

해설

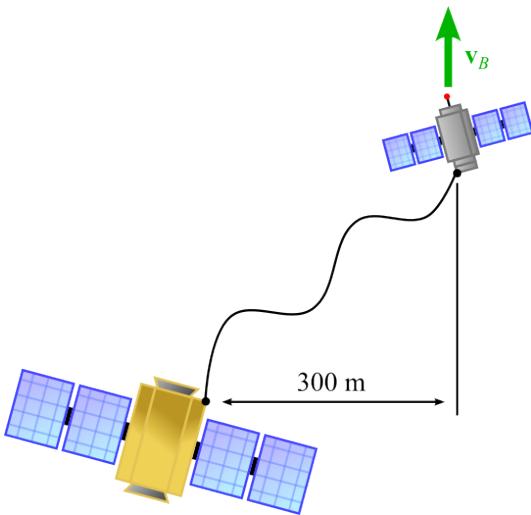
동역학(이동훈 교수님) 2025-1 기말고사

시험 실시 : 2025-06-10 18:30-20:30+ α (120분+ α)

2026-01-01



Question 1 — Prob. 14.45 in textbook



The 2-kg sub-satellite B has an initial velocity $\mathbf{v}_B = 3 \text{ m/s} \mathbf{j}$. It is connected to the 20-kg base-satellite A by a 500-m space tether. Determine the velocity of the base satellite and sub-satellite immediately after the tether becomes taut (assuming no rebound).

$$m_A = 20 \text{ kg}, \quad m_B = 2 \text{ kg}$$

$$\mathbf{L} = m_B \mathbf{v}_B = m_A \mathbf{v}'_A + m_B \mathbf{v}'_B$$

$$\mathbf{v}'_A = v'_A \angle \arctan \frac{4}{3} = \frac{3}{5} v'_A \mathbf{i} + \frac{4}{5} v'_A \mathbf{j}$$

$$L_x = 0 = \frac{3}{5} m_A v'_A + m_B v'_{B,x}$$

$$L_y = m_B v_B = \frac{4}{5} m_A v'_A + m_B v'_{B,y}$$

$$\Rightarrow v'_{B,x} = -\frac{3m_A}{5m_B} v'_A = -\frac{3(20)}{5(2)} v'_A = -6v'_A$$

$$\Rightarrow v'_{B,y} = v_B - \frac{4m_A}{5m_B} v'_A = 3 \text{ m/s} - \frac{4(20)}{5(2)} v'_A = 3 \text{ m/s} - 8v'_A$$

$$\mathbf{v}_{B/A} = \mathbf{v}'_B - \mathbf{v}'_A = \left(v_{B,x} - \frac{3}{5} v'_A \right) \mathbf{i} + \left(v_{B,y} - \frac{4}{5} v'_A \right) \mathbf{j} = v'_{B/A} \angle \left(\arctan \frac{3}{4} + 90^\circ \right)$$

$$\Rightarrow -\frac{v'_{B,y} - \frac{4}{5} v'_A}{v'_{B,x} - \frac{3}{5} v'_A} = -\frac{3 \text{ m/s} - 8v'_A}{-6.6v'_A} = \frac{3}{4}$$

$$4(3 \text{ m/s} - 8v'_A) = 3(6.6v'_A)$$

$$v'_A = \frac{12}{4(8.8) + 3(6.6)} \text{ m/s} = \frac{12}{55} \text{ m/s} = 0.218 \text{ m/s}$$

$$\theta_A = \arctan \frac{3}{4} = 36.9^\circ$$

$$v'_B = \sqrt{v'^2_{B,x} + v'^2_{B,y}} = \sqrt{(6v'_A)^2 + (3 \text{ m/s} - 8v'_A)^2} = 1.813 \text{ m/s}$$

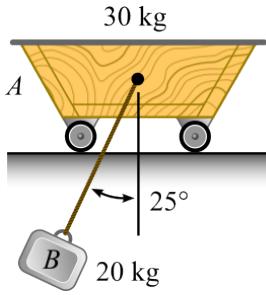
$$\theta_B = \arctan \frac{v'_{B,y}}{-v'_{B,x}} = \arctan \frac{3 \text{ m/s} - 8v'_A}{6v'_A} = 43.8^\circ$$

$$180^\circ - \theta_B = 136.2^\circ$$

$$\mathbf{v}'_A = 0.218 \text{ m/s} \angle 36.9^\circ \quad \blacktriangleleft$$

$$\mathbf{v}'_B = 1.813 \text{ m/s} \angle 136.2^\circ \quad \blacktriangleleft$$

Question 2 — Prob. 14.40 in textbook



A 20-kg block B is suspended from a 2-m cord attached to a 30-kg cart A , which may roll freely on a frictionless, horizontal track. If the system is released from rest in the position shown, determine the velocities of A and B as B passes directly under A .

Let the state 1 be the initial state and the state 2 be when B passes directly under A .

$$\begin{aligned} \cancel{\vec{P}_1^0 + V_{g1}} &= T_2 + \cancel{\vec{V}_{g2}^0} \\ m_B gl(1 - \cos 25^\circ) &= \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \\ m_A v_A^2 + m_B v_B^2 &= 2m_B gl(1 - \cos 25^\circ) \end{aligned} \quad (1)$$

Impulses by gravity and ground are independent to horizontal component of momentum of A and B .

$$\begin{aligned} \sum p_{x1} &= \sum p_{x2} \\ 0 &= -m_A v_A + m_B v_B \end{aligned} \quad (2)$$

$$\begin{cases} m_A v_A^2 + m_B v_B^2 = 2m_B gl(1 - \cos 25^\circ) \\ -m_A v_A + m_B v_B = 0 \end{cases} \quad (1 \text{ \& } 2)$$

$$m_A v_A = m_B v_B$$

$$m_A m_B v_A^2 + m_A^2 v_A^2 = 2m_B^2 gl(1 - \cos 25^\circ)$$

$$\frac{m_A}{m_B} v_A^2 + \left(\frac{m_A}{m_B}\right)^2 v_A^2 = 2gl(1 - \cos 25^\circ)$$

$$k(k+1)v_A^2 = 2gl(1 - \cos 25^\circ), \quad k = \frac{m_A}{m_B} = 1.5$$

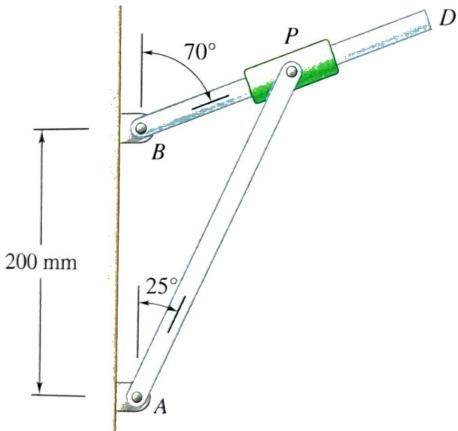
$$v_A = \sqrt{\frac{2gl(1 - \cos 25^\circ)}{k(k+1)}} = \sqrt{\frac{2(9.81)(2)(1 - \cos 25^\circ)}{(1.5)(2.5)}} \text{ m/s} = 0.990 \text{ m/s}$$

$$v_B = \frac{m_A}{m_B} v_A = kv_A = (1.5)(0.990) \text{ m/s} = 1.485 \text{ m/s}$$

$$\mathbf{v}_A = 0.990 \text{ m/s} \leftarrow \blacktriangleleft$$

$$\mathbf{v}_B = 1.485 \text{ m/s} \rightarrow \blacktriangleright$$

Question 3 — variation of Prob. 15.176 in textbook



Knowing that at the instant shown the rod attached at A has an angular velocity of 5 rad/s counterclockwise and an angular acceleration of 2 rad/s^2 clockwise, determine the acceleration of point D . (Length of the boom BD is 1000 mm .)

$$\overline{BP} = \frac{0.2 \text{ m}}{\sin 45^\circ} \cdot \sin 25^\circ = 119.5345 \text{ mm}$$

$$\overline{AP} = \frac{0.2 \text{ m}}{\sin 45^\circ} \cdot \sin 110^\circ = 265.785 \text{ mm}$$

$$\boldsymbol{\omega} = 5 \mathbf{k} \text{ s}^{-1}, \quad \dot{\boldsymbol{\omega}} = -2 \mathbf{k} \text{ s}^{-2}$$

$$\mathbf{v}_P = \boldsymbol{\omega} \times \mathbf{r}_{P/A} = 1.328926 \text{ m/s} \angle 155^\circ$$

$$v_{P,\theta} = v_P \cos(155^\circ - 110^\circ) = 0.939693 \text{ m/s}$$

$$\omega_{BD} = \frac{v_{P,\theta}}{r_{P/B}} = 7.86127 \text{ s}^{-1} (\circlearrowleft)$$

$$\begin{aligned} \mathbf{a}_P &= \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{P/A}) + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{P/A} = 6.64463 \text{ m/s}^2 \angle (-115^\circ) + 0.531570 \text{ m/s}^2 \angle 155^\circ \\ &= 4.32259 \text{ m/s}^2 \angle (-70^\circ) + 5.07434 \text{ m/s}^2 \angle (-160^\circ) \end{aligned}$$

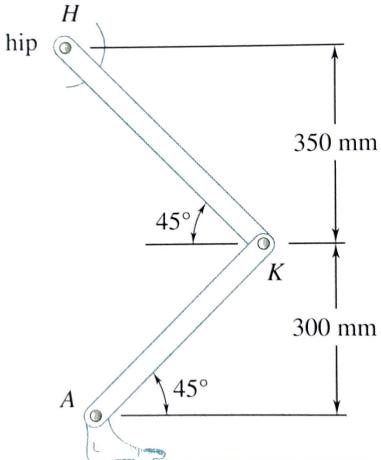
$$a_{P,\theta} = 4.32259 \text{ m/s}^2$$

$$\dot{\omega}_{BD} = \frac{a_{P,\theta}}{r_{P/B}} = 16.26346 \text{ s}^{-2} (\circlearrowleft)$$

$$\begin{aligned} \ddot{\mathbf{r}}_D &= \boldsymbol{\omega}_{BD} \times (\boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B}) + \dot{\boldsymbol{\omega}}_{BD} \times \mathbf{r}_{D/B} = 61.7995 \text{ m/s}^2 \angle (-160^\circ) + 16.26346 \text{ m/s}^2 \angle (-70^\circ) \\ &= 63.904 \text{ m/s}^2 \angle (-174.744^\circ) \end{aligned}$$

$$\mathbf{a}_D = 63.904 \text{ m/s}^2 \angle (-174.744^\circ) \quad \blacktriangleleft$$

Question 4 — Prob. 15.171 in textbook



The human leg can be crudely approximated as two rigid bars (the femur and the tibia) connected with a pin joint. At the instant shown the velocity and acceleration of the ankle is zero. During a jump, the velocity of the ankle A is zero, the tibia AK has an angular velocity of 1.5 rad/s counterclockwise and an angular acceleration of 1 rad/s^2 counterclockwise. Determine the relative angular velocity and angular acceleration of the femur KH with respect to AK so that the velocity and acceleration of H are both straight up at the instant shown.

*femur : 대퇴골, ankle : 발목, tibia : 정강이 뼈

Let angluars be positive when they are conterclockwise. The origin of all fixed frame is point A .

$$\text{Given : } l = 50 \text{ mm}, \quad v_A = 0, \quad \Omega = \omega_{AK} = 1.5 \text{ rad/s}, \quad \dot{\Omega} = \dot{\omega}_{AK} = 1 \text{ rad/s}^2$$

$$\theta_{AK} = 45^\circ, \quad \theta_{KH} = 135^\circ, \quad \dot{r}_{Hx} = 0, \quad \ddot{r}_{Hx} = 0$$

$$\text{Determine : } \omega = \omega_{KH/AK}, \quad \dot{\omega} = \dot{\omega}_{KH/AK}$$

$$\mathbf{r}_{K/A} = l(6\mathbf{i} + 6\mathbf{j}), \quad \mathbf{r}_{H/K} = l(-7\mathbf{i} + 7\mathbf{j}), \quad \mathbf{r}_{H/A} = \mathbf{r}_{K/A} + \mathbf{r}_{H/K} = l(-1\mathbf{i} + 13\mathbf{j})$$

$$\dot{\mathbf{r}}_H = \dot{\mathbf{r}}'_H + \dot{\mathbf{r}}_H|_{AK} = \boldsymbol{\Omega} \times \mathbf{r}_{H/A} + \boldsymbol{\omega} \times \mathbf{r}_{H/K}$$

$$= \Omega l(-13\mathbf{i} - \mathbf{j}) + \omega l(-7\mathbf{i} - 7\mathbf{j}) = l(-13\Omega - 7\omega)\mathbf{i} + l(-\Omega - 7\omega)\mathbf{j}$$

$$\dot{r}_{Hx} = l(-13\Omega - 7\omega) = 0, \quad \omega = -\frac{13}{7}\Omega = -\frac{39}{14} \text{ rad/s} = -2.786 \text{ rad/s}$$

$$\dot{r}_{Hy} = l(-\Omega - 7\omega)$$

$$\dot{\mathbf{r}}_H|_{AK} = \boldsymbol{\omega} \times \mathbf{r}_{H/K}$$

$$\ddot{\mathbf{r}}_H = \ddot{\mathbf{r}}'_H + \ddot{\mathbf{r}}_H|_{AK}$$

$$= [\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{H/A}) + 2\boldsymbol{\Omega} \times \dot{\mathbf{r}}_H|_{AK} + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{H/A}] + [\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{H/K}) + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{H/K}]$$

$$= [\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{H/A}) + 2\boldsymbol{\Omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{H/K}) + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{H/A}] + [\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{H/K}) + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{H/K}]$$

$$= [\Omega^2 l(\mathbf{i} - 13\mathbf{j}) + 2\Omega\omega l(-7\mathbf{i} - 7\mathbf{j}) + \dot{\Omega}l(-13\mathbf{i} - \mathbf{j})] + [\omega^2 l(7\mathbf{i} - 7\mathbf{j}) + \dot{\omega}l(-7\mathbf{i} - 7\mathbf{j})]$$

$$= (\Omega^2 l - 14\Omega\omega l - 13\dot{\Omega}l + 7\omega^2 l - 7\dot{\omega}l) \mathbf{i} + (-13\Omega^2 l - 14\Omega\omega l - \dot{\Omega}l - 7\omega^2 l - 7\dot{\omega}l) \mathbf{j}$$

$$\ddot{r}_{Hx} = \Omega^2 l - 14\Omega\omega l - 13\dot{\Omega}l + 7\omega^2 l - 7\dot{\omega}l = 0$$

$$\dot{\omega} = \frac{\Omega^2 l - 14\Omega\omega l - 13\dot{\Omega}l + 7\omega^2 l}{7l} = \frac{\Omega^2 - 13\dot{\Omega}}{7} - 2\Omega\omega + \omega^2 = -2.133 \text{ rad/s}^2$$

$$\omega = 2.786 \text{ rad/s} \quad \blacktriangleleft$$

$$\dot{\omega} = 2.133 \text{ rad/s}^2 \quad \blacktriangleleft$$