# Hierarchical Health Assessment of Equipment with Uncertain Fault Diagnosis Result

Shigang Zhang, Xu Luo\*, Lei Li, Yongmin Yang

Laboratory of Science and Technology on Integrated Logistics Support, College of Intelligence Science and Technology, National University of Defense Technology, Changsha 410073, P.R.C

\* Corresponding author: luoxu2009@163.com

Abstract—Monitoring health status of equipment is very important for risk avoiding and maintenance decision making, especially for complex safety-critical systems. Most of existing fault diagnosis systems can only generate the state of a specific system level. Models should be developed to assess the health states of the equipment in different hierarchical levels. In this paper, a model based on Bayesian networks is proposed, where determined fault diagnosis result and the fault diagnosis result with uncertainty can all be used. The model structure, how to set uncertain diagnosis result by virtual nodes and how to represent multi-states are formulated and discussed in detail. An application example on a diesel engine combustion system is given, which shows that the method proposed in this paper can realize hierarchical health assessment, including the scenarios that the diagnosis result is uncertain.

Keywords-Hierarchical health assessment; PHM; fault diagnosis; Bayesian networks; virtual node

# I. INTRODUCTION

In fault diagnosis systems, it is possible that the fault state of components in the lowest level of the system hierarchy is obtained. Since there may be complex relationships between different modules, a fault in the lowest level of hierarchy may have different influence on the system. In other words, in many application scenarios, status of the modules in the higher level of system hierarchy is our prime concern. Then, how to calculate the status of modules in different levels based on the diagnostic result becomes an interesting problem to study.

For this problem, Fan *et al* proposed a health state evaluation method based on state value calculation for a satellite control system[1]. Wang studied the aeroengine health assessment problem, where a model combining fuzzy analytic hierarchy process (fuzzy AHP), fuzzy preference programming (FPP) and technique for order performance by similarity to ideal solution (TOPSIS) is proposed. Health indexes can be obtained for the equipment and decisions can then be made based on the information[2]. Actually, AHP method and those based on it have also been used by other researchers to solve the similar problems, such as airplane health assessment[3], bridge state assessment[4], risk assessment[5, 6], fire and explosion accidents for steel oil storage tanks[7] and so on.

In addition to the health assessment for equipment, Wu *et al* studied the health assessment problem for equipment cluster based on grey clustering and fuzzy synthetic evaluation method[3]. Furthermore, equipment may have the ability to execute different tasks, which means that the health state and its effect on the mission may be different. For this problem, Geng *et al* proposed a mission-oriented health assessment method and applied it to a ship[8].

The methods mentioned above can be used to assess the health state of the equipment given the health indexes in the lower level. However, they cannot be integrated with the fault diagnosis system, especially when the diagnosis results have different forms. In the real world systems, it is possible that fault states cannot be accurately determined by the fault diagnosis system because of unreliable sensors, environmental influence, poor testability design, etc. For example, an ambiguity group may be obtained after fault diagnosis. In this paper, we propose a hierarchical health assessment method based on Bayesian networks with the ability that can deal with the health state assessment problem with uncertain fault diagnosis results. In fact, Bayesian networks has been used a lot in fault diagnosis[9]–[11]. Developing a hierarchical health assessment model will further increase its application range.

The remainder of this paper is organized as follows. In section II, common forms of diagnosis result are introduced. In section III, a hierarchical health assessment model is presented, including the structure and parameters associated with it. In section IV, how to input different diagnosis results to the model and generate a health assessment result is discussed. In section V, an application case to a diesel engine combustion system is studied. The result validates effectiveness of the method. In section VI, the paper is concluded.

#### II. FORMS OF DIAGNOSIS RESULTS

In fault diagnosis systems, it is possible that not all the faults can be isolated to a single one because of lack of sensors or inappropriate diagnostic strategies. In this section, common fault diagnosis result forms are presented.

This study was supported by the National Natural Science Foundation of China under grant No.61503398

#### A. A Fault Can be Exactly Isolated

In this scenario, the fault status can be absolutely determined by the fault diagnosis system, which means that the failure mode of the system is very clear to us. The problem of hierarchical health assessment becomes analysis of fault effect on different levels of the system given a determined status in the lower level. This will be formulated as evidences in the Bayesian networks.

### B. An Ambiguity Group is Obtained

In some cases, it is possible that only the most possible fault set can be obtained after fault diagnosis because of lack of test information. Formally, the set is called an ambiguity group. For example, if a lamp does not work, the diagnostic result may be: light bulb burned out, power failure, or line fault, which is an ambiguity group consists of three faults. However, if we know that the power source works properly by testing the voltage, the power failure can be excluded. The ambiguity group becomes light bulb burned out and line fault. Unlike the determined fault status, the number of possible faults and variations of fault status should be considered in the health assessment process.

#### C. Fault Status with Probabilities

In real world systems, sensors may be unreliable. Consequently, fault diagnosis systems with uncertain reasoning ability are needed. Some models such as Bayesian networks can realize this kind of fault diagnosis. The result generated by this system is fault status with a probability. For example, the result shown in Table I is a diagnostic result with probabilities, which means that there may be three faults in the system, i.e., starting injection pressure low, inferior atomizing ability of fuel injectors and fuel injectors jam, with probabilities 80%, 70% and 54%, respectively.

TABLE I. DIAGNOSTIC RESULTS WITH PROBABILITIES

Fault No.	Failure mode	Fault Probability	
1	starting injection pressure low	80%	
2	inferior atomizing ability of fuel injectors	70%	
3	fuel injectors jam	54%	

#### III. HIERARCHICAL HEALTH ASSESSMENT MODEL

#### A. Structure of the Model

In order to assess the health status in different levels of equipment, the model should represent impact relationships between different levels. Here, a multi-layer Bayesian networks for hierarchical health assessment is proposed. The structure of the model is shown in Fig.1.

In Fig.1, the system is divided into four layers, i.e., failure source, LRU (Line Replaceable Unit), subsystem and system. Each module or failure mode is represented by a node. Nodes in the lower layer are the father nodes of those in the higher layer, which means that the state of the equipment in a higher layer is determined by the components contained by it. Note that the number of layers may be different in real world systems. It should be set according to the actual situation.

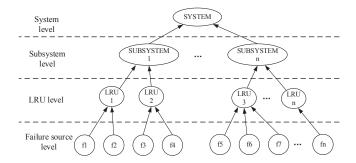


Figure 1. Structure of the hierarchical health assessment model

#### B. Parameters of the Model

In the Bayesian networks, each node has several states representing health status of the corresponding equipment. Conditional probabilities are associated with each directed edge denoting the strength of a dependency relationship. Take LRU1 in Fig.1 as an example. The part of its network is shown in Fig.2.

Let's assume that each node has two states: *good* and *bad*, then the parameters can be set to the value shown in Table II and Table III. Table II are the prior fault probabilities of f1 and f2. Table III is the parameter of LRU1, which includes several conditional probabilities. For example, the probability that LRU1 is *bad* is 0.01 given the condition that both f1 and f2 are *good*. Other parameters have a similar meaning.

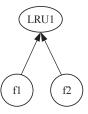


Figure 2. LRU1 and its parent nodes

TABLE II. PARAMETERS OF NODE f1 and f2

state	f1	f2
good	0.95	0.9
bad	0.05	0.1

TABLE III. PARAMETERS OF LRU1

f1		good	good	bad	bad
f2		good	bad	good	bad
LRU1	good	0.99	0	0	0
	bad	0.01	1	1	1

#### IV. HIERARCHICAL HEALTH ASSESSMENT METHOD

#### A. Inference with Determined Diagnosis Results

A determined diagnosis result means that states of some nodes can be known exactly. In the health assessment Bayesian networks, this result can be converted to evidences. Then, probabilities of other nodes can be calculated using the inference algorithms such as message-passing algorithm[12].

#### B. Inference with Ambiguity Groups

Unlike the scenario that a fault can be isolated with 100%, each fault in an ambiguity group may has the possibility to be *good* or *bad*. This information should be added to the model to get a correct health assessment result. In this section, we will discuss how to construct this model given the ambiguity group with fault probabilities.

According to the meaning of the diagnostic result with probabilities, evidences with probabilities should be set to the health assessment Bayesian networks. Here, we realize this by adding virtual nodes. For a node denoting a component (take LRU1 as an example) in the Bayesian networks, the virtual node V can be added as its father node or as its child node, as shown in Fig.3 and Fig.4 respectively.

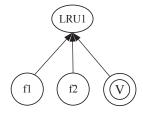


Figure 3. Virtual node as a father node

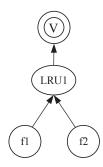


Figure 4. Virtual node as a child node

For LRU1, it is evident that the number of its parameters increases exponentially with the number of its father nodes. Specifically, the number of conditional probabilities is  $2^n$ , where n denotes the number of its father nodes. Consequently, the model in Fig.4 needs fewer parameters that the one shown in Fig.3. Furthermore, this will also increase complexity of inference. Therefore, the second model shown in Fig.4 is chosen as the one to deal with the health assessment problem with ambiguity groups.

Part of a hierarchical health assessment model is shown in Fig.5. Since each node in the model denotes a component in the system, we will not distinguish them in the following formulation. Let's assume that node C is one component in the ambiguity group.

Assume that C has two states, and the prior probability before fault diagnosis is  $P(C=1) = \phi_1$  and  $P(C=0) = \phi_0$ , where the values 0 and 1 correspond to the state *good* and *bad* mentioned above. It is evident that  $\phi_1 + \phi_0 = 1$ . Let  $\overline{P}(C=1) = v$  denote the diagnosis result of C in the ambiguity

group. The problem becomes how to change P(C=1) from  $\phi$ , to  $\gamma$  by adding a virtual node V.

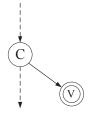


Figure 5. Part of the hierarchy health assessment model

Let's assume that V has two states, i.e., 1 and 0, and define

$$P(V = 1 \mid C = 1) = x_1 \tag{1}$$

$$P(V = 1 \mid C = 0) = x_0 \tag{2}$$

If  $\overline{P}(C=1) = y$  can be obtained by setting node V to state 1, then  $\overline{P}(C=1) = y$  can be written to

$$\overline{P}(C=1) = P(C=1 | V=1)$$
 (3)

According to the Bayes rule,

$$P(C=1|V=1) = \frac{P(V=1|C=1)P(C=1)}{P(V=1)}$$

$$= \frac{P(V=1|C=1)P(C=1)}{\sum_{C} P(V=1,C)}$$
(4)

Substituting (1) and (2) into (4), we obtain

$$P(C=1|V=1) = \frac{\phi_1 x_1}{\phi_1 x_1 + \phi_0 x_0} = y$$
 (5)

Then,

$$\frac{x_1}{x_0} = \frac{y\phi_0}{\phi_1 - y\phi_1} \tag{6}$$

which means that the target probability can be obtained by setting  $x_1$  and  $x_0$  proportionally according to (6). It is evident that infinitely many solutions exist in (6). In application, we can set the larger one to an absolute value in [0,1], and then get the other one via (6).

The method discussed above can be used to the node with two states. However, a node may have several states denoting different health status.

Let n denote the number of states, and the prior probabilities before fault diagnosis are

$$P(C = C_i) = \phi_i, i = 1, \dots, n \tag{7}$$

Let  $P(C=C_i | V=1) = y_i$ ,  $i=1,\cdots,n$  denote the fault probabilities in the ambiguity group after fault diagnosis. It is evident that

$$\sum_{i=1}^{n} y_i = 1 \tag{8}$$

The objective is to calculate the conditional probabilities  $P(V=1 | C=C_i) = x_i$ ,  $i=1,\dots,n$ .

According to the Bayes rule,

$$P(C = C_i | V = 1) = \frac{P(V = 1 | C = C_i)P(C = C_i)}{P(V = 1)}$$

$$= \frac{P(V = 1 | C = C_i)P(C = C_i)}{\sum_{C} P(V = 1, C)}$$
(9)

Then,

$$y_i = \frac{\phi_i x_i}{\sum_{j=1}^n \phi_j x_j} \tag{10}$$

Since  $\sum_{i=1}^{n} y_i = 1$ , the number of linearly independent equations shown in (10) is n-1. Like the one shown in (6), (10) has infinitely many solutions. However, a unique solution can be obtained if a variable is designated before calculation.

Specifically, according to (10),

$$\frac{\phi_i x_i}{\phi_i x_j} = \frac{y_i}{y_j} \tag{11}$$

$$\frac{x_i}{x_j} = \frac{y_i \phi_j}{y_j \phi_i} \tag{12}$$

$$x_i = \frac{y_i \phi_j x_j}{y_j \phi_i} \tag{13}$$

Let  $x_1 = 1$ , then we can get a solution

$$x' = \{1, x_2', \dots, x_n'\}$$
 (14)

In application, take a number  $\alpha \in (0,1)$ , then the solution can be obtained as

$$\overline{x} = \frac{\alpha x'}{\max(x')} \tag{15}$$

where  $\max(x')$  returns the maximum value in x'.

# V. EXPERIMENTS AND VALIDATION

#### A. Model of the example

The method proposed in this paper is validated in a diesel engine combustion system, which is divided into four levels, as shown in Table IV. Note that a larger number denotes a component in the lower level. In this example, not all the components in the system are presented. Specifically, the fuel system is modeled in detail. States of intake and exhaust

system and cylinder components are denoted by two nodes. This which will not impact validation of the method.

TABLE IV. LEVELS OF THE DIESEL ENGINE COMBUSTION SYSTEM

No.	Failure / Module names	Level
1	combustion system	1
2	intake and exhaust system	2
3	cylinder components	2
4	fuel system	2
5	fuel pipeline	3
6	injector	3
7	fuel pipeline blockage	4
8	fuel supply advance angle	4
9	starting injection pressure	4
10	inferior atomizing ability	4
11	fuel injectors jam	4

All the modules are assumed to have two states: *good* and *bad*. The model is constructed using the Bayesian networks modeling tool GeNIe[13], as shown in Fig.6.

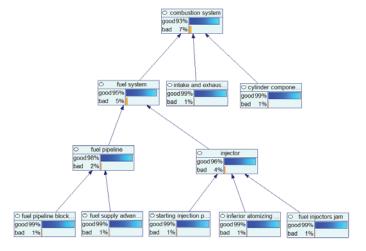


Figure 6. Health Assessment Model of a Diesel Engine Combustion System

#### B. Assessment with a determined diagnosis result

Given a diagnosis result, i.e., inferior atomizing ability, health state can be obtained by inference with the model, as shown in Fig.7. It means that probabilities of *bad* state of injector, fuel system and combustion system are all 0.85.



Figure 7. The model with a determined failure mode

# C. Assessment with a binary uncertain diagnosis result

However, if the fault inferior atomizing ability cannot be 100 percent isolated, an uncertain evidence should be added to the model. Assume that the fault probability of inferior atomizing ability is 0.8, according to the method discussed above, a virtual node V1 is added to the model. In the calculation, the prior probabilities are 0.99 and 0.01, as shown in Fig.6. The target fault probability is 0.8. Then, the conditional probabilities can be obtained via (6). Here, the result obtained is  $x_1 = 0.0023$ ,  $x_0 = 0.9$ . After inference calculation, probabilities of *bad* state of injector, fuel system and combustion system are 0.69,0.69 and 0.7, which are different from the determined diagnosis result, as shown in Fig.8.

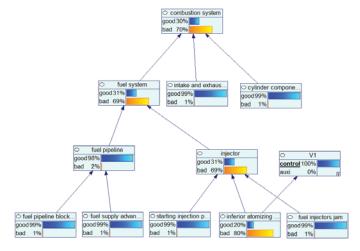


Figure 8. Health Assessment Result with an Uncertain Diagnosis Result

#### D. Assessment with multi-states

In addition to the model with binary states, the method proposed in this paper can also be used to those with multistates. For example, the modules may have a deterioration state between *good* and *bad*. In this scenario, more accurate result may be obtained, as shown in Fig.9. Take the injector as an example, the deterioration state with probability 0.2 can be obtained, which is more reasonable compared with Fig.7.

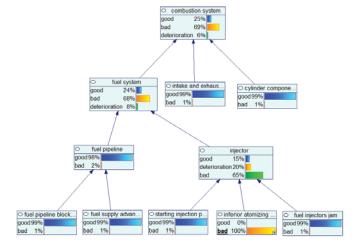


Figure 9. Health Assessment Result of Multi-states with a Determined Diagnosis Result

# E. Assessment with multi-states and uncertain diagnosis results

In real world systems, it is possible that the health state of component in a higher level can be monitored. If we can get a diagnosis result of the injector, and the probabilities of *good*, *deterioration* and *bad* are 0.3,0.6 and 0.1 respectively. Assume the prior probabilities are those shown in Fig.10.

According to the method proposed in this paper. A Virtual node should be added as a child node of injector. Let  $\alpha = 0.9$ , the parameter of the virtual node can be set as  $\overline{x} = \{0.0478, 0.9, 0.45\}$  via (15). Then, the health assessment result after inference calculation is shown in Fig.11. We can see that the state is exactly the expected one. In other words, it is the same as the diagnosis result. Furthermore, the state probabilities of the injector's father nodes also have updated, which can be used to analysis the reasons causing the failure of the injector.

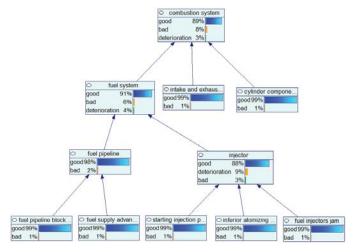


Figure 10. Prior Probabilities of the health state

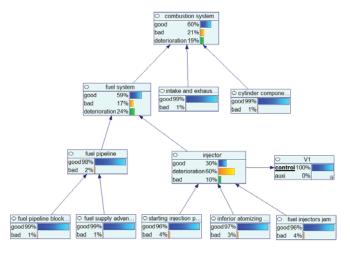


Figure 11. Health Assessment Result with an Uncertain Higher Level
Diagnosis Result

#### VI. CONCLUSIONS

In this study, a hierarchical health assessment model based on Bayesian networks was proposed. The model structure and parameters were discussed. Since a diagnosis system may generate uncertain diagnosis results, i.e., ambiguity groups, how to use this kind of uncertain information was formulated. By adding virtual nodes to the model, the probability of a node state can be exactly regulated to match the diagnosis result. Besides the nodes with binary states, multi-states health assessment can also be realized applying this model. Uncertain diagnosis result of the components in the middle level can also be represented. An application example to a diesel engine combustion system was presented. The scenarios that the nodes with binary states, multi-states, uncertain diagnosis result were all validated, which showed that the method proposed in this paper can realize hierarchical health assessment for complex equipment with uncertain diagnosis result.

#### REFERENCES

- [1] G. Fan, J. Zhou, and Y. Zhou, "A health state evaluation for satellite control system based on state value calculating," Fire Control & Command Control, vol. 37, no. 3, pp. 39-42.
- [2] J. Wang, K. Fan, and W. Wang, "Integration of fuzzy AHP and FPP with TOPSIS methodology for aeroengine health assessment," *EXPERT SYST. APPL.*, vol. 37, no. 12, pp. 8516-8526.
- [3] B. Wu, X. Jia, and L. Xia, "Health Assessment of Equipment-equipment Cluster Based on Grey Clustering and Fuzzy Synthetic Evaluation," Journal of Ordance Engineering College, vol. 21, no. 5, pp. 1-5.

- [4] J. Fan, H. Yuan, W. Liu, and D. Zeng, "Fuzzy Comprehensive Assessment in Bridge State Based on the Uncertain Analytic Hierarchy Process," JOURNAL OF WUHAN UNIVERSITY OF TECHNOLOGY, vol. 27, no. 04, pp. 57-60.
- [5] A. W. Dawotola, P. Van Gelder, and J. K. Vrijling, "Risk assessment of petroleum pipelines using a combined analytical hierarchy process fault tree analysis (AHP-FTA)," in *Proc.* Proceedings of the 7th international probabilistic workshop, Delft, 2009.
- [6] K. Hyun, S. Min, H. Choi, J. Park, and I. Lee, "Risk analysis using fault-tree analysis (FTA) and analytic hierarchy process (AHP) applicable to shield TBM tunnels," *TUNN. UNDERGR. SP. TECH.*, vol. 49, pp. 121-129
- [7] L. Shi, J. Shuai, and K. Xu, "Fuzzy fault tree assessment based on improved AHP for fire and explosion accidents for steel oil storage tanks," J. HAZARD. MATER., no. 278, pp. 529-538.
- [8] J. Geng, S. Huang, J. Jin, and W. Liu, "An integrated evaluation method for complex systems condition based on mission sections," J. Huazhong Univ. of Sci. Tech. (Nature Science Edition), vol. 34, no. 01, pp. 32-34.
- [9] D. Montenegro and G. A. Ramos, "Smart diagnosis of power quality disturbances using Bayesian networks," in *Proc.* 2012 Sixth IEEE/PES Transmission and Distribution: Latin America Conference and Exposition (T&D-LA), Montevideo, Uruguay, 2012, pp. 1-5.
- [10] O. J. Mengshoel, S. Poll, and T. Kurtoglu, "Developing large-scale Bayesian networks by composition: fault diagnosis of electrical power systems in aircraft and spacecraft," in *Proc.* Twenty-first International Joint Conference on Artificial, Pasadena, CA, USA, 2009, pp. 59-66.
- [11] S. Bottone, D. Lee, M. O'Sullivan, and M. Spivack, "Failure prediction and diagnosis for satellite monitoring systems using Bayesian networks," in *Proc.* IEEE Military Communications Conference 2008 (MILCOM 2008), San Diego, CA, USA, 2008, pp. 1-7.
- [12] R. E. Neapolitan, Learning Bayesian networks. Upper Saddle River, NJ: Pearson Prentice Hall, 2004.
- [13] GeNIe [Online]. Available: http://www.genie.sis.pitt.edu