해설

2023-1 동역학(이동훈 교수님) 기말고사

2025–12–30

숭실대학교
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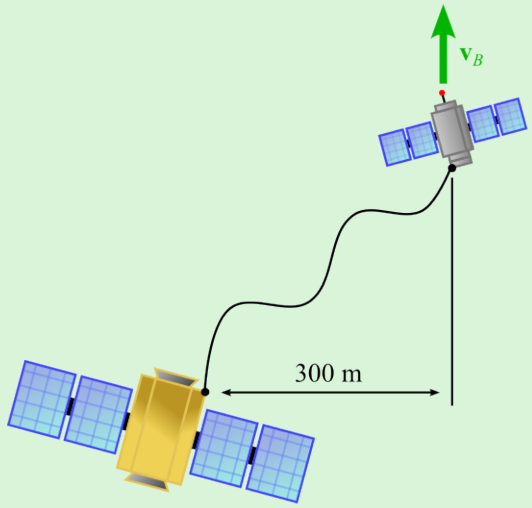
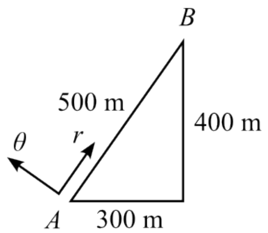
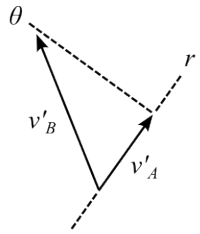
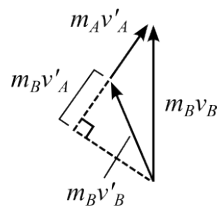
Question 1



The 2-kg sub-satellite B has an initial velocity $\mathbf{v}_B = 3\,\mathrm{m/s}\,\mathbf{j}$. It is connected to the 20-kg base-satellite A by a 500-m space tether. Determine the velocity of the base satellite and sub-satellite immediately after the tether becomes taut (assuming no rebound).







$$\frac{m_B v_B}{5} = \frac{m_A v'_A + m_B v'_A}{4} = \frac{\sqrt{(m_B v'_B)^2 - (m_B v'_A)^2}}{3}$$

$$\begin{cases} 4m_B v_B = 5v'_A(m_A + m_B) \\ 3v_B = 5\sqrt{v'^2_B - v'^2_A} \end{cases}$$

$$v'_A = \frac{4m_B v_B}{5(m_A + m_B)} = \frac{4(2)(3)}{5(20+2)}\,\mathrm{m/s} = \frac{12}{55}\,\mathrm{m/s} = 0.218\,\mathrm{m/s}$$

$$v'_B = \sqrt{\frac{9}{25}v_B^2 + v'^2_A} = \sqrt{\frac{9}{25}(3)^2 + \left(\frac{12}{55}\right)^2}\,\mathrm{m/s} = 1.813\,\mathrm{m/s}$$

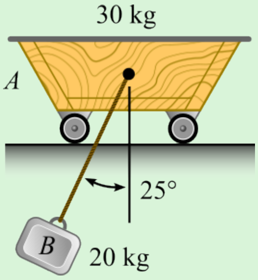
$$\theta_A = \arctan\frac{400\,\mathrm{m}}{300\,\mathrm{m}} = \arctan\frac{4}{3} = 53.1^\circ$$

$$\theta_B = \theta_A + \arccos\frac{v'_A}{v'_B} = 53.1^\circ + \arccos\frac{0.218}{1.813} = 136.2^\circ$$

$\mathbf{v}_A = 0.218\,\mathrm{m/s}\,\measuredangle 53.1^\circ$ ◀

$\mathbf{v}_B = 1.813\,\mathrm{m/s}\,\measuredangle 136.2^\circ$ ◀

Question 2



A 20-kg block B is suspended from a 2-m cord attached to a 30-kg cart A, which may roll freely on a frictionless, horizontal track. If the system is released from rest in the position shown, determine the velocities of A and B as B passes directly under A.

Let the state 1 be the initial state and the state 2 be when B passes directly under A.

$$\cancelto{0}{T_1} + V_{g1} = T_2 + \cancelto{0}{V_{g2}}$$

$$m_B g l(1 - \cos 25^\circ) = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$m_A v_A^2 + m_B v_B^2 = 2 m_B g l(1 - \cos 25^\circ) \tag{1}$$

Impulses by gravity and ground are independent to horizontal component of momentum of A and B.

$$\sum p_{x1} = \sum p_{x2}$$

$$0 = -m_A v_A + m_B v_B \tag{2}$$

$$\begin{cases} m_A v_A^2 + m_B v_B^2 = 2 m_B g l(1 - \cos 25^\circ) \\ -m_A v_A + m_B v_B = 0 \end{cases} \tag{1 \& 2}$$

$$m_A v_A = m_B v_B$$

$$m_A m_B v_A^2 + m_A^2 v_A^2 = 2 m_B^2 g l(1 - \cos 25^\circ)$$

$$\frac{m_A}{m_B} v_A^2 + \left(\frac{m_A}{m_B}\right)^2 v_A^2 = 2 g l(1 - \cos 25^\circ)$$

$$k(k+1) v_A^2 = 2 g l(1 - \cos 25^\circ), \qquad k = \frac{m_A}{m_B} = 1.5$$

$$v_A = \sqrt{\frac{2gl(1 - \cos 25^\circ)}{k(k+1)}} = \sqrt{\frac{2(9.81)(2)(1 - \cos 25^\circ)}{(1.5)(2.5)}}\,\mathrm{m/s} = 0.990\,\mathrm{m/s}$$

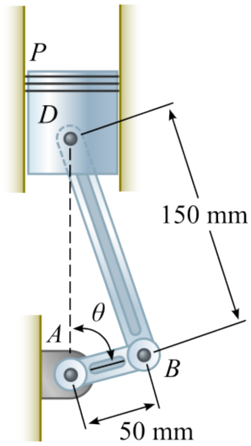
$$v_B = \frac{m_A}{m_B} v_A = k v_A = (1.5)(0.990)\,\mathrm{m/s} = 1.485\,\mathrm{m/s}$$

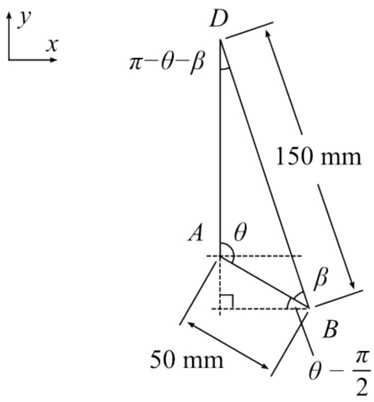
$$\mathbf{v}_A = 0.990\,\mathrm{m/s} \leftarrow \quad ◀$$

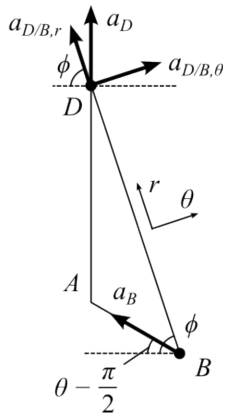
$$\mathbf{v}_B = 1.485\,\mathrm{m/s} \rightarrow \quad ◀$$

Question 3

A crank AB is rotating clockwise with constant angular speed 900 rpm. Determine the acceleration of piston P when $\theta = 120°$.







$$L = 0.15\,\text{m}, \quad l = 0.05\,\text{m}, \quad \dot{\theta} = 900\,\text{rpm} \times \frac{1\,\text{min}}{60\,\text{s}} \times \frac{2\pi\,(\text{rad})}{1\,\text{rev}} = 30\pi\,\text{s}^{-1}$$

$$\frac{\sin(\pi - \theta - \beta)}{50\,\text{mm}} = \frac{\sin\theta}{150\,\text{mm}}$$

$$\sin(\theta + \beta) = \frac{1}{3}\sin\theta$$

$$\phi = \theta + \beta - \frac{\pi}{2}$$

$$\sin\left(\phi + \frac{\pi}{2}\right) = \frac{1}{3}\sin\theta$$

$$3\cos\phi = \sin\theta$$

$$\phi = \arccos\left(\frac{\sin\theta}{3}\right) = \arccos\left(\frac{\sin 120°}{3}\right) = 73.2213°$$

$$-3\dot{\phi}\sin\phi = \dot{\theta}\cos\theta$$

$$\dot{\phi} = -\frac{\dot{\theta}\cos\theta}{3\sin\phi} = -\frac{(30\pi\,\text{s}^{-1})\cos 120°}{3\sin 73.2213°} = 16.40644\,\text{s}^{-1}$$

$$-3\ddot{\phi}\sin\phi - 3\dot{\phi}^2\cos\phi = -\dot{\theta}^2\sin\theta \qquad \left(\because \ddot{\theta} = 0\right)$$

$$\ddot{\phi} = \frac{-\dot{\theta}^2\sin\theta + 3\dot{\phi}^2\cos\phi}{-3\sin\phi} = \frac{-(30\pi\,\text{s}^{-1})^2\sin 120° + 3(16.40644\,\text{s}^{-1})^2\cos 120°}{-3\sin 73.2213°} = 2597.06\,\text{s}^{-2}$$

$$a_B = l\dot{\theta}^2$$

$$a_{D/B,r} = -L\dot{\phi}^2$$

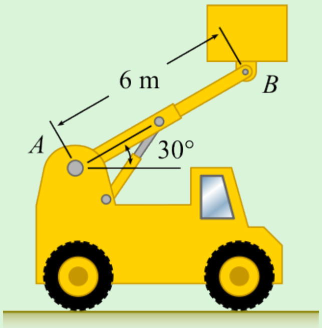
$$a_{D/B,\theta} = L\ddot{\phi}$$

$$\mathbf{a}_D = \mathbf{a}_{D/B} + \mathbf{a}_B$$

$$\begin{aligned} a_D &= a_{D/B,r}\sin\phi + a_{D/B,\theta}\cos\phi + a_B\cos\theta = -L\dot{\phi}^2\sin\phi + L\ddot{\phi}\cos\phi + l\dot{\theta}^2\sin\left(\theta - \frac{\pi}{2}\right) \\ &= -(0.15)(16.40644)^2\sin 73.2213° + (0.15)(2597.06)\cos 73.2213° + (0.05)(30\pi)^2\sin 30° \\ &= 296\,\text{m/s}^2 \end{aligned}$$

$296\,\text{m/s}^2 \uparrow$ ◀

Question 4



At the instant shown the length of the boom AB is being increased at the constant rate of 0.2 m/s, the boom is being lowered at the constant rate of 0.08 rad/s, and it is moving forward with a speed of 0.1 m/s and is slowing down at 0.05 m/s^2. Determine (a) the velocity of Point B, (b) the acceleration of Point B.

$$r = 6\,\text{m}, \quad \dot{r} = 0.2\,\text{m/s}, \quad \ddot{r} = 0,$$
$$V = 0.1\,\text{m/s}, \quad \dot{V} = -0.05\,\text{m/s}^2$$
$$\theta = 30^\circ, \quad \dot{\theta} = -0.08\,\text{s}^{-1}, \quad \ddot{\theta} = 0$$
$$v_{B/A,r} = \dot{r}, \quad v_{B/A,\theta} = r\dot{\theta}$$
$$v_{B,x} = \dot{r}\cos\theta - r\dot{\theta}\sin\theta + V = (0.2)\cos 30^\circ - (6)(-0.08)\sin 30^\circ + 0.1\,[\text{m/s}] = 0.513\,\text{m/s}$$
$$v_{B,y} = \dot{r}\sin\theta + r\dot{\theta}\cos\theta = (0.2)\sin 30^\circ + (6)(-0.08)\cos 30^\circ\,[\text{m/s}] = -0.316\,\text{m/s}$$
$$v_B = \sqrt{v_{B,x}^2 + v_{B,y}^2} = 0.603\,\text{m/s}$$
$$\phi_v = \arctan\frac{v_{B,y}}{v_{B,x}} = -31.6^\circ$$

(a) $0.603\,\text{m/s} \measuredangle - 31.6^\circ$ ◀

$$a_{B/A,r} = \ddot{r} - r\dot{\theta}^2 = -r\dot{\theta}^2, \quad a_{B/A,\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2\dot{r}\dot{\theta}$$
$$a_{B,x} = -r\dot{\theta}^2\cos\theta - 2\dot{r}\dot{\theta}\sin\theta + \dot{V} = -(6)(-0.08)^2\cos 30^\circ - 2(0.2)(-0.08)\sin 30^\circ - 0.05\,[\text{m/s}^2]$$
$$= -0.0672554\,\text{m/s}^2$$
$$a_{B,y} = -r\dot{\theta}^2\sin\theta + 2\dot{r}\dot{\theta}\cos\theta = -(6)(-0.08)^2\sin 30^\circ + 2(0.2)(-0.08)\cos 30^\circ\,[\text{m/s}^2]$$
$$= -0.0469128\,\text{m/s}^2$$
$$a_B = \sqrt{a_{B,x} + a_{B,y}} = 0.0820\,\text{m/s}^2$$
$$\phi_a = \arctan\frac{a_{B,y}}{a_{B,x}} = 34.9^\circ$$

(b) $0.0820\,\text{m/s}^2 \measuredangle - 145.1^\circ$ ◀

Question 4 — Alternative Solution

The system xy on point A. The system XY is fixed framed.

$$\mathbf{r}|_A = \mathbf{r}|_{xy} = 6\mathbf{e}_r\,\mathrm{m}, \quad \dot{\mathbf{r}}|_{xy} = 0.2\mathbf{e}_r\,\mathrm{m/s} = \text{const.} \quad \Rightarrow \quad \ddot{\mathbf{r}}|_{xy} = \mathbf{0}$$
$$\dot{\mathbf{r}}_A = 0.1\mathbf{i}\,\mathrm{m/s}, \quad \ddot{\mathbf{r}}_A = -0.05\mathbf{i}\,\mathrm{m/s^2}$$
$$\theta = 30^\circ, \quad \mathbf{e}_r = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}, \quad \boldsymbol{\Omega} = -0.08\mathbf{k}\,\mathrm{s^{-1}} = \text{const.} \quad \Rightarrow \quad \dot{\boldsymbol{\Omega}} = \mathbf{0}$$
$$\begin{aligned}
\dot{\mathbf{r}}|_{XY} &= \dot{\mathbf{r}}_A + \dot{\mathbf{r}}|_A = \dot{\mathbf{r}}_A + \dot{\mathbf{r}}|_{xy} + \boldsymbol{\Omega} \times \mathbf{r}|_A \\
&= (0.1)(\rightarrow) + (0.2)\measuredangle 30^\circ + (0.08\cdot 6)\measuredangle(-60^\circ)\ [\mathrm{m/s}] \\
&= \{0.1 + 0.2\cos 30^\circ + (0.08)(6)\cos(-60^\circ)\}\,\mathbf{i} + \{0.2\sin 30^\circ + (0.08)(6)\sin(-60^\circ)\}\,\mathbf{j}\ [\mathrm{m/s}] \\
&= (0.513205\mathbf{i} - 0.315692\mathbf{j})\,\mathrm{m/s}
\end{aligned}$$
$$|\dot{\mathbf{r}}|_{XY}| = \sqrt{0.513205^2 + 0.315692^2}\,\mathrm{m/s} = 0.603\,\mathrm{m/s}$$
$$\theta_v = -\arctan\frac{0.315692}{0.513205} = -31.6^\circ$$

(a) $\quad 0.603\,\mathrm{m/s}\measuredangle -31.6^\circ$ ◀

$$\begin{aligned}
\ddot{\mathbf{r}}|_{XY} &= \ddot{\mathbf{r}}_A + \cancelto{0}{\ddot{\mathbf{r}}|_{xy}} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}|_A) + \cancelto{0}{\dot{\boldsymbol{\Omega}} \times \mathbf{r}|_A} + 2\boldsymbol{\Omega} \times \dot{\mathbf{r}}|_{xy} \\
&= (0.05)(\leftarrow) + (6)(0.08)^2\measuredangle(-150^\circ) + 2(0.08)(0.2)\measuredangle(-60^\circ)\ [\mathrm{m/s^2}] \\
&= \left\{-0.05 + (6)(0.08)^2\cos(-150^\circ) + 2(0.08)(0.2)\cos(-60^\circ)\right\}\mathbf{i} \\
&\qquad + \left\{(6)(0.08)^2\sin(-150^\circ) + 2(0.08)(0.2)\sin(-60^\circ)\right\}\mathbf{j}\ [\mathrm{m/s^2}] \\
&= (-0.0672554\mathbf{i} - 0.0469128\mathbf{j})\,\mathrm{m/s^2}
\end{aligned}$$
$$|\ddot{\mathbf{r}}|_{XY}| = \sqrt{0.0672554^2 + 0.0469128^2}\,\mathrm{m/s} = 0.0820\,\mathrm{m/s}$$
$$\theta_a = \arctan\frac{0.0469128}{0.0672554} - 180^\circ = -145.1^\circ$$

(b) $\quad 0.0820\,\mathrm{m/s^2}\measuredangle -145.1^\circ$ ◀