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Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

Course Number	ELE 532 - Section 12
Course Title	Signals and Systems 1
Semester / Year	F2023

Instructor	Dr. Javad Alirezaie

Assignment Number	3

Assignment Title	Fourier Series Analysis using Matlab

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*By signing above you attest that you have contributed to this written lab report and confirm that all work you have contributed to this lab report is your own work. Any suspicion of copying or plagiarism in this work will result in an investigation of Academic Misconduct and may result in a "0" on the work, an "F" in the course, or possibly more severe penalties, as well as a Disciplinary Notice on your academic record under the Student Code of Academic Conduct, which can be found online at: www.ryerson.ca/senate/current/pol60.pdf.

ELE532 - Lab 2

Problem A.1

Problem A.1

$$x_1(t) = \cos\frac{3\pi}{10}t + \frac{1}{2}\cos\frac{\pi}{10}t$$

$$x_1(t) = \left(\frac{1}{2}e^{\frac{3\pi}{10}jt} + \frac{1}{2}e^{-\frac{3\pi}{10}jt}\right) + \frac{1}{2}\left(\frac{1}{2}e^{\frac{\pi}{10}jt} + \frac{1}{2}e^{\frac{\pi}{10}jt}\right)$$

$$= \frac{1}{2}e^{\frac{3\pi}{10}jt} + \frac{1}{2}e^{-\frac{3\pi}{10}jt} + \frac{1}{4}e^{\frac{\pi}{10}jt} + \frac{1}{4}e^{-\frac{\pi}{10}jt}$$

$\frac{3\pi}{10} \Big/ \frac{\pi}{10} = 3 \rightarrow$ Rational , $\omega_{0_1} = \frac{3\pi}{10}$. $\omega_{0_2} = \frac{\pi}{10}$

$\hookrightarrow \omega_0 = \frac{\pi}{10}$ $\quad T_0 = \frac{2\pi}{\pi/10} = 20$

$\sum_{n=-\infty}^{\infty} D_n e^{j\omega_0 t} \rightarrow e^{\frac{3\pi}{10}jt} \Rightarrow n=3$

$\hookrightarrow \boxed{D_3 = \frac{1}{2}}$

$e^{\frac{\pi}{10}jt} \Rightarrow n=1$

$\hookrightarrow \boxed{D_1 = \frac{1}{4}}$

$e^{-\frac{\pi}{10}jt} \Rightarrow n=-1$

$\hookrightarrow \boxed{D_{-1} = \frac{1}{4}}$

$e^{-\frac{3\pi}{10}jt} \Rightarrow n=-3$

$\hookrightarrow \boxed{D_{-3} = \frac{1}{2}}$

$$D_n = \frac{1}{20}\int_{-10}^{10}\left\{\frac{1}{2}e^{\frac{3\pi}{10}jt} + \frac{1}{2}e^{-\frac{3\pi}{10}jt} + \frac{1}{4}e^{\frac{\pi}{10}jt} + \frac{1}{4}e^{-\frac{\pi}{10}jt}\right] dt$$

$$= \frac{1}{20}\left[\frac{e^{(3-n)\pi j} - e^{-(3-n)\pi j}}{2j(3-n)\frac{\pi}{10}} + \frac{e^{(3+n)\pi j} - e^{-(3+n)\pi j}}{2j(3+n)\frac{\pi}{10}} + \frac{e^{(1+n)\pi j} - e^{-j(1+n)\pi}}{4j(1+n)\pi/10} + \frac{e^{(1-n)\pi j} - e^{-j(1-n)\pi}}{4j(1-n)\pi/10}\right.$$

$$= \frac{1}{2}\left\{\sin((3-n)\pi) + \sin((3+n)\pi) + \frac{1}{2}\sin(1+n)\pi + \frac{1}{2}\sin(1-n)\pi\right\}$$

Problem A.2

Problem A.2.

$$x_2(t) = T_0 = 20, \quad \omega_0 = \frac{2\pi}{20} = \frac{\pi}{10}$$

$$D_n = \frac{1}{20}\int_{-5}^{5} (1)\, e^{-j\frac{n\pi}{10}t}\, dt = \frac{1}{20}\left\{\frac{-1}{j\frac{n\pi}{10}}\, e^{-j\frac{n\pi}{10}t}\right]_{-5}^{5} = \frac{1}{20}\left[\frac{-10}{jn\pi}e^{-j\frac{n\pi}{2}} + \frac{10}{jn\pi}e^{j\frac{n\pi}{2}}\right]$$

$$\Rightarrow \boxed{\frac{1}{n\pi}\sin\left(\frac{\pi n}{2}\right)}$$

$$x_3(t) = T_0 = 40, \quad \omega_0 = \frac{2\pi}{40} = \frac{\pi}{20}$$

$$D_n = \frac{1}{40}\int_{-5}^{5} (1)\, e^{-j\frac{n\pi}{20}t}\, dt = \frac{1}{40}\left\{\frac{-1}{j\frac{n\pi}{20}}\, e^{-j\frac{n\pi}{20}t}\right]_{-5}^{5} = \frac{1}{40}\left[\frac{-20}{jn\pi}e^{-j\frac{n\pi}{4}} + \frac{20}{jn\pi}e^{j\frac{n\pi}{4}}\right]$$

$$\Rightarrow \boxed{\frac{1}{n\pi}\sin\left(\frac{\pi n}{4}\right)}$$

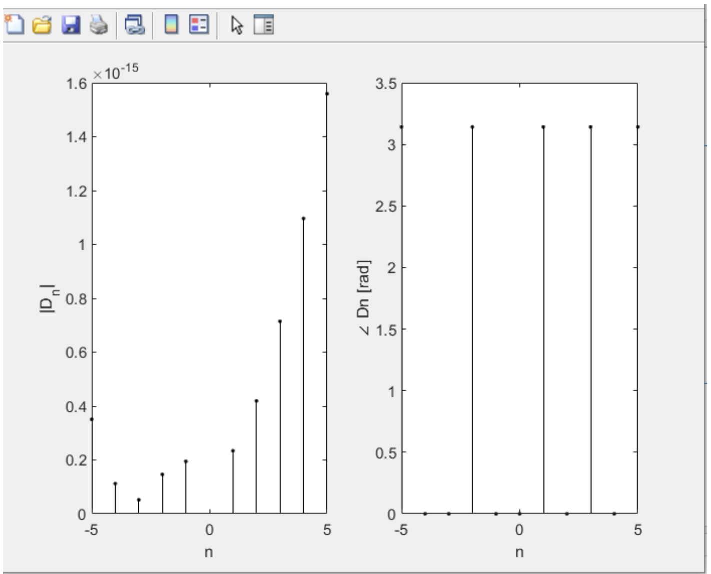
Problem A.3

Code:

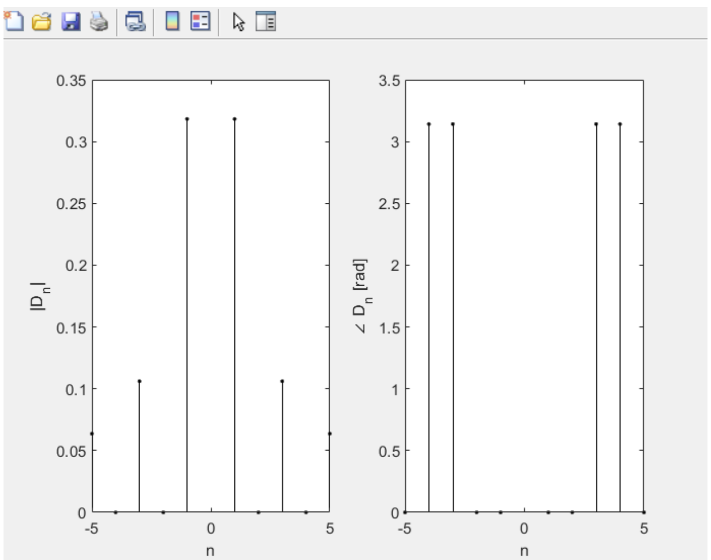
```
%% A.3
function [D] = a3(D,n)
D1 = (1/2)*(abs(n)==3)+(1/4)*(abs(n)==1);
D2 = (1/(n.*pi)*sin((n*pi)/2));
D3 = (1/(n.*pi)*sin((n*pi)/4));
if (d == 1)
D = D1;
end
if (d == 2)
D = D2;
end
if (d == 3)
D = D3;
end
```

Problem A.4

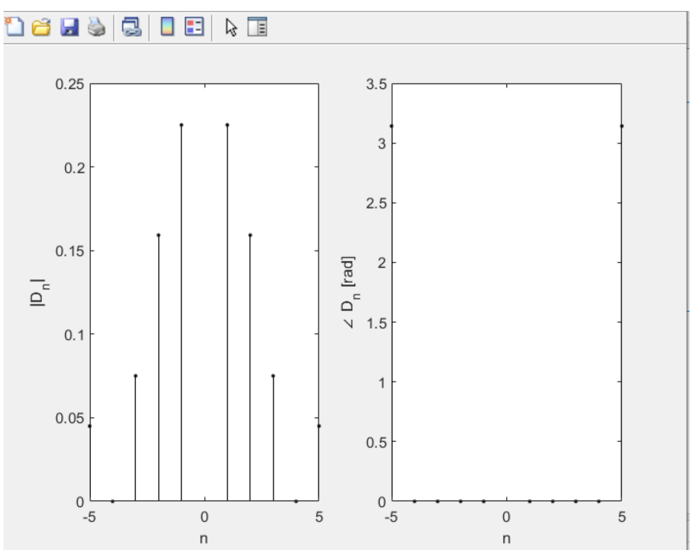
Code: Part A




```
%x_1(t)
clf;
n = (-5:5);
D_n = 1./2.*((1./(pi.*n))
.*sin((3-n).*pi )) +(1./pi.*n).
*sin((3+n).*pi) ...
+ (1./(2.*n.*pi).*sin((1+n).*pi)) +
(1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2);
stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle Dn
[rad]');
%%
```

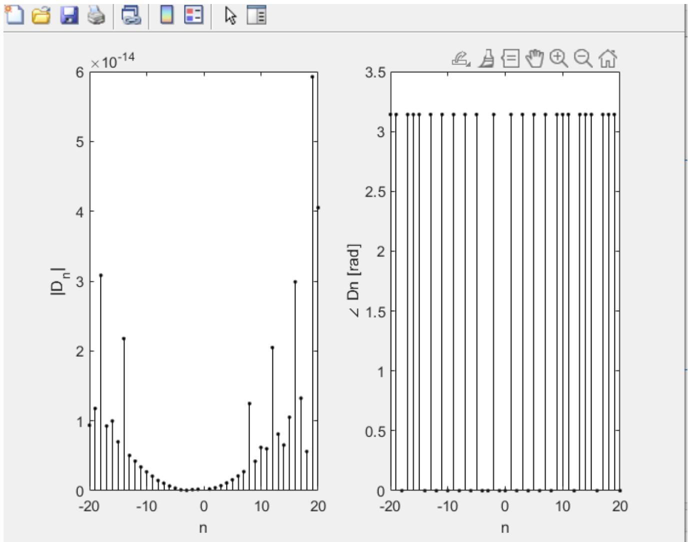



```
%x_2(t)
clf;
n = (-5:5);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2);
stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n
[rad]');
%%
```

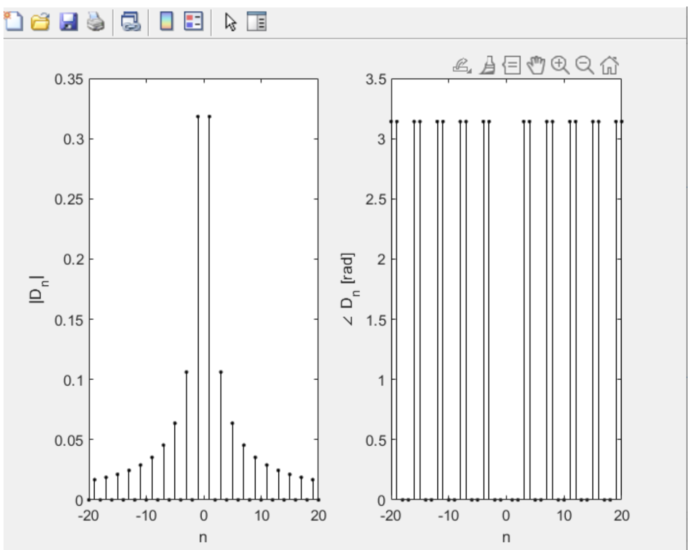



```
%x_3(t)
clf;
n = (-5:5);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2);
stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n
[rad]');
```

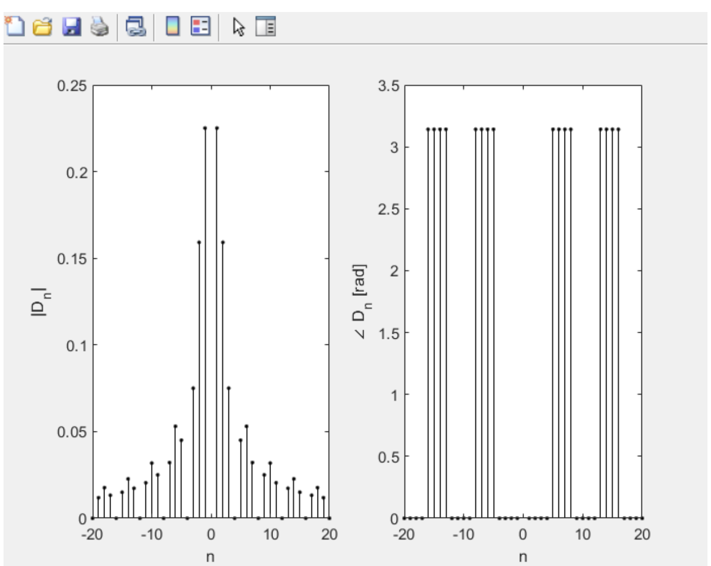
Code:Part B




```
%x_1(t)
clf;
n = (-20:20);
D_n =
1./2.*((1./(pi.*n)).*sin((3-n).*pi )) +
(1./pi.*n).*sin((3+n).*pi) ...
+ (1./(2.*n.*pi).*sin((1+n).*pi)) +
(1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2);
stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle Dn [rad]');
%%
```

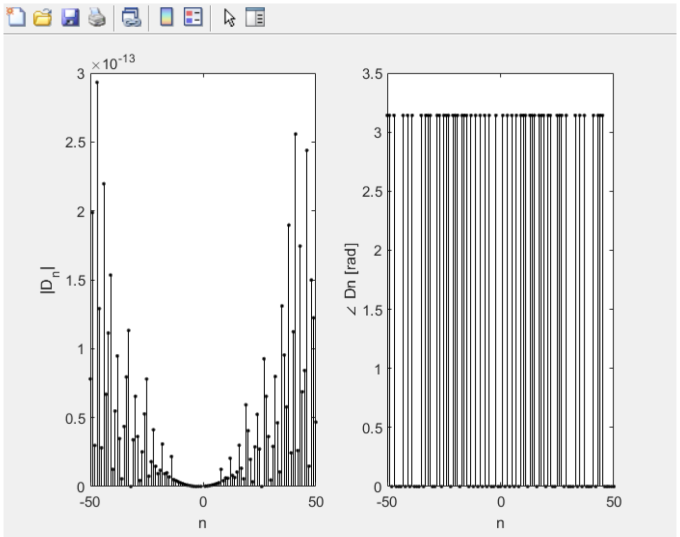



```
%x_2(t)
clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2);
stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n
[rad]');
%%
```

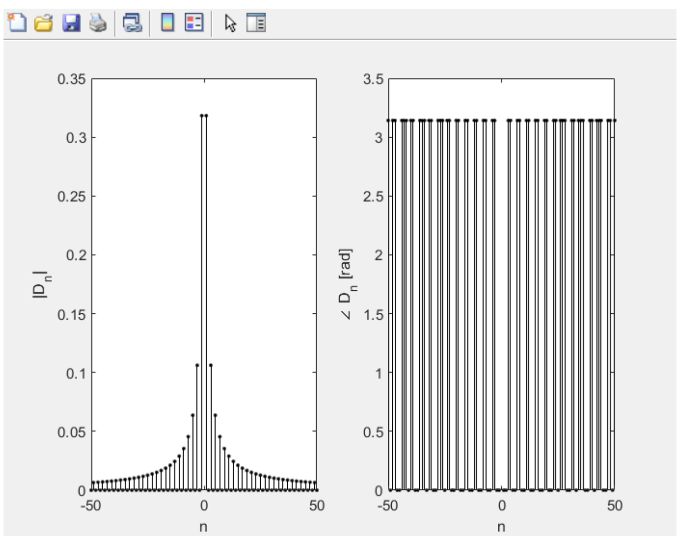



```
%x_3(t)
clf;
n = (-20:20);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2);
stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n
[rad]');
```

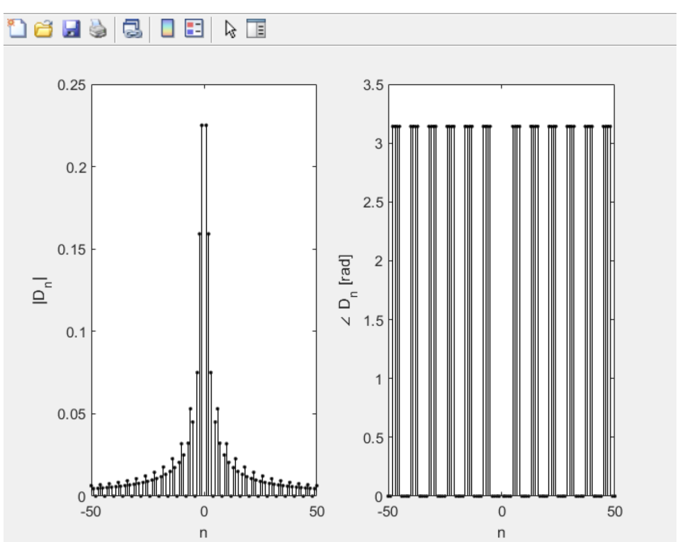
Code: Part C




```
%x_1(t)
clf;
n = (-50:50);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi
)) + (1./pi.*n).*sin((3+n).*pi) ...
+ (1./(2.*n.*pi).*sin((1+n).*pi)) +
(1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle Dn [rad]');
%%
```

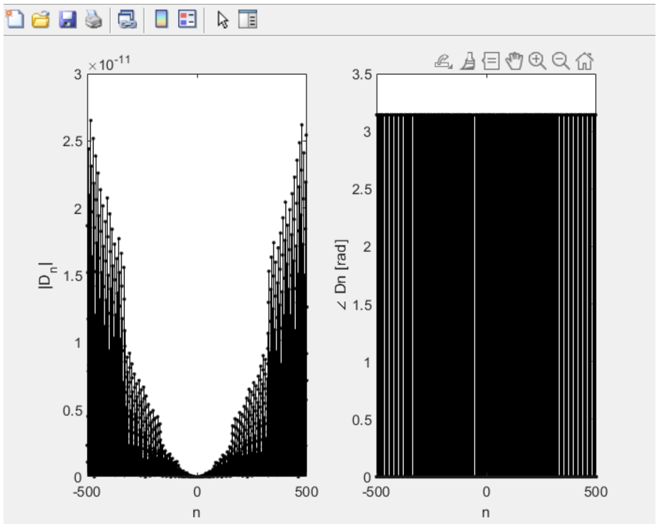



```
%x_2(t)
clf;
n = (-50:50);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
%%
```

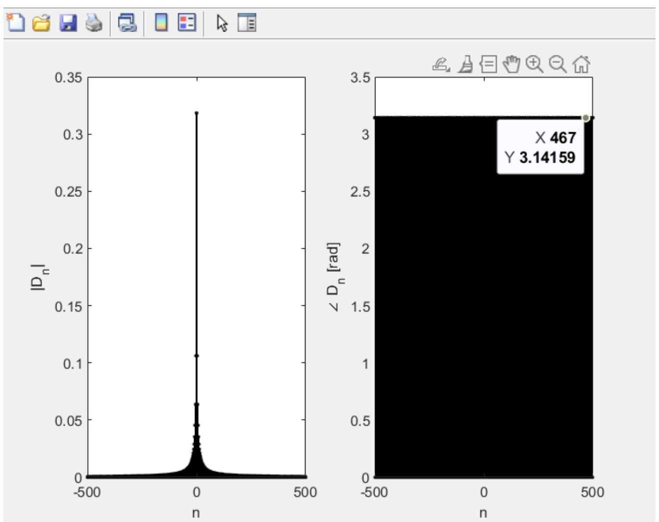



```
%x_3(t)
clf;
n = (-50:50);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

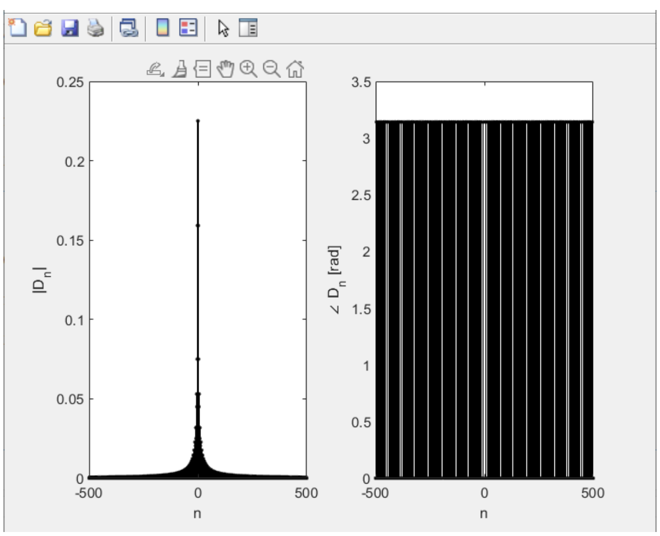
Code: Part D




```
%x_1(t)
clf;
n = (-500:500);
D_n = 1./2.*((1./(pi.*n)).*sin((3-n).*pi
)) + (1./pi.*n).*sin((3+n).*pi) ...
+ (1./(2.*n.*pi).*sin((1+n).*pi)) +
(1./(2.*n.*pi).*sin((1-n).*pi)) ;
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle Dn [rad]');
%%
```




```
%x_2(t)
clf;
n = (-500:500);
D_n = (1./(n.*pi).*sin((n.*pi)./2));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
%%
```




```
%x_3(t)
clf;
n = (-500:500);
D_n = (1./(n.*pi).*sin((n.*pi)./4));
subplot(1,2,1); stem(n,abs(D_n),'.k');
xlabel('n'); ylabel('|D_n|');
subplot(1,2,2); stem(n,angle(D_n),'.k');
xlabel('n'); ylabel('\angle D_n [rad]');
```

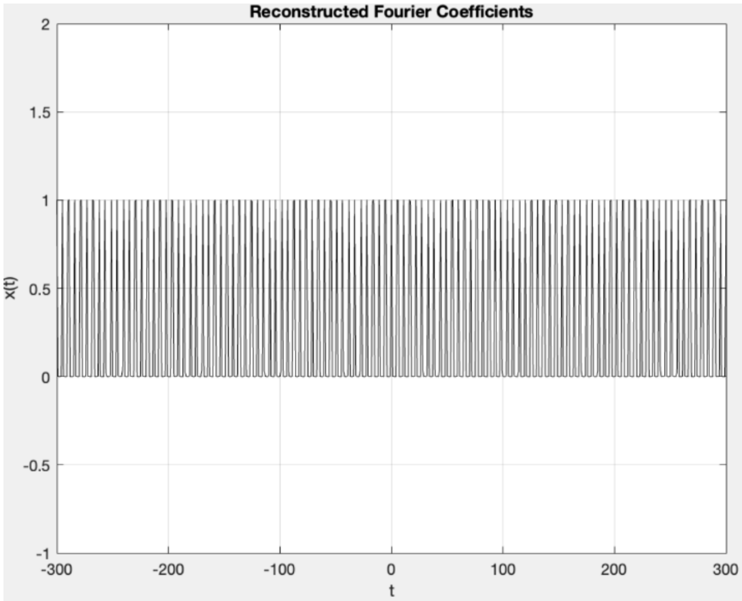
Problem A.5

A function is created using the Matlab code displayed below.

Code:

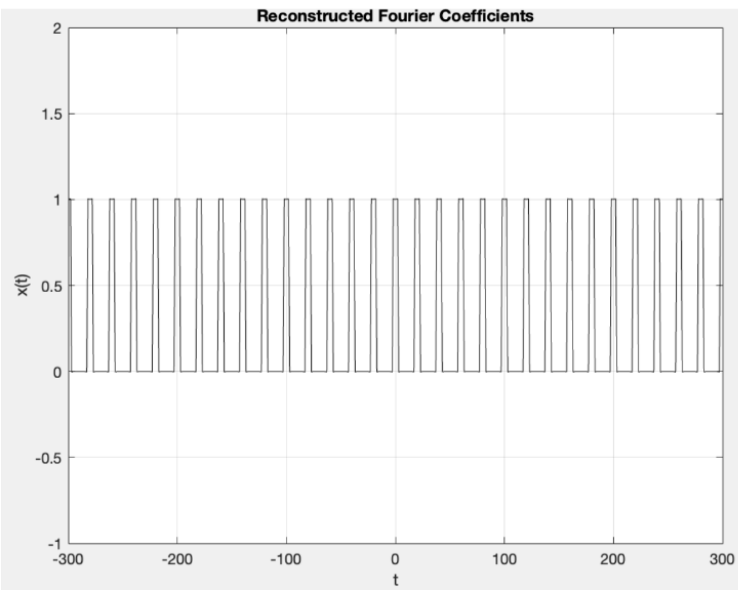
```
%% A5
function x = a5(d ,Dn)
if(d == 1)
w = pi/10;
elseif (d == 2)
w = pi/10;
elseif (d == 3)
w = pi/20;
end
t = -300:1:300;
x = zeros(size(t));
for i = 1:length(x)
total = 0;
j = 1;
for n = -500:500
total = total + Dn(j) * exp(1i* n * w * t(i));
j = j+1;
end
x(i) = total;
end
figure(1);
plot(t, x, 'b')
xlabel('t (s)');
ylabel('x(t)');
if(d ~= 1)
axis([-300 300 -1 2]);
end
title('Reconstructed Fourier Coefficients');
grid;
end
```

Problem A.6



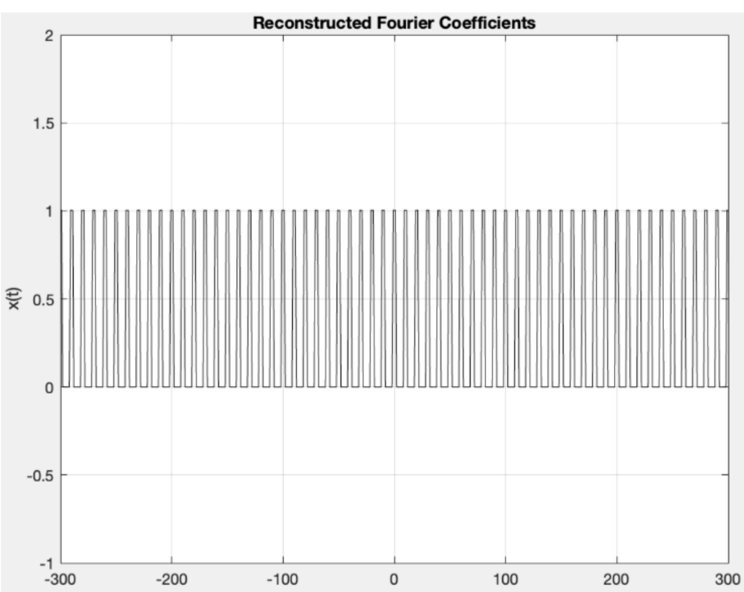
Code:

```
%% A.6 for x1(t)
clear;
clf;
n = (-500:500);
D_n = zeros(size(n));
a0 = 0;
for i = 1:length(n)
D_n(i) = a3(1, n(i), a0);
end
a5(1, D_n);
```



Code:

```
%% A.6 for x2(t)
clear;
clf;
n = (-500:500);
D_n = zeros(size(n));
a0 = 0.5;
for i = 1:length(n)
D_n(i) = a3(2, n(i), a0);
end
a5(2, D_n);
```



Code:

```
%% A.6 for x3(t)
clear;
clf;
n = (-500:500);
D_n = zeros(size(n));
a0 = 0.25;
for i = 1:length(n)
D_n(i) = a3(3, n(i), a0);
end
a5(3, D_n);
```

Problem B.1

$$x_1(t) = \cos\left(\frac{\pi}{10}\right)t + \frac{1}{2}\cos\left(\frac{\pi}{10}\right)t$$

$$W_{o_1} = \frac{3\pi}{10}, \quad W_{o_2} = \frac{\pi}{10}, \quad w_0 = \frac{GCF}{LCM} = \frac{\pi}{10}$$

$$x_2(t) = T_0 = 20t \qquad x_3(t) = T_0 = 40t$$

$$W_0 = \frac{2\pi}{20} = \boxed{\frac{\pi}{10}} \qquad W_0 = \frac{2\pi}{40} = \boxed{\frac{\pi}{20}}$$

Problem B.2

What is the main difference between the Fourier coefficients of x1(t) and x2(t)?

The main difference between the Fourier coefficients of X1(t) and X2(t) is that the function X1(t) is derived from the equation while X2(t) is derived from a graph where the Dn value is significantly different.

Problem B.3

Signals x2(t) and x3(t) have the same rectangular pulse shape but different periods. How are these characteristics reflected in their respective Fourier coefficients?

The fourier coefficients reflect the pulse which are depicted in the X2(t) and X3(t) functions. The X3(t) has a smaller fundamental frequency than the X2(t) signal.

Problem B.4

The Fourier coefficient D0 represents the DC value of the signal. Let x4(t) be the periodic waveform shown in Figure 2. Derive D0 of x4(t) from D0 of x2(t)

While looking at the created function graph of X4(t), it can be seen that the waveform for X2(t) and X4(t) are quite similar. The value of D0 of X4(t) can be derived from the equation of X2(t), where D0 is 0.5

Problem B.5

Using the results of Problem A.6, explain how the reconstructed signal changes as you increase the number of Fourier coefficients used in the reconstruction. Discuss for both x1(t) and x2(t).

Theoretically, increasing the number of Fourier coefficients in the reconstructed signal will result in the new graphs having a greater accuracy. In addition, the X1(t) signal has a greater 'W' value than the X2(t). Since the W value is greater, this will result in the reconstructed Fourier coefficients to occur more frequently as compared to X1(t), which is depicted in the graph.

Problem B.6

How many Fourier coefficients do you need to **perfectly** reconstruct the periodic waveforms discussed in this lab experiment?

In order to reconstruct a perfect periodic waveform discussed in this lab, we would need an infinite number of Dn values. Without an infinite number of Dn values, there will be no perfectly reconstructed periodic waveform.

Problem B.7

Let x(t) be an arbitrary periodic signal. Instead of storing x(t) on a computer, we consider storing the corresponding Fourier coefficients. When we need to access x(t), we read the Fourier coefficients stored on the computer hard drive and reconstruct the signal. Is this a viable scenario? Explain your answer.

This is not a viable scenario as the periodic signal will have an infinite number of Dn values. Only if there is a finite number of Dn values, they can be stored as it will take a certain amount of storage. At the same time, if the Dn values are finite, it may take up too much space on the drive and it may not be optimal .