A multi-stage preventive maintenance optimization model for products with rectangular warranty region

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***Abstract*—According to the age and the usage, a rectangular warranty region is divided into four nonoverlapping sub-regions. Preventive maintenance activities with different levels are performed over these sub-regions. In the sub-regions, which cover the early and last stages of the warranty regions, the maintenance is minimal. In the second sub-region, an imperfect and periodic two-dimensional preventive maintenance strategy is performed. While in the third sub-region, perfect and periodic two-dimensional preventive maintenance actions are scheduled. The age and usage intervals for the two periodic two-dimensional strategies are optimized by minimizing the associated cost. A numerical example is given to illustrate the application of the strategy.**

Keywords-two-dimensional warranty; preventive maintenance; multi-stage; expected costs

I. INTRODUCTION

It is crucial for manufacturers to making scientific and reasonable decisions on maintenance strategies[1]. Modelling and optimization of maintenance strategies for products with two-dimensional warranty region has received more attention in the field of reliability management. Most researches focus on multi-stage or imperfect preventive maintenance([2][3]).

In [4], the given rectangular warranty region Ω is divided into three disjoint sub-regions. They assume that repairs in the first and last sub-regions are minimal. While the first repair in the middle sub-region is complete and the others are minimal. The partitioning of the whole warranty region is optimized by minimizing the warranty cost under the condition that $U_1/W_1 = U_2/W_2$. The model is extended by [5]. Recently, a great deal of decision researches on preventive maintenance(PM) strategies for products sold with two-dimensional warranty servicing has been done([6-8]). In the most literatures, PM activities are optimized from the dimension of item age or usage. In [9], a periodic two-dimensional preventive maintenance strategy was proposed. Under the policy, the interval of PMs is characterized by usage or age(whichever occur first).

By dividing the rectangular warranty region into four nonoverlapping sub-regions, a multi-stage preventive maintenance strategy is introduced in this paper. Minimal, perfect and imperfect preventive maintenance are adopted in these sub-regions. This paper extends the one-stage two-dimensional preventive maintenance strategy into multi-stage case.

The paper is organized as follows. In Section 2 assumptions of the model are presented. The cost incurred during the whole warranty period is obtained in Section 3. A numerical example is given to illustrate the application of the strategy in Section 4. Section 5 concludes the paper.

Notations used in this paper are listed in the following.

t, u	age and usage of the product
W, U	age and usage limits of 2D warranty strategy
$W_i, U_i \ (i=1,2,3)$	age and usage limits of Ω_i period
K, L_1	age and usage intervals of imperfect 2D PM strategy over Ω_2
K, L_2	age and usage intervals of perfect 2D PM strategy over Ω_3
r	usage rate
$\lambda(t \mid r)$	conditional intensity function given usage rate r
m	level of imperfect PM effort $(0 < m < M)$
$C_p(m)$	imperfect PM cost with level m
C_t	perfect PM cost
C_r	expected minimal repair cost

II. Model formulation

Assumptions of the model are as below.

(1) A two-dimensional rectangular warranty region of a product, Ω, is divided into four stages, Ω_1 Ω_2, and Ω_3, Ω_4, where $\eta_1 = U_1/W_1, \eta_2 = U_2/W_2, \eta_3 = U_3/W_3, U/W = \eta,$ and $\eta_1 = \eta_2 = \eta_3 = \eta$, as shown in Fig. 1.

(2) The manufacturer offers a multi-stage maintenance

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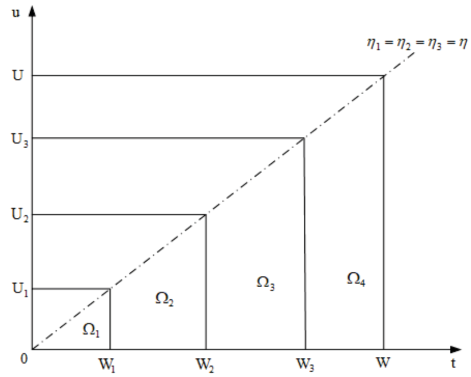

Figure 1. Regions characterizing the multi-stage preventive maintenance strategy

strategy and paid for it. All failures in Ω_1 and Ω_4 are minimally repaired. PM activities are performed every K units of age or L_1 units of usage and K age or L_2 usage (whichever occurs first) in Ω_2 and Ω_3, respectively. Furthermore the maintenance activities in Ω_2 are imperfect and those in Ω_3 are perfect. The repair for failures which occur between two successive PMs is minimal. The time for minimal, imperfect and perfect maintenance and PM actions is assumed to be negligible.

(3)A non-homogeneous Poisson process is used to model the product failures process. The intensity function of the process depends on both age and usage. Let $U(t)$ be the total usage of a product at age t. Given the usage rate $R=r$ and $U(t)=rt$, the conditional intensity function is assumed to be a polynomial function as follows(see [10][11])

$$\lambda(t\,|\,r)=\theta_0+\theta_1 r+(\theta_2+\theta_3 r)t, \theta_i>0(i=0,1,2,3)$$

(4) Suppose that PM actions are performed at actual age $\tau_1, \tau_2, \cdots \tau_j, \cdots$, with $\tau_0=0$. If a PM with level m $(m=0,1,\cdots,M)$ is performed at the j th PM, the product's virtual age immediately after it

$$v_j=v_{j-1}+\delta(m)(\tau_j-\tau_{j-1})$$

where $\delta(m)=(1+m)e^{-m}$ ([12]). Assume that constant effort level is u over Ω_2.

III. Cost Analysis

A. Age and virtual age analysis

Given the usage rate $R=r$, a product's age at $\Omega_i(i=1,2,3)$ expiry is $W_i^r=\min\{W_i, U_i/r\}$. The age at Ω_4 expiry $W^r=\min\{W, U/r\}$. Under the proposed PM strategy, imperfect PM interval over Ω_2 is given by $K_1^r=\min\{K, L_1/r\}$. Therefore the number of imperfect PM actions performed over Ω_2 can be obtained as $n_1^r=\max\{j\,|\,W_1^r+jK_1^r\le W_2^r, j\ge 0\}$. Similarly, the number of perfect PM actions over Ω_3 is given by $n_2^r=\max\{j\,|\,W_2^r+jK_2^r\le W_3^r, j\ge 0\}$, where $K_2^r=\min\{K, L_2/r\}$.

Over Ω_2, the virtual age of the product after the j th PM action is $v_j^r = j\delta(m)K_1^r$, where $j=1,2,\cdots,n_1^r$ ([11]).

B. Cost analysis for various cases

Suppose that $C(\Omega)$, $C(\Omega_i)(i=1,2,3,4)$ is the warranty cost over Ω, $\Omega_i(i=1,2,3,4)$ respectively. C_r, C_p and C_t are costs for minimal repair, imperfect PM cost with level m and perfect PM cost respectively. Let $c_1=L_1/K$, $c_2=L_2/K$ and assume that $c_1<c_2$. Considering all possible orderings among η, c_1 and c_2, the following three cases are needed to be investigated, $c_1<c_2\le\eta$ (case 1), $c_1\le\eta<c_2$ (case 2) and $\eta<c_1<c_2$ (case 3). For case 1, four subcases are needed to be investigated. They are $r<c_1$, $c_1\le r<c_2$, $c_2\le r<\eta$ and $\eta\le r$ respectively.

In the case $r<c_1$, $W_1^r=W_1$. Due to $r<\eta$, the mean warranty cost over Ω_1

$$E[C(\Omega_1)\,|\,r<c_1]=C_r\int_0^{W_1}\lambda(t\,|\,r)dt$$

The number of imperfect PM actions performed during Ω_2

$$n_1=\max\{j\,|\,W_1+jK\le W_2, j\ge 0\}$$

Therefore, the conditional number of failures over Ω_2

$$E\left[N_1(\Omega_2)\,|\,r<c_1\right]=\sum_{j=0}^{n_1-1}\int_{W_1+j\delta(m)K_1}^{W_1+j\delta(m)K_1+K_1}\lambda(t\,|\,r)dt + \int_{W_1+n_1\delta(m)K_1}^{W_1+n_1\delta(m)K_1+W_2-n_1K_1}\lambda(t\,|\,r)dt$$

Combing the aforementioned costs, the cost over Ω_2, given $R=r<c_1$,

$$E\left[C(\Omega_2)\,|\,r<c_1\right]=C_r E\left[N_1(\Omega_2)\,|\,r<c_1\right]+n_1 C_p(m)$$

In an similar manner, the number of PM activities, the conditional number of failures and the cost over Ω_3, given $R=r<c_1$, can be given as

$$n_2 = \max\{j \mid W_2 + jK_2 \le W_3, j \ge 0\}$$

$$E[N_1(\Omega_3) \mid r < c_1] = \int_{v(W_2)}^{v(W_2)+K_2} \lambda(t \mid r)dt + (n_2 - 1)\int_0^{K_2} \lambda(t \mid r)dt + \int_0^{W_3 - W_2 - n_2 K_2} \lambda(t \mid r)dt$$

$$E[C_1(\Omega_3) \mid r < c_1] = C_r E[N_1(\Omega_3) \mid r < c_1] + n_2 C_t$$

where $v(W_2) = W_1 + n_1\delta(m)K_1 + W_2 - n_1 K_1$ is the virtual of the product at expire of Ω_2. The expected warranty cost over Ω_4, conditional on $R = r < c_1$,

$$E[C_1(\Omega_4) \mid r < c_1] = C_r \int_{W_3 - W_2 - n_2 K}^{W - W_2 - n_2 K} \lambda(t \mid r)dt$$

From Equations (3)-(6), the warranty servicing cost over Ω, given $R = r < c_1$, $E[C(\Omega) \mid r < c_1]$ can be given.

Let $E[C(\Omega) \mid c_1 \le r < c_2]$, $E[C(\Omega) \mid c_2 \le r < \eta]$, $E[C(\Omega) \mid \eta \le r]$ be the associated conditional servicing cost over Ω. They can be presented in an manner to that of $E[C(\Omega) \mid r < c_1]$. Furthermore, the total warranty cost for case 1 can be calculated by

$$\begin{aligned} &E[C_1(\Omega)] \\ &= \int_0^{c_1} E[C(\Omega) \mid r < c_1] g(r)dr + \int_{c_1}^{c_2} E[C(\Omega) \mid c_1 \le r < c_2] g(r)dr \\ &+ \int_{c_2}^{\eta} E[C(\Omega) \mid c_2 \le r < \eta] g(r)dr + \int_{\eta}^{\infty} E[C(\Omega) \mid \eta \le r] g(r)dr \end{aligned}$$

Similarly, the cost for the other two cases can be proposed.

IV. NUMERICAL EXAMPLE

A numerical example is presented to illustrate the effectiveness of the proposed multi-stage PM optimization model. The limits for Ω, Ω_1, Ω_2, Ω_3 and Ω_4 and the other parameters are shown in Table 1.

Let $\lambda(t \mid r) = 0.1 + 0.2r + (0.7 + 0.7r)t$. The usage rate R is assumed to be uniformly distributed over $[0.1, 2.9]$. The decision variables are K, L_1 and L_2. We perform a grid search for the minimization of $E[C(\Omega)]$. See Table 2 for detail.

From Table 2, the optimal multi-stage PM strategy is $(K^*, L_1^*, L_2^*) = (3.6, 1.5, 1.8)$ and the associated cost is $E[C^*(\Omega)] = 235.20(\$)$. This indicates that the warranty cost is minimized when imperfect PM activities are performed according to 3.6 months interval or 1.5×10^3 usage interval, whichever occurs first within Ω_2, and 3.6 months interval or 1.8×10^3 usage interval, whichever occurs first within Ω_3.

TABLE I. THE PARAMETER SETTINGS(W and $W_i (i = 1,2,3)$ in year, U and $U_i (i = 1,2,3)$ in 10^4 km)

Parameter	Value
$W = U$	3
$W_1 = U_1$	0.5
$W_2 = U_2$	2
$W_3 = U_3$	2.5
m	3
C_r	40
C_p	60
C_t	100

TABLE II. OPTIMAL MULTI-STAGE PM STRATEGIES FOR DIFFERENT AGE INTERVALS(in K month, $L_i^* (i = 1,2)$ in 10^3 km)

K	L_1^*	L_2^*	$E[C^*(\Omega)]$
1.2	0.1	1.1	654.19
2.4	1.0	1.2	269.00
3.6	1.5	1.8	235.20
4.8	2.8	3.2	236.64
6.0	2.0	2.5	251.18

V. CONCLUSIONS

A multi-stage and 2D PM strategy for products with two-dimensional rectangle warranty period is proposed. The strategy is characterized by providing periodic imperfect and perfect two-dimensional PM activities over nonoverlapping sub-regions. The numerical example illustrates its effectiveness.

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