanced contact during the mesh. Therefore, it must check whether the teeth of the driving wheel contact with the teeth of the driven pinion in each simulation step. There are three cases listed as below:

1) If the paired teeth contact normally with each other, the driving wheel is set to rotate a pre-defined small increment $\Delta\theta_w$ and the driven pinion is set to rotate a corresponding angle

$\Delta\theta_p = \Delta\theta_w \dfrac{N_w}{N_p}$ to continue the next simulation step.

2) If the teeth of the wheel and pinion lose the contact, it means there is a delayed mesh caused by the gear error. Then, the driving wheel is set to rotate with the pre-defined small increment $\Delta\theta_w$ and the driven pinion is set to keep stationary to continue the next simulation step.

3) If the virtual teeth of the wheel and pinion overlap to some extent, it hints there is an advanced mesh caused by the gear error. In this case, both the driving wheel and the driven pinion must be remounted to their original angle in the last step. Then, the driving wheel is set to rotate with a new increment $\Delta\theta_w' = \dfrac{\Delta\theta_w}{2}$ and the driven pinion is set to rotate with the

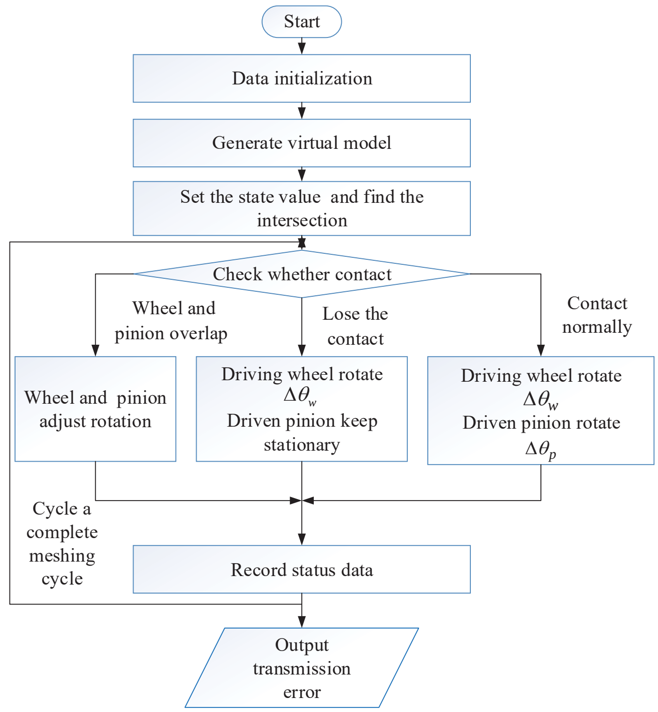

Figure 7. Software flow chart

corresponding angle $\Delta\theta_p' = \Delta\theta_w' \dfrac{N_w}{N_p}$.

Note that in the presented version of the two-dimensional virtual model, it assumes that the gear teeth are rigid and only rotational degree of freedom is taken into the consideration.

IV. SIMULATED RESULTS AND DISCUSSIONS

In this section, the simulated static transmission error from the virtual model is compared with the published experimental results in Ref. [16].

A. *System Parameters of the Simulated Gear Pair*

To make the simulated results comparable with the existing experimental results, the system parameters of the simulated gear pair are chosen as the same as the tested gears in Ref. [16]. The geometric parameters of the simulated gear pair are given in Table. I. In this case, two identical gears with 50 teeth are set as the driving wheel and driven pinion respectively. Therefore,

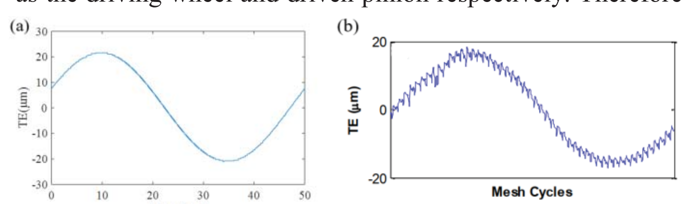

Figure 8. Simulated transmission error (a) a complete rotation with 20μm run-out error ,(b) measured quasi-static transmission error with 20μm run-out error reproduced from Ref. [16].

the gear ratio of the simulated gear pair is 1 and the contact ratio equals to 1.8.

TABLE I. GEAR AND PINION PARAMETERS

Parameter	Value
Number of teeth	50
Normal module [mm]	3
Pressure angle [deg]	20
Pitch diameter [mm]	150
Base diameter [mm]	140.954
Major diameter [mm]	156
Minor diameter [mm]	140.68
Circular tooth thickness [mm]	4.64

B. *Simulated Results*

Four different scenarios of the indexing errors, summarized in Table. Ⅱ, were simulated and studied in this work. Besides the indexing errors, a 20μm run-out error was also seeded in

TABLE II. THE TRANSMISSION ERROR SCENARIO MATRIX

Scenario #	Indexing Error Sequence
1	0-15-0
2	0-30-0
3	0-15-15-0
4	0-15-30-0

each scenario to mimic the practical common installing error. Figure.8(a) compares the simulated static transmission error due to the 20μm run-out error and the measured quasi-static transmission error from the existing literature [16]. It can be seen from Fig. 8(b) that the proposed virtual model is able to well predict the transmission error caused by the run-out error.

In scenario #1 and #2, a tooth with a 15μm indexing error and a 30μm indexing error was virtually seeded on the driving wheel, respectively. The driven pinion was set free of the indexing error in those scenarios. The simulated static transmission errors and the existing measured quasi-static transmission errors are shown in Fig. 7 and 8, in which the transmission errors significantly fluctuate in the presence of the indexing error. Compared with the measured quasi-static transmission error, it can be concluded that the proposed virtual model is capable to give a quick assessment of the rough magnitude and shape of the static transmission error. However, the duration of the influence of indexing error on the transmission errors in the quasi-static experiment (lasting nearly 2 mesh circles) are longer than the simulated results. This phenomenon may be due to the deformation of teeth under the quasi-static operational condition. The details will be investigated in the future work.

Similar trend can also be observed in scenario #3 and #4 as illustrated in Fig. 9 and 10, respectively. In scenario #3 and #4, two consecutive teeth on the driving wheel were both virtually set to have a 15μm indexing error. The driven pinion was set

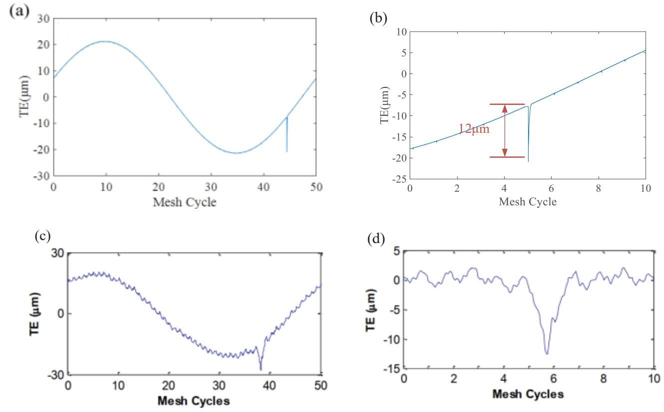

Figure 9. Simulated transmission error for scenario #1 in Table II (a) simulated transmission error from the virtual model(b) a zoomed view of 10 mesh cycles of the simulated result (c) measured quasi-static transmission error reproduced from Ref. [16], (d) a zoomed view of 10 mesh cycles reproduced from Ref. [16].

free of the indexing error in scenario #3. A 15μm indexing error was seeded on a tooth of the driven pinion in scenario #4, which meshed with the two consecutive wheel teeth having the indexing error.

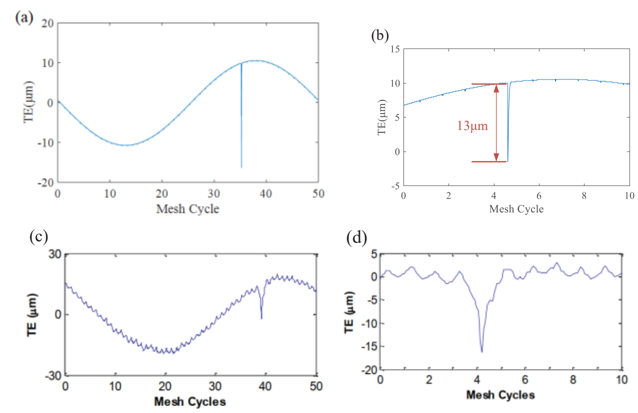

Figure 10. Simulated transmission error for scenario #2 in Table II (a) simulated transmission error from the virtual model(b) a zoomed view of 10 mesh cycles of the simulated result (c) measured quasi-static transmission error reproduced from Ref. [16], (d) a zoomed view of 10 mesh cycles reproduced from Ref. [16].

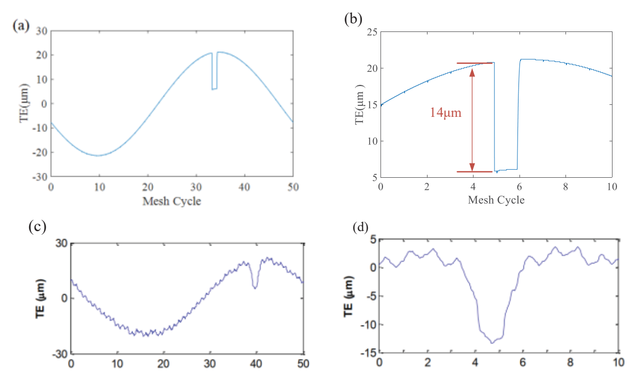

Figure 11. Simulated transmission error for scenario #3 in Table II (a) simulated transmission error from the virtual model(b) a zoomed 1 view of 10 mesh cycles of the simulated result (c) measured quasi-static transmission error reproduced from Ref. [16], (d) a zoomed view of 10 mesh cycles reproduced from Ref. [16].

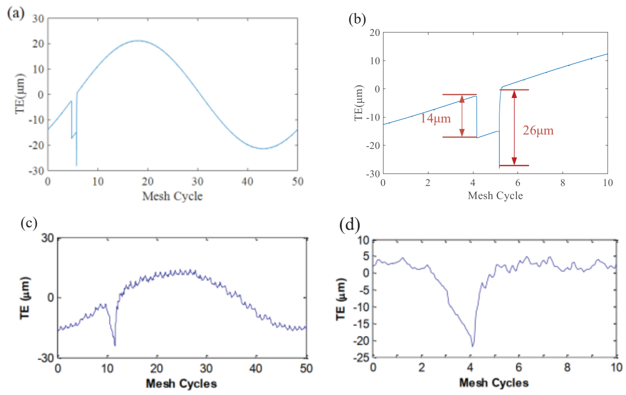

Figure 12. Simulated transmission error for scenario #4 in Table II (a) simulated transmission error from the virtual model(b) a zoomed view of 10 mesh cycles of the simulated result (c) measured quasi-static transmission error reproduced from Ref. [16], (d) a zoomed view of 10 mesh cycles reproduced from Ref. [16].

V. Conclusion

A virtual gear model is proposed in this work that is based on the mathematical formulation of the involute profile to numerically generate the teeth flanks of spur gear pairs. The proposed virtual model is capable to give a quick evaluation of the static transmission error caused by the indexing errors and run-out errors. The simulated static transmission errors from the virtual model successfully predict the magnitudes and trends of the measured quasi-static transmission errors that are published in previously research work. Therefore, the proposed virtual model can aid in the in-depth analysis of dynamic behavior of gear transmissions and the optimization of design of gearboxes.

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