Numerical Solutions of Differential Equations Project #2

due 2023 APR 17, 10:00 a.m.

1 The Assignment

(A) (50 points) Section 9.5 in the notes.

(B) (30 points) Repeat Section 9.5 in the notes to solve the two-dimensional Poisson equation in Definition 7.4 on $\Omega = (0,1)^2$ by a straightforward generalization of the one-dimensional multigrid method. In addition to errors and convergence rates, you should also compare the CPU time between your multigrid implementation and your LU-factorization implementation in Project 1.

Extra credits:

(a) (10 points) Use template programming throughout your package with `int dim` as the template parameter so that the main program and the unit tests are the only places where `dim`, the dimensionality of the domain, is specialized to 1 and 2.

(b) (20 points) Give a complete and coherent presentation of the theory to analyze the convergence of your two-dimensional multigrid method on $\Omega = (0,1)^2$. Your report must be written in LaTeX and must follow the style of the notes. (**hint**: use Kronecker product in Section 7.6.1)

(c) (50 points) Change the problem domain $\Omega = (0,1)^2$ to $\tilde{\Omega}$, which is the same boundary as Ω except that the lower boundary is changed to

$$\left\{(x,y) : y = \frac{1}{16}\sin(\pi x); x \in [0,1]\right\}.$$

Design a two-dimensional multigrid method for simply-connected irregular domains and carry out (B) and (b) for your method.

To get any extra credit for (b) and (c), your report must hit at least half of the main points on the answer sheet.

Additional 10% credits will be given to you if you typeset your report in LaTeX.

2 How to submit

Send to `NumPDEs@163.com` your report, your `C++` package, and other documentation in a single gzipped tar ball named as `YourName_Project2.tar.gz`.