Reliability Assessment by a Wiener Process that Integrates Different Degradation Datasets

Zhongyi Cai, Zezhou Wang
Equipment Management & UAV Engineering College,
Air Force Engineering University, Xi'an, 710051, China
afeuczy@163.com

Lin Deng
No. 29 Research Institute of China Electronics
Technology Group Corporation, Chengdu, 610063, China

***Abstract*—A Wiener process based on field degradation data is used to describe the product degradation process. By integrating the accelerated degradation data of similar products with target-product field-measured degradation data, a new reliability assessment method is proposed. Given the difference between field and laboratory stress environments, a Wiener process model with calibration factors is built. A degradation model for similar products and a target product is built to obtain estimates of the distribution parameters under each type of stress. An accelerated factor is used to convert the estimates obtained under accelerated stress into estimates representative of regular stress, which constitutes the data sample of the prior distribution's parameters. A Bayesian inference method is used to obtain the posterior distribution parameters using the field degradation data of a target product. A Markov chain Monte Carlo algorithm is used to obtain the estimates of the posterior distribution parameters. The accuracy of the proposed method is verified by an example.**

Keywords—Bayesian inference; Wiener process; degradation data integration; stress environment difference; MCMC method.

I. INTRODUCTION

Highly reliable products cannot produce enough failure data in field stress environments, which renders traditional failure assessment methods inadequate. However, by measuring key product performance data, product reliability and lifetime data can be obtained.

At the same time, the accelerated degradation test (ADT) data of similar products can be used to enrich degradation data samples and improve assessment accuracy; however, the following problems should be taken into account.

The first aspect is the differences in the stress environments of the field and laboratory. The solution is to use a generalized accelerated model to describe field stress environments so as to modify the product distribution parameter [2-5]. Wang et al. [2] put forward a modified degradation model parameter based on a generalized accelerated model to measure stress environment differences. Tan et al. [5] proposed a modified lifetime distribution parameter to measure the Weibull distribution.

The second aspect is variation in degradation among individual products. Because of random factors in manufacturing, degradation rates vary among different products. The solution is to randomize the Wiener process parameters [6-10]. Wang et al. [8] randomized two Wiener process parameters and assumed that prior distribution parameters obey a normal-inverse gamma distribution. Tang et al. [10] thought that individual degradation variation is mainly caused by a drift coefficient, which they assumed obeys a normal distribution. Although this can be calculated easily, it ignores the influence of a diffusion coefficient on the product degradation process.

The third aspect is the integration of field-measured data with ADT data. The solution is to use Bayesian methods to build a parameter estimation model and use expectation maximization (EM) or a Markov chain Monte Carlo (MCMC) algorithm to obtain the unknown parameter estimates. Wang et al. [2] assumed that the prior distribution parameter is known and its posterior parameter mean is obtained by the MCMC algorithm by integrating ADT and accelerated life test (ALT) data with field data. The disadvantage of this method is its lack of consideration of choosing a prior distribution.

This paper proposes a reliability assessment method that integrates the accelerated degradation data of similar products with target-product degradation data obtained in the field.

II. WIENER PROCESS WITH CALIBRATION FACTORS

Assume that $X(t)$ is the performance degradation value at time t. $X(t)$ is a monohydric continuous random process and obeys a Wiener process. It can be written as:

$$X(t) = \mu \cdot t + \sigma \cdot B(t) \quad (1)$$

where μ is the drift coefficient; σ is the diffusion coefficient; $B(t)$ represents standard Brownian motion; and x_0 is the initial value of performance degradation.

As for accelerated testing, we often assume that μ is relative to stress S, so an accelerated model can be written as follows:

$$\mu = \exp[a + b \cdot \varphi(S)] \quad (2)$$

where a, b are unknown constants and $\varphi(S)$ is a function of S.

Compared with the laboratory environment, product

978-1-7281-0860-5/19/$31.00 ©2019 IEEE

failure mechanisms in the field are more complicated and caused by multiple stresses. So, the distribution parameter should be modified as follows:

$$\mu_f = \mu_A \cdot k_1 \tag{3}$$

$$\sigma_f^2 = \sigma_A^2 \cdot k_2 \tag{4}$$

where k_1, k_2 are the modified coefficients; μ_f, σ_f are the drift coefficient and diffusion coefficient in the field stress environment; and μ_A, σ_A are the drift coefficient and diffusion coefficient in the laboratory stress environment.

Let l be the failure threshold, such that the product fails when $X(t) < l$. The product first hitting time (FHT) t to l has an inverse Gaussian distribution. The reliability function (RF) and probability density function (PDF) can be written as:

$$\begin{aligned} R_f(t) = \Phi(\frac{l - x_0 - k_1 \cdot \mu_A t}{\sqrt{k_2 \cdot \sigma_A^2 t}}) - \\ \exp[\frac{2k_1 \cdot \mu_A (l - x_0)}{k_2 \cdot \sigma_A^2}]\Phi(-\frac{l - x_0 + k_1 \cdot \mu_A t}{\sqrt{k_2 \cdot \sigma_A^2 t}}) \end{aligned} \tag{5}$$

$$f_f(t) = \frac{l - x_0}{\sqrt{2\pi t^3 \sigma_A^2 \cdot k_2}} \exp[-\frac{(l - x_0 - k_1 \cdot \mu_A t)^2}{2k_2 \cdot \sigma_A^2 t}] \tag{6}$$

III. Degradation Data Modeling

Now, there are m samples under n accelerated stresses undergoing step-stress ADT. The initial degradation value under each new stress is equivalent to the last degradation value under the former stress. So, degradation under each stress in step-stress ADT can be expressed as:

$$X(t) = \begin{cases} \mu_1 t + \sigma_1 B(t) + x_0 & 0 \le t < t_1 \\ \mu_2 (t - t_1) + \mu_1 t_1 + \sigma_2 B(t) + x_0 & t_1 \le t < t_2 \\ \vdots & \\ \sum_{i=1}^{n-1} \mu_i (t_i - t_{i-1}) + & \\ \mu_n (t - t_{n-1}) + \sigma_n B(t) + x_0 & t_{n-1} \le t < t_n \end{cases} \tag{7}$$

The degradation parameter of samples under each stress is measured K times. So, we denote sample j at measurement time k under stress S_i as $t_{i,k}^j$ and denote its degradation value as $X(t_{i,k}^j)$ ($i = 1,2,\cdots,n$, $j = 1,2,\cdots,m$, $k = 1,2,\cdots,K$). The degradation increment of sample j under stress S_i at measurement time k to $k-1$ is $\Delta X(t_{i,k}^j) = X(t_{i,k}^j) - X(t_{i,k-1}^j)$ and the time increment is $\Delta t_{i,k}^j = t_{i,k}^j - t_{i,k-1}^j$.

The drift and diffusion coefficients of sample j under stress S_i are μ_{ij} and σ_{ij}.

According to the features of the Wiener process, the degradation increment obeys a normal distribution. The likelihood function of the distribution parameters $(\mu_{ij}, \sigma_{ij}^2)$ is written as:

$$\begin{aligned} L[\Delta X(t_{ij}^k)/\mu_{ij}, \sigma_{ij}^2] = \prod_{k=1}^{K} h(\Delta X_{ij}^k) = \\ \prod_{k=1}^{K} \frac{1}{\sqrt{2\pi \Delta t_{ij} \sigma_{ij}^2}} \exp[-\frac{(\Delta X_{ij}^k - \mu_{ij} \Delta t_{ij}^k)^2}{2\sigma_{ij}^2 \Delta t_{ij}}] \end{aligned} \tag{8}$$

From (8), the estimates of the distribution parameters $(\hat{\mu}_{ij}, \hat{\sigma}_{ij}^2)$ can be determined.

For a target product under a field stress environment, its degradation value is denoted as X_f.

$$X_f = [X_f(t_1), X_f(t_2), \cdots, X_f(t_{K'})]$$

The likelihood function of the distribution parameters (μ_f, σ_f^2) is written as:

$$\begin{aligned} L(X_f) = \prod_{k=1}^{K'} h[\Delta X_f(t_j)] = \\ \prod_{k=1}^{K'} \frac{1}{\sqrt{2\pi \Delta t_j \sigma_f^2}} \exp[-\frac{(\Delta X_f(t_j) - \mu_f \Delta t_j)^2}{2\sigma_f^2 \Delta t_j}] \end{aligned} \tag{9}$$

From (9), the estimates of the distribution parameter $(\hat{\mu}_{ij}, \hat{\sigma}_{ij}^2)$ can be determined.

IV. Bayesian Inferences

We treat field-measured degradation data X_f as the assessment object. A Bayesian inference model is used to build the likelihood function. ADT data of similar products is treated as prior information to derive the unknown parameter set Θ and obtain its prior distribution. The posterior distribution of the unknown parameters can be determined as:

$$\pi(\Theta / X_f) \propto L(X_f / \Theta) \cdot f(\Theta) \tag{10}$$

where $\pi(\Theta / X_f)$ is the posterior distribution function of parameter set Θ; $L(X_f / \Theta)$ is the likelihood function; and $f(\Theta)$ is the prior distribution function.

A. Prior distributions

1) Prior distribution type

The relationship between accelerated stress and Wiener parameters can be expressed as follows:

$$A_{pq} = \frac{\mu_p}{\mu_q} = \frac{\sigma_p^2}{\sigma_q^2} \tag{11}$$

$$A_{pq} = exp[b(\varphi(S_p) - \varphi(S_q))] \tag{12}$$

According to the estimates $(\hat{\mu}_{ij}, \hat{\sigma}_{ij}^2)$ and Equations (11) and (12), the least-squares (LS) method is used to obtain $\hat{b}$ and $\hat{A}_{pq}$. So, the accelerated factors $\hat{A}_{10}, \hat{A}_{20}, \cdots, \hat{A}_{n0}$ of each accelerated stress S_i to common stress S_0 can be obtained.

Furthermore, the estimates $(\hat{\mu}_{ij},\hat{\sigma}_{ij}^2)$ under accelerated stress S_i can be converted into $(\hat{\mu}_{0j},\hat{\sigma}_{0j}^2)$ under common stress S_0, which can be expressed as:

$$\begin{array}{llll} S_1 \to S_0: & (\hat{\mu}_{01},\hat{\sigma}_{01}^2) & (\hat{\mu}_{02},\hat{\sigma}_{02}^2) & \cdots & (\hat{\mu}_{0m},\hat{\sigma}_{0m}^2) \\ S_2 \to S_0: & (\hat{\mu}_{01},\hat{\sigma}_{01}^2) & (\hat{\mu}_{02},\hat{\sigma}_{02}^2) & \cdots & (\hat{\mu}_{0m},\hat{\sigma}_{0m}^2) \\ \vdots & \vdots & \vdots & & \vdots \\ S_n \to S_0: & (\hat{\mu}_{01},\hat{\sigma}_{01}^2) & (\hat{\mu}_{02},\hat{\sigma}_{02}^2) & \cdots & (\hat{\mu}_{0m},\hat{\sigma}_{0m}^2) \end{array}$$

According to Equations (3) and (4), the estimates $(\hat{\mu}_f,\hat{\sigma}_f^2)$ and $(\hat{\mu}_{0j},\hat{\sigma}_{0j}^2)$ are used as sample data of the modified coefficients k_1,k_2.

Goodness-of-fit test methods include Kolmogorov-Smirnov (KS) tests, Cramer-von Mises (CvM) tests and Anderson-Darling (AD) tests. These are used to determine the prior distribution type of the unknown parameter from several candidate distributions, such as normal, Weibull, gamma and inverse-gamma. This paper chooses the CvM test method, whose formula is:

$$W_n^2 = n\int_{-\infty}^{+\infty}[F(x)-F_n(x)]^2 dF(x) \tag{13}$$

where $F_n(x)$ is an experiential distribution function; and $F(x)$ is the assumed distribution function.

As for the certain sample number n, CvM statistics can be written as:

$$W_n^2 = \frac{1}{12n}+\sum_{i=1}^{n}[F(x_i)-\frac{2i-1}{2n}]^2 \tag{14}$$

By comparing W_n^2 from the candidate distributions, the prior distribution type is determined.

2) Hyper-parameter estimation

Assuming that the prior distribution type of σ_0^2 is inverse-gamma, namely $\frac{1}{\sigma_0^2} \sim \Gamma(\alpha,\beta)$, where (α,β are hyper-parameters). Its likelihood function is as follows:

$$\mathrm{L}(\frac{1}{\sigma_{0j}^2}/\alpha,\beta)=\prod_{oj=1}^{n\times m}\frac{1}{\beta^{\alpha}\Gamma(\alpha)}(\frac{1}{\sigma_{0j}^2})^{\alpha-1}\exp(-\frac{1}{\beta\sigma_{0j}^2}) \tag{15}$$

Put the estimates $\hat{\sigma}_{0j}^2(0j=1,2,\cdots,n\times m)$ under common stress S_0 into (15), and estimates of $\hat{\alpha},\hat{\beta}$ can be obtained.

B. Posterior distributions

After determining the prior distribution of all parameters, according to (10), the posterior distribution of Θ under field degradation data is expressed as:

$$\begin{aligned} &\pi(\Theta \mid X_{\mathrm{f}})=\pi(\mu_0,\sigma_0^2,k_1,k_2 \mid X_{\mathrm{f}}) \propto \\ &L(X_{\mathrm{f}} \mid \mu_0,\sigma_0^2,k_1,k_2)\cdot f(\mu_0,\sigma_0^2,k_1,k_2)= \\ &\prod_{k=1}^{K^{'}}\frac{1}{\sqrt{2\pi\Delta t_j\sigma_0^2\cdot k_2}}\exp[-\frac{(\Delta X_f(t_j)-\mu_0\cdot k_1\cdot\Delta t_j)^2}{2\sigma_0^2\cdot k_2\cdot\Delta t_j}]\cdot \\ &f(\mu_0)\cdot f(\sigma_0^2)\cdot f(k_1)\cdot f(k_2) \end{aligned} \tag{16}$$

From (16), the parameter set Θ cannot be solved by the traditional maximum likelihood estimation (MLE) method. So, this paper uses the MCMC method to solve (16) and obtain its posterior means. We put these posterior means into (5) and (6) to obtain the RF and PDF of the target product. When the field degradation data of the target product is enriched, its reliability function is updated by updating the posterior means of the unknown parameters.

V. EXAMPLES

We use data from the literature as an example [2]. According to the presented method, the target field reliability is assessed by integration with prior ADT data and field-measured degradation data. The detailed process is as follows.

We collect degradation data of a target product under a field temperature level of S_0 = 25 °C for 5000 h. The observation time interval is 5 h, resulting in 1000 observations. Field data of the target product is shown in Figure 1.

At the same time, there are four similar products tested at accelerated temperature levels of S_1 = 60 °C, S_2 = 70 °C, S_3 = 80 °C, S_4 = 100 °C, and S_5 = 120 °C. The testing time at each level is 1250 h, 750 h, 500 h, 500 h, and 500 h. The initial value of degradation, x_0, is 100. The failure threshold, l, is 50. The observation time interval is 5 h, resulting in 600 observations. Data of the four similar products is shown in Figure 2.

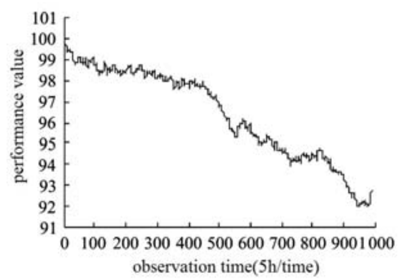

Figure 1 Field degradation data

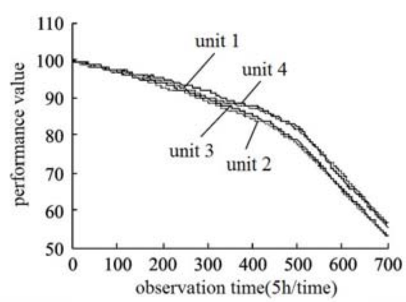

Figure 2 Step-stress ADT data

A. Distribution parameter estimation

According to (8) and (9), the MLE method is used to obtain the parameter estimates of the target product and similar products. $(\hat{\mu}_f, \hat{\sigma}_f^2)$ is (2.52×10^{-3}, 18.19×10^{-4}). $(\hat{\mu}_{ij}, \hat{\sigma}_{ij}^2)$ is shown in Table I.

TABLE I ESTIMATE OF DISTRIBUTION PARAMETERS UNDER ADT STRESS ($\times 10^{-4}$)

Stress	Sample 1		Sample 2		Sample 3		Sample 4	
	μ	σ^2	μ	σ^2	μ	σ^2	μ	σ^2
60 °C	0.69	3.44	0.72	4.13	0.74	3.82	0.57	3.88
70 °C	1.49	7.28	0.89	6.64	1.18	5.52	1.33	8.03
80 °C	2.47	12.41	2.81	8.68	1.48	13.60	2.41	12.68
100 °C	2.82	14.72	2.78	18.57	3.52	15.84	2.58	15.58
120 °C	4.76	33.10	6.58	35.28	6.52	35.92	6.01	27.79

TABLE II ESTIMATES OF DISTRIBUTION PARAMETERS UNDER ADT STRESS CONVERTED INTO COMMON STRESS ($\times 10^{-4}$)

Stress	Sample 1		Sample 2		Sample 3		Sample 4	
	μ_0	σ_0^2	μ_0	σ_0^2	μ_0	σ_0^2	μ_0	σ_0^2
60→25 °C	0.18	8.38	0.17	9.95	0.19	9.18	0.16	9.47
70→25 °C	0.19	9.84	0.13	8.91	0.17	7.42	0.19	10.85
80→25 °C	0.22	11.48	0.27	8.08	0.15	12.62	0.24	11.81
100→25 °C	0.16	7.98	0.16	10.09	0.20	8.72	0.15	8.57
120→25 °C	0.14	8.92	0.15	9.52	0.18	9.58	0.17	7.52

B. Prior distribution determination

An Arrhenius model is used to describe the relationship between temperature and the drift coefficient. According to $(\hat{\mu}_{ij}, \hat{\sigma}_{ij}^2)$ and (11), the MLE method is used to derive the estimates $\hat{b}$ (-4785) and accelerated factors $\hat{A}_{10}, \hat{A}_{20}, \hat{A}_{30}, \hat{A}_{40}, \hat{A}_{50}$ (4.09, 7.42, 10.68, 18.27, 37.20). Estimated values of the distribution parameters under ADT stress are converted into common stress, as shown in Table II. According to (3) (4), sample data of the modified factors k_1, k_2 are shown in Table III.

TABLE III SAMPLE DATA OF CALIBRATION FACTORS k_1, k_2

Sample 1		Sample 2		Sample 3		Sample 4	
k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2
1.12	1.58	1.04	2.24	1.82	1.44	1.06	1.54
1.18	1.85	1.88	2.03	1.48	2.47	1.41	1.69
2.05	2.05	1.38	1.92	1.50	1.89	1.62	2.44
1.72	2.27	1.57	1.79	1.28	2.1	1.77	2.13
1.52	2.17	1.48	1.82	1.38	1.98	1.69	1.91

The CvM test method is used to calculate CvM statistics W_n^2 of $\mu_0, \sigma_0^2, k_1, k_2$ from the candidate distributions shown in Table IV.

TABLE IV CVM STATISTICS OF CANDIDATE DISTRIBUTION PARAMETERS

Parameter	Normal	Weibull	Gamma	Inverse-gamma
μ_0	0.52	1.88	0.67	1.42
σ_0^2	0.48	1.81	0.77	0.44
k_1	0.73	3.31	0.92	1.75
k_2	0.51	0.86	0.28	0.57

From Table IV, the goodness-of-fit distribution of $\mu_0, \sigma_0^2, k_1, k_2$ are normal, inverse-gamma, normal and gamma distributions, respectively. According to the PDF of the prior distribution of $\mu_0, \sigma_0^2, k_1, k_2$, the likelihood function of hyper-parameters is built and the MLE method is used to obtain its estimates as follows:

$$\mu_0 \sim N(\mu_{\mu_0}, \sigma_{\mu_0}^2) = N(1.75\times10^{-3}, 125\times10^{-5})$$

$$1/\sigma_0^2 \sim \Gamma(a,b) = \Gamma(0.75, 6.98)$$

$$k_1 \sim N(\mu_{k_1}, \sigma_{k_1}^2) = N(1.51, 0.078)$$

$$k_2 \sim \Gamma(a_{k_2}, b_{k_2}) = \Gamma(49.15, 24.95)$$

C. Posterior distribution determination

According to (16), the posterior function for Θ is written as:

$$\pi(\Theta \mid X_f) \propto L(X_f \mid \mu_0, \sigma_0^2, k_1, k_2) \cdot f(\mu_0, \sigma_0^2, k_1, k_2) = \prod_{k=1}^{K'} \frac{1}{\sqrt{2\pi\sigma_0^2 \cdot k_2}} \exp[-\frac{\Delta X_f(t_j) - \mu_0 \cdot k_1 \cdot \Delta t_j}{2\sigma_0^2 \cdot k_2 \cdot \Delta t_j}] \Phi(\frac{\mu_0 - \mu_{\mu_0}}{\sigma_{\mu_0}}) \cdot \Phi(\frac{k_1 - \mu_{k_1}}{\sigma_{k_1}}) \frac{b^a}{\Gamma(a)} (\frac{1}{\sigma_0^2})^{a+1} \exp(\frac{b}{\sigma_0^2}) \cdot \frac{b_{k_2}^{a_{k_2}}}{\Gamma(a_{k_2})} k_2^{a_{k_2}+1} \exp(k_2 b_{k_2}) \quad (17)$$

WinBUGS software was used to solve (17) by the MCMC method and posterior means of Θ. According to the field degradation data of the target product, posterior means of Θ at different observation times are obtained and compared with real values from the literature [2] in Table V.

TABLE V POSTERIOR MEANS OF PARAMETER SETS AT DIFFERENT OBSERVATION TIMES

t /shots	μ_0 ×10-3	σ_0^2 ×10-4	k_1	k_2
600	1.59	10.15	1.18	1.35
800	1.41	8.82	1.35	1.48
1000	1.11	7.08	1.56	1.72
Real value	1.00	6.25	1.50	1.96

From Table V, when the field degradation data of the target product is enriched, the posterior means of Θ with integrated ADT and field-measured data approaches the real value. Hence, its estimation accuracy is better.

D. Estimation result analysis

We denote the present method that using ADT data and field degradation data as M1 and denote assessment method that only using field degradation data as M2. Put posterior means by M1 and M2 into (9) (10). Estimation value of distribution parameter in the field stress environment is shown in Tab.□.

TABLE VI ESTIMATES OF DISTRIBUTION PARAMETERS BY DIFFERENT METHODS

Parameter	M2	M1			Real value
		600	800	1000	
μ_f ×10-3	2.51	2.01	1.79	1.65	1.50
σ_f^2 ×10-4	18.16	14.08	13.18	11.60	12.25

We then put the estimated distribution parameters into (5) and (6). The RF and PDF of M1, M2, and the real values are given in Figure3 and Figure4.

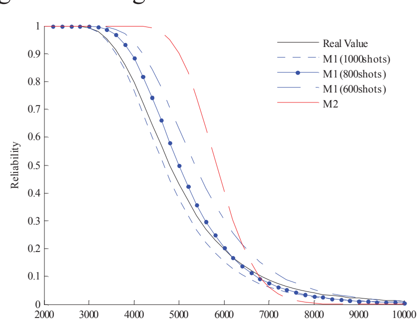

Figure 3 Reliability of different methods

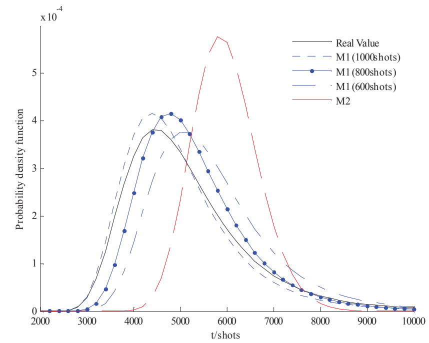

Figure 4 PDF of different methods

From Figure 3 and Figure 4, it can be seen that M1 is better than M2.

VI. CONCLUSIONS

Given the differences between field and laboratory stress environments, a modified Wiener process was built with calibration factors. An accelerated factor was used to convert the estimates obtained under accelerated stress into estimates obtained under regular stress. The CvM test method was used to derive the prior distribution type of unknown parameters from candidate distributions. The MCMC algorithm was used to obtain the posterior parameters. When the target product's field degradation data is enriched, its reliability function is updated by updating its posterior means.

REFERENCES

[1] T. M. Jiang, "Reliability test technology," BEIHANG University Press, Beijing: 2012, pp. 52-55 (in Chinese).

[2] L. Z.Wang, T. M. Jiang, X. Y. Li and X. H. Wang, "Lifetime evaluation method with integrated accelerated testing and field information," Journal of Beijing University of Aeronautics and Astronautics,vol. 39, pp. 947-951, July 2013 (in Chinese).

[3] L. Z. Wang, R. Pan, X. Y. Li, and T. M. Jiang, "A Bayesian reliability evaluation method with integrated accelerated degradation testing and field information," Reliability Engineering and System Safety,vol. 112, pp. 38-47, March 2014.

[4] R. Pan, "Bayes approach to reliability prediction utilizing data from accelerated life tests and field failure observations," Quality and Reliability Engineering International, vol. 25, pp. 229-240, Febuary 2009.

[5] Y. T. Tan, C. H. Zhang, X. Chen and Y. S. Wang, "Remaining Life Evaluation Based on Accelerated Life Testing," . Journal of mechanical engineering, vol. 46, pp. 150-154, Febuary 2010.

[6] G. A. Whitmore, "Normal- gamma mixtures of inverse Gaussian distribution," Scandinavian Journal of Statistics, vol. 13, pp. 211-220, January 1986.

[7] X. Wang, "Wiener processes with random effects for degradation data," Journal of Multivariate Analysis, vol. 101, pp. 340-351, Febuary 2010.

[8] X. L. Wang, B. Guo, and Z. J. Cheng, "Reliability Assessment of Products with Wiener Process Degradation by Fusing Multiple Information," Acta Electronica Sinica, vol. 40, pp. 977-982, May 2012(in Chinese).

[9] C. Y. Peng and S. T. Tseng, "Mis-specification analysis of linear degradation models," IEEE Transactions on Reliability, vol. 58, pp. 444-455, March 2009.

[10] S. J. Tang, X. S. Guo, H. F. Zhou, and X. S. Si, "Step Stress Accelerated Degradation Process Modeling and Remaining Useful Life Estimation," Journal of mechanical engineering,vol. 50, pp. 33-40, August 2014.