RYERSON UNIVERSITY

Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

Course Number	ELE 532 - Section 12
Course Title	Signals and Systems 1
Semester / Year	F2023

Instructor	Luella Marcos

Assignment Number	2

Assignment Title	System Properties and Convolution

Submission Date	2023/10/12
Due Date	2023/10/22

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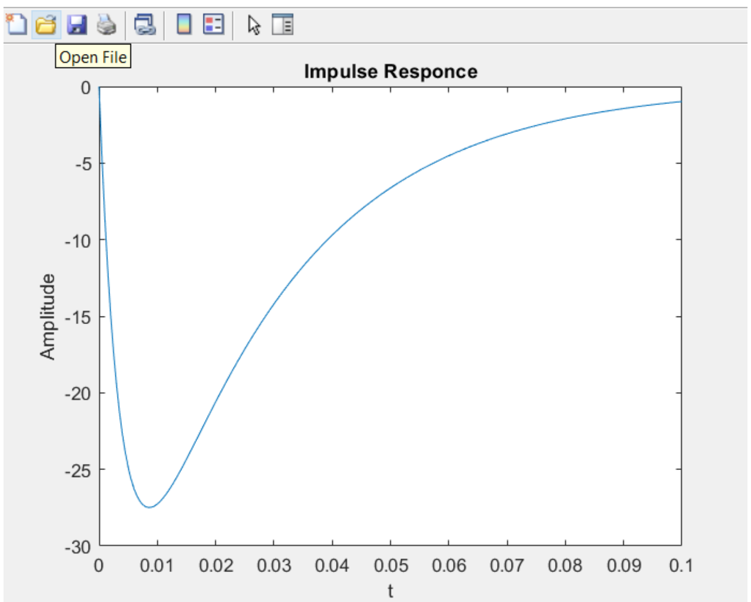
ELE532 - Lab 2

Problem A.1

Code:

```
%% Part A1
% Set component values:
R = [1e4, 1e4, 1e4];
C = [1e-9, 1e-6];
% Determine coefficients for characteristic equation:
A = [1, (1/R(1) +1/R(2) + 1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
%Characteristic root
Lambda = roots(A)
%Root of the matrix and returns the origional polynomial equation.
poly(Lambda)
```

Problem A.2



Code:

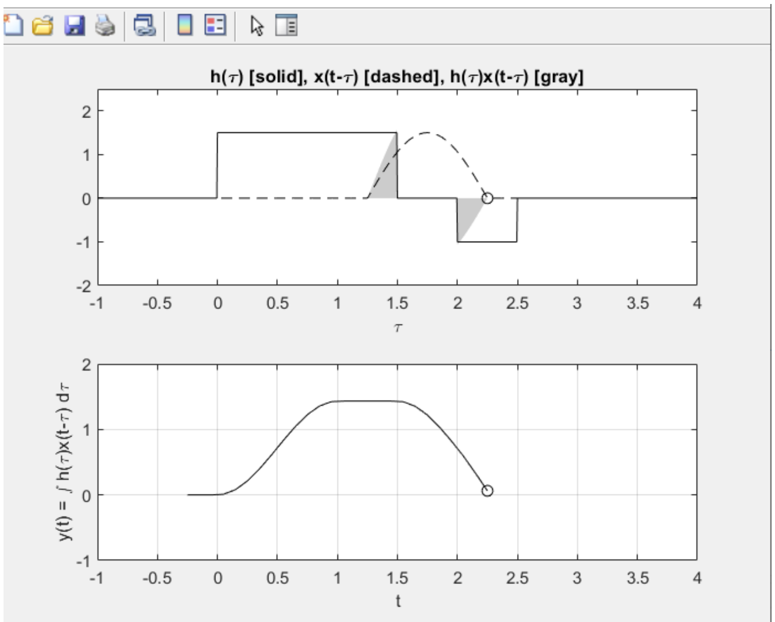
```
syms y(t)
dy = 1*diff(y,t,2) +
300*diff(y,t) + 10000*y ==0;
conditions = [y(0) == 0,
subs(diff(y,t),t,0) ==1];
h0 = dsolve(dy, conditions);
h1 = -10000*h0;
syms t
tm = 0:0.0005:0.1;
%u = @(t) 1.0* (t>=0);
u = 1.0*(tm>=0);
h = subs(h1,t,tm);
plot(tm,h.*u);
xlabel('t');
ylabel('Amplitude');
title('Impulse Responce');
```

Problem A.3

Code:

```
function [lambda] = CH2MP2(R,C)
% coefficients for the characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% characteristic roots:
lambda = roots(A)
end
```

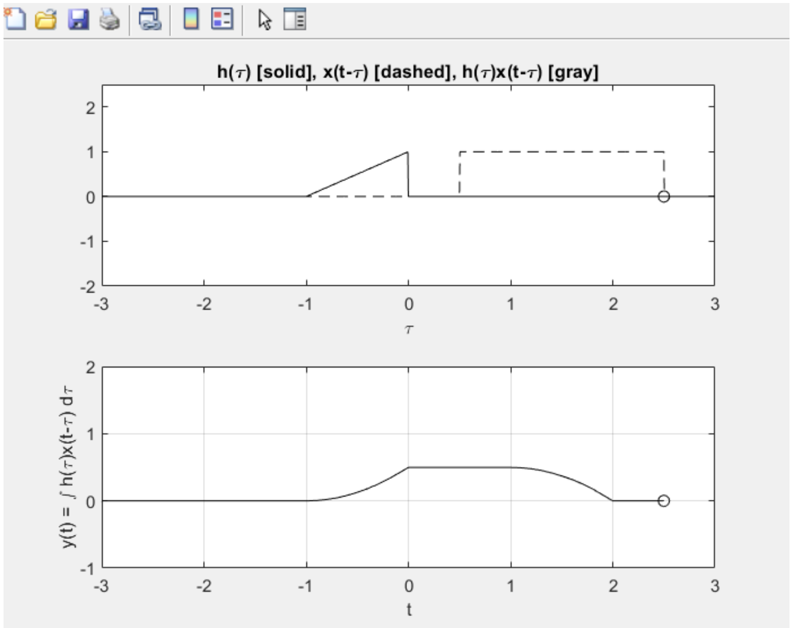
Problem B.1



Code:

```
%% Part B1
% CH2MP4.m : Chapter 2, MATLAB Program 4
% Script M-file graphically demonstrates the convolution process.
figure(1); % Create figure window
u = @(t) 1.0*(t>=0);
x = @(t) 1.5*sin(pi*t).*(u(t)-u(t-1));
h = @(t) 1.5*(u(t)-u(t-1.5))-u(t-2)+u(t-2.5);
dtau = 0.005; tau = -1:dtau:4;
ti = 0; tvec = -.25:.1:3.75;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
figure
for t = tvec
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
pause;
end
```

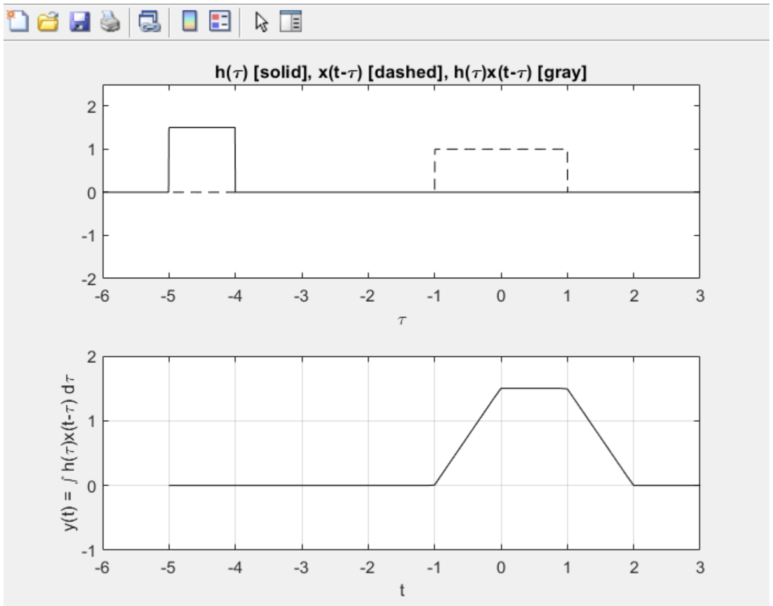
Problem B.2



Code:

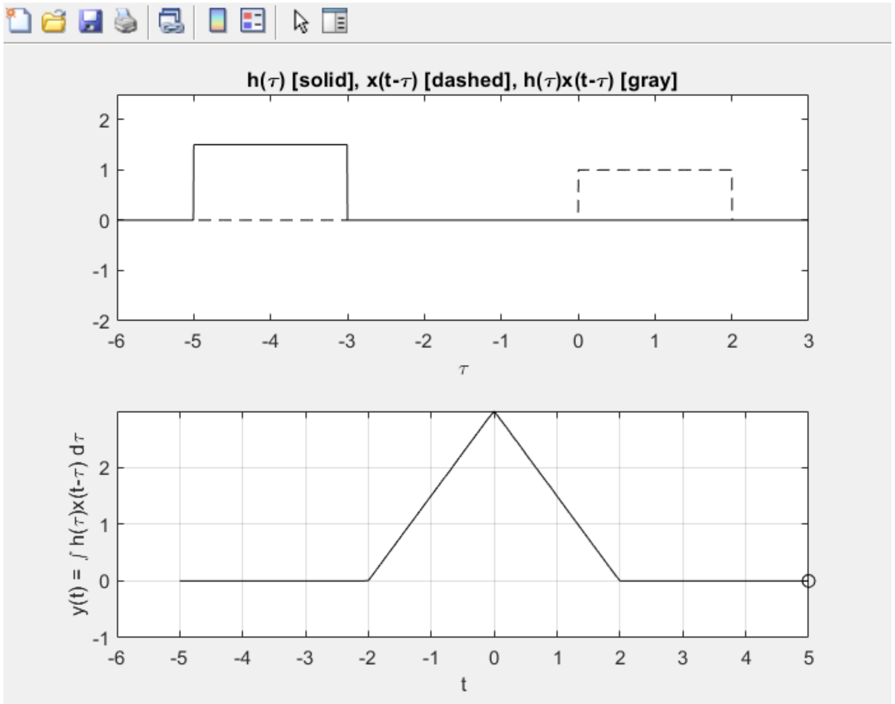
```
%% Part B2
figure(2);
u = @(t) 1.0*(t>=0);
x = @(t) u(t)-u(t-2);
h = @(t) (t+1).*(u(t+1)-u(t));
dtau = 0.005; tau = -2:dtau:2;
ti = 0;
tvec = -5:.1:5; y =
NaN*zeros(1,length(tvec));
% Pre-allocate memory
for t = tvec
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh =
length(xh);
y(ti) = sum(xh.*dtau); %
Trapezoidal approximation of
convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau)
[gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
pause;
end
```

Problem B.3



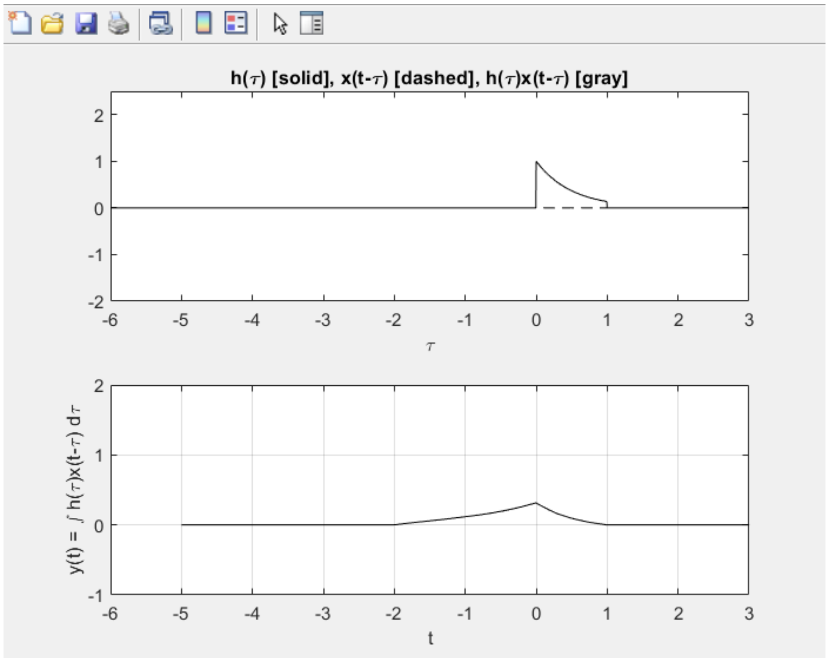
a) **Code:**

```
%% Part B3-a
figure(3);
u = @(t) 1.0*(t>=0);
%Value of A is not given So the following assumptions are made:
A = 1; B = 1.5;
x = @(t) A*(u(t-4)-u(t-6));
h = @(t) B*(u(t+5)-u(t+4));
dtau = 0.005; tau = -6:dtau:3;
ti = 0; tvec = -5:.1:5;
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```



b) Code:

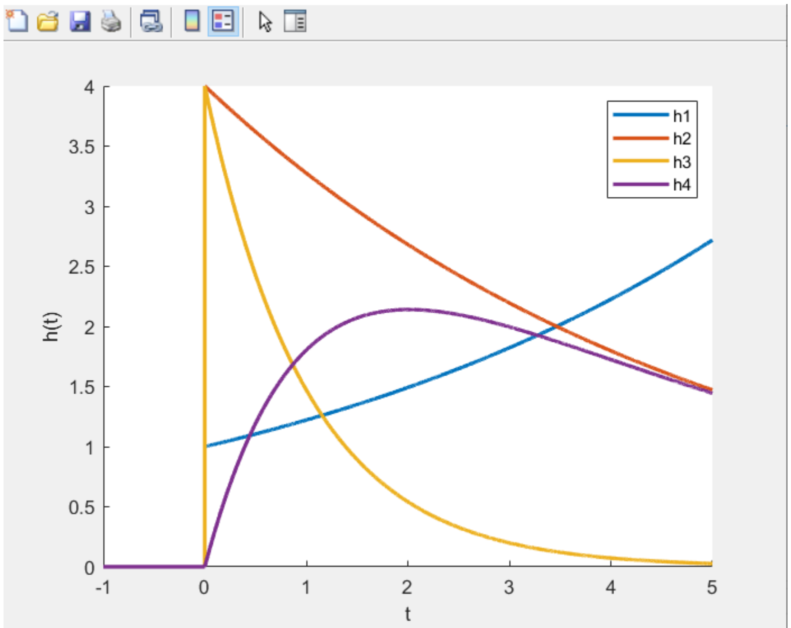
```
%% Part B3-b
figure(4);
u = @(t) 1.0*(t>=0);
%Value of A is not
given So the
following assumptions
are made:
A = 1; B = 1.5;
x = @(t)
A*(u(t-3)-u(t-5));
h = @(t)
B*(u(t+5)-u(t+3));
dtau = 0.005; tau =
-6:dtau:3;
ti = 0; tvec =
-5:.1:5;
y =
NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution
integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed],
h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```



c) Code:

```
%% Part B3-b
figure(5);
u = @(t) 1.0*(t>=0);
x = @(t)
exp(t).*(u(t+2)-u(t));
h = @(t) exp(-2*t)
.*(u(t)-u(t-1));
dtau = 0.005; tau =
-6:dtau:3;
ti = 0; tvec = -5:.1:5;
y = NaN*zeros
(1,length(tvec)); %
Pre-allocate memory
for t = tvec
ti = ti+1; % Time index
xh = x(t-tau).*h(tau);
lxh = length(xh);
y(ti) = sum(xh.*dtau); %
Trapezoidal
approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau)
[gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```

Problem C.1:



Code:
```
%% Part C1
t = [-1:0.001:5];
% Function
u = @(t) 1.0.* (t>=0);
h1 = @(t) exp(t/5).*u(t);
h2 = @(t) 4*exp(-t/5).*u(t);
h3 = @(t) 4*exp(-t).*u(t);
h4 = @(t) 4*(exp(-t/5) -
exp(-t)).*u(t);
xlabel("t"); ylabel("h(t)");
hold on;
plot(t,h1(t), "LineWidth",2);
plot(t,h2(t), "LineWidth",2);
plot(t,h3(t),"LineWidth",2);
plot(t,h4(t), "LineWidth",2);
legend("h1", "h2", "h3","h4");
hold off;
```

Problem C.2:

The characteristic values (eigenvalues) of systems S1–S are as follows:

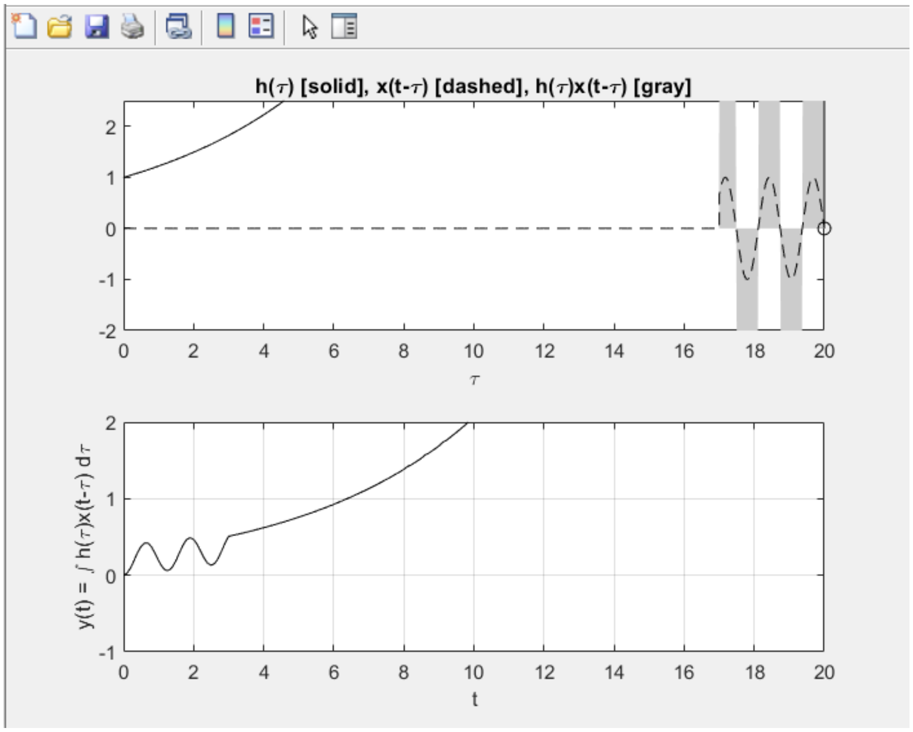
S1: $\lambda 1 = \frac{1}{5}$

S2: $\lambda 1 = -\frac{1}{5}$

S3: $\lambda 1 = -1$

S4: $\lambda 1 = -\frac{1}{5}, \lambda 2 = -1$

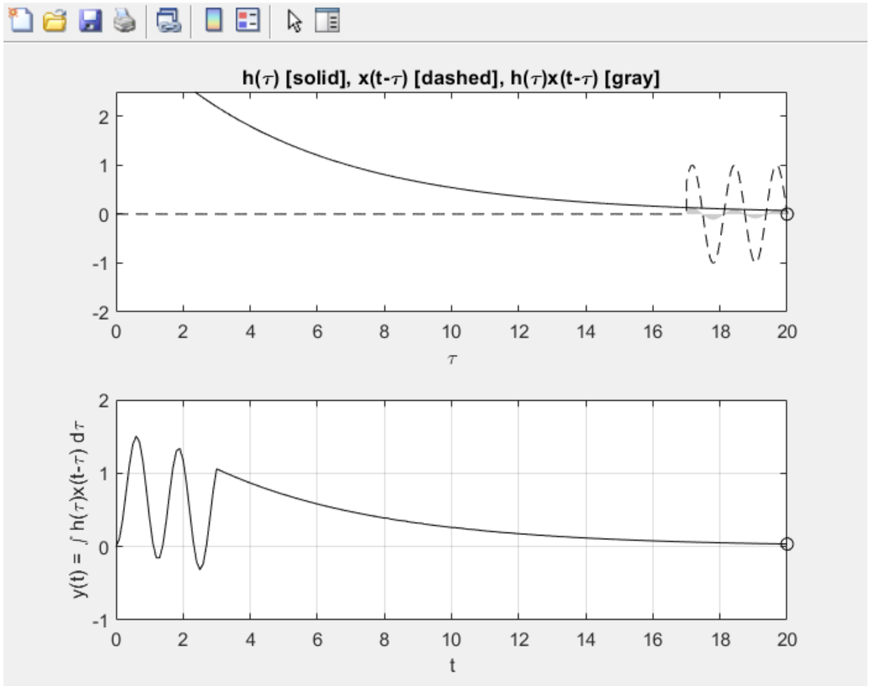
Problem C.3:



a) Code (h1):

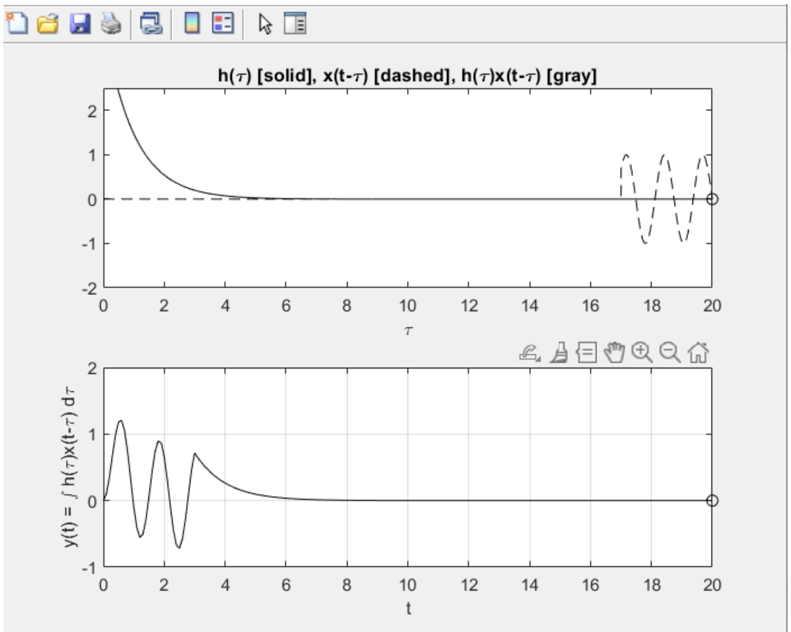
```
%% C3 - a
u = @(t)
1.0*(t>=0);
%x(t) function
x = @(t)
sin(5*t).*(u(t) -
u(t-3));
%Truncate inpulse
responce function
h = @(t)
exp(t/5).*(u(t) -
u(t-20));
%Modified CH2MP2
section
dtau = 0.005; tau =
0:dtau:20;
ti = 0; tvec =
0:0.1:20;
```

```
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau)
[gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```



b) Code (h2):

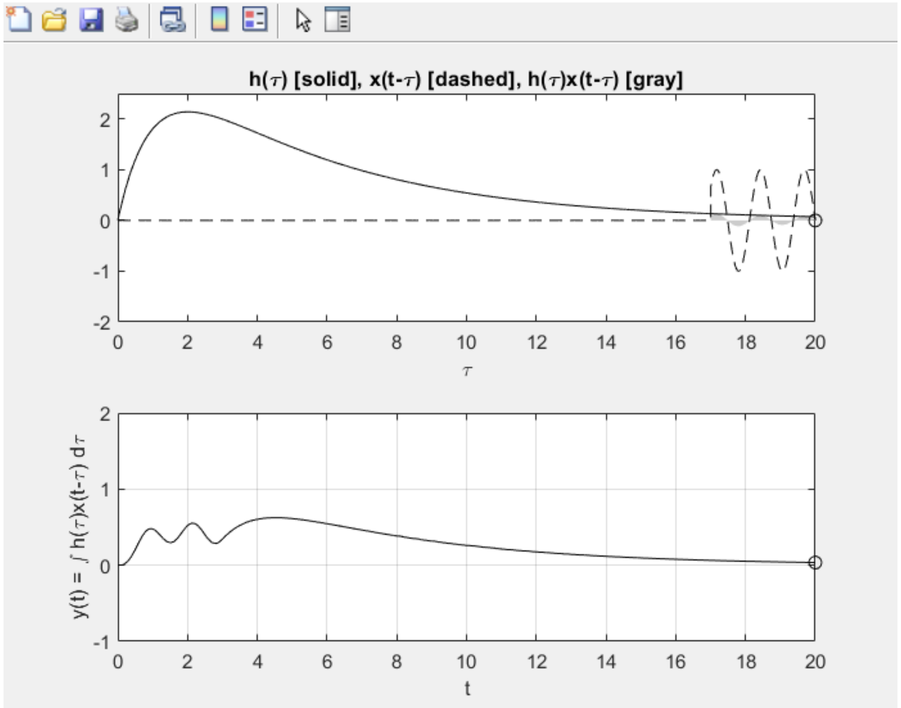
```
%% C3 - b
u = @(t) 1.0*(t>=0);
%x(t) function
x = @(t)
sin(5*t).*(u(t) -
u(t-3));
%Truncate inpulse
responce function
h2(t)
h = @(t)
4*exp(-t/5).*(u(t) -
u(t-20));
%Modified CH2MP2
section
dtau = 0.005; tau =
0:dtau:20;
ti = 0; tvec =
0:0.1:20;
y =
NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution
integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed],
h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```



c) Code (h3)

```
%% C3 - c
u = @(t) 1.0*(t>=0);
%x(t) function
x = @(t) sin(5*t).*(u(t) -
u(t-3));
%Truncate inpulse responce
function h3(t)
h = @(t) 4*exp(-t).*(u(t) -
u(t-20));
%Modified CH2MP2 section
dtau = 0.005; tau =
0:dtau:20;
ti = 0; tvec = 0:0.1:20;
y =
NaN*zeros(1,length(tvec)); %
Pre-allocate memory
for t = tvec
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh =
length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution
integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed],
h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```

d) Code h(4)




```
%% C3 - d
u = @(t) 1.0*(t>=0);
%x(t) function
x = @(t) sin(5*t).*(u(t) -
u(t-3));
%Truncate inpulse responce
function h4(t)
h = @(t)
4*(exp(-t/5)-exp(-t)).*(u(t)
- u(t-20));
%Modified CH2MP2 section
dtau = 0.005; tau =
0:dtau:20;
ti = 0; tvec = 0:0.1:20;
y =
NaN*zeros(1,length(tvec)); %
Pre-allocate memory
for t = tvec
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh =
length(xh);
y(ti) = sum(xh.*dtau); %
Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau)
[gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
End
```

The elements within each of tne convolution ahev similarities with the sin(t) function. Furthermore, it became apparent that the duration of the signal resulting from the convolution of two signals equates to the sum of the durations of the individual functions. There is a relationship among S2, S3, and S4. S2 and S3 exhibit a similar convolution, while S4 displays convolution patterns similar to those in S2 and S3.

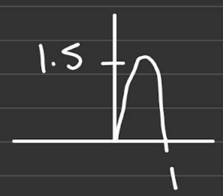
Discussion

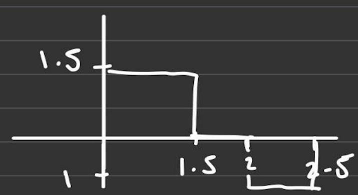
D.1

B1

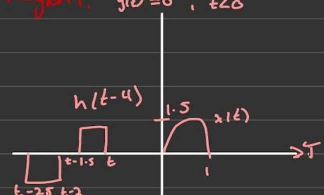
$$x(t) = 1.5\sin(\pi t)(u(t) - u(t-1))$$

$$h(t) = 1.5(u(t) - u(t-1.5)) - (u(t-2) + u(t-2.5))$$





Region 1: $y(t) = 0$, $t < 0$



Region 2: $0 \le t \le 1$

$$y(t) = \int_0^t (1.5)(1.5\sin \pi t)\,dt$$

$$\Rightarrow -2.25\frac{\cos \pi t}{\pi}\Big|_0^t - 2.25\frac{\cos \pi t}{\pi} + \frac{2.25\cos(0)}{\pi}$$

$$\Rightarrow 2.25\left(\frac{1}{\pi} - \frac{\cos(\pi t)}{\pi}\right)$$

Region 3: $t \ge 1,\ t - 1.5 \le 0$

$t \le t \le 1.5$

$$y(t) = \int_0^1 (1.5)(1.5\sin(\pi t))\,dt \Rightarrow \frac{-2.25\cos(\pi t)}{\pi}\Big|_0^1 \Rightarrow \frac{-2.25\cos(\pi)}{\pi} + \frac{2.25\cos(0)}{\pi}$$

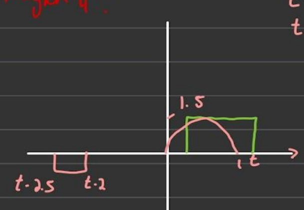
$$\Rightarrow \frac{2.25}{\pi} + \frac{2.25}{\pi} = 1.43241$$

Region 4:

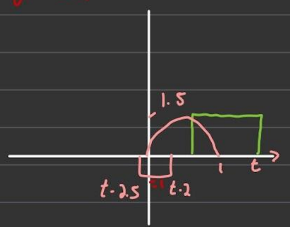
$$\left.\begin{array}{l} t - 1.5 \le 0.5 \rightarrow t \le 2 \\ t - 1.5 \ge 0 \end{array}\right\} 1.5 \le t \le 2$$

$$y(t) = \int_{t-1.5}^1 (1.5)(1.5\sin(\pi t))\,dt \Rightarrow \frac{-2.25\cos(\pi t)}{\pi}\Big|_{t-1.5}^1$$

$$\Rightarrow \frac{-2.25\cos(\pi)}{\pi} + \frac{2.25\cos(\pi(t-1.5))}{\pi} = \frac{2.25}{\pi}(1 + \cos(\pi(t-1.5)))$$



Region 5:



$$\left.\begin{matrix} t-2 \geq 0 \\ t-1.5 \leq 1 \end{matrix}\right\} 2 \leq t \leq 2.5$$

$$y(t) = \int_0^{t-2} (-1)(1.5\sin(\pi t))\,dt + \int_{t-1.5}^{1} (1.5)(1.5\sin(\pi t))\,dt$$

$$\Rightarrow \left.\frac{1.5\cos(\pi t)}{\pi}\right|_0^{t-2} - \left.\frac{2.25\cos(\pi t)}{\pi}\right|_{t-1.5}^{1}$$

$$\Rightarrow \frac{1.5}{\pi}\left(\cos(\pi(t-2)) - 1\right) + \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5))\right)$$

Region 6:

$$\left.\begin{matrix} t-2 \leq 1 \rightarrow t \leq 3 \\ t-2.5 \geq 0 \end{matrix}\right\} 2.5 \leq t \leq 3$$

$$y(t) = \int_{t-2.5}^{t-2} (-1)(1.5)\sin(\pi t)\,dt \Rightarrow \left.\frac{1.5\cos(\pi t)}{\pi}\right|_{t-2.5}^{t-2}$$

$$\Rightarrow \underline{1.5\cos(\pi(t-2))} - \frac{1.5\cos(\pi(t-2.5))}{\pi}$$

$$\Rightarrow \frac{1.5}{\pi}\left(\cos(\pi(t-2)) - \cos(\pi(t-2.5))\right)$$

Region 7: $3 \leq t \leq 3.5$

$$\int_{t-2.5}^{1} (-1)(1.5)\sin(\pi t)\,dt \Rightarrow \left.\frac{1.5\cos(\pi t)}{\pi}\right|_{t-2.5}^{1}$$

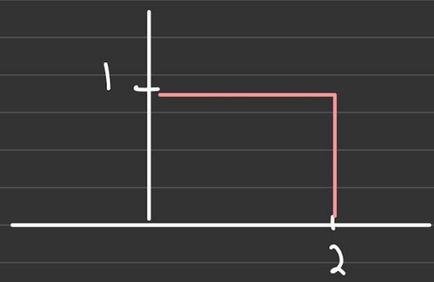
$$\Rightarrow \frac{1.5}{\pi}\left(\cos - 1 - \cos(\pi(t-2.5))\right)$$

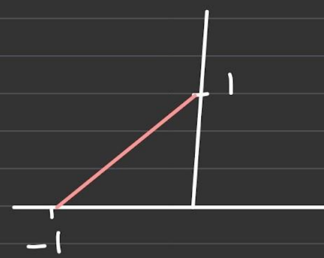
Region 8: $y(t) = 0$

$$y(t) = \begin{cases} 0 & t \leq 0 \\ 2.25\left(\frac{t}{\pi} - \frac{\cos(\pi t)}{\pi}\right) & 0 \leq t \leq 1 \\ 1.43241 & 1 \leq t \leq 1.5 \\ \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5))\right) & 1.5 \leq t \leq 2 \\ \frac{1.5}{\pi}\left(\cos(\pi(t-2)) - 1\right) + \frac{2.25}{\pi}\left(1 + \cos(\pi(t-1.5))\right) & 2 \leq t \leq 2.5 \\ \frac{1.5}{\pi}\left(\cos(\pi(t-2)) - \cos(\pi(t-2.5))\right) & 2.5 \leq t \leq 3 \\ \frac{1.5}{\pi}\left(\cos - 1 - \cos(\pi(t-2.5))\right) & 3 \leq t \leq 3.5 \\ 0 & t \geq 3.5 \end{cases}$$

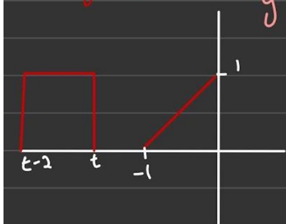
B2

$x(t) = u(t) - u(t-2)$ $\qquad$ $h(t) = (t+1)(u(t+1) - u(t))$





Region 1: $y(t) = 0$
$t \le -1$

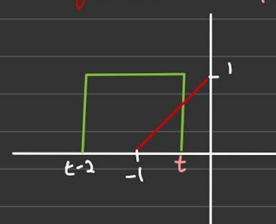


Region 2: $-1 \le t \le 0$

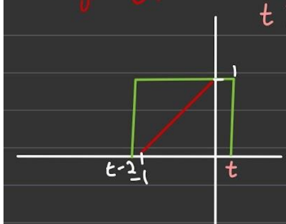
$y(t) = \int_{-1}^{t} (1)(t+1)\,dt$

$\Rightarrow T\left(\frac{t}{2} + 1\right)\Big|_{-1}^{t} = \left(\frac{t^2}{2} + t\right)\left(\frac{1}{2} - 1\right)$

$\Rightarrow \frac{t^2}{2} + t + \frac{1}{2}$



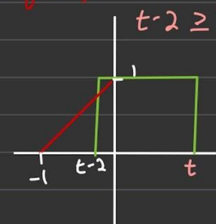
Region 3: $\left.\begin{matrix} t-2 \le 1 \\ t \ge 0 \end{matrix}\right\} 0 \le t \le 1$



$y(t) = \int_{-1}^{0} (t+1)\,dt \Rightarrow \frac{t^2}{2} + t\Big|_{-1}^{0}$

$\Rightarrow \frac{1}{2} - 1 = -0.5$

Region 4: $\left.\begin{matrix} t-2 \le 0 \\ t-2 \ge 1 \end{matrix}\right\} 1 \le t \le 2$



$y(t) = \int_{t-2}^{0} (1)(t+1)\,dt$

$\Rightarrow \frac{t^2}{2} + t\Big|_{t-2}^{0}$

$\Rightarrow -\left\{\frac{(t-2)^2}{2} + t - 2\right\}$

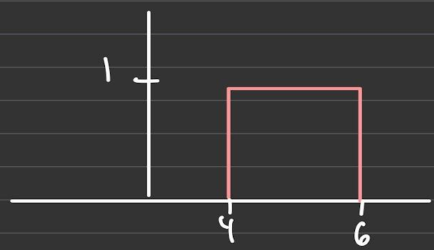
Region 5: $y(t) = 0 \quad t-2 \ge 0 \rightarrow t \ge 2$

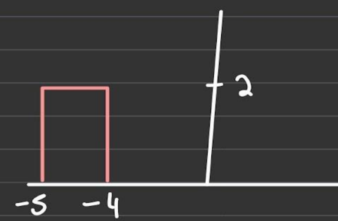
$$y(t) \begin{cases} 0 & t \le -1 \\ \frac{t^2}{2} + t + \frac{1}{2} & -1 \le t \le 0 \\ -0.5 & 0 \le t \le 1 \\ -\left\{\frac{(t-2)^2}{2} + t - 2\right\} & 1 \le t \le 2 \\ 0 & t \ge 2 \end{cases}$$

B3

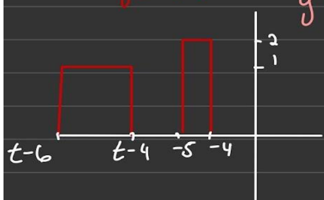
Assume $A = 1,\ B = 2$

$x(t) = u(t-4) - u(t-6)$ $\quad$ $h(t) = 2(u(t+5) - u(t+4))$





Region 1:

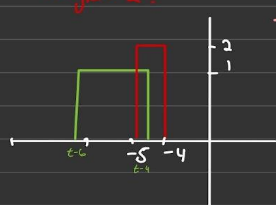


$y(t) = 0$

$t - 4 \leq 5$

$t \leq -1$

Region 2:



$-5 \leq t-4 \leq 4$

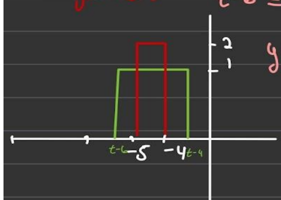
$-1 \leq t \leq 0$

$y(t) = \int_{-5}^{t-4} (1)(2)\,dt \Rightarrow 2t \Big|_{-5}^{t-4}$

$\Rightarrow 2(t-4) - (-10)$

$\Rightarrow 2t - 8 + 10 = 2t + 2$

Region 3: $\left.\begin{array}{l} t-4 \geq -4 \\ t-6 \leq -5 \end{array}\right\} 0 \leq t \leq 1$

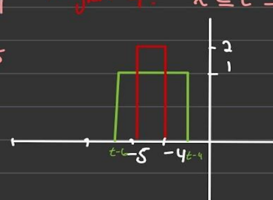


$y(t) = \int_{-5}^{-4} 2\,dt = 2\tau \Big|_{-5}^{-4}$

$\Rightarrow (-8) - (-10)$

$= 2$

Region 4: $x \leq t \leq 2$



$y(t) = \int_{t-6}^{-4} 2\tau \Rightarrow 2\tau \Big|_{t-6}^{-1}$

$\Rightarrow -8 - (2t - 12)$

$= -2t + 4$

Region 5: $y(t) = 0$

$t - 6 \geq -4$

$t \geq 2$

$$y(t) \begin{cases} 0 & t \leq -1 \\ 2t + 2 & -1 \leq t \leq 0 \\ 2 & 0 \leq t \leq 1 \\ -2t + 4 & 1 \leq t \leq 2 \\ 0 & t \geq 2 \end{cases}$$

D.2

While observing the width and duration of the 2 convolution functions, it can be seen that the restaurant convolution is the sum of each of the individual durations of the 2 signals.