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Assignment 2

1. a. $f(n) = 4^n,\ g(n) = 6^n,\ \lim_{n \to \infty} (\frac{4}{6})^n = 0$,then $f(n) = \Omega(g(n))$

   b. $f(n) = 9\ log_3 n,\ g(n) = 3log_9 n,\ \lim_{n \to \infty} (\frac{9\ log_3 n}{3log_9 n}) = 6$,then $f(n) = \theta(g(n))$

   c. $f(n) = 4n^2 + 7n,\ g(n) = 12n^2 + 9n + n^3,\ \lim_{n \to \infty} (\frac{4n^2+7n}{12n^2+9n+n^3}) = 0$,then

   $f(n) = \Omega(g(n))$

   d. $f(n) = 8n^7,\ g(n) = (3n^5 + 5n^2)/2,\ \lim_{n \to \infty} (\frac{8n^7}{(3/2)n^5+(5/2)n^2}) = \infty$, then

   $f(n) = O(g(n))$

   e. $f(n) = 4log_5 n,\ g(n) = 8log_5 n,\ \lim_{n \to \infty} (\frac{4log_5 n}{8log_5 n}) = 1/2$, then $f(n) = \theta(g(n))$

2. Suppose that the running time of algorithm A is $2800n^2$and the running time of algorithm B is $40n^4$.In order to find the largest integer value of $n$such that the running time of A will be larger than B, we must satisfy the following inequality: $2800n^2 > 40n^4$. This inequality simplifies to $\pm\sqrt{70} > n$, but we are only concerned with the largest integer value, so we must take the floor function in order to obtain the largest integer value. $\lfloor\sqrt{70}\rfloor = 8$, therefore the running time of algorithm A will be larger than that of algorithm B for $n \leq 8$.

3. Suppose that $f(n) = 8n^2 + 11n + 6,\ g(n) = n^2$ and there exists 2 constants $a = 26$ and $b = 7$, which are positive. By the definition of big-O, $f(n) = O(g(n))$if $0 \leq f(n) \leq a * g(n)$for all $n \geq b$. If $n > 1$, then $n^2 > n$; the same relationship will

hold for $n \geq 7$. By the relationship $n^2 > n$, then $9n^2 > 8n^2$, $11n^2 > 11n$,and $6n^2 > 6$ each holds true. Thus, $8n^2 + 11n + 6 < 9n^2 + 11n^2 + 6n^2$ which simplifies to $8n^2 + 11n + 6 < 26n^2$ for all $n \geq 7$. Therefore, by the definition of big-O, $f(n) = O(g(n))$.

4. Suppose that $f(n) = 8n^2 - 11n - 6$, $g(n) = n^2$ and there exists 2 constants $a = 1$, $b = 2$, which are positive. By the definition of big-Ω, $f(n) = \Omega(g(n))$ if $0 \leq a * g(n) \leq f(n)$ for all $n \geq b$. $8n^2 - 11n - 6 \geq n^2$ simplifies to $(7n + 3)(n - 2) \geq 0$ which yields intersections at $n =- 7/3,\ 2$. This means that when $n \geq 2$, $8n^2 - 11n - 6 \geq n^2$;therefore, $f(n) = \Omega(g(n))$.

5. We are given the following: $f(n) = \Omega(g(n))$, $h(n) = (f(n))^2$. Suppose that each of these functions are non-negative and for the purpose of obtaining a contradiction that $h(n) \neq \Omega((g(n))^2)$. By the definition of big-Ω, $a(n) = \Omega(b(n))$ if $0 \leq A * b(n) \leq a(n)$ for all $n \geq B$ if there exists constants A, B. If $f(n) = \Omega(g(n))$, then there exists constants a, b which satisfies the definition, proving that the following inequality holds true: $a * g(n) \leq f(n)$ for all $n \geq b$. Since each function is non-negative the following inequality also holds true: $a(g(n))^2 \leq a * g(n) * f(n) \leq (f(n))^2$ for all $n \geq b$. However, if this is true, then $A = a$, $B = b$,and since $h(n) = (f(n))^2$,then we have a contradiction in our conclusion. Therefore, $h(n) = \Omega((g(n))^2)$.

6. a.

Constant	Times
c1	1
c2	1
c3	$\sum_{i=1}^{n+1} 1$= n+1
c4	$\sum_{i=1}^{n} \sum_{j=2}^{n-i+3} 1$=$(n^2/2) + (3n/2)$
c5	$\sum_{i=1}^{n} \sum_{j=2}^{n-i+2} 1$=$(n^2/2) + (3n/2) - 1$
c6	$\sum_{i=1}^{n} \sum_{j=2}^{n-i+2} \sum_{k=j}^{j+i+1} t_k$(variable may be 0 or 1)
c7	$\sum_{i=1}^{n} \sum_{j=2}^{n-i+2} \sum_{k=j}^{j+i} t_k$(variable may be 0 or 1)
c8	$\sum_{i=1}^{n} \sum_{j=2}^{n-i+2} \sum_{k=j}^{j+i} t_k$(variable may be 0 or 1)
c9	1

General Running Time

$$T(n) = c1 + c2 + c3(n + 1) + c4((n^2/2) + (3n/2)) + c5((n^2/2) + (n/2)) + \sum_{i=1}^{n} \sum_{j=2}^{n-i+2} \sum_{k=j}^{j+i+1} t_k(c6) + 2\sum_{i=1}^{n} \sum_{j=2}^{n-i+2} \sum_{k=j}^{j+i} t_k + c9$$

b. Best-Case Running Time:

$$T(n) = c1 + c2 + c3(n + 1) + c4((n^2/2) + (3n/2)) + c5((n^2/2) + (n/2)) + c9$$

which simplifies to a format of $An^2 + Bn + C$where A, B, C are constants.

Fastest Growing Term: n^2

c. Worst-Case Running Time:

$T(n) = c1 + c2 + c3(n + 1) + c4((n^2/2) + (3n/2)) + c5((n^2/2) + (3n/2) - 1) + c6((n^3/3) + (n^2/2) + (5n/3)) + c7((n^3/3) + (n^2/2) + (5n/3) - 1) + c8((n^3/3) + (n^2/2) + (5n/3) - 1)$

which simplifies to a format of $An^3 + Bn^2 + Cn + D$where A, B, C, D are constants.

Fastest Growing Term: n^3