# Reliability Assessment Methodology of Power Devices Based on Physics of Failure and Multivariate Copulas

Wendi Guo, Guicui Fu\*, Bo Wan, Yutai Su School of Reliability and Systems Engineering Beihang University

Beijing, China fuguicui@buaa.edu.cn

Abstract—The reliability assessment of power devices based on Physics of Failure (PoF) involves factors such as performance, structural materials, and operating conditions. However, PoF methods are mainly used for the specific failure mechanisms (FMs) of the device. Now a fusion multiple FMs reliability assessment is needed. This paper proposes a reliability assessment methodology considering the failure correlation of different structures of power devices based on multivariate copulas. We illustrate the assessment process by taking a FS discrete device as a case. Firstly, the FMs and PoF models of power devices are studied. The parameter values in PoF models are determined; The unknown parameters are obtained by finite element analysis (FEA). Secondly, the multivariate copulas are used to derive the reliability function with multi-FMs, and the unknown parameters in copulas are estimated. Finally, comparing the Monte Carlo-Competition (MCC) method, the results show that the proposed method can consider the crossinfluence factors between PoF models, and the correlation of different structural lifetime distributions is not negligible. Moreover, the computational efficiency is also improved.

Keywords-reliability assessment; power devices; Physics of Failure; multivariate copulas

## I. INTRODUCTION

As power devices are moving toward small size, large scale, and high performance, higher requirements are placed on their reliability assessment. The PoF methods for power devices can analyze the potential FMs and describe physical and chemical processes, which is one of essential research directions to improve the reliability of electronic products. Pin [1] used the Two-layer viscoelastoplastic model to assess the reliability of the die attachment under thermal cycling. Shohji [2] used a modified Coffin-Manson model to evaluate the lifetime of solder joints in chip scale packages (CSP). Nakagawa [3] established a thermal resistance model to investigate thermal runaway of SiC-SBD and described its robustness. These studies have used a single PoF model to obtain the reliability for a specific structure in a device. In practice, application environment profiles of devices always contain multiple stresses. Reliability assessments for single

stress are relatively inaccurate. In addition, the FM is in one-toone correspondence with the PoF model. Different PoF models can be considered independent. Therefore, competition failure is widely used. Wan [4] proposed a computer simulation method to assess the reliability of microelectronic devices with multi-FMs. Su [5] considered the FMs of structures in systemin-package (SiP) including chips, substrates, solder joints, etc. They all used the competition failure to obtain the lifetime prediction. However, they did not consider the cross-influence factors between FMs. For example, the temperature can affect the reliability of multi-structures. Therefore, it is necessary to carry out reliability research based on n-dependent lifetime distributions. According to Liu [6], a copula model was used to assess the reliability of dependent degradation processes. However, using the inverse Gaussian probability to describe failure processes may not simulate its root cause.

This paper proposed a reliability assessment methodology of power devices combining different FMs and different failure points. Taking an IGBT discrete device as a case, we used this method to evaluate the device's reliability. The MCC method was compared to illustrate the advantages of this proposed method. The remaining of this paper is organized as follows: Section 2 introduces the assessment process and copula theory, section 3 is the case study, and mainly introduces the process of assessing the device's reliability. And section 4 summarizes the concluding remarks in the end.

# II. METHODOLOGY

## A. Frame

The reliability assessment method based on PoF and multivariate copulas has three steps: 1) Information collection, including operating conditions, structures and FMs of the device; 2) PoF model application, including estimation of different structures' lifetime and their lifetime distributions; 3) Reliability assessment, including multivariate copulas modeling, model selection and parameter estimation, and finally we can predict the mean time to failure (MTTF).

In step 1, structures of a power device are analyzed, including chips, solder layers, substrates, package housing, bonding wires and so on. The working conditions include temperature, electricity, etc., which decide potential FMs.

In step 2, PoF models are used to obtain the lifetimes of different structures. We use FEA to obtain the structural temperature, stress, and strain data. The Monte Carlo (MC) method is used to randomize PoF model parameters to obtain lifetime distributions of different structures.

In step 3, because there exist cross-influence parameters in PoF models, the lifetime distribution can be regarded as dependent. A reliability expression is derived based on the series model by using multivariate copulas, and the assessment result, MTTF, is obtained.

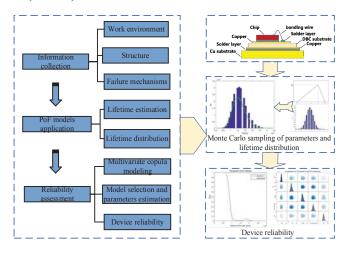


Figure 1. reliability assessment process

# B. PoF method

Excessive current, voltage, and junction temperature during operation can cause failure, which is closely related to the temperature factor. Therefore, this paper mainly studies the effect of temperature on power devices. Firstly, we analyzed the FMs, considered the failure of both chips and interconnects as shown in Fig. 2, and introduced the temperature-dependent PoF models.

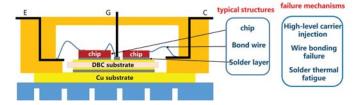


Figure 2. Failure indication of chip and interconnect structures

The residual carrier concentration increases with increasing temperature. When the majority carrier concentration increases, the minority carrier recombination speed becomes slower, which causes the shutdown rate to decrease. The remaining carrier lifetime [7] in a chip is subject to (1):

$$\tau = \begin{bmatrix} \left(400 + 11.76 \times 10^{-13}\right) N_t \left(\frac{T}{300}\right)^{-0.57} \\ +2.78 \times 10^{-31} p^2 \left(\frac{T}{300}\right)^{-0.72} \end{bmatrix} + \begin{bmatrix} \left(400 + 3 \times 10^{-13}\right) N_t \left(\frac{T}{300}\right)^{-1.77} \\ +1.83 \times 10^{-31} p^2 \left(\frac{T}{300}\right)^{1.18} \end{bmatrix}$$
(1)

where  $\tau$  is High-level carrier lifetime (s),  $N_t$  is Recovery center density (cm<sup>-3</sup>), p is Hole concentration (cm<sup>-3</sup>), and T is temperature (K).

The emitter is connected to the DCB copper layer through bonding wires. Under repeated thermal cycling, thermal stress is continuously accumulated in the contact area between the wires and the DCB, and the bonding wire is detached. The stress at the root of the bonding wire is expressed as [8]:

$$\sigma = 6E_w \frac{r}{D} (L/D - 1)^{1/2} \left( 2\alpha_s + \frac{\alpha_s - \alpha_w}{1 - D/L} \right) \Delta T$$
 (2)

where  $\alpha_w$  and  $\alpha_s$  correspond to the coefficients of thermal expansion (CTE) of the bonding wire and the substrate materials (°C<sup>-1</sup>);  $\Delta T$  is the temperature variation range (°C). 2L is the wire length (mm); 2D is the wire transconductance (mm); r is the cross-sectional radius of wires (mm),  $E_W$  is the elastic modulus of the wire material (Pa);

The thermal fatigue life (cycles) of the bonding wire is expressed as:

$$N_f = C_w \sigma^{-m_w} \tag{3}$$

where  $C_w$  is the tensile fatigue coefficient, usually 5 to 7;  $m_w$  is the tensile fatigue coefficient.

Due to the unevenness of the solder layer and the CTE mismatch of materials on both sides, the thermal stress caused by temperature fluctuation may lead to the solder layer slightly cracked. The Engelmaier equation can calculate the thermal fatigue life of solder layers [9]:

$$N_f = \frac{1}{2} \left( \frac{\Delta \gamma}{2\varepsilon_f'} \right)^{\frac{1}{c}} \tag{4}$$

where  $N_f$  is the thermal fatigue life;  $\Delta \gamma$  is the plastic shear strain;  $\varepsilon'_f$  is the fatigue ductility coefficient; c is the fatigue ductility index as (5):

$$c = -0.442 - 0.0006T_{sj} + 0.0174 \ln\left(1 + \frac{360}{T_D}\right)$$
 (5)

where  $T_D$  is the temperature soaking time (minutes). The plastic shear strain  $\Delta \gamma$  is as:

$$\Delta \gamma = 0.5FI \frac{K_D}{200 Ah} (\Delta \alpha L T_s - \Delta \alpha L T_c)^2$$
 (6)

where,  $\Delta \alpha LT_c$ ,  $\Delta \alpha LT_s$  respectively characterize the thermal diffusion of chips and copper substrates as (7) (8):

$$\Delta \alpha L T_c = \sqrt{\left(L_x \alpha_{cx}\right)^2 + \left(L_y \alpha_{cy}\right)^2} \left(T_c^{\text{max}} - T_c^{\text{min}}\right) \tag{7}$$

$$\Delta \alpha L T_s = \sqrt{\left(L_x \alpha_{sx}\right)^2 + \left(L_y \alpha_{sy}\right)^2} \left(T_s^{\text{max}} - T_s^{\text{min}}\right)$$
 (8)

where  $L_x$ ,  $L_y$  are the maximum pitches in x and y direction (mm);  $\alpha_{cx}$ ,  $\alpha_{cy}$  are the CTEs of the chip in x and y direction (ppm/°C);  $\alpha_{sx}$ ,  $\alpha_{sy}$  are the CTEs of the substrate in x and y direction (ppm/°C);  $T_c^{max}$ ,  $T_c^{min}$  are the highest and lowest temperatures of the chip (°C);  $T_s^{max}$ ,  $T_s^{min}$  are the highest and lowest temperatures of the substrate (°C).

# Reliability modeling using multivariate copulas

In many studies, PoF model results are independent. But in fact, there are crossovers among input parameters. As shown in Fig. 3, the activation energy of common materials and surrounding temperature may affect the life prediction values of PoF models, so the calculation results cannot be considered completely independent.

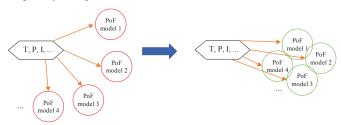


Figure 3. Schematic diagram of cross-influencing factors

Copulas can connect marginal distributions to obtain a joint distribution, and can describe the correlation between variables. Among them, the Archimedean Copula family is a commonly used multivariate copulas, mainly Clayton, Gumbel, and Frank. If PoF models results of a power device are considered to be random variables after parameters randomization, and obey the continuous distribution  $F_1(t_1)$ ,  $F_2(t_2)$ , ...  $F_n(t_n)$ ; According to Sklar's theorem, the cumulative probability distribution function (CDF) obtained from n-PoF models exist a unique copula C:

$$H(t_1, t_2, ..., t_n) = C(F_1(t_1), F_2(t_2), ..., F_n(t_n))$$
 (9)

TTFs are obtained using MC sampling, and tail-related features are shown due to cross-impact factors. A joint CDF model of power devices' lifetime is determined based on the Akaike Information Criterion (AIC). Because FMs are independent, the reliability containing n-FMs satisfies the series model:

$$R(t) = P_{f}(t_{1} > t, t_{2} > t, ..., t_{n} > t)$$

$$= 1 - P_{f}\left(\bigcup_{i=1}^{n} t_{i} \leq t\right)$$

$$= 1 - \sum_{i=1}^{n} P_{fi}(t_{i} \leq t) + \sum_{1 \leq i < j \leq n} C(P_{fi}(t_{i} \leq t), P_{fj}(t_{j} \leq t)) \quad (10)$$

$$- \sum_{1 \leq i < g < j \leq n} C(P_{fi}(t_{i} \leq t), P_{fg}(t_{g} \leq t), P_{fj}(t_{j} \leq t)) + ...$$

$$+ (-1)^{n} C(P_{fi}(t_{1} \leq t), P_{f2}(t_{2} \leq t), ..., P_{fn}(t_{n} \leq t))$$

## D. Parameter estimation and model selection

We used the relationship between Kendall rank correlation  $\tau_{\alpha}$  and model parameter  $\alpha$  to achieve the parameter evaluation.

Given that there are n continuous random vectors (X, Y), the copula function of (X, Y) is  $C_{\alpha}$ . According to the definition of Kendall rank correlation,  $\tau_{\alpha}$  is defined as:

$$\hat{\tau}_{\alpha} = t = \sum_{1 \le i < j \le n} sign((x_i - x_j)(y_i - y_j)) / \binom{n}{2}$$
 (11)

where sign(.) is a symbolic function; The parameter  $\alpha$  can be obtained from the  $\tau_{\alpha}$  value. The estimated value of  $\alpha$  in Clayton

is 
$$\hat{\alpha} = \frac{2\hat{\tau}_{\alpha}}{1 - \hat{\tau}_{\alpha}}$$
, and in Gumbel is  $\hat{\alpha} = \frac{1}{1 - \hat{\tau}_{\alpha}}$ , and in Frank is

obtained with maximum likelihood estimation (MLE).

We used the AIC value to determine the most appropriate copula. If the marginal distribution of the random vector  $(t_1, t_2,$ ...,  $t_n$ ) are  $F_1(t)$ ,  $F_2(t)$ , ...,  $F_n(t)$ , the AIC is calculated as:

$$AIC = -2\sum_{i=1}^{N} \ln c(F_{1i}, F_{2i}, ..., F_{ni}) + 2k$$
 (12)

where N is the sample data size; k is the number of parameters in copulas. Among all alternative Copulas, the best fitting model has the smallest AIC value.

#### III. CASE STUDY

## Simulation

In this paper, an IGBT device, IRGB4059DPbF, was used as the case study. We considered possible failures of the device under a thermal cycling condition. The PoF models in section 2 were applied.

The FEA was used to obtain parameters in PoF models, including the temperature, stress, and strain. The material parameters for simulation were shown in Table I:

TABLE I. MATERIAL PRAMETERS OF THE CASE

	Thermal Parameters			
Material	Density Thermal Conductivity (Kg/m³) (W/(m·k))		CTE×10 <sup>-6</sup> (K <sup>-1</sup> )	
Si	2330	118	2.9	
SnAg	7360	33	30	
Cu	8700	380	17	
Al	2700	238	23	
Epoxy	1211	0.67	59	
Material	Specific heat (J/(Kg·K))	Young's modulus (GPa)	Poisson's ratio	
Si	690	130	0.28	
SnAg	180	13.8	0.35	
Cu	385	110	0.34	
Al	900	70	0.33	
Epoxy	500	9.3	0.38	

We selected the Anand model to describe solder material properties. The values of Anand parameters were shown in Table II.

TABLE II. ANAND MODEL PARAMETERS

symbol	S <sub>\theta</sub> (Pa)	Q/R (K)	A (s <sup>-1</sup> )	ξ	m
value	4.57×10 <sup>7</sup>	7460	5.87×10 <sup>6</sup>	2	0.0942
symbol	h <sub>θ</sub> (Pa)		Ŝ (Pa)	а	n
value	9.35×10 <sup>9</sup>	5.83×10 <sup>7</sup>		1.5	0.015

The internal power consumptions of the IGBT chip and diode chip were 4W and 0.1W. Referring to the thermal cycling test condition (shown in Fig. 4) in MIL-STD-750H, we applied the transient thermal module to simulation.

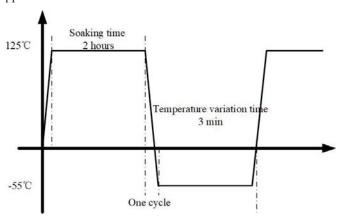


Figure 4. Temperature profile used in the simulation

The heat dissipation of the heat sink was equivalent to the forced convection on the bottom surface. A fixed support was applied to the bolt connection and pins to match the actual installation conditions, as shown in Fig. 5.

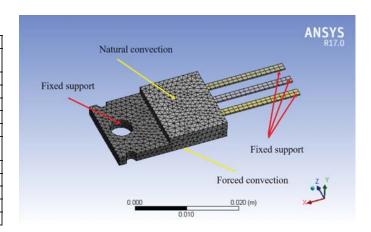


Figure 5. Schematic diagram of the set constraints

According to the thermal resistance of the radiator, the forced convection coefficient could be determined as:

$$U = \frac{1}{RA} = \frac{1}{17.9^{\circ}\text{C}/W \times 295.15mm^{2}} = 190W/(mm^{2} \cdot ^{\circ}\text{C}) (13)$$

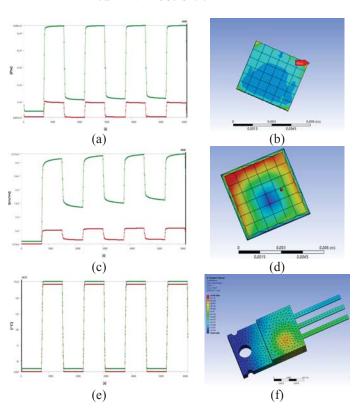


Figure 6. Simulation results under temperature cycling condition (a) (b) the equivalent stress curve and stress distribution (c) (d) the equivalent strain curve and strain distribution (e) (f) the temperature curve and temperature distribution

The temperature of the device, the equivalent stress and plastic strain distribution of the solder layer are shown in Fig. 6 (f) (b) (d). The curves of temperature, equivalent stress and plastic strain with time are shown in Fig. 6 (e) (a) (c).

The structural temperature, equivalent stress, and plastic strain data were shown in Table III.

Structure	Average Cycle Temperature (°C)	Temperature Variation Range (°C)
IGBT chip	90.85	150.79
Diode chip	87.66	148.57
Solder layer (IGBT chip)	90.37	149.76
Solder layer (diode chip)	87.58	148.59
Packaging	90.02	150.79
substrate	89.82	149.67
Bonding wire	90.38	150.53
Pin1	89.03	150.46
Pin2	88.55	150.10
Pin3	86.00	146.72

## B. Lifetime distribution

The geometry and material thermal parameters were sampled as triangular distributions, and the temperature parameters were sampled as normal distributions. The lifetime distribution corresponding to each FM is obtained by the MC algorithm, and Gaussian kernel density function were used to describe their characteristics. The density function and distribution function are expressed as (14) (15):

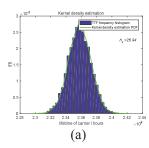
$$f_X(x) = \frac{1}{\sqrt{2\pi}nh_X} \sum_{i=1}^n e^{-\frac{1}{2}\left(\frac{x-x_i}{h_X}\right)^2}$$
 (14)

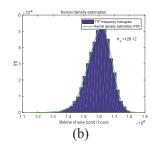
$$F_{X}(x) = \int_{-\infty}^{x} f_{X}(x)$$
 (15)

where  $h_x$  represents the bandwidth, and it can be calculated as:

$$h_x = 0.9 \times N^{-0.2} \times \min(std, iqr/1.34)$$
 (16)

where, N is the sample size; std is the sample standard deviation; iqr is the quartile in the sample sequence, and the lifetime distributions of three FMs are shown in Fig. 7.





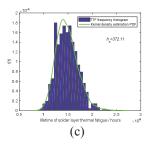


Figure 7. Lifetime distribution under three FMs (a) residual carrier lifetime distribution (b) bonding wire lifetime distribution (c) solder layer thermal fatigue lifetime distribution

# C. Reliability assessment

After obtaining three marginal distributions, we used Gumbel, Clayton, and Frank copulas to assess the reliability. We used (11) to obtain the  $\tau_{\alpha}$  and the parameters  $\alpha$ , and AIC values among three marginal distributions are as Table. IV.

TABLE IV. COMPARISON OF THREE COPULAS

Copulas	$ au_{lpha}$	α	AIC
Clayton		0.444	-3162.86
Gumbel	0.1816	1.222	Inf+Infi
Frank		1.175	229.0301

The AIC results showed that Clayton copula was the optimal function, and the reliability assessment of the case was calculated according to (10). The reliability calculation results of this method compared with the MCC method were shown in Fig. 8.

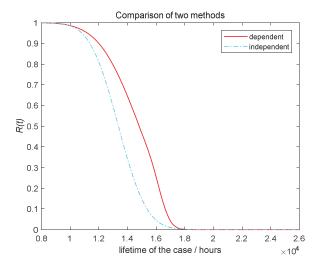


Figure 8. Overall reliability with three PoF models

It can be seen from Fig. 8 that the reliability of considering different structure's lifetime correlation is higher than that of considering the independent hypothesis. MTTFs of two methods are shown in Table V, and it indicates that ignoring the correlation of PoF methods results may misjudge the lifetime of the case. Comparing the operation time of the two methods when the number of samples is 50000, it can be seen that the multivariate copula method is faster than the MCC.

TABLE V. COMPARISON OF CALCULATION RESULTS BETWEEN THE TWO METHODS

Assessment Method	MTTF (hours)	Total Operation Time (s)
Multivariate copula	14800	0.378
MCC	13300	0.818

## CONCLUSION

Based on PoF methods, this paper studied the reliability assessment of power devices mainly under thermal conditions by using multivariate copulas. Firstly, the internal structures of a device were used to analyze FMs and the corresponding PoF models. For the unknown temperature, stress, and strain parameters in PoF models, we used the FEA. Secondly, considering the failure of multi-structures and the common influencing factors of multi-FMs, an assessment methodology based on multivariate copulas was established. The innovation of this paper is to regard the device as a system, and the FMs are connected in series. At the same time, considering the correlation introduced by each FM due to the cross-influence factors, the reliability function of the device is derived by using multivariate copulas. The results show that the reliability of considering the correlation of PoF model results is higher than that of the MCC method. The completely independent hypothesis may misjudge the lifetime. And the operation time of this proposed method is shorter than the MCC. It provides a new idea for power devices' reliability assessment.

However, this paper only evaluates the reliability of a discrete device. We will continue applying this method to

evaluate the reliability of power modules to aid the design improvement of device structures.

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