

# SSerxhs 的 ICPC 模板

SSerxhs

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# 1 前言

此模板的初衷是个人使用，因此已有的模板可能未列出。建议结合 Heltion 模板和 HDU 模板使用。

模板需要的版本为 cpp17 或 cpp20。

大部分情况下，涉及取模的都需要使用 unsigned long long，即使类型名是 ll。这是因为值域较大有利于合理减少取模次数。

optional 的用法：一个 optional 变量 r 可以用 if (r) 判断其是否有值。取出值的方法是 \*r。常见于包含无解又包含空集解的代码中，便于区分无解和空集解。

常见的被漏掉的初始代码：

```
#define all(x) (x).begin(),(x).end()
using lll=__int128;
template<class T1, class T2> bool cmin(T1 &x, const T2 &y) { if (y<x) { x=y; return 1; } return 0; }
template<class T1, class T2> bool cmax(T1 &x, const T2 &y) { if (x<y) { x=y; return 1; } return 0; }
template<class typC> void read(typC &x)
{
    int c=getchar(),fh=1;
    while ((c<48)|| (c>57))
    {
        if (c=='-') {c=getchar();fh=-1;break;}
        c=getchar();
    }
    x=c^48;c=getchar();
    while ((c>=48)&&(c<=57))
    {
        x=x*10+(c^48);
        c=getchar();
    }
    x*=fh;
}
```

常见的缺漏算法：

回文自动机。

## 2 数据结构

### 2.1 树状数组

支持单点修改、求前缀和、二分前缀和大于等于  $x$  的第一个位置。

二分这部分没有验证过。

```
template<typename typC> struct bit
{
    vector<typC> a;
    int n;
    bit() { }
    bit(int nn):n(nn), a(nn+1) { }
    template<typename T> bit(int nn, T *b):n(nn), a(nn+1)
    {
        for (int i=1; i<=n; i++) a[i]=b[i];
        for (int i=1; i<=n; i++) if (i+(i&-i)<=n) a[i+(i&-i)]+=a[i];
    }
    void add(int x, typC y)
    {
        //cerr<<"add "<<x<<" by "<<y<<endl;
        assert(1<=x&&x<=n);
        a[x]+=y;
        while ((x+=x&-x)<=n) a[x]+=y;
    }
    typC sum(int x)
    {
        //cerr<<"sum "<<x;
        assert(0<=x&&x<=n);
        typC r=a[x];
        while (x^=x&-x) r+=a[x];
        //cerr<<"= "<<r<<endl;
        return r;
    }
    typC sum(int x, int y)
    {
        return sum(y)-sum(x-1);
    }
    int lower_bound(typC x)
    {
        if (n==0) return 0;
        int i=__lg(n), j=0;
        for (; i>=0; i--) if ((1<<i|j)<=n&&a[1<<i|j]<x) j|=1<<i, x-=a[j];
        return j+1;
    }
};
```

### 2.2 线段树

包含标记的线段树，支持线段树上二分，采用左闭右闭。但只支持求左侧第一个符合条件的下标。

要求：具有  $\text{info}+\text{info}$ ,  $\text{info}+=\text{tag}$ ,  $\text{tag}+=\text{tag}$ 。info, tag 需要有默认构造，但不必有正确的值。

```
template<class info, class tag> struct sgt
```

```

{
    int n, shift;
    info *a;
    info tmp;
    vector<info> s;
    vector<tag> tg;
    vector<int> lz;
    bool flg;
    void build(int x, int l, int r)
    {
        if (l==r)
        {
            s[x]=(flg?tmp:a[l]);
            return;
        }
        int c=x*2, m=l+r>>1;
        build(c, l, m); build(c+1, m+1, r);
        s[x]=s[c]+s[c+1];
    }
    sgt(info *b, int L, int R):n(R-L+1), shift(L-1), a(b+L-1), s(R-L+1<<2), tg(R-L+1<<2), lz(R-L+1<<2)
    {
        flg=0;
        build(1, 1, n);
    }//[L,R]
    sgt(info b, int L, int R):n(R-L+1), shift(L-1), s(R-L+1<<2), tg(R-L+1<<2), lz(R-L+1<<2)
    {
        tmp=b;
        flg=1;
        build(1, 1, n);
    }//[L,R]
    int z, y;
    info res;
    tag dt;
    bool fir;
private:
    void _modify(int x, int l, int r)
    {
        if (z<=l&&r<=y)
        {
            s[x]+=dt;
            if (lz[x]) tg[x]+=dt; else tg[x]=dt;
            lz[x]=1;
            return;
        }
        int c=x*2, m=l+r>>1;
        if (lz[x])
        {
            if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
            lz[c]=1; s[c]+=tg[x]; c^=1;
            if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
            lz[c]=1; s[c]+=tg[x]; c^=1;
            lz[x]=0;
        }
        if (z<=m) _modify(c, l, m);
        if (m<y) _modify(c+1, m+1, r);
        s[x]=s[c]+s[c+1];
    }
}

```



```

}
void ask(int x, int l, int r)
{
    if (z<=l&&r<=y)
    {
        res=fir?s[x]:res+s[x];
        fir=0;
        return;
    }
    int c=x*2, m=l+r>>1;
    if (lz[x])
    {
        if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
        lz[c]=1; s[c]+=tg[x]; c^=1;
        if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
        lz[c]=1; s[c]+=tg[x]; c^=1;
        lz[x]=0;
    }
    if (z<=m) ask(c, l, m);
    if (m<y) ask(c+1, m+1, r);
}
function<bool>(info) check;
void find_left_most(int x, int l, int r)
{
    if (r<z||!check(s[x])) return;
    if (l==r) { y=l; res=s[x]; return; }
    int c=x*2, m=l+r>>1;
    if (lz[x])
    {
        if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
        lz[c]=1; s[c]+=tg[x]; c^=1;
        if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
        lz[c]=1; s[c]+=tg[x]; c^=1;
        lz[x]=0;
    }
    find_left_most(c, l, m);
    if (y==n+1) find_left_most(c+1, m+1, r);
}
void find_right_most(int x, int l, int r)
{
    if (l>y||!check(s[x])) return;
    if (l==r) { z=l; res=s[x]; return; }
    int c=x*2, m=l+r>>1;
    if (lz[x])
    {
        if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
        lz[c]=1; s[c]+=tg[x]; c^=1;
        if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
        lz[c]=1; s[c]+=tg[x]; c^=1;
        lz[x]=0;
    }
    find_right_most(c+1, m+1, r);
    if (z==0) find_right_most(c, l, m);
}
public:
void modify(int l, int r, const tag &x)//[l,r]
{

```

```

    z=l-shift; y=r-shift; dt=x;
    // cerr<<"modify ["<<l<<', '<<r<<" " <<'\\n';
    assert(1<=z&&z<=y&&y<=n);
    _modify(1, 1, n);
}
void modify(int pos, const info &o)
{
    pos-=shift;
    int l=1, r=n, m, c, x=1;
    while (l<r)
    {
        c=x*2; m=l+r>>1;
        if (lz[x])
        {
            if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
            lz[c]=1; s[c]+=tg[x]; c^=1;
            if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
            lz[c]=1; s[c]+=tg[x]; c^=1;
            lz[x]=0;
        }
        if (pos<=m) x=c, r=m; else x=c+1, l=m+1;
    }
    s[x]=o;
    while (x>>=1) s[x]=s[x*2]+s[x*2+1];
}
info ask(int l, int r)//[l,r]
{
    z=l-shift; y=r-shift; fir=1;
    // cerr<<"ask ["<<l<<', '<<r<<" " <<'\\n';
    assert(1<=z&&z<=y&&y<=n);
    ask(1, 1, n);
    return res;
}
pair<int, info> find_left_most(int l, const function<bool(info)> &_check)
{
    check=_check;
    z=l-shift; y=n+1;
    assert(1<=z&&z<=n+1);
    find_left_most(1, 1, n);
    return {y+shift, res};
}
pair<int, info> find_right_most(int r, const function<bool(info)> &_check)
{
    check=_check;
    z=0; y=r-shift;
    assert(0<=y&&y<=n);
    find_right_most(1, 1, n);
    return {z+shift, res};
}
};

```

## 2.3 哈希表

支持如同 map 一样使用 [] 访问。default 指的是未赋值情形的值。新版本未验证。

```

template<class Tx, class Ty> struct hashtable //定义域, 值域
{

```

```

const static int N=2e6+5,p=1e6+7;//元素个数,模数
Tx X[N];
Ty Y[N],default;
int fir[p],nxt[N],sz;
ht(Ty default=Ty{}):val(default):sz(0){memset(fir,-1,sizeof fir);}
Ty &operator[](T x)
{
    int index=(x%p+p)%p;
    for (int i=fir[index];i!=-1;i=nxt[i]) if (X[i]==x) return Y[i];//若 x 不重复,可以省略这个
    for
    X[cnt]=x;
    Y[cnt]=default;
    nxt[cnt]=fir[index];
    fir[index]=cnt++;
    return Y[cnt-1];
}
void clear()
{
    cnt=0;
    while (sz) fir[((X[--sz])%p+p)%p]=0;
}
void iterate()//遍历。用于自行修改
{
    for (int i=0;i<sz;i++)
    {
        T x=X[i];
        TT y=Y[i];
        //(x,y)
    }
}
};

```

## 2.4 珂朵莉树

支持区间赋值、单点访问。维护每个连续段的范围和值。

如果希望维护所有连续段的整体信息（如长度的最大值），修改 `add` 和 `del` 函数即可，分别表示连续段被加入和被删去。

特别注意一开始 `insert` 的不会触发 `add`，只有 `modify` 会触发。

```

namespace chtholly_tree
{
    using T=int;//可以把 T 修改为任意想要的类型。
    struct node
    {
        int l;
        mutable int r;
        mutable T v;
        int len() const { return r-l+1; }
        bool operator<(const node &x) const { return l<x.l; }
    };
    void add(const node &a) {}
    void del(const node &a) {}
    class odt: public set<node>
    {
    public:
        typedef odt::iterator iter;
    };
}

```

```

    iter split(int x)
    {
        iter it=lower_bound({x});
        if (it!=end()&&it->l==x) return it;
        node t=*--it,a={t.l,x-1,t.v},b={x,t.r,t.v};
        del(*it); add(a); add(b);
        erase(it); insert(a);
        return insert(b).first;
    }
    void modify(int l,int r,T v)//[l,r]
    {
        iter lt,rt,it;
        rt=r==rbegin()->r?end():split(r+1); lt=split(l);//[lt,rt)
        while (lt!=begin()&&(it=prev(lt))->v==v) l=(lt=it)->l;
        while (rt!=end()&&rt->v==v) r=(rt++)->r;
        for (it=lt; it!=rt; it++) del(*it);
        add({l,r,v});
        erase(lt,rt); insert({l,r,v});
    }
    T operator[](const int x) const { return prev(upper_bound({x}))->v; }//直接访问单点
    iter find(int x) const {return prev(upper_bound({x}));}//找到对应的线段
};
}
using chtholly_tree::node,chtholly_tree::odt;
typedef odt::iterator iter;
int main()
{
    odt s;
    s.insert({0,5,1}); // 先 insert({L,R,x}) 表示整个下标范围和初始值。 左闭右闭。
                        // s={1,1,1,1,1,1}
    s.modify(2,3,2); // 左闭右闭。 s={1,1,2,2,1,1}
    for (auto [l,r,v]:s)
    {
        //(l,r,v)=(0,1,1)
        //(l,r,v)=(2,3,2)
        //(l,r,v)=(4,5,1)
    }
}

```

## 2.5 带删堆

本质是额外维护一个堆  $q$  表示要被删除的元素，当  $p$  的最值和  $q$  一样时删除。

需要保证每次  $\text{pop}$  的元素都存在于堆中。

本代码的用法和 `priority_queue` 一致。

```

template<class T, class T1=vector<T>, class T2=less<T>> struct heap
{
private:
    priority_queue<T, T1, T2> p, q;
public:
    void push(const T &x)
    {
        if (!q.empty()&&q.top()==x)
        {
            q.pop();
            while (!q.empty()&&q.top()==p.top()) p.pop(), q.pop();
        }
    }
}

```

```

    }
    else p.push(x);
}
void pop()
{
    p.pop();
    while (!q.empty() && p.top() == q.top()) p.pop(), q.pop();
}
void pop(const T &x)
{
    if (p.top() == x)
    {
        p.pop();
        while (!q.empty() && p.top() == q.top()) p.pop(), q.pop();
    }
    else q.push(x);
}
T top() const { return p.top(); }
int size() const { return p.size() - q.size(); }
bool empty() const { return p.empty(); }
vector<T> to_vector() const
{
    vector<T> a;
    auto P=p, Q=q;
    while (P.size())
    {
        a.push_back(P.top()); P.pop();
        while (Q.size() && P.top() == Q.top()) P.pop(), Q.pop();
    }
    return a;
}
};

```

## 2.6 前 $k$ 大的和

本质是用小根堆维护前  $k$  大的数，用大根堆维护其余数。

如果需要支持删除，结合前面一个使用，或者直接用 `multiset` 进行 `extract`。

为了方便起见，直接给出支持删除的版本，并且使用 `long long`。如果不需要支持删除，类型改为优先队列并去掉 `pop` 函数即可。

注意：复杂度为  $O(k - k')$ ，其中  $k'$  是上一次询问的  $k$ 。也就是说，多组询问时询问的  $k$  的差值应该尽可能小。

其用法与 `priority_queue` 保持一致，可以用同样的方法改写成前  $k$  小。

```

using ll=long long;
template<class T, class T1=vector<T>, class T2=less<T>> struct ksum_pop
{
private:
    struct __cmp
    {
        bool operator()(const T &x, const T &y) const
        {
            return x!=y&&!T2()(x, y);
        }
    };
};
heap<T, T1, __cmp> p;

```

```

heap<T, T1, T2> q;
ll cur;
public:
ksum_pop():cur(0) { }
void push(const T &x)
{
    if (!q.size()||!T2()(x, q.top())) p.push(x), cur+=x; else q.push(x);
}
int size() const { return p.size()+q.size(); }
void pop(const T &x)
{
    if (q.size()&&!T2()(q.top(), x)) q.pop(x);
    else p.pop(x), cur-=x;
}
ll sum(int k)
{
    while (p.size()<k)
    {
        cur+=q.top();
        p.push(q.top());
        q.pop();
    }
    while (p.size()>k)
    {
        cur-=p.top();
        q.push(p.top());
        p.pop();
    }
    return cur;
}
};

```

## 2.7 可持久化数组

历史遗留产物，无意义，仅作留存，不会更新。

$O((n+q)\log(n))$ ,  $O((n+q)\log(n))$ 。

```

struct arr
{
    int c[M][2],rt[0],s[M],b[N];
    int ds,n,ver,v,p,i;
    void build(int &x,int l,int r)
    {
        x==ds;
        if (l==r) {s[x]=b[l];return;}
        build(c[x][0],l,l+r>>1);
        build(c[x][1],(l+r>>1)+1,r);
    }
    void rebuild(int &x,int pre)
    {
        x==ds;int l=1,r=n,mid,now=x;
        while (l<r)
        {
            mid=l+r>>1;
            if (mid>=p){c[now][1]=c[pre][1];now=c[now][0]==ds;r=mid;pre=c[pre][0];} else {c[now][0]=c[pre][0];now=c[now][1]==ds;l=mid+1;pre=c[pre][1];}
        }
    }
};

```

```

    }
    s[now]=v;
}
void init(int *a,int nn)
{
    n=nn;
    for (i=1;i<=n;i++) b[i]=a[i];
    build(rt[0],1,n);
}
int mdf(int pv,int pos,int val)
{
    p=pos,v=val;
    rebuild(rt[++ver],rt[pv]);
    return ver;
}
int ask(int ve,int pos)
{
    int l=1,r=n,x=rt[ve],mid;
    rt[++ver]=rt[ve];
    while (l<r)
    {
        mid=l+r>>1;
        if (mid>=pos) {x=c[x][0];r=mid;} else {x=c[x][1];l=mid+1;}
    }
    return s[x];
}
};

```

## 2.8 左偏树/可并堆

建议不要使用。pb\_ds 可以替代这个功能。我完全没有使用过这个板子。

$O((n+q)\log n)$ ,  $O(n)$ 。

```

struct left_tree//小根堆，大根堆需要改的地方注释了
{
    int jl[N],v[N],f[N],c[N][2],tf[N],n;//tf只有删非堆顶才用
    bool ed[N];
    void init(const int nn,const int *a)
    {
        jl[0]=-1;n=nn;
        memset(jl+1,0,n<<2);
        memset(tf+1,0,n<<2);//同上
        memset(c+1,0,n<<3);
        memset(ed+1,0,n);
        for (int i=1;i<=n;i++) v[f[i]=i]=a[i];
    }
    int mg(int x,int y)
    {
        if (!(x&& y)) return x|y;
        if (v[x]>v[y]||v[x]==v[y]&&x>y) swap(x,y);//改
        tf[c[x][1]=mg(c[x][1],y)]=x;//同上
        if (jl[c[x][0]]<jl[c[x][1]]) swap(c[x][0],c[x][1]);
        jl[x]=jl[c[x][1]]+1;
        return x;
    }
    int getf(int x)

```

```

{
    if (f[x]==x) return x;
    return f[x]=getf(f[x]);
}
int merge(int x,int y)
{
    if (ed[x]||ed[y]||(x=getf(x))==y=getf(y)) return x;
    int z=mg(x,y);return f[x]=f[y]=z;
}
int getv(int x)//需要自行判断是否存在
{
    return v[getf(x)];
}
int del(int x)//删除堆内最值
{
    tf[c[x][0]]=tf[c[x][1]]=0;
    f[c[x][0]]=f[c[x][1]]=f[x]=mg(c[x][0],c[x][1]);
    ed[x]=1;c[x][0]=c[x][1]=tf[x]=0;return f[x];
}
int del_all(int x)//删除堆内非最值（没验证过）
{
    int fa=tf[x];
    if (f[c[x][0]]==x) f[c[x][0]]=getf(tf[x]);
    if (f[c[x][1]]==x) f[c[x][1]]=f[tf[x]];
    tf[x]=tf[c[x][0]]=tf[c[x][1]]=0;
    tf[c[fa][c[fa][1]]==x]=mg(c[x][0],c[x][1])=fa;
    c[x][0]=c[x][1]=0;
    while (j1[c[fa][0]]<j1[c[fa][1]])
    {
        swap(c[fa][0],c[fa][1]);
        j1[fa]=j1[c[fa][1]]+1;
        fa=tf[fa];
    }
}
void out(int n)
{
    for (int i=1;i<=n;i++) printf("%d: c%d&%d,f%d,v%d\n",i,c[i][0],c[i][1],f[i],v[i]);
}
};

```

## 2.9 树状数组区间加区间求和

本质： $a_n$  区间加等价于差分数组  $d_n$  的单点加。

$$\sum_{i=1}^m a_i = \sum_{i=1}^m \sum_{j=1}^i d_j = \sum_{j=1}^m d_j (m-j+1) = ((m+1) \sum_{j=1}^m d_j) - (\sum_{j=1}^m j d_j)。$$

分别维护  $d_j$  和  $j d_j$  的前缀和。

$O(n) \sim O(q \log n)$ ,  $O(n)$ 。

```

struct bit
{
    ll a[N],b[N],s[N];//有初始值
    int n;
    void init(int nn,int *a)//初始值
    {
        n=nn;s[0]=0;
        for (int i=1;i<=n;i++) s[i]=s[i-1]+a[i];
    }
};

```



```

}
void mdf(int l,int r,ll dt)
{
    int i; ++r;
    ll j=dt*l;
    a[l]+=dt;b[l]+=j;
    while ((l+=l&-l)<=n)
    {
        a[l]+=dt;
        b[l]+=j;
    }
    if (r<=n)
    {
        j=dt*r;
        a[r]-=dt;b[r]-=j;
        while ((r+=r&-r)<=n)
        {
            a[r]-=dt;
            b[r]-=j;
        }
    }
}
ll presum(int x)
{
    ll r=a[x],rr=b[x];
    int y=x;
    while (x^=x&-x)
    {
        r+=a[x];
        rr+=b[x];
    }
    return r*(y+1)-rr+s[y];
}
ll sum(int l,int r)
{
    return presum(r)-presum(l-1);
}
};

```

## 2.10 二维树状数组矩形加矩形求和

本质还是差分，只不过这次要维护  $d_{i,j}, d_{i,j}i, d_{i,j}j, d_{i,j}ij$ 。

$O(n^2) \sim O(q \log^2 n), O(n^2)$

```

struct bit2
{
    ll a[2050][2050],b[2050][2050],c[2050][2050],d[2050][2050];
    int n,m;
private:
    void cha(ll a[][2050],int x,int y,int z)
    {
        int i,j;
        for (i=x;i<=n;i+=(i&(-i))) for (j=y;j<=m;j+=(j&(-j))) a[i][j]+=z;
    }
    ll he(int x,int y)
    {

```

```

    if ((x<=0)|| (y<=0)) return 0;
    int i,j;
    ll z=0,w=0;
    for (i=x;i; i--=(i&(-i))) for (j=y;j; j--=(j&(-j))) z+=a[i][j];
    z*=(x+1)*(y+1);
    w=0;
    for (i=x;i; i--=(i&(-i))) for (j=y;j; j--=(j&(-j))) w+=b[i][j];
    z-=w*(y+1);
    w=0;
    for (i=x;i; i--=(i&(-i))) for (j=y;j; j--=(j&(-j))) w+=c[i][j];
    z-=w*(x+1);
    for (i=x;i; i--=(i&(-i))) for (j=y;j; j--=(j&(-j))) z+=d[i][j];
    return z;
}
public:
void init(int x,int y)
{
    n=x;m=y;
}
void add(int u,int v,int x,int y,int z)//(x1,y1,x2,y2,dt)
{
    cha(a,u,v,z);
    cha(b,u,v,u*z); //小心乘爆
    cha(c,u,v,v*z);
    cha(d,u,v,u*v*z);
    ++x;++y;
    if (x<=n)
    {
        cha(a,x,v,-z);
        cha(b,x,v,-z*x);
        cha(c,x,v,-z*v);
        cha(d,x,v,-z*x*v);
    }
    if (y<=m)
    {
        cha(a,u,y,-z);
        cha(b,u,y,-z*u);
        cha(c,u,y,-z*y);
        cha(d,u,y,-z*u*y);
        if (x<=n)
        {
            cha(a,x,y,z);
            cha(b,x,y,z*x);
            cha(c,x,y,z*y);
            cha(d,x,y,z*x*y);
        }
    }
}
ll sum(int u,int v,int x,int y)//(x1,y1,x2,y2)
{
    --u;--v;
    return (he(x,y)+he(u,v)-he(u,y)-he(x,v));
}
};

```

## 2.11 带修莫队（功能：区间数有多少种不同的数字）

按照  $n^{\frac{2}{3}}$  分块，排序关键字是  $l, r, t$  所在的块（ $t$  是版本号，每次修改都会增加一个版本），可以奇偶分块优化。

相比于传统莫队多了一个 `modify`。

$O(n^{\frac{5}{3}})$ ,  $O(n)$ 。

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
#define all(x) (x).begin(),(x).end()
const int N=1.4e5,M=1e6+2;
int a[N],ans[N],bel[N],cnt[M],sum,z,y,cur;
struct P
{
    int p,v;
};
struct Q
{
    int l,r,t,p;
    bool operator<(const Q &o) const
    {
        if (bel[l]!=bel[o.l]) return bel[l]<bel[o.l];
        if (bel[r]!=bel[o.r]) return (bel[l]&1)^bel[r]<bel[o.r];
        return (bel[r]&1)?t<o.t:t>o.t;
    }
};
Q b[N];
P d[N];
void add(const int &x) {sum+=!(cnt[a[x]]++);}
void del(const int &x) {sum-=!(--cnt[a[x]]);}
void mdf(const int &x)
{
    auto &[p,v]=d[x];
    if (z<=p&&p<=y) del(p);
    swap(a[p],v);
    if (z<=p&&p<=y) add(p);
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m,q1=0,q2=0,i,ksiz;
    cin>>n>>m;
    for (i=1;i<=n;i++) cin>>a[i];
    for (i=1;i<=m;i++)
    {
        char c;
        int l,r;
        cin>>c>>l>>r;
        if (c=='Q') ++q1,b[q1]={l,r,q2,q1};
        else d[++q2]={l,r};
    }
    ksiz=max(1.0,round(cbrt((ll)n*n)));
    for (i=1;i<=n;i++) bel[i]=i/ksiz;
    sort(b+1,b+q1+1);
    z=b[1].l;y=z-1;cur=0;
    for (i=1;i<=q1;i++)
```

```

{
    auto [l,r,t,p]=b[i];
    while (z>l) add(--z);
    while (y<r) add(++y);
    while (z<l) del(z++);
    while (y>r) del(y--);
    while (cur<t) mdf(++cur);
    while (cur>t) mdf(cur--);
    ans[p]=sum;
}
for (i=1;i<=q1;i++) cout<<ans[i]<<'\n';
}

```

## 2.12 二次离线莫队

直接摘录题解，用途不大。

$O(n\sqrt{n})$ ,  $O(n)$ 。

珂朵莉给了你一个序列  $a$ ，每次查询给一个区间  $[l, r]$ ，查询  $l \leq i < j \leq r$ ，且  $a_i \oplus a_j$  的二进制表示下有  $k$  个 1 的二元组  $(i, j)$  的个数。 $\oplus$  是指按位异或。

二次离线莫队，通过扫描线，再次将更新答案的过程离线处理，降低时间复杂度。假设更新答案的复杂度为  $O(k)$ ，它将莫队的复杂度从  $O(nk\sqrt{n})$  降到了  $O(nk + n\sqrt{n})$ ，大大简化了计算。设  $x$  对区间  $[l, r]$  的贡献为  $f(x, [l, r])$ ，我们考虑区间端点变化对答案的影响：以  $[l..r]$  变成  $[l..(r+k)]$  为例， $\forall x \in [r+1, r+k]$  求  $f(x, [l, x-1])$ 。我们可以进行差分： $f(x, [l, x-1]) = f(x, [1, x-1]) - f(x, [1, l-1])$ ，这样转化为了一个数对一个前缀的贡献。保存下来所有这样的询问，从左到右扫描数组计算就可以了。但是这样做，空间是  $O(n\sqrt{n})$  的，不太优秀，而且时间常数巨大。。这样的贡献分为两类：

1. 减号左边的贡献永远是一个前缀和它后面一个数的贡献。这可以预处理出来。2. 减号右边的贡献对于一次移动中所有的  $x$  来说，都是不变的。我们打标记的时候，可以只标记左右端点。

这样，减小时间常数的同时，空间降为了  $O(n)$  级别。是一个很优秀的算法了。处理前缀询问的时候，我们利用异或运算的交换律，即  $a \text{ xor } b = c \iff a \text{ xor } c = b$  开一个桶  $t$ ， $t[i]$  表示当前前缀中与  $i$  异或有  $k$  个数位为 1 的数有多少个。则每加入一个数  $a[i]$ ，对于所有  $\text{popcount}(x) = k$  的  $x$ ， $t[a[i] \text{ xor } x] \leftarrow t[a[i] \text{ xor } x] + 1$  即可。

```

typedef long long ll;
const int N=1e5+2,M=1<<14;
ll f[N],ans[N],ta[N];
int a[N],cnt[M],bel[N],pc[M],st[N];
int n,m,ksiz;
struct Q
{
    int z,y,wz;
    bool operator<(const Q& x) const {return (bel[z]<bel[x.z])||(bel[z]==bel[x.z])&&((y<x.y)&&(bel[z]&1)|| (y>x.y)&&(1^bel[z]&1));}
};
Q mq(const int x,const int y,const int z)
{
    Q a;
    a.z=x;a.y=y;a.wz=z;
    return a;
}
Q q[N];
vector<Q> b[N];
int main()
{

```

```

int i,j,k,l=1,r=0,tp=0,x,na;
read(n);read(m);read(k);ksiz=sqrt(n);
for (i=1;i<=n;i++) {read(a[i]);bel[i]=(i-1)/ksiz+1;}
if (k==0) st[++tp]=0;
for (i=1;i<16384;i++)
{
    if (i&1) pc[i]=pc[i>>1]+1; else pc[i]=pc[i>>1];
    if (pc[i]==k) st[++tp]=i;
}
for (i=1;i<=n;i++)
{
    j=tp+1;f[i]=f[i-1];
    while (--j) f[i]+=cnt[st[j]^a[i]];
    ++cnt[a[i]];
}
for (i=1;i<=m;i++) {read(q[i].z);read(q[q[i].wz=i].y);}
sort(q+1,q+m+1);
for (i=1;i<=m;i++)
{
    ans[i]=f[q[i].y]-f[r]+f[q[i].z-1]-f[l-1];
    if (k==0) ans[i]+=q[i].z-1;
    if (r<q[i].y)
    {
        b[l-1].push_back(mq(r+1,q[i].y,-i));
        r=q[i].y;
    }
    if (l>q[i].z)
    {
        b[r].push_back(mq(q[i].z,l-1,i));
        l=q[i].z;
    }
    if (r>q[i].y)
    {
        b[l-1].push_back(mq(q[i].y+1,r,i));
        r=q[i].y;
    }
    if (l<q[i].z)
    {
        b[r].push_back(mq(l,q[i].z-1,-i));
        l=q[i].z;
    }
}
memset(cnt,0,sizeof(cnt));
for (i=1;i<=n;i++)
{
    j=tp+1;x=a[i];
    while (--j) ++cnt[x^st[j]];
    for (j=0;j<b[i].size();j++)
    {
        na=0;l=b[i][j].z;r=b[i][j].y;
        for (k=1;k<=r;k++) na+=cnt[a[k]];
        if (b[i][j].wz>0) ans[b[i][j].wz]+=na; else ans[-b[i][j].wz]-=na;
    }
}
for (i=2;i<=m;i++) ans[i]+=ans[i-1];
for (i=1;i<=m;i++) ta[q[i].wz]=ans[i];
for (i=1;i<=m;i++) printf("%lld\n",ta[i]);

```

}

## 2.13 回滚莫队

不删除的莫队，比如求  $\max$ 。

做法：块内询问暴力。对于  $l$  所在块相同的询问，按照  $r$  升序排序，并且将左指针固定在  $l$  所在块的最右侧。（由于块内询问暴力，这不会导致左指针更大）

回答每个询问的时候，先右端点右移到  $r$ ，然后左端点左移到  $l$ 。询问完成后，把左端点移回去。移回去的过程虽然涉及删除，但不需要维护答案变成什么了（因为在左端点左移之前已经求过了）。换句话说，相当于“撤销”而不是删除，完全可以记录移动过程中的所有变化来撤销。

$O(n\sqrt{n})$ ,  $O(n)$ 。

```
#include <bits/stdc++.h>
using namespace std;
const int N=2e5+2;
int a[N],z[N],y[N],wz[N],b[N],d[N],bel[N],ans[N],st[N][2],pos[N][2];
void qs(int l,int r)
{
    int i=l,j=r,m=bel[z[l+r>>1]],mm=y[l+r>>1];
    while (i<=j)
    {
        while ((bel[z[i]]<m)||((bel[z[i]]==m)&&(y[i]<mm))) ++i;
        while ((bel[z[j]]>m)||((bel[z[j]]==m)&&(y[j]>mm))) --j;
        if (i<=j)
        {
            swap(wz[i],wz[j]);
            swap(z[i],z[j]);
            swap(y[i++],y[j--]);
        }
    }
    if (i<r) qs(i,r);
    if (l<j) qs(l,j);
}
int main()
{
    read(n);ksiz=sqrt(n);
    for (i=1;i<=n;i++) {read(a[i]);b[i]=a[i];bel[i]=(i-1)/ksiz+1;}
    sort(b+1,b+n+1);
    d[gs=1]=b[1];
    for (i=2;i<=n;i++) if (b[i]!=b[i-1]) d[++gs]=b[i];
    for (i=1;i<=n;i++) a[i]=lower_bound(d+1,d+gs+1,a[i])-d;
    read(m);assert(int(n/sqrt(m)));
    for (i=1;i<=m;i++) {read(z[i]);read(y[wz[i]=i]);}
    qs(1,m);
    for (i=1;i<=m;i++)
    {
        if (bel[z[i]]>bel[z[i-1]])
        {
            while (l<=r) {pos[a[l]][0]=pos[a[l]][1]=0;++l;}na=0;
            if (bel[z[i]]==bel[y[i]])
            {
                for (j=z[i];j<=y[i];j++) if (pos[a[j]][0]) na=max(na,j-pos[a[j]][0]); else pos[a[j]][0]=j;
                ans[wz[i]]=na;for (j=z[i];j<=y[i];j++) pos[a[j]][0]=0;na=0;l=ksiz*bel[z[i]];r=l-1;
                continue;
            }
        }
    }
}
```

```

    }
    l=ksiz*bel[z[i]];r=l-1;na=0;
}
if (bel[z[i]]==bel[y[i]])
{
    while (l<=r) {pos[a[l]][0]=pos[a[l]][1]=0;++l;}na=0;
    for (j=z[i];j<=y[i];j++) if (pos[a[j]][0]) na=max(na,j-pos[a[j]][0]); else pos[a[j]]
        [0]=j;
    ans[wz[i]]=na;for (j=z[i];j<=y[i];j++) pos[a[j]][0]=0;
    l=ksiz*bel[z[i]];r=l-1;na=0;
    continue;
}
while (r<y[i])
{
    x=a[++r];pos[x][1]=r;
    if (!pos[x][0]) pos[x][0]=r; else na=max(na,r-pos[x][0]);
}c=na;
while (l>z[i])
{
    x=a[--l];st[++tp][0]=x;st[tp][1]=pos[x][0];
    pos[x][0]=1;
    if (!pos[x][1])
    {
        st[++tp][0]=x+n;st[tp][1]=0;
        pos[x][1]=1;
    } else na=max(na,pos[x][1]-1);
}
ans[wz[i]]=na;na=c;++tp;l=ksiz*bel[z[i]];
while (--tp) if (st[tp][0]<=n) pos[st[tp][0]][0]=st[tp][1]; else pos[st[tp][0]-n][1]=st[tp]
    [1];
}
for (i=1;i<=m;i++) printf("%d\n",ans[i]);
}

```

## 2.14 李超树

题意：插入线段，查询某个  $x$  的最大  $y$ （输出最小编号）

算法核心：修改时，线段树每个点只维护在中点取值最大的线段，中点取值较小的线段只会在至多一侧有用，递归下去插入，复杂度  $O(\log^2)$ 。查询时询问线段树上  $\log$  个点的线段中最大的。

```

struct Q
{
    int x0,y0,dx,dy,id;
    Q():x0(0),y0(-1),dx(1),dy(0),id(-1){} //y>=0
    Q(int a,int b,int c,int d,int e):x0(a),y0(b),dx(c),dy(d),id(e){}
    bool contains(const int &x) const {return x0<=x&&x<=x0+dx;}
};

bool cmp(const Q &a,const Q &b,int x)//小心数值爆炸
{
    ll A=((ll)a.y0*a.dx+(ll)(x-a.x0)*a.dy)*b.dx,B=((ll)b.y0*b.dx+(ll)(x-b.x0)*b.dy)*a.dx;
    if (A!=B) return A<B;
    return a.id>b.id;
}

bool cmp2(const Q &a,const Q &b)
{
    if (a.y0+a.dy!=b.y0+b.dy) return a.y0+a.dy<b.y0+b.dy;
}

```

```

    return a.id>b.id;
}
const int inf=1e9;
int ans;
namespace seg
{
    const int N=4e4+2,M=N*4;
    Q s[M],X[N];
    int n,z,y;
    void init(int nn) {n=nn;for (int i=1;i<=n*4;i++) s[i]=Q();}
    void insert(int x,int l,int r,Q dt)
    {
        int c=x*2,m=l+r>>1;
        if (z<=l&&r<=y)
        {
            if (cmp(s[x],dt,m)) swap(s[x],dt);
            if (l==r) return;
            if (cmp(s[x],dt,l)) insert(c,l,m,dt);
            else if (cmp(s[x],dt,r)) insert(c+1,m+1,r,dt);
            return;
        }
        if (z<=m) insert(c,l,m,dt);
        if (y>m) insert(c+1,m+1,r,dt);
    }
    void insert(const Q &o)
    {
        z=o.x0;y=z+o.dx;
        assert(1<=z&&z<=y&&y<=n);
        if (z==y)
        {
            if (cmp2(X[z],o)) X[z]=o;
            return;
        }
        insert(1,1,n,o);
    }
    Q askmax(int p)
    {
        Q ans=s[1].contains(p)?s[1]:Q();
        int x=1,l=1,r=n,c,m;
        while (l<r)
        {
            c=x*2,m=l+r>>1;
            if (p<=m) x=c,r=m; else x=c+1,l=m+1;
            if (s[x].contains(p)&&cmp(ans,s[x],p)) ans=s[x];
        }
        Q o(X[p].x0,X[p].y0+X[p].dy,1,0,0);
        return cmp(ans,o,p)?X[p]:ans;
    }
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(15);
    int n=4e4,m,i;
    seg::init(n);
    cin>>m;
    while (m--)

```



```

{
    int op;
    cin>>op;
    if (op)
    {
        int x[2],y[2];
        cin>>x[0]>>y[0]>>x[1]>>y[1];
        for (int &v:x) v=(v+ans-1)%39989+1;
        for (int &v:y) v=(v+ans-1)%inf+1;
        if (x[0]>x[1]||x[0]==x[1]&&y[0]>y[1]) swap(x[0],x[1]),swap(y[0],y[1]);
        static int id;
        seg::insert({x[0],y[0],x[1]-x[0],y[1]-y[0],++id});
    }
    else
    {
        int x;
        cin>>x;
        x=(x+ans-1)%39989+1;
        cout<<(ans=max(0,seg::askmax(x).id))<<'\\n';
    }
}
}

```

## 2.15 李超树（动态开点）

```

struct Q
{
    int k;
    ll b;
    ll y(const int &x) const {return (ll)k*x+b;}
};

const int inf=1e9;
const ll INF=1e18;
struct seg//可以析构，不能并行
{
    const static int N=4e5+2,M=N*8*8+(1<<23);
    const static ll npos=9e18;
    static Q s[M];
    static int c[M][2],id;
    int z,y,L,R;
    seg(int l,int r)
    {
        L=l;R=r;id=1;
        s[1]={0,npos};
        assert(L<=R&&(ll)R-L<1ll<<32);
    }
private:
    void insert(int &x,int l,int r,Q o)
    {
        if (!x)
        {
            x=++id;
            assert(id<M);
            s[x]={0,npos};
        }
        int m=l+(r-l>>1);

```

```

    if (z<=l&& r<=y)
    {
        if (s[x].y(m)>o.y(m)) swap(s[x],o);
        if (s[x].y(l)>o.y(l)) insert(c[x][0],l,m,o);
        else if (s[x].y(r)>o.y(r)) insert(c[x][1],m+1,r,o);
        return;
    }
    if (z<=m) insert(c[x][0],l,m,o);
    if (y>m) insert(c[x][1],m+1,r,o);
}

public:
void insert(const Q &x,const int &l,const int &r)//[l,r]
{
    z=l;y=r;int tmp=1;
    insert(tmp,L,R,x);
    assert(tmp==1);
}

ll askmin(const int &p)
{
    ll res=s[1].y(p);
    int l=L,r=R,m,x=1;
    while (l<r)
    {
        m=l+(r-l>>1);
        if (p<=m) x=c[x][0],r=m; else x=c[x][1],l=m+1;
        if (!x) return res;
        res=min(res,s[x].y(p));
    }
    return res;
}

~seg()
{
    ++id;
    while (--id) c[id][0]=c[id][1]=0;
}

};

Q seg::s[seg::M];
int seg::c[seg::M][2],seg::id;

```

## 2.16 splay

没啥用。

$O(n)$ ,  $O((n+q)\log n)$ 。

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef unsigned int ui;
const int N=1e6+20,p=998244353;
void inc(int &x,const int y){if ((x+=y)>=p) x-=p;}
void dec(int &x,const int y){if ((x-=y)<0) x+=p;}
void mul(int &x,const int y){x=(ll)x*y%p;}
template<int N> struct _splay
{
    int c[N][2],plz[N],clz[N],st[N],siz[N],s[N],v[N],f[N];
    bool fg[N],flz[N];

```

```

int tp,rt;
void allout(int x)
{
    if (!x) return;
    pushdown(x);
    allout(c[x][0]);
    if (x>2) printf("%d_",v[x]);
    allout(c[x][1]);
}
void out(int x)
{
    printf("%d:_%c_%d_%d_%d_%d_%d_%d_%d\n",x,c[x][0],c[x][1],f[x],s[x],v[x],siz[x]);
    if (c[x][0]) out(c[x][0]);
    if (c[x][1]) out(c[x][1]);
    if (x==rt) puts("-----");
}
void iinit()
{
    for (int i=1;i<N;i++) st[N-i]=i;
    tp=N-1;
}
void init()
{
    tp=N-3;
    c[1][0]=c[1][1]=flz[1]=plz[1]=fg[1]=v[1]=f[1]=s[1]=0;clz[1]=1;
    c[2][0]=c[2][1]=flz[2]=plz[2]=fg[2]=v[2]=f[2]=s[2]=0;clz[2]=1;
    c[1][1]=2;f[2]=1;rt=1;siz[2]=1;siz[1]=2;
}
void pushup(int x)
{
    s[x]=((ui)s[c[x][0]]+s[c[x][1]]+v[x])%p;
    siz[x]=siz[c[x][0]]+siz[c[x][1]]+1;
}
void pushdown(int x)
{
    int lc=c[x][0],rc=c[x][1];
    if (flz[x])
    {
        if (lc) flz[lc]^=1,swap(c[lc][0],c[lc][1]);
        if (rc) flz[rc]^=1,swap(c[rc][0],c[rc][1]);
        flz[x]=0;
    }
    if (fg[x])
    {
        clz[x]=1;plz[x]=0;
        if (lc) fg[lc]=1,v[lc]=v[x],s[lc]=(ll)v[x]*siz[lc]%p;
        if (rc) fg[rc]=1,v[rc]=v[x],s[rc]=(ll)v[x]*siz[rc]%p;
        fg[x]=0;
    }
    else
    {
        if (clz[x]!=1)
        {
            if (lc) mul(clz[lc],clz[x]),mul(s[lc],clz[x]),mul(plz[lc],clz[x]),mul(v[lc],clz[x])
                ;
            if (rc) mul(clz[rc],clz[x]),mul(s[rc],clz[x]),mul(plz[rc],clz[x]),mul(v[rc],clz[x])
                ;
        }
    }
}

```

```

        clz[x]=1;
    }
    if (plz[x])
    {
        if (lc) inc(plz[lc],plz[x]),inc(v[lc],plz[x]),s[lc]=(s[lc]+(ll)siz[lc]*plz[x])%p;
        if (rc) inc(plz[rc],plz[x]),inc(v[rc],plz[x]),s[rc]=(s[rc]+(ll)siz[rc]*plz[x])%p;
        plz[x]=0;
    }
}
}
void zigzag(int x)
{
    int y=f[x],z=f[y],typ=(c[y][0]==x);
    if (z) c[z][c[z][1]==y]=x;
    f[x]=z;f[y]=x;c[y][typ^1]=c[x][typ];
    if (c[x][typ]) f[c[x][typ]]=y;
    c[x][typ]=y;
    pushup(y);
}
void allpd(int x)
{
    static int st[N],tp;
    st[tp]=x;
    while (x=f[x]) st[++tp]=x;
    while (tp) pushdown(st[tp--]);
}
void splay(int x,int tar)
{
    if (!tar) rt=x;
    int y;
    while ((y=f[x])!=tar)
    {
        if (f[y]!=tar) zigzag(c[f[y]][0]==y^c[y][0]==x?x:y);
        zigzag(x);
    }
    pushup(x);
}
void find(int kth,int tar)
{
    int x=rt;
    while (siz[c[x][0]]+1!=kth)
    {
        pushdown(x);
        if (siz[c[x][0]]>=kth) x=c[x][0]; else
        {
            kth-=siz[c[x][0]]+1;
            x=c[x][1];
        }
    }
    pushdown(x);
    splay(x,tar);
}
int rk(int x)
{
    allpd(x);
    splay(x,0);
    return siz[c[x][0]];
}

```

```

}
void split(int x,int y)
{
    find(x,0);find(y+2,rt);
}
int npt()
{
    int x=st[tp--];
    c[x][0]=c[x][1]=plz[x]=siz[x]=s[x]=v[x]=fg[x]=flz[x]=0;
    clz[x]=1;
    return x;
}
int build(int *a,int l,int r)
{
    if (l>r) return 0;
    int m=l+r>>1,x;
    v[x=npt()]=a[m];
    //printf("build %d %d %d\n",l,r,x);
    if (l==r)
    {
        siz[x]=1;
        s[x]=v[x];
        return x;
    }
    c[x][0]=build(a,l,m-1);
    c[x][1]=build(a,m+1,r);
    if (c[x][0]) f[c[x][0]]=x;
    if (c[x][1]) f[c[x][1]]=x;
    pushup(x);
    return x;
}
void ins(int pos,int *a,int n)//在pos后插入
{
    if (!n) return;
    split(pos+1,pos);
    // out(rt);
    int x=c[rt][1];
    c[x][0]=build(a,1,n);
    // printf("%d %d\n",x,c[x][0]);
    f[c[x][0]]=x;
    pushup(x);pushup(rt);
}
void del(int l,int r)//删除[l,r]
{
    split(l,r);
    c[c[rt][1]][0]=0;
    pushup(c[rt][1]);
    pushup(rt);
}
void rev(int l,int r)
{
    split(l,r);
    int x=c[c[rt][1]][0];
    swap(c[x][0],c[x][1]);
    flz[x]^=1;
}
void add(int l,int r,int val)

```

```

{
    split(l,r);
    int x=c[c[rt][1]][0];
    inc(v[x],val);inc(plz[x],val);
    s[x]=(s[x]+(l1)val*siz[x])%p;
    pushup(f[x]);pushup(rt);
}
void multi(int l,int r,int val)
{
    split(l,r);
    int x=c[c[rt][1]][0];
    mul(v[x],val);mul(plz[x],val);
    mul(s[x],val);mul(clz[x],val);
    pushup(f[x]);pushup(rt);
}
void mov(int l1,int r1,int l2)//都是原下标
{
    if (l2>l1) l2-=r1-l1+1;
    split(l1,r1);int x=c[c[rt][1]][0];
    allpd(x);c[f[x]][0]=0;
    pushup(f[x]);pushup(rt);
    split(l2+1,l2);
    allpd(c[rt][1]);
    c[c[rt][1]][0]=x;f[x]=c[rt][1];
    pushup(f[x]);pushup(rt);
}
int sum(int l,int r)
{
    split(l,r);puts("spe ");out(rt);
    return s[c[c[rt][1]][0]];
}
};
_splay<N> s;
int a[N];
int n,q,i,x,y,z;
int main()
{
    read(n);read(q);s.iinit();
    for (i=1;i<=n;i++) a[i]=i;
    s.init();s.ins(0,a,n);//s.out(s.rt);
    while (q--)
    {
        read(x);read(y);s.rev(x,y);
    }
    s.allout(s.rt);
}

```

## 2.17 区间线性基

$O((n+q)\log a)$ ,  $O(n\log a)$ 。

```

template<class T,int M=sizeof(T)*8> struct base//线性基
{
    array<T,M> a;
    base():a{ } { }
    bool insert(T x)//线性基插入
    {

```

```

    if (x==0) return 0;
    for (int i=__lg(x); x; i=__lg(x))
    {
        if (!a[i])
        {
            a[i]=x;
            return 1;
        }
        x^=a[i];
    }
    return 0;
}
base &operator+=(const base &o)//合并线性基
{
    for (ll x:o.a) if (x) insert(x);
    return *this;
}
base operator+(base o) const { return o+*this; }//合并线性基
bool contains(T x) const//查询是否能 xor 出 x
{
    if (x==0) return 1;
    for (int i=__lg(x); x; i=__lg(x))
    {
        if (!a[i]) return 0;
        x^=a[i];
    }
    return 1;
}
T max(T x=0) const//查询子集 xor 的最大值。若有传入参数 x, 表示子集 xor x 的最大值。
{
    for (int i=M-1; i>=0; i--) if (1^x>>i&1) x^=a[i];
    return x;
}
};
template<class T=ll,int M=sizeof(T)*8> struct rangebase//[0,...)
{
    vector<array<pair<T,int>,M>> a;
    rangebase():a{{ }} { }
    rangebase(const vector<T> &b):a{{ }} { for (T x:b) insert(x); }//直接用一个 vector 构造
    void push_back(T x)//在最后插入 x
    {
        int n=a.size()-1;
        a.push_back(a.back());
        if (x==0) return;
        for (int i=__lg(x); x; i=__lg(x))
        {
            auto &[v,p]=a.back()[i];
            if (v)
            {
                if (n>p)
                {
                    swap(x,v);
                    swap(n,p);
                }
                x^=v;
            }
            else

```

```

        {
            v=x;
            p=n;
            return;
        }
    }
}
base<T,M> ask(int l,int r)//查询  $[l,r]$  元素构成的线性基。下标从 0 开始 (同 vector)
{
    assert(0<=l&&l<=r&&r<=a.size());
    base<T,M> res;
    for (int i=0; i<M; i++)
    {
        auto [v,p]=a[r][i];
        if (v&&p>=1) res.a[i]=v;
    }
    return res;
}
};

```

## 2.18 splay 重构

$O(n)$ ,  $O((n+q)\log n)$ 。

```

template<class info,class tag> struct splay
{
#define _rev
    struct node
    {
        node *c[2],*f;
        int siz;
        info s,v;
        tag t;
        node():c{},f(0),siz(1),s(),v(),t() {}
        node(info x):c{},f(0),siz(1),s(x),v(x),t() {}
        void operator+=(const tag &o)
        {
            s+=o; v+=o; t+=o;
#ifdef _rev
            if (o.rev) swap(c[0],c[1]);
#endif
        }
        void pushup()
        {
            if (c[0]) s=c[0]->s+v,siz=c[0]->siz+1; else s=v,siz=1;
            if (c[1]) s=s+c[1]->s,siz+=c[1]->siz;
        }
        void pushdown()
        {
            for (auto x:c) if (x) *x+=t;
            t={};
        }
        void zigzag()
        {
            node *y=f,*z=y->f;
            int typ=y->c[0]==this;
            if (z) z->c[z->c[1]==y]=this;

```



```

        f=z; y->f=this;
        y->c[typ^1]=c[typ];
        if (c[typ]) c[typ]->f=y;
        c[typ]=y;
        y->pushup();
    }
    void splay(node *tar)//不要在 makeroot 以外调用
    {
        for (node *y=f; y!=tar; zigzag(),y=f) if (node *z=y->f; z!=tar) (z->c[1]==y^y->c[1]==
            this?this:y)->zigzag();
        pushup();
    }
    void clear()
    {
        for (node *x:c) if (x) x->clear();
        delete this;
    }
};
node *rt;
void debug()
{
    map<node *,int> id;
    id[0]=0; id[rt]=1;
    int cnt=1;
    function<void(node *)> out=[&](node *x)
    {
        if (!x) return;
        for (auto y:x->c) if (!id.count(y)) id[y]=++cnt;
        cerr<<id[x]<<'_ '<<id[x->c[0]]<<'_ '<<id[x->c[1]]<<'_ '<<id[x->f]<<'_ '<<x->siz<<'\n';
        for (auto y:x->c) out(y);
    };
    out(rt);
}
node *build(info *a,int n)
{
    if (n==0) return 0;
    int m=n-1>>1;
    node *x=new node(a[m]);
    x->c[0]=build(a,m);
    x->c[1]=build(a+m+1,n-1-m);
    for (node *y:x->c) if (y) y->f=x;
    x->pushup();
    return x;
}
splay()
{
    rt=new node;
    rt->c[1]=new node;
    rt->c[1]->f=rt;
    rt->siz=2;
}
int shift;
splay(info *a,int l,int r)//[l,r)
{
    shift=l-1;
    rt=new node;
    rt->c[1]=new node;

```

```

    rt->c[1]->f=rt;
    if (l<r)
    {
        rt->c[1]->c[0]=build(a+l,r-1);
        rt->c[1]->c[0]->f=rt->c[1];
    }
    rt->c[1]->pushup();
    rt->pushup();
}
void makeroot(node *u,node *tar)
{
    if (!tar) rt=u;
    u->splay();
}
void findnth(int k,node *tar)
{
    node *x=rt;
    while (1)
    {
        x->pushdown();
        int v=x->c[0]?x->c[0]->siz:0;
        if (v+1==k) { x->splay(tar); if (!tar) rt=x; return; }
        if (v>=k) x=x->c[0]; else x=x->c[1],k-=v+1;
    }
}
void split(int l,int r)
{
    assert(1<=l&&r<=rt->siz-2&&l-1<=r);
    findnth(l,0);
    findnth(r+2,rt);
}
#ifdef _rev
void reverse(int l,int r)
{
    l-=shift; r-=shift+1;
    if (l-1==r) return;
    assert(1<=l&&l<=r&&r<=rt->siz-2);
    split(l,r);
    *(rt->c[1]->c[0])+=tag(1);
}
#endif
void insert(int pos,info x)//insert before pos
{
    pos-=shift;
    assert(1<=pos&&pos<=rt->siz-1);
    split(pos,pos-1);
    rt->c[1]->c[0]=new node(x);
    rt->c[1]->c[0]->f=rt->c[1];
    rt->c[1]->pushup();
    rt->pushup();
}
void insert(int pos,info *a,int n)//insert before pos, [1,n]
{
    pos-=shift;
    assert(1<=pos&&pos<=rt->siz-1);
    split(pos,pos-1);
    rt->c[1]->c[0]=build(a,n);

```

```

    rt->c[1]->c[0]->f=rt->c[1];
    rt->c[1]->pushup();
    rt->pushup();
}
void erase(int pos)
{
    pos-=shift;
    assert(1<=pos&&pos<=rt->siz-2);
    split(pos,pos);
    delete rt->c[1]->c[0];
    rt->c[1]->c[0]=0;
    rt->c[1]->pushup();
    rt->pushup();
}
void erase(int l,int r)
{
    l-=shift; r-=shift+1;
    if (l-1==r) return;
    assert(1<=l&&l<=r&&r<=rt->siz-2);
    split(l,r);
    rt->c[1]->c[0]->clear();
    rt->c[1]->c[0]=0;
    rt->c[1]->pushup();
    rt->pushup();
}
void modify(int pos,info x)//not checked
{
    pos-=shift;
    assert(1<=pos&&pos<=rt->siz-2);
    findnth(pos+1,0);
    rt->v=x; rt->pushup();
}
void modify(int l,int r,tag w)
{
    l-=shift; r-=shift+1;
    if (l-1==r) return;
    assert(1<=l&&l<=r&&r<=rt->siz-2);
    split(l,r);
    node *x=rt->c[1]->c[0];
    *x+=w;
    rt->c[1]->pushup();
    rt->pushup();
}
info ask(int l,int r)
{
    l-=shift; r-=shift+1;
    assert(1<=l&&l<=r&&r<=rt->siz-2);
    split(l,r);
    return rt->c[1]->c[0]->s;
}
~splay() { rt->clear(); }
#undef _rev
};
struct Q
{
    bool rev;
    Q():rev(0) {}

```

```

Q(bool c):rev(c) {}
void operator+=(const Q &o)
{
    rev^=o.rev;
}
};
struct P
{
    ll s;
    void operator+=(const Q &o) const
    {
    }
    P operator+(const P &o) const { return{s+o.s}; }
};

```

## 2.19 第 $k$ 大线性基

注意数字大于  $2^{50}$  时可能要修改循环范围。

$O((n+q)\log a)$ ,  $O(\log a)$ 。

```

void ins(ll x)
{
    if (x==0) return con=1,void();//con=1:有0
    int i;
    for (i=50;x;i--) if (x>>i&1)
    {
        if (!ji[i]) {ji[i]=x;i=-1;break;}x^=ji[i];
    }
    if (!x) con=1;
}
ll kmax(ll x)//查询第 k 大（本质不同，不允许空集）的 xor 结果，若有初始值改 r 即可
{
    ll r=0;
    int m=0,i;
    for (i=50;~i;i--) if (ji[i]) a[++m]=i;
    if (1ll<<m<=x-con) return -1;//个数少于k
    x=(1ll<<m)-x;
    for (i=1;i<=m;i++) if ((x>>m-i^r>>a[i])&1) r^=ji[a[i]];
    return r;
}
ll kmin(ll x)//查询第 k 小（本质不同，不允许空集）的 xor 结果，若有初始值改 r 即可
{
    ll r=0;
    int m=0,i;
    for (i=50;~i;i--) if (ji[i]) a[++m]=i;
    x=-con;
    if (1ll<<m<=x) return -1;//个数少于k
    for (i=1;i<=m;i++) if ((x>>m-i^r>>a[i])&1) r^=ji[a[i]];
    return r;
}

```

## 2.20 fhq-treap

洛谷模板：普通平衡树。

$O((n+q)\log n)$ ,  $O(n)$ 。

```

const int N=1.1e6+2;
int c[N][2],v[N],w[N],s[N];
int n,i,x,y,ds,val,kth,p,q,z,rt,la,m,ans;
void pushup(const int x)
{
    s[x]=s[c[x][0]]+s[c[x][1]]+1;
}
void split_val(int now,int &x,int &y)//调用外部val,相等归入y
{
    if (!now) return x=y=0,void();
    if (val<=v[now]) split_val(c[y=now][0],x,c[now][0]);
    else split_val(c[x=now][1],c[now][1],y);
    pushup(now);
}
void split_kth(int now,int &x,int &y)//调用外部kth, 左子树大小为 kth
{
    if (!now) return x=y=0,void();
    if (kth<=s[c[now][0]]) split_kth(c[y=now][0],x,c[now][0]);
    else kth-=s[c[now][0]]+1,split_kth(c[x=now][1],c[now][1],y);
    pushup(now);
}
int merge(int x,int y)//小根ver.
{
    if (!(x&& y)) return x|y;
    if (w[x]<w[y]) {c[x][1]=merge(c[x][1],y);pushup(x);return x;}
    else {c[y][0]=merge(x,c[y][0]);pushup(y);return y;}
}
int main()
{
    read(n);read(m);srand(998244353);
    for (i=1;i<=n;i++)
    {
        read(x);val=v[++ds]=x;w[ds]=rand();s[ds]=1;split_val(rt,p,q);rt=merge(merge(p,ds),q);
    }
    while (m--)
    {
        read(y);read(x);x^=la;
        if (y==4)//找到第 x 小的
        {
            kth=x;split_kth(rt,p,q);x=p;
            while (c[x][1]) x=c[x][1];
            ans^=(la=v[x]);rt=merge(p,q);
            continue;
        }
        val=x;//注意这一步
        if (y==1)//插入 x
        {
            v[++ds]=x;w[ds]=rand();s[ds]=1;
            split_val(rt,p,q);rt=merge(merge(p,ds),q);
            continue;
        }
        if (y==2)//删除一个 x
        {
            split_val(rt,p,q);kth=1;split_kth(q,i,z);
            rt=merge(p,z);continue;
        }
    }
}

```

```

    if (y==3)//询问 x 的排名 (比 x 小的数字个数 +1)
    {
        split_val(rt,p,q);ans^=(la=s[p]+1);
        rt=merge(p,q);continue;
    }
    if (y==5)//询问比 x 小的最大值
    {
        split_val(rt,p,q);x=p;
        while (c[x][1]) x=c[x][1];ans^=(la=v[x]);
        rt=merge(p,q);continue;
    }
    ++val;split_val(rt,p,q);x=q;//询问比 x 大的最小值
    while (c[x][0]) x=c[x][0];
    ans^=(la=v[x]);rt=merge(p,q);
}
printf("%d",ans);
}

```

## 2.21 笛卡尔树的线性建树

$p[1, 2, \dots, n]$  是原序列,  $c$  表示子结点。

笛卡尔树满足堆性质 (权值小于等于子结点权值), 并且中序遍历是原序列。

$O(n)$ ,  $O(n)$ 。

```

int c[N][2],p[N],st[N];
int main()
{
    int i,n,tp=0;
    ll la=0,ra=0;
    read(n);
    for (i=1;i<=n;i++)
    {
        read(p[i]);st[tp+1]=0;
        while ((tp)&&(p[st[tp]]>p[i])) --tp;
        c[c[st[tp]][1]=i][0]=st[tp+1];st[++tp]=i;
    }
    for (i=1;i<=n;i++) la^=(ll)i*(c[i][0]+1);
    for (i=1;i<=n;i++) ra^=(ll)i*(c[i][1]+1);
    printf("%lld_%lld",la,ra);
}

```

## 2.22 扫描线

求矩形并的面积和周长 (包括内周长)

$O((n+q)\log n)$ ,  $O(n+q)$ 。

```

using T=ll;
vector<T> fun(vector<tuple<T, T, T, T>> &a)
{
    vector<T> x;
    for (auto [x1, y1, x2, y2]:a)
    {
        x.push_back(x1);
        x.push_back(x2);
    }
}

```

```

    sort(all(x)); x.resize(unique(all(x))-x.begin());
    for (auto &[x1, y1, x2, y2]:a)
    {
        x1=lower_bound(all(x), x1)-x.begin();
        x2=lower_bound(all(x), x2)-x.begin();
    }
    return x;
}

struct sgt
{
    int n, z, y, d;
    vector<T> cnt, &p;
    vector<int> mn, lz;
    void build(int x, int l, int r)
    {
        cnt[x]=p[min(r, n-1)]-p[l];
        if (l+1==r) return;
        int c=x*2, m=l+r>>1;
        build(c, l, m); build(c+1, m, r);
    }
    sgt(vector<T> &p):n(p.size()), p(p), cnt(n*4), mn(n*4), lz(n*4) { build(1, 0, n); }
    void dfs(int x, int l, int r)
    {
        if (z<=l&&r<=y)
        {
            mn[x]+=d;
            lz[x]+=d;
            return;
        }
        int c=x*2, m=l+r>>1;
        if (lz[x])
        {
            lz[c]+=lz[x]; lz[c+1]+=lz[x];
            mn[c]+=lz[x]; mn[c+1]+=lz[x];
            lz[x]=0;
        }
        if (z<m) dfs(c, l, m);
        if (m<y) dfs(c+1, m, r);
        mn[x]=min(mn[c], mn[c+1]);
        cnt[x]=cnt[c]*(mn[x]==mn[c])+cnt[c+1]*(mn[x]==mn[c+1]);
    }
    void modify(int l, int r, int dt)
    {
        z=l;
        y=r;
        d=dt;
        dfs(1, 0, n);
    }
};

T area(vector<tuple<T, T, T, T>> a)//[x1,y1,x2,y2], x1<y1, x2<y2
{
    int n=a.size(), i;
    auto X=fun(a);
    vector<tuple<T, int, T, T>> b(n*2);
    for (i=0; i<n; i++)
    {
        auto [x1, y1, x2, y2]=a[i];

```

```

        b[i]={y1, -1, x1, x2};
        b[i+n]={y2, 1, x1, x2};
    }
    sort(all(b), greater<>());
    sgt s(X);
    T lst=0, ans=0;
    for (auto [y, d, l, r]:b)
    {
        ans+=(lst-y)*(X.back()-X[0]-s.cnt[1]);
        s.modify(l, r, d);
        lst=y;
    }
    return ans;
}
T perimeter_x(vector<tuple<T, T, T, T>> a)
{
    int n=a.size(), i;
    auto X=fun(a);
    vector<tuple<T, int, T, T>> b(n*2);
    for (i=0; i<n; i++)
    {
        auto [x1, y1, x2, y2]=a[i];
        b[i]={y1, -1, x1, x2};
        b[i+n]={y2, 1, x1, x2};
    }
    sort(all(b), greater<>());
    sgt s(X);
    T lst=s.cnt[1], ans=0;
    for (auto [y, d, l, r]:b)
    {
        s.modify(l, r, d);
        T cur=s.cnt[1];
        ans+=abs(lst-cur);
        lst=cur;
    }
    return ans;
}
T perimeter(vector<tuple<T, T, T, T>> a)//[x1,y1,x2,y2], x1<y1, x2<y2
{
    T ansx=perimeter_x(a);
    for (auto &[x1, y1, x2, y2]:a)
    {
        swap(x1, y1);
        swap(x2, y2);
    }
    T ansy=perimeter_x(a);
    return ansx+ansy;
}

```

## 2.23 Segmenttree Beats!

核心是 P (tag) 和 Q (info) 的维护。线段树部分是套的模板，并非全都有用。

1.  $l, r, k$ : 对于所有的  $i \in [l, r]$ ，将  $A_i$  加上  $k$  ( $k$  可以为负数)。
2.  $l, r, v$ : 对于所有的  $i \in [l, r]$ ，将  $A_i$  变成  $\min(A_i, v)$ 。



3.  $l, r$ : 求  $\sum_{i=l}^r A_i$ 。
4.  $l, r$ : 对于所有的  $i \in [l, r]$ , 求  $A_i$  的最大值。
5.  $l, r$ : 对于所有的  $i \in [l, r]$ , 求  $B_i$  的最大值。

其中  $B_i$  是  $A_i$  的历史最大值。

```

struct P
{
    ll tg,L,R;
    P(ll a=0,ll b=-inf,ll c=inf):tg(a),L(b),R(c) { }
    void operator+=(P o)
    {
        o.L-=tg; o.R-=tg; tg+=o.tg;
        if (L>=o.R) L=R=o.R;
        else if (R<=o.L) L=R=o.L;
        else cmax(L,o.L),cmin(R,o.R);
    }
};

struct Q
{
    ll mx0,cmx,mx1,mn0,cmn,mn1,cnt,sum;
    Q():mx0(-inf),cmx(0),mx1(-inf),mn0(inf),cmn(0),mn1(inf),cnt(0),sum(0) { }
    Q(ll x):mx0(x),cmx(1),mx1(-inf),mn0(x),cmn(1),mn1(inf),cnt(1),sum(x) { }
    bool operator+=(const P &o)
    {
        if (o.L==o.R)
        {
            ll c=cnt;
            *this=Q(o.L+o.tg);
            cnt=cmx=cmn=c;
            sum=cnt*(o.L+o.tg);
            return 1;
        }
        if (o.L>=mn1 || o.R<=mx1) return 0;
        if (mx0==mn0)
        {
            mn0=min(o.R,max(mx0,o.L));
            sum+=cnt*(mn0-mx0);
            mx0=mn0;
        }
        else
        {
            if (o.L>mn0)
            {
                sum+=(o.L-mn0)*cmn;
                mn0=o.L;
                cmax(mx1,o.L);
            }
            if (o.R<mx0)
            {
                sum+=(o.R-mx0)*cmx;
                mx0=o.R;
                cmin(mn1,o.R);
            }
        }
    }
};

```

```

        if (o.tg)
        {
            sum+=o.tg*cnt;
            mx0+=o.tg;
            mx1+=o.tg;
            mn0+=o.tg;
            mn1+=o.tg;
        }
        return 1;
    }
};
Q operator+(const Q &a,const Q &b)
{
    Q res;
    res.sum=a.sum+b.sum;
    res.cnt=a.cnt+b.cnt;
    res.mx0=max(a.mx0,b.mx0);
    res.mx1=max(a.mx1,b.mx1);
    if (res.mx0==a.mx0) res.cmx+=a.cmx; else cmx(res.mx1,a.mx0);
    if (res.mx0==b.mx0) res.cmx+=b.cmx; else cmx(res.mx1,b.mx0);

    res.mn0=min(a.mn0,b.mn0);
    res.mn1=min(a.mn1,b.mn1);
    if (res.mn0==a.mn0) res.cmn+=a.cmn; else cmin(res.mn1,a.mn0);
    if (res.mn0==b.mn0) res.cmn+=b.cmn; else cmin(res.mn1,b.mn0);

    return res;
}
template<class info,class tag> struct sgt
{
    int n,shift;
    vector<info> s;
    vector<tag> tg;
    vector<char> lz;
    template<class T> void build(T *a,int x,int l,int r)
    {
        if (l==r)
        {
            s[x]=a[l];
            return;
        }
        int c=x*2,m=l+r>>1;
        build(a,c,l,m); build(a,c+1,m+1,r);
        s[x]=s[c]+s[c+1];
    }
    template<class T> sgt(T *b,int L,int R):n(R-L+1),shift(L-1),s(R-L+1<<2),tg(R-L+1<<2),lz(R-L
        +1<<2)
    {
        build(b+L-1,1,1,n);
    } // [L,R]
    int z,y;
    info res;
    tag dt;
    bool fir;
private:
    void pushdown(int x)
    {

```

```

    int c=x*2;
    if (lz[x])
    {
        if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
        lz[c]=1;
        if (!(s[c]+=tg[x]))
        {
            pushdown(c);
            s[c]=s[c*2]+s[c*2+1];
        }
        c^=1;
        if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
        lz[c]=1;
        if (!(s[c]+=tg[x]))
        {
            pushdown(c);
            s[c]=s[c*2]+s[c*2+1];
        }
        c^=1;
        lz[x]=0;
    }
}

void _modify(int x,int l,int r)
{
    if (z<=l&&r<=y)
    {
        if (lz[x]) tg[x]+=dt; else tg[x]=dt;
        lz[x]=1;
        if (!(s[x]+=dt))
        {
            pushdown(x);
            s[x]=s[x*2]+s[x*2+1];
        }
        return;
    }
    int c=x*2,m=l+r>>1;
    pushdown(x);
    if (z<=m) _modify(c,l,m);
    if (m<y) _modify(c+1,m+1,r);
    s[x]=s[c]+s[c+1];
}

void ask(int x,int l,int r)
{
    if (z<=l&&r<=y)
    {
        res=fir?s[x]:res+s[x];
        fir=0;
        return;
    }
    int c=x*2,m=l+r>>1;
    pushdown(x);
    if (z<=m) ask(c,l,m);
    if (m<y) ask(c+1,m+1,r);
}

function<bool>(info)> check;
void find_left_most(int x,int l,int r)
{

```

```

    if (r<z||!check(s[x])) return;
    if (l==r) { y=l; res=s[x]; return; }
    int c=x*2,m=l+r>>1;
    pushdown(x);
    find_left_most(c,l,m);
    if (y==n+1) find_left_most(c+1,m+1,r);
}

void find_right_most(int x,int l,int r)
{
    if (l>y||!check(s[x])) return;
    if (l==r) { z=l; res=s[x]; return; }
    int c=x*2,m=l+r>>1;
    pushdown(x);
    find_right_most(c+1,m+1,r);
    if (z==0) find_right_most(c,l,m);
}

public:
void modify(int l,int r,const tag &x)//[l,r]
{
    z=l-shift; y=r-shift; dt=x;
    // cerr<<"modify ["<<l<<','<<r<<"] "<<'\n';
    assert(1<=z&&z<=y&&y<=n);
    _modify(1,1,n);
}

void modify(int pos,const info &o)
{
    pos-=shift;
    int l=1,r=n,m,c,x=1;
    while (l<r)
    {
        c=x*2; m=l+r>>1;
        pushdown(x);
        if (pos<=m) x=c,r=m; else x=c+1,l=m+1;
    }
    s[x]=o;
    while (x>>=1) s[x]=s[x*2]+s[x*2+1];
}

info ask(int l,int r)//[l,r]
{
    z=l-shift; y=r-shift; fir=1;
    // cerr<<"ask ["<<l<<','<<r<<"] "<<'\n';
    assert(1<=z&&z<=y&&y<=n);
    ask(1,1,n);
    return res;
}

pair<int,info> find_left_most(int l,const function<bool(info)> &_check)//y=n+1 第二个参数是乱给的
{
    check=_check;
    z=l-shift; y=n+1;
    assert(1<=z&&z<=n+1);
    find_left_most(1,1,n);
    return {y+shift,res};
}

pair<int,info> find_right_most(int r,const function<bool(info)> &_check)//z=0 第二个参数是乱给的
{

```

```

        check=_check;
        z=0; y=r-shift;
        assert(0<=y&&y<=n);
        find_right_most(1,1,n);
        return {z+shift,res};
    }
};
//要求: 具有 info+info, info+=tag, tag+=tag。info, tag 需要拥有默认构造, 但不必拥有正确的值。
//采用左闭右闭
mt19937 rnd(345);
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout<<fixed<<setprecision(15);
    int n,q,i;
    cin>>n>>q;
    vector<ll> a(n);
    cin>>a;
    sgt<Q,P> s(a.data(),0,n-1);
    while (q-->0)
    {
        int op,l,r;
        cin>>op>>l>>r;
        --r;
        if (op==3)
        {
            ll res=s.ask(l,r).sum;
            cout<<res<<'\\n';
        }
        else
        {
            ll b;
            cin>>b;
            if (op==0) s.modify(l,r,{0,-inf,b});
            else if (op==1) s.modify(l,r,{0,b});
            else s.modify(l,r,{b});
        }
    }
}

```

## 2.24 $k$ -d 树 (二进制分组)

均摊  $O(\log^2 n)$  插入,  $O(\sqrt{n})$  矩形查询。

```

#define tml template<class T>
typedef long long ll;
tml struct P
{
    ll x,y;
    T v;
};
tml struct Q
{
    ll x[2],y[2];
    bool t;
    T s;
    Q() {}
}

```

```

Q(const P<T> &a)
{
    x[0]=x[1]=a.x;
    y[0]=y[1]=a.y;
    s=a.v;
}
};

tmpl bool cmp0(const P<T> &a,const P<T> &b) { return a.x<b.x; }
tmpl bool cmp1(const P<T> &a,const P<T> &b) { return a.y<b.y; }
tmpl struct kdt
{
    vector<P<T>> c;
    vector<Q<T>> a;
    ll m,u,d,l,r;
    T ans;
    bool fir;
    void build(int x,P<T> *b,int n)
    {
        if (x==1)
        {
            a.resize(m=n<<1);
            a[x].t=0;
            c.resize(n);
            for (int i=0; i<n; i++) c[i]=b[i];
        }
        if (n==1)
        {
            a[x]=Q<T>(b[0]);
            return;
        }
        int mid=n>>1,c=x<<1;
        nth_element(b,b+mid,b+n,a[x].t?cmp1<T>:cmp0<T>);
        a[c].t=a[c|1].t=a[x].t^1;
        build(c,b,mid);
        build(c|1,b+mid,n-mid);
        a[x].s=a[c].s+a[c|1].s;
        a[x].x[0]=min(a[c].x[0],a[c|1].x[0]);
        a[x].x[1]=max(a[c].x[1],a[c|1].x[1]);
        a[x].y[0]=min(a[c].y[0],a[c|1].y[0]);
        a[x].y[1]=max(a[c].y[1],a[c|1].y[1]);
    }
    void find(int x)
    {
        if (x>=m||a[x].x[1]<u||a[x].x[0]>d||a[x].y[1]<l||a[x].y[0]>r) return;
        if (u<=a[x].x[0]&&a[x].x[1]<=d&&l<=a[x].y[0]&&a[x].y[1]<=r)
        {
            ans=fir?a[x].s:ans+a[x].s;
            fir=0;
            return;
        }
        find(x<<1); find(x<<1|1);
    }
    pair<bool,T> find(ll x1,ll y1,ll x2,ll y2)
    {
        fir=1;
        ans={};
        u=x1; d=x2;
    }

```

```

        l=y1; r=y2;
        find(1);
        return {!fir,ans};
    }
};
const int N=2e5+2,M=18;
templ struct KDT
{
    kdt<T> s[M];
    P<T> a[N];
    int n,m,i;
    KDT() { n=0; }
    KDT(int N,ll *x,ll *y,T *w)//[0,n)
    {
        n=N;
        int i,j;
        for (i=0; i<n; i++) a[i]={x[i],y[i],w[i]};
        for (i=j=0; n>>i; i++) if (n>>i&1) s[i].build(1,a+j,1<<i),j+=1<<i;
    }
    void insert(ll x,ll y,T w)//插入 (x,y) 的一个数 w
    {
        a[0]={x,y,w}; m=1;
        for (i=0; n&1<<i; i++) for (auto u:s[i].c) a[m++]=u;
        s[i].build(1,a,m);
        ++n;
    }
    pair<bool,T> ask(ll x,ll y,ll xx,ll yy)//查询 [x,xx]*[y,yy] 的和
    {
        T ans;
        bool fir=1;
        for (i=0; 1<<i<=n; i++) if (1<<i&n)
        {
            auto [_,tmp]=s[i].find(x,y,xx,yy);
            if (!_) continue;
            ans=fir?tmp:ans+tmp;
            fir=0;
        }
        return {!fir,ans};
    }
};
int x[N],y[N],w[N];
int main()
{
    ios::sync_with_stdio(0); cin.tie(0); cout.tie(0);
    int n,q,i;
    cin>>n>>q;
    for (i=0; i<n; i++) cin>>x[i]>>y[i]>>w[i];
    KDT<ll> s(n,x,y,w);
    while (q--)
    {
        int op,x,y,w;
        cin>>op>>x>>y>>w;
        if (op==0) s.insert(x,y,w); else
        {
            cin>>op;
            cout<<s.ask(x,y,w-1,op-1)<<'\n';
        }
    }
}

```

```

    }
    return 0;
}

```

## 2.25 双端队列全局查询

对一个支持结合律的信息  $T$ ，维护 deque 内信息的和。总复杂度线性。

```

template<class T> struct dq
{
    vector<T> l,sl,r,sr;
    void push_front(const T &o)
    {
        sl.push_back(sl.size()?o+sl.back():o);
        l.push_back(o);
    }
    void push_back(const T &o)
    {
        sr.push_back(sr.size()?sr.back()+o:o);
        r.push_back(o);
    }
    void pop_front()
    {
        if (l.size()) sl.pop_back(),l.pop_back();
        else
        {
            assert(r.size());
            int n=r.size(),m,i;
            if (m=n-1>>1)
            {
                l.resize(m); sl.resize(m);
                for (i=1; i<=m; i++) l[m-i]=r[i];
                sl[0]=l[0];
                for (i=1; i<m; i++) sl[i]=l[i]+sl[i-1];
            }
            for (i=m+1; i<n; i++) r[i-(m+1)]=r[i];
            m=n-(m+1);
            r.resize(m); sr.resize(m);
            if (m)
            {
                sr[0]=r[0];
                for (i=1; i<m; i++) sr[i]=sr[i-1]+r[i];
            }
        }
    }
    void pop_back()
    {
        if (r.size()) sr.pop_back(),r.pop_back();
        else
        {
            assert(l.size());
            int n=l.size(),m,i;
            if (m=n-1>>1)
            {
                r.resize(m); sr.resize(m);
                for (i=1; i<=m; i++) r[m-i]=l[i];
                sr[0]=r[0];
            }
        }
    }
}

```



```

        for (i=1; i<m; i++) sr[i]=sr[i-1]+r[i];
    }
    for (i=m+1; i<n; i++) l[i-(m+1)]=l[i];
    m=n-(m+1);
    l.resize(m); sl.resize(m);
    if (m)
    {
        sl[0]=l[0];
        for (i=1; i<m; i++) sl[i]=l[i]+sl[i-1];
    }
}
}
template<class TT> TT ask(TT r)
{
    if (sl.size()) r=r+sl.back();
    if (sr.size()) r=r+sr.back();
    return r;
}
T ask()
{
    assert(sl.size()||sr.size());
    if (sl.size() && sr.size()) return sl.back()+sr.back();
    return sl.size()?sl.back():sr.back();
}
}; //参数: 类型。结合使用 + 运算符

```

## 2.26 静态矩形加矩形和

```

const ll p=998244353;
struct Q
{
    int n,m;
    ll w;
    int typ;
    bool operator<(const Q &o) const
    {
        if (n!=o.n) return n<o.n;
        return typ<o.typ;
    }
};
template<class T> struct tork
{
    vector<T> a;
    int n;
    tork(const vector<T> &b):a(all(b))
    {
        sort(all(a));
        a.resize(unique(all(a))-a.begin());
        n=a.size();
    }
    tork(const T *first,const T *last):a(first,last)
    {
        sort(all(a));
        a.resize(unique(all(a))-a.begin());
        n=a.size();
    }
}

```

```

void get(T &x) { x=lower_bound(all(a),x)-a.begin()+1; }
T operator[](const int &x) { return a[x]; }
};

struct bit
{
    vector<ll> a;
    int n;
    bit() {}
    bit(int nn):n(nn),a(nn+1) {}
    template<class T> bit(int nn,T *b):n(nn),a(nn+1)
    {
        for (int i=1; i<=n; i++) a[i]=b[i];
        for (int i=1; i<=n; i++) if (i+(i&-i)<=n) a[i+(i&-i)]+=a[i];
    }
    void add(int x,ll y)
    {
        // cerr<<"add "<<x<<" by "<<y<<endl;
        assert(1<=x&&x<=n);
        if ((a[x]+=y)>=p) a[x]-=p;
        while ((x+=x&-x)<=n) if ((a[x]+=y)>=p) a[x]-=p;
    }
    ll sum(int x)
    {
        // cerr<<"sum "<<x;
        assert(0<=x&&x<=n);
        ll r=a[x];
        while (x^=x&-x) r+=a[x];
        // cerr<<"= "<<r<<endl;
        return r%p;
    }
    ll sum(int x,int y)
    {
        return (sum(y)+p-sum(x-1))%p;
    }
};

struct matrix
{
    int l,d,r,u;
    ll w;
};

vector<ll> rec_add_rec_sum(const vector<matrix> &op,const vector<matrix> &query)
{
    vector<Q> a[4];
    int n=op.size(),m=query.size(),i;
    for (auto &v:a) v.reserve(n+m<<2);
    for (auto [l,d,r,u,w]:op)//[l,r]*[d,u] += w
    {
        a[0].push_back({l,d,w*1%p*d%p,-1});
        a[1].push_back({l,d,w*1%p,-1});
        a[2].push_back({l,d,w*d%p,-1});
        a[3].push_back({l,d,w,-1});
        w=(p-w)%p;
        a[0].push_back({l,u,w*1%p*u%p,-1});
        a[1].push_back({l,u,w*1%p,-1});
        a[2].push_back({l,u,w*u%p,-1});
        a[3].push_back({l,u,w,-1});
        a[0].push_back({r,d,w*r%p*d%p,-1});
    }
}

```

```

    a[1].push_back({r,d,w*r%p,-1});
    a[2].push_back({r,d,w*d%p,-1});
    a[3].push_back({r,d,w,-1});
    w=(p-w)%p;
    a[0].push_back({r,u,w*r%p*u%p,-1});
    a[1].push_back({r,u,w*r%p,-1});
    a[2].push_back({r,u,w*u%p,-1});
    a[3].push_back({r,u,w,-1});
}
i=0;
for (auto [l,d,r,u,w]:query)//ask sum of [l,r)*[d,u)
{
    a[0].push_back({l,d,1,i});
    a[1].push_back({l,d,(p*2-d)%p,i});
    a[2].push_back({l,d,(p*2-1)%p,i});
    a[3].push_back({l,d,(1l)*d%p,i});
    a[0].push_back({l,u,p-1,i});
    a[1].push_back({l,u,u%p,i});
    a[2].push_back({l,u,1%p,i});
    a[3].push_back({l,u,(p*2-1)*u%p,i});
    a[0].push_back({r,u,1,i});
    a[1].push_back({r,u,(p*2-u)%p,i});
    a[2].push_back({r,u,(p*2-r)%p,i});
    a[3].push_back({r,u,(1l)*u*r%p,i});
    a[0].push_back({r,d,p-1,i});
    a[1].push_back({r,d,d%p,i});
    a[2].push_back({r,d,r%p,i});
    a[3].push_back({r,d,(p*2-d)*r%p,i});
    ++i;
}
assert(a[0].size()==n+m<<2);
vector<ll> ans(m);
auto cal=[&](vector<Q> a)
{
    int n=a.size(),i;
    vector<int> b(n);
    for (i=0; i<n; i++) b[i]=(a[i].m==a[i].typ>=0),a[i].n-=a[i].typ>=0;
    sort(all(a));
    tork t(b);
    for (i=0; i<n; i++) t.get(a[i].m);
    int m=t.a.size();
    bit s(m);
    for (auto [n,m,w,typ]:a) if (typ>=0) ans[typ]=(ans[typ]+s.sum(m)*w)%p; else s.add(m,w);
};
for (auto &v:a) cal(v);
return ans;
}
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(15);
    int n,m,i;
    cin>>n>>m;
    vector<matrix> a(n),b(m);
    for (auto &[l,d,r,u,w]:a) cin>>l>>d>>r>>u>>w;
    for (auto &[l,d,r,u,w]:b) cin>>l>>d>>r>>u;
    auto ans=rec_add_rec_sum(a,b);

```

```

    for (i=0; i<m; i++) cout<<ans[i]<<'\n';
}

```

## 2.27 线段树分裂

```

namespace sgt
{
#define ask_kth
    int L=0,R=1e9;
    void set_bound(int l,int r) { L=l; R=r; }
    typedef ll info;
    const info E=0;//找不到会返回 E
    const int N=8e6+5;
#define lc(x) (a[x].lc)
#define rc(x) (a[x].rc)
#define s(x) (a[x].s)
    struct node
    {
        int lc,rc;
        info s;
    };
    node a[N];
    vector<int> id;
    int ids=0,pos,z,y;
    bool fir;
    info tmp;
    int npt()
    {
        int x;
        if (id.size()) x=id.back(),id.pop_back();
        else x=++ids;
        lc(x)=rc(x)=0;
        return x;
    }
    void pushup(int &x)
    {
        if (lc(x)&&rc(x)) s(x)=s(lc(x))+s(rc(x));
        else if (lc(x)) s(x)=s(lc(x));
        else if (rc(x)) s(x)=s(rc(x));
        else id.push_back(x),x=0;
    }
    void insert(int &x,int l,int r)
    {
        if (l==r)
        {
            if (!x) x=npt(),s(x)=tmp;
            else s(x)=s(x)+tmp;
            return;
        }
        if (!x) x=npt();
        int mid=l+r>>1;
        if (pos<=mid)
        {
            insert(lc(x),l,mid);
            if (rc(x)) s(x)=s(lc(x))+s(rc(x)); else s(x)=s(lc(x));
        }
    }
}

```

```

    else
    {
        insert(rc(x),mid+1,r);
        if (lc(x)) s(x)=s(lc(x))+s(rc(x)); else s(x)=s(rc(x));
    }
}
void modify(int &x,int l,int r)
{
    if (!x) x=npt();
    if (l==r)
    {
        s(x)=tmp;
        return;
    }
    int mid=l+r>>1;
    if (pos<=mid)
    {
        insert(lc(x),l,mid);
        if (rc(x)) s(x)=s(lc(x))+s(rc(x)); else s(x)=s(lc(x));
    }
    else
    {
        insert(rc(x),mid+1,r);
        if (lc(x)) s(x)=s(lc(x))+s(rc(x)); else s(x)=s(rc(x));
    }
}
int merge(int x1,int x2,int l,int r)
{
    if (!(x1&&x2)) return x1|x2;
    if (l==r) { s(x1)=s(x1)+s(x2); return x1; }
    int mid=l+r>>1;
    lc(x1)=merge(lc(x1),lc(x2),l,mid);
    rc(x1)=merge(rc(x1),rc(x2),mid+1,r);
    pushup(x1);
    return x1;
}
void ask(int x,int l,int r)
{
    if (!x) return;
    if (z<=l&&r<=y)
    {
        if (fir) tmp=s(x),fir=0; else tmp=tmp+s(x);
        return;
    }
    int mid=l+r>>1;
    if (z<=mid) ask(lc(x),l,mid);
    if (y>mid) ask(rc(x),mid+1,r);
}
void split(int &x1,int &x2,int l,int r)
{
    assert(!x1);
    if (!x2) return;
    if (z<=l&&r<=y) { x1=x2; x2=0; return; }
    x1=npt();
    int mid=l+r>>1;
    if (z<=mid) split(lc(x1),lc(x2),l,mid);
    if (y>mid) split(rc(x1),rc(x2),mid+1,r);
}

```

```

    pushup(x1); pushup(x2);
}
info *b;
void build(int &x,int l,int r)
{
    x=npt();
    if (l==r) { s(x)=b[l]; return; }
    int mid=l+r>>1;
    build(lc(x),l,mid); build(rc(x),mid+1,r);
    s(x)=s(lc(x))+s(rc(x));
}
struct set
{
    int rt;
    set():rt(0) {}
    set(info *a):rt(0) { b=a; build(rt,L,R); }
    void modify(int p,const info &o) { pos=p; tmp=o; sgt::modify(rt,L,R); }
    void insert(int p,const info &o) { pos=p; tmp=o; sgt::insert(rt,L,R); }
    void join(const set &o) { rt=merge(rt,o.rt,L,R); }
    info ask(int l,int r)
    {
        z=l; y=r; fir=1;
        sgt::ask(rt,L,R);
        return fir?E:tmp;
    }
    set split(int l,int r)
    {
        z=l; y=r; set p;
        sgt::split(p.rt,rt,L,R);
        return p;
    }
#ifdef ask_kth
    int kth(info k)
    {
        int x=rt,l=L,r=R,mid;
        if (k>s(x)) return -1;
        s(0)=0;
        while (l<r)
        {
            mid=l+r>>1;
            if (s(lc(x))>=k) x=lc(x),r=mid;
            else k-=s(lc(x)),x=rc(x),l=mid+1;
        }
        return l;
    }
#endif
};
#undef lc
#undef rc
#undef s
}
typedef sgt::set tree;

```

## 2.28 bitset (手写, 未验证)

```
struct Bitset
```

```

{
    typedef unsigned int ui;
    typedef unsigned long long ll;
#define all(x) (x).begin(),(x).end()
    const static ll B=-1llu;
    vector<ll> a;
    int n;
    Bitset() { }
    Bitset(int _n):n(_n), a(_n+63>>6) { }
    bool test(int x) const { assert(x>=0&&x<n); return a[x>>6]>>(x&63)&1; }
    bool operator[](int x) const { return test(x); }
    void set(int x, bool y) { assert(x>=0&&x<n); a[x>>6]=(a[x>>6]&(B^1llu<<(x&63)))|((ll)y<<(x&63)); }
    void set(int x) { assert(x>=0&&x<n); a[x>>6]|=1llu<<(x&63); }
    void set() { memset(a.data(), 0xff, a.size()*sizeof a[0]); a.back()&=(1llu<<1+(n-1&63))-1; }
    void reset(int x) { assert(x>=0&&x<n); a[x>>6]&=~(1llu<<(x&63)); }
    void reset() { memset(a.data(), 0, a.size()*sizeof a[0]); }
    int count() const
    {
        int r=0;
        for (ll x:a) r+=__builtin_popcountll(x);
        return r;
    }
    Bitset &operator|=(const Bitset &o)
    {
        assert(n==o.n);
        for (int i=0; i<a.size(); i++) a[i]|=o.a[i];
        return *this;
    }
    Bitset operator|(Bitset o) { o|=*this; return o; }
    Bitset &operator&=(const Bitset &o)
    {
        assert(n==o.n);
        for (int i=0; i<a.size(); i++) a[i]&=o.a[i];
        return *this;
    }
    Bitset operator&(Bitset o) { o&=*this; return o; }
    Bitset &operator^=(const Bitset &o)
    {
        assert(n==o.n);
        for (int i=0; i<a.size(); i++) a[i]^=o.a[i];
        return *this;
    }
    Bitset operator^(Bitset o) { o^=*this; return o; }
    Bitset operator~() const
    {
        auto r=*this;
        for (ll &x:r.a) x=~x;
        return r;
    }
    Bitset &operator<<=(int x)
    {
        if (x>=n)
        {
            fill(all(a), 0);
            return *this;
        }
    }
}

```

```

    assert(x>=0);
    int y=x>>6;
    x&=63;
    if (x==0)
    {
        for (int i=(int)a.size()-1; i>=y; i--) a[i]=a[i-y]<<x;
        if (n&63) a.back()&=(1llu<<1+(n-1&63))-1;
        memset(a.data(), 0, y*sizeof a[0]);
        return *this;
    }
    for (int i=(int)a.size()-1; i>y; i--) a[i]=a[i-y]<<x|a[i-y-1]>>64-x;
    a[y]=a[0]<<x;
    memset(a.data(), 0, y*sizeof a[0]);
    // fill_n(a.begin(),y,0);
    if (n&63) a.back()&=(1llu<<1+(n-1&63))-1;
    return *this;
}

Bitset operator<<(int x)
{
    auto r=*this;
    r<<=x;
    return r;
}

Bitset &operator>>=(int x)
{
    if (x>=n)
    {
        fill(all(a), 0);
        return *this;
    }
    assert(x>=0);
    int y=x>>6, R=(int)a.size()-y-1;
    x&=63;
    for (int i=0; i<R; i++) a[i]=a[i+y]>>x|a[i+y+1]<<64-x;
    a[R]=a.back()>>x;
    memset(a.data()+R+1, 0, y*sizeof a[0]);
    // fill(R+1+all(a),0);
    return *this;
}

Bitset operator>>(int x)
{
    auto r=*this;
    r>>=x;
    return r;
}

void range_set(int l, int r)//[l,r) to 1
{
    if (l>>6==r>>6)
    {
        a[l>>6]|=(1llu<<r-l)-1<<(1&63);
        return;
    }
    if (1&63)
    {
        a[l>>6]|=~((1llu<<(1&63))-1);//[1&63,64)
        l=(l>>6)+1<<6;
    }
}

```



```

        if (r&63)
        {
            a[r>>6]|=(1llu<<(r&63))-1;
            r=(r>>6)-1<<6;
        }
        memset(a.data()+(l>>6), 0xff, (r-l>>6)*sizeof a[0]);
    }
    void range_reset(int l, int r)//[l,r) to 0
    {
        if (l>>6==r>>6)
        {
            a[l>>6]&=~((1llu<<(r-l)-1<<(l&63)));
            return;
        }
        if (l&63)
        {
            a[l>>6]&=(1llu<<(l&63))-1;//[l&63,64)
            l=(l>>6)+1<<6;
        }
        if (r&63)
        {
            a[r>>6]&=~((1llu<<(r&63))-1);
            r=(r>>6)-1<<6;
        }
        memset(a.data()+(l>>6), 0, (r-l>>6)*sizeof a[0]);
    }
    void range_set(int l, int r, bool x)//[l,r)
    {
        if (x) range_set(l, r);
        else range_reset(l, r);
    }
    int size() const { return n; }
    int _Find_first() const
    {
        for (int i=0; i<a.size(); i++) if (a[i]) return i*64+__lg(a[i]&-a[i]);
        return n;
    }
};

istream &operator>>(istream &cin, Bitset &o)
{
    string s;
    cin>>s;
    int n=s.size(), i;
    assert(n<=o.size());
    for (i=0; i<n; i++) o.set(i, s[n-i-1]-'0');
    return cin;
}

ostream &operator<<(ostream &cout, const Bitset &o)
{
    int n=o.size(), i;
    string s(n, '0');
    for (i=0; i<n; i++) s[n-i-1]+=o.test(i);
    return cout;
}

```

## 2.29 区间众数

```

template<class T> struct mode//[0,n)
{
    int n,ksz,m;
    vector<T> b;
    vector<vector<int>>> pos,f;
    vector<int> a,blk,id,l;
    mode(const vector<T> &c):n(c.size()),ksz(max<int>(1,sqrt(n))),m((n+ksz-1)/ksz),b(c),
        pos(n),f(m,vector<int>(m)),a(n),blk(n),id(n),l(m+1)
    {
        int i,j,k;
        sort(all(b)); b.resize(unique(all(b))-b.begin());
        for (i=0; i<n; i++)
        {
            a[i]=lower_bound(all(b),c[i])-b.begin();
            id[i]=pos[a[i]].size();
            pos[a[i]].push_back(i);
        }
        for (i=0; i<n; i++) blk[i]=i/ksz;
        for (i=0; i<=m; i++) l[i]=min(i*ksz,n);
        vector<int> cnt(b.size());
        for (i=0; i<m; i++)
        {
            fill(all(cnt),0);
            pair<int,int> cur={0,0};
            for (j=i; j<m; j++)
            {
                for (k=l[j]; k<l[j+1]; k++) cmax(cur,pair{++cnt[a[k]],a[k]});
                f[i][j]=cur.second;
            }
        }
    }
}

pair<T,int> ask(int L,int R)//返回最大众数
{
    assert(0<=L&&L<R&&R<=n);
    int val=blk[L]==blk[R-1]?0:f[blk[L]+1][blk[R-1]-1],i;
    int cnt=lower_bound(all(pos[val]),R)-lower_bound(all(pos[val]),L);
    for (i=min(R,l[blk[L]+1])-1; i>=L; i--)
    {
        auto &v=pos[a[i]];
        while (id[i]+cnt<v.size()&&v[id[i]+cnt]<R) ++cnt,val=a[i];
        if (a[i]>val&&id[i]+cnt-1<v.size()&&v[id[i]+cnt-1]<R) val=a[i];
    }
    for (i=max(L,l[blk[R-1]]); i<R; i++)
    {
        auto &v=pos[a[i]];
        while (id[i]>=cnt&&v[id[i]-cnt]>=L) ++cnt,val=a[i];
        if (a[i]>val&&id[i]>=cnt-1&&v[id[i]-cnt+1]>=L) val=a[i];
    }
    return {b[val],cnt};
}
};

```

## 2.30 表达式树

传入表达式，输出表达式树。

输入的第二个参数是全体括号以外的运算符，每个运算符要记录字符优先级和是否右结合。优先级数字越大，越优先计算，且优先级必须为正整数。

输出的第一个参数是子节点数组，第二个参数是每个结点对应的字符，第三个参数是根。结点编号从 1 开始。

输出的表达式树满足每个结点对应一个字符。若包含数字串，则视为相邻数码之间加一个井号，表示“数码链接”这个运算符。你不需要，也不应该手动加入这个井号。

如果表达式非法，将返回根为 0。不允许一元运算符（负号），不允许省略乘号，不允许出现字母（除非字母是运算符）。

如果需要支持字母作为数字，修改所有包含 `isdigit` 的部分。

由于存在“数码链接”，在 `dfs` 树的时候最好记录一下子树大小，便于链接时计算（你不能在链接时直接看右子树的数字大小，因为有可能有前导 0）。

```
struct Q
{
    char ch;
    int prec;
    bool right;
};

tuple<vector<array<int, 2>>, vector<char>, int> parse_expr(string s, vector<Q> op) {
    static int idx[128];
    int maxp = 0, pos = 0, n, err = 0, i;
    {
        string t;
        for (char c : s)
        {
            if (t.size() && isdigit(t.back()) && isdigit(c)) t += '#';
            t += c;
        }
        swap(s, t);
        n = s.size();
    }
    for (i = 0; i < op.size(); ++i)
    {
        idx[op[i].ch] = i + 1;
        cmax(maxp, op[i].prec);
    }
    op.push_back({'#', ++maxp, 0});
    idx['#'] = op.size();
    vector<array<int, 2>> c(1);
    vector<char> ch(1);
    auto node = [&](char x) {
        c.push_back({0, 0});
        ch.push_back(x);
        return c.size() - 1;
    };
    function<int(int)> parse = [&](int lv) -> int {
        int u;
        if (lv > maxp)
        {
            if (pos < n && s[pos] == '(')
            {
                pos++;
                u = parse(1);
            }
        }
    };
}
```

```

        if (err != (pos >= n || s[pos++] != '(')) return 0;
        return u;
    }
    else if (pos < n && isdigit(s[pos])) return u = node(s[pos++]);
    else return err = 1, 0;
}
else
{
    u = parse(lv + 1);
    while (!err && pos < n)
    {
        char ch = s[pos];
        int i = idx[ch] - 1;
        if (i >= 0 && op[i].prec == lv)
        {
            ++pos;
            int v = node(ch), w = parse(lv + !op[i].right);
            c[v] = {u, w};
            u = v;
        }
        else break;
    }
    return u;
}
};
int root = parse(0);
for (auto [ch, _, __] : op) idx[ch] = 0;
if (err || pos != n) return {{ }, { }, 0};
return {c, ch, root};
}
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    string s;
    getline(cin, s);
    vector<Q> op = {
        {'|', 1, 0},
        {'&', 2, 0},
    };
    auto [c, ch, root] = parse_expr(s, op);
    assert(root);
    function<array<int, 3>(int)> dfs = [&](int u)->array<int, 3> {
        if (isdigit(ch[u])) return {ch[u] - '0', 0, 0};
        auto [l, r1, r2] = dfs(c[u][0]);
        if (ch[u] == '|')
        {
            if (l) return {1, r1, r2 + 1};
            auto [r, r3, r4] = dfs(c[u][1]);
            return {r, r1 + r3, r2 + r4};
        }
        else
        {
            if (!l) return {0, r1 + 1, r2};
            auto [r, r3, r4] = dfs(c[u][1]);
            return {r, r1 + r3, r2 + r4};
        }
    };
}

```

```
};  
auto [r0, r1, r2] = dfs(root);  
cout << r0 << endl << r1 << ' ' << r2 << endl;  
}
```

## 3 数学

### 3.1 单情况矩阵 (+)

没啥用。特殊的 ddp 有用。

```
template<class T,int n> struct matrix
{
    #define all(x) (x).begin(),(x).end()
    array<pair<int,T>,n> a;
    matrix(char c='E')
    {
        int i;
        if (c=='E') for (i=0;i<n;i++) a[i]={i,0};
        else assert(0);
    }
    matrix(char c,int x)
    {

    }
    matrix operator+(const matrix &o) const
    {
        matrix r;
        int i,j,k;
        for (i=0;i<n;i++)
        {
            auto [x,y]=a[i];
            r.a[i]={o.a[x].first,o.a[x].second+y};
        }
        return r;
    }
};
```

### 3.2 矩阵求逆（要求质数）

一种原地算法，总体效率更高。

$O(n^3)$ ,  $O(n)$ 。

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=402,p=1e9+7;
void inv(int &x)
{
    int y=p-2,r=1;
    while (y)
    {
        if (y&1) r=(ll)r*x%p;
        x=(ll)x*x%p;
        y>>=1;
    }
    x=r;
}
int a[N][N],ih[N],jh[N];
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
```

```

int i,j,k,n;
cin>>n;
for (i=0;i<n;i++) for (j=0;j<n;j++) cin>>a[i][j];
memset(ih,-1,sizeof ih);
memset(jh,-1,sizeof jh);
for (k=0;k<n;k++)
{
    //ih,jh要清空
    for (i=k;i<n;i++) if (ih[k]==-1) for (j=k;j<n;j++) if (a[i][j])
    {
        ih[k]=i;
        jh[k]=j;
        break;
    }
    if (ih[k]==-1) return cout<<"No Solution"<<endl,0;
    for (j=0;j<n;j++) swap(a[k][j],a[ih[k]][j]);
    for (i=0;i<n;i++) swap(a[i][k],a[i][jh[k]]);
    if (!a[k][k]) return cout<<"No Solution"<<endl,0;inv(a[k][k]);
    for (i=0;i<n;i++) if (i!=k) a[k][i]=(ll)a[k][i]*a[k][k]%p;
    for (i=0;i<n;i++) if (i!=k) for (j=0;j<n;j++) if (j!=k) a[i][j]=(a[i][j]+(ll)(p-a[i][k])*a[k][j])%p;
    for (i=0;i<n;i++) if (i!=k) a[i][k]=(ll)(p-a[i][k])*a[k][k]%p;
}
for (k=n-1;k>=0;k--)
{
    for (j=0;j<n;j++) swap(a[k][j],a[jh[k]][j]);
    for (i=0;i<n;i++) swap(a[i][k],a[i][ih[k]]);
}
}
/*
输入
3
1 2 8
2 5 6
5 1 2
输出
718750005 718750005 968750007
171875001 671875005 296875002
117187501 867187506 429687503
*/

```

### 3.3 任意模数矩阵求逆（未验证）

$O(n^3)$ ,  $O(n^2)$ 。

原理和任意模数行列式类似，辗转相除。注意仍然要求对角线元素是有逆的。

```

int ksm(int x,int y)
{
    int r=1;
    while (y)
    {
        if (y&1) r=(ll)r*x%p;
        y>>=1;x=(ll)x*x%p;
    }
    return r;
}
int phi(int n)

```

```

{
    int r=n;
    for (int i=2;i*i<=n;i++) if (n%i==0)
    {
        r=r/i*(i-1);n/=i;
        while (n%i==0) n/=i;
    }
    if (n>1) r=r/n*(n-1);
    return r;
}

void cal(int a[][N],int b[][N],int n)
{
    int i,j,k,r,ph=phi(p);
    for (i=1;i<=n;i++) memset(b+1,0,n<<2);
    for (i=1;i<=n;i++) b[i][i]=1;
    for (i=1;i<=n;i++)
    {
        k=i;
        for (j=i+1;j<=n;j++) if (a[j][i]&&a[j][i]<a[k][i]) k=j;
        if (!a[k][i]) {puts("No_Solution");exit(0);}
        swap(a[i],a[k]);swap(b[i],b[k]);
        for (j=i+1;j<=n;j++) if (a[j][i])
        {
            r=p-a[j][i]/a[i][i];
            for (k=i;k<=n;k++) a[j][k]=(a[j][k]+(ll)r*a[i][k])%p;
            for (k=1;k<=n;k++) b[j][k]=(b[j][k]+(ll)r*b[i][k])%p;
            while (a[j][i])
            {
                swap(a[i],a[j]);swap(b[i],b[j]);
                r=p-a[j][i]/a[i][i];
                for (k=i;k<=n;k++) a[j][k]=(a[j][k]+(ll)r*a[i][k])%p;
                for (k=1;k<=n;k++) b[j][k]=(b[j][k]+(ll)r*b[i][k])%p;
            }
        }
        if (__gcd(a[i][i],p)!=1) {puts("No_Solution");exit(0);}
        r=ksm(a[i][i],ph-1);
        for (j=i;j<=n;j++) a[i][j]=(ll)a[i][j]*r%p;
        for (j=1;j<=n;j++) b[i][j]=(ll)b[i][j]*r%p;
        assert(a[i][i]==1);
        for (j=1;j<i;j++)
        {
            r=p-a[j][i];
            for (k=i;k<=n;k++) a[j][k]=(a[j][k]+(ll)r*a[i][k])%p;
            for (k=1;k<=n;k++) b[j][k]=(b[j][k]+(ll)r*b[i][k])%p;
        }
    }
}
}

```

### 3.4 矩阵的特征多项式

$O(n^3)$ ,  $O(n^2)$ 。

封装版本见矩阵类。

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;

```



```

const int N=502,p=998244353;
int a[N][N],f[N];
int n,i,j,k,x,y,r;
void inc(int &x,const int y)
{
    if ((x+=y)>=p) x-=p;
}
void dec(int &x,const int y)
{
    if ((x-=y)<0) x+=p;
}
int ksm(int x,int y)
{
    int r=1;
    while (y)
    {
        if (y&1) r=(ll)r*x%p;
        x=(ll)x*x%p;y>>=1;
    }
    return r;
}
void calmatrix(int a[N][N],int n)
{
    int i,j,k,r;
    for (i=2;i<=n;i++)
    {
        for (j=i;j<=n;&&!a[j][i-1];j++);
        if (j>n) continue;
        if (j>i)
        {
            swap(a[i],a[j]);
            for (k=1;k<=n;k++) swap(a[k][j],a[k][i]);
        }
        r=a[i][i-1];
        for (j=1;j<=n;j++) a[j][i]=(ll)a[j][i]*r%p;
        r=ksm(r,p-2);
        for (j=i-1;j<=n;j++) a[i][j]=(ll)a[i][j]*r%p;
        for (j=i+1;j<=n;j++)
        {
            r=a[j][i-1];
            for (k=1;k<=n;k++) a[k][i]=(a[k][i]+(ll)a[k][j]*r)%p;
            r=p-r;
            for (k=i-1;k<=n;k++) a[j][k]=(a[j][k]+(ll)a[i][k]*r)%p;
        }
    }
}
void calpoly(int a[N][N],int n,int *f)
{
    static int g[N][N];
    memset(g,0,sizeof(g));
    g[0][0]=1;
    int i,j,k,r,rr;
    for (i=1;i<=n;i++)
    {
        r=p-1;
        for (j=i;j;j--)//第 j 行选第 n 列
        {

```

```

        rr=(ll)r*a[j][i]%p;
        for (k=0;k<j;k++) g[i][k]=(g[i][k]+(ll)rr*g[j-1][k])%p;
        r=(ll)r*a[j][j-1]%p;
    }
    for (k=1;k<=i;k++) inc(g[i][k],g[i-1][k-1]);
}
memcpy(f,g[n],n+1<<2);
//if (n&1) for (i=0;i<=n;i++) if (f[i]) f[i]=p-f[i]; //若注释掉则为 |kE-A|
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cin>>n;
    for (i=1;i<=n;i++) for (j=1;j<=n;j++) cin>>a[i][j];
    calmatrix(a,n);calpoly(a,n,f);
    for (i=0;i<=n;i++) cout<<f[i]<<"x^"<<i<<"+"\n"[i==n];
}
/*
3
1 2 3
4 5 6
7 8 9
输出: 0x^0+998244335x^1+998244338x^2+1x^3
*/

```

### 3.5 矩阵类（较新）

```

typedef unsigned long long ll;
const ll p=998244353;
ll ksm(ll x, ll y)
{
    ll r=1;
    while (y)
    {
        if (y&1) r=r*x%p;
        x=x*x%p; y>>=1;
    }
    return r;
}
struct matrix:vector<vector<ll>>
{
    explicit matrix(int n=0, int m=0):vector(n, vector<ll>(m)) { }
    pair<int, int> sz() const { if (size()) return {size(), back().size()}; return {0, 0}; }
    int rank() const//秩
    {
        vector<vector<ll>> a=*this;
        auto [n, m]=sz();
        int i, j, k, l, r=0;
        for (i=0, j=0; i<n&&j<m; j++)
        {
            for (k=i; k<n; k++) if (a[k][j]) break;
            if (k==n) continue;
            ::swap(a[i], a[k]);
            ll iv=ksm(a[i][j], p-2);
            for (k=j; k<m; k++) a[i][k]=a[i][k]*iv%p;
            for (k=i+1; k<n; k++) for (l=j+1; l<m; l++) a[k][l]=(a[k][l]+(p-a[k][j])*a[i][l])%p;
        }
    }
}

```

```

        ++i; ++r;
    }
    return r;
}
vector<ll> poly()//特征多项式
{
    auto [n, m]=sz();
    vector<vector<ll>> a=*this;
    assert(n==m);
    int i, j, k;
    for (i=1; i<n; i++)
    {
        for (j=i; j<n&&!a[j][i-1]; j++);
        if (j==n) continue;
        if (j>i)
        {
            ::swap(a[i], a[j]);
            for (k=0; k<n; k++) ::swap(a[k][j], a[k][i]);
        }
        ll r=a[i][i-1];
        for (j=0; j<n; j++) a[j][i]=a[j][i]*r%p;
        r=ksm(r, p-2);
        for (j=i-1; j<n; j++) a[i][j]=a[i][j]*r%p;
        for (j=i+1; j<n; j++)
        {
            r=a[j][i-1];
            for (k=0; k<n; k++) a[k][i]=(a[k][i]+a[k][j]*r)%p;
            r=p-r;
            for (k=i-1; k<n; k++) a[j][k]=(a[j][k]+a[i][k]*r)%p;
        }
    }
    vector g(n+1, vector<ll>(n+1));
    g[0][0]=1;
    for (i=0; i<n; i++)
    {
        ll r=p-1, rr;
        for (j=i; j>=0; j--)//第 j 行选第 n 列
        {
            rr=r*a[j][i]%p;
            for (k=0; k<=j; k++) g[i+1][k]=(g[i+1][k]+rr*g[j][k])%p;
            if (j) r=r*a[j][j-1]%p;
        }
        for (k=1; k<=i+1; k++) (g[i+1][k]+=g[i][k-1])%=p;
    }
    auto f=g[n];
    //if (n&1) for (i=0;i<=n;i++) if (f[i]) f[i]=p-f[i];//若注释掉则为 |kE-A|
    return f;
}
};
istream &operator>>(istream &cin, matrix &r) { for (auto &v:r) for (ll &x:v) cin>>x; return cin;
}
ostream &operator<<(ostream &cout, const matrix &r) { auto [n, m]=r.sz(); for (int i=0; i<n; i++)
    for (int j=0; j<m; j++) cout<<r[i][j]<<"\n"[j+1==m]; return cout; }
matrix &operator+=(matrix &a, const matrix &b)
{
    assert(a.size()==b.size());
    auto [n, m]=a.sz();

```

```

    for (int i=0; i<n; i++) for (int j=0; j<m; j++) (a[i][j]+=b[i][j])%=p;
    return a;
}
matrix &operator--(matrix &a, const matrix &b)
{
    assert(a.size()==b.size());
    auto [n, m]=a.sz();
    for (int i=0; i<n; i++) for (int j=0; j<m; j++) (a[i][j]+=p-b[i][j])%=p;
    return a;
}
matrix operator*(const matrix &a, const matrix &b)
{
    auto [n, m]=a.sz();
    auto [_, q]=b.sz();
    assert(m==_);
    int i, j, k;
    matrix c(n, q);
    for (k=0; k<m; k++)
    {
        for (i=0; i<n; i++) for (j=0; j<q; j++) c[i][j]+=a[i][k]*b[k][j];
        if (!(k^q-1)&15)) for (auto &v:c) for (ll &x:v) x%=p;
    }
    return c;
}
matrix operator+(matrix a, const matrix &b) { return a+=b; }
matrix operator-(matrix a, const matrix &b) { return a-=b; }
matrix &operator*=(matrix &a, const matrix &b) { return a=a*b; }
matrix &operator*=(matrix &a, ll k) { for (auto &v:a) for (ll &x:v) x=x*k%p; return a; }
matrix operator*(matrix a, ll k) { return a*=k; }
matrix E(int n) { matrix r(n, n); for (int i=0; i<n; i++) r[i][i]=1; return r; }
matrix pow(matrix a, long long k)//普通的快速幂
{
    assert(k>=0);
    auto [n, m]=a.sz();
    assert(n==m);
    matrix r=k&1?a:E(n);
    k>>=1;
    while (k)
    {
        a*=a;
        if (k&1) r*=a;
        k>>=1;
    }
    return r;
}
matrix pow2(matrix a, long long k)//较快的快速幂。运用了一些技巧。
{
    vector<ll> f=a.poly();
    int n=f.size()-1, i, j;
    if (!n) return matrix();
    if (n==1) return E(1)*ksm(a[0][0], k);
    assert(f[n]==1);
    vector<ll> r(n), x(n), t(n*2);
    r[0]=x[1]=1;
    for (ll &x:f) x=(p-x)%p;
    reverse(all(f));
    fill(all(t), 0);

```

```

if (k&1)
{
    for (i=0; i<n; i++) for (j=0; j<n; j++) t[i+j]=(t[i+j]+r[i]*x[j])%p;
    for (i=n*2-2; i>=n; i--) for (j=1; j<=n; j++) t[i-j]=(t[i-j]+f[j]*t[i])%p;
    for (i=0; i<n; i++) r[i]=t[i];
}
k>>=1;
while (k)
{
    fill(all(t), 0);
    for (i=0; i<n; i++) for (j=0; j<n; j++) t[i+j]=(t[i+j]+x[i]*x[j])%p;
    for (i=n*2-2; i>=n; i--) for (j=1; j<=n; j++) t[i-j]=(t[i-j]+f[j]*t[i])%p;
    for (i=0; i<n; i++) x[i]=t[i];
    if (k&1)
    {
        fill(all(t), 0);
        for (i=0; i<n; i++) for (j=0; j<n; j++) t[i+j]=(t[i+j]+r[i]*x[j])%p;
        for (i=n*2-2; i>=n; i--) for (j=1; j<=n; j++) t[i-j]=(t[i-j]+f[j]*t[i])%p;
        for (i=0; i<n; i++) r[i]=t[i];
    }
    k>>=1;
}
matrix res(n, n);
int b=ceil(sqrt(n));
vector<matrix> s(b+1);
s[0]=E(n); s[1]=a;
for (i=2; i<=b; i++) s[i]=s[i-1]*a;
for (i=b-1; i>=0; i--)
{
    res*=s[b];
    for (j=min(n, (i+1)*b)-1; j>=i*b; j--) res+=s[j-i*b]*r[j];
}
return res;
}

```

### 3.6 最短递推式 (BM 算法)

给定  $\{a\}$ , 求最短的  $\{r\}$  满足  $\sum_{j=0}^{m-1} a_{i-j-1}r_j = a_i$ 。

```

vector<ui> bm(const vector<ui> &a)
{
    vector<ui> r,lst;
    int n=a.size(),m=0,q=0,i,j,k=-1;
    ui D=0;
    for (i=0;i<n;i++)
    {
        ui cur=0;
        for (j=0;j<m;j++) cur=(cur+(ll)a[i-j-1]*r[j])%p;
        cur=(a[i]+p-cur)%p;
        if (!cur) continue;
        if (k===-1)
        {
            k=i;
            D=cur;
            r.resize(m=i+1);
            continue;
        }
    }
}

```

```

    }
    auto v=r;
    ui x=(ll)cur*ksm(D,p-2)%p;
    if (m<q+i-k) r.resize(m=q+i-k);
    (r[i-k-1]+=x)%=p;
    ui *b=r.data()+i-k;
    x=(p-x)%p;
    for (j=0;j<q;j++) b[j]=(b[j]+(ll)x*lst[j])%p;
    if (v.size()+k<lst.size()+i)
    {
        lst=v;
        q=v.size();
        k=i;
        D=cur;
    }
}
return r;
}

```

### 3.7 在线 $O(1)$ 逆元

预处理复杂度为  $O(p^{\frac{2}{3}})$ 。

```

namespace online_inv
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    const ll p=1e9+7,n=1010,m=n*n,N=m+2;
    int l[N],r[N];
    ll y[N];
    bool s[N];
    ll _inv[N*2],i,j,k;
    void init_inv()
    {
        assert(n*n*n>p);
        _inv[1]=1;
        for (i=2;i<m*2;i++)
        {
            j=p/i;
            _inv[i]=(p-j)*_inv[p-i*j]%p;
        }
        s[0]=y[0]=1;
        for (i=1;i<n;i++) for (j=i;j<n;j++) if (!s[k=i*m/j])
        {
            y[k]=j;
            s[k]=1;
        }
        l[0]=1;
        for (i=1;i<=m;i++) l[i]=s[i]?y[i]:l[i-1];
        r[m]=1;
        for (i=m-1;~i;i--) r[i]=s[i]?y[i]:r[i+1];
        for (i=0;i<=m;i++) y[i]=min(l[i],r[i]);
    }
    inline ll inv(const ll &x)
    {
        assert(x&& x<p);
        if (x<m*2) return _inv[x];
    }
}

```

```

        k=x*m/p;
        j=y[k]*x%p;
        return (j<m*2?_inv[j]:p-_inv[p-j])*y[k]%p;
    }
}
using online_inv::init_inv,online_inv::inv,online_inv::p;

```

### 3.8 Strassen 矩阵乘法

没用，不如卡常。 $O(n^{\log_2 7})$ 。

```

#include <bits/stdc++.h>
using namespace std;
typedef unsigned int ui;
typedef unsigned long long ull;
const ui p=998244353;
const ull fh=1ull<<31;
struct Q
{
    ui **a;
    int n;
    Q(){n=0;}
    void clear()
    {
        for (int i=0;i<n;i++) delete a[i];
        if (n) delete a;n=0;
    }
    Q(int nn)//不能传入不是 2 的幂的数!
    {
        n=nn;
        assert(n==(n&-n));
        a=new ui*[n];
        for (int i=0;i<n;i++) a[i]=new ui[n],memset(a[i],0,n*sizeof a[0][0]);
    }
    const Q & operator=(const Q& b)
    {
        clear();n=b.n;
        a=new ui*[n];
        for (int i=0;i<n;i++) a[i]=new ui[n],memcpy(a[i],b.a[i],n*sizeof a[0][0]);
        return *this;
    }
    ~Q(){clear();}
    Q operator+(const Q &b)
    {
        Q c(n);
        for (int i=0;i<n;i++) for (int j=0;j<n;j++) if ((c.a[i][j]=a[i][j]+b.a[i][j])>=p) c.a[i][j]
            -=p;
        return c;
    }
    Q operator-(const Q &b)
    {
        Q c(n);
        for (int i=0;i<n;i++) for (int j=0;j<n;j++) if ((c.a[i][j]=a[i][j]-b.a[i][j])&fh) c.a[i][j]
            +=p;
        return c;
    }
    Q operator*(Q &b)

```

```

{
    Q c(n);
    if (n<=128)
    {
        for (int i=0;i<n;i++) for (int k=0;k<n;k++) for (int j=0;j<n;j++) c.a[i][j]=(c.a[i][j]
            +(ull)a[i][k]*b.a[k][j])%p;
        return c;
    }
    Q A[2][2],B[2][2],s[10],p[5];
    n>>=1;
    int i,j,k,l;
    for (i=0;i<2;i++) for (j=0;j<2;j++)
    {
        A[i][j]=Q(n);
        for (k=0;k<n;k++) memcpy(A[i][j].a[k],a[k+i*n]+j*n,n*sizeof a[0][0]);
        B[i][j]=Q(n);
        for (k=0;k<n;k++) memcpy(B[i][j].a[k],b.a[k+i*n]+j*n,n*sizeof a[0][0]);
    }
    s[0]=B[0][1]-B[1][1];
    s[1]=A[0][0]+A[0][1];
    s[2]=A[1][0]+A[1][1];
    s[3]=B[1][0]-B[0][0];
    s[4]=A[0][0]+A[1][1];
    s[5]=B[0][0]+B[1][1];
    s[6]=A[0][1]-A[1][1];
    s[7]=B[1][0]+B[1][1];
    s[8]=A[0][0]-A[1][0];
    s[9]=B[0][0]+B[0][1];
    p[0]=A[0][0]*s[0];
    p[1]=s[1]*B[1][1];
    p[2]=s[2]*B[0][0];
    p[3]=A[1][1]*s[3];
    p[4]=s[4]*s[5];
    A[0][0]=p[4]+p[3]-p[1]+s[6]*s[7];
    A[0][1]=p[0]+p[1];
    A[1][0]=p[2]+p[3];
    A[1][1]=p[4]+p[0]-p[2]-s[8]*s[9];
    for (i=0;i<2;i++) for (j=0;j<2;j++) for (k=0;k<n;k++) memcpy(c.a[k+i*n]+j*n,A[i][j].a[k],n
        *sizeof a[0][0]);
    n<<=1;
    return c;
}
};
int main()
{
    int i,j,n,m,k;
    ios::sync_with_stdio(0);cin.tie(0);
    cin>>n>>m>>k;
    int N=1<<32-min({__builtin_clz(n-1),__builtin_clz(m-1),__builtin_clz(k-1)});
    Q a(N),b(N);
    for (i=0;i<n;i++) for (j=0;j<m;j++) cin>>a.a[i][j];
    for (i=0;i<m;i++) for (j=0;j<k;j++) cin>>b.a[i][j];
    a=a*b;
    for (i=0;i<n;i++) for (j=0;j<k;j++) cout<<a.a[i][j]<<"\n"[j+1==k];
}

```



### 3.9 扩展欧拉定理

求  $a \uparrow\uparrow b \bmod c$ 。前面的 Prime 命名空间只是求  $\varphi$  用的。

```
namespace Prime
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    const int N=1e6+2;
    const ll M=(ll)(N-1)*(N-1);
    ui pr[N],mn[N],phi[N],cnt;
    int mu[N];
    void init_prime()
    {
        ui i,j,k;
        phi[1]=mu[1]=1;
        for (i=2;i<N;i++)
        {
            if (!mn[i])
            {
                pr[cnt++]=i;
                phi[i]=i-1;mu[i]=-1;
                mn[i]=i;
            }
            for (j=0;(k=i*pr[j])<N;j++)
            {
                mn[k]=pr[j];
                if (i%pr[j]==0)
                {
                    phi[k]=phi[i]*pr[j];
                    break;
                }
                phi[k]=phi[i]*(pr[j]-1);
                mu[k]=-mu[i];
            }
        }
        //for (i=2;i<N;i++) if (mu[i]<0) mu[i]+=p;
    }
    template<class T> T getphi(T x)
    {
        assert(M>=x);
        T r=x;
        for (ui i=0;i<cnt&&(T)pr[i]*pr[i]<=x&&x>=N;i++) if (x%pr[i]==0)
        {
            ui y=pr[i],tmp;
            x/=y;
            while (x==(tmp=x/y)*y) x=tmp;
            r=r/y*(y-1);
        }
        if (x>=N) return r/x*(x-1);
        while (x>1)
        {
            ui y=mn[x],tmp;
            x/=y;
            while (x==(tmp=x/y)*y) x=tmp;
            r=r/y*(y-1);
        }
        return r;
    }
}
```

```

}
template<class T> vector<pair<T,ui>> getw(T x)
{
    assert(M>=x);
    vector<pair<T,ui>> r;
    for (ui i=0;i<cnt&&(T)pr[i]*pr[i]<=x&&x>=N;i++) if (x%pr[i]==0)
    {
        ui y=pr[i],z=1,tmp;
        x/=y;
        while (x==(tmp=x/y)*y) x=tmp,++z;
        r.push_back({y,z});
    }
    if (x>=N)
    {
        r.push_back({x,1});
        return r;
    }
    while (x>1)
    {
        ui y=mn[x],z=1,tmp;
        x/=y;
        while (x==(tmp=x/y)*y) x=tmp,++z;
        r.push_back({y,z});
    }
    return r;
}
}
using Prime::pr,Prime::phi,Prime::getw,Prime::getphi;
using Prime::mu,Prime::init_prime;
ui ksm(ll x,ui y,ui p)
{
    x=x%p+(x>=p)*p;
    ll r=1;
    while (y)
    {
        if (y&1)
        {
            if ((r*=x)>=p) r=r%p+p; else r%=p;
        }
        if ((x*=x)>=p) x=x%p+p; else x%=p;
        y>>=1;
    }
    return r;
}
}
struct Q
{
    vector<ui> p;
    Q(const ui &P)
    {
        p.push_back(P);
        while (p.back()>1) p.push_back(getphi(p.back()));
    }
    ui operator()(ll a,ll b)
    {
        if (!a) return (1~b&1)%p[0];
        ui r=1;
        int i=min(b,(ll)p.size());

```

```

        while ((--i)>=0) r=ksm(a,r,p[i]);
        return r%p[0];
    }
};
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(15);
    int n,i;
    init_prime();
    int T;
    cin>>T;
    while (T--)
    {
        ui a,b,c;
        cin>>a>>b>>c;
        cout<<Q(c)(a,b)<<'\n';
    }
}

```

### 3.10 exgcd

$O(\log p)$ ,  $O(\log p)$ 。

递归版：

```

int exgcd(int a,int b,int c)//ax+by=c,return x
{
    if (a==0) return c/b;
    return (c-(ll)b*exgcd(b%a,a,c))/a%b;
}

```

递推重构版：

```

pair<ll,ll> exgcd(ll a,ll b,ll c)//ax+by=c, {-1,-1} 无解, b=0 返回 {c/a,0}, 否则返回最小非负 x
{
    assert(a||b);
    if (!b) return {c/a,0};
    if (a<0) a=-a,b=-b,c=-c;
    ll d=gcd(a,b);
    if (c%d) return {-1,-1};
    ll x=1,x1=0,p=a,q=b,k;
    b=abs(b);
    while (b)
    {
        k=a/b;
        x-=k*x1;a-=k*b;
        swap(x,x1);
        swap(a,b);
    }
    b=abs(q/d);
    x=x*(c/d)%b;
    if (x<0) x+=b;
    return {x, (ll)((c-(ll)p*x)/q)};
}
ll fun(ll a, ll b, ll p)//ax=b(mod p)
{
    return exgcd(a, -p, b).first%p;
}

```

}

### 3.11 exCRT

实现了一个类 Q，表示一条方程，支持合并。

```
namespace CRT
{
    typedef long long ll;
    pair<ll,ll> exgcd(ll a,ll b,ll c)
    {
        assert(a||b);
        if (!b) return {c/a,0};
        ll d=gcd(a,b);
        if (c%d) return {-1,-1};
        ll x=1,x1=0,p=a,q=b,k;
        b=abs(b);
        while (b)
        {
            k=a/b;
            x-=k*x1;a-=k*b;
            swap(x,x1);
            swap(a,b);
        }
        b=abs(q/d);
        x=x*(c/d)%b;
        if (x<0) x+=b;
        return {x,(c-p*x)/q};
    }
    struct Q
    {
        ll p,r;//0<=r<p
        Q operator+(const Q &o) const
        {
            if (p==0||o.p==0) return {0,0};
            auto [x,y]=exgcd(p,-o.p,r-o.r);
            if (x==-1&&y==-1) return {0,0};
            ll q=lcm(p,o.p);
            return {q,((r-x*p)%q+q)%q};
        }
    };
};
using CRT::Q;
```

### 3.12 exBSGS

$O(\sqrt{n})$ 。哈希表 ht 可以用 map 代替。

```
namespace BSGS
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    template<int N,class T,class TT> struct ht//个数，定义域，值域
    {
        const static int p=1e6+7,M=p+2;
        TT a[N];
    };
};
```

```

T v[N];
int fir[p+2],nxt[N],st[p+2];//和模数相适应
int tp,ds;//自定义模数
ht(){memset(fir,0,sizeof fir);tp=ds=0;}
void mdf(T x,TT z)//位置, 值
{
    ui y=x%p;
    for (int i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i]=z,void();//若不可能重复不需要 for
    v[++ds]=x;a[ds]=z;
    if (!fir[y]) st[++tp]=y;
    nxt[ds]=fir[y];fir[y]=ds;
}
TT find(T x)
{
    ui y=x%p;
    int i;
    for (i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i];
    return 0;//返回值和是否判断依据要求决定
}
void clear()
{
    ++tp;
    while (--tp) fir[st[tp]]=0;
    ds=0;
}
};
const int N=5e4;
ht<N,ui,ui> s;
int exgcd(int a,int b)
{
    if (a==1) return 1;
    return (1-(long long)b*exgcd(b%a,a))/a;//not ll
}
int bsgs(ui a,ui b,ui p)
{
    s.clear();
    a%=p;b%=p;
    if (!a) return 1-min((int)b,2);//舍 -1
    ui i,j,k,x,y;
    x=sqrt(p)+2;
    for (i=0,j=1;i<x;i++,j=(ll)j*a%p)
    {
        if (j==b) return i;
        s.mdf((ll)j*b%p,i+1);
    }
    k=j;
    for (i=1;i<=x;i++,j=(ll)j*k%p) if (y=s.find(j)) return (ll)i*x-y+1;
    return -1;
}
bool isprime(ui p)
{
    if (p<=1) return 0;
    for (ui i=2;i*i<=p;i++) if (p%i==0) return 0;
    return 1;
}
int exbsgs(ui a,ui b,ui p)//a^x=b(mod p)
{

```

```

//if (isprime(p)) return bsgs(a,b,p);
a%=p;b%=p;
ui i,j,k,x,y=__lg(p),cnt=0;
for (i=0,j=1%p;i<=y;i++,j=(ll)j*a%p) if (j==b) return i;
y=1;
while (1)
{
    if ((x=gcd(a,p))==1) break;
    if (b%x) return -1;//no sol
    ++cnt;
    p/=x;b/=x;
    y=(ll)y*(a/x)%p;
}
a%=p;
b=(ll)b*(p+exgcd(y,p))%p;
int r=bsgs(a,b,p);
return r==-1?-1:r+cnt;
}
}
using BSGS::bsgs,BSGS::exbsgs;

```

### 3.13 exLucas

求组合数。包含多个不同的版本，按需使用。

```

namespace exlucas
{
    typedef long long ll;
    typedef pair<int,int> pa;
    int P,p,q,i;
    vector<pa> a;
    vector<vector<int>> > b;
    vector<int> ph;
    vector<int> xs;
    int ksm(unsigned int x,ll y,const unsigned int p)
    {
        unsigned int r=1;
        while (y)
        {
            if (y&1) r=(unsigned long long)r*x%p;
            x=(unsigned long long)x*x%p;
            y>>=1;
        }
        return r;
    }
    void init(int x)//分解质因数，如有必要可以使用更快的方法
    {
        a.clear();b.clear();
        int i,y,z;
        vector<int> v;
        for (i=2;i*i<=x;i++) if (x%i==0)
        {
            z=i;x/=i;
            while (1)
            {
                y=x/i;
                if (i*y==x) x=y; else break;
            }
        }
    }
}

```

```

        z*=i;
    }
    a.push_back(pa(i,z));
    b.push_back(v);
}
if (x>1) a.push_back(pa(x,x)),b.push_back(v);
ph.resize(a.size());
xs.resize(a.size());
for (int k=0;k<a.size();k++)
{
    tie(q,p)=a[k];
    ph[k]=p/q*(q-1);
    xs[k]=(ll)ksm(P/p,ph[k]-1,p)*(P/p)%P;
}
}
void spinit(int x)//O(p) space
{
    for (int k=0;k<a.size();k++)
    {
        int q,p;
        tie(q,p)=a[k];
        b[k].resize(p);
        b[k][0]=1;
        for (int i=1,j=q;i<p;i++) if (i==j) j+=q,b[k][i]=b[k][i-1]; else b[k][i]=(ll)b[k][i-1]*
            i%p;
    }
}
ll g(ll n)
{
    ll r=0,s;
    while (n>=q)
    {
        n/=q;
        r+=n;
    }
    return r;
}
// int f(ll n)
// {
//     if (n==0) return 1;
//     int r=1;//若 p>1e9 j 要 unsigned
//     for (int i=1,j=q;i<p;i++) if (i==j) j+=q; else r=(ll)r*i%p;
//     r=(ll)ksm(r,n/p,p)*f(n/q)%p;
//     n%=p;
//     for (int i=1,j=q;i<=n;i++) if (i==j) j+=q; else r=(ll)r*i%p;
//     return r;
// }//O(T\sum p) time,O(1) space ver.
int f(ll n)
{
    int r=1;
    ll cs=0;
    while (n)
    {
        r=(ll)r*b[i][n%p]%p;
        cs+=n/p;
        n/=q;
    }
}

```

```

    return (ll)ksm(b[i][p-1],cs%ph[i],p)*r%p;
} // O(\sum p) time, O(p) space ver.
int C(ll n, ll m, int M)
{
    if (n < m) return 0;
    int r=0, w;
    if (P!=M) init(P=M), spinit(P); // sp for O(p) space
    for (i=0; i<a.size(); i++)
    {
        tie(q,p)=a[i];
        w=(ll)ksm(q, g(n)-g(m)-g(n-m), p)*f(n)%p*ksm((ll)f(m)*f(n-m)%p, ph[i]-1, p)%p;
        r=(r+(ll)xs[i]*w)%M;
    }
    return r;
}
#define C(x,y,z) exlucas::C(x,y,z)

```

### 3.14 杜教筛

求  $\varphi(n)$  的前缀和。

核心：构造  $g$  满足  $h(n) = \sum_{d|n} f(d)g(\frac{n}{d})$  容易计算，

则有  $\sum_{i=1}^n h(i) = \sum_{i=1}^n g(i) \sum_{j=1}^{\lfloor n/i \rfloor} f(j)$ ,

故  $g(1) \sum_{j=1}^n f(j) = \sum_{i=1}^n h(i) - \sum_{i=2}^n g(i) \sum_{j=1}^{\lfloor n/i \rfloor} f(j)$ ,

则  $f$  前缀和可以递归求解。

```

namespace du_seive
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    unordered_map<ll, ui> mp;
    const int N=1e7+2;
    const ui p=998244353;
    ui pr[N], phi[N];
    ui cnt;
    void init()
    {
        cnt=0; phi[1]=1;
        int i, j;
        for (i=2; i<N; i++)
        {
            if (!phi[i])
            {
                pr[cnt++]=i;
                phi[i]=i-1;
            }
            for (j=0; i*pr[j]<N; j++)
            {
                if (i%pr[j]==0)
                {
                    phi[i*pr[j]]=phi[i]*pr[j];
                    break;
                }
            }
        }
    }
}

```



```

        }
        phi[i*pr[j]]=phi[i]*(pr[j]-1);
    }
    if ((phi[i]+=phi[i-1])>=p) phi[i]-=p;
}
}
ui get_phi_sum(ll n)
{
    if (n<N) return phi[n];
    if (mp.count(n)) return mp[n];
    ui sum=0;
    for (ll i=2,j,k;i<=n;i=j+1)
    {
        j=n/(k=n/i);
        sum=(sum+(ll)get_phi_sum(k)*(j-i+1))%p;
    }
    ui nn=n%p;
    sum=(nn*(nn+1ll)/2+p-sum)%p;
    mp[n]=sum;
    return sum;
}
}
using du_seive::init,du_seive::get_phi_sum;

```

### 3.15 线性规划

用法：构造函数指明目标函数系数，add 函数增加限制。额外的限制是  $x_i \geq 0$ 。

```

typedef long double db;//__float128
struct linear
{
    static const int N=45;//n+m
    db r[N][N];
    int col[N],row[N];
    const db eps=1e-10,inf=1e9;//1e-17
    int n,m;
    template<class T> linear(const vector<T> &a)//target: maximize \sum a(i-1)xi
    {
        memset(r,0,sizeof r);
        memset(col,0,sizeof col);
        memset(row,0,sizeof row);
        n=a.size();m=0;
        for (int i=1;i<=n;i++) r[0][i]=-a[i-1];
    }
    template<class T> void add(const vector<T> &a,db b)//limit: \sum a(i-1)xi<=b
    {
        assert(a.size()==n);
        ++m;
        for (int i=1;i<=n;i++) r[m][i]=-a[i-1];
        r[m][0]=b;
    }
    void pivot(int k, int t)
    {
        swap(row[k+n],row[t]);
        db rkt=-r[k][t];
        int i,j;
        for (i=0;i<=n;i++) r[k][i]/=rkt;
    }
}

```

```

    r[k][t]=-1/rkt;
    for (i=0;i<=m;i++) if (i!=k)
    {
        db rit=r[i][t];
        if (rit>=-eps&&rit<=eps) continue;
        for (j=0;j<=n;j++) if (j!=t) r[i][j]+=rit*r[k][j];
        r[i][t]=r[k][t]*rit;
    }
}
bool init()
{
    int i;
    for (i=1;i<=n+m;i++) row[i]=i;
    while(1)
    {
        int q=1;
        auto b_min=r[1][0];
        for (i=2;i<=m;i++) if (r[i][0]<b_min) b_min=r[i][0],q=i;
        if (b_min+eps>=0) return 1;
        int p=0;
        for (i=1;i<=n;i++) if (r[q][i]>eps&&(!p||row[i]>row[p])) p=i;
        if (!p) break;
        pivot(q,p);
    }
    return 0;
}
bool simplex()
{
    while (1)
    {
        int t=1,k=0,i;
        for (i=2;i<=n;i++) if (r[0][i]<r[0][t]) t=i;
        if (r[0][t]>=-eps) return 1;
        db ratio_min=inf;
        for (i=1;i<=m;i++) if (r[i][t]<-eps)
        {
            db ratio=-r[i][0]/r[i][t];
            if (!k||ratio<ratio_min||ratio<=ratio_min+eps&&row[i]>row[k])
            {
                ratio_min=ratio;
                k=i;
            }
        }
        if (!k) break;
        pivot(k,t);
    }
    return 0;
}
void solve(int type)
{
    if (!init())
    {
        cout<<"Infeasible\n";
        return;
    }
    if (!simplex())
    {

```

```

        cout<<"Unbounded\n";
        return;
    }
    cout<<(long double)(-r[0][0])<<"\n";
    if (type)
    {
        int i;
        memset(col+1,0,n*sizeof col[0]);
        for (i=n+1;i<=n+m;i++) col[row[i]]=i;
        for (i=1;i<=n;i++) cout<<(long double)(col[i]?r[col[i]-n][0]:0)<<"\n"[i==n];
    }
}
};

```

### 3.16 斐波那契数列

使用生日攻击的方法寻找循环节，一种更通用的方法是 bsgs。

```

const int NN=3e7+2,M=4e5,N=1e6+10;
char c[NN];
ll n;
ll y,mo,x,z;
int p,i,j,k;
struct Q
{
    int a[2][2];
    Q(int b=0,int c=0,int d=0,int e=0){a[0][0]=b,a[0][1]=c,a[1][0]=d,a[1][1]=e;}
    Q operator*(const Q &o)
    {
        return Q(((ll)a[0][0]*o.a[0][0]+(ll)a[0][1]*o.a[1][0])%p,
            ((ll)a[0][0]*o.a[0][1]+(ll)a[0][1]*o.a[1][1])%p,
            ((ll)a[1][0]*o.a[0][0]+(ll)a[1][1]*o.a[1][0])%p,
            ((ll)a[1][0]*o.a[0][1]+(ll)a[1][1]*o.a[1][1])%p);
    }
};
struct ht
{
    ll v[N],a[N];
    int fir[N],nxt[N],st[N]; //和模数相适应
    int tp,p,ds; //自定义模数
    ht(){tp=0,p=1e6+7,ds=0;}
    void mdf(const ll x,const ll z) //位置, 值
    {
        const int y=x%p;
        for (int i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i]=z,void(); //若不可能重复不需要这一步
        if, 但需要for?
        v[++ds]=x;a[ds]=z;if (!fir[y]) st[++tp]=y;
        nxt[ds]=fir[y];fir[y]=ds;
    }
    ll find(const ll x)
    {
        const int y=x%p;int i;
        for (i=fir[y];i;i=nxt[i]) if (v[i]==x) break;
        if (!i) return 0; //返回值和是否判断依据要求决定
        return a[i];
    }
    void clear()

```

```

    {
        ++tp;
        while (--tp) fir[st[tp]]=0;ds=0;
    }
};
ht mp;
Q f[M],g[M],ji;
int fib(ll n)
{
    Q x=f[n/k]*g[n/k];
    return x.a[0][1];
}
ll spefib(ll n)
{
    Q x=f[n/k]*g[n/k];
    return (ll)x.a[0][1]*p+x.a[1][1];
}
ll sj()
{
    ll x=rand();
    x=x<<15^rand();
    x=x<<15^rand();
    x=x<<15^rand();
    return x>0?x:-x;
}
ll ab(ll x)
{
    return x>0?x:-x;
}
int main()
{
    srand(383778817);
    scanf("%s\n%d",c+1,&p);
    k=sqrt((ll)20*p)+1;ji=Q(0,1,1,1);
    f[0]=Q(1,0,0,1);for (i=1;i<=k;i++) f[i]=f[i-1]*ji;
    g[0]=Q(1,0,0,1);for (i=1;i<=k;i++) g[i]=g[i-1]*f[k];
    while (1)
    {
        x=sj()%(20ll*p)+1;y=spefib(x);
        if (z=mp.find(y))
        {
            if (z!=x)
            {
                mo=ab(x-z);
                break;
            }
        } else mp.mdf(y,x);
    }
    n=0;
    for (i=1;c[i]>=48&& c[i]<=57;i++) n=(n*10+(c[i]^48))%mo;
    printf("%d",fib(n));
}

```

### 3.17 线性插值 ( $k$ 次幂和)

$O(m)$ ,  $O(m)$ 。

```

ll interpolation(vector<ll> a, ll n)
{
    int m = a.size(), i;
    vector<ll> ans(2);
    n %= p;
    if (n < m) return a[n];
    ll k = ifac[m - 1];
    for (i = m - 1; i >= 0; i--)
    {
        (a[i] *= k) %= p;
        (k *= n - i) %= p;
    }
    k = 1;
    for (i = 0; i < m; i++)
    {
        (ans[(m ^ i) & 1] += a[i] * k) %= p;
        k = k * inv[i + 1] % p * (n - i) % p * (m - i - 1) % p;
    }
    return (ans[1] + p - ans[0]) % p;
}

ll sum_of_kth_power(ll n, ll k)
{
    if (n == 0) return 0;
    ll m = min(n + 1, k + 2);
    int i;
    vector<ll> s(m);
    vector<int> pr, ed(m); pr.reserve(m / 4);
    s[1] = 1;
    for (i = 2; i < m; i++)
    {
        if (!ed[i]) s[i] = ksm(i, k);
        for (int j : pr) if (i * j < m)
        {
            s[i * j] = s[i] * s[j] % p;
            if (i % j == 0) break;
        }
        else break;
    }
    for (i = 1; i < m; i++) (s[i] += s[i - 1]) %= p;
    return interpolation(s, n);
}

```

### 3.18 单原根（仅手动验证质数）

```

namespace get_root
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    ui ksm(ui x, ui y, ui p)
    {
        ui r = 1;
        while (y)
        {
            if (y & 1) r = (ll)r * x % p;
            x = (ll)x * x % p; y >>= 1;
        }
    }
}

```

```

    }
    return r;
}
vector<ui> getw(ui n)
{
    vector<ui> w;
    for (ui i=2;i*i<=n;i++) if (n%i==0)
    {
        w.push_back(i);
        n/=i;
        for (ui j=n/i;n==i*j;j=n/i) n/=i;
    }
    if (n>1) w.push_back(n);
    return w;
}
int getrt(ui n)
{
    if (n<=2) return n-1;
    auto w=getw(n);
    ui ph=n;
    for (ui x:w) ph=ph/x*(x-1);
    w=getw(ph);
    for (ui &x:w) x=ph/x;
    for (ui i=2;i<n;i++) if (gcd(i,n)==1)
    {
        for (ui x:w) if (ksm(i,x,n)==1) goto no;
        return i;
        no:;
    }
    return -1;
}
}
using get_root::getrt;

```

### 3.19 稍快单原根（仅验证质数）

```

namespace get_root
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    bool ied=0;
    const int N=1e5+5;
    vector<ui> pr;
    bool ed[N];
    void init()
    {
        pr.reserve(N);
        for (ui i=2;i<N;i++)
        {
            if (!ed[i]) pr.push_back(i);
            for (ui x:pr)
            {
                if (i*x>=N) break;
                ed[i*x]=1;
                if (i%x==0) break;
            }
        }
    }
}

```

```

    }
}
ui ksm(ui x,ui y,ui p)
{
    ui r=1;
    while (y)
    {
        if (y&1) r=(ll)r*x%p;
        x=(ll)x*x%p;y>>=1;
    }
    return r;
}
vector<ui> getw(ui n)
{
    vector<ui> w;
    for (ui x:pr)
    {
        if (x*x>n) break;
        if (n%x==0)
        {
            w.push_back(x);
            n/=x;
            for (ui i=n/x;n==x*i;i=n/x) n/=x;
        }
    }
    if (n>1) w.push_back(n);
    return w;
}
int getrt(ui n)
{
    if (n<=2) return n-1;
    if (!ed[4]) init();
    auto w=getw(n);
    ui ph=n;
    for (ui x:w) ph=ph/x*(x-1);
    w=getw(ph);
    for (ui &x:w) x=ph/x;
    for (ui i=2;i<n;i++) if (gcd(i,n)==1)
    {
        for (ui x:w) if (ksm(i,x,n)==1) goto no;
        return i;
    }
    no;;
    return -1;
}
}
using get_root::getrt;

```

### 3.20 筛全部原根

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=1e6+2;
int ss[N],mn[N],fmn[N],phi[N];
int t,n,gs,i,d;

```

```

bool ed[N],av[N],yg[N],hv[N];
double inv[N];
void getfac(int x,int *a,int &n)
{
    int y=x,z;
    if (1^x&1)
    {
        a[n=1]=2;x>>=1;while (1^x&1) x>>=1;
    }
    while (x>1)
    {
        x=1e-9+(x*inv[a[++n]=z=mn[x]]);
        while (x%z==0) x=1e-9+x*inv[z];
    }
    for (i=1;i<=n;i++) av[a[i]]=0,a[i]=1e-9+(y*inv[a[i]]);
}
int ksm(int x,int y,int p)
{
    int r=1;
    while (y)
    {
        if (y&1) r=(ll)r*x%p;
        x=(ll)x*x%p;y>>=1;
    }
    return r;
}
bool ck(int x,int *a,int n,int p)
{
    for (int i=1;i<=n;i++) if (ksm(x,a[i],p)==1) return 0;
    return 1;
}
void getrt(int x,int d)
{
    if (!hv[x]) return puts("0\n"),void();
    static int a[30];
    int n=0,y,i,g=0,c=d;y=phi[x];
    fill(av+1,av+y+1,1);
    getfac(y,a,n);
    for (i=1;i<x;i++) if (__gcd(i,x)==1&&ck(i,a,n,x)) break;
    yg[g=i]=1;//g就是最小原根
    int j=(ll)g*g%x;
    for (i=2;i<y;i++,j=(ll)j*g%x) yg[j]=av[i]=av[mn[i]]&av[fmn[i]];
    printf("%d\n",phi[y]);
    for (i=1;i<x;i++) if (yg[i])
    {
        yg[i]=0;
        if (--c==0) printf("%d□",i),c=d;
    }puts("");
}
void init()
{
    int i,j,k,n=N-1;
    mn[1]=phi[1]=1;
    for (i=1;i<=n;i++) inv[i]=1.0/i;
    for (i=2;i<=n;i++)
    {
        if (!ed[i]) phi[mn[i]=ss[++gs]=i]=i-1,hv[i]=1;
    }
}

```



```

    for (j=1;j<=gs&&(k=ss[j]*i)<=n;j++)
    {
        ed[k]=1;mn[k]=ss[j];
        if (i%ss[j]==0) {phi[k]=phi[i]*ss[j];hv[k]=hv[i];break;}
        phi[k]=phi[i]*(ss[j]-1);
    }
}
for (i=n;i;i--) fmn[i]=1e-9+(i*inv[mn[i]]),hv[i]|=(1^i&1)&&hv[i]>>1;
for (i=8;i<=n;i<=1) hv[i]=0;
}
int main()
{
    init();
    scanf("%d",&t);
    while (t--)
    {
        scanf("%d%d",&n,&d);
        getrt(n,d);
    }
}

```

### 3.21 高斯消元（通解）

返回方程的一组解和自由元。

```

tuple<int,vector<ui>,vector<vector<ui>>> gauss(vector<vector<ui>> a)//sum = a[i][m], rank of base
, one sol, base
{
    int n=a.size(),m=a[0].size()-1,i,j,k,R=m;
    vector<int> fix(m,-1);
    for (i=k=0;i<m;i++)
    {
        for (j=k;j<n;j++) if (a[j][i]) break;
        if (j==n) continue;
        fix[i]=k;--R;
        swap(a[k],a[j]);
        ui *u=a[k].data();
        ui x=ksm(u[i],p-2);
        for (j=i;j<=m;j++) u[j]=(ll)u[j]*x%p;
        for (auto &v:a) if (v.data()!=a[k].data())
        {
            x=p-v[i];
            for (j=i;j<=m;j++) v[j]=(v[j]+(ll)x*u[j])%p;
        }
        ++k;
    }
    for (i=k;i<n;i++) if (a[i][m]) return {-1,{},{}};
    vector<ui> r(m);
    vector<vector<ui>> c;
    for (i=0;i<m;i++) if (fix[i]!=-1) r[i]=a[fix[i]][m];
    for (i=0;i<m;i++) if (fix[i]==-1)
    {
        vector<ui> r(m);
        r[i]=1;
        for (j=0;j<m;j++) if (fix[j]!=-1) r[j]=(p-a[fix[j]][i])%p;
        c.push_back(r);
    }
}

```

```

    return {R,r,c};
}

```

### 3.22 高斯消元（列主元）

$O(n^3)$ ,  $O(n^2)$ 。

浮点数的版本。

```

namespace Gauss
{
    typedef double db;
    const db eps=1e-8;
    template<class T> pair<vector<db>,int> solve(const vector<vector<T>> &A)//和为 0。返回秩，负数
        无解
    {
        assert(A.size());
        int n=A.size(),m=A[0].size()-1,i,j,k,l,r,fg=1;
        db a[n][m+1],b;
        for (i=0;i<n;i++) for (j=0;j<=m;j++) a[i][j]=A[i][j];
        for (i=l=r=0;i<n&&l<m;i++,l++)
        {
            k=i;
            for (j=i+1;j<n;j++) if (fabs(a[j][l])>fabs(a[k][l])) k=j;
            if (fabs(a[k][l])<eps) {--i;continue;}
            if (i!=k) for (j=l;j<=m;j++) swap(a[i][j],a[k][j]);
            b=1/a[i][l];++r;a[i][l]=1;
            for (j=l+1;j<=m;j++) a[i][j]*=b;
            for (j=0;j<n;j++) if (i!=j)
            {
                b=a[j][l];a[j][l]=0;
                for (k=l+1;k<=m;k++) a[j][k]-=b*a[i][k];
            }
        }
        vector<db> X(m);
        for (j=0;j<l;j++) for (k=0;k<i;k++) if (a[k][j]==1)
        {
            X[j]=-a[k][m];
            break;
        }
        for (j=i;j<n&&~fg;j++)
        {
            b=a[j][m];
            for (k=0;k<m;k++) b+=X[k]*a[j][k];
            if (fabs(b)>eps) fg=-1;
        }
        return {X,r*fg};
    }
}

```

### 3.23 行列式求值（任意模数）

$O(n^3)$ ,  $O(n^2)$ 。

原理：辗转相除。注意这个  $\log p$  并不在  $n^3$  上。

```

#include <bits/stdc++.h>

```

```

using namespace std;
typedef long long ll;
const int N=502,p=998244353;
int cal(int a[][N],int n)
{
    int i,j,k,r=1,fh=0,l;
    for (i=1;i<=n;i++)
    {
        k=i;
        for (j=i+1;j<=n;j++) if (a[j][i]) {k=j;break;}
        if (a[k][i]==0) return 0;
        if (i!=k) {swap(a[k],a[i]);fh^=1;}
        for (j=i+1;j<=n;j++)
        {
            if (a[j][i]>a[i][i]) swap(a[j],a[i]),fh^=1;
            while (a[j][i])
            {
                l=a[i][i]/a[j][i];
                for (k=i;k<=n;k++) a[i][k]=(a[i][k]+(ll)(p-l)*a[j][k])%p;
                swap(a[j],a[i]);fh^=1;
            }
        }
        r=(ll)r*a[i][i]%p;
    }
    if (fh) return (p-r)%p;
    return r;
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,i,j;
    static int a[N][N];
    cin>>n;
    for (i=1;i<=n;i++) for (j=1;j<=n;j++) cin>>a[i][j];
    cout<<cal(a,n)<<endl;
}

```

### 3.24 行列式求值（质数模数）

$O(n^3)$ ,  $O(n^2)$ 。

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=502,p=998244353;
int ksm(int x,int y)
{
    int r=1;
    while (y)
    {
        if (y&1) r=(ll)r*x%p;
        y>>=1;x=(ll)x*x%p;
    }
    return r;
}
int cal(int a[][N],int n)
{

```

```

int i,j,k,r=1,fh=0,l;
for (i=1;i<=n;i++)
{
    for (j=i;j<=n;j++) if (a[j][i]) break;
    if (j>n) return 0;
    if (i!=j) swap(a[j],a[i]),fh^=1;
    r=(ll)r*a[i][i]%p;
    k=ksm(a[i][i],p-2);
    for (j=i;j<=n;j++) a[i][j]=(ll)a[i][j]*k%p;
    for (j=i+1;j<=n;j++)
    {
        a[j][i]=p-a[j][i];
        for (k=i+1;k<=n;k++) a[j][k]=(a[j][k]+(ll)a[j][i]*a[i][k])%p;
        a[j][i]=0;
    }
}
if (fh) return (p-r)%p;
return r;
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,i,j;
    static int a[N][N];
    cin>>n;
    for (i=1;i<=n;i++) for (j=1;j<=n;j++) cin>>a[i][j];
    cout<<cal(a,n)<<endl;
}
/*
3
3 1 4
1 5 9
2 6 5
998244263
*/

```

### 3.25 稀疏矩阵系列

safe 宏用于验证结果正确性，可不定义。实现了稀疏矩阵的行列式和求解方程组。

```

vector<ui> bm(const vector<ui> &a)
{
    vector<ui> r,lst;
    int n=a.size(),m=0,q=0,i,j,k=-1;
    ui D=0;
    for (i=0;i<n;i++)
    {
        ui cur=0;
        for (j=0;j<m;j++) cur=(cur+(ll)a[i-j-1]*r[j])%p;
        cur=(a[i]+p-cur)%p;
        if (!cur) continue;
        if (k==--1)
        {
            k=i;
            D=cur;
            r.resize(m=i+1);
            continue;
        }
    }
}

```

```

    }
    auto v=r;
    ui x=(ll)cur*ksm(D,p-2)%p;
    if (m<q+i-k) r.resize(m=q+i-k);
    (r[i-k-1]+=x)%=p;
    ui *b=r.data()+i-k;
    x=(p-x)%p;
    for (j=0;j<q;j++) b[j]=(b[j]+(ll)x*lst[j])%p;
    if (v.size()+k<lst.size()+i)
    {
        lst=v;
        q=v.size();
        k=i;
        D=cur;
    }
}
return r;
}
#define safe
struct Q
{
    int x,y;
    ui w;
};
mt19937_64 rnd(9980);
vector<ui> minpoly(int n,const vector<Q> &a)//[0,n),max:1
{
    for (auto [x,y,w]:a) assert(min(x,y)>=0&&max(x,y)<n);
    vector<ui> u(n),v(n),b(n*2+1),tmp(n);
    int i;
    for (ui &x:u) x=rnd()%p;
    for (ui &x:v) x=rnd()%p;
    assert(*min_element(all(u))&&*min_element(all(v)));
    for (ui &r:b)
    {
        for (i=0;i<n;i++) r=(r+(ll)u[i]*v[i])%p;
        fill(all(tmp),0);
        for (auto [x,y,w]:a) tmp[x]=(tmp[x]+(ll)w*v[y])%p;
        swap(v,tmp);
    }
    auto r=bm(b);
#ifdef safe
    for (ui &x:u) x=rnd()%p;
    for (ui &x:v) x=rnd()%p;
    for (ui &r:b)
    {
        for (i=0;i<n;i++) r=(r+(ll)u[i]*v[i])%p;
        fill(all(tmp),0);
        for (auto [x,y,w]:a) tmp[x]=(tmp[x]+(ll)w*v[y])%p;
        swap(v,tmp);
    }
    auto rr=bm(b);
    assert(r==rr);
#endif
    reverse(all(r));
    for (ui &x:r) if (x) x=p-x;
    r.push_back(1);

```

```

    return r;
}
ui det(int n,vector<Q> a)//[0,m]
{
    vector<ui> b(n);
    for (ui &x:b) x=rnd()%p;
    assert(*min_element(all(b)));
    for (auto &[x,y,w]:a) w=(ll)w*b[x]%p;
    ui r=minpoly(n,a)[0],tmp=1;
    for (ui x:b) tmp=(ll)tmp*x%p;
    r=(ll)r*ksm(tmp,p-2)%p;
#ifdef safe
    for (ui &x:b) x=rnd()%p;
    assert(*min_element(all(b)));
    for (auto &[x,y,w]:a) w=(ll)w*b[x]%p;
    ui rr=minpoly(n,a)[0],tmpp=1;
    for (ui x:b) tmpp=(ll)tmpp*x%p;
    rr=(ll)rr*ksm(tmpp,p-2)%p*ksm(tmp,p-2)%p;
    assert(r==rr);
#endif
    return n&1?(p-r)%p:r;
}
vector<ui> gauss(const vector<Q> &a,vector<ui> v)
{
    int n=v.size(),i,j;
    for (auto [x,y,w]:a) assert(0<=x&&x<n&&0<=y&&y<n);
    vector<ui> u(n),b(2*n+1),tmp(n),tv=v;
    for (ui &x:u) x=rnd()%p;
    assert(*min_element(all(u)));
    for (ui &r:b)
    {
        for (i=0;i<n;i++) r=(r+(ll)u[i]*v[i])%p;
        fill(all(tmp),0);
        for (auto [x,y,w]:a) tmp[x]=(tmp[x]+(ll)w*v[y])%p;
        swap(v,tmp);
    }
    auto f=bm(b);
    f.insert(f.begin(),p-1);
    int m=(int)f.size()-2;
    v=tv;fill(all(u),0);
    ui x;
    for (i=0;i<=m;i++)
    {
        x=f[m-i];
        for (j=0;j<n;j++) u[j]=(u[j]+(ll)v[j]*x)%p;
        fill(all(tmp),0);
        for (auto [x,y,w]:a) tmp[x]=(tmp[x]+(ll)w*v[y])%p;
        swap(v,tmp);
    }
    x=ksm((p-f.back())%p,p-2);
    for (ui &y:u) y=(ll)y*x%p;
#ifdef safe
    for (auto [x,y,w]:a) tv[x]=(tv[x]+(ll)(p-w)*u[y])%p;
    assert(!*min_element(all(tv)));
#endif
    return u;
}

```

## 3.26 Min\_25 筛

$f(p^k) = p^k(p^k - 1)$ , 求  $\sum_{i=1}^n f(i)$ 。这个的原理我了解的不多, 因此没有更多注释。

```
const int N=1e5+2,p=1e9+7,i6=166666668;
ll fs[N<<1],m;
int ss[N],ys[N<<1],s[N],f[N<<1],g[N<<1],ls[N<<1],cs[N<<1];
int gs,n,i,j,k,cnt,ct,ans,sq;
bool ed[N];
int S(ll n,int x)
{
    int r,i,j,l;
    ll k;
    if (ss[x]>=n) return 0;
    if (n>sq) r=g[ys[m/n]]; else r=g[n];
    if ((r=r-s[x])<0) r+=p;
    for (i=x+1;(ll)ss[i]*ss[i]<=n;i++) for (j=1,k=ss[i];k<=n;j++,k*=ss[i])
    {
        l=(k-1)%p;
        r=(r+(ll)l*(l+1)%p*((j!=1)+S(n/k,i)))%p;
    }
    return r;
}
int main()
{
    n=1e5;
    for (i=2;i<=n;i++)
    {
        if (!ed[i]) ss[++gs]=i;
        for (j=1;(j<=gs)&&(i*ss[j]<=n);j++)
        {
            ed[i*ss[j]]=1;
            if (i%ss[j]==0) break;
        }
    }
    ss[gs+1]=1e6;
    s[1]=ss[1]*ss[1];
    for (i=2;i<=gs;i++) s[i]=(s[i-1]+(ll)ss[i]*ss[i])%p; //s 是多项式在素数位置的前缀和
    memcpy(cs,s,sizeof(s));
    ll i,j,k,x,z; scanf("%lld",&m);
    sq=n=sqrt(m);while ((ll)(n+1)*(n+1)<=m) ++n;
    cnt=n-1;
    for (i=n;i<=m;i=j+1) {j=m/(m/i);++cnt;}ct=cnt++;
    for (i=1;i<=m;i=j+1)
    {
        j=m/(k=m/i);
        if (k<=n) g[fs[k]=k]=(k*(k+1)*(k<<1|1)/6-1)%p; //这里是多项式前缀和 (不含1)
        else
        {
            z=k%p; //一样
            g[ys[j]=--cnt]=(z*(z+1)%p*(z<<1|1)%p+p-6)*i6%p; fs[cnt]=k;
        }
    }
    cnt=ct;
    for (j=1;(j<=gs)&&(z=(ll)ss[j]*ss[j]);j++) for (i=cnt;z<=fs[i];i--)
    {
        x=fs[i]/ss[j];if (x>n) x=ys[m/x];
        g[i]=(g[i]+(ll)(p-ss[j])*ss[j]%p*(g[x]-s[j-1]+p))%p; //另一处需要修改的
    }
}
```

```

memcpy(ls,g,sizeof(g));
s[1]=ss[1];
for (i=2;i<=gs;i++) s[i]=s[i-1]+ss[i];
cnt=n-1;
for (i=n;i<=m;i=j+1) {j=m/(m/i);++cnt;}ct=cnt++;
for (i=1;i<=m;i=j+1)
{
    j=m/(k=m/i);
    if (k<=n) g[fs[k]=k]=((k*(k+1)>>1)-1)%p;
    else
    {
        z=k%p;
        g[ys[j]=--cnt]=(z*(z+1)-2>>1)%p;fs[cnt]=k;
    }
}
cnt=ct;
for (j=1;(j<=gs)&&(z=(ll)ss[j]*ss[j]);j++) for (i=cnt;z<=fs[i];i--)
{
    x=fs[i]/ss[j];if (x>n) x=ys[m/x];
    g[i]=(g[i]+(ll)(p-ss[j])*(g[x]-s[j-1]+p))%p;
}
for (i=1;i<=cnt;i++) if ((g[i]=ls[i]-g[i])<0) g[i]+=p;
for (i=1;i<=gs;i++) if ((s[i]=cs[i]-s[i])<0) s[i]+=p;
ans=S(m,0)+1;if (ans==p) ans=0;printf("%d",ans);
}

```

### 3.27 Min\_25 筛（卡常，素数个数，注意评测机 double 性能）

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=3.2e5+2;
ll s[N];
int ss[N],ys[N],gs=0;
bool ed[N];
ll cal(ll m)
{
    static ll g[N<<1],fs[N<<1];
    ll i,j,k,x;
    int n;
    int p,q,cnt;
    n=round(sqrt(m));
    q=lower_bound(ss+1,ss+gs+1,n)-ss;
    memset(g,0,sizeof(g));memset(ys,0,sizeof(ys));cnt=n-1;
    for (i=n;i<=m;i=j+1) {j=m/(m/i);++cnt;}int ct=cnt++;
    for (i=1;i<=m;i=j+1)
    {
        j=m/(k=m/i);
        if (k<=n) g[fs[k]=k]=k-1; else {g[ys[j]=--cnt]=k-1;fs[cnt]=k;}
    }cnt=ct;
    for (j=1;j<=q;j++) for (i=cnt;(ll)ss[j]*ss[j]<=fs[i];i--)
    {
        x=fs[i]/ss[j];if (x>n) x=ys[m/x];
        g[i]-=g[x]-j+1;
    }
    return g[cnt];//这里 g[cnt-i+1] 表示的是 [1,m/i] 的答案
}

```



```

}
int main()
{
    int n,i,j,t;
    n=3.2e5;
    for (i=2;i<=n;i++)
    {
        if (!ed[i]) ss[++gs]=i;
        for (j=1;(j<=gs)&&(i*ss[j]<=n);j++)
        {
            ed[i*ss[j]]=1;
            if (i%ss[j]==0) break;
        }
    }
    s[1]=ss[1];
    for (i=2;i<=gs;i++) s[i]=s[i-1]+ss[i];
    t=1;
    ll m;
    while (t--) cin>>m,cout<<cal(m)<<"\n";
}

```

### 3.28 扩展 min-max 容斥（重返现世）

$$k\text{-th max}\{S\} = \sum_{T \subset S} (-1)^{|T|-k} \binom{|T|-1}{k-1} \min\{T\}$$

```

scanf("%d%d%d",&n,&q,&m); inv[1]=1;q=n+1-q;
for (i=2;i<=m;i++) inv[i]=p-(ll)p/i*inv[p%i]%p;
for (i=1;i<=n;i++) scanf("%d",a+i);f[0][0]=1;
for (j=1;j<=n;j++) for (i=q;i;i--) for (k=m;k>=a[j];k--) if ((f[i][k]=f[i][k]+f[i-1][k-a[j]]-f[i][k-a[j]])>=p) f[i][k]-=p; else if (f[i][k]<0) f[i][k]+=p;
for (i=1;i<=m;i++) ans=(ans+(ll)f[q][i]*inv[i])%p;
ans=(ll)ans*m%p;printf("%d",ans);

```

### 3.29 模数为偶数 FWT & 光速乘

$O(n2^n)$ ,  $O(2^n)$ 。

原理：让模数变为  $p2^n$ ，就可以正常做除法了。

```

const int N=1<<20,M=21;
int x[M];
ll p,f[N],g[N];
int n,m,c;
ll mul(ll x,ll y)
{
    x=x*y-(ll)((ldb)x/p*y+1e-8)*p;
    if (x<0) return x+p;return x;
}
void dft(ll *a)
{
    int i,j,k,l;
    ll b;
    for (i=1;i<n;i=1)
    {
        l=i<<1;
        for (j=0;j<n;j+=1) for (k=0;k<i;k++)

```

```

        {
            b=a[j|k|i];
            a[j|k|i]=(a[j|k]-b+p)%p;
            a[j|k]=(a[j|k]+b)%p;
        }
    }
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    ll t;int i;
    cin>>m>>t>>p;p*=(n=1<<m);
    for (i=0;i<n;i++) cin>>f[i];
    dft(f);
    for (i=0;i<=m;i++) cin>>x[i];
    for (i=1;i<n;i++) g[i]=g[i>>1]+(i&1);
    for (i=0;i<n;i++) g[i]=x[g[i]];dft(g);
    while (t)
    {
        if (t&1) for (i=0;i<n;i++) f[i]=mul(f[i],g[i]);
        for (i=0;i<n;i++) g[i]=mul(g[i],g[i]);t>>=1;
    }
    dft(f);
    for (i=0;i<n;i++) cout<<(f[i]>>m)<<'\\n';
}

```

### 3.30 二次剩余

```

namespace cipolla
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    ui p,w;
    struct Q
    {
        ll x,y;
        Q operator*(const Q &o) const {return {(x*o.x+y*o.y%p*w)%p,(x*o.y+y*o.x)%p};}
    };
    ui ksm(ll x,ui y)
    {
        ll r=1;
        while (y)
        {
            if (y&1) r=r*x%p;
            x=x*x%p;y>>=1;
        }
        return r;
    }
    Q ksm(Q x,ui y)
    {
        Q r={1,0};
        while (y)
        {
            if (y&1) r=r*x;
            x=x*x;y>>=1;
        }
    }
}

```

```

    return r;
}
ui mosqrt(ui x, ui P) // 0 <= x < P
{
    if (x == 0 || P == 2) return x;
    p = P;
    if (ksm(x, p-1 >> 1) != 1) return -1;
    ui y;
    mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    do y = rnd() % p, w = ((ll)y * y + p - x) % p; while (ksm(w, p-1 >> 1) <= 1); // not for p=2
    y = ksm(y, 1}, p+1 >> 1).x;
    if (y * 2 > p) y = p - y; // 两解取小
    return y;
}
}
using cipolla::mosqrt;

```

### 3.31 $k$ 次剩余

```

namespace get_root
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    bool ied = 0;
    const int N = 1e5 + 5;
    vector<ui> pr;
    bool ed[N];
    void init()
    {
        pr.reserve(N);
        for (ui i = 2; i < N; i++)
        {
            if (!ed[i]) pr.push_back(i);
            for (ui x : pr)
            {
                if (i * x >= N) break;
                ed[i * x] = 1;
                if (i % x == 0) break;
            }
        }
    }
    ui ksm(ui x, ui y, ui p)
    {
        ui r = 1;
        while (y)
        {
            if (y & 1) r = (ll)r * x % p;
            x = (ll)x * x % p; y >>= 1;
        }
        return r;
    }
    vector<ui> getw(ui n)
    {
        vector<ui> w;
        for (ui x : pr)
        {

```

```

        if (x*x>n) break;
        if (n%x==0)
        {
            w.push_back(x);
            n/=x;
            for (ui i=n/x;n==x*i;i=n/x) n/=x;
        }
    }
    if (n>1) w.push_back(n);
    return w;
}
int getrt(ui n)
{
    if (n<=2) return n-1;
    if (!ed[4]) init();
    auto w=getw(n);
    ui ph=n;
    for (ui x:w) ph=ph/x*(x-1);
    w=getw(ph);
    for (ui &x:w) x=ph/x;
    for (ui i=2;i<n;i++) if (gcd(i,n)==1)
    {
        for (ui x:w if (ksm(i,x,n)==1) goto no;
        return i;
    }
    no:;
    return -1;
}
}
namespace BSGS
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    template<int N,class T,class TT> struct ht//个数, 定义域, 值域
    {
        const static int p=1e6+7,M=p+2;
        TT a[N];
        T v[N];
        int fir[p+2],nxt[N],st[p+2];//和模数相适应
        int tp,ds;//自定义模数
        ht(){memset(fir,0,sizeof fir);tp=ds=0;}
        void mdf(T x,TT z)//位置, 值
        {
            ui y=x%p;
            for (int i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i]=z,void();//若不可能重复不需要 for
            v[++ds]=x;a[ds]=z;
            if (!fir[y]) st[++tp]=y;
            nxt[ds]=fir[y];fir[y]=ds;
        }
        TT find(T x)
        {
            ui y=x%p;
            int i;
            for (i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i];
            return 0;//返回值和是否判断依据要求决定
        }
    }
    void clear()

```

```

    {
        ++tp;
        while (--tp) fir[st[tp]]=0;
        ds=0;
    }
};
const int N=5e4;
ht<N,ui,ui> s;
int exgcd(int a,int b)
{
    if (a==1) return 1;
    return (1-(long long)b*exgcd(b%a,a))/a;//not ll
}
int bsgs(ui a,ui b,ui p)
{
    s.clear();
    a%=p;b%=p;
    if (!a) return 1-min((int)b,2);//舍 -1
    ui i,j,k,x,y;
    x=sqrt(p)+2;
    for (i=0,j=1;i<x;i++,j=(ll)j*a%p)
    {
        if (j==b) return i;
        s.mdf((ll)j*b%p,i+1);
    }
    k=j;
    for (i=1;i<=x;i++,j=(ll)j*k%p) if (y=s.find(j)) return (ll)i*x-y+1;
    return -1;
}
bool isprime(ui p)
{
    if (p<=1) return 0;
    for (ui i=2;i*i<=p;i++) if (p%i==0) return 0;
    return 1;
}
int exbsgs(ui a,ui b,ui p)//a^x=b(mod p)
{
    //if (isprime(p)) return bsgs(a,b,p);
    a%=p;b%=p;
    ui i,j,k,x,y=__lg(p),cnt=0;
    for (i=0,j=1%p;i<=y;i++,j=(ll)j*a%p) if (j==b) return i;
    y=1;
    while (1)
    {
        if ((x=gcd(a,p))==1) break;
        if (b%x) return -1;//no sol
        ++cnt;
        p/=x;b/=x;
        y=(ll)y*(a/x)%p;
    }
    a%=p;
    b=(ll)b*(p+exgcd(y,p))%p;
    int r=bsgs(a,b,p);
    return r==-1?-1:r+cnt;
}
pair<ll,ll> exgcd(ll a,ll b,ll c)//ax+by=c, {-1,-1} 无解, b=0 返回 {c/a,0}, 否则返回最小非负 x

```

```

{
    assert(a||b);
    if (!b) return {c/a,0};
    if (a<0) a=-a,b=-b,c=-c;
    ll d=gcd(a,b);
    if (c%d) return {-1,-1};
    ll x=1,x1=0,p=a,q=b,k;
    b=abs(b);
    while (b)
    {
        k=a/b;
        x-=k*x1;a-=k*b;
        swap(x,x1);
        swap(a,b);
    }
    b=abs(q/d);
    x=x*(c/d)%b;
    if (x<0) x+=b;
    return {x,(c-p*x)/q};
}
ll fun(ll a,ll b,ll p)//ax=b(mod p)
{
    return exgcd(-p,a,b).second%p;
}
using get_root::getrt;
using BSGS::bsgs,BSGS::exbsgs;
int nth_root(ui k,ui y,ui p)//x^k=y(mod p)
{
    if (k==0) return y==1?0:-1;
    if (y==0) return 0;
    ui g=getrt(p);
    ui z=bsgs(g,y,p);
    ll x=fun(k,z,p-1);
    if (x==-1) return -1;
    return get_root::ksm(g,x,p);
}

```

网上的超快版本

```

#define popcount __builtin_popcount
using namespace std;
typedef long long int ll;
//using ll=__int128_t;
typedef pair<ll, int> P;
ll gcd(ll a, ll b){
    if (b==0) return a;
    return gcd(b, a%b);
}
ll powmod(ll a, ll k, ll mod){
    ll ap=a, ans=1;
    while(k){
        if (k&1){
            ans*=ap;
            ans%=mod;
        }
        ap=ap*ap;
        ap%=mod;
        k>>=1;
    }
}

```

```

    }
    return ans;
}
ll inv(ll a, ll m){
    ll b=m, x=1, y=0;
    while(b>0){
        ll t=a/b;
        swap(a-=t*b, b);
        swap(x-=t*y, y);
    }
    return (x%m+m)%m;
}
vector<P> fac(ll x){
    vector<P> ret;
    for(ll i=2; i*i<=x; i++){
        if (x%i==0){
            int e=0;
            while(x%i==0){
                x/=i;
                e++;
            }
            ret.push_back({i, e});
        }
    }
    if (x>1) ret.push_back({x, 1});
    return ret;
}
//mt19937_64 mt(334);
mt19937 mt(334);
ll solve1(ll p, ll q, int e, ll a){
    int s=0;
    ll r=p-1, qs=1, qp=1;
    while(r%q==0){
        r/=q;
        qs*=q;
        s++;
    }
    for(int i=0; i<e; i++) qp*=q;
    ll d=qp-inv(r%qp, qp);
    ll t=(d*r+1)/qp;
    ll at=powmod(a, t, p), inva=inv(a, p);
    if (e>=s){
        if (powmod(at, qp, p)!=a) return -1;
        else return at;
    }
    //uniform_int_distribution<long long> rnd(1, p-1);
    uniform_int_distribution<> rnd(1, p-1);
    ll rv;
    while(1){
        rv=powmod(rnd(mt), r, p);
        if (powmod(rv, qs/q, p)!=1) break;
    }
    int i=0;
    ll qi=1, sq=1;
    while(sq*sq<q) sq++;
    while(i<s-e){
        ll qq=qs/qp/qi/q;

```

```

vector<P> v(sq);
ll rvi=powmod(rv, qp*qq*(p-2)%(p-1), p), rvp=powmod(rv, sq*qp*qq, p);
ll x=powmod(powmod(at, qp, p)*inva%p, qq*(p-2)%(p-1), p), y=1;
for(int j=0; j<sq; j++){
    v[j]=P(x, j);
    (x*=rvi)%=p;
}
sort(v.begin(), v.end());
ll z=-1;
for(int j=0; j<sq; j++){
    int l=lower_bound(v.begin(), v.end(), P(y, 0))-v.begin();
    if (v[l].first==y){
        z=v[l].second+j*sq;
        break;
    }
    (y*=rvp)%=p;
}
if (z==-1) return -1;
(at*=powmod(rv, z, p))%=p;
i++;
qi*=q;
rv=powmod(rv, q, p);
}
return at;
}
ll solve0(ll p, ll q, ll r, ll a){
    ll d=q-inv(r%q, q);
    ll t=(d*r+1)/q;
    ll at=powmod(a, t, p), inva=inv(a, p);
    if (powmod(at, q, p)!=a) return -1;
    else return at;
}
ll solve(ll p, ll k, ll a)//p k y
{
    if (k==0)
    {
        if (a==1) return 1;
        return -1;
    }
    if (a==0) return 0;
    if (p==2 || a==1) return 1;
    ll a1=a;
    ll g=gcd(p-1, k);
    ll c=inv(k/g%((p-1)/g), (p-1)/g);
    a=powmod(a, c, p);
    if (g==1){
        if (powmod(a, k, p)==a1) return a;
        else return -1;
    }
    ll g1=gcd(g, (p-1)/g), g2=g;
    vector<P> f1=fac(g1), f;
    for(auto r:f1){
        ll q=r.first;
        int e=0;
        while(g2%q==0){
            g2/=q;
            e++;

```



```

    }
    f.push_back({q, e});
}
ll ret=1, gp=1;
if (g2>1){
    ll x=solve0(p, g2, (p-1)/g2, a);
    if (x==-1) return -1;
    ret=x, gp*=g2;
}
for(auto r:f){
    ll qp=1;
    for(int i=0; i<r.second; i++) qp*=r.first;
    ll x=solve1(p, r.first, r.second, a);
    if (x==-1) return -1;
    if (gp==1){
        ret=x, gp*=qp;
        continue;
    }
    ll s=inv(gp%qp, qp), t=(1-gp*s)/qp;
    if (t>=0) ret=powmod(ret, t, p);
    else ret=powmod(ret, p-1+t%(p-1), p);
    if (s>=0) x=powmod(x, s, p);
    else x=powmod(x, p-1+s%(p-1), p);
    (ret*=x)%=p;
    gp*=qp;
}
if (powmod(ret, k, p)!=a1) return -1;
return ret;
}

```

### 3.32 FWT/子集卷积

$O(n2^n)$ ,  $O(2^n)$ 。注意全都是无符号的。

这里混合了两个版本的代码，但只有 ui 和 ull 的差异。容易自行调整。

```

void fwt_and(vector<ll> &A)//本质：母集和
{
    ll n=A.size(), *a=A.data(), i, j, k, l, *f, *g;
    for (i=1; i<n; i=1)
    {
        l=i*2;
        for (j=0; j<n; j+=1)
        {
            f=a+j; g=a+j+i;
            for (k=0; k<i; k++) f[k]+=g[k];
        }
        if (l==n||i==1<<10) for (ll &x:A) x%=p;
    }
}
void ifwt_and(vector<ll> &A)
{
    ll n=A.size(), *a=A.data(), i, j, k, l, *f, *g;
    for (i=1; i<n; i=1)
    {
        l=i*2;
        for (j=0; j<n; j+=1)

```

```

    {
        f=a+j; g=a+j+i;
        for (k=0; k<i; k++) f[k]+=p*i-g[k];
    }
    if (l==n||i==1<<10) for (ll &x:A) x%=p;
}
}
void fwt_or(vector<ll> &A)//本质: 子集和
{
    ll n=A.size(), *a=A.data(), i, j, k, l, *f, *g;
    for (i=1; i<n; i=1)
    {
        l=i*2;
        for (j=0; j<n; j+=l)
        {
            f=a+j; g=a+j+i;
            for (k=0; k<i; k++) g[k]+=f[k];
        }
        if (l==n||i==1<<10) for (ll &x:A) x%=p;
    }
}
void ifwt_or(vector<ll> &A)
{
    ll n=A.size(), *a=A.data(), i, j, k, l, *f, *g;
    for (i=1; i<n; i=1)
    {
        l=i*2;
        for (j=0; j<n; j+=l)
        {
            f=a+j; g=a+j+i;
            for (k=0; k<i; k++) g[k]+=p*i-f[k];
        }
        if (l==n||i==1<<10) for (ll &x:A) x%=p;
    }
}
void fwt_xor(vector<ui> &A)
{
    ui n=A.size(),*a=A.data(),i,j,k,l,*f,*g;
    for (i=1;i<n;i=1)
    {
        l=i*2;
        for (j=0;j<n;j+=l)
        {
            f=a+j;g=a+j+i;
            for (k=0;k<i;k++)
            {
                if ((f[k]+=g[k])>=p) f[k]-=p;
                g[k]=(f[k]+2*(p-g[k]))%p;
            }
        }
    }
}
void ifwt_xor(vector<ui> &A)
{
    ui n=A.size(),*a=A.data(),i,j,k,l,*f,*g,x=p+1>>1,y=1;
    for (i=1;i<n;i=1)
    {

```

```

    l=i*2;
    for (j=0;j<n;j+=1)
    {
        f=a+j;g=a+j+i;
        for (k=0;k<i;k++)
        {
            if ((f[k]+=g[k])>=p) f[k]-=p;
            g[k]=(f[k]+2*(p-g[k]))%p;
        }
    }
    y=(1l)y*x%p;
}
for (i=0;i<n;i++) a[i]=(1l)a[i]*y%p;
}
vector<ui> fst(const vector<ui> &s,const vector<ui> &t)
{
    int n=s.size(),m=__builtin_ctz(n),i,j,k;
    vector<ui> a[m+1],b[m+1],c[m+1],r(n);
    for (i=0;i<=m;i++) a[i].resize(n),b[i].resize(n),c[i].resize(n);
    for (i=0;i<n;i++)
    {
        k=__builtin_popcount(i);
        a[k][i]=s[i];
        b[k][i]=t[i];
    }
    for (i=0;i<m;i++) fwt_or(a[i]),fwt_or(b[i]);
    for (i=0;i<=m;i++) for (j=0;j<=i;j++) for (k=0;k<n;k++) c[i][k]=(c[i][k]+(1l)a[j][k]*b[i-j][k])%p;
    for (i=1;i<=m;i++) ifwt_or(c[i]);
    for (i=0;i<n;i++) r[i]=c[__builtin_popcount(i)][i];
    return r;
}

```

### 3.33 NTT

一种较快的 NTT（尤其是对于卷积以外的用途），但不推荐在不熟悉的情况下直接使用。一般的卷积可以参照字符串部分通配符的字符串匹配，其余的用途可以参照其他板子。

如果确实需要卡常，建议先抄写需要的函数，并递归地找到需要补的内容。

注意事项：所有 1l 为无符号。始终保证数组大小为  $2^n$ ，不应当使用 `resize` 而应该使用取模来调整长度。三种卷积对应的运算符见注释。

需要特别小心其长度的变化，注意不要越界。如果修改模数，`dft` 和 `hf_dft` 处有一个参数也要修改。

常见函数如下（带 new 的基本上都是较快但较长的）：

卷积 `operator*`，循环卷积 `operator&`，差卷积 `operator^`，求逆 `operator~/`（包含一个较短版，被注释了），分治 `cdq`，对数 `ln`，指数 `exp`,`exp_cdq`,`exp_new`，开方 `sqrt`,`sqrt_new`，幂函数 `pow(Q,1l)`,`pow(Q,string)`,`pow2(Q,1l)`,`pow(Q,1l,Q)`，整除与取模 `div`,`mod`,`div_mod`，线性递推 `recurrent`,`recurrent_new`,`recurrent_interval`，连乘 `prod`,`prod_new`，多点求值 `evaluation`,`evaluation_new`，阶乘 `factorial`，快速插值 `interpolation`，复合（逆）`comp`,`comp_inv`，多项式平移 `shift`，区间点值平移 `shift`，Z 变换 `Z_transform`，贝尔数（ $[n]$  划分等价类方案数）`Bell`，斯特林数 `S1_row`,`S1_column`,`S2_row`,`S2_column`,`signed_S1_row`，伯努利数 `Bernoulli`，划分数 `Partition`，最大公因式 `gcd`，求根 `root`，模多项式意义的逆 `inverse`。

```
#include <optional>
```

```

namespace NTT
{
    using ll = unsigned long long;
    const ll g = 3, p = 998244353;
    const int N = 1 << 22; // 务必修改
    ll inv[N], fac[N], ifac[N]; // 非必要
    void getfac(int n) // 非必要
    {
        static int pre = -1;
        if (pre == -1) pre = 1, ifac[0] = fac[0] = fac[1] = ifac[1] = inv[1] = 1;
        if (n <= pre) return;
        for (int i = pre + 1, j; i <= n; i++)
        {
            j = p / i;
            inv[i] = (p - j) * inv[p - i * j] % p;
            fac[i] = fac[i - 1] * i % p;
            ifac[i] = ifac[i - 1] * inv[i] % p;
        }
        pre = n;
    }
    ll w[N];
    int r[N];
    ll ksm(ll x, ll y)
    {
        ll r = 1;
        while (y)
        {
            if (y & 1) r = r * x % p;
            x = x * x % p;
            y >>= 1;
        }
        return r;
    }
    void init(int n)
    {
        static int pr = 0, pw = 0;
        if (pr == n) return;
        int b = __lg(n) - 1, i, j, k;
        for (i = 1; i < n; i++) r[i] = r[i >> 1] >> 1 | (i & 1) << b;
        if (pw < n)
        {
            for (j = 1; j < n; j = k)
            {
                k = j * 2;
                ll wn = ksm(g, (p - 1) / k);
                w[j] = 1;
                for (i = j + 1; i < k; i++) w[i] = w[i - 1] * wn % p;
            }
            pw = n;
        }
        pr = n;
    }
    int cal(int x) { return 1 << __lg(max(x, 1) * 2 - 1); }
    struct Q : vector<ll>
    {
        bool flag;
        Q& operator%=(int n) { assert((n & -n) == n); resize(n); return *this; }
    }
}

```

```

Q operator%(int n) const
{
    assert((n & -n) == n);
    if (size() <= n)
    {
        auto f = *this;
        return f %= n;
    }
    return Q(vector(begin(), begin() + n));
}
int deg() const
{
    int n = size() - 1;
    while (n >= 0 && begin()[n] == 0) --n;
    return n;
}
explicit Q(int x = 1, bool f = 0) :flag(f), vector<ll>(cal(x)) { }//小心: {}会调用这条而非
    下一条
Q(const vector<ll>& o, bool f = 0) :Q(o.size(), f) { copy(all(o), begin()); }
Q(const initializer_list<ll>& o, bool f = 0) :Q(vector(o), f) { }
ll fx(ll x)
{
    ll r = 0;
    for (auto it = rbegin(); it != rend(); ++it) r = (r * x + *it) % p;
    return r;
}
void dft()
{
    int n = size(), i, j, k;
    ll y, * f, * g, * wn, * a = data();
    init(n);
    for (i = 1; i < n; i++) if (i < r[i]) ::swap(a[i], a[r[i]]);
    for (k = 1; k < n; k *= 2)
    {
        wn = w + k;
        for (i = 0; i < n; i += k * 2)
        {
            g = (f = a + i) + k;
            for (j = 0; j < k; j++)
            {
                y = g[j] * wn[j] % p;
                g[j] = f[j] + p - y;
                f[j] += y;
            }
        }
        //此处要求 12*p*p<=2^64。如果调整模数，需要修改 12。
        if (__lg(n / k) % 12 == 1) for (i = 0; i < n; i++) a[i] %= p;
    }
    if (flag)
    {
        y = ksm(n, p - 2);
        for (i = 0; i < n; i++) a[i] = a[i] * y % p;
        reverse(a + 1, a + n);
    }
    flag ^= 1;
}
void hf_dft()
{

```

```

assert(size() >= 2 && flag);
int n = size() / 2, i, j, k;
ll x, y, * f, * g, * wn, * a = data();
init(n);
for (i = 1; i < n; i++) if (i < r[i]) ::swap(a[i], a[r[i]]);
for (k = 1; k < n; k *= 2)
{
    wn = w + k;
    for (i = 0; i < n; i += k * 2)
    {
        g = (f = a + i) + k;
        for (j = 0; j < k; j++)
        {
            y = g[j] * wn[j] % p;
            g[j] = f[j] + p - y;
            f[j] += y;
        }
    }
    if (__lg(n / k) % 12 == 1) for (i = 0; i < n; i++) a[i] %= p;
}
if (flag)
{
    x = ksm(n, p - 2);
    for (i = 0; i < n; i++) a[i] = a[i] * x % p;
    reverse(a + 1, a + n);
}
flag ^= 1;
}
Q operator<<(int m) const
{
    int n = deg(), i;
    Q r(n + m + 1);
    for (i = 0; i <= n; i++) r[i + m] = at(i);
    return r;
}
Q operator>>(int m) const
{
    int n = deg(), i;
    if (n < m) return Q();
    Q r(n + 1 - m);
    for (i = m; i <= n; i++) r[i - m] = at(i);
    return r;
}
};
Q shrink(Q f) { return f %= cal(f.deg() + 1); }
ostream& operator<<(ostream& cout, const Q& o)
{
    int n = o.deg();
    if (n < 0) return cout << "[0]";
    cout << "[" << o[n];
    for (int i = n - 1; i >= 0; i--) cout << ", " << o[i];
    return cout << "]";
}
Q der(const Q& f)
{
    ll n = f.size(), i;
    Q r(n);

```

```

    for (i = 1; i < n; i++) r[i - 1] = f[i] * i % p;
    return r;
}
Q integral(const Q& f)
{
    ll n = f.size(), i;
    getfac(n);
    Q r(n);
    for (i = 1; i < n; i++) r[i] = f[i - 1] * inv[i] % p;
    return r;
}
Q& operator+=(Q& f, ll x) { (f[0] += x) %= p; return f; }
Q operator+(Q f, ll x) { return f += x; }
Q& operator-=(Q& f, ll x) { (f[0] += p - x) %= p; return f; }
Q operator-(Q f, ll x) { return f -= x; }
Q& operator*=(Q& f, ll x) { for (ll& y : f) (y *= x) %= p; return f; }
Q operator*(Q f, ll x) { return f *= x; }
Q& operator+=(Q& f, const Q& g)
{
    f %= max(f.size(), g.size());
    for (int i = 0; i < g.size(); i++) f[i] = (f[i] + g[i]) % p;
    return f;
}
Q operator+(Q f, const Q& g) { return f += g; }
Q& operator-=(Q& f, const Q& g)
{
    f %= max(f.size(), g.size());
    for (int i = 0; i < g.size(); i++) f[i] = (f[i] + p - g[i]) % p;
    return f;
}
Q operator-(Q f, const Q& g) { return f -= g; }
Q& operator*=(Q& f, Q g) //卷积
{
    if (f.flag | g.flag)
    {
        int n = f.size(), i;
        assert(n == g.size());
        if (!f.flag) f.dft();
        if (!g.flag) g.dft();
        for (i = 0; i < n; i++) (f[i] *= g[i]) %= p;
        f.dft();
    }
    else
    {
        int n = cal(f.size() + g.size() - 1), i, j;
        int m1 = f.deg(), m2 = g.deg();
        if ((ll)m1 * m2 > (ll)n * __lg(n) * 8)
        {
            (f %= n).dft(); (g %= n).dft();
            for (i = 0; i < n; i++) (f[i] *= g[i]) %= p;
            f.dft();
        }
        else
        {
            vector<ll> r(max(0, m1 + m2 + 1));
            for (i = 0; i <= m1; i++) for (j = 0; j <= m2; j++) (r[i + j] += f[i] * g[j]) %= p;
            f = Q(n);

```

```

        copy(all(r), f.begin());
    }
}
return f;
}
Q operator*(Q f, const Q& g) { return f *= g; }
Q& operator+=(Q& f, Q g) // 循环卷积
{
    assert(f.size() == g.size());
    int n = f.size(), i;
    if (!f.flag) f.dft();
    if (!g.flag) g.dft();
    for (i = 0; i < n; i++) (f[i] += g[i]) %= p;
    f.dft();
    return f;
}
Q operator&(Q f, const Q& g) { return f &= g; }
Q& operator^=(Q& f, Q g) // 差卷积
{
    int n = f.size();
    g %= n;
    reverse(all(g));
    f *= g;
    rotate(f.begin(), n - 1 + all(f));
    return f %= n;
}
Q operator^(Q f, const Q& g) { return f ^= g; }
Q sqr(Q f)
{
    assert(!f.flag);
    int n = f.size() * 2, i;
    (f %= n).dft();
    for (i = 0; i < n; i++) f[i] = f[i] * f[i] % p;
    f.dft();
    return f;
}
/*Q operator~(const Q &f)
{
    Q r;
    r[0]=ksm(f[0],p-2);
    for (int i=1; i<=f.size(); i*=2) r=(-((f%i)*r-2)*r)%i;
    return r;
}*/trivial, 5e5 750ms*/
Q operator~(const Q& f)
{
    Q q, r, g;
    int n = f.size(), i, j, k;
    r[0] = ksm(f[0], p - 2);
    for (j = 2; j <= n; j *= 2)
    {
        k = j / 2;
        g = (r %= j) % k;
        r.dft();
        q = f % j * r;
        fill_n(q.begin(), k, 0);
        r *= q;
        copy(all(g), r.begin());
    }
}

```



```

    for (i = k; i < j; i++) r[i] = (p - r[i]) % p;
}
return r;
} //5e5 200ms, inv(1 6 3 4 9)=(1 998244347 33 998244169 1020)
Q&& operator/=(Q& f, const Q& g) { int n = f.size(); return (f *= ~g) %= n; }
Q operator/(Q f, const Q& g) { return f /= g; }
void cdq(Q& f, Q& g, int l, int r) //g_0=1,i*g_i=g_{i-j}*f_j,use for cdq
{
    static vector<Q> cd;
    int i, m = l + r >> 1, n = r - l, nn = n >> 1;
    if (r - l == f.size())
    {
        getfac(n - 1);
        g = Q(n);
        cd.clear();
        for (i = 2; i <= n; i *= 2)
        {
            cd.emplace_back(i);
            Q& h = cd.back();
            h %= i;
            copy_n(f.begin(), i, h.begin());
            h.dft();
        }
    }
    if (l + 1 == r)
    {
        g[l] = 1 ? g[l] * inv[l] % p : 1;
        return;
    }
    cdq(f, g, l, m);
    Q h(n);
    copy_n(g.begin() + 1, nn, h.begin());
    h *= cd[_lg(n) - 1];
    for (i = m; i < r; i++) (g[i] += h[i - 1]) %= p;
    cdq(f, g, m, r);
}
Q exp_cdq(Q f)
{
    Q g;
    int n = f.size(), i;
    for (i = 1; i < n; i++) f[i] = f[i] * i % p;
    cdq(f, g, 0, n);
    return g;
} //5e5 455ms
Q ln(const Q& f) { return integral(der(f) / f); }
//5e5 330ms, ln(1 2 3 4 5)=(0 2 1 665496236 499122177)
Q exp(Q f)
{
    Q r; r[0] = 1;
    for (int i = 1; i <= f.size(); i *= 2) (r *= f % i - ln(r % i) + 1) %= i;
    return r;
} //5e5 700ms, exp(0 4 2 3 5)=(1 4 10 665496257 665496281)
Q exp_new(Q b)
{
    Q h, f, r, u, v, bj;
    int n = b.size(), i, j, k;
    r[0] = h[0] = 1;

```

```

for (j = 2; j <= n; j *= 2)
{
    f = bj = der(b % j); k = j / 2; fill(k + all(bj), 0);
    h.dft(); u = der(r) & h;
    v = (r & h) % j - 1 & bj;
    for (i = 0; i < k; i++) f[i + k] = (p * p + u[i] - v[i] - f[i] - f[i + k]) % p, f[i] =
        0;
    f[k - 1] = (f[j - 1] + v[k - 1]) % p;
    u = (r %= j) & integral(f);
    for (i = k; i < j; i++) r[i] = (p - u[i]) % p;
    if (j < n) h = ~r;
}
return r;
} //5e5 420ms
optional<ll> mosqrt(ll x)
{
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    static ll W;
    struct P
    {
        ll x, y;
        P operator*(const P& a) const
        {
            return {(x * a.x + y * a.y % p * W) % p, (x * a.y + y * a.x) % p};
        }
    };
    if (x == 0) return {0};
    if (ksm(x, p - 1 >> 1) != 1) return { };
    ll y;
    do y = rnd() % p; while (ksm(W = (y * y % p + p - x) % p, p - 1 >> 1) <= 1); //not for p=2
    y = [&](P x, ll y)
    {
        P r{1, 0};
        while (y)
        {
            if (y & 1) r = r * x;
            x = x * x; y >>= 1;
        }
        return r.x;
    }({y, 1}, p + 1 >> 1);
    return {y * 2 < p ? y : p - y};
}
optional<Q> sqrt(Q f)
{
    const static ll i2 = p + 1 >> 1;
    Q r;
    int n = f.size(), i, l;

    for (i = 0; i < n; i++) if (f[i]) break;
    if (i == n) return f;
    if (i & 1) return { };
    l = i / 2;
    copy(i + all(f), f.begin());
    fill(n - i + all(f), 0);

    auto rt = mosqrt(f[0]);
    if (rt) r[0] = rt.value(); else return { };
}

```

```

    for (i = 2; i <= n; i *= 2) r = (sqr(r) + f % i) / (r % i) % i * i2;

    copy_backward(all(r) - 1, r.end());
    fill_n(r.begin(), 1, 0);

    return {r};
} //5e5 530ms, sqrt(0 0 4 2 3)=(0 2 499122177 311951361 171573248)
optional<Q> sqrt_new(Q f)
{
    const static ll i2 = p + 1 >> 1;
    Q q, r;
    int n = f.size(), i, j, k, l;

    for (i = 0; i < n; i++) if (f[i]) break;
    if (i == n) return f;
    if (i & 1) return { };
    l = i / 2;
    copy(i + all(f), f.begin());
    fill(n - i + all(f), 0);

    auto rt = mosqrt(f[0]);
    if (rt) r[0] = rt.value(); else return { };
    for (j = 2; j <= n; j *= 2)
    {
        k = j / 2; (q = r).dft(); (q &= q) %= j;
        for (i = k; i < j; i++) q[i] = (q[i - k] + p * 2 - f[i] - f[i - k]) * i2 % p, q[i - k]
            = 0;
        q &= ~r % j; r %= j;
        for (i = k; i < j; i++) r[i] = (p - q[i]) % p;
    }

    copy_backward(all(r) - 1, r.end());
    fill_n(r.begin(), 1, 0);

    return {r};
} //5e5 280ms
Q pow(Q b, ll m) //不应传入超过 int 内容
{
    assert(m <= 1llu << 32);
    int n = b.size(), i, j = n, k;
    for (i = 0; i < n; i++) if (b[i]) { j = i; break; }
    if (j == n) return b[0] = !m, b;
    if (j * m >= n) return Q(n);
    copy(j + all(b), b.begin());
    fill(n - j + all(b), 0);
    k = b[0]; j *= m;
    b = exp_new(ln(b * ksm(k, p - 2)) * m) * ksm(k, m);
    copy_backward(all(b) - j, b.end());
    fill_n(b.begin(), j, 0);
    return b;
}
Q pow(Q b, string s)
{
    int n = b.size(), i, j = n, k;
    for (i = 0; i < n; i++) if (b[i]) { j = i; break; }
    if (j == n) return b[0] = s == "0", b;
    if (j && (s.size() > 8 || j * stoll(s) >= n)) return Q(n);

```

```

    ll m0 = 0, m1 = 0;
    for (auto c : s) m0 = (m0 * 10 + c - '0') % p, m1 = (m1 * 10 + c - '0') % (p - 1);
    copy(j + all(b), b.begin());
    fill(n - j + all(b), 0);
    k = b[0]; j *= m0;
    b = exp_new(ln(b * ksm(k, p - 2)) * m0) * ksm(k, m1);
    copy_backward(all(b) - j, b.end());
    fill_n(b.begin(), j, 0);
    return b;
} // 5e5 1e18 700ms
Q pow2(Q b, ll m)
{
    int n = b.size();
    Q r(n); r[0] = 1;
    while (m)
    {
        if (m & 1) (r *= b) %= n;
        if (m >>= 1) b = sqr(b) % n;
    }
    return r;
} // 5e5 1e18 7425ms
Q div(Q f, Q g)
{
    int n = 0, m = 0, i;
    for (i = f.size() - 1; i >= 0; i--) if (f[i]) { n = i + 1; break; }
    for (i = g.size() - 1; i >= 0; i--) if (g[i]) { m = i + 1; break; }
    assert(m);
    if (n < m) return Q(1);
    reverse(f.begin(), f.begin() + n);
    reverse(g.begin(), g.begin() + m);
    n = n - m + 1; m = cal(n);
    f = (f % m) / (g % m) % m;
    fill(n + all(f), 0);
    reverse(f.begin(), f.begin() + n);
    return f;
}
Q mod(const Q& a, const Q& b)
{
    if (a.deg() < b.deg()) return shrink(a);
    Q r = (a - b * div(a, b));
    return shrink(r %= min(r.size(), b.size()));
}
Q pow(Q x, ll y, Q f)
{
    Q r(1);
    r[0] = 1;
    while (y)
    {
        if (y & 1) r = mod(r * x, f);
        if (y >>= 1) x = mod(sqr(x), f);
    }
    return r;
}
pair<Q, Q> div_mod(const Q& a, const Q& b) { Q q = div(a, b); Q r = (a - b * q); return {q, r
    %= min(r.size(), b.size())}; }
// 5e5 430ms (1 2 3 4)=(916755018 427819009)*(5 6 7)+(407446676 346329673)
// Q cdq_inv(const Q &f) { return (~(f-1))*(p-1); } // g_0=1, g_i=g_{i-j}*f_j ?

```

```

11 recurrent(const vector<ll>& f, const vector<ll>& a, ll m)//常系数齐次线性递推, find  $a_m, a_n = a_{n-i} * f_i, f_1 \dots k, a_0 \dots k-1$ 
{
    if (m < a.size()) return a[m];
    assert(f.size() == a.size() + 1 && f[0] == 0);
    int k = a.size(), n = cal(k + 1) * 2, i;
    ll ans = 0;
    Q h(n), g(2);
    for (i = 1; i <= k; i++) h[k - i] = (p - f[i]) % p;
    h[k] = g[1] = 1;
    Q r = pow(g, m, h);
    k = min(k, (int)r.size());
    for (i = 0; i < k; i++) ans = (ans + a[i] * r[i]) % p;
    return ans;
} //1e5 1e18 8500ms

11 recurrent_new(const vector<ll>& f, const vector<ll>& a, ll m)//常系数齐次线性递推, find  $a_m, a_n = a_{n-i} * f_i, f_1 \dots k, a_0 \dots k-1$ 
{
    const static ll i2 = p + 1 >> 1;
    if (m < a.size()) return a[m];
    assert(f.size() == a.size() + 1 && f[0] == 0);
    int k = a.size(), n = cal(k + 1), i;
    Q g(n * 2), h(n * 2);
    for (h[0] = i = 1; i <= k; i++) h[i] = (p - f[i]) % p;
    copy(all(a), g.begin());
    g &= h; fill(k++ + all(g), 0);
    vector<ll> res(n);
    while (m)
    {
        if (m & 1)
        {
            ll x = p - g[0];
            for (i = 1; i < k; i += 2) res[i >> 1] = x * h[i] % p;
            copy_n(g.begin() + 1, k - 1, g.begin());
            g[k - 1] = 0;
        }
        g.dft(); h.dft();
        ll* a = g.data(), * b = h.data(), * c = a + n, * d = b + n;
        for (i = 0; i < n; i++) g[i] = (a[i] * d[i] + b[i] * c[i]) % p * i2 % p;
        for (i = 0; i < n; i++) h[i] = h[i] * h[i ^ n] % p;
        g.hf_dft(); h.hf_dft();
        fill(k + all(g), 0);
        if (m & 1) for (i = 0; i < k; i++) (g[i] += res[i]) %= p;
        fill(k + all(h), 0);
        m >>= 1;
    }
    assert(h[0] == 1);
    return g[0];
} //1e5 1e18 1000ms

vector<ll> recurrent_interval(const vector<ll>& f, const vector<ll>& a, ll L, ll R)//常系数齐次线性递推, find  $a_{[L,R]}, a_n = a_{n-i} * f_i, f_1 \dots k, a_0 \dots k-1$ 
{
    assert(f.size() == a.size() + 1 && f[0] == 0);
    int k = a.size(), n = cal(k + 1) * 2, i, len = R - L;
    ll ans = 0, m = L;
    Q h(n), g(2), r;
    for (i = 1; i <= k; i++) h[k - i] = (p - f[i]) % p;

```

```

    h[k] = g[1] = r[0] = 1;
    while (m)
    {
        if (m & 1) r = mod(r * g, h);
        if (m >>= 1) g = mod(sqr(g), h);
    }
    Q F(f), A(a);
    F[0] = p - 1;
    A *= F;
    A %= cal(k);
    fill(k + all(A), 0);
    n = cal(len + k);
    F %= n;
    A *= ~F;
    r %= cal(k);
    reverse(r.begin(), r.begin() + k);
    r *= A;
    r.erase(r.begin(), r.begin() + k - 1);
    r.resize(len);
    return r;
} // 1e5 1e18 5e5 10000ms
Q prod(const vector<Q>& a)
{
    if (!a.size()) return {1};
    function<Q(int, int)> dfs = [&](int l, int r)
    {
        if (r - l == 1) return a[l];
        int m = l + r >> 1;
        return shrink(dfs(l, m) * dfs(m, r));
    };
    return dfs(0, a.size());
} // not check
Q prod_new(const vector<Q>& a)
{
    if (!a.size()) return {1};
    struct cmp
    {
        bool operator()(const Q& f, const Q& g) const { return f.size() > g.size(); }
    };
    priority_queue<Q, vector<Q>, cmp> q(all(a));
    while (q.size() > 1)
    {
        auto f = q.top(); q.pop();
        f = shrink(f * q.top()); q.pop();
        q.push(f);
    }
    return q.top();
} // not check
vector<ll> evaluation(const Q& f, const vector<ll>& X)
{
    int m = X.size(), n = f.size() - 1, i, j;
    vector<Q> pro(m * 4 + 4);
    while (n > 1 && !f[n]) --n;
    vector<ll> y(m);
    function<void(int, int, int)> build = [&](int x, int l, int r)
    {
        if (l + 1 == r)

```

```

    {
        pro[x] = Q(vector{(p - X[l]) % p, 1llu});
        return;
    }
    int mid = l + r >> 1, c = x * 2;
    build(c, l, mid); build(c + 1, mid, r);
    pro[x] = shrink(pro[c] * pro[c + 1]);
};

function<void(int, int, int, Q, int)> dfs = [&](int x, int l, int r, Q f, int d)
{
    const static int limit = 256;
    if (d >= r - l) f = shrink(mod(f, pro[x]));
    if (r - l < limit)
    {
        for (int i = l; i < r; i++) y[i] = f.fx(X[i]);
        return;
    }
    int mid = l + r >> 1, c = x * 2;
    dfs(c, l, mid, f, d);
    dfs(c + 1, mid, r, f, d);
};

build(1, 0, m);
dfs(1, 0, m, f, n);
return y;
} //131072 880ms
vector<ll> evaluation_new(Q f, const vector<ll>& X) //多项式多点求值
{
    int m = X.size(), i, j;
    vector<ll> y(m);
    if (X.size() <= 10)
    {
        for (i = 0; i < m; i++) y[i] = f.fx(X[i]);
        return y;
    }
    int n = f.size();
    while (n > 1 && !f[n - 1]) --n;
    f.resize(cal(n));
    vector<Q> pro(m * 4 + 4);
    function<void(int, int, int)> build = [&](int x, int l, int r)
    {
        if (l == r)
        {
            pro[x] = Q(vector{1llu, (p - X[l]) % p});
            return;
        }
        int m = l + r >> 1, c = x * 2;
        build(c, l, m); build(c + 1, m + 1, r);
        pro[x] = shrink(pro[c] * pro[c + 1]);
    };
    function<void(int, int, int, Q)> dfs = [&](int x, int l, int r, Q f)
    {
        const static int limit = 30;
        if (r - l + 1 <= limit)
        {
            int m = r - l + 1, m1, m2, mid = l + r >> 1, i, j, k;
            static ll g[limit + 2], g1[limit + 2], g2[limit + 2];
            m1 = m2 = r - l;

```

```

    copy_n(f.data(), m, g1);
    copy_n(g1, m, g2);
    for (i = mid + 1; i <= r; i++, --m1) for (k = 0; k < m1; k++) g1[k] = (g1[k] +
        g1[k + 1] * (p - X[i])) % p;
    for (i = 1; i <= mid; i++, --m2) for (k = 0; k < m2; k++) g2[k] = (g2[k] + g2[k
        + 1] * (p - X[i])) % p;
    for (i = 1; i <= mid; i++)
    {
        copy_n(g1, (m = m1) + 1, g);
        for (j = 1; j <= mid; j++) if (i != j)
        {
            for (k = 0; k < m; k++) g[k] = (g[k] + g[k + 1] * (p - X[j])) % p;
            --m;
        }
        y[i] = g[0];
    }
    for (i = mid + 1; i <= r; i++)
    {
        copy_n(g2, (m = m2) + 1, g);
        for (j = mid + 1; j <= r; j++) if (i != j)
        {
            for (k = 0; k < m; k++) g[k] = (g[k] + g[k + 1] * (p - X[j])) % p;
            --m;
        }
        y[i] = g[0];
    }
    return;
}
int mid = l + r >> 1, c = x * 2, n = f.size();
f.dft();
for (auto [x, len] : {pair{c, r - mid}, {c + 1, mid - 1 + 1}})
{
    pro[x] %= n;
    reverse(all(pro[x])); pro[x] &= f;
    rotate(all(pro[x]) - 1, pro[x].end());
    pro[x] %= cal(len);
    fill(len + all(pro[x]), 0);
}
dfs(c, 1, mid, pro[c + 1]);
dfs(c + 1, mid + 1, r, pro[c]);
};
build(1, 0, m - 1);
pro[1] %= f.size();
(f ^= ~pro[1]) %= cal(m);
fill(min(m, n) + all(f), 0);
dfs(1, 0, m - 1, f);
return y;
} //131072 460ms
ll factorial(ll n)
{
    if (n >= p) return 0;
    if (n <= 1) return 1 % p;
    ll B = ::sqrt(n), i;
    vector F(B, Q({0, 1}));
    for (i = 0; i < B; i++) F[i][0] = i + 1;
    auto f = prod(F);
    vector<ll> x(B);

```



```

    for (i = 0; i < B; i++) x[i] = i * B;
    ll r = 1;
    auto y = evaluation(f, x);
    for (i = 0; i < B; i++) r = r * y[i] % p;
    for (i = B * B + 1; i <= n; i++) r = r * i % p;
    return r;
} // 998244352 170ms
vector<ll> getinvs(vector<ll> a)
{
    int n = a.size(), i;
    if (n <= 2)
    {
        for (i = 0; i < n; i++) a[i] = ksm(a[i], p - 2);
        return a;
    }
    vector<ll> l(n), r(n);
    l[0] = a[0]; r[n - 1] = a[n - 1];
    for (i = 1; i < n; i++) l[i] = l[i - 1] * a[i] % p;
    for (i = n - 2; i; i--) r[i] = r[i + 1] * a[i] % p;
    ll x = ksm(l[n - 1], p - 2);
    a[0] = x * r[1] % p; a[n - 1] = x * l[n - 2] % p;
    for (i = 1; i < n - 1; i++) a[i] = x * l[i - 1] % p * r[i + 1] % p;
    return a;
}
Q interpolation(const vector<ll>& X, const vector<ll>& y) // 多项式快速插值
{
    assert(X.size() == y.size());
    int n = X.size(), i, j;
    if (n <= 1) return Q(y);
    if (1)
    {
        auto vv = X; sort(all(vv));
        assert(unique(all(vv)) - vv.begin() == n);
    }
    vector<Q> sum(4 * n + 4), pro(4 * n + 4);
    function<void(int, int, int)> build = [&](int x, int l, int r)
    {
        if (l == r)
        {
            sum[x] = Q(vector{(p - X[l]) % p, 1llu});
            return;
        }
        int mid = l + r >> 1, c = x * 2;
        build(c, l, mid); build(c + 1, mid + 1, r);
        sum[x] = shrink(sum[c] * sum[c + 1]);
    };
    build(1, 0, n - 1);
    auto v = evaluation_new(sum[1] = der(sum[1]), X);
    assert(v.size() == n);
    auto Y = getinvs(v);
    for (i = 0; i < n; i++) Y[i] = Y[i] * y[i] % p;
    function<void(int, int, int)> dfs = [&](int x, int l, int r)
    {
        if (l == r)
        {
            pro[x][0] = Y[l];
            return;

```

```

    }
    int c = x * 2, mid = l + r >> 1;
    dfs(c, l, mid); dfs(c | 1, mid + 1, r);
    pro[x] = shrink((pro[c] * sum[c | 1]) + (pro[c | 1] * sum[c]));
};
dfs(1, 0, n - 1);
return pro[1] %= cal(n);
} //131072 1150ms
Q comp(const Q& f, Q g) //多项式复合  $f(g(x)) = [x^i]f(x)g(x)^i$ 
{
    int n = f.size(), l = ceil(::sqrt(n)), i, j;
    assert(n >= g.size()); //返回  $n-1$  次多项式
    vector<Q> a(l + 1), b(1);
    a[0] %= n; a[0][0] = 1; a[1] = g;
    g %= n * 2;
    Q u = g, v(n);
    g.dft();
    for (i = 2; i <= l; i++) a[i] = ((u &= g) %= n), u %= n * 2;
    for (i = 2; i < l; i++)
    {
        u.dft(); b[i - 1] = u;
        u &= b[1]; fill(n + all(u), 0);
    }
    u.dft(); b[l - 1] = u;
    for (i = 0; i < l; i++)
    {
        fill(all(v), 0);
        for (j = 0; j < l; j++) if (i * l + j < n) v += a[j] * f[i * l + j];
        if (i == 0) u = v; else u += ((v %= n * 2) &= b[i]) %= n;
    }
    return u;
} //n^2+n*sqrt n*log n, 8000 350ms
Q comp_inv(Q f) //多项式复合逆  $g(f(x))=x$ , 求  $g$ ,  $[x^n]g = ([x^{n-1}](x/f)^n)/n$ , 要求常数 0 一次非 0
{
    assert(!f[0] && f[1]);
    int n = f.size(), l = ceil(::sqrt(n)), i, j, k, m; //l>=2
    rotate(f.begin(), 1 + all(f));
    f = ~f;
    getfac(n * 2);
    vector<Q> a(l + 1), b(1);
    Q u, v;
    u = a[1] = f;
    u %= n * 2; (v = u).dft();
    for (i = 2; i <= l; i++)
    {
        u &= v;
        fill(n + all(u), 0);
        a[i] = u;
    }
    b[0] %= n; b[0][0] = 1; b[1] = u; (v = u).dft();
    for (i = 2; i < l; i++)
    {
        u &= v;
        fill(n + all(u), 0);
        b[i] = u;
    }
    u %= n; u[0] = 0;

```

```

for (i = 0; i < l; i++) for (j = 1; j <= l; j++) if (i * l + j < n)
{
    m = i * l + j - 1;
    ll r = 0, * f = b[i].data(), * g = a[j].data();
    for (k = 0; k <= m; k++) r = (r + f[k] * g[m - k]) % p;
    u[m + 1] = r * inv[m + 1] % p;
}
return u;
} // 8000 200ms
Q shift(Q f, ll c) // get f(x+c), c \in [0, p)
{
    int n = f.size(), i, j;
    Q g(n);
    getfac(n);
    for (i = 0; i < n; i++) (f[i] *= fac[i]) %= p;
    g[0] = 1;
    for (i = 1; i < n; i++) g[i] = g[i - 1] * c % p;
    for (i = 0; i < n; i++) (g[i] *= ifac[i]) %= p;
    f ^= g;
    for (i = 0; i < n; i++) (f[i] *= ifac[i]) %= p;
    return f;
} // 5e5 200ms (1 2 3 4 5) 3 -> (547 668 309 64 5)
vector<ll> shift(vector<ll> y, ll c, ll m) // [0, n) 点值 -> [c, c+m) 点值
{
    assert(y.size());
    if (y.size() == 1) return vector(m, y[0]);
    vector<ll> r, res;
    r.reserve(m);
    int n = y.size(), i, j, mm = m;
    while (c < n && m) r.push_back(y[c++]), --m;
    if (c + m > p)
    {
        res = shift(y, 0, c + m - p);
        m = p - c;
    }
    if (!m) { r.insert(r.end(), all(res)); return r; }
    int len = cal(m + n - 1), l = m + n - 1;
    for (i = n & 1; i < n; i += 2) y[i] = (p - y[i]) % p;
    getfac(n);
    for (i = 0; i < n; i++) y[i] = y[i] * ifac[i] % p * ifac[n - 1 - i] % p;
    y.resize(len);
    Q f, g;
    vector<ll> v(m + n - 1);
    c -= n - 1;
    for (i = 0; i < l; i++) v[i] = (c + i) % p;
    f = Q(y); g = Q(getinvs(v)) % len;
    f *= g;
    vector<ll> u(m);
    for (i = n - 1; i < l; i++) u[i - (n - 1)] = f[i];
    v.resize(m);
    for (i = 0; i < m; i++) v[i] = c + i;
    v = getinvs(v); c += n;
    ll tmp = 1;
    for (i = c - n; i < c; i++) tmp = tmp * i % p;
    for (i = 0; i < m; i++) (u[i] *= tmp) %= p, tmp = tmp * (c + i) % p * v[i] % p;
    r.insert(r.end(), all(u));
    r.insert(r.end(), all(res));
}

```

```

    assert(r.size() == mm);
    return r;
} // 5e5 430ms, (1 4 9 16) 3 5 -> (16 25 36 49 64)
vector<ll> Z_transform(Q f, ll c, ll m) // 求  $f(c^{[0,m]})$ 。核心  $ij=C(i+j,2)-C(i,2)-C(j,2)$ 
{
    const static ll B = 1e5;
    static ll a[B + 2], b[B + 2];
    int i, n = f.size();
    if (n * m < B * 5)
    {
        vector<ll> r(m);
        ll j;
        for (i = 0, j = 1; i < m; i++) r[i] = f.fx(j), j = j * c % p;
        return r;
    }
    auto mic = [&](ll x) { return a[x % B] * b[x / B] % p; };
    ll l = cal(m += n - 1);
    Q g(l);
    assert(B * B > p);
    a[0] = b[0] = g[0] = g[1] = 1;
    for (i = 1; i <= B; i++) a[i] = a[i - 1] * c % p;
    for (i = 1; i <= B; i++) b[i] = b[i - 1] * a[B] % p;
    for (i = 2; i < n; i++) f[i] = f[i] * mic((p * 2 - 2 - i) * (i - 1) / 2 % (p - 1)) % p;
    for (i = 2; i < m; i++) g[i] = mic(i * (i - 1llu) / 2 % (p - 1));
    reverse(all(f)); (f %= 1) &= g;
    vector<ll> r(f.begin() + n - 1, f.begin() + m); m -= n - 1;
    for (i = 2; i < m; i++) r[i] = r[i] * mic((p * 2 - 2 - i) * (i - 1) / 2 % (p - 1)) % p;
    return r;
} // luogu 1e6 500ms
vector<ll> Bell(int n) // B(0...n)
{
    ++n;
    getfac(n - 1);
    Q f(n);
    int i;
    for (i = 1; i < n; i++) f[i] = ifac[i];
    f = exp_new(f);
    for (i = 2; i < n; i++) f[i] = f[i] * fac[i] % p;
    return vector<ll>(f.begin(), f.begin() + n);
} // not check
vector<ll> S1_row(int n, int m) // S1(n, 0...m), 0(nlogn), unsigned
{
    int cm = cal(++m);
    if (n == 0)
    {
        vector<ll> r(m);
        r[0] = 1;
        return r;
    }
    function<Q(int)> dfs = [&](int n)
    {
        if (n == 1)
        {
            Q f(2);
            f[1] = 1;
            return f;
        }
    }

```

```

    Q f = dfs(n / 2);
    f *= shift(f, n / 2);
    if (n & 1)
    {
        f %= cal(n + 1);
        for (int i = n; i; i--) f[i] = f[i - 1];
        // for (int i=1; i<=n; i++) f[i]=f[i-1];
        --n;
        for (int i = 0; i <= n; i++) f[i] = (f[i] + f[i + 1] * n) % p;
    }
    if (f.size() > cm) f %= cm;
    return f;
};
Q f = dfs(n);
if (f.size() < cm) f %= cm;
return vector<ll>(f.begin(), f.begin() + m);
}
vector<ll> S1_column(int n, int m)//S1(0...n,m),0(nlogn)
{
    if (m == 0)
    {
        vector<ll> r(n + 1);
        r[0] = 1;
        return r;
    }
    Q f(n + 1);
    getfac(max(n, m));
    int i;
    for (i = 1; i <= n; i++) f[i] = inv[i];
    f = pow(f, m);
    for (i = m; i <= n; i++) f[i] = f[i] * fac[i] % p * ifac[m] % p;
    return vector<ll>(f.begin(), f.begin() + n + 1);
}
vector<ll> S2_row(int n, int m)//S2(n,0...m),0(mlogm)
{
    int tm = ++m, i, j, cnt = 0;
    if (n == 0)
    {
        vector<ll> r(m);
        r[0] = 1;
        return r;
    }
    m = min(m, n + 1);
    vector<ll> pr(m), pw(m);
    pw[1] = 1;
    for (i = 2; i < m; i++)
    {
        if (!pw[i]) pr[cnt++] = i, pw[i] = ksm(i, n);
        for (j = 0; i * pr[j] < m; j++)
        {
            pw[i * pr[j]] = pw[i] * pw[pr[j]] % p;
            if (i % pr[j] == 0) break;
        }
    }
    getfac(m - 1);
    Q f(m), g(m);
    for (i = 0; i < m; i += 2) f[i] = ifac[i];

```

```

    for (i = 1; i < m; i += 2) f[i] = p - ifac[i];
    // for (i=1; i<m; i++) g[i]=pw[i]*ifac[i]%p;
    for (i = 1; i < m; i++) g[i] = ksm(i, n) * ifac[i] % p;
    f *= g;
    vector<ll> r(f.begin(), f.begin() + m);
    r.resize(tm);
    return r;
} //5e5 150ms
vector<ll> S2_column(int n, int m) //S2(0...n,m), 0(nlogn)
{
    if (m == 0)
    {
        vector<ll> r(n + 1);
        r[0] = 1;
        return r;
    }
    Q f(n + 1);
    getfac(max(n, m));
    int i;
    for (i = 1; i <= n; i++) f[i] = ifac[i];
    f = pow(f, m);
    for (i = m; i <= n; i++) f[i] = f[i] * fac[i] % p * ifac[m] % p;
    return vector<ll>(f.begin(), f.begin() + n + 1);
} //5e5 640ms
vector<ll> signed_S1_row(int n, int m)
{
    auto v = S1_row(n, m);
    for (int i = 1 ^ n & 1; i <= m; i += 2) v[i] = (p - v[i]) % p;
    return v;
} //5e5 190ms
vector<ll> Bernoulli(int n) //B(0...n)
{
    getfac(++n);
    int i;
    Q f(n);
    for (i = 0; i < n; i++) f[i] = ifac[i + 1];
    f = ~f;
    for (i = 0; i < n; i++) f[i] = f[i] * fac[i] % p;
    return vector<ll>(f.begin(), f.begin() + n);
} //5e5 180ms
vector<ll> Partition(int n) //P(0...n), 拆分数
{
    Q f(++n);
    int i, l = 0, r = 0;
    while (--l) if (3 * l * l - l >= n * 2) break;
    while (++r) if (3 * r * r - r >= n * 2) break;
    ++l;
    for (i = 1 + abs(l) % 2; i < r; i += 2) f[3 * i * i - i >> 1] = 1;
    for (i = 1 + abs(l + 1) % 2; i < r; i += 2) f[3 * i * i - i >> 1] = p - 1;
    f = ~f;
    return vector<ll>(f.begin(), f.begin() + n);
} //5e5 150ms
struct reg
{
    Q a00, a01, a10, a11;
    reg operator*(const reg& o) const
    {

```

```

    return {
        shrink(a00 * o.a00 + a01 * o.a10),
        shrink(a00 * o.a01 + a01 * o.a11),
        shrink(a10 * o.a00 + a11 * o.a10),
        shrink(a10 * o.a01 + a11 * o.a11)};
}
pair<Q, Q> operator*(const pair<Q, Q>& o) const
{
    const auto& [b0, b1] = o;
    return {shrink(a00 * b0 + a01 * b1), shrink(a10 * b0 + a11 * b1)};
}
} E = {{vector{1llu}}, Q(), Q(), {vector{1llu}}};
ostream& operator<<(ostream& cout, const reg& o)
{
    return cout << "[" << o.a00 << ",□" << o.a01 << "]\n"
        << "[" << o.a10 << ",□" << o.a11 << "]\n";
}
reg hgcd(Q a, Q b)
{
    int m = a.deg() + 1 >> 1;
    if (b.deg() < m) return E;
    reg r = hgcd(a >> m, b >> m);
    auto [c, d] = r * pair{a, b};
    if (d.deg() < m) return r;
    auto [q, e] = div_mod(c, d);
    r.a00 -= shrink(q * r.a10);
    r.a01 -= shrink(q * r.a11);
    swap(r.a00, r.a10);
    swap(r.a01, r.a11);
    if (e.deg() < m) return r;
    int k = 2 * m - d.deg();
    auto s = hgcd(d >> k, e >> k);
    return s * r;
}
Q gcd(Q a, Q b)
{
    if (a.deg() < b.deg()) swap(a, b);
    while (b.deg() >= 0)
    {
        a = mod(a, b);
        swap(a, b);
        auto tmp = hgcd(a, b);
        tie(a, b) = tmp * pair{a, b};
    }
    if (a.deg() == -1) return a;
    ll k = ksm(a[a.deg()], p - 2);
    for (int i = 0; i < a.size(); i++) a[i] = a[i] * k % p;
    return a;
}
vector<ll> root(Q f)
{
    Q x(2);
    x[1] = 1;
    x = pow(x, p, f);
    if (x.size() < 2) x %= 2;
    (x[1] += p - 1) %= p;
    f = gcd(f, x);
}

```

```

vector<ll> res;
static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
function<void(Q)> dfs = [&](Q f)
{
    int n = f.deg(), i;
    if (n <= 0) return;
    if (n == 1)
    {
        res.push_back((p - f[0]) % p);
        return;
    }
    Q g(n);
    for (i = 0; i < n; i++) g[i] = rnd() % p;
    g = gcd(pow(g, (p - 1) / 2, f) - 1, f);
    dfs(g); dfs(div(f, g));
};
dfs(f);
sort(all(res));
assert(unique(all(res)) == res.end());
return res;
} // 4000 950ms
optional<Q> inverse(Q a, Q m)
{
    Q b = m;
    vector<pair<reg, Q>> buf;
    a = mod(a, b);
    swap(a, b);
    while (b.deg() >= 0)
    {
        auto [q, r] = div_mod(a, b);
        swap(a, r); swap(a, b);
        auto tmp = hgcd(a, b);
        tie(a, b) = tmp * pair{a, b};
        buf.emplace_back(move(tmp), q);
    }
    if (a.deg()) return { };
    reg res = E;
    reverse(all(buf));
    for (const auto& [tmp, q] : buf)
    {
        res = res * tmp;
        res.a00 -= shrink(q * res.a01);
        res.a10 -= shrink(q * res.a11);
        swap(res.a00, res.a01);
        swap(res.a10, res.a11);
    }
    return {res.a01 * ksm(a[0], p - 2)};
} // 5e4 950ms
}
using NTT::p;
using poly = NTT::Q;

```

### 3.34 MTT

```

namespace MTT
{

```



```

template<ll p> constexpr ll ksm(ll x,ll y=p-2)
{
    ll r=1;
    while (y)
    {
        if (y&1) r=r*x%p;
        x=x*x%p;
        y>>=1;
    }
    return r;
}
int cal(int x) { return 1<<__lg(max(x,1)*2-1); }
const int N=1<<22;
const ll p=1e9+7,g=3,
    p1=469'762'049,p2=998'244'353,p3=1004'535'809, //三模, 原根都是 3, 非常好
    inv_p1=ksm<p2>(p1),inv_p12=ksm<p3>(p1*p2%p3),_p12=p1*p2%p; //三模, 1 关于 2 逆, 1*2 关于 3
    逆, 1*2 mod 3
int r[N];
struct P
{
    ll v1,v2,v3;
    P operator+(const P &o) const { return {v1+o.v1,v2+o.v2,v3+o.v3}; }
    P operator-(const P &o) const { return {v1+p1-o.v1,v2+p2-o.v2,v3+p3-o.v3}; }
    P operator*(const P &o) const { return {v1*o.v1,v2*o.v2,v3*o.v3}; }
    void operator+=(const P &o) { v1+=o.v1,v2+=o.v2,v3+=o.v3; }
    void operator-=(const P &o) { v1+=p1-o.v1,v2+=p2-o.v2,v3+=p3-o.v3; }
    void operator*=(const P &o) { v1*=o.v1,v2*=o.v2,v3*=o.v3; }
    void mod() { v1%=p1,v2%=p2,v3%=p3; }
};
P w[N];
void init(int n)
{
    static int pr=0,pw=0;
    if (pr==n) return;
    int b=__lg(n)-1,i,j,k;
    for (i=1; i<n; i++) r[i]=r[i>>1]>>1|(i&1)<<b;
    if (pw<n)
    {
        for (j=1; j<n; j=k)
        {
            k=j*2;
            P wn={ksm<p1>(g,(p1-1)/k),ksm<p2>(g,(p2-1)/k),ksm<p3>(g,(p3-1)/k)};
            w[j]={1,1,1};
            for (i=j+1; i<k; i++) w[i]=w[i-1]*wn,w[i].mod();
        }
        pw=n;
    }
    pr=n;
}
void dft(vector<P> &a,int o=0)
{
    int n=a.size(),i,j,k;
    P *f,*g,*wn,*b=a.data(),x,y;
    init(n);
    for (i=1; i<n; i++) if (i<r[i]) swap(a[i],a[r[i]]);
    for (k=1; k<n; k*=2)
    {

```

```

        wn=w+k;
        for (i=0; i<n; i+=k*2)
        {
            f=b+i; g=b+i+k;
            for (j=0; j<k; j++)
            {
                y=g[j]*wn[j];
                y.mod();
                g[j]=f[j]-y;
                f[j]+=y;
            }
        }
        if (k*2==n || k==1<<14) for (P &x:a) x.mod();
    }
    if (o)
    {
        x={ksm<p1>(n),ksm<p2>(n),ksm<p3>(n)};
        for (P &y:a) y*=x,y.mod();
        reverse(1+all(a));
    }
}
struct Q:vector<ll>
{
    Q(int x=1):vector(x) { }
    Q &operator%=(int n) { resize(n); return *this; }
};
Q &operator*=(Q &f,const Q &g)
{
    int n=f.size()+g.size()-1,m=cal(n),i;
    vector<P> F(m,{0,0,0}),G(m,{0,0,0});
    for (i=0; i<f.size(); i++) F[i]={f[i]%p1,f[i]%p2,f[i]%p3};
    for (i=0; i<g.size(); i++) G[i]={g[i]%p1,g[i]%p2,g[i]%p3};
    dft(F); dft(G);
    for (i=0; i<m; i++) F[i]*=G[i],F[i].mod();
    dft(F,1);
    f%=n;
    ll x;
    for (i=0; i<n; i++)
    {
        auto [r1,r2,r3]=F[i];
        x=(r2+p2-r1)*inv_p1%p2*p1+r1;
        f[i]=((x+p3-r3)%p3*(p3-inv_p12)%p3*_p12+x)%p;
    }
    return f;
} //5e5 440ms
Q operator*(Q f,const Q &g) { return f*=g; }
}
using MTT::p;
using poly=MTT::Q;

```

### 3.35 FFT

```

namespace FFT
{
    #define all(x) (x).begin(),(x).end()
    typedef double db;

```

```

const int N=1<<21;
const db pi=3.14159265358979323846;
struct comp
{
    db x,y;
    comp operator+(const comp &o) const {return {x+o.x,y+o.y};}
    comp operator-(const comp &o) const {return {x-o.x,y-o.y};}
    comp operator*(const comp &o) const {return {x*o.x-y*o.y,o.x*y+x*o.y};}
    comp operator*(const db &o) const {return {x*o,y*o};}
    void operator*=(const comp &o) {*this={x*o.x-y*o.y,o.x*y+x*o.y};}
    void operator*=(const db &o) {x*=o;y*=o;}
    void operator/=(const db &o) {x/=o;y/=o;}
    comp operator/(const comp &o) const
    {
        db z=1/(o.x*o.x+o.y*o.y);
        return {z*(x*o.x+y*o.y),z*(o.x*y-x*o.y)};
    } //not necessary, no check
};
long long dtol(const double &x) {return fabs(round(x));}
const comp I{0,-1};
ostream & operator<<(ostream &cout,const comp &o) {cout<<o.x;if (o.y>=0) cout<<'+';return cout
    <<o.y<<'i';}
int r[N];
char c;
comp Wn[N];
void init(int n)
{
    static int preone=-1;
    if (n==preone) return;
    preone=n;
    int b,i;
    b=__builtin_ctz(n)-1;
    for (i=1;i<n;i++) r[i]=r[i>>1]>>1|(i&1)<<b;
    for (i=0;i<n;i++) Wn[i]={cos(pi*i/n),sin(pi*i/n)};
}
int cal(int x) {return 1u<<32-__builtin_clz(max(x,2)-1);}
struct Q
{
    vector<comp> a;
    int deg;
    comp* pt() {return a.data();}
    Q(int n=0)
    {
        deg=n;
        a.resize(cal(n));
    }
    void dft(int xs=0)//1,0
    {
        int i,j,k,l,n=a.size(),d;
        comp w,wn,b,c,*f=pt(),*g,*a=f;
        init(n);
        if (xs) reverse(a+1,a+n); //spe
        for (i=0;i<n;i++) if (i<r[i]) swap(a[i],a[r[i]]);
        for (i=1,d=0;i<n;i=l,d++)
        {
            //wn={cos(pi/i),(xs?-1:1)*sin(pi/i)};
            l=i<<1;

```

```

        for (j=0;j<n;j+=1)
        {
            //w={1,0};
            f=a+j;g=f+i;
            for (k=0;k<i;k++)
            {
                w=Wn[k*(n>>d)];
                b=f[k];c=g[k]*w;
                f[k]=b+c;
                g[k]=b-c;
                //w*=wn;
            }
        }
    }
    if (xs) for (i=0;i<n;i++) a[i]/=n;
}
void operator|=(Q o)
{
    int n=deg+o.deg-1,m=cal(n),i;
    a.resize(m);o.a.resize(m);
    dft();o.dft();
    for (i=0;i<m;i++) a[i]*=o.a[i];
    dft(1);
    for (i=n;i<m;i++) a[i]={};
    deg=n;
}
Q operator|(Q o) const {o|=*this;return o;}
};
Q mul(Q a,const Q &b)//三次变两次, 仅实数, 注意精度
{
    int n=a.deg+b.deg-1,m=cal(n),i;
    a.a.resize(m);
    for (i=0;i<b.deg;i++) a.a[i]={a.a[i].x,b.a[i].x};
    a.dft();
    for (i=0;i<m;i++) a.a[i]*=a.a[i];
    a.dft(1);
    for (i=0;i<n;i++) a.a[i]={a.a[i].y*.5};
    for (i=n;i<m;i++) a.a[i]={};
    a.deg=n;
    return a;
}
void ddt(Q &a,Q &b)//double dft, 仅实数, 注意精度
{
    comp x,y;
    int n=a.a.size(),i;
    assert(n==b.a.size());
    for (i=0;i<n;i++) a.a[i]={a.a[i].x,b.a[i].x};
    a.dft();
    for (i=0;i<n;i++) b.a[i]={a.a[i].x,-a.a[i].y};
    reverse(b.pt()+1,b.pt()+n);
    for (i=0;i<n;i++)
    {
        x=a.a[i];y=b.a[i];
        a.a[i]=(x+y)*.5;
        b.a[i]=(y-x)*.5*I;
    }
}

```

```
}
using FFT::dtol;
```

### 3.36 约数个数和

$O(\sqrt[3]{n} \log n)$ 。

```
#include<bits/stdc++.h>
#define ll long long
#define lll __int128
using namespace std;

void myw(lll x){
    if(!x) return;
    myw(x/10);printf("%d", (int)(x%10));
}

struct vec{
    ll x,y;
    vec (ll x0=0,ll y0=0){x=x0,y=y0;}
    vec operator +(const vec b){return vec(x+b.x,y+b.y);}
};

ll N;
vec stk[1000005];int len;
vec P;
vec L,R;

bool ninR(vec a){return N<(lll)a.x*a.y;}
bool steep(ll x,vec a){return (lll)N*a.x<=(lll)x*x*a.y;}

lll Solve(){
    len=0;
    ll cbr=cbrt(N),sqr=sqrt(N);
    P.x=N/sqr,P.y=sqr+1;
    lll ans=0;
    stk[++len]=vec(1,0);stk[++len]=vec(1,1);
    while(1){
        L=stk[len--];
        while(ninR(vec(P.x+L.x,P.y-L.y)))
            ans+=(lll)P.x*L.y+(lll)(L.y+1)*(L.x-1)/2,
            P.x+=L.x,P.y-=L.y;
        if(P.y<=cbr) break;
        R=stk[len];
        while(!ninR(vec(P.x+R.x,P.y-R.y))) L=R,R=stk[--len];
        while(1){
            vec mid=L+R;
            if(ninR(vec(P.x+mid.x,P.y-mid.y))) R=stk[++len]=mid;
            else if(steep(P.x+mid.x,R)) break;
            else L=mid;
        }
    }
    for(int i=1;i<P.y;i++) ans+=N/i;
    return ans*2-sqr*sqr;
}

int T;
```

```
int main(){
    scanf("%d",&T);
    while(T--){
        scanf("%lld",&N);
        myw(Solve());printf("\n");
    }
}
```

### 3.37 万能欧几里得

题意:  $\sum_{i=0}^{n-1} \lfloor \frac{ai+b}{m} \rfloor$  ( $0 \leq a, b$ )

注意若  $b \geq m$  需要增加先往上走一步。

原理: 考虑紧贴着斜线的折线的答案。每个 `nd` 表示的是一段折线, 你需要实现 `operator+` 来计算出拼接两个折线之后的答案。除此以外的原理不必了解。

你需要传入的 `a` 和 `b` 表示向上和向右的折线的答案 (也就是边界)。

如果你发现横竖反了, 自行调整。你可以通过在 `nd` 中加一个 `string` 记录当前是什么折线来确认这一点。

```
struct nd
{
    ll x,y,sy;
    nd operator+(const nd &o) const
    {
        return {x+o.x,y+o.y,sy+o.sy+y*o.x};
    }
};
nd ksm (nd a,int k)
{
    nd res{};
    while (k)
    {
        if (k&1) res=res+a;
        a=a+a;k>>=1;
    }
    return res;
}
nd sol (int p,int q,int r,int l,nd a,nd b)//(0,l],(pi+r)/q
{
    if (!l) return {};
    if (p>=q) return sol(p%q,q,r,l,a,ksm(a,p/q)+b);
    int m=((ll)l*p+r)/q;
    if (!m) return ksm(b,l);
    int cnt=l-((ll)q*m-r-1)/p;
    return ksm(b,(q-r-1)/p)+a+sol(q,p,(q-r-1)%p,m-1,b,a)+ksm(b,cnt);
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(15);
    int T;cin>>T;
    while (T--)
    {
        int n,m,a,b;
        cin>>n>>m>>a>>b;
```

```

    nd nx={1,0,0},ny={0,1,0};
    nd ans=sol(a,m,b,n-1,ny,nx);
    cout<<ans.sy<<'\\n';
}
}

```

### 3.38 高斯整数类

圆上整点的基础。

```

ll roundiv(ll x,ll y)
{
    return x>=0?(x+y/2)/y:(x-y/2)/y;
}
struct Q
{
    ll x,y;
    Q operator~() const { return {x,-y}; }
    ll len2() const { return x*x+y*y; }
    Q operator+(const Q &o) const { return {x+o.x,y+o.y}; }
    Q operator-(const Q &o) const { return {x-o.x,y-o.y}; }
    Q operator*(const Q &o) const { return {x*o.x-y*o.y,x*o.y+y*o.x}; }
    Q operator/(const Q &o) const
    {
        Q t=*this*~o;
        ll l=o.len2();
        return {roundiv(t.x,l),roundiv(t.y,l)};
    }
    Q operator%(const Q &o) const { return *this-*this/o*o; }
};
Q gcd(Q a,Q b)
{
    if (a.len2()>b.len2()) swap(a,b);
    while (a.len2())
    {
        b=b%a;
        swap(a,b);
    }
    return b;
}

```

### 3.39 Miller Rabin/Pollard Rho

1s: 200 组  $10^{18}$ 。

如果你只需要做 `int` 以内的分解，你可以改为

```

typedef int ll;
typedef long long lll;

```

```

namespace pr
{
    typedef long long ll;
    typedef __int128 lll;
    typedef pair<ll,int> pa;
    ll ksm(ll x,ll y,const ll p)

```

```

{
    ll r=1;
    while (y)
    {
        if (y&1) r=(lll)r*x%p;
        x=(lll)x*x%p; y>>=1;
    }
    return r;
}
namespace miller
{
    const int p[7]={2,3,5,7,11,61,24251};
    ll s,t;
    bool test(ll n,int p)
    {
        if (p>=n) return 1;
        ll r=ksm(p,t,n),w;
        for (int j=0; j<s&&r!=1; j++)
        {
            w=(lll)r*r%n;
            if (w==1&&r!=n-1) return 0;
            r=w;
        }
        return r==1;
    }
    bool prime(ll n)
    {
        if (n<2||n==46'856'248'255'981) return 0;
        for (int i=0; i<7; ++i) if (n%p[i]==0) return n==p[i];
        s=__builtin_ctz(n-1); t=n-1>>s;
        for (int i=0; i<7; ++i) if (!test(n,p[i])) return 0;
        return 1;
    }
}
using miller::prime;
mt19937_64 rnd(chrono::steady_clock::now().time_since_epoch().count());
namespace rho
{
    void nxt(ll &x,ll &y,ll &p) { x=((lll)x*x+y)%p; }
    ll find(ll n,ll C)
    {
        ll l,r,d,p=1;
        l=rnd()%(n-2)+2,r=1;
        nxt(r,C,n);
        int cnt=0;
        while (l^r)
        {
            p=(lll)p*llabs(l-r)%n;
            if (!p) return gcd(n,llabs(l-r));
            ++cnt;
            if (cnt==127)
            {
                cnt=0;
                d=gcd(llabs(l-r),n);
                if (d>1) return d;
            }
            nxt(l,C,n); nxt(r,C,n); nxt(r,C,n);
        }
    }
}

```



```

    }
    return gcd(n,p);
}
vector<pa> w;
vector<ll> d;
void dfs(ll n,int cnt)
{
    if (n==1) return;
    if (prime(n)) return w.emplace_back(n,cnt),void();
    ll p=n,C=rnd()%(n-1)+1;
    while (p==1||p==n) p=find(n,C++);
    int r=1; n/=p;
    while (n%p==0) n/=p,++r;
    dfs(p,r*cnt); dfs(n,cnt);
}
vector<pa> getw(ll n)
{
    w=vector<pa>(0); dfs(n,1);
    if (n==1) return w;
    sort(w.begin(),w.end());
    int i,j;
    for (i=1,j=0; i<w.size(); i++) if (w[i].first==w[j].first) w[j].second+=w[i].second;
        else w[++j]=w[i];
    w.resize(j+1);
    return w;
}
void dfss(int x,ll n)
{
    if (x==w.size()) return d.push_back(n),void();
    dfss(x+1,n);
    for (int i=1; i<=w[x].second; i++) dfss(x+1,n*=w[x].first);
}
vector<ll> getd(ll n)
{
    getw(n); d=vector<ll>(0); dfss(0,1);
    sort(d.begin(),d.end());
    return d;
}
}
using rho::getw,rho::getd;
using miller::prime;
}
using pr::getw,pr::getd,pr::prime;

```

## 4 字符串

### 4.1 字典树 (trie 树)

```
struct trie
{
    const static int N=3e6+2, M=62;
    int c[N][M], sz[N]; //sz 维护有多少个以当前字符串为前缀的字符串。
    int cnt;
    void insert(string s)
    {
        int u=0;
        ++sz[u];
        for (char ch:s)
        {
            assert(ch>=0&&ch<M);
            int &v=c[u][ch];
            if (!v) v=++cnt;
            u=v;
            ++sz[u];
        }
        //此时 u 是字符串结束位置。你可以在此存储结点信息。
    }
    int match(string s) //返回字符串结束位置。可能为 0。
    {
        int u=0;
        for (char ch:s)
        {
            assert(ch>=0&&ch<M);
            u=c[u][ch];
            if (!u) return 0;
        }
        return u;
    }
    void clear()
    {
        memset(c, 0, (cnt+1)*sizeof c[0]);
        memset(sz, 0, (cnt+1)*sizeof sz[0]);
        cnt=0;
    }
} s;
```

### 4.2 AC 自动机

注意 AC 自动机与 trie 不同的地方在于，根必须是 0。

题意：给你一个文本串  $S$  和  $n$  个模式串  $T_1 \sim T_n$ ，请你分别求出每个模式串  $T_i$  在  $S$  中出现的次数。

```
struct AC
{
    const static int N=3e6+2, M=26;
    int c[N][M], sz[N], pos[N], f[N], app[N]; //sz 维护有多少个以当前字符串为前缀的字符串。
    int cnt=0, id=0;
    vector<int> q;
    void insert(string s)
    {
```

```

    int u=0;
    ++sz[u];
    for (char ch:s)
    {
        assert(ch>=0&&ch<M);
        int &v=c[u][ch];
        if (!v) v=++cnt;
        u=v;
        ++sz[u];
    }
    pos[id++]=u;
    //此时 u 是字符串结束位置。你可以在此存储结点信息。
}
vector<int> match(string s)//返回答案。复杂度 O(结点数)
{
    int u=0, i;
    for (char ch:s)
    {
        assert(ch>=0&&ch<M);
        u=c[u][ch];
        ++app[u];
    }
    for (int u:q) app[f[u]]+=app[u];
    vector<int> r(id);
    for (i=0; i<id; i++) r[i]=app[pos[i]];
    memset(app, 0, (cnt+1)*sizeof app[0]);
    return r;
}
void clear()
{
    memset(c, 0, (cnt+1)*sizeof c[0]);
    memset(f, 0, (cnt+1)*sizeof f[0]);
    memset(sz, 0, (cnt+1)*sizeof sz[0]);
    cnt=id=0;
}
void build()
{
    q.clear();
    int ql=0;
    for (int i=0; i<M; i++) if (c[0][i]) q.push_back(c[0][i]);
    while (ql<q.size())
    {
        int u=q[ql++];
        for (int i=0; i<M; i++) if (c[u][i])
        {
            q.push_back(c[u][i]);
            f[c[u][i]]=c[f[u]][i];
        }
        else c[u][i]=c[f[u]][i];
    }
    reverse(all(q));
}
} s;
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int n, i;

```

```

cin>>n;
while (n--)
{
    string t;
    cin>>t;
    for (char &c:t) c-='a';
    s.insert(t);
}
s.build();
string t;
cin>>t;
for (char &c:t) c-='a';
auto res=s.match(t);
for (int x:res) cout<<x<<"\n";
}

```

### 4.3 hash

在调试时，可以把 base 设置为 10 的幂方便输出。可能建议把第一个模数也设置为 1，但未测试是否有奇怪的问题。但要注意，此时不应当使用接近 10 的幂次的模数。

$O(n)$ ,  $O(n)$ 。

双模数版本：注意使用的是无符号数，效率比 int128 高，但不卡常建议抄 int128 版本。

特别注意这里 m 数组预处理的不是幂次，而是幂次的相反数。如果有复杂的变换需要建议用 int128 版本。

其返回值是两个 32 位数拼接而成的，要改动比较麻烦。

```

namespace sh
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    const int N=1e6+5;
    const ll p1=2'034'452'107, p2=2'013'074'419;
    struct pa
    {
        ll v1, v2;
        pa(ll v=0):v1(v), v2(v) { }
        pa(ll v1, ll v2):v1(v1), v2(v2) { }
        pa operator*(const pa &o) const { return {v1*o.v1%p1, v2*o.v2%p2}; }
    };
    pa fma(const pa &a, const pa &b, const pa &c) { return {(a.v1*b.v1+c.v1)%p1, (a.v2*b.v2+c.v2)%p2}; }
    const pa b={137, 149}, inv={1'603'801'661, 1'024'053'074};
    pa m[N];
    void init()
    {
        m[0]={p1-1, p2-1};
        for (int i=1; i<N; i++) m[i]=m[i-1]*b;
    }
    int i=(init(), 0);
    struct str
    {
        int n;
        vector<pa> a;
        template<class T> str(const vector<T> &s):n(s.size()), a(n+1)
        {

```

```

        for (i=0; i<n; i++) a[i+1]=fma(a[i], b, s[i]);
    }
    template<class T> str(const basic_string<T> &s):n(s.size()), a(n+1)//直接去掉模板换成
        string 也可以
    {
        for (i=0; i<n; i++) a[i+1]=fma(a[i], b, s[i]);
    }
    l1l getv(int l, int r)//[l,r)
    {
        auto [x, y]=fma(a[l], m[r-1], a[r]);
        return x<<32|y;
    }
};
}
using sh::str;
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int T; cin>>T;
    set<ull> s;
    while (T--)
    {
        string t;
        cin>>t;
        s.insert(str(t).getv(0, t.size()));
    }
    cout<<s.size()<<endl;
}

```

\_\_int128 版本:

```

namespace sh
{
    typedef __uint128_t l1l;
    const int N=1e6+5;
    const l1l p=1'80'143'985'094'819'841, b=137;
    l1l m[N];
    void init()
    {
        m[0]=1;
        for (int i=1; i<N; i++) m[i]=m[i-1]*b%p;
    }
    int i=(init(), 0);
    struct str
    {
        int n;
        vector<l1l> a;
        template<class T> str(const vector<T> &s):n(s.size()), a(n+1)
        {
            for (i=0; i<n; i++) a[i+1]=(a[i]*b+s[i])%p;
        }
        template<class T> str(const basic_string<T> &s):n(s.size()), a(n+1)//直接去掉模板换成
            string 也可以
        {
            for (i=0; i<n; i++) a[i+1]=(a[i]*b+s[i])%p;
        }
        l1l getv(int l, int r)//[l,r)
        {

```

```

        return (a[r]+(p-a[l])*m[r-l])%p;
    }
};
}
using sh::str;

```

## 4.4 KMP

$O(n)$ ,  $O(n)$ 。

```

struct str
{
    vector<int> nxt,s;
    int n;
    str(int *S,int _n)//[1,n]
    {
        n=_n;
        nxt.resize(n+1);
        s=vector<int>(S,S+n+1);
        int i,j=0;
        nxt[1]=0;
        for (i=2;i<=n;i++)
        {
            while (j&& s[i]!=s[j+1]) j=nxt[j];
            nxt[i]=j+s[i]==s[j+1];
        }
    }
    vector<int> match(int *t,int m)//find s(str) in t (start pos)
    {
        vector<int> r;
        int i,j=0;
        for (i=1;i<=m;i++)
        {
            while (j&& t[i]!=s[j+1]) j=nxt[j];
            if ((j+t[i]==s[j+1])==n) j=nxt[j],r.push_back(i-n+1);
        }
        return r;
    }
};
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    string s,t;
    cin>>s>>t;
    int n=s.size(),m=t.size(),i;
    vector<int> a(n+1),b(m+1);
    for (i=1;i<=n;i++) a[i]=s[i-1];
    for (i=1;i<=m;i++) b[i]=t[i-1];
    str q(b.data(),m);
    auto r=q.match(a.data(),n);
    for (int x:r) cout<<x<<'\\n';
    for (i=1;i<=m;i++) cout<<q.nxt[i]<<"\\n"[i==m];
}

```

## 4.5 KMP (重构, 未验证)

$O(n)$ ,  $O(n)$ 。

```
struct str//[0,n)
{
    vector<int> nxt,s;
    int n;
    str(const vector<int> &_s):nxt(_s.size(),-1),s(all(_s)),n(_s.size())
    {
        int i,j=-1;
        for (i=1;i<n;i++)
        {
            while (j!=-1&&s[i]!=s[j+1]) j=nxt[j];
            nxt[i]=j+s[i]==s[j+1];
        }
    }
    vector<int> match(const vector<int> &t)//find s(str) in t (start pos)
    {
        int m=t.size();
        vector<int> r;
        int i,j=-1;
        for (i=0;i<m;i++)
        {
            while (j!=-1&&t[i]!=s[j+1]) j=nxt[j];
            if ((j+t[i]==s[j+1])==n-1) j=nxt[j],r.push_back(i-n+1);
        }
        return r;
    }
};
```

## 4.6 manacher

$O(n)$ ,  $O(n)$ 。

```
vector<int> manacher(const string &t)//ex[i](total length) centered at i/2
{
    string S="$#";
    int n=t.size(),i,r=1,m=0;
    for (i=0;i<n;i++) S+=t[i],S+= '#';
    S+= '#';
    char *s=S.data()+2;
    n=n*2-1;
    vector<int> ex(n);
    ex[0]=2;
    for (i=1;i<n;i++)
    {
        ex[i]=i<r?min(ex[m*2-i],r-i+1):1;
        while (s[i+ex[i]]==s[i-ex[i]]) ++ex[i];
        if (i+ex[i]-1>r) r=i+ex[i]-1;
    }
    for (i=0;i<n;i++) --ex[i];
    return ex;
}
```

## 4.7 SA

$O((n + \sum) \log n)$ ,  $O(n + \sum)$ 。

功能：查询两个后缀的 lcp。单次询问复杂度  $O(1)$ 。

下标从 1 开始。

```
//这回只花了114514min就打完了。
//真好。记得多手造几组。ACM拍什么拍。
#include "bits/stdc++.h"
using namespace std;
using ui = unsigned; using db = long double; using ll = long long; using ull = unsigned long long
; using lll = __int128;
template<class T1, class T2> istream &operator>>(istream &cin, pair<T1, T2> &a) { return cin >> a
.first >> a.second; }
template<std::size_t Index = 0, typename... Ts> typename std::enable_if<Index == sizeof...(Ts),
void>::type tuple_read(std::istream &is, std::tuple<Ts...> &t) { }
template<std::size_t Index = 0, typename... Ts> typename std::enable_if<Index < sizeof...(Ts),
void>::type tuple_read(std::istream &is, std::tuple<Ts...> &t) { is >> std::get<Index>(t);
tuple_read<Index + 1>(is, t); }
template<typename... Ts> std::istream &operator>>(std::istream &is, std::tuple<Ts...> &t) {
tuple_read(is, t); return is; }
template<class T1> istream &operator>>(istream &cin, valarray<T1> &a);
template<class T1> istream &operator>>(istream &cin, vector<T1> &a) { for (auto &x : a) cin >> x;
return cin; }
template<class T1> istream &operator>>(istream &cin, valarray<T1> &a) { for (auto &x : a) cin >>
x; return cin; }
template<class T1, class T2> bool cmin(T1 &x, const T2 &y) { if (y < x) { x = y; return 1; }
return 0; }
template<class T1, class T2> bool cmax(T1 &x, const T2 &y) { if (x < y) { x = y; return 1; }
return 0; }
template<class T1> vector<T1> range(T1 l, T1 r, T1 step = 1) { assert(step > 0); int n = (r - l +
step - 1) / step, i; vector<T1> res(n); for (i = 0; i < n; i++) res[i] = l + step * i;
return res; }
template<class T1> basic_string<T1> operator*(const basic_string<T1> &s, int m) { auto r = s; m
*= s.size(); r.resize(m); for (int i = s.size(); i < m; i++) r[i] = r[i - s.size()]; return r
; }
istream &operator>>(istream &cin, lll &x) { bool flg = 0; x = 0; static string s; cin >> s; if (s
[0] == '-') flg = 1, s = s.substr(1); for (char c : s) x = x * 10 + (c - '0'); if (flg) x = -
x; return cin; }
ostream &operator<<(ostream &cout, lll x) { static char s[60]; if (x < 0) cout << '-', x = -x;
int tp = 1; s[0] = '0' + (x % 10); while (x /= 10) s[tp++] = '0' + (x % 10); while (tp--)
cout << s[tp]; return cout; }
#ifdef ONLINE_JUDGE
#include "my_header/debug.h"
#else
#define dbg(...) ;
#endif
template<class T1, class T2> ostream &operator<<(ostream &cout, const pair<T1, T2> &a) { return
cout << a.first << ' ' << a.second; }
template<class T1, class T2> ostream &operator<<(ostream &cout, const vector<pair<T1, T2>> &a) {
for (auto &x : a) cout << x << '\n'; return cout; }
template<class T1> ostream &operator<<(ostream &cout, vector<T1> &a) { int n = a.size(); if (!n)
return cout; cout << a[0]; for (int i = 1; i < n; i++) cout << ' ' << a[i]; return cout; }
template<class T1> ostream &operator<<(ostream &cout, const valarray<T1> &a) { int n = a.size();
if (!n) return cout; cout << a[0]; for (int i = 1; i < n; i++) cout << ' ' << a[i]; return
cout; }
template<class T1> ostream &operator<<(ostream &cout, const vector<valarray<T1>> &a) { int n = a.
size(); if (!n) return cout; cout << a[0]; for (int i = 1; i < n; i++) cout << '\n' << a[i];
```



```

    return cout; }
template<class T1> ostream &operator<<(ostream &cout, const vector<vector<T1>> &a) { int n = a.
    size(); if (!n) return cout; cout << a[0]; for (int i = 1; i < n; i++) cout << '\n' << a[i];
    return cout; }
#define all(x) (x).begin(),(x).end()
#define print(...) cout<<format(__VA_ARGS__)
#define println(...) cout<<format(__VA_ARGS__)<<'\n'
#define err(...) cerr<<format(__VA_ARGS__)
#define errln(...) cerr<<format(__VA_ARGS__)<<'\n'
struct SA
{
    int n;
    vector<vector<int>> st;
    vector<int> sa, rk, h;
    int lcp(int x, int y)
    {
        if (x == y) return n - x;
        x = rk[x]; y = rk[y];
        if (x > y) swap(x, y);
        ++x;
        int z = __lg(y - x + 1);
        return min(st[z][x], st[z][y - (1 << z) + 1]);
    }
    SA(vector<int> a) :n(a.size()), st(__lg(n) + 1, vector<int>(n + 1)), sa(n), h(n)
    {
        const static int N = 2e6 + 2;
        static int s[N];
        int i, j, m, cnt;
        m = *min_element(all(a));
        for (int &x : a) x -= m;
        m = *max_element(all(a)) + 1;
        assert(max(n, m) < N);
        a.resize(n * 2);
        for (i = 0; i < n; i++) a[i + n] = -i - 1;
        vector<int> id(n * 2);
        rk = a;
        for (i = 0; i < n; i++) ++s[a[i]];
        for (i = 1; i < m; i++) s[i] += s[i - 1];
        for (i = n - 1; i >= 0; i--) sa[--s[rk[i]]] = i;
        memset(s, 0, m * sizeof s[0]);
        for (j = 1; j <= n; j <= 1)
        {
            cnt = 0;
            for (i = n - j; i < n; i++) id[cnt++] = i;
            for (i = 0; i < n; i++) if (sa[i] >= j) id[cnt++] = sa[i] - j;
            for (i = 0; i < n; i++) ++s[rk[i]];
            for (i = 1; i < m; i++) s[i] += s[i - 1];
            for (i = n - 1; i >= 0; i--) sa[--s[rk[id[i]]]] = id[i];
            id[sa[0]] = cnt = 0;
            memset(s, 0, m * sizeof s[0]);
            for (i = 1; i < n; i++)
                if (rk[sa[i]] == rk[sa[i - 1]] && rk[sa[i] + j] == rk[sa[i - 1] + j])
                    id[sa[i]] = cnt;
                else
                    id[sa[i]] = ++cnt;
            swap(rk, id);
            if ((m = cnt + 1) == n) break;
        }
    }
};

```

```

    }
    j = 0;
    for (i = 0; i < n; i++) if (rk[i])
    {
        cnt = sa[rk[i] - 1];
        while (a[i + j] == a[cnt + j]) ++j;
        h[rk[i]] = j;
        if (j) --j;
    }
    st[0] = h;
    for (j = 0; j < __lg(n); j++)
        for (i = 0, m = n - (1 << j + 1); i <= m; i++)
            st[j + 1][i] = min(st[j][i], st[j][i + (1 << j)]);
}
};

```

## 4.8 SAM

$O(n \sum)$ ,  $O(2n \sum)$ 。

```

template<int M> struct sam//M: 字符集大小
{
    vector<array<int,M>> c;
    vector<int> len,fa,ep;
    int np,cd;
    sam():c(2),len(2),fa(2),ep(2),np(1),cd(0) { }
    void insert(int ch)
    {
        int p=np,q,nq;
        np=c.size();
        len.push_back(++cd);
        fa.push_back(0);
        c.push_back({ });
        ep.push_back(cd);
        while (p&&!c[p][ch]) c[p][ch]=np,p=fa[p];
        if (!p)
        {
            fa[np]=1;
            return;
        }
        q=c[p][ch];
        if (len[q]==len[p]+1)
        {
            fa[np]=q;
            return;
        }
        nq=c.size();
        len.push_back(len[p]+1);
        c.push_back(c[q]);
        fa.push_back(fa[q]);
        ep.push_back(ep[q]);
        fa[np]=fa[q]=nq;
        c[p][ch]=nq;
        while (c[p=fa[p]][ch]==q) c[p][ch]=nq;
    }
    vector<int> match(const string &s)//返回每个前缀最长匹配长度
    {

```

```

    vector<int> r;
    r.reserve(s.size());
    int p=1,nl=0;
    for (auto ch:s)
    {
        if (c[p][ch]) ++nl,p=c[p][ch];
        else
        {
            while (p&& c[p][ch]==0) p=fa[p];
            if (p==0) p=1,nl=0; else nl=len[p]+1,p=c[p][ch];
        }
        r.push_back(nl);
    }
    return r;
}
array<int,3> max_match(const string &s)//返回长度, 结尾(开)
{
    array<int,3> r{0,0,0};
    int p=1,nl=0,i=0;
    for (auto ch:s)
    {
        if (c[p][ch]) ++nl,p=c[p][ch];
        else
        {
            while (p&& c[p][ch]==0) p=fa[p];
            if (p==0) p=1,nl=0; else nl=len[p]+1,p=c[p][ch];
        }
        cmax(r,array{nl,ep[p],i+1});
        ++i;
    }
    if (r[0]==0) return { };
    return r;
}
};

```

## 4.9 SqAM

$O(n \sum)$ ,  $O(n \sum)$ 。

```

struct sqam
{
    int c[N][26],ds,i,j,lst[26],pre[N];
    void csh()
    {
        ds=1;
    }
    void ins(int zf)
    {
        ++ds;
        for (i=0;i<=25;i++) if (lst[i]) for (j=lst[i];(j)&&(c[j][zf]==0);j=pre[j]) c[j][zf]=ds;
        if (!lst[zf]) c[1][zf]=ds; else pre[ds]=lst[zf];
        lst[zf]=ds;
    }
};

```

## 4.10 ukkonen 后缀树

 $O(n), O(2n \sum)$ 。

```

void dfs(int x,int lf)
{
    if (!fir[x])
    {
        siz[x][1]=1;
        return;
    }
    int i,j;
    for (i=fir[x];i;i=nxt[i])
    {
        j=c[x][lj[i]];
        if ((f[j]<=m)&&(t[j]>=m)) ++siz[x][0];
        dfs(zd[j],t[j]-f[j]+1);
        siz[x][0]+=siz[zd[j]][0];
        siz[x][1]+=siz[zd[j]][1];
        if ((t[j]==n)&&(f[j]<=m)) --siz[x][1];
    }
    ans+=(ll)siz[x][0]*siz[x][1]*lf;
}

void add(int a,int b,int cc,int d)
{
    zd[++bbs]=b;
    t[bbs]=d;
    c[a][s[f[bbs]=cc]]=bbs;
}

void add(int x,int y)
{
    lj[++bs]=y;
    nxt[bs]=fir[x];
    fir[x]=bs;
}

s[++m]=26;
fa[1]=point=ds=1;
for (i=1;i<=m;i++)
{
    ad=0;++remain;
    while (remain)
    {
        if (r==0) edge=i;
        if ((j=c[point][s[edge]]))==0)
        {
            fa[++ds]=1;
            fa[ad]=point;
            add(ad=point,ds,edge,m);
            add(point,s[edge]);
        }
        else
        {
            if ((t[j]!=m)&&(t[j]-f[j]+1<=r))
            {
                r-=t[j]-f[j]+1;
                edge+=t[j]-f[j]+1;
                point=zd[j];
                continue;
            }
        }
    }
}

```

```

    }
    if (s[f[j]+r]==s[i]) {++r;fa[ad]=point;break;}
    fa[fa[ad]=++ds]=1;
    add(ad=ds,zd[j],f[j]+r,t[j]);
    add(ds,s[i]);add(ds,s[f[j]+r]);fa[++ds]=1;
    add(ds-1,ds,i,m);
    zd[j]=ds-1;t[j]=f[j]+r-1;
}
--remain;
if ((r)&&(point==1))
{
    --r;edge=i-remain+1;
} else point=fa[point];
}
}
for (i=1;i<=ds;i++) for (j=fir[i];j;j=nxt[j]) {len[j]=t[c[i][lj[j]]]-f[c[i][lj[j]]]+1;lj[j]=zd
[c[i][lj[j]]];}

```

## 4.11 ukkonen 后缀树（重构）

```

struct suffixtree
{
    const static int M=27;
    struct P
    {
        int v,w;
    };
    struct Q
    {
        int f,t,v;//t=0: n
    };
    vector<Q> edges;
    vector<vector<P>> e;
    vector<array<int,M>> c;
    vector<int> s,fa,dep,siz;
    int n,point,ds,remain,r,edge;
    bool bd;
    suffixtree():c(2),fa({0,1}),edges(1),e(2)
    {
        n=remain=r=edge=bd=0;
        point=ds=1;
    }
    suffixtree(const string &s):c(2),fa({0,1}),edges(1),e(2)
    {
        n=remain=r=edge=bd=0;
        point=ds=1;
        reserve(s.size());
        for (auto c:s) insert(c-'a');
        insert(26);
    }
    void reserve(int len)
    {
        ++len;
        s.reserve(len);
        len=len*2+2;
        c.reserve(len);
    }
}

```

```

    fa.reserve(len);
    e.reserve(len);
}
inline void add(int a,int b,int cc,int d)
{
    assert(edges.size());
    c[a][s[cc]]=edges.size();
    edges.push_back({cc,d,b});
}
void insert(int ch)//[0,|S|)
{
    assert(ds==fa.size()-1&&ds==c.size()-1&&n==s.size()&&ds==e.size()-1);
    assert(ch>=0&&ch<M);
    s.push_back(ch);
    int ad=0;
    ++remain;
    while (remain)
    {
        if (!r) edge=n;
        if (int m=c[point][s[edge]];!m)
        {
            assert(!m);
            fa.push_back(1);c.push_back({});e.push_back({});
            fa[ad]=point;
            add(ad=point,++ds,edge,-1);
            e[point].push_back({s[edge]});
            //add(point,s[edge]);
        }
        else
        {
            assert(m);
            auto [f,t,v]=edges[m];
            if (t>=0&&t-f+1<=r)
            {
                assert(t!=n);
                r-=t-f+1;
                edge+=t-f+1;
                point=v;
                continue;
            }
            assert(f+r<=n);
            if (s[f+r]==s[n])
            {
                ++r;
                fa[ad]=point;
                break;
            }
            fa.push_back(1);c.push_back({});e.push_back({});
            fa.push_back(1);c.push_back({});e.push_back({});
            fa[ad]=++ds;
            add(ad=ds,v,f+r,t);
            e[ds].push_back({s[n]});
            e[ds].push_back({s[f+r]});
            //add(ds,s[n]);add(ds,s[f+r]);
            ++ds;add(ds-1,ds,n,-1);
            edges[m]={f,f+r-1,ds-1};
        }
    }
}

```

```

        --remain;
        if (r&&point==1)
        {
            --r;
            edge=n-remain+1;
        } else point=fa[point];
    }
    ++n;
}

void build_edge()
{
    bd=1;

    //其余信息
    dep.resize(ds+1);
    siz.resize(ds+1);

    int i,j;
    for (i=1;i<=ds;i++) for (auto &[v,w]:e[i])
    {
        j=c[i][v];
        v=edges[j].v;
        w=(edges[j].t>=0?edges[j].t:n-1)-edges[j].f+1;
    }
}

void out()
{
    int i;
    for (i=1;i<=ds;i++) for (int j:c[i]) if (j)
    {
        auto [f,t,v]=edges[j];
        if (t==-1) t=n-1;
        cerr<<i<<"<<v<<"<<endl;
        //cerr<<i<<" -> "<<v<<"<<endl;
        for (int k=f;k<=t;k++) cerr<<char('a'+s[k]);
        cerr<<endl;
    }
}

ll ans;
void dfs(int u)
{
    assert(bd);
    ++ans;
    for (auto [v,w]:e[u])
    {
        //dep[v]=dep[u]+w;
        dfs(v);
        ans+=w-1;
    }
}

ll fun()
{
    ans=0;
    build_edge();
    dfs(1);
    return ans-n;
}

```

```
};
```

## 4.12 Z 函数

表示每个后缀和母串的 lcp。

```
vector<int> Z(const string &s)
{
    int n=s.size(),i,l,r;
    vector<int> z(n);
    z[0]=n;
    for (i=1,l=r=0; i<n; i++)
    {
        if (i<=r&&z[i-l]<r-i+1) z[i]=z[i-l];
        else
        {
            z[i]=max(0,r-i+1);
            while (i+z[i]<n&&s[i+z[i]]==s[z[i]]) ++z[i];
        }
        if (i+z[i]-1>r) l=i,r=i+z[i]-1;
    }
    return z;
}
```

## 4.13 最小表示法

找到一个串的循环同构串中字典序最小的那个，将这个串直接变过去。常见应用：环哈希（基环树哈希）。

如果只需要找到起点下标，在 rotate 前返回  $\min\{i, j\}$  即可。

$O(n)$ ,  $O(1)$ 。

```
template<class T> void min_order(vector<T>& a)
{
    int n = a.size(), i, j, k;
    a.resize(n * 2);
    for (i = 0; i < n; i++) a[i + n] = a[i];
    i = k = 0; j = 1;
    while (i < n && j < n && k < n)
    {
        T x = a[i + k], y = a[j + k];
        if (x == y) ++k; else
        {
            (x > y ? i : j) += k + 1;
            j += (i == j);
            k = 0;
        }
    }
    a.resize(n);
    // [min(i,j), n) + [0, min(i,j))
    rotate(a.begin(), min(i, j) + all(a));
}
```

## 4.14 带通配符的字符串匹配

原理：匹配等价于  $\sum (f_i - g_i)^2 = 0$ 。带通配符等价于  $\sum f_i g_i (f_i - g_i)^2 = 0$ ，展开即可。



这里也是较为推荐的 NTT 版本，直接实现任意长度的多项式相乘，便于一般情况的运用。不需要提前做任何 init。

```
namespace NTT
{
    typedef unsigned ui;
    typedef unsigned long long ll;
    const int N=1<<22;
    const ui p=998244353, g=3;
    inline ui ksm(ui x, ui y)
    {
        ui ans=1;
        while (y)
        {
            if (y&1) ans=1llu*ans*x%p;
            y>>=1; x=1llu*x*x%p;
        }
        return ans;
    }
    ui r[N], w[N];
    void ntt(vector<ui> &a)
    {
        int n=a.size(), i, j, k;
        for (i=0; i<n; i++) if (i<r[i]) swap(a[i], a[r[i]]);
        for (k=1; k<n; k<=<=1)
        {
            for (i=0; i<n; i+=k<<1)
            {
                for (j=0; j<k; j++)
                {
                    ui x=a[i+j], y=1llu*a[i+j+k]*w[j+k]%p;
                    a[i+j]=(x+y)%p; a[i+j+k]=(x-p-y)%p;
                }
            }
        }
    }
    vector<ui> mul(vector<ui> a, vector<ui> b)
    {
        if (a.size()==0||b.size()==0) return { };
        int m=a.size()+b.size()-1;
        int n=1<<__lg(m*2-1);
        int i, j, base=__lg(n)-1;
        ui inv=ksm(n, p-2);
        for (i=1; i<n; i++) r[i]=r[i>>1]>>1|(i&1)<<base;
        for (j=1; j<n; j<=<=1)
        {
            ui wn=ksm(3, (p-1)/(j<<1));
            w[j]=1;
            for (i=1; i<j; i++) w[j+i]=1llu*w[j+i-1]*wn%p;
        }
        a.resize(n); b.resize(n);
        ntt(a); ntt(b);
        for (i=0; i<n; i++) a[i]=1llu*a[i]*b[i]%p;
        ntt(a); reverse(1+all(a)); a.resize(n=m);
        for (i=0; i<n; i++) a[i]=1llu*a[i]*inv%p;
        return a;
    }
}
```

```

vector<int> match(const string &s, const string &t)
{
    using NTT::p, NTT::mul;
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    static array<ui, 256> c;
    static bool initied=0;
    if (!initied)
    {
        initied=1;
        for (ui &x:c) x=rnd()%NTT::p;
        c['*']=0; //通配符
    }
    int n=s.size(), m=t.size(), i, j;
    if (n<m) return { };
    vector<int> ans;
    vector<ui> f(n), ff(n), fff(n), g(m), gg(m), ggg(m);
    for (i=0; i<n; i++)
    {
        f[i]=c[s[i]];
        ff[i]=1llu*f[i]*f[i]%p;
        fff[i]=1llu*ff[i]*f[i]%p;
    }
    for (i=0; i<m; i++)
    {
        g[i]=c[t[m-i-1]];
        gg[i]=1llu*g[i]*g[i]%p;
        ggg[i]=1llu*gg[i]*g[i]%p;
    }
    auto fffg=mul(fff, g), ffgg=mul(ff, gg), fgfg=mul(f, ggg);
    for (i=0; i<=n-m; i++) if ((fffg[m-1+i]+fgfg[m-1+i]+2*(NTT::p-ffgg[m-1+i]))%NTT::p==0) ans.
        push_back(i);
    return ans;
}

```

快一些的版本，手动拆开了多项式乘法。

```

const int N=1<<22;
const ui p=998244353, g=3;
inline ui ksm(ui x, ui y)
{
    ui ans=1;
    while (y)
    {
        if (y&1) ans=1llu*ans*x%p;
        y>>=1; x=1llu*x*x%p;
    }
    return ans;
}
ui r[N], w[N];
void ntt(vector<ui> &a)
{
    int n=a.size(), i, j, k;
    for (k=1; k<n; k<=<=1)
    {
        for (i=0; i<n; i+=k<<=1)
        {
            for (j=0; j<k; j++)
            {

```

```

        ui x=a[i+j], y=1llu*a[i+j+k]*w[j+k]%p;
        a[i+j]=(x+y)%p; a[i+j+k]=(x+p-y)%p;
    }
}
}
vector<int> match(string s, string t, char ch='*')
{
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    static array<ui, 256> c;
    static bool initied=0;
    if (!initied)
    {
        initied=1;
        for (ui &x:c) x=rnd()%p;
        // for (int i=0; i<256; i++) c[i]=i-96;
        c[ch]=0; //通配符
    }
    int n=s.size(), m=t.size(), i, j;
    if (n<m) return { };
    vector<int> ans;
    int N=1<<__lg(n*2-1), base=__lg(N)-1;
    vector<ui> f(N), ff(N), fff(N), g(N), gg(N), ggg(N);
    reverse(all(t));
    s.resize(N, ch), t.resize(N, ch);
    for (i=0; i<N; i++)
    {
        r[i]=r[i>>1]>>1|(i&1)<<base;
        if (i<r[i])
        {
            swap(s[i], s[r[i]]);
            swap(t[i], t[r[i]]);
        }
    }
    for (j=1; j<N; j<=<=1)
    {
        ui wn=ksm(3, (p-1)/(j<<1));
        w[j]=1;
        for (i=1; i<j; i++) w[j+i]=1llu*w[j+i-1]*wn%p;
    }
    for (i=0; i<N; i++)
    {
        f[i]=c[s[i]];
        ff[i]=1llu*f[i]*f[i]%p;
        fff[i]=1llu*ff[i]*f[i]%p;
        g[i]=c[t[i]];
        gg[i]=1llu*g[i]*g[i]%p;
        ggg[i]=1llu*gg[i]*g[i]%p;
    }
    ntt(f); ntt(ff); ntt(fff); ntt(g); ntt(gg); ntt(ggg);
    for (i=0; i<N; i++) f[i]=(1llu*fff[i]*g[i]+1llu*f[i]*ggg[i]+2llu*(p-ff[i])*gg[i])%p;
    for (i=0; i<N; i++) if (i<r[i]) swap(f[i], f[r[i]]);
    ntt(f); reverse(1+all(f));
    for (i=0; i<=n-m; i++) if (f[m+i-1]==0) ans.push_back(i);
    return ans;
}

```

## 5 图论

### 5.1 最小密度环

求所有环中边权和除以边数最少的,  $O(nm)$ 。更常用的做法是二分 spfa。

```
#include <bits/stdc++.h>
using namespace std;
const int N=3e3+5,M=1e4+5;
const double inf=1e18;
int u[M],v[M];
double f[N][N],w[M];
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(8);
    int n,m,i,j;
    cin>>n>>m;
    for (i=1;i<=m;i++) cin>>u[i]>>v[i]>>w[i];
    ++n;
    for (i=1;i<=n;i++)
    {
        fill_n(f[i]+1,n,inf);
        for (j=1;j<=m;j++) f[i][v[j]]=min(f[i][v[j]],f[i-1][u[j]]+w[j]);
    }
    double ans=inf;
    for (i=1;i<n;i++) if (f[n][i]!=inf)
    {
        double r=-inf;
        for (j=1;j<n;j++) r=max(r,(f[n][i]-f[j][i])/(n-j));
        ans=min(ans,r);
    }
    cout<<ans<<endl;
}
```

### 5.2 全源最短路与判负环

使用 floyd 实现全源最短路与判负环。注意边权较大时可能需要考虑 int128。

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int,int> pa;
typedef tuple<int,int,int> tp;
const int N=152;
const ll inf=5e8;
ll dis[N][N],d[N][N];
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    while (1)
    {
        int n,m,q,i,j,k;
        cin>>n>>m>>q;
        if (tp(n,m,q)==tp(0,0,0)) return 0;
        for (i=0;i<n;i++) fill_n(dis[i],n,inf*inf);
        for (i=0;i<n;i++) dis[i][i]=0;
```

```

while (m--)
{
    int u,v,w;
    cin>>u>>v>>w;
    dis[u][v]=min(dis[u][v],(ll)w);
}
for (k=0;k<n;k++) for (i=0;i<n;i++) for (j=0;j<n;j++) dis[i][j]=max(min(dis[i][j],dis[i][k]
    ]+dis[k][j]),-inf*2);
for (i=0;i<n;i++) copy_n(dis[i],n,d[i]);
for (k=0;k<n;k++) for (i=0;i<n;i++) for (j=0;j<n;j++) dis[i][j]=max(min(dis[i][j],dis[i][k]
    ]+dis[k][j]),-inf*2);
while (q--)
{
    int u,v;
    cin>>u>>v;
    if (d[u][v]>inf) cout<<"Impossible\n"; else if (dis[u][v]!=d[u][v]||d[u][v]<=-inf) cout
        <<"-Infinity\n"; else cout<<d[u][v]<<"\n";
}
cout<<"\n";
}
}

```

### 5.3 三/四元环计数

不能处理有重边和自环的情况。

$O(m\sqrt{m})$ ,  $O(n+m)$ 。

注意四元环数的是边四元环。点四元环需要去掉四点完全图个数 \*2, 似乎不太能做?

三元环是可以枚举的, 你可以在 ans 改变处记录三元环  $(i, u, v)$ 。

```

ll triple(const vector<pair<int,int>> &edges)//start from 0
{
    int n=0,i;
    for (auto [u,v]:edges) n=max({n,u,v});
    ++n;
    vector<int> d(n),id(n),rk(n),cnt(n);
    vector<vector<int>> e(n);
    for (auto [u,v]:edges) ++d[u],++d[v];
    iota(all(id),0); sort(all(id),[&](int x,int y) { return d[x]<d[y]; });
    for (i=0; i<n; i++) rk[id[i]]=i;
    for (auto [u,v]:edges)
    {
        if (rk[u]>rk[v]) swap(u,v);
        e[u].push_back(v);
    }
    ll ans=0;
    for (i=0; i<n; i++)
    {
        for (int u:e[i]) cnt[u]=1;
        for (int u:e[i]) for (int v:e[u]) ans+=cnt[v];
        for (int u:e[i]) cnt[u]=0;
    }
    return ans;
}
ll quadruple(const vector<pair<int,int>> &edges)
{
    int n=0,i;

```

```

for (auto [u,v]:edges) n=max({n,u,v});
++n;
vector<int> d(n),id(n),rk(n),cnt(n);
vector<vector<int>> e(n),lk(n);
for (auto [u,v]:edges) ++d[u],++d[v];
iota(all(id),0); sort(all(id), [&](int x,int y) { return d[x]<d[y]; });
for (i=0; i<n; i++) rk[id[i]]=i;
for (auto [u,v]:edges)
{
    if (rk[u]>rk[v]) swap(u,v);
    e[u].push_back(v);
    lk[u].push_back(v);
    lk[v].push_back(u);
}
ll ans=0;
for (i=0; i<n; i++)
{
    for (int u:lk[i]) for (int v:e[u]) if (rk[v]>rk[i]) ans+=cnt[v]++;
    for (int u:lk[i]) for (int v:e[u]) cnt[v]=0;
}
return ans;
}
map<pair<int, int>, ll> quadruple(vector<pair<int, int>> edges)
{
    int n = 0, i;
    for (auto [u, v] : edges) n = max({n, u, v});
    ++n;
    map<pair<int, int>, int> ec;
    for (auto [u, v] : edges)
    {
        if (u > v) swap(u, v);
        ++ec[{u, v}];
    }
    vector<ll> c;
    edges.clear();
    for (auto [_, cc] : ec) edges.push_back(_), c.push_back(cc);
    vector d(n, 0), id(d), rk(d);
    vector<ll> cnt(n);
    vector<vector<pair<int, int>>> e(n), lk(n);
    for (auto [u, v] : edges) ++d[u], ++d[v];
    iota(all(id), 0); sort(all(id), [&](int x, int y) { return d[x] < d[y]; });
    for (i = 0; i < n; i++) rk[id[i]] = i;
    i = 0;
    for (auto [u, v] : edges)
    {
        if (rk[u] > rk[v]) swap(u, v);
        e[u].push_back({v, i});
        lk[u].push_back({v, i});
        lk[v].push_back({u, i});
        ++i;
    }
    int m = edges.size();
    vector<ll> ans(m);
    for (i = 0; i < n; i++)
    {
        for (auto [u, w1] : lk[i]) for (auto [v, w2] : e[u]) if (rk[v] > rk[i])
        {

```

```

        cnt[v] += c[w1] * c[w2];
    }
    for (auto [u, w1] : lk[i]) for (auto [v, w2] : e[u]) if (rk[v] > rk[i])
    {
        ans[w1] += (cnt[v] - c[w1] * c[w2]) * c[w2];
        ans[w2] += (cnt[v] - c[w1] * c[w2]) * c[w1];
    }
    for (auto [u, w1] : lk[i]) for (auto [v, w2] : e[u]) if (rk[v] > rk[i]) cnt[v] = 0;
}
map<pair<int, int>, ll> mp;
for (i = 0; i < m; i++) mp[edges[i]] = ans[i];
return mp;
}
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int n, m, i;
    cin >> n >> m;
    vector<pair<int, int>> eg(m);
    cin >> eg;
    auto mp = quadruple(eg);
    for (i = 0; i < m; i++)
    {
        auto [u, v] = eg[i];
        if (u > v) swap(u, v);
        cout << mp[{u, v}] << "\n"[i + 1 == m];
    }
}

```

## 5.4 最短路系列

Johnson 不适用于图中存在负环的情况，因为负环不一定是可以经过的。

$O(nm \log m)$ ,  $O(n + m)$ 。

```

vector<ll> spfa(const vector<vector<pair<int, ll>>> &e, int s)
{
    int n=e.size(), i;
    assert(n);
    queue<int> q;
    vector<int> len(n), ed(n);
    vector<ll> dis(n, inf);
    q.push(s); dis[s]=0;
    while (q.size())
    {
        int u=q.front(); q.pop();
        ed[u]=0;
        for (auto [v, w]:e[u]) if (cmin(dis[v], dis[u]+w))
        {
            len[v]=len[u]+1;
            if (len[v]>n) return { };
            if (!ed[v])
            {
                ed[v]=1;
                q.push(v);
            }
        }
    }
}

```

```

    }
}
return dis;
}
vector<ll> spfa(const vector<vector<pair<int, ll>>> &e)
{
    int n=e.size(), i;
    assert(n);
    queue<int> q;
    vector<int> len(n), ed(n, 1);
    vector<ll> dis(n);
    for (i=0; i<n; i++) q.push(i);
    while (q.size())
    {
        int u=q.front(); q.pop();
        ed[u]=0;
        for (auto [v, w]:e[u] if (cmin(dis[v], dis[u]+w))
        {
            len[v]=len[u]+1;
            if (len[v]>n) return { };
            if (!ed[v])
            {
                ed[v]=1;
                q.push(v);
            }
        }
    }
    return dis;
}
vector<ll> dijk(const vector<vector<pair<int, ll>>> &e, int s)
{
    int n=e.size();
    using pa=pair<ll, int>;
    vector<ll> d(n, inf);
    vector<int> ed(n);
    priority_queue<pa, vector<pa>, greater<pa>> q;
    d[s]=0; q.push({0, s});
    while (q.size())
    {
        int u=q.top().second; q.pop();
        ed[u]=1;
        for (auto [v, w]:e[u] if (cmin(d[v], d[u]+w)) q.push({d[v], v});
        while (q.size() && ed[q.top().second]) q.pop();
    }
    return d;
}
vector<vector<ll>> dijk(const vector<vector<pair<int, ll>>> &e)
{
    vector<vector<ll>> r;
    for (int i=0; i<e.size(); i++) r.push_back(dijk(e, i));
    return r;
}
vector<vector<ll>> john(vector<vector<pair<int, ll>>> e)
{
    int n=e.size(), i, j;
    assert(n);
    auto h=spfa(e);

```



```

if (!h.size()) return { };
for (i=0; i<n; i++) for (auto &[v, w]:e[i]) w+=h[i]-h[v];
auto r=dijk(e);
for (i=0; i<n; i++) for (j=0; j<n; j++) if (r[i][j]!=inf) r[i][j]-=h[i]-h[j];
return r;
}

```

## 5.5 弦图

单纯点： $v$  和  $v$  邻点构成团。

完美消除序列： $v_i$  在  $\{v_i, v_{i+1}, \dots, v_n\}$  为单纯点。

$N(v_i) = \{v_j | j > i \wedge (v_i, v_j) \in E\}$ ,  $next(v_i)$  为  $N(v_i)$  最靠前的点。

极大团一定是  $\{v\} \cup N(v)$ 。

最大团大小等于色数。

弦图判定：等价于是否存在完美消除序列。首先求出一个完美消除序列，然后判定是否合法。

判定方法：设  $v_{i+1}, \dots, v_n$  中与  $v_i$  相邻的依次为  $v'_1, \dots, v'_m$ 。只需判断是否  $v'_1$  与  $v'_2, \dots, v'_m$  相邻。

LexBFS 算法（我不会写）

每个点有一个字符串 label，初始为 0。从  $i = n$  到  $i = 1$  确定，选 label 字典序最大的  $u$ ，再把  $u$  邻点的 label 后面接一个  $i$ 。

最大势算法：从  $v_n$  求到  $v_1$ ，设  $label_i$  表示  $i$  与多少个已选点相邻，每次选  $label_i$  最大的点。

弦图极大团： $\{v | \forall next(w) = v, |N(v)| \geq |N(w)|\}$ 。选出的集合为基本点，按上述极大团构造。

弦图染色：从  $v_n$  到  $v_1$  依次选最小可染的色。

最大独立集：从  $v_1$  到  $v_n$  能选就选。

最小团覆盖：设最大独立集为  $\{p_m\}$ ，最小团覆盖为  $\{\{p_i\} \cup N(p_i)\}$ 。

区间图：两个区间有边当且仅当交集非空。

区间图是弦图。

### 5.5.1 代码

```

namespace chordal_graph//下标从 1 开始
{
    const int N=1e5+2;//点数
    bool ed[N];
    vector<int> e[N];
    int n;
    void init(const vector<pair<int,int>> &edges)
    {
        n=0;
        for (auto [u,v]:edges) n=max({n,u,v});
        for (int i=1;i<=n;i++) e[i].clear();
        for (auto [u,v]:edges) e[u].push_back(v),e[v].push_back(u);
    }
    vector<int> perfect_seq(const vector<pair<int,int>> &edges)//MCS
    {
        init(edges);
        static int d[N];
        static vector<int> buc[N];
        int i,mx=0;
        memset(d+1,0,n*sizeof d[0]);
        memset(ed+1,0,n*sizeof ed[0]);
        for (i=1;i<=n;i++) buc[i].clear();
    }
}

```

```

    buc[0].resize(n);
    iota(all(buc[0]),1);
    vector<int> r(n);
    for (i=n-1;i>=0;i--)
    {
        int u=0;
        while (!u)
        {
            while (buc[mx].size() if (ed[buc[mx].back()]) buc[mx].pop_back();
            else
            {
                ed[u=buc[mx].back()]=1;
                buc[mx].pop_back();
                goto yes;
            }
            --mx;
        }
        yes;;
        r[i]=u;
        for (int v:e[u]) if (!ed[v]) buc[++d[v]].push_back(v),mx=max(mx,d[v]);
    }
    return r;
}

bool check_perfect_seq(vector<int> a)
{
    static bool ee[N];
    memset(ed+1,0,n*sizeof ed[0]);
    memset(ee+1,0,n*sizeof ee[0]);
    reverse(all(a));
    for (int u:a)
    {
        ed[u]=1;
        int w=0;
        for (int v:e[u]) if (ed[v]) {w=v;break;}
        if (!w) continue;
        ee[w]=1;
        for (int v:e[w]) ee[v]=1;
        for (int v:e[u]) if (ed[v]&&!ee[v]) return 0;
        ee[w]=0;
        for (int v:e[w]) ee[v]=0;
    }
    return 1;
}

bool check_chordal(const vector<pair<int,int>> &edges) {return check_perfect_seq(perfect_seq(
    edges));}

vector<int> color(int _n,const vector<pair<int,int>> &edges)//返回长度为 _n+1。其中 0 无意义
{
    auto a=perfect_seq(edges);
    reverse(all(a));
    memset(ed+1,0,n*sizeof ed[0]);
    vector<int> r(_n+1);
    for (int u:a)
    {
        for (int v:e[u]) ed[r[v]]=1;
        int x=1;
        while (ed[x]) ++x;
        r[u]=x;
    }
}

```

```

        for (int v:e[u]) ed[r[v]]=0;
    }
    for (int i=n+1;i<=_n;i++) r[i]=1;
    return r;
}
vector<int> max_independent(int _n,const vector<pair<int,int>> &edges)//注意有孤立点这种奇怪东西
{
    auto a=perfect_seq(edges);
    memset(ed+1,0,n*sizeof ed[0]);
    vector<int> r;
    for (int u:a) if (!ed[u])
    {
        r.push_back(u);
        for (int v:e[u]) ed[v]=1;
    }
    for (int i=n+1;i<=_n;i++) r.push_back(i);
    return r;
}
}
using chordal_graph::check_chordal,chordal_graph::color,chordal_graph::max_independent;

```

## 5.6 最小割树

结论：两个点之间的最小割等于最小割树上两点间最小边权。

直接返回任意两点最小割。

```

template<class T> vector<vector<T>> min_cut(int n, const vector<tuple<int, int, T>> &edges)//[0,n)
{
    int m=edges.size(), i, s, t, cnt=0;
    vector<int> fir(n, -1), nxt(m*2, -1), fc(n), q(n);
    vector<pair<int, T>> e(m*2);
    vector<tuple<T, int, int>> eg;
    auto add=[&](int u, int v, T w)
    {
        e[cnt]={v, w};
        nxt[cnt]=fir[u];
        fir[u]=cnt++;
    };
    for (auto [u, v, w]:edges) add(u, v, w), add(v, u, w);
    auto E=e;
    auto bfs=[&]()
    {
        fill(all(fc), 0);
        int ql=0, qr=0, u, i;
        fc[q[0]=s]=1;
        while (ql<=qr)
        {
            u=q[ql++];
            for (int i=fir[u]; i!=-1; i=nxt[i])
                if (auto &[v, w]=e[i]; w&&!fc[v]) fc[q[++qr]=v]=fc[u]+1;
        }
        return fc[t];
    };
    function<T(int, T)> dfs=[&](int u, T maxf)

```

```

{
    if (u==t) return maxf;
    T j=0, k;
    for (int i=fir[u]; i!=-1; i=nxt[i])
        if (auto &[v, w]=e[i]; w&&fc[v]==fc[u]+1&&(k=dfs(v, min(maxf-j, w))))
        {
            j+=k;
            w-=k;
            e[i^1].second+=k;
            if (j==maxf) return j;
        }
    fc[u]=0;
    return j;
};

function<void(vector<int>>> solve=[&](vector<int> id)
{
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    if (id.size()<=1) return;
    vector<int> u(2);
    sample(all(id), u.begin(), 2, rnd);
    s=u[0], t=u[1], e=E;
    T ans=0;
    while (bfs()) ans+=dfs(s, numeric_limits<T>::max());
    auto it=partition(all(id), [&](int u) { return fc[u]; });
    eg.emplace_back(ans, s, t);
    solve(vector(id.begin(), it));
    solve(vector(it, id.end()));
};

solve(range(0, n));
sort(all(eg), greater<>());
vector<basic_string<int>>> ver(n);
vector ans(n, vector<T>(n));
vector<int> f(n);
for (i=0; i<n; i++) ver[i]={f[i]=i};
function<int(int)> getf=[&](int u) { return f[u]==u?f[u]=getf(f[u]); };
for (auto [w, u, v]:eg)
{
    u=getf(u);
    v=getf(v);
    for (int w1:ver[u]) for (int w2:ver[v]) ans[w1][w2]=ans[w2][w1]=w;
    ver[u]+=ver[v];
    f[v]=u;
}
return ans;
}

```

## 5.7 二分图与网络流建图

以下约定，若为二分图则  $n, m$  表示两侧点数，否则仅  $n$  表示全图点数。

### 5.7.1 二分图边染色

留坑待填。

结论： $\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1$ ，二分图时  $\chi'(G) = \Delta(G)$ 。 $\Delta(G)$  为图的最大度。

### 5.7.2 二分图最小点集覆盖

$ans = \text{maxmatch}$ , 方案如下。

```
#include <bits/stdc++.h>
using namespace std;
const int N=5e3+2;
vector<int> e[N];
int ed[N],lk[N],kl[N],flg[N],now;
bool dfs(int u)
{
    for (int v:e[u]) if (ed[v]!=now)
    {
        ed[v]=now;
        if (!lk[v]||dfs(lk[v])) return lk[v]=u;
    }
    return 0;
}
void dfs2(int u)
{
    for (int v:e[u]) if (!flg[v]) flg[v]=1,dfs2(lk[v]);
}
int main()
{
    int n,m,i,r=0;
    cin>>n>>m;
    while (m--)
    {
        int u,v;
        cin>>u>>v;
        e[u].push_back(v);
    }
    for (i=1;i<=n;i++) dfs(now=i);
    for (i=1;i<=n;i++) kl[lk[i]]=i;
    for (i=1;i<=n;i++) if (!kl[i]) dfs2(i);
    vector<int> A[2];
    for (i=1;i<=n;i++) if (lk[i])
    {
        if (flg[i]) A[1].push_back(i); else A[0].push_back(lk[i]);
    }
    for (int j=0;j<2;j++)
    {
        cout<<A[j].size();
        for (int x:A[j]) cout<<' ';<x;cout<<'\\n';
    }
}
```

### 5.7.3 二分图最大独立集

$ans = n + m - \text{maxmatch}$ , 方案是最小点集覆盖的补集。

### 5.7.4 二分图最小边覆盖

$ans = n + m - \text{maxmatch}$ , 方案是最大匹配加随便一些边（用于覆盖失配点）。无解当且仅当有孤立点，算法会视为单选孤立点（无边）。这个定理对一般图也成立。

### 5.7.5 有向无环图最小不相交链覆盖

$ans = n - \text{maxmatch}$ , 其中二分图建图方法是拆入点和出点 (实现时直接跑一次二分图就行, 不用额外处理), 注意不需要传递闭包。方案如下。

```
#include <bits/stdc++.h>
using namespace std;
const int N=152;
vector<int> e[N];
int lk[N],kl[N],ed[N],now;
bool dfs(int u)
{
    for (int v:e[u]) if (ed[v]!=now)
    {
        ed[v]=now;
        if (!lk[v]||dfs(lk[v])) return lk[v]=u;
    }
    return 0;
}
int main()
{
    int n,m,i;
    ios::sync_with_stdio(0);cin.tie(0);
    cin>>n>>m;
    while (m--)
    {
        int u,v;
        cin>>u>>v;
        e[u].push_back(v);
    }
    int r=0;
    for (i=1;i<=n;i++) r+=dfs(now=i);
    for (i=1;i<=n;i++) kl[lk[i]]=i;
    for (i=1;i<=n;i++) if (ed[i]!=-1&&!lk[i])
    {
        vector<int> ans;
        int u=i;
        while (u)
        {
            ed[u]=-1;
            ans.push_back(u);
            u=kl[u];
        }
        for (int j=0;j<ans.size();j++) cout<<ans[j]<<"\n"[j+1==ans.size()];
    }
    cout<<n-r<<endl;
}
```

### 5.7.6 有向无环图最大互不可达集

$ans = n - \text{maxmatch}$ , 其中二分图建图方法是拆入点和出点 (实现时直接跑一次二分图就行, 不用额外处理), 注意需要传递闭包。方案?

### 5.7.7 最大权闭合子图

若  $v_i > 0$ ,  $s \rightarrow i$  流量  $v_i$ ; 若  $v_i < 0$ ,  $i \rightarrow t$  流量  $-v_i$ 。若原图  $u \rightarrow v$  可花费  $w$  代价违抗, 流量  $w$ , 否则  $+\infty$ 。答案为  $\sum_{v_i > 0} v_i - \text{maxflow}$ 。方案?

## 5.8 二分图匹配 (时间戳写法)

```
bool dfs(int u)
{
    for (int v:e[u]) if (ed[v]!=now)
    {
        ed[v]=now;
        if (!lk[v]||dfs(lk[v])) return lk[v]=u;
    }
    return 0;
}
```

## 5.9 二分图最大权匹配

```
namespace KM
{
    const int N=405;//点数
    typedef long long ll;//答案范围
    const ll inf=1e16;
    int lk[N],kl[N],pre[N],q[N],n,h,t;
    ll sl[N],e[N][N],lx[N],ly[N];
    bool edx[N],edy[N];
    bool ck(int v)
    {
        if (edy[v]=1,kl[v]) return edx[q[++t]=kl[v]]=1;
        while (v) swap(v,lk[kl[v]=pre[v]]);
        return 0;
    }
    void bfs(int u)
    {
        fill_n(sl+1,n,inf);
        memset(edx+1,0,n*sizeof edx[0]);
        memset(edy+1,0,n*sizeof edy[0]);
        q[h=t=1]=u;edx[u]=1;
        while (1)
        {
            while (h<=t)
            {
                int u=q[h++],v;
                ll d;
                for (v=1;v<=n;v++) if (!edy[v]&&sl[v]>=(d=lx[u]+ly[v]-e[u][v])) if (pre[v]=u,d) sl[v]=d; else if (!ck(v)) return;
            }
            int i;
            ll m=inf;
            for (i=1;i<=n;i++) if (!edy[i]) m=min(m,sl[i]);
            for (i=1;i<=n;i++)
            {
                if (edx[i]) lx[i]-=m;
                if (edy[i]) ly[i]+=m; else sl[i]-=m;
            }
        }
    }
}
```

```

    }
    for (i=1;i<=n;i++) if (!edy[i]&&!sl[i]&&!ck(i)) return;
}
}
template<class TT> ll max_weighted_match(int N,const vector<tuple<int,int,TT>> &edges)//lk[[1,
n]]->[1,n]
{
    int i;n=N;
    memset(lk+1,0,n*sizeof lk[0]);
    memset(kl+1,0,n*sizeof kl[0]);
    memset(ly+1,0,n*sizeof ly[0]);
    for (i=1;i<=n;i++) fill_n(e[i]+1,n,0);//若不需保证匹配边最多,置 0 即可,否则 -inf/N
    for (auto [u,v,w]:edges) e[u][v]=max(e[u][v],(ll)w);
    for (i=1;i<=n;i++) lx[i]=*max_element(e[i]+1,e[i]+n+1);
    for (i=1;i<=n;i++) bfs(i);
    ll r=0;
    for (i=1;i<=n;i++) r+=e[i][lk[i]];
    return r;
}
}
using KM::max_weighted_match,KM::lk,KM::kl,KM::e;

```

## 5.10 一般图最大匹配

```

namespace blossom_tree
{
    const int N=1005;
    vector<int> e[N];
    int lk[N],rt[N],f[N],dfn[N],typ[N],q[N];
    int id,h,t,n;
    int lca(int u,int v)
    {
        ++id;
        while (1)
        {
            if (u)
            {
                if (dfn[u]==id) return u;
                dfn[u]=id;u=rt[f[lk[u]]];
            }
            swap(u,v);
        }
    }
    void blm(int u,int v,int a)
    {
        while (rt[u]!=a)
        {
            f[u]=v;
            v=lk[u];
            if (typ[v]==1) typ[q[++t]=v]=0;
            rt[u]=rt[v]=a;
            u=f[v];
        }
    }
    void aug(int u)
    {

```



```

    while (u)
    {
        int v=lk[f[u]];
        lk[lk[u]=f[u]]=u;
        u=v;
    }
}

void bfs(int root)
{
    memset(typ+1,-1,n*sizeof typ[0]);
    iota(rt+1,rt+n+1,1);
    typ[q[h=t=1]=root]=0;
    while (h<=t)
    {
        int u=q[h++];
        for (int v:e[u])
        {
            if (typ[v]==-1)
            {
                typ[v]=1;f[v]=u;
                if (!lk[v]) return aug(v);
                typ[q[++t]=lk[v]]=0;
            } else if (!typ[v]&&rt[u]!=rt[v])
            {
                int a=lca(rt[u],rt[v]);
                blm(v,u,a);blm(u,v,a);
            }
        }
    }
}

int max_general_match(int N,vector<pair<int,int>> edges)//[1,n]
{
    n=N;id=0;
    memset(f+1,0,n*sizeof f[0]);
    memset(dfn+1,0,n*sizeof dfn[0]);
    memset(lk+1,0,n*sizeof lk[0]);
    int i;
    for (i=1;i<=n;i++) e[i].clear();
    mt19937 rnd(114);
    shuffle(all(edges),rnd);
    for (auto [u,v]:edges)
    {
        e[u].push_back(v),e[v].push_back(u);
        if (!(lk[u]||lk[v])) lk[u]=v,lk[v]=u;
    }
    int r=0;
    for (i=1;i<=n;i++) if (!lk[i]) bfs(i);
    for (i=1;i<=n;i++) r+=!!lk[i];
    return r/2;
}

using blossom_tree::max_general_match,blossom_tree::lk;

```

## 5.11 一般图最大权匹配

$n = 400$ : UOJ 600ms, Luogu 135ms

```

#include<bits/stdc++.h>
using namespace std;
#define all(x) (x).begin(),(x).end()
namespace weighted_blossom_tree
{
    #define d(x) (lab[x.u]+lab[x.v]-e[x.u][x.v].w*2)
    const int N=403*2;//两倍点数
    typedef long long ll;//总和大小
    typedef int T;//权值大小
    //均不允许无符号
    const T inf=numeric_limits<int>::max()>>1;
    struct Q
    {
        int u,v;
        T w;
    } e[N][N];
    T lab[N];
    int n,m=0,id,h,t,lk[N],sl[N],st[N],f[N],b[N][N],s[N],ed[N],q[N];
    vector<int> p[N];
    void upd(int u,int v) {if (!sl[v]||d(e[u][v])<d(e[sl[v]][v])) sl[v]=u;}
    void ss(int v)
    {
        sl[v]=0;
        for (int u=1;u<=n;u++) if (e[u][v].w>0&&st[u]!=v&&!s[st[u]]) upd(u,v);
    }
    void ins(int u) {if (u<=n) q[++t]=u; else for (int v:p[u]) ins(v);}
    void mdf(int u,int w)
    {
        st[u]=w;
        if (u>n) for (int v:p[u]) mdf(v,w);
    }
    int gr(int u,int v)
    {
        if ((v=find(all(p[u]),v)-p[u].begin())&1)
        {
            reverse(1+all(p[u]));
            return (int)p[u].size()-v;
        }
        return v;
    }
    void stm(int u,int v)
    {
        lk[u]=e[u][v].v;
        if (u<=n) return;
        Q w=e[u][v];
        int x=b[u][w.u],y=gr(u,x),i;
        for (i=0;i<y;i++) stm(p[u][i],p[u][i^1]);
        stm(x,v);
        rotate(p[u].begin(),y+all(p[u]));
    }
    void aug(int u,int v)
    {
        int w=st[lk[u]];
        stm(u,v);
        if (!w) return;
        stm(w,st[f[w]]);
    }
}

```

```

    aug(st[f[w]],w);
}
int lca(int u,int v)
{
    for (++id;u|v;swap(u,v))
    {
        if (!u) continue;
        if (ed[u]==id) return u;
        ed[u]=id;//????????v?? 这是原作者的注释,我也不知道是啥
        if (u=st[lk[u]]) u=st[f[u]];
    }
    return 0;
}
void add(int u,int a,int v)
{
    int x=n+1,i,j;
    while (x<=m&&st[x]) ++x;
    if (x>m) ++m;
    lab[x]=s[x]=st[x]=0;lk[x]=lk[a];
    p[x].clear();p[x].push_back(a);
    for (i=u;i!=a;i=st[f[j]]) p[x].push_back(i),p[x].push_back(j=st[lk[i]]),ins(j);//复制,改一
    处
    reverse(1+all(p[x]));
    for (i=v;i!=a;i=st[f[j]]) p[x].push_back(i),p[x].push_back(j=st[lk[i]]),ins(j);
    mdf(x,x);
    for (i=1;i<=m;i++) e[x][i].w=e[i][x].w=0;
    memset(b[x]+1,0,n*sizeof b[0][0]);
    for (int u:p[x])
    {
        for (v=1;v<=m;v++) if (!e[x][v].w||d(e[u][v])<d(e[x][v])) e[x][v]=e[u][v],e[v][x]=e[v][
            u];
        for (v=1;v<=n;v++) if (b[u][v]) b[x][v]=u;
    }
    ss(x);
}
void ex(int u) // s[u] == 1
{
    for (int x:p[u]) mdf(x,x);
    int a=b[u][e[u][f[u]].u],r=gr(u,a),i;
    for (i=0;i<r;i+=2)
    {
        int x=p[u][i],y=p[u][i+1];
        f[x]=e[y][x].u;
        s[x]=1;s[y]=0;
        sl[x]=0;ss(y);
        ins(y);
    }
    s[a]=1;f[a]=f[u];
    for (i=r+1;i<p[u].size();i++) s[p[u][i]]=-1,ss(p[u][i]);
    st[u]=0;
}
bool on(const Q &e)
{
    int u=st[e.u],v=st[e.v],a;
    if(s[v]==-1)
    {
        f[v]=e.u;s[v]=1;
    }
}

```

```

        a=st[lk[v]];
        sl[v]=sl[a]=s[a]=0;
        ins(a);
    }
    else if(!s[v])
    {
        a=lca(u,v);
        if (!a) return aug(u,v),aug(v,u),1;
        else add(u,a,v);
    }
    return 0;
}
bool bfs()
{
    memset(s+1,-1,m*sizeof s[0]);
    memset(sl+1,0,m*sizeof sl[0]);
    h=1;t=0;
    int i,j;
    for (i=1;i<=m;i++) if (st[i]==i&&!lk[i]) f[i]=s[i]=0,ins(i);
    if (h>t) return 0;
    while (1)
    {
        while (h<=t)
        {
            int u=q[h++],v;
            if (s[st[u]]!=1) for (v=1; v<=n;v++) if (e[u][v].w>0&&st[u]!=st[v])
            {
                if (d(e[u][v])) upd(u,st[v]); else if (on(e[u][v])) return 1;
            }
        }
        T x=inf;
        for (i=n+1;i<=m;i++) if (st[i]==i&&s[i]==1) x=min(x,lab[i]>>1);
        for (i=1;i<=m;i++) if (st[i]==i&&sl[i]&&s[i]!=1) x=min(x,d(e[sl[i]] [i])>>s[i]+1);
        for (i=1;i<=n;i++) if (~s[st[i]]) if ((lab[i]+=(s[st[i]]*2-1)*x)<=0) return 0;
        for (i=n+1;i<=m;i++) if (st[i]==i&&~s[st[i]]) lab[i]+=(2-s[st[i]]*4)*x;
        h=1;t=0;
        for (i=1;i<=m;i++) if (st[i]==i&&sl[i]&&st[sl[i]]!=i&&!d(e[sl[i]] [i])&&on(e[sl[i]] [i]))
            return 1;
        for (i=n+1;i<=m;i++) if (st[i]==i&&s[i]==1&&!lab[i]) ex(i);
    }
    return 0;
}
template<class TT> ll max_weighted_general_match(int N,const vector<tuple<int,int,TT>> &edges)
    //[1,n], 返回权值
{
    memset(ed+1,0,m*sizeof ed[0]);
    memset(lk+1,0,m*sizeof lk[0]);
    n=m=N;id=0;
    iota(st+1,st+n+1,1);
    int i,j;
    T wm=0;
    ll r=0;
    for (i=1;i<=n;i++) for (j=1;j<=n;j++) e[i][j]={i,j,0};
    for (auto [u,v,w]:edges) wm=max(wm,e[v][u].w=e[u][v].w=max(e[u][v].w,(T)w));
    for (i=1;i<=n;i++) p[i].clear();
    for (i=1;i<=n;i++) for (j=1;j<=n;j++) b[i][j]=i*(i==j);
    fill_n(lab+1,n,wm);
}

```

```

        while (bfs());
        for (i=1;i<=n;i++) if (lk[i]) r+=e[i][lk[i]].w;
        return r/2;
    }
    #undef d
}
using weighted_blossom_tree::max_weighted_general_match,weighted_blossom_tree::lk;
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m;
    cin>>n>>m;
    vector<tuple<int,int,long long>> edges(m);
    for (auto &[u,v,w]:edges) cin>>u>>v>>w;
    cout<<max_weighted_general_match(n,edges)<<'\n';
    for (int i=1;i<=n;i++) cout<<lk[i]<<"\n"[i==n];
}

```

## 5.12 网络流代码

```

namespace net
{
    const int N = 4e5 + 50; //number of nodes
    namespace flow
    {
        const ll inf = 4e18;
        struct Q
        {
            int v;
            ll w;
            int id;
        };
        vector<Q> e[N];
        vector<Q>::iterator fir[N];
        int fc[N], q[N];
        int n, s, t;
        int bfs()
        {
            for (int i = 0; i < n; i++)
            {
                fir[i] = e[i].begin();
                fc[i] = 0;
            }
            int p1 = 0, p2 = 0, u;
            fc[s] = 1; q[0] = s;
            while (p1 <= p2)
            {
                int u = q[p1++];
                for (auto [v, w, id] : e[u]) if (w && !fc[v])
                {
                    q[++p2] = v;
                    fc[v] = fc[u] + 1;
                }
            }
            return fc[t];
        }
    }
}

```

```

11 dfs(int u, ll maxf)
{
    if (u == t) return maxf;
    ll j = 0, k;
    for (auto& it = fir[u]; it != e[u].end(); ++it)
    {
        auto& [v, w, id] = *it;
        if (w && fc[v] == fc[u] + 1 && (k = dfs(v, min(maxf - j, w))))
        {
            j += k;
            w -= k;
            e[v][id].w += k;
            if (j == maxf) return j;
        }
    }
    fc[u] = 0;
    return j;
}

11 max_flow(int _n, const vector<tuple<int, int, ll>>& edges, int _s, int _t)//[0,n]
{
    s = _s; t = _t; n = _n + 1;
    for (int i = 0; i < n; i++) e[i].clear();
    for (auto [u, v, w] : edges) if (u != v)
    {
        e[u].push_back({v, w, (int)e[v].size()});
        e[v].push_back({u, 0, (int)e[u].size() - 1});
    }
    ll r = 0;
    while (bfs()) r += dfs(s, inf);
    return r;
}

}

using flow::max_flow, flow::fc;
namespace match
{
    int lk[N], kl[N], ed[N];
    vector<int> e[N];
    int max_match(int n, int m, const vector<pair<int, int>>& edges)//lk[[0,n]]->[0,m]
    {
        ++n; ++m;
        int s = n + m, t = n + m + 1, i;
        vector<tuple<int, int, ll>> eg;
        eg.reserve(n + m + edges.size());
        for (i = 0; i < n; i++) eg.push_back({s, i, 1});
        for (i = 0; i < m; i++) eg.push_back({i + n, t, 1});
        for (auto [u, v] : edges) eg.push_back({u, v + n, 1});
        int r = max_flow(t, eg, s, t);
        fill_n(lk, n, -1);
        for (i = 0; i < n; i++) for (auto [v, w, id] : flow::e[i]) if (v < s && !w)
        {
            lk[i] = v - n;
            break;
        }
        return r;
    }

    void dfs(int u)
    {

```

```

    for (int v : e[u]) if (!ed[v]) ed[v] = 1, dfs(kl[v]);
}
pair<vector<int>, vector<int>> min_cover(int n, int m, const vector<pair<int, int>>& edges
    )//[0,n]-[0,m]
{
    max_match(n, m, edges);
    ++n; ++m;
    fill_n(kl, m, -1); fill_n(ed, m, 0);
    int i;
    for (i = 0; i < n; i++)
    {
        e[i].clear();
        if (lk[i] != -1) kl[lk[i]] = i;
    }
    for (auto [u, v] : edges) e[u].push_back(v);
    for (i = 0; i < n; i++) if (lk[i] == -1) dfs(i);
    vector<int> r[2];
    for (i = 0; i < m; i++) if (kl[i] != -1)
    {
        if (ed[i]) r[1].push_back(i); else r[0].push_back(kl[i]);
    }
    sort(all(r[0]));
    return {r[0], r[1]};
}
}
using match::max_match, match::min_cover, match::lk, match::kl;
namespace cost_flow
{
    const ll inf = 4e18;
    struct Q
    {
        int v;
        ll w, c;
        int id;
    };
    vector<Q> e[N];
    ll dis[N];
    int pre[N], pid[N], ipd[N];
    bool ed[N];
    int n, s, t;
    pair<ll, ll> spfa()
    {
        queue<int> q;
        fill_n(dis, n, inf);
        memset(ed, 0, n * sizeof ed[0]);
        q.push(s); dis[s] = 0;
        while (q.size())
        {
            int u = q.front(); q.pop(); ed[u] = 0;
            for (auto [v, w, c, id] : e[u]) if (w && dis[v] > dis[u] + c)
            {
                dis[v] = dis[u] + c;
                pre[v] = u;
                pid[v] = e[v][id].id;
                ipd[v] = id;
                if (!ed[v]) q.push(v), ed[v] = 1;
            }
        }
    }
}

```

```

    }
    if (dis[t] == inf) return {0, 0};
    ll mw = 9e18;
    for (int i = t; i != s; i = pre[i]) mw = min(mw, e[pre[i]][pid[i]].w);
    for (int i = t; i != s; i = pre[i]) e[pre[i]][pid[i]].w -= mw, e[i][ipd[i]].w += mw;
    return {mw, mw * dis[t]};
}

pair<ll, ll> mcmf_spfa(int _n, const vector<tuple<int, int, ll, ll>>& edges, int _s, int
    _t)//[0,n]
{
    s = _s; t = _t; n = _n + 1;
    for (int i = 0; i < n; i++) e[i].clear();
    for (auto [u, v, w, c] : edges) if (u != v)
    {
        e[u].push_back({v, w, c, (int)e[v].size()});
        e[v].push_back({u, 0, -c, (int)e[u].size() - 1});
    }
    pair<ll, ll> r{0, 0}, rr;
    while ((rr = spfa()).first) r = {r.first + rr.first, r.second + rr.second};
    return r;
}

pair<ll, ll> mcmf_dijk(int _n, const vector<tuple<int, int, ll, ll>>& edges, int _s, int
    _t)//[0,n]
{
    s = _s; t = _t; n = _n + 1;
    for (int i = 0; i < n; i++) e[i].clear();
    for (auto [u, v, w, c] : edges) if (u != v)
    {
        e[u].push_back({v, w, c, (int)e[v].size()});
        e[v].push_back({u, 0, -c, (int)e[u].size() - 1});
    }
    static ll h[N];
    auto get_h = [&]()
    {
        fill_n(h, n, inf);
        memset(ed, 0, n * sizeof ed[0]);
        queue<int> q;
        q.push(s); h[s] = 0;
        while (q.size())
        {
            int u = q.front(); q.pop(); ed[u] = 0;
            for (auto [v, w, c, id] : e[u]) if (w && h[v] > h[u] + c)
            {
                h[v] = h[u] + c;
                if (!ed[v]) q.push(v), ed[v] = 1;
            }
        }
        return;
    };
    auto dijkstra = [&]() -> pair<ll, ll>
    {
        static int fl[N], zl[N];
        int i;
        memset(ed, 0, n * sizeof ed[0]);
        fill_n(dis, n, inf);
        typedef pair<ll, int> pa;
        priority_queue<pa, vector<pa>, greater<pa>> q;

```



```

        dis[s] = 0; q.push({0, s});
        while (q.size())
        {
            int u = q.top().second;
            q.pop(); ed[u] = 1;
            i = 0;
            for (auto [v, w, c, id] : e[u])
            {
                if (w && dis[v] > dis[u] + c) fl[v] = id, zl[v] = i, q.push({dis[v] = dis
                    [pre[v] = u] + c, v});
                ++i;
            }
            while (q.size() && ed[q.top().second]) q.pop();
        }
        if (dis[t] == inf) return {0, 0};
        ll tf = numeric_limits<ll>::max();
        for (i = t; i != s; i = pre[i]) tf = min(tf, e[pre[i]][zl[i]].w);
        for (i = t; i != s; i = pre[i]) e[pre[i]][zl[i]].w -= tf, e[i][fl[i]].w += tf;
        for (int u = 0; u < n; u++) for (auto& [v, w, c, id] : e[u]) c += dis[u] - dis[v]
            ];
        return {tf, tf * (h[t] += dis[t])};
    };
    get_h();
    for (int u = 0; u < n; u++) for (auto& [v, w, c, id] : e[u]) c += h[u] - h[v];
    pair<ll, ll> r{0, 0}, rr;
    while ((rr = dijkstra()).first) r = {r.first + rr.first, r.second + rr.second};
    return r;
}

}

using cost_flow::mcmf_spfa, cost_flow::mcmf_dijk;
namespace bounded_flow
{
    bool valid_flow(int n, const vector<tuple<int, int, ll, ll>>& edges)//方案需加上 1
    {
        if (!edges.size()) return 1;
        ++n;
        int i;
        ll tot = 0;
        static ll cd[N];
        memset(cd, 0, n * sizeof cd[0]);
        for (auto [u, v, l, r] : edges) cd[u] += l, cd[v] -= l;
        vector<tuple<int, int, ll>> eg;
        eg.reserve(n + edges.size());
        for (i = 0; i < n; i++) if (cd[i] > 0) eg.push_back({i, n + 1, cd[i]}), tot += cd[i];
        else if (cd[i] < 0) eg.push_back({n, i, -cd[i]});
        for (auto [u, v, l, r] : edges) eg.push_back({u, v, r - l});
        return tot == flow::max_flow(n + 1, eg, n, n + 1);
    }

    ll valid_flow_st(int n, vector<tuple<int, int, ll, ll>> edges, int s, int t)//-1 invalid,
        ll=ll
    {
        ll tot = 0;
        for (auto [u, v, l, r] : edges) tot += (u == s) * r;
        edges.push_back({t, s, 0, tot});
        if (!valid_flow(n, edges)) return -1;
        assert(flow::e[s].back().v == t);
        assert(flow::e[t].back().v == s);
    }
}

```

```

        return tot - flow::e[t].back().w;
    }
ll valid_max_flow(int n, const vector<tuple<int, int, ll, ll>>& edges, int s, int t)//-1
    invalid, ll=ll
{
    ll r = valid_flow_st(n, edges, s, t);
    if (r < 0) return r;
    flow::s = s; flow::t = t;
    flow::e[s].pop_back(); flow::e[t].pop_back();
    while (flow::bfs()) r += flow::dfs(s, flow::inf);
    return r;
}
ll valid_min_flow(int n, const vector<tuple<int, int, ll, ll>>& edges, int s, int t)//-1
    invalid, ll=ll
{
    ll r = valid_flow_st(n, edges, s, t);
    if (r < 0) return r;
    flow::s = t; flow::t = s;
    flow::e[s].pop_back(); flow::e[t].pop_back();
    while (flow::bfs()) r -= flow::dfs(t, flow::inf);
    return r;
}
}
using bounded_flow::valid_flow, bounded_flow::valid_flow_st, bounded_flow::valid_max_flow,
    bounded_flow::valid_min_flow;
namespace bounded_cost_flow
{
    pair<ll, ll> valid_mcf(int n, const vector<tuple<int, int, ll, ll, ll>>& edges, int s, int
        t)//[u,v,l,r,c],mincost flow
    {
        ++n;
        int ss = n, tt = n + 1;
        static ll cd[N];
        memset(cd, 0, n * sizeof cd[0]);
        for (auto [u, v, l, r, c] : edges) cd[u] += l, cd[v] -= l;
        vector<tuple<int, int, ll, ll>> e;
        ll t1 = 0, t2 = 0;
        for (int i = 0; i < n; i++) if (cd[i] > 0) e.push_back({i, tt, cd[i], 0}), t2 += cd[i];
        else if (cd[i] < 0) e.push_back({ss, i, -cd[i], 0});
        for (auto [u, v, l, r, c] : edges) e.push_back({u, v, r - l, c});
        for (auto [u, v, w, c] : e) t1 += (u == s) * w;
        e.push_back({t, s, t1, 0});
        auto res = mcmf_spfa(tt, e, ss, tt);//checked dijk
        if (res.first != t2) return {-1, -1};
        res.first = cost_flow::e[s].back().w;
        for (auto [u, v, l, r, c] : edges) res.second += l * c;
        return res;
    }
    pair<ll, ll> valid_mcmf(int n, const vector<tuple<int, int, ll, ll, ll>>& edges, int s,
        int t)//[u,v,l,r,c],mincost max_flow
    {
        auto r = valid_mcf(n, edges, s, t);
        if (r.first < 0) return {-1, -1};
        cost_flow::e[s].pop_back();
        cost_flow::e[t].pop_back();
        cost_flow::s = s; cost_flow::t = t;
        pair<ll, ll> rr;
    }
}

```

```

        while ((rr = cost_flow::spfa()).first) r = {r.first + rr.first, r.second + rr.second};
        //spfa ver. not checked dijk
        return r;
    }
}
using bounded_cost_flow::valid_mcf, bounded_cost_flow::valid_mcmf;
namespace ne_cost_flow
{
    pair<ll, ll> ne_mcmf(int n, const vector<tuple<int, int, ll, ll>>& edges, int s, int t)
    {
        vector<tuple<int, int, ll, ll, ll>> e;
        for (auto [u, v, w, c] : edges) if (c >= 0) e.push_back({u, v, 0, w, c}); else
        {
            e.push_back({u, v, w, w, c});
            e.push_back({v, u, 0, w, -c});
        }
        return valid_mcmf(n, e, s, t);
    }
}
using ne_cost_flow::ne_mcmf;
}

```

### 5.13 费用流 (SPFA)

```

bool dfs()
{
    memset(jl, -0x3f, sizeof(jl));
    jl[d1[tou=wei]=0]=0;
    while (tou<=wei)
    {
        ed[x=d1[tou++]]=0;
        for (i=fir[x]; i; i=nxt[i]) if ((lj[i][1])&&(jl[lj[i][0]]<jl[x]+lj[i][2]))
        {
            jl[lj[i][0]]=jl[x]+lj[i][2];
            qq[lj[i][0]]=x;
            dy[lj[i][0]]=i;
            if (!ed[lj[i][0]]) ed[d1[++wei]=lj[i][0]]=1;
        }
    }
    zg=m;
    if (jl[t]==jl[t+1]) return 0;
    for (i=t; i; i=qq[i]) zg=min(zg, lj[dy[i]][1]);
    for (i=t; i; i=qq[i])
    {
        lj[dy[i]][1]-=zg;
        ans+=zg*lj[dy[i]][2];
        if (dy[i]&1) lj[dy[i]+1][1]+=zg; else lj[dy[i]-1][1]+=zg;
    }
    return 1;
}
while (dfs());

```

### 5.14 费用流 (Dijkstra)

```

priority_queue<pa,vector<pa>,greater<pa> > heap;
const int N=5e3+2,M=1e5+2;
pa ans;
int lj[M][3],nxt[M],fir[N],dis[N],h[N],pre[N],f1[N];
int n,m,s,t,bs,x,y,z,w,ans1,ans2;
bool ed[N];
void add(const int u,const int v,const int x,const int y)
{
    lj[++bs][0]=v;
    lj[bs][1]=x;
    lj[bs][2]=y;
    nxt[bs]=fir[u];
    fir[u]=bs;
    lj[++bs][0]=u;
    lj[bs][1]=0;
    lj[bs][2]=-y;
    nxt[bs]=fir[v];
    fir[v]=bs;
}
void spfa()//本题中用dijkstra代替,目的是处理 h 数组。
{
    int x,i,j;
    memset(h,0x3f,sizeof(h));h[s]=0;
    heap.push(make_pair(0,s));
    while (!heap.empty())
    {
        ed[x=heap.top().second]=1;heap.pop();
        for (i=fir[x];i;i=nxt[i]) if ((lj[i][1]&&(h[lj[i][0]]>h[x]+lj[i][2])))
            heap.push(make_pair(h[lj[i][0]]=h[x]+lj[i][2],lj[i][0]));
        while ((!heap.empty())&&(ed[heap.top().second])) heap.pop();
    }
    for (i=1;i<=n;i++) for (j=fir[i];j;j=nxt[j]) lj[j][2]+=h[i]-h[lj[j][0]];
    memset(ed,0,sizeof(ed));
}
pa dijkstra()
{
    int i,j,x,tf=1e9;
    memset(dis,0x3f,sizeof(dis));memset(pre,0,sizeof(pre));dis[s]=0;heap.push(make_pair(0,s));
    while (!heap.empty())
    {
        ed[x=heap.top().second]=1;heap.pop();
        for (i=fir[x];i;i=nxt[i]) if ((lj[i][1]&&(dis[lj[i][0]]>dis[x]+lj[i][2])))
            heap.push(make_pair(dis[lj[i][0]]=dis[x]+lj[i][2],lj[i][0]));f1[lj[i][0]]=i;
        while ((!heap.empty())&&(ed[heap.top().second])) heap.pop();
    }
    if (dis[t]==dis[t+1]) return make_pair(0,0);
    for (i=t;i!=s;i=pre[i]) tf=min(tf,lj[f1[i]][1]);
    for (i=t;i!=s;i=pre[i]) lj[f1[i]][1]-=tf,lj[f1[i]^1][1]+=tf;
    for (i=1;i<=n;i++) for (j=fir[i];j;j=nxt[j]) lj[j][2]+=dis[i]-dis[lj[j][0]];
    h[t]+=dis[t];memset(ed,0,sizeof(ed));
    return make_pair(tf,tf*h[t]);
}
signed main()
{
    while (!heap.empty()) heap.pop();

```

```

read(n);read(m);read(s);read(t);bs=1;
while (m--)
{
    read(x);read(y);read(z);read(w);
    add(x,y,z,w);
}
spfa();
while ((ans=dijkstra()).first) ans1+=ans.first,ans2+=ans.second;
printf("%d_%d",ans1,ans2);
}

```

### 5.15 假花树

一种错误的一般图最大匹配算法，但较难卡掉。推荐在时间不足时作为乱搞使用。

```

mt19937 rnd(3214);
vector<int> lj[N];
int lk[N],ed[N];
int n,m,cnt,i,t,x,y,ans,la;
bool dfs(int x)
{
    ed[x]=cnt;int v;
    shuffle(lj[x].begin(),lj[x].end(),rnd);
    for (auto u:lj[x]) if (ed[v=lk[u]]!=cnt)
    {
        lk[v]=0,lk[u]=x,lk[x]=u;
        if (!v||dfs(v)) return 1;
        lk[v]=u,lk[u]=v,lk[x]=0;
    }
    return 0;
}
int main()
{
    srand(time(0));la=-1;
    read(n);read(m);
    while (m--) read(x),read(y),lj[x].push_back(y),lj[y].push_back(x);
    while (la!=ans)
    {
        memset(ed+1,0,n<<2);la=ans;
        for (i=1;i<=n;i++) if (!lk[i]) ans+=dfs(cnt=i);
    }
    printf("%d\n",ans);
    for (i=1;i<=n;i++) printf("%d_",lk[i]);
}

```

### 5.16 Stoer-Wagner 全局最小割

无向图  $G$  的最小割为：一个去掉后可以使  $G$  变成两个连通分量，且边权和最小的边集的边权和。

$O(n^3)$ 。可优化到  $O(nm \log n)$ 。

```

#include <bits/stdc++.h>
using namespace std;
namespace StoerWagner
{

```

```

const int N=602;//点数
typedef int T;//边权和
T e[N][N],w[N];
int ed[N],p[N],f[N];//f 仅输出方案用
int getf(int u){return f[u]==u?f[u]:getf(f[u]);}
template<class TT> pair<T,vector<int>> mincut(int n,const vector<tuple<int,int,TT>> &edges)//
    [1,n], 返回某一集合
{
    vector<int> ans;ans.reserve(n);
    int i,j,m;
    T r;
    r=numeric_limits<T>::max();
    for (i=1;i<=n;i++) memset(e[i]+1,0,n*sizeof e[0][0]);
    for (auto [u,v,w]:edges) e[u][v]+=w,e[v][u]+=w;
    fill_n(ed+1,n,0);
    iota(f+1,f+n+1,1);
    for (m=n;m>1;m--)
    {
        fill_n(w+1,n,0);
        for (i=1;i<=n;i++) ed[i]&=2;
        for (i=1;i<=m;i++)
        {
            int x=0;
            for (j=1;j<=n;j++) if (!ed[j]) break;x=j;
            for (j++;j<=n;j++) if (!ed[j]*w[j]>w[x]) x=j;
            ed[p[i]=x]=1;
            for (j=1;j<=n;j++) w[j]+=!ed[j]*e[x][j];
        }
        int s=p[m-1],t=p[m];
        if (r>w[t])
        {
            r=w[t];ans.clear();
            for (i=1;i<=n;i++) if (getf(i)==getf(t)) ans.push_back(i);
        }
        for (i=1;i<=n;i++) e[i][s]=e[s][i]+e[t][i];
        ed[t]=2;
        f[getf(s)]=getf(t);
    }
    return {r,ans};
}
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m;
    cin>>n>>m;
    vector<tuple<int,int,int>> e(m);
    for (auto &[u,v,w]:e) cin>>u>>v>>w;
    auto [_ ,v]=StoerWagner::mincut(n,e);
    cout<<_<<endl;
    static int ed[602];
    for (int x:v) ed[x]=1;
    for (auto [u,v,w]:e) _-=w*(ed[u]^ed[v]);
    assert(!_);
}

```

### 5.17 点双

一些结论：

判定一个图里是否有（点不重复）偶环：看其所有点双，若存在点数为偶数的或边数多于点数的点双，则存在偶环。

（无自环时）点双的边不交，边双的点不交。点双内的总点数  $O(n)$ ，总边数为  $m$ ，边双内的总点数为  $n$ ，总边数不超过  $m$ 。

构造函数传入邻接表和边数，其中 `pair` 的 `second` 是边的标号。

所有标号从 0 开始。

不能处理有自环的情况，因为此时点双内的总边数不是线性的。

`bcc_node`：每个点双包含的点（已验证）；`bcc_edge`：每个点双包含的边；`bcc_n`：新图点数；`ct`：是否割点（已验证）；`blk`：边所属点双标号。

```
struct node_bcc
{
    int n, id, tp, bcc_n;
    vector<vector<pair<int, int>>> e;
    vector<vector<int>> bcc_node, bcc_edge;
    vector<int> dfn, low, st, ed, blk, ct;
    node_bcc(const vector<vector<pair<int, int>>> &e, int m) :
        n(e.size()), id(0), tp(0), bcc_n(0), e(e), dfn(n, -1), low(n, -1), st(m), ed(m), blk(m),
        ct(n)
    {
        for (int i = 0; i < n; i++) if (dfn[i] == -1) dfs(i, 1);
        bcc_node.resize(bcc_n);
        for (int i = 0; i < n; i++) for (auto [v, w] : e[i]) bcc_node[blk[w]].push_back(i);
        vector<int> flg(n);
        for (auto &v : bcc_node)
        {
            vector<int> t;
            for (int x : v) if (!exchange(flg[x], 1)) t.push_back(x);
            swap(t, v);
            for (int x : v) flg[x] = 0;
        }
        for (int i = 0; i < n; i++) if (e[i].size() == 0)
        {
            bcc_node.push_back({i});
            bcc_edge.push_back({ });
            ++bcc_n;
        }
    }
}

void dfs(int u, bool rt)
{
    dfn[u] = low[u] = id++;
    int cnt = 0;
    for (auto [v, w] : e[u]) if (!ed[w])
    {
        st[tp++] = w;
        ed[w] = 1;
        if (dfn[v] == -1)
        {
            dfs(v, 0);
            ++cnt;
            cmin(low[u], low[v]);
            if (dfn[u] <= low[v])
            {

```

```

        ct[u] = cnt > rt;
        bcc_edge.push_back({ });
        do
        {
            bcc_edge[bcc_n].push_back(st[--tp]);
            blk[st[tp]] = bcc_n;
        } while (st[tp] != w);
        ++bcc_n;
    }
}
else cmin(low[u], dfn[v]);
}
}
};

```

## 5.18 边双

$O(n + m)$ ,  $O(n + m)$ 。

构造函数传入邻接表和边数，其中 pair 的 second 是边的标号。

所有标号从 0 开始。

bcc\_node: 每个边双包含的点（已验证）；bcc\_edge: 每个边双包含的边；bcc\_n: 新图点数；cur\_e: 新图边表；ct: 是否割边；blk: 点所属边双标号。

```

struct edge_bcc
{
    int n, id, tp, bcc_n;
    vector<vector<pair<int, int>>> e, cur_e;
    vector<vector<int>> bcc_node, bcc_edge;
    vector<int> dfn, low, st, blk, ct;
    edge_bcc(const vector<vector<pair<int, int>>> &e, int m) :
        n(e.size()), id(0), tp(0), bcc_n(0), e(e), dfn(n, -1), low(n, -1), st(n), blk(n), ct(m)
    {
        for (int i = 0; i < n; i++) if (dfn[i] == -1) dfs(i, -1);
        cur_e.resize(bcc_n);
        for (int i = 0; i < n; i++) for (auto [v, w] : e[i]) if (ct[w]) cur_e[blk[i]].push_back({blk[v], w});
        else bcc_edge[blk[i]].push_back(w);
        vector<int> flg(m);
        for (auto &v : bcc_edge)
        {
            vector<int> t;
            for (int x:v) if (!exchange(flg[x], 1)) t.push_back(x);
            swap(t, v);
        }
    }
    void dfs(int u, int fw)
    {
        dfn[u] = low[u] = id++;
        st[tp++] = u;
        for (auto [v, w] : e[u]) if (w != fw)
        {
            if (dfn[v] == -1)
            {
                dfs(v, w);
                cmin(low[u], low[v]);
                ct[w] = (dfn[u] < low[v]);
            }
        }
    }
};

```



```

    }
    else cmin(low[u], dfn[v]);
}
if (dfn[u] == low[u])
{
    bcc_node.push_back({ });
    bcc_edge.push_back({ });
    do
    {
        bcc_node[bcc_n].push_back(st[--tp]);
        blk[st[tp]] = bcc_n;
    } while (st[tp] != u);
    ++bcc_n;
}
}
};
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n, m, i;
    cin >> n >> m;
    vector<vector<pair<int, int>>> e(n);
    for (i = 0; i < m; i++)
    {
        int u, v;
        cin >> u >> v;
        --u, --v;
        e[u].push_back({v, i});
        e[v].push_back({u, i});
    }
    edge_bcc s(e, m);
    cout << s.bcc_n << '\n';
    for (auto &v : s.bcc_node)
    {
        for (int &x : v) ++x;
        cout << v.size() << '□' << v << '\n';
    }
}

```

## 5.19 双极分解

无向图，图点双连通时对任意  $s, t$  存在。

含义：确定一个拓扑序，使得按这个拓扑序定向后，入度为 0 的只有  $s$ ，出度为 0 的只有  $t$ 。

```

vector<int> bipolar_orientation(const vector<pair<int, int>> &edges, int n, int s, int t)//[0,n)
{
    assert(s!=t);
    vector e(n, vector<int>());
    for (auto [u, v]:edges)
    {
        e[u].push_back(v);
        e[v].push_back(u);
    }
    int cur=1, i;
    vector<int> pre(n), low(n), p(n);
    pre[s]=1;

```

```

vector<int> id;
bool flg=0;
function<void(int)> dfs=[&](int x)
{
    pre[x]=++cur;
    low[x]=x;
    for (int y:e[x])
    {
        flg|=y==s;
        if (pre[y]==0)
        {
            id.push_back(y);
            dfs(y);
            p[y]=x;
            if (pre[low[y]]<pre[low[x]]) low[x]=low[y];
        }
        else if (pre[y]!=0&&pre[y]<pre[low[x]]) low[x]=y;
    }
};
dfs(t);
if (!flg) return { };
vector<int> sign(n, -1);
vector<int> l(n), r(n);
r[s]=t;
l[t]=s;
for (int v:id)
{
    if (sign[low[v]]== -1)
    {
        l[v]=l[p[v]];
        r[l[v]]=v;
        l[p[v]]=v;
        r[v]=p[v];
        sign[p[v]]=1;
    }
    else
    {
        r[v]=r[p[v]];
        l[r[v]]=v;
        r[p[v]]=v;
        l[v]=p[v];
        sign[p[v]]=-1;
    }
}
vector<int> a(n);
int x;
for (i=0, x=s; i<n; x=r[x], i++) a[i]=x;
vector<int> ia(n, -1), rd(n), cd(n);
for (i=0; i<n; i++) ia[a[i]]=i;
if (count(all(ia), -1)) return { };
for (auto [u, v]:edges)
{
    if (ia[u]>ia[v]) swap(u, v);
    ++cd[u]; ++rd[v];
}
for (i=0; i<n; i++) if (i!=s&&i!=t&&(!cd[i]||!rd[i])) return { };
return a;

```

```
}

```

## 5.20 输出负环

```
#include <bits/stdc++.h>
using namespace std;
const int N=34;
struct Q
{
    int v,w,c;
    Q(){}
    Q(int x,int y,int z):v(x),w(y),c(z){}
};
vector<Q> lj[N];
int dis[N],cnt[N],pt[N],S;
Q pre[N],st[N];
int n,m,ans,tp;
bool ed[N];
int main()
{
    freopen("arbitrage.in","r",stdin);
    freopen("arbitrage.out","w",stdout);
    ios::sync_with_stdio(0);cin.tie(0);
    cin>>n>>m;
    while (m--)
    {
        int x,y,z,w;
        cin>>x>>y>>z>>w;
        lj[x].emplace_back(y,w,z);
        lj[y].emplace_back(x,0,-z);
    }
    for (int i=1;i<=n;i++) lj[0].emplace_back(i,1,0);
    while (1)
    {
        memset(dis,-0x3f,sizeof dis);dis[0]=0;
        for (int i=0;i<=n;i++) ed[i]=cnt[i]=0;S=-1;
        queue<int> q;q.push(0);
        while (!q.empty())
        {
            int u=q.front();q.pop();ed[u]=0;
            for (auto &[v,w,c]:lj[u]) if (w&&dis[v]<dis[u]+c)
            {
                dis[v]=dis[u]+c;pre[v]=Q(u,w,c);
                if (!ed[v])
                {
                    if (++cnt[v]>n+1) {S=v;goto aa;}
                    ed[v]=1;q.push(v);
                }
            }
        }
        aa:;
        if (S==-1) break;
        {
            static bool ed[N];
            memset(ed,0,sizeof ed);
            while (!ed[S]) ed[S]=1,S=pre[S].v;

```

```

    }
    st[tp=1]=pre[S];pt[1]=S;
    int x=pre[S].v;
    while (x!=S)
    {
        st[++tp]=pre[x];pt[tp]=x;
        x=pre[x].v;
        assert(tp<=n+5);
    }
    int fl=1e9;
    for (int j=1;j<=tp;j++) fl=min(fl,st[j].w);
    assert(fl);
    for (int j=1;j<=tp;j++)
    {
        ans+=fl*st[j].c;
        int nn=0;
        for (auto &[v,w,c]:lj[st[j].v]) if (v==pt[j]&&st[j].c==c&&st[j].w==w) {++nn;w-=fl;break;}
        for (auto &[v,w,c]:lj[pt[j]]) if (v==st[j].v&&st[j].c+c==0) {++nn;w+=fl;break;}assert(
            nn==2);
    }
}
cout<<ans<<endl;
}

```

## 5.21 (基环) 树哈希

有根树返回每个子树的哈希值，无根树返回树的哈希值（长度至多为 2 的 vector），基环树返回图的哈希值（长度等于环长的 vector）。

```

vector<int> tree_hash(const vector<vector<int>>& e, int root)//[0,n)
{
    int n = e.size();
    static map<vector<int>, int> mp;
    static int id = 0;
    vector<int> h(n), ed(n);
    function<void(int)> dfs = [&](int u)
    {
        ed[u] = 1;
        vector<int> c;
        for (int v : e[u]) if (!ed[v])
        {
            dfs(v);
            c.push_back(h[v]);
        }
        sort(all(c));
        if (!mp.count(c)) mp[c] = id++;
        h[u] = mp[c];
    };
    dfs(root);
    return h;
}
vector<int> tree_hash(const vector<vector<int>>& e)//[0,n)
{
    int n = e.size();
    if (n == 0) return { };
}

```

```

vector<int> sz(n), mx(n);
function<void(int)> dfs = [&](int u)
{
    sz[u] = 1;
    for (int v : e[u]) if (!sz[v])
    {
        dfs(v);
        sz[u] += sz[v];
        cmax(mx[u], sz[v]);
    }
    cmax(mx[u], n - sz[u]);
};
dfs(0);
int m = *min_element(all(mx)), i;
vector<int> rt;
for (i = 0; i < n; i++) if (mx[i] == m) rt.push_back(i);
for (int& u : rt) u = tree_hash(e, u)[u];
sort(all(rt));
return rt;
}
template<class T> void min_order(vector<T>& a)
{
    int n = a.size(), i, j, k;
    a.resize(n * 2);
    for (i = 0; i < n; i++) a[i + n] = a[i];
    i = k = 0; j = 1;
    while (i < n && j < n && k < n)
    {
        T x = a[i + k], y = a[j + k];
        if (x == y) ++k; else
        {
            (x > y ? i : j) += k + 1;
            j += (i == j);
            k = 0;
        }
    }
    a.resize(n);
    //[min(i,j),n)+[0,min(i,j))
    rotate(a.begin(), min(i, j) + all(a));
}
vector<int> pseudotree_hash(const vector<vector<int>>& e)//[0,n)
{
    int n = e.size();
    static map<vector<int>, int> mp;
    static int id = 0;
    vector<int> f(n), ed(n), h(n);
    pair lp{-1, -1};
    function<void(int)> dfs = [&](int u)
    {
        ed[u] = 1;
        for (int v : e[u]) if (!ed[v])
        {
            f[v] = u;
            dfs(v);
        }
        else if (v != f[u]) lp = {u, v};
    };
};

```

```

dfs(0);
auto [x, y] = lp;
vector<int> node = {y};
do node.push_back(y = f[y]); while (y != x);
fill(all(ed), 0);
for (int u : node) ed[u] = 1;
dfs = [&](int u)
{
    ed[u] = 1;
    vector<int> c;
    for (int v : e[u]) if (!ed[v])
    {
        dfs(v);
        c.push_back(h[v]);
    }
    sort(all(c));
    if (!mp.count(c)) mp[c] = id++;
    h[u] = mp[c];
};
vector<int> r0;
for (int u : node)
{
    dfs(u);
    r0.push_back(h[u]);
}
auto r1 = r0;
reverse(all(r1));
min_order(r0);
min_order(r1);
return min(r0, r1);
}

```

## 5.22 无向图最小环

原理：floyd 外层循环本质是计算只经过  $\leq k$  的点的最短路。因此枚举环上标号最大的，在做这一轮转移之前正好是不经过它的最短路。

$O(n^3)$ ,  $O(n^2)$ 。

```

int f[N][N], j1[N][N];
int n, m, c, ans = inf, i, j, k, x, y, z;
int main()
{
    read(n); read(m);
    memset(f, 0x3f, sizeof(f));
    memset(j1, 0x3f, sizeof(j1));
    while (m--)
    {
        read(x); read(y); read(z);
        j1[x][y] = j1[y][x] = f[x][y] = f[y][x] = min(f[y][x], z);
    }
    for (k = 1; k <= n; k++)
    {
        for (i = 1; i < k; i++) if (j1[k][i] != j1[0][0]) for (j = 1; j < i; j++)
            if (j1[k][j] != j1[0][0]) ans = min(ans, j1[k][i] + j1[k][j] + f[i][j]);
        for (i = 1; i <= n; i++) if (i != k) for (j = 1; j <= n; j++)
            if ((j != i) && (j != k)) f[i][j] = min(f[i][j], f[i][k] + f[k][j]);
    }
}

```

```

}
if (ans==inf) puts("No solution."); else printf("%d",ans);
}

```

### 5.23 切比雪夫距离最小生成树

原理：先转曼哈顿距离，再用曼哈顿的板子。

$O(n \log n)$ ,  $O(n)$ 。

```

const int N=3e5+2,M=N<<2;
struct P
{
    int u,v,w;
    P(int a=0,int b=0,int c=0):u(a),v(b),w(c){}
    bool operator<(const P &o) const {return w<o.w;}
};
struct Q
{
    int x,y,id;
    Q(int a=0,int b=0,int c=0):x(a),y(b),id(c){}
    bool operator<(const Q &o) const {return x!=o.x?x>o.x:y>o.y;}
};
ll ans;
P lb[M];
Q a[N],b[N];
int f[N],c[N];
int n,m,i,x,y;
struct bit
{
    int a[N],pos[N],n;
    void init(int &nn)
    {
        memset(a+1,0x7f,(n=nn)*sizeof a[0]);
        memset(pos+1,0,n*sizeof pos[0]);
    }
    void mdf(int x,const int y,const int z)
    {
        if (a[x]>y) a[x]=y,pos[x]=z;
        while (x-=x&-x) if (a[x]>y) a[x]=y,pos[x]=z;
    }
    int sum(int x)
    {
        int r=a[x],rr=pos[x];
        while ((x+=x&-x)<=n) if (a[x]<r) r=a[x],rr=pos[x];
        return rr;
    }
};
bit s;
void cal()
{
    int i,x,y;
    s.init(n);
    memcpy(b+1,a+1,sizeof(Q)*n);
    sort(a+1,a+n+1);
    for (i=1;i<=n;i++) c[i]=a[i].y-a[i].x;
    sort(c+1,c+n+1);
}

```

```

for (i=1;i<=n;i++)
{
    if (x=s.sum(y=lower_bound(c+1,c+n+1,a[i].y-a[i].x)-c))
        lb[++m]=P(a[x].id,a[i].id,a[x].x+a[x].y-a[i].x-a[i].y); //谨防 int 爆
    s.mdf(y,a[i].y+a[i].x,i);
}
memcpy(a+1,b+1,sizeof(Q)*n);
}
int getf(int x) {return f[x]==x?f[x]:f[x]=getf(f[x]);}
int main()
{
    read(n);
    for (i=1;i<=n;i++) {read(a[f[i]=a[i].id=i].x);read(a[i].y);
        swap(a[i].x,a[i].y);a[i]=Q(a[i].x+a[i].y,a[i].x-a[i].y,i);}
    cal();for (i=1;i<=n;i++) swap(a[i].x,a[i].y);
    cal();for (i=1;i<=n;i++) a[i].y=-a[i].y;
    cal();for (i=1;i<=n;i++) swap(a[i].x,a[i].y);
    cal();sort(lb+1,lb+m+1);
    for (i=1;i<=m;i++) if ((x=getf(lb[i].u))!=(y=getf(lb[i].v))) f[x]=y,ans+=lb[i].w;
    printf("%lld\n",ans>>1);
}

```

## 5.24 点分治

点分治板子的参考意义不大。

$O(n \log n)$ ,  $O(n)$ 。

```

int siz[N], dep[N];
int n, ksiz, md, rt, mn;
bool ed[N];
void find(int u)
{
    ed[u] = 1; siz[u] = 1;
    int mx = 0;
    for (int v : e[u]) if (!ed[v])
    {
        find(v);
        siz[u] += siz[v];
        mx = max(mx, siz[v]);
    }
    mx = max(mx, ksiz - siz[u]);
    if (mn > mx) mn = mx, rt = u;
    ed[u] = 0;
}
void cal(int u)
{
    md = max(md, dep[u]);
    ed[u] = 1; ++cnt[dep[u]];
    for (int v : e[u]) if (!ed[v])
    {
        dep[v] = dep[u] + 1;
        cal(v);
    }
    ed[u] = 0;
}
void solve(int u)

```



```

{
    mn = 1e9;
    find(u);
    ed[rt] = 1;
    vector<int> c;
    for (int v : e[rt]) if (!ed[v])
    {
        c.push_back(v);
        if (siz[v] >= siz[rt]) siz[v] = siz[u] - siz[rt];
    }
    sort(all(c), [&](const int &a, const int &b) {return siz[a] < siz[b]; });
    NTT::Q a(vector<ui>{1});
    NT::Q b(vector<ui>{1});
    for (int v : c)
    {
        md = 0; dep[v] = 1;
        cal(v); ++md;
        vector<ui> d(cnt, cnt + md);
        NTT::Q e(d);
        NT::Q f(d);
        auto g = e & a;
        auto h = f & b;
        for (int i = 0; i < g.a.size(); i++) r1[i] = (r1[i] + g.a[i]) % NTT::p;
        for (int i = 0; i < h.a.size(); i++) r2[i] = (r2[i] + h.a[i]) % NT::p;
        a += e; b += f;
        fill_n(cnt, md, 0);
    }
    for (int v : c)
    {
        ksiz = siz[v];
        solve(v);
    }
}
}

```

## 5.25 点分树

核心结论：点分树上 lca 出现在原树路径上。

$O(n \log^2 n)$ ,  $O(n \log n)$ 。

```

template<typename typC> struct bit
{
    vector<typC> a;
    int n;
    bit() { }
    bit(int nn) : n(nn), a(nn + 1) { }
    template<typename T> bit(int nn, T *b) : n(nn), a(nn + 1)
    {
        for (int i = 1; i <= n; i++) a[i] = b[i - 1];
        for (int i = 1; i <= n; i++) if (i + (i & -i) <= n) a[i + (i & -i)] += a[i];
    }
    void add(int x, typC y)
    {
        //cerr<<"add "<<x<<" by "<<y<<endl;
        ++x;
        x = clamp(x, 1, n + 1);
        if (x > n) return;
    }
}

```

```

    assert(1 <= x && x <= n);
    a[x] += y;
    while ((x += x & -x) <= n) a[x] += y;
}
typC sum(int x)
{
    //cerr<<"sum "<<x;
    ++x;
    x = clamp(x, 0, n);
    assert(0 <= x && x <= n);
    typC r = a[x];
    while (x ^= x & -x) r += a[x];
    //cerr<<"=" <<r<<endl;
    return r;
}
typC sum(int x, int y)
{
    return sum(y) - sum(x - 1);
}
int lower_bound(typC x)
{
    if (n == 0) return 0;
    int i = __lg(n), j = 0;
    for (; i >= 0; i--) if ((1 << i | j) <= n && a[1 << i | j] < x) j |= 1 << i, x -= a[j];
    return j + 1;
}
};
namespace DFS
{
    typedef long long ll;
    const int N = 1e5 + 5, M = 18;
    ll a[N];
    int st[M][N * 2], lg[N * 2];
    int dep[N], dfn[N], siz[N], f[N], szp[N], szn[N];
    vector<int> e[N], c[N], rg[N];
    bool ed[N];
    int n, ksiz, rt, mn, id;
    int lca(int u, int v)
    {
        u = dfn[u]; v = dfn[v];
        if (u > v) swap(u, v);
        int z = lg[v - u + 1];
        return dep[st[z][u]] < dep[st[z][v - (1 << z) + 1]] ? st[z][u] : st[z][v - (1 << z) + 1];
    }
    int dis(int u, int v)
    {
        return dep[u] + dep[v] - dep[lca(u, v)] * 2;
    }
    void findroot(int u)
    {
        ed[u] = siz[u] = 1;
        int mx = 0;
        for (int v : e[u]) if (!ed[v])
        {
            findroot(v);
            siz[u] += siz[v];
            mx = max(mx, siz[v]);
        }
    }
}

```

```

    }
    mx = max(mx, ksiz - siz[u]);
    ed[u] = 0;
    if (mn > mx) mn = mx, rt = u;
}
int dfs(int u)
{
    mn = 1e9;
    findroot(u);
    u = rt;
    ed[u] = 1;
    for (int v : e[u]) if (!ed[v] && siz[v] > siz[u]) siz[v] = ksiz - siz[u];
    for (int v : e[u]) if (!ed[v])
    {
        ksiz = siz[v];
        c[u].push_back(dfs(v));
        f[c[u].back()] = u;
    }
    return u;
}
void pre_dfs(int u)
{
    st[0][dfn[u] = ++id] = u;
    ed[u] = 1;
    for (int v : e[u]) if (!ed[v])
    {
        dep[v] = dep[u] + 1;
        pre_dfs(v);
        st[0][++id] = u;
    }
    ed[u] = 0;
}
void init(int _n)
{
    n = _n; id = 0;
    int i;
    for (int i = 1; i <= n; i++)
    {
        e[i].clear();
        a[i] = f[i] = ed[i] = 0;
    }
}
void new_dfs(int u)
{
    siz[u] = 1;
    for (int v : c[u]) new_dfs(v), siz[u] += siz[v];
    vector<int> &q = rg[u];
    q = {u};
    int ql = 0;
    while (ql < q.size())
    {
        int x = q[ql++];
        for (int v : c[x]) q.push_back(v);
    }
}
void fun()
{

```

```

    pre_dfs(1);
    int i, j;
    for (i = 2; i <= id; i++) lg[i] = lg[i >> 1] + 1;
    for (j = 0; j < lg[id]; j++)
    {
        int R = id - (2 << j) + 1;
        for (i = 1; i <= R; i++) st[j + 1][i] = dep[st[j][i]] < dep[st[j][i + (1 << j)]] ? st[j]
            [i] : st[j][i + (1 << j)];
    }
    ksiz = n;
    rt = dfs(1);
    new_dfs(rt);
}
vector<int> get(int u)
{
    vector<int> st = {u};
    while (u = f[u]) st.push_back(u);
    return st;
}
}
using DFS::init, DFS::fun, DFS::e, DFS::dis, DFS::rg, DFS::get;

```

圆环修改和单点查询:

```

int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int n, m, i;
    cin >> n >> m;
    vector<int> a(n + 1);
    for (i = 1; i <= n; i++) cin >> a[i];
    DFS::init(n);
    for (i = 1; i < n; i++)
    {
        int u, v;
        cin >> u >> v;
        ++u; ++v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    DFS::fun();
    vector<bit<ll>> inc(n + 1), dec(n + 1);
    for (i = 1; i <= n; i++)
    {
        int mx = 0;
        for (int v : rg[i]) cmax(mx, dis(i, v));
        inc[i] = bit<ll>(mx + 1);
        if (i != DFS::rt)
        {
            mx = 0;
            for (int v : rg[i]) cmax(mx, dis(DFS::f[i], v));
            dec[i] = bit<ll>(mx + 1);
        }
    }
    while (m--)
    {
        int op, u;

```

```

cin >> op >> u; ++u;
if (op == 0)
{
    int l, r, x;
    cin >> l >> r >> x;
    auto v = get(u);
    int m = v.size();
    for (i = 0; i < m; i++)
    {
        inc[v[i]].add(l - dis(v[i], u), x);
        inc[v[i]].add(r - dis(v[i], u), -x);
    }
    for (i = 0; i + 1 < m; i++)
    {
        dec[v[i]].add(l - dis(v[i + 1], u), x);
        dec[v[i]].add(r - dis(v[i + 1], u), -x);
    }
}
else
{
    ll res = a[u];
    auto v = get(u);
    int m = v.size();
    for (i = 0; i < m; i++) res += inc[v[i]].sum(dis(v[i], u));
    for (i = 0; i + 1 < m; i++) res -= dec[v[i]].sum(dis(v[i + 1], u));
    cout << res << '\n';
}
}
}

```

单点修改和圆环查询:

```

int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int n, m, i;
    cin >> n >> m;
    vector<int> a(n + 1);
    for (i = 1; i <= n; i++) cin >> a[i];
    DFS::init(n);
    for (i = 1; i < n; i++)
    {
        int u, v;
        cin >> u >> v;
        ++u; ++v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    DFS::fun();
    vector<bit<ll>> inc(n + 1), dec(n + 1);
    vector<ll> tmp(n + 1);
    for (i = 1; i <= n; i++)
    {
        int mx = 0;
        for (int v : rg[i])
        {
            int d = dis(i, v);

```

```

        cmax(mx, d);
        tmp[d] += a[v];
    }
    inc[i] = bit<ll>(mx + 1, tmp.data());
    fill_n(tmp.begin(), mx + 1, 0);
    if (i != DFS::rt)
    {
        mx = 0;
        for (int v : rg[i])
        {
            int d = dis(DFS::f[i], v);
            cmax(mx, d);
            tmp[d] += a[v];
        }
        dec[i] = bit<ll>(mx + 1, tmp.data());
        fill_n(tmp.begin(), mx + 1, 0);
    }
}
while (m--)
{
    int op, u;
    cin >> op >> u; ++u;
    if (op == 0)
    {
        int x;
        cin >> x;
        auto v = get(u);
        int m = v.size();
        for (i = 0; i < m; i++) inc[v[i]].add(dis(v[i], u), x);
        for (i = 0; i + 1 < m; i++) dec[v[i]].add(dis(v[i + 1], u), x);
    }
    else
    {
        int l, r;
        cin >> l >> r;
        --r;
        ll res = 0;
        auto v = get(u);
        int m = v.size();
        for (i = 0; i < m; i++) res += inc[v[i]].sum(l - dis(v[i], u), r - dis(v[i], u));
        for (i = 0; i + 1 < m; i++) res -= dec[v[i]].sum(l - dis(v[i + 1], u), r - dis(v[i + 1], u));
        cout << res << '\n';
    }
}
}

```

## 5.26 prufer 与树的互相转化

$O(n)$ ,  $O(n)$ 。

```

vector<int> edges_to_prufer(const vector<pair<int,int>> &eg)//[1,n], 定根为 n
{
    int n=eg.size()+1,i,j,k;
    vector<int> fir(n+1),nxt(n*2+1),e(n*2+1),rd(n+1);
    int cnt=0;
    for (auto [u,v]:eg)

```

```

{
    e[++cnt]=v;nxt[cnt]=fir[u];fir[u]=cnt;++rd[v];
    e[++cnt]=u;nxt[cnt]=fir[v];fir[v]=cnt;++rd[u];
}
for (i=1;i<=n;i++) if (rd[i]==1) break;
int u=i;
vector<int> r;r.reserve(n-2);
for (j=1;j<n-1;j++)
{
    for (k=fir[u],u=rd[u]=0;k=k=nxt[k]) if (rd[e[k]])
    {
        r.push_back(e[k]);
        if ((--rd[e[k]]==1)&&(e[k]<i)) u=e[k];
    }
    if (!u) { while (rd[i]!=1) ++i;u=i;}
}
return r;
}
vector<pair<int,int>> prufer_to_edges(const vector<int> &p)//[1,n], 定根为 n
{
    int n=p.size(),i,j,k;
    int m=n+3;
    vector<int> cs(m);
    for (i=0;i<n;i++) ++cs[p[i]];
    i=0;
    while (cs[++i]);
    int u=i,v;
    vector<pair<int,int>> r;
    r.reserve(n-2);
    for (j=0;j<n;j++)
    {
        cs[u]=1e9;
        r.push_back({u,v=p[j]});
        if ((--cs[v]==0)&&(v<i)) u=v;
        if (v!=u) {while (cs[i]) ++i;u=i;}
    }
    r.push_back({u,n+2});
    return r;
}

```

## 5.27 LCT

$O(n \log n)$ ,  $O(n)$ 。

makeroot 会变根, split 会把  $y$  变根, findroot 会把根变根, link 会把  $x, y$  变根 ( $y$  是新的), cut 会把  $x, y$  变根 ( $x$  是新的), 注意 swap 子节点可能要 pushup。

```

template<int N,class Q> struct LCT
{
    int f[N],c[N][2],siz[N],st[N];
    Q s[N],v[N];
#ifdef Rev
    Q rs[N];
#endif
    //heap g[N]; //虚子树
    bool lz[N];
    void init(int n)

```

```

{
    ++n;
    for (int i=0;i<n;i++)
    {
        f[i]=c[i][0]=c[i][1]=lz[i]=0;
        s[i]=v[i]=Q();
        #ifdef Rev
        rs[i]=Q();
        #endif
        siz[i]=!!i;
    }
}

void modify(int x,const Q &o)
{
    makeroot(x);
    v[x]=o;
    pushup(x);
}

bool nroot(int x) const
{
    return c[f[x]][0]==x||c[f[x]][1]==x;
}

void pushup(int x)
{
    int lc=c[x][0],rc=c[x][1];
    s[x]=v[x];siz[x]=1;
    #ifdef Rev
    rs[x]=v[x];
    #endif
    if (lc)
    {
        s[x]=s[lc]+s[x];
        siz[x]+=siz[lc];
        #ifdef Rev
        rs[x]=rs[x]+rs[lc];
        #endif
    }
    if (rc)
    {
        s[x]=s[x]+s[rc];
        siz[x]+=siz[rc];
        #ifdef Rev
        rs[x]=rs[rc]+rs[x];
        #endif
    }
}

void swp(int x)
{
    swap(c[x][0],c[x][1]);
    #ifdef Rev
    swap(s[x],rs[x]);
    #endif
    lz[x]^=1;
}

void pushdown(int x)
{
    int lc=c[x][0],rc=c[x][1];

```



```

    if (lz[x])
    {
        if (lc) swp(lc);
        if (rc) swp(rc);
        lz[x]=0;
    }
}

void zigzag(int x)
{
    int y=f[x],z=f[y],typ=(c[y][0]==x);
    if (nroot(y)) c[z][c[z][1]==y]=x;
    f[x]=z;f[y]=x;
    if (c[x][typ]) f[c[x][typ]]=y;
    c[y][typ^1]=c[x][typ];c[x][typ]=y;
    pushup(y);
}

void splay(int x)
{
    int y,tp=0;
    st[tp+1]=y=x;
    while (nroot(y)) st[++tp]=y=f[y];
    while (tp) pushdown(st[tp--]);
    for (;nroot(x);zigzag(x)) if (!nroot(f[x])) continue; else zigzag((c[f[x]][0]==x)^(c[f[f[x]
    ]][0]==f[x]) ? x:f[x]);
    pushup(x);
}

void access(int x)
{
    for (int y=0;x;x=f[y=x])
    {
        splay(x);
        //g[x].ins(s[c[x][1]]);g[x].del(s[y]);虚子树变化
        c[x][1]=y;pushup(x);
    }
}

int findroot(int x)
{
    access(x);splay(x);pushdown(x);
    while (c[x][0]) pushdown(x=c[x][0]);
    splay(x);
    return x;
}

void split(int x,int y)//x 为树新根, y 为 splay 新根
{
    makeroot(x);
    access(y);
    splay(y);
}

void makeroot(int x)
{
    access(x);splay(x);
    swp(x);
}

void link(int x,int y)//y 为新根
{
    makeroot(x);
    if (x!=findroot(y))//可能已经连通

```

```

    {
        makeroot(y); f[x]=y; //虚子树变化
    }
}
void cut(int x,int y)
{
    makeroot(x);
    if (x==findroot(y))//可能本不连通
    {
        pushdown(x);
        if (c[x][1]==y&&!c[y][0]&&!c[y][1])//可能连通但无边
        {
            c[x][1]=f[y]=0; //可能需要修改
            pushup(x);
        }
    }
}
};

```

## 5.28 LCT（重构，代码为动态割边割点）

```

#include "bits/stdc++.h"
using namespace std;
template<int N,class info,class tag> struct LCT
{
    int f[N],c[N][2];
    info s[N],v[N];
#ifdef Rev
    info rs[N];
#endif
    tag tg[N];
    bool rev[N],lz[N];
    void init(int n,info *a)
    {
        for (int i=0; i<=n; i++)
        {
            rev[i]=lz[i]=0;
            f[i]=c[i][0]=c[i][1]=0;
            s[i]=v[i]=a[i];
#ifdef Rev
            rs[i]=a[i];
#endif
        }
    }
    bool nroot(int x) const
    {
        return c[f[x]][0]==x || c[f[x]][1]==x;
    }
    void pushup(int x)
    {
        int lc=c[x][0],rc=c[x][1];
        s[x]=v[x];
#ifdef Rev
        rs[x]=v[x];
#endif
        if (lc)

```

```

    {
        s[x]=s[lc]+s[x];
#ifdef Rev
        rs[x]=rs[x]+rs[lc];
#endif
    }
    if (rc)
    {
        s[x]=s[x]+s[rc];
#ifdef Rev
        rs[x]=rs[rc]+rs[x];
#endif
    }
}
void swp(int x)
{
    swap(c[x][0],c[x][1]);
#ifdef Rev
    swap(s[x],rs[x]);
#endif
    rev[x]^=1;
}
void pushdown(int x)
{
    if (rev[x])
    {
        for (int y:c[x]) if (y) swp(y);
        rev[x]=0;
    }
    if (lz[x])
    {
        for (int y:c[x]) if (y)
        {
            if (lz[y]) tg[y]+=tg[x]; else tg[y]=tg[x],lz[y]=1;
            s[y]+=tg[x];
        }
        lz[x]=0;
    }
}
void zigzag(int x)
{
    int y=f[x],z=f[y],typ=(c[y][0]==x);
    if (nroot(y)) c[z][c[z][1]==y]=x;
    f[x]=z; f[y]=x;
    if (c[x][typ]) f[c[x][typ]]=y;
    c[y][typ^1]=c[x][typ]; c[x][typ]=y;
    pushup(y);
}
void splay(int x)
{
    static int st[N];
    int y,tp;
    st[tp=1]=y=x;
    while (nroot(y)) st[++tp]=y=f[y];
    while (tp) pushdown(st[tp--]);
    for (; nroot(x); zigzag(x)) if (nroot(y=f[x])) zigzag((c[y][0]==x)^(c[f[y]][0]==y)?x:f[x])
;
}

```

```

    pushup(x);
}
int access(int x)
{
    int y=0;
    for (; x=x=f[y=x]) splay(x),c[x][1]=y,pushup(x);
    return y;
}
int findroot(int x)//splay 根为树根, splay 维护树根到 x 的链
{
    access(x); splay(x); pushdown(x);
    while (c[x][0]) pushdown(x=c[x][0]);
    splay(x); return x;
}
void split(int x,int y)//x 为树新根, y 为 splay 新根
{ makeroot(x); access(y); splay(y); }
void makeroot(int x)//x 为树、splay 新根
{ access(x); splay(x); swp(x); }
void modify(int x,const info &o)
{ makeroot(x); v[x]=o; pushup(x); }
void modify(int x,int y,const tag &o)
{
    split(x,y); s[y]+=o;
    if (lz[y]) tg[y]+=o; else tg[y]=o,lz[y]=1;
}
info ask(int x,int y) { split(x,y); return s[y]; }
bool connected(int x,int y)//注意会改变形态
{ makeroot(x); return findroot(y)==x; }
void link(int x,int y)//y 为新根
{ if (!connected(x,y)) makeroot(f[x]=y); }
void cut(int x,int y)
{
    if (connected(x,y))//可能本不连通
    {
        pushdown(x);
        if (c[x][1]==y&&!c[y][0]&&!c[y][1])//可能连通但无边
        {
            c[x][1]=f[y]=0;
            pushup(x);
        }
    }
}
int lca(int x,int y) { access(x); return access(y); }
vector<int> res;
void dfs(int x)
{
    if (!x) return;
    pushdown(x);
    dfs(c[x][0]); res.push_back(x); dfs(c[x][1]);
}
vector<int> get_path(int x,int y)
{
    res.clear(); split(x,y); dfs(y);
    if (res[0]!=x) return {};
    return res;
}
};

```

```

const int N=2e5+5,M=4e5+5;
struct Q
{
    void operator+=(const Q &o) const {}
};
void operator+=(int &x,const Q &o) { x=0; }
LCT<N,int,Q> s;
LCT<M,int,Q> t;
int a[N],b[M];
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int n,m,i,r=0;
    cin>>n>>m;
    fill_n(a+n+1,n,1);
    fill_n(b+1,n,1);
    s.init(n*2,a);
    t.init(n+m,b);
    int bs=n,ds=n;
    while (m--)
    {
        int op,u,v;
        cin>>op>>u>>v;
        u^=r; v^=r;
        // dbg(op,u,v);
        if (u<1||u>n||v<1||v>n) return 0;
        if (op==1)
        {
            if (s.connected(u,v))
            {
                s.modify(u,v,{});
                auto c=t.get_path(u,v);
                for (i=1; i<c.size(); i++) t.cut(c[i-1],c[i]);
                ++ds;
                for (int x:c) t.link(ds,x);
            }
            else
            {
                s.link(++bs,u);
                s.link(bs,v);
                t.link(++ds,u);
                t.link(ds,v);
            }
        }
        else
        {
            if (!s.connected(u,v))
            {
                cout<<"-1\n";
                continue;
            }
            r=op==2?s.ask(u,v):t.ask(u,v);
            cout<<r<<"\n";
        }
    }
}

```

## 5.29 带子树的 LCT

$O(n \log n)$ ,  $O(n)$ 。

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
template<int N> struct LCT
{
    ll s[N], v[N], sg[N];
    int f[N], c[N][2], siz[N], st[N];
    //heap g[N]; //虚子树
    bool lz[N];
    void init(int n)
    {
        memset(f, 0, n+1<<2);
        memset(c, 0, n+1<<3);
        memset(s, 0, n+1<<3);
        memset(v, 0, n+1<<3);
        memset(lz, 0, n+1);
    }
    bool nroot(int x)
    {
        return c[f[x]][0]==x || c[f[x]][1]==x;
    }
    void pushup(int x)
    {
        s[x]=s[c[x][0]]+s[c[x][1]]+v[x]+sg[x];
        siz[x]=siz[c[x][0]]+siz[c[x][1]]+1;
    }
    void pushdown(int x)
    {
        if (lz[x])
        {
            swap(c[c[x][0]][0], c[c[x][0]][1]);
            swap(c[c[x][1]][0], c[c[x][1]][1]);
            lz[c[x][0]]^=1;
            lz[c[x][1]]^=1;
            lz[x]=0;
        }
    }
    void zigzag(int x)
    {
        int y=f[x], z=f[y], typ=(c[y][0]==x);
        if (nroot(y)) c[z][c[z][1]==y]=x;
        f[x]=z; f[y]=x;
        if (c[x][typ]) f[c[x][typ]]=y;
        c[y][typ^1]=c[x][typ]; c[x][typ]=y;
        pushup(y);
    }
    void splay(int x)
    {
        int y, tp=0;
        st[tp]=y=x;
        while (nroot(y)) st[++tp]=y=f[y];
        while (tp) pushdown(st[tp--]);
        for (; nroot(x); zigzag(x)) if (!nroot(f[x])) continue; else zigzag((c[f[x]][0]==x)^(c[f[f[x]
        ]][0]==f[x]) ? x:f[x]);
    }
};
```

```

    pushup(x);
}
void access(int x)
{
    for (int y=0;x;x=f[y=x])
    {
        splay(x);sg[x]-=s[y];s[x]-=s[y];
        sg[x]+=s[c[x][1]];s[x]+=s[c[x][1]];
        //g[x].ins(s[c[x][1]]);g[x].del(s[y]);虚子树变化
        c[x][1]=y;pushup(x);
    }
}
int findroot(int x)
{
    access(x);splay(x);pushdown(x);
    while (c[x][0]) pushdown(x=c[x][0]);
    splay(x);
    return x;
}
void split(int x,int y)
{
    makeroot(x);
    access(y);
    splay(y);
}
void makeroot(int x)
{
    access(x);splay(x);lz[x]^=1;swap(c[x][0],c[x][1]); pushup(x);
}
void link(int x,int y)
{
    makeroot(x);
    if (x!=findroot(y))//可能已经连通
    {
        makeroot(y);f[x]=y;//虚子树变化
        sg[y]+=s[x];s[y]+=s[x];
    }
}
void cut(int x,int y)
{
    makeroot(x);
    if (x==findroot(y))//可能本不连通
    {
        pushdown(x);
        if (c[x][1]==y&&!c[y][0]&&!c[y][1])//可能连通但无边
        {
            c[x][1]=f[y]=0;//可能需要修改
            pushup(x);
        }
    }
}
};
const int N=2e5+2;
LCT<N> s;
int n,q,i,x,y,z,w;
int main()
{

```

```

read(n);read(q);s.init(n);
for (i=1;i<=n;i++) read(x),s.s[i]=s.v[i]=x;
for (i=1;i<n;i++)
{
    read(x);read(y);++x;++y;
    s.link(x,y);
}
while (q--)
{
    read(x);read(y);read(z);++y;
    if (x==0)
    {
        read(x);read(w);
        ++z;++x;++w;
        s.cut(y,z);s.link(x,w);
        continue;
    }
    if (x==1)
    {
        s.split(y,y);
        s.s[y]=(s.v[y]+=z);
    }
    else
    {
        ++z;
        s.split(y,z);
        printf("%lld\n",s.s[y]);
    }
}
}

```

### 5.30 轻重链剖分/DFS 序 LCA

首先 `init(n)`，然后正常存边  $([1, n])$ ，然后 `fun(root)`。  
`get_path` 会返回这条路径上的 `dfn` 区间。

```

namespace HLD
{
    const int N = 5e5 + 2;
    vector<int> e[N];
    int dfn[N], nfd[N], dep[N], f[N], siz[N], hc[N], top[N];
    int id, n;
    void dfs1(int u)
    {
        siz[u] = 1;
        for (int v : e[u]) if (v != f[u])
        {
            dep[v] = dep[f[v] = u] + 1;
            dfs1(v);
            siz[u] += siz[v];
            if (siz[v] > siz[hc[u]]) hc[u] = v;
        }
    }
    void dfs2(int u)
    {
        dfn[u] = ++id;
    }
}

```



```

    nfd[id] = u;
    if (hc[u])
    {
        top[hc[u]] = top[u];
        dfs2(hc[u]);
        for (int v : e[u]) if (v != hc[u] && v != f[u]) dfs2(top[v] = v);
    }
}

int lca(int u, int v)
{
    while (top[u] != top[v])
    {
        if (dep[top[u]] < dep[top[v]]) swap(u, v);
        u = f[top[u]];
    }
    if (dep[u] > dep[v]) swap(u, v);
    return u;
}

int dis(int u, int v)
{
    return dep[u] + dep[v] - (dep[lca(u, v)] << 1);
}

void init(int _n)
{
    n = _n;
    for (int i = 1; i <= n; i++)
    {
        e[i].clear();
        f[i] = hc[i] = 0;
    }
    id = 0;
}

void fun(int root)
{
    dep[root] = 1; dfs1(root); dfs2(top[root] = root);
}

vector<pair<int, int>> get_path(int u, int v) //u->v, 注意可能出现 [r>1] (表示反过来走)
{
    //cerr<<"path from "<<u<<" to "<<v<<": ";
    vector<pair<int, int>> v1, v2;
    while (top[u] != top[v])
    {
        if (dep[top[u]] > dep[top[v]]) v1.push_back({dfn[u], dfn[top[u]]}), u = f[top[u]];
        else v2.push_back({dfn[top[v]], dfn[v]}), v = f[top[v]];
    }
    v1.reserve(v1.size() + v2.size() + 1);
    v1.push_back({dfn[u], dfn[v]});
    reverse(v2.begin(), v2.end());
    for (auto v : v2) v1.push_back(v);
    //for (auto [x,y]:v1) cerr<<"["<<x<<','<<y<<"] ";cerr<<endl;
    return v1;
}

}

using HLD::e, HLD::dfn, HLD::nfd, HLD::dep, HLD::f, HLD::siz, HLD::get_path;
using HLD::init; //5e5
namespace LCA
{

```

```

using HLD::N, HLD::n;
int st[__lg(N) + 1][N];
int cmp(const int &x, const int &y) { return dep[x] < dep[y] ? x : y; }
void fun(int rt)
{
    HLD::fun(rt);
    assert(f[rt] == 0);
    for (int i = 1; i <= n; i++) st[0][dfn[i] - 1] = f[i];
    for (int j = 0; j < __lg(n); j++)
        for (int i = 1, k = n - (1 << j + 1); i <= k; i++) st[j + 1][i] = cmp(st[j][i], st[j][i
            + (1 << j)]);
}
int lca(int u, int v)
{
    if (u == v) return u;
    u = dfn[u], v = dfn[v];
    if (u > v) swap(u, v);
    int g = __lg(v - u);
    return cmp(st[g][u], st[g][v - (1 << g)]);
}
int dis(int u, int v)
{
    return dep[u] + dep[v] - (dep[lca(u, v)] << 1);
}
}
using LCA::lca, LCA::fun, LCA::dis;

```

### 5.31 换根树剖

本质是对普通树剖在换根后的子树进行分类讨论。

设预处理的根是  $u$ ，当前根是  $v$ ，那么  $w$  的子树如下：

1.  $w = v$ ，dfn 区间为  $[1, n]$ 。
2.  $w$  在  $u, v$  之间，dfn 区间为  $[1, n]$  去掉  $w$  前往  $v$  方向的子树。找到这个子树的方法见 find 函数。
3. 其余情况，dfn 区间和原来一致。

$O(n + q \log n)$ ， $O(n)$ 。

```

void dfs1(int x)
{
    int i;
    siz[x]=1;
    for (i=fir[x];i;nxt[i]) if (lj[i]!=f[x])
    {
        dep[lj[i]]=dep[f[lj[i]]=x]+1;
        dfs1(lj[i]);
        siz[x]+=siz[lj[i]];
        if (siz[hc[x]]<siz[lj[i]]) hc[x]=lj[i];
    }
}
void dfs2(int x)
{
    nfd[dfn[x]=++bs]=x;
}

```

```

if (hc[x])
{
    int i;
    top[hc[x]]=top[x];
    dfs2(hc[x]);
    for (i=fir[x];i;i=nxt[i]) if ((lj[i]!=f[x])&&(lj[i]!=hc[x])) dfs2(top[lj[i]]=lj[i]);
}
}

void mdf(int xx,int yy)
{
    while (top[xx]!=top[yy])
    {
        if (dep[top[xx]]<dep[top[yy]]) swap(xx,yy);
        z=dfn[top[xx]];y=dfn[xx];xdsmdf(1);
        xx=f[top[xx]];
    }
    if (dep[xx]>dep[yy]) swap(xx,yy);
    z=dfn[xx];y=dfn[yy];
    xdsmdf(1);
}

int find(int x,int y)//找到 y 向 x 的子树
{
    while ((top[x]!=top[y])&&(f[top[x]]!=y)) x=f[top[x]];
    if (top[x]==top[y]) return hc[y];
    return top[x];
}

int main()
{
    read(n);read(m);
    for (i=2;i<=n;i++)
    {
        read(x);read(y);
        add();
    }bs=0;
    for (i=1;i<=n;i++) read(v[i]);
    dfs1(dep[1]=1);dfs2(top[1]=1);
    read(rt);r[l[1]=1]=n;build(1);
    while (m--)
    {
        read(x);read(y);
        if (x==1) {rt=y;continue;}
        if (x==2)
        {
            read(x);read(dt);
            mdf(x,y);continue;
        }
        x=y;dt=inf;
        if (x==rt)
        {
            z=1;y=n;sum(1);
        }
        else if ((dfn[x]<dfn[rt])&&(dfn[x]+siz[x]>dfn[rt]))
        {
            c=find(rt,x);
            z=1;y=dfn[c]-1;if (z<=y) sum(1);
            z=dfn[c]+siz[c];y=n;if (z<=y) sum(1);
        }
    }
}

```

```

        else
        {
            z=dfn[x];y=z+siz[x]-1;sum(1);
        }
        printf("%d\n",dt);
    }
}

```

### 5.32 树上启发式合并, DSU on tree

一种过时的、基于两次 dfs 的写法, 在复杂度要求不严时不如直接存储 set。  
流程:

1. dfs 轻子树计算答案, 并清空全局统计信息。
2. dfs 重子树统计答案和全局信息。
3. dfs 轻子树统计全局信息。

```

void dfs1(int x)
{
    siz[x]=zdep[x]=1;
    int i;
    for (i=fir[x];i;i=nxt[i]) if (lj[i]!=f[x])
    {
        dep[lj[i]]=dep[f[lj[i]]]=x+1;
        dfs1(lj[i]);
        siz[x]+=siz[lj[i]];
        if (siz[hc[x]]<siz[lj[i]]) hc[x]=lj[i];
        zdep[x]=max(zdep[x],zdep[lj[i]]+1);
    }
}

void cal(int x)
{
    int i;
    dl[tou=wei=1]=x;
    while (tou<=wei)
    {
        ++dp[dep[x=dl[tou++]]];
        if ((dp[dep[x]]>dp[zd])||(dp[dep[x]]==dp[zd]&&(dep[x]<zd)) zd=dep[x];
        for (i=fir[x];i;i=nxt[i]) if (lj[i]!=f[x]) dl[++wei]=lj[i];
    }
}

void dfs2(int x)
{
    if (!hc[x])
    {
        if (++dp[dep[x]]>dp[zd]) zd=dep[x];
        return;
    }
    int i;
    for (i=fir[x];i;i=nxt[i]) if ((lj[i]!=f[x])&&(lj[i]!=hc[x]))
    {
        dfs2(lj[i]);
        memset(dp+dep[lj[i]],0,zdep[lj[i]]<<2);
    }
}

```

```

}
dfs2(hc[x]);
dp[dep[x]]=1;
if (dp[zd]<=1) zd=dep[x];
for (i=fir[x];i;i=nxt[i]) if ((lj[i]!=f[x])&&(lj[i]!=hc[x])) cal(lj[i]);
ans[x]=zd-dep[x];
}

```

### 5.33 长链剖分 ( $k$ 级祖先)

$O(n \log n + q)$ ,  $O(n)$ 。

```

void dfs1(int x)
{
    int i;
    for (i=1;i<=er[dep[x]-1];i++) f[x][i]=f[f[x][i-1]][i-1];md[x]=dep[x];
    for (i=fir[x];i;i=nxt[i]) {dep[lj[i]]=dep[x]+1;dfs1(lj[i]);if (md[lj[i]]>md[dc[x]]) dc[x]=lj[i];}
    if (dc[x]) md[x]=md[dc[x]];
}
void dfs2(int x)
{
    int i;
    if (dc[x])
    {
        top[dc[x]]=top[x];
        dfs2(dc[x]);
        for (i=fir[x];i;i=nxt[i]) if (lj[i]!=dc[x]) dfs2(top[lj[i]]=lj[i]);
    }
    if (x==top[x])
    {
        c=md[x]-dep[x];y=x;up[x].push_back(x);down[x].push_back(x);
        for (i=1;(i<=c)&&(y=f[y][0]);i++) up[x].push_back(y);y=x;
        for (i=1;i<=c;i++) down[x].push_back(y=dc[y]);
    }
}
int main()
{
    int n,q,ans=0,x,y,c,i;
    ll ta=0;
    read(n);read(q);read(s);
    for (i=1;i<=n;i++) {read(f[i][0]);if (f[i][0]==0) rt=i; else add(f[i][0],i);}
    for (i=2;i<=n;i++) er[i]=er[i>>1]+1;dep[rt]=1;
    dfs1(rt);dfs2(top[rt]=rt);
    for (i=1;i<=q;i++)
    {
        x=(get(s)^ans)%n+1;y=(get(s)^ans)%dep[x];
        //此时计算 x 的 y 级祖先。结果在 ans 中。
        if (y==0) {ans=x;ta^=(ll)i*ans;continue;}
        c=dep[x]-y;x=top[f[x][er[y]]];
        if (dep[x]>c) ans=up[x][dep[x]-c]; else ans=down[x][c-dep[x]];
        ta^=(ll)i*ans;
    }
    printf("%lld",ta);
}

```

### 5.34 长链剖分 (dp 合并)

一种常见的实现方法是用指针指向同一片数组区域,使得从链头到链尾正好指向连续的一段数组,就不需要计算偏移量了。

$O(n)$ ,  $O(n)$ 。

```
void dfs1(int x)
{
    top[x]=1;
    for (int i=fir[x];i;i=nxt[i]) if (!top[lj[i]])
    {
        dfs1(lj[i]);
        if (len[lj[i]]>len[hc[x]]) hc[x]=lj[i];
    }
    len[x]=len[hc[x]]+1;top[hc[x]]=0;
}
void dfs2(int x)
{
    *f[x]=1;gs[x]=1;
    if (!hc[x]) return;
    ed[x]=1;f[hc[x]]=f[x]+1;
    for (int i=fir[x];i;i=nxt[i]) if (!ed[lj[i]]) dfs2(lj[i]);
    ans[x]=ans[hc[x]]+1;gs[x]=gs[hc[x]];
    if (gs[x]==1) ans[x]=0;
    for (int i=fir[x];i;i=nxt[i]) if ((!ed[lj[i]])&&(lj[i]!=hc[x]))
    {
        int v=lj[i],*p;
        for (int j=0;j<len[v];j++)
        {
            *(p=f[x]+j+1)+=*(f[v]+j);
            if (j+1==ans[x]) {gs[x]=*p;continue;}
            if ((*p>gs[x])||(*p==gs[x])&&(j+1<ans[x])) {gs[x]=*p;ans[x]=j+1;}
        }
    }
    gs[x]=*(f[x]+ans[x]);
    ed[x]=0;
}
```

### 5.35 动态 dp (全局平衡二叉树)

意义不大。

$O((n+q)\log n)$ ,  $O(n)$ 。

```
#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <fstream>
using namespace std;
const int N=1e6+2,M=6e7+2,INF=-1e9;
struct matrix
{
    int a[2][2];
};
matrix s[N],js;
matrix operator *(matrix x,matrix y)
{
    js.a[0][0]=max(x.a[0][0]+y.a[0][0],x.a[0][1]+y.a[1][0]);
```

```

js.a[0][1]=max(x.a[0][0]+y.a[0][1],x.a[0][1]+y.a[1][1]);
js.a[1][0]=max(x.a[1][0]+y.a[0][0],x.a[1][1]+y.a[1][0]);
js.a[1][1]=max(x.a[1][0]+y.a[0][1],x.a[1][1]+y.a[1][1]);
return js;
}
int st[N],c[N][2],hc[N],lj[N<<1],nxt[N<<1],fir[N],siz[N],v[N],g[N][2],fa[N],f[N],val[N];
int n,m,i,j,x,y,z,ntp,stp,tp,fh,bs,rt,aaa,la;
char dr[M+5],sc[M];
void pushup(int x)
{
    s[x].a[0][0]=s[x].a[0][1]=g[x][0];
    s[x].a[1][0]=g[x][1];s[x].a[1][1]=INF;
    if (c[x][0]) s[x]=s[c[x][0]]*s[x];
    if (c[x][1]) s[x]=s[x]*s[c[x][1]];
}
void add(int x,int y)
{
    lj[++bs]=y;
    nxt[bs]=fir[x];
    fir[x]=bs;
    lj[++bs]=x;
    nxt[bs]=fir[y];
    fir[y]=bs;
}
bool nroot(int x)
{
    return ((c[f[x]][0]==x)||(c[f[x]][1]==x));
}
void dfs1(int x)
{
    siz[x]=1;
    int i;
    for (i=fir[x];i;i=nxt[i]) if (lj[i]!=fa[x])
    {
        fa[lj[i]]=x;
        dfs1(lj[i]);
        siz[x]+=siz[lj[i]];
        if (siz[hc[x]]<siz[lj[i]]) hc[x]=lj[i];
    }
}
int build(int l,int r)
{
    if (l>r) return 0;
    int i,tot=0,upn=0;
    for (i=1;i<=r;i++) tot+=val[i];tot>>=1;
    for (i=1;i<=r;i++)
    {
        upn+=val[i];
        if (upn>=tot)
        {
            f[c[st[i]][0]]=build(l,i-1)=st[i];
            f[c[st[i]][1]]=build(i+1,r)=st[i];
            pushup(st[i]);
            ++aaa;
            return st[i];
        }
    }
}

```

```

}
int dfs2(int x)
{
    int i,j;
    for (i=x;i;i=hc[i]) for (j=fir[i];j;j=nxt[j]) if ((lj[j]!=fa[i])&&(lj[j]!=hc[i]))
    {
        f[y=dfs2(lj[j])]=i;
        g[i][0]+=max(s[y].a[0][0],s[y].a[1][0]);
        g[i][1]+=s[y].a[0][0];
    }
    tp=0;
    for (i=x;i;i=hc[i]) st[++tp]=i;
    for (i=1;i<tp;i++) val[i]=siz[st[i]]-siz[st[i+1]];
    val[tp]=siz[st[tp]];
    return build(1,tp);
}
void change(int x,int y)
{
    g[x][1]+=y-v[x];v[x]=y;
    while (f[x])
    {
        if (nroot(x)) pushup(x);
        else
        {
            g[f[x]][0]-=max(s[x].a[0][0],s[x].a[1][0]);
            g[f[x]][1]-=s[x].a[0][0];
            pushup(x);
            g[f[x]][0]+=max(s[x].a[0][0],s[x].a[1][0]);
            g[f[x]][1]+=s[x].a[0][0];
        }
        x=f[x];
    }
    pushup(x);
}
int main()
{
    scanf("%d%d",&n,&m);
    fread(dr+1,1,min(M,n*20+m*20),stdin);
    for (i=1;i<=n;i++)
    {
        read(g[i][1]);
        v[i]=g[i][1];
    }
    for (i=1;i<n;i++)
    {
        read(x);read(y);
        add(x,y);
    }
    dfs1(1);
    rt=dfs2(1);tp=0;
    while (m--)
    {
        read(x);read(y);
        change(x^1a,y);
        x=1a=max(s[rt].a[0][0],s[rt].a[1][0]);
        while (x)
        {

```



```

        st[++tp]=x%10;
        x/=10;
    }
    while (tp) sc[++stp]=st[tp--]|48;
    sc[++stp]=10;
}
fwrite(sc+1,1,stp,stdout);
}

```

### 5.36 全局平衡二叉树（修改版）

$O((n+q)\log n)$ ,  $O(n)$ 。

```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int,int> pa;
const int N=1e6+2,M=1e6+2;
ll ans;
pa w[N];
int c[N][2],f[N],fa[N],v[N],s[N],lz[N],lj[M],nxt[M],siz[N],hc[N],fir[N],st[N];
int a[N],top[N];
int n,i,x,y,z,bs,tp,rt;
void add()
{
    lj[++bs]=y;nxt[bs]=fir[x];fir[x]=bs;
    lj[++bs]=x;nxt[bs]=fir[y];fir[y]=bs;
}
void pushup(int &x)
{
    s[x]=min(v[x],min(s[c[x][0]],s[c[x][1]]));
}
void pushdown(int &x)
{
    if (lz[x]<0)
    {
        int cc=c[x][0];
        if (cc)
        {
            lz[cc]+=lz[x];s[cc]+=lz[x];v[cc]+=lz[x];
        }
        cc=c[x][1];
        if (cc)
        {
            v[cc]+=lz[x];lz[cc]+=lz[x];s[cc]+=lz[x];
        }
        lz[x]=0;
        return;
    }
}
bool nroot(int &x) {return c[f[x]][0]==x||c[f[x]][1]==x;}
bool cmp(pa &o,pa &p) {return o>p;}
void dfs1(int x)
{
    siz[x]=1;
    for (int i=fir[x];i;i=nxt[i]) if (lj[i]!=fa[x])
    {
        fa[lj[i]]=x;dfs1(lj[i]);siz[x]+=siz[lj[i]];
    }
}

```

```

        if (siz[hc[x]]<siz[lj[i]]) hc[x]=lj[i];
    }
}
int build(int l,int r)
{
    if (l>r) return 0;
    if (l==r)
    {
        l=st[l];s[l]=v[l]=siz[l]>>1;
        return l;
    }
    int x=lower_bound(a+l,a+r+1,a[l]+a[r]>>1)-a,y=st[x];
    c[y][0]=build(l,x-1);
    c[y][1]=build(x+1,r);
    v[y]=siz[y]>>1;
    if (c[y][0]) f[c[y][0]]=y;
    if (c[y][1]) f[c[y][1]]=y;
    pushup(y);
    return y;
}
void dfs2(int x)
{
    if (!hc[x]) return;
    int i;
    top[hc[x]]=top[x];
    if (top[x]==x)
    {
        st[tp=1]=x;
        for (i=hc[x];i;i=hc[i]) st[++tp]=i;
        for (i=1;i<=tp;i++) a[i]=siz[st[i]]-siz[hc[st[i]]]+a[i-1];
        f[build(1,tp)]=fa[x];
    }
    dfs2(hc[x]);
    for (i=fir[x];i;i=nxt[i]) if (lj[i]!=fa[x]&&lj[i]!=hc[x]) dfs2(top[lj[i]]=lj[i]);
}
void mdf(int x)
{
    int y=x;
    st[tp=1]=x;
    while (y=f[y]) st[++tp]=y;y=x;
    while (tp) pushdown(st[tp--]);
    while (x)
    {
        --v[x];--lz[c[x][0]];--v[c[x][0]];--s[c[x][0]];
        while (c[f[x]][0]==x) x=f[x];x=f[x];
    }
    pushup(y);
    while (y=f[y]) pushup(y);
}
int ask(int x)
{
    int y=x;
    st[tp=1]=x;
    while (y=f[y]) st[++tp]=y;
    while (tp) pushdown(st[tp--]);
    int r=v[x];
    while (x)

```

```

{
    r=min(r,min(v[x],s[c[x][0]]));
    while (c[f[x]][0]==x) x=f[x];x=f[x];
}
return r;
}
signed main()
{
    read(n);s[0]=1e9;
    for (i=1;i<=n;i++) read(w[w[i].second=i].first);
    for (i=1;i<=n;i++) read(x),read(y),add();
    sort(w+1,w+n+1,cmp);dfs1(1);dfs2(top[1]=1);rt=1;while (f[rt]) rt=f[rt];
    for (i=1;i<=n&&v[rt];i++) if (ask(w[i].second)) mdf(w[i].second),ans+=w[i].first;
    printf("%lld",ans);
}

```

### 5.37 虚树

传入点标号列表，返回虚树边表。自动认为 1 是根，标号从 1 开始。

需要注意的是：在清空的时候需要同时考虑点列表和边表，都清空一下。

你需要提供的是：dep, lca, dfn。

$O(n + \sum k \log n)$ ,  $O(n)$ 。

```

vector<pair<int, int>> get_tree(vector<int> a)
{
    vector<pair<int, int>> edges;
    sort(all(a), [&](int u, int v) { return dfn[u]<dfn[v]; });
    vector<int> st(a.size()+2);
    int tp=0;
    auto ins=[&](int u)
    {
        if (tp==0)
        {
            st[tp=1]=u;
            return;
        }
        int v=lca(st[tp], u);
        while (tp>1&&dep[v]<dep[st[tp-1]])
        {
            edges.emplace_back(st[tp-1], st[tp]);
            --tp;
        }
        if (dep[v]<dep[st[tp]]) edges.emplace_back(v, st[tp--]);
        if (!tp||st[tp]!=v) st[++tp]=v;
        st[++tp]=u;
    };
    if (a[0]!=1) st[tp=1]=1; //先行添加根节点
    for (int u:a) ins(u);
    if (tp) while (--tp) edges.emplace_back(st[tp], st[tp+1]); //回溯
    return edges;
}

```

### 5.38 圆方树

题意：求仙人掌上两点最短路。

$O(n+m)$ ,  $O(n+m)$ 。

```
#include <bits/stdc++.h>
using namespace std;
#if !defined(ONLINE_JUDGE)&&defined(LOCAL)
#include "my_header\debug.h"
#else
#define dbg(...); 1;
#endif
typedef unsigned int ui;
typedef long long ll;
#define all(x) (x).begin(),(x).end()
const int N=3e4+2,M=3e4+2;//M 包括方点
struct P
{
    int v,w,id;
    P(int a,int b,int c):v(a),w(b),id(c){}
};
struct Q
{
    int v,w;
    Q(int a,int b):v(a),w(b){}
};
vector<P> e[N];
vector<Q> fe[M];
int dfn[M],low[N],st[N],len[M],top[M],siz[M],hc[M],dep[M],f[M],rb[N];
bool ed[M];//ed,dfn,loop,sum,fe,hc,tp,id,cnt,dep[1] 需初始化(注意倍率), ed 大小为边数
int tp,id,cnt,n;
void dfs1(int u)
{
    dfn[u]=low[u]=++id;
    st[++tp]=u;
    for (auto [v,w,id]:e[u]) if (!ed[id])
    {
        if (dfn[v]) low[u]=min(low[u],dfn[v]),rb[v]=w; else
        {
            ed[id]=1;
            dfs1(v);
            if (dfn[u]>low[v]) low[u]=min(low[u],low[v]),rb[v]=w; else
            {
                int ntp=tp;
                while (st[ntp]!=v) --ntp;
                if (ntp==tp)//圆圆边
                {
                    --tp;
                    fe[u].emplace_back(v,w);
                    f[v]=u;
                    continue;
                }
                ++cnt;f[cnt]=u;
                for (int i=ntp;i<=tp;i++) f[st[i]]=cnt;
                len[st[ntp]]=w;
                for (int i=ntp+1;i<=tp;i++) len[st[i]]=len[st[i-1]]+rb[st[i]];
                len[cnt]=len[st[tp]]+rb[u];
                fe[u].emplace_back(cnt,0);
                for (int i=ntp;i<=tp;i++) fe[cnt].emplace_back(st[i],min(len[st[i]],len[cnt]-len[st[i]]));
                tp=ntp-1;
            }
        }
    }
}
```

```

    }
    }
}
void dfs2(int u)
{
    siz[u]=1;
    for (auto [v,w]:fe[u])
    {
        dep[v]=dep[u]+w;
        dfs2(v);
        siz[u]+=siz[v];
        if (siz[v]>siz[hc[u]]) hc[u]=v;
    }
}
void dfs3(int u)
{
    dfn[u]=++id;
    if (hc[u])
    {
        top[hc[u]]=top[u];
        dfs3(hc[u]);
        for (auto [v,w]:fe[u]) if (v!=hc[u]) dfs3(top[v]=v);
    }
}
int lca(int u,int v)
{
    while (top[u]!=top[v]) if (dfn[top[u]]>dfn[top[v]]) u=f[top[u]]; else v=f[top[v]]; //注意不能用
    dep
    return dfn[u]<dfn[v]?u:v;
}
int find(int u,int v)//u 是根
{
    if (dfn[hc[u]]+siz[hc[u]]>dfn[v]) return hc[u];
    while (f[top[v]]!=u) v=f[top[v]];
    return top[v];
}
int dis(int u,int v)
{
    int o=lca(u,v),r=dep[u]+dep[v];
    if (o<=n) return r-(dep[o]<<1);
    u=find(o,u);v=find(o,v);
    if (len[u]>len[v]) swap(u,v);
    return r+min(len[v]-len[u],len[o]-(len[v]-len[u]))-dep[u]-dep[v];
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int m,q,i;
    cin>>n>>m>>q;cnt=n;
    for (i=1;i<=m;i++)
    {
        int u,v,w;
        cin>>u>>v>>w;
        e[u].emplace_back(v,w,i);
        e[v].emplace_back(u,w,i);
    }
}

```

```

mt19937 rnd(time(0));
for (i=1;i<=n;i++) shuffle(all(e[i]),rnd);
dfs1(1);id=0;
dfs2(1);
dfs3(top[1]=1);
while (q--)
{
    int u,v;
    cin>>u>>v;
    cout<<dis(u,v)<<'\n';
}
}

```

### 5.39 广义圆方树

$O(n+m)$ ,  $O(n+m)$ 。

```

void dfs(int u)
{
    dfn[u]=low[u]=++id;
    st[++tp]=u;
    for (int v:e[u]) if (dfn[v]) low[u]=min(low[u],dfn[v]); else
    {
        dfs(v);
        low[u]=min(low[u],low[v]);
        if (dfn[u]<=low[v])
        {
            vector cur={u};
            do
            {
                cur.push_back(st[tp]);
            } while (st[tp--]!=v);
            ans.push_back(cur);
        }
    }
}
}

```

### 5.40 支配树 (DAG 版)

其定义见一般图版。

$O(m \log n)$ ,  $O(n \log n)$ 。

```

int lca(int x,int y)
{
    int i;
    if (dep[x]<dep[y]) swap(x,y);
    for (i=lm[x];dep[x]!=dep[y];i--) if (dep[f[x][i]]>=dep[y]) x=f[x][i];
    if (x==y) return x;
    for (i=lm[x];f[x][0]!=f[y][0];i--) if (f[x][i]!=f[y][i])
    {
        x=f[x][i];y=f[y][i];
    }
    return f[x][0];
}
void dfs(int x)
{

```

```

s[x]=1;
int i;
for (i=sfir[x];i;i=snext[i])
{
    dfs(slj[i]);
    s[x]+=s[slj[i]];
}
}
int main()
{
    dep[0]=-1;
    read(n);
    for (i=1;i<=n;i++)
    {
        read(x);
        while (x)
        {
            add(x,i);
            read(x);
        }
    }
    dl[tou=wei=1]=++n;
    for (i=1;i<n;i++) if (!rd[i]) add(n,i);
    while (tou<=wei)
    {
        for (i=fir[x=dl[tou++]];i;i=nxt[i]) if (--rd[lj[i]]==0) dl[++wei]=lj[i];
        if (i=ffir[x])
        {
            y=flj[i];
            while (i=fnxt[i]) y=lca(y,flj[i]);
            f[x][0]=y;
        } else y=0;
        sadd(y,x);
        f[x][0]=y;
        for (i=1;i<=16;i++) if (0==(f[x][i]=f[f[x][i-1]][i-1]))
        {
            lm[x]=i;
            break;
        }
        dep[x]=dep[y]+1;
    }
    dfs(n);
    for (i=1;i<n;i++) printf("%d\n",s[i]-1);
}

```

### 5.41 支配树（一般图）

$u$  支配  $v$  指的是从  $S$  到  $v$  的路径必然经过  $u$ 。支配树是保持支配关系不变的树，其中  $S$  是根， $idom[u]$  是  $u$  的父节点。

```

#include <bits/stdc++.h>
using namespace std;
const int N=2e5+2;
vector<int> lj[N],llj[N],fl[N],tl[N],buc[N],c[N];
int f[N],mn[N],siz[N],sdom[N],idom[N],dfn[N],nfd[N],pv[N];
int n,m,cnt,i,j,x,y,na;

```

```

bool reach[N];
void dfs1(int x)
{
    nfd[dfn[x]=++cnt]=x;
    for (auto v:l[j[x]] if (!dfn[v]) dfs1(v),c[x].push_back(v);
}
int getf(int x)
{
    if (f[x]==x) return x;
    int u=getf(f[x]);
    mn[x]=dfn[sdom[mn[x]]]<dfn[sdom[mn[f[x]]]]?mn[x]:mn[f[x]];
    return f[x]=u;
}
void dfs0(int u)
{
    reach[u]=1;
    for (auto &v:l[j[u]] if (!reach[v]) dfs0(v);
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int S;
    cin>>n>>m>>S; ++S;
    while (m--) cin>>x>>y, ++x, ++y, l[j[x]].push_back(y);
    for (i=1; i<=n; i++) mn[i]=f[i]=i;
    dfs0(S);
    for (i=1; i<=n; i++) if (reach[i]) for (auto &v:l[j[i]] if (reach[v]) llj[i].push_back(v), fl[v].
        push_back(i);
    for (i=1; i<=n; i++) lj[i]=llj[i];
    dfs1(S); dfn[0]=1e9;
    for (i=cnt; i; i--)
    {
        x=nfd[i]; na=0;
        for (auto v:fl[x])
        {
            sdom[x]=dfn[sdom[x]]<dfn[v]?sdom[x]:v;
            if (dfn[v]>dfn[x])
            {
                getf(v);
                na=dfn[sdom[na]]<dfn[sdom[mn[v]]]?na:mn[v];
            }
        }
        sdom[x]=dfn[sdom[x]]<dfn[sdom[na]]?sdom[x]:sdom[na];
        buc[sdom[x]].push_back(x);
        for (auto v:buc[x]) getf(v), pv[v]=mn[v];
        for (auto v:c[x]) f[v]=x, mn[v]=dfn[sdom[mn[v]]]<dfn[sdom[mn[x]]]?mn[v]:mn[x];
    }
    for (i=1; i<=n; i++) idom[nfd[i]]=(sdom[pv[nfd[i]]]==sdom[nfd[i]])?sdom[nfd[i]]:idom[pv[nfd[i]]];
    for (i=1; i<=n; i++) cout<<(i==S?S:idom[i])-1<<"\n"[i==n];
}

```

## 5.42 最小乘积生成树

题意：每条边有两个属性  $x_i, y_i$ ，你需要最小化  $(\sum x_i)(\sum y_i)$ 。

你需要实现的是 sol1，即按照  $val$  为权值的答案。 $val_i$  是根据  $x_i, y_i$  计算的。



```

#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=202,M=10002;
struct P
{
    int x,y;
    P(int a=0,int b=0):x(a),y(b){}
    bool operator<(const P &o) const {return (ll)x*y<(ll)o.x*o.y||((ll)x*y==(ll)o.x*o.y&& x<o.x);}
};
struct Q
{
    int u,v,x,y,val;
    bool operator<(const Q &o) const {return val<o.val;}
};
P ans=P(1e9,1e9),l,r;
Q a[M];
int f[N];
int n,m,i;
int getf(int x)
{
    if (f[x]==x) return x;
    return f[x]=getf(f[x]);
}
P sol1()
{
    P r=P(0,0);
    for (i=1;i<=n;i++) f[i]=i;
    sort(a+1,a+m+1);
    for (i=1;i<=m;i++) if (getf(a[i].u)!=getf(a[i].v))
    {
        f[f[a[i].u]]=f[a[i].v];
        r.x+=a[i].x,r.y+=a[i].y;
    }
    return r;
}
void sol2(P l,P r)
{
    for (i=1;i<=m;i++) a[i].val=(r.x-l.x)*a[i].y+(l.y-r.y)*a[i].x;
    P np=sol1();
    ans=min(ans,np);
    if ((ll)(r.x-l.x)*(np.y-l.y)-(ll)(r.y-l.y)*(np.x-l.x)>=0) return;
    sol2(l,np);sol2(np,r);
}
int main()
{
    read(n);read(m);
    for (i=1;i<=m;i++) read(a[i].u),read(a[i].v),read(a[i].x),read(a[i].y),++a[i].u,++a[i].v;
    for (i=1;i<=m;i++) a[i].val=a[i].x;l=sol1();
    for (i=1;i<=m;i++) a[i].val=a[i].y;r=sol1();
    ans=min(ans,min(l,r));sol2(l,r);
    printf("%d_%d",ans.x,ans.y);
}

```

### 5.43 最小斯坦纳树

题意：让给定点集连通的最小生成树（不要求全图连通）

$O(3^k n + 2^k m \log m)$ 。

```
const int N=102,M=1002,K=1024;
typedef long long ll;
typedef pair<ll,int> pa;
priority_queue<pa,vector<pa>,greater<pa> > heap;
pa cr;
ll f[K][N],inf;
int lj[M],len[M],nxt[M],fir[N];
int n,m,q,i,j,k,x,y,z,bs,c;
void add()
{
    lj[++bs]=y;
    len[bs]=z;
    nxt[bs]=fir[x];
    fir[x]=bs;
    lj[++bs]=x;
    len[bs]=z;
    nxt[bs]=fir[y];
    fir[y]=bs;
}
void dijk(int s)
{
    int i;
    while (!heap.empty())
    {
        x=heap.top().second;heap.pop();
        for (i=fir[x];i;i=nxt[i]) if (f[s][lj[i]]>f[s][x]+len[i])
        {
            cr.first=f[s][cr.second=lj[i]]=f[s][x]+len[i];
            heap.push(cr);
        }
        while ((!heap.empty())&&(heap.top().first!=f[s][heap.top().second])) heap.pop();
    }
}
int main()
{
    memset(f,0x3f,sizeof(f));inf=f[0][0];
    read(n);read(m);read(q);
    while (m--)
    {
        read(x);read(y);read(z);
        add();
    }
    for (i=1;i<=q;i++)
    {
        read(x);
        f[1<<i-1][x]=0;
    }
    q=(1<<q)-1;
    for (i=1;i<=q;i++)
    {
        for (k=1;k<=n;k++)
        {
            for (j=i&(i-1);j;j=i&(j-1)) f[i][k]=min(f[i][k],f[j][k]+f[i^j][k]);
        }
    }
}
```

```

        if (f[i][k]<inf) heap.push(pa(f[i][k],k));
    }
    dijk(i);
}
for (i=1;i<=n;i++) inf=min(inf,f[q][i]);
printf("%lld",inf);
}

```

## 5.44 2-sat

支持添加一个条件  $\text{add}(u, x, v, y)$ , 表示  $a_u = x \Rightarrow a_v = y$ 。支持设定一个变量的值。

$O(n + m)$ ,  $O(n + m)$ 。

```

struct sat
{
    vector<vector<int>> e;
    vector<int> dfn, low, st, f, ed;
    int fs, tp, id, n;
    sat(int n):n(n), e(n*2), dfn(n*2, -1), low(n*2), st(n*2), f(n*2, -1), ed(n*2), fs(0), tp(-1), id(0){}
    void dfs(int u)
    {
        dfn[u]=low[u]=id++;
        ed[u]=1; st[++tp]=u;
        for (int v:e[u]) if (dfn[v]==-1)
        {
            if (ed[v]) low[u]=min(low[u], dfn[v]);
        } else dfs(v), low[u]=min(low[u], low[v]);
        if (dfn[u]==low[u])
        {
            do
            {
                f[st[tp]]=fs;
                ed[st[tp]]=0;
            } while (st[tp--]!=u);
            ++fs;
        }
    }
    void add(int u, bool x, int v, bool y)
    {
        assert(u>=0&&u<n&&v>=0&&v<n);
        e[u+x*n].push_back(v+y*n);
        e[v+(y^1)*n].push_back(u+(x^1)*n);
    }
    void set(int u, bool x)
    {
        assert(u>=0&&u<n);
        e[u+(x^1)*n].push_back(u+x*n);
    }
    vector<int> getans()
    {
        int i;
        for (i=0; i<n*2; i++) if (dfn[i]==-1) dfs(i);
        vector<int> r(n);
        for (i=0; i<n; i++)
        {
            if (f[i]==f[i+n]) return {};
        }
    }
}

```

```

        r[i]=f[i]>f[i+n];
    }
    return r;
}
};

```

### 5.45 Kosaraju 强连通分量 (bitset 优化)

实用意义不大。

$O(\frac{n^2}{w})$ ,  $O(\frac{n^2}{w})$ 。

```

void dfs1(int x)
{
    int i;ed[x]=0;
    for (i=(lj[x]&ed)._Find_first();i<=n;i=(lj[x]&ed)._Find_next(i)) dfs1(i);
    sx[--tp]=x;
}
void dfs2(int x)
{
    int i;ed[x]=0;tv[f[x]=f[0]]+=v[x];
    for (i=(fj[x]&ed)._Find_first();i<=n;i=(fj[x]&ed)._Find_next(i)) dfs2(i);
}
int main()
{
    read(n);read(m);tp=n+1;
    for (i=1;i<=n;i++) {ed[i]=1;read(v[i]);}
    for (i=1;i<=m;i++)
    {
        read(x);read(y);lj[x][y]=1;fj[y][x]=1;lb[i][0]=x;lb[i][1]=y;
    }
    for (i=1;i<=n;i++) if (ed[i]) dfs1(i);
    ed.set();
    for (i=1;i<=n;i++) if (ed[sx[i]]) {++f[0];dfs2(sx[i]);}
    for (i=1;i<=m;i++) if (f[lb[i][0]]!=f[lb[i][1]])
    {
        flj[f[lb[i][0]]].push_back(f[lb[i][1]]);++rd[f[lb[i][1]]];
    }
    for (i=1;i<=f[0];i++) if (!rd[i]) dl[++wei]=i;
    while (tou<=wei)
    {
        x=dl[tou++];g[x]+=tv[x];
        for (i=0;i<flj[x].size();i++)
        {
            g[flj[x][i]]=max(g[flj[x][i]],g[x]);
            if (--rd[flj[x][i]]==0) dl[++wei]=flj[x][i];
        }
    }
    for (i=1;i<=f[0];i++) ans=max(ans,g[i]);printf("%d",ans);
}

```

### 5.46 Tarjan 强连通分量

$O(n+m)$ ,  $O(n+m)$ 。

```

int dfn[N],low[N],st[N],f[N],fs,tp,id;
bool ed[N];

```

```

void tarjan(int u)
{
    dfn[u]=low[u]=++id;
    ed[u]=1;st[++tp]=u;
    for (int v:e[u]) if (dfn[v])
    {
        if (ed[v]) low[u]=min(low[u],dfn[v]);
    } else tarjan(v),low[u]=min(low[u],low[v]);
    if (dfn[u]==low[u])
    {
        ++fs;
        do
        {
            f[st[tp]]=fs;
            ed[st[tp]]=0;
        } while (st[tp--]!=u);
    }
}

```

### 5.47 动态强连通分量

给出一个加边序列，solve 会返回每个时间进入强连通分量的边。点标号范围是  $[0, n)$

```

struct union_set
{
    vector<int> f;
    int n;
    union_set() { }
    union_set(int nn) :n(nn), f(nn+1)
    {
        iota(all(f), 0);
    }
    int getf(int u) { return f[u]==u ? u : f[u] = getf(f[u]); }
    bool merge(int u, int v)
    {
        u = getf(u); v = getf(v);
        if (u==v) return 0;
        f[u] = v;
        return 1;
    }
    bool connected(int u, int v) { return getf(u)==getf(v); }
};

struct edge
{
    int u, v, t;
};

vector<vector<edge>> solve(int n, const auto& eg)//[0,n)
{
    int m = eg.size(), tp = -1, id = 0, fs = 0;
    vector<vector<edge>> res(m);
    vector e(n, vector<int>());
    vector<int> dfn(n, -1), low(n, -1), st(n), ed(n), blk(n), node;
    union_set s(n-1);
    function<void(int)> dfs = [&](int u)
    {
        dfn[u] = low[u] = id++;
    }
}

```

```

    ed[st[++tp] = u] = 1;
    for (int v : e[u]) if (dfn[v] != -1)
    {
        if (ed[v]) cmin(low[u], dfn[v]);
    }
    else dfs(v), cmin(low[u], low[v]);
    if (dfn[u] == low[u])
    {
        do
        {
            ed[st[tp]] = 0;
            blk[st[tp]] = fs;
        } while (st[tp--] != u);
        ++fs;
    }
};

auto ztef = [&](auto ztef, int l, int r, const vector<edge>& q)
{
    if (eg.size() == 0) return;
    if (l + 1 == r)
    {
        if (l < m)
        {
            res[l].insert(res[l].end(), all(q));
            for (auto [u, v, t] : q) s.merge(u, v);
        }
        return;
    }
    int m = (l + r) / 2;
    node.clear();
    for (auto [u, v, t] : q) if (t < m)
    {
        u = s.getf(u);
        v = s.getf(v);
        e[u].push_back(v);
        node.push_back(u);
        node.push_back(v);
    }
    else break;
    for (int u : node) if (dfn[u] == -1) dfs(u);
    vector<vector<edge>> g(2);
    for (auto [u, v, t] : q) g[t < m && blk[s.f[u]] == blk[s.f[v]]].push_back({u, v, t});
    for (int u : node)
    {
        e[u].clear();
        dfn[u] = low[u] = -1;
    }
    id = fs = 0;
    ztef(ztef, l, m, g[1]);
    ztef(ztef, m, r, g[0]);
};

ztef(ztef, 0, m + 1, eg);
return res;
}

int main()
{
    ios::sync_with_stdio(0); cin.tie(0);

```

```

cout<<fixed<<setprecision(15);
int n, m, i, j;
cin>>n>>m;
vector<ll> x(n);
cin>>x;
vector<edge> edges(m);
for (i = 0;i<m;i++)
{
    auto& [u, v, t] = edges[i];
    cin>>u>>v;
    t = i;
}
auto event = solve(n, edges);
union_set s(n-1);
ll ans = 0;
for (auto e:event)
{
    for (auto [u, v, t]:e)
    {
        u = s.getf(u);
        v = s.getf(v);
        if (u==v) continue;
        s.f[v] = u;
        (ans += x[u]*x[v]) %= p;
        (x[u] += x[v]) %= p;
    }
    cout<<ans<<'\n';
}
}

```

### 5.48 欧拉路径（字典序最小）

```

#include <bits/stdc++.h>
using namespace std;
#if !defined(ONLINE_JUDGE)&&defined(LOCAL)
#include "my_header\debug.h"
#else
#define dbg(...) 1;
#endif
typedef unsigned int ui;
typedef long long ll;
#define all(x) (x).begin(),(x).end()
const int N=1e5+2;
vector<int> e[N];
int rd[N],cd[N];
vector<int> ans;
void dfs(int u)
{
    while (e[u].size())
    {
        int v=e[u].back();
        e[u].pop_back();
        dfs(v);
        ans.push_back(v);
    }
}

```

```

int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m,i,x=0;
    cin>>n>>m;ans.reserve(m);
    while (m--)
    {
        int u,v;
        cin>>u>>v;
        e[u].push_back(v);
        ++cd[u];++rd[v];
    }
    for (i=1;i<=n;i++) if (cd[i]!=rd[i])
    {
        if (abs(cd[i]-rd[i])>1) goto no;
        ++x;
    }
    if (x>2) goto no;x=1;
    for (i=1;i<=n;i++) if (cd[i]>rd[i]) {x=i;break;}
    for (i=1;i<=n;i++) sort(all(e[i])),reverse(all(e[i])));
    dfs(x);ans.push_back(x);reverse(all(ans));
    for (i=0;i<ans.size();i++) cout<<ans[i]<<"\n"[i+1==ans.size()];
    return 0;
    no:cout<<"No"<<endl;
}

```

## 5.49 欧拉回/通路构造

$O(n+m)$ ,  $O(n+m)$ 。

```

optional<vector<int>> undirected_euler_cycle(int n,const vector<pair<int,int>> &edges)//[1,n]/[1,
m], 正数表示正向, 负数表示反向
{
    int i=0;
    vector<int> rd(n+1),ed(edges.size()+1),r;
    vector<vector<pair<int,int>>> e(n+1);
    for (auto [u,v]:edges)
    {
        ++rd[u],++rd[v];
        e[u].push_back({v,++i});
        e[v].push_back({u,-i});
    }
    for (i=1;i<=n;i++) if (rd[i]&1) return {};
    function<void(int)> dfs=[&](int u)
    {
        while (e[u].size())
        {
            auto [v,w]=e[u].back();
            e[u].pop_back();
            if (ed[abs(w)]) continue;
            ed[abs(w)]=1;
            dfs(v);
            r.push_back(w);
        }
    };
    for (i=1;i<=n;i++) if (rd[i]) {dfs(i);break;}
    reverse(all(r));
}

```



```

    if (r.size()!=edges.size()) return {};
    return {r};
}
optional<vector<int>> directed_euler_cycle(int n,const vector<pair<int,int>> &edges)//[1,n]/[1,m]
{
    int i=0;
    vector<int> rd(n+1),cd(n+1),r;
    vector<vector<pair<int,int>>> e(n+1);
    for (auto [u,v]:edges)
    {
        ++cd[u],++rd[v];
        e[u].push_back({v,++i});
    }
    for (i=1;i<=n;i++) if (rd[i]!=cd[i]) return {};
    function<void(int)> dfs=[&](int u)
    {
        while (e[u].size())
        {
            auto [v,w]=e[u].back();
            e[u].pop_back();
            dfs(v);
            r.push_back(w);
        }
    };
    for (i=1;i<=n;i++) if (cd[i]) {dfs(i);break;}
    reverse(all(r));
    if (r.size()!=edges.size()) return {};
    return {r};
}
optional<vector<int>> undirected_euler_trail(int n,const vector<pair<int,int>> &edges)//[1,n]/[1,
m], 正数表示正向, 负数表示反向
{
    int i=0;
    vector<int> rd(n+1),ed(edges.size()+1),r;
    vector<vector<pair<int,int>>> e(n+1);
    for (auto [u,v]:edges)
    {
        ++rd[u],++rd[v];
        e[u].push_back({v,++i});
        e[v].push_back({u,-i});
    }
    int odd=0;
    for (i=1; i<=n; i++) odd+=rd[i]&1;
    if (odd>2) return { };
    function<void(int)> dfs=[&](int u)
    {
        while (e[u].size())
        {
            auto [v,w]=e[u].back();
            e[u].pop_back();
            if (ed[abs(w)]) continue;
            ed[abs(w)]=1;
            dfs(v);
            r.push_back(w);
        }
    };
    for (i=1; i<=n; i++) if (rd[i]&1) { dfs(i); break; }
}

```

```

    if (i>n)
    {
        for (i=1; i<=n; i++) if (rd[i]) { dfs(i); break; }
    }
    reverse(all(r));
    if (r.size()!=edges.size()) return { };
    return {r};
}

optional<vector<int>> directed_euler_trail(int n,const vector<pair<int,int>> &edges)//[1,n]/[1,m]
{
    int i=0;
    vector<int> rd(n+1),cd(n+1),r;
    vector<vector<pair<int,int>>> e(n+1);
    for (auto [u,v]:edges)
    {
        ++cd[u],++rd[v];
        e[u].push_back({v,++i});
    }
    int diff=0;
    for (i=1; i<=n; i++)
    {
        if (abs(rd[i]-cd[i])>1) return { };
        if (rd[i]!=cd[i]) ++diff;
    }
    if (diff>2) return { };
    function<void(int)> dfs=[&](int u)
    {
        while (e[u].size())
        {
            auto [v,w]=e[u].back();
            e[u].pop_back();
            dfs(v);
            r.push_back(w);
        }
    };
    for (i=1; i<=n; i++) if (cd[i]>rd[i]) { dfs(i); break; }
    if (i>n)
    {
        for (i=1; i<=n; i++) if (cd[i]) { dfs(i); break; }
    }
    reverse(all(r));
    if (r.size()!=edges.size()) return { };
    return {r};
}

```

### 5.50 有向图欧拉回路计数 (BEST 定理) / 生成树计数

$O(n^3)$ ,  $O(n^2)$ 。

以  $u$  为起点的欧拉回路个数  $sum = T(u) \times \prod_{v=1}^n (out(v) - 1)!$ , 其中  $T(u)$  是以  $u$  为根的内向树个数 (出度矩阵-邻接矩阵),  $out(v)$  是  $v$  的出度。若允许循环同构 (如  $1 \rightarrow 2 \rightarrow 1 \rightarrow 3 \rightarrow 1$  与  $1 \rightarrow 3 \rightarrow 1 \rightarrow 2 \rightarrow 1$ ), 还需多乘  $out(u)$ 。

这里的部分代码是未经验证的。

```

11 det(vector<vector<ll>> b)
{

```

```

11 r=1;
int n=b.size(), i, j, k;
for (i=0; i<n; i++)
{
    for (j=i; j<n; j++) if (b[j][i]) break;
    if (j==n) return 0;
    swap(b[j], b[i]);
    if (j!=i) r=(p-r)%p;
    r=r*b[i][i]%p;
    b[i][i]=ksm(b[i][i], p-2);
    for (j=n-1; j>=i; j--) b[i][j]=b[i][j]*b[i][i]%p;
    for (j=i+1; j<n; j++) for (k=n-1; k>=i; k--) b[j][k]=(b[j][k]+(p-b[j][i])*b[i][k])%p;
}
return r;
}

11 euler_path_count(vector<vector<int>> a, int s, int t)
{
    int n=a.size(), i, j, k;
    ++a[t][s]; s=t;
    vector<int> rd(n), cd(n);
    for (i=0; i<n; i++) for (j=0; j<n; j++) cd[i]+=a[i][j], rd[j]+=a[i][j];
    for (i=0; i<n; i++) if (cd[i]!=rd[i]) return 0;
    vector<int> f(n);
    iota(all(f), 0);
    function<int(int)> getf=[&](int u) { return f[u]==u?u:f[u]=getf(f[u]); };
    for (i=0; i<n; i++) for (j=0; j<n; j++) if (a[i][j]) f[getf(i)]=getf(j);
    11 r=1;
    vector<int> id;
    for (i=0; i<n; i++) if (cd[i])
    {
        if (getf(i)!=getf(s)) return 0;
        r=r*fac[cd[i]-1]%p;
        if (i!=s) id.push_back(i);
    }
    n=id.size();
    vector b(n, vector<ll>(n));
    for (i=0; i<n; i++)
    {
        b[i][i]=cd[id[i]]-a[id[i]][id[i]];
        for (j=0; j<n; j++) if (i!=j) b[i][j]=(p-a[id[i]][id[j]])%p;
    }
    return r*det(b)%p;
}

11 euler_path_count(vector<vector<int>> a)
{
    int n=a.size(), i, j, s=-1, t=-1;
    vector<int> rd(n), cd(n), d(n);
    for (i=0; i<n; i++) for (j=0; j<n; j++) cd[i]+=a[i][j], rd[j]+=a[i][j];
    if (count(all(cd), 0)==n) return 1;
    for (i=0; i<n; i++) d[i]=cd[i]-rd[i];
    s=max_element(all(d))-d.begin();
    t=min_element(all(d))-d.begin();
    11 r=0;
    if (s==t)
    {
        for (i=0; i<n; i++) if (cd[i]) r+=euler_path_count(a, i, i);
    }
}

```

```

    else r=eular_path_count(a, s, t);
    return r%p;
}
ll eular_circuit_count(vector<vector<int>> a)
{
    int n=a.size(), i, j;
    for (i=0; i<n; i++) for (j=0; j<n; j++) if (a[i][j]) return eular_path_count(a, i, i)*ksm(
        accumulate(all(a[i]), 0llu)%p, p-2)%p;
    return 1;
}
ll directed_spanning_tree_count(vector<vector<int>> a, int s)
{
    int n=a.size(), i, j;
    vector b(n-1, vector<ll>(n-1));
    for (i=0; i<n; i++) a[i][i]=0;
    for (i=0; i<n; i++) if (i!=s) for (j=0; j<n; j++) if (j!=s&&i!=j) b[i-(i>s)][j-(j>s)]=(p-a[i][j])%p;
    for (i=0; i<n; i++) if (i!=s) for (j=0; j<n; j++) (b[i-(i>s)][i-(i>s)]+=a[j][i])%p;
    return det(b);
} //外向
ll undirected_spanning_tree_count(vector<vector<int>> a)
{
    int n=a.size(), i, j;
    --n;
    vector b(n, vector<ll>(n));
    for (i=0; i<n; i++) a[i][i]=0;
    for (i=0; i<n; i++) for (j=0; j<n; j++) if (i!=j) b[i][j]=(p-a[i][j])%p;
    for (i=0; i<n; i++) b[i][i]=reduce(all(a[i]), 0llu)%p;
    return det(b);
}

```

### 5.51 点染色

结论:  $\chi(G) \leq \Delta(G) + 1$ , 其中  $\Delta(G)$  是图的最大度。只有奇圈和完全图取等。  
构造方案只能爆搜。

```

vector<int> chromatic_number(int n, const vector<pair<int, int>> &edges) // [0, n)
{
    vector r(n, -1), cur(n, -1);
    vector<vector<int>> e(n);
    int ans=0, i;
    for (auto [u, v]: edges) e[u].push_back(v), e[v].push_back(u);
    for (i=0; i<n; i++) ans=max(ans, (int)e[i].size());
    ans+=2;
    vector p(n, vector(ans, 0));
    function<void(int)> dfs=[&](int u)
    {
        int col=u?*max_element(cur.begin(), cur.begin()+u)+1:0;
        if (col>=ans) return;
        if (u==n)
        {
            r=cur;
            ans=col;
            return;
        }
        int i;
    }
    dfs(0);
}

```

```

    for (int i=0;i<=col;i++) if (!p[u][i])
    {
        cur[u]=i;
        for (int v:e[u]) ++p[v][i];
        dfs(u+1);
        for (int v:e[u]) --p[v][i];
    }
};
dfs(0);
return r;
}

```

## 5.52 最大独立集

爆搜。

```

vector<int> indep_set(int n,const vector<pair<int,int>> &edges)//[0,n)
{
    vector<vector<int>> e(n);
    mt19937 rnd(998);
    vector<int> p(n),q(n),ed(n);
    iota(all(p),0);
    shuffle(all(p),rnd);
    for (int i=0;i<n;i++) q[p[i]]=i;
    for (auto [u,v]:edges)
    {
        e[p[u]].push_back(p[v]);
        e[p[v]].push_back(p[u]);
    }
    vector<int> r,cur;
    function<void(int)> dfs=[&](int u)
    {
        if (cur.size()+n-u<=r.size()) return;
        if (u==n)
        {
            r=cur;
            return;
        }
        if (!ed[u])
        {
            cur.push_back(u);
            for (int v:e[u]) ++ed[v];
            dfs(u+1);
            for (int v:e[u]) --ed[v];
            cur.pop_back();
        }
        if (ed[u]||e[u].size()) dfs(u+1);
    };dfs(0);
    for (int &x:r) x=q[x];
    sort(all(r));
    return r;
}

```

## 6 计算几何

### 6.1 自适应 simpson 法

sim(l,r) 计算  $\int_l^r f(x) dx$

```
const db eps=1e-7;
db sl,sr,sm,a;
db f(db x)
{
    return pow(x,a/x-x);
}
db g(db l,db r)
{
    db mid=(l+r)*0.5;
    return (f(l)+f(r)+f(mid)*4)/6*(r-l);
}
db sim(db l,db r)
{
    db mid=(l+r)*0.5;
    sl=g(l,mid);sr=g(mid,r);sm=g(l,r);
    if (abs(sl+sr-sm)<eps) return sl+sr;
    return sim(l,mid)+sim(mid,r);
}
```

### 6.2 计算几何全

功能其实比较少，因为实际遇到的几何题不多。最有用的可能是闵可夫斯基和合并凸包，和常规的线段判交之类的。其余功能最好直接使用 HDU 板。

```
namespace geometry//不要用 int!
{
#define tml template<class T>
    typedef long long ll;
    typedef long double db;
    const db eps=1e-6;
#define all(x) (x).begin(),(x).end()
    inline int sgn(const ll &x)
    {
        if (x<0) return -1;
        return x>0;
    }
    inline int sgn(const db &x)
    {
        if (fabs(x)<eps) return 0;
        return x>0?1:-1;
    }
    tml struct point/* 为叉乘, & 为点乘, 只允许使用 (long )double 和 ll
    {
        T x, y;
        point() { }
        point(T a, T b):x(a), y(b) { }
        operator point<ll>() const { return point<ll>(x, y); }
        operator point<db>() const { return point<db>(x, y); }
        point<T> operator+(const point<T> &o) const { return point(x+o.x, y+o.y); }
        point<T> operator-(const point<T> &o) const { return point(x-o.x, y-o.y); }
        point<T> operator*(const T &k) const { return point(x*k, y*k); }
```

```

    point<T> operator/(const T &k) const { return point(x/k, y/k); }
    T operator*(const point<T> &o) const { return x*o.y-y*o.x; }
    T operator&(const point<T> &o) const { return x*o.x+y*o.y; }
    void operator+=(const point<T> &o) { x+=o.x; y+=o.y; }
    void operator-=(const point<T> &o) { x-=o.x; y-=o.y; }
    void operator*=(const T &k) { x*=k; y*=k; }
    void operator/=(const T &k) { x/=k; y/=k; }
    bool operator==(const point<T> &o) const { return x==o.x&&y==o.y; }
    bool operator!=(const point<T> &o) const { return x!=o.x||y!=o.y; }
    db len() const { return sqrt(len2()); } //模长
    T len2() const { return x*x+y*y; }
};

const point<db> npos=point<db>(514e194, 9810e191), apos=point<db>(145e174, 999e180);
const int DS[4]={1, 2, 4, 3};
templ int quad(const point<T> &o) //坐标轴归右上象限, 返回值 [1,4]
{
    return DS[(sgn(o.y)<0)*2+(sgn(o.x)<0)];
}

templ bool angle_cmp(const point<T> &a, const point<T> &b)
{
    int c=quad(a), d=quad(b);
    if (c!=d) return c<d;
    return a*b>0;
}

templ db dis(const point<T> &a, const point<T> &b) { return (a-b).len(); }
templ T dis2(const point<T> &a, const point<T> &b) { return (a-b).len2(); }
templ point<T> operator*(const T &k, const point<T> &o) { return point<T>(k*o.x, k*o.y); }
templ bool operator<(const point<T> &a, const point<T> &b)
{
    int s=sgn(a*b);
    return s>0||s==0&&sgn(a.len2()-b.len2())<0;
}

istream &operator>>(istream &cin, point<ll> &o) { return cin>>o.x>>o.y; }
istream &operator>>(istream &cin, point<db> &o)
{
    string s;
    cin>>s;
    o.x=stod(s);
    cin>>s;
    o.y=stod(s);
    return cin;
}

templ ostream &operator<<(ostream &cout, const point<T> &o)
{
    if ((point<db>)o==apos) return cout<<"all_position";
    if ((point<db>)o==npos) return cout<<"no_position";
    return cout<<'('<<o.x<<','<<o.y<<')';
}

templ struct line
{
    point<T> o, d;
    line() { }
    line(const point<T> &a, const point<T> &b, int twopoint);
    bool operator!=(const line<T> &m) { return !(*this==m); }
};

template<> line<ll>::line(const point<ll> &a, const point<ll> &b, int twopoint)
{

```

```

    o=a;
    d=twopoint?b-a:b;
    ll tmp=gcd(d.x, d.y);
    assert(tmp);
    if (d.x<0||d.x==0&& d.y<0) tmp=-tmp;
    d.x/=tmp; d.y/=tmp;
}

template<> line<db>::line(const point<db> &a, const point<db> &b, int twopoint)
{
    o=a;
    d=twopoint?b-a:b;
    int s=sgn(d.x);
    if (s<0||!s&& d.y<0) d.x=-d.x, d.y=-d.y;
}

tpl line<T> rotate_90(const line<T> &m) { return line(m.o, point(m.d.y, -m.d.x), 0); }
tpl line<db> rotate(const line<T> &m, db angle)
{
    return {(point<db>)m.o, {m.d.x*cos(angle)-m.d.y*sin(angle), m.d.x*sin(angle)+m.d.y*cos(
        angle)}, 0};
}

tpl db get_angle(const line<T> &m, const line<T> &n) { return asin((m.d*n.d)/(m.d.len()*n.d.
    len())); }

tpl bool operator<(const line<T> &m, const line<T> &n)
{
    int s=sgn(m.d*n.d);
    return s?s>0:m.d*m.o<n.d*n.o;
}

bool operator==(const line<ll> &m, const line<ll> &n) { return m.d==n.d&&(m.o-n.o)*m.d==0; }
bool operator==(const line<db> &m, const line<db> &n) { return fabs(m.d*n.d)<eps&&fabs((n.o-m.
    o)*m.d)<eps; }

tpl ostream &operator<<(ostream &cout, const line<T> &o) { return cout<<'('<<o.d.x<<"k_+<<
    o.o.x<<"_<<o.d.y<<"k_+<<o.o.y<<")"; }

tpl point<db> intersect(const line<T> &m, const line<T> &n)
{
    if (!sgn(m.d*n.d))
    {
        if (!sgn(m.d*(n.o-m.o))) return apos;
        return npos;
    }
    return (point<db>)m.o+(n.o-m.o)*n.d/(db)(m.d*n.d)*(point<db>)m.d;
}

tpl db dis(const line<T> &m, const point<T> &o) { return abs(m.d*(o-m.o)/m.d.len()); }
tpl db dis(const point<T> &o, const line<T> &m) { return abs(m.d*(o-m.o)/m.d.len()); }

struct circle
{
    point<db> o;
    db r;
    circle() { }
    circle(const point<db> &o, const db &R=0):o(point<db>((db)0.x, (db)0.y)), r(R) { }//圆心半
        径构造
    circle(const point<db> &a, const point<db> &b)//直径构造
    {
        o=(a+b)*0.5;
        r=dis(b, o);
    }
    circle(const point<db> &a, const point<db> &b, const point<db> &c)//三点构造外接圆(非最小
        圆)

```



```

{
    auto A=(b+c)*0.5, B=(a+c)*0.5;
    o=intersect(rotate_90(line(A, c, 1)), rotate_90(line(B, c, 1)));
    r=dis(o, c);
}
circle(vector<point<db>> a)
{
    int n=a.size(), i, j, k;
    mt19937 rnd(75643);
    shuffle(all(a), rnd);
    *this=circle(a[0]);
    for (i=1; i<n; i++) if (!cover(a[i]))
    {
        *this=circle(a[i]);
        for (j=0; j<i; j++) if (!cover(a[j]))
        {
            *this=circle(a[i], a[j]);
            for (k=0; k<j; k++) if (!cover(a[k])) *this=circle(a[i], a[j], a[k]);
        }
    }
}
circle(const vector<point<ll>> &b)
{
    vector<point<db>> a(b.size());
    int n=a.size(), i, j, k;
    for (i=0; i<a.size(); i++) a[i]=(point<db>)b[i];
    *this=circle(a);
}
templ bool cover(const point<T> &a) { return sgn(dis((point<db>)a, o)-r)<=0; }
};
templ struct segment
{
    point<T> a, b;
    segment() { }
    segment(point<T> o, point<T> p)
    {
        int s=sgn(o.x-p.x);
        if (s>0||!s&&o.y>p.y) swap(o, p);
        a=o; b=p;
    }
};
templ bool intersect(const segment<T> &m, const segment<T> &n)
{
    auto a=n.b-n.a, b=m.b-m.a;
    auto d=n.a-m.a;
    if (sgn(n.b.x-m.a.x)<0||sgn(m.b.x-n.a.x)<0) return 0;
    if (sgn(max(n.a.y, n.b.y)-min(m.a.y, m.b.y))<0||sgn(max(m.a.y, m.b.y)-min(n.a.y, n.b.y))
        <0) return 0;
    return sgn(b*d)*sgn((n.b-m.a)*b)>=0&&sgn(a*d)*sgn((m.b-n.a)*a)<=0;
}
templ struct convex
{
    vector<point<T>> p;
    convex(vector<point<T>> a);
    db peri()//周长
    {
        int i, n=p.size();

```

```

    db C=(p[n-1]-p[0]).len();
    for (i=1; i<n; i++) C+=(p[i-1]-p[i]).len();
    return C;
}
db area() { return area2()*0.5; } //面积
T area2() //两倍面积
{
    int i, n=p.size();
    T S=p[n-1]*p[0];
    for (i=1; i<n; i++) S+=p[i-1]*p[i];
    return abs(S);
}
db diam() { return sqrt(diam2()); }
T diam2() //直径平方
{
    T r=0;
    int n=p.size(), i, j;
    if (n<=2)
    {
        for (i=0; i<n; i++) for (j=i+1; j<n; j++) r=max(r, dis2(p[i], p[j]));
        return r;
    }
    p.push_back(p[0]);
    for (i=0, j=1; i<n; i++)
    {
        while ((p[i+1]-p[i])*(p[j]-p[i])<=(p[i+1]-p[i])*(p[j+1]-p[i])) if (++j==n) j=0;
        r=max({r, dis2(p[i], p[j]), dis2(p[i+1], p[j])});
    }
    p.pop_back();
    return r;
}
bool cover(const point<T> &o) const //点是否在凸包内
{
    if (o.x<p[0].x||o.x==p[0].x&&o.y<p[0].y) return 0;
    if (o==p[0]) return 1;
    if (p.size()==1) return 0;
    ll tmp=(o-p[0])*(p.back()-p[0]);
    if (tmp==0) return dis2(o, p[0])<=dis2(p.back(), p[0]);
    if (tmp<0||p.size()==2) return 0;
    int x=upper_bound(1+all(p), o, [&](const point<T> &a, const point<T> &b) { return (a-p[0])*(b-p[0])>0; })-p.begin()-1;
    return (o-p[x])*(p[x+1]-p[x])<=0;
}
convex<T> operator+(const convex<T> &A) const
{
    int n=p.size(), m=A.p.size(), i, j;
    vector<point<T>> c;
    if (min(n, m)<=2)
    {
        c.reserve(n*m);
        for (i=0; i<n; i++) for (j=0; j<m; j++) c.push_back(p[i]+A.p[j]);
        return convex<T>(c);
    }
    point<T> a[n], b[m];
    for (i=0; i+1<n; i++) a[i]=p[i+1]-p[i];
    a[n-1]=p[0]-p[n-1];
    for (i=0; i+1<m; i++) b[i]=A.p[i+1]-A.p[i];

```

```

        b[m-1]=A.p[0]-A.p[m-1];
        c.reserve(n+m);
        c.push_back(p[0]+A.p[0]);
        for (i=j=0; i<n&&j<m;) c.push_back(c.back()+(a[i]*b[j]>0?a[i++]:b[j++]));
        while (i<n-1) c.push_back(c.back()+a[i++]);
        while (j<m-1) c.push_back(c.back()+b[j++]);
        return convex<T>(c);
    }

    void operator+=(const convex &a) { *this=*this+a; }
};

tpl convex<T>::convex(vector<point<T>> a)
{
    int n=a.size(), i;
    if (!n) return;
    p=a;
    for (i=1; i<n; i++) if (p[i].x<p[0].x||p[i].x==p[0].x&& p[i].y<p[0].y) swap(p[0], p[i]);
    a.resize(0); a.reserve(n);
    for (i=1; i<n; i++) if (p[i]!=p[0]) a.push_back(p[i]-p[0]);
    sort(all(a));
    for (i=0; i<a.size(); i++) a[i]+=p[0];
    point<T> *st=p.data()-1;
    int tp=1;
    for (auto &v:a)
    {
        while (tp>1&&sgn((st[tp]-st[tp-1])*(v-st[tp-1]))<=0) --tp;
        st[++tp]=v;
    }
    p.resize(tp);
}

template<> bool convex<db>::cover(const point<db> &o) const//点是否在凸包内
{
    if (o.x<p[0].x||o.x==p[0].x&&o.y<p[0].y) return 0;
    if (o==p[0]) return 1;
    if (p.size()==1) return 0;
    ll tmp=(o-p[0])*(p.back()-p[0]);
    if (tmp==0) return dis2(o, p[0])<=dis2(p.back(), p[0]);
    if (tmp<0||p.size()==2) return 0;
    int x=upper_bound(1+all(p), o, [&](const point<db> &a, const point<db> &b) { return (a-p[0])*(b-p[0])>eps; })-p.begin()-1;
    return (o-p[x])*(p[x+1]-p[x])<=0;
}

tpl struct half_plane//默认左侧
{
    point<T> o, d;
    operator half_plane<ll>() const { return {(point<ll>)o, (point<ll>)d, 0}; }
    operator half_plane<db>() const { return {(point<db>)o, (point<db>)d, 0}; }
    half_plane() { }
    half_plane(const point<T> &a, const point<T> &b, bool twopoint)
    {
        o=a;
        d=twopoint?b-a:b;
    }
    bool operator<(const half_plane<T> &a) const
    {
        int p=quad(d), q=quad(a.d);
        if (p!=q) return p<q;
        p=sgn(d*a.d);

```

```

        if (p) return p>0;
        return sgn(d*(a.o-o))>0;
    }
};

tmpl ostream &operator<<(ostream &cout, half_plane<T> &m) { return cout<<m.o<<"|_|"<<m.d; }
tmpl point<db> intersect(const half_plane<T> &m, const half_plane<T> &n)
{
    if (!sgn(m.d*n.d))
    {
        if (!sgn(m.d*(n.o-m.o))) return apos;
        return npos;
    }
    return (point<db>)m.o+(n.o-m.o)*n.d/(db)(m.d*n.d)*(point<db>)m.d;
}

const db inf=1e9;
tmpl convex<db> intersect(vector<half_plane<T>> a)
{
    T I=inf;
    a.push_back({{-I, -I}, {I, -I}, 1});
    a.push_back({{I, -I}, {I, I}, 1});
    a.push_back({{I, I}, {-I, I}, 1});
    a.push_back({{-I, I}, {-I, -I}, 1});
    sort(all(a));
    int n=a.size(), i, h=0, t=-1;
    half_plane<db> q[n];
    point<db> p[n];
    vector<point<db>> r;
    for (i=0; i<n; i++) if (i==n-1||sgn(a[i].d*a[i+1].d))
    {
        auto x=(half_plane<db>)a[i];
        while (h<t&&sgn((p[t-1]-x.o)*x.d)>=0) --t;
        while (h<t&&sgn((p[h]-x.o)*x.d)>=0) ++h;
        q[++t]=x;
        if (h<t) p[t-1]=intersect(q[t-1], q[t]);
    }
    while (h<t&&sgn((p[t-1]-q[h].o)*q[h].d)>=0) --t;
    if (h==t) return convex<db>(vector<point<db>>(0));
    p[t]=intersect(q[h], q[t]);
    return convex<db>(vector<point<db>>(p+h, p+t+1));
}

tmpl db dis(const point<db> &o, const segment<T> &l)
{
    if ((l.b-l.a&o-l.a)<0||(l.a-l.b&o-l.b)<0) return min(dis(o, l.a), dis(o, l.b));
    return dis(o, line(l.a, l.b, 1));
}

tmpl db dis(const segment<T> &l, const point<db> &o)
{
    if ((l.b-l.a&o-l.a)<0||(l.a-l.b&o-l.b)<0) return min(dis(o, l.a), dis(o, l.b));
    return dis(o, line(l.a, l.b, 1));
}

pair<ll, ll> __sqrt1(ll x)
{
    ll y=sqrt1(x);
    while (y*y>x) --y;
    while ((y+1)*(y+1)<=x) ++y;
    return {y, y+(y*y<x)};
}

```

```

pair<int, int> closest_pair(const vector<point<ll>> &a)
{
    int n=a.size(), i, j;
    assert(n>=2);
    auto b=a;
    sort(all(b), [&](auto p, auto q)
        {
            return p.x==q.x?p.y<q.y:p.x<q.x;
        });
    tuple<ll, int, int> ans={dis2(b[0], b[1]), 0, 1};
    set<pair<ll, int>> s;
    for (i=j=0; i<n; i++)
    {
        auto [x, y]=b[i];
        ll d=__sqrt(get<0>(ans)).first;
        if (d==0) break;
        for (auto it=s.lower_bound({y-d, 0}); it!=s.end(); ++it)
        {
            auto [q, k]=*it;
            cmin(ans, tuple{dis2(b[k], b[i]), i, k});
        }
        s.emplace(y, i);
        while (b[j].x<x-d) s.erase({b[j].y, j}), ++j;
    }
    auto [_, j1, j2]=ans;
    int i1, i2;
    for (i1=0; i1<n; i1++) if (a[i1]==b[j1]) break;
    for (i2=0; i2<n; i2++) if (i2!=i1&&a[i2]==b[j2]) break;
    return {i1, i2};
}

pair<int, int> furthest_pair(const vector<point<ll>> &a)
{
    int n=a.size(), i, j;
    assert(n>=2);
    auto b=convex(a).p;
    int m=b.size();
    if (m==1) return {0, 1};
    b.push_back(b[0]);
    tuple<ll, int, int> ans={dis2(b[0], b[1]), 0, 1};
    for (i=0, j=1; i<m; i++)
    {
        while (abs((b[i+1]-b[i])*(b[j]-b[i]))<abs((b[i+1]-b[i])*(b[(j+1)%m]-b[i]))) j=(j+1)%m;
        cmax(ans, tuple{dis2(b[i], b[j]), i, j});
        cmax(ans, tuple{dis2(b[i+1], b[j]), i+1, j});
    }
    auto [_, j1, j2]=ans;
    int i1, i2;
    for (i1=0; i1<n; i1++) if (a[i1]==b[j1]) break;
    for (i2=0; i2<n; i2++) if (i2!=i1&&a[i2]==b[j2]) break;
    return {i1, i2};
}

#undef tpl
}

using geometry::point, geometry::line, geometry::circle, geometry::convex, geometry::half_plane;
using geometry::db, geometry::sgn, geometry::eps, geometry::segment;
using geometry::intersect, geometry::dis;

```

## 7 公式与杂项

### 7.1 枚举大小为 $k$ 的集合

思路：通过进位创造 1，再把一串 1 移到最后。

```
for (int s=(1<<k)-1,t;s<1<<n;t=s+(s&-s),s=(s&~t)>>__lg(s&-s)+1|t)
{}

```

### 7.2 min plus 卷积

计算  $c_i = \min_{j=0}^i a_j + b_{i-j}$ 。

要求  $b$  是凸的，即  $b_{i+1} - b_i$  不降。

```
template <class T> vector<T> min_plus_convolution(const vector<T> &a,const vector<T> &b)
{
    int n=a.size(),m=b.size(),i;
    vector<T> c(n+m-1);
    function<void(int,int,int,int)> dfs=[&](int l,int r,int ql,int qr)
    {
        if (l>r) return;
        int mid=l+r>>1;
        while (ql+m<=l) ++ql;
        while (qr>r) --qr;
        int qmid=-1;
        c[mid]=inf;
        for (int i=ql; i<=qr; i++) if (mid-i>=0&&mid-i<m&&cmin(c[mid],a[i]+b[mid-i])) qmid=i;
        dfs(l,mid-1,ql,qmid);
        dfs(mid+1,r,qmid,qr);
    };
    dfs(0,n+m-2,0,n-1);
    return c;
}

```

### 7.3 所有区间 GCD

需要自定义 fun，如 gcd，and，or。

```
template<class T> struct GCD
{
    vector<pair<int, T>> res;
    GCD(const vector<T> &a):res(n)
    {
        int n=a.size(), i, j;
        vector<ll> v(n);
        vector<int> l(n);
        vector<vector<pair<int, T>> res(n);
        for (i=0; i<n; i++)
        {
            for (v[i]=a[i], j=l[i]=i; j>=0; j=l[j]-1)
            {
                v[j]=fun(v[j], a[i]);
                while (l[j]&&fun(a[i], v[l[j]-1])==fun(a[i], v[j])) l[j]=l[l[j]-1];
                //l[j]..j,i 区间内的值求 fun 均为 v[j]
            }
        }
    }
}

```

```

        for (j=i; j>=0; j=l[j]) res[i].push_back({l[j], v[j]});
        reverse(all(res[i]));
    }
}
T ask(int l, int r)//[l,r]
{
    return res[r].prev(upper_bound(l))->second;
}
};

```

## 7.4 整体二分（区间 $k$ -th）

$O((n+q)\log a)$ ,  $O(n+q)$ 。

```

struct cz
{
    int x,y,kth,pos,typ;
};
cz q[M],st1[M],st2[M];
int a[N],b[N],d[N],ans[N],s[N];
int n,m,t1,t2,i,j,c,gs;
int lb(int x)
{
    return x&&(-x);
}
void add(int x,int y)
{
    for (;x<=n;x+=lb(x)) s[x]+=y;
}
int sum(int x)
{
    int ans=0;
    for (;x;x-=lb(x)) ans+=s[x];
    return ans;
}
void ztef(int ql,int qr,int l,int r)
{
    if (ql>qr) return;
    int mid=l+r>>1,i,midd;
    t1=t2=0;
    if (l==r)
    {
        for (i=ql;i<=qr;i++) if (q[i].typ) ans[q[i].pos]=d[l];
        return;
    }
    for (i=ql;i<=qr;i++) if (q[i].typ)
    {
        midd=sum(q[i].y)-sum(q[i].x-1);
        if (midd>=q[i].kth) st1[++t1]=q[i]; else
        {
            st2[++t2]=q[i];
            st2[t2].kth-=midd;
        }
    }
    else if (q[i].pos<=mid)
    {
        add(q[i].x,1);
    }
}

```

```

        st1[++t1]=q[i];
    }
    else st2[++t2]=q[i];
    for (i=1;i<=t1;i++) if (!st1[i].typ) add(st1[i].x,-1);
    for (i=1;i<=t1;i++) q[i+ql-1]=st1[i];
    midd=ql+t1-1;
    for (i=1;i<=t2;i++) q[i+midd]=st2[i];
    ztef(ql,midd,l,mid);ztef(midd+1,qr,mid+1,r);
}
int main()
{
    read(n);read(m);
    for (i=1;i<=n;i++)
    {
        read(a[i]);b[i]=a[i];
    }
    sort(b+1,b+n+1);
    d[gs=1]=b[1];
    for (i=2;i<=n;i++) if (b[i]!=b[i-1]) d[++gs]=b[i];
    for (i=1;i<=n;i++) a[i]=lower_bound(d+1,d+gs+1,a[i])-d;
    for (i=1;i<=n;i++)
    {
        q[i].x=i;q[i].pos=a[i];q[i].typ=0;
    }
    for (i=1;i<=m;i++)
    {
        read(q[i+n].x);read(q[i+n].y);read(q[i+n].kth);q[i+n].pos=i;q[i+n].typ=1;
    }
    ztef(1,n+m,1,gs);
    for (i=1;i<=m;i++) printf("%d\n",ans[i]);
}

```

## 7.5 cdq 分治（三维偏序）

本质：统计跨越区间中点的贡献，此时左右就不必保持原本的下标顺序了，可以按下一个维度排序。

$O(n \log^2 n)$ ,  $O(n)$ 。

```

int lb(int x)
{
    return x&&(-x);
}
void add(int x,int y)
{
    for (;x<=mx;x+=lb(x)) a[x]+=y;
}
int sum(int x)
{
    int ans=0;
    for (;x;x^=lb(x)) ans+=a[x];
    return ans;
}
void gb(int l,int r)
{
    int i=l,m=l+r>>1,j=m+1,p=1;
    if (i<m) gb(i,m);

```



```

if (j<r) gb(j,r);
while ((i<=m)|| (j<=r)) if ((j>r)|| (i<=m)&&(q[i].x<=q[j].x))
{
    if (!q[i].typ) add(q[i].y,1);
    qq[p++]=q[i++];
}
else
{
    if (q[j].typ) ans[q[j].pos]+=q[j].typ*sum(q[j].y);
    qq[p++]=q[j++];
}
for (i=1;i<=m;i++) if (!q[i].typ) add(q[i].y,-1);
for (i=1;i<=r;i++) q[i]=qq[i];
}
int main()
{
    read(n);read(m);
    for (i=1;i<=n;i++)
    {
        read(q[i].x);read(q[i].y);++q[i].y;
        yc[i]=q[i].y;
        if (q[i].y>mx) mx=q[i].y;
    }
    qs=ys=n;
    for (i=1;i<=m;i++)
    {
        read(x);read(y);read(z);read(j);
        q[++qs].x=x-1;q[qs].y=y;q[qs].pos=i;q[qs].typ=1;
        q[++qs].x=z;q[qs].y=y;q[qs].pos=i;q[qs].typ=-1;
        q[++qs].x=x-1;q[qs].y=j+1;q[qs].pos=i;q[qs].typ=-1;
        q[++qs].x=z;q[qs].y=j+1;q[qs].pos=i;q[qs].typ=1;
        if (j+1>mx) mx=j+1;
    }
    gb(1,qs);
    for (i=1;i<=m;i++) printf("%d\n",ans[i]);
}

```

## 7.6 高精度

除法和取模有点问题，但 gcd 是对的。

```

struct bigint
{
    using ll=unsigned long long;
    using lll=unsigned __int128;
    const static ll base=1e6;
    const static ll sign=1llu<<63;
    const static lll p=4179340454199820289;
    const static lll g=5;
    const static int N=1<<23;
    static int r[N];
    static lll w[N];
    bool neg;
    vector<ll> a;
private:
    static lll ksm(lll x,ll y)
    {

```

```

    lll r=1;
    while (y)
    {
        if (y&1) r=r*x%p;
        x=x*x%p; y>>=1;
    }
    return r;
}

static void init(int n)
{
    static int pr=0,pw=0;
    if (pr==n) return;
    int b=__lg(n)-1,i,j,k;
    for (i=1; i<n; i++) r[i]=r[i>>1]>>1|(i&1)<<b;
    if (pw<n)
    {
        for (j=1; j<n; j=k)
        {
            k=j*2;
            ll wn=ksm(g,(p-1)/k);
            w[j]=1;
            for (i=j+1; i<k; i++) w[i]=w[i-1]*wn%p;
        }
        pw=n;
    }
    pr=n;
}

static void dft(vector<lll> &a,int o=0)
{
    int n=a.size(),i,j,k;
    lll y,*f,*g,*wn,*A=a.data();
    init(n);
    for (i=1; i<n; i++) if (i<r[i]) swap(A[i],A[r[i]]);
    for (k=1; k<n; k*=2)
    {
        wn=w+k;
        for (i=0; i<n; i+=k*2)
        {
            f=A+i; g=A+i+k;
            for (j=0; j<k; j++)
            {
                y=g[j]*wn[j]%p;
                g[j]=f[j]+p-y;
                f[j]+=y;
            }
        }
        if (k*2==n||k==1<<10) for (lll &x:a) x%=p;
    }
    if (o)
    {
        y=ksm(n,p-2);
        for (lll &x:a) x=x*y%p;
        reverse(1+all(a));
    }
}

ll &operator[](const int &x) { return a[x]; }
const ll &operator[](const int &x) const { return a[x]; }

```

```

static void plus_by(vector<ll> &a,const vector<ll> &b)
{
    int n=a.size(),m=b.size(),i,j;
    cmax(n,m);
    a.resize(++n);
    for (i=0; i<m; i++) if ((a[i]+=b[i])>=base) a[i]-=base,++a[i+1];
    for (i=m; i<n&& a[i]>=base; i++) a[i]-=base,++a[i+1];
    if (a[n-1]==0) a.pop_back();
}
static void minus_by(vector<ll> &a,const vector<ll> &b)
{
    int n=a.size(),m=b.size(),i,j;
    for (i=0; i<m; i++) if (!(a[i]&sign)&&a[i]>=b[i]) a[i]-=b[i];
    else --a[i+1],a[i]+=base-b[i];
    for (; i<n&&(a[i]&sign); i++) --a[i+1],a[i]+=base-b[i];
    while (a.size()>1&&!a.back()) a.pop_back();
}
static bool less(const vector<ll> &a,const vector<ll> &b)
{
    if (a.size()!=b.size()) return a.size()<b.size();
    for (int i=a.size()-1; i>=0; i--) if (a[i]!=b[i]) return a[i]<b[i];
    return 0;
}
static int cal(int x) { return 1<<_lg(max(x,1)*2-1); }
public:
bigint &operator+=(const bigint &o)
{
    if (neg==o.neg) plus_by(a,o.a);
    else if (neg)
    {
        if (less(o.a,a)) minus_by(a,o.a);
        else
        {
            neg=0;
            auto t=o.a;
            swap(a,t);
            minus_by(a,t);
        }
    }
    else
    {
        if (less(a,o.a))
        {
            neg=1;
            auto t=o.a;
            swap(a,t);
            minus_by(a,t);
        }
        else minus_by(a,o.a);
    }
    return *this;
}
bigint &operator-=(const bigint &o)
{
    neg^=1;
    *this+=o;
    neg^=1;
}

```

```

    if (a==vector<ll>{0}) neg=0;
    return *this;
}
bigint &operator*=(const bigint &o)
{
    neg^=o.neg;
    int n=a.size(),m=o.a.size(),i,j;
    assert(min(n,m)<=p/((base-1)*(base-1)));
    if (min(n,m)<=64)
    {
        vector<ll> c(n+m);
        for (i=0; i<n; i++) for (j=0; j<m; j++) c[i+j]+=a[i]*o[j];
        for (i=0; i<n+m-1; i++)
        {
            c[i+1]+=c[i]/base;
            c[i]%=base;
        }
        swap(a,c);
        while (a.size()>1&&!a.back()) a.pop_back();
        if (a==vector<ll>{0}) neg=0;
        return *this;
    }
    int len=cal(n+m-1);
    vector<lll> f(len),g(len);
    copy_n(a.begin(),n,f.begin());
    copy_n(o.a.begin(),m,g.begin());
    dft(f); dft(g);
    for (i=0; i<len; i++) f[i]=f[i]*g[i]%p;
    dft(f,1);
    a.resize(n+m-1);
    copy_n(f.begin(),n+m-1,a.begin());
    for (i=0; i<n+m-2; i++)
    {
        a[i+1]+=a[i]/base;
        a[i]%=base;
    }
    while (a.size()>1&&!a.back()) a.pop_back();
    if (a==vector<ll>{0}) neg=0;
    return *this;
}
bigint &operator/=(long long x)//to zero
{
    if (x<0) x=-x,neg^=1;
    for (int i=a.size()-1; i; i--)
    {
        a[i-1]+=a[i]%x*base;
        a[i]/=x;
    }
    a[0]/=x;
    while (a.size()>1&&!a.back()) a.pop_back();
    if (a==vector<ll>{0}) neg=0;
    return *this;
}
bigint operator+(bigint o) const { return o+*this; }
bigint operator-(bigint o) const { o=*this; if (o.a!=vector<ll>{0}) o.neg^=1; return o; }
bigint operator*(bigint o) const { return o*~*this; }
bigint operator/(long long x) const { auto res=*this; return res/=x; }

```

```

long long operator%(long long x) const
{
    bool flg=neg;
    if (x<0) flg^=1,x=-x;
    ll res=0;
    for (int i=(base%x==0?0:a.size()-1); i>=0; i--) res=(res*base+a[i])%x;
    return (long long)res*(flg?-1:1);
}

bigint(long long x=0):neg(0)
{
    if (x<0) x=-x,neg=1;
    a.push_back(x%base);
    while (x/=base) a.push_back(x%base);
}

bool operator<(const bigint &o) const
{
    if (neg!=o.neg) return neg;
    if (neg) return less(o.a,a);
    return less(a,o.a);
}

bool operator>(const bigint &o) const { return o<*this; }
bool operator==(const bigint &o) const { return neg==o.neg&& a==o.a; }
bool operator!=(const bigint &o) const { return neg!=o.neg||a!=o.a; }
bool operator<=(const bigint &o) const { return !(*this>o); }
bool operator>=(const bigint &o) const { return !(*this<o); }
};

istream &operator>>(istream &cin,bigint &x)
{
    x.neg=0;
    x.a.clear();
    string s;
    cin>>s;
    const int length=round(log10(bigint::base));
    if (s[0]=='-') x.neg=1,s.erase(s.begin());
    reverse(all(s));
    ll base=1;
    for (int i=0; i<s.size(); i++)
    {
        if (i%length==0) x.a.push_back(0),base=1;
        x.a.back()=x.a.back()+(s[i]-'0')*base;
        base*=10;
    }
    return cin;
}

ostream &operator<<(ostream &cout,const bigint &x)
{
    if (x.neg) cout<<"-";
    cout<<x.a.back();
    int length=round(log10(bigint::base));
    for (int i=x.a.size()-2; i>=0; i--) cout<<setfill('0')<<setw(length)<<x.a[i];
    return cout;
}

bigint abs(bigint x)
{
    x.neg=0;
    return x;
}

```

```

bigint gcd(bigint x, bigint y)
{
    x.neg=y.neg=0;
    if (x==bigint(0)) return y;
    if (y==bigint(0)) return x;
    int c1=0, c2=0;
    while (x%2==0) x/=2, ++c1;
    while (y%2==0) y/=2, ++c2;
    cmin(c1, c2);
    if (x>y) swap(x, y);
    while (x!=y)
    {
        y-=x;
        y/=2;
        while (y%2==0) y/=2;
        if (x>y) swap(x, y);
    }
    while (c1--) y*=bigint(2);
    return y;
}
bigint::l1l bigint::w[bigint::N];
int bigint::r[bigint::N];

```

## 7.7 分散层叠算法 (Fractional Cascading)

$O(n + q(k + \log n))$ ,  $O(n)$ 。

给出  $k$  个长度为  $n$  的有序数组。

现在有  $q$  个查询: 给出数  $x$ , 分别求出每个数组中大于等于  $x$  的最小的数 (非严格后继)。

若后继不存在, 则定义为 0。你需要在线地回答这些询问。

```

int a[M][N], b[M][N<<1], c[M][N<<1][2], len[M], ans[M];
int n, m, qs, p, q, d, i, j, x, y, la;
int main()
{
    read(n); read(m); read(qs); read(d);
    for (j=1; j<=m; j++) for (i=0; i<n; i++) read(a[j][i]);
    for (j=1; j<=m; j++) a[j][n]=inf+j; ++n;
    for (i=0; i<n; i++) b[m][i]=a[m][i], c[m][i][0]=i;
    len[m]=n;
    for (j=m-1; j; j--)
    {
        p=0, q=1;
        while (p<n&&q<len[j+1])
            if (a[j][p]<b[j+1][q]) b[j][len[j]]=a[j][p], c[j][len[j]][0]=p++, c[j][len[j]][1]=q;
            else b[j][len[j]]=b[j+1][q], c[j][len[j]][0]=p, c[j][len[j]][1]=q, q+=2;
        while (p<n) b[j][len[j]]=a[j][p], c[j][len[j]][0]=p++, c[j][len[j]][1]=q;
        while (q<len[j+1]) b[j][len[j]]=b[j+1][q], c[j][len[j]][0]=p, c[j][len[j]][1]=q, q+=2;
    }
    for (int ii=1; ii<=qs; ii++)
    {
        read(x); x^=la;
        y=lower_bound(b[1], b[1]+len[1], x)-b[1];
        ans[1]=a[1][c[1][y][0]]; y=c[1][y][1]; //下标是c[1][y][0]
        for (j=2; j<=m; j++)
        {
            if (y&&b[j][y-1]>=x) --y;

```

```

        ans[j]=a[j][c[j][y][0]]; //下标是c[j][y][0]
        y=c[j][y][1];
    }
    la=0;
    for (i=1;i<=m;i++) la^=ans[i]>inf?0:ans[i];
    if (ii%d==0) printf("%d\n",la);
}
}

```

## 7.8 圆上整点（二平方和定理）

$x^2 + y^2 = n$  的整数解的数目的四分之一  $f(n)$  是积性数论函数，且对于素数幂有： $f(p^k) =$

$$\begin{cases} 1 & p = 2 \\ k + 1 & p \equiv 1 \pmod{4} \\ (k + 1) \bmod 2 & p \equiv 3 \pmod{4} \end{cases}$$

以下代码给出所有的非负整数解。注意非负整数解个数不等于  $f(n)$ 。

时间复杂度为  $O(n^{\frac{1}{4}} + f(n))$ ，其中  $O(n^{\frac{1}{4}})$  是 pollard-rho 的复杂度。

$f(n)$  的量级不好分析，但不会超过约数个数  $O(d(n)) \approx O(n^{\frac{1}{3}})$ ，且可以推测不能达到。

```

namespace pr
{
    typedef long long ll;
    typedef __int128 lll;
    typedef pair<ll, int> pa;
    ll ksm(ll x, ll y, const ll p)
    {
        ll r=1;
        while (y)
        {
            if (y&1) r=(lll)r*x%p;
            x=(lll)x*x%p; y>>=1;
        }
        return r;
    }
}
namespace miller
{
    const int p[7]={2, 3, 5, 7, 11, 61, 24251};
    ll s, t;
    bool test(ll n, int p)
    {
        if (p>=n) return 1;
        ll r=ksm(p, t, n), w;
        for (int j=0; j<s&&r!=1; j++)
        {
            w=(lll)r*r%n;
            if (w==1&&r!=n-1) return 0;
            r=w;
        }
        return r==1;
    }
    bool prime(ll n)
    {
        if (n<2||n==46'856'248'255'98111) return 0;
        for (int i=0; i<7; ++i) if (n%p[i]==0) return n==p[i];
        s=__builtin_ctz(n-1); t=n-1>>s;
    }
}

```

```

        for (int i=0; i<7; ++i) if (!test(n, p[i])) return 0;
        return 1;
    }
}
using miller::prime;
mt19937_64 rnd(chrono::steady_clock::now().time_since_epoch().count());
namespace rho
{
    void nxt(ll &x, ll &y, ll &p) { x=((lll)x*x+y)%p; }
    ll find(ll n, ll C)
    {
        ll l, r, d, p=1;
        l=rnd()%(n-2)+2, r=1;
        nxt(r, C, n);
        int cnt=0;
        while (l^r)
        {
            p=(lll)p*llabs(l-r)%n;
            if (!p) return gcd(n, llabs(l-r));
            ++cnt;
            if (cnt==127)
            {
                cnt=0;
                d=gcd(llabs(l-r), n);
                if (d>1) return d;
            }
            nxt(l, C, n); nxt(r, C, n); nxt(r, C, n);
        }
        return gcd(n, p);
    }
}
vector<pa> w;
vector<ll> d;
void dfs(ll n, int cnt)
{
    if (n==1) return;
    if (prime(n)) return w.emplace_back(n, cnt), void();
    ll p=n, C=rnd()%(n-1)+1;
    while (p==1||p==n) p=find(n, C++);
    int r=1; n/=p;
    while (n%p==0) n/=p, ++r;
    dfs(p, r*cnt); dfs(n, cnt);
}
vector<pa> getw(ll n)
{
    w=vector<pa>(0); dfs(n, 1);
    if (n==1) return w;
    sort(w.begin(), w.end());
    int i, j;
    for (i=1, j=0; i<w.size(); i++) if (w[i].first==w[j].first) w[j].second+=w[i].second;
        else w[++j]=w[i];
    w.resize(j+1);
    return w;
}
void dfss(int x, ll n)
{
    if (x==w.size()) return d.push_back(n), void();
    dfss(x+1, n);
}

```



```

        for (int i=1; i<=w[x].second; i++) dfss(x+1, n*=w[x].first);
    }
    vector<ll> getd(ll n)
    {
        getw(n); d=vector<ll>(0); dfss(0, 1);
        sort(d.begin(), d.end());
        return d;
    }
}
using rho::getw, rho::getd;
using miller::prime;
}
using pr::getw, pr::getd, pr::prime;
lll roundiv(lll x, lll y)
{
    return x>=0?(x+y/2)/y:(x-y/2)/y;
}
struct G
{
    lll x, y;
    G operator~() const { return {x, -y}; }
    lll len2() const { return x*x+y*y; }
    G operator+(const G &o) const { return {x+o.x, y+o.y}; }
    G operator-(const G &o) const { return {x-o.x, y-o.y}; }
    G operator*(const G &o) const { return {x*o.x-y*o.y, x*o.y+y*o.x}; }
    G operator/(const G &o) const
    {
        G t=*this*~o;
        lll l=o.len2();
        return {roundiv(t.x, l), roundiv(t.y, l)};
    }
    G operator%(const G &o) const { return *this-*this/o*o; }
};
G gcd(G a, G b)
{
    if (a.len2()>b.len2()) swap(a, b);
    while (a.len2())
    {
        b=b%a;
        swap(a, b);
    }
    return b;
}
namespace cipolla
{
    typedef unsigned long long ui;
    typedef __uint128_t ll;
    ui p, w;
    struct Q
    {
        ll x, y;
        Q operator*(const Q &o) const { return {(x*o.x+y*o.y%p*w)%p, (x*o.y+y*o.x)%p}; }
    };
    ui ksm(ll x, ui y)
    {
        ll r=1;
        while (y)

```

```

    {
        if (y&1) r=r*x%p;
        x=x*x%p; y>>=1;
    }
    return r;
}
Q ksm(Q x, ui y)
{
    Q r={1, 0};
    while (y)
    {
        if (y&1) r=r*x;
        x=x*x; y>>=1;
    }
    return r;
}
ui mosqrt(ui x, ui P)//0<=x<P
{
    if (x==0||P==2) return x;
    p=P;
    if (ksm(x, p-1>>1)!=1) return -1;
    ui y;
    mt19937_64 rnd(chrono::steady_clock::now().time_since_epoch().count());
    do y=rnd()%p, w=((1l)y*y+p-x)%p; while (ksm(w, p-1>>1)<=1);//not for p=2
    y=ksm({y, 1}, p+1>>1).x;
    if (y*2>p) y=p-y;//两解取小
    return y;
}
}
using cipolla::mosqrt;
vector<pair<ll, ll>> two_sqr_sum(ll n)//只会返回非负解, 按照字典序排序
{
    if (n<0) return { };
    if (n==0) return {{0, 0}};
    ll m=__lg(n&-n), d=1<<m/2, i;
    n>>=m;
    auto w=getw(n);
    vector<G> r((m&1)?vector{G{1, 1}}:vector{G{0, 1}, G{1, 0}});
    for (auto [p, k]:w) if (p%4==1)
    {
        vector<G> pw(k+1);
        pw[0]={1, 0};
        pw[1]=gcd(G(p, 0), G(mosqrt(p-1, p), 1));
        assert(pw[1].len2()==p);
        for (i=2; i<=k; i++) pw[i]=pw[i-1]*pw[1];
        vector<G> rr; rr.reserve(r.size()*(k+1));
        for (i=0; i<=k; i++)
        {
            G x=pw[i]*~pw[k-i];
            for (G y:r) rr.push_back(x*y);
        }
        swap(r, rr);
    }
    else
    {
        if (k%2) return { };
        k/=2;
    }
}

```

```

    while (k--) d*=p;
}
vector<pair<ll, ll>> ans;
ans.reserve(r.size());
for (auto [x, y]:r) ans.push_back({abs((ll)x*d), abs((ll)y*d)});
sort(all(ans));
ans.resize(unique(all(ans))-ans.begin());
return ans;
}

```

## 7.9 模意义真分数还原

没啥用。

$$q \equiv \frac{x}{a} \pmod{p}, \quad |a| \leq A.$$

```

pair<int, int> approx(int p, int q, int A)
{
    int x=q, y=p, a=1, b=0;
    while (x>A)
    {
        swap(x, y); swap(a, b);
        a-=x/y*b; x%=y;
    }
    return make_pair(x, a);
}

```

## 7.10 快速取模

```

__uint128_t brt=((__uint128_t)1<<64)/mod;
for(int i=1; i<=n; i++)
{
    ans*=i;
    ans=ans-mod*(brt*ans>>64);
    while(ans>=mod) ans-=mod; //可以替换为 if, 但据说会变慢。如果循环展开则需要替换
}

struct barret{
    ll p, m; //p 表示上面的模数, m 为取模参数
    int c=0;
    inline void init(ll t){
        c=48+log2(t), p=t;
        m=(ll)((ull1(1)<<c)/t));
    }
    friend inline ll operator % (ll n, const barret &d) { // get n % d
        return n-((ull1(n)*d.m)>>d.c)*d.p;
    }
}modp;

```

## 7.11 IO 优化

### 7.11.1 WDOI

```

class fast_istream{
private:
    const int MAXBF = 1 << 20; FILE *inf, *ouf;
    char *inbuf, *inst, *ined;
    char *obuf, *oust, *oued;
    inline void _flush(){fwrite(obuf, 1, oued - oust, ouf);}
    inline char _getchar(){
        if(inst == ined) inst = inbuf, ined = inbuf + fread(inbuf, 1, MAXBF, inf);
        return inst == ined ? EOF : *inst++;
    }
    inline void _putchar(char c){
        if(oued == oust + MAXBF) _flush(), oued = oubuf;
        *oued++ = c;
    }
public:
    fast_istream(FILE *_inf = stdin, FILE *_ouf = stdout)
    :inbuf(new char[MAXBF]), inf(_inf), inst(inbuf), ined(inbuf),
    oubuf(new char[MAXBF]), ouf(_ouf), oust(obuf), oued(obuf){}
    ~fast_istream(){_flush(); delete inbuf; delete oubuf;}
    template <class Int>
    fast_istream& operator >> (Int &n){
        static char c;
        while((c = _getchar()) < '0' || c > '9'); n = c - '0';
        while((c = _getchar()) >='0' && c <='9') n = n * 10 + c - '0';
        return *this;
    }
    template <class Int>
    fast_istream& operator << (Int n){
        if(n < 0) _putchar('-'), n = -n; static char S[20]; int t = 0;
        do{S[t++] = '0' + n % 10, n /= 10;} while(n);
        for(int i = 0; i < t; ++i) _putchar(S[t - i - 1]);
        return *this;
    }
    fast_istream& operator << (char c){_putchar(c); return *this;}
    fast_istream& operator << (const char *s){
        for(int i = 0; s[i]; ++i) _putchar(s[i]); return *this;
    }
}fio;//unsigned

```

### 7.11.2 自用

```

c[fread(c+1,1,N,stdin)+1]=0;char *cc=c;
void read(int &x)
{
    char *c=cc;
    while ((*c<48)||(*c>57)) ++c;
    x=*(c++)^48;
    while ((*c>=48)&&(*c<=57)) x=x*10+(*(c++)^48);cc=c;
}
void read(int &x)
{
    char *c=cc;fh=1;
    while ((*c<48)||(*c>57)){if (*c=='-') {++c;fh=-1;break;}++c;}
    x=*(c++)^48;
    while ((*c>=48)&&(*c<=57)) x=x*10+(*(c++)^48);
}

```

```

    x*=fh;cc=c;
}
void write(const int x)
{
    while (x)
    {
        st[++tp]=x%10;
        x/=10;
    }
    char *c=nc;
    while (tp) *(++c)=st[tp--]|48;
    *(++c)=10;nc=c;
}
char *nc=sc;
fwrite(sc+1,1,stp,stdout);

```

## 7.12 手动开栈

一种写法是文件开头放，但部分 OJ 会失效。

```
#pragma comment(linker, "/STACK:102400000,102400000")
```

另一种写法是在 main 开头写，但必须以 `exit(0)` 结束程序。

```

{
    static int OP=0;
    if (OP++==0)
    {
        int size=128<<20;//128MB
        char* p=new char[size]+size;
        __asm__ __volatile__ ("movq %0, %%rsp\n" "pushq $exit\n" "jmp _main\n" :: "r"(p));
    }
}

```

## 7.13 德扑

`solve` 返回按照出现次数排序的 `vector<int>` (0 下标处为牌型)，这样就可以字典序比较了。

```

struct Q
{
    int suit, rank;
    bool operator<(const Q &o) const { return pair{rank, suit}<pair{o.rank, o.suit}; }
    bool operator==(const Q &o) const { return pair{rank, suit}==pair{o.rank, o.suit}; }
};
auto solve=[&](vector<Q> a)
{
    vector<int> res;
    vector<int> cnt(15);
    for (auto [s, r]:a) ++cnt[r];
    sort(all(a));
    int i;
    bool is_flush=1, is_str=0;
    for (i=1; i<5; i++) is_flush&=a[i].suit==a[0].suit;
    is_str=*max_element(all(cnt))==1&&a[0].rank+4==a[4].rank;
    vector<int> b(6);
    for (i=1; i<6; i++) b[i]=a[i-1].rank;
}

```

```
sort(1+all(b), [&](int x, int y)
{
    return pair{cnt[x], x}>pair{cnt[y], y};
});
if (b==vector{0, 12, 3, 2, 1, 0}) is_str=1, b[1]=0;
if (is_flush&&is_str) return b[0]=9, b;
if (cnt[b[1]]==4) return b[0]=8, b;
if (cnt[b[1]]==3&&cnt[b[4]]==2) return b[0]=7, b;
if (is_flush) return b[0]=6, b;
if (is_str) return b[0]=5, b;
if (cnt[b[1]]==3) return b[0]=4, b;
if (cnt[b[1]]==2&&cnt[b[3]]==2) return b[0]=3, b;
if (cnt[b[1]]==2) return b[0]=2, b;
return b;
};
auto turn=[&](string s)
{
    Q res=Q{"SHDC"s.find(s[0]), "23456789TJQKA"s.find(s[1])};
    return res;
};
```

7.14 约数个数表

$n$	$n$ 前第一个质数	$n$ 后第一个质数	$\max\{\omega(n)\}$	$\max\{d(n)\}$	$\pi(n)$
$10^1$	$10^1 - 3$	$10^1 + 1$	2	$d(6) = 4$	4
$10^2$	$10^2 - 3$	$10^2 + 1$	3	$d(60) = 12$	25
$10^3$	$10^3 - 3$	$10^3 + 13$	4	$d(840) = 32$	168
$10^4$	$10^4 - 27$	$10^4 + 7$	5	$d(7560) = 64$	1229
$10^5$	$10^5 - 9$	$10^5 + 3$	6	$d(83160) = 128$	9592
$10^6$	$10^6 - 17$	$10^6 + 3$	7	$d(720720) = 240$	$7.9 \times 10^4$
$10^7$	$10^7 - 9$	$10^7 + 19$	8	$d(8648640) = 448$	$6.7 \times 10^5$
$10^8$	$10^8 - 11$	$10^8 + 7$	8	$d(73513440) = 768$	$5.8 \times 10^6$
$10^9$	$10^9 - 63$	$10^9 + 7$	9	$d(735134400) = 1344$	$5.1 \times 10^7$
$10^{10}$	$10^{10} - 33$	$10^{10} + 19$	10	$d(6983776800) = 2304$	$4.6 \times 10^8$
$10^{11}$	$10^{11} - 23$	$10^{11} + 3$	10	$d(97772875200) = 4032$	$4.2 \times 10^8$
$10^{12}$	$10^{12} - 11$	$10^{12} + 39$	11	$d(963761198400) = 6720$	$3.8 \times 10^9$
$10^{13}$	$10^{13} - 29$	$10^{13} + 37$	12	$d(9316358251200) = 10752$	$3.5 \times 10^{10}$
$10^{14}$	$10^{14} - 27$	$10^{14} + 31$	12	$d(97821761637600) = 17280$	$3.3 \times 10^{11}$
$10^{15}$	$10^{15} - 11$	$10^{15} + 37$	13	$d(866421317361600) = 26880$	$3 \times 10^{12}$
$10^{16}$	$10^{16} - 63$	$10^{16} + 61$	13	$d(8086598962041600) = 41472$	$2.8 \times 10^{13}$
$10^{17}$	$10^{17} - 3$	$10^{17} + 3$	14	$d(74801040398884800) = 64512$	
$10^{18}$	$10^{18} - 11$	$10^{18} + 3$	15	$d(897612484786617600) = 103680$	
$10^{19}$	$10^{19} - 39$	$10^{19} + 51$	16	$d(9200527969062830400) = 161280$	

7.15 NTT 质数

$p = r \times 2^k + 1$	$r$	$k$	$g$ (最小原根)
17	1	4	3
97	3	5	5
193	3	6	5
257	1	8	3
7681	15	9	17
12289	3	12	11
40961	5	13	3
65537	1	16	3
786433	3	18	10
5767169	11	19	3
7340033	7	20	3
23068673	11	21	3
104857601	25	22	3
167772161	5	25	3
469762049	7	26	3
998244353	119	23	3
1004535809	479	21	3
2013265921	15	27	31
2281701377	17	27	3
3221225473	3	30	5
75161927681	35	31	3
77309411329	9	33	7
206158430209	3	36	22
2061584302081	15	37	7
2748779069441	5	39	3
6597069766657	3	41	5
39582418599937	9	42	5
79164837199873	9	43	5
263882790666241	15	44	7
1231453023109121	35	45	3
1337006139375617	19	46	3
3799912185593857	27	47	5
4222124650659841	15	48	19
7881299347898369	7	50	6
31525197391593473	7	52	3
180143985094819841	5	55	6
1945555039024054273	27	56	5
4179340454199820289	29	57	3

7.16 公式

向上取整的整除分块  $[i, \lfloor \frac{n-1}{\lceil \frac{n}{i} \rceil - 1} \rfloor]$

$n$  个点  $k$  个连通块的生成树方案  $n^{k-2} \prod_{i=1}^k siz_i$

$(x, y)$  曼哈顿距离  $\rightarrow (x+y, x-y)$  切比雪夫距离  $(x, y)$  切比雪夫距离  $\rightarrow (\frac{x+y}{2}, \frac{x-y}{2})$  曼哈顿距离

错排数  $= [0.5 + \frac{n!}{e}]$

Kummer's Theorem:  $\binom{n+m}{n}$  含  $p$  ( $p \in \text{prime}$ ) 的次数是  $n+m$  在  $p$  进制下的进位数

$$\ln(1-x^V) = -\sum_{i \geq 1} \frac{x^{Vi}}{i}$$

$$x^{\bar{n}} = \sum_i S_1(n, i) x^i$$

$$\begin{cases} x \equiv a_1 \pmod{m_1} \\ x \equiv a_2 \pmod{m_2} \\ \dots \\ x \equiv a_n \pmod{m_n} \end{cases}$$

$m_i$  为不同的质数。设  $M = \prod_{i=1}^n m_i$ ,  $t_i \times \frac{M}{m_i} \equiv 1 \pmod{m_i}$ , 则  $x \equiv \sum_{i=1}^n a_i t_i \frac{M}{m_i}$ 。

$V - E + F = 2$ ,  $S = n + \frac{s}{2} - 1$ 。 ( $n$  为内部,  $s$  为边上)

用途: 对于相邻的不相等的值, 在中间画一条线 (最外也画), 连通块个数  $= 1 + E - V +$  内部框个数

注意全都是不含矩形边界上的。

五边形数 GF:  $\frac{x(2x+1)}{(1-x)^3}$

五边形数:  $\frac{3n^2-n}{2}$ , 广义含非正, 逆为分拆数 GF (注意系数正负和  $n$  取值奇偶性相同)

贝尔数 (划分集合方案数) EGF:  $\exp(e^x - 1)$ ,  $B_n = \sum_{i=0}^n S_2(n, i)$ , 伯努利数 EGF:  $\frac{x}{e^x - 1}$

$$S_1(i, m) \text{ EGF: } \frac{(\sum_{i \geq 0} \frac{x^i}{i})^m}{m!}, S_2(i, m) \text{ EGF: } \frac{(e^x - 1)^m}{m!}$$

多项式牛顿迭代: 如果已知  $G(F(x)) \equiv 0 \pmod{x^{2n}}$ ,  $G(F_*(x)) \equiv 0 \pmod{x^n}$ , 则有  $F(x) \equiv F_*(x) - \frac{G(F_*(x))}{G'(F_*(x))} \pmod{x^{2n}}$ 。求导时孤立的多项式视为常数。

$$\int_0^1 t^a (1-t)^b dt = \frac{a!b!}{(a+b+1)!}, \sum_{i=0}^{n-1} i^k = \frac{n^{k+1}}{k+1}$$

Burnside 引理: 等价类数量为  $\sum_{g \in G} \frac{X^g}{|G|}$ ,  $X^g$  表示  $g$  变换下不动点的数量。

Polya 定理: 染色方案数为  $\sum_{g \in G} \frac{m^{c(g)}}{|G|}$ , 其中  $c(g)$  表示  $g$  变换下环的数量。

矩阵树定理: 有向图内向生成树个数计算用出度矩阵-邻接矩阵

假设已经只保留了一个牛人酋长, 其名字为  $A = a_1 a_2 \cdots a_l$ 。

假设王国旁边开了一座赌场, 每单位时间 (就称为“秒”吧) 会有一个赌徒带着 1 铜币进入赌场。

赌场规则很简单: 支付  $x$  铜币赌下一秒会唱出  $y$ , 如果猜对了就返还  $nx$  铜币, 否则钱就没了。

每个赌徒会如下行动: 支付 1 铜币赌下一秒会唱出  $a_1$ , 如果赌对了就支付得到的  $n$  铜币赌下一秒会唱出  $a_2$ , 如果还对了就支付得到的  $n^2$  铜币赌下一秒会唱出  $a_3$ , 等等, 以此类推, 最后支付  $n^{l-1}$  铜币赌下一秒会唱出  $a_l$ 。

一旦连续唱出了  $a_1 a_2 \cdots a_l$ , 赌场老板就会认为自己亏大了而关门, 并驱散所有赌徒。

那么关门前发生了什么呢? 以  $A = \{1, 4, 1, 5, 1, 1, 4, 1\}, n = 5$  为例:

- 最后一位赌徒拿着 5 铜币离开; - 倒数第三位赌徒拿着  $5^3$  铜币离开; - 倒数第八位赌徒拿着  $5^8$  铜币离开; - 其他所有赌徒空手而归。



我们可以发现 1,3 恰好是原序列的所有 border 的长度，而且对于其他的名字也有这样的规律。  
这时候最神奇的一步来了：由于这个赌博游戏是公平的，因此赌场应该期望下不赚不赔，因此关门时期望来了  $5 + 5^3 + 5^8$  个赌徒，因此期望需要  $5 + 5^3 + 5^8$  单位时间唱出这个名字。  
同理，即可知道对于一般的  $A$ ，答案为：

$$\sum_{a_1 a_2 \cdots a_c = a_{l-c+1} a_{l-c+2} \cdots a_l} n^c$$

## 8 语言基础

### 8.1 Makefile

```
%.cpp %.in
g++ $< -o $@ -std=c++17 -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC
./$@ < $@.in
```

### 8.2 初始代码

```
#include "bits/stdc++.h"
using namespace std;
typedef long long ll;
#define all(x) (x).begin(), (x).end()
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int T; cin>>T;
    while (T--)
    {

    }
}
```

### 8.3 bitset

```
#include <bits/stdc++.h>
using namespace std;
bitset<10> f(12);
char s2[]="100101";
bitset<10> g(s2);
string s="100101";//reverse 了
bitset<10> h(s);
int main()
{
    for (int i=0;i<=9;i++) if (f[i]) printf("1"); else printf("0");puts("");
    for (int i=0;i<=9;i++) if (g[i]) printf("1"); else printf("0");puts("");
    for (int i=0;i<=9;i++) if (h[i]) printf("1"); else printf("0");puts("");
    cout<<h<<endl;
    foo.count();//1的个数
    foo.flip();//全部翻转
    foo.set();//变1
    foo.reset();//变0
    foo.to_string();
    foo.to_ulong();
    foo.to_ullong();
    foo._Find_first();
    foo._Find_next();
    //位运算: << 变大, >> 变小
}
```

输出:

```

0011000000
1010010000
1010010000
0000100101

```

## 8.4 pb\_ds 和一些奇怪的用法

```

#pragma GCC optimize("Ofast")
#pragma GCC target("popcnt","sse3","sse2","sse","avx","sse4","sse4.1","sse4.2","ssse3","f16c","
    fma","avx2","xop","fma4")
#pragma GCC optimize("inline","fast-math","unroll-loops","no-stack-protector")
#include "bits/stdc++.h"
#include "ext/pb_ds/assoc_container.hpp"
#include "ext/pb_ds/tree_policy.hpp" //balanced tree
#include "ext/pb_ds/hash_policy.hpp" //hash table
#include "ext/pb_ds/priority_queue.hpp" //priority_queue
using namespace __gnu_pbds;
using namespace std;
typedef tree<int,null_type,less<int>,rb_tree_tag,tree_order_statistics_node_update> rbtree;
cc_hash_table<string,int>mp1;//拉链法
gp_hash_table<string,int>mp2;//查探法
rbtree s1,s2;//注意是不可重的
//null_type无映射(低版本g++为null_mapped_type)
//less<int>从小到大排序
//插入t.insert();
//删除t.erase();
//求有多少个数比 k 小:t.order_of_key(k);
//求树中第 k+1 小:t.find_by_order(k);
//a.join(b) b并入a, 前提是两棵树的 key 的取值范围不相交, b 会清空但迭代器没事, 如不满足会抛出异常。我
    听说复杂度是线性???
//a.split(v,b) key 小于等于 v 的元素属于 a, 其余的属于 b
//T.lower_bound(x) >=x 的 min 的迭代器
//T.upper_bound(x) >x 的 min 的迭代器
__gnu_pbds::priority_queue<int,greater<int>,pairing_heap_tag> pq;
//join(priority_queue &other) //合并两个堆,other会被清空
//split(Pred prd,priority_queue &other) //分离出两个堆
//modify(point_iterator it,const key) //修改一个节点的值
inline char gc()
{
    static char buf[1048576], *p1, *p2;
    return p1 == p2 && (p2 = (p1 = buf) + fread(buf, 1, 1048576, stdin),
        p1 == p2) ? EOF : *p1++;
}
inline int read()
{
    char ch = gc(); int r = 0, w = 1;
    for (; ch < '0' || ch > '9'; ch = gc()) if (ch == '-') w = -1;
    for (; '0' <= ch && ch <= '9'; ch = gc()) r = r * 10 + (ch - '0');
    return r * w;
}
struct my_bit
{
    // ll v[Len];
    __m256i V[Len/4];
    void reset()
    {

```

```

V[0]=_mm256_set_epi64x(0, 0, 0, 0);
V[1]=_mm256_set_epi64x(0, 0, 0, 0);
V[2]=_mm256_set_epi64x(0, 0, 0, 0);
V[3]=_mm256_set_epi64x(0, 0, 0, 0);
V[4]=_mm256_set_epi64x(0, 0, 0, 0);
V[5]=_mm256_set_epi64x(0, 0, 0, 0);
V[6]=_mm256_set_epi64x(0, 0, 0, 0);
V[7]=_mm256_set_epi64x(0, 0, 0, 0);
V[8]=_mm256_set_epi64x(0, 0, 0, 0);
V[9]=_mm256_set_epi64x(0, 0, 0, 0);
V[10]=_mm256_set_epi64x(0, 0, 0, 0);
V[11]=_mm256_set_epi64x(0, 0, 0, 0);
V[12]=_mm256_set_epi64x(0, 0, 0, 0);
V[13]=_mm256_set_epi64x(0, 0, 0, 0);
}
void set(int u)
{
    switch (u>>6&3)
    {
    case 0:
        V[u>>8]=_mm256_set_epi64x(1ull<<(u&63), 0, 0, 0);
        break;
    case 1:
        V[u>>8]=_mm256_set_epi64x(0, 1ull<<(u&63), 0, 0);
        break;
    case 2:
        V[u>>8]=_mm256_set_epi64x(0, 0, 1ull<<(u&63), 0);
        break;
    case 3:
        V[u>>8]=_mm256_set_epi64x(0, 0, 0, 1ull<<(u&63));
        break;
    }
    // v[u>>6]!=(1ull<<(u&63));
}
void operator |= (const my_bit &B)
{
    V[0]=B.V[0];
    V[1]=B.V[1];
    V[2]=B.V[2];
    V[3]=B.V[3];
    V[4]=B.V[4];
    V[5]=B.V[5];
    V[6]=B.V[6];
    V[7]=B.V[7];
    V[8]=B.V[8];
    V[9]=B.V[9];
    V[10]=B.V[10];
    V[11]=B.V[11];
    V[12]=B.V[12];
    V[13]=B.V[13];
    // V[6]=B.V[6];
    // V[7]=B.V[7];
    // V[8]=B.V[8];
    // V[9]=B.V[9];
    // V[10]=B.V[10];
    // V[11]=B.V[11];

```

```

// V[12] |= B.V[12];
// V[13] |= B.V[13];
// V[14] |= B.V[14];
// V[15] |= B.V[15];
// V[16] |= B.V[16];
// V[17] |= B.V[17];
// V[18] |= B.V[18];
// V[19] |= B.V[19];
// V[20] |= B.V[20];
// V[21] |= B.V[21];
// V[22] |= B.V[22];
// V[23] |= B.V[23];
}
int count()
{
    return
        __builtin_popcountll(((1l *)&(V[0]))[0]) + __builtin_popcountll(((1l *)&(V[0]))[1])
        + __builtin_popcountll(((1l *)&(V[0]))[2]) + __builtin_popcountll(((1l *)&(V[0]))[3])
        + __builtin_popcountll(((1l *)&(V[1]))[0]) + __builtin_popcountll(((1l *)&(V[1]))[1])
        + __builtin_popcountll(((1l *)&(V[1]))[2]) + __builtin_popcountll(((1l *)&(V[1]))[3])
        + __builtin_popcountll(((1l *)&(V[2]))[0]) + __builtin_popcountll(((1l *)&(V[2]))[1])
        + __builtin_popcountll(((1l *)&(V[2]))[2]) + __builtin_popcountll(((1l *)&(V[2]))[3])
        + __builtin_popcountll(((1l *)&(V[3]))[0]) + __builtin_popcountll(((1l *)&(V[3]))[1])
        + __builtin_popcountll(((1l *)&(V[3]))[2]) + __builtin_popcountll(((1l *)&(V[3]))[3])
        + __builtin_popcountll(((1l *)&(V[4]))[0]) + __builtin_popcountll(((1l *)&(V[4]))[1])
        + __builtin_popcountll(((1l *)&(V[4]))[2]) + __builtin_popcountll(((1l *)&(V[4]))[3])
        + __builtin_popcountll(((1l *)&(V[5]))[0]) + __builtin_popcountll(((1l *)&(V[5]))[1])
        + __builtin_popcountll(((1l *)&(V[5]))[2]) + __builtin_popcountll(((1l *)&(V[5]))[3])
        + __builtin_popcountll(((1l *)&(V[6]))[0]) + __builtin_popcountll(((1l *)&(V[6]))[1])
        + __builtin_popcountll(((1l *)&(V[6]))[2]) + __builtin_popcountll(((1l *)&(V[6]))[3])
        + __builtin_popcountll(((1l *)&(V[7]))[0]) + __builtin_popcountll(((1l *)&(V[7]))[1])
        + __builtin_popcountll(((1l *)&(V[7]))[2]) + __builtin_popcountll(((1l *)&(V[7]))[3])
        + __builtin_popcountll(((1l *)&(V[8]))[0]) + __builtin_popcountll(((1l *)&(V[8]))[1])
        + __builtin_popcountll(((1l *)&(V[8]))[2]) + __builtin_popcountll(((1l *)&(V[8]))[3])
        + __builtin_popcountll(((1l *)&(V[9]))[0]) + __builtin_popcountll(((1l *)&(V[9]))[1])
        + __builtin_popcountll(((1l *)&(V[9]))[2]) + __builtin_popcountll(((1l *)&(V[9]))[3])
        + __builtin_popcountll(((1l *)&(V[10]))[0]) + __builtin_popcountll(((1l *)&(V[10]))[1])
        + __builtin_popcountll(((1l *)&(V[10]))[2]) + __builtin_popcountll(((1l *)&(V[10]))[3])
        + __builtin_popcountll(((1l *)&(V[11]))[0]) + __builtin_popcountll(((1l *)&(V[11]))[1])
        + __builtin_popcountll(((1l *)&(V[11]))[2]) + __builtin_popcountll(((1l *)&(V[11]))[3])
        + __builtin_popcountll(((1l *)&(V[12]))[0]) + __builtin_popcountll(((1l *)&(V[12]))[1])
        + __builtin_popcountll(((1l *)&(V[12]))[2]) + __builtin_popcountll(((1l *)&(V[12]))[3])
        + __builtin_popcountll(((1l *)&(V[13]))[0]) + __builtin_popcountll(((1l *)&(V[13]))[1])
        + __builtin_popcountll(((1l *)&(V[13]))[2]) + __builtin_popcountll(((1l *)&(V[13]))[3]);
    // int ans=0;
    // return __builtin_popcountll(v[0])
    // + __builtin_popcountll(v[1])
    // + __builtin_popcountll(v[2])
    // + __builtin_popcountll(v[3])
    // + __builtin_popcountll(v[4])
    // + __builtin_popcountll(v[5])
    // + __builtin_popcountll(v[6])
    // + __builtin_popcountll(v[7])
    // + __builtin_popcountll(v[8])
    // + __builtin_popcountll(v[9])
    // + __builtin_popcountll(v[10])
    // + __builtin_popcountll(v[11])

```

```

        // __builtin_popcountll(v[12])
        // __builtin_popcountll(v[13])
        // __builtin_popcountll(v[14])
        // __builtin_popcountll(v[15])
        // __builtin_popcountll(v[16])
        // __builtin_popcountll(v[17])
        // __builtin_popcountll(v[18])
        // __builtin_popcountll(v[19])
        // __builtin_popcountll(v[20])
        // __builtin_popcountll(v[21])
        // __builtin_popcountll(v[22])
        // __builtin_popcountll(v[23]);
        // return ans;
    }
}r[N];
int main()
{
    __builtin_clz();//前导 0
    __builtin_ctz();//后面的 0
    ios::sync_with_stdio(0);cin.tie(0);
    mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    cout<<fixed<<setprecision(15);
    rbtree::iterator it;
    string s="abc",t="dabce";
    boyer_moore_horspool_searcher S(all(s));
    if (search(all(t),S)!=t.end())
    {
        cout<<"find\n";
    }
    uniform_real_distribution<> a(1,2);
    numeric_limits<int>::max();
}

```

## 8.5 python 使用方法

注意事项：python 容易爆栈，且引用与赋值较为混乱。注意局部变量的 global 怎么写（如果需要修改全局内容）。

文件操作

```

fi = open("discuss.in", "r")
fo = open("discuss.out", "w")
n=int(fi.readline())
fo.write(str(ans))

```

类的构造，重载运算符

```

class Q:
    def __init__(self,x,y):
        self.x=x
        self.y=y
    def __add__(self,o):
        r=Q(self.x+o.x,self.y+o.y)
        return r
    def __sub__(self,o):
        r=Q(self.x-o.x,self.y-o.y)
        return r
    def __mul__(self,o):

```

```
        return self.x*o.y-self.y*o.x
    def __lt__(self,o):
        if self.x!=o.x:
            return self.x<o.x
        return self.y<o.y
n,m=map(int,input().split())
c=list(map(int,input().split()))
print(*c)
a=Q(0,0)
b=Q(1,1)
if a<b-a:
    pass
```

## 9 其他人的板子（补充）

### 9.1 MTT+exp

```
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef double db;
int read(){
    int res=0;
    char c=getchar(),f=1;
    while(c<48||c>57){if(c=='-')f=0;c=getchar();}
    while(c>=48&& c<=57)res=(res<<3)+(res<<1)+(c&15),c=getchar();
    return f?res:-res;
}

const int L=1<<19,mod=1e9+7;
const db pi2=3.141592653589793*2;
int inc(int x,int y){return x+y>=mod?x+y-mod:x+y;}
int dec(int x,int y){return x-y<0?x-y+mod:x-y;}
int mul(int x,int y){return (ll)x*y%mod;}
int qpow(int x,int y){
    int res=1;
    for(;y;y>>=1)res=y&1?mul(res,x):res,x=mul(x,x);
    return res;
}
int inv(int x){return qpow(x,mod-2);}

struct cp{
    db x,y;
    cp(){}
    cp(db a,db b){x=a,y=b;}
    cp operator+(const cp& p)const{return cp(x+p.x,y+p.y);}
    cp operator-(const cp& p)const{return cp(x-p.x,y-p.y);}
    cp operator*(const cp& p)const{return cp(x*p.x-y*p.y,x*p.y+y*p.x);}
    cp conj(){return cp(x,-y);}
}w[L];
int re[L];
int getre(int n){
    int len=1,bit=0;
    while(len<n)++bit,len<=1;
    for(int i=1;i<len;++i)re[i]=(re[i>>1]>>1)|((i&1)<<(bit-1));
    return len;
}
void getw(){
    for(int i=0;i<L;++i)w[i]=cp(cos(pi2/L*i),sin(pi2/L*i));
}
void fft(cp* a,int len,int m){
    for(int i=1;i<len;++i)if(i<re[i])swap(a[i],a[re[i]]);
    for(int k=1,r=L>>1;k<len;k<=1,r>>=1)
        for(int i=0;i<len;i+=k<<1)
            for(int j=0;j<k;++j){
                cp &L=a[i+j],&R=a[i+j+k],t=w[r*j]*R;
                R=L-t,L=L+t;
            }
    if(!~m){
        reverse(a+1,a+len);
    }
}
```



```

        cp tmp=cp(1.0/len,0);
        for(int i=0;i<len;++i)a[i]=a[i]*tmp;
    }
}
void mul(int* a,int* b,int* c,int n1,int n2,int n){
    static cp f1[L],f2[L],f3[L],f4[L];
    int len=getre(n1+n2-1);
    for(int i=0;i<len;++i){
        f1[i]=i<n1?cp(a[i]>>15,a[i]&32767):cp(0,0);
        f2[i]=i<n2?cp(b[i]>>15,b[i]&32767):cp(0,0);
    }
    fft(f1,len,1),fft(f2,len,1);
    cp t1=cp(0.5,0),t2=cp(0,-0.5),r=cp(0,1);
    cp x1,x2,x3,x4;
    for(int i=0;i<len;++i){
        int j=(len-i)&(len-1);
        x1=(f1[i]+f1[j].conj())*t1;
        x2=(f1[i]-f1[j].conj())*t2;
        x3=(f2[i]+f2[j].conj())*t1;
        x4=(f2[i]-f2[j].conj())*t2;
        f3[i]=x1*(x3+x4*r);
        f4[i]=x2*(x3+x4*r);
    }
    fft(f3,len,-1),fft(f4,len,-1);
    ll c1,c2,c3,c4;
    for(int i=0;i<n;++i){
        c1=(ll)(f3[i].x+0.5)%mod,c2=(ll)(f3[i].y+0.5)%mod;
        c3=(ll)(f4[i].x+0.5)%mod,c4=(ll)(f4[i].y+0.5)%mod;
        c[i]=((((c1<15)+c2+c3)<<15)+c4)%mod;
    }
}
void inv(int* a,int* b,int n){
    if(n==1){b[0]=1;return;}
    static int c[L];
    int l=(n+1)>>1;
    inv(a,b,l);
    mul(a,b,c,n,l,n);
    for(int i=0;i<n;++i)c[i]=mod-c[i];
    c[0]+=2;
    mul(b,c,b,n,n,n);
}
void der(int* a,int n){
    for(int i=1;i<n;++i)a[i-1]=mul(a[i],i);
    a[n-1]=0;
}
void its(int* a,int n){
    for(int i=n-1;i--i)a[i]=mul(a[i-1],inv(i));
    a[0]=0;
}
void ln(int* a,int* b,int n){
    static int c[L];
    for(int i=0;i<n;++i)c[i]=a[i];
    der(c,n);
    inv(a,b,n);
    mul(b,c,b,n,n,n);
    its(b,n);
}

```

```

void exp(int* a,int* b,int n){
    if(n==1){b[0]=1;return;}
    static int c[L];
    int l=(n+1)>>1;
    exp(a,b,l);
    ln(b,c,n);
    for(int i=0;i<n;++i)c[i]=dec(a[i],c[i]);
    ++c[0];
    mul(b,c,b,l,n,n);
    for(int i=0;i<n;++i)c[i]=0;
}

int n,k,a[L],f[L],g[L];
int main(){
    getw();
    n=read(),k=read();
    for(int i=1;i<=k;++i)a[i]=inv(i);
    for(int i=2;i<=n;++i)
        for(int j=1;i*j<=k;++j)
            f[i*j]=inc(f[i*j],a[j]);
    for(int i=1;i<=k;++i)f[i]=mod-f[i];
    for(int i=1;i<=k;++i)f[i]=inc(f[i],mul(n-1,a[i]));
    exp(f,g,k+1);
    printf("%d\n",g[k]);
}

```

## 9.2 半平面交

```

const int N=305;
const db inf=1e15,eps=1e-10;
int sign(db x){
    if(fabs(x)<eps)return 0;
    return x>0?1:-1;
}

struct vec{
    db x,y;
    vec(){
    }
    vec(db a,db b){x=a,y=b;}
    vec operator+(const vec& p)const{
        return vec(x+p.x,y+p.y);
    }
    vec operator-(const vec& p)const{
        return vec(x-p.x,y-p.y);
    }
    db operator*(const vec& p)const{
        return x*p.y-y*p.x;
    }
    vec operator*(const db& p)const{
        return vec(x*p,y*p);
    }
}p1[N],p2[N];

struct line{
    vec s,t;
    line(){
    }
}

```

```

    line(vec a,vec b){s=a,t=b;}
}a[N],q[N];
db ang(vec v){
    return atan2(v.y,v.x);
}
db ang(line l){
    return ang(l.t-l.s);
}
bool cmp(line x,line y){
    int s=sign(ang(x)-ang(y));
    return s?s<0:sign((x.t-x.s)*(y.t-x.s))>0;
}

vec inter(line x,line y){
    vec a=y.s-x.s,b=x.t-x.s,c=y.t-y.s;
    return y.s+c*((a*b)/(b*c));
}
bool out(line l,vec p){
    return sign((l.t-l.s)*(p-l.s))<0;
}

int n,tot=0;
db ans=inf;
int main(){
    scanf("%d",&n);
    for(int i=1;i<=n;++i)scanf("%lf",&p1[i].x);
    for(int i=1;i<=n;++i)scanf("%lf",&p1[i].y);
    for(int i=1;i<n;++i)a[i]=line(p1[i],p1[i+1]);
    a[n]=line(vec(p1[1].x,inf),vec(p1[1].x,p1[1].y));
    a[n+1]=line(vec(p1[n].x,p1[n].y),vec(p1[n].x,inf));

    sort(a+1,a+n+2,cmp);
    for(int i=1;i<=n;++i){
        if(!sign(ang(a[i])-ang(a[i+1])))continue;
        a[++tot]=a[i];
    }a[++tot]=a[n+1];

    int l=1,r=0;
    q[++r]=a[1],q[++r]=a[2];
    for(int i=3;i<=tot;++i){
        while(l<r&&out(a[i],inter(q[r],q[r-1])))--r;
        while(l<r&&out(a[i],inter(q[l],q[l+1])))++l;
        q[++r]=a[i];
    }
    while(l<r&&out(q[l],inter(q[r],q[r-1])))--r;
    while(l<r&&out(q[r],inter(q[l],q[l+1])))++l;
    //.....
}

```

### 9.3 旋转卡壳

```

if(top==3)return !printf("%d\n",dis(a[sta[1]],a[sta[2]]));
for(int i=1,j=2;i<top;++i){
    while(area(a[sta[i]],a[sta[i+1]],a[sta[j]])>=area(a[sta[i]],a[sta[i+1]],a[sta[j%top+1]]))j=j%
        top+1;
    ans=max(ans,max(dis(a[sta[i]],a[sta[j]]),dis(a[sta[i+1]],a[sta[j]])));
}

```

```
}printf("%d\n",ans);
```

## 9.4 多项式复合 (yurzhang)

$O(n \log n \sqrt{n \log n})$ , 奇慢无比, 慎用

```
#pragma GCC optimize("Ofast,inline")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,sse4.1,sse4.2,popcnt,abm,mmx,avx,avx2,tune=native")
#include <cstdio>
#include <cstring>
#include <cmath>
#include <algorithm>

#define MOD 998244353
#define G 332748118
#define N 262210
#define re register
#define gc pa==pb&&(pb=(pa=buf)+fread(buf,1,100000,stdin),pa==pb)?EOF:*pa++
typedef long long ll;
static char buf[100000],*pa(buf),*pb(buf);
static char pbuf[3000000],*pp(pbuf),st[15];
int read() {
    re int x(0);re char c(gc);
    while(c<'0' || c>'9')c=gc;
    while(c>='0'&&c<='9')
        x=x*10+c-48,c=gc;
    return x;
}
void write(re int v) {
    if(v==0)
        *pp++=48;
    else {
        re int tp(0);
        while(v)
            st[++tp]=v%10+48,v/=10;
        while(tp)
            *pp++=st[tp--];
    }
    *pp++=32;
}

int pow(re int a,re int b) {
    re int ans(1);
    while(b)
        ans=b&1?(ll)ans*a%MOD:ans,a=(ll)a*a%MOD,b>>=1;
    return ans;
}

int inv[N],ifac[N];
void pre(re int n) {
    inv[1]=ifac[0]=1;
    for(re int i(2);i<=n;++i)
        inv[i]=(ll)(MOD-MOD/i)*inv[MOD%i]%MOD;
    for(re int i(1);i<=n;++i)
        ifac[i]=(ll)ifac[i-1]*inv[i]%MOD;
}
```

```

int getLen(re int t) {
    return 1<<(32-__builtin_clz(t));
}

int lmt(1),r[N],w[N];
void init(re int n) {
    re int l(0);
    while(lmt<=n)
        lmt<<=1,++l;
    for(re int i(1);i<lmt;++i)
        r[i]=(r[i>>1]>>1)|((i&1)<<(l-1));
    re int wn(pow(3,(MOD-1)/lmt));
    w[lmt>>1]=1;
    for(re int i((lmt>>1)+1);i<lmt;++i)
        w[i]=(ll)w[i-1]*wn%MOD;
    for(re int i((lmt>>1)-1);i--i)
        w[i]=w[i<<1];
}

void DFT(int*a,re int l) {
    static unsigned long long tmp[N];
    re int u(__builtin_ctz(lmt)-__builtin_ctz(l)),t;
    for(re int i(0);i<l;++i)
        tmp[i]=(a[r[i]>>u])%MOD;
    for(re int i(1);i<l;i<=1)
        for(re int j(0),step(i<<1);j<l;j+=step)
            for(re int k(0);k<i;++k)
                t=(ll)w[i+k]*tmp[i+j+k]%MOD,
                tmp[i+j+k]=tmp[j+k]+MOD-t,
                tmp[j+k]+=t;
    for(re int i(0);i<l;++i)
        a[i]=tmp[i]%MOD;
}

void IDFT(int*a,re int l) {
    std::reverse(a+1,a+l);DFT(a,l);
    re int bk(MOD-(MOD-1)/l);
    for(re int i(0);i<l;++i)
        a[i]=(ll)a[i]*bk%MOD;
}

int n,m;
int a[N],b[N],c[N];

void getInv(int*a,int*b,int deg) {
    if(deg==1)
        b[0]=pow(a[0],MOD-2);
    else {
        static int tmp[N];
        getInv(a,b,(deg+1)>>1);
        re int l(getLen(deg<<1));
        for(re int i(0);i<l;++i)
            tmp[i]=i<deg?a[i]:0;
        DFT(tmp,l),DFT(b,l);
        for(re int i(0);i<l;++i)
            b[i]=(211-(ll)tmp[i]*b[i]%MOD+MOD)%MOD*b[i]%MOD;
        IDFT(b,l);
    }
}

```

```

        for(re int i(deg);i<l;++i)
            b[i]=0;
    }
}

void getDer(int*a,int*b,int deg) {
    for(re int i(0);i+1<deg;++i)
        b[i]=(1l)a[i+1]*(i+1)%MOD;
    b[deg-1]=0;
}

void getComp(int*a,int*b,int k,int m,int&n,int*c,int*d) {
    if(k==1) {
        for(re int i(0);i<m;++i)
            c[i]=0,d[i]=b[i];
        n=m,c[0]=a[0];
    } else {
        static int t1[N],t2[N];
        int nl(n),nr(n),*cl,*cr,*dl,*dr;
        getComp(a,b,k>>1,m,nl,cl=c,dl=d);
        getComp(a+(k>>1),b,(k+1)>>1,m,nr,cr=c+nl,dr=d+nl);
        n=std::min(n,nl+nr-1);
        re int _l(getLen(nl+nr));
        for(re int i(0);i<_l;++i)
            t1[i]=i<nl?dl[i]:0;
        for(re int i(0);i<_l;++i)
            t2[i]=i<nr?cr[i]:0;
        DFT(t1,_l),DFT(t2,_l);
        for(re int i(0);i<_l;++i)
            t2[i]=(1l)t1[i]*t2[i]%MOD;
        IDFT(t2,_l);
        for(re int i(0);i<n;++i)
            c[i]=((i<nl?cl[i]:0)+t2[i])%MOD;
        for(re int i(0);i<_l;++i)
            t2[i]=i<nr?dr[i]:0;
        DFT(t2,_l);
        for(re int i(0);i<_l;++i)
            t2[i]=(1l)t1[i]*t2[i]%MOD;
        IDFT(t2,_l);
        for(re int i(0);i<n;++i)
            d[i]=t2[i];
    }
}

void getComp(int*a,int*b,int*c,int deg) {
    static int ts[N],ps[N],c0[N],_t1[N],idM[N];
    int M(std::max((int)ceil(sqrt(deg/log2(deg))*2.5),2)),_n(deg+deg/M);
    getComp(a,b,deg,M,_n,c0,_t1);
    re int _l(getLen(_n+deg));
    for(re int i(_n);i<_l;++i)
        c0[i]=0;
    for(re int i(0);i<_l;++i)
        ps[i]=i==0;
    for(re int i(0);i<_l;++i)
        ts[i]=M<=i&& i<deg?b[i]:0;
    getDer(b,_t1,M);
    for(re int i(M-1);i<deg;++i)

```

```

    _t1[i]=0; /// Important!!!
    getInv(_t1,idM,deg);
    for(int i=deg;i<_l;++i)
        idM[i]=0;
    DFT(ts,_l),DFT(idM,_l);
    for(re int t(0);t*M<deg;++t) {
        for(re int i(0);i<_l;++i)
            _t1[i]=i<deg?c0[i]:0;
        DFT(ps,_l),DFT(_t1,_l);
        for(re int i(0);i<_l;++i)
            _t1[i]=(ll)_t1[i]*ps[i]%MOD,
            ps[i]=(ll)ps[i]*ts[i]%MOD;
        IDFT(ps,_l),IDFT(_t1,_l);
        for(re int i(deg);i<_l;++i)
            ps[i]=0;
        for(re int i(0);i<deg;++i)
            c[i]=((ll)_t1[i]*ifac[t]+c[i])%MOD;
        getDer(c0,c0,_n);
        for(re int i(_n-1);i<_l;++i)
            c0[i]=0;
        DFT(c0,_l);
        for(re int i(0);i<_l;++i)
            c0[i]=(ll)c0[i]*idM[i]%MOD;
        IDFT(c0,_l);
        for(re int i(_n-1);i<_l;++i)
            c0[i]=0;
    }
}

int main() {
    n=read(),m=read();
    for(re int i(0);i<=n;++i)
        a[i]=read();
    for(re int i(0);i<=m;++i)
        b[i]=read();

    m=(n>m?n:m)+1;
    pre(m);init(m*5);
    getComp(a,b,c,m);

    for(re int i(0);i<=n;++i)
        write(c[i]);
    fwrite(pbuf,1,pp-pbuf,stdout);
    return 0;
}

```

## 9.5 下降幂多项式乘法

$O(n \log n)$ 。

```

#include<cstdio>
#include<algorithm>
const int N=524288,md=998244353,g3=(md+1)/3;
typedef long long LL;
int n,m,A[N],B[N],fac[N],iv[N],rev[N],C[N],g[20][N],lim,M;
int pow(int a,int b){
    int ret=1;

```

```

    for(;b;b>>=1,a=(LL)a*a%md)if(b&1)ret=(LL)ret*a%md;
    return ret;
}
void upd(int&a){a+=a>>31&md;}
void init(int n){
    int l=-1;
    for(lim=1;lim<n;lim<=<=1)++l;M=l+1;
    for(int i=1;i<lim;++i)
        rev[i]=((rev[i>>1])>>1)|((i&1)<<1);
}
void NTT(int*a,int f){
    for(int i=1;i<lim;++i)if(i<rev[i])std::swap(a[i],a[rev[i]]);
    for(int i=0;i<M;++i){
        const int*G=g[i],c=1<<i;
        for(int j=0;j<lim;j+=c<<1)
            for(int k=0;k<c;++k){
                const int x=a[j+k],y=a[j+k+c]*(LL)G[k]%md;
                upd(a[j+k]+=y-md),upd(a[j+k+c]=x-y);
            }
    }
    if(!f){
        const int iv=pow(lim,md-2);
        for(int i=0;i<lim;++i)a[i]=(LL)a[i]*iv%md;
        std::reverse(a+1,a+lim);
    }
}
int main(){
    scanf("%d%d",&n,&m);++n,++m;
    for(int i=0;i<20;++i){
        int*G=g[i];
        G[0]=1;
        const int gi=G[1]=pow(3,(md-1)/(1<<i+1));
        for(int j=2;j<1<<i;++j)G[j]=(LL)G[j-1]*gi%md;
    }
    for(int i=0;i<n;++i)scanf("%d",A+i);
    for(int i=0;i<m;++i)scanf("%d",B+i);
    for(int i=*fac=1;i<N;++i)
        fac[i]=fac[i-1]*(LL)i%md;
    iv[N-1]=pow(fac[N-1],md-2);
    for(int i=N-2;~i;--i)iv[i]=(i+1LL)*iv[i+1]%md;
    init(n+m<<1);
    for(int i=0;i<n+m-1;++i)C[i]=iv[i];
    NTT(A,1),NTT(B,1),NTT(C,1);
    for(int i=0;i<lim;++i)A[i]=(LL)A[i]*C[i]%md,B[i]=(LL)B[i]*C[i]%md;
    NTT(A,0),NTT(B,0);
    for(int i=0;i<lim;++i)C[i]=0;
    for(int i=0;i<n+m-1;++i)
        C[i]=(i&1)?md-iv[i]:iv[i];
    for(int i=0;i<lim;++i)A[i]=(LL)A[i]*B[i]%md*fac[i]%md;
    for(int i=n+m-1;i<lim;++i)A[i]=0;
    NTT(A,1),NTT(C,1);
    for(int i=0;i<lim;++i)A[i]=(LL)A[i]*C[i]%md;
    NTT(A,0);
    for(int i=0;i<n+m-1;++i)printf("%d%c",A[i],"\n"[i==n+m-2]);
    return 0;
}

```



## 9.6 平面欧几里得距离最小生成树

$10^5$ , 400ms。

By Claris.

```
#include<cstdio>
#include<algorithm>
#include<cmath>
using namespace std;
typedef long long ll;
const int N=100010;
const ll inf=2000000000000000000LL;
const double eps=1e-9;
inline int sgn(double x){
    if(x>eps)return 1;
    if(x<eps)return -1;
    return 0;
}
struct P{
    double x,y;
    P(){}
    P(double _x,double _y){x=_x,y=_y;}
    bool operator<(const P&a)const{return sgn(x-a.x)<0||sgn(x-a.x)==0&&sgn(y-a.y)<0;}
    P operator-(const P&a)const{return P(x-a.x,y-a.y);}
    double operator&(const P&a)const{return x*a.y-y*a.x;}
    double operator|(const P&a)const{return x*a.x+y*a.y;}
}p[N];
struct PI{
    ll x,y;
    PI(){}
    PI(ll _x,ll _y){x=_x,y=_y;}
}loc[N],pool[N];
inline double check(const P&a,const P&b,const P&c){return (b-a)&(c-a);}
inline double dis2(const P&a){return a.x*a.x+a.y*a.y;}
inline bool cross(int a,int b,int c,int d){
    return sgn(check(p[a],p[c],p[d]))<0&&sgn(check(p[b],p[c],p[d]))<0&&sgn(check(p[a],p[b],p[d]))<0;
}
inline ll dis(const PI&a,const PI&b){return (a.x-b.x)*(a.x-b.x)+(a.y-b.y)*(a.y-b.y);}
inline bool cmpx(const PI&a,const PI&b){return a.x<b.x;}
inline bool cmpy(int a,int b){return pool[a].y<pool[b].y;}
struct P3{
    double x,y,z;
    P3(){}
    P3(double _x,double _y,double _z){x=_x,y=_y,z=_z;}
    bool operator<(const P3&a)const{return sgn(x-a.x)<0||sgn(x-a.x)==0&&sgn(y-a.y)<0;}
    P3 operator-(const P3&a)const{return P3(x-a.x,y-a.y,z-a.z);}
    double operator|(const P3&a)const{return x*a.x+y*a.y+z*a.z;}
    P3 operator&(const P3&a)const{return P3(y*a.z-z*a.y,z*a.x-x*a.z,x*a.y-y*a.x);}
}ori[N];
inline P3 check(const P3&a,const P3&b,const P3&c){return (b-a)&(c-a);}
inline P3 gp3(const P&a){return P3(a.x,a.y,a.x*a.x+a.y*a.y);}
inline int cal(double x){
    int y=x;
    for(int i=y-2;i<=y+2;i++)if(!sgn(x-i))return i;
}
bool incir(int a,int b,int c,int d){
    P3 aa=gp3(p[a]),bb=gp3(p[b]),cc=gp3(p[c]),dd=gp3(p[d]);
```

```

    if(sgn(check(p[a],p[b],p[c]))<0)swap(bb,cc);
    return sgn(check(aa,bb,cc)|(dd-aa)<0);
}
int n,i,j,et,la[N],tot,l,r,q[N<<2];
struct E{
    int to,l,r;
    E(){}
    E(int _to,int _l,int _r=0){to=_to,l=_l,r=_r;}
}e[N<<5];
inline void add(int x,int y){
    e[++et]=E(y,la[x]),e[la[x]].r=et,la[x]=et;
    e[++et]=E(x,la[y]),e[la[y]].r=et,la[y]=et;
}
inline void del(int x){
    e[e[x].r].l=e[x].l;
    e[e[x].l].r=e[x].r;
    la[e[x^1].to]==x?la[e[x^1].to]=e[x].l:1;
}
void delaunay(int l,int r){
    if(r-l<=2){
        for(int i=l;i<r;i++)for(int j=i+1;j<=r;j++)add(i,j);
        return;
    }
    int i,j,mid=(l+r)>>1,ld=0,rd=0,id,op;
    delaunay(l,mid),delaunay(mid+1,r);
    for(tot=0,i=1;i<=r;q[++tot]=i++)
        while(tot>1&&sgn(check(p[q[tot-1]],p[q[tot]],p[i]))<0)tot--;
    for(i=1;i<tot&&!ld;i++)if(q[i]<=mid&&mid<q[i+1])ld=q[i],rd=q[i+1];
    for(;add(ld,rd),1;){
        id=op=0;
        for(i=la[ld];i;i=e[i].l)
            if(sgn(check(p[ld],p[rd],p[e[i].to]))>0)
                if(!id||incir(ld,rd,id,e[i].to))op=-1,id=e[i].to;
        for(i=la[rd];i;i=e[i].l)
            if(sgn(check(p[rd],p[ld],p[e[i].to]))<0)
                if(!id||incir(ld,rd,id,e[i].to))op=1,id=e[i].to;
        if(op==0)break;
        if(op==-1){
            for(i=la[ld];i;i=e[i].l)
                if(cross(rd,id,ld,e[i].to))del(i),del(i^1),i=e[i].r;
            ld=id;
        }else{
            for(i=la[rd];i;i=e[i].l)
                if(cross(ld,id,rd,e[i].to))del(i),del(i^1),i=e[i].r;
            rd=id;
        }
    }
}
namespace DS{
int m,tot,a[N],f[N],g[N],v[N<<1],nxt[N<<1],ed,col[N];ll w[N<<1];
double ans;
struct E{int x,y;ll w;E(){}E(int _x,int _y,ll _w){x=_x,y=_y,w=_w;}}e[N<<3];
inline bool cmp(const E&a,const E&b){return a.w<b.w;}
inline void newedge(int x,int y,ll z){e[++tot]=E(x,y,z);}
int F(int x){return f[x]==x?f[x]=F(f[x]);}
inline void merge(int x,int y,ll z){
    if(F(x)==F(y))return;

```

```

f[f[x]]=f[y];
v[++ed]=y;w[ed]=z;nxt[ed]=g[x];g[x]=ed;
v[++ed]=x;w[ed]=z;nxt[ed]=g[y];g[y]=ed;
ans+=sqrt(z);
}
inline void work(){
    sort(e+1,e+tot+1,cmp);
    for(ed=0,i=1;i<=n;i++)f[i]=i,g[i]=0;
    for(i=1;i<=tot;i++)merge(e[i].x,e[i].y,e[i].w);
    printf("%.15f\n",ans);
}
}
int main(){
    while(~scanf("%d",&n)){
        for(i=0;i<=n+1;i++)la[i]=0;
        et=1;
        DS::tot=0;
        for(i=1;i<=n;i++){
            ll x,y;
            scanf("%lld%lld",&x,&y);
            p[i]=P(x,y);
            loc[i]=PI(x,y);
            ori[i]=P3(x,y,i);
        }
        sort(p+1,p+n+1);
        sort(ori+1,ori+n+1);
        delaunay(1,n);
        for(i=1;i<=n;i++)for(j=la[i];j;j=e[j].l){
            int x=cal(ori[i].z),y=cal(ori[e[j].to].z);
            DS::newedge(x,y,dis(loc[x],loc[y]));
        }
        DS::work();
    }
}

```

## 9.7 弦图找错

```

#include <bits/stdc++.h>
using namespace std;
const int MAXN = 200005;
using lint = long long;
using pi = pair<int, int>;
// the algorithm may be wrong. if you have any ideas for proving / disproving this, please
// contact me.
vector<int> gph[MAXN];
int n, m, cnt[MAXN], idx[MAXN];
int mark[MAXN], vis[MAXN], par[MAXN];
void report(int x, int y){
    gph[x].erase(find(gph[x].begin(), gph[x].end(), y));
    gph[y].erase(find(gph[y].begin(), gph[y].end(), x));
    for(int i=1; i<=n; i++){
        if(binary_search(gph[i].begin(), gph[i].end(), x) &&
            binary_search(gph[i].begin(), gph[i].end(), y)){
            mark[i] = 1;
        }
    }
}

```

```

queue<int> que;
vis[x] = 1;
que.push(x);
while(!que.empty()){
    int x = que.front(); que.pop();
    for(auto &i : gph[x]){
        if(!mark[i] && !vis[i]){
            par[i] = x;
            vis[i] = 1;
            que.push(i);
        }
    }
}
assert(vis[y]);
vector<int> v;
while(y){
    v.push_back(y);
    y = par[y];
}
printf("NO\n%d\n", v.size());
for(auto &i : v) printf("%d_", i-1);
}

int main(){
    scanf("%d_%d",&n,&m);
    for(int i=0; i<m; i++){
        int s, e; scanf("%d_%d",&s,&e);
        s++, e++;
        gph[s].push_back(e);
        gph[e].push_back(s);
    }
    for(int i=1; i<=n; i++) sort(gph[i].begin(), gph[i].end());
    priority_queue<pi> pq;
    for(int i=1; i<=n; i++) pq.emplace(cnt[i], i);
    vector<int> ord;
    while(!pq.empty()){
        int x = pq.top().second, y = pq.top().first;
        pq.pop();
        if(cnt[x] != y || idx[x]) continue;
        ord.push_back(x);
        idx[x] = n + 1 - ord.size();
        for(auto &i : gph[x]){
            if(!idx[i]){
                cnt[i]++;
                pq.emplace(cnt[i], i);
            }
        }
    }
    reverse(ord.begin(), ord.end());
    for(auto &i : ord){
        int minBef = 1e9;
        for(auto &j : gph[i]){
            if(idx[j] > idx[i]) minBef = min(minBef, idx[j]);
        }
        minBef--;
        if(minBef < n){
            minBef = ord[minBef];
        }
    }
}

```

```

        for(auto &j : gph[i]){
            if(idx[j] > idx[minBef] && !binary_search(gph[minBef].begin(), gph[minBef].end(), j)
                ){
                report(minBef, i);
                return 0;
            }
        }
    }
}
puts("YES");
for(auto &i : ord) printf("%d□", i-1);
}

```

## 9.8 最长公共子序列

复杂度  $O(\frac{nm}{\omega})$ 。

```

/*
 * Author : _Wallace_
 * Source : https://www.cnblogs.com/-Wallace-/
 * Problem : LOJ #6564. 最长公共子序列
 * Standard : GNU C++ 03
 * Optimal : -Ofast
 */
#include <algorithm>
#include <cstdio>
#include <cstring>
#include <string>

typedef unsigned long long ULL;

const int N = 7e4 + 5;
int n, m, u;

struct bitset {
    ULL t[N / 64 + 5];

    bitset() {
        memset(t, 0, sizeof(t));
    }
    bitset(const bitset &rhs) {
        memcpy(t, rhs.t, sizeof(t));
    }

    bitset& set(int p) {
        t[p >> 6] |= 1llu << (p & 63);
        return *this;
    }
    bitset& shift() {
        ULL last = 0llu;
        for (int i = 0; i < u; i++) {
            ULL cur = t[i] >> 63;
            (t[i] <= 1) |= last, last = cur;
        }
        return *this;
    }
    int count() {

```

```

    int ret = 0;
    for (int i = 0; i < u; i++)
        ret += __builtin_popcountll(t[i]);
    return ret;
}

bitset& operator = (const bitset &rhs) {
    memcpy(t, rhs.t, sizeof(t));
    return *this;
}

bitset& operator &= (const bitset &rhs) {
    for (int i = 0; i < u; i++) t[i] &= rhs.t[i];
    return *this;
}

bitset& operator |= (const bitset &rhs) {
    for (int i = 0; i < u; i++) t[i] |= rhs.t[i];
    return *this;
}

bitset& operator ^= (const bitset &rhs) {
    for (int i = 0; i < u; i++) t[i] ^= rhs.t[i];
    return *this;
}

friend bitset operator - (const bitset &lhs, const bitset &rhs) {
    ULL last = 0llu; bitset ret;
    for (int i = 0; i < u; i++){
        ULL cur = (lhs.t[i] < rhs.t[i] + last);
        ret.t[i] = lhs.t[i] - rhs.t[i] - last;
        last = cur;
    }
    return ret;
}
} p[N], f, g;

signed main() {
    scanf("%d%d", &n, &m), u = n / 64 + 1;
    for (int i = 1, c; i <= n; i++)
        scanf("%d", &c), p[c].set(i);
    for (int i = 1, c; i <= m; i++) {
        scanf("%d", &c), (g = f) |= p[c];
        f.shift(), f.set(0);
        ((f = g - f) ^= g) &= g;
    }
    printf("%d\n", f.count());
    return 0;
}

```

### 另一个实现

```

#include <bits/stdc++.h>
#pragma GCC target("popcnt,bmi")

using namespace std;
using ull = uint64_t;

const int N = 70005, M = 1136;

int n, m;

```

```

ull g[N][M], f[M];

int read() {
    const int M = 1e6;
    static streambuf *in = cin.rdbuf();
#define gc (p1 == p2 && (p2 = (p1 = buf) + in -> sgetn(buf, M), p1 == p2) ? -1 : *p1++)
    static char buf[M], *p1, *p2;
    int c = gc, r = 0;

    while (c < 48)
        c = gc;

    while (c > 47)
        r = r * 10 + (c & 15), c = gc;

    return r;
}

int main() {
    cin.tie(0)->sync_with_stdio(0);
    cin >> n >> m;

    for (int i = 0; i < n; i++)
        g[read()][i / 62] |= 1ULL << (i % 62);

    int lim = (n - 1) / 62;

    for (int i = 0; i < m; i++) {
        int c = 1;
        auto can = g[read()];

        for (int j = 0; j <= lim; j++) {
            ull x = f[j], y = x | can[j];
            x += x + c + (~y & (1ULL << 62) - 1);
            f[j] = x & y, c = x >> 62;
        }
    }

    int ans = 0;

    for (int i = 0; i <= lim; i++)
        ans += __builtin_popcountll(f[i]);

    cout << ans;
}

```

## 9.9 区间 LIS (排列)

```

#include<bits/stdc++.h>
using namespace std;
//dengyaotriangle!

const int maxn=100005;

int pool[(int)5e7];int ps;
inline int *alloc(int x){
    ps+=x;return pool+ps-x;
}

```

```

}
void unit_monge_mult(int *a,int *b,int *r,int n){
    if(n==2){
        if(a[0]==0&&b[0]==0)r[0]=0,r[1]=1;
        else r[0]=1,r[1]=0;
        return;
    }
    if(n==1){r[0]=0;return;}
    int lps=ps;
    int d=n/2;
    int *a1=alloc(d),*a2=alloc(n-d),*b1=alloc(d),*b2=alloc(n-d);
    int *mpa1=alloc(d),*mpa2=alloc(n-d),*mpb1=alloc(d),*mpb2=alloc(n-d);
    int p[2]={0,0};
    for(int i=0;i<n;i++){
        if(a[i]<d)a1[p[0]]=a[i],mpa1[p[0]]=i,p[0]++;
        else a2[p[1]]=a[i]-d,mpa2[p[1]]=i,p[1]++;
    }
    p[0]=p[1]=0;
    for(int i=0;i<n;i++){
        if(b[i]<d)b1[p[0]]=b[i],mpb1[p[0]]=i,p[0]++;
        else b2[p[1]]=b[i]-d,mpb2[p[1]]=i,p[1]++;
    }
    int *c1=alloc(d),*c2=alloc(n-d);
    unit_monge_mult(a1,b1,c1,d),unit_monge_mult(a2,b2,c2,n-d);
    int *cpx=alloc(n),*cpy=alloc(n),*cqx=alloc(n),*cqy=alloc(n);
    for(int i=0;i<d;i++)cpx[mpa1[i]]=mpb1[c1[i]],cpy[mpa1[i]]=0;
    for(int i=0;i<n-d;i++)cpx[mpa2[i]]=mpb2[c2[i]],cpy[mpa2[i]]=1;
    for(int i=0;i<n;i++)r[i]=cpx[i];
    for(int i=0;i<n;i++)cqx[cpx[i]]=i,cqy[cpx[i]]=cpy[i];
    int hi=n,lo=n,his=0,los=0;
    for(int i=0;i<n;i++){
        if(cqy[i]^(cqx[i]>=hi))his--;
        while(hi>0&&his<0){
            hi--;
            if(cpy[hi]^(cpx[hi]>i))his++;
        }
        while(lo>0&&los<=0){
            lo--;
            if(cpy[lo]^(cpx[lo]>=i))los++;
        }
        if(los>0&&hi==lo)r[lo]=i;
        if(cqy[i]^(cqx[i]>=lo))los--;
    }
    ps=lps;
}
void subunit_monge_mult(int*a,int*b,int*c,int n){
    int lps=ps;
    int *za=alloc(n),*zb=alloc(n),*res=alloc(n),*vis=alloc(n),*mpa=alloc(n),*mpb=alloc(n),*rb=alloc(n);
    memset(vis,0,sizeof(int)*n);
    memset(mpa,-1,sizeof(int)*n);
    memset(mpb,-1,sizeof(int)*n);
    memset(rb,-1,sizeof(int)*n);
    int ca=n;
    for(int i=n-1;i>=0;i--)if(a[i]!=-1){
        vis[a[i]]=1;ca--;za[ca]=a[i];mpa[ca]=i;
    }
    for(int i=n-1;i>=0;i--)if(!vis[i])za[--ca]=i;
}

```



```

memset(vis,-1,sizeof(int)*n);
for(int i=0;i<n;i++)if(b[i]!=-1)vis[b[i]]=i;
ca=0;
for(int i=0;i<n;i++)if(vis[i]!=-1){
    mpb[ca]=i;rb[vis[i]]=ca++;
}
for(int i=0;i<n;i++)if(rb[i]==-1)rb[i]=ca++;
for(int i=0;i<n;i++)zb[rb[i]]=i;
unit_monge_mult(za,zb,res,n);
memset(c,-1,sizeof(int)*n);
for(int i=0;i<n;i++)if(mpa[i]!=-1&&mpb[res[i]]!=-1)c[mpa[i]]=mpb[res[i]];
ps=lps;
}

void solve(int *p,int *ret,int n){
    if(n==1){ret[0]=-1;return;}
    int lps=ps,d=n/2;
    int *pl=alloc(d),*pr=alloc(n-d);
    for(int i=0;i<d;i++)pl[i]=p[i];
    for(int i=0;i<n-d;i++)pr[i]=p[i+d];
    int *vis=alloc(n);memset(vis,-1,sizeof(int)*n);
    for(int i=0;i<d;i++)vis[pl[i]]=i;
    int *tl=alloc(d),*tr=alloc(n-d),*mpl=alloc(d),*mpr=alloc(n-d);
    int ca=0;
    for(int i=0;i<n;i++)if(vis[i]!=-1)mpl[ca]=i,tl[vis[i]]=ca++;
    ca=0;memset(vis,-1,sizeof(int)*n);
    for(int i=0;i<n-d;i++)vis[pr[i]]=i;
    for(int i=0;i<n;i++)if(vis[i]!=-1)mpr[ca]=i,tr[vis[i]]=ca++;
    int *vl=alloc(d),*vr=alloc(n-d);
    solve(tl,vl,d),solve(tr,vr,n-d);
    int *sl=alloc(n),*sr=alloc(n);
    iota(sl,sl+n,0);iota(sr,sr+n,0);
    for(int i=0;i<d;i++)sl[mpl[i]]=(vl[i]==-1?-1:mpl[vl[i]]);
    for(int i=0;i<n-d;i++)sr[mpr[i]]=(vr[i]==-1?-1:mpr[vr[i]]);
    subunit_monge_mult(sl,sr,ret,n);
    ps=lps;
}

int invp[maxn],res_monge[maxn];
int main(){
    ios::sync_with_stdio(0);cin.tie(0);
    int n,q;
    cin>>n>>q;
    vector<int> a(n);
    for(int i=0;i<n;i++)cin>>a[i],invp[a[i]]=i;
    solve(invp,res_monge,n);
    vector<int> fwk(n+1),ans(q);
    vector<vector<pair<int,int> > > qry(n+1);
    for(int i=0;i<q;i++){
        int l,r;
        cin>>l>>r;
        qry[l].push_back({r,i});
        ans[i]=r-1;
    }
    for(int i=n-1;i>=0;i--){
        if(res_monge[i]!=-1){
            for(int p=res_monge[i]+1;p<=n;p+=p&-p)fwk[p]++;
        }
    }
}

```

```

    }
    for(auto& z:qry[i]){
        int id,c;tie(id,c)=z;
        for(int p=id;p;p-=p&-p)ans[c]-=fwk[p];
    }
}
for(int i=0;i<q;i++)cout<<ans[i]<<'\n';
return 0;
}

```

## 9.10 区间 LCS

$s_{[0,a]}$  和  $t_{[b,c]}$  的 LCS

```

#include<bits/stdc++.h>
using namespace std;
//dengyaotriangle!

const int maxn=1005;
const int maxq=500005;
int n,m,q;
char a[maxn],b[maxn];
struct qryt{
    int x,nxt;
}z[maxq];
int qry[maxn][maxn];
int ans[maxq];
int r[maxn];
int bit[maxn];

int main(){
    ios::sync_with_stdio(0);cin.tie(0);
    cin>>q>>b>>a;n=strlen(a);m=strlen(b);
    //q,s,t
    for(int i=1;i<=q;i++){
        int a,b,c;
        cin>>a>>b>>c;
        if(a){
            ans[i]=c-b;
            z[i].x=b;z[i].nxt=qry[a][c];
            qry[a][c]=i;
        }
    }
    for(int i=0;i<n;i++)r[i]=i;
    for(int i=0;i<m;i++){
        int lp=-1;
        for(int j=0;j<n;j++)if(a[j]==b[i]){lp=j;break;}
        if(lp!=-1){
            for(int j=lp+1;j<n;j++){
                if(a[j]!=b[i]){
                    if(r[j-1]<r[j])swap(r[j-1],r[j]);
                }
            }
            for(int i=n-1;i>lp;i--)r[i]=r[i-1];
            r[lp]=-1;
        }
    }
    for(int i=0;i<=n;i++)bit[i]=0;

```

```

    for(int j=0;j<n;j++){
        if(r[j]!=-1){
            for(int p=n-r[j];p<=n;p+=p&-p)bit[p]++;
        }
        for(int y=qry[i+1][j+1];y;y=z[y].nxt){
            for(int p=n-z[y].x;p;p-=p&-p)ans[y]-=bit[p];
        }
    }
}
for(int i=1;i<=q;i++)cout<<ans[i]<<'\n';
return 0;
}

```

## 9.11 毛毛虫剖分

毛毛虫剖分, 一种由轻重链剖分 (HLD) 推广而成的树上结点重标号方法, 支持修改 / 查询一只毛毛虫的信息, 并且可以对毛毛虫的身体和足分别修改 / 查询不同信息.

严格强于树剖, 而且复杂度和树剖一样哦!

一些定义 (默认在一棵树上):

毛毛虫: 一条链和与这条链邻接的所有结点构成的集合. 虫身 (身体): 毛毛虫的链部分. 虫足 (足): 毛毛虫除虫身的部分. 重标号方法首先重剖求出重链. DFS, 若现在处理到结点  $u$ : 若  $u$  还未被标号, 则为其标号. 若  $u$  是重链头, 遍历这条重链, 将邻接这条链的结点依次标号. 先递归重儿子, 再递归轻儿子. 重标号性质对于重链, 除链头外的结点标号连续. 对于任意结点, 其轻儿子标号连续. 对于以重链头为根的子树, 与这条重链邻接的所有结点标号连续. 这样就可以随便维护毛毛虫信息了, 顺便还能维护链信息, 子树信息等.

时间复杂度同轻重链剖分.

以 SAM 为例, 若我们只保留所有的转移边  $(u, v)$ , 满足到达  $u$  的路径数目大于到达  $v$  的路径数目一半, 且从  $v$  出发的路径数目大于从  $u$  出发的路径数目一半, 这样剩余的子图显然会形成若干条链, 且每个点恰好在一条链上. 这样, 我们容易证明, 从根结点出发的任何一条路径, 至多经过  $O(\log n)$  条不在链上的转移边 (也意味着至多经过  $O(\log n)$  条链).

以下是一段示例代码, 展示了将一条链对应区间取出来的过程

```

vector<int> e[N];
vector<pair<int, int>> seg[N], qu[N];
int ans[Q];
int dfn[N], dep[N], nfd[N], top[N], f[N], sz[N], hc[N], pre[N], fir[N], lst2[N], rt[N];
int
void insert()
void dfs1(int u)
{
    sz[u] = 1;
    for (int v : e[u]) if (v != f[u])
    {
        dep[v] = dep[u] + 1;
        f[v] = u;
        dfs1(v);
        sz[u] += sz[v];
        if (sz[v] > sz[hc[u]]) hc[u] = v;
    }
    if (f[u]) erase(e[u], f[u]);
}
void dfs2(int u)

```

```

{
    static int id = 0;
    //dbg(u);
    if (!dfn[u])
    {
        dfn[u] = ++id;
        nfd[id] = u;
    }
    if (top[u] == u)
    {
        vector<int> stk;
        for (int v = u; v; v = hc[v])
        {
            for (int w : e[v]) if (w != hc[v])
            {
                dfn[w] = ++id;
                nfd[id] = w;
                pre[v] = id;
                cmin(fir[v], id);
                lst2[v] = id;
            }
            stk.push_back(v);
        }
        for (int i = (int)stk.size() - 2; i >= 0; i--)
        {
            cmin(fir[stk[i]], fir[stk[i + 1]]);
            cmax(lst2[stk[i]], lst2[stk[i + 1]]);
        }
        for (int i = 1; i < stk.size(); i++)
        {
            cmax(pre[stk[i]], pre[stk[i - 1]]);
        }
    }
    //dbg(u);
    top[hc[u]] = top[u];
    if (hc[u]) dfs2(hc[u]);
    for (int v : e[u]) if (v != hc[u]) dfs2(top[v] = v);
}

mt19937 rnd(245);
int main()
{
    memset(fir, 0x3f, sizeof fir);
    ios::sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(15);
    int n, m, q, i, j;
    cin >> n >> m >> q;
    for (i = 1; i < n; i++)
    {
        int u, v;
        //cin >> u >> v;
        u = i + 1;
        v = rnd() % i + 1;
        //v = (i + 1) / 2;
        //v = i / 2 + 1;
        //dbg(u, v);
        e[u].push_back(v);
        e[v].push_back(u);
    }
}

```

```

}
dfs1(dep[1] = 1);
//dbg("??");
dfs2(top[1] = 1);
//for (i = 1;i <= n;i++) cerr << i << ": " << dfn[i] << endl;
for (i = 1;i <= m;i++)
{
    int u, v;
    //cin >> u >> v;
    u = rnd() % n + 1;
    v = rnd() % n + 1;
    int uu = u, vv = v;
    //dbg(uu, vv);
    auto& w = seg[i];
    while (top[u] != top[v])
    {
        if (dep[top[u]] < dep[top[v]]) swap(u, v);
        w.push_back({fir[top[u]], pre[u]});
        //else w.push_back({fir[top[u]], lst2[top[u]]});
        if (hc[u]) w.push_back({dfn[hc[top[u]]], dfn[hc[u]]});
        else if (top[u] != u) w.push_back({dfn[hc[top[u]]], dfn[u]});
        //dbg(u, v, w);
        //[fir[top[u]],lst[u]]
        u = f[top[u]];
    }
    if (dep[u] < dep[v]) swap(u, v);
    w.push_back({fir[v], pre[u]});
    //else if (!hc[u]) w.push_back({fir[v], lst2[v]});
    //dbg(v, lst2[v], fir[v]);
    if (hc[u]) w.push_back({dfn[hc[v]], dfn[hc[u]]});
    else if (u != v) w.push_back({dfn[hc[v]], dfn[u]});
    //dbg(w);
    w.push_back({dfn[v], dfn[v]});
    if (f[v]) w.push_back({dfn[f[v]], dfn[f[v]]});
    erase_if(w, [&](const auto& x) {return x.first > x.second;});
    //int len = 0;
    //for (auto [l, r] : w) len += r - l + 1;
    //for (auto [l, r] : w)
    //{
    //    for (int j = l;j <= r;j++) cerr << nfd[j] << ' ';cerr << " | ";
    //}
    //cerr << endl;
    //int tl = 0;
    //set<int> s = {uu, vv};
    //while (uu != vv)
    //{
    //    if (dep[uu] < dep[vv]) swap(uu, vv);
    //    s.insert(all(e[uu]));s.insert(f[uu]);uu = f[uu];
    //}
    //s.insert(all(e[uu]));
    //if (f[uu]) s.insert(f[uu]);
    ////dbg(s);
    //assert(len == s.size());
}
for (i = 1;i <= q;i++)
{
    int l, r;

```

```

        cin >> l >> r;
        qu[l].push_back({r, i});
    }
    for (i = m; i; i--)
    {

    }
    for (i = 1; i <= q; i++) cout << ans[i] << '\n';
    //cerr << "??\n";
}

```

## 9.12 所有区间 GCD

```

template<typename T> struct GCD
{
    vector<pair<int, T>> res;
    GCD(const vector<T> &a) :res(n)
    {
        int n = a.size(), i, j;
        vector<ll> v(n);
        vector<int> l(n);
        vector < vector<pair<int, T>> res(n);
        for (i = 0; i < n; i++)
        {
            for (v[i] = a[i], j = l[i] = i; j >= 0; j = l[j] - 1)
            {
                v[j] = fun(v[j], a[i]);
                while (l[j] && fun(a[i], v[l[j] - 1]) == fun(a[i], v[j])) l[j] = l[l[j] - 1];
                // [l[j]..j,i] 区间内的值求fun均为v[j]
            }
            for (j = i; j >= 0; j = l[j] - 1) res[i].push_back({l[j], v[j]});
            reverse(all(res[i]));
        }
    }
    T ask(int l, int r) //[l,r]
    {
        return res[r].prev(upper_bound(1))->second;
    }
};

```