Selection of fundamental period components of vibration modeling for non-faulty planetary gearboxes

Xinguang Ma
Nuclear Power Institute of China
Chengdu, China
guang880@qq.com

Li Zhan
Nuclear Power Institute of China
Chengdu, China
78517996@qq.com

Jie Liu
Nuclear Power Institute of China
Chengdu, China
liujie_cult@163.com

Lin Zhang
Nuclear Power Institute of China
Chengdu, China
castalzhang@126.com

Jun Liu
1. Tianjin Key Laboratory of the Design and Intelligent Control of the Advanced Mechanical System, Tianjin University of Technology, China
2. National Demonstration Center for Experimental Mechanical and Electrical Engineering Education, Binshuixidao 391, Xiqing District, Tianjin, 300384, China
liujunjp@tjut.edu.cn

Mian Zhang*
1. Tianjin Key Laboratory of the Design and Intelligent Control of the Advanced Mechanical System, Tianjin University of Technology, China
2. National Demonstration Center for Experimental Mechanical and Electrical Engineering Education, Binshuixidao 391, Xiqing District, Tianjin, 300384, China
zoommian@foxmail.com

***Abstract*—Planetary gearboxes are 'hearts' of transmission train among heavy industrials. The health condition of planetary gearboxes is the key to maintain reliable operation of industrial and promote harmonious social development. Frequency spectrum analysis of vibration measurements is a popular way for condition assessing. As prior guidance for spectrum analyzing, phenomenological modeling emerges as a simple and effective mathematical description of the vibration measurements referring to the sensor. A reasonable model of a non-faulty planetary gearbox is the essential work, either in understanding the motion behavior, or in condition monitoring and even for fault diagnosis. In this paper, traditional models are reexamined and investigated: we found the contradicting results from two classical references on the selection of fundamental period components when constructing models. The rationality of the traditional models is mathematically and experimentally studied, the more reasonable one is indicated.**

Keywords-Planetary gearboxes; Phenomenological model; Fundamental period components; Vibration analysis

I. Introduction

Assembling multiple planet gears archives planetary gearboxes with merits of compactness, large transmission ratio, and high capacity [1,2]. They are widely employed to transmit power among a broad range of industrial and military applications [3,4]. However, planetary gearboxes are vulnerable part due to the complicated structural and harsh working environment [5,6], resulting in tragedies or catastrophic failures of the entire system, in turn, generate enormous economic loss and social panic. Condition monitoring of planetary gearboxes is a heated topic to maintain reliable operation of the industry and promote harmonious social development.

Analysis and measures of vibrations from planetary gearboxes are of vital importance, either in a viewing of reducing equipment noise, or monitoring the mechanical condition and fault diagnosis. Vibration modeling is widely used in condition monitoring, which provides the theoretical guidance for the understanding of a system motion behavior, the vibration features, and the spectrum analyzing. Phenomenological modeling is a typical pattern to describe the vibration measurements with regard to the sensor directly. With the construction of some periodic functions, phenomenological modeling provides a simple and effective way of representing the system response.

The investigation of phenomenological modeling of non-faulty planetary gearboxes began early in the 1980s: McFadden and Smith [7] built a model to study the asymmetrical phenomena of the frequency spectrum. Several years later, McNames [8], Mosher [9], Vicuna and Parra [10] further studied the model in Ref. [7] to investigate the connection between the mechanical structure and the distribution of the sidebands in the frequency spectrum. On the basis of the non-faulty model, some researchers considering the mechanisms of gear fault and inducing the fault effect to explore the characteristics of fault induced vibrations [12]. Additionally, Zhang et al. [6,14] deduced the theoretical mechanism of a frequency domain indicator based on the phenomenological model, and successfully applied the indicator to diagnose faults occurring on sun gear or planet gear.

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In light of the above review, researchers have made valuable progress on the modeling and spectrum analysis. A reasonable phenomenological model of a non-faulty planetary gearbox is the foundation of the understanding of system vibration behaviors and fault diagnosis. Typically, the selection of the fundamental periodic functions is the critical process for constructing a proper model. According to our literature survey, McNames in Ref. [8] pick N (N represents the number of planet gears) copies rotational frequency of the planet carrier as the fundamental components. However, the fundamental periods of McNames model [8] are contradicting with the model of McFadden and Smith [7]. In this paper, we will mainly discuss the rationality of the two models through mathematical derivation and experimental data.

The structure of this paper is organized as follows: Section II gives the discussion of phenomenological modeling, including basic concepts and analysis of fundamental frequency components; Section III provides experimental studies to demonstrate the rationality of the selection of fundamental frequency components; Finally, Section IV concludes the whole paper.

II. Discussion on Phenomenological modeling

A. Basic conceptrual

Normally, gear meshing processes and transfer effects are two main factors in phenomenological modeling of planetary gearboxes. When a planetary gear system contains one planet gear, as is shown in Figure 1 :

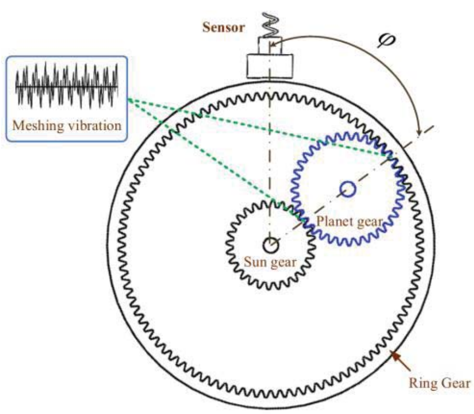

Figure 1. Schematic of the planetary gear system with one planet gear

Fig. 1 exhibits a planet gear system consisting of one planet gear, a sun gear, and a fixed ring gear. The sensor is mounted on the housing for data acquiring. $\varphi \in [0, 2\pi)$ denotes the initial phase angle between the planet gear and the sensor. According to the existing ref. [7], due to the sensor mounts close to the ring gear, the vibration from the planet gear and the ring gear is dominated components captured by the sensor. Therefore, we also focus on the meshing vibration generated by the planet gear and ring gear.

Along with the operating of the system, the planet gear will constantly mesh with the ring gear, that is the meshing frequency, which can be expressed as follow [7]:

$$f_m = Z_r f_c \quad (1)$$

where f_m represents the meshing frequency; Z_r represents the teeth number of ring gear; f_c represents the rotational frequency of the carrier.

Due to the imperfect profile from errors in design or manufacture on gear teeth, the meshing vibration will generate both the meshing frequency and its harmonics. In this regard, the meshing function can be represented via Fourier series [8,9]:

$$m_0(t) = \sum_{l=-\infty}^{\infty} m_{0,l} e^{-j2\pi l f_m t} \quad (2)$$

where $m_0(t)$ denotes the meshing function with respect to the time series; $m_{0,l}$ denotes the Fourier coefficient of frequency component, lf_m; l is integer number.

Once the gear meshing vibration generated, the vibration will transmit a length of distance through ring gear to the sensor. The transmission distance depends on the phase angle, φ. Additionally, relative to the one revolution of the planet gear, the planet gear will arrive at its initial position. Therefore, the period of transfer function relies on the rotational frequency of the carrier, that is the revolution frequency of the planet gear. Consequently, the transfer function can also be represented via Fourier series [8,9]:

$$w_0(t) = \sum_{k=-\infty}^{\infty} w_{0,k} e^{-j2\pi k f_c t} \quad (3)$$

where $w_0(t)$ means the transfer function refer to the time series; $w_{0,k}$ means the Fourier coefficient of the frequency component, kf_c.

Referring to the sensor, the captured vibrations is the product result of $m_0(t)$ and $w_0(t)$. Furthermore, considering any position of the planet gear, the model of sensor captured vibration can be represented as:

$$v_1(t) = m_0(t - \frac{\varphi}{w_c}) w_0(t - \frac{\varphi}{w_c}) = m_1(t) w_1(t) \quad (4)$$

where $v_1(t)$ denotes the model of senor captured vibration caused by one planet gear; w_c denotes the angular speed of planet carrier. $\frac{\varphi}{w_c}$ can get the time delay of any value of φ.

When considering the system equally assembles N planet gears, that is the phase differences of each planet gear is $\frac{2\pi}{N}$. Assume that the pattern of vibration and transfer function of each planet gear is same, but with different phase. In such a scenario, the vibration generated by i^{th} ($i \neq 1$) planet gear can be represent by the 1^{st} planet gear with time delay t_i:

$$v_i(t) = m_1(t - t_i)w_1(t - t_i) \tag{5}$$

where $v_i(t)$ is the sensor captured vibration of i^{th} planet gear; $t_i = \dfrac{2\pi(i-1)}{Nw_c}$.

Generally, modeling of multiple planet gears could be represented as the linear summation of the vibration generated by each planet gear [7-9]:

$$v(t) = \sum_{i=1}^{N} v_i(t) \tag{6}$$

where $v(t)$ represents the phenomenological model of multiple planet gears.

B. Selection of fundamental period components

McFadden and Smith in Ref. [7] mainly introduce the conceptual of the phenomenological model process. However, they did not analysis the frequency components and the amplitudes in the model.

Considering the Fourier coefficient $m_{0,l}$ and $w_{0,k}$ in Eq. (2) and Eq. (3), for i^{th} planet gear with the time delay t_i, which can be further represented as follows:

$$m_{i,l} = | m_{i,l} | e^{-j2\pi l f_m t_i} = | m_{1,l} | e^{-j2\pi l Z_r \frac{(i-1)}{N}} \tag{7}$$

$$w_{i,k} = | w_{i,k} | e^{-j2\pi k f_c t_i} = | w_{1,k} | e^{-j2\pi k \frac{(i-1)}{N}} \tag{8}$$

where $|\cdot|$ means the modulus process; $m_{i,l}$ and $w_{i,k}$ mean the Fourier coefficient of meshing function and transfer function of i^{th} planet gear. As we have mentioned that the vibration patterns are deemed identical for each planet gear, therefore we can get: $| w_{i,k} | = | w_{1,k} |$ and $| m_{i,l} | = | m_{1,l} |$.

In light of the above 2 equations, the vibration model of $v_i(t)$ in Eq. (5) can be further developed as:

$$v_i(t) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} | m_{1,l} | | w_{1,k} | e^{-j2\pi(lZ_r+k) f_c t} e^{-j2\pi(lZ_r+k)\frac{(i-1)}{N}} \tag{9}$$

Eq. (9) tells that the fundamental frequency generated by i^{th} planet gear is $(lZ_r + k) f_c$, with the initial phase of $2\pi(lZ_r + k)\dfrac{(i-1)}{N}$, and amplitude of $| m_{1,l} | | w_{1,k} |$ correspondingly. Furthermore, the vibration model of multiple planet gears, $v(t)$, is the linear summation of i^{th} planet gear:

$$\begin{aligned} v(t) &= \sum_{i=1}^{N} v_i(t) \\ &= \sum_{i=1}^{N} \sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} | m_{1,l} | | w_{1,k} | e^{-j2\pi(lZ_r+k) f_c t} e^{-j2\pi(lZ_r+k)(\frac{i-1}{N})} \end{aligned} \tag{10}$$

From Eq. (10), after interpretation of the model with multiple planet gears proposed by McFadden and Smith [7], $v(t)$ also contains the fundamental frequency of $(lZ_r + k) f_c$ with the amplitude of $| m_{1,l} | | w_{1,k} |$. The only difference with $v_i(t)$ is the initial phase of the fundamental period components.

On the other hand, McNames [8] further developed the model of McFadden and Smith [7]. McNames recognized that the model of multiple planet gears should contain N copies of each planet gear vibration. In such a scenario, the model of multiple planet gears should be built with the fundamental frequency of Nf_c, that is [8],

$$v(t)_{McN} = N \sum_{k=-\infty}^{\infty} \alpha_{1,k} e^{j2\pi kNf_c t} \tag{11}$$

where $v(t)_{McN}$ denotes the model proposed by McNames [8]; $\alpha_{1,k}$ denotes the Fourier coefficient of frequency components kNf_c; k is an integer number. According to Eq. (8) and Eq. (11), fundamental frequency components of the two models are listed in the following table:

TABLE I. SELECTION OF FUNDAMENTAL FREQUENCY

Model types	Fundamental frequency	Comments
$v(t)$	$(lZ_r + k) f_c$	Derivation of McFadden and Smith [7]
$v(t)_{McN}$	kNf_c	Proposed by McNames [8]

From TABLE I , it is not difficult to find that when $\dfrac{lZ_r + k}{N}$ is an integer, Nf_c is the particular case of $(lZ_r + k) f_c$. In order to validate the correctness of the fundamental frequency selection of the two models, the frequency spectrum will be analyzed through experimental data.

III. EXPERIMENTAL STUDY

The experimental studies operated on a planetary gearbox from a Drivetrain Dynamics Simulator (DDS) platform, at University of Electronic Science and Technology of China (UESTC), laboratory of Equipment Reliability and Prognostic and Health Management (ERPHM). The configuration of the test rig is shown in the below:

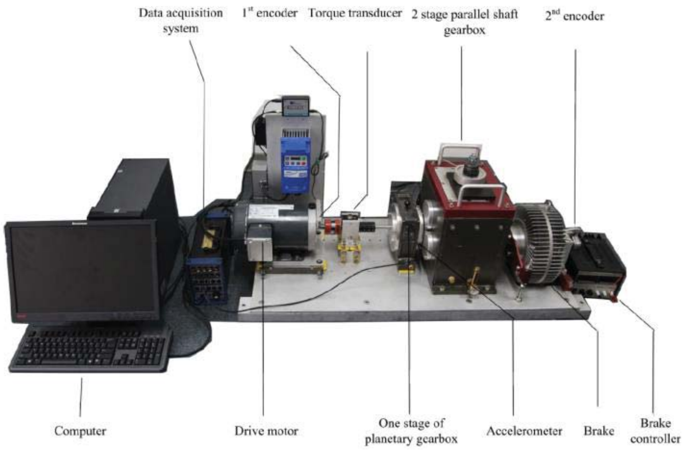

Figure 2. Configuration of DDS platform.

Fig. 2 shows that the DDS platform consisting of a 2.24kW three-phase electronic drive motor controlling the input speed, connecting one stage of a planetary gearbox, then linking a parallel shaft gearbox, and a brake at the end for load application. Two accelerometers are mounted on the planetary gearbox to collect vibration data. The assemble parameters of the planetary gearbox are listed as follow:

TABLE II. ASSEMBLED PARAMETERS OF THE PLANETARY GEARBOX

Item	Value	Symbol
Teeth number of sun gear	28	Z_s
Teeth number of plaent gear	36	Z_p
Teeth number of ring gear	100	Z_r
Number of planet gear	4	N

The rotational frequency of the motor was setting as 30Hz. The meshing frequency and the rotational frequency of the planet carrier can be calculated based on the expression in Ref. [6]:

TABLE III. VALUE OF CHARACTERISTIC FREQUENCIES

Item	Expression &Value	Symbol
Rotational frequency of sun gear	Setting=30Hz	f_{shaft}
Rotational frequency of planet carrier	$\frac{Z_s}{Z_s+Z_r} f_{shaft} = 6.5625Hz$	f_c
Meshing frequency	$f_c Z_r = 656.25Hz$	f_m

TABLE IV. VALUES OF FUNDAMENTAL FREQUENCY COMPONENTS OF TWO MODELS

Model types	Values of Fundamental frequency components
$v(t)$	$(100l+k)6.5625$ Hz
$v(t)_{McN}$	$k26.25$ Hz

According to TABLE II and TABLE III the fundamental frequency components of TABLE I can be determined in the following table:

Four groups of vibration data from the planetary gearbox are collected with the sampling frequency of 10240Hz, with the time length of 10 seconds. Take the captured data from the vertical direction for analyzing, the frequency spectrum of four groups is exhibited in the below:

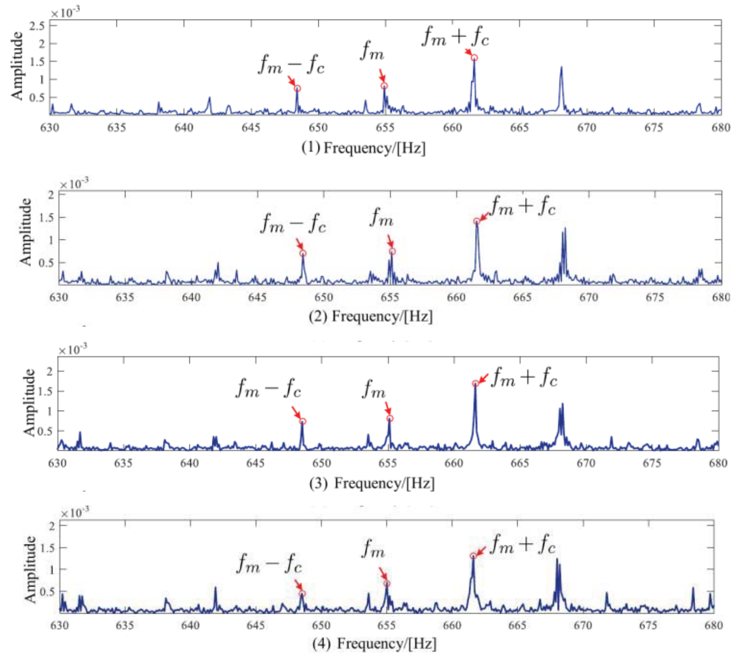

Figure 3. Frequency spectrum of 4 groups data

Fig. 3 demonstrates that f_m and $f_m \pm f_c$ are dominated in all 4 spectral of the expreimental data, indicating the characteristic frequency components of the vibration response of the planetary gear system. Referring to TABLE IV. $f_m + f_c$ equals to 662.8125Hz; and $f_m - f_c$ equals to 649.6875Hz. According to TABLE III. the fundamental frequency of $x(t)_{McN}$ proposed by McNames, namely k26.25Hz, we can not find any integer value of k that equal to $f_m \pm f_c$. Therefore, $x(t)_{McN}$ loses its ability to represent the vibration response of the planetary gear system.

On the other hand, the fundamental frequency of $v(t)$, which derived from McFadden and Smith [7], *(100l+k)*6.5625 Hz: It can be found that when l=1, k=$\pm$1 which are suitble to the frequency components of $f_m \pm f_c$. Consequently, $v(t)$ is more reasonable than $x(t)_{McN}$ in the vibration description of planetary gear system.

IV. CONCLUSION

Selection of fundamental period components is the critical process when constructing a phenomenological model of planetary gearboxes. In this paper, traditional models of McFadden and Smith [7], and McNames [8] are reexamined and mathematically analyzed. The fundamental period components selection of the two models are evaluated based on experimental study. The results show that the model of McFadden and Smith is more reasonable than McNames's model.

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