SSerxhs 的 ICPC 板子

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目录

1 数据结构

1.1 哈希表

支持如同 map 一样使用 [] 访问。default 指的是未赋值情形的值。新版本未验证。

```cpp
template<class Tx,class Ty> struct hashtable //定义域，值域
{
    const static int N=2e6+5,p=1e6+7;//元素个数，模数
    Tx X[N];
    Ty Y[N],default;
    int fir[p],nxt[N],sz;
    ht(Ty default=Ty{}):val(default):sz(0),{memset(fir,-1,sizeof fir);}
    Ty &operator[](T x)
    {
        int index=(x%p+p)%p;
        for (int i=fir[index];i!=-1;i=nxt[i]) if (X[i]==x) return Y[i];//若 x 不
            重复，可以省略这个 for
        X[cnt]=x;
        Y[cnt]=default;
        nxt[cnt]=fir[index];
        fir[index]=cnt++;
        return Y[cnt-1];
    }
    void clear()
    {
        cnt=0;
        while (sz) fir[((X[--sz])%p+p)%p]=0;
    }
    void iterate()//遍历。用于自行修改
    {
        for (int i=0;i<sz;i++)
        {
            T x=X[i];
            TT y=Y[i];
            //(x,y)
        }
    }
};
```

1.2 珂朵莉树

支持区间赋值、单点访问。维护每个连续段的范围和值。

如果希望维护所有连续段的整体信息（如长度的最大值），修改 add 和 del 函数即可，分别表示连续段被加入和被删去。

特别注意一开始 insert 的不会触发 add，只有 modify 会触发。

```cpp
namespace chtholly_tree
{
    using T=int;//可以把 T 修改为任意想要的类型。
    struct node
    {
        int l;
        mutable int r;
        mutable T v;
```

```cpp
        int len() const { return r-l+1; }
        bool operator<(const node &x) const { return l<x.l; }
    };
    void add(const node &a) {}
    void del(const node &a) {}
    class odt: public set<node>
    {
    public:
        typedef odt::iterator iter;
        iter split(int x)
        {
            iter it=lower_bound({x});
            if (it!=end()&&it->l==x) return it;
            node t=*--it,a={t.l,x-1,t.v},b={x,t.r,t.v};
            del(*it); add(a); add(b);
            erase(it); insert(a);
            return insert(b).first;
        }
        void modify(int l,int r,T v)//[l,r]
        {
            iter lt,rt,it;
            rt=r==rbegin()->r?end():split(r+1); lt=split(l);//[lt,rt)
            while (lt!=begin()&&(it=prev(lt))->v==v) l=(lt=it)->l;
            while (rt!=end()&&rt->v==v) r=(rt++)->r;
            for (it=lt; it!=rt; it++) del(*it);
            add({l,r,v});
            erase(lt,rt); insert({l,r,v});
        }
        T operator[](const int x) const { return prev(upper_bound({x}))->v; }//
            直接访问单点
        iter find(int x) const {return prev(upper_bound({x}));}//找到对应的线段
    };
}
using chtholly_tree::node,chtholly_tree::odt;
typedef odt::iterator iter;
int main()
{
    odt s;
    s.insert({0,5,1});  // 先 insert({L,R,x}) 表示整个下标范围和初始值。 左闭右
        闭。
                      // s={1,1,1,1,1,1}
    s.modify(2,3,2);  // 左闭右闭。s={1,1,2,2,1,1}
    for (auto [l,r,v]:s)
    {
        //(l,r,v)=(0,1,1)
        //(l,r,v)=(2,3,2)
        //(l,r,v)=(4,5,1)
    }
}
```

1.3 带删堆

本质是额外维护一个堆 q 表示要被删除的元素，当 p 的最值和 q 一样时删除。

需要保证每次 pop 的元素都存在于堆中。

本代码的用法和 priority queue 一致。

```cpp
template<typename T, typename T1=vector<T>, typename T2=less<T>> struct heap
{
private:
    priority_queue<T, T1, T2> p, q;
public:
    void push(const T &x)
    {
        if (!q.empty()&&q.top()==x)
        {
            q.pop();
            while (!q.empty()&&q.top()==p.top()) p.pop(), q.pop();
        }
        else p.push(x);
    }
    void pop()
    {
        p.pop();
        while (!q.empty()&&p.top()==q.top()) p.pop(), q.pop();
    }
    void pop(const T &x)
    {
        if (p.top()==x)
        {
            p.pop();
            while (!q.empty()&&p.top()==q.top()) p.pop(), q.pop();
        }
        else q.push(x);
    }
    T top() const { return p.top(); }
    int size() const { return p.size()-q.size(); }
    bool empty() const { return p.empty(); }
    vector<T> to_vector() const
    {
        vector<T> a;
        auto P=p, Q=q;
        while (P.size())
        {
            a.push_back(P.top()); P.pop();
            while (Q.size()&&P.top()==Q.top()) P.pop(), Q.pop();
        }
        return a;
    }
};
```

1.4   前 k 大的和

本质是用小根堆维护前 k 大的数，用大根堆维护其余数。

如果需要支持删除，结合前面一个使用，或者直接用 multiset 进行 erase。

为了方便起见，直接给出支持删除的版本，并且使用 long long。如果不需要支持删除，类型改为优先队列并去掉 pop 函数即可。

注意：复杂度为 $O(k - k')$，其中 k' 是上一次询问的 k。也就是说，多组询问时询问的 k 的差值应该尽可能小。

其用法与 priority queue 保持一致，可以用同样的方法改写成前 k 小。

```cpp
using ll=long long;
template<typename T, typename T1=vector<T>, typename T2=less<T>> struct
    ksum_pop
{
private:
    struct __cmp
    {
        bool operator()(const T &x, const T &y) const
        {
            return x!=y&&!T2()(x, y);
        }
    };
    heap<T, T1, __cmp> p;
    heap<T, T1, T2> q;
    ll cur;
public:
    ksum_pop():cur(0) { }
    void push(const T &x)
    {
        if (!q.size()||!T2()(x, q.top())) p.push(x), cur+=x; else q.push(x);
    }
    int size() const { return p.size()+q.size(); }
    void pop(const T &x)
    {
        if (q.size()&&!T2()(q.top(), x)) q.pop(x);
        else p.pop(x), cur-=x;
    }
    ll sum(int k)
    {
        while (p.size()<k)
        {
            cur+=q.top();
            p.push(q.top());
            q.pop();
        }
        while (p.size()>k)
        {
            cur-=p.top();
            q.push(p.top());
            p.pop();
        }
        return cur;
    }
};
```

1.5 可持久化数组

历史遗留产物，无意义，仅作留存，不会更新。

$O((n+q)\log(n))$，$O((n+q)\log(n))$。

```cpp
struct arr
{
    int c[M][2],rt[O],s[M],b[N];
    int ds,n,ver,v,p,i;
    void build(int &x,int l,int r)
    {
```

```cpp
        x=++ds;
        if (l==r) {s[x]=b[l];return;}
        build(c[x][0],l,l+r>>1);
        build(c[x][1],(l+r>>1)+1,r);
    }
    void rebuild(int &x,int pre)
    {
        x=++ds;int l=1,r=n,mid,now=x;
        while (l<r)
        {
            mid=l+r>>1;
            if (mid>=p){c[now][1]=c[pre][1];now=c[now][0]=++ds;r=mid;pre=c[pre
                ][0];} else {c[now][0]=c[pre][0];now=c[now][1]=++ds;l=mid+1;pre=c
                [pre][1];}
        }
        s[now]=v;
    }
    void init(int *a,int nn)
    {
        n=nn;
        for (i=1;i<=n;i++) b[i]=a[i];
        build(rt[0],1,n);
    }
    int mdf(int pv,int pos,int val)
    {
        p=pos,v=val;
        rebuild(rt[++ver],rt[pv]);
        return ver;
    }
    int ask(int ve,int pos)
    {
        int l=1,r=n,x=rt[ve],mid;
        rt[++ver]=rt[ve];
        while (l<r)
        {
            mid=l+r>>1;
            if (mid>=pos) {x=c[x][0];r=mid;} else {x=c[x][1];l=mid+1;}
        }
        return s[x];
    }
};
```

1.6 左偏树/可并堆

建议不要使用。pbds 可以替代这个功能。我完全没有使用过这个板子。
$O((n+q)\log n)$，$O(n)$。

```cpp
struct left_tree//小根堆，大根堆需要改的地方注释了
{
    int jl[N],v[N],f[N],c[N][2],tf[N],n;//tf只有删非堆顶才用
    bool ed[N];
    void init(const int nn,const int *a)
    {
        jl[0]=-1;n=nn;
        memset(jl+1,0,n<<2);
        memset(tf+1,0,n<<2);//同上
```

```cpp
        memset(c+1,0,n<<3);
        memset(ed+1,0,n);
        for (int i=1;i<=n;i++) v[f[i]=i]=a[i];
    }
    int mg(int x,int y)
    {
        if (!(x&&y)) return x|y;
        if (v[x]>v[y]||v[x]==v[y]&&x>y) swap(x,y);//改
        tf[c[x][1]=mg(c[x][1],y)]=x;//同上
        if (jl[c[x][0]]<jl[c[x][1]]) swap(c[x][0],c[x][1]);
        jl[x]=jl[c[x][1]]+1;
        return x;
    }
    int getf(int x)
    {
        if (f[x]==x) return x;
        return f[x]=getf(f[x]);
    }
    int merge(int x,int y)
    {
        if (ed[x]||ed[y]||(x=getf(x))==(y=getf(y))) return x;
        int z=mg(x,y);return f[x]=f[y]=z;
    }
    int getv(int x)//需要自行判断是否存在
    {
        return v[getf(x)];
    }
    int del(int x)//删除堆内最值
    {
        tf[c[x][0]]=tf[c[x][1]]=0;
        f[c[x][0]]=f[c[x][1]]=f[x]=mg(c[x][0],c[x][1]);
        ed[x]=1;c[x][0]=c[x][1]=tf[x]=0;return f[x];
    }
    int del_all(int x)//删除堆内非最值（没验证过）
    {
        int fa=tf[x];
        if (f[c[x][0]]==x) f[c[x][0]]=getf(tf[x]);
        if (f[c[x][1]]==x) f[c[x][1]]=f[tf[x]];
        tf[x]=tf[c[x][0]]=tf[c[x][1]]=0;
        tf[c[fa][c[fa][1]==x]=mg(c[x][0],c[x][1])]=fa;
        c[x][0]=c[x][1]=0;
        while (jl[c[fa][0]]<jl[c[fa][1]])
        {
            swap(c[fa][0],c[fa][1]);
            jl[fa]=jl[c[fa][1]]+1;
            fa=tf[fa];
        }
    }
    void out(int n)
    {
        for (int i=1;i<=n;i++) printf("%d:␣c%d&%d␣f%d␣v%d\n",i,c[i][0],c[i][1],f
            [i],v[i]);
    }
};
```

1.7 树状数组区间加区间求和

本质：a_n 区间加等价于差分数组 d_n 的单点加。

$$\sum_{i=1}^{m} a_i = \sum_{i=1}^{m}\sum_{j=1}^{i} d_j = \sum_{j=1}^{m} d_j(m-j+1) = ((m+1)\sum_{j=1}^{m} d_j) - (\sum_{j=1}^{m} jd_j)。$$

分别维护 d_j 和 jd_j 的前缀和。

$O(n) \sim O(q\log n)，\ O(n)。$

```cpp
struct bit
{
    ll a[N],b[N],s[N];//有初始值
    int n;
    void init(int nn,int *a)//初始值
    {
        n=nn;s[0]=0;
        for (int i=1;i<=n;i++) s[i]=s[i-1]+a[i];
    }
    void mdf(int l,int r,ll dt)
    {
        int i;++r;
        ll j=dt*l;
        a[l]+=dt;b[l]+=j;
        while ((l+=l&-l)<=n)
        {
            a[l]+=dt;
            b[l]+=j;
        }
        if (r<=n)
        {
            j=dt*r;
            a[r]-=dt;b[r]-=j;
            while ((r+=r&-r)<=n)
            {
                a[r]-=dt;
                b[r]-=j;
            }
        }
    }
    ll presum(int x)
    {
        ll r=a[x],rr=b[x];
        int y=x;
        while (x^=x&-x)
        {
            r+=a[x];
            rr+=b[x];
        }
        return r*(y+1)-rr+s[y];
    }
    ll sum(int l,int r)
    {
        return presum(r)-presum(l-1);
    }
};
```

1.8   二维树状数组矩形加矩形求和

本质还是差分，只不过这次要维护 $d_{i,j}, d_{i,j}i, d_{i,j}j, d_{i,j}ij$。

$O(n^2) \sim O(q\log^2 n)$，$O(n^2)$

```cpp
struct bit2
{
    ll a[2050][2050],b[2050][2050],c[2050][2050],d[2050][2050];
    int n,m;
    private:
    void cha(ll a[][2050],int x,int y,int z)
    {
        int i,j;
        for (i=x;i<=n;i+=(i&(-i))) for (j=y;j<=m;j+=(j&(-j))) a[i][j]+=z;
    }
    ll he(int x,int y)
    {
        if ((x<=0)||(y<=0)) return 0;
        int i,j;
        ll z=0,w=0;
        for (i=x;i;i-=(i&(-i))) for (j=y;j;j-=(j&(-j))) z+=a[i][j];
        z*=(x+1)*(y+1);
        w=0;
        for (i=x;i;i-=(i&(-i))) for (j=y;j;j-=(j&(-j))) w+=b[i][j];
        z-=w*(y+1);
        w=0;
        for (i=x;i;i-=(i&(-i))) for (j=y;j;j-=(j&(-j))) w+=c[i][j];
        z-=w*(x+1);
        for (i=x;i;i-=(i&(-i))) for (j=y;j;j-=(j&(-j))) z+=d[i][j];
        return z;
    }
    public:
    void init(int x,int y)
    {
        n=x;m=y;
    }
    void add(int u,int v,int x,int y,int z)//(x1,y1,x2,y2,dt)
    {
        cha(a,u,v,z);
        cha(b,u,v,u*z);//小心乘爆
        cha(c,u,v,v*z);
        cha(d,u,v,u*v*z);
        ++x;++y;
        if (x<=n)
        {
            cha(a,x,v,-z);
            cha(b,x,v,-z*x);
            cha(c,x,v,-z*v);
            cha(d,x,v,-z*x*v);
        }
        if (y<=m)
        {
            cha(a,u,y,-z);
            cha(b,u,y,-z*u);
            cha(c,u,y,-z*y);
            cha(d,u,y,-z*u*y);
            if (x<=n)
            {
```

```
                    cha(a,x,y,z);
                    cha(b,x,y,z*x);
                    cha(c,x,y,z*y);
                    cha(d,x,y,z*x*y);
                }
            }
        }
    ll sum(int u,int v,int x,int y)//(x1,y1,x2,y2)
    {
        --u;--v;
        return (he(x,y)+he(u,v)-he(u,y)-he(x,v));
    }
};
```

1.9 带修莫队（功能：区间数有多少种不同的数字）

按照 $n^{\frac{2}{3}}$ 分块，排序关键字是 l,r,t 所在的块（t 是版本号，每次修改都会增加一个版本），可以奇偶分块优化。

相比于传统莫队多了一个 modify。

$O(n^{\frac{5}{3}})$，$O(n)$。

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
#define all(x) (x).begin(),(x).end()
const int N=1.4e5,M=1e6+2;
int a[N],ans[N],bel[N],cnt[M],sum,z,y,cur;
struct P
{
    int p,v;
};
struct Q
{
    int l,r,t,p;
    bool operator<(const Q &o) const
    {
        if (bel[l]!=bel[o.l]) return bel[l]<bel[o.l];
        if (bel[r]!=bel[o.r]) return (bel[l]&1)^bel[r]<bel[o.r];
        return (bel[r]&1)?t<o.t:t>o.t;
    }
};
Q b[N];
P d[N];
void add(const int &x) {sum+=!(cnt[a[x]]++);}
void del(const int &x) {sum-=!(--cnt[a[x]]);}
void mdf(const int &x)
{
    auto &[p,v]=d[x];
    if (z<=p&&p<=y) del(p);
    swap(a[p],v);
    if (z<=p&&p<=y) add(p);
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m,q1=0,q2=0,i,ksiz;
```

```cpp
    cin>>n>>m;
    for (i=1;i<=n;i++) cin>>a[i];
    for (i=1;i<=m;i++)
    {
        char c;
        int l,r;
        cin>>c>>l>>r;
        if (c=='Q') ++q1,b[q1]={l,r,q2,q1};
        else d[++q2]={l,r};
    }
    ksiz=max(1.0,round(cbrt((ll)n*n)));
    for (i=1;i<=n;i++) bel[i]=i/ksiz;
    sort(b+1,b+q1+1);
    z=b[1].l;y=z-1;cur=0;
    for (i=1;i<=q1;i++)
    {
        auto [l,r,t,p]=b[i];
        while (z>l) add(--z);
        while (y<r) add(++y);
        while (z<l) del(z++);
        while (y>r) del(y--);
        while (cur<t) mdf(++cur);
        while (cur>t) mdf(cur--);
        ans[p]=sum;
    }
    for (i=1;i<=q1;i++) cout<<ans[i]<<'\n';
}
```

1.10　二次离线莫队

直接摘录题解，用途不大。

$O(n\sqrt{n})$，$O(n)$。

珂朵莉给了你一个序列 a，每次查询给一个区间 $[l,r]$，查询 $l \le i < j \le r$，且 $a_i \oplus a_j$ 的二进制表示下有 k 个 1 的二元组 (i,j) 的个数。$\oplus$ 是指按位异或。

二次离线莫队，通过扫描线，再次将更新答案的过程离线处理，降低时间复杂度。假设更新答案的复杂度为 $O(k)$，它将莫队的复杂度从 $O(nk\sqrt{n})$ 降到了 $O(nk+n\sqrt{n})$，大大简化了计算。设 x 对区间 $[l,r]$ 的贡献为 $f(x,[l,r])$，我们考虑区间端点变化对答案的影响：以 $[l..r]$ 变成 $[l..(r+k)]$ 为例，$\forall x \in [r+1,r+k]$ 求 $f(x,[l,x-1])$。我们可以进行差分：$f(x,[l,x-1]) = f(x,[1,x-1]) - f(x,[1,l-1])$，这样转化为了一个数对一个前缀的贡献。保存下来所有这样的询问，从左到右扫描数组计算就可以了。但是这样做，空间是 $O(n\sqrt{n})$ 的，不太优秀，而且时间常数巨大。。这样的贡献分为两类：

1. 减号左边的贡献永远是一个前缀和它后面一个数的贡献。这可以预处理出来。2. 减号右边的贡献对于一次移动中所有的 x 来说，都是不变的。我们打标记的时候，可以只标记左右端点。

这样，减小时间常数的同时，空间降为了 $O(n)$ 级别。是一个很优秀的算法了。处理前缀询问的时候，我们利用异或运算的交换律，即 $a \operatorname{xor} b = c \iff a \operatorname{xor} c = b$ 开一个桶 t，$t[i]$ 表示当前前缀中与 i 异或有 k 个数位为 1 的数有多少个。则每加入一个数 $a[i]$，对于所有 $\operatorname{popcount}(x) = k$ 的 x，$t[a[i] \operatorname{xor} x] \leftarrow t[a[i] \operatorname{xor} x] + 1$ 即可。

```cpp
typedef long long ll;
const int N=1e5+2,M=1<<14;
```

```cpp
ll f[N],ans[N],ta[N];
int a[N],cnt[M],bel[N],pc[M],st[N];
int n,m,ksiz;
struct Q
{
    int z,y,wz;
    bool operator<(const Q& x) const {return (bel[z]<bel[x.z])||(bel[z]==bel[x.z
        ])&&((y<x.y)&&(bel[z]&1)||(y>x.y)&&(1^bel[z]&1));}
};
Q mq(const int x,const int y,const int z)
{
    Q a;
    a.z=x;a.y=y;a.wz=z;
    return a;
}
Q q[N];
vector<Q> b[N];
void read(int &x)
{
    int c=getchar();
    while ((c<48)||(c>57)) c=getchar();
    x=c^48;c=getchar();
    while ((c>=48)&&(c<=57))
    {
        x=x*10+(c^48);
        c=getchar();
    }
}
int main()
{
    int i,j,k,l=1,r=0,tp=0,x,na;
    read(n);read(m);read(k);ksiz=sqrt(n);
    for (i=1;i<=n;i++) {read(a[i]);bel[i]=(i-1)/ksiz+1;}
    if (k==0) st[++tp]=0;
    for (i=1;i<16384;i++)
    {
        if (i&1) pc[i]=pc[i>>1]+1; else pc[i]=pc[i>>1];
        if (pc[i]==k) st[++tp]=i;
    }
    for (i=1;i<=n;i++)
    {
        j=tp+1;f[i]=f[i-1];
        while (--j) f[i]+=cnt[st[j]^a[i]];
        ++cnt[a[i]];
    }
    for (i=1;i<=m;i++) {read(q[i].z);read(q[q[i].wz=i].y);}
    sort(q+1,q+m+1);
    for (i=1;i<=m;i++)
    {
        ans[i]=f[q[i].y]-f[r]+f[q[i].z-1]-f[l-1];
        if (k==0) ans[i]+=q[i].z-l;
        if (r<q[i].y)
        {
            b[l-1].push_back(mq(r+1,q[i].y,-i));
            r=q[i].y;
        }
        if (l>q[i].z)
```

```
        {
            b[r].push_back(mq(q[i].z,l-1,i));
            l=q[i].z;
        }
        if (r>q[i].y)
        {
            b[l-1].push_back(mq(q[i].y+1,r,i));
            r=q[i].y;
        }
        if (l<q[i].z)
        {
            b[r].push_back(mq(l,q[i].z-1,-i));
            l=q[i].z;
        }
    }
    memset(cnt,0,sizeof(cnt));
    for (i=1;i<=n;i++)
    {
        j=tp+1;x=a[i];
        while (--j) ++cnt[x^st[j]];
        for (j=0;j<b[i].size();j++)
        {
            na=0;l=b[i][j].z;r=b[i][j].y;
            for (k=l;k<=r;k++) na+=cnt[a[k]];
            if (b[i][j].wz>0) ans[b[i][j].wz]+=na; else ans[-b[i][j].wz]-=na;
        }
    }
    for (i=2;i<=m;i++) ans[i]+=ans[i-1];
    for (i=1;i<=m;i++) ta[q[i].wz]=ans[i];
    for (i=1;i<=m;i++) printf("%lld\n",ta[i]);
}
```

1.11 回滚莫队

不删除的莫队，比如求 max。

做法：块内询问暴力。对于 l 所在块相同的询问，按照 r 升序排序，并且将左指针固定在 l 所在块的最右侧。（由于块内询问暴力，这不会导致左指针更大）

回答每个询问的时候，先右端点右移到 r，然后左端点左移到 l。询问完成后，把左端点移回去。移回去的过程虽然涉及删除，但不需要维护答案变成什么了（因为在左端点左移之前已经求过了）。换句话说，相当于"撤销"而不是删除，完全可以记录移动过程中的所有变化来撤销。

$O(n\sqrt{n})$，$O(n)$。

```
#include <bits/stdc++.h>
using namespace std;
const int N=2e5+2;
int a[N],z[N],y[N],wz[N],b[N],d[N],bel[N],ans[N],st[N][2],pos[N][2];
int n,m,i,j,x,c,ksiz,gs,l=1,r,tp,na,ca;
void read(int &x)
{
    c=getchar();
    while ((c<48)||(c>57)) c=getchar();
    x=c^48;c=getchar();
    while ((c>=48)&&(c<=57))
    {
```

```
            x=x*10+(c^48);
            c=getchar();
        }
}
void qs(int l,int r)
{
    int i=l,j=r,m=bel[z[l+r>>1]],mm=y[l+r>>1];
    while (i<=j)
    {
        while ((bel[z[i]]<m)||(bel[z[i]]==m)&&(y[i]<mm)) ++i;
        while ((bel[z[j]]>m)||(bel[z[j]]==m)&&(y[j]>mm)) --j;
        if (i<=j)
        {
            swap(wz[i],wz[j]);
            swap(z[i],z[j]);
            swap(y[i++],y[j--]);
        }
    }
    if (i<r) qs(i,r);
    if (l<j) qs(l,j);
}
int main()
{
    read(n);ksiz=sqrt(n);
    for (i=1;i<=n;i++) {read(a[i]);b[i]=a[i];bel[i]=(i-1)/ksiz+1;}
    sort(b+1,b+n+1);
    d[gs=1]=b[1];
    for (i=2;i<=n;i++) if (b[i]!=b[i-1]) d[++gs]=b[i];
    for (i=1;i<=n;i++) a[i]=lower_bound(d+1,d+gs+1,a[i])-d;
    read(m);assert(int(n/sqrt(m)));
    for (i=1;i<=m;i++) {read(z[i]);read(y[wz[i]=i]);}
    qs(1,m);
    for (i=1;i<=m;i++)
    {
        if (bel[z[i]]>bel[z[i-1]])
        {
            while (l<=r) {pos[a[l]][0]=pos[a[l]][1]=0;++l;}na=0;
            if (bel[z[i]]==bel[y[i]])
            {
                for (j=z[i];j<=y[i];j++) if (pos[a[j]][0]) na=max(na,j-pos[a[j
                    ]][0]); else pos[a[j]][0]=j;
                ans[wz[i]]=na;for (j=z[i];j<=y[i];j++) pos[a[j]][0]=0;na=0;l=ksiz
                    *bel[z[i]];r=l-1;
                continue;
            }
            l=ksiz*bel[z[i]];r=l-1;na=0;
        }
        if (bel[z[i]]==bel[y[i]])
        {
            while (l<=r) {pos[a[l]][0]=pos[a[l]][1]=0;++l;}na=0;
            for (j=z[i];j<=y[i];j++) if (pos[a[j]][0]) na=max(na,j-pos[a[j]][0]);
                 else pos[a[j]][0]=j;
            ans[wz[i]]=na;for (j=z[i];j<=y[i];j++) pos[a[j]][0]=0;
            l=ksiz*bel[z[i]];r=l-1;na=0;
            continue;
        }
        while (r<y[i])
```

```
    {
        x=a[++r];pos[x][1]=r;
        if (!pos[x][0]) pos[x][0]=r; else na=max(na,r-pos[x][0]);
    }c=na;
    while (l>z[i])
    {
        x=a[--l];st[++tp][0]=x;st[tp][1]=pos[x][0];
        pos[x][0]=l;
        if (!pos[x][1])
        {
            st[++tp][0]=x+n;st[tp][1]=0;
            pos[x][1]=l;
        } else na=max(na,pos[x][1]-l);
    }
    ans[wz[i]]=na;na=c;++tp;l=ksiz*bel[z[i]];
    while (--tp) if (st[tp][0]<=n) pos[st[tp][0]][0]=st[tp][1]; else pos[st[
        tp][0]-n][1]=st[tp][1];
    }
    for (i=1;i<=m;i++) printf("%d\n",ans[i]);
}
```

1.12   李超树

题意：插入线段，查询某个 x 的最大 y（输出最小编号）

算法核心：修改时，线段树每个点只维护在中点取值最大的线段，中点取值较小的线段只会在至多一侧有用，递归下去插入，复杂度 $O(\log^2)$。查询时询问线段树上 log 个点的线段中最大的。

```
struct Q
{
    int x0,y0,dx,dy,id;
    Q():x0(0),y0(-1),dx(1),dy(0),id(-1){}//y>=0
    Q(int a,int b,int c,int d,int e):x0(a),y0(b),dx(c),dy(d),id(e){}
    bool contains(const int &x) const {return x0<=x&&x<=x0+dx;}
};
bool cmp(const Q &a,const Q &b,int x)//小心数值爆炸
{
    ll A=((ll)a.y0*a.dx+(ll)(x-a.x0)*a.dy)*b.dx,B=((ll)b.y0*b.dx+(ll)(x-b.x0)*b.
        dy)*a.dx;
    if (A!=B) return A<B;
    return a.id>b.id;
}
bool cmp2(const Q &a,const Q &b)
{
    if (a.y0+a.dy!=b.y0+b.dy) return a.y0+a.dy<b.y0+b.dy;
    return a.id>b.id;
}
const int inf=1e9;
int ans;
namespace seg
{
    const int N=4e4+2,M=N*4;
    Q s[M],X[N];
    int n,z,y;
    void init(int nn) {n=nn;for (int i=1;i<=n*4;i++) s[i]=Q();}
    void insert(int x,int l,int r,Q dt)
```

```cpp
    {
        int c=x*2,m=l+r>>1;
        if (z<=l&&r<=y)
        {
            if (cmp(s[x],dt,m)) swap(s[x],dt);
            if (l==r) return;
            if (cmp(s[x],dt,l)) insert(c,l,m,dt);
            else if (cmp(s[x],dt,r)) insert(c+1,m+1,r,dt);
            return;
        }
        if (z<=m) insert(c,l,m,dt);
        if (y>m) insert(c+1,m+1,r,dt);
    }
    void insert(const Q &o)
    {
        z=o.x0;y=z+o.dx;
        assert(1<=z&&z<=y&&y<=n);
        if (z==y)
        {
            if (cmp2(X[z],o)) X[z]=o;
            return;
        }
        insert(1,1,n,o);
    }
    Q askmax(int p)
    {
        Q ans=s[1].contains(p)?s[1]:Q();
        int x=1,l=1,r=n,c,m;
        while (l<r)
        {
            c=x*2,m=l+r>>1;
            if (p<=m) x=c,r=m; else x=c+1,l=m+1;
            if (s[x].contains(p)&&cmp(ans,s[x],p)) ans=s[x];
        }
        Q o(X[p].x0,X[p].y0+X[p].dy,1,0,0);
        return cmp(ans,o,p)?X[p]:ans;
    }
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(15);
    int n=4e4,m,i;
    seg::init(n);
    cin>>m;
    while (m--)
    {
        int op;
        cin>>op;
        if (op)
        {
            int x[2],y[2];
            cin>>x[0]>>y[0]>>x[1]>>y[1];
            for (int &v:x) v=(v+ans-1)%39989+1;
            for (int &v:y) v=(v+ans-1)%inf+1;
            if (x[0]>x[1]||x[0]==x[1]&&y[0]>y[1]) swap(x[0],x[1]),swap(y[0],y[1])
                ;
```

```
            static int id;
            seg::insert({x[0],y[0],x[1]-x[0],y[1]-y[0],++id});
        }
        else
        {
            int x;
            cin>>x;
            x=(x+ans-1)%39989+1;
            cout<<(ans=max(0,seg::askmax(x).id))<<'\n';
        }
    }
}
```

1.13   李超树（动态开点）

```
struct Q
{
    int k;
    ll b;
    ll y(const int &x) const {return (ll)k*x+b;}
};
const int inf=1e9;
const ll INF=1e18;
struct seg//可以析构，不能并行
{
    const static int N=4e5+2,M=N*8*8+(1<<23);
    const static ll npos=9e18;
    static Q s[M];
    static int c[M][2],id;
    int z,y,L,R;
    seg(int l,int r)
    {
        L=l;R=r;id=1;
        s[1]={0,npos};
        assert(L<=R&&(ll)R-L<1ll<<32);
    }
private:
    void insert(int &x,int l,int r,Q o)
    {
        if (!x)
        {
            x=++id;
            assert(id<M);
            s[x]={0,npos};
        }
        int m=l+(r-l>>1);
        if (z<=l&&r<=y)
        {
            if (s[x].y(m)>o.y(m)) swap(s[x],o);
            if (s[x].y(l)>o.y(l)) insert(c[x][0],l,m,o);
            else if (s[x].y(r)>o.y(r)) insert(c[x][1],m+1,r,o);
            return;
        }
        if (z<=m) insert(c[x][0],l,m,o);
        if (y>m) insert(c[x][1],m+1,r,o);
    }
```

```cpp
public:
    void insert(const Q &x,const int &l,const int &r)//[l,r]
    {
        z=l;y=r;int tmp=1;
        insert(tmp,L,R,x);
        assert(tmp==1);
    }
    ll askmin(const int &p)
    {
        ll res=s[1].y(p);
        int l=L,r=R,m,x=1;
        while (l<r)
        {
            m=l+(r-l>>1);
            if (p<=m) x=c[x][0],r=m; else x=c[x][1],l=m+1;
            if (!x) return res;
            res=min(res,s[x].y(p));
        }
        return res;
    }
    ~seg()
    {
        ++id;
        while (--id) c[id][0]=c[id][1]=0;
    }
};
Q seg::s[seg::M];
int seg::c[seg::M][2],seg::id;
```

1.14   splay

$O(n)$，$O((n+q)\log n)$。

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef unsigned int ui;
const int N=1e6+20,p=998244353;
void inc(int &x,const int y){if ((x+=y)>=p) x-=p;}
void dec(int &x,const int y){if ((x-=y)<0) x+=p;}
void mul(int &x,const int y){x=(ll)x*y%p;}
template<int N> struct _splay
{
    int c[N][2],plz[N],clz[N],st[N],siz[N],s[N],v[N],f[N];
    bool fg[N],flz[N];
    int tp,rt;
    void allout(int x)
    {
        if (!x) return;
        pushdown(x);
        allout(c[x][0]);
        if (x>2) printf("%d␣",v[x]);
        allout(c[x][1]);
    }
    void out(int x)
    {
```

```
        printf("%d:␣c␣%d␣&␣%d␣f␣%d␣s␣%d␣v␣%d␣siz␣%d\n",x,c[x][0],c[x][1],f[x],s[
            x],v[x],siz[x]);
        if (c[x][0]) out(c[x][0]);
        if (c[x][1]) out(c[x][1]);
        if (x==rt) puts("-------------------");
    }
    void iinit()
    {
        for (int i=1;i<N;i++) st[N-i]=i;
        tp=N-1;
    }
    void init()
    {
        tp=N-3;
        c[1][0]=c[1][1]=flz[1]=plz[1]=fg[1]=v[1]=f[1]=s[1]=0;clz[1]=1;
        c[2][0]=c[2][1]=flz[2]=plz[2]=fg[2]=v[2]=f[2]=s[2]=0;clz[2]=1;
        c[1][1]=2;f[2]=1;rt=1;siz[2]=1;siz[1]=2;
    }
    void pushup(int x)
    {
        s[x]=((ui)s[c[x][0]]+s[c[x][1]]+v[x])%p;
        siz[x]=siz[c[x][0]]+siz[c[x][1]]+1;
    }
    void pushdown(int x)
    {
        int lc=c[x][0],rc=c[x][1];
        if (flz[x])
        {
            if (lc) flz[lc]^=1,swap(c[lc][0],c[lc][1]);
            if (rc) flz[rc]^=1,swap(c[rc][0],c[rc][1]);
            flz[x]=0;
        }
        if (fg[x])
        {
            clz[x]=1;plz[x]=0;
            if (lc) fg[lc]=1,v[lc]=v[x],s[lc]=(ll)v[x]*siz[lc]%p;
            if (rc) fg[rc]=1,v[rc]=v[x],s[rc]=(ll)v[x]*siz[rc]%p;
            fg[x]=0;
        }
        else
        {
            if (clz[x]!=1)
            {
                if (lc) mul(clz[lc],clz[x]),mul(s[lc],clz[x]),mul(plz[lc],clz[x])
                    ,mul(v[lc],clz[x]);
                if (rc) mul(clz[rc],clz[x]),mul(s[rc],clz[x]),mul(plz[rc],clz[x])
                    ,mul(v[rc],clz[x]);
                clz[x]=1;
            }
            if (plz[x])
            {
                if (lc) inc(plz[lc],plz[x]),inc(v[lc],plz[x]),s[lc]=(s[lc]+(ll)
                    siz[lc]*plz[x])%p;
                if (rc) inc(plz[rc],plz[x]),inc(v[rc],plz[x]),s[rc]=(s[rc]+(ll)
                    siz[rc]*plz[x])%p;
                plz[x]=0;
            }
```

```
        }
    }
    void zigzag(int x)
    {
        int y=f[x],z=f[y],typ=(c[y][0]==x);
        if (z) c[z][c[z][1]==y]=x;
        f[x]=z;f[y]=x;c[y][typ^1]=c[x][typ];
        if (c[x][typ]) f[c[x][typ]]=y;
        c[x][typ]=y;
        pushup(y);
    }
    void allpd(int x)
    {
        static int st[N],tp;
        st[tp=1]=x;
        while (x=f[x]) st[++tp]=x;
        while (tp) pushdown(st[tp--]);
    }
    void splay(int x,int tar)
    {
        if (!tar) rt=x;
        int y;
        while ((y=f[x])!=tar)
        {
            if (f[y]!=tar) zigzag(c[f[y]][0]==y^c[y][0]==x?x:y);
            zigzag(x);
        }
        pushup(x);
    }
    void find(int kth,int tar)
    {
        int x=rt;
        while (siz[c[x][0]]+1!=kth)
        {
            pushdown(x);
            if (siz[c[x][0]]>=kth) x=c[x][0]; else
            {
                kth-=siz[c[x][0]]+1;
                x=c[x][1];
            }
        }
        pushdown(x);
        splay(x,tar);
    }
    int rk(int x)
    {
        allpd(x);
        splay(x,0);
        return siz[c[x][0]];
    }
    void split(int x,int y)
    {
        find(x,0);find(y+2,rt);
    }
    int npt()
    {
        int x=st[tp--];
```

```
        c[x][0]=c[x][1]=plz[x]=siz[x]=s[x]=v[x]=fg[x]=flz[x]=0;
        clz[x]=1;
        return x;
    }
    int build(int *a,int l,int r)
    {
        if (l>r) return 0;
        int m=l+r>>1,x;
        v[x=npt()]=a[m];
        //printf("build %d %d %d\n",l,r,x);
        if (l==r)
        {
            siz[x]=1;
            s[x]=v[x];
            return x;
        }
        c[x][0]=build(a,l,m-1);
        c[x][1]=build(a,m+1,r);
        if (c[x][0]) f[c[x][0]]=x;
        if (c[x][1]) f[c[x][1]]=x;
        pushup(x);
        return x;
    }
    void ins(int pos,int *a,int n)//在pos后插入
    {
        if (!n) return;
        split(pos+1,pos);
        // out(rt);
        int x=c[rt][1];
        c[x][0]=build(a,1,n);
        // printf("%d %d\n",x,c[x][0]);
        f[c[x][0]]=x;
        pushup(x);pushup(rt);
    }
    void del(int l,int r)//删除[l,r]
    {
        split(l,r);
        c[c[rt][1]][0]=0;
        pushup(c[rt][1]);
        pushup(rt);
    }
    void rev(int l,int r)
    {
        split(l,r);
        int x=c[c[rt][1]][0];
        swap(c[x][0],c[x][1]);
        flz[x]^=1;
    }
    void add(int l,int r,int val)
    {
        split(l,r);
        int x=c[c[rt][1]][0];
        inc(v[x],val);inc(plz[x],val);
        s[x]=(s[x]+(ll)val*siz[x])%p;
        pushup(f[x]);pushup(rt);
    }
    void multi(int l,int r,int val)
```

```
        {
            split(l,r);
            int x=c[c[rt][1]][0];
            mul(v[x],val);mul(plz[x],val);
            mul(s[x],val);mul(clz[x],val);
            pushup(f[x]);pushup(rt);
        }
        void mov(int l1,int r1,int l2)//都是原下标
        {
            if (l2>l1) l2-=r1-l1+1;
            split(l1,r1);int x=c[c[rt][1]][0];
            allpd(x);c[f[x]][0]=0;
            pushup(f[x]);pushup(rt);
            split(l2+1,l2);
            allpd(c[rt][1]);
            c[c[rt][1]][0]=x;f[x]=c[rt][1];
            pushup(f[x]);pushup(rt);
        }
        int sum(int l,int r)
        {
            split(l,r);//puts("spe ");out(rt);
            return s[c[c[rt][1]][0]];
        }
};
_splay<N> s;
int a[N];
int n,q,i,x,y,z;
void read(int &x)
{
    int c=getchar();
    while (c<48||c>57) c=getchar();
    x=c^48;c=getchar();
    while (c>=48&&c<=57) x=x*10+(c^48),c=getchar();
}
int main()
{
    read(n);read(q);s.iinit();
    for (i=1;i<=n;i++) a[i]=i;
    s.init();s.ins(0,a,n);//s.out(s.rt);
    while (q--)
    {
        read(x);read(y);s.rev(x,y);
    }
    s.allout(s.rt);
}
```

1.15 区间线性基

$O((n+q)\log a)$，$O(n \log a)$。

```
template<class T,int M=sizeof(T)*8> struct base//线性基
{
    array<T,M> a;
    base():a{ } { }
    bool insert(T x)//线性基插入
    {
        if (x==0) return 0;
```

```
        for (int i=__lg(x); x; i=__lg(x))
        {
            if (!a[i])
            {
                a[i]=x;
                return 1;
            }
            x^=a[i];
        }
        return 0;
    }
    base &operator+=(const base &o)//合并线性基
    {
        for (ll x:o.a) if (x) insert(x);
        return *this;
    }
    base operator+(base o) const { return o+=*this; }//合并线性基
    bool contains(T x) const//查询是否能 xor 出 x
    {
        if (x==0) return 1;
        for (int i=__lg(x); x; i=__lg(x))
        {
            if (!a[i]) return 0;
            x^=a[i];
        }
        return 1;
    }
    T max(T x=0) const//查询子集 xor 的最大值。若有传入参数 x，表示子集 xor x
        的最大值。
    {
        for (int i=M-1; i>=0; i--) if (1^x>>i&1) x^=a[i];
        return x;
    }
};
template<class T=ll,int M=sizeof(T)*8> struct rangebase//[0,...)
{
    vector<array<pair<T,int>,M>> a;
    rangebase():a{{ }} { }
    rangebase(const vector<T> &b):a{{ }} { for (T x:b) insert(x); }//直接用一
        个 vector 构造
    void push_back(T x)//在最后插入 x
    {
        int n=a.size()-1;
        a.push_back(a.back());
        if (x==0) return;
        for (int i=__lg(x); x; i=__lg(x))
        {
            auto &[v,p]=a.back()[i];
            if (v)
            {
                if (n>p)
                {
                    swap(x,v);
                    swap(n,p);
                }
                x^=v;
            }
```

```
            else
            {
                v=x;
                p=n;
                return;
            }
        }
    }
    base<T,M> ask(int l,int r)//查询 $[l,r)$ 元素构成的线性基。下标从 0 开始
        （同 vector）
    {
        assert(0<=l&&l<=r&&r<=a.size());
        base<T,M> res;
        for (int i=0; i<M; i++)
        {
            auto [v,p]=a[r][i];
            if (v&&p>=l) res.a[i]=v;
        }
        return res;
    }
};
```

1.16 splay 重构

$O(n)$，$O((n+q)\log n)$。

```
template<typename info,typename tag> struct splay
{
#define _rev
    struct node
    {
        node *c[2],*f;
        int siz;
        info s,v;
        tag t;
        node():c{},f(0),siz(1),s(),v(),t() {}
        node(info x):c{},f(0),siz(1),s(x),v(x),t() {}
        void operator+=(const tag &o)
        {
            s+=o; v+=o; t+=o;
#ifdef _rev
            if (o.rev) swap(c[0],c[1]);
#endif
        }
        void pushup()
        {
            if (c[0]) s=c[0]->s+v,siz=c[0]->siz+1; else s=v,siz=1;
            if (c[1]) s=s+c[1]->s,siz+=c[1]->siz;
        }
        void pushdown()
        {
            for (auto x:c) if (x) *x+=t;
            t={};
        }
        void zigzag()
        {
```

```cpp
            node *y=f,*z=y->f;
            int typ=y->c[0]==this;
            if (z) z->c[z->c[1]==y]=this;
            f=z; y->f=this;
            y->c[typ^1]=c[typ];
            if (c[typ]) c[typ]->f=y;
            c[typ]=y;
            y->pushup();
        }
        void splay(node *tar)//不要在 makeroot 以外调用
        {
            for (node *y=f; y!=tar; zigzag(),y=f) if (node *z=y->f; z!=tar) (z->c
                [1]==y^y->c[1]==this?this:y)->zigzag();
            pushup();
        }
        void clear()
        {
            for (node *x:c) if (x) x->clear();
            delete this;
        }
    };
    node *rt;
    void debug()
    {
        map<node *,int> id;
        id[0]=0; id[rt]=1;
        int cnt=1;
        function<void(node *)> out=[&](node *x)
        {
            if (!x) return;
            for (auto y:x->c) if (!id.count(y)) id[y]=++cnt;
            cerr<<id[x]<<' '<<id[x->c[0]]<<' '<<id[x->c[1]]<<' '<<id[x->f]<<' '<<
                x->siz<<'\n';
            for (auto y:x->c) out(y);
        };
        out(rt);
    }
    node *build(info *a,int n)
    {
        if (n==0) return 0;
        int m=n-1>>1;
        node *x=new node(a[m]);
        x->c[0]=build(a,m);
        x->c[1]=build(a+m+1,n-1-m);
        for (node *y:x->c) if (y) y->f=x;
        x->pushup();
        return x;
    }
    splay()
    {
        rt=new node;
        rt->c[1]=new node;
        rt->c[1]->f=rt;
        rt->siz=2;
    }
    int shift;
    splay(info *a,int l,int r)//[l,r)
```

```
    {
        shift=l-1;
        rt=new node;
        rt->c[1]=new node;
        rt->c[1]->f=rt;
        if (l<r)
        {
            rt->c[1]->c[0]=build(a+1,r-1);
            rt->c[1]->c[0]->f=rt->c[1];
        }
        rt->c[1]->pushup();
        rt->pushup();
    }
    void makeroot(node *u,node *tar)
    {
        if (!tar) rt=u;
        u->splay();
    }
    void findnth(int k,node *tar)
    {
        node *x=rt;
        while (1)
        {
            x->pushdown();
            int v=x->c[0]?x->c[0]->siz:0;
            if (v+1==k) { x->splay(tar); if (!tar) rt=x; return; }
            if (v>=k) x=x->c[0]; else x=x->c[1],k-=v+1;
        }
    }
    void split(int l,int r)
    {
        assert(1<=l&&r<=rt->siz-2&&l-1<=r);
        findnth(l,0);
        findnth(r+2,rt);
    }
#ifdef _rev
    void reverse(int l,int r)
    {
        l-=shift; r-=shift+1;
        if (l-1==r) return;
        assert(1<=l&&l<=r&&r<=rt->siz-2);
        split(l,r);
        *(rt->c[1]->c[0])+=tag(1);
    }
#endif
    void insert(int pos,info x)//insert before pos
    {
        pos-=shift;
        assert(1<=pos&&pos<=rt->siz-1);
        split(pos,pos-1);
        rt->c[1]->c[0]=new node(x);
        rt->c[1]->c[0]->f=rt->c[1];
        rt->c[1]->pushup();
        rt->pushup();
    }
    void insert(int pos,info *a,int n)//insert before pos, [1,n]
    {
```

```cpp
            pos-=shift;
            assert(1<=pos&&pos<=rt->siz-1);
            split(pos,pos-1);
            rt->c[1]->c[0]=build(a,n);
            rt->c[1]->c[0]->f=rt->c[1];
            rt->c[1]->pushup();
            rt->pushup();
        }
        void erase(int pos)
        {
            pos-=shift;
            assert(1<=pos&&pos<=rt->siz-2);
            split(pos,pos);
            delete rt->c[1]->c[0];
            rt->c[1]->c[0]=0;
            rt->c[1]->pushup();
            rt->pushup();
        }
        void erase(int l,int r)
        {
            l-=shift; r-=shift+1;
            if (l-1==r) return;
            assert(1<=l&&l<=r&&r<=rt->siz-2);
            split(l,r);
            rt->c[1]->c[0]->clear();
            rt->c[1]->c[0]=0;
            rt->c[1]->pushup();
            rt->pushup();
        }
        void modify(int pos,info x)//not checked
        {
            pos-=shift;
            assert(1<=pos&&pos<=rt->siz-2);
            findnth(pos+1,0);
            rt->v=x; rt->pushup();
        }
        void modify(int l,int r,tag w)
        {
            l-=shift; r-=shift+1;
            if (l-1==r) return;
            assert(1<=l&&l<=r&&r<=rt->siz-2);
            split(l,r);
            node *x=rt->c[1]->c[0];
            *x+=w;
            rt->c[1]->pushup();
            rt->pushup();
        }
        info ask(int l,int r)
        {
            l-=shift; r-=shift+1;
            assert(1<=l&&l<=r&&r<=rt->siz-2);
            split(l,r);
            return rt->c[1]->c[0]->s;
        }
        ~splay() { rt->clear(); }
#undef _rev
};
```

```cpp
struct Q
{
    bool rev;
    Q():rev(0) {}
    Q(bool c):rev(c) {}
    void operator+=(const Q &o)
    {
        rev^=o.rev;
    }
};
struct P
{
    ll s;
    void operator+=(const Q &o) const
    {
    }
    P operator+(const P &o) const { return{s+o.s}; }
};
```

1.17 第 k 大线性基

注意数字大于 2^50 时可能要修改循环范围。

$O((n+q)\log a)$，$O(\log a)$。

```cpp
void ins(ll x)
{
    if (x==0) return con=1,void();//con=1:有0
    int i;
    for (i=50;x;i--) if (x>>i&1)
    {
        if (!ji[i]) {ji[i]=x;i=-1;break;}x^=ji[i];
    }
    if (!x) con=1;
}
ll kmax(ll x)//查询第 k 大（本质不同，不允许空集）的 xor 结果，若有初始值改 r 即可
{
    ll r=0;
    int m=0,i;
    for (i=50;~i;i--) if (ji[i]) a[++m]=i;
    if (1ll<<m<=x-con) return -1;//个数少于k
    x=(1ll<<m)-x;
    for (i=1;i<=m;i++) if ((x>>m-i^r>>a[i])&1) r^=ji[a[i]];
    return r;
}
ll kmin(ll x)//查询第 k 小（本质不同，不允许空集）的 xor 结果，若有初始值改 r 即可
{
    ll r=0;
    int m=0,i;
    for (i=50;~i;i--) if (ji[i]) a[++m]=i;
    x-=con;
    if (1ll<<m<=x) return -1;//个数少于k
    for (i=1;i<=m;i++) if ((x>>m-i^r>>a[i])&1) r^=ji[a[i]];
    return r;
}
```

1.18 fhq-treap

洛谷模板：普通平衡树。

$O((n+q)\log n)$，$O(n)$。

```cpp
const int N=1.1e6+2;
int c[N][2],v[N],w[N],s[N];
int n,i,x,y,ds,val,kth,p,q,z,rt,la,m,ans;
void pushup(const int x)
{
    s[x]=s[c[x][0]]+s[c[x][1]]+1;
}
void split_val(int now,int &x,int &y)//调用外部val,相等归入y
{
    if (!now) return x=y=0,void();
    if (val<=v[now]) split_val(c[y=now][0],x,c[now][0]);
    else split_val(c[x=now][1],c[now][1],y);
    pushup(now);
}
void split_kth(int now,int &x,int &y)//调用外部kth, 左子树大小为 kth
{
    if (!now) return x=y=0,void();
    if (kth<=s[c[now][0]]) split_kth(c[y=now][0],x,c[now][0]);
    else kth-=s[c[now][0]]+1,split_kth(c[x=now][1],c[now][1],y);
    pushup(now);
}
int merge(int x,int y)//小根ver.
{
    if (!(x&&y)) return x|y;
    if (w[x]<w[y]) {c[x][1]=merge(c[x][1],y);pushup(x);return x;}
    else {c[y][0]=merge(x,c[y][0]);pushup(y);return y;}
}
int main()
{
    read(n);read(m);srand(998244353);
    for (i=1;i<=n;i++)
    {
        read(x);val=v[++ds]=x;w[ds]=rand();s[ds]=1;split_val(rt,p,q);rt=merge(
            merge(p,ds),q);
    }
    while (m--)
    {
        read(y);read(x);x^=la;
        if (y==4)//找到第 x 小的
        {
            kth=x;split_kth(rt,p,q);x=p;
            while (c[x][1]) x=c[x][1];
            ans^=(la=v[x]);rt=merge(p,q);
            continue;
        }
        val=x;//注意这一步
        if (y==1)//插入 x
        {
            v[++ds]=x;w[ds]=rand();s[ds]=1;
            split_val(rt,p,q);rt=merge(merge(p,ds),q);
            continue;
        }
        if (y==2)//删除一个 x
```

```
        {
            split_val(rt,p,q);kth=1;split_kth(q,i,z);
            rt=merge(p,z);continue;
        }
        if (y==3)//询问 x 的排名（比 x 小的数字个数 +1）
        {
            split_val(rt,p,q);ans^=(la=s[p]+1);
            rt=merge(p,q);continue;
        }
        if (y==5)//询问比 x 小的最大值
        {
            split_val(rt,p,q);x=p;
            while (c[x][1]) x=c[x][1];ans^=(la=v[x]);
            rt=merge(p,q);continue;
        }
        ++val;split_val(rt,p,q);x=q;//询问比 x 大的最小值
        while (c[x][0]) x=c[x][0];
        ans^=(la=v[x]);rt=merge(p,q);
    }
    printf("%d",ans);
}
```

1.19　笛卡尔树的线性建树

$p[1, 2, \ldots, n]$ 是原序列，c 表示子结点。

笛卡尔树满足堆性质（权值小于等于子结点权值），并且中序遍历是原序列。

$O(n)$，$O(n)$。

```
int c[N][2],p[N],st[N];
int main()
{
    int i,n,tp=0;
    ll la=0,ra=0;
    read(n);
    for (i=1;i<=n;i++)
    {
        read(p[i]);st[tp+1]=0;
        while ((tp)&&(p[st[tp]]>p[i])) --tp;
        c[c[st[tp]][1]=i][0]=st[tp+1];st[++tp]=i;
    }
    for (i=1;i<=n;i++) la^=(ll)i*(c[i][0]+1);
    for (i=1;i<=n;i++) ra^=(ll)i*(c[i][1]+1);
    printf("%lld %lld",la,ra);
}
```

1.20　扫描线

求矩形并的面积和周长（包括内周长）

$O((n + q) \log n)$，$O(n + q)$。

```
using T=ll;
vector<T> fun(vector<tuple<T, T, T, T>> &a)
{
    vector<T> x;
    for (auto [x1, y1, x2, y2]:a)
```

```cpp
        {
            x.push_back(x1);
            x.push_back(x2);
        }
        sort(all(x)); x.resize(unique(all(x))-x.begin());
        for (auto &[x1, y1, x2, y2]:a)
        {
            x1=lower_bound(all(x), x1)-x.begin();
            x2=lower_bound(all(x), x2)-x.begin();
        }
        return x;
}
struct sgt
{
        int n, z, y, d;
        vector<T> cnt, &p;
        vector<int> mn, lz;
        void build(int x, int l, int r)
        {
            cnt[x]=p[min(r, n-1)]-p[l];
            if (l+1==r) return;
            int c=x*2, m=l+r>>1;
            build(c, l, m); build(c+1, m, r);
        }
        sgt(vector<T> &p):n(p.size()), p(p), cnt(n*4), mn(n*4), lz(n*4) { build
            (1, 0, n); }
        void dfs(int x, int l, int r)
        {
            if (z<=l&&r<=y)
            {
                mn[x]+=d;
                lz[x]+=d;
                return;
            }
            int c=x*2, m=l+r>>1;
            if (lz[x])
            {
                lz[c]+=lz[x]; lz[c+1]+=lz[x];
                mn[c]+=lz[x]; mn[c+1]+=lz[x];
                lz[x]=0;
            }
            if (z<m) dfs(c, l, m);
            if (m<y) dfs(c+1, m, r);
            mn[x]=min(mn[c], mn[c+1]);
            cnt[x]=cnt[c]*(mn[x]==mn[c])+cnt[c+1]*(mn[x]==mn[c+1]);
        }
        void modify(int l, int r, int dt)
        {
            z=l;
            y=r;
            d=dt;
            dfs(1, 0, n);
        }
};
T area(vector<tuple<T, T, T, T>> a)//[x1,y1,x2,y2], x1<y1, x2<y2
{
        int n=a.size(), i;
```

```cpp
    auto X=fun(a);
    vector<tuple<T, int, T, T>> b(n*2);
    for (i=0; i<n; i++)
    {
        auto [x1, y1, x2, y2]=a[i];
        b[i]={y1, -1, x1, x2};
        b[i+n]={y2, 1, x1, x2};
    }
    sort(all(b), greater<>());
    sgt s(X);
    T lst=0, ans=0;
    for (auto [y, d, l, r]:b)
    {
        ans+=(lst-y)*(X.back()-X[0]-s.cnt[1]);
        s.modify(l, r, d);
        lst=y;
    }
    return ans;
}
T perimeter_x(vector<tuple<T, T, T, T>> a)
{
    int n=a.size(), i;
    auto X=fun(a);
    vector<tuple<T, int, T, T>> b(n*2);
    for (i=0; i<n; i++)
    {
        auto [x1, y1, x2, y2]=a[i];
        b[i]={y1, -1, x1, x2};
        b[i+n]={y2, 1, x1, x2};
    }
    sort(all(b), greater<>());
    sgt s(X);
    T lst=s.cnt[1], ans=0;
    for (auto [y, d, l, r]:b)
    {
        s.modify(l, r, d);
        T cur=s.cnt[1];
        ans+=abs(lst-cur);
        lst=cur;
    }
    return ans;
}
T perimeter(vector<tuple<T, T, T, T>> a)//[x1,y1,x2,y2], x1<y1, x2<y2
{
    T ansx=perimeter_x(a);
    for (auto &[x1, y1, x2, y2]:a)
    {
        swap(x1, y1);
        swap(x2, y2);
    }
    T ansy=perimeter_x(a);
    return ansx+ansy;
}
```

1.21 Segmenttree Beats!

核心是 P（tag）和 Q（info）的维护。线段树部分是套的模板，并非全都有用。

1. l, r, k：对于所有的 $i \in [l, r]$，将 A_i 加上 k（k 可以为负数）。

2. l, r, v：对于所有的 $i \in [l, r]$，将 A_i 变成 $\min(A_i, v)$。

3. l, r：求 $\sum\limits_{i=l}^{r} A_i$。

4. l, r：对于所有的 $i \in [l, r]$，求 A_i 的最大值。

5. l, r：对于所有的 $i \in [l, r]$，求 B_i 的最大值。

```cpp
struct P
{
    ll tg,L,R;
    P(ll a=0,ll b=-inf,ll c=inf):tg(a),L(b),R(c) { }
    void operator+=(P o)
    {
        o.L-=tg; o.R-=tg; tg+=o.tg;
        if (L>=o.R) L=R=o.R;
        else if (R<=o.L) L=R=o.L;
        else cmax(L,o.L),cmin(R,o.R);
    }
};
struct Q
{
    ll mx0,cmx,mx1,mn0,cmn,mn1,cnt,sum;
    Q():mx0(-inf),cmx(0),mx1(-inf),mn0(inf),cmn(0),mn1(inf),cnt(0),sum(0) { }
    Q(ll x):mx0(x),cmx(1),mx1(-inf),mn0(x),cmn(1),mn1(inf),cnt(1),sum(x) { }
    bool operator+=(const P &o)
    {
        if (o.L==o.R)
        {
            ll c=cnt;
            *this=Q(o.L+o.tg);
            cnt=cmx=cmn=c;
            sum=cnt*(o.L+o.tg);
            return 1;
        }
        if (o.L>=mn1||o.R<=mx1) return 0;
        if (mx0==mn0)
        {
            mn0=min(o.R,max(mx0,o.L));
            sum+=cnt*(mn0-mx0);
            mx0=mn0;
        }
        else
        {
            if (o.L>mn0)
            {
                sum+=(o.L-mn0)*cmn;
                mn0=o.L;
                cmax(mx1,o.L);
            }
```

```cpp
                if (o.R<mx0)
                {
                    sum+=(o.R-mx0)*cmx;
                    mx0=o.R;
                    cmin(mn1,o.R);
                }
            }
            if (o.tg)
            {
                sum+=o.tg*cnt;
                mx0+=o.tg;
                mx1+=o.tg;
                mn0+=o.tg;
                mn1+=o.tg;
            }
            return 1;
        }
};
Q operator+(const Q &a,const Q &b)
{
    Q res;
    res.sum=a.sum+b.sum;
    res.cnt=a.cnt+b.cnt;
    res.mx0=max(a.mx0,b.mx0);
    res.mx1=max(a.mx1,b.mx1);
    if (res.mx0==a.mx0) res.cmx+=a.cmx; else cmax(res.mx1,a.mx0);
    if (res.mx0==b.mx0) res.cmx+=b.cmx; else cmax(res.mx1,b.mx0);

    res.mn0=min(a.mn0,b.mn0);
    res.mn1=min(a.mn1,b.mn1);
    if (res.mn0==a.mn0) res.cmn+=a.cmn; else cmin(res.mn1,a.mn0);
    if (res.mn0==b.mn0) res.cmn+=b.cmn; else cmin(res.mn1,b.mn0);

    return res;
}
template<class info,class tag> struct sgt
{
    int n,shift;
    vector<info> s;
    vector<tag> tg;
    vector<char> lz;
    template<class T> void build(T *a,int x,int l,int r)
    {
        if (l==r)
        {
            s[x]=a[l];
            return;
        }
        int c=x*2,m=l+r>>1;
        build(a,c,l,m); build(a,c+1,m+1,r);
        s[x]=s[c]+s[c+1];
    }
    template<class T> sgt(T *b,int L,int R):n(R-L+1),shift(L-1),s(R-L+1<<2),tg(R
        -L+1<<2),lz(R-L+1<<2)
    {
        build(b+L-1,1,1,n);
    }//[L,R]
```

```
    int z,y;
    info res;
    tag dt;
    bool fir;
private:
    void pushdown(int x)
    {
        int c=x*2;
        if (lz[x])
        {
            if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
            lz[c]=1;
            if (!(s[c]+=tg[x]))
            {
                pushdown(c);
                s[c]=s[c*2]+s[c*2+1];
            }
            c^=1;
            if (lz[c]) tg[c]+=tg[x]; else tg[c]=tg[x];
            lz[c]=1;
            if (!(s[c]+=tg[x]))
            {
                pushdown(c);
                s[c]=s[c*2]+s[c*2+1];
            }
            c^=1;
            lz[x]=0;
        }
    }
    void _modify(int x,int l,int r)
    {
        if (z<=l&&r<=y)
        {
            if (lz[x]) tg[x]+=dt; else tg[x]=dt;
            lz[x]=1;
            if (!(s[x]+=dt))
            {
                pushdown(x);
                s[x]=s[x*2]+s[x*2+1];
            }
            return;
        }
        int c=x*2,m=l+r>>1;
        pushdown(x);
        if (z<=m) _modify(c,l,m);
        if (m<y) _modify(c+1,m+1,r);
        s[x]=s[c]+s[c+1];
    }
    void ask(int x,int l,int r)
    {
        if (z<=l&&r<=y)
        {
            res=fir?s[x]:res+s[x];
            fir=0;
            return;
        }
        int c=x*2,m=l+r>>1;
```

```cpp
        pushdown(x);
        if (z<=m) ask(c,l,m);
        if (m<y) ask(c+1,m+1,r);
    }
    function<bool(info)> check;
    void find_left_most(int x,int l,int r)
    {
        if (r<z||!check(s[x])) return;
        if (l==r) { y=l; res=s[x]; return; }
        int c=x*2,m=l+r>>1;
        pushdown(x);
        find_left_most(c,l,m);
        if (y==n+1) find_left_most(c+1,m+1,r);
    }
    void find_right_most(int x,int l,int r)
    {
        if (l>y||!check(s[x])) return;
        if (l==r) { z=l; res=s[x]; return; }
        int c=x*2,m=l+r>>1;
        pushdown(x);
        find_right_most(c+1,m+1,r);
        if (z==0) find_right_most(c,l,m);
    }
public:
    void modify(int l,int r,const tag &x)//[l,r]
    {
        z=l-shift; y=r-shift; dt=x;
        // cerr<<"modify ["<<l<<','<<r<<"] "<<'\n';
        assert(1<=z&&z<=y&&y<=n);
        _modify(1,1,n);
    }
    void modify(int pos,const info &o)
    {
        pos-=shift;
        int l=1,r=n,m,c,x=1;
        while (l<r)
        {
            c=x*2; m=l+r>>1;
            pushdown(x);
            if (pos<=m) x=c,r=m; else x=c+1,l=m+1;
        }
        s[x]=o;
        while (x>>=1) s[x]=s[x*2]+s[x*2+1];
    }
    info ask(int l,int r)//[l,r]
    {
        z=l-shift; y=r-shift; fir=1;
        // cerr<<"ask ["<<l<<','<<r<<"] "<<'\n';
        assert(1<=z&&z<=y&&y<=n);
        ask(1,1,n);
        return res;
    }
    pair<int,info> find_left_most(int l,const function<bool(info)> &_check)//y=n
        +1 第二个参数是乱给的
    {
        check=_check;
        z=l-shift; y=n+1;
```

```
        assert(1<=z&&z<=n+1);
        find_left_most(1,1,n);
        return {y+shift,res};
    }
    pair<int,info> find_right_most(int r,const function<bool(info)> &_check)//z
        =0 第二个参数是乱给的
    {
        check=_check;
        z=0; y=r-shift;
        assert(0<=y&&y<=n);
        find_right_most(1,1,n);
        return {z+shift,res};
    }
};
//要求：具有 info+info, info+=tag, tag+=tag。info, tag 需要拥有默认构造，但不必拥
    有正确的值。
//采用左闭右闭
mt19937 rnd(345);
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout<<fixed<<setprecision(15);
    int n,q,i;
    cin>>n>>q;
    vector<ll> a(n);
    cin>>a;
    sgt<Q,P> s(a.data(),0,n-1);
    while (q--)
    {
        int op,l,r;
        cin>>op>>l>>r;
        --r;
        if (op==3)
        {
            ll res=s.ask(l,r).sum;
            cout<<res<<'\n';
        }
        else
        {
            ll b;
            cin>>b;
            if (op==0) s.modify(l,r,{0,-inf,b});
            else if (op==1) s.modify(l,r,{0,b});
            else s.modify(l,r,{b});
        }
    }
}
```

1.22 k-d 树（二进制分组）

均摊 $O(\log^2 n)$ 插入，$O(\sqrt{n})$ 矩形查询。

```
#define tmpl template<typename T>
typedef long long ll;
tmpl struct P
{
    ll x,y;
```

```cpp
        T v;
};
tmpl struct Q
{
    ll x[2],y[2];
    bool t;
    T s;
    Q() {}
    Q(const P<T> &a)
    {
        x[0]=x[1]=a.x;
        y[0]=y[1]=a.y;
        s=a.v;
    }
};
tmpl bool cmp0(const P<T> &a,const P<T> &b) { return a.x<b.x; }
tmpl bool cmp1(const P<T> &a,const P<T> &b) { return a.y<b.y; }
tmpl struct kdt
{
    vector<P<T>> c;
    vector<Q<T>> a;
    ll m,u,d,l,r;
    T ans;
    bool fir;
    void build(int x,P<T> *b,int n)
    {
        if (x==1)
        {
            a.resize(m=n<<1);
            a[x].t=0;
            c.resize(n);
            for (int i=0; i<n; i++) c[i]=b[i];
        }
        if (n==1)
        {
            a[x]=Q<T>(b[0]);
            return;
        }
        int mid=n>>1,c=x<<1;
        nth_element(b,b+mid,b+n,a[x].t?cmp1<T>:cmp0<T>);
        a[c].t=a[c|1].t=a[x].t^1;
        build(c,b,mid);
        build(c|1,b+mid,n-mid);
        a[x].s=a[c].s+a[c|1].s;
        a[x].x[0]=min(a[c].x[0],a[c|1].x[0]);
        a[x].x[1]=max(a[c].x[1],a[c|1].x[1]);
        a[x].y[0]=min(a[c].y[0],a[c|1].y[0]);
        a[x].y[1]=max(a[c].y[1],a[c|1].y[1]);
    }
    void find(int x)
    {
        if (x>=m||a[x].x[1]<u||a[x].x[0]>d||a[x].y[1]<l||a[x].y[0]>r) return;
        if (u<=a[x].x[0]&&a[x].x[1]<=d&&l<=a[x].y[0]&&a[x].y[1]<=r)
        {
            ans=fir?a[x].s:ans+a[x].s;
            fir=0;
            return;
```

```
        }
        find(x<<1); find(x<<1|1);
    }
    pair<bool,T> find(ll x1,ll y1,ll x2,ll y2)
    {
        fir=1;
        ans={};
        u=x1; d=x2;
        l=y1; r=y2;
        find(1);
        return {!fir,ans};
    }
};
const int N=2e5+2,M=18;
tmpl struct KDT
{
    kdt<T> s[M];
    P<T> a[N];
    int n,m,i;
    KDT() { n=0; }
    KDT(int N,ll *x,ll *y,T *w)//[0,n)
    {
        n=N;
        int i,j;
        for (i=0; i<n; i++) a[i]={x[i],y[i],w[i]};
        for (i=j=0; n>>i; i++) if (n>>i&1) s[i].build(1,a+j,1<<i),j+=1<<i;
    }
    void insert(ll x,ll y,T w)//插入 (x,y) 的一个数 w
    {
        a[0]={x,y,w}; m=1;
        for (i=0; n&1<<i; i++) for (auto u:s[i].c) a[m++]=u;
        s[i].build(1,a,m);
        ++n;
    }
    pair<bool,T> ask(ll x,ll y,ll xx,ll yy)//查询 [x,xx]*[y,yy] 的和
    {
        T ans;
        bool fir=1;
        for (i=0; 1<<i<=n; i++) if (1<<i&n)
        {
            auto [_,tmp]=s[i].find(x,y,xx,yy);
            if (!_) continue;
            ans=fir?tmp:ans+tmp;
            fir=0;
        }
        return {!fir,ans};
    }
};
int x[N],y[N],w[N];
int main()
{
    ios::sync_with_stdio(0); cin.tie(0); cout.tie(0);
    int n,q,i;
    cin>>n>>q;
    for (i=0; i<n; i++) cin>>x[i]>>y[i]>>w[i];
    KDT<ll> s(n,x,y,w);
    while (q--)
```

```
    {
        int op,x,y,w;
        cin>>op>>x>>y>>w;
        if (op==0) s.insert(x,y,w); else
        {
            cin>>op;
            cout<<s.ask(x,y,w-1,op-1)<<'\n';
        }
    }
    return 0;
}
```

1.23 双端队列全局查询

对一个支持结合律的信息 T，维护 deque 内信息的和。总复杂度线性。

```cpp
template<typename T> struct dq
{
    vector<T> l,sl,r,sr;
    void push_front(const T &o)
    {
        sl.push_back(sl.size()?o+sl.back():o);
        l.push_back(o);
    }
    void push_back(const T &o)
    {
        sr.push_back(sr.size()?sr.back()+o:o);
        r.push_back(o);
    }
    void pop_front()
    {
        if (l.size()) sl.pop_back(),l.pop_back();
        else
        {
            assert(r.size());
            int n=r.size(),m,i;
            if (m=n-1>>1)
            {
                l.resize(m); sl.resize(m);
                for (i=1; i<=m; i++) l[m-i]=r[i];
                sl[0]=l[0];
                for (i=1; i<m; i++) sl[i]=l[i]+sl[i-1];
            }
            for (i=m+1; i<n; i++) r[i-(m+1)]=r[i];
            m=n-(m+1);
            r.resize(m); sr.resize(m);
            if (m)
            {
                sr[0]=r[0];
                for (i=1; i<m; i++) sr[i]=sr[i-1]+r[i];
            }
        }
    }
    void pop_back()
    {
        if (r.size()) sr.pop_back(),r.pop_back();
```

```cpp
        else
        {
            assert(l.size());
            int n=l.size(),m,i;
            if (m=n-1>>1)
            {
                r.resize(m); sr.resize(m);
                for (i=1; i<=m; i++) r[m-i]=l[i];
                sr[0]=r[0];
                for (i=1; i<m; i++) sr[i]=sr[i-1]+r[i];
            }
            for (i=m+1; i<n; i++) l[i-(m+1)]=l[i];
            m=n-(m+1);
            l.resize(m); sl.resize(m);
            if (m)
            {
                sl[0]=l[0];
                for (i=1; i<m; i++) sl[i]=l[i]+sl[i-1];
            }
        }
    }
    template<typename TT> TT ask(TT r)
    {
        if (sl.size()) r=r+sl.back();
        if (sr.size()) r=r+sr.back();
        return r;
    }
    T ask()
    {
        assert(sl.size()||sr.size());
        if (sl.size()&&sr.size()) return sl.back()+sr.back();
        return sl.size()?sl.back():sr.back();
    }
};//参数：类型。结合使用 + 运算符
```

1.24 静态矩形加矩形和

```cpp
const ll p=998244353;
struct Q
{
    int n,m;
    ll w;
    int typ;
    bool operator<(const Q &o) const
    {
        if (n!=o.n) return n<o.n;
        return typ<o.typ;
    }
};
template<typename T> struct tork
{
    vector<T> a;
    int n;
    tork(const vector<T> &b):a(all(b))
    {
        sort(all(a));
```

```cpp
        a.resize(unique(all(a))-a.begin());
        n=a.size();
    }
    tork(const T *first,const T *last):a(first,last)
    {
        sort(all(a));
        a.resize(unique(all(a))-a.begin());
        n=a.size();
    }
    void get(T &x) { x=lower_bound(all(a),x)-a.begin()+1; }
    T operator[](const int &x) { return a[x]; }
};
struct bit
{
    vector<ll> a;
    int n;
    bit() {}
    bit(int nn):n(nn),a(nn+1) {}
    template<typename T> bit(int nn,T *b):n(nn),a(nn+1)
    {
        for (int i=1; i<=n; i++) a[i]=b[i];
        for (int i=1; i<=n; i++) if (i+(i&-i)<=n) a[i+(i&-i)]+=a[i];
    }
    void add(int x,ll y)
    {
        // cerr<<"add "<<x<<" by "<<y<<endl;
        assert(1<=x&&x<=n);
        if ((a[x]+=y)>=p) a[x]-=p;
        while ((x+=x&-x)<=n) if ((a[x]+=y)>=p) a[x]-=p;
    }
    ll sum(int x)
    {
        // cerr<<"sum "<<x;
        assert(0<=x&&x<=n);
        ll r=a[x];
        while (x^=x&-x) r+=a[x];
        // cerr<<"= "<<r<<endl;
        return r%p;
    }
    ll sum(int x,int y)
    {
        return (sum(y)+p-sum(x-1))%p;
    }
};
struct matrix
{
    int l,d,r,u;
    ll w;
};
vector<ll> rec_add_rec_sum(const vector<matrix> &op,const vector<matrix> &query
    )
{
    vector<Q> a[4];
    int n=op.size(),m=query.size(),i;
    for (auto &v:a) v.reserve(n+m<<2);
    for (auto [l,d,r,u,w]:op)//[l,r)*[d,u) += w
    {
```

```
        a[0].push_back({l,d,w*l%p*d%p,-1});
        a[1].push_back({l,d,w*l%p,-1});
        a[2].push_back({l,d,w*d%p,-1});
        a[3].push_back({l,d,w,-1});
        w=(p-w)%p;
        a[0].push_back({l,u,w*l%p*u%p,-1});
        a[1].push_back({l,u,w*l%p,-1});
        a[2].push_back({l,u,w*u%p,-1});
        a[3].push_back({l,u,w,-1});
        a[0].push_back({r,d,w*r%p*d%p,-1});
        a[1].push_back({r,d,w*r%p,-1});
        a[2].push_back({r,d,w*d%p,-1});
        a[3].push_back({r,d,w,-1});
        w=(p-w)%p;
        a[0].push_back({r,u,w*r%p*u%p,-1});
        a[1].push_back({r,u,w*r%p,-1});
        a[2].push_back({r,u,w*u%p,-1});
        a[3].push_back({r,u,w,-1});
    }
    i=0;
    for (auto [l,d,r,u,w]:query)//ask sum of [l,r)*[d,u)
    {
        a[0].push_back({l,d,1,i});
        a[1].push_back({l,d,(p*2-d)%p,i});
        a[2].push_back({l,d,(p*2-l)%p,i});
        a[3].push_back({l,d,(ll)l*d%p,i});
        a[0].push_back({l,u,p-1,i});
        a[1].push_back({l,u,u%p,i});
        a[2].push_back({l,u,l%p,i});
        a[3].push_back({l,u,(p*2-l)*u%p,i});
        a[0].push_back({r,u,1,i});
        a[1].push_back({r,u,(p*2-u)%p,i});
        a[2].push_back({r,u,(p*2-r)%p,i});
        a[3].push_back({r,u,(ll)u*r%p,i});
        a[0].push_back({r,d,p-1,i});
        a[1].push_back({r,d,d%p,i});
        a[2].push_back({r,d,r%p,i});
        a[3].push_back({r,d,(p*2-d)*r%p,i});
        ++i;
    }
    assert(a[0].size()==n+m<<2);
    vector<ll> ans(m);
    auto cal=[&](vector<Q> a)
    {
        int n=a.size(),i;
        vector<int> b(n);
        for (i=0; i<n; i++) b[i]=(a[i].m-=a[i].typ>=0),a[i].n-=a[i].typ>=0;
        sort(all(a));
        tork t(b);
        for (i=0; i<n; i++) t.get(a[i].m);
        int m=t.a.size();
        bit s(m);
        for (auto [n,m,w,typ]:a) if (typ>=0) ans[typ]=(ans[typ]+s.sum(m)*w)%p;
            else s.add(m,w);
    };
    for (auto &v:a) cal(v);
    return ans;
```

```cpp
}
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(15);
    int n,m,i;
    cin>>n>>m;
    vector<matrix> a(n),b(m);
    for (auto &[l,d,r,u,w]:a) cin>>l>>d>>r>>u>>w;
    for (auto &[l,d,r,u,w]:b) cin>>l>>d>>r>>u;
    auto ans=rec_add_rec_sum(a,b);
    for (i=0; i<m; i++) cout<<ans[i]<<'\n';
}
```

1.25　线段树分裂

```cpp
namespace sgt
{
#define ask_kth
    int L=0,R=1e9;
    void set_bound(int l,int r) { L=l; R=r; }
    typedef ll info;
    const info E=0;//找不到会返回 E
    const int N=8e6+5;
#define lc(x) (a[x].lc)
#define rc(x) (a[x].rc)
#define s(x) (a[x].s)
    struct node
    {
        int lc,rc;
        info s;
    };
    node a[N];
    vector<int> id;
    int ids=0,pos,z,y;
    bool fir;
    info tmp;
    int npt()
    {
        int x;
        if (id.size()) x=id.back(),id.pop_back();
        else x=++ids;
        lc(x)=rc(x)=0;
        return x;
    }
    void pushup(int &x)
    {
        if (lc(x)&&rc(x)) s(x)=s(lc(x))+s(rc(x));
        else if (lc(x)) s(x)=s(lc(x));
        else if (rc(x)) s(x)=s(rc(x));
        else id.push_back(x),x=0;
    }
    void insert(int &x,int l,int r)
    {
        if (l==r)
```

```cpp
    {
        if (!x) x=npt(),s(x)=tmp;
        else s(x)=s(x)+tmp;
        return;
    }
    if (!x) x=npt();
    int mid=l+r>>1;
    if (pos<=mid)
    {
        insert(lc(x),l,mid);
        if (rc(x)) s(x)=s(lc(x))+s(rc(x)); else s(x)=s(lc(x));
    }
    else
    {
        insert(rc(x),mid+1,r);
        if (lc(x)) s(x)=s(lc(x))+s(rc(x)); else s(x)=s(rc(x));
    }
}
void modify(int &x,int l,int r)
{
    if (!x) x=npt();
    if (l==r)
    {
        s(x)=tmp;
        return;
    }
    int mid=l+r>>1;
    if (pos<=mid)
    {
        insert(lc(x),l,mid);
        if (rc(x)) s(x)=s(lc(x))+s(rc(x)); else s(x)=s(lc(x));
    }
    else
    {
        insert(rc(x),mid+1,r);
        if (lc(x)) s(x)=s(lc(x))+s(rc(x)); else s(x)=s(rc(x));
    }
}
int merge(int x1,int x2,int l,int r)
{
    if (!(x1&&x2)) return x1|x2;
    if (l==r) { s(x1)=s(x1)+s(x2); return x1; }
    int mid=l+r>>1;
    lc(x1)=merge(lc(x1),lc(x2),l,mid);
    rc(x1)=merge(rc(x1),rc(x2),mid+1,r);
    pushup(x1);
    return x1;
}
void ask(int x,int l,int r)
{
    if (!x) return;
    if (z<=l&&r<=y)
    {
        if (fir) tmp=s(x),fir=0; else tmp=tmp+s(x);
        return;
    }
    int mid=l+r>>1;
```

```cpp
        if (z<=mid) ask(lc(x),l,mid);
        if (y>mid) ask(rc(x),mid+1,r);
    }
    void split(int &x1,int &x2,int l,int r)
    {
        assert(!x1);
        if (!x2) return;
        if (z<=l&&r<=y) { x1=x2; x2=0; return; }
        x1=npt();
        int mid=l+r>>1;
        if (z<=mid) split(lc(x1),lc(x2),l,mid);
        if (y>mid) split(rc(x1),rc(x2),mid+1,r);
        pushup(x1); pushup(x2);
    }
    info *b;
    void build(int &x,int l,int r)
    {
        x=npt();
        if (l==r) { s(x)=b[l]; return; }
        int mid=l+r>>1;
        build(lc(x),l,mid); build(rc(x),mid+1,r);
        s(x)=s(lc(x))+s(rc(x));
    }
    struct set
    {
        int rt;
        set():rt(0) {}
        set(info *a):rt(0) { b=a; build(rt,L,R); }
        void modify(int p,const info &o) { pos=p; tmp=o; sgt::modify(rt,L,R); }
        void insert(int p,const info &o) { pos=p; tmp=o; sgt::insert(rt,L,R); }
        void join(const set &o) { rt=merge(rt,o.rt,L,R); }
        info ask(int l,int r)
        {
            z=l; y=r; fir=1;
            sgt::ask(rt,L,R);
            return fir?E:tmp;
        }
        set split(int l,int r)
        {
            z=l; y=r; set p;
            sgt::split(p.rt,rt,L,R);
            return p;
        }
#ifdef ask_kth
        int kth(info k)
        {
            int x=rt,l=L,r=R,mid;
            if (k>s(x)) return -1;
            s(0)=0;
            while (l<r)
            {
                mid=l+r>>1;
                if (s(lc(x))>=k) x=lc(x),r=mid;
                else k-=s(lc(x)),x=rc(x),l=mid+1;
            }
            return l;
        }
```

```
#endif
    };
#undef lc
#undef rc
#undef s
}
typedef sgt::set tree;
```

1.26 bitset（手写，未验证）

```cpp
struct Bitset
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
#define all(x) (x).begin(),(x).end()
    const static ll B=-1llu;
    vector<ll> a;
    int n;
    Bitset() { }
    Bitset(int _n):n(_n),a(_n+63>>6) { }
    bool operator[](int x) const { assert(x>=0&&x<n); return a[x>>6]>>(x&63)&1;
        }
    void set(int x,bool y) { assert(x>=0&&x<n); a[x>>6]=(a[x>>6]&(B^1<<(x&63)))
        |((ll)y<<(x&63)); }
    void set(int x) { assert(x>=0&&x<n); a[x>>6]|=1llu<<(x&63); }
    void set() { memset(a.data(),0xff,a.size()*sizeof a[0]); a.back()&=(1llu
        <<1+(n-1&63))-1; }
    void reset(int x) { a[x>>6]&=~(1llu<<(x&63)); }
    void reset() { memset(a.data(),0,a.size()*sizeof a[0]); }
    int count() const
    {
        int r=0;
        for (ll x:a) r+=__builtin_popcountll(x);
        return r;
    }
    Bitset &operator|=(const Bitset &o)
    {
        assert(n==o.n);
        for (int i=0; i<a.size(); i++) a[i]|=o.a[i];
        return *this;
    }
    Bitset operator|(Bitset o) { o|=*this; return o; }
    Bitset &operator&=(const Bitset &o)
    {
        assert(n==o.n);
        for (int i=0; i<a.size(); i++) a[i]&=o.a[i];
        return *this;
    }
    Bitset operator&(Bitset o) { o&=*this; return o; }
    Bitset &operator^=(const Bitset &o)
    {
        assert(n==o.n);
        for (int i=0; i<a.size(); i++) a[i]^=o.a[i];
        return *this;
    }
```

```cpp
Bitset operator^(Bitset o) { o^=*this; return o; }
Bitset operator~() const
{
    auto r=*this;
    for (ll &x:r.a) x=~x;
    return r;
}
Bitset &operator<<=(int x)
{
    if (x>=n)
    {
        fill(all(a),0);
        return *this;
    }
    assert(x>=0);
    int y=x>>6;
    x&=63;
    if (x==0)
    {
        for (int i=(int)a.size()-1; i>=y; i--) a[i]=a[i-y]<<x;
        if (n&63) a.back()&=(1llu<<1+(n-1&63))-1;
        memset(a.data(),0,y*sizeof a[0]);
        return *this;
    }
    for (int i=(int)a.size()-1; i>y; i--) a[i]=a[i-y]<<x|a[i-y-1]>>64-x;
    a[y]=a[0]<<x;
    memset(a.data(),0,y*sizeof a[0]);
    // fill_n(a.begin(),y,0);
    if (n&63) a.back()&=(1llu<<1+(n-1&63))-1;
    return *this;
}
Bitset operator<<(int x)
{
    auto r=*this;
    r<<=x;
    return r;
}
Bitset &operator>>=(int x)
{
    if (x>=n)
    {
        fill(all(a),0);
        return *this;
    }
    assert(x>=0);
    int y=x>>6,R=(int)a.size()-y-1;
    x&=63;
    for (int i=0; i<R; i++) a[i]=a[i+y]>>x|a[i+y+1]<<64-x;
    a[R]=a.back()>>x;
    memset(a.data()+R+1,0,y*sizeof a[0]);
    // fill(R+1+all(a),0);
    return *this;
}
Bitset operator>>(int x)
{
    auto r=*this;
    r>>=x;
```

```
            return r;
        }
    void range_set(int l,int r)//[l,r) to 1
    {
        if (l>>6==r>>6)
        {
            a[l>>6]|=(1llu<<r-l)-1<<(l&63);
            return;
        }
        if (l&63)
        {
            a[l>>6]|=~((1llu<<(l&63))-1);//[l&63,64)
            l=(l>>6)+1<<6;
        }
        if (r&63)
        {
            a[r>>6]|=(1llu<<(r&63))-1;
            r=(r>>6)-1<<6;
        }
        memset(a.data()+(l>>6),0xff,(r-l>>6)*sizeof a[0]);
    }
    void range_reset(int l,int r)//[l,r) to 0
    {
        if (l>>6==r>>6)
        {
            a[l>>6]&=~((1llu<<r-l)-1<<(l&63));
            return;
        }
        if (l&63)
        {
            a[l>>6]&=(1llu<<(l&63))-1;//[l&63,64)
            l=(l>>6)+1<<6;
        }
        if (r&63)
        {
            a[r>>6]&=~((1llu<<(r&63))-1);
            r=(r>>6)-1<<6;
        }
        memset(a.data()+(l>>6),0,(r-l>>6)*sizeof a[0]);
    }
    void range_set(int l,int r,bool x)//[l,r)
    {
        if (x) range_set(l,r);
        else range_reset(l,r);
    }
};
```

1.27 区间众数

```
template<class T> struct mode//[0,n)
{
    int n,ksz,m;
    vector<T> b;
    vector<vector<int>> pos,f;
    vector<int> a,blk,id,l;
```

```cpp
    mode(const vector<T> &c):n(c.size()),ksz(max<int>(1,sqrt(n))),m((n+ksz-1)/
       ksz),b(c),
       pos(n),f(m,vector<int>(m)),a(n),blk(n),id(n),l(m+1)
    {
        int i,j,k;
        sort(all(b)); b.resize(unique(all(b))-b.begin());
        for (i=0; i<n; i++)
        {
            a[i]=lower_bound(all(b),c[i])-b.begin();
            id[i]=pos[a[i]].size();
            pos[a[i]].push_back(i);
        }
        for (i=0; i<n; i++) blk[i]=i/ksz;
        for (i=0; i<=m; i++) l[i]=min(i*ksz,n);
        vector<int> cnt(b.size());
        for (i=0; i<m; i++)
        {
            fill(all(cnt),0);
            pair<int,int> cur={0,0};
            for (j=i; j<m; j++)
            {
                for (k=l[j]; k<l[j+1]; k++) cmax(cur,pair{++cnt[a[k]],a[k]});
                f[i][j]=cur.second;
            }
        }
    }
    pair<T,int> ask(int L,int R)//返回最大众数
    {
        assert(0<=L&&L<R&&R<=n);
        int val=blk[L]==blk[R-1]?0:f[blk[L]+1][blk[R-1]-1],i;
        int cnt=lower_bound(all(pos[val]),R)-lower_bound(all(pos[val]),L);
        for (i=min(R,l[blk[L]+1])-1; i>=L; i--)
        {
            auto &v=pos[a[i]];
            while (id[i]+cnt<v.size()&&v[id[i]+cnt]<R) ++cnt,val=a[i];
            if (a[i]>val&&id[i]+cnt-1<v.size()&&v[id[i]+cnt-1]<R) val=a[i];
        }
        for (i=max(L,l[blk[R-1]]); i<R; i++)
        {
            auto &v=pos[a[i]];
            while (id[i]>=cnt&&v[id[i]-cnt]>=L) ++cnt,val=a[i];
            if (a[i]>val&&id[i]>=cnt-1&&v[id[i]-cnt+1]>=L) val=a[i];
        }
        return {b[val],cnt};
    }
};
```

2 数学

2.1 单情况矩阵 (+)

没啥用。特殊的 ddp 有用。

```cpp
template<typename T,int n> struct matrix
{
    #define all(x) (x).begin(),(x).end()
    array<pair<int,T>,n> a;
    matrix(char c='E')
    {
        int i;
        if (c=='E') for (i=0;i<n;i++) a[i]={i,0};
        else assert(0);
    }
    matrix(char c,int x)
    {

    }
    matrix operator+(const matrix &o) const
    {
        matrix r;
        int i,j,k;
        for (i=0;i<n;i++)
        {
            auto [x,y]=a[i];
            r.a[i]={o.a[x].first,o.a[x].second+y};
        }
        return r;
    }
};
```

2.2 矩阵求逆（要求质数）

一种原地算法，总体效率更高。

$O(n^3)$，$O(n)$。

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=402,p=1e9+7;
void inv(int &x)
{
    int y=p-2,r=1;
    while (y)
    {
        if (y&1) r=(ll)r*x%p;
        x=(ll)x*x%p;
        y>>=1;
    }
    x=r;
}
int a[N][N],ih[N],jh[N];
int main()
{
```

```cpp
    ios::sync_with_stdio(0);cin.tie(0);
    int i,j,k,n;
    cin>>n;
    for (i=0;i<n;i++) for (j=0;j<n;j++) cin>>a[i][j];
    memset(ih,-1,sizeof ih);
    memset(jh,-1,sizeof jh);
    for (k=0;k<n;k++)
    {//ih,jh要清空
        for (i=k;i<n;i++) if (ih[k]==-1) for (j=k;j<n;j++) if (a[i][j])
        {
            ih[k]=i;
            jh[k]=j;
            break;
        }
        if (ih[k]==-1) return cout<<"No Solution"<<endl,0;
        for (j=0;j<n;j++) swap(a[k][j],a[ih[k]][j]);
        for (i=0;i<n;i++) swap(a[i][k],a[i][jh[k]]);
        if (!a[k][k]) return cout<<"No Solution"<<endl,0;inv(a[k][k]);
        for (i=0;i<n;i++) if (i!=k) a[k][i]=(ll)a[k][i]*a[k][k]%p;
        for (i=0;i<n;i++) if (i!=k) for (j=0;j<n;j++) if (j!=k) a[i][j]=(a[i][j
            ]+(ll)(p-a[i][k])*a[k][j])%p;
        for (i=0;i<n;i++) if (i!=k) a[i][k]=(ll)(p-a[i][k])*a[k][k]%p;
    }
    for (k=n-1;k>=0;k--)
    {
        for (j=0;j<n;j++) swap(a[k][j],a[jh[k]][j]);
        for (i=0;i<n;i++) swap(a[i][k],a[i][ih[k]]);
    }
}
/*
输入
3
1 2 8
2 5 6
5 1 2
输出
718750005 718750005 968750007
171875001 671875005 296875002
117187501 867187506 429687503
*/
```

2.3 任意模数矩阵求逆（未验证）

$O(n^3)$，$O(n^2)$。

原理和任意模数行列式类似，辗转相除。注意仍然要求对角线元素是有逆的。

```cpp
int ksm(int x,int y)
{
    int r=1;
    while (y)
    {
        if (y&1) r=(ll)r*x%p;
        y>>=1;x=(ll)x*x%p;
    }
    return r;
}
```

```cpp
int phi(int n)
{
    int r=n;
    for (int i=2;i*i<=n;i++) if (n%i==0)
    {
        r=r/i*(i-1);n/=i;
        while (n%i==0) n/=i;
    }
    if (n>1) r=r/n*(n-1);
    return r;
}
void cal(int a[][N],int b[][N],int n)
{
    int i,j,k,r,ph=phi(p);
    for (i=1;i<=n;i++) memset(b+1,0,n<<2);
    for (i=1;i<=n;i++) b[i][i]=1;
    for (i=1;i<=n;i++)
    {
        k=i;
        for (j=i+1;j<=n;j++) if (a[j][i]&&a[j][i]<a[k][i]) k=j;
        if (!a[k][i]) {puts("No Solution");exit(0);}
        swap(a[i],a[k]);swap(b[i],b[k]);
        for (j=i+1;j<=n;j++) if (a[j][i])
        {
            r=p-a[j][i]/a[i][i];
            for (k=i;k<=n;k++) a[j][k]=(a[j][k]+(ll)r*a[i][k])%p;
            for (k=1;k<=n;k++) b[j][k]=(b[j][k]+(ll)r*b[i][k])%p;
            while (a[j][i])
            {
                swap(a[i],a[j]);swap(b[i],b[j]);
                r=p-a[j][i]/a[i][i];
                for (k=i;k<=n;k++) a[j][k]=(a[j][k]+(ll)r*a[i][k])%p;
                for (k=1;k<=n;k++) b[j][k]=(b[j][k]+(ll)r*b[i][k])%p;
            }
        }
        if (__gcd(a[i][i],p)!=1) {puts("No Solution");exit(0);}
        r=ksm(a[i][i],ph-1);
        for (j=i;j<=n;j++) a[i][j]=(ll)a[i][j]*r%p;
        for (j=1;j<=n;j++) b[i][j]=(ll)b[i][j]*r%p;
        assert(a[i][i]==1);
        for (j=1;j<i;j++)
        {
            r=p-a[j][i];
            for (k=i;k<=n;k++) a[j][k]=(a[j][k]+(ll)r*a[i][k])%p;
            for (k=1;k<=n;k++) b[j][k]=(b[j][k]+(ll)r*b[i][k])%p;
        }
    }
}
```

2.4 矩阵的特征多项式

$O(n^3)$，$O(n^2)$。
封装版本见矩阵类。

```cpp
#include <bits/stdc++.h>
using namespace std;
```

```cpp
typedef long long ll;
const int N=502,p=998244353;
int a[N][N],f[N];
int n,i,j,k,x,y,r;
void inc(int &x,const int y)
{
    if ((x+=y)>=p) x-=p;
}
void dec(int &x,const int y)
{
    if ((x-=y)<0) x+=p;
}
int ksm(int x,int y)
{
    int r=1;
    while (y)
    {
        if (y&1) r=(ll)r*x%p;
        x=(ll)x*x%p;y>>=1;
    }
    return r;
}
void calmatrix(int a[N][N],int n)
{
    int i,j,k,r;
    for (i=2;i<=n;i++)
    {
        for (j=i;j<=n&&!a[j][i-1];j++);
        if (j>n) continue;
        if (j>i)
        {
            swap(a[i],a[j]);
            for (k=1;k<=n;k++) swap(a[k][j],a[k][i]);
        }
        r=a[i][i-1];
        for (j=1;j<=n;j++) a[j][i]=(ll)a[j][i]*r%p;
        r=ksm(r,p-2);
        for (j=i-1;j<=n;j++) a[i][j]=(ll)a[i][j]*r%p;
        for (j=i+1;j<=n;j++)
        {
            r=a[j][i-1];
            for (k=1;k<=n;k++) a[k][i]=(a[k][i]+(ll)a[k][j]*r)%p;
            r=p-r;
            for (k=i-1;k<=n;k++) a[j][k]=(a[j][k]+(ll)a[i][k]*r)%p;
        }
    }
}
void calpoly(int a[N][N],int n,int *f)
{
    static int g[N][N];
    memset(g,0,sizeof(g));
    g[0][0]=1;
    int i,j,k,r,rr;
    for (i=1;i<=n;i++)
    {
        r=p-1;
        for (j=i;j;j--)//第 j 行选第 n 列
```

```
        {
            rr=(ll)r*a[j][i]%p;
            for (k=0;k<j;k++) g[i][k]=(g[i][k]+(ll)rr*g[j-1][k])%p;
            r=(ll)r*a[j][j-1]%p;
        }
        for (k=1;k<=i;k++) inc(g[i][k],g[i-1][k-1]);
    }
    memcpy(f,g[n],n+1<<2);
    //if (n&1) for (i=0;i<=n;i++) if (f[i]) f[i]=p-f[i];//若注释掉则为 |kE-A|
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cin>>n;
    for (i=1;i<=n;i++) for (j=1;j<=n;j++) cin>>a[i][j];
    calmatrix(a,n);calpoly(a,n,f);
    for (i=0;i<=n;i++) cout<<f[i]<<"x^"<<i<<"+\n"[i==n];
}
/*
3
1 2 3
4 5 6
7 8 9
输出：0x^0+998244335x^1+998244338x^2+1x^3
*/
```

2.5 矩阵类（较新）

```
typedef unsigned long long ll;
const ll p=998244353;
ll ksm(ll x, ll y)
{
    ll r=1;
    while (y)
    {
        if (y&1) r=r*x%p;
        x=x*x%p; y>>=1;
    }
    return r;
}
struct matrix:vector<vector<ll>>
{
    explicit matrix(int n=0, int m=0):vector(n, vector<ll>(m)) { }
    pair<int, int> sz() const { if (size()) return {size(), back().size()};
        return {0, 0}; }
    int rank() const//秩
    {
        vector<vector<ll>> a=*this;
        auto [n, m]=sz();
        int i, j, k, l, r=0;
        for (i=0, j=0; i<n&&j<m; j++)
        {
            for (k=i; k<n; k++) if (a[k][j]) break;
            if (k==n) continue;
            ::swap(a[i], a[k]);
            ll iv=ksm(a[i][j], p-2);
```

```cpp
                for (k=j; k<m; k++) a[i][k]=a[i][k]*iv%p;
                for (k=i+1; k<n; k++) for (l=j+1; l<m; l++) a[k][l]=(a[k][l]+(p-a[k][
                    j])*a[i][l])%p;
                ++i; ++r;
            }
            return r;
        }
        vector<ll> poly()//特征多项式
        {
            auto [n, m]=sz();
            vector<vector<ll>> a=*this;
            assert(n==m);
            int i, j, k;
            for (i=1; i<n; i++)
            {
                for (j=i; j<n&&!a[j][i-1]; j++);
                if (j==n) continue;
                if (j>i)
                {
                    ::swap(a[i], a[j]);
                    for (k=0; k<n; k++) ::swap(a[k][j], a[k][i]);
                }
                ll r=a[i][i-1];
                for (j=0; j<n; j++) a[j][i]=a[j][i]*r%p;
                r=ksm(r, p-2);
                for (j=i-1; j<n; j++) a[i][j]=a[i][j]*r%p;
                for (j=i+1; j<n; j++)
                {
                    r=a[j][i-1];
                    for (k=0; k<n; k++) a[k][i]=(a[k][i]+a[k][j]*r)%p;
                    r=p-r;
                    for (k=i-1; k<n; k++) a[j][k]=(a[j][k]+a[i][k]*r)%p;
                }
            }
            vector g(n+1, vector<ll>(n+1));
            g[0][0]=1;
            for (i=0; i<n; i++)
            {
                ll r=p-1, rr;
                for (j=i; j>=0; j--)//第 j 行选第 n 列
                {
                    rr=r*a[j][i]%p;
                    for (k=0; k<=j; k++) g[i+1][k]=(g[i+1][k]+rr*g[j][k])%p;
                    if (j) r=r*a[j][j-1]%p;
                }
                for (k=1; k<=i+1; k++) (g[i+1][k]+=g[i][k-1])%=p;
            }
            auto f=g[n];
            //if (n&1) for (i=0;i<=n;i++) if (f[i]) f[i]=p-f[i];//若注释掉则为 |kE-A|
            return f;
        }
};
istream &operator>>(istream &cin, matrix &r) { for (auto &v:r) for (ll &x:v)
    cin>>x; return cin; }
ostream &operator<<(ostream &cout, const matrix &r) { auto [n, m]=r.sz(); for (
    int i=0; i<n; i++) for (int j=0; j<m; j++) cout<<r[i][j]<<" \n"[j+1==m];
    return cout; }
```

```cpp
matrix &operator+=(matrix &a, const matrix &b)
{
    assert(a.size()==b.size());
    auto [n, m]=a.sz();
    for (int i=0; i<n; i++) for (int j=0; j<m; j++) (a[i][j]+=b[i][j])%=p;
    return a;
}
matrix &operator-=(matrix &a, const matrix &b)
{
    assert(a.size()==b.size());
    auto [n, m]=a.sz();
    for (int i=0; i<n; i++) for (int j=0; j<m; j++) (a[i][j]+=p-b[i][j])%=p;
    return a;
}
matrix operator*(const matrix &a, const matrix &b)
{
    auto [n, m]=a.sz();
    auto [_, q]=b.sz();
    assert(m==_);
    int i, j, k;
    matrix c(n, q);
    for (k=0; k<m; k++)
    {
        for (i=0; i<n; i++) for (j=0; j<q; j++) c[i][j]+=a[i][k]*b[k][j];
        if (!((k^q-1)&15)) for (auto &v:c) for (ll &x:v) x%=p;
    }
    return c;
}
matrix operator+(matrix a, const matrix &b) { return a+=b; }
matrix operator-(matrix a, const matrix &b) { return a-=b; }
matrix &operator*=(matrix &a, const matrix &b) { return a=a*b; }
matrix &operator*=(matrix &a, ll k) { for (auto &v:a) for (ll &x:v) x=x*k%p;
    return a; }
matrix operator*(matrix a, ll k) { return a*=k; }
matrix E(int n) { matrix r(n, n); for (int i=0; i<n; i++) r[i][i]=1; return r;
    }
matrix pow(matrix a, long long k)//普通的快速幂
{
    assert(k>=0);
    auto [n, m]=a.sz();
    assert(n==m);
    matrix r=k&1?a:E(n);
    k>>=1;
    while (k)
    {
        a*=a;
        if (k&1) r*=a;
        k>>=1;
    }
    return r;
}
matrix pow2(matrix a, long long k)//较快的快速幂。运用了一些技巧。
{
    vector<ll> f=a.poly();
    int n=f.size()-1, i, j;
    if (!n) return matrix();
    if (n==1) return E(1)*ksm(a[0][0], k);
```

```
    assert(f[n]==1);
    vector<ll> r(n), x(n), t(n*2);
    r[0]=x[1]=1;
    for (ll &x:f) x=(p-x)%p;
    reverse(all(f));
    fill(all(t), 0);
    if (k&1)
    {
        for (i=0; i<n; i++) for (j=0; j<n; j++) t[i+j]=(t[i+j]+r[i]*x[j])%p;
        for (i=n*2-2; i>=n; i--) for (j=1; j<=n; j++) t[i-j]=(t[i-j]+f[j]*t[i])%
            p;
        for (i=0; i<n; i++) r[i]=t[i];
    }
    k>>=1;
    while (k)
    {
        fill(all(t), 0);
        for (i=0; i<n; i++) for (j=0; j<n; j++) t[i+j]=(t[i+j]+x[i]*x[j])%p;
        for (i=n*2-2; i>=n; i--) for (j=1; j<=n; j++) t[i-j]=(t[i-j]+f[j]*t[i])%
            p;
        for (i=0; i<n; i++) x[i]=t[i];
        if (k&1)
        {
            fill(all(t), 0);
            for (i=0; i<n; i++) for (j=0; j<n; j++) t[i+j]=(t[i+j]+r[i]*x[j])%p;
            for (i=n*2-2; i>=n; i--) for (j=1; j<=n; j++) t[i-j]=(t[i-j]+f[j]*t[i
                ])%p;
            for (i=0; i<n; i++) r[i]=t[i];
        }
        k>>=1;
    }
    matrix res(n, n);
    int b=ceil(sqrt(n));
    vector<matrix> s(b+1);
    s[0]=E(n); s[1]=a;
    for (i=2; i<=b; i++) s[i]=s[i-1]*a;
    for (i=b-1; i>=0; i--)
    {
        res*=s[b];
        for (j=min(n, (i+1)*b)-1; j>=i*b; j--) res+=s[j-i*b]*r[j];
    }
    return res;
}
```

2.6 最短递推式（BM 算法）

给定 $\{a\}$，求最短的 $\{r\}$ 满足 $\sum\limits_{j=0}^{m-1} a_{i-j-1} r_j = a_i$。

```
vector<ui> bm(const vector<ui> &a)
{
    vector<ui> r,lst;
    int n=a.size(),m=0,q=0,i,j,k=-1;
    ui D=0;
    for (i=0;i<n;i++)
    {
        ui cur=0;
```

```cpp
        for (j=0;j<m;j++) cur=(cur+(ll)a[i-j-1]*r[j])%p;
        cur=(a[i]+p-cur)%p;
        if (!cur) continue;
        if (k==-1)
        {
            k=i;
            D=cur;
            r.resize(m=i+1);
            continue;
        }
        auto v=r;
        ui x=(ll)cur*ksm(D,p-2)%p;
        if (m<q+i-k) r.resize(m=q+i-k);
        (r[i-k-1]+=x)%=p;
        ui *b=r.data()+i-k;
        x=(p-x)%p;
        for (j=0;j<q;j++) b[j]=(b[j]+(ll)x*lst[j])%p;
        if (v.size()+k<lst.size()+i)
        {
            lst=v;
            q=v.size();
            k=i;
            D=cur;
        }
    }
    return r;
}
```

2.7 在线 $O(1)$ 逆元

    预处理复杂度为 $O(p^{\frac{2}{3}})$。

```cpp
namespace online_inv
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    const ll p=1e9+7,n=1010,m=n*n,N=m+2;
    int l[N],r[N];
    ll y[N];
    bool s[N];
    ll _inv[N*2],i,j,k;
    void init_inv()
    {
        assert(n*n*n>p);
        _inv[1]=1;
        for (i=2;i<m*2;i++)
        {
            j=p/i;
            _inv[i]=(p-j)*_inv[p-i*j]%p;
        }
        s[0]=y[0]=1;
        for (i=1;i<n;i++) for (j=i;j<n;j++) if (!s[k=i*m/j])
        {
            y[k]=j;
            s[k]=1;
        }
```

```
        l[0]=1;
        for (i=1;i<=m;i++) l[i]=s[i]?y[i]:l[i-1];
        r[m]=1;
        for (i=m-1;~i;i--) r[i]=s[i]?y[i]:r[i+1];
        for (i=0;i<=m;i++) y[i]=min(l[i],r[i]);
    }
    inline ll inv(const ll &x)
    {
        assert(x&&x<p);
        if (x<m*2) return _inv[x];
        k=x*m/p;
        j=y[k]*x%p;
        return (j<m*2?_inv[j]:p-_inv[p-j])*y[k]%p;
    }
}
using online_inv::init_inv,online_inv::inv,online_inv::p;
```

2.8 Strassen 矩阵乘法

没用，不如卡常。$O(n^{\log_2 7})$。

```
#include <bits/stdc++.h>
using namespace std;
typedef unsigned int ui;
typedef unsigned long long ull;
const ui p=998244353;
const ull fh=1ull<<31;
struct Q
{
    ui **a;
    int n;
    Q(){n=0;}
    void clear()
    {
        for (int i=0;i<n;i++) delete a[i];
        if (n) delete a;n=0;
    }
    Q(int nn)//不能传入不是 2 的幂的数!
    {
        n=nn;
        assert(n==(n&-n));
        a=new ui*[n];
        for (int i=0;i<n;i++) a[i]=new ui[n],memset(a[i],0,n*sizeof a[0][0]);
    }
    const Q & operator=(const Q& b)
    {
        clear();n=b.n;
        a=new ui*[n];
        for (int i=0;i<n;i++) a[i]=new ui[n],memcpy(a[i],b.a[i],n*sizeof a
            [0][0]);
        return *this;
    }
    ~Q(){clear();}
    Q operator+(const Q &b)
    {
        Q c(n);
```

```cpp
            for (int i=0;i<n;i++) for (int j=0;j<n;j++) if ((c.a[i][j]=a[i][j]+b.a[i
                ][j])>=p) c.a[i][j]-=p;
            return c;
        }
        Q operator-(const Q &b)
        {
            Q c(n);
            for (int i=0;i<n;i++) for (int j=0;j<n;j++) if ((c.a[i][j]=a[i][j]-b.a[i
                ][j])&fh) c.a[i][j]+=p;
            return c;
        }
        Q operator*(Q &b)
        {
            Q c(n);
            if (n<=128)
            {
                for (int i=0;i<n;i++) for (int k=0;k<n;k++) for (int j=0;j<n;j++) c.a
                    [i][j]=(c.a[i][j]+(ull)a[i][k]*b.a[k][j])%p;
                return c;
            }
            Q A[2][2],B[2][2],s[10],p[5];
            n>>=1;
            int i,j,k,l;
            for (i=0;i<2;i++) for (j=0;j<2;j++)
            {
                A[i][j]=Q(n);
                for (k=0;k<n;k++) memcpy(A[i][j].a[k],a[k+i*n]+j*n,n*sizeof a[0][0]);
                B[i][j]=Q(n);
                for (k=0;k<n;k++) memcpy(B[i][j].a[k],b.a[k+i*n]+j*n,n*sizeof a
                    [0][0]);
            }
            s[0]=B[0][1]-B[1][1];
            s[1]=A[0][0]+A[0][1];
            s[2]=A[1][0]+A[1][1];
            s[3]=B[1][0]-B[0][0];
            s[4]=A[0][0]+A[1][1];
            s[5]=B[0][0]+B[1][1];
            s[6]=A[0][1]-A[1][1];
            s[7]=B[1][0]+B[1][1];
            s[8]=A[0][0]-A[1][0];
            s[9]=B[0][0]+B[0][1];
            p[0]=A[0][0]*s[0];
            p[1]=s[1]*B[1][1];
            p[2]=s[2]*B[0][0];
            p[3]=A[1][1]*s[3];
            p[4]=s[4]*s[5];
            A[0][0]=p[4]+p[3]-p[1]+s[6]*s[7];
            A[0][1]=p[0]+p[1];
            A[1][0]=p[2]+p[3];
            A[1][1]=p[4]+p[0]-p[2]-s[8]*s[9];
            for (i=0;i<2;i++) for (j=0;j<2;j++) for (k=0;k<n;k++) memcpy(c.a[k+i*n]+
                j*n,A[i][j].a[k],n*sizeof a[0][0]);
            n<<=1;
            return c;
        }
};
int main()
```

```
{
    int i,j,n,m,k;
    ios::sync_with_stdio(0);cin.tie(0);
    cin>>n>>m>>k;
    int N=1<<32-min({__builtin_clz(n-1),__builtin_clz(m-1),__builtin_clz(k-1)});
    Q a(N),b(N);
    for (i=0;i<n;i++) for (j=0;j<m;j++) cin>>a.a[i][j];
    for (i=0;i<m;i++) for (j=0;j<k;j++) cin>>b.a[i][j];
    a=a*b;
    for (i=0;i<n;i++) for (j=0;j<k;j++) cout<<a.a[i][j]<<"␣\n"[j+1==k];
}
```

2.9 扩展欧拉定理

求 $a \uparrow\uparrow b \bmod c$。前面的 Prime 命名空间只是求 φ 用的。

```
namespace Prime
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    const int N=1e6+2;
    const ll M=(ll)(N-1)*(N-1);
    ui pr[N],mn[N],phi[N],cnt;
    int mu[N];
    void init_prime()
    {
        ui i,j,k;
        phi[1]=mu[1]=1;
        for (i=2;i<N;i++)
        {
            if (!mn[i])
            {
                pr[cnt++]=i;
                phi[i]=i-1;mu[i]=-1;
                mn[i]=i;
            }
            for (j=0;(k=i*pr[j])<N;j++)
            {
                mn[k]=pr[j];
                if (i%pr[j]==0)
                {
                    phi[k]=phi[i]*pr[j];
                    break;
                }
                phi[k]=phi[i]*(pr[j]-1);
                mu[k]=-mu[i];
            }
        }
        //for (i=2;i<N;i++) if (mu[i]<0) mu[i]+=p;
    }
    template<typename T> T getphi(T x)
    {
        assert(M>=x);
        T r=x;
        for (ui i=0;i<cnt&&(T)pr[i]*pr[i]<=x&&x>=N;i++) if (x%pr[i]==0)
        {
            ui y=pr[i],tmp;
```

```cpp
            x/=y;
            while (x==(tmp=x/y)*y) x=tmp;
            r=r/y*(y-1);
        }
        if (x>=N) return r/x*(x-1);
        while (x>1)
        {
            ui y=mn[x],tmp;
            x/=y;
            while (x==(tmp=x/y)*y) x=tmp;
            r=r/y*(y-1);
        }
        return r;
    }
    template<typename T> vector<pair<T,ui>> getw(T x)
    {
        assert(M>=x);
        vector<pair<T,ui>> r;
        for (ui i=0;i<cnt&&(T)pr[i]*pr[i]<=x&&x>=N;i++) if (x%pr[i]==0)
        {
            ui y=pr[i],z=1,tmp;
            x/=y;
            while (x==(tmp=x/y)*y) x=tmp,++z;
            r.push_back({y,z});
        }
        if (x>=N)
        {
            r.push_back({x,1});
            return r;
        }
        while (x>1)
        {
            ui y=mn[x],z=1,tmp;
            x/=y;
            while (x==(tmp=x/y)*y) x=tmp,++z;
            r.push_back({y,z});
        }
        return r;
    }
}
using Prime::pr,Prime::phi,Prime::getw,Prime::getphi;
using Prime::mu,Prime::init_prime;
ui ksm(ll x,ui y,ui p)
{
    x=x%p+(x>=p)*p;
    ll r=1;
    while (y)
    {
        if (y&1)
        {
            if ((r*=x)>=p) r=r%p+p; else r%=p;
        }
        if ((x*=x)>=p) x=x%p+p; else x%=p;
        y>>=1;
    }
    return r;
}
```

```
struct Q
{
    vector<ui> p;
    Q(const ui &P)
    {
        p.push_back(P);
        while (p.back()>1) p.push_back(getphi(p.back()));
    }
    ui operator()(ll a,ll b)
    {
        if (!a) return (1^b&1)%p[0];
        ui r=1;
        int i=min(b,(ll)p.size());
        while ((--i)>=0) r=ksm(a,r,p[i]);
        return r%p[0];
    }
};
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(15);
    int n,i;
    init_prime();
    int T;
    cin>>T;
    while (T--)
    {
        ui a,b,c;
        cin>>a>>b>>c;
        cout<<Q(c)(a,b)<<'\n';
    }
}
```

2.10 exgcd

$O(\log p)$，$O(\log p)$。

递归版：

```
int exgcd(int a,int b,int c)//ax+by=c,return x
{
    if (a==0) return c/b;
    return (c-(ll)b*exgcd(b%a,a,c))/a%b;
}
```

递推重构版：

```
pair<ll,ll> exgcd(ll a,ll b,ll c)//ax+by=c, {-1,-1} 无解, b=0 返回 {c/a,0}, 否则
    返回最小非负 x
{
    assert(a||b);
    if (!b) return {c/a,0};
    if (a<0) a=-a,b=-b,c=-c;
    ll d=gcd(a,b);
    if (c%d) return {-1,-1};
    ll x=1,x1=0,p=a,q=b,k;
    b=abs(b);
    while (b)
```

```cpp
    {
        k=a/b;
        x-=k*x1;a-=k*b;
        swap(x,x1);
        swap(a,b);
    }
    b=abs(q/d);
    x=x*(c/d)%b;
    if (x<0) x+=b;
    return {x, (ll)((c-(lll)p*x)/q)};
}
ll fun(ll a, ll b, ll p)//ax=b(mod p)
{
    return exgcd(a, -p, b).first%p;
}
```

2.11 exCRT

实现了一个类 Q，表示一条方程，支持合并。

```cpp
namespace CRT
{
    typedef long long ll;
    pair<ll,ll> exgcd(ll a,ll b,ll c)
    {
        assert(a||b);
        if (!b) return {c/a,0};
        ll d=gcd(a,b);
        if (c%d) return {-1,-1};
        ll x=1,x1=0,p=a,q=b,k;
        b=abs(b);
        while (b)
        {
            k=a/b;
            x-=k*x1;a-=k*b;
            swap(x,x1);
            swap(a,b);
        }
        b=abs(q/d);
        x=x*(c/d)%b;
        if (x<0) x+=b;
        return {x,(c-p*x)/q};
    }
    struct Q
    {
        ll p,r;//0<=r<p
        Q operator+(const Q &o) const
        {
            if (p==0||o.p==0) return {0,0};
            auto [x,y]=exgcd(p,-o.p,r-o.r);
            if (x==-1&&y==-1) return {0,0};
            ll q=lcm(p,o.p);
            return {q,((r-x*p)%q+q)%q};
        }
    };
}
using CRT::Q;
```

2.12 exBSGS

$O(\sqrt{n})$。哈希表 ht 可以用 map 代替。

```cpp
namespace BSGS
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    template<int N,typename T,typename TT> struct ht//个数，定义域，值域
    {
        const static int p=1e6+7,M=p+2;
        TT a[N];
        T v[N];
        int fir[p+2],nxt[N],st[p+2];//和模数相适应
        int tp,ds;//自定义模数
        ht(){memset(fir,0,sizeof fir);tp=ds=0;}
        void mdf(T x,TT z)//位置，值
        {
            ui y=x%p;
            for (int i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i]=z,void();//若
                不可能重复不需要 for
            v[++ds]=x;a[ds]=z;
            if (!fir[y]) st[++tp]=y;
            nxt[ds]=fir[y];fir[y]=ds;
        }
        TT find(T x)
        {
            ui y=x%p;
            int i;
            for (i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i];
            return 0;//返回值和是否判断依据要求决定
        }
        void clear()
        {
            ++tp;
            while (--tp) fir[st[tp]]=0;
            ds=0;
        }
    };
    const int N=5e4;
    ht<N,ui,ui> s;
    int exgcd(int a,int b)
    {
        if (a==1) return 1;
        return (1-(long long)b*exgcd(b%a,a))/a;//not ll
    }
    int bsgs(ui a,ui b,ui p)
    {
        s.clear();
        a%=p;b%=p;
        if (!a) return 1-min((int)b,2);//含 -1
        ui i,j,k,x,y;
        x=sqrt(p)+2;
        for (i=0,j=1;i<x;i++,j=(ll)j*a%p)
        {
```

```
                if (j==b) return i;
                s.mdf((ll)j*b%p,i+1);
            }
            k=j;
            for (i=1;i<=x;i++,j=(ll)j*k%p) if (y=s.find(j)) return (ll)i*x-y+1;
            return -1;
        }
        bool isprime(ui p)
        {
            if (p<=1) return 0;
            for (ui i=2;i*i<=p;i++) if (p%i==0) return 0;
            return 1;
        }
        int exbsgs(ui a,ui b,ui p)//a^x=b(mod p)
        {
            //if (isprime(p)) return bsgs(a,b,p);
            a%=p;b%=p;
            ui i,j,k,x,y=__lg(p),cnt=0;
            for (i=0,j=1%p;i<=y;i++,j=(ll)j*a%p) if (j==b) return i;
            y=1;
            while (1)
            {
                if ((x=gcd(a,p))==1) break;
                if (b%x) return -1;//no sol
                ++cnt;
                p/=x;b/=x;
                y=(ll)y*(a/x)%p;
            }
            a%=p;
            b=(ll)b*(p+exgcd(y,p))%p;
            int r=bsgs(a,b,p);
            return r==-1?-1:r+cnt;
        }
    }
    using BSGS::bsgs,BSGS::exbsgs;
```

2.13 exLucas

求组合数。包含多个不同的版本，按需使用。

```
namespace exlucas
{
    typedef long long ll;
    typedef pair<int,int> pa;
    int P,p,q,i;
    vector<pa> a;
    vector<vector<int> > b;
    vector<int> ph;
    vector<int> xs;
    int ksm(unsigned int x,ll y,const unsigned int p)
    {
        unsigned int r=1;
        while (y)
        {
            if (y&1) r=(unsigned long long)r*x%p;
            x=(unsigned long long)x*x%p;
            y>>=1;
```

```cpp
    }
    return r;
}
void init(int x)//分解质因数，如有必要可以使用更快的方法
{
    a.clear();b.clear();
    int i,y,z;
    vector<int> v;
    for (i=2;i*i<=x;i++) if (x%i==0)
    {
        z=i;x/=i;
        while (1)
        {
            y=x/i;
            if (i*y==x) x=y; else break;
            z*=i;
        }
        a.push_back(pa(i,z));
        b.push_back(v);
    }
    if (x>1) a.push_back(pa(x,x)),b.push_back(v);
    ph.resize(a.size());
    xs.resize(a.size());
    for (int k=0;k<a.size();k++)
    {
        tie(q,p)=a[k];
        ph[k]=p/q*(q-1);
        xs[k]=(ll)ksm(P/p,ph[k]-1,p)*(P/p)%P;
    }
}
void spinit(int x)//O(p) space
{
    for (int k=0;k<a.size();k++)
    {
        int q,p;
        tie(q,p)=a[k];
        b[k].resize(p);
        b[k][0]=1;
        for (int i=1,j=q;i<p;i++) if (i==j) j+=q,b[k][i]=b[k][i-1]; else b[k
            ][i]=(ll)b[k][i-1]*i%p;
    }
}
ll g(ll n)
{
    ll r=0,s;
    while (n>=q)
    {
        n/=q;
        r+=n;
    }
    return r;
}
// int f(ll n)
// {
//   if (n==0) return 1;
//   int r=1;//若 p>1e9 j 要 unsigned
//   for (int i=1,j=q;i<p;i++) if (i==j) j+=q; else r=(ll)r*i%p;
```

```cpp
//  r=(ll)ksm(r,n/p,p)*f(n/q)%p;
//  n%=p;
//  for (int i=1,j=q;i<=n;i++) if (i==j) j+=q; else r=(ll)r*i%p;
//  return r;
// }//O(T\sum p) time,O(1) space ver.
int f(ll n)
{
    int r=1;
    ll cs=0;
    while (n)
    {
        r=(ll)r*b[i][n%p]%p;
        cs+=n/p;
        n/=q;
    }
    return (ll)ksm(b[i][p-1],cs%ph[i],p)*r%p;
}//O(\sum p) time,O(p) space ver.
int C(ll n,ll m,int M)
{
    if (n<m) return 0;
    int r=0,w;
    if (P!=M) init(P=M),spinit(P);//sp for O(p) space
    for (i=0;i<a.size();i++)
    {
        tie(q,p)=a[i];
        w=(ll)ksm(q,g(n)-g(m)-g(n-m),p)*f(n)%p*ksm((ll)f(m)*f(n-m)%p,ph[i]-1,
            p)%p;
        r=(r+(ll)xs[i]*w)%M;
    }
    return r;
}
}
#define C(x,y,z) exlucas::C(x,y,z)
```

2.14 杜教筛

求 $\varphi(n)$ 的前缀和。

核心：构造 g 满足 $h(n) = \sum_{d|n} f(d)g(\dfrac{n}{d})$ 容易计算，则有 $\sum_{i=1}^{n} h(i) = \sum_{i=1}^{n} g(i) \sum_{j=1}^{\lfloor n/i \rfloor} f(j)$，

故 $g(1) \sum_{j=1}^{n} f(j) = \sum_{i=1}^{n} h(i) - \sum_{i=2}^{n} g(i) \sum_{j=1}^{\lfloor n/i \rfloor} f(j)$，则 f 前缀和可以递归求解。

```cpp
namespace du_seive
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    unordered_map<ll,ui> mp;
    const int N=1e7+2;
    const ui p=998244353;
    ui pr[N],phi[N];
    ui cnt;
    void init()
    {
        cnt=0;phi[1]=1;
        int i,j;
        for (i=2;i<N;i++)
```

```
        {
            if (!phi[i])
            {
                pr[cnt++]=i;
                phi[i]=i-1;
            }
            for (j=0;i*pr[j]<N;j++)
            {
                if (i%pr[j]==0)
                {
                    phi[i*pr[j]]=phi[i]*pr[j];
                    break;
                }
                phi[i*pr[j]]=phi[i]*(pr[j]-1);
            }
            if ((phi[i]+=phi[i-1])>=p) phi[i]-=p;
        }
    }
    ui get_phi_sum(ll n)
    {
        if (n<N) return phi[n];
        if (mp.count(n)) return mp[n];
        ui sum=0;
        for (ll i=2,j,k;i<=n;i=j+1)
        {
            j=n/(k=n/i);
            sum=(sum+(ll)get_phi_sum(k)*(j-i+1))%p;
        }
        ui nn=n%p;
        sum=(nn*(nn+1ll)/2+p-sum)%p;
        mp[n]=sum;
        return sum;
    }
}
using du_seive::init,du_seive::get_phi_sum;
```

2.15   线性规划

用法：构造函数指明目标函数系数，add 函数增加限制。额外的限制是 $x_i \geq 0$。

```
typedef long double db;//__float128
struct linear
{
    static const int N=45;//n+m
    db r[N][N];
    int col[N],row[N];
    const db eps=1e-10,inf=1e9;//1e-17
    int n,m;
    template<typename T> linear(const vector<T> &a)//target: maximize \sum a(i
        -1)xi
    {
        memset(r,0,sizeof r);
        memset(col,0,sizeof col);
        memset(row,0,sizeof row);
        n=a.size();m=0;
        for (int i=1;i<=n;i++) r[0][i]=-a[i-1];
    }
```

```cpp
template<typename T> void add(const vector<T> &a,db b)//limit: \sum a(i-1)xi
    <=b
{
    assert(a.size()==n);
    ++m;
    for (int i=1;i<=n;i++) r[m][i]=-a[i-1];
    r[m][0]=b;
}
void pivot(int k, int t)
{
    swap(row[k+n],row[t]);
    db rkt=-r[k][t];
    int i,j;
    for (i=0;i<=n;i++) r[k][i]/=rkt;
    r[k][t]=-1/rkt;
    for (i=0;i<=m;i++) if (i!=k)
    {
        db rit=r[i][t];
        if (rit>=-eps&&rit<=eps) continue;
        for (j=0;j<=n;j++) if (j!=t) r[i][j]+=rit*r[k][j];
        r[i][t]=r[k][t]*rit;
    }
}
bool init()
{
    int i;
    for (i=1;i<=n+m;i++) row[i]=i;
    while(1)
    {
        int q=1;
        auto b_min=r[1][0];
        for (i=2;i<=m;i++) if (r[i][0]<b_min) b_min=r[i][0],q=i;
        if (b_min+eps>=0) return 1;
        int p=0;
        for (i=1;i<=n;i++) if (r[q][i]>eps&&(!p||row[i]>row[p])) p=i;
        if (!p) break;
        pivot(q,p);
    }
    return 0;
}
bool simplex()
{
    while (1)
    {
        int t=1,k=0,i;
        for (i=2;i<=n;i++) if (r[0][i]<r[0][t]) t=i;
        if (r[0][t]>=-eps) return 1;
        db ratio_min=inf;
        for (i=1;i<=m;i++) if (r[i][t]<-eps)
        {
            db ratio=-r[i][0]/r[i][t];
            if (!k||ratio<ratio_min||ratio<=ratio_min+eps&&row[i]>row[k])
            {
                ratio_min=ratio;
                k=i;
            }
        }
```

```cpp
            if (!k) break;
            pivot(k,t);
        }
        return 0;
    }
    void solve(int type)
    {
        if (!init())
        {
            cout<<"Infeasible\n";
            return;
        }
        if (!simplex())
        {
            cout<<"Unbounded\n";
            return;
        }
        cout<<(long double)(-r[0][0])<<'\n';
        if (type)
        {
            int i;
            memset(col+1,0,n*sizeof col[0]);
            for (i=n+1;i<=n+m;i++) col[row[i]]=i;
            for (i=1;i<=n;i++) cout<<(long double)(col[i]?r[col[i]-n][0]:0)<<"␣\n
                "[i==n];
        }
    }
};
```

2.16 斐波那契数列

使用生日攻击的方法寻找循环节，一种更通用的方法是 bsgs。

```cpp
const int NN=3e7+2,M=4e5,N=1e6+10;
char c[NN];
ll n;
ll y,mo,x,z;
int p,i,j,k;
struct Q
{
    int a[2][2];
    Q(int b=0,int c=0,int d=0,int e=0){a[0][0]=b,a[0][1]=c,a[1][0]=d,a[1][1]=e;}
    Q operator*(const Q &o)
    {
        return Q(((ll)a[0][0]*o.a[0][0]+(ll)a[0][1]*o.a[1][0])%p,
                 ((ll)a[0][0]*o.a[0][1]+(ll)a[0][1]*o.a[1][1])%p,
                 ((ll)a[1][0]*o.a[0][0]+(ll)a[1][1]*o.a[1][0])%p,
                 ((ll)a[1][0]*o.a[0][1]+(ll)a[1][1]*o.a[1][1])%p);
    }
};
struct ht
{
    ll v[N],a[N];
    int fir[N],nxt[N],st[N];//和模数相适应
    int tp,p,ds;//自定义模数
    ht(){tp=0,p=1e6+7,ds=0;}
    void mdf(const ll x,const ll z)//位置，值
```

```cpp
    {
        const int y=x%p;
        for (int i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i]=z,void();//若不可
            能重复不需要这一步if，但需要for?
        v[++ds]=x;a[ds]=z;if (!fir[y]) st[++tp]=y;
        nxt[ds]=fir[y];fir[y]=ds;
    }
    ll find(const ll x)
    {
        const int y=x%p;int i;
        for (i=fir[y];i;i=nxt[i]) if (v[i]==x) break;
        if (!i) return 0;//返回值和是否判断依据要求决定
        return a[i];
    }
    void clear()
    {
        ++tp;
        while (--tp) fir[st[tp]]=0;ds=0;
    }
};
ht mp;
Q f[M],g[M],ji;
int fib(ll n)
{
    Q x=f[n%k]*g[n/k];
    return x.a[0][1];
}
ll spefib(ll n)
{
    Q x=f[n%k]*g[n/k];
    return (ll)x.a[0][1]*p+x.a[1][1];
}
ll sj()
{
    ll x=rand();
    x=x<<15^rand();
    x=x<<15^rand();
    x=x<<15^rand();
    return x>0?x:-x;
}
ll ab(ll x)
{
    return x>0?x:-x;
}
int main()
{
    srand(383778817);
    scanf("%s\n%d",c+1,&p);
    k=sqrt((ll)20*p)+1;ji=Q(0,1,1,1);
    f[0]=Q(1,0,0,1);for (i=1;i<=k;i++) f[i]=f[i-1]*ji;
    g[0]=Q(1,0,0,1);for (i=1;i<=k;i++) g[i]=g[i-1]*f[k];
    while (1)
    {
        x=sj()%(20ll*p)+1;y=spefib(x);
        if (z=mp.find(y))
        {
            if (z!=x)
```

```
            {
                mo=ab(x-z);
                break;
            }
        } else mp.mdf(y,x);
    }
    n=0;
    for (i=1;c[i]>=48&&c[i]<=57;i++) n=(n*10+(c[i]^48))%mo;
    printf("%d",fib(n));
}
```

2.17 线性插值（k 次幂和）

$O(m)$，$O(m)$。

```
int f(int *a,int n,int m)//这种写法不包含0处取值，n是值，m-1是次数，至少需要 m
    项。换而言之，f(1,2,...,m)=a[1],a[2],...,a[m]
{
    if (n<=m) return a[n];
    static int inv[N],l[N],r[N],ifac[N];
    int i;
    ifac[0]=inv[1]=1;
    for (i=2;i<=m;i++) inv[i]=p-(ll)p/i*inv[p%i]%p;
    for (i=1;i<=m;i++) ifac[i]=(ll)ifac[i-1]*inv[i]%p;//以上可以预跑
    int ans=0,rr=0;
    l[0]=1;r[m+1]=1;
    for (i=1;i<m;i++) l[i]=(ll)l[i-1]*(n-i)%p;
    for (i=m;i;i--) r[i]=(ll)r[i+1]*(n-i)%p;
    for (i=1;i<=m;i++)
    {
        if ((m^i)&1) rr=p-a[i]; else rr=a[i];
        ans=(ans+(ll)rr*ifac[i-1]%p*ifac[m-i]%p*l[i-1]%p*r[i+1])%p;
    }
    return ans;
}
```

2.18 单原根（仅手动验证质数）

```
namespace get_root
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    ui ksm(ui x,ui y,ui p)
    {
        ui r=1;
        while (y)
        {
            if (y&1) r=(ll)r*x%p;
            x=(ll)x*x%p;y>>=1;
        }
        return r;
    }
    vector<ui> getw(ui n)
    {
        vector<ui> w;
```

```cpp
        for (ui i=2;i*i<=n;i++) if (n%i==0)
        {
            w.push_back(i);
            n/=i;
            for (ui j=n/i;n==i*j;j=n/i) n/=i;
        }
        if (n>1) w.push_back(n);
        return w;
    }
    int getrt(ui n)
    {
        if (n<=2) return n-1;
        auto w=getw(n);
        ui ph=n;
        for (ui x:w) ph=ph/x*(x-1);
        w=getw(ph);
        for (ui &x:w) x=ph/x;
        for (ui i=2;i<n;i++) if (gcd(i,n)==1)
        {
            for (ui x:w) if (ksm(i,x,n)==1) goto no;
            return i;
            no:;
        }
        return -1;
    }
}
using get_root::getrt;
```

2.19   稍快单原根（仅验证质数）

```cpp
namespace get_root
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    bool ied=0;
    const int N=1e5+5;
    vector<ui> pr;
    bool ed[N];
    void init()
    {
        pr.reserve(N);
        for (ui i=2;i<N;i++)
        {
            if (!ed[i]) pr.push_back(i);
            for (ui x:pr)
            {
                if (i*x>=N) break;
                ed[i*x]=1;
                if (i%x==0) break;
            }
        }
    }
    ui ksm(ui x,ui y,ui p)
    {
        ui r=1;
        while (y)
```

```cpp
        {
            if (y&1) r=(ll)r*x%p;
            x=(ll)x*x%p;y>>=1;
        }
        return r;
    }
    vector<ui> getw(ui n)
    {
        vector<ui> w;
        for (ui x:pr)
        {
            if (x*x>n) break;
            if (n%x==0)
            {
                w.push_back(x);
                n/=x;
                for (ui i=n/x;n==x*i;i=n/x) n/=x;
            }
        }
        if (n>1) w.push_back(n);
        return w;
    }
    int getrt(ui n)
    {
        if (n<=2) return n-1;
        if (!ed[4]) init();
        auto w=getw(n);
        ui ph=n;
        for (ui x:w) ph=ph/x*(x-1);
        w=getw(ph);
        for (ui &x:w) x=ph/x;
        for (ui i=2;i<n;i++) if (gcd(i,n)==1)
        {
            for (ui x:w) if (ksm(i,x,n)==1) goto no;
            return i;
            no:;
        }
        return -1;
    }
}
using get_root::getrt;
```

2.20   筛全部原根

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=1e6+2;
int ss[N],mn[N],fmn[N],phi[N];
int t,n,gs,i,d;
bool ed[N],av[N],yg[N],hv[N];
double inv[N];
void getfac(int x,int *a,int &n)
{
    int y=x,z;
    if (1^x&1)
```

```cpp
    {
        a[n=1]=2;x>>=1;while (1^x&1) x>>=1;
    }
    while (x>1)
    {
        x=1e-9+(x*inv[a[++n]=z=mn[x]]);
        while (x%z==0) x=1e-9+x*inv[z];
    }
    for (i=1;i<=n;i++) av[a[i]]=0,a[i]=1e-9+(y*inv[a[i]]);
}
int ksm(int x,int y,int p)
{
    int r=1;
    while (y)
    {
        if (y&1) r=(ll)r*x%p;
        x=(ll)x*x%p;y>>=1;
    }
    return r;
}
bool ck(int x,int *a,int n,int p)
{
    for (int i=1;i<=n;i++) if (ksm(x,a[i],p)==1) return 0;
    return 1;
}
void getrt(int x,int d)
{
    if (!hv[x]) return puts("0\n"),void();
    static int a[30];
    int n=0,y,i,g=0,c=d;y=phi[x];
    fill(av+1,av+y+1,1);
    getfac(y,a,n);
    for (i=1;i<x;i++) if (__gcd(i,x)==1&&ck(i,a,n,x)) break;
    yg[g=i]=1;//g就是最小原根
    int j=(ll)g*g%x;
    for (i=2;i<y;i++,j=(ll)j*g%x) yg[j]=av[i]=av[mn[i]]&av[fmn[i]];
    printf("%d\n",phi[y]);
    for (i=1;i<x;i++) if (yg[i])
    {
        yg[i]=0;
        if (--c==0) printf("%d␣",i),c=d;
    }puts("");
}
void init()
{
    int i,j,k,n=N-1;
    mn[1]=phi[1]=1;
    for (i=1;i<=n;i++) inv[i]=1.0/i;
    for (i=2;i<=n;i++)
    {
        if (!ed[i]) phi[mn[i]=ss[++gs]=i]=i-1,hv[i]=1;
        for (j=1;j<=gs&&(k=ss[j]*i)<=n;j++)
        {
            ed[k]=1;mn[k]=ss[j];
            if (i%ss[j]==0) {phi[k]=phi[i]*ss[j];hv[k]=hv[i];break;}
            phi[k]=phi[i]*(ss[j]-1);
        }
    }
```

```
    }
    for (i=n;i;i--) fmn[i]=1e-9+(i*inv[mn[i]]),hv[i]|=(1^i&1)&&hv[i>>1];
    for (i=8;i<=n;i<<=1) hv[i]=0;
}
int main()
{
    init();
    scanf("%d",&t);
    while (t--)
    {
        scanf("%d%d",&n,&d);
        getrt(n,d);
    }
}
```

2.21 高斯消元（通解）

返回方程的一组解和自由元。

```
tuple<int,vector<ui>,vector<vector<ui>>> gauss(vector<vector<ui>> a)//sum = a[i
    ][m], rank of base, one sol, base
{
    int n=a.size(),m=a[0].size()-1,i,j,k,R=m;
    vector<int> fix(m,-1);
    for (i=k=0;i<m;i++)
    {
        for (j=k;j<n;j++) if (a[j][i]) break;
        if (j==n) continue;
        fix[i]=k;--R;
        swap(a[k],a[j]);
        ui *u=a[k].data();
        ui x=ksm(u[i],p-2);
        for (j=i;j<=m;j++) u[j]=(ll)u[j]*x%p;
        for (auto &v:a) if (v.data()!=a[k].data())
        {
            x=p-v[i];
            for (j=i;j<=m;j++) v[j]=(v[j]+(ll)x*u[j])%p;
        }
        ++k;
    }
    for (i=k;i<n;i++) if (a[i][m]) return {-1,{},{}};
    vector<ui> r(m);
    vector<vector<ui>> c;
    for (i=0;i<m;i++) if (fix[i]!=-1) r[i]=a[fix[i]][m];
    for (i=0;i<m;i++) if (fix[i]==-1)
    {
        vector<ui> r(m);
        r[i]=1;
        for (j=0;j<m;j++) if (fix[j]!=-1) r[j]=(p-a[fix[j]][i])%p;
        c.push_back(r);
    }
    return {R,r,c};
}
```

2.22 高斯消元（列主元）

$O(n^3)$，$O(n^2)$。

浮点数的版本。

```cpp
namespace Gauss
{
    typedef double db;
    const db eps=1e-8;
    template<typename T> pair<vector<db>,int> solve(const vector<vector<T>> &A)
        //和为 0。返回秩，负数无解
    {
        assert(A.size());
        int n=A.size(),m=A[0].size()-1,i,j,k,l,r,fg=1;
        db a[n][m+1],b;
        for (i=0;i<n;i++) for (j=0;j<=m;j++) a[i][j]=A[i][j];
        for (i=l=r=0;i<n&&l<m;i++,l++)
        {
            k=i;
            for (j=i+1;j<n;j++) if (fabs(a[j][l])>fabs(a[k][l])) k=j;
            if (fabs(a[k][l])<eps) {--i;continue;}
            if (i!=k) for (j=l;j<=m;j++) swap(a[i][j],a[k][j]);
            b=1/a[i][l];++r;a[i][l]=1;
            for (j=l+1;j<=m;j++) a[i][j]*=b;
            for (j=0;j<n;j++) if (i!=j)
            {
                b=a[j][l];a[j][l]=0;
                for (k=l+1;k<=m;k++) a[j][k]-=b*a[i][k];
            }
        }
        vector<db> X(m);
        for (j=0;j<l;j++) for (k=0;k<i;k++) if (a[k][j]==1)
        {
            X[j]=-a[k][m];
            break;
        }
        for (j=i;j<n&&~fg;j++)
        {
            b=a[j][m];
            for (k=0;k<m;k++) b+=X[k]*a[j][k];
            if (fabs(b)>eps) fg=-1;
        }
        return {X,r*fg};
    }
}
```

2.23 行列式求值（任意模数）

$O(n^3)$，$O(n^2)$。

原理：辗转相除。注意这个 $\log p$ 并不在 n^3 上。

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=502,p=998244353;
int cal(int a[][N],int n)
{
```

```
    int i,j,k,r=1,fh=0,l;
    for (i=1;i<=n;i++)
    {
        k=i;
        for (j=i+1;j<=n;j++) if (a[j][i]) {k=j;break;}
        if (a[k][i]==0) return 0;
        if (i!=k) {swap(a[k],a[i]);fh^=1;}
        for (j=i+1;j<=n;j++)
        {
            if (a[j][i]>a[i][i]) swap(a[j],a[i]),fh^=1;
            while (a[j][i])
            {
                l=a[i][i]/a[j][i];
                for (k=i;k<=n;k++) a[i][k]=(a[i][k]+(ll)(p-l)*a[j][k])%p;
                swap(a[j],a[i]);fh^=1;
            }
        }
        r=(ll)r*a[i][i]%p;
    }
    if (fh) return (p-r)%p;
    return r;
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,i,j;
    static int a[N][N];
    cin>>n;
    for (i=1;i<=n;i++) for (j=1;j<=n;j++) cin>>a[i][j];
    cout<<cal(a,n)<<endl;
}
```

2.24 行列式求值（质数模数）

$O(n^3)$，$O(n^2)$。

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=502,p=998244353;
int ksm(int x,int y)
{
    int r=1;
    while (y)
    {
        if (y&1) r=(ll)r*x%p;
        y>>=1;x=(ll)x*x%p;
    }
    return r;
}
int cal(int a[][N],int n)
{
    int i,j,k,r=1,fh=0,l;
    for (i=1;i<=n;i++)
    {
        for (j=i;j<=n;j++) if (a[j][i]) break;
        if (j>n) return 0;
```

```
        if (i!=j) swap(a[j],a[i]),fh^=1;
        r=(ll)r*a[i][i]%p;
        k=ksm(a[i][i],p-2);
        for (j=i;j<=n;j++) a[i][j]=(ll)a[i][j]*k%p;
        for (j=i+1;j<=n;j++)
        {
            a[j][i]=p-a[j][i];
            for (k=i+1;k<=n;k++) a[j][k]=(a[j][k]+(ll)a[j][i]*a[i][k])%p;
            a[j][i]=0;
        }
    }
    if (fh) return (p-r)%p;
    return r;
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,i,j;
    static int a[N][N];
    cin>>n;
    for (i=1;i<=n;i++) for (j=1;j<=n;j++) cin>>a[i][j];
    cout<<cal(a,n)<<endl;
}
/*
3
3 1 4
1 5 9
2 6 5
998244263
*/
```

2.25 稀疏矩阵系列

safe 宏用于验证结果正确性，可不定义。实现了稀疏矩阵的行列式和求解方程组。

```
vector<ui> bm(const vector<ui> &a)
{
    vector<ui> r,lst;
    int n=a.size(),m=0,q=0,i,j,k=-1;
    ui D=0;
    for (i=0;i<n;i++)
    {
        ui cur=0;
        for (j=0;j<m;j++) cur=(cur+(ll)a[i-j-1]*r[j])%p;
        cur=(a[i]+p-cur)%p;
        if (!cur) continue;
        if (k==-1)
        {
            k=i;
            D=cur;
            r.resize(m=i+1);
            continue;
        }
        auto v=r;
        ui x=(ll)cur*ksm(D,p-2)%p;
```

```cpp
        if (m<q+i-k) r.resize(m=q+i-k);
        (r[i-k-1]+=x)%=p;
        ui *b=r.data()+i-k;
        x=(p-x)%p;
        for (j=0;j<q;j++) b[j]=(b[j]+(ll)x*lst[j])%p;
        if (v.size()+k<lst.size()+i)
        {
            lst=v;
            q=v.size();
            k=i;
            D=cur;
        }
    }
    return r;
}
#define safe
struct Q
{
    int x,y;
    ui w;
};
mt19937_64 rnd(9980);
vector<ui> minpoly(int n,const vector<Q> &a)//[0,n),max:1
{
    for (auto [x,y,w]:a) assert(min(x,y)>=0&&max(x,y)<n);
    vector<ui> u(n),v(n),b(n*2+1),tmp(n);
    int i;
    for (ui &x:u) x=rnd()%p;
    for (ui &x:v) x=rnd()%p;
    assert(*min_element(all(u))&&*min_element(all(v)));
    for (ui &r:b)
    {
        for (i=0;i<n;i++) r=(r+(ll)u[i]*v[i])%p;
        fill(all(tmp),0);
        for (auto [x,y,w]:a) tmp[x]=(tmp[x]+(ll)w*v[y])%p;
        swap(v,tmp);
    }
    auto r=bm(b);
    #ifdef safe
        for (ui &x:u) x=rnd()%p;
        for (ui &x:v) x=rnd()%p;
        for (ui &r:b)
        {
            for (i=0;i<n;i++) r=(r+(ll)u[i]*v[i])%p;
            fill(all(tmp),0);
            for (auto [x,y,w]:a) tmp[x]=(tmp[x]+(ll)w*v[y])%p;
            swap(v,tmp);
        }
        auto rr=bm(b);
        assert(r==rr);
    #endif
    reverse(all(r));
    for (ui &x:r) if (x) x=p-x;
    r.push_back(1);
    return r;
}
ui det(int n,vector<Q> a)//[0,m)
```

```cpp
{
    vector<ui> b(n);
    for (ui &x:b) x=rnd()%p;
    assert(*min_element(all(b)));
    for (auto &[x,y,w]:a) w=(ll)w*b[x]%p;
    ui r=minpoly(n,a)[0],tmp=1;
    for (ui x:b) tmp=(ll)tmp*x%p;
    r=(ll)r*ksm(tmp,p-2)%p;
    #ifdef safe
        for (ui &x:b) x=rnd()%p;
        assert(*min_element(all(b)));
        for (auto &[x,y,w]:a) w=(ll)w*b[x]%p;
        ui rr=minpoly(n,a)[0],tmpp=1;
        for (ui x:b) tmpp=(ll)tmpp*x%p;
        rr=(ll)rr*ksm(tmpp,p-2)%p*ksm(tmp,p-2)%p;
        assert(r==rr);
    #endif
    return n&1?(p-r)%p:r;
}
vector<ui> gauss(const vector<Q> &a,vector<ui> v)
{
    int n=v.size(),i,j;
    for (auto [x,y,w]:a) assert(0<=x&&x<n&&0<=y&&y<n);
    vector<ui> u(n),b(2*n+1),tmp(n),tv=v;
    for (ui &x:u) x=rnd()%p;
    assert(*min_element(all(u)));
    for (ui &r:b)
    {
        for (i=0;i<n;i++) r=(r+(ll)u[i]*v[i])%p;
        fill(all(tmp),0);
        for (auto [x,y,w]:a) tmp[x]=(tmp[x]+(ll)w*v[y])%p;
        swap(v,tmp);
    }
    auto f=bm(b);
    f.insert(f.begin(),p-1);
    int m=(int)f.size()-2;
    v=tv;fill(all(u),0);
    ui x;
    for (i=0;i<=m;i++)
    {
        x=f[m-i];
        for (j=0;j<n;j++) u[j]=(u[j]+(ll)v[j]*x)%p;
        fill(all(tmp),0);
        for (auto [x,y,w]:a) tmp[x]=(tmp[x]+(ll)w*v[y])%p;
        swap(v,tmp);
    }
    x=ksm((p-f.back())%p,p-2);
    for (ui &y:u) y=(ll)y*x%p;
    #ifdef safe
        for (auto [x,y,w]:a) tv[x]=(tv[x]+(ll)(p-w)*u[y])%p;
        assert(!*min_element(all(tv)));
    #endif
    return u;
}
```

2.26 Min_25 筛

$f(p^k) = p^k(p^k - 1)$，求 $\sum\limits_{i=1}^{n} f(i)$。这个的原理我了解的不多，因此没有更多注释。

```cpp
const int N=1e5+2,p=1e9+7,i6=166666668;
ll fs[N<<1],m;
int ss[N],ys[N<<1],s[N],f[N<<1],g[N<<1],ls[N<<1],cs[N<<1];
int gs,n,i,j,k,cnt,ct,ans,sq;
bool ed[N];
int S(ll n,int x)
{
    int r,i,j,l;
    ll k;
    if (ss[x]>=n) return 0;
    if (n>sq) r=g[ys[m/n]]; else r=g[n];
    if ((r=r-s[x])<0) r+=p;
    for (i=x+1;(ll)ss[i]*ss[i]<=n;i++) for (j=1,k=ss[i];k<=n;j++,k*=ss[i])
    {
        l=(k-1)%p;
        r=(r+(ll)l*(l+1)%p*((j!=1)+S(n/k,i)))%p;
    }
    return r;
}
int main()
{
    n=1e5;
    for (i=2;i<=n;i++)
    {
        if (!ed[i]) ss[++gs]=i;
        for (j=1;(j<=gs)&&(i*ss[j]<=n);j++)
        {
            ed[i*ss[j]]=1;
            if (i%ss[j]==0) break;
        }
    }ss[gs+1]=1e6;
    s[1]=ss[1]*ss[1];
    for (i=2;i<=gs;i++) s[i]=(s[i-1]+(ll)ss[i]*ss[i])%p;//s 是多项式在素数位置的
        前缀和
    memcpy(cs,s,sizeof(s));
    ll i,j,k,x,z; scanf("%lld",&m);
    sq=n=sqrt(m);while ((ll)(n+1)*(n+1)<=m) ++n;
    cnt=n-1;
    for (i=n;i<=m;i=j+1) {j=m/(m/i);++cnt;}ct=cnt++;
    for (i=1;i<=m;i=j+1)
    {
        j=m/(k=m/i);
        if (k<=n) g[fs[k]=k]=(k*(k+1)*(k<<1|1)/6-1)%p;//这里是多项式前缀和（不含1
            ）
        else
        {
            z=k%p;//一样
            g[ys[j]=--cnt]=(z*(z+1)%p*(z<<1|1)%p+p-6)*i6%p;fs[cnt]=k;
        }
    }
    cnt=ct;
    for (j=1;(j<=gs)&&(z=(ll)ss[j]*ss[j]);j++) for (i=cnt;z<=fs[i];i--)
```

```
    {
        x=fs[i]/ss[j];if (x>n) x=ys[m/x];
        g[i]=(g[i]+(ll)(p-ss[j])*ss[j]%p*(g[x]-s[j-1]+p))%p;//另一处需要修改的
    }
    memcpy(ls,g,sizeof(g));
    s[1]=ss[1];
    for (i=2;i<=gs;i++) s[i]=s[i-1]+ss[i];
    cnt=n-1;
    for (i=n;i<=m;i=j+1) {j=m/(m/i);++cnt;}ct=cnt++;
    for (i=1;i<=m;i=j+1)
    {
        j=m/(k=m/i);
        if (k<=n) g[fs[k]=k]=((k*(k+1)>>1)-1)%p;
        else
        {
            z=k%p;
            g[ys[j]=--cnt]=(z*(z+1)-2>>1)%p;fs[cnt]=k;
        }
    }
    cnt=ct;
    for (j=1;(j<=gs)&&(z=(ll)ss[j]*ss[j]);j++) for (i=cnt;z<=fs[i];i--)
    {
        x=fs[i]/ss[j];if (x>n) x=ys[m/x];
        g[i]=(g[i]+(ll)(p-ss[j])*(g[x]-s[j-1]+p))%p;
    }
    for (i=1;i<=cnt;i++) if ((g[i]=ls[i]-g[i])<0) g[i]+=p;
    for (i=1;i<=gs;i++) if ((s[i]=cs[i]-s[i])<0) s[i]+=p;
    ans=S(m,0)+1;if (ans==p) ans=0;printf("%d",ans);
}
```

2.27 Min_25 筛（卡常，素数个数，注意评测机 double 性能）

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=3.2e5+2;
ll s[N];
int ss[N],ys[N],gs=0;
bool ed[N];
ll cal(ll m)
{
    static ll g[N<<1],fs[N<<1];
    ll i,j,k,x;
    int n;
    int p,q,cnt;
    n=round(sqrt(m));
    q=lower_bound(ss+1,ss+gs+1,n)-ss;
    memset(g,0,sizeof(g));memset(ys,0,sizeof(ys));cnt=n-1;
    for (i=n;i<=m;i=j+1) {j=m/(m/i);++cnt;}int ct=cnt++;
    for (i=1;i<=m;i=j+1)
    {
        j=m/(k=m/i);
        if (k<=n) g[fs[k]=k]=k-1; else {g[ys[j]=--cnt]=k-1;fs[cnt]=k;}
    }cnt=ct;
    for (j=1;j<=q;j++) for (i=cnt;(ll)ss[j]*ss[j]<=fs[i];i--)
    {
```

```
            x=fs[i]/ss[j];if (x>n) x=ys[m/x];
            g[i]-=g[x]-j+1;
        }
        return g[cnt];//这里 g[cnt-i+1] 表示的是 [1,m/i] 的答案
}
int main()
{
        int n,i,j,t;
        n=3.2e5;
        for (i=2;i<=n;i++)
        {
                if (!ed[i]) ss[++gs]=i;
                for (j=1;(j<=gs)&&(i*ss[j]<=n);j++)
                {
                        ed[i*ss[j]]=1;
                        if (i%ss[j]==0) break;
                }
        }
        s[1]=ss[1];
        for (i=2;i<=gs;i++) s[i]=s[i-1]+ss[i];
        t=1;
        ll m;
        while (t--) cin>>m,cout<<cal(m)<<'\n';
}
```

2.28 扩展 min-max 容斥（重返现世）

$$k\text{-th}\max\{S\} = \sum_{T \subset S} (-1)^{|T|-k} \binom{|T|-1}{k-1} \min\{T\}$$

```
        scanf("%d%d%d",&n,&q,&m);inv[1]=1;q=n+1-q;
        for (i=2;i<=m;i++) inv[i]=p-(ll)p/i*inv[p%i]%p;
        for (i=1;i<=n;i++) scanf("%d",a+i);f[0][0]=1;
        for (j=1;j<=n;j++) for (i=q;i;i--) for (k=m;k>=a[j];k--) if ((f[i][k]=f[i][k
            ]+f[i-1][k-a[j]]-f[i][k-a[j]])>=p) f[i][k]-=p; else if (f[i][k]<0) f[i][
            k]+=p;
        for (i=1;i<=m;i++) ans=(ans+(ll)f[q][i]*inv[i])%p;
        ans=(ll)ans*m%p;printf("%d",ans);
```

2.29 模数为偶数 FWT & 光速乘

$O(n2^n)$，$O(2^n)$。

原理：让模数变为 $p2^n$，就可以正常做除法了。

```
const int N=1<<20,M=21;
int x[M];
ll p,f[N],g[N];
int n,m,c;
ll mul(ll x,ll y)
{
        x=x*y-(ll)((ldb)x/p*y+1e-8)*p;
        if (x<0) return x+p;return x;
}
void read(int &x)
{
        c=getchar();
```

```
    while ((c<48)||(c>57)) c=getchar();
    x=c^48;c=getchar();
    while ((c>=48)&&(c<=57))
    {
        x=x*10+(c^48);
        c=getchar();
    }
}
void dft(ll *a)
{
    int i,j,k,l;
    ll b;
    for (i=1;i<n;i=l)
    {
        l=i<<1;
        for (j=0;j<n;j+=l) for (k=0;k<i;k++)
        {
            b=a[j|k|i];
            a[j|k|i]=(a[j|k]-b+p)%p;
            a[j|k]=(a[j|k]+b)%p;
        }
    }
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    ll t;int i;
    cin>>m>>t>>p;p*=(n=1<<m);
    for (i=0;i<n;i++) cin>>f[i];
    dft(f);
    for (i=0;i<=m;i++) cin>>x[i];
    for (i=1;i<n;i++) g[i]=g[i>>1]+(i&1);
    for (i=0;i<n;i++) g[i]=x[g[i]];dft(g);
    while (t)
    {
        if (t&1) for (i=0;i<n;i++) f[i]=mul(f[i],g[i]);
        for (i=0;i<n;i++) g[i]=mul(g[i],g[i]);t>>=1;
    }
    dft(f);
    for (i=0;i<n;i++) cout<<(f[i]>>m)<<'\n';
}
```

2.30 二次剩余

```
namespace cipolla
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    ui p,w;
    struct Q
    {
        ll x,y;
        Q operator*(const Q &o) const {return {(x*o.x+y*o.y%p*w)%p,(x*o.y+y*o.x)
            %p};}
    };
    ui ksm(ll x,ui y)
```

```
    {
        ll r=1;
        while (y)
        {
            if (y&1) r=r*x%p;
            x=x*x%p;y>>=1;
        }
        return r;
    }
    Q ksm(Q x,ui y)
    {
        Q r={1,0};
        while (y)
        {
            if (y&1) r=r*x;
            x=x*x;y>>=1;
        }
        return r;
    }
    ui mosqrt(ui x,ui P)//0<=x<P
    {
        if (x==0||P==2) return x;
        p=P;
        if (ksm(x,p-1>>1)!=1) return -1;
        ui y;
        mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
        do y=rnd()%p,w=((ll)y*y+p-x)%p; while (ksm(w,p-1>>1)<=1);//not for p=2
        y=ksm({y,1},p+1>>1).x;
        if (y*2>p) y=p-y;//两解取小
        return y;
    }
}
using cipolla::mosqrt;
```

2.31 k 次剩余

```
namespace get_root
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    bool ied=0;
    const int N=1e5+5;
    vector<ui> pr;
    bool ed[N];
    void init()
    {
        pr.reserve(N);
        for (ui i=2;i<N;i++)
        {
            if (!ed[i]) pr.push_back(i);
            for (ui x:pr)
            {
                if (i*x>=N) break;
                ed[i*x]=1;
                if (i%x==0) break;
            }
        }
```

```cpp
        }
    }
    ui ksm(ui x,ui y,ui p)
    {
        ui r=1;
        while (y)
        {
            if (y&1) r=(ll)r*x%p;
            x=(ll)x*x%p;y>>=1;
        }
        return r;
    }
    vector<ui> getw(ui n)
    {
        vector<ui> w;
        for (ui x:pr)
        {
            if (x*x>n) break;
            if (n%x==0)
            {
                w.push_back(x);
                n/=x;
                for (ui i=n/x;n==x*i;i=n/x) n/=x;
            }
        }
        if (n>1) w.push_back(n);
        return w;
    }
    int getrt(ui n)
    {
        if (n<=2) return n-1;
        if (!ed[4]) init();
        auto w=getw(n);
        ui ph=n;
        for (ui x:w) ph=ph/x*(x-1);
        w=getw(ph);
        for (ui &x:w) x=ph/x;
        for (ui i=2;i<n;i++) if (gcd(i,n)==1)
        {
            for (ui x:w) if (ksm(i,x,n)==1) goto no;
            return i;
            no:;
        }
        return -1;
    }
}
namespace BSGS
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    template<int N,typename T,typename TT> struct ht//个数，定义域，值域
    {
        const static int p=1e6+7,M=p+2;
        TT a[N];
        T v[N];
        int fir[p+2],nxt[N],st[p+2];//和模数相适应
        int tp,ds;//自定义模数
```

```cpp
    ht(){memset(fir,0,sizeof fir);tp=ds=0;}
    void mdf(T x,TT z)//位置，值
    {
        ui y=x%p;
        for (int i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i]=z,void();//若
            不可能重复不需要 for
        v[++ds]=x;a[ds]=z;
        if (!fir[y]) st[++tp]=y;
        nxt[ds]=fir[y];fir[y]=ds;
    }
    TT find(T x)
    {
        ui y=x%p;
        int i;
        for (i=fir[y];i;i=nxt[i]) if (v[i]==x) return a[i];
        return 0;//返回值和是否判断依据要求决定
    }
    void clear()
    {
        ++tp;
        while (--tp) fir[st[tp]]=0;
        ds=0;
    }
};
const int N=5e4;
ht<N,ui,ui> s;
int exgcd(int a,int b)
{
    if (a==1) return 1;
    return (1-(long long)b*exgcd(b%a,a))/a;//not ll
}
int bsgs(ui a,ui b,ui p)
{
    s.clear();
    a%=p;b%=p;
    if (!a) return 1-min((int)b,2);//含 -1
    ui i,j,k,x,y;
    x=sqrt(p)+2;
    for (i=0,j=1;i<x;i++,j=(ll)j*a%p)
    {
        if (j==b) return i;
        s.mdf((ll)j*b%p,i+1);
    }
    k=j;
    for (i=1;i<=x;i++,j=(ll)j*k%p) if (y=s.find(j)) return (ll)i*x-y+1;
    return -1;
}
bool isprime(ui p)
{
    if (p<=1) return 0;
    for (ui i=2;i*i<=p;i++) if (p%i==0) return 0;
    return 1;
}
int exbsgs(ui a,ui b,ui p)//a^x=b(mod p)
{
    //if (isprime(p)) return bsgs(a,b,p);
    a%=p;b%=p;
```

```cpp
        ui i,j,k,x,y=__lg(p),cnt=0;
        for (i=0,j=1%p;i<=y;i++,j=(ll)j*a%p) if (j==b) return i;
        y=1;
        while (1)
        {
            if ((x=gcd(a,p))==1) break;
            if (b%x) return -1;//no sol
            ++cnt;
            p/=x;b/=x;
            y=(ll)y*(a/x)%p;
        }
        a%=p;
        b=(ll)b*(p+exgcd(y,p))%p;
        int r=bsgs(a,b,p);
        return r==-1?-1:r+cnt;
    }
}
pair<ll,ll> exgcd(ll a,ll b,ll c)//ax+by=c，{-1,-1} 无解，b=0 返回 {c/a,0}，否则
    返回最小非负 x
{
    assert(a||b);
    if (!b) return {c/a,0};
    if (a<0) a=-a,b=-b,c=-c;
    ll d=gcd(a,b);
    if (c%d) return {-1,-1};
    ll x=1,x1=0,p=a,q=b,k;
    b=abs(b);
    while (b)
    {
        k=a/b;
        x-=k*x1;a-=k*b;
        swap(x,x1);
        swap(a,b);
    }
    b=abs(q/d);
    x=x*(c/d)%b;
    if (x<0) x+=b;
    return {x,(c-p*x)/q};
}
ll fun(ll a,ll b,ll p)//ax=b(mod p)
{
    return exgcd(-p,a,b).second%p;
}
using get_root::getrt;
using BSGS::bsgs,BSGS::exbsgs;
int nth_root(ui k,ui y,ui p)//x^k=y(mod p)
{
    if (k==0) return y==1?0:-1;
    if (y==0) return 0;
    ui g=getrt(p);
    ui z=bsgs(g,y,p);
    ll x=fun(k,z,p-1);
    if (x==-1) return -1;
    return get_root::ksm(g,x,p);
}
```

网上的超快版本

```cpp
#define popcount __builtin_popcount
using namespace std;
typedef long long int ll;
//using ll=__int128_t;
typedef pair<ll, int> P;
ll gcd(ll a, ll b){
    if (b==0) return a;
    return gcd(b, a%b);
}
ll powmod(ll a, ll k, ll mod){
    ll ap=a, ans=1;
    while(k){
        if (k&1){
            ans*=ap;
            ans%=mod;
        }
        ap=ap*ap;
        ap%=mod;
        k>>=1;
    }
    return ans;
}
ll inv(ll a, ll m){
    ll b=m, x=1, y=0;
    while(b>0){
        ll t=a/b;
        swap(a-=t*b, b);
        swap(x-=t*y, y);
    }
    return (x%m+m)%m;
}
vector<P> fac(ll x){
    vector<P> ret;
    for(ll i=2; i*i<=x; i++){
        if (x%i==0){
            int e=0;
            while(x%i==0){
                x/=i;
                e++;
            }
            ret.push_back({i, e});
        }
    }
    if (x>1) ret.push_back({x, 1});
    return ret;
}
//mt19937_64 mt(334);
mt19937 mt(334);
ll solve1(ll p, ll q, int e, ll a){
    int s=0;
    ll r=p-1, qs=1, qp=1;
    while(r%q==0){
        r/=q;
        qs*=q;
        s++;
    }
    for(int i=0; i<e; i++) qp*=q;
```

```cpp
        ll d=qp-inv(r%qp, qp);
        ll t=(d*r+1)/qp;
        ll at=powmod(a, t, p), inva=inv(a, p);
        if (e>=s){
            if (powmod(at, qp, p)!=a) return -1;
            else return at;
        }
        //uniform_int_distribution<long long> rnd(1, p-1);
        uniform_int_distribution<> rnd(1, p-1);
        ll rv;
        while(1){
            rv=powmod(rnd(mt), r, p);
            if (powmod(rv, qs/q, p)!=1) break;
        }
        int i=0;
        ll qi=1, sq=1;
        while(sq*sq<q) sq++;
        while(i<s-e){
            ll qq=qs/qp/qi/q;
            vector<P> v(sq);
            ll rvi=powmod(rv, qp*qq*(p-2)%(p-1), p), rvp=powmod(rv, sq*qp*qq, p);
            ll x=powmod(powmod(at, qp, p)*inva%p, qq*(p-2)%(p-1), p), y=1;
            for(int j=0; j<sq; j++){
                v[j]=P(x, j);
                (x*=rvi)%=p;
            }
            sort(v.begin(), v.end());
            ll z=-1;
            for(int j=0; j<sq; j++){
                int l=lower_bound(v.begin(), v.end(), P(y, 0))-v.begin();
                if (v[l].first==y){
                    z=v[l].second+j*sq;
                    break;
                }
                (y*=rvp)%=p;
            }
            if (z==-1) return -1;
            (at*=powmod(rv, z, p))%=p;
            i++;
            qi*=q;
            rv=powmod(rv, q, p);
        }
        return at;
}
ll solve0(ll p, ll q, ll r, ll a){
        ll d=q-inv(r%q, q);
        ll t=(d*r+1)/q;
        ll at=powmod(a, t, p), inva=inv(a, p);
        if (powmod(at, q, p)!=a) return -1;
        else return at;
}
ll solve(ll p, ll k, ll a)//p k y
{
        if (k==0)
        {
            if (a==1) return 1;
            return -1;
```

```
    }
    if (a==0) return 0;
    if (p==2 || a==1) return 1;
    ll a1=a;
    ll g=gcd(p-1, k);
    ll c=inv(k/g%((p-1)/g), (p-1)/g);
    a=powmod(a, c, p);
    if (g==1){
        if (powmod(a, k, p)==a1) return a;
        else return -1;
    }
    ll g1=gcd(g, (p-1)/g), g2=g;
    vector<P> f1=fac(g1), f;
    for(auto r:f1){
        ll q=r.first;
        int e=0;
        while(g2%q==0){
            g2/=q;
            e++;
        }
        f.push_back({q, e});
    }
    ll ret=1, gp=1;
    if (g2>1){
        ll x=solve0(p, g2, (p-1)/g2, a);
        if (x==-1) return -1;
        ret=x, gp*=g2;
    }
    for(auto r:f){
        ll qp=1;
        for(int i=0; i<r.second; i++) qp*=r.first;
        ll x=solve1(p, r.first, r.second, a);
        if (x==-1) return -1;
        if (gp==1){
            ret=x, gp*=qp;
            continue;
        }
        ll s=inv(gp%qp, qp), t=(1-gp*s)/qp;
        if (t>=0) ret=powmod(ret, t, p);
        else ret=powmod(ret, p-1+t%(p-1), p);
        if (s>=0) x=powmod(x, s, p);
        else x=powmod(x, p-1+s%(p-1), p);
        (ret*=x)%=p;
        gp*=qp;
    }
    if (powmod(ret, k, p)!=a1) return -1;
    return ret;
}
```

2.32 FWT/子集卷积

$O(n2^n)$，$O(2^n)$。注意全都是无符号的。

这里混合了两个版本的代码，但只有 ui 和 ull 的差异。容易自行调整。

```
void fwt_and(vector<ll> &A)//本质：母集和
{
```

```
    ll n=A.size(), *a=A.data(), i, j, k, l, *f, *g;
    for (i=1; i<n; i=l)
    {
        l=i*2;
        for (j=0; j<n; j+=l)
        {
            f=a+j; g=a+j+i;
            for (k=0; k<i; k++) f[k]+=g[k];
        }
        if (l==n||i==1<<10) for (ll &x:A) x%=p;
    }
}
void ifwt_and(vector<ll> &A)
{
    ll n=A.size(), *a=A.data(), i, j, k, l, *f, *g;
    for (i=1; i<n; i=l)
    {
        l=i*2;
        for (j=0; j<n; j+=l)
        {
            f=a+j; g=a+j+i;
            for (k=0; k<i; k++) f[k]+=p*i-g[k];
        }
        if (l==n||i==1<<10) for (ll &x:A) x%=p;
    }
}
void fwt_or(vector<ll> &A)//本质：子集和
{
    ll n=A.size(), *a=A.data(), i, j, k, l, *f, *g;
    for (i=1; i<n; i=l)
    {
        l=i*2;
        for (j=0; j<n; j+=l)
        {
            f=a+j; g=a+j+i;
            for (k=0; k<i; k++) g[k]+=f[k];
        }
        if (l==n||i==1<<10) for (ll &x:A) x%=p;
    }
}
void ifwt_or(vector<ll> &A)
{
    ll n=A.size(), *a=A.data(), i, j, k, l, *f, *g;
    for (i=1; i<n; i=l)
    {
        l=i*2;
        for (j=0; j<n; j+=l)
        {
            f=a+j; g=a+j+i;
            for (k=0; k<i; k++) g[k]+=p*i-f[k];
        }
        if (l==n||i==1<<10) for (ll &x:A) x%=p;
    }
}
void fwt_xor(vector<ui> &A)
{
    ui n=A.size(),*a=A.data(),i,j,k,l,*f,*g;
```

```
    for (i=1;i<n;i=l)
    {
        l=i*2;
        for (j=0;j<n;j+=l)
        {
            f=a+j;g=a+j+i;
            for (k=0;k<i;k++)
            {
                if ((f[k]+=g[k])>=p) f[k]-=p;
                g[k]=(f[k]+2*(p-g[k]))%p;
            }
        }
    }
}
void ifwt_xor(vector<ui> &A)
{
    ui n=A.size(),*a=A.data(),i,j,k,l,*f,*g,x=p+1>>1,y=1;
    for (i=1;i<n;i=l)
    {
        l=i*2;
        for (j=0;j<n;j+=l)
        {
            f=a+j;g=a+j+i;
            for (k=0;k<i;k++)
            {
                if ((f[k]+=g[k])>=p) f[k]-=p;
                g[k]=(f[k]+2*(p-g[k]))%p;
            }
        }
        y=(ll)y*x%p;
    }
    for (i=0;i<n;i++) a[i]=(ll)a[i]*y%p;
}
vector<ui> fst(const vector<ui> &s,const vector<ui> &t)
{
    int n=s.size(),m=__builtin_ctz(n),i,j,k;
    vector<ui> a[m+1],b[m+1],c[m+1],r(n);
    for (i=0;i<=m;i++) a[i].resize(n),b[i].resize(n),c[i].resize(n);
    for (i=0;i<n;i++)
    {
        k=__builtin_popcount(i);
        a[k][i]=s[i];
        b[k][i]=t[i];
    }
    for (i=0;i<m;i++) fwt_or(a[i]),fwt_or(b[i]);
    for (i=0;i<=m;i++) for (j=0;j<=i;j++) for (k=0;k<n;k++) c[i][k]=(c[i][k]+(ll
        )a[j][k]*b[i-j][k])%p;
    for (i=1;i<=m;i++) ifwt_or(c[i]);
    for (i=0;i<n;i++) r[i]=c[__builtin_popcount(i)][i];
    return r;
}
```

2.33　NTT

　　一种较快的 NTT（尤其是对于卷积以外的用途），但不推荐在不熟悉的情况下直接使用。一般的卷积可以参照字符串部分，其余的用途可以参照其他板子。

如果确实需要卡常，建议先抄写需要的函数，并递归地找到需要补的内容。

注意事项：所有 ll 为无符号。始终保证数组大小为 2^n，不应当使用 resize 而应该使用取模来调整长度。三种卷积对应的运算符见注释。

需要特别小心其长度的变化，注意不要越界。

```cpp
#include <optional>
namespace NTT
{
    const ll g=3, p=998244353;
    const int N=1<<22;//务必修改
    ll inv[N], fac[N], ifac[N];//非必要
    void getfac(int n)//非必要
    {
        static int pre=-1;
        if (pre==-1) pre=1, ifac[0]=fac[0]=fac[1]=ifac[1]=inv[1]=1;
        if (n<=pre) return;
        for (int i=pre+1, j; i<=n; i++)
        {
            j=p/i;
            inv[i]=(p-j)*inv[p-i*j]%p;
            fac[i]=fac[i-1]*i%p;
            ifac[i]=ifac[i-1]*inv[i]%p;
        }
        pre=n;
    }
    ll w[N];
    int r[N];
    ll ksm(ll x, ll y)
    {
        ll r=1;
        while (y)
        {
            if (y&1) r=r*x%p;
            x=x*x%p;
            y>>=1;
        }
        return r;
    }
    void init(int n)
    {
        static int pr=0, pw=0;
        if (pr==n) return;
        int b=__lg(n)-1, i, j, k;
        for (i=1; i<n; i++) r[i]=r[i>>1]>>1|(i&1)<<b;
        if (pw<n)
        {
            for (j=1; j<n; j=k)
            {
                k=j*2;
                ll wn=ksm(g, (p-1)/k);
                w[j]=1;
                for (i=j+1; i<k; i++) w[i]=w[i-1]*wn%p;
            }
            pw=n;
        }
        pr=n;
```

```
        }
        int cal(int x) { return 1<<__lg(max(x, 1)*2-1); }
        struct Q:vector<ll>
        {
            bool flag;
            Q &operator%=(int n) { assert((n&-n)==n); resize(n); return *this; }
            Q operator%(int n) const
            {
                assert((n&-n)==n);
                if (size()<=n)
                {
                    auto f=*this;
                    return f%=n;
                }
                return Q(vector(begin(), begin()+n));
            }
            int deg() const
            {
                int n=size()-1;
                while (n>=0&&begin()[n]==0) --n;
                return n;
            }
            explicit Q(int x=1, bool f=0):flag(f), vector<ll>(cal(x)) { }//小心:
                {}会调用这条而非下一条
            Q(const vector<ll> &o, bool f=0):Q(o.size(), f) { copy(all(o), begin
                ()); }
            Q(const initializer_list<ll> &o, bool f=0):Q(vector(o), f) { }
            ll fx(ll x)
            {
                ll r=0;
                for (auto it=rbegin(); it!=rend(); ++it) r=(r*x+*it)%p;
                return r;
            }
            void dft()
            {
                int n=size(), i, j, k;
                ll y, *f, *g, *wn, *a=data();
                init(n);
                for (i=1; i<n; i++) if (i<r[i]) ::swap(a[i], a[r[i]]);
                for (k=1; k<n; k*=2)
                {
                    wn=w+k;
                    for (i=0; i<n; i+=k*2)
                    {
                        g=(f=a+i)+k;
                        for (j=0; j<k; j++)
                        {
                            y=g[j]*wn[j]%p;
                            g[j]=f[j]+p-y;
                            f[j]+=y;
                        }
                    }
                    if (k*2==n||k==1<<14) for (i=0; i<n; i++) a[i]%=p;
                }
                if (flag)
                {
                    y=ksm(n, p-2);
```

```cpp
            for (i=0; i<n; i++) a[i]=a[i]*y%p;
            reverse(a+1, a+n);
        }
        flag^=1;
    }
    void hf_dft()
    {
        assert(size()>=2&&flag);
        int n=size()/2, i, j, k;
        ll x, y, *f, *g, *wn, *a=data();
        init(n);
        for (i=1; i<n; i++) if (i<r[i]) ::swap(a[i], a[r[i]]);
        for (k=1; k<n; k*=2)
        {
            wn=w+k;
            for (i=0; i<n; i+=k*2)
            {
                g=(f=a+i)+k;
                for (j=0; j<k; j++)
                {
                    y=g[j]*wn[j]%p;
                    g[j]=f[j]+p-y;
                    f[j]+=y;
                }
            }
            if (k*2==n||k==1<<14) for (i=0; i<n; i++) a[i]%=p;
        }
        if (flag)
        {
            x=ksm(n, p-2);
            for (i=0; i<n; i++) a[i]=a[i]*x%p;
            reverse(a+1, a+n);
        }
        flag^=1;
    }
    Q operator<<(int m) const
    {
        int n=deg(), i;
        Q r(n+m+1);
        for (i=0; i<=n; i++) r[i+m]=at(i);
        return r;
    }
    Q operator>>(int m) const
    {
        int n=deg(), i;
        if (n<m) return { };
        Q r(n+1-m);
        for (i=m; i<=n; i++) r[i-m]=at(i);
        return r;
    }
};
Q shrink(Q f) { return f%=cal(f.deg()+1); }
ostream &operator<<(ostream &cout, const Q &o)
{
    int n=o.deg();
    if (n<0) return cout<<"[0]";
    cout<<"["<<o[n];
```

```cpp
        for (int i=n-1; i>=0; i--) cout<<", "<<o[i];
        return cout<<"]";
    }
    Q der(const Q &f)
    {
        ll n=f.size(), i;
        Q r(n);
        for (i=1; i<n; i++) r[i-1]=f[i]*i%p;
        return r;
    }
    Q integral(const Q &f)
    {
        ll n=f.size(), i;
        getfac(n);
        Q r(n);
        for (i=1; i<n; i++) r[i]=f[i-1]*inv[i]%p;
        return r;
    }
    Q operator-(Q f) { for (ll &x:f) if (x) x=p-x; return f; }
    Q &operator+=(Q &f, ll x) { (f[0]+=x)%=p; return f; }
    Q operator+(Q f, ll x) { return f+=x; }
    Q &operator-=(Q &f, ll x) { (f[0]+=p-x)%=p; return f; }
    Q operator-(Q f, ll x) { return f-=x; }
    Q &operator*=(Q &f, ll x) { for (ll &y:f) (y*=x)%=p; return f; }
    Q operator*(Q f, ll x) { return f*=x; }
    Q &operator+=(Q &f, const Q &g) { f%=max(f.size(), g.size()); for (int i
        =0; i<g.size(); i++) f[i]=(f[i]+g[i])%p; return f; }
    Q operator+(Q f, const Q &g) { return f+=g; }
    Q &operator-=(Q &f, const Q &g) { f%=max(f.size(), g.size()); for (int i
        =0; i<g.size(); i++) f[i]=(f[i]+p-g[i])%p; return f; }
    Q operator-(Q f, const Q &g) { return f-=g; }
    Q &operator*=(Q &f, Q g)//卷积
    {
        if (f.flag|g.flag)
        {
            int n=f.size(), i;
            assert(n==g.size());
            if (!f.flag) f.dft();
            if (!g.flag) g.dft();
            for (i=0; i<n; i++) (f[i]*=g[i])%=p;
            f.dft();
        }
        else
        {
            int n=cal(f.size()+g.size()-1), i, j;
            int m1=f.deg(), m2=g.deg();
            if ((ll)m1*m2>(ll)n*__lg(n)*8)
            {
                (f%=n).dft(); (g%=n).dft();
                for (i=0; i<n; i++) (f[i]*=g[i])%=p;
                f.dft();
            }
            else
            {
                vector<ll> r(max(0, m1+m2+1));
                for (i=0; i<=m1; i++) for (j=0; j<=m2; j++) (r[i+j]+=f[i]*g[j
                    ])%=p;
```

```
                f=Q(n);
                copy(all(r), f.begin());
            }
        }
        return f;
    }
    Q operator*(Q f, const Q &g) { return f*=g; }
    Q &operator&=(Q &f, Q g)//循环卷积
    {
        assert(f.size()==g.size());
        int n=f.size(), i;
        if (!f.flag) f.dft();
        if (!g.flag) g.dft();
        for (i=0; i<n; i++) (f[i]*=g[i])%=p;
        f.dft();
        return f;
    }
    Q operator&(Q f, const Q &g) { return f&=g; }
    Q &operator^=(Q &f, Q g)//差卷积
    {
        int n=f.size();
        g%=n;
        reverse(all(g));
        f*=g;
        rotate(f.begin(), n-1+all(f));
        return f%=n;
    }
    Q operator^(Q f, const Q &g) { return f^=g; }
    Q sqr(Q f)
    {
        assert(!f.flag);
        int n=f.size()*2, i;
        (f%=n).dft();
        for (i=0; i<n; i++) f[i]=f[i]*f[i]%p;
        f.dft();
        return f;
    }
    /*Q operator~(const Q &f)
    {
        Q r;
        r[0]=ksm(f[0],p-2);
        for (int i=1; i<=f.size(); i*=2) r=(-((f%i)*r-2)*r)%i;
        return r;
    }//trivial, 5e5 750ms*/
    Q operator~(const Q &f)
    {
        Q q, r, g;
        int n=f.size(), i, j, k;
        r[0]=ksm(f[0], p-2);
        for (j=2; j<=n; j*=2)
        {
            k=j/2;
            g=(r%=j)%k;
            r.dft();
            q=f%j*r;
            fill_n(q.begin(), k, 0);
            r*=q;
```

```
            copy(all(g), r.begin());
            for (i=k; i<j; i++) r[i]=(p-r[i])%p;
        }
        return r;
}//5e5 200ms, inv(1 6 3 4 9)=(1 998244347 33 998244169 1020)
Q &operator/=(Q &f, const Q &g) { int n=f.size(); return (f*=~g)%=n; }
Q operator/(Q f, const Q &g) { return f/=g; }
void cdq(Q &f, Q &g, int l, int r)//g_0=1,i*g_i=g_{i-j}*f_j,use for cdq
{
    static vector<Q> cd;
    int i, m=l+r>>1, n=r-l, nn=n>>1;
    if (r-l==f.size())
    {
        getfac(n-1);
        g=Q(n);
        cd.clear();
        for (i=2; i<=n; i*=2)
        {
            cd.emplace_back(i);
            Q &h=cd.back();
            h%=i;
            copy_n(f.begin(), i, h.begin());
            h.dft();
        }
    }
    if (l+1==r)
    {
        g[l]=l?g[l]*inv[l]%p:1;
        return;
    }
    cdq(f, g, l, m);
    Q h(n);
    copy_n(g.begin()+l, nn, h.begin());
    h*=cd[__lg(n)-1];
    for (i=m; i<r; i++) (g[i]+=h[i-l])%=p;
    cdq(f, g, m, r);
}
Q exp_cdq(Q f)
{
    Q g;
    int n=f.size(), i;
    for (i=1; i<n; i++) f[i]=f[i]*i%p;
    cdq(f, g, 0, n);
    return g;
}//5e5 455ms
Q ln(const Q &f) { return integral(der(f)/f); }
//5e5 330ms, ln(1 2 3 4 5)=(0 2 1 665496236 499122177)
Q exp(Q f)
{
    Q r; r[0]=1;
    for (int i=1; i<=f.size(); i*=2) (r*=f%i-ln(r%i)+1)%=i;
    return r;
}//5e5 700ms, exp(0 4 2 3 5)=(1 4 10 665496257 665496281)
Q exp_new(Q b)
{
    Q h, f, r, u, v, bj;
    int n=b.size(), i, j, k;
```

```
        r[0]=h[0]=1;
        for (j=2; j<=n; j*=2)
        {
            f=bj=der(b%j); k=j/2; fill(k+all(bj), 0);
            h.dft(); u=der(r)&h;
            v=(r&h)%j-1&bj;
            for (i=0; i<k; i++) f[i+k]=(p*p+u[i]-v[i]-f[i]-f[i+k])%p, f[i]=0;
            f[k-1]=(f[j-1]+v[k-1])%p;
            u=(r%=j)&integral(f);
            for (i=k; i<j; i++) r[i]=(p-u[i])%p;
            if (j<n) h=~r;
        }
        return r;
}//5e5 420ms
optional<ll> mosqrt(ll x)
{
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().
        count());
    static ll W;
    struct P
    {
        ll x, y;
        P operator*(const P &a) const
        {
            return {(x*a.x+y*a.y%p*W)%p, (x*a.y+y*a.x)%p};
        }
    };
    if (x==0) return {0};
    if (ksm(x, p-1>>1)!=1) return { };
    ll y;
    do y=rnd()%p; while (ksm(W=(y*y%p+p-x)%p, p-1>>1)<=1);//not for p=2
    y=[&](P x, ll y)
        {
            P r{1, 0};
            while (y)
            {
                if (y&1) r=r*x;
                x=x*x; y>>=1;
            }
            return r.x;
        }({y, 1}, p+1>>1);
    return {y*2<p?y:p-y};
}
optional<Q> sqrt(Q f)
{
    const static ll i2=p+1>>1;
    Q r;
    int n=f.size(), i, l;

    for (i=0; i<n; i++) if (f[i]) break;
    if (i==n) return f;
    if (i&1) return { };
    l=i/2;
    copy(i+all(f), f.begin());
    fill(n-i+all(f), 0);

    auto rt=mosqrt(f[0]);
```

```
        if (rt) r[0]=rt.value(); else return { };
        for (i=2; i<=n; i*=2) r=(sqr(r)+f%i)/(r%i)%i*i2;

        copy_backward(all(r)-l, r.end());
        fill_n(r.begin(), l, 0);

        return {r};
}//5e5 530ms, sqrt(0 0 4 2 3)=(0 2 499122177 311951361 171573248)
optional<Q> sqrt_new(Q f)
{
        const static ll i2=p+1>>1;
        Q q, r;
        int n=f.size(), i, j, k, l;

        for (i=0; i<n; i++) if (f[i]) break;
        if (i==n) return f;
        if (i&1) return { };
        l=i/2;
        copy(i+all(f), f.begin());
        fill(n-i+all(f), 0);

        auto rt=mosqrt(f[0]);
        if (rt) r[0]=rt.value(); else return { };
        for (j=2; j<=n; j*=2)
        {
            k=j/2; (q=r).dft(); (q&=q)%=j;
            for (i=k; i<j; i++) q[i]=(q[i-k]+p*2-f[i]-f[i-k])*i2%p, q[i-k]=0;
            q&=~r%j; r%=j;
            for (i=k; i<j; i++) r[i]=(p-q[i])%p;
        }

        copy_backward(all(r)-l, r.end());
        fill_n(r.begin(), l, 0);

        return {r};
}//5e5 280ms
Q pow(Q b, ll m)//不应传入超过 int 内容
{
        assert(m<=1llu<<32);
        int n=b.size(), i, j=n, k;
        for (i=0; i<n; i++) if (b[i]) { j=i; break; }
        if (j==n) return b[0]=!m, b;
        if (j*m>=n) return Q(n);
        copy(j+all(b), b.begin());
        fill(n-j+all(b), 0);
        k=b[0]; j*=m;
        b=exp_new(ln(b*ksm(k, p-2))*m)*ksm(k, m);
        copy_backward(all(b)-j, b.end());
        fill_n(b.begin(), j, 0);
        return b;
}
Q pow(Q b, string s)
{
        int n=b.size(), i, j=n, k;
        for (i=0; i<n; i++) if (b[i]) { j=i; break; }
        if (j==n) return b[0]=s=="0", b;
        if (j&&(s.size()>8||j*stoll(s)>=n)) return Q(n);
```

```
        ll m0=0, m1=0;
        for (auto c:s) m0=(m0*10+c-'0')%p, m1=(m1*10+c-'0')%(p-1);
        copy(j+all(b), b.begin());
        fill(n-j+all(b), 0);
        k=b[0]; j*=m0;
        b=exp_new(ln(b*ksm(k, p-2))*m0)*ksm(k, m1);
        copy_backward(all(b)-j, b.end());
        fill_n(b.begin(), j, 0);
        return b;
}//5e5 1e18 700ms
Q pow2(Q b, ll m)
{
        int n=b.size();
        Q r(n); r[0]=1;
        while (m)
        {
                if (m&1) (r*=b)%=n;
                if (m>>=1) b=sqr(b)%n;
        }
        return r;
}//5e5 1e18 7425ms
Q div(Q f, Q g)
{
        int n=0, m=0, i;
        for (i=f.size()-1; i>=0; i--) if (f[i]) { n=i+1; break; }
        for (i=g.size()-1; i>=0; i--) if (g[i]) { m=i+1; break; }
        assert(m);
        if (n<m) return Q(1);
        reverse(f.begin(), f.begin()+n);
        reverse(g.begin(), g.begin()+m);
        n=n-m+1; m=cal(n);
        f=(f%m)/(g%m)%m;
        fill(n+all(f), 0);
        reverse(f.begin(), f.begin()+n);
        return f;
}
Q mod(const Q &a, const Q &b)
{
        if (a.deg()<b.deg()) return shrink(a);
        Q r=(a-b*div(a, b));
        return shrink(r%=min(r.size(), b.size()));
}
Q pow(Q x, ll y, Q f)
{
        Q r(1);
        r[0]=1;
        while (y)
        {
                if (y&1) r=mod(r*x, f);
                if (y>>=1) x=mod(sqr(x), f);
        }
        return r;
}
pair<Q, Q> div_mod(const Q &a, const Q &b) { Q q=div(a, b); Q r=(a-b*q);
        return {q, r%=min(r.size(), b.size())}; }
//5e5 430ms (1 2 3 4)=(916755018 427819009)*(5 6 7)+(407446676
        346329673)
```

```cpp
// Q cdq_inv(const Q &f) { return (~(f-1))*(p-1); }//g_0=1,g_i=g_{i-j}*
    f_j ?
ll recurrent(const vector<ll> &f, const vector<ll> &a, ll m)//常系数齐次
    线性递推, find a_m,a_n=a_{n-i}*f_i,f_1...k,a_0...k-1
{
    if (m<a.size()) return a[m];
    assert(f.size()==a.size()+1&&f[0]==0);
    int k=a.size(), n=cal(k+1)*2, i;
    ll ans=0;
    Q h(n), g(2);
    for (i=1; i<=k; i++) h[k-i]=(p-f[i])%p;
    h[k]=g[1]=1;
    Q r=pow(g, m, h);
    k=min(k, (int)r.size());
    for (i=0; i<k; i++) ans=(ans+a[i]*r[i])%p;
    return ans;
}//1e5 1e18 8500ms
ll recurrent_new(const vector<ll> &f, const vector<ll> &a, ll m)//常系数
    齐次线性递推, find a_m,a_n=a_{n-i}*f_i,f_1...k,a_0...k-1
{
    const static ll i2=p+1>>1;
    if (m<a.size()) return a[m];
    assert(f.size()==a.size()+1&&f[0]==0);
    int k=a.size(), n=cal(k+1), i;
    Q g(n*2), h(n*2);
    for (h[0]=i=1; i<=k; i++) h[i]=(p-f[i])%p;
    copy(all(a), g.begin());
    g&=h; fill(k+++all(g), 0);
    vector<ll> res(n);
    while (m)
    {
        if (m&1)
        {
            ll x=p-g[0];
            for (i=1; i<k; i+=2) res[i>>1]=x*h[i]%p;
            copy_n(g.begin()+1, k-1, g.begin());
            g[k-1]=0;
        }
        g.dft(); h.dft();
        ll *a=g.data(), *b=h.data(), *c=a+n, *d=b+n;
        for (i=0; i<n; i++) g[i]=(a[i]*d[i]+b[i]*c[i])%p*i2%p;
        for (i=0; i<n; i++) h[i]=h[i]*h[i^n]%p;
        g.hf_dft(); h.hf_dft();
        fill(k+all(g), 0);
        if (m&1) for (i=0; i<k; i++) (g[i]+=res[i])%=p;
        fill(k+all(h), 0);
        m>>=1;
    }
    assert(h[0]==1);
    return g[0];
}//1e5 1e18 1000ms
vector<ll> recurrent_interval(const vector<ll> &f, const vector<ll> &a,
    ll L, ll R)//常系数齐次线性递推, find a_[L,R),a_n=a_{n-i}*f_i,f_1...k
    ,a_0...k-1
{
    assert(f.size()==a.size()+1&&f[0]==0);
    int k=a.size(), n=cal(k+1)*2, i, len=R-L;
```

```
        ll ans=0, m=L;
        Q h(n), g(2), r;
        for (i=1; i<=k; i++) h[k-i]=(p-f[i])%p;
        h[k]=g[1]=r[0]=1;
        while (m)
        {
            if (m&1) r=mod(r*g, h);
            if (m>>=1) g=mod(sqr(g), h);
        }
        Q F(f), A(a);
        F[0]=p-1;
        A*=F;
        A%=cal(k);
        fill(k+all(A), 0);
        n=cal(len+k);
        F%=n;
        A*=~F;
        r%=cal(k);
        reverse(r.begin(), r.begin()+k);
        r*=A;
        r.erase(r.begin(), r.begin()+k-1);
        r.resize(len);
        return r;
    }//1e5 1e18 5e5 10000ms
    Q prod(const vector<Q> &a)
    {
        if (!a.size()) return {1};
        function<Q(int, int)> dfs=[&](int l, int r)
            {
                if (r-l==1) return a[l];
                int m=l+r>>1;
                return shrink(dfs(l, m)*dfs(m, r));
            };
        return dfs(0, a.size());
    }//not check
    Q prod_new(const vector<Q> &a)
    {
        if (!a.size()) return {1};
        struct cmp
        {
            bool operator()(const Q &f, const Q &g) const { return f.size()>g
                .size(); }
        };
        priority_queue<Q, vector<Q>, cmp> q(all(a));
        while (q.size()>1)
        {
            auto f=q.top(); q.pop();
            f=shrink(f*q.top()); q.pop();
            q.push(f);
        }
        return q.top();
    }//not check
    vector<ll> evaluation(const Q &f, const vector<ll> &X)
    {
        int m=X.size(), n=f.size()-1, i, j;
        vector<Q> pro(m*4+4);
        while (n>1&&!f[n]) --n;
```

```cpp
        vector<ll> y(m);
        function<void(int, int, int)> build=[&](int x, int l, int r)
            {
                if (l+1==r)
                {
                    pro[x]=Q(vector{(p-X[l])%p, 1llu});
                    return;
                }
                int mid=l+r>>1, c=x*2;
                build(c, l, mid); build(c+1, mid, r);
                pro[x]=shrink(pro[c]*pro[c+1]);
            };
        function<void(int, int, int, Q, int)> dfs=[&](int x, int l, int r, Q
            f, int d)
            {
                const static int limit=256;
                if (d>=r-l) f=shrink(mod(f, pro[x]));
                if (r-l<limit)
                {
                    for (int i=l; i<r; i++) y[i]=f.fx(X[i]);
                    return;
                }
                int mid=l+r>>1, c=x*2;
                dfs(c, l, mid, f, d);
                dfs(c+1, mid, r, f, d);
            };
        build(1, 0, m);
        dfs(1, 0, m, f, n);
        return y;
}//131072 880ms
vector<ll> evaluation_new(Q f, const vector<ll> &X)//多项式多点求值
{
        int m=X.size(), i, j;
        vector<ll> y(m);
        if (X.size()<=10)
        {
            for (i=0; i<m; i++) y[i]=f.fx(X[i]);
            return y;
        }
        int n=f.size();
        while (n>1&&!f[n-1]) --n;
        f.resize(cal(n));
        vector<Q> pro(m*4+4);
        function<void(int, int, int)> build=[&](int x, int l, int r)
            {
                if (l==r)
                {
                    pro[x]=Q(vector{1llu, (p-X[l])%p});
                    return;
                }
                int m=l+r>>1, c=x*2;
                build(c, l, m); build(c+1, m+1, r);
                pro[x]=shrink(pro[c]*pro[c+1]);
            };
        function<void(int, int, int, Q)> dfs=[&](int x, int l, int r, Q f)
            {
                const static int limit=30;
```

```cpp
                    if (r-l+1<=limit)
                    {
                        int m=r-l+1, m1, m2, mid=l+r>>1, i, j, k;
                        static ll g[limit+2], g1[limit+2], g2[limit+2];
                        m1=m2=r-l;
                        copy_n(f.data(), m, g1);
                        copy_n(g1, m, g2);
                        for (i=mid+1; i<=r; i++, --m1) for (k=0; k<m1; k++) g1[k]=(
                            g1[k]+g1[k+1]*(p-X[i]))%p;
                        for (i=l; i<=mid; i++, --m2) for (k=0; k<m2; k++) g2[k]=(g2
                            [k]+g2[k+1]*(p-X[i]))%p;
                        for (i=l; i<=mid; i++)
                        {
                            copy_n(g1, (m=m1)+1, g);
                            for (j=l; j<=mid; j++) if (i!=j)
                            {
                                for (k=0; k<m; k++) g[k]=(g[k]+g[k+1]*(p-X[j]))%p;
                                --m;
                            }
                            y[i]=g[0];
                        }
                        for (i=mid+1; i<=r; i++)
                        {
                            copy_n(g2, (m=m2)+1, g);
                            for (j=mid+1; j<=r; j++) if (i!=j)
                            {
                                for (k=0; k<m; k++) g[k]=(g[k]+g[k+1]*(p-X[j]))%p;
                                --m;
                            }
                            y[i]=g[0];
                        }
                        return;
                    }
                    int mid=l+r>>1, c=x*2, n=f.size();
                    f.dft();
                    for (auto [x, len]:{pair{c, r-mid}, {c+1, mid-l+1}})
                    {
                        pro[x]%=n;
                        reverse(all(pro[x])); pro[x]&=f;
                        rotate(all(pro[x])-1, pro[x].end());
                        pro[x]%=cal(len);
                        fill(len+all(pro[x]), 0);
                    }
                    dfs(c, l, mid, pro[c+1]);
                    dfs(c+1, mid+1, r, pro[c]);
                };
            build(1, 0, m-1);
            pro[1]%=f.size();
            (f^=~pro[1])%=cal(m);
            fill(min(m, n)+all(f), 0);
            dfs(1, 0, m-1, f);
            return y;
        }//131072 460ms
        ll factorial(ll n)
        {
            if (n>=p) return 0;
            if (n<=1) return 1%p;
```

```
        ll B=::sqrt(n), i;
        vector F(B, Q({0, 1}));
        for (i=0; i<B; i++) F[i][0]=i+1;
        auto f=prod(F);
        vector<ll> x(B);
        for (i=0; i<B; i++) x[i]=i*B;
        ll r=1;
        auto y=evaluation(f, x);
        for (i=0; i<B; i++) r=r*y[i]%p;
        for (i=B*B+1; i<=n; i++) r=r*i%p;
        return r;
}//998244352 170ms
vector<ll> getinvs(vector<ll> a)
{
        int n=a.size(), i;
        if (n<=2)
        {
                for (i=0; i<n; i++) a[i]=ksm(a[i], p-2);
                return a;
        }
        vector<ll> l(n), r(n);
        l[0]=a[0]; r[n-1]=a[n-1];
        for (i=1; i<n; i++) l[i]=l[i-1]*a[i]%p;
        for (i=n-2; i; i--) r[i]=r[i+1]*a[i]%p;
        ll x=ksm(l[n-1], p-2);
        a[0]=x*r[1]%p; a[n-1]=x*l[n-2]%p;
        for (i=1; i<n-1; i++) a[i]=x*l[i-1]%p*r[i+1]%p;
        return a;
}
Q interpolation(const vector<ll> &X, const vector<ll> &y)//多项式快速插值
{
        assert(X.size()==y.size());
        int n=X.size(), i, j;
        if (n<=1) return Q(y);
        if (1)
        {
                auto vv=X; sort(all(vv));
                assert(unique(all(vv))-vv.begin()==n);
        }
        vector<Q> sum(4*n+4), pro(4*n+4);
        function<void(int, int, int)> build=[&](int x, int l, int r)
                {
                        if (l==r)
                        {
                                sum[x]=Q(vector{(p-X[l])%p, 1llu});
                                return;
                        }
                        int mid=l+r>>1, c=x*2;
                        build(c, l, mid); build(c+1, mid+1, r);
                        sum[x]=shrink(sum[c]*sum[c+1]);
                };
        build(1, 0, n-1);
        auto v=evaluation_new(sum[1]=der(sum[1]), X);
        assert(v.size()==n);
        auto Y=getinvs(v);
        for (i=0; i<n; i++) Y[i]=Y[i]*y[i]%p;
        function<void(int, int, int)> dfs=[&](int x, int l, int r)
```

```cpp
        {
            if (l==r)
            {
                pro[x][0]=Y[l];
                return;
            }
            int c=x*2, mid=l+r>>1;
            dfs(c, l, mid); dfs(c|1, mid+1, r);
            pro[x]=shrink((pro[c]*sum[c|1])+(pro[c|1]*sum[c]));
        };
        dfs(1, 0, n-1);
        return pro[1]%=cal(n);
}//131072 1150ms
Q comp(const Q &f, Q g)//多项式复合 f(g(x))=[x^i]f(x)g(x)^i
{
    int n=f.size(), l=ceil(::sqrt(n)), i, j;
    assert(n>=g.size());//返回 n-1 次多项式
    vector<Q> a(l+1), b(l);
    a[0]%=n; a[0][0]=1; a[1]=g;
    g%=n*2;
    Q u=g, v(n);
    g.dft();
    for (i=2; i<=l; i++) a[i]=((u&=g)%=n), u%=n*2;
    for (i=2; i<l; i++)
    {
        u.dft(); b[i-1]=u;
        u&=b[1]; fill(n+all(u), 0);
    }
    u.dft(); b[l-1]=u;
    for (i=0; i<l; i++)
    {
        fill(all(v), 0);
        for (j=0; j<l; j++) if (i*l+j<n) v+=a[j]*f[i*l+j];
        if (i==0) u=v; else u+=((v%=n*2)&=b[i])%=n;
    }
    return u;
}//n^2+n\sqrt n\log n, 8000 350ms
Q comp_inv(Q f)//多项式复合逆 g(f(x))=x, 求 g, [x^n]g=([x^{n-1}](x/f)^n)/
    n, 要求常数 0 一次非 0
{
    assert(!f[0]&&f[1]);
    int n=f.size(), l=ceil(::sqrt(n)), i, j, k, m;//l>=2
    rotate(f.begin(), 1+all(f));
    f=~f;
    getfac(n*2);
    vector<Q> a(l+1), b(l);
    Q u, v;
    u=a[1]=f;
    u%=n*2; (v=u).dft();
    for (i=2; i<=l; i++)
    {
        u&=v;
        fill(n+all(u), 0);
        a[i]=u;
    }
    b[0]%=n; b[0][0]=1; b[1]=u; (v=u).dft();
    for (i=2; i<l; i++)
```

```
        {
            u&=v;
            fill(n+all(u), 0);
            b[i]=u;
        }
        u%=n; u[0]=0;
        for (i=0; i<l; i++) for (j=1; j<=l; j++) if (i*l+j<n)
        {
            m=i*l+j-1;
            ll r=0, *f=b[i].data(), *g=a[j].data();
            for (k=0; k<=m; k++) r=(r+f[k]*g[m-k])%p;
            u[m+1]=r*inv[m+1]%p;
        }
        return u;
}//8000 200ms
Q shift(Q f, ll c)//get f(x+c),c\in [0,p)
{
        int n=f.size(), i, j;
        Q g(n);
        getfac(n);
        for (i=0; i<n; i++) (f[i]*=fac[i])%=p;
        g[0]=1;
        for (i=1; i<n; i++) g[i]=g[i-1]*c%p;
        for (i=0; i<n; i++) (g[i]*=ifac[i])%=p;
        f^=g;
        for (i=0; i<n; i++) (f[i]*=ifac[i])%=p;
        return f;
}//5e5 200ms (1 2 3 4 5) 3 -> (547 668 309 64 5)
vector<ll> shift(vector<ll> y, ll c, ll m)//[0,n) 点值 -> [c,c+m) 点值
{
        assert(y.size());
        if (y.size()==1) return vector(m, y[0]);
        vector<ll> r, res;
        r.reserve(m);
        int n=y.size(), i, j, mm=m;
        while (c<n&&m) r.push_back(y[c++]), --m;
        if (c+m>p)
        {
            res=shift(y, 0, c+m-p);
            m=p-c;
        }
        if (!m) { r.insert(r.end(), all(res)); return r; }
        int len=cal(m+n-1), l=m+n-1;
        for (i=n&1; i<n; i+=2) y[i]=(p-y[i])%p;
        getfac(n);
        for (i=0; i<n; i++) y[i]=y[i]*ifac[i]%p*ifac[n-1-i]%p;
        y.resize(len);
        Q f, g;
        vector<ll> v(m+n-1);
        c-=n-1;
        for (i=0; i<l; i++) v[i]=(c+i)%p;
        f=Q(y); g=Q(getinvs(v))%len;
        f*=g;
        vector<ll> u(m);
        for (i=n-1; i<l; i++) u[i-(n-1)]=f[i];
        v.resize(m);
        for (i=0; i<m; i++) v[i]=c+i;
```

```
        v=getinvs(v); c+=n;
        ll tmp=1;
        for (i=c-n; i<c; i++) tmp=tmp*i%p;
        for (i=0; i<m; i++) (u[i]*=tmp)%=p, tmp=tmp*(c+i)%p*v[i]%p;
        r.insert(r.end(), all(u));
        r.insert(r.end(), all(res));
        assert(r.size()==mm);
        return r;
}//5e5 430ms, (1 4 9 16) 3 5 -> (16 25 36 49 64)
vector<ll> Z_transform(Q f, ll c, ll m)//求 f(c^[0,m])。核心 ij=C(i+j,2)-
    C(i,2)-C(j,2)
{
        const static ll B=1e5;
        static ll a[B+2], b[B+2];
        int i, n=f.size();
        if (n*m<B*5)
        {
            vector<ll> r(m);
            ll j;
            for (i=0, j=1; i<m; i++) r[i]=f.fx(j), j=j*c%p;
            return r;
        }
        auto mic=[&](ll x) { return a[x%B]*b[x/B]%p; };
        ll l=cal(m+=n-1);
        Q g(l);
        assert(B*B>p);
        a[0]=b[0]=g[0]=g[1]=1;
        for (i=1; i<=B; i++) a[i]=a[i-1]*c%p;
        for (i=1; i<=B; i++) b[i]=b[i-1]*a[B]%p;
        for (i=2; i<n; i++) f[i]=f[i]*mic((p*2-2-i)*(i-1)/2%(p-1))%p;
        for (i=2; i<m; i++) g[i]=mic(i*(i-1llu)/2%(p-1));
        reverse(all(f)); (f%=l)&=g;
        vector<ll> r(f.begin()+n-1, f.begin()+m); m-=n-1;
        for (i=2; i<m; i++) r[i]=r[i]*mic((p*2-2-i)*(i-1)/2%(p-1))%p;
        return r;
}//luogu 1e6 500ms
vector<ll> get_Bell(int n)//B(0...n)
{
        ++n;
        getfac(n-1);
        Q f(n);
        int i;
        for (i=1; i<n; i++) f[i]=ifac[i];
        f=exp_new(f);
        for (i=2; i<n; i++) f[i]=f[i]*fac[i]%p;
        return vector<ll>(f.begin(), f.begin()+n);
}//not check
vector<ll> S1_row(int n, int m)//S1(n,0...m),O(nlogn),unsigned
{
        int cm=cal(++m);
        if (n==0)
        {
            vector<ll> r(m);
            r[0]=1;
            return r;
        }
        function<Q(int)> dfs=[&](int n)
```

```
            {
                if (n==1)
                {
                    Q f(2);
                    f[1]=1;
                    return f;
                }
                Q f=dfs(n/2);
                f*=shift(f, n/2);
                if (n&1)
                {
                    f%=cal(n+1);
                    for (int i=n; i; i--) f[i]=f[i-1];
                    // for (int i=1; i<=n; i++) f[i]=f[i-1];
                    --n;
                    for (int i=0; i<=n; i++) f[i]=(f[i]+f[i+1]*n)%p;
                }
                if (f.size()>cm) f%=cm;
                return f;
            };
        Q f=dfs(n);
        if (f.size()<cm) f%=cm;
        return vector<ll>(f.begin(), f.begin()+m);
    }
    vector<ll> S1_column(int n, int m)//S1(0...n,m),O(nlogn)
    {
        if (m==0)
        {
            vector<ll> r(n+1);
            r[0]=1;
            return r;
        }
        Q f(n+1);
        getfac(max(n, m));
        int i;
        for (i=1; i<=n; i++) f[i]=inv[i];
        f=pow(f, m);
        for (i=m; i<=n; i++) f[i]=f[i]*fac[i]%p*ifac[m]%p;
        return vector<ll>(f.begin(), f.begin()+n+1);
    }
    vector<ll> S2_row(int n, int m)//S2(n,0...m),O(mlogm)
    {
        int tm=++m, i, j, cnt=0;
        if (n==0)
        {
            vector<ll> r(m);
            r[0]=1;
            return r;
        }
        m=min(m, n+1);
        vector<ll> pr(m), pw(m);
        pw[1]=1;
        for (i=2; i<m; i++)
        {
            if (!pw[i]) pr[cnt++]=i, pw[i]=ksm(i, n);
            for (j=0; i*pr[j]<m; j++)
            {
```

```cpp
                pw[i*pr[j]]=pw[i]*pw[pr[j]]%p;
                if (i%pr[j]==0) break;
            }
        }
        getfac(m-1);
        Q f(m), g(m);
        for (i=0; i<m; i+=2) f[i]=ifac[i];
        for (i=1; i<m; i+=2) f[i]=p-ifac[i];
        // for (i=1; i<m; i++) g[i]=pw[i]*ifac[i]%p;
        for (i=1; i<m; i++) g[i]=ksm(i, n)*ifac[i]%p;
        f*=g;
        vector<ll> r(f.begin(), f.begin()+m);
        r.resize(tm);
        return r;
}//5e5 150ms
vector<ll> S2_column(int n, int m)//S2(0...n,m),O(nlogn)
{
        if (m==0)
        {
            vector<ll> r(n+1);
            r[0]=1;
            return r;
        }
        Q f(n+1);
        getfac(max(n, m));
        int i;
        for (i=1; i<=n; i++) f[i]=ifac[i];
        f=pow(f, m);
        for (i=m; i<=n; i++) f[i]=f[i]*fac[i]%p*ifac[m]%p;
        return vector<ll>(f.begin(), f.begin()+n+1);
}//5e5 640ms
vector<ll> signed_S1_row(int n, int m)
{
        auto v=S1_row(n, m);
        for (int i=1^n&1; i<=m; i+=2) v[i]=(p-v[i])%p;
        return v;
}//5e5 190ms
vector<ll> Bernoulli(int n)//B(0...n)
{
        getfac(++n);
        int i;
        Q f(n);
        for (i=0; i<n; i++) f[i]=ifac[i+1];
        f=~f;
        for (i=0; i<n; i++) f[i]=f[i]*fac[i]%p;
        return vector<ll>(f.begin(), f.begin()+n);
}//5e5 180ms
vector<ll> Partition(int n)//P(0...n),分拆数
{
        Q f(++n);
        int i, l=0, r=0;
        while (--l) if (3*l*l-l>=n*2) break;
        while (++r) if (3*r*r-r>=n*2) break;
        ++l;
        for (i=l+abs(l)%2; i<r; i+=2) f[3*i*i-i>>1]=1;
        for (i=l+abs(l+1)%2; i<r; i+=2) f[3*i*i-i>>1]=p-1;
        f=~f;
```

```cpp
        return vector<ll>(f.begin(), f.begin()+n);
}//5e5 150ms
struct reg
{
    Q a00, a01, a10, a11;
    reg operator*(const reg &o) const
    {
        return {
            shrink(a00*o.a00+a01*o.a10),
            shrink(a00*o.a01+a01*o.a11),
            shrink(a10*o.a00+a11*o.a10),
            shrink(a10*o.a01+a11*o.a11)};
    }
    pair<Q, Q> operator*(const pair<Q, Q> &o) const
    {
        const auto &[b0, b1]=o;
        return {shrink(a00*b0+a01*b1), shrink(a10*b0+a11*b1)};
    }
};
ostream &operator<<(ostream &cout, const reg &o)
{
    return cout<<"["<<o.a00<<", "<<o.a01<<"]\n"
        <<"["<<o.a10<<", "<<o.a11<<"]\n";
}
reg hgcd(Q a, Q b)
{
    int m=a.deg()+1>>1;
    if (b.deg()<m) return {{1}, { }, { }, {1}};
    reg r=hgcd(a>>m, b>>m);
    auto [c, d]=r*pair{a, b};
    if (d.deg()<m) return r;
    auto [q, e]=div_mod(c, d);
    // reg rq({}, {1}, {1}, -q);
    r.a00-=shrink(q*r.a10);
    r.a01-=shrink(q*r.a11);
    swap(r.a00, r.a10);
    swap(r.a01, r.a11);
    if (e.deg()<m) return r;
    int k=2*m-d.deg();
    auto s=hgcd(d>>k, e>>k);
    return s*r;
}
Q gcd(Q a, Q b)
{
    if (a.deg()<b.deg()) swap(a, b);
    while (b.deg()>=0)
    {
        a=mod(a, b);
        swap(a, b);
        auto tmp=hgcd(a, b);
        tie(a, b)=tmp*pair{a, b};
    }
    if (a.deg()==-1) return a;
    ll k=ksm(a[a.deg()], p-2);
    for (int i=0; i<a.size(); i++) a[i]=a[i]*k%p;
    return a;
}
```

```cpp
vector<ll> root(Q f)
{
    Q x(2);
    x[1]=1;
    x=pow(x, p, f);
    if (x.size()<2) x%=2;
    (x[1]+=p-1)%=p;
    f=gcd(f, x);
    vector<ll> res;
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().
        count());
    function<void(Q)> dfs=[&](Q f)
        {
            int n=f.deg(), i;
            if (n<=0) return;
            if (n==1)
            {
                res.push_back((p-f[0])%p);
                return;
            }
            Q g(n);
            for (i=0; i<n; i++) g[i]=rnd()%p;
            g=gcd(pow(g, (p-1)/2, f)-1, f);
            dfs(g); dfs(div(f, g));
        };
    dfs(f);
    sort(all(res));
    assert(unique(all(res))==res.end());
    return res;
}//4000 950ms
optional<Q> inverse(Q a, Q m)
{
    Q b=m;
    vector<pair<reg, Q>> buf;
    a=mod(a, b);
    swap(a, b);
    while (b.deg()>=0)
    {
        auto [q, r]=div_mod(a, b);
        swap(a, r); swap(a, b);
        auto tmp=hgcd(a, b);
        tie(a, b)=tmp*pair{a, b};
        buf.emplace_back(move(tmp), q);
    }
    if (a.deg()) return { };
    reg res{{1}, { }, { }, {1}};
    reverse(all(buf));
    for (const auto &[tmp, q]:buf)
    {
        res=res*tmp;
        res.a00-=shrink(q*res.a01);
        res.a10-=shrink(q*res.a11);
        swap(res.a00, res.a01);
        swap(res.a10, res.a11);
    }
    return {res.a01*ksm(a[0], p-2)};
}//5e4 950ms
```

```
    }
using NTT::p;
using poly=NTT::Q;
/*
函数名称: sqr, cdq, exp_cdq, sqrt, ln, exp, exp_new, sqrt_new, pow, pow2,
    div,
recurrent, recurrent_new, prod, evaluation, evaluation_new, factorial,
    interpolation,
comp, comp_inv, shift, Z_transform, Bell, S1_row, S1_column, S2_row,
S2_column, signed_S1_row, Bernoulli, Partition, gcd, root, inverse.
*/
```

2.34 MTT

```
namespace MTT
{
    template<ll p> constexpr ll ksm(ll x,ll y=p-2)
    {
        ll r=1;
        while (y)
        {
            if (y&1) r=r*x%p;
            x=x*x%p;
            y>>=1;
        }
        return r;
    }
    int cal(int x) { return 1<<__lg(max(x,1)*2-1); }
    const int N=1<<22;
    const ll p=1e9+7,g=3,
        p1=469'762'049,p2=998'244'353,p3=1004'535'809,//三模，原根都是 3，非常好
        inv_p1=ksm<p2>(p1),inv_p12=ksm<p3>(p1*p2%p3),_p12=p1*p2%p;//三模，1 关于
            2 逆，1*2 关于 3 逆，1*2 mod 3
    int r[N];
    struct P
    {
        ll v1,v2,v3;
        P operator+(const P &o) const { return {v1+o.v1,v2+o.v2,v3+o.v3}; }
        P operator-(const P &o) const { return {v1+p1-o.v1,v2+p2-o.v2,v3+p3-o.v3
            }; }
        P operator*(const P &o) const { return {v1*o.v1,v2*o.v2,v3*o.v3}; }
        void operator+=(const P &o) { v1+=o.v1,v2+=o.v2,v3+=o.v3; }
        void operator-=(const P &o) { v1+=p1-o.v1,v2+=p2-o.v2,v3+=p3-o.v3; }
        void operator*=(const P &o) { v1*=o.v1,v2*=o.v2,v3*=o.v3; }
        void mod() { v1%=p1,v2%=p2,v3%=p3; }
    };
    P w[N];
    void init(int n)
    {
        static int pr=0,pw=0;
        if (pr==n) return;
        int b=__lg(n)-1,i,j,k;
        for (i=1; i<n; i++) r[i]=r[i>>1]>>1|(i&1)<<b;
        if (pw<n)
        {
            for (j=1; j<n; j=k)
```

```
                {
                    k=j*2;
                    P wn={ksm<p1>(g,(p1-1)/k),ksm<p2>(g,(p2-1)/k),ksm<p3>(g,(p3-1)/k)
                    };
                    w[j]={1,1,1};
                    for (i=j+1; i<k; i++) w[i]=w[i-1]*wn,w[i].mod();
                }
                pw=n;
            }
            pr=n;
        }
        void dft(vector<P> &a,int o=0)
        {
            int n=a.size(),i,j,k;
            P *f,*g,*wn,*b=a.data(),x,y;
            init(n);
            for (i=1; i<n; i++) if (i<r[i]) swap(a[i],a[r[i]]);
            for (k=1; k<n; k*=2)
            {
                wn=w+k;
                for (i=0; i<n; i+=k*2)
                {
                    f=b+i; g=b+i+k;
                    for (j=0; j<k; j++)
                    {
                        y=g[j]*wn[j];
                        y.mod();
                        g[j]=f[j]-y;
                        f[j]+=y;
                    }
                }
                if (k*2==n||k==1<<14) for (P &x:a) x.mod();
            }
            if (o)
            {
                x={ksm<p1>(n),ksm<p2>(n),ksm<p3>(n)};
                for (P &y:a) y*=x,y.mod();
                reverse(1+all(a));
            }
        }
        struct Q:vector<ll>
        {
            Q(int x=1):vector(x) { }
            Q &operator%=(int n) { resize(n); return *this; }
        };
        Q &operator*=(Q &f,const Q &g)
        {
            int n=f.size()+g.size()-1,m=cal(n),i;
            vector<P> F(m,{0,0,0}),G(m,{0,0,0});
            for (i=0; i<f.size(); i++) F[i]={f[i]%p1,f[i]%p2,f[i]%p3};
            for (i=0; i<g.size(); i++) G[i]={g[i]%p1,g[i]%p2,g[i]%p3};
            dft(F); dft(G);
            for (i=0; i<m; i++) F[i]*=G[i],F[i].mod();
            dft(F,1);
            f%=n;
            ll x;
            for (i=0; i<n; i++)
```

```
        {
            auto [r1,r2,r3]=F[i];
            x=(r2+p2-r1)*inv_p1%p2*p1+r1;
            f[i]=((x+p3-r3)%p3*(p3-inv_p12)%p3*_p12+x)%p;
        }
        return f;
    }//5e5 440ms
    Q operator*(Q f,const Q &g) { return f*=g; }
}
using MTT::p;
using poly=MTT::Q;
```

2.35   FFT

```
namespace FFT
{
    #define all(x) (x).begin(),(x).end()
    typedef double db;
    const int N=1<<21;
    const db pi=3.14159265358979323846;
    struct comp
    {
        db x,y;
        comp operator+(const comp &o) const {return {x+o.x,y+o.y};}
        comp operator-(const comp &o) const {return {x-o.x,y-o.y};}
        comp operator*(const comp &o) const {return {x*o.x-y*o.y,o.x*y+x*o.y};}
        comp operator*(const db &o) const {return {x*o,y*o};}
        void operator*=(const comp &o) {*this={x*o.x-y*o.y,o.x*y+x*o.y};}
        void operator*=(const db &o) {x*=o;y*=o;}
        void operator/=(const db &o) {x/=o;y/=o;}
        comp operator/(const comp &o) const
        {
            db z=1/(o.x*o.x+o.y*o.y);
            return {z*(x*o.x+y*o.y),z*(o.x*y-x*o.y)};
        }//not necessary, no check
    };
    long long dtol(const double &x) {return fabs(round(x));}
    const comp I{0,-1};
    ostream & operator<<(ostream &cout,const comp &o) {cout<<o.x;if (o.y>=0)
        cout<<'+';return cout<<o.y<<'i';}
    int r[N];
    char c;
    comp Wn[N];
    void init(int n)
    {
        static int preone=-1;
        if (n==preone) return;
        preone=n;
        int b,i;
        b=__builtin_ctz(n)-1;
        for (i=1;i<n;i++) r[i]=r[i>>1]>>1|(i&1)<<b;
        for (i=0;i<n;i++) Wn[i]={cos(pi*i/n),sin(pi*i/n)};
    }
    int cal(int x) {return 1u<<32-__builtin_clz(max(x,2)-1);}
    struct Q
    {
    {
```

```cpp
    vector<comp> a;
    int deg;
    comp* pt() {return a.data();}
    Q(int n=0)
    {
        deg=n;
        a.resize(cal(n));
    }
    void dft(int xs=0)//1,0
    {
        int i,j,k,l,n=a.size(),d;
        comp w,wn,b,c,*f=pt(),*g,*a=f;
        init(n);
        if (xs) reverse(a+1,a+n);//spe
        for (i=0;i<n;i++) if (i<r[i]) swap(a[i],a[r[i]]);
        for (i=1,d=0;i<n;i=l,d++)
        {
            //wn={cos(pi/i),(xs?-1:1)*sin(pi/i)};
            l=i<<1;
            for (j=0;j<n;j+=l)
            {
                //w={1,0};
                f=a+j;g=f+i;
                for (k=0;k<i;k++)
                {
                    w=Wn[k*(n>>d)];
                    b=f[k];c=g[k]*w;
                    f[k]=b+c;
                    g[k]=b-c;
                    //w*=wn;
                }
            }
        }
        if (xs) for (i=0;i<n;i++) a[i]/=n;
    }
    void operator|=(Q o)
    {
        int n=deg+o.deg-1,m=cal(n),i;
        a.resize(m);o.a.resize(m);
        dft();o.dft();
        for (i=0;i<m;i++) a[i]*=o.a[i];
        dft(1);
        for (i=n;i<m;i++) a[i]={};
        deg=n;
    }
    Q operator|(Q o) const {o|=*this;return o;}
};
Q mul(Q a,const Q &b)//三次变两次，仅实数，注意精度
{
    int n=a.deg+b.deg-1,m=cal(n),i;
    a.a.resize(m);
    for (i=0;i<b.deg;i++) a.a[i]={a.a[i].x,b.a[i].x};
    a.dft();
    for (i=0;i<m;i++) a.a[i]*=a.a[i];
    a.dft(1);
    for (i=0;i<n;i++) a.a[i]={a.a[i].y*.5};
    for (i=n;i<m;i++) a.a[i]={};
```

```
        a.deg=n;
        return a;
    }
    void ddt(Q &a,Q &b)//double dft，仅实数，注意精度
    {
        comp x,y;
        int n=a.a.size(),i;
        assert(n==b.a.size());
        for (i=0;i<n;i++) a.a[i]={a.a[i].x,b.a[i].x};
        a.dft();
        for (i=0;i<n;i++) b.a[i]={a.a[i].x,-a.a[i].y};
        reverse(b.pt()+1,b.pt()+n);
        for (i=0;i<n;i++)
        {
            x=a.a[i];y=b.a[i];
            a.a[i]=(x+y)*.5;
            b.a[i]=(y-x)*.5*I;
        }
    }
}
using FFT::dtol;
```

2.36   约数个数和

$O(\sqrt[3]{n}\log n)$。

```
#include<bits/stdc++.h>
#define ll long long
#define lll __int128
using namespace std;

void myw(lll x){
    if(!x) return;
    myw(x/10);printf("%d",(int)(x%10));
}

struct vec{
    ll x,y;
    vec (ll x0=0,ll y0=0){x=x0,y=y0;}
    vec operator +(const vec b){return vec(x+b.x,y+b.y);}
};

ll N;
vec stk[1000005];int len;
vec P;
vec L,R;

bool ninR(vec a){return N<(lll)a.x*a.y;}
bool steep(ll x,vec a){return (lll)N*a.x<=(lll)x*x*a.y;}

lll Solve(){
    len=0;
    ll cbr=cbrt(N),sqr=sqrt(N);
    P.x=N/sqr,P.y=sqr+1;
    lll ans=0;
    stk[++len]=vec(1,0);stk[++len]=vec(1,1);
    while(1){
```

```
        L=stk[len--];
        while(ninR(vec(P.x+L.x,P.y-L.y)))
            ans+=(lll)P.x*L.y+(lll)(L.y+1)*(L.x-1)/2,
            P.x+=L.x,P.y-=L.y;
        if(P.y<=cbr) break;
        R=stk[len];
        while(!ninR(vec(P.x+R.x,P.y-R.y))) L=R,R=stk[--len];
        while(1){
            vec mid=L+R;
            if(ninR(vec(P.x+mid.x,P.y-mid.y))) R=stk[++len]=mid;
            else if(steep(P.x+mid.x,R)) break;
            else L=mid;
        }
    }
    for(int i=1;i<P.y;i++) ans+=N/i;
    return ans*2-sqr*sqr;
}

int T;

int main(){
    scanf("%d",&T);
    while(T--){
        scanf("%lld",&N);
        myw(Solve());printf("\n");
    }
}
```

2.37 万能欧几里得

题意：$\sum_{i=0}^{n-1}\lfloor\dfrac{ai+b}{m}\rfloor$ （$0\le a,b$）

注意若 $b\ge m$ 需要增加先往上走一步。

原理：考虑紧贴着斜线的折线的答案。每个 nd 表示的是一段折线，你需要实现 operator+ 来计算出拼接两个折线之后的答案。除此以外的原理不必了解。

你需要传入的 a 和 b 表示向上和向右的折线的答案（也就是边界）。

如果你发现横竖反了，自行调整。你可以通过在 nd 中加一个 string 记录当前是什么折线来确认这一点。

```
struct nd
{
    ll x,y,sy;
    nd operator+(const nd &o) const
    {
        return {x+o.x,y+o.y,sy+o.sy+y*o.x};
    }
};
nd ksm (nd a,int k)
{
    nd res{};
    while (k)
    {
        if (k&1) res=res+a;
        a=a+a;k>>=1;
    }
    return res;
```

```cpp
}
nd sol (int p,int q,int r,int l,nd a,nd b)//(0,l],(pi+r)/q
{
    if (!l) return {};
    if (p>=q) return sol(p%q,q,r,l,a,ksm(a,p/q)+b);
    int m=((ll)l*p+r)/q;
    if (!m) return ksm(b,l);
    int cnt=l-((ll)q*m-r-1)/p;
    return ksm(b,(q-r-1)/p)+a+sol(q,p,(q-r-1)%p,m-1,b,a)+ksm(b,cnt);
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(15);
    int T;cin>>T;
    while (T--)
    {
        int n,m,a,b;
        cin>>n>>m>>a>>b;
        nd nx={1,0,0},ny={0,1,0};
        nd ans=sol(a,m,b,n-1,ny,nx);
        cout<<ans.sy<<'\n';
    }
}
```

2.38 高斯整数类

圆上整点的基础。

```cpp
ll roundiv(ll x,ll y)
{
    return x>=0?(x+y/2)/y:(x-y/2)/y;
}
struct Q
{
    ll x,y;
    Q operator~() const { return {x,-y}; }
    ll len2() const { return x*x+y*y; }
    Q operator+(const Q &o) const { return {x+o.x,y+o.y}; }
    Q operator-(const Q &o) const { return {x-o.x,y-o.y}; }
    Q operator*(const Q &o) const { return {x*o.x-y*o.y,x*o.y+y*o.x}; }
    Q operator/(const Q &o) const
    {
        Q t=*this*~o;
        ll l=o.len2();
        return {roundiv(t.x,l),roundiv(t.y,l)};
    }
    Q operator%(const Q &o) const { return *this-*this/o*o; }
};
Q gcd(Q a,Q b)
{
    if (a.len2()>b.len2()) swap(a,b);
    while (a.len2())
    {
        b=b%a;
        swap(a,b);
    }
```

```
    return b;
}
```

3 字符串

3.1 AC 自动机

```
scanf("%d",&n);
    for (i=1;i<=n;i++)
    {
        x=0;cc=getchar();
        while ((cc<'a')||(cc>'z')) cc=getchar();
        while ((cc>='a')&&(cc<='z'))
        {
            cc-='a';
            if (c[x][cc]==0) c[x][cc]=++ds;
            x=c[x][cc];
            cc=getchar();
        }
        ys[i]=x;
    }tou=1;wei=0;
    for (int v:c[0]) if (v) dl[++wei]=v;
    while (tou<=wei)
    {
        x=dl[tou++];
        for (i=0;i<=25;i++) if (c[x][i]) f[dl[++wei]=c[x][i]]=c[f[x]][i]; else c
            [x][i]=c[f[x]][i];
    }
    x=0;cc=getchar();
    while ((cc<'a')||(cc>'z')) cc=getchar();
    while ((cc>='a')&&(cc<='z'))
    {
        ++cs[x=c[x][cc-'a']];cc=getchar();
    }++wei;
    while (--wei) cs[f[dl[wei]]]+=cs[dl[wei]];
    for (i=1;i<=n;i++) printf("%d\n",cs[ys[i]]);
```

3.2 hash

$O(n)$，$O(n)$。

```
typedef unsigned int ui;
typedef unsigned long long ull;
namespace sh
{
    const int N=1e6+5;
    const ll p1=2'034'452'107,p2=2'013'074'419;
    struct pa
    {
        ll v1,v2;
        pa(ll v=0):v1(v),v2(v) { }
        pa(ll v1,ll v2):v1(v1),v2(v2) { }
        pa operator*(const pa &o) const { return {v1*o.v1%p1,v2*o.v2%p2}; }
    };
    pa fma(const pa &a,const pa &b,const pa &c) { return {(a.v1*b.v1+c.v1)%p1,(a
        .v2*b.v2+c.v2)%p2}; }
    const pa b={137,149},inv={1'603'801'661,1'024'053'074};
    pa m[N];
    void init()
```

```
    {
        m[0]={p1-1,p2-1};
        for (int i=1; i<N; i++) m[i]=m[i-1]*b;
    }
    int i=(init(),0);
    struct str
    {
        int n;
        vector<pa> a;
        vector<ll> s;
        template<class T> str(const vector<T> &_):n(_.size()),a(n+1),s(n)
        {
            a[0]={0,0};
            for (i=0; i<n; i++) a[i+1]=fma(a[i],b,s[i]=_[i]);
        }
        template<class T> str(const basic_string<T> &_):n(_.size()), a(n+1), s(n
            )
        {
            a[0]={0, 0};
            for (i=0; i<n; i++) a[i+1]=fma(a[i], b, s[i]=_[i]);
        }
        ll getv(int l,int r)//[l,r)
        {
            auto [x,y]=fma(a[l],m[r-l],a[r]);
            return x<<32|y;
        }
    };
}
using sh::str;
```

3.3 KMP

$O(n)$，$O(n)$。

```
struct str
{
    vector<int> nxt,s;
    int n;
    str(int *S,int _n)//[1,n]
    {
        n=_n;
        nxt.resize(n+1);
        s=vector<int>(S,S+n+1);
        int i,j=0;
        nxt[1]=0;
        for (i=2;i<=n;i++)
        {
            while (j&&s[i]!=s[j+1]) j=nxt[j];
            nxt[i]=j+=s[i]==s[j+1];
        }
    }
    vector<int> match(int *t,int m)//find s(str) in t (start pos)
    {
        vector<int> r;
        int i,j=0;
        for (i=1;i<=m;i++)
        {
```

```cpp
            while (j&&t[i]!=s[j+1]) j=nxt[j];
            if ((j+=t[i]==s[j+1])==n) j=nxt[j],r.push_back(i-n+1);
        }
        return r;
    }
};
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    string s,t;
    cin>>s>>t;
    int n=s.size(),m=t.size(),i;
    vector<int> a(n+1),b(m+1);
    for (i=1;i<=n;i++) a[i]=s[i-1];
    for (i=1;i<=m;i++) b[i]=t[i-1];
    str q(b.data(),m);
    auto r=q.match(a.data(),n);
    for (int x:r) cout<<x<<'\n';
    for (i=1;i<=m;i++) cout<<q.nxt[i]<<" \n"[i==m];
}
```

3.4 KMP（重构，未验证）

$O(n)$，$O(n)$。

```cpp
struct str//[0,n)
{
    vector<int> nxt,s;
    int n;
    str(const vector<int> &_s):nxt(_s.size(),-1),s(all(_s)),n(_s.size())
    {
        int i,j=-1;
        for (i=1;i<n;i++)
        {
            while (j!=-1&&s[i]!=s[j+1]) j=nxt[j];
            nxt[i]=j+=s[i]==s[j+1];
        }
    }
    vector<int> match(const vector<int> &t)//find s(str) in t (start pos)
    {
        int m=t.size();
        vector<int> r;
        int i,j=-1;
        for (i=0;i<m;i++)
        {
            while (j!=-1&&t[i]!=s[j+1]) j=nxt[j];
            if ((j+=t[i]==s[j+1])==n-1) j=nxt[j],r.push_back(i-n+1);
        }
        return r;
    }
};
```

3.5 manacher

$O(n)$，$O(n)$。

```cpp
vector<int> manacher(const string &t)//ex[i](total length) centered at i/2
{
    string S="$#";
    int n=t.size(),i,r=1,m=0;
    for (i=0;i<n;i++) S+=t[i],S+='#';
    S+='#';
    char *s=S.data()+2;
    n=n*2-1;
    vector<int> ex(n);
    ex[0]=2;
    for (i=1;i<n;i++)
    {
        ex[i]=i<r?min(ex[m*2-i],r-i+1):1;
        while (s[i+ex[i]]==s[i-ex[i]]) ++ex[i];
        if (i+ex[i]-1>r) r=i+ex[m=i]-1;
    }
    for (i=0;i<n;i++) --ex[i];
    return ex;
}
```

3.6 SA

$O((n+\sum)\log n)$，$O(n+\sum)$。

```cpp
namespace SA
{
    const int N=1e6+2;
    int x[N],y[N],s[N];
    int m,cnt;
    struct Q
    {
        vector<vector<int>> st;
        vector<int> sa,rk,h;
        int lcp(int x,int y)
        {
            assert(x^y);
            x=rk[x];y=rk[y];
            if (x>y) swap(x,y);
            ++x;
            int z=__lg(y-x+1);
            return min(st[z][x],st[z][y-(1<<z)+1]);
        }
        Q(int *a,int n):sa(n+1),rk(n+1),h(n+1),st(__lg(n)+1,vector<int>(n+1))//
            [1,n]
        {
            int i,j,k;
            m=*min_element(a+1,a+n+1);--m;
            for (i=1;i<=n;i++) a[i]-=m;
            m=*max_element(a+1,a+n+1);
            assert(n<N);assert(m<N);
            memset(s+1,0,m*sizeof s[0]);
            for (i=1;i<=n;i++) ++s[x[i]=a[i]];
            for (i=2;i<=m;i++) s[i]+=s[i-1];
            for (i=n;i;i--) sa[s[x[i]]--]=i;
            memset(s+1,0,m*sizeof s[0]);
            for (j=1;j<=n;j<<=1)
```

```
            {
                cnt=0;
                for (i=n-j+1;i<=n;i++) y[++cnt]=i;
                for (i=1;i<=n;i++) if (sa[i]>j) y[++cnt]=sa[i]-j;
                for (i=1;i<=n;i++) ++s[x[i]];
                for (i=2;i<=m;i++) s[i]+=s[i-1];
                for (i=n;i;i--) sa[s[x[y[i]]]--]=y[i];
                y[sa[1]]=cnt=1;
                memset(s+1,0,m*sizeof s[0]);
                for (i=2;i<=n;i++) if (x[sa[i]]==x[sa[i-1]]&&sa[i]<=n-j&&sa[i
                    -1]<=n-j&&x[sa[i]+j]==x[sa[i-1]+j]) y[sa[i]]=cnt; else y[sa[i
                    ]]=++cnt;
                memcpy(x,y,sizeof(y));
                if ((m=cnt)==n) break;
            }
            for (i=1;i<=n;i++) rk[sa[i]]=i;
            j=0;
            for (i=1;i<=n;i++) if (x[i]>1)
            {
                cnt=sa[x[i]-1];
                while (i+j<=n&&cnt+j<=n&&a[i+j]==a[cnt+j]) ++j;
                h[x[i]]=j;
                if (j) --j;
            }
            for (i=1;i<=n;i++) st[0][i]=h[i];
            for (j=1;j<=__lg(n);j++) for (i=1,k=n-(1<<j)+1;i<=k;i++) st[j][i]=min
                (st[j-1][i],st[j-1][i+(1<<j-1)]);
        }
    };
}
using str=SA::Q;
```

3.7 SAM

$O(n \sum)$，$O(2n \sum)$。

```
template<int M> struct sam//M: 字符集大小
{
    vector<array<int,M>> c;
    vector<int> len,fa,ep;
    int np,cd;
    sam():c(2),len(2),fa(2),ep(2),np(1),cd(0) { }
    void insert(int ch)
    {
        int p=np,q,nq;
        np=c.size();
        len.push_back(++cd);
        fa.push_back(0);
        c.push_back({ });
        ep.push_back(cd);
        while (p&&!c[p][ch]) c[p][ch]=np,p=fa[p];
        if (!p)
        {
            fa[np]=1;
            return;
        }
        q=c[p][ch];
```

```cpp
            if (len[q]==len[p]+1)
            {
                fa[np]=q;
                return;
            }
            nq=c.size();
            len.push_back(len[p]+1);
            c.push_back(c[q]);
            fa.push_back(fa[q]);
            ep.push_back(ep[q]);
            fa[np]=fa[q]=nq;
            c[p][ch]=nq;
            while (c[p=fa[p]][ch]==q) c[p][ch]=nq;
    }
    vector<int> match(const string &s)//返回每个前缀最长匹配长度
    {
        vector<int> r;
        r.reserve(s.size());
        int p=1,nl=0;
        for (auto ch:s)
        {
            if (c[p][ch]) ++nl,p=c[p][ch];
            else
            {
                while (p&&c[p][ch]==0) p=fa[p];
                if (p==0) p=1,nl=0; else nl=len[p]+1,p=c[p][ch];
            }
            r.push_back(nl);
        }
        return r;
    }
    array<int,3> max_match(const string &s)//返回长度，结尾（开）
    {
        array<int,3> r{0,0,0};
        int p=1,nl=0,i=0;
        for (auto ch:s)
        {
            if (c[p][ch]) ++nl,p=c[p][ch];
            else
            {
                while (p&&c[p][ch]==0) p=fa[p];
                if (p==0) p=1,nl=0; else nl=len[p]+1,p=c[p][ch];
            }
            cmax(r,array{nl,ep[p],i+1});
            ++i;
        }
        if (r[0]==0) return { };
        return r;
    }
};
```

3.8 SqAM

$O(n \sum)$，$O(n \sum)$。

```cpp
struct sqam
{
```

```
    int c[N][26],ds,i,j,lst[26],pre[N];
    void csh()
    {
        ds=1;
    }
    void ins(int zf)
    {
        ++ds;
        for (i=0;i<=25;i++) if (lst[i]) for (j=lst[i];(j)&&(c[j][zf]==0);j=pre[j
            ]) c[j][zf]=ds;
        if (!lst[zf]) c[1][zf]=ds; else pre[ds]=lst[zf];
        lst[zf]=ds;
    }
};
```

3.9 ukkonen 后缀树

$O(n)$，$O(2n\sum)$。

```
void dfs(int x,int lf)
{
    if (!fir[x])
    {
        siz[x][1]=1;
        return;
    }
    int i,j;
    for (i=fir[x];i;i=nxt[i])
    {
        j=c[x][lj[i]];
        if ((f[j]<=m)&&(t[j]>=m)) ++siz[x][0];
        dfs(zd[j],t[j]-f[j]+1);
        siz[x][0]+=siz[zd[j]][0];
        siz[x][1]+=siz[zd[j]][1];
        if ((t[j]==n)&&(f[j]<=m)) --siz[x][1];
    }
    ans+=(ll)siz[x][0]*siz[x][1]*lf;
}
void add(int a,int b,int cc,int d)
{
    zd[++bbs]=b;
    t[bbs]=d;
    c[a][s[f[bbs]=cc]]=bbs;
}
void add(int x,int y)
{
    lj[++bs]=y;
    nxt[bs]=fir[x];
    fir[x]=bs;
}
    s[++m]=26;
    fa[1]=point=ds=1;
    for (i=1;i<=m;i++)
    {
        ad=0;++remain;
        while (remain)
        {
```

```
            if (r==0) edge=i;
            if ((j=c[point][s[edge]])==0)
            {
                fa[++ds]=1;
                fa[ad]=point;
                add(ad=point,ds,edge,m);
                add(point,s[edge]);
            }
            else
            {
                if ((t[j]!=m)&&(t[j]-f[j]+1<=r))
                {
                    r-=t[j]-f[j]+1;
                    edge+=t[j]-f[j]+1;
                    point=zd[j];
                    continue;
                }
                if (s[f[j]+r]==s[i]) {++r;fa[ad]=point;break;}
                fa[fa[ad]=++ds]=1;
                add(ad=ds,zd[j],f[j]+r,t[j]);
                add(ds,s[i]);add(ds,s[f[j]+r]);fa[++ds]=1;
                add(ds-1,ds,i,m);
                zd[j]=ds-1;t[j]=f[j]+r-1;
            }
            --remain;
            if ((r)&&(point==1))
            {
                --r;edge=i-remain+1;
            } else point=fa[point];
        }
    }
    for (i=1;i<=ds;i++) for (j=fir[i];j;j=nxt[j]) {len[j]=t[c[i][lj[j]]]-f[c[i][
        lj[j]]]+1;lj[j]=zd[c[i][lj[j]]];}
```

3.10   ukkonen 后缀树（重构）

```
struct suffixtree
{
    const static int M=27;
    struct P
    {
        int v,w;
    };
    struct Q
    {
        int f,t,v;//t=0: n
    };
    vector<Q> edges;
    vector<vector<P>> e;
    vector<array<int,M>> c;
    vector<int> s,fa,dep,siz;
    int n,point,ds,remain,r,edge;
    bool bd;
    suffixtree():c(2),fa({0,1}),edges(1),e(2)
    {
        n=remain=r=edge=bd=0;
```

```cpp
        point=ds=1;
    }
    suffixtree(const string &s):c(2),fa({0,1}),edges(1),e(2)
    {
        n=remain=r=edge=bd=0;
        point=ds=1;
        reserve(s.size());
        for (auto c:s) insert(c-'a');
        insert(26);
    }
    void reserve(int len)
    {
        ++len;
        s.reserve(len);
        len=len*2+2;
        c.reserve(len);
        fa.reserve(len);
        e.reserve(len);
    }
    inline void add(int a,int b,int cc,int d)
    {
        assert(edges.size());
        c[a][s[cc]]=edges.size();
        edges.push_back({cc,d,b});
    }
    void insert(int ch)//[0,|S|)
    {
        assert(ds==fa.size()-1&&ds==c.size()-1&&n==s.size()&&ds==e.size()-1);
        assert(ch>=0&&ch<M);
        s.push_back(ch);
        int ad=0;
        ++remain;
        while (remain)
        {
            if (!r) edge=n;
            if (int m=c[point][s[edge]];!m)
            {
                assert(!m);
                fa.push_back(1);c.push_back({});e.push_back({});
                fa[ad]=point;
                add(ad=point,++ds,edge,-1);
                e[point].push_back({s[edge]});
                //add(point,s[edge]);
            }
            else
            {
                assert(m);
                auto [f,t,v]=edges[m];
                if (t>=0&&t-f+1<=r)
                {
                    assert(t!=n);
                    r-=t-f+1;
                    edge+=t-f+1;
                    point=v;
                    continue;
                }
                assert(f+r<=n);
```

```cpp
            if (s[f+r]==s[n])
            {
                ++r;
                fa[ad]=point;
                break;
            }
            fa.push_back(1);c.push_back({});e.push_back({});
            fa.push_back(1);c.push_back({});e.push_back({});
            fa[ad]=++ds;
            add(ad=ds,v,f+r,t);
            e[ds].push_back({s[n]});
            e[ds].push_back({s[f+r]});
            //add(ds,s[n]);add(ds,s[f+r]);
            ++ds;add(ds-1,ds,n,-1);
            edges[m]={f,f+r-1,ds-1};
        }
        --remain;
        if (r&&point==1)
        {
            --r;
            edge=n-remain+1;
        } else point=fa[point];
    }
    ++n;
}
void build_edge()
{
    bd=1;

    //其余信息
    dep.resize(ds+1);
    siz.resize(ds+1);

    int i,j;
    for (i=1;i<=ds;i++) for (auto &[v,w]:e[i])
    {
        j=c[i][v];
        v=edges[j].v;
        w=(edges[j].t>=0?edges[j].t:n-1)-edges[j].f+1;
    }
}
void out()
{
    int i;
    for (i=1;i<=ds;i++) for (int j:c[i]) if (j)
    {
        auto [f,t,v]=edges[j];
        if (t==-1) t=n-1;
        cerr<<i<<'␣'<<v<<'␣';
        //cerr<<i<<" -> "<<v<<": ";
        for (int k=f;k<=t;k++) cerr<<char('a'+s[k]);
        cerr<<endl;
    }
}
ll ans;
void dfs(int u)
{
```

```
        assert(bd);
        ++ans;
        for (auto [v,w]:e[u])
        {
            //dep[v]=dep[u]+w;
            dfs(v);
            ans+=w-1;
        }
    }
    ll fun()
    {
        ans=0;
        build_edge();
        dfs(1);
        return ans-n;
    }
};
```

3.11　Z 函数

表示每个后缀和母串的 lcp。

```
vector<int> Z(const string &s)
{
    int n=s.size(),i,l,r;
    vector<int> z(n);
    z[0]=n;
    for (i=1,l=r=0; i<n; i++)
    {
        if (i<=r&&z[i-l]<r-i+1) z[i]=z[i-l];
        else
        {
            z[i]=max(0,r-i+1);
            while (i+z[i]<n&&s[i+z[i]]==s[z[i]]) ++z[i];
        }
        if (i+z[i]-1>r) l=i,r=i+z[i]-1;
    }
    return z;
}
```

3.12　最小表示法

$O(n)$，$O(1)$。

```
template<typename T> void min_order(T *a,int n)//[0,n)
{
    int i,j,k;
    T x,y;
    i=k=0;j=1;
    while (i<n&&j<n&&k<n)
    {
        x=a[(i+k)%n];y=a[(j+k)%n];
        if (x==y) ++k; else
        {
            if (x>y) i+=k+1; else j+=k+1;
            if (i==j) ++j;
```

```
            k=0;
        }
    }
    if (j>i) j=i;
    //[j,n)+[0,j)
    rotate(a,a+j,a+n);
}
```

3.13   带通配符的字符串匹配

```
namespace NTT
{
    typedef unsigned ui;
    typedef unsigned long long ll;
    const int N=1<<22;
    const ui p=998244353, g=3;
    inline ui ksm(ui x, ui y)
    {
        ui ans=1;
        while (y)
        {
            if (y&1) ans=1llu*ans*x%p;
            y>>=1; x=1llu*x*x%p;
        }
        return ans;
    }
    ui r[N], w[N];
    void ntt(vector <ui> &a)
    {
        int n=a.size(), i, j, k;
        for (i=0; i<n; i++) if (i<r[i]) swap(a[i], a[r[i]]);
        for (k=1; k<n; k<<=1)
        {
            for (i=0; i<n; i+=k<<1)
            {
                for (j=0; j<k; j++)
                {
                    ui x=a[i+j], y=1llu*a[i+j+k]*w[j+k]%p;
                    a[i+j]=(x+y)%p; a[i+j+k]=(x+p-y)%p;
                }
            }
        }
    }
    vector<ui> mul(vector <ui> a, vector <ui> b)
    {
        if (a.size()==0||b.size()==0) return { };
        int m=a.size()+b.size()-1;
        int n=1<<__lg(m*2-1);
        int i, j, base=__lg(n)-1;
        ui inv=ksm(n, p-2);
        for (i=1; i<n; i++) r[i]=r[i>>1]>>1|(i&1)<<base;
        for (j=1; j<n; j<<=1)
        {
            ui wn=ksm(3, (p-1)/(j<<1));
            w[j]=1;
            for (i=1; i<j; i++) w[j+i]=1llu*w[j+i-1]*wn%p;
```

```cpp
        }
        a.resize(n); b.resize(n);
        ntt(a); ntt(b);
        for (i=0; i<n; i++) a[i]=1llu*a[i]*b[i]%p;
        ntt(a); reverse(1+all(a)); a.resize(n=m);
        for (i=0; i<n; i++) a[i]=1llu*a[i]*inv%p;
        return a;
    }
}
vector<int> match(const string &s, const string &t)
{
    using NTT::p, NTT::mul;
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    static array<ui, 256> c;
    static bool inited=0;
    if (!inited)
    {
        inited=1;
        for (ui &x:c) x=rnd()%NTT::p;
        c['*']=0;//通配符
    }
    int n=s.size(), m=t.size(), i, j;
    if (n<m) return { };
    vector<int> ans;
    vector<ui> f(n), ff(n), fff(n), g(m), gg(m), ggg(m);
    for (i=0; i<n; i++)
    {
        f[i]=c[s[i]];
        ff[i]=1llu*f[i]*f[i]%p;
        fff[i]=1llu*ff[i]*f[i]%p;
    }
    for (i=0; i<m; i++)
    {
        g[i]=c[t[m-i-1]];
        gg[i]=1llu*g[i]*g[i]%p;
        ggg[i]=1llu*gg[i]*g[i]%p;
    }
    auto fffg=mul(fff, g), ffgg=mul(ff, gg), fggg=mul(f, ggg);
    for (i=0; i<=n-m; i++) if ((fffg[m-1+i]+fggg[m-1+i]+2*(NTT::p-ffgg[m-1+i]))%
        NTT::p==0) ans.push_back(i);
    return ans;
}
```

快一些的版本

```cpp
const int N=1<<22;
const ui p=998244353, g=3;
inline ui ksm(ui x, ui y)
{
    ui ans=1;
    while (y)
    {
        if (y&1) ans=1llu*ans*x%p;
        y>>=1; x=1llu*x*x%p;
    }
    return ans;
}
```

```cpp
ui r[N], w[N];
void ntt(vector <ui> &a)
{
    int n=a.size(), i, j, k;
    for (k=1; k<n; k<<=1)
    {
        for (i=0; i<n; i+=k<<1)
        {
            for (j=0; j<k; j++)
            {
                ui x=a[i+j], y=1llu*a[i+j+k]*w[j+k]%p;
                a[i+j]=(x+y)%p; a[i+j+k]=(x+p-y)%p;
            }
        }
    }
}\
vector<int> match(string s, string t, char ch='*')
{
    static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    static array<ui, 256> c;
    static bool inited=0;
    if (!inited)
    {
        inited=1;
        for (ui &x:c) x=rnd()%p;
        // for (int i=0; i<256; i++) c[i]=i-96;
        c[ch]=0;//通配符
    }
    int n=s.size(), m=t.size(), i, j;
    if (n<m) return { };
    vector<int> ans;
    int N=1<<__lg(n*2-1), base=__lg(N)-1;
    vector<ui> f(N), ff(N), fff(N), g(N), gg(N), ggg(N);
    reverse(all(t));
    s.resize(N, ch), t.resize(N, ch);
    for (i=0; i<N; i++)
    {
        r[i]=r[i>>1]>>1|(i&1)<<base;
        if (i<r[i])
        {
            swap(s[i], s[r[i]]);
            swap(t[i], t[r[i]]);
        }
    }
    for (j=1; j<N; j<<=1)
    {
        ui wn=ksm(3, (p-1)/(j<<1));
        w[j]=1;
        for (i=1; i<j; i++) w[j+i]=1llu*w[j+i-1]*wn%p;
    }
    for (i=0; i<N; i++)
    {
        f[i]=c[s[i]];
        ff[i]=1llu*f[i]*f[i]%p;
        fff[i]=1llu*ff[i]*f[i]%p;
        g[i]=c[t[i]];
        gg[i]=1llu*g[i]*g[i]%p;
```

```
        ggg[i]=1llu*gg[i]*g[i]%p;
    }
    ntt(f); ntt(ff); ntt(fff); ntt(g); ntt(gg); ntt(ggg);
    for (i=0; i<N; i++) f[i]=(1llu*fff[i]*g[i]+1llu*f[i]*ggg[i]+2llu*(p-ff[i])*
        gg[i])%p;
    for (i=0; i<N; i++) if (i<r[i]) swap(f[i], f[r[i]]);
    ntt(f); reverse(1+all(f));
    for (i=0; i<=n-m; i++) if (f[m+i-1]==0) ans.push_back(i);
    return ans;
}
```

4 图论

4.1 最小密度环

$O(nm)$。

```cpp
#include <bits/stdc++.h>
using namespace std;
const int N=3e3+5,M=1e4+5;
const double inf=1e18;
int u[M],v[M];
double f[N][N],w[M];
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(8);
    int n,m,i,j;
    cin>>n>>m;
    for (i=1;i<=m;i++) cin>>u[i]>>v[i]>>w[i];
    ++n;
    for (i=1;i<=n;i++)
    {
        fill_n(f[i]+1,n,inf);
        for (j=1;j<=m;j++) f[i][v[j]]=min(f[i][v[j]],f[i-1][u[j]]+w[j]);
    }
    double ans=inf;
    for (i=1;i<n;i++) if (f[n][i]!=inf)
    {
        double r=-inf;
        for (j=1;j<n;j++) r=max(r,(f[n][i]-f[j][i])/(n-j));
        ans=min(ans,r);
    }
    cout<<ans<<endl;
}
```

4.2 全源最短路与判负环

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int,int> pa;
typedef tuple<int,int,int> tp;
const int N=152;
const ll inf=5e8;
ll dis[N][N],d[N][N];
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    while (1)
    {
        int n,m,q,i,j,k;
        cin>>n>>m>>q;
        if (tp(n,m,q)==tp(0,0,0)) return 0;
        for (i=0;i<n;i++) fill_n(dis[i],n,inf*inf);
        for (i=0;i<n;i++) dis[i][i]=0;
        while (m--)
```

```
        {
            int u,v,w;
            cin>>u>>v>>w;
            dis[u][v]=min(dis[u][v],(ll)w);
        }
        for (k=0;k<n;k++) for (i=0;i<n;i++) for (j=0;j<n;j++) dis[i][j]=max(min(
            dis[i][j],dis[i][k]+dis[k][j]),-inf*2);
        for (i=0;i<n;i++) copy_n(dis[i],n,d[i]);
        for (k=0;k<n;k++) for (i=0;i<n;i++) for (j=0;j<n;j++) dis[i][j]=max(min(
            dis[i][j],dis[i][k]+dis[k][j]),-inf*2);
        while (q--)
        {
            int u,v;
            cin>>u>>v;
            if (d[u][v]>inf) cout<<"Impossible\n"; else if (dis[u][v]!=d[u][v]||d
                [u][v]<-inf) cout<<"-Infinity\n"; else cout<<d[u][v]<<'\n';
        }
        cout<<'\n';
    }
}
```

4.3 三/四元环计数

$O(m\sqrt{m})$，$O(n+m)$。

注意四元环数的是边四元环。点四元环需要去掉四点完全图个数 *2，似乎不太能做？

```
ll triple(const vector<pair<int,int>> &edges)//start from 0
{
    int n=0,i;
    for (auto [u,v]:edges) n=max({n,u,v});
    ++n;
    vector d(n,0),id(d),rk(d),cnt(d);
    vector e(n,vector(0,0));
    for (auto [u,v]:edges) ++d[u],++d[v];
    iota(all(id),0); sort(all(id),[&](int x,int y) { return d[x]<d[y]; });
    for (i=0; i<n; i++) rk[id[i]]=i;
    for (auto [u,v]:edges)
    {
        if (rk[u]>rk[v]) swap(u,v);
        e[u].push_back(v);
    }
    ll ans=0;
    for (i=0; i<n; i++)
    {
        for (int u:e[i]) cnt[u]=1;
        for (int u:e[i]) for (int v:e[u]) ans+=cnt[v];
        for (int u:e[i]) cnt[u]=0;
    }
    return ans;
}
ll quadruple(const vector<pair<int,int>> &edges)
{
    int n=0,i;
    for (auto [u,v]:edges) n=max({n,u,v});
    ++n;
```

```cpp
    vector d(n,0),id(d),rk(d),cnt(d);
    vector e(n,vector(0,0)),lk(n,vector(0,0));
    for (auto [u,v]:edges) ++d[u],++d[v];
    iota(all(id),0); sort(all(id),[&](int x,int y) { return d[x]<d[y]; });
    for (i=0; i<n; i++) rk[id[i]]=i;
    for (auto [u,v]:edges)
    {
        if (rk[u]>rk[v]) swap(u,v);
        e[u].push_back(v);
        lk[u].push_back(v);
        lk[v].push_back(u);
    }
    ll ans=0;
    for (i=0; i<n; i++)
    {
        for (int u:lk[i]) for (int v:e[u]) if (rk[v]>rk[i]) ans+=cnt[v]++;
        for (int u:lk[i]) for (int v:e[u]) cnt[v]=0;
    }
    return ans;
}
```

4.4 Johnson 全源带负权最短路

$O(nm\log m)$，$O(n+m)$。

```cpp
for (int u=1;u<=n;u++) for (auto &[v,w]:e[u]) w+=dis[u]-dis[v];
```

4.5 弦图

单纯点：v 和 v 邻点构成团。

完美消除序列：v_i 在 $\{v_i, v_{i+1}, \cdots, v_n\}$ 为单纯点。

$N(v_i) = \{v_j | j > i \wedge (v_i, v_j) \in E\}$，$next(v_i)$ 为 $N(v_i)$ 最靠前的点。

极大团一定是 $\{v\} \cup N(v)$。

最大团大小等于色数。

弦图判定：等价于是否存在完美消除序列。首先求出一个完美消除序列，然后判定是否合法。

判定方法：设 $v_{i+1}, \cdots, v_n$ 中与 v_i 相邻的依次为 $v_1', \cdots, v_m'$。只需判断是否 v_1' 与 $v_2', \cdots, v_m'$ 相邻。

LexBFS 算法（我不会写）

每个点有一个字符串 label，初始为 0。从 $i = n$ 到 $i = 1$ 确定，选 label 字典序最大的 u，再把 u 邻点的 label 后面接一个 i。

最大势算法：从 v_n 求到 v_1，设 $label_i$ 表示 i 与多少个已选点相邻，每次选 $label_i$ 最大的点。

弦图极大团：$\{v | \forall next(w) = v, |N(v)| \geq |N(w)|\}$。选出的集合为基本点，按上述极大团构造。

弦图染色：从 v_n 到 v_1 依次选最小可染的色。

最大独立集：从 v_1 到 v_n 能选就选。

最小团覆盖：设最大独立集为 $\{p_m\}$，最小团覆盖为 $\{\{p_i\} \cup N(p_i)\}$。

区间图：两个区间有边当且仅当交集非空。

区间图是弦图。

4.5.1 代码

```cpp
namespace chordal_graph//下标从 1 开始
{
    const int N=1e5+2;//点数
    bool ed[N];
    vector<int> e[N];
    int n;
    void init(const vector<pair<int,int>> &edges)
    {
        n=0;
        for (auto [u,v]:edges) n=max({n,u,v});
        for (int i=1;i<=n;i++) e[i].clear();
        for (auto [u,v]:edges) e[u].push_back(v),e[v].push_back(u);
    }
    vector<int> perfect_seq(const vector<pair<int,int>> &edges)//MCS
    {
        init(edges);
        static int d[N];
        static vector<int> buc[N];
        int i,mx=0;
        memset(d+1,0,n*sizeof d[0]);
        memset(ed+1,0,n*sizeof ed[0]);
        for (i=1;i<=n;i++) buc[i].clear();
        buc[0].resize(n);
        iota(all(buc[0]),1);
        vector<int> r(n);
        for (i=n-1;i>=0;i--)
        {
            int u=0;
            while (!u)
            {
                while (buc[mx].size()) if (ed[buc[mx].back()]) buc[mx].pop_back()
                    ;
                else
                {
                    ed[u=buc[mx].back()]=1;
                    buc[mx].pop_back();
                    goto yes;
                }
                --mx;
            }
            yes:;
            r[i]=u;
            for (int v:e[u]) if (!ed[v]) buc[++d[v]].push_back(v),mx=max(mx,d[v])
                ;
        }
        return r;
    }
    bool check_perfect_seq(vector<int> a)
    {
        static bool ee[N];
        memset(ed+1,0,n*sizeof ed[0]);
        memset(ee+1,0,n*sizeof ee[0]);
        reverse(all(a));
        for (int u:a)
        {
```

```
            ed[u]=1;
            int w=0;
            for (int v:e[u]) if (ed[v]) {w=v;break;}
            if (!w) continue;
            ee[w]=1;
            for (int v:e[w]) ee[v]=1;
            for (int v:e[u]) if (ed[v]&&!ee[v]) return 0;
            ee[w]=0;
            for (int v:e[w]) ee[v]=0;
        }
        return 1;
    }
    bool check_chordal(const vector<pair<int,int>> &edges) {return
        check_perfect_seq(perfect_seq(edges));}
    vector<int> color(int _n,const vector<pair<int,int>> &edges)//返回长度为 _n
        +1。其中 0 无意义
    {
        auto a=perfect_seq(edges);
        reverse(all(a));
        memset(ed+1,0,n*sizeof ed[0]);
        vector<int> r(_n+1);
        for (int u:a)
        {
            for (int v:e[u]) ed[r[v]]=1;
            int x=1;
            while (ed[x]) ++x;
            r[u]=x;
            for (int v:e[u]) ed[r[v]]=0;
        }
        for (int i=n+1;i<=_n;i++) r[i]=1;
        return r;
    }
    vector<int> max_independent(int _n,const vector<pair<int,int>> &edges)//注意
        有孤立点这种奇怪东西
    {
        auto a=perfect_seq(edges);
        memset(ed+1,0,n*sizeof ed[0]);
        vector<int> r;
        for (int u:a) if (!ed[u])
        {
            r.push_back(u);
            for (int v:e[u]) ed[v]=1;
        }
        for (int i=n+1;i<=_n;i++) r.push_back(i);
        return r;
    }
}
using chordal_graph::check_chordal,chordal_graph::color,chordal_graph::
    max_independent;
```

4.6 最小割树

```
    template<class T> vector<vector<T>> min_cut(int n, const vector<tuple<int,
        int, T>> &edges)//[0,n)
{
    int m=edges.size(), i, s, t, cnt=0;
```

```cpp
vector<int> fir(n, -1), nxt(m*2, -1), fc(n), q(n);
vector<pair<int, T>> e(m*2);
vector<tuple<T, int, int>> eg;
auto add=[&](int u, int v, T w)
    {
        e[cnt]={v, w};
        nxt[cnt]=fir[u];
        fir[u]=cnt++;
    };
for (auto [u, v, w]:edges) add(u, v, w), add(v, u, w);
auto E=e;
auto bfs=[&]()
    {
        fill(all(fc), 0);
        int ql=0, qr=0, u, i;
        fc[q[0]=s]=1;
        while (ql<=qr)
        {
            u=q[ql++];
            for (int i=fir[u]; i!=-1; i=nxt[i])
                if (auto &[v, w]=e[i]; w&&!fc[v]) fc[q[++qr]=v]=fc[u]+1;
        }
        return fc[t];
    };
function<T(int, T)> dfs=[&](int u, T maxf)
    {
        if (u==t) return maxf;
        T j=0, k;
        for (int i=fir[u]; i!=-1; i=nxt[i])
            if (auto &[v, w]=e[i]; w&&fc[v]==fc[u]+1&&(k=dfs(v, min(maxf-j, w
                ))))
            {
                j+=k;
                w-=k;
                e[i^1].second+=k;
                if (j==maxf) return j;
            }
        fc[u]=0;
        return j;
    };
function<void(vector<int>)> solve=[&](vector<int> id)
    {
        static mt19937 rnd(chrono::steady_clock::now().time_since_epoch().
            count());
        if (id.size()<=1) return;
        vector<int> u(2);
        sample(all(id), u.begin(), 2, rnd);
        s=u[0], t=u[1], e=E;
        T ans=0;
        while (bfs()) ans+=dfs(s, numeric_limits<T>::max());
        auto it=partition(all(id), [&](int u) { return fc[u]; });
        eg.emplace_back(ans, s, t);
        solve(vector(id.begin(), it));
        solve(vector(it, id.end()));
    };
solve(range(0, n));
sort(all(eg), greater<>());
```

```cpp
    vector<basic_string<int>> ver(n);
    vector ans(n, vector<T>(n));
    vector<int> f(n);
    for (i=0; i<n; i++) ver[i]={f[i]=i};
    function<int(int)> getf=[&](int u) { return f[u]==u?u:f[u]=getf(f[u]); };
    for (auto [w, u, v]:eg)
    {
        u=getf(u);
        v=getf(v);
        for (int w1:ver[u]) for (int w2:ver[v]) ans[w1][w2]=ans[w2][w1]=w;
        ver[u]+=ver[v];
        f[v]=u;
    }
    return ans;
}
```

4.7 二分图与网络流建图

以下约定，若为二分图则 n, m 表示两侧点数，否则仅 n 表示全图点数。

4.7.1 二分图边染色

留坑待填。

结论：$\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1$，二分图时 $\chi'(G) = \Delta(G)$。$\Delta(G)$ 为图的最大度。

4.7.2 二分图最小点集覆盖

$ans = \text{maxmatch}$，方案如下。

```cpp
#include <bits/stdc++.h>
using namespace std;
const int N=5e3+2;
vector<int> e[N];
int ed[N],lk[N],kl[N],flg[N],now;
bool dfs(int u)
{
    for (int v:e[u]) if (ed[v]!=now)
    {
        ed[v]=now;
        if (!lk[v]||dfs(lk[v])) return lk[v]=u;
    }
    return 0;
}
void dfs2(int u)
{
    for (int v:e[u]) if (!flg[v]) flg[v]=1,dfs2(lk[v]);
}
int main()
{
    int n,m,i,r=0;
    cin>>n>>m;
    while (m--)
    {
        int u,v;
        cin>>u>>v;
```

```
        e[u].push_back(v);
    }
    for (i=1;i<=n;i++) dfs(now=i);
    for (i=1;i<=n;i++) kl[lk[i]]=i;
    for (i=1;i<=n;i++) if (!kl[i]) dfs2(i);
    vector<int> A[2];
    for (i=1;i<=n;i++) if (lk[i])
    {
        if (flg[i]) A[1].push_back(i); else A[0].push_back(lk[i]);
    }
    for (int j=0;j<2;j++)
    {
        cout<<A[j].size();
        for (int x:A[j]) cout<<'␣'<<x;cout<<'\n';
    }
}
```

4.7.3　二分图最大独立集

$ans = n + m - \mathrm{maxmatch}$，方案是最小点集覆盖的补集。

4.7.4　二分图最小边覆盖

$ans = n + m - \mathrm{maxmatch}$，方案是最大匹配加随便一些边。无解当且仅当有孤立点，算法会视为单选孤立点（无边）。这个定理对一般图也成立。

4.7.5　有向无环图最小不相交链覆盖

$ans = n - \mathrm{maxmatch}$，其中二分图建图方法是拆入点和出点（实现时直接跑一次二分图就行，不用额外处理），注意**不**需要传递闭包。方案如下。

```cpp
#include <bits/stdc++.h>
using namespace std;
const int N=152;
vector<int> e[N];
int lk[N],kl[N],ed[N],now;
bool dfs(int u)
{
    for (int v:e[u]) if (ed[v]!=now)
    {
        ed[v]=now;
        if (!lk[v]||dfs(lk[v])) return lk[v]=u;
    }
    return 0;
}
int main()
{
    int n,m,i;
    ios::sync_with_stdio(0);cin.tie(0);
    cin>>n>>m;
    while (m--)
    {
        int u,v;
        cin>>u>>v;
        e[u].push_back(v);
    }
```

```
    int r=0;
    for (i=1;i<=n;i++) r+=dfs(now=i);
    for (i=1;i<=n;i++) kl[lk[i]]=i;
    for (i=1;i<=n;i++) if (ed[i]!=-1&&!lk[i])
    {
        vector<int> ans;
        int u=i;
        while (u)
        {
            ed[u]=-1;
            ans.push_back(u);
            u=kl[u];
        }
        for (int j=0;j<ans.size();j++) cout<<ans[j]<<" \n"[j+1==ans.size()];
    }
    cout<<n-r<<endl;
}
```

4.7.6　有向无环图最大互不可达集

$ans = n - \mathrm{maxmatch}$，其中二分图建图方法是拆入点和出点（实现时直接跑一次二分图就行，不用额外处理），注意**需要**传递闭包。方案？

4.7.7　最大权闭合子图

若 $v_i > 0$，$s \to i$ 流量 v_i；若 $v_i < 0$，$i \to t$ 流量 $-v_i$。若原图 $u \to v$ 可花费 w 代价违抗，流量 w，否则 $+\infty$ 。答案为 $\sum\limits_{v_i > 0} v_i - \mathrm{maxflow}$。方案？

4.8　二分图匹配（时间戳写法）

```
bool dfs(int u)
{
    for (int v:e[u]) if (ed[v]!=now)
    {
        ed[v]=now;
        if (!lk[v]||dfs(lk[v])) return lk[v]=u;
    }
    return 0;
}
```

4.9　二分图最大权匹配

```
namespace KM
{
    const int N=405;//点数
    typedef long long ll;//答案范围
    const ll inf=1e16;
    int lk[N],kl[N],pre[N],q[N],n,h,t;
    ll sl[N],e[N][N],lx[N],ly[N];
    bool edx[N],edy[N];
    bool ck(int v)
    {
        if (edy[v]=1,kl[v]) return edx[q[++t]=kl[v]]=1;
```

```
            while (v) swap(v,lk[kl[v]=pre[v]]);
            return 0;
    }
    void bfs(int u)
    {
        fill_n(sl+1,n,inf);
        memset(edx+1,0,n*sizeof edx[0]);
        memset(edy+1,0,n*sizeof edy[0]);
        q[h=t=1]=u;edx[u]=1;
        while (1)
        {
            while (h<=t)
            {
                int u=q[h++],v;
                ll d;
                for (v=1;v<=n;v++) if (!edy[v]&&sl[v]>=(d=lx[u]+ly[v]-e[u][v]))
                    if (pre[v]=u,d) sl[v]=d; else if (!ck(v)) return;
            }
            int i;
            ll m=inf;
            for (i=1;i<=n;i++) if (!edy[i]) m=min(m,sl[i]);
            for (i=1;i<=n;i++)
            {
                if (edx[i]) lx[i]-=m;
                if (edy[i]) ly[i]+=m; else sl[i]-=m;
            }
            for (i=1;i<=n;i++) if (!edy[i]&&!sl[i]&&!ck(i)) return;
        }
    }
    template<typename TT> ll max_weighted_match(int N,const vector<tuple<int,int
        ,TT>> &edges)//lk[[1,n]]->[1,n]
    {
        int i;n=N;
        memset(lk+1,0,n*sizeof lk[0]);
        memset(kl+1,0,n*sizeof kl[0]);
        memset(ly+1,0,n*sizeof ly[0]);
        for (i=1;i<=n;i++) fill_n(e[i]+1,n,0);//若不需保证匹配边最多，置 0 即可，
            否则 -inf/N
        for (auto [u,v,w]:edges) e[u][v]=max(e[u][v],(ll)w);
        for (i=1;i<=n;i++) lx[i]=*max_element(e[i]+1,e[i]+n+1);
        for (i=1;i<=n;i++) bfs(i);
        ll r=0;
        for (i=1;i<=n;i++) r+=e[i][lk[i]];
        return r;
    }
}
using KM::max_weighted_match,KM::lk,KM::kl,KM::e;
```

4.10 一般图最大匹配

```
namespace blossom_tree
{
    const int N=1005;
    vector<int> e[N];
    int lk[N],rt[N],f[N],dfn[N],typ[N],q[N];
    int id,h,t,n;
```

```cpp
int lca(int u,int v)
{
    ++id;
    while (1)
    {
        if (u)
        {
            if (dfn[u]==id) return u;
            dfn[u]=id;u=rt[f[lk[u]]];
        }
        swap(u,v);
    }
}
void blm(int u,int v,int a)
{
    while (rt[u]!=a)
    {
        f[u]=v;
        v=lk[u];
        if (typ[v]==1) typ[q[++t]=v]=0;
        rt[u]=rt[v]=a;
        u=f[v];
    }
}
void aug(int u)
{
    while (u)
    {
        int v=lk[f[u]];
        lk[lk[u]=f[u]]=u;
        u=v;
    }
}
void bfs(int root)
{
    memset(typ+1,-1,n*sizeof typ[0]);
    iota(rt+1,rt+n+1,1);
    typ[q[h=t=1]=root]=0;
    while (h<=t)
    {
        int u=q[h++];
        for (int v:e[u])
        {
            if (typ[v]==-1)
            {
                typ[v]=1;f[v]=u;
                if (!lk[v]) return aug(v);
                typ[q[++t]=lk[v]]=0;
            } else if (!typ[v]&&rt[u]!=rt[v])
            {
                int a=lca(rt[u],rt[v]);
                blm(v,u,a);blm(u,v,a);
            }
        }
    }
}
int max_general_match(int N,vector<pair<int,int>> edges)//[1,n]
```

```
    {
        n=N;id=0;
        memset(f+1,0,n*sizeof f[0]);
        memset(dfn+1,0,n*sizeof dfn[0]);
        memset(lk+1,0,n*sizeof lk[0]);
        int i;
        for (i=1;i<=n;i++) e[i].clear();
        mt19937 rnd(114);
        shuffle(all(edges),rnd);
        for (auto [u,v]:edges)
        {
            e[u].push_back(v),e[v].push_back(u);
            if (!(lk[u]||lk[v])) lk[u]=v,lk[v]=u;
        }
        int r=0;
        for (i=1;i<=n;i++) if (!lk[i]) bfs(i);
        for (i=1;i<=n;i++) r+=!!lk[i];
        return r/2;
    }
}
using blossom_tree::max_general_match,blossom_tree::lk;
```

4.11 一般图最大权匹配

$n = 400$：UOJ 600ms, Luogu 135ms

```
#include<bits/stdc++.h>
using namespace std;
#define all(x) (x).begin(),(x).end()
namespace weighted_blossom_tree
{
    #define d(x) (lab[x.u]+lab[x.v]-e[x.u][x.v].w*2)
    const int N=403*2;//两倍点数
    typedef long long ll;//总和大小
    typedef int T;//权值大小
    //均不允许无符号
    const T inf=numeric_limits<int>::max()>>1;
    struct Q
    {
        int u,v;
        T w;
    } e[N][N];
    T lab[N];
    int n,m=0,id,h,t,lk[N],sl[N],st[N],f[N],b[N][N],s[N],ed[N],q[N];
    vector<int> p[N];
    void upd(int u,int v) {if (!sl[v]||d(e[u][v])<d(e[sl[v]][v])) sl[v]=u;}
    void ss(int v)
    {
        sl[v]=0;
        for (int u=1;u<=n;u++) if (e[u][v].w>0&&st[u]!=v&&!s[st[u]]) upd(u,v);
    }
    void ins(int u) {if (u<=n) q[++t]=u; else for (int v:p[u]) ins(v);}
    void mdf(int u,int w)
    {
        st[u]=w;
        if (u>n) for (int v:p[u]) mdf(v,w);
    }
```

```cpp
int gr(int u,int v)
{
    if ((v=find(all(p[u]),v)-p[u].begin())&1)
    {
        reverse(1+all(p[u]));
        return (int)p[u].size()-v;
    }
    return v;
}
void stm(int u,int v)
{
    lk[u]=e[u][v].v;
    if (u<=n) return;
    Q w=e[u][v];
    int x=b[u][w.u],y=gr(u,x),i;
    for (i=0;i<y;i++) stm(p[u][i],p[u][i^1]);
    stm(x,v);
    rotate(p[u].begin(),y+all(p[u]));
}
void aug(int u,int v)
{
    int w=st[lk[u]];
    stm(u,v);
    if (!w) return;
    stm(w,st[f[w]]);
    aug(st[f[w]],w);
}
int lca(int u,int v)
{
    for (++id;u|v;swap(u,v))
    {
        if (!u) continue;
        if (ed[u]==id) return u;
        ed[u]=id;//??????????v?? 这是原作者的注释，我也不知道是啥
        if (u=st[lk[u]]) u=st[f[u]];
    }
    return 0;
}
void add(int u,int a,int v)
{
    int x=n+1,i,j;
    while (x<=m&&st[x]) ++x;
    if (x>m) ++m;
    lab[x]=s[x]=st[x]=0;lk[x]=lk[a];
    p[x].clear();p[x].push_back(a);
    for (i=u;i!=a;i=st[f[j]]) p[x].push_back(i),p[x].push_back(j=st[lk[i]]),
        ins(j);//复制，改一处
    reverse(1+all(p[x]));
    for (i=v;i!=a;i=st[f[j]]) p[x].push_back(i),p[x].push_back(j=st[lk[i]]),
        ins(j);
    mdf(x,x);
    for (i=1;i<=m;i++) e[x][i].w=e[i][x].w=0;
    memset(b[x]+1,0,n*sizeof b[0][0]);
    for (int u:p[x])
    {
        for (v=1;v<=m;v++) if (!e[x][v].w||d(e[u][v])<d(e[x][v])) e[x][v]=e[u
            ][v],e[v][x]=e[v][u];
```

```cpp
        for (v=1;v<=n;v++) if (b[u][v]) b[x][v]=u;
    }
    ss(x);
}
void ex(int u) // s[u] == 1
{
    for (int x:p[u]) mdf(x,x);
    int a=b[u][e[u][f[u]].u],r=gr(u,a),i;
    for (i=0;i<r;i+=2)
    {
        int x=p[u][i],y=p[u][i+1];
        f[x]=e[y][x].u;
        s[x]=1;s[y]=0;
        sl[x]=0;ss(y);
        ins(y);
    }
    s[a]=1;f[a]=f[u];
    for (i=r+1;i<p[u].size();i++) s[p[u][i]]=-1,ss(p[u][i]);
    st[u]=0;
}
bool on(const Q &e)
{
    int u=st[e.u],v=st[e.v],a;
    if(s[v]==-1)
    {
        f[v]=e.u;s[v]=1;
        a=st[lk[v]];
        sl[v]=sl[a]=s[a]=0;
        ins(a);
    }
    else if(!s[v])
    {
        a=lca(u,v);
        if (!a) return aug(u,v),aug(v,u),1;
        else add(u,a,v);
    }
    return 0;
}
bool bfs()
{
    memset(s+1,-1,m*sizeof s[0]);
    memset(sl+1,0,m*sizeof sl[0]);
    h=1;t=0;
    int i,j;
    for (i=1;i<=m;i++) if (st[i]==i&&!lk[i]) f[i]=s[i]=0,ins(i);
    if (h>t) return 0;
    while (1)
    {
        while (h<=t)
        {
            int u=q[h++],v;
            if (s[st[u]]!=1) for (v=1; v<=n;v++) if (e[u][v].w>0&&st[u]!=st[v
                ])
            {
                if (d(e[u][v])) upd(u,st[v]); else if (on(e[u][v])) return 1;
            }
        }
```

```cpp
                T x=inf;
                for (i=n+1;i<=m;i++) if (st[i]==i&&s[i]==1) x=min(x,lab[i]>>1);
                for (i=1;i<=m;i++) if (st[i]==i&&sl[i]&&s[i]!=1) x=min(x,d(e[sl[i]][i
                    ])>>s[i]+1);
                for (i=1;i<=n;i++) if (~s[st[i]]) if ((lab[i]+=(s[st[i]]*2-1)*x)<=0)
                    return 0;
                for (i=n+1;i<=m;i++) if (st[i]==i&&~s[st[i]]) lab[i]+=(2-s[st[i]]*4)*
                    x;
                h=1;t=0;
                for (i=1;i<=m;i++) if (st[i]==i&&sl[i]&&st[sl[i]]!=i&&!d(e[sl[i]][i])
                    &&on(e[sl[i]][i])) return 1;
                for (i=n+1;i<=m;i++) if (st[i]==i&&s[i]==1&&!lab[i]) ex(i);
            }
            return 0;
        }
        template<typename TT> ll max_weighted_general_match(int N,const vector<tuple
            <int,int,TT>> &edges)//[1,n], 返回权值
        {
            memset(ed+1,0,m*sizeof ed[0]);
            memset(lk+1,0,m*sizeof lk[0]);
            n=m=N;id=0;
            iota(st+1,st+n+1,1);
            int i,j;
            T wm=0;
            ll r=0;
            for (i=1;i<=n;i++) for (j=1;j<=n;j++) e[i][j]={i,j,0};
            for (auto [u,v,w]:edges) wm=max(wm,e[v][u].w=e[u][v].w=max(e[u][v].w,(T)
                w));
            for (i=1;i<=n;i++) p[i].clear();
            for (i=1;i<=n;i++) for (j=1;j<=n;j++) b[i][j]=i*(i==j);
            fill_n(lab+1,n,wm);
            while (bfs());
            for (i=1;i<=n;i++) if (lk[i]) r+=e[i][lk[i]].w;
            return r/2;
        }
    #undef d
}
using weighted_blossom_tree::max_weighted_general_match,weighted_blossom_tree::
    lk;
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m;
    cin>>n>>m;
    vector<tuple<int,int,long long>> edges(m);
    for (auto &[u,v,w]:edges) cin>>u>>v>>w;
    cout<<max_weighted_general_match(n,edges)<<'\n';
    for (int i=1;i<=n;i++) cout<<lk[i]<<" \n"[i==n];
}
```

4.12 网络流代码

```cpp
namespace net
{
    const int N=4e5+50;//number of points
    namespace flow
```

```cpp
{
    typedef ll wT;//single flow
    typedef ll cT;//total flow
    const cT inf=numeric_limits<cT>::max()/2;//maximum
    struct Q
    {
        int v;
        wT w;
        int id;
    };
    vector<Q> e[N];
    int fc[N],q[N];
    int n,s,t;
    int bfs()
    {
        fill_n(fc,n,0);
        int p1=0,p2=0,u;
        fc[s]=1; q[0]=s;
        while (p1<=p2)
        {
            int u=q[p1++];
            for (auto [v,w,id]:e[u]) if (w&&!fc[v]) fc[q[++p2]=v]=fc[u]+1;
        }
        return fc[t];
    }
    cT dfs(int u,cT maxf)
    {
        if (u==t) return maxf;
        cT j=0,k;
        for (auto &[v,w,id]:e[u]) if (w&&fc[v]==fc[u]+1&&(k=dfs(v,min(maxf-j
            ,(cT)w))))
        {
            j+=k;
            w-=k;
            e[v][id].w+=k;
            if (j==maxf) return j;
        }
        fc[u]=0;
        return j;
    }
    cT max_flow(const vector<tuple<int,int,wT>> &edges,int S,int T)//[0,n]
    {
        s=S; t=T; n=max(s,t);
        for (auto [u,v,w]:edges) n=max({n,u,v});
        ++n;
        assert(n<N);
        for (int i=0; i<n; i++) e[i].clear();
        for (auto [u,v,w]:edges) if (u!=v)
        {
            e[u].push_back({v,w,(int)e[v].size()});
            e[v].push_back({u,0,(int)e[u].size()-1});
        }
        cT r=0;
        while (bfs()) r+=dfs(s,inf);
        return r;
    }
}
```

```cpp
using flow::max_flow,flow::fc;
namespace match
{
    int lk[N];
    int max_match(int n,int m,const vector<pair<int,int>> &edges)//lk[[0,n
        ]]->[0,m]
    {
        ++n; ++m;
        assert(max(n,m)<N);
        int s=n+m,t=n+m+1,i;
        vector<tuple<int,int,ll>> eg;
        eg.reserve(n+m+edges.size());
        for (i=0; i<n; i++) eg.push_back({s,i,1});
        for (i=0; i<m; i++) eg.push_back({i+n,t,1});
        for (auto [u,v]:edges) eg.push_back({u,v+n,1});
        int r=max_flow(eg,s,t);
        fill_n(lk,n,-1);
        for (i=0; i<n; i++) for (auto [v,w,id]:flow::e[i]) if (v<s&&!w) { lk[
            i]=v-n; break; }
        return r;
    }
    bool ed[N];
    int kl[N];
    vector<int> e[N];
    void dfs(int u)
    {
        for (int v:e[u]) if (!ed[v]) ed[v]=1,dfs(kl[v]);
    }
    pair<vector<int>,vector<int>> min_cover(int n,int m,const vector<pair<
        int,int>> &edges)//[0,n]-[0,m]
    {
        max_match(n,m,edges);
        ++n; ++m;
        fill_n(kl,m,-1); fill_n(ed,m,0);
        int i;
        for (i=0; i<n; i++)
        {
            e[i].clear();
            if (lk[i]!=-1) kl[lk[i]]=i;
        }
        for (auto [u,v]:edges) e[u].push_back(v);
        for (i=0; i<n; i++) if (lk[i]==-1) dfs(i);
        vector<int> r[2];
        for (i=0; i<m; i++) if (kl[i]!=-1)
        {
            if (ed[i]) r[1].push_back(i); else r[0].push_back(kl[i]);
        }
        sort(all(r[0]));
        return {r[0],r[1]};
    }
}
using match::max_match,match::min_cover,match::lk,match::kl;
namespace cost_flow
{
    typedef ll wT;
    typedef ll cT;
    const cT inf=numeric_limits<cT>::max()/2;
```

```cpp
struct Q
{
    int v;
    wT w;
    cT c;
    int id;
};
vector<Q> e[N];
cT dis[N];
int pre[N],pid[N],ipd[N];
bool ed[N];
int n,s,t;
pair<wT,cT> spfa()
{
    queue<int> q;
    fill_n(dis,n,inf);
    memset(ed,0,n*sizeof ed[0]);
    q.push(s); dis[s]=0;
    while (q.size())
    {
        int u=q.front(); q.pop(); ed[u]=0;
        for (auto [v,w,c,id]:e[u]) if (w&&dis[v]>dis[u]+c)
        {
            dis[v]=dis[u]+c;
            pre[v]=u;
            pid[v]=e[v][id].id;
            ipd[v]=id;
            if (!ed[v]) q.push(v),ed[v]=1;
        }
    }
    if (dis[t]==inf) return {0,0};
    wT mw=numeric_limits<wT>::max();
    for (int i=t; i!=s; i=pre[i]) mw=min(mw,e[pre[i]][pid[i]].w);
    for (int i=t; i!=s; i=pre[i]) e[pre[i]][pid[i]].w-=mw,e[i][ipd[i]].w
        +=mw;
    return {mw,(cT)mw*dis[t]};
}
pair<wT,cT> mcmf_spfa(const vector<tuple<int,int,wT,cT>> &edges,int S,
    int T)//[0,n]
{
    s=S; t=T; n=max(s,t);
    for (auto [u,v,w,c]:edges) n=max({n,u,v});
    ++n;
    assert(n<N);
    for (int i=0; i<n; i++) e[i].clear();
    for (auto [u,v,w,c]:edges) if (u!=v)
    {
        e[u].push_back({v,w,c,(int)e[v].size()});
        e[v].push_back({u,0,-c,(int)e[u].size()-1});
    }
    pair<wT,cT> r{0,0},rr;
    while ((rr=spfa()).first) r={r.first+rr.first,r.second+rr.second};
    return r;
}
pair<wT,cT> mcmf_dijk(const vector<tuple<int,int,wT,cT>> &edges,int S,
    int T)//[0,n]
{
```

```cpp
s=S; t=T; n=max(s,t);
for (auto [u,v,w,c]:edges) n=max({n,u,v});
++n;
assert(n<N);
for (int i=0; i<n; i++) e[i].clear();
for (auto [u,v,w,c]:edges) if (u!=v)
{
    e[u].push_back({v,w,c,(int)e[v].size()});
    e[v].push_back({u,0,-c,(int)e[u].size()-1});
}
static cT h[N];
auto get_h=[&]()
{
    fill_n(h,n,inf);
    memset(ed,0,n*sizeof ed[0]);
    queue<int> q;
    q.push(s); h[s]=0;
    while (q.size())
    {
        int u=q.front(); q.pop(); ed[u]=0;
        for (auto [v,w,c,id]:e[u]) if (w&&h[v]>h[u]+c)
        {
            h[v]=h[u]+c;
            if (!ed[v]) q.push(v),ed[v]=1;
        }
    }
    return;
};
auto dijkstra=[&]() -> pair<wT,cT>
{
    static int fl[N],zl[N];
    int i;
    memset(ed,0,n*sizeof ed[0]);
    fill_n(dis,n,inf);
    typedef pair<cT,int> pa;
    priority_queue<pa,vector<pa>,greater<pa>> q;
    dis[s]=0; q.push({0,s});
    while (q.size())
    {
        int u=q.top().second;
        q.pop(); ed[u]=1;
        i=0;
        for (auto [v,w,c,id]:e[u])
        {
            if (w&&dis[v]>dis[u]+c) fl[v]=id,zl[v]=i,q.push({dis[v]=dis
                [pre[v]=u]+c,v});
            ++i;
        }
        while (q.size()&&ed[q.top().second]) q.pop();
    }
    if (dis[t]==inf) return {0,0};
    wT tf=numeric_limits<wT>::max();
    for (i=t; i!=s; i=pre[i]) tf=min(tf,e[pre[i]][zl[i]].w);
    for (i=t; i!=s; i=pre[i]) e[pre[i]][zl[i]].w-=tf,e[i][fl[i]].w+=
        tf;
    for (int u=0; u<n; u++) for (auto &[v,w,c,id]:e[u]) c+=dis[u]-dis
        [v];
```

```
                return {tf,tf*(h[t]+=dis[t])};
            };
            get_h();
            for (int u=0; u<n; u++) for (auto &[v,w,c,id]:e[u]) c+=h[u]-h[v];
            pair<wT,cT> r{0,0},rr;
            while ((rr=dijkstra()).first) r={r.first+rr.first,r.second+rr.second
                };
            return r;
        }
    }
}
using cost_flow::mcmf_spfa,cost_flow::mcmf_dijk;
namespace bounded_flow
{
    typedef ll wT;//single flow
    typedef ll cT;//total flow
    bool valid_flow(const vector<tuple<int,int,wT,wT>> &edges)//方案需加上 l
    {
        if (!edges.size()) return 1;
        int n=0,i;
        cT tot=0;
        for (auto [u,v,l,r]:edges)
        {
            n=max({n,u,v});
            if (l>r) return 0;
        }
        ++n;
        static cT cd[N];
        memset(cd,0,n*sizeof cd[0]);
        for (auto [u,v,l,r]:edges) cd[u]+=l,cd[v]-=l;
        vector<tuple<int,int,wT>> eg;
        eg.reserve(n+edges.size());
        for (i=0; i<n; i++) if (cd[i]>0) eg.push_back({i,n+1,cd[i]}),tot+=cd[
            i];
        else if (cd[i]<0) eg.push_back({n,i,-cd[i]});
        for (auto [u,v,l,r]:edges) eg.push_back({u,v,r-l});
        return tot==flow::max_flow(eg,n,n+1);
    }
    cT valid_flow_st(vector<tuple<int,int,wT,wT>> edges,int s,int t)//-1
         invalid, wT=cT
    {
        int n=max(s,t);
        cT tot=0;
        for (auto [u,v,l,r]:edges) n=max({n,u,v}),tot+=(u==s)*r;
        ++n;
        edges.push_back({t,s,0,tot});
        if (!valid_flow(edges)) return -1;
        assert(flow::e[s].back().v==t);
        assert(flow::e[t].back().v==s);
        return tot-flow::e[t].back().w;
    }
    cT valid_max_flow(const vector<tuple<int,int,wT,wT>> &edges,int s,int t)
         //-1 invalid, wT=cT
    {
        cT r=valid_flow_st(edges,s,t);
        if (r<0) return r;
        flow::s=s; flow::t=t;
        flow::e[s].pop_back(); flow::e[t].pop_back();
```

```cpp
        while (flow::bfs()) r+=flow::dfs(s,flow::inf);
        return r;
    }
    cT valid_min_flow(const vector<tuple<int,int,wT,wT>> &edges,int s,int t)
        //-1 invalid, wT=cT
    {
        cT r=valid_flow_st(edges,s,t);
        if (r<0) return r;
        flow::s=t; flow::t=s;
        flow::e[s].pop_back(); flow::e[t].pop_back();
        while (flow::bfs()) r-=flow::dfs(t,flow::inf);
        return r;
    }//not check
}
using bounded_flow::valid_flow,bounded_flow::valid_flow_st,bounded_flow::
    valid_max_flow,bounded_flow::valid_min_flow;
namespace bounded_cost_flow
{
    pair<ll,ll> valid_mcf(const vector<tuple<int,int,ll,ll,ll>> &edges,int s
        ,int t)//[u,v,l,r,c],mincost flow
    {
        int n=max(s,t);
        for (auto [u,v,l,r,c]:edges) n=max({n,u,v});
        ++n;
        int ss=n,tt=n+1;
        static ll cd[N];
        memset(cd,0,n*sizeof cd[0]);
        for (auto [u,v,l,r,c]:edges) cd[u]+=l,cd[v]-=l;
        vector<tuple<int,int,ll,ll>> e;
        ll t1=0,t2=0;
        for (int i=0; i<n; i++) if (cd[i]>0) e.push_back({i,tt,cd[i],0}),t2+=
            cd[i];
        else if (cd[i]<0) e.push_back({ss,i,-cd[i],0});
        for (auto [u,v,l,r,c]:edges) e.push_back({u,v,r-l,c});
        for (auto [u,v,w,c]:e) t1+=(u==s)*w;
        e.push_back({t,s,t1,0});
        auto res=mcmf_spfa(e,ss,tt);//checked dijk
        if (res.first!=t2) return {-1,-1};
        res.first=cost_flow::e[s].back().w;
        for (auto [u,v,l,r,c]:edges) res.second+=l*c;
        return res;
    }
    pair<ll,ll> valid_mcmf(const vector<tuple<int,int,ll,ll,ll>> &edges,int
        s,int t)//[u,v,l,r,c],mincost max_flow
    {
        auto r=valid_mcf(edges,s,t);
        if (r.first<0) return {-1,-1};
        cost_flow::e[s].pop_back();
        cost_flow::e[t].pop_back();
        cost_flow::s=s; cost_flow::t=t;
        pair<ll,ll> rr;
        while ((rr=cost_flow::spfa()).first) r={r.first+rr.first,r.second+rr.
            second};//spfa ver. not checked dijk
        return r;
    }
}
using bounded_cost_flow::valid_mcf,bounded_cost_flow::valid_mcmf;
```

```cpp
    namespace ne_cost_flow
    {
        pair<ll,ll> ne_mcmf(const vector<tuple<int,int,ll,ll>> &edges,int s,int
            t)
        {
            vector<tuple<int,int,ll,ll,ll>> e;
            for (auto [u,v,w,c]:edges) if (c>=0) e.push_back({u,v,0,w,c}); else
            {
                e.push_back({u,v,w,w,c});
                e.push_back({v,u,0,w,-c});
            }
            return valid_mcmf(e,s,t);
        }
    }
    using ne_cost_flow::ne_mcmf;
}
```

4.13 费用流（SPFA）

```cpp
bool dfs()
{
    memset(jl,-0x3f,sizeof(jl));
    jl[dl[tou=wei=1]=0]=0;
    while (tou<=wei)
    {
        ed[x=dl[tou++]]=0;
        for (i=fir[x];i;i=nxt[i]) if ((lj[i][1])&&(jl[lj[i][0]]<jl[x]+lj[i][2]))
        {
            jl[lj[i][0]]=jl[x]+lj[i][2];
            qq[lj[i][0]]=x;
            dy[lj[i][0]]=i;
            if (!ed[lj[i][0]]) ed[dl[++wei]=lj[i][0]]=1;
        }
    }
    zg=m;
    if (jl[t]==jl[t+1]) return 0;
    for (i=t;i;i=qq[i]) zg=min(zg,lj[dy[i]][1]);
    for (i=t;i;i=qq[i])
    {
        lj[dy[i]][1]-=zg;
        ans+=zg*lj[dy[i]][2];
        if (dy[i]&1) lj[dy[i]+1][1]+=zg; else lj[dy[i]-1][1]+=zg;
    }
    return 1;
}
while (dfs());
```

4.14 费用流（Dijkstra）

```cpp
priority_queue<pa,vector<pa>,greater<pa> > heap;
const int N=5e3+2,M=1e5+2;
pa ans;
int lj[M][3],nxt[M],fir[N],dis[N],h[N],pre[N],fl[N];
int n,m,s,t,bs,x,y,z,w,ans1,ans2;
```

```cpp
bool ed[N];
void add(const int u,const int v,const int x,const int y)
{
    lj[++bs][0]=v;
    lj[bs][1]=x;
    lj[bs][2]=y;
    nxt[bs]=fir[u];
    fir[u]=bs;
    lj[++bs][0]=u;
    lj[bs][1]=0;
    lj[bs][2]=-y;
    nxt[bs]=fir[v];
    fir[v]=bs;
}
void spfa()//本题中用dijkstra代替
{
    int x,i,j;
    memset(h,0x3f,sizeof(h));h[s]=0;
    heap.push(make_pair(0,s));
    while (!heap.empty())
    {
        ed[x=heap.top().second]=1;heap.pop();
        for (i=fir[x];i;i=nxt[i]) if ((lj[i][1])&&(h[lj[i][0]]>h[x]+lj[i][2]))
            heap.push(make_pair(h[lj[i][0]]=h[x]+lj[i][2],lj[i][0]));
        while ((!heap.empty())&&(ed[heap.top().second])) heap.pop();
    }
    for (i=1;i<=n;i++) for (j=fir[i];j;j=nxt[j]) lj[j][2]+=h[i]-h[lj[j][0]];
    memset(ed,0,sizeof(ed));
}
pa dijkstra()
{
    int i,j,x,tf=1e9;
    memset(dis,0x3f,sizeof(dis));memset(pre,0,sizeof(pre));dis[s]=0;heap.push(
        make_pair(0,s));
    while (!heap.empty())
    {
        ed[x=heap.top().second]=1;heap.pop();
        for (i=fir[x];i;i=nxt[i]) if ((lj[i][1])&&(dis[lj[i][0]]>dis[x]+lj[i
            ][2]))
            heap.push(make_pair(dis[lj[i][0]]=dis[pre[lj[i][0]]=x]+lj[i][2],lj[i
                ][0]))),fl[lj[i][0]]=i;
        while ((!heap.empty())&&(ed[heap.top().second])) heap.pop();
    }
    if (dis[t]==dis[t+1]) return make_pair(0,0);
    for (i=t;i!=s;i=pre[i]) tf=min(tf,lj[fl[i]][1]);
    for (i=t;i!=s;i=pre[i]) lj[fl[i]][1]-=tf,lj[fl[i]^1][1]+=tf;
    for (i=1;i<=n;i++) for (j=fir[i];j;j=nxt[j]) lj[j][2]+=dis[i]-dis[lj[j][0]];
    h[t]+=dis[t];memset(ed,0,sizeof(ed));
    return make_pair(tf,tf*h[t]);
}
signed main()
{
    while (!heap.empty()) heap.pop();
    read(n);read(m);read(s);read(t);bs=1;
    while (m--)
    {
        read(x);read(y);read(z);read(w);
```

```
        add(x,y,z,w);
    }
    spfa();
    while ((ans=dijkstra()).first) ans1+=ans.first,ans2+=ans.second;
    printf("%d␣%d",ans1,ans2);
}
```

4.15 假花树

```
vector<int> lj[N];
int lk[N],ed[N];
int n,m,cnt,i,t,x,y,ans,la;
bool dfs(int x)
{
    ed[x]=cnt;int v;
    random_shuffle(lj[x].begin(),lj[x].end());
    for (auto u:lj[x]) if (ed[v=lk[u]]!=cnt)
    {
        lk[v]=0,lk[u]=x,lk[x]=u;
        if (!v||dfs(v)) return 1;
        lk[v]=u,lk[u]=v,lk[x]=0;
    }
    return 0;
}
int main()
{
    srand(time(0));la=-1;
    read(n);read(m);
    while (m--) read(x),read(y),lj[x].push_back(y),lj[y].push_back(x);
    while (la!=ans)
    {
        memset(ed+1,0,n<<2);la=ans;
        for (i=1;i<=n;i++) if (!lk[i]) ans+=dfs(cnt=i);
    }
    printf("%d\n",ans);
    for (i=1;i<=n;i++) printf("%d␣",lk[i]);
}
```

4.16 Stoer-Wagner 全局最小割

$O(n^3)$。可优化到 $O(nm\log n)$。

```
namespace StoerWagner
{
    const int N=602;//点数
    typedef int T;//边权和
    T e[N][N],w[N];
    int ed[N],p[N],f[N];//f 仅输出方案用
    int getf(int u){return f[u]==u?u:f[u]=getf(f[u]);}
    template<typename TT> pair<T,vector<int>> mincut(int n,const vector<tuple<
        int,int,TT>> &edges)//[1,n]，返回某一半点集
    {
        vector<int> ans;ans.reserve(n);
        int i,j,m;
        T r;
```

```
        r=numeric_limits<T>::max();
        for (i=1;i<=n;i++) memset(e[i]+1,0,n*sizeof e[0][0]);
        for (auto [u,v,w]:edges) e[u][v]+=w,e[v][u]+=w;
        fill_n(ed+1,n,0);
        iota(f+1,f+n+1,1);
        for (m=n;m>1;m--)
        {
            fill_n(w+1,n,0);
            for (i=1;i<=n;i++) ed[i]&=2;
            for (i=1;i<=m;i++)
            {
                int x=0;
                for (j=1;j<=n;j++) if (!ed[j]) break;x=j;
                for (j++;j<=n;j++) if (!ed[j]*w[j]>w[x]) x=j;
                ed[p[i]=x]=1;
                for (j=1;j<=n;j++) w[j]+=!ed[j]*e[x][j];
            }
            int s=p[m-1],t=p[m];
            if (r>w[t])
            {
                r=w[t];ans.clear();
                for (i=1;i<=n;i++) if (getf(i)==getf(t)) ans.push_back(i);
            }
            for (i=1;i<=n;i++) e[i][s]=e[s][i]+=e[t][i];
            ed[t]=2;
            f[getf(s)]=getf(t);
        }
        return {r,ans};
    }
}
```

4.17 点双

$O(n+m)$，$O(n+m)$。

ans 存放每个点双包含的边。ct 为 1 表示是割点。没有自环。

```
struct Q
{
    int v,w;
};
vector<vector<int>> ans;
vector<int> cur;
vector<Q> e[N];
int dfn[N],low[N],ct[N],st[N];
bool ed[N],eed[N];
int id,tp;
void dfs(int u,bool rt)
{
    dfn[u]=low[u]=++id;
    int cnt=0;
    for (auto [v,w]:e[u]) if (!ed[w])
    {
        st[tp++]=w;ed[w]=1;
        if (dfn[v]) low[u]=min(low[u],dfn[v]);
        else
        {
```

```
            dfs(v,0);
            ++cnt;
            low[u]=min(low[u],low[v]);
            if (dfn[u]<=low[v])
            {
                ct[u]=cnt>rt;
                cur.clear();
                do cur.push_back(st[--tp]); while (st[tp]!=w);
                ans.push_back(cur);
            }
        }
    }
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m,i;
    cin>>n>>m;
    for (i=0;i<m;i++)
    {
        int u,v;
        cin>>u>>v;
        e[u].push_back({v,i});
        e[v].push_back({u,i});
    }
    for (i=0;i<n;i++) if (!dfn[i]) dfs(i,1);
    cout<<ans.size()<<'\n';
    for (auto &v:ans) cout<<v.size()<<'␣'<<v<<'\n';
}
```

ans 存放每个点双包含的点。可以自环。

```
const int N=5e5+5;
struct Q
{
    int v,w;
};
vector<vector<int>> ans;
vector<int> cur;
vector<int> e[N];
int dfn[N],low[N],st[N];
int id,tp;
void dfs(int u)
{
    dfn[u]=low[u]=++id;
    st[++tp]=u;
    for (int v:e[u]) if (dfn[v]) low[u]=min(low[u],dfn[v]); else
    {
        dfs(v);
        low[u]=min(low[u],low[v]);
        if (dfn[u]<=low[v])
        {
            vector cur={u};
            do
            {
                cur.push_back(st[tp]);
            } while (st[tp--]!=v);
            ans.push_back(cur);
```

```cpp
        }
    }
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(15);
    int n,m,i;
    cin>>n>>m;
    for (i=0;i<m;i++)
    {
        int u,v;
        cin>>u>>v;
        e[u].push_back(v);
        e[v].push_back(u);
    }
    for (i=0;i<n;i++) if (!dfn[i]) dfs(i);
    for (i=0;i<n;i++) if (count(all(e[i]),i)==e[i].size()) ans.push_back({i});
    cout<<ans.size()<<'\n';
    for (auto &v:ans) cout<<v.size()<<'␣'<<v<<'\n';
}
```

4.18 边双

$O(n+m)$，$O(n+m)$。

ans 存放每个边双包含的点。ct 为 1 表示是割边。

```cpp
struct Q
{
    int v,w;
};
vector<vector<int>> ans;
vector<int> cur;
vector<Q> e[N];
int dfn[N],low[N],ed[N];
bool ct[N];
int id;
void dfs(int u,int fw)
{
    dfn[u]=low[u]=++id;
    for (auto [v,w]:e[u]) if (w!=fw)
    {
        if (!dfn[v])
        {
            dfs(v,w);
            low[u]=min(low[u],low[v]);
            ct[w]=dfn[u]<low[v];
        } else low[u]=min(low[u],dfn[v]);
    }
}
void dfs(int u)
{
    cur.push_back(u);ed[u]=1;
    for (auto [v,w]:e[u]) if (!ct[w]&&!ed[v]) dfs(v);
}
int main()
```

```cpp
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m,i;
    cin>>n>>m;
    for (i=0;i<m;i++)
    {
        int u,v;
        cin>>u>>v;
        e[u].push_back({v,i});
        e[v].push_back({u,i});
    }
    for (i=0;i<n;i++) if (!dfn[i]) dfs(i,-1);
    for (i=0;i<n;i++) if (!ed[i])
    {
        cur.clear();
        dfs(i);
        ans.push_back(cur);
    }
    cout<<ans.size()<<'\n';
    for (auto &v:ans) cout<<v.size()<<' '<<v<<'\n';
}
```

4.19 双极分解

图点双连通时对任意 s, t 存在。

```cpp
vector<int> bipolar_orientation(const vector<pair<int, int>> &edges, int n,
    int s, int t)//[0,n)
{
    assert(s!=t);
    vector e(n, vector<int>());
    for (auto [u, v]:edges)
    {
        e[u].push_back(v);
        e[v].push_back(u);
    }
    int cur=1, i;
    vector<int> pre(n), low(n), p(n);
    pre[s]=1;
    vector<int> id;
    function<void(int)> dfs=[&](int x)
        {
            pre[x]=++cur;
            low[x]=x;
            for (int y:e[x])
            {
                if (pre[y]==0)
                {
                    id.push_back(y);
                    dfs(y);
                    p[y]=x;
                    if (pre[low[y]]<pre[low[x]]) low[x]=low[y];
                }
                else if (pre[y]!=0&&pre[y]<pre[low[x]]) low[x]=y;
            }
        };
    dfs(t);
```

```cpp
        vector<int> sign(n, -1);
        vector<int> l(n), r(n);
        r[s]=t;
        l[t]=s;
        for (int v:id)
        {
            if (sign[low[v]]==-1)
            {
                l[v]=l[p[v]];
                r[l[v]]=v;
                l[p[v]]=v;
                r[v]=p[v];
                sign[p[v]]=1;
            }
            else
            {
                r[v]=r[p[v]];
                l[r[v]]=v;
                r[p[v]]=v;
                l[v]=p[v];
                sign[p[v]]=-1;
            }
        }
        vector<int> a(n);
        int x;
        for (i=0, x=s; i<n; x=r[x], i++) a[i]=x;
        vector<int> ia(n, -1), rd(n), cd(n);
        for (i=0; i<n; i++) ia[a[i]]=i;
        if (count(all(ia), -1)) return { };
        for (auto [u, v]:edges)
        {
            if (ia[u]>ia[v]) swap(u, v);
            ++cd[u]; ++rd[v];
        }
        for (i=0; i<n; i++) if (i!=s&&i!=t&&(!cd[i]||!rd[i])) return { };
        return a;
    }
```

4.20 输出负环

```cpp
#include <bits/stdc++.h>
using namespace std;
const int N=34;
struct Q
{
    int v,w,c;
    Q(){}
    Q(int x,int y,int z):v(x),w(y),c(z){}
};
vector<Q> lj[N];
int dis[N],cnt[N],pt[N],S;
Q pre[N],st[N];
int n,m,ans,tp;
bool ed[N];
int main()
{
```

```cpp
freopen("arbitrage.in","r",stdin);
freopen("arbitrage.out","w",stdout);
ios::sync_with_stdio(0);cin.tie(0);
cin>>n>>m;
while (m--)
{
    int x,y,z,w;
    cin>>x>>y>>z>>w;
    lj[x].emplace_back(y,w,z);
    lj[y].emplace_back(x,0,-z);
}
for (int i=1;i<=n;i++) lj[0].emplace_back(i,1,0);
while (1)
{
    memset(dis,-0x3f,sizeof dis);dis[0]=0;
    for (int i=0;i<=n;i++) ed[i]=cnt[i]=0;S=-1;
    queue<int> q;q.push(0);
    while (!q.empty())
    {
        int u=q.front();q.pop();ed[u]=0;
        for (auto &[v,w,c]:lj[u]) if (w&&dis[v]<dis[u]+c)
        {
            dis[v]=dis[u]+c;pre[v]=Q(u,w,c);
            if (!ed[v])
            {
                if (++cnt[v]>n+1) {S=v;goto aa;}
                ed[v]=1;q.push(v);
            }
        }
    }
    aa:;
    if (S==-1) break;
    {
        static bool ed[N];
        memset(ed,0,sizeof ed);
        while (!ed[S]) ed[S]=1,S=pre[S].v;
    }
    st[tp=1]=pre[S];pt[1]=S;
    int x=pre[S].v;
    while (x!=S)
    {
        st[++tp]=pre[x];pt[tp]=x;
        x=pre[x].v;
        assert(tp<=n+5);
    }
    int fl=1e9;
    for (int j=1;j<=tp;j++) fl=min(fl,st[j].w);
    assert(fl);
    for (int j=1;j<=tp;j++)
    {
        ans+=fl*st[j].c;
        int nn=0;
        for (auto &[v,w,c]:lj[st[j].v]) if (v==pt[j]&&st[j].c==c&&st[j].w==w)
            {++nn;w-=fl;break;}
        for (auto &[v,w,c]:lj[pt[j]]) if (v==st[j].v&&st[j].c+c==0) {++nn;w+=
            fl;break;}assert(nn==2);
    }
```

```
    }
    cout<<ans<<endl;
}
```

4.21 DAG 删点最长路

$O((n+m)\log n)$，$O(n+m)$。

```
priority_queue<int> hp1,hp2,del1,del2;
int lj[M],nxt[M],fir[N],flj[M],fnxt[M],ffir[N],dl[N],rd[N],cd[N],dis1[N],dis2[N
    ];
int dtp;
char c[M*15+1];
int main()
{
    int n,m,i,j,x,y,tou,wei,zd=0,ans=M,cur,pos=0;
    scanf("%d%d",&n,&m);
    fread(c+1,1,m*15,stdin);
    for (i=1;i<=m;i++)
    {
        read(x);read(y);++cd[x];
        lj[i]=y;nxt[i]=fir[x];fir[x]=i;++rd[y];
        flj[i]=x;fnxt[i]=ffir[y];ffir[y]=i;
    }
    tou=1;wei=0;
    for (i=1;i<=n;i++) if (!cd[i]) dl[++wei]=i;
    while (tou<=wei) for (i=ffir[x=dl[tou++]];i;i=fnxt[i])
    {
        dis2[flj[i]]=max(dis2[flj[i]],dis2[x]+1);
        if (--cd[flj[i]]==0) dl[++wei]=flj[i];
    }
    tou=1;wei=0;
    for (i=1;i<=n;i++) if (!rd[i]) dl[++wei]=i;
    while (tou<=wei) for (i=fir[x=dl[tou++]];i;i=nxt[i])
    {
        dis1[lj[i]]=max(dis1[lj[i]],dis1[x]+1);
        if (--rd[lj[i]]==0) dl[++wei]=lj[i];
    }
    for (i=1;i<=n;i++) hp1.push(dis2[i]);hp1.push(0);hp2.push(0);
    for (j=1;j<=wei;j++)
    {
        x=dl[j];
        if (dis2[x]==hp1.top())
        {
            hp1.pop();
            while ((!del1.empty())&&(del1.top()==hp1.top())) {hp1.pop();del1.pop
                ();}
        } else del1.push(dis2[x]);
        for (i=ffir[x];i;i=fnxt[i]) del2.push(dis1[flj[i]]+dis2[x]+1);
        while ((!del2.empty())&&(del2.top()==hp2.top())) {hp2.pop();del2.pop();}
        cur=max(zd,max(hp1.top(),hp2.top()));
        if (cur<ans)
        {
            pos=dl[j];ans=cur;
        }
        zd=max(zd,dis1[x]);
        for (i=fir[x];i;i=nxt[i]) hp2.push(dis1[x]+dis2[lj[i]]+1);
```

```
        if (ans<=zd) break;
    }
    printf("%d␣%d",pos,ans);
}
```

4.22 树哈希

```cpp
vector<int> tree_hash(const vector<pair<int,int>> &edges,int root)//[0,n)
{
    int n=edges.size()+1;
    vector e(n,vector<int>());
    for (auto [u,v]:edges)
    {
        e[u].push_back(v);
        e[v].push_back(u);
    }
    map<vector<int>,int> mp;
    vector<int> h(n),ed(n);
    int id=0;
    function<void(int)> dfs=[&](int u)
        {
            ed[u]=1;
            vector<int> c;
            for (int v:e[u]) if (!ed[v])
            {
                dfs(v);
                c.push_back(h[v]);
            }
            sort(all(c));
            if (!mp.count(c)) mp[c]=id++;
            h[u]=mp[c];
        };
    dfs(root);
    return h;
}
```

4.23 （基环）树哈希

```cpp
#include <bits/stdc++.h>
using namespace std;
namespace tree_hash
{
    typedef unsigned int ui;
    typedef unsigned long long ll;
    const int N=1e6+2;
    const ui p1=2034452107,p2=2013074419,B=(1ll<<32)-1;
    mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    ui bas1[N],bas2[N],lst;
    ui uni1,uni2;
    vector<int> e[N];
    vector<ll> rt;
    ll g[N];
    int siz[N],h[N],f[N],num[N*2];
    int n,m;
```

```cpp
void init()
{
    uni1=rnd()%(p1/2)+p1/2;uni2=rnd()%(p2/2)+p2/2;
    lst=0;
}
void dfs1(int u)
{
    siz[u]=1;
    int mx=0;
    for (auto &v:e[u]) if (v!=f[u])
    {
        f[v]=u;dfs1(v);siz[u]+=siz[v];
        mx=max(mx,siz[v]);
    }
    mx=max(mx,n-siz[u]);
    if (mx*2<=n) rt.push_back(u);
}
void dfs2(int u)
{
    for (auto &v:e[u]) if (v!=f[u]) f[v]=u,dfs2(v),h[u]=max(h[u],h[v]);
    ++h[u];
    int n=0;
    static ll a[N];
    for (auto &v:e[u]) if (v!=f[u]) a[n++]=g[v];
    sort(a,a+n);
    ll r1=0,r2=0;
    a[n++]=1ll<<32|1;
    for (int i=0;i<n;i++) r1=(r1*bas1[h[u]]+(a[i]>>32))%p1,r2=(r2*bas2[h[u
        ]]+(a[i]&B))%p2;
    g[u]=r1<<32|r2;
}
void get_e(vector<pair<int,int>> &E)
{
    int i;
    n=E.size()+1;m=0;
    for (auto &[u,v]:E) num[m++]=u,num[m++]=v;
    sort(num,num+m);m=unique(num,num+m)-num;
    for (i=0;i<m;i++) e[num[i]].clear();
    for (auto &[u,v]:E) e[u].push_back(v),e[v].push_back(u);
    while (lst<n) bas1[++lst]=rnd()%(p1/2)+p1/2,bas2[lst]=rnd()%(p2/2)+p2/2;
}
ll rooted_tree_hash(int u)
{
    if (n==1) return 1ll<<32|1;
    for (int i=0;i<m;i++) f[num[i]]=0,h[num[i]]=0;
    dfs2(u);
    return g[u];
}
ll t_h(vector<pair<int,int>> &E)
{
    int i;
    get_e(E);
    for (i=0;i<m;i++) f[num[i]]=0;
    rt.clear();dfs1(1);
    ll r1=0,r2=0;
    for (auto &u:rt) u=rooted_tree_hash(u);
    sort(rt.begin(),rt.end());
```

```cpp
            for (auto &u:rt) r1=(r1*uni1+(u>>32))%p1,r2=(r2*uni2+(u&B))%p2;
            return r1<<32|r2;
        }
    }
}
using tree_hash::get_e;
using tree_hash::rooted_tree_hash;
using tree_hash::t_h;
typedef pair<int,int> pa;
typedef unsigned int ui;
typedef unsigned long long ull;
const ui mod1=2034452107,mod2=2013074419,B=(1ll<<32)-1;
ui b1,b2;
const int N=1e6+2;
vector<int> e[N];
int f[N];
vector<int> lp;
int getf(int u) {return f[u]==u?u:f[u]=getf(f[u]);}
void dfs1(int u)
{
    for (auto &v:e[u]) if (v!=f[u]) f[v]=u,dfs1(v);
}
bool ed[N];
void dfs2(int u,vector<pa> &E)
{
    for (auto &v:e[u]) if (!ed[v]) ed[v]=1,E.emplace_back(u,v),dfs2(v,E);
}
void min_order(ull *a,int n)
{
    int i,j,k;
    ull x,y;
    i=k=0;j=1;
    while ((i<n)&&(j<n)&&(k<n))
    {
        x=a[(i+k)%n];y=a[(j+k)%n];
        if (x==y) ++k; else
        {
            if (x>y) i+=k+1; else j+=k+1;
            if (i==j) ++j;
            k=0;
        }
    }
    if (j>i) j=i;
    //[j,n)+[0,j)
    rotate(a,a+j,a+n);
}
int cal()
{
    int n,m,p1,p2;
    cin>>m;
    vector<pair<ull,ull>> a(m);
    for (auto &V:a)
    {
        int i;
        cin>>n;
        for (i=1;i<=n;i++) e[i].clear();
        iota(f+1,f+n+1,1);
        for (i=1;i<=n;i++)
```

```
        {
            int u,v;
            cin>>u>>v;
            if (getf(u)==getf(v)) {p1=u;p2=v;continue;}
            e[u].push_back(v);
            e[v].push_back(u);
            f[f[u]]=f[v];
        }
        memset(f+1,0,n*sizeof f[0]);
        dfs1(p1);
        static int st[N];
        memset(ed+1,0,n*sizeof ed[0]);
        int tp=1;st[1]=p2;
        while (p2!=p1) st[++tp]=p2=f[p2];
        for (i=1;i<=tp;i++) ed[st[i]]=1;
        vector<pa> E;
        static ull ans[N];
        E.reserve(n);
        for (i=1;i<=tp;i++)
        {
            dfs2(st[i],E);
            get_e(E);
            ans[i]=rooted_tree_hash(st[i]);
            E.clear();
        }
        min_order(ans+1,tp);
        ull r1=0,r2=0,r,rr;
        for (int i=1;i<=tp;i++) r1=(r1*b1+(ans[i]>>32))%mod1,r2=(r2*b2+(ans[i]&B
            ))%mod2;
        r=r1<<32|r2;
        reverse(ans+1,ans+tp+1);
        min_order(ans+1,tp);r1=r2=0;
        for (int i=1;i<=tp;i++) r1=(r1*b1+(ans[i]>>32))%mod1,r2=(r2*b2+(ans[i]&B
            ))%mod2;
        rr=r1<<32|r2;
        if (r>rr) swap(r,rr);
        V=make_pair(r,rr);
    }
    sort(a.begin(),a.end());
    return unique(a.begin(),a.end())-a.begin();
}
int main()
{
    b1=tree_hash::rnd()%(mod1/2)+mod1/2;
    b2=tree_hash::rnd()%(mod2/2)+mod2/2;
    tree_hash::init();
    ios::sync_with_stdio(0);cin.tie(0);
    int n,T;
    cin>>T;
    while (T--) cout<<cal()<<'\n';
}
```

4.24 无向图最小环

$O(n^3)$，$O(n^2)$。

```
int f[N][N],jl[N][N];
```

```
int n,m,c,ans=inf,i,j,k,x,y,z;
int main()
{
    read(n);read(m);
    memset(f,0x3f,sizeof(f));
    memset(jl,0x3f,sizeof(jl));
    while (m--)
    {
        read(x);read(y);read(z);
        jl[x][y]=jl[y][x]=f[x][y]=f[y][x]=min(f[y][x],z);
    }
    for (k=1;k<=n;k++)
    {
        for (i=1;i<k;i++) if (jl[k][i]!=jl[0][0]) for (j=1;j<i;j++)
            if (jl[k][j]!=jl[0][0]) ans=min(ans,jl[k][i]+jl[k][j]+f[i][j]);
        for (i=1;i<=n;i++) if (i!=k) for (j=1;j<=n;j++)
            if ((j!=i)&&(j!=k)) f[i][j]=min(f[i][j],f[i][k]+f[k][j]);
    }
    if (ans==inf) puts("No solution."); else printf("%d",ans);
}
```

4.25　切比雪夫距离最小生成树

$O(n \log n)$，$O(n)$。

```
const int N=3e5+2,M=N<<2;
struct P
{
    int u,v,w;
    P(int a=0,int b=0,int c=0):u(a),v(b),w(c){}
    bool operator<(const P &o) const {return w<o.w;}
};
struct Q
{
    int x,y,id;
    Q(int a=0,int b=0,int c=0):x(a),y(b),id(c){}
    bool operator<(const Q &o) const {return x!=o.x?x>o.x:y>o.y;}
};
ll ans;
P lb[M];
Q a[N],b[N];
int f[N],c[N];
int n,m,i,x,y;
struct bit
{
    int a[N],pos[N],n;
    void init(int &nn)
    {
        memset(a+1,0x7f,(n=nn)*sizeof a[0]);
        memset(pos+1,0,n*sizeof pos[0]);
    }
    void mdf(int x,const int y,const int z)
    {
        if (a[x]>y) a[x]=y,pos[x]=z;
        while (x-=x&-x) if (a[x]>y) a[x]=y,pos[x]=z;
    }
    int sum(int x)
```

```
    {
        int r=a[x],rr=pos[x];
        while ((x+=x&-x)<=n) if (a[x]<r) r=a[x],rr=pos[x];
        return rr;
    }
};
bit s;
void cal()
{
    int i,x,y;
    s.init(n);
    memcpy(b+1,a+1,sizeof(Q)*n);
    sort(a+1,a+n+1);
    for (i=1;i<=n;i++) c[i]=a[i].y-a[i].x;
    sort(c+1,c+n+1);
    for (i=1;i<=n;i++)
    {
        if (x=s.sum(y=lower_bound(c+1,c+n+1,a[i].y-a[i].x)-c))
            lb[++m]=P(a[x].id,a[i].id,a[x].x+a[x].y-a[i].x-a[i].y);//谨防 int 爆
        s.mdf(y,a[i].y+a[i].x,i);
    }
    memcpy(a+1,b+1,sizeof(Q)*n);
}
int getf(int x) {return f[x]==x?x:f[x]=getf(f[x]);}
int main()
{
    read(n);
    for (i=1;i<=n;i++) {read(a[f[i]=a[i].id=i].x);read(a[i].y);
        swap(a[i].x,a[i].y);a[i]=Q(a[i].x+a[i].y,a[i].x-a[i].y,i);}
    cal();for (i=1;i<=n;i++) swap(a[i].x,a[i].y);
    cal();for (i=1;i<=n;i++) a[i].y=-a[i].y;
    cal();for (i=1;i<=n;i++) swap(a[i].x,a[i].y);
    cal();sort(lb+1,lb+m+1);
    for (i=1;i<=m;i++) if ((x=getf(lb[i].u))!=(y=getf(lb[i].v))) f[x]=y,ans+=lb[
        i].w;
    printf("%lld\n",ans>>1);
}
```

4.26 点分治

$O(n \log n)$，$O(n)$。

```
int siz[N],dep[N];
    int n,ksiz,md,rt,mn;
    bool ed[N];
    void find(int u)
    {
        ed[u]=1;siz[u]=1;
        int mx=0;
        for (int v:e[u]) if (!ed[v])
        {
            find(v);
            siz[u]+=siz[v];
            mx=max(mx,siz[v]);
        }
        mx=max(mx,ksiz-siz[u]);
        if (mn>mx) mn=mx,rt=u;
```

```cpp
        ed[u]=0;
    }
    void cal(int u)
    {
        md=max(md,dep[u]);
        ed[u]=1;++cnt[dep[u]];
        for (int v:e[u]) if (!ed[v])
        {
            dep[v]=dep[u]+1;
            cal(v);
        }
        ed[u]=0;
    }
    void solve(int u)
    {
        mn=1e9;
        find(u);
        ed[rt]=1;
        vector<int> c;
        for (int v:e[rt]) if (!ed[v])
        {
            c.push_back(v);
            if (siz[v]>=siz[rt]) siz[v]=siz[u]-siz[rt];
        }
        sort(all(c),[&](const int &a,const int &b){return siz[a]<siz[b];});
        NTT::Q a(vector<ui>{1});
        NT::Q b(vector<ui>{1});
        for (int v:c)
        {
            md=0;dep[v]=1;
            cal(v);++md;
            vector<ui> d(cnt,cnt+md);
            NTT::Q e(d);
            NT::Q f(d);
            auto g=e&a;
            auto h=f&b;
            for (int i=0;i<g.a.size();i++) r1[i]=(r1[i]+g.a[i])%NTT::p;
            for (int i=0;i<h.a.size();i++) r2[i]=(r2[i]+h.a[i])%NT::p;
            a+=e;b+=f;
            fill_n(cnt,md,0);
        }
        for (int v:c)
        {
            ksiz=siz[v];
            solve(v);
        }
    }
```

4.27 点分树

$O(n\log^2 n)$，$O(n\log n)$。

```cpp
namespace DFS
{
    template<typename typC> struct bit0
    {
        vector<typC> a;
```

```cpp
    int n;
    bit0() {}
    bit0(int nn):n(nn),a(nn+1) {}
    template<typename T> bit0(int nn,T *b):n(nn),a(nn+1)
    {
        for (int i=1; i<=n; i++) a[i]=b[i-1];
        for (int i=1; i<=n; i++) if (i+(i&-i)<=n) a[i+(i&-i)]+=a[i];
    }
    void add(int x,typC y)
    {
        ++x;
        //cerr<<"add "<<x<<" by "<<y<<endl;
        assert(1<=x&&x<=n);
        a[x]+=y;
        while ((x+=x&-x)<=n) a[x]+=y;
    }
    typC sum(int x)
    {
        ++x;
        //cerr<<"sum "<<x;
        if (x>n) x=n;
        assert(0<=x&&x<=n);
        typC r=a[x];
        while (x^=x&-x) r+=a[x];
        //cerr<<"= "<<r<<endl;
        return r;
    }
    typC sum(int x,int y)
    {
        return sum(y)-sum(x-1);
    }
};
typedef long long ll;
const int N=1e5+5,M=18;
ll a[N];
bit0<ll> s[N],rc[N];
int st[M][N*2],lg[N*2];
int dep[N],dfn[N],siz[N],f[N];
vector<int> e[N],c[N];
bool ed[N];
int n,ksiz,rt,mn,id;
int lca(int u,int v)
{
    u=dfn[u]; v=dfn[v];
    if (u>v) swap(u,v);
    int z=lg[v-u+1];
    return dep[st[z][u]]<dep[st[z][v-(1<<z)+1]]?st[z][u]:st[z][v-(1<<z)+1];
}
int dis(int u,int v)
{
    return dep[u]+dep[v]-dep[lca(u,v)]*2;
}
void findroot(int u)
{
    ed[u]=siz[u]=1;
    int mx=0;
    for (int v:e[u]) if (!ed[v])
```

```
        {
            findroot(v);
            siz[u]+=siz[v];
            mx=max(mx,siz[v]);
        }
        mx=max(mx,ksiz-siz[u]);
        ed[u]=0;
        if (mn>mx) mn=mx,rt=u;
    }
    int fun(int u)
    {
        mn=1e9;
        findroot(u);
        u=rt;
        ed[u]=1;
        for (int v:e[u]) if (!ed[v]&&siz[v]>siz[u]) siz[v]=ksiz-siz[u];
        for (int v:e[u]) if (!ed[v])
        {
            ksiz=siz[v];
            c[u].push_back(fun(v));
            f[c[u].back()]=u;
        }
        return u;
    }
    void pre_dfs(int u)
    {
        st[0][dfn[u]=++id]=u;
        ed[u]=1;
        for (int v:e[u]) if (!ed[v])
        {
            dep[v]=dep[u]+1;
            pre_dfs(v);
            st[0][++id]=u;
        }
        ed[u]=0;
    }
    void clear(int _n)
    {
        n=_n; id=0;
        int i;
        for (int i=1; i<=n; i++)
        {
            e[i].clear();
            a[i]=f[i]=ed[i]=0;
        }
    }
    void new_dfs(int u)
    {
        siz[u]=1;
        for (int v:c[u]) new_dfs(v),siz[u]+=siz[v];
        static int nd[N];
        vector<int> q={u};
        int mx=0,ql=0;
        nd[u]=0;
        while (ql<q.size())
        {
            int x=q[ql++];
```

```
            for (int v:c[x]) q.push_back(v),mx=max(mx,nd[v]=nd[u]+1);
        }
        static ll tmp[N];
        static int ds[N];
        mx=0;
        for (int v:q) mx=max(mx,ds[v]=dis(u,v));
        ++mx;
        fill_n(tmp,mx,0);
        for (int v:q) tmp[ds[v]]+=a[v];
        s[u]=bit0<ll>(mx,tmp);
        if (u!=rt)
        {
            mx=0;
            for (int v:q) mx=max(mx,ds[v]=dis(f[u],v));
            ++mx;
            fill_n(tmp,mx,0);
            for (int v:q) tmp[ds[v]]+=a[v];
            rc[u]=bit0<ll>(mx,tmp);
        }
    }
    void init()
    {
        pre_dfs(1);
        int i,j;
        for (i=2; i<=id; i++) lg[i]=lg[i>>1]+1;
        for (j=0; j<lg[id]; j++)
        {
            int R=id-(2<<j)+1;
            for (i=1; i<=R; i++) st[j+1][i]=dep[st[j][i]]<dep[st[j][i+(1<<j)]]?st
                [j][i]:st[j][i+(1<<j)];
        }
        ksiz=n;
        int tmp=fun(1);
        rt=tmp;
        new_dfs(rt);
    }
    void add(int u,ll val)
    {
        static int st[N],ds[N];
        int tp=1,i; st[1]=u;
        while (u=f[u]) st[++tp]=u;
        u=st[1];
        for (i=1; i<=tp; i++) ds[i]=dis(u,st[i]);
        for (i=1; i<=tp; i++)
        {
            s[st[i]].add(ds[i],val);
            if (i<tp) rc[st[i]].add(ds[i+1],val);
        }
    }
    ll ask(int u,int k)
    {
        ll res=0;
        static int st[N],ds[N];
        int tp=1,i; st[1]=u;
        while (u=f[u]) st[++tp]=u;
        u=st[1];
        for (i=1; i<=tp; i++) ds[i]=dis(u,st[i]);
```

```
        for (i=1; i<=tp; i++)
        {
            if (ds[i]<=k) res+=s[st[i]].sum(k-ds[i]);
            if (i<tp&&ds[i+1]<=k) res-=rc[st[i]].sum(k-ds[i+1]);
        }
        return res;
    }
}
//核心: 点分树上 lca 出现在原树路径上
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    cout<<setiosflags(ios::fixed)<<setprecision(15);
    int n,m,i;
    cin>>n>>m;
    DFS::clear(n);
    for (i=1; i<=n; i++) cin>>DFS::a[i];
    for (i=1; i<n; i++)
    {
        int u,v;
        cin>>u>>v;
        DFS::e[u].push_back(v);
        DFS::e[v].push_back(u);
    }
    DFS::init();
    ll ans=0;
    while (m--)
    {
        int op,x,y;
        cin>>op>>x>>y; x^=ans; y^=ans;
        if (op==1) DFS::add(x,y-DFS::a[x]),DFS::a[x]=y;
        else cout<<(ans=DFS::ask(x,y))<<'\n';
    }
}
```

4.28   prufer 与树的互相转化

$O(n)$，$O(n)$。

```
vector<int> edges_to_prufer(const vector<pair<int,int>> &eg)//[1,n], 定根为 n
{
    int n=eg.size()+1,i,j,k;
    int fir[n+1],nxt[n*2+1],e[n*2+1];
    int rd[n+1],cnt=0;
    memset(rd,0,sizeof rd);memset(nxt,0,sizeof nxt);memset(fir,0,sizeof fir);
    for (auto [u,v]:eg)
    {
        e[++cnt]=v;nxt[cnt]=fir[u];fir[u]=cnt;++rd[v];
        e[++cnt]=u;nxt[cnt]=fir[v];fir[v]=cnt;++rd[u];
    }
    for (i=1;i<=n;i++) if (rd[i]==1) break;
    int u=i;
    vector<int> r;r.reserve(n-2);
    for (j=1;j<n-1;j++)
    {
        for (k=fir[u],u=rd[u]=0;k;k=nxt[k]) if (rd[e[k]])
        {
```

```
            r.push_back(e[k]);
            if ((--rd[e[k]]==1)&&(e[k]<i)) u=e[k];
        }
        if (!u) { while (rd[i]!=1) ++i;u=i;}
    }
    return r;
}
vector<pair<int,int>> prufer_to_edges(const vector<int> &p)//[1,n]，定根为 n
{
    int n=p.size(),i,j,k;
    int m=n+3;
    int cs[m];memset(cs,0,sizeof cs);
    for (i=0;i<n;i++) ++cs[p[i]];
    i=0;
    while (cs[++i]);
    int u=i,v;
    vector<pair<int,int>> r;
    r.reserve(n-2);
    for (j=0;j<n;j++)
    {
        cs[u]=1e9;
        r.push_back({u,v=p[j]});
        if ((--cs[v]==0)&&(v<i)) u=v;
        if (v!=u) {while (cs[i]) ++i;u=i;}
    }
    r.push_back({u,n+2});
    return r;
}
```

4.29   LCT

$O(n\log n)$，$O(n)$。

makeroot 会变根，split 会把 y 变根，findroot 会把根变根，link 会把 x,y 变根（y 是新的），cut 会把 x,y 变根（x 是新的），注意 swap 子节点可能要 pushup。

```
template<int N,typename Q> struct LCT
{
    int f[N],c[N][2],siz[N],st[N];
    Q s[N],v[N];
    #ifdef Rev
    Q rs[N];
    #endif
    //heap g[N]; //虚子树
    bool lz[N];
    void init(int n)
    {
        ++n;
        for (int i=0;i<n;i++)
        {
            f[i]=c[i][0]=c[i][1]=lz[i]=0;
            s[i]=v[i]=Q();
            #ifdef Rev
            rs[i]=Q();
            #endif
            siz[i]=!!i;
        }
    }
```

```
    }
    void modify(int x,const Q &o)
    {
        makeroot(x);
        v[x]=o;
        pushup(x);
    }
    bool nroot(int x) const
    {
        return c[f[x]][0]==x||c[f[x]][1]==x;
    }
    void pushup(int x)
    {
        int lc=c[x][0],rc=c[x][1];
        s[x]=v[x];siz[x]=1;
        #ifdef Rev
        rs[x]=v[x];
        #endif
        if (lc)
        {
            s[x]=s[lc]+s[x];
            siz[x]+=siz[lc];
            #ifdef Rev
            rs[x]=rs[x]+rs[lc];
            #endif
        }
        if (rc)
        {
            s[x]=s[x]+s[rc];
            siz[x]+=siz[rc];
            #ifdef Rev
            rs[x]=rs[rc]+rs[x];
            #endif
        }
    }
    void swp(int x)
    {
        swap(c[x][0],c[x][1]);
        #ifdef Rev
        swap(s[x],rs[x]);
        #endif
        lz[x]^=1;
    }
    void pushdown(int x)
    {
        int lc=c[x][0],rc=c[x][1];
        if (lz[x])
        {
            if (lc) swp(lc);
            if (rc) swp(rc);
            lz[x]=0;
        }
    }
    void zigzag(int x)
    {
        int y=f[x],z=f[y],typ=(c[y][0]==x);
        if (nroot(y)) c[z][c[z][1]==y]=x;
```

```cpp
        f[x]=z;f[y]=x;
        if (c[x][typ]) f[c[x][typ]]=y;
        c[y][typ^1]=c[x][typ];c[x][typ]=y;
        pushup(y);
    }
    void splay(int x)
    {
        int y,tp=0;
        st[tp=1]=y=x;
        while (nroot(y)) st[++tp]=y=f[y];
        while (tp) pushdown(st[tp--]);
        for (;nroot(x);zigzag(x)) if (!nroot(f[x])) continue; else zigzag((c[f[x
            ]][0]==x)^(c[f[f[x]]][0]==f[x]) ? x:f[x]);
        pushup(x);
    }
    void access(int x)
    {
        for (int y=0;x;x=f[y=x])
        {
            splay(x);
            //g[x].ins(s[c[x][1]]);g[x].del(s[y]);虚子树变化
            c[x][1]=y;pushup(x);
        }
    }
    int findroot(int x)
    {
        access(x);splay(x);pushdown(x);
        while (c[x][0]) pushdown(x=c[x][0]);
        splay(x);
        return x;
    }
    void split(int x,int y)//x 为树新根，y 为 splay 新根
    {
        makeroot(x);
        access(y);
        splay(y);
    }
    void makeroot(int x)
    {
        access(x);splay(x);
        swp(x);
    }
    void link(int x,int y)//y 为新根
    {
        makeroot(x);
        if (x!=findroot(y))//可能已经连通
        {
            makeroot(y);f[x]=y;//虚子树变化
        }
    }
    void cut(int x,int y)
    {
        makeroot(x);
        if (x==findroot(y))//可能本不连通
        {
            pushdown(x);
            if (c[x][1]==y&&!c[y][0]&&!c[y][1])//可能连通但无边
```

```
            {
                c[x][1]=f[y]=0;//可能需要修改
                pushup(x);
            }
        }
    }
};
```

4.30 LCT（重构，代码为动态割边割点）

```cpp
#include "bits/stdc++.h"
using namespace std;
template<int N,typename info,typename tag> struct LCT
{
    int f[N],c[N][2];
    info s[N],v[N];
#ifdef Rev
    info rs[N];
#endif
    tag tg[N];
    bool rev[N],lz[N];
    void init(int n,info *a)
    {
        for (int i=0; i<=n; i++)
        {
            rev[i]=lz[i]=0;
            f[i]=c[i][0]=c[i][1]=0;
            s[i]=v[i]=a[i];
#ifdef Rev
            rs[i]=a[i];
#endif
        }
    }
    bool nroot(int x) const
    {
        return c[f[x]][0]==x||c[f[x]][1]==x;
    }
    void pushup(int x)
    {
        int lc=c[x][0],rc=c[x][1];
        s[x]=v[x];
#ifdef Rev
        rs[x]=v[x];
#endif
        if (lc)
        {
            s[x]=s[lc]+s[x];
#ifdef Rev
            rs[x]=rs[x]+rs[lc];
#endif
        }
        if (rc)
        {
            s[x]=s[x]+s[rc];
#ifdef Rev
            rs[x]=rs[rc]+rs[x];
```

```cpp
#endif
    }
    }
    void swp(int x)
    {
        swap(c[x][0],c[x][1]);
#ifdef Rev
        swap(s[x],rs[x]);
#endif
        rev[x]^=1;
    }
    void pushdown(int x)
    {
        if (rev[x])
        {
            for (int y:c[x]) if (y) swp(y);
            rev[x]=0;
        }
        if (lz[x])
        {
            for (int y:c[x]) if (y)
            {
                if (lz[y]) tg[y]+=tg[x]; else tg[y]=tg[x],lz[y]=1;
                s[y]+=tg[x];
            }
            lz[x]=0;
        }
    }
    void zigzag(int x)
    {
        int y=f[x],z=f[y],typ=(c[y][0]==x);
        if (nroot(y)) c[z][c[z][1]==y]=x;
        f[x]=z; f[y]=x;
        if (c[x][typ]) f[c[x][typ]]=y;
        c[y][typ^1]=c[x][typ]; c[x][typ]=y;
        pushup(y);
    }
    void splay(int x)
    {
        static int st[N];
        int y,tp;
        st[tp=1]=y=x;
        while (nroot(y)) st[++tp]=y=f[y];
        while (tp) pushdown(st[tp--]);
        for (; nroot(x); zigzag(x)) if (nroot(y=f[x])) zigzag((c[y][0]==x)^(c[f[
            y]][0]==y)?x:f[x]);
        pushup(x);
    }
    int access(int x)
    {
        int y=0;
        for (; x; x=f[y=x]) splay(x),c[x][1]=y,pushup(x);
        return y;
    }
    int findroot(int x)//splay 根为树根，splay 维护树根到 x 的链
    {
        access(x); splay(x); pushdown(x);
```

```cpp
            while (c[x][0]) pushdown(x=c[x][0]);
            splay(x); return x;
        }
        void split(int x,int y)//x 为树新根，y 为 splay 新根
        { makeroot(x); access(y); splay(y); }
        void makeroot(int x)//x 为树、splay 新根
        { access(x); splay(x); swp(x); }
        void modify(int x,const info &o)
        { makeroot(x); v[x]=o; pushup(x); }
        void modify(int x,int y,const tag &o)
        {
            split(x,y); s[y]+=o;
            if (lz[y]) tg[y]+=o; else tg[y]=o,lz[y]=1;
        }
        info ask(int x,int y) { split(x,y); return s[y]; }
        bool connected(int x,int y)//注意会改变形态
        { makeroot(x); return findroot(y)==x; }
        void link(int x,int y)//y 为新根
        { if (!connected(x,y)) makeroot(f[x]=y); }
        void cut(int x,int y)
        {
            if (connected(x,y))//可能本不连通
            {
                pushdown(x);
                if (c[x][1]==y&&!c[y][0]&&!c[y][1])//可能连通但无边
                {
                    c[x][1]=f[y]=0;
                    pushup(x);
                }
            }
        }
        int lca(int x,int y) { access(x); return access(y); }
        vector<int> res;
        void dfs(int x)
        {
            if (!x) return;
            pushdown(x);
            dfs(c[x][0]); res.push_back(x); dfs(c[x][1]);
        }
        vector<int> get_path(int x,int y)
        {
            res.clear(); split(x,y); dfs(y);
            if (res[0]!=x) return {};
            return res;
        }
};
const int N=2e5+5,M=4e5+5;
struct Q
{
    void operator+=(const Q &o) const {}
};
void operator+=(int &x,const Q &o) { x=0; }
LCT<N,int,Q> s;
LCT<M,int,Q> t;
int a[N],b[M];
int main()
{
```

```
    ios::sync_with_stdio(0); cin.tie(0);
    int n,m,i,r=0;
    cin>>n>>m;
    fill_n(a+n+1,n,1);
    fill_n(b+1,n,1);
    s.init(n*2,a);
    t.init(n+m,b);
    int bs=n,ds=n;
    while (m--)
    {
        int op,u,v;
        cin>>op>>u>>v;
        u^=r; v^=r;
        // dbg(op,u,v);
        if (u<1||u>n||v<1||v>n) return 0;
        if (op==1)
        {
            if (s.connected(u,v))
            {
                s.modify(u,v,{});
                auto c=t.get_path(u,v);
                for (i=1; i<c.size(); i++) t.cut(c[i-1],c[i]);
                ++ds;
                for (int x:c) t.link(ds,x);
            }
            else
            {
                s.link(++bs,u);
                s.link(bs,v);
                t.link(++ds,u);
                t.link(ds,v);
            }
        }
        else
        {
            if (!s.connected(u,v))
            {
                cout<<"-1\n";
                continue;
            }
            r=op==2?s.ask(u,v):t.ask(u,v);
            cout<<r<<'\n';
        }
    }
}
```

4.31   带子树的 LCT

$O(n \log n)$，$O(n)$。

```
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
template<int N> struct LCT
{
    ll s[N],v[N],sg[N];
    int f[N],c[N][2],siz[N],st[N];
```

```cpp
//heap g[N]; //虚子树
bool lz[N];
void init(int n)
{
    memset(f,0,n+1<<2);
    memset(c,0,n+1<<3);
    memset(s,0,n+1<<3);
    memset(v,0,n+1<<3);
    memset(lz,0,n+1);
}
bool nroot(int x)
{
    return c[f[x]][0]==x||c[f[x]][1]==x;
}
void pushup(int x)
{
    s[x]=s[c[x][0]]+s[c[x][1]]+v[x]+sg[x];
    siz[x]=siz[c[x][0]]+siz[c[x][1]]+1;
}
void pushdown(int x)
{
    if (lz[x])
    {
        swap(c[c[x][0]][0],c[c[x][0]][1]);
        swap(c[c[x][1]][0],c[c[x][1]][1]);
        lz[c[x][0]]^=1;
        lz[c[x][1]]^=1;
        lz[x]=0;
    }
}
void zigzag(int x)
{
    int y=f[x],z=f[y],typ=(c[y][0]==x);
    if (nroot(y)) c[z][c[z][1]==y]=x;
    f[x]=z;f[y]=x;
    if (c[x][typ]) f[c[x][typ]]=y;
    c[y][typ^1]=c[x][typ];c[x][typ]=y;
    pushup(y);
}
void splay(int x)
{
    int y,tp=0;
    st[tp=1]=y=x;
    while (nroot(y)) st[++tp]=y=f[y];
    while (tp) pushdown(st[tp--]);
    for (;nroot(x);zigzag(x)) if (!nroot(f[x])) continue; else zigzag((c[f[x
        ]][0]==x)^(c[f[f[x]]][0]==f[x]) ? x:f[x]);
    pushup(x);
}
void access(int x)
{
    for (int y=0;x;x=f[y=x])
    {
        splay(x);sg[x]-=s[y];s[x]-=s[y];
        sg[x]+=s[c[x][1]];s[x]+=s[c[x][1]];
        //g[x].ins(s[c[x][1]]);g[x].del(s[y]);虚子树变化
        c[x][1]=y;pushup(x);
```

```
        }
    }
    int findroot(int x)
    {
        access(x);splay(x);pushdown(x);
        while (c[x][0]) pushdown(x=c[x][0]);
        splay(x);
        return x;
    }
    void split(int x,int y)
    {
        makeroot(x);
        access(y);
        splay(y);
    }
    void makeroot(int x)
    {
        access(x);splay(x);lz[x]^=1;swap(c[x][0],c[x][1]); pushup(x);
    }
    void link(int x,int y)
    {
        makeroot(x);
        if (x!=findroot(y))//可能已经连通
        {
            makeroot(y);f[x]=y;//虚子树变化
            sg[y]+=s[x];s[y]+=s[x];
        }
    }
    void cut(int x,int y)
    {
        makeroot(x);
        if (x==findroot(y))//可能本不连通
        {
            pushdown(x);
            if (c[x][1]==y&&!c[y][0]&&!c[y][1])//可能连通但无边
            {
                c[x][1]=f[y]=0;//可能需要修改
                pushup(x);
            }
        }
    }
};
const int N=2e5+2;
LCT<N> s;
int n,q,i,x,y,z,w;
void read(int &x)
{
    int c=getchar();
    while (c<48||c>57) c=getchar();
    x=c^48;c=getchar();
    while (c>=48&&c<=57) x=x*10+(c^48),c=getchar();
}
int main()
{
    read(n);read(q);s.init(n);
    for (i=1;i<=n;i++) read(x),s.s[i]=s.v[i]=x;
    for (i=1;i<n;i++)
```

```
    {
        read(x);read(y);++x;++y;
        s.link(x,y);
    }
    while (q--)
    {
        read(x);read(y);read(z);++y;
        if (x==0)
        {
            read(x);read(w);
            ++z;++x;++w;
            s.cut(y,z);s.link(x,w);
            continue;
        }
        if (x==1)
        {
            s.split(y,y);
            s.s[y]=(s.v[y]+=z);
        }
        else
        {
            ++z;
            s.split(y,z);
            printf("%lld\n",s.s[y]);
        }
    }
}
```

4.32 轻重链剖分

```
namespace HLD
{
    const int N=5e5+2;
    vector<int> e[N];
    int dfn[N],dep[N],f[N],siz[N],hc[N],top[N];
    int id;
    void dfs1(int u)
    {
        siz[u]=1;
        for (int v:e[u]) if (v!=f[u])
        {
            dep[v]=dep[f[v]=u]+1;
            dfs1(v);
            siz[u]+=siz[v];
            if (siz[v]>siz[hc[u]]) hc[u]=v;
        }
    }
    void dfs2(int u)
    {
        dfn[u]=++id;
        if (hc[u])
        {
            top[hc[u]]=top[u];
            dfs2(hc[u]);
            for (int v:e[u]) if (v!=hc[u]&&v!=f[u]) dfs2(top[v]=v);
```

```cpp
        }
    }
    int lca(int u,int v)
    {
        while (top[u]!=top[v])
        {
            if (dep[top[u]]<dep[top[v]]) swap(u,v);
            u=f[top[u]];
        }
        if (dep[u]>dep[v]) swap(u,v);
        return u;
    }
    int dis(int u,int v)
    {
        return dep[u]+dep[v]-(dep[lca(u,v)]<<1);
    }
    void init(int n)
    {
        for (int i=1;i<=n;i++)
        {
            e[i].clear();
            f[i]=hc[i]=0;
        }
        id=0;
    }
    void fun(int root)
    {
        dep[root]=1;dfs1(root);dfs2(top[root]=root);
    }
    vector<pair<int,int>> get_path(int u,int v)//u->v，注意可能出现 [r>l]（表示反
        过来走）
    {
        //cerr<<"path from "<<u<<" to "<<v<<": ";
        vector<pair<int,int>> v1,v2;
        while (top[u]!=top[v])
        {
            if (dep[top[u]]>dep[top[v]]) v1.push_back({dfn[u],dfn[top[u]]}),u=f[
                top[u]];
            else v2.push_back({dfn[top[v]],dfn[v]}),v=f[top[v]];
        }
        v1.reserve(v1.size()+v2.size()+1);
        v1.push_back({dfn[u],dfn[v]});
        reverse(v2.begin(),v2.end());
        for (auto v:v2) v1.push_back(v);
        //for (auto [x,y]:v1) cerr<<"["<<x<<','<<y<<"] ";cerr<<endl;
        return v1;
    }
}
using HLD::e,HLD::lca,HLD::dis,HLD::dfn,HLD::dep,HLD::f,HLD::siz,HLD::get_path;
using HLD::fun;//5e5
```

4.33 换根树剖

$O(n+q\log n)$，$O(n)$。

```cpp
void dfs1(int x)
{
```

```
    int i;
    siz[x]=1;
    for (i=fir[x];i;i=nxt[i]) if (lj[i]!=f[x])
    {
        dep[lj[i]]=dep[f[lj[i]]=x]+1;
        dfs1(lj[i]);
        siz[x]+=siz[lj[i]];
        if (siz[hc[x]]<siz[lj[i]]) hc[x]=lj[i];
    }
}
void dfs2(int x)
{
    nfd[dfn[x]=++bs]=x;
    if (hc[x])
    {
        int i;
        top[hc[x]]=top[x];
        dfs2(hc[x]);
        for (i=fir[x];i;i=nxt[i]) if ((lj[i]!=f[x])&&(lj[i]!=hc[x])) dfs2(top[lj
            [i]]=lj[i]);
    }
}
void mdf(int xx,int yy)
{
    while (top[xx]!=top[yy])
    {
        if (dep[top[xx]]<dep[top[yy]]) swap(xx,yy);
        z=dfn[top[xx]];y=dfn[xx];xdsmdf(1);
        xx=f[top[xx]];
    }
    if (dep[xx]>dep[yy]) swap(xx,yy);
    z=dfn[xx];y=dfn[yy];
    xdsmdf(1);
}
int find(int x,int y)
{
    while ((top[x]!=top[y])&&(f[top[x]]!=y)) x=f[top[x]];
    if (top[x]==top[y]) return hc[y];
    return top[x];
}
int main()
{
    read(n);read(m);
    for (i=2;i<=n;i++)
    {
        read(x);read(y);
        add();
    }bs=0;
    for (i=1;i<=n;i++) read(v[i]);
    dfs1(dep[1]=1);dfs2(top[1]=1);
    read(rt);r[l[1]=1]=n;build(1);
    while (m--)
    {
        read(x);read(y);
        if (x==1) {rt=y;continue;}
        if (x==2)
        {
```

```
            read(x);read(dt);
            mdf(x,y);continue;
        }
        x=y;dt=inf;
        if (x==rt)
        {
            z=1;y=n;sum(1);
        }
        else if ((dfn[x]<dfn[rt])&&(dfn[x]+siz[x]>dfn[rt]))
        {
            c=find(rt,x);
            z=1;y=dfn[c]-1;if (z<=y) sum(1);
            z=dfn[c]+siz[c];y=n;if (z<=y) sum(1);
        }
        else
        {
            z=dfn[x];y=z+siz[x]-1;sum(1);
        }
        printf("%d\n",dt);
    }
}
```

4.34 树上启发式合并，DSU on tree

```
void dfs1(int x)
{
    siz[x]=zdep[x]=1;
    int i;
    for (i=fir[x];i;i=nxt[i]) if (lj[i]!=f[x])
    {
        dep[lj[i]]=dep[f[lj[i]]=x]+1;
        dfs1(lj[i]);
        siz[x]+=siz[lj[i]];
        if (siz[hc[x]]<siz[lj[i]]) hc[x]=lj[i];
        zdep[x]=max(zdep[x],zdep[lj[i]]+1);
    }
}
void cal(int x)
{
    int i;
    dl[tou=wei=1]=x;
    while (tou<=wei)
    {
        ++dp[dep[x=dl[tou++]]];
        if ((dp[dep[x]]>dp[zd])||(dp[dep[x]]==dp[zd])&&(dep[x]<zd)) zd=dep[x];
        for (i=fir[x];i;i=nxt[i]) if (lj[i]!=f[x]) dl[++wei]=lj[i];
    }
}
void dfs2(int x)
{
    if (!hc[x])
    {
        if (++dp[dep[x]]>dp[zd]) zd=dep[x];
        return;
    }
    int i;
```

```
    for (i=fir[x];i;i=nxt[i]) if ((lj[i]!=f[x])&&(lj[i]!=hc[x]))
    {
        dfs2(lj[i]);
        memset(dp+dep[lj[i]],0,zdep[lj[i]]<<2);
    }
    dfs2(hc[x]);
    dp[dep[x]]=1;
    if (dp[zd]<=1) zd=dep[x];
    for (i=fir[x];i;i=nxt[i]) if ((lj[i]!=f[x])&&(lj[i]!=hc[x])) cal(lj[i]);
    ans[x]=zd-dep[x];
}
```

4.35 长链剖分（k 级祖先）

$O(n+q)$，$O(n)$。

```
void dfs1(int x)
{
    int i;
    for (i=1;i<=er[dep[x]-1];i++) f[x][i]=f[f[x][i-1]][i-1];md[x]=dep[x];
    for (i=fir[x];i;i=nxt[i]) {dep[lj[i]]=dep[x]+1;dfs1(lj[i]);if (md[lj[i]]>md[
        dc[x]]) dc[x]=lj[i];}
    if (dc[x]) md[x]=md[dc[x]];
}
void dfs2(int x)
{
    int i;
    if (dc[x])
    {
        top[dc[x]]=top[x];
        dfs2(dc[x]);
        for (i=fir[x];i;i=nxt[i]) if (lj[i]!=dc[x]) dfs2(top[lj[i]]=lj[i]);
    }
    if (x==top[x])
    {
        c=md[x]-dep[x];y=x;up[x].push_back(x);down[x].push_back(x);
        for (i=1;(i<=c)&&(y=f[y][0]);i++) up[x].push_back(y);y=x;
        for (i=1;i<=c;i++) down[x].push_back(y=dc[y]);
    }
}
int main()
{
    int n,q,ans=0,x,y,c,i;
    ll ta=0;
    read(n);read(q);read(s);
    for (i=1;i<=n;i++) {read(f[i][0]);if (f[i][0]==0) rt=i; else add(f[i][0],i)
        ;}
    for (i=2;i<=n;i++) er[i]=er[i>>1]+1;dep[rt]=1;
    dfs1(rt);dfs2(top[rt]=rt);
    for (i=1;i<=q;i++)
    {
        x=(get(s)^ans)%n+1;y=(get(s)^ans)%dep[x];
        if (y==0) {ans=x;ta^=(ll)i*ans;continue;}
        c=dep[x]-y;x=top[f[x][er[y]]];
        if (dep[x]>c) ans=up[x][dep[x]-c]; else ans=down[x][c-dep[x]];
        ta^=(ll)i*ans;
    }
```

```
        printf("%lld",ta);
}
```

4.36　长链剖分（dp 合并）

$O(n)$，$O(n)$。

```
void dfs1(int x)
{
    top[x]=1;
    for (int i=fir[x];i;i=nxt[i]) if (!top[lj[i]])
    {
        dfs1(lj[i]);
        if (len[lj[i]]>len[hc[x]]) hc[x]=lj[i];
    }
    len[x]=len[hc[x]]+1;top[hc[x]]=0;
}
void dfs2(int x)
{
    *f[x]=1;gs[x]=1;
    if (!hc[x]) return;
    ed[x]=1;f[hc[x]]=f[x]+1;
    for (int i=fir[x];i;i=nxt[i]) if (!ed[lj[i]]) dfs2(lj[i]);
    ans[x]=ans[hc[x]]+1;gs[x]=gs[hc[x]];
    if (gs[x]==1) ans[x]=0;
    for (int i=fir[x];i;i=nxt[i]) if ((!ed[lj[i]])&&(lj[i]!=hc[x]))
    {
        int v=lj[i],*p;
        for (int j=0;j<len[v];j++)
        {
            *(p=f[x]+j+1)+=*(f[v]+j);
            if (j+1==ans[x]) {gs[x]=*p;continue;}
            if ((*p>gs[x])||(*p==gs[x])&&(j+1<ans[x])) {gs[x]=*p;ans[x]=j+1;}
        }
    }
    gs[x]=*(f[x]+ans[x]);
    ed[x]=0;
}
```

4.37　动态 dp（全局平衡二叉树）

$O((n+q)\log n)$，$O(n)$。

```
#include <stdio.h>
#include <string.h>
#include <algorithm>
#include <fstream>
using namespace std;
const int N=1e6+2,M=6e7+2,INF=-1e9;
struct matrix
{
    int a[2][2];
};
matrix s[N],js;
matrix operator *(matrix x,matrix y)
{
```

```cpp
        js.a[0][0]=max(x.a[0][0]+y.a[0][0],x.a[0][1]+y.a[1][0]);
        js.a[0][1]=max(x.a[0][0]+y.a[0][1],x.a[0][1]+y.a[1][1]);
        js.a[1][0]=max(x.a[1][0]+y.a[0][0],x.a[1][1]+y.a[1][0]);
        js.a[1][1]=max(x.a[1][0]+y.a[0][1],x.a[1][1]+y.a[1][1]);
        return js;
}
int st[N],c[N][2],hc[N],lj[N<<1],nxt[N<<1],fir[N],siz[N],v[N],g[N][2],fa[N],f[N
    ],val[N];
int n,m,i,j,x,y,z,dtp,stp,tp,fh,bs,rt,aaa,la;
char dr[M+5],sc[M];
void pushup(int x)
{
    s[x].a[0][0]=s[x].a[0][1]=g[x][0];
    s[x].a[1][0]=g[x][1];s[x].a[1][1]=INF;
    if (c[x][0]) s[x]=s[c[x][0]]*s[x];
    if (c[x][1]) s[x]=s[x]*s[c[x][1]];
}
void read(int &x)
{
    ++dtp;fh=0;
    while ((dr[dtp]<48)||(dr[dtp]>57))
    {
        if (dr[dtp++]=='-')
        {
            fh=1;
            break;
        }
    }
    x=dr[dtp++]^48;
    while ((dr[dtp]>=48)&&(dr[dtp]<=57)) x=x*10+(dr[dtp++]^48);
    if (fh) x=-x;
}
void add(int x,int y)
{
    lj[++bs]=y;
    nxt[bs]=fir[x];
    fir[x]=bs;
    lj[++bs]=x;
    nxt[bs]=fir[y];
    fir[y]=bs;
}
bool nroot(int x)
{
    return ((c[f[x]][0]==x)||(c[f[x]][1]==x));
}
void dfs1(int x)
{
    siz[x]=1;
    int i;
    for (i=fir[x];i;i=nxt[i]) if (lj[i]!=fa[x])
    {
        fa[lj[i]]=x;
        dfs1(lj[i]);
        siz[x]+=siz[lj[i]];
        if (siz[hc[x]]<siz[lj[i]]) hc[x]=lj[i];
    }
}
```

```c
int build(int l,int r)
{
    if (l>r) return 0;
    int i,tot=0,upn=0;
    for (i=l;i<=r;i++) tot+=val[i];tot>>=1;
    for (i=l;i<=r;i++)
    {
        upn+=val[i];
        if (upn>=tot)
        {
            f[c[st[i]][0]=build(l,i-1)]=st[i];
            f[c[st[i]][1]=build(i+1,r)]=st[i];
            pushup(st[i]);
            ++aaa;
            return st[i];
        }
    }
}
int dfs2(int x)
{
    int i,j;
    for (i=x;i;i=hc[i]) for (j=fir[i];j;j=nxt[j]) if ((lj[j]!=fa[i])&&(lj[j]!=hc
        [i]))
    {
        f[y=dfs2(lj[j])]=i;
        g[i][0]+=max(s[y].a[0][0],s[y].a[1][0]);
        g[i][1]+=s[y].a[0][0];
    }
    tp=0;
    for (i=x;i;i=hc[i]) st[++tp]=i;
    for (i=1;i<tp;i++) val[i]=siz[st[i]]-siz[st[i+1]];
    val[tp]=siz[st[tp]];
    return build(1,tp);
}
void change(int x,int y)
{
    g[x][1]+=y-v[x];v[x]=y;
    while (f[x])
    {
        if (nroot(x)) pushup(x);
        else
        {
            g[f[x]][0]-=max(s[x].a[0][0],s[x].a[1][0]);
            g[f[x]][1]-=s[x].a[0][0];
            pushup(x);
            g[f[x]][0]+=max(s[x].a[0][0],s[x].a[1][0]);
            g[f[x]][1]+=s[x].a[0][0];
        }
        x=f[x];
    }
    pushup(x);
}
int main()
{
    scanf("%d%d",&n,&m);
    fread(dr+1,1,min(M,n*20+m*20),stdin);
    for (i=1;i<=n;i++)
```

```
    {
        read(g[i][1]);
        v[i]=g[i][1];
    }
    for (i=1;i<n;i++)
    {
        read(x);read(y);
        add(x,y);
    }
    dfs1(1);
    rt=dfs2(1);tp=0;
    while (m--)
    {
        read(x);read(y);
        change(x^la,y);
        x=la=max(s[rt].a[0][0],s[rt].a[1][0]);
        while (x)
        {
            st[++tp]=x%10;
            x/=10;
        }
        while (tp) sc[++stp]=st[tp--]|48;
        sc[++stp]=10;
    }
    fwrite(sc+1,1,stp,stdout);
}
```

4.38   全局平衡二叉树（修改版）

$O((n+q)\log n)$，$O(n)$。

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef pair<int,int> pa;
const int N=1e6+2,M=1e6+2;
ll ans;
pa w[N];
int c[N][2],f[N],fa[N],v[N],s[N],lz[N],lj[M],nxt[M],siz[N],hc[N],fir[N],st[N];
int a[N],top[N];
int n,i,x,y,z,bs,tp,rt;
void read(int &x)
{
    int c=getchar();
    while (c<48||c>57) c=getchar();
    x=c^48;c=getchar();
    while (c>=48&&c<=57) x=x*10+(c^48),c=getchar();
}
void add()
{
    lj[++bs]=y;nxt[bs]=fir[x];fir[x]=bs;
    lj[++bs]=x;nxt[bs]=fir[y];fir[y]=bs;
}
void pushup(int &x)
{
    s[x]=min(v[x],min(s[c[x][0]],s[c[x][1]]));
}
```

```
void pushdown(int &x)
{
    if (lz[x]<0)
    {
        int cc=c[x][0];
        if (cc)
        {
            lz[cc]+=lz[x];s[cc]+=lz[x];v[cc]+=lz[x];
        }
        cc=c[x][1];
        if (cc)
        {
            v[cc]+=lz[x];lz[cc]+=lz[x];s[cc]+=lz[x];
        }lz[x]=0;
        return;
    }
}
bool nroot(int &x) {return c[f[x]][0]==x||c[f[x]][1]==x;}
bool cmp(pa &o,pa &p) {return o>p;}
void dfs1(int x)
{
    siz[x]=1;
    for (int i=fir[x];i;i=nxt[i]) if (lj[i]!=fa[x])
    {
        fa[lj[i]]=x;dfs1(lj[i]);siz[x]+=siz[lj[i]];
        if (siz[hc[x]]<siz[lj[i]]) hc[x]=lj[i];
    }
}
int build(int l,int r)
{
    if (l>r) return 0;
    if (l==r)
    {
        l=st[l];s[l]=v[l]=siz[l]>>1;
        return l;
    }
    int x=lower_bound(a+l,a+r+1,a[l]+a[r]>>1)-a,y=st[x];
    c[y][0]=build(l,x-1);
    c[y][1]=build(x+1,r);
    v[y]=siz[y]>>1;
    if (c[y][0]) f[c[y][0]]=y;
    if (c[y][1]) f[c[y][1]]=y;
    pushup(y);
    return y;
}
void dfs2(int x)
{
    if (!hc[x]) return;
    int i;
    top[hc[x]]=top[x];
    if (top[x]==x)
    {
        st[tp=1]=x;
        for (i=hc[x];i;i=hc[i]) st[++tp]=i;
        for (i=1;i<=tp;i++) a[i]=siz[st[i]]-siz[hc[st[i]]]+a[i-1];
        f[build(1,tp)]=fa[x];
    }
```

```
        dfs2(hc[x]);
        for (i=fir[x];i;i=nxt[i]) if (lj[i]!=fa[x]&&lj[i]!=hc[x]) dfs2(top[lj[i]]=lj
            [i]);
}
void mdf(int x)
{
    int y=x;
    st[tp=1]=x;
    while (y=f[y]) st[++tp]=y;y=x;
    while (tp) pushdown(st[tp--]);
    while (x)
    {
        --v[x];--lz[c[x][0]];--v[c[x][0]];--s[c[x][0]];
        while (c[f[x]][0]==x) x=f[x];x=f[x];
    }
    pushup(y);
    while (y=f[y]) pushup(y);
}
int ask(int x)
{
    int y=x;
    st[tp=1]=x;
    while (y=f[y]) st[++tp]=y;
    while (tp) pushdown(st[tp--]);
    int r=v[x];
    while (x)
    {
        r=min(r,min(v[x],s[c[x][0]]));
        while (c[f[x]][0]==x) x=f[x];x=f[x];
    }
    return r;
}
signed main()
{
    read(n);s[0]=1e9;
    for (i=1;i<=n;i++) read(w[w[i].second=i].first);
    for (i=1;i<n;i++) read(x),read(y),add();
    sort(w+1,w+n+1,cmp);dfs1(1);dfs2(top[1]=1);rt=1;while (f[rt]) rt=f[rt];
    for (i=1;i<=n&&v[rt];i++) if (ask(w[i].second)) mdf(w[i].second),ans+=w[i].
        first;
    printf("%lld",ans);
}
```

4.39   单调队列优化树上背包

```
#include <stdio.h>
#include <string.h>
#include <algorithm>
using namespace std;
const int N=502,M=4002,inf=-1e9;
int lj[N<<1],nxt[N<<1],fir[N],siz[N],v[N],p[N],l[N],f[N][M],num[M],dl[M];
int n,m,i,j,x,y,c,bs,t,ksiz,rt,zx,ans,tou,wei;
bool ed[N];
void add()
{
    lj[++bs]=y;
```

```cpp
        nxt[bs]=fir[x];
        fir[x]=bs;
        lj[++bs]=x;
        nxt[bs]=fir[y];
        fir[y]=bs;
}
void read(int &x)
{
        c=getchar();
        while ((c<48)||(c>57)) c=getchar();
        x=c^48;c=getchar();
        while ((c>=48)&&(c<=57))
        {
                x=x*10+(c^48);
                c=getchar();
        }
}
void dfs2(int x)
{
        ed[x]=siz[x]=1;
        int i,zd=0;
        for (i=fir[x];i;i=nxt[i]) if (!ed[lj[i]])
        {
                dfs2(lj[i]);
                siz[x]+=siz[lj[i]];
                zd=max(zd,siz[lj[i]]);
        }
        zd=max(zd,ksiz-siz[x]);
        if (zd<zx)
        {
                zx=zd;
                rt=x;
        }
        ed[x]=0;
        for (i=1;i<=m;i++) f[x][i]=inf;
}
void dfs3(int x)
{
        if (p[x]>m) return;
        ed[x]=1;
        f[x][0]=max(f[x][0],0);
        int i;
        if (!l[x])
        {
                for (i=fir[x];i;i=nxt[i]) if (!ed[lj[i]])
                {
                        for (j=0;j<=m;j++) f[lj[i]][j]=f[x][j];
                        dfs3(lj[i]);
                        for (j=m-p[x];~j;j--) f[x][j]=max(f[x][j],f[lj[i]][j]);
                }
                for (i=m;i>=p[x];i--) f[x][i]=f[x][i-p[x]]+v[x];
                for (i=0;i<p[x];i++) f[x][i]=inf;
                ed[x]=0;
                return;
        }
        for (i=0;i<p[x];i++)
        {
```

```
        y=(m-i)/p[x];
        num[dl[tou=wei=1]=0]=f[x][i];
        for (j=1;j<=y;j++)
        {
            while ((tou<wei)&&(j-dl[tou]>l[x])) ++tou;
            f[x][i+j*p[x]]=max(num[j]=f[x][i+j*p[x]],num[dl[tou]]+(j-dl[tou])*v[x
                ]);
            while ((tou<=wei)&&(num[dl[wei]]+(j-dl[wei])*v[x]<=num[j])) --wei;
            dl[++wei]=j;
        }
    }
    for (i=fir[x];i;i=nxt[i]) if (!ed[lj[i]])
    {
        for (j=0;j<=m;j++) f[lj[i]][j]=f[x][j];
        dfs3(lj[i]);
        for (j=m-p[x];~j;j--) f[x][j]=max(f[x][j],f[lj[i]][j]);
    }
    for (i=m;i>=p[x];i--) f[x][i]=f[x][i-p[x]]+v[x];
    for (i=0;i<p[x];i++) f[x][i]=inf;
    ed[x]=0;
}
void dfs1(int x)
{
    int i,j=ksiz;
    rt=x;zx=n;
    dfs2(x);
    dfs3(x=rt);
    for (i=p[x];i<=m;i++) ans=max(ans,f[x][i]);
    ed[x]=1;
    for (i=fir[x];i;i=nxt[i]) if (!ed[lj[i]])
    {
        if (j>siz[lj[i]]) ksiz=siz[lj[i]]; else ksiz=j-siz[x];
        dfs1(lj[i]);
    }
}
int main()
{
    read(t);
    while (t--)
    {
        ans=0;
        read(n);read(m);
        for (i=1;i<=n;i++) read(v[i]);
        for (i=1;i<=n;i++) read(p[i]);
        for (i=1;i<=n;i++) read(l[i]);
        for (i=1;i<=n;i++) --l[i];
        memset(f,0,sizeof(f));
        ksiz=n;
        for (i=1;i<n;i++)
        {
            read(x);read(y);add();
        }
        dfs1(1);
        printf("%d\n",ans);
        memset(fir,0,sizeof(fir));bs=0;
        memset(ed,0,sizeof(ed));
    }
```

```
}
```

4.40 树上背包

```cpp
void dfs(int x)
{
    int i;
    for (i=fir[x];i;i=nxt[i])
    {
        for (j=1;j<=m;j++) f[lj[i]][j]=f[x][j-1]+v[lj[i]];
        dfs(lj[i]);
        for (j=0;j<=m;j++) f[x][j]=max(f[x][j],f[lj[i]][j]);
    }
}
```

4.41 虚树

$O(n + \sum k \log n)$，$O(n)$。

```cpp
    vector<pair<int, int>> get_tree(vector<int> a)
    {
        vector<pair<int, int>> edges;
        sort(all(a), [&](int u, int v) { return dfn[u]<dfn[v]; });
        vector<int> st(a.size()+2);
        int tp=0;
        auto ins=[&](int u)
            {
                if (tp==0)
                {
                    st[tp=1]=u;
                    return;
                }
                int v=lca(st[tp], u);
                while (tp>1&&dep[v]<dep[st[tp-1]])
                {
                    edges.emplace_back(st[tp-1], st[tp]);
                    --tp;
                }
                if (dep[v]<dep[st[tp]]) edges.emplace_back(v, st[tp--]);
                if (!tp||st[tp]!=v) st[++tp]=v;
                st[++tp]=u;
            };
        if (a[0]!=1) st[tp=1]=1;//先行添加根节点
        for (int u:a) ins(u);
        if (tp) while (--tp) edges.emplace_back(st[tp], st[tp+1]);//回溯
        return edges;
    }
```

4.42 圆方树

$O(n + m)$，$O(n + m)$。

```cpp
#include <bits/stdc++.h>
using namespace std;
```

```cpp
#if !defined(ONLINE_JUDGE)&&defined(LOCAL)
#include "my_header\debug.h"
#else
#define dbg(...); 1;
#endif
typedef unsigned int ui;
typedef long long ll;
#define all(x) (x).begin(),(x).end()
const int N=3e4+2,M=3e4+2;//M 包括方点
struct P
{
    int v,w,id;
    P(int a,int b,int c):v(a),w(b),id(c){}
};
struct Q
{
    int v,w;
    Q(int a,int b):v(a),w(b){}
};
vector<P> e[N];
vector<Q> fe[M];
int dfn[M],low[N],st[N],len[M],top[M],siz[M],hc[M],dep[M],f[M],rb[N];
bool ed[M];//ed,dfn,loop,sum,fe,hc,tp,id,cnt,dep[1] 需初始化（注意倍率），ed 大
    小为边数
int tp,id,cnt,n;
void dfs1(int u)
{
    dfn[u]=low[u]=++id;
    st[++tp]=u;
    for (auto [v,w,id]:e[u]) if (!ed[id])
    {
        if (dfn[v]) low[u]=min(low[u],dfn[v]),rb[v]=w; else
        {
            ed[id]=1;
            dfs1(v);
            if (dfn[u]>low[v]) low[u]=min(low[u],low[v]),rb[v]=w; else
            {
                int ntp=tp;
                while (st[ntp]!=v) --ntp;
                if (ntp==tp)//圆圆边
                {
                    --tp;
                    fe[u].emplace_back(v,w);
                    f[v]=u;
                    continue;
                }
                ++cnt;f[cnt]=u;
                for (int i=ntp;i<=tp;i++) f[st[i]]=cnt;
                len[st[ntp]]=w;
                for (int i=ntp+1;i<=tp;i++) len[st[i]]=len[st[i-1]]+rb[st[i]];
                len[cnt]=len[st[tp]]+rb[u];
                fe[u].emplace_back(cnt,0);
                for (int i=ntp;i<=tp;i++) fe[cnt].emplace_back(st[i],min(len[st[i
                    ]],len[cnt]-len[st[i]]));
                tp=ntp-1;
            }
        }
    }
```

```cpp
    }
}
void dfs2(int u)
{
    siz[u]=1;
    for (auto [v,w]:fe[u])
    {
        dep[v]=dep[u]+w;
        dfs2(v);
        siz[u]+=siz[v];
        if (siz[v]>siz[hc[u]]) hc[u]=v;
    }
}
void dfs3(int u)
{
    dfn[u]=++id;
    if (hc[u])
    {
        top[hc[u]]=top[u];
        dfs3(hc[u]);
        for (auto [v,w]:fe[u]) if (v!=hc[u]) dfs3(top[v]=v);
    }
}
int lca(int u,int v)
{
    while (top[u]!=top[v]) if (dfn[top[u]]>dfn[top[v]]) u=f[top[u]]; else v=f[
        top[v]];//注意不能用 dep
    return dfn[u]<dfn[v]?u:v;
}
int find(int u,int v)//u 是根
{
    if (dfn[hc[u]]+siz[hc[u]]>dfn[v]) return hc[u];
    while (f[top[v]]!=u) v=f[top[v]];
    return top[v];
}
int dis(int u,int v)
{
    int o=lca(u,v),r=dep[u]+dep[v];
    if (o<=n) return r-(dep[o]<<1);
    u=find(o,u);v=find(o,v);
    if (len[u]>len[v]) swap(u,v);
    return r+min(len[v]-len[u],len[o]-(len[v]-len[u]))-dep[u]-dep[v];
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int m,q,i;
    cin>>n>>m>>q;cnt=n;
    for (i=1;i<=m;i++)
    {
        int u,v,w;
        cin>>u>>v>>w;
        e[u].emplace_back(v,w,i);
        e[v].emplace_back(u,w,i);
    }
    mt19937 rnd(time(0));
    for (i=1;i<=n;i++) shuffle(all(e[i]),rnd);
```

```
    dfs1(1);id=0;
    dfs2(1);
    dfs3(top[1]=1);
    while (q--)
    {
        int u,v;
        cin>>u>>v;
        cout<<dis(u,v)<<'\n';
    }
}
```

4.43　广义圆方树

$O(n+m)$，$O(n+m)$。

```
void dfs(int u)
{
    dfn[u]=low[u]=++id;
    st[++tp]=u;
    for (int v:e[u]) if (dfn[v]) low[u]=min(low[u],dfn[v]); else
    {
        dfs(v);
        low[u]=min(low[u],low[v]);
        if (dfn[u]<=low[v])
        {
            vector cur={u};
            do
            {
                cur.push_back(st[tp]);
            } while (st[tp--]!=v);
            ans.push_back(cur);
        }
    }
}
```

4.44　支配树（DAG 版）

$O(m\log n)$，$O(n\log n)$。

```
int lca(int x,int y)
{
    int i;
    if (dep[x]<dep[y]) swap(x,y);
    for (i=lm[x];dep[x]!=dep[y];i--) if (dep[f[x][i]]>=dep[y]) x=f[x][i];
    if (x==y) return x;
    for (i=lm[x];f[x][0]!=f[y][0];i--) if (f[x][i]!=f[y][i])
    {
        x=f[x][i];y=f[y][i];
    }
    return f[x][0];
}
void dfs(int x)
{
    s[x]=1;
    int i;
    for (i=sfir[x];i;i=snxt[i])
```

```
    {
        dfs(slj[i]);
        s[x]+=s[slj[i]];
    }
}
int main()
{
    dep[0]=-1;
    read(n);
    for (i=1;i<=n;i++)
    {
        read(x);
        while (x)
        {
            add(x,i);
            read(x);
        }
    }
    dl[tou=wei=1]=++n;
    for (i=1;i<n;i++) if (!rd[i]) add(n,i);
    while (tou<=wei)
    {
        for (i=fir[x=dl[tou++]];i;i=nxt[i]) if (--rd[lj[i]]==0) dl[++wei]=lj[i];
        if (i=ffir[x])
        {
            y=flj[i];
            while (i=fnxt[i]) y=lca(y,flj[i]);
            f[x][0]=y;
        } else y=0;
        sadd(y,x);
        f[x][0]=y;
        for (i=1;i<=16;i++) if (0==(f[x][i]=f[f[x][i-1]][i-1]))
        {
            lm[x]=i;
            break;
        }
        dep[x]=dep[y]+1;
    }
    dfs(n);
    for (i=1;i<n;i++) printf("%d\n",s[i]-1);
}
```

4.45   支配树（一般图）

```
#include <bits/stdc++.h>
using namespace std;
const int N=2e5+2;
vector<int> lj[N],llj[N],fl[N],tl[N],buc[N],c[N];
int f[N],mn[N],siz[N],sdom[N],idom[N],dfn[N],nfd[N],pv[N];
int n,m,cnt,i,j,x,y,na;
bool reach[N];
void dfs1(int x)
{
    nfd[dfn[x]=++cnt]=x;
    for (auto v:lj[x]) if (!dfn[v]) dfs1(v),c[x].push_back(v);
}
```

```cpp
int getf(int x)
{
    if (f[x]==x) return x;
    int u=getf(f[x]);
    mn[x]=dfn[sdom[mn[x]]]<dfn[sdom[mn[f[x]]]]?mn[x]:mn[f[x]];
    return f[x]=u;
}
void dfs0(int u)
{
    reach[u]=1;
    for (auto &v:lj[u]) if (!reach[v]) dfs0(v);
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int S;
    cin>>n>>m>>S;++S;
    while (m--) cin>>x>>y,++x,++y,lj[x].push_back(y);
    for (i=1;i<=n;i++) mn[i]=f[i]=i;
    dfs0(S);
    for (i=1;i<=n;i++) if (reach[i]) for (auto &v:lj[i]) if (reach[v]) llj[i].
        push_back(v),fl[v].push_back(i);
    for (i=1;i<=n;i++) lj[i]=llj[i];
    dfs1(S);dfn[0]=1e9;
    for (i=cnt;i;i--)
    {
        x=nfd[i];na=0;
        for (auto v:fl[x])
        {
            sdom[x]=dfn[sdom[x]]<dfn[v]?sdom[x]:v;
            if (dfn[v]>dfn[x])
            {
                getf(v);
                na=dfn[sdom[na]]<dfn[sdom[mn[v]]]?na:mn[v];
            }
        }
        sdom[x]=dfn[sdom[x]]<dfn[sdom[na]]?sdom[x]:sdom[na];
        buc[sdom[x]].push_back(x);
        for (auto v:buc[x]) getf(v),pv[v]=mn[v];
        for (auto v:c[x]) f[v]=x,mn[v]=dfn[sdom[mn[v]]]<dfn[sdom[mn[x]]]?mn[v]:
            mn[x];
    }
    for (i=1;i<=n;i++) idom[nfd[i]]=(sdom[pv[nfd[i]]]==sdom[nfd[i]])?sdom[nfd[i
        ]]:idom[pv[nfd[i]]];
    for (i=1;i<=n;i++) cout<<(i==S?S:idom[i])-1<<" \n"[i==n];
}
```

4.46 最小树形图（朱刘算法，无方案）

$O(nm)$，$O(n+m)$。

```cpp
int main()
{
    read(n);read(m);read(rt);
    for (i=1;i<=m;i++)
    {
        read(lj[i][1]);read(lj[i][2]);read(lj[i][0]);
```

```cpp
    }
    while (1)
    {
        memset(infl,0x3f,sizeof(infl));
        memset(ed,0,sizeof(ed));
        memset(fa,0,sizeof(fa));
        for (i=1;i<=m;i++) if ((lj[i][1]!=lj[i][2])&&(lj[i][2]!=rt)&&(infl[lj[i
            ][2]]>lj[i][0]))
        {
            infl[lj[i][2]]=lj[i][0];
            pre[lj[i][2]]=lj[i][1];
        }
        for (i=1;i<=n;i++) if (i!=rt)
        {
            if (infl[i]==infl[0])
            {
                puts("-1");return 0;
            }
            ans+=infl[i];
            for (j=i;(ed[j]!=i)&&(fa[j]==0)&&(j!=rt);j=pre[j]) ed[j]=i;
            if (ed[j]==i)
            {
                ++cnt;
                while (fa[j]==0)
                {
                    fa[j]=cnt;
                    j=pre[j];
                }
            }
        }
        if (!cnt)
        {
            printf("%d",ans);return 0;
        }
        for (i=1;i<=n;i++) if (!fa[i]) fa[i]=++cnt;
        for (i=1;i<=m;i++)
        {
            lj[i][0]-=infl[lj[i][2]];
            lj[i][1]=fa[lj[i][1]];
            lj[i][2]=fa[lj[i][2]];
        }
        rt=fa[rt];
        n=cnt;cnt=0;
    }
}
```

4.47　最小乘积生成树

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
const int N=202,M=10002;
template<typename typC> void read(typC &x)
{
    int c=getchar(),fh=1;
    while ((c<48)||(c>57))
```

```
    {
        if (c=='-') {c=getchar();fh=-1;break;}
        c=getchar();
    }
    x=c^48;c=getchar();
    while ((c>=48)&&(c<=57))
    {
        x=x*10+(c^48);
        c=getchar();
    }
    x*=fh;
}
struct P
{
    int x,y;
    P(int a=0,int b=0):x(a),y(b){}
    bool operator<(const P &o) const {return (ll)x*y<(ll)o.x*o.y||(ll)x*y==(ll)o
        .x*o.y&&x<o.x;}
};
struct Q
{
    int u,v,x,y,val;
    bool operator<(const Q &o) const {return val<o.val;}
};
P ans=P(1e9,1e9),l,r;
Q a[M];
int f[N];
int n,m,i;
int getf(int x)
{
    if (f[x]==x) return x;
    return f[x]=getf(f[x]);
}
P sol1()
{
    P r=P(0,0);
    for (i=1;i<=n;i++) f[i]=i;
    sort(a+1,a+m+1);
    for (i=1;i<=m;i++) if (getf(a[i].u)!=getf(a[i].v))
    {
        f[f[a[i].u]]=f[a[i].v];
        r.x+=a[i].x,r.y+=a[i].y;
    }
    return r;
}
void sol2(P l,P r)
{
    for (i=1;i<=m;i++) a[i].val=(r.x-l.x)*a[i].y+(l.y-r.y)*a[i].x;
    P np=sol1();
    ans=min(ans,np);
    if ((ll)(r.x-l.x)*(np.y-l.y)-(ll)(r.y-l.y)*(np.x-l.x)>=0) return;
    sol2(l,np);sol2(np,r);
}
int main()
{
    read(n);read(m);
    for (i=1;i<=m;i++) read(a[i].u),read(a[i].v),read(a[i].x),read(a[i].y),++a[i
```

```
    ].u,++a[i].v;
  for (i=1;i<=m;i++) a[i].val=a[i].x;l=sol1();
  for (i=1;i<=m;i++) a[i].val=a[i].y;r=sol1();
  ans=min(ans,min(l,r));sol2(l,r);
  printf("%d␣%d",ans.x,ans.y);
}
```

4.48　最小斯坦纳树

$O(3^k n + 2^k m \log m)$。

```
const int N=102,M=1002,K=1024;
typedef long long ll;
typedef pair<ll,int> pa;
priority_queue<pa,vector<pa>,greater<pa> > heap;
pa cr;
ll f[K][N],inf;
int lj[M],len[M],nxt[M],fir[N];
int n,m,q,i,j,k,x,y,z,bs,c;
void add()
{
    lj[++bs]=y;
    len[bs]=z;
    nxt[bs]=fir[x];
    fir[x]=bs;
    lj[++bs]=x;
    len[bs]=z;
    nxt[bs]=fir[y];
    fir[y]=bs;
}
void read(int &x)
{
    c=getchar();
    while ((c<48)||(c>57)) c=getchar();
    x=c^48;c=getchar();
    while ((c>=48)&&(c<=57))
    {
        x=x*10+(c^48);
        c=getchar();
    }
}
void dijk(int s)
{
    int i;
    while (!heap.empty())
    {
        x=heap.top().second;heap.pop();
        for (i=fir[x];i;i=nxt[i]) if (f[s][lj[i]]>f[s][x]+len[i])
        {
            cr.first=f[s][cr.second=lj[i]]=f[s][x]+len[i];
            heap.push(cr);
        }
        while ((!heap.empty())&&(heap.top().first!=f[s][heap.top().second]))
            heap.pop();
    }
}
int main()
```

```
{
    memset(f,0x3f,sizeof(f));inf=f[0][0];
    read(n);read(m);read(q);
    while (m--)
    {
        read(x);read(y);read(z);
        add();
    }
    for (i=1;i<=q;i++)
    {
        read(x);
        f[1<<i-1][x]=0;
    }
    q=(1<<q)-1;
    for (i=1;i<=q;i++)
    {
        for (k=1;k<=n;k++)
        {
            for (j=i&(i-1);j;j=i&(j-1)) f[i][k]=min(f[i][k],f[j][k]+f[i^j][k]);
            if (f[i][k]<inf) heap.push(pa(f[i][k],k));
        }
        dijk(i);
    }
    for (i=1;i<=n;i++) inf=min(inf,f[q][i]);
    printf("%lld",inf);
}
```

4.49 2-sat

$O(n+m)$，$O(n+m)$。

```
struct sat
{
    vector<vector<int>> e;
    vector<int> dfn,low,st,f,ed;
    int fs,tp,id,n;
    sat(int n):n(n),e(n*2),dfn(n*2,-1),low(n*2),st(n*2),f(n*2,-1),ed(n*2),fs(0),
        tp(-1),id(0){}
    void dfs(int u)
    {
        dfn[u]=low[u]=id++;
        ed[u]=1;st[++tp]=u;
        for (int v:e[u]) if (dfn[v]!=-1)
        {
            if (ed[v]) low[u]=min(low[u],dfn[v]);
        } else dfs(v),low[u]=min(low[u],low[v]);
        if (dfn[u]==low[u])
        {
            do
            {
                f[st[tp]]=fs;
                ed[st[tp]]=0;
            } while (st[tp--]!=u);
            ++fs;
        }
    }
    void add(int u,bool x,int v,bool y)//d:dif
```

```
    {
        assert(u>=0&&u<n&&v>=0&&v<n);
        e[u+x*n].push_back(v+y*n);
        e[v+(y^1)*n].push_back(u+(x^1)*n);
    }
    void set(int u,bool x)
    {
        assert(u>=0&&u<n);
        e[u+(x^1)*n].push_back(u+x*n);
    }
    vector<int> getans()
    {
        int i;
        for (i=0;i<n*2;i++) if (dfn[i]==-1) dfs(i);
        vector<int> r(n);
        for (i=0;i<n;i++)
        {
            if (f[i]==f[i+n]) return {};
            r[i]=f[i]>f[i+n];
        }
        return r;
    }
};
```

4.50 Kosaraju 强连通分量（bitset 优化）

$O(\frac{n^2}{w})$，$O(\frac{n^2}{w})$。

```
void dfs1(int x)
{
    int i;ed[x]=0;
    for (i=(lj[x]&ed)._Find_first();i<=n;i=(lj[x]&ed)._Find_next(i)) dfs1(i);
    sx[--tp]=x;
}
void dfs2(int x)
{
    int i;ed[x]=0;tv[f[x]=f[0]]+=v[x];
    for (i=(fj[x]&ed)._Find_first();i<=n;i=(fj[x]&ed)._Find_next(i)) dfs2(i);
}
int main()
{
    read(n);read(m);tp=n+1;
    for (i=1;i<=n;i++) {ed[i]=1;read(v[i]);}
    for (i=1;i<=m;i++)
    {
        read(x);read(y);lj[x][y]=1;fj[y][x]=1;lb[i][0]=x;lb[i][1]=y;
    }
    for (i=1;i<=n;i++) if (ed[i]) dfs1(i);
    ed.set();
    for (i=1;i<=n;i++) if (ed[sx[i]]) {++f[0];dfs2(sx[i]);}
    for (i=1;i<=m;i++) if (f[lb[i][0]]!=f[lb[i][1]])
    {
        flj[f[lb[i][0]]].push_back(f[lb[i][1]]);++rd[f[lb[i][1]]];
    }
    for (i=1;i<=f[0];i++) if (!rd[i]) dl[++wei]=i;
    while (tou<=wei)
    {
```

```
        x=dl[tou++];g[x]+=tv[x];
        for (i=0;i<flj[x].size();i++)
        {
            g[flj[x][i]]=max(g[flj[x][i]],g[x]);
            if (--rd[flj[x][i]]==0) dl[++wei]=flj[x][i];
        }
    }
    for (i=1;i<=f[0];i++) ans=max(ans,g[i]);printf("%d",ans);
}
```

4.51 Tarjan 强连通分量

$O(n+m)$，$O(n+m)$。

```
int dfn[N],low[N],st[N],f[N],fs,tp,id;
bool ed[N];
void tarjan(int u)
{
    dfn[u]=low[u]=++id;
    ed[u]=1;st[++tp]=u;
    for (int v:e[u]) if (dfn[v])
    {
        if (ed[v]) low[u]=min(low[u],dfn[v]);
    } else tarjan(v),low[u]=min(low[u],low[v]);
    if (dfn[u]==low[u])
    {
        ++fs;
        do
        {
            f[st[tp]]=fs;
            ed[st[tp]]=0;
        } while (st[tp--]!=u);
    }
}
```

4.52 欧拉路径（字典序最小）

```
#include <bits/stdc++.h>
using namespace std;
#if !defined(ONLINE_JUDGE)&&defined(LOCAL)
#include "my_header\debug.h"
#else
#define dbg(...); 1;
#endif
typedef unsigned int ui;
typedef long long ll;
#define all(x) (x).begin(),(x).end()
const int N=1e5+2;
vector<int> e[N];
int rd[N],cd[N];
vector<int> ans;
void dfs(int u)
{
    while (e[u].size())
    {
```

```
            int v=e[u].back();
            e[u].pop_back();
            dfs(v);
            ans.push_back(v);
        }
}
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    int n,m,i,x=0;
    cin>>n>>m;ans.reserve(m);
    while (m--)
    {
        int u,v;
        cin>>u>>v;
        e[u].push_back(v);
        ++cd[u];++rd[v];
    }
    for (i=1;i<=n;i++) if (cd[i]!=rd[i])
    {
        if (abs(cd[i]-rd[i])>1) goto no;
        ++x;
    }
    if (x>2) goto no;x=1;
    for (i=1;i<=n;i++) if (cd[i]>rd[i]) {x=i;break;}
    for (i=1;i<=n;i++) sort(all(e[i])),reverse(all(e[i]));
    dfs(x);ans.push_back(x);reverse(all(ans));
    for (i=0;i<ans.size();i++) cout<<ans[i]<<" \n"[i+1==ans.size()];
    return 0;
    no:cout<<"No"<<endl;
}
```

4.53 欧拉回/通路构造

$O(n+m)$，$O(n+m)$。

```
optional<vector<int>> undirected_euler_cycle(int n,const vector<pair<int,int>>
    &edges)//[1,n]/[1,m], 正数表示正向, 负数表示反向
{
    int i=0;
    vector<int> rd(n+1),ed(edges.size()+1),r;
    vector<vector<pair<int,int>>> e(n+1);
    for (auto [u,v]:edges)
    {
        ++rd[u],++rd[v];
        e[u].push_back({v,++i});
        e[v].push_back({u,-i});
    }
    for (i=1;i<=n;i++) if (rd[i]&1) return {};
    function<void(int)> dfs=[&](int u)
    {
        while (e[u].size())
        {
            auto [v,w]=e[u].back();
            e[u].pop_back();
            if (ed[abs(w)]) continue;
            ed[abs(w)]=1;
```

```cpp
            dfs(v);
            r.push_back(w);
        }
    };
    for (i=1;i<=n;i++) if (rd[i]) {dfs(i);break;}
    reverse(all(r));
    if (r.size()!=edges.size()) return {};
    return {r};
}
optional<vector<int>> directed_euler_cycle(int n,const vector<pair<int,int>> &
    edges)//[1,n]/[1,m]
{
    int i=0;
    vector<int> rd(n+1),cd(n+1),r;
    vector<vector<pair<int,int>>> e(n+1);
    for (auto [u,v]:edges)
    {
        ++cd[u],++rd[v];
        e[u].push_back({v,++i});
    }
    for (i=1;i<=n;i++) if (rd[i]!=cd[i]) return {};
    function<void(int)> dfs=[&](int u)
    {
        while (e[u].size())
        {
            auto [v,w]=e[u].back();
            e[u].pop_back();
            dfs(v);
            r.push_back(w);
        }
    };
    for (i=1;i<=n;i++) if (cd[i]) {dfs(i);break;}
    reverse(all(r));
    if (r.size()!=edges.size()) return {};
    return {r};
}
optional<vector<int>> undirected_euler_trail(int n,const vector<pair<int,int>>
    &edges)//[1,n]/[1,m]，正数表示正向，负数表示反向
{
    int i=0;
    vector<int> rd(n+1),ed(edges.size()+1),r;
    vector<vector<pair<int,int>>> e(n+1);
    for (auto [u,v]:edges)
    {
        ++rd[u],++rd[v];
        e[u].push_back({v,++i});
        e[v].push_back({u,-i});
    }
    int odd=0;
    for (i=1; i<=n; i++) odd+=rd[i]&1;
    if (odd>2) return { };
    function<void(int)> dfs=[&](int u)
        {
            while (e[u].size())
            {
                auto [v,w]=e[u].back();
                e[u].pop_back();
```

```
                if (ed[abs(w)]) continue;
                ed[abs(w)]=1;
                dfs(v);
                r.push_back(w);
            }
        };
    for (i=1; i<=n; i++) if (rd[i]&1) { dfs(i); break; }
    if (i>n)
    {
        for (i=1; i<=n; i++) if (rd[i]) { dfs(i); break; }
    }
    reverse(all(r));
    if (r.size()!=edges.size()) return { };
    return {r};
}
optional<vector<int>> directed_euler_trail(int n,const vector<pair<int,int>> &
    edges)//[1,n]/[1,m]
{
    int i=0;
    vector<int> rd(n+1),cd(n+1),r;
    vector<vector<pair<int,int>>> e(n+1);
    for (auto [u,v]:edges)
    {
        ++cd[u],++rd[v];
        e[u].push_back({v,++i});
    }
    int diff=0;
    for (i=1; i<=n; i++)
    {
        if (abs(rd[i]-cd[i])>1) return { };
        if (rd[i]!=cd[i]) ++diff;
    }
    if (diff>2) return { };
    function<void(int)> dfs=[&](int u)
        {
            while (e[u].size())
            {
                auto [v,w]=e[u].back();
                e[u].pop_back();
                dfs(v);
                r.push_back(w);
            }
        };
    for (i=1; i<=n; i++) if (cd[i]>rd[i]) { dfs(i); break; }
    if (i>n)
    {
        for (i=1; i<=n; i++) if (cd[i]) { dfs(i); break; }
    }
    reverse(all(r));
    if (r.size()!=edges.size()) return { };
    return {r};
}
```

4.54   有向图欧拉回路计数（BEST 定理）/生成树计数

$O(n^3)$，$O(n^2)$。

以 u 为起点的欧拉回路个数 $sum = T(u) \times \prod_{v=1}^{n} (out(v)-1)!$，其中 $T(u)$ 是以 u 为根的内向树个数（出度矩阵-邻接矩阵），$out(v)$ 是 v 的出度。若允许循环同构（如 $1 \to 2 \to 1 \to 3 \to 1$ 与 $1 \to 3 \to 1 \to 2 \to 1$），还需多乘 $out(u)$。

```cpp
    ll det(vector<vector<ll>> b)
{
    ll r=1;
    int n=b.size(), i, j, k;
    for (i=0; i<n; i++)
    {
        for (j=i; j<n; j++) if (b[j][i]) break;
        if (j==n) return 0;
        swap(b[j], b[i]);
        if (j!=i) r=(p-r)%p;
        r=r*b[i][i]%p;
        b[i][i]=ksm(b[i][i], p-2);
        for (j=n-1; j>=i; j--) b[i][j]=b[i][j]*b[i][i]%p;
        for (j=i+1; j<n; j++) for (k=n-1; k>=i; k--) b[j][k]=(b[j][k]+(p-b[j][i
            ])*b[i][k])%p;
    }
    return r;
}
ll eular_path_count(vector<vector<int>> a, int s, int t)
{
    int n=a.size(), i, j, k;
    ++a[t][s]; s=t;
    vector<int> rd(n), cd(n);
    for (i=0; i<n; i++) for (j=0; j<n; j++) cd[i]+=a[i][j], rd[j]+=a[i][j];
    for (i=0; i<n; i++) if (cd[i]!=rd[i]) return 0;
    vector<int> f(n);
    iota(all(f), 0);
    function<int(int)> getf=[&](int u) { return f[u]==u?u:f[u]=getf(f[u]); };
    for (i=0; i<n; i++) for (j=0; j<n; j++) if (a[i][j]) f[getf(i)]=getf(j);
    ll r=1;
    vector<int> id;
    for (i=0; i<n; i++) if (cd[i])
    {
        if (getf(i)!=getf(s)) return 0;
        r=r*fac[cd[i]-1]%p;
        if (i!=s) id.push_back(i);
    }
    n=id.size();
    vector b(n, vector<ll>(n));
    for (i=0; i<n; i++)
    {
        b[i][i]=cd[id[i]]-a[id[i]][id[i]];
        for (j=0; j<n; j++) if (i!=j) b[i][j]=(p-a[id[i]][id[j]])%p;
    }
    return r*det(b)%p;
}
ll eular_path_count(vector<vector<int>> a)
{
    int n=a.size(), i, j, s=-1, t=-1;
    vector<int> rd(n), cd(n), d(n);
    for (i=0; i<n; i++) for (j=0; j<n; j++) cd[i]+=a[i][j], rd[j]+=a[i][j];
    if (count(all(cd), 0)==n) return 1;
    for (i=0; i<n; i++) d[i]=cd[i]-rd[i];
```

```
        s=max_element(all(d))-d.begin();
        t=min_element(all(d))-d.begin();
        ll r=0;
        if (s==t)
        {
            for (i=0; i<n; i++) if (cd[i]) r+=eular_path_count(a, i, i);
        }
        else r=eular_path_count(a, s, t);
        return r%p;
}
ll eular_circuit_count(vector<vector<int>> a)
{
        int n=a.size(), i, j;
        for (i=0; i<n; i++) for (j=0; j<n; j++) if (a[i][j]) return eular_path_count
            (a, i, i)*ksm(accumulate(all(a[i]), 0llu)%p, p-2)%p;
        return 1;
}
ll directed_spanning_tree_count(vector<vector<int>> a, int s)
{
        int n=a.size(), i, j;
        vector b(n-1, vector<ll>(n-1));
        for (i=0; i<n; i++) a[i][i]=0;
        for (i=0; i<n; i++) if (i!=s) for (j=0; j<n; j++) if (j!=s&&i!=j) b[i-(i>s)
            ][j-(j>s)]=(p-a[i][j])%p;
        for (i=0; i<n; i++) if (i!=s) for (j=0; j<n; j++) (b[i-(i>s)][i-(i>s)]+=a[j
            ][i])%=p;
        return det(b);
}//外向
ll undirected_spanning_tree_count(vector<vector<int>> a)
{
        int n=a.size(), i, j;
        --n;
        vector b(n, vector<ll>(n));
        for (i=0; i<n; i++) a[i][i]=0;
        for (i=0; i<n; i++) for (j=0; j<n; j++) if (i!=j) b[i][j]=(p-a[i][j])%p;
        for (i=0; i<n; i++) b[i][i]=reduce(all(a[i]), 0llu)%p;
        return det(b);
}
```

4.55 点染色

结论：$\chi(G) \le \Delta(G) + 1$，其中 $\Delta(G)$ 是图的最大度。只有奇圈和完全图取等。

```
vector<int> chromatic_number(int n,const vector<pair<int,int>> &edges)//[0,n)
{
        vector r(n,-1),cur(n,-1);
        vector<vector<int>> e(n);
        int ans=0,i;
        for (auto [u,v]:edges) e[u].push_back(v),e[v].push_back(u);
        for (i=0;i<n;i++) ans=max(ans,(int)e[i].size());
        ans+=2;
        vector p(n,vector(ans,0));
        function<void(int)> dfs=[&](int u)
        {
            int col=u?*max_element(cur.begin(),cur.begin()+u)+1:0;
```

```
            if (col>=ans) return;
            if (u==n)
            {
                r=cur;
                ans=col;
                return;
            }
            int i;
            for (int i=0;i<=col;i++) if (!p[u][i])
            {
                cur[u]=i;
                for (int v:e[u]) ++p[v][i];
                dfs(u+1);
                for (int v:e[u]) --p[v][i];
            }
        };
        dfs(0);
        return r;
}
```

4.56   最大独立集

```
vector<int> indep_set(int n,const vector<pair<int,int>> &edges)//[0,n)
{
    vector<vector<int>> e(n);
    mt19937 rnd(998);
    vector<int> p(n),q(n),ed(n);
    iota(all(p),0);
    shuffle(all(p),rnd);
    for (int i=0;i<n;i++) q[p[i]]=i;
    for (auto [u,v]:edges)
    {
        e[p[u]].push_back(p[v]);
        e[p[v]].push_back(p[u]);
    }
    vector<int> r,cur;
    function<void(int)> dfs=[&](int u)
    {
        if (cur.size()+n-u<=r.size()) return;
        if (u==n)
        {
            r=cur;
            return;
        }
        if (!ed[u])
        {
            cur.push_back(u);
            for (int v:e[u]) ++ed[v];
            dfs(u+1);
            for (int v:e[u]) --ed[v];
            cur.pop_back();
        }
        if (ed[u]||e[u].size()) dfs(u+1);
    };dfs(0);
    for (int &x:r) x=q[x];
    sort(all(r));
```

```
    return r;
}
```

4.57 2-sat

```cpp
struct sat
{
    vector<vector<int>> e;
    vector<int> dfn,low,st,f,ed;
    int fs,tp,id,n;
    sat(int n):n(n),e(n*2),dfn(n*2,-1),low(n*2),st(n*2),f(n*2,-1),ed(n*2),fs(0),
        tp(-1),id(0){}
    void dfs(int u)
    {
        dfn[u]=low[u]=id++;
        ed[u]=1;st[++tp]=u;
        for (int v:e[u]) if (dfn[v]!=-1)
        {
            if (ed[v]) low[u]=min(low[u],dfn[v]);
        } else dfs(v),low[u]=min(low[u],low[v]);
        if (dfn[u]==low[u])
        {
            do
            {
                f[st[tp]]=fs;
                ed[st[tp]]=0;
            } while (st[tp--]!=u);
            ++fs;
        }
    }
    void add(int u,bool x,int v,bool y)//d:dif
    {
        assert(u>=0&&u<n&&v>=0&&v<n);
        e[u+x*n].push_back(v+y*n);
        e[v+(y^1)*n].push_back(u+(x^1)*n);
    }
    void set(int u,bool x)
    {
        assert(u>=0&&u<n);
        e[u+(x^1)*n].push_back(u+x*n);
    }
    vector<int> getans()
    {
        int i;
        for (i=0;i<n*2;i++) if (dfn[i]==-1) dfs(i);
        vector<int> r(n);
        for (i=0;i<n;i++)
        {
            if (f[i]==f[i+n]) return {};
            r[i]=f[i]>f[i+n];
        }
        return r;
    }
};
```

4.58 树剖

```cpp
namespace HLD
{
    const int N=5e5+2;
    vector<int> e[N];
    int dfn[N],nfd[N],dep[N],f[N],siz[N],hc[N],top[N];
    int id;
    void dfs1(int u)
    {
        siz[u]=1;
        for (int v:e[u]) if (v!=f[u])
        {
            dep[v]=dep[f[v]=u]+1;
            dfs1(v);
            siz[u]+=siz[v];
            if (siz[v]>siz[hc[u]]) hc[u]=v;
        }
    }
    void dfs2(int u)
    {
        dfn[u]=++id;
        nfd[id]=u;
        if (hc[u])
        {
            top[hc[u]]=top[u];
            dfs2(hc[u]);
            for (int v:e[u]) if (v!=hc[u]&&v!=f[u]) dfs2(top[v]=v);
        }
    }
    int lca(int u,int v)
    {
        while (top[u]!=top[v])
        {
            if (dep[top[u]]<dep[top[v]]) v=f[top[v]];
            else u=f[top[u]];
        }
        if (dep[u]>dep[v]) swap(u,v);
        return u;
    }
    int dis(int u,int v)
    {
        return dep[u]+dep[v]-(dep[lca(u,v)]<<1);
    }
    void init(int n)
    {
        for (int i=1;i<=n;i++)
        {
            e[i].clear();
            f[i]=hc[i]=0;
        }
        id=0;
    }
    void fun(int root)
    {
        dep[root]=1;dfs1(root);dfs2(top[root]=root);
    }
```

```cpp
    vector<pair<int,int>> get_path(int u,int v)//u->v，注意可能出现 [r>l]（表示反
        过来走）
    {
        //cerr<<"path from "<<u<<" to "<<v<<": ";
        vector<pair<int,int>> v1,v2;
        while (top[u]!=top[v])
        {
            if (dep[top[u]]>dep[top[v]]) v1.push_back({dfn[u],dfn[top[u]]}),u=f[
                top[u]];
            else v2.push_back({dfn[top[v]],dfn[v]}),v=f[top[v]];
        }
        v1.reserve(v1.size()+v2.size()+1);
        v1.push_back({dfn[u],dfn[v]});
        reverse(v2.begin(),v2.end());
        for (auto v:v2) v1.push_back(v);
        //for (auto [x,y]:v1) cerr<<"["<<x<<','<<y<<"] ";cerr<<endl;
        return v1;
    }
}
using HLD::e,HLD::lca,HLD::dis,HLD::dfn,HLD::nfd,HLD::dep,HLD::f,HLD::siz,HLD::
    get_path;
using HLD::fun,HLD::init;//5e5
```

5   计算几何

5.1   自适应 simpson 法

```cpp
const db eps=1e-7;
db sl,sr,sm,a;
db f(db x)
{
    return pow(x,a/x-x);
}
db g(db l,db r)
{
    db mid=(l+r)*0.5;
    return (f(l)+f(r)+f(mid)*4)/6*(r-l);
}
db ab(db x)
{
    if (x>0) return x;
    return -x;
}
db sim(db l,db r)
{
    db mid=(l+r)*0.5;
    sl=g(l,mid);sr=g(mid,r);sm=g(l,r);
    if (ab(sl+sr-sm)<eps) return sl+sr;
    return sim(l,mid)+sim(mid,r);
}
```

5.2   板子

```cpp
namespace geometry//不要用 int!
{
#define tmpl template<typename T>
    typedef long long ll;
    typedef long double db;
    const db eps=1e-6;
#define all(x) (x).begin(),(x).end()
    inline int sgn(const ll &x)
    {
        if (x<0) return -1;
        return x>0;
    }
    inline int sgn(const db &x)
    {
        if (fabs(x)<eps) return 0;
        return x>0?1:-1;
    }
    tmpl struct point//* 为叉乘, & 为点乘, 只允许使用 double 和 ll
    {
        T x, y;
        point() { }
        point(T a, T b):x(a), y(b) { }
        operator point<ll>() const { return point<ll>(x, y); }
        operator point<db>() const { return point<db>(x, y); }
```

```cpp
        point<T> operator+(const point<T> &o) const { return point(x+o.x, y+o
            .y); }
        point<T> operator-(const point<T> &o) const { return point(x-o.x, y-o
            .y); }
        point<T> operator*(const T &k) const { return point(x*k, y*k); }
        point<T> operator/(const T &k) const { return point(x/k, y/k); }
        T operator*(const point<T> &o) const { return x*o.y-y*o.x; }
        T operator&(const point<T> &o) const { return x*o.x+y*o.y; }
        void operator+=(const point<T> &o) { x+=o.x; y+=o.y; }
        void operator-=(const point<T> &o) { x-=o.x; y-=o.y; }
        void operator*=(const T &k) { x*=k; y*=k; }
        void operator/=(const T &k) { x/=k; y/=k; }
        bool operator==(const point<T> &o) const { return x==o.x&&y==o.y; }
        bool operator!=(const point<T> &o) const { return x!=o.x||y!=o.y; }
        db len() const { return sqrt(len2()); }//模长
        T len2() const { return x*x+y*y; }
};
const point<db> npos=point<db>(514e194, 9810e191), apos=point<db>(145
    e174, 999e180);
const int DS[4]={1, 2, 4, 3};
tmpl int quad(const point<T> &o)//坐标轴归右上象限，返回值 [1,4]
{
    return DS[(sgn(o.y)<0)*2+(sgn(o.x)<0)];
}
tmpl bool angle_cmp(const point<T> &a, const point<T> &b)
{
    int c=quad(a), d=quad(b);
    if (c!=d) return c<d;
    return a*b>0;
}
tmpl db dis(const point<T> &a, const point<T> &b) { return (a-b).len();
    }
tmpl T dis2(const point<T> &a, const point<T> &b) { return (a-b).len2();
     }
tmpl point<T> operator*(const T &k, const point<T> &o) { return point<T
    >(k*o.x, k*o.y); }
tmpl bool operator<(const point<T> &a, const point<T> &b)
{
    int s=sgn(a*b);
    return s>0||s==0&&sgn(a.len2()-b.len2())<0;
}
istream &operator>>(istream &cin, point<ll> &o) { return cin>>o.x>>o.y;
    }
istream &operator>>(istream &cin, point<db> &o)
{
    string s;
    cin>>s;
    o.x=stod(s);
    cin>>s;
    o.y=stod(s);
    return cin;
}
tmpl ostream &operator<<(ostream &cout, const point<T> &o)
{
    if ((point<db>)o==apos) return cout<<"all␣position";
    if ((point<db>)o==npos) return cout<<"no␣position";
    return cout<<'('<<o.x<<','<<o.y<<')';
```

```cpp
}
tmpl struct line
{
    point<T> o, d;
    line() { }
    line(const point<T> &a, const point<T> &b, int twopoint);
    bool operator!=(const line<T> &m) { return !(*this==m); }
};
template<> line<ll>::line(const point<ll> &a, const point<ll> &b, int
    twopoint)
{
    o=a;
    d=twopoint?b-a:b;
    ll tmp=gcd(d.x, d.y);
    assert(tmp);
    if (d.x<0||d.x==0&&d.y<0) tmp=-tmp;
    d.x/=tmp; d.y/=tmp;
}
template<> line<db>::line(const point<db> &a, const point<db> &b, int
    twopoint)
{
    o=a;
    d=twopoint?b-a:b;
    int s=sgn(d.x);
    if (s<0||!s&&d.y<0) d.x=-d.x, d.y=-d.y;
}
tmpl line<T> rotate_90(const line<T> &m) { return line(m.o, point(m.d.y,
    -m.d.x), 0); }
tmpl line<db> rotate(const line<T> &m, db angle)
{
    return {(point<db>)m.o, {m.d.x*cos(angle)-m.d.y*sin(angle), m.d.x*sin
        (angle)+m.d.y*cos(angle)}, 0};
}
tmpl db get_angle(const line<T> &m, const line<T> &n) { return asin((m.d
    *n.d)/(m.d.len()*n.d.len())); }
tmpl bool operator<(const line<T> &m, const line<T> &n)
{
    int s=sgn(m.d*n.d);
    return s?s>0:m.d*m.o<n.d*n.o;
}
bool operator==(const line<ll> &m, const line<ll> &n) { return m.d==n.d
    &&(m.o-n.o)*m.d==0; }
bool operator==(const line<db> &m, const line<db> &n) { return fabs(m.d*
    n.d)<eps&&fabs((n.o-m.o)*m.d)<eps; }
tmpl ostream &operator<<(ostream &cout, const line<T> &o) { return cout
    <<'('<<o.d.x<<"␣k␣+␣"<<o.o.x<<"␣,␣"<<o.d.y<<"␣k␣+␣"<<o.o.y<<")"; }
tmpl point<db> intersect(const line<T> &m, const line<T> &n)
{
    if (!sgn(m.d*n.d))
    {
        if (!sgn(m.d*(n.o-m.o))) return apos;
        return npos;
    }
    return (point<db>)m.o+(n.o-m.o)*n.d/(db)(m.d*n.d)*(point<db>)m.d;
}
tmpl db dis(const line<T> &m, const point<T> &o) { return abs(m.d*(o-m.o
    )/m.d.len()); }
```

```
tmpl db dis(const point<T> &o, const line<T> &m) { return abs(m.d*(o-m.o
    )/m.d.len()); }
struct circle
{
    point<db> o;
    db r;
    circle() { }
    circle(const point<db> &O, const db &R=0):o(point<db>((db)O.x, (db)O.
        y)), r(R) { }//圆心半径构造
    circle(const point<db> &a, const point<db> &b)//直径构造
    {
        o=(a+b)*0.5;
        r=dis(b, o);
    }
    circle(const point<db> &a, const point<db> &b, const point<db> &c)//
        三点构造外接圆（非最小圆）
    {
        auto A=(b+c)*0.5, B=(a+c)*0.5;
        o=intersect(rotate_90(line(A, c, 1)), rotate_90(line(B, c, 1)));
        r=dis(o, c);
    }
    circle(vector<point<db>> a)
    {
        int n=a.size(), i, j, k;
        mt19937 rnd(75643);
        shuffle(all(a), rnd);
        *this=circle(a[0]);
        for (i=1; i<n; i++) if (!cover(a[i]))
        {
            *this=circle(a[i]);
            for (j=0; j<i; j++) if (!cover(a[j]))
            {
                *this=circle(a[i], a[j]);
                for (k=0; k<j; k++) if (!cover(a[k])) *this=circle(a[i], a[
                    j], a[k]);
            }
        }
    }
    circle(const vector<point<ll>> &b)
    {
        vector<point<db>> a(b.size());
        int n=a.size(), i, j, k;
        for (i=0; i<a.size(); i++) a[i]=(point<db>)b[i];
        *this=circle(a);
    }
    tmpl bool cover(const point<T> &a) { return sgn(dis((point<db>)a, o)-
        r)<=0; }
};
tmpl struct segment
{
    point<T> a, b;
    segment() { }
    segment(point<T> o, point<T> p)
    {
        int s=sgn(o.x-p.x);
        if (s>0||!s&&o.y>p.y) swap(o, p);
        a=o; b=p;
```

```cpp
    }
};
tmpl bool intersect(const segment<T> &m, const segment<T> &n)
{
    auto a=n.b-n.a, b=m.b-m.a;
    auto d=n.a-m.a;
    if (sgn(n.b.x-m.a.x)<0||sgn(m.b.x-n.a.x)<0) return 0;
    if (sgn(max(n.a.y, n.b.y)-min(m.a.y, m.b.y))<0||sgn(max(m.a.y, m.b.y)
        -min(n.a.y, n.b.y))<0) return 0;
    return sgn(b*d)*sgn((n.b-m.a)*b)>=0&&sgn(a*d)*sgn((m.b-n.a)*a)<=0;
}
tmpl struct convex
{
    vector<point<T>> p;
    convex(vector<point<T>> a);
    db peri()//周长
    {
        int i, n=p.size();
        db C=(p[n-1]-p[0]).len();
        for (i=1; i<n; i++) C+=(p[i-1]-p[i]).len();
        return C;
    }
    db area() { return area2()*0.5; }//面积
    T area2()//两倍面积
    {
        int i, n=p.size();
        T S=p[n-1]*p[0];
        for (i=1; i<n; i++) S+=p[i-1]*p[i];
        return abs(S);
    }
    db diam() { return sqrt(diam2()); }
    T diam2()//直径平方
    {
        T r=0;
        int n=p.size(), i, j;
        if (n<=2)
        {
            for (i=0; i<n; i++) for (j=i+1; j<n; j++) r=max(r, dis2(p[i],
                p[j]));
            return r;
        }
        p.push_back(p[0]);
        for (i=0, j=1; i<n; i++)
        {
            while ((p[i+1]-p[i])*(p[j]-p[i])<=(p[i+1]-p[i])*(p[j+1]-p[i]))
                if (++j==n) j=0;
            r=max({r, dis2(p[i], p[j]), dis2(p[i+1], p[j])});
        }
        p.pop_back();
        return r;
    }
    bool cover(const point<T> &o) const//点是否在凸包内
    {
        if (o.x<p[0].x||o.x==p[0].x&&o.y<p[0].y) return 0;
        if (o==p[0]) return 1;
        if (p.size()==1) return 0;
        ll tmp=(o-p[0])*(p.back()-p[0]);
```

```
        if (tmp==0) return dis2(o, p[0])<=dis2(p.back(), p[0]);
        if (tmp<0||p.size()==2) return 0;
        int x=upper_bound(1+all(p), o, [&](const point<T> &a, const point
            <T> &b) { return (a-p[0])*(b-p[0])>0; })-p.begin()-1;
        return (o-p[x])*(p[x+1]-p[x])<=0;
    }
    convex<T> operator+(const convex<T> &A) const
    {
        int n=p.size(), m=A.p.size(), i, j;
        vector<point<T>> c;
        if (min(n, m)<=2)
        {
            c.reserve(n*m);
            for (i=0; i<n; i++) for (j=0; j<m; j++) c.push_back(p[i]+A.p[j
                ]);
            return convex<T>(c);
        }
        point<T> a[n], b[m];
        for (i=0; i+1<n; i++) a[i]=p[i+1]-p[i];
        a[n-1]=p[0]-p[n-1];
        for (i=0; i+1<m; i++) b[i]=A.p[i+1]-A.p[i];
        b[m-1]=A.p[0]-A.p[m-1];
        c.reserve(n+m);
        c.push_back(p[0]+A.p[0]);
        for (i=j=0; i<n&&j<m;) c.push_back(c.back()+(a[i]*b[j]>0?a[i++]:b
            [j++]));
        while (i<n-1) c.push_back(c.back()+a[i++]);
        while (j<m-1) c.push_back(c.back()+b[j++]);
        return convex<T>(c);
    }
    void operator+=(const convex &a) { *this=*this+a; }
};
tmpl convex<T>::convex(vector<point<T>> a)
{
    int n=a.size(), i;
    if (!n) return;
    p=a;
    for (i=1; i<n; i++) if (p[i].x<p[0].x||p[i].x==p[0].x&&p[i].y<p[0].y)
        swap(p[0], p[i]);
    a.resize(0); a.reserve(n);
    for (i=1; i<n; i++) if (p[i]!=p[0]) a.push_back(p[i]-p[0]);
    sort(all(a));
    for (i=0; i<a.size(); i++) a[i]+=p[0];
    point<T> *st=p.data()-1;
    int tp=1;
    for (auto &v:a)
    {
        while (tp>1&&sgn((st[tp]-st[tp-1])*(v-st[tp-1]))<=0) --tp;
        st[++tp]=v;
    }
    p.resize(tp);
}
template<> bool convex<db>::cover(const point<db> &o) const//点是否在凸包
    内
{
    if (o.x<p[0].x||o.x==p[0].x&&o.y<p[0].y) return 0;
    if (o==p[0]) return 1;
```

```cpp
        if (p.size()==1) return 0;
        ll tmp=(o-p[0])*(p.back()-p[0]);
        if (tmp==0) return dis2(o, p[0])<=dis2(p.back(), p[0]);
        if (tmp<0||p.size()==2) return 0;
        int x=upper_bound(1+all(p), o, [&](const point<db> &a, const point<db
            > &b) { return (a-p[0])*(b-p[0])>eps; })-p.begin()-1;
        return (o-p[x])*(p[x+1]-p[x])<=0;
    }
    tmpl struct half_plane//默认左侧
    {
        point<T> o, d;
        operator half_plane<ll>() const { return {(point<ll>)o, (point<ll>)d,
            0}; }
        operator half_plane<db>() const { return {(point<db>)o, (point<db>)d,
            0}; }
        half_plane() { }
        half_plane(const point<T> &a, const point<T> &b, bool twopoint)
        {
            o=a;
            d=twopoint?b-a:b;
        }
        bool operator<(const half_plane<T> &a) const
        {
            int p=quad(d), q=quad(a.d);
            if (p!=q) return p<q;
            p=sgn(d*a.d);
            if (p) return p>0;
            return sgn(d*(a.o-o))>0;
        }
    };
    tmpl ostream &operator<<(ostream &cout, half_plane<T> &m) { return cout
        <<m.o<<"␣|␣"<<m.d; }
    tmpl point<db> intersect(const half_plane<T> &m, const half_plane<T> &n)
    {
        if (!sgn(m.d*n.d))
        {
            if (!sgn(m.d*(n.o-m.o))) return apos;
            return npos;
        }
        return (point<db>)m.o+(n.o-m.o)*n.d/(db)(m.d*n.d)*(point<db>)m.d;
    }
    const db inf=1e9;
    tmpl convex<db> intersect(vector<half_plane<T>> a)
    {
        T I=inf;
        a.push_back({{-I, -I}, {I, -I}, 1});
        a.push_back({{I, -I}, {I, I}, 1});
        a.push_back({{I, I}, {-I, I}, 1});
        a.push_back({{-I, I}, {-I, -I}, 1});
        sort(all(a));
        int n=a.size(), i, h=0, t=-1;
        half_plane<db> q[n];
        point<db> p[n];
        vector<point<db>> r;
        for (i=0; i<n; i++) if (i==n-1||sgn(a[i].d*a[i+1].d))
        {
            auto x=(half_plane<db>)a[i];
```

```
                while (h<t&&sgn((p[t-1]-x.o)*x.d)>=0) --t;
                while (h<t&&sgn((p[h]-x.o)*x.d)>=0) ++h;
                q[++t]=x;
                if (h<t) p[t-1]=intersect(q[t-1], q[t]);
        }
        while (h<t&&sgn((p[t-1]-q[h].o)*q[h].d)>=0) --t;
        if (h==t) return convex<db>(vector<point<db>>(0));
        p[t]=intersect(q[h], q[t]);
        return convex<db>(vector<point<db>>(p+h, p+t+1));
}
tmpl db dis(const point<db> &o, const segment<T> &l)
{
        if ((l.b-l.a&o-l.a)<0||(l.a-l.b&o-l.b)<0) return min(dis(o, l.a), dis
            (o, l.b));
        return dis(o, line(l.a, l.b, 1));
}
tmpl db dis(const segment<T> &l, const point<db> &o)
{
        if ((l.b-l.a&o-l.a)<0||(l.a-l.b&o-l.b)<0) return min(dis(o, l.a), dis
            (o, l.b));
        return dis(o, line(l.a, l.b, 1));
}
pair<ll, ll> __sqrt(ll x)
{
        ll y=sqrtl(x);
        while (y*y>x) --y;
        while ((y+1)*(y+1)<=x) ++y;
        return {y, y+(y*y<x)};
}
pair<int, int> closest_pair(const vector<point<ll>> &a)
{
        int n=a.size(), i, j;
        assert(n>=2);
        auto b=a;
        sort(all(b), [&](auto p, auto q)
            {
                return p.x==q.x?p.y<q.y:p.x<q.x;
            });
        tuple<ll, int, int> ans={dis2(b[0], b[1]), 0, 1};
        set<pair<ll, int>> s;
        for (i=j=0; i<n; i++)
        {
                auto [x, y]=b[i];
                ll d=__sqrt(get<0>(ans)).first;
                if (d==0) break;
                for (auto it=s.lower_bound({y-d, 0}); it!=s.end(); ++it)
                {
                        auto [q, k]=*it;
                        cmin(ans, tuple{dis2(b[k], b[i]), i, k});
                }
                s.emplace(y, i);
                while (b[j].x<x-d) s.erase({b[j].y, j}), ++j;
        }
        auto [_, j1, j2]=ans;
        int i1, i2;
        for (i1=0; i1<n; i1++) if (a[i1]==b[j1]) break;
        for (i2=0; i2<n; i2++) if (i2!=i1&&a[i2]==b[j2]) break;
```

```cpp
        return {i1, i2};
    }
    pair<int, int> furthest_pair(const vector<point<ll>> &a)
    {
        int n=a.size(), i, j;
        assert(n>=2);
        auto b=convex(a).p;
        int m=b.size();
        if (m==1) return {0, 1};
        b.push_back(b[0]);
        tuple<ll, int, int> ans{dis2(b[0], b[1]), 0, 1};
        for (i=0, j=1; i<m; i++)
        {
            while (abs((b[i+1]-b[i])*(b[j]-b[i]))<abs((b[i+1]-b[i])*(b[(j+1)%
                m]-b[i]))) j=(j+1)%m;
            cmax(ans, tuple{dis2(b[i], b[j]), i, j});
            cmax(ans, tuple{dis2(b[i+1], b[j]), i+1, j});
        }
        auto [_, j1, j2]=ans;
        int i1, i2;
        for (i1=0; i1<n; i1++) if (a[i1]==b[j1]) break;
        for (i2=0; i2<n; i2++) if (i2!=i1&&a[i2]==b[j2]) break;
        return {i1, i2};

    }
#undef tmpl
}
using geometry::point, geometry::line, geometry::circle, geometry::convex,
    geometry::half_plane;
using geometry::db, geometry::sgn, geometry::eps, geometry::segment;
using geometry::intersect, geometry::dis;
```

6 公式与杂项

6.1 枚举大小为 k 的集合

思路：通过进位创造 1，再把一串 1 移到最后

```
for (int s=(1<<k)-1,t;s<1<<n;t=s+(s&-s),s=(s&~t)>>__lg(s&-s)+1|t)
{
}
```

6.2 min plus 卷积

```cpp
template <class T> vector<T> min_plus_convolution(const vector<T> &a,const
    vector<T> &b)
{
    //a arbitrary
    //b convex (delta increase)
    int n=a.size(),m=b.size(),i;
    vector<T> c(n+m-1);
    function<void(int,int,int,int)> dfs=[&](int l,int r,int ql,int qr)
        {
            if (l>r) return;
            int mid=l+r>>1;
            while (ql+m<=l) ++ql;
            while (qr>r) --qr;
            int qmid=-1;
            c[mid]=inf;
            for (int i=ql; i<=qr; i++) if (mid-i>=0&&mid-i<m&&cmin(c[mid],a[i]+b[
                mid-i])) qmid=i;
            dfs(l,mid-1,ql,qmid);
            dfs(mid+1,r,qmid,qr);
        };
    dfs(0,n+m-2,0,n-1);
    return c;
}
```

6.3 所有区间 GCD

```cpp
    template<typename T> struct GCD
{
    vector<pair<int, T>> res;
    GCD(const vector<T> &a):res(n)
    {
        int n=a.size(), i, j;
        vector<ll> v(n);
        vector<int> l(n);
        vector<vector<pair<int, T>> res(n);
        for (i=0; i<n; i++)
        {
            for (v[i]=a[i], j=l[i]=i; j>=0; j=l[j]-1)
            {
                v[j]=fun(v[j], a[i]);
                while (l[j]&&fun(a[i], v[l[j]-1])==fun(a[i], v[j])) l[j]=l[l[j
                    ]-1];
```

```
                //[l[j]..j,i]区间内的值求fun均为v[j]
            }
            for (j=i; j>=0; j=l[j]) res[i].push_back({l[j], v[j]});
            reverse(all(res[i]));
        }
    }
    T ask(int l, int r)//[l,r]
    {
        return res[r].prev(upper_bound(l))->second;
    }
};
//需要自定义 fun，如 gcd，and，or。
```

6.4 整体二分（区间 k-**th**）

$O((n+q)\log a)$，$O(n+q)$。

```cpp
struct cz
{
    int x,y,kth,pos,typ;
};
cz q[M],st1[M],st2[M];
int a[N],b[N],d[N],ans[N],s[N];
int n,m,t1,t2,i,j,c,gs;
int lb(int x)
{
    return x&(-x);
}
void add(int x,int y)
{
    for (;x<=n;x+=lb(x)) s[x]+=y;
}
int sum(int x)
{
    int ans=0;
    for (;x;x-=lb(x)) ans+=s[x];
    return ans;
}
void ztef(int ql,int qr,int l,int r)
{
    if (ql>qr) return;
    int mid=l+r>>1,i,midd;
    t1=t2=0;
    if (l==r)
    {
        for (i=ql;i<=qr;i++) if (q[i].typ) ans[q[i].pos]=d[l];
        return;
    }
    for (i=ql;i<=qr;i++) if (q[i].typ)
    {
        midd=sum(q[i].y)-sum(q[i].x-1);
        if (midd>=q[i].kth) st1[++t1]=q[i]; else
        {
            st2[++t2]=q[i];
            st2[t2].kth-=midd;
        }
    }
```

```
         else if (q[i].pos<=mid)
         {
             add(q[i].x,1);
             st1[++t1]=q[i];
         }
         else st2[++t2]=q[i];
         for (i=1;i<=t1;i++) if (!st1[i].typ) add(st1[i].x,-1);
         for (i=1;i<=t1;i++) q[i+ql-1]=st1[i];
         midd=ql+t1-1;
         for (i=1;i<=t2;i++) q[i+midd]=st2[i];
         ztef(ql,midd,l,mid);ztef(midd+1,qr,mid+1,r);
}
int main()
{
     read(n);read(m);
     for (i=1;i<=n;i++)
     {
         read(a[i]);b[i]=a[i];
     }
     sort(b+1,b+n+1);
     d[gs=1]=b[1];
     for (i=2;i<=n;i++) if (b[i]!=b[i-1]) d[++gs]=b[i];
     for (i=1;i<=n;i++) a[i]=lower_bound(d+1,d+gs+1,a[i])-d;
     for (i=1;i<=n;i++)
     {
         q[i].x=i;q[i].pos=a[i];q[i].typ=0;
     }
     for (i=1;i<=m;i++)
     {
         read(q[i+n].x);read(q[i+n].y);read(q[i+n].kth);q[i+n].pos=i;q[i+n].typ
             =1;
     }
     ztef(1,n+m,1,gs);
     for (i=1;i<=m;i++) printf("%d\n",ans[i]);
}
```

6.5 cdq 分治（三维偏序）

$O(n\log^2 n)$，$O(n)$。

```
int lb(int x)
{
    return x&(-x);
}
void add(int x,int y)
{
    for (;x<=mx;x+=lb(x)) a[x]+=y;
}
int sum(int x)
{
    int ans=0;
    for (;x;x^=lb(x)) ans+=a[x];
    return ans;
}
void gb(int l,int r)
{
    int i=l,m=l+r>>1,j=m+1,p=l;
```

```
        if (i<m) gb(i,m);
        if (j<r) gb(j,r);
        while ((i<=m)||(j<=r)) if ((j>r)||(i<=m)&&(q[i].x<=q[j].x))
        {
            if (!q[i].typ) add(q[i].y,1);
            qq[p++]=q[i++];
        }
        else
        {
            if (q[j].typ) ans[q[j].pos]+=q[j].typ*sum(q[j].y);
            qq[p++]=q[j++];
        }
        for (i=l;i<=m;i++) if (!q[i].typ) add(q[i].y,-1);
        for (i=l;i<=r;i++) q[i]=qq[i];
}
int main()
{
    read(n);read(m);
    for (i=1;i<=n;i++)
    {
        read(q[i].x);read(q[i].y);++q[i].y;
        yc[i]=q[i].y;
        if (q[i].y>mx) mx=q[i].y;
    }
    qs=ys=n;
    for (i=1;i<=m;i++)
    {
        read(x);read(y);read(z);read(j);
        q[++qs].x=x-1;q[qs].y=y;q[qs].pos=i;q[qs].typ=1;
        q[++qs].x=z;q[qs].y=y;q[qs].pos=i;q[qs].typ=-1;
        q[++qs].x=x-1;q[qs].y=j+1;q[qs].pos=i;q[qs].typ=-1;
        q[++qs].x=z;q[qs].y=j+1;q[qs].pos=i;q[qs].typ=1;
        if (j+1>mx) mx=j+1;
    }
    gb(1,qs);
    for (i=1;i<=m;i++) printf("%d\n",ans[i]);
}
```

6.6 k 阶差分（$[L, R]$ 加 $\binom{j-L+k}{k}$）

$O((n+q)k)$，$O(nk)$。

```
int main()
{
    read(n);read(m);
    for (i=1;i<=n;i++) read(b[i]);
    C[0][0]=1;
    for (i=1;i<=n+100;i++)
    {
        C[i][0]=1;
        for (j=1;j<=min(i,100);j++)
        {
            C[i][j]=C[i-1][j-1]+C[i-1][j];
            if (C[i][j]>=p) C[i][j]-=p;
        }
    }
    while (m--)
```

```
    {
        read(x);read(y);read(z);
        ++a[x][z];
        for (i=0;i<=z;i++)
        {
            a[y+1][z-i]-=C[y-x+i][i];
            if (a[y+1][z-i]<0) a[y+1][z-i]+=p;
        }
    }
    for (i=100;i>=0;i--) for (j=1;j<=n;j++)
    {
        a[j][i]+=a[j-1][i];
        if (a[j][i]>=p) a[j][i]-=p;
        a[j][i]+=a[j][i+1];
        if (a[j][i]>=p) a[j][i]-=p;
    }
    for (i=1;i<=n;i++) printf("%d␣",(b[i]+a[i][0])%p);
}
```

6.7 高精度

```
struct bigint
{
    using ll=unsigned long long;
    using lll=unsigned __int128;
    const static ll base=1e6;
    const static ll sign=1llu<<63;
    const static lll p=4179340454199820289;
    const static lll g=5;
    const static int N=1<<23;
    static int r[N];
    static lll w[N];
    bool neg;
    vector<ll> a;
private:
    static lll ksm(lll x,ll y)
    {
        lll r=1;
        while (y)
        {
            if (y&1) r=r*x%p;
            x=x*x%p; y>>=1;
        }
        return r;
    }
    static void init(int n)
    {
        static int pr=0,pw=0;
        if (pr==n) return;
        int b=__lg(n)-1,i,j,k;
        for (i=1; i<n; i++) r[i]=r[i>>1]>>1|(i&1)<<b;
        if (pw<n)
        {
            for (j=1; j<n; j=k)
            {
                k=j*2;
```

```
            ll wn=ksm(g,(p-1)/k);
            w[j]=1;
            for (i=j+1; i<k; i++) w[i]=w[i-1]*wn%p;
        }
        pw=n;
    }
    pr=n;
}
static void dft(vector<lll> &a,int o=0)
{
    int n=a.size(),i,j,k;
    lll y,*f,*g,*wn,*A=a.data();
    init(n);
    for (i=1; i<n; i++) if (i<r[i]) swap(A[i],A[r[i]]);
    for (k=1; k<n; k*=2)
    {
        wn=w+k;
        for (i=0; i<n; i+=k*2)
        {
            f=A+i; g=A+i+k;
            for (j=0; j<k; j++)
            {
                y=g[j]*wn[j]%p;
                g[j]=f[j]+p-y;
                f[j]+=y;
            }
        }
        if (k*2==n||k==1<<10) for (lll &x:a) x%=p;
    }
    if (o)
    {
        y=ksm(n,p-2);
        for (lll &x:a) x=x*y%p;
        reverse(1+all(a));
    }
}
ll &operator[](const int &x) { return a[x]; }
const ll &operator[](const int &x) const { return a[x]; }
static void plus_by(vector<ll> &a,const vector<ll> &b)
{
    int n=a.size(),m=b.size(),i,j;
    cmax(n,m);
    a.resize(++n);
    for (i=0; i<m; i++) if ((a[i]+=b[i])>=base) a[i]-=base,++a[i+1];
    for (i=m; i<n&&a[i]>=base; i++) a[i]-=base,++a[i+1];
    if (a[n-1]==0) a.pop_back();
}
static void minus_by(vector<ll> &a,const vector<ll> &b)
{
    int n=a.size(),m=b.size(),i,j;
    for (i=0; i<m; i++) if (!(a[i]&sign)&&a[i]>=b[i]) a[i]-=b[i];
    else --a[i+1],a[i]+=base-b[i];
    for (; i<n&&(a[i]&sign); i++) --a[i+1],a[i]+=base-b[i];
    while (a.size()>1&&!a.back()) a.pop_back();
}
static bool less(const vector<ll> &a,const vector<ll> &b)
{
```

```
        if (a.size()!=b.size()) return a.size()<b.size();
        for (int i=a.size()-1; i>=0; i--) if (a[i]!=b[i]) return a[i]<b[i];
        return 0;
    }
    static int cal(int x) { return 1<<__lg(max(x,1)*2-1); }
public:
    bigint &operator+=(const bigint &o)
    {
        if (neg==o.neg) plus_by(a,o.a);
        else if (neg)
        {
            if (less(o.a,a)) minus_by(a,o.a);
            else
            {
                neg=0;
                auto t=o.a;
                swap(a,t);
                minus_by(a,t);
            }
        }
        else
        {
            if (less(a,o.a))
            {
                neg=1;
                auto t=o.a;
                swap(a,t);
                minus_by(a,t);
            }
            else minus_by(a,o.a);
        }
        return *this;
    }
    bigint &operator-=(const bigint &o)
    {
        neg^=1;
        *this+=o;
        neg^=1;
        if (a==vector<ll>{0}) neg=0;
        return *this;
    }
    bigint &operator*=(const bigint &o)
    {
        neg^=o.neg;
        int n=a.size(),m=o.a.size(),i,j;
        assert(min(n,m)<=p/((base-1)*(base-1)));
        if (min(n,m)<=64)
        {
            vector<ll> c(n+m);
            for (i=0; i<n; i++) for (j=0; j<m; j++) c[i+j]+=a[i]*o[j];
            for (i=0; i<n+m-1; i++)
            {
                c[i+1]+=c[i]/base;
                c[i]%=base;
            }
            swap(a,c);
            while (a.size()>1&&!a.back()) a.pop_back();
```

```cpp
            if (a==vector<ll>{0}) neg=0;
            return *this;
        }
        int len=cal(n+m-1);
        vector<lll> f(len),g(len);
        copy_n(a.begin(),n,f.begin());
        copy_n(o.a.begin(),m,g.begin());
        dft(f); dft(g);
        for (i=0; i<len; i++) f[i]=f[i]*g[i]%p;
        dft(f,1);
        a.resize(n+m-1);
        copy_n(f.begin(),n+m-1,a.begin());
        for (i=0; i<n+m-2; i++)
        {
            a[i+1]+=a[i]/base;
            a[i]%=base;
        }
        while (a.size()>1&&!a.back()) a.pop_back();
        if (a==vector<ll>{0}) neg=0;
        return *this;
    }
    bigint &operator/=(long long x)//to zero
    {
        if (x<0) x=-x,neg^=1;
        for (int i=a.size()-1; i; i--)
        {
            a[i-1]+=a[i]%x*base;
            a[i]/=x;
        }
        a[0]/=x;
        while (a.size()>1&&!a.back()) a.pop_back();
        if (a==vector<ll>{0}) neg=0;
        return *this;
    }
    bigint operator+(bigint o) const { return o+=*this; }
    bigint operator-(bigint o) const { o-=*this; if (o.a!=vector<ll>{0}) o.neg
        ^=1; return o; }
    bigint operator*(bigint o) const { return o*=*this; }
    bigint operator/(long long x) const { auto res=*this; return res/=x; }
    long long operator%(long long x) const
    {
        bool flg=neg;
        if (x<0) flg^=1,x=-x;
        ll res=0;
        for (int i=(base%x==0?0:a.size()-1); i>=0; i--) res=(res*base+a[i])%x;
        return (long long)res*(flg?-1:1);
    }
    bigint(long long x=0):neg(0)
    {
        if (x<0) x=-x,neg=1;
        a.push_back(x%base);
        while (x/=base) a.push_back(x%base);
    }
    bool operator<(const bigint &o) const
    {
        if (neg!=o.neg) return neg;
        if (neg) return less(o.a,a);
```

```cpp
        return less(a,o.a);
    }
    bool operator>(const bigint &o) const { return o<*this; }
    bool operator==(const bigint &o)const { return neg==o.neg&&a==o.a; }
    bool operator!=(const bigint &o)const { return neg!=o.neg||a!=o.a; }
    bool operator<=(const bigint &o) const { return !(*this>o); }
    bool operator>=(const bigint &o) const { return !(*this<o); }
};
istream &operator>>(istream &cin,bigint &x)
{
    x.neg=0;
    x.a.clear();
    string s;
    cin>>s;
    const int length=round(log10(bigint::base));
    if (s[0]=='-') x.neg=1,s.erase(s.begin());
    reverse(all(s));
    ll base=1;
    for (int i=0; i<s.size(); i++)
    {
        if (i%length==0) x.a.push_back(0),base=1;
        x.a.back()=x.a.back()+(s[i]-'0')*base;
        base*=10;
    }
    return cin;
}
ostream &operator<<(ostream &cout,const bigint &x)
{
    if (x.neg) cout<<"-";
    cout<<x.a.back();
    int length=round(log10(bigint::base));
    for (int i=x.a.size()-2; i>=0; i--) cout<<setfill('0')<<setw(length)<<x.a[i
        ];
    return cout;
}
bigint abs(bigint x)
{
    x.neg=0;
    return x;
}
bigint gcd(bigint x,bigint y)
{
    x.neg=y.neg=0;
    if (x==bigint(0)) return y;
    if (y==bigint(0)) return x;
    int c1=0,c2=0;
    while (x%2==0) x/=2,++c1;
    while (y%2==0) y/=2,++c2;
    cmin(c1,c2);
    if (x>y) swap(x,y);
    while (x!=y)
    {
        y-=x;
        y/=2;
        while (y%2==0) y/=2;
        if (x>y) swap(x,y);
    }
```

```
    while (c1--) y*=bigint(2);
    return y;
}
bigint::lll bigint::w[bigint::N];
int bigint::r[bigint::N];
```

6.8 分散层叠算法（Fractional Cascading）

$O(n + q(k + \log n))$，$O(n)$。

给出 k 个长度为 n 的有序数组。

现在有 q 个查询: 给出数 x，分别求出每个数组中大于等于 x 的最小的数 (非严格后继)。

若后继不存在，则定义为 0。你需要在线地回答这些询问。

```
int a[M][N],b[M][N<<1],c[M][N<<1][2],len[M],ans[M];
int n,m,qs,p,q,d,i,j,x,y,la;
int main()
{
    read(n);read(m);read(qs);read(d);
    for (j=1;j<=m;j++) for (i=0;i<n;i++) read(a[j][i]);
    for (j=1;j<=m;j++) a[j][n]=inf+j;++n;
    for (i=0;i<n;i++) b[m][i]=a[m][i],c[m][i][0]=i;
    len[m]=n;
    for (j=m-1;j;j--)
    {
        p=0,q=1;
        while (p<n&&q<len[j+1])
if (a[j][p]<b[j+1][q]) b[j][len[j]]=a[j][p],c[j][len[j]][0]=p++,c[j][len[j
    ]++][1]=q;
        else b[j][len[j]]=b[j+1][q],c[j][len[j]][0]=p,c[j][len[j]++][1]=q,q+=2;
        while (p<n) b[j][len[j]]=a[j][p],c[j][len[j]][0]=p++,c[j][len[j]++][1]=q
            ;
        while (q<len[j+1]) b[j][len[j]]=b[j+1][q],c[j][len[j]][0]=p,c[j][len[j
            ]++][1]=q,q+=2;
    }
    for (int ii=1;ii<=qs;ii++)
    {
        read(x);x^=la;
        y=lower_bound(b[1],b[1]+len[1],x)-b[1];
        ans[1]=a[1][c[1][y][0]];y=c[1][y][1];//下标是c[1][y][0]
        for (j=2;j<=m;j++)
        {
            if (y&&b[j][y-1]>=x) --y;
            ans[j]=a[j][c[j][y][0]];//下标是c[j][y][0]
            y=c[j][y][1];
        }
        la=0;
        for (i=1;i<=m;i++) la^=ans[i]>inf?0:ans[i];
        if (ii%d==0) printf("%d\n",la);
    }
}
```

6.9 圆上整点（二平方和定理）

$x^2 + y^2 = n$ 的整数解的数目的四分之一 $f(n)$ 是积性数论函数，且对于**素数**

幂有：$f(p^k) = \begin{cases} 1 & p = 2 \\ k + 1 & p \equiv 1 \pmod{4} \\ (k + 1) \bmod 2 & p \equiv 3 \pmod{4} \end{cases}$

以下代码给出所有的非负整数解。注意非负整数解个数不等于 $f(n)$。

时间复杂度为 $O(n^{\frac{1}{4}} + f(n))$，其中 $O(n^{\frac{1}{4}})$ 是 pollard-rho 的复杂度。

$f(n)$ 的量级不好分析，但不会超过约数个数 $O(d(n)) \approx O(n^{\frac{1}{3}})$，且可以推测不能达到。

```cpp
namespace pr
{
    typedef long long ll;
    typedef __int128 lll;
    typedef pair<ll, int> pa;
    ll ksm(ll x, ll y, const ll p)
    {
        ll r=1;
        while (y)
        {
            if (y&1) r=(lll)r*x%p;
            x=(lll)x*x%p; y>>=1;
        }
        return r;
    }
    namespace miller
    {
        const int p[7]={2, 3, 5, 7, 11, 61, 24251};
        ll s, t;
        bool test(ll n, int p)
        {
            if (p>=n) return 1;
            ll r=ksm(p, t, n), w;
            for (int j=0; j<s&&r!=1; j++)
            {
                w=(lll)r*r%n;
                if (w==1&&r!=n-1) return 0;
                r=w;
            }
            return r==1;
        }
        bool prime(ll n)
        {
            if (n<2||n==46'856'248'255'981ll) return 0;
            for (int i=0; i<7; ++i) if (n%p[i]==0) return n==p[i];
            s=__builtin_ctz(n-1); t=n-1>>s;
            for (int i=0; i<7; ++i) if (!test(n, p[i])) return 0;
            return 1;
        }
    }
    using miller::prime;
    mt19937_64 rnd(chrono::steady_clock::now().time_since_epoch().count());
    namespace rho
```

```
{
    void nxt(ll &x, ll &y, ll &p) { x=((lll)x*x+y)%p; }
    ll find(ll n, ll C)
    {
        ll l, r, d, p=1;
        l=rnd()%(n-2)+2, r=l;
        nxt(r, C, n);
        int cnt=0;
        while (l^r)
        {
            p=(lll)p*llabs(l-r)%n;
            if (!p) return gcd(n, llabs(l-r));
            ++cnt;
            if (cnt==127)
            {
                cnt=0;
                d=gcd(llabs(l-r), n);
                if (d>1) return d;
            }
            nxt(l, C, n); nxt(r, C, n); nxt(r, C, n);
        }
        return gcd(n, p);
    }
    vector<pa> w;
    vector<ll> d;
    void dfs(ll n, int cnt)
    {
        if (n==1) return;
        if (prime(n)) return w.emplace_back(n, cnt), void();
        ll p=n, C=rnd()%(n-1)+1;
        while (p==1||p==n) p=find(n, C++);
        int r=1; n/=p;
        while (n%p==0) n/=p, ++r;
        dfs(p, r*cnt); dfs(n, cnt);
    }
    vector<pa> getw(ll n)
    {
        w=vector<pa>(0); dfs(n, 1);
        if (n==1) return w;
        sort(w.begin(), w.end());
        int i, j;
        for (i=1, j=0; i<w.size(); i++) if (w[i].first==w[j].first) w[j].
            second+=w[i].second; else w[++j]=w[i];
        w.resize(j+1);
        return w;
    }
    void dfss(int x, ll n)
    {
        if (x==w.size()) return d.push_back(n), void();
        dfss(x+1, n);
        for (int i=1; i<=w[x].second; i++) dfss(x+1, n*=w[x].first);
    }
    vector<ll> getd(ll n)
    {
        getw(n); d=vector<ll>(0); dfss(0, 1);
        sort(d.begin(), d.end());
        return d;
```

```cpp
        }
    }
    using rho::getw, rho::getd;
    using miller::prime;
}
using pr::getw, pr::getd, pr::prime;
lll roundiv(lll x, lll y)
{
    return x>=0?(x+y/2)/y:(x-y/2)/y;
}
struct G
{
    lll x, y;
    G operator~() const { return {x, -y}; }
    lll len2() const { return x*x+y*y; }
    G operator+(const G &o) const { return {x+o.x, y+o.y}; }
    G operator-(const G &o) const { return {x-o.x, y-o.y}; }
    G operator*(const G &o) const { return {x*o.x-y*o.y, x*o.y+y*o.x}; }
    G operator/(const G &o) const
    {
        G t=*this*~o;
        lll l=o.len2();
        return {roundiv(t.x, l), roundiv(t.y, l)};
    }
    G operator%(const G &o) const { return *this-*this/o*o; }
};
G gcd(G a, G b)
{
    if (a.len2()>b.len2()) swap(a, b);
    while (a.len2())
    {
        b=b%a;
        swap(a, b);
    }
    return b;
}
namespace cipolla
{
    typedef unsigned long long ui;
    typedef __uint128_t ll;
    ui p, w;
    struct Q
    {
        ll x, y;
        Q operator*(const Q &o) const { return {(x*o.x+y*o.y%p*w)%p, (x*o.y+y*o.
            x)%p}; }
    };
    ui ksm(ll x, ui y)
    {
        ll r=1;
        while (y)
        {
            if (y&1) r=r*x%p;
            x=x*x%p; y>>=1;
        }
        return r;
    }
```

```cpp
    Q ksm(Q x, ui y)
    {
        Q r={1, 0};
        while (y)
        {
            if (y&1) r=r*x;
            x=x*x; y>>=1;
        }
        return r;
    }
    ui mosqrt(ui x, ui P)//0<=x<P
    {
        if (x==0||P==2) return x;
        p=P;
        if (ksm(x, p-1>>1)!=1) return -1;
        ui y;
        mt19937_64 rnd(chrono::steady_clock::now().time_since_epoch().count());
        do y=rnd()%p, w=((ll)y*y+p-x)%p; while (ksm(w, p-1>>1)<=1);//not for p=2
        y=ksm({y, 1}, p+1>>1).x;
        if (y*2>p) y=p-y;//两解取小
        return y;
    }
}
using cipolla::mosqrt;
vector<pair<ll, ll>> two_sqr_sum(ll n)//只会返回非负解，按照字典序排序
{
    if (n<0) return { };
    if (n==0) return {{0, 0}};
    ll m=__lg(n&-n), d=1<<m/2, i;
    n>>=m;
    auto w=getw(n);
    vector<G> r((m&1)?vector{G{1, 1}}:vector{G{0, 1}, G{1, 0}});
    for (auto [p, k]:w) if (p%4==1)
    {
        vector<G> pw(k+1);
        pw[0]={1, 0};
        pw[1]=gcd(G(p, 0), G(mosqrt(p-1, p), 1));
        assert(pw[1].len2()==p);
        for (i=2; i<=k; i++) pw[i]=pw[i-1]*pw[1];
        vector<G> rr; rr.reserve(r.size()*(k+1));
        for (i=0; i<=k; i++)
        {
            G x=pw[i]*~pw[k-i];
            for (G y:r) rr.push_back(x*y);
        }
        swap(r, rr);
    }
    else
    {
        if (k%2) return { };
        k/=2;
        while (k--) d*=p;
    }
    vector<pair<ll, ll>> ans;
    ans.reserve(r.size());
    for (auto [x, y]:r) ans.push_back({abs((ll)x*d), abs((ll)y*d)});
    sort(all(ans));
```

```
        ans.resize(unique(all(ans))-ans.begin());
        return ans;
}
```

6.10 模意义真分数还原

$$q \equiv \frac{x}{a} \pmod{p}, \ |a| \le A。$$

```
pair<int, int> approx(int p,int q,int A)
{
    int x=q,y=p,a=1,b=0;
    while (x>A)
    {
        swap(x,y);swap(a,b);
        a-=x/y*b;x%=y;
    }
    return make_pair(x,a);
}
```

6.11 快速取模

```
__uint128_t brt=((__uint128_t)1<<64)/mod;
for(int i=1;i<=n;i++)
{
    ans*=i;
    ans=ans-mod*(brt*ans>>64);
    while(ans>=mod) ans-=mod;//可以替换为 if，但据说会变慢。如果循环展开则需要替换
}

struct barret{
    ll p,m; //p 表示上面的模数，m 为取模参数
    int c=0;
    inline void init(ll t){
        c=48+log2(t),p=t;
        m=(ll((ulll(1)<<c)/t));
    }
    friend inline ll operator % (ll n,const barret &d) { // get n % d
        return n-((ulll(n)*d.m)>>d.c)*d.p;
    }
}modp;
```

6.12 IO 优化

6.12.1 WDOI

```
class fast_iostream{
private:
    const int MAXBF = 1 << 20; FILE *inf, *ouf;
    char *inbuf, *inst, *ined;
    char *oubuf, *oust, *oued;
    inline void _flush(){fwrite(oubuf, 1, oued - oust, ouf);}
    inline char _getchar(){
        if(inst == ined) inst = inbuf, ined = inbuf + fread(inbuf, 1, MAXBF, inf
            );
```

```
        return inst == ined ? EOF : *inst++;
    }
    inline void _putchar(char c){
        if(oued == oust + MAXBF) _flush(), oued = oubuf;
        *oued++ = c;
    }
public:
    fast_iostream(FILE *_inf = stdin, FILE * _ouf = stdout)
    :inbuf(new char[MAXBF]), inf(_inf), inst(inbuf), ined(inbuf),
     oubuf(new char[MAXBF]), ouf(_ouf), oust(oubuf), oued(oubuf){}
    ~fast_iostream(){_flush(); delete inbuf; delete oubuf;}
    template <typename Int>
    fast_iostream& operator >> (Int &n){
        static char c;
        while((c = _getchar()) < '0' || c > '9');n = c - '0';
        while((c = _getchar()) >='0' && c <='9') n = n * 10 + c - '0';
        return *this;
    }
    template <typename Int>
    fast_iostream& operator << (Int n){
        if(n < 0) _putchar('-'), n = -n; static char S[20]; int t = 0;
        do{S[t++] = '0' + n % 10, n /= 10;} while(n);
        for(int i = 0;i < t;++i) _putchar(S[t - i - 1]);
        return *this;
    }
    fast_iostream& operator << (char c){_putchar(c); return *this;}
    fast_iostream& operator << (const char *s){
        for(int i = 0;s[i];++i) _putchar(s[i]); return *this;
    }
}fio;//unsigned
```

6.12.2 自用

```
    c[fread(c+1,1,N,stdin)+1]=0;char *cc=c;
void read(int &x)
{
    char *c=cc;
    while ((*c<48)||(*c>57)) ++c;
    x=*(c++)^48;
    while ((*c>=48)&&(*c<=57)) x=x*10+(*(c++)^48);cc=c;
}
void read(int &x)
{
    char *c=cc;fh=1;
    while ((*c<48)||(*c>57)){if (*c=='-') {++c;fh=-1;break;}++c;}
    x=*(c++)^48;
    while ((*c>=48)&&(*c<=57)) x=x*10+(*(c++)^48);
    x*=fh;cc=c;
}
void write(const int x)
{
    while (x)
    {
        st[++tp]=x%10;
        x/=10;
    }
```

```
        char *c=nc;
        while (tp) *(++c)=st[tp--]|48;
        *(++c)=10;nc=c;
}
        char *nc=sc;
        fwrite(sc+1,1,stp,stdout);
```

6.13 手动开栈

```
//#pragma comment(linker, "/STACK:102400000,102400000") 偶尔没用
    {
        static int OP=0;
        if (OP++==0)
        {
            int size=128<<20;//128MB
            char* p=new char[size]+size;
            __asm__ __volatile__("movq␣%0,␣%%rsp\n""pushq␣$exit\n""jmp␣main\n"::"
                r"(p));
        }
    }//main 开头，需要配合 exit(0) 食用
```

6.14 德扑

```
struct Q
{
    int x,y;
    bool operator<(const Q &o) const { return x>o.x; }
};
const ll inf=1e18;
ll getrk(vector<Q> a)
{
    assert(a.size()==5);
    int i,j;
    bool isf=1,iss=1,spe=0;
    sort(all(a));//decrease
    for (i=1; i<5; i++) if (a[i].y!=a[0].y) { isf=0; break; }
    for (i=1; i<5; i++) if (a[0].x!=i+a[i].x) { iss=0; break; }
    if (a[0].x==14)
    {
        for (i=1; i<5; i++) if (a[i].x!=6-i) break;
        if (i==5) iss=1,spe=1;
    }
    if (iss&&isf&&a[4].x==10) return 6*inf;
    if (iss&&isf) return 5*inf+a[4].x*!spe;
    static int cnt[15];
    static ll hash[5];
    for (auto [x,y]:a) ++cnt[x];
    memset(hash,0,5*sizeof hash[0]);
    for (auto [x,y]:a) if (cnt[x]) hash[cnt[x]]=hash[cnt[x]]*15+x,cnt[x]=0;
    if (hash[4]) return 4*inf+hash[4]*15+hash[1];
    if (hash[3]&&hash[2]) return 3*inf+hash[3]*225+hash[2];
    return hash[3]*170'859'375+hash[2]*759'375+hash[1]+iss*(inf+a[4].x*!spe)+isf
        *2*inf;
```

```
}
Q␣stoq(const␣string␣&s)
{
␣␣␣␣static␣string␣num="␣?23456789TJQKA",col="␣SHCD";
␣␣␣␣return␣{(int)num.find(s[0]),(int)col.find(s[1])};
}
```

6.15 质数，$\omega(n)$，$d(n)$，$\pi(n)$

n	n 前第一个质数	n 后第一个质数	$\max\{\omega(n)\}$	$\max\{d(n)\}$	$\pi(n) \le$
10^1	$10^1 - 3$	$10^1 + 1$	2	4	4
10^2	$10^2 - 3$	$10^2 + 1$	3	12	25
10^3	$10^3 - 3$	$10^3 + 13$	4	32	168
10^4	$10^4 - 27$	$10^4 + 7$	5	64	1229
10^5	$10^5 - 9$	$10^5 + 3$	6	128	9592
10^6	$10^6 - 17$	$10^6 + 3$	7	240	7.9×10^4
10^7	$10^7 - 9$	$10^7 + 19$	8	448	6.7×10^5
10^8	$10^8 - 11$	$10^8 + 7$	8	768	5.8×10^6
10^9	$10^9 - 63$	$10^9 + 7$	9	1344	5.1×10^7
10^{10}	$10^{10} - 33$	$10^{10} + 19$	10	2304	4.6×10^8
10^{11}	$10^{11} - 23$	$10^{11} + 3$	10	4032	4.2×10^8
10^{12}	$10^{12} - 11$	$10^{12} + 39$	11	6720	3.8×10^9
10^{13}	$10^{13} - 29$	$10^{13} + 37$	12	10752	3.5×10^{10}
10^{14}	$10^{14} - 27$	$10^{14} + 31$	12	17280	3.3×10^{11}
10^{15}	$10^{15} - 11$	$10^{15} + 37$	13	26880	3×10^{12}
10^{16}	$10^{16} - 63$	$10^{16} + 61$	13	41472	2.8×10^{13}
10^{17}	$10^{17} - 3$	$10^{17} + 3$	14	64512	
10^{18}	$10^{18} - 11$	$10^{18} + 3$	15	103680	
10^{19}	$10^{19} - 39$	$10^{19} + 51$	16	161280	

6.16 NTT 质数

$p = r \times 2^k + 1$	r	k	g（最小原根）
17	1	4	3
97	3	5	5
193	3	6	5
257	1	8	3
7681	15	9	17
12289	3	12	11
40961	5	13	3
65537	1	16	3
786433	3	18	10
5767169	11	19	3
7340033	7	20	3
23068673	11	21	3
104857601	25	22	3
167772161	5	25	3
469762049	7	26	3
998244353	119	23	3
1004535809	479	21	3
2013265921	15	27	31
2281701377	17	27	3
3221225473	3	30	5
75161927681	35	31	3
77309411329	9	33	7
206158430209	3	36	22
2061584302081	15	37	7
2748779069441	5	39	3
6597069766657	3	41	5
39582418599937	9	42	5
79164837199873	9	43	5
263882790666241	15	44	7
1231453023109121	35	45	3
1337006139375617	19	46	3
3799912185593857	27	47	5
4222124650659841	15	48	19
7881299347898369	7	50	6
31525197391593473	7	52	3
180143985094819841	5	55	6
1945555039024054273	27	56	5
4179340454199820289	29	57	3

6.17 公式

向上取整整除分块 $[i, \lfloor \frac{n-1}{\lceil \frac{n}{i} \rceil - 1} \rfloor]$

n 个点 k 个连通块的生成树方案 $n^{k-2} \prod_{i=1}^{k} siz_i$

杜教筛 $g(1)S(n) = \sum_{i=1}^{n}(f*g)(i) - \sum_{j=2}^{n}g(j)S(\lfloor\frac{n}{j}\rfloor)$

(x,y) 曼哈顿距离 $\to (x+y, x-y)$ 切比雪夫距离 (x,y) 切比雪夫距离 $\to (\frac{x+y}{2}, \frac{x-y}{2})$ 曼哈顿距离

错排数 $= \lceil 0.5 + \frac{n!}{e} \rceil$

Kummer's Theorem: $\binom{n+m}{n}$ 含 p ($p \in$ prime) 的次数是 $n+m$ 在 p 进制下的进位数

$\ln(1-x^V) = -\sum_{i \ge 1} \frac{x^{Vi}}{i}$

$x^{\bar{n}} = \sum_i S_1(n,i)x^i$

$$\begin{cases} x \equiv a_1 \pmod{m_1} \\ x \equiv a_2 \pmod{m_2} \\ \cdots \\ x \equiv a_n \pmod{m_n} \end{cases}$$

m_i 为不同的质数。设 $M = \prod_{i=1}^{n} m_i$, $t_i \times \frac{M}{m_i} \equiv 1 \pmod{m_i}$, 则 $x \equiv \sum_{i=1}^{n} a_i t_i \frac{M}{m_i}$。

$V - E + F = 2$, $S = n + \frac{s}{2} - 1$。(n 为内部, s 为边上)

用途:对于相邻的不相等的值,在中间画一条线(最外也画),连通块个数 $= 1 + E - V +$ 内部框个数

注意全都是不含矩形边界上的。

π^{-1} 最小时 π 最小, π 最大等价于 π^{-1} 最大?

五边形数 GF:$\frac{x(2x+1)}{(1-x)^3}$

五边形数:$\frac{3n^2-n}{2}$, 广义含非正,逆为分拆数 GF(注意系数正负和 n 取值奇偶性相同)

贝尔数(划分集合方案数)EGF:$\exp(e^x - 1)$, $B_n = \sum_{i=0}^{n} S_2(n,i)$, 伯努利数 EGF:$\frac{x}{e^x - 1}$

$S_1(i,m)$ EGF:$\frac{(\sum_{i \ge 0} \frac{x^i}{i})^m}{m!}$, $S_2(i,m)$ EGF:$\frac{(e^x - 1)^m}{m!}$

多项式牛顿迭代:如果已知 $G(F(x)) \equiv 0 \pmod{x^{2n}}$, $G(F_*(x)) \equiv 0 \pmod{x^n}$, 则有 $F(x) \equiv F_*(x) - \frac{G(F_*(x))}{G'(F_*(x))} \pmod{x^{2n}}$。求导时孤立的多项式视为常数。

$\int_0^1 t^a(1-t)^b \mathrm{d}t = \frac{a!b!}{(a+b+1)!}$, $\sum_{i=0}^{n-1} i^{\underline{k}} = \frac{n^{\underline{k+1}}}{k+1}$

Burnside 引理:等价类数量为 $\sum_{g \in G} \frac{X^g}{|G|}$, X^g 表示 g 变换下不动点的数量。

Polya 定理:染色方案数为 $\sum_{g \in G} \frac{m^{c(g)}}{|G|}$, 其中 $c(g)$ 表示 g 变换下环的数量。

矩阵树定理:有向图内向生成树个数计算用出度矩阵-邻接矩阵

假设已经只保留了一个牛人酋长,其名字为 $A = a_1 a_2 \cdots a_l$。

假设王国旁边开了一座赌场,每单位时间(就称为"秒"吧)会有一个赌徒带着 1 铜币进入赌场。

赌场规则很简单:支付 x 铜币赌下一秒会唱出 y, 如果猜对了就返还 nx 铜币,否则钱就没了。

每个赌徒会如下行动:支付 1 铜币赌下一秒会唱出 a_1, 如果赌对了就支付得到的 n 铜币赌下一秒会唱出 a_2, 如果还对了就支付得到的 n^2 铜币赌下一秒会

唱出 a_3，等等，以此类推，最后支付 n^{l-1} 铜币赌下一秒会唱出 a_l。

一旦连续唱出了 $a_1a_2 \cdots a_l$，赌场老板就会认为自己亏大了而关门，并驱散所有赌徒。

那么关门前发生了什么呢？以 $A = \{1,4,1,5,1,1,4,1\}, n = 5$ 为例：

- 最后一位赌徒拿着 5 铜币离开；- 倒数第三位赌徒拿着 5^3 铜币离开；- 倒数第八位赌徒拿着 5^8 铜币离开；- 其他所有赌徒空手而归。

我们可以发现 $1, 3$ 恰好是原序列的所有 border 的长度，而且对于其他的名字也有这样的规律。

这时候最神奇的一步来了：由于这个赌博游戏是公平的，因此赌场应该期望下不赚不赔，因此关门时期望来了 $5 + 5^3 + 5^8$ 个赌徒，因此期望需要 $5 + 5^3 + 5^8$ 单位时间唱出这个名字。

同理，即可知道对于一般的 A，答案为：

$$\sum_{a_1 a_2 \cdots a_c = a_{l-c+1} a_{l-c+2} \cdots a_l} n^c$$

7   语言基础

7.1   Makefile

```
%:%.cpp %.in
    g++ $< -o $@ -std=c++17 -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC
    ./$@ < $@.in
```

7.2   初始代码

```cpp
#include "bits/stdc++.h"
using namespace std;
typedef long long ll;
#define all(x) (x).begin(),(x).end()
int main()
{
    ios::sync_with_stdio(0); cin.tie(0);
    int T; cin>>T;
    while (T--)
    {

    }
}
```

7.3   bitset

```cpp
#include <bits/stdc++.h>
using namespace std;
bitset<10> f(12);
char s2[]="100101";
bitset<10> g(s2);
string s="100101";//reverse 了
bitset<10> h(s);
int main()
{
    for (int i=0;i<=9;i++) if (f[i]) printf("1"); else printf("0");puts("");
    for (int i=0;i<=9;i++) if (g[i]) printf("1"); else printf("0");puts("");
    for (int i=0;i<=9;i++) if (h[i]) printf("1"); else printf("0");puts("");
    cout<<h<<endl;
    foo.count();//1的个数
    foo.flip();//全部翻转
    foo.set();//变1
    foo.reset();//变0
    foo.to_string();
    foo.to_ulong();
    foo.to_ullong();
    foo._Find_first();
    foo._Find_next();
    //位运算: << 变大, >> 变小
    __builtin_clz();//前导 0
    __builtin_ctz();//后面的 0
}
```

输出：

```
0011000000
1010010000
1010010000
0000100101
```

7.4 pb_ds 和一些奇怪的用法

```cpp
#pragma GCC optimize("Ofast")
#pragma GCC target("popcnt","sse3","sse2","sse","avx","sse4","sse4.1","sse4.2",
    "ssse3","f16c","fma","avx2","xop","fma4")
#pragma GCC optimize("inline","fast-math","unroll-loops","no-stack-protector")
#pragma GCC diagnostic error "-fwhole-program"
#pragma GCC diagnostic error "-fcse-skip-blocks"
#pragma GCC diagnostic error "-funsafe-loop-optimizations"
#include "bits/stdc++.h"
#include "ext/pb_ds/assoc_container.hpp"
#include "ext/pb_ds/tree_policy.hpp" //balanced tree
#include "ext/pb_ds/hash_policy.hpp" //hash table
#include "ext/pb_ds/priority_queue.hpp" //priority_queue
using namespace __gnu_pbds;
using namespace std;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> rbtree;
cc_hash_table<string,int>mp1;//拉链法
gp_hash_table<string,int>mp2;//查探法
rbtree s1,s2;//注意是不可重的
//null_type无映射(低版本g++为null_mapped_type)
//less<int>从小到大排序
//插入t.insert();
//删除t.erase();
//求有多少个数比 k 小:t.order_of_key(k);
//求树中第 k+1 小:t.find_by_order(k);
//a.join(b) b并入a, 前提是两棵树的 key 的取值范围不相交, b 会清空但迭代器没事, 如
    不满足会抛出异常。我听说复杂度是线性???
//a.split(v,b) key 小于等于 v 的元素属于 a, 其余的属于 b
//T.lower_bound(x) >=x 的 min 的迭代器
//T.upper_bound(x) >x 的 min 的迭代器
__gnu_pbds::priority_queue<int,greater<int>,pairing_heap_tag> pq;
//join(priority_queue &other) //合并两个堆,other会被清空
//split(Pred prd,priority_queue &other) //分离出两个堆
//modify(point_iterator it,const key) //修改一个节点的值
inline char gc()
{
    static char buf[1048576], *p1, *p2;
    return p1 == p2 && (p2 = (p1 = buf) + fread(buf, 1, 1048576, stdin),
    p1 == p2) ? EOF : *p1++;
}
inline int read()
{
    char ch = gc(); int r = 0, w = 1;
    for (; ch < '0' || ch > '9'; ch = gc()) if (ch == '-') w = -1;
    for (; '0' <= ch && ch <= '9'; ch = gc()) r = r * 10 + (ch - '0');
    return r * w;
}
struct my_bit
{
```

```cpp
// ll v[Len];
__m256i V[Len/4];
void reset()
{

    V[0]=_mm256_set_epi64x(0, 0, 0, 0);
    V[1]=_mm256_set_epi64x(0, 0, 0, 0);
    V[2]=_mm256_set_epi64x(0, 0, 0, 0);
    V[3]=_mm256_set_epi64x(0, 0, 0, 0);
    V[4]=_mm256_set_epi64x(0, 0, 0, 0);
    V[5]=_mm256_set_epi64x(0, 0, 0, 0);
    V[6]=_mm256_set_epi64x(0, 0, 0, 0);
    V[7]=_mm256_set_epi64x(0, 0, 0, 0);
    V[8]=_mm256_set_epi64x(0, 0, 0, 0);
    V[9]=_mm256_set_epi64x(0, 0, 0, 0);
    V[10]|=_mm256_set_epi64x(0, 0, 0, 0);
    V[11]|=_mm256_set_epi64x(0, 0, 0, 0);
    V[12]|=_mm256_set_epi64x(0, 0, 0, 0);
    V[13]|=_mm256_set_epi64x(0, 0, 0, 0);
}
void set(int u)
{
    switch (u>>6&3)
    {
    case 0:
        V[u>>8]|=_mm256_set_epi64x(1ull<<(u&63), 0, 0, 0);
        break;
    case 1:
        V[u>>8]|=_mm256_set_epi64x(0, 1ull<<(u&63), 0, 0);
        break;
    case 2:
        V[u>>8]|=_mm256_set_epi64x(0, 0, 1ull<<(u&63), 0);
        break;
    case 3:
        V[u>>8]|=_mm256_set_epi64x(0, 0, 0, 1ull<<(u&63));
        break;
    }
        // v[u>>6]|=(1ull<<(u&63));
}
void operator |= (const my_bit &B)
{
    V[0]|=B.V[0];
    V[1]|=B.V[1];
    V[2]|=B.V[2];
    V[3]|=B.V[3];
    V[4]|=B.V[4];
    V[5]|=B.V[5];
    V[6]|=B.V[6];
    V[7]|=B.V[7];
    V[8]|=B.V[8];
    V[9]|=B.V[9];
    V[10]|=B.V[10];
    V[11]|=B.V[11];
    V[12]|=B.V[12];
    V[13]|=B.V[13];
    // V[6]|=B.V[6];
    // V[7]|=B.V[7];
```

```cpp
    // V[8]|=B.V[8];
    // V[9]|=B.V[9];
    // V[10]|=B.V[10];
    // V[11]|=B.V[11];
    // V[12]|=B.V[12];
    // V[13]|=B.V[13];
    // V[14]|=B.V[14];
    // V[15]|=B.V[15];
    // V[16]|=B.V[16];
    // V[17]|=B.V[17];
    // V[18]|=B.V[18];
    // V[19]|=B.V[19];
    // V[20]|=B.V[20];
    // V[21]|=B.V[21];
    // V[22]|=B.V[22];
    // V[23]|=B.V[23];
}
int count()
{
    return
        __builtin_popcountll(((ll *)&(V[0]))[0])+__builtin_popcountll(((ll *)
            &(V[0]))[1])
        +__builtin_popcountll(((ll *)&(V[0]))[2])+__builtin_popcountll(((ll
            *)&(V[0]))[3])
        +__builtin_popcountll(((ll *)&(V[1]))[0])+__builtin_popcountll(((ll
            *)&(V[1]))[1])
        +__builtin_popcountll(((ll *)&(V[1]))[2])+__builtin_popcountll(((ll
            *)&(V[1]))[3])
        +__builtin_popcountll(((ll *)&(V[2]))[0])+__builtin_popcountll(((ll
            *)&(V[2]))[1])
        +__builtin_popcountll(((ll *)&(V[2]))[2])+__builtin_popcountll(((ll
            *)&(V[2]))[3])
        +__builtin_popcountll(((ll *)&(V[3]))[0])+__builtin_popcountll(((ll
            *)&(V[3]))[1])
        +__builtin_popcountll(((ll *)&(V[3]))[2])+__builtin_popcountll(((ll
            *)&(V[3]))[3])
        +__builtin_popcountll(((ll *)&(V[4]))[0])+__builtin_popcountll(((ll
            *)&(V[4]))[1])
        +__builtin_popcountll(((ll *)&(V[4]))[2])+__builtin_popcountll(((ll
            *)&(V[4]))[3])
        +__builtin_popcountll(((ll *)&(V[5]))[0])+__builtin_popcountll(((ll
            *)&(V[5]))[1])
        +__builtin_popcountll(((ll *)&(V[5]))[2])+__builtin_popcountll(((ll
            *)&(V[5]))[3])
        +__builtin_popcountll(((ll *)&(V[6]))[0])+__builtin_popcountll(((ll
            *)&(V[6]))[1])
        +__builtin_popcountll(((ll *)&(V[6]))[2])+__builtin_popcountll(((ll
            *)&(V[6]))[3])
        +__builtin_popcountll(((ll *)&(V[7]))[0])+__builtin_popcountll(((ll
            *)&(V[7]))[1])
        +__builtin_popcountll(((ll *)&(V[7]))[2])+__builtin_popcountll(((ll
            *)&(V[7]))[3])
        +__builtin_popcountll(((ll *)&(V[8]))[0])+__builtin_popcountll(((ll
            *)&(V[8]))[1])
        +__builtin_popcountll(((ll *)&(V[8]))[2])+__builtin_popcountll(((ll
            *)&(V[8]))[3])
        +__builtin_popcountll(((ll *)&(V[9]))[0])+__builtin_popcountll(((ll
```

```
                *)&(V[9]))[1])
            +__builtin_popcountll(((ll *)&(V[9]))[2])+__builtin_popcountll(((ll
                *)&(V[9]))[3])
            +__builtin_popcountll(((ll *)&(V[10]))[0])+__builtin_popcountll(((ll
                *)&(V[10]))[1])
            +__builtin_popcountll(((ll *)&(V[10]))[2])+__builtin_popcountll(((ll
                *)&(V[10]))[3])
            +__builtin_popcountll(((ll *)&(V[11]))[0])+__builtin_popcountll(((ll
                *)&(V[11]))[1])
            +__builtin_popcountll(((ll *)&(V[11]))[2])+__builtin_popcountll(((ll
                *)&(V[11]))[3])
            +__builtin_popcountll(((ll *)&(V[12]))[0])+__builtin_popcountll(((ll
                *)&(V[12]))[1])
            +__builtin_popcountll(((ll *)&(V[12]))[2])+__builtin_popcountll(((ll
                *)&(V[12]))[3])
            +__builtin_popcountll(((ll *)&(V[13]))[0])+__builtin_popcountll(((ll
                *)&(V[13]))[1])
            +__builtin_popcountll(((ll *)&(V[13]))[2])+__builtin_popcountll(((ll
                *)&(V[13]))[3]);
        // int ans=0;
        // return __builtin_popcountll(v[0])
        //  +__builtin_popcountll(v[1])
        //  +__builtin_popcountll(v[2])
        //  +__builtin_popcountll(v[3])
        //  +__builtin_popcountll(v[4])
        //  +__builtin_popcountll(v[5])
        //  +__builtin_popcountll(v[6])
        //  +__builtin_popcountll(v[7])
        //  +__builtin_popcountll(v[8])
        //  +__builtin_popcountll(v[9])
        //  +__builtin_popcountll(v[10])
        //  +__builtin_popcountll(v[11])
        //  +__builtin_popcountll(v[12])
        //  +__builtin_popcountll(v[13])
        //  +__builtin_popcountll(v[14])
        //  +__builtin_popcountll(v[15])
        //  +__builtin_popcountll(v[16])
        //  +__builtin_popcountll(v[17])
        //  +__builtin_popcountll(v[18])
        //  +__builtin_popcountll(v[19])
        //  +__builtin_popcountll(v[20])
        //  +__builtin_popcountll(v[21])
        //  +__builtin_popcountll(v[22])
        //  +__builtin_popcountll(v[23]);
            // return ans;
    }
}r[N];
int main()
{
    ios::sync_with_stdio(0);cin.tie(0);
    mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
    cout<<setiosflags(ios::fixed)<<setprecision(15);
    rbtree::iterator it;
    string s="abc",t="dabce";
    boyer_moore_horspool_searcher S(all(s));
    if (search(all(t),S)!=t.end())
    {
```

```cpp
        cout<<"find\n";
    }
uniform_real_distribution<> a(1,2);
numeric_limits<int>::max();
for (int i=1;i<=10;i++) s1.insert(i*2);
//it=s2.lower_bound(35);template<class T> vector<vector<T>> min_cut(int n,
    const vector<tuple<int, int, T>> &edges)//[0,n)
{
    int m=edges.size(), i, s, t, cnt=0;
    vector<int> fir(n, -1), nxt(m*2, -1), fc(n), q(n);
    vector<pair<int, T>> e(m*2);
    vector<tuple<T, int, int>> eg;
    auto add=[&](int u, int v, T w)
        {
            e[cnt]={v, w};
            nxt[cnt]=fir[u];
            fir[u]=cnt++;
        };
    for (auto [u, v, w]:edges) add(u, v, w), add(v, u, w);
    auto E=e;
    auto bfs=[&]()
        {
            fill(all(fc), 0);
            int ql=0, qr=0, u, i;
            fc[q[0]=s]=1;
            while (ql<=qr)
            {
                u=q[ql++];
                for (int i=fir[u]; i!=-1; i=nxt[i])
                    if (auto &[v, w]=e[i]; w&&!fc[v]) fc[q[++qr]=v]=fc[u]+1;
            }
            return fc[t];
        };
    function<T(int, T)> dfs=[&](int u, T maxf)
        {
            if (u==t) return maxf;
            T j=0, k;
            for (int i=fir[u]; i!=-1; i=nxt[i])
                if (auto &[v, w]=e[i]; w&&fc[v]==fc[u]+1&&(k=dfs(v, min(maxf-j
                    , w))))
                {
                    j+=k;
                    w-=k;
                    e[i^1].second+=k;
                    if (j==maxf) return j;
                }
            fc[u]=0;
            return j;
        };
    function<void(vector<int>)> solve=[&](vector<int> id)
        {
            static mt19937 rnd(chrono::steady_clock::now().time_since_epoch()
                .count());
            if (id.size()<=1) return;
            vector<int> u(2);
            sample(all(id), u.begin(), 2, rnd);
            s=u[0], t=u[1], e=E;
```

```cpp
            T ans=0;
            while (bfs()) ans+=dfs(s, numeric_limits<T>::max());
            auto it=partition(all(id), [&](int u) { return fc[u]; });
            eg.emplace_back(ans, s, t);
            solve(vector(id.begin(), it));
            solve(vector(it, id.end()));
        };
    solve(range(0, n));
    sort(all(eg), greater<>());
    vector<basic_string<int>> ver(n);
    vector ans(n, vector<T>(n));
    vector<int> f(n);
    for (i=0; i<n; i++) ver[i]={f[i]=i};
    function<int(int)> getf=[&](int u) { return f[u]==u?u:f[u]=getf(f[u]);
        };
    for (auto [w, u, v]:eg)
    {
        u=getf(u);
        v=getf(v);
        for (int w1:ver[u]) for (int w2:ver[v]) ans[w1][w2]=ans[w2][w1]=w;
        ver[u]+=ver[v];
        f[v]=u;
    }
    return ans;
}
for (auto u:s1) printf("%d\n",u);puts("");
printf("%d\n",*s1.find_by_order(10));
//printf("%d\n",*it);
}
```

7.5　python 使用方法

```python
fi = open("discuss.in", "r")
fo = open("discuss.out", "w")
n=int(fi.readline())
fo.write(str(ans))
```

8 其他板子（补充）

8.1 MTT+exp

```cpp
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
typedef double db;
int read(){
    int res=0;
    char c=getchar(),f=1;
    while(c<48||c>57){if(c=='-')f=0;c=getchar();}
    while(c>=48&&c<=57)res=(res<<3)+(res<<1)+(c&15),c=getchar();
    return f?res:-res;
}

const int L=1<<19,mod=1e9+7;
const db pi2=3.141592653589793*2;
int inc(int x,int y){return x+y>=mod?x+y-mod:x+y;}
int dec(int x,int y){return x-y<0?x-y+mod:x-y;}
int mul(int x,int y){return (ll)x*y%mod;}
int qpow(int x,int y){
    int res=1;
    for(;y;y>>=1)res=y&1?mul(res,x):res,x=mul(x,x);
    return res;
}
int inv(int x){return qpow(x,mod-2);}

struct cp{
    db x,y;
    cp(){}
    cp(db a,db b){x=a,y=b;}
    cp operator+(const cp& p)const{return cp(x+p.x,y+p.y);}
    cp operator-(const cp& p)const{return cp(x-p.x,y-p.y);}
    cp operator*(const cp& p)const{return cp(x*p.x-y*p.y,x*p.y+y*p.x);}
    cp conj(){return cp(x,-y);}
}w[L];
int re[L];
int getre(int n){
    int len=1,bit=0;
    while(len<n)++bit,len<<=1;
    for(int i=1;i<len;++i)re[i]=(re[i>>1]>>1)|((i&1)<<(bit-1));
    return len;
}
void getw(){
    for(int i=0;i<L;++i)w[i]=cp(cos(pi2/L*i),sin(pi2/L*i));
}
void fft(cp* a,int len,int m){
    for(int i=1;i<len;++i)if(i<re[i])swap(a[i],a[re[i]]);
    for(int k=1,r=L>>1;k<len;k<<=1,r>>=1)
        for(int i=0;i<len;i+=k<<1)
            for(int j=0;j<k;++j){
                cp &L=a[i+j],&R=a[i+j+k],t=w[r*j]*R;
                R=L-t,L=L+t;
            }
    if(!~m){
        reverse(a+1,a+len);
```

```cpp
            cp tmp=cp(1.0/len,0);
            for(int i=0;i<len;++i)a[i]=a[i]*tmp;
        }
}
void mul(int* a,int* b,int* c,int n1,int n2,int n){
    static cp f1[L],f2[L],f3[L],f4[L];
    int len=getre(n1+n2-1);
    for(int i=0;i<len;++i){
        f1[i]=i<n1?cp(a[i]>>15,a[i]&32767):cp(0,0);
        f2[i]=i<n2?cp(b[i]>>15,b[i]&32767):cp(0,0);
    }
    fft(f1,len,1),fft(f2,len,1);
    cp t1=cp(0.5,0),t2=cp(0,-0.5),r=cp(0,1);
    cp x1,x2,x3,x4;
    for(int i=0;i<len;++i){
        int j=(len-i)&(len-1);
        x1=(f1[i]+f1[j].conj())*t1;
        x2=(f1[i]-f1[j].conj())*t2;
        x3=(f2[i]+f2[j].conj())*t1;
        x4=(f2[i]-f2[j].conj())*t2;
        f3[i]=x1*(x3+x4*r);
        f4[i]=x2*(x3+x4*r);
    }
    fft(f3,len,-1),fft(f4,len,-1);
    ll c1,c2,c3,c4;
    for(int i=0;i<n;++i){
        c1=(ll)(f3[i].x+0.5)%mod,c2=(ll)(f3[i].y+0.5)%mod;
        c3=(ll)(f4[i].x+0.5)%mod,c4=(ll)(f4[i].y+0.5)%mod;
        c[i]=((((c1<<15)+c2+c3)<<15)+c4)%mod;
    }
}
void inv(int* a,int* b,int n){
    if(n==1){b[0]=1;return;}
    static int c[L];
    int l=(n+1)>>1;
    inv(a,b,l);
    mul(a,b,c,n,l,n);
    for(int i=0;i<n;++i)c[i]=mod-c[i];
    c[0]+=2;
    mul(b,c,b,n,n,n);
}
void der(int* a,int n){
    for(int i=1;i<n;++i)a[i-1]=mul(a[i],i);
    a[n-1]=0;
}
void its(int* a,int n){
    for(int i=n-1;i;--i)a[i]=mul(a[i-1],inv(i));
    a[0]=0;
}
void ln(int* a,int* b,int n){
    static int c[L];
    for(int i=0;i<n;++i)c[i]=a[i];
    der(c,n);
    inv(a,b,n);
    mul(b,c,b,n,n,n);
    its(b,n);
}
```

```cpp
void exp(int* a,int* b,int n){
    if(n==1){b[0]=1;return;}
    static int c[L];
    int l=(n+1)>>1;
    exp(a,b,l);
    ln(b,c,n);
    for(int i=0;i<n;++i)c[i]=dec(a[i],c[i]);
    ++c[0];
    mul(b,c,b,l,n,n);
    for(int i=0;i<n;++i)c[i]=0;
}

int n,k,a[L],f[L],g[L];
int main(){
    getw();
    n=read(),k=read();
    for(int i=1;i<=k;++i)a[i]=inv(i);
    for(int i=2;i<=n;++i)
        for(int j=1;i*j<=k;++j)
            f[i*j]=inc(f[i*j],a[j]);
    for(int i=1;i<=k;++i)f[i]=mod-f[i];
    for(int i=1;i<=k;++i)f[i]=inc(f[i],mul(n-1,a[i]));
    exp(f,g,k+1);
    printf("%d\n",g[k]);
}
```

8.2 多项式

```cpp
#include<bits/stdc++.h>
using namespace std;
typedef long long ll;
int read(){
    int res=0;
    char c=getchar(),f=1;
    while(c<48||c>57){if(c=='-')f=0;c=getchar();}
    while(c>=48&&c<=57)res=(res<<3)+(res<<1)+(c&15),c=getchar();
    return f?res:-res;
}
void write(int x){
    char c[21];
    int len=0;
    if(!x)return putchar('0'),void();
    if(x<0)x=-x,putchar('-');
    while(x)c[++len]=x%10,x/=10;
    while(len)putchar(c[len--]+48);
}
#define space(x) write(x),putchar(' ')
#define enter(x) write(x),putchar('\n')

const int mod=998244353;
struct M{
    int x;
    M(int a=0):x(a){}
    M operator+(const M& p)const{return x+p.x>=mod?x+p.x-mod:x+p.x;}
    M operator-()const{return x?mod-x:0;}
    M operator-(const M& p)const{return x-p.x<0?x-p.x+mod:x-p.x;}
```

```cpp
    M operator*(const M& p)const{return (ll)x*p.x%mod;}
    bool operator==(const int& p)const{return x==p;}
    void operator+=(const M& p){*this=*this+p;}
    void operator-=(const M& p){*this=*this-p;}
    void operator*=(const M& p){*this=*this*p;}
};
void write(const M& x){write(x.x);}

M qpow(M x,int y){
    M res(1);
    for(;y;y>>=1)res=y&1?res*x:res,x=x*x;
    return res;
}
M inv(M x){return qpow(x,mod-2);}

const int N=1<<21|7;
namespace NTT{
int re[N];
M w[2][N];
int getre(int n){
    int len=1,bit=0;
    while(len<n)len<<=1,++bit;
    for(int i=1;i<len;++i)re[i]=(re[i>>1]>>1)|((i&1)<<(bit-1));
    w[0][0]=w[1][0]=1,w[0][1]=qpow(3,(mod-1)/len),w[1][1]=inv(w[0][1]);
    for(int o=0;o<2;++o)for(int i=2;i<=len;++i)
        w[o][i]=w[o][i-1]*w[o][1];
    return len;
}
void NTT(M* a,int n,int o=0){
    for(int i=1;i<n;++i)if(i<re[i])swap(a[i],a[re[i]]);
    M L,R;
    for(int k=1;k<n;k<<=1)
        for(int i=0,st=n/(k<<1);i<n;i+=k<<1)
            for(int j=0,nw=0;j<k;++j,nw+=st){
                L=a[i+j],R=a[i+j+k]*w[o][nw];
                a[i+j]=L+R,a[i+j+k]=L-R;
            }
    if(o){
        L=inv(n);
        for(int i=0;i<n;++i)a[i]=a[i]*L;
    }
}

M t0[N],t1[N],t2[N];
void mul(const M* a,const M* b,M* c,int n,int m){
    int len=getre(n+m+1);
    memset(t0,0,sizeof(int)*len),memcpy(t0,a,sizeof(int)*(n+1));
    memset(t1,0,sizeof(int)*len),memcpy(t1,b,sizeof(int)*(m+1));
    NTT(t0,len),NTT(t1,len);
    for(int i=0;i<len;++i)t0[i]=t0[i]*t1[i];
    NTT(t0,len,1);
    memcpy(c,t0,sizeof(int)*(n+m+1));
}
void inv(const M* a,M* b,int n){
    int len=1;
    while(len<=n)len<<=1;
    memset(t0,0,sizeof(int)*len),memcpy(t0,a,sizeof(int)*(n+1));
```

```
        memset(t1,0,sizeof(int)*(len<<1));
        memset(t2,0,sizeof(int)*(len<<1));
        t2[0]=inv(t0[0]);
        for(int k=1;k<=len;k<<=1){
            memcpy(t1,t0,sizeof(int)*k);
            getre(k<<1);
            NTT(t1,k<<1),NTT(t2,k<<1);
            for(int i=0;i<(k<<1);++i)t2[i]*=(-t1[i]*t2[i]+2);
            NTT(t2,k<<1,1);
            for(int i=k;i<(k<<1);++i)t2[i]=0;
        }
        memcpy(b,t2,sizeof(int)*(n+1));
    }
} //namespace NTT

struct poly:public vector<M>{
    int time()const{return size()-1;}
    poly(int tim=0,int c=0){
        resize(tim+1);
        if(tim>=0)at(0)=c;
    }
    poly operator%(const int& n)const{
        poly r(*this);
        r.resize(n);
        return r;
    }
    poly operator%=(const int& n){
        resize(n);
        return *this;
    }
    poly operator+(const poly& p)const{
        int n=time(),m=p.time();
        poly r(*this);
        if(n<m)r.resize(m+1);
        for(int i=0;i<=m;++i)r[i]+=p[i];
        return r;
    }
    poly operator-(const poly& p)const{
        int n=time(),m=p.time();
        poly r(*this);
        if(n<m)r.resize(m+1);
        for(int i=0;i<=m;++i)r[i]-=p[i];
        return r;
    }
    poly operator*(const poly& p)const{
        poly r(time()+p.time());
        NTT::mul(&((*this)[0]),&p[0],&r[0],time(),p.time());
        return r;
    }
};

poly inv(const poly& a){
    poly r(a.time());
    NTT::inv(&a[0],&r[0],a.time());
    return r;
}
```

```
poly der(const poly& a){
    int n=a.time();
    poly r(n-1);
    for(int i=1;i<=n;++i)r[i-1]=a[i]*i;
    return r;
}
M _[N];
poly itr(const poly& a){
    int n=a.time();
    poly r(n+1);
    _[1]=1;
    for(int i=2;i<=n+1;++i)_[i]=_[mod%i]*(mod-mod/i);
    for(int i=0;i<=n;++i)r[i+1]=a[i]*_[i+1];
    return r;
}
poly ln(const poly& a){
    return itr(der(a)*inv(a)%a.time());
}

poly exp(const poly& a){
    poly r(0,1);
    int n=a.time(),k=1;
    while(r.time()<n)
        r%=k,r=r*(a%k-ln(r)+poly(0,1))%k,k<<=1;
    return r%(n+1);
}

void read(poly& a,int n=-1){
    if(!~n)n=a.time();
    else a.resize(n+1);
    for(int i=0;i<=n;++i)a[i]=read();
}
void write(const poly& a,int n=-1){
    if(!~n)n=a.time();
    else n=min(n,a.time());
    for(int i=0;i<n;++i)space(a[i]);
    enter(a[n]);
}
```

8.3 Miller Rabin/Pollard Rho

1s：200 组 10^{18}。

```
namespace pr
{
    typedef long long ll;
    typedef __int128 lll;
    typedef pair<ll,int> pa;
    ll ksm(ll x,ll y,const ll p)
    {
        ll r=1;
        while (y)
        {
            if (y&1) r=(lll)r*x%p;
            x=(lll)x*x%p; y>>=1;
        }
        return r;
```

```
    }
namespace miller
{
    const int p[7]={2,3,5,7,11,61,24251};
    ll s,t;
    bool test(ll n,int p)
    {
        if (p>=n) return 1;
        ll r=ksm(p,t,n),w;
        for (int j=0; j<s&&r!=1; j++)
        {
            w=(lll)r*r%n;
            if (w==1&&r!=n-1) return 0;
            r=w;
        }
        return r==1;
    }
    bool prime(ll n)
    {
        if (n<2||n==46'856'248'255'981ll) return 0;
        for (int i=0; i<7; ++i) if (n%p[i]==0) return n==p[i];
        s=__builtin_ctz(n-1); t=n-1>>s;
        for (int i=0; i<7; ++i) if (!test(n,p[i])) return 0;
        return 1;
    }
}
using miller::prime;
mt19937_64 rnd(chrono::steady_clock::now().time_since_epoch().count());
namespace rho
{
    void nxt(ll &x,ll &y,ll &p) { x=((lll)x*x+y)%p; }
    ll find(ll n,ll C)
    {
        ll l,r,d,p=1;
        l=rnd()%(n-2)+2,r=l;
        nxt(r,C,n);
        int cnt=0;
        while (l^r)
        {
            p=(lll)p*llabs(l-r)%n;
            if (!p) return gcd(n,llabs(l-r));
            ++cnt;
            if (cnt==127)
            {
                cnt=0;
                d=gcd(llabs(l-r),n);
                if (d>1) return d;
            }
            nxt(l,C,n); nxt(r,C,n); nxt(r,C,n);
        }
        return gcd(n,p);
    }
    vector<pa> w;
    vector<ll> d;
    void dfs(ll n,int cnt)
    {
        if (n==1) return;
```

```
            if (prime(n)) return w.emplace_back(n,cnt),void();
            ll p=n,C=rnd()%(n-1)+1;
            while (p==1||p==n) p=find(n,C++);
            int r=1; n/=p;
            while (n%p==0) n/=p,++r;
            dfs(p,r*cnt); dfs(n,cnt);
        }
        vector<pa> getw(ll n)
        {
            w=vector<pa>(0); dfs(n,1);
            if (n==1) return w;
            sort(w.begin(),w.end());
            int i,j;
            for (i=1,j=0; i<w.size(); i++) if (w[i].first==w[j].first) w[j].
                second+=w[i].second; else w[++j]=w[i];
            w.resize(j+1);
            return w;
        }
        void dfss(int x,ll n)
        {
            if (x==w.size()) return d.push_back(n),void();
            dfss(x+1,n);
            for (int i=1; i<=w[x].second; i++) dfss(x+1,n*=w[x].first);
        }
        vector<ll> getd(ll n)
        {
            getw(n); d=vector<ll>(0); dfss(0,1);
            sort(d.begin(),d.end());
            return d;
        }
    }
    using rho::getw,rho::getd;
    using miller::prime;
}
using pr::getw,pr::getd,pr::prime;
```

8.4 半平面交

```
const int N=305;
const db inf=1e15,eps=1e-10;
int sign(db x){
    if(fabs(x)<eps)return 0;
    return x>0?1:-1;
}

struct vec{
    db x,y;
    vec(){}
    vec(db a,db b){x=a,y=b;}
    vec operator+(const vec& p)const{
        return vec(x+p.x,y+p.y);
    }
    vec operator-(const vec& p)const{
        return vec(x-p.x,y-p.y);
    }
    db operator*(const vec& p)const{
```

```
            return x*p.y-y*p.x;
        }
        vec operator*(const db& p)const{
            return vec(x*p,y*p);
        }
}p1[N],p2[N];

struct line{
    vec s,t;
    line(){}
    line(vec a,vec b){s=a,t=b;}
}a[N],q[N];
db ang(vec v){
    return atan2(v.y,v.x);
}
db ang(line l){
    return ang(l.t-l.s);
}
bool cmp(line x,line y){
    int s=sign(ang(x)-ang(y));
    return s?s<0:sign((x.t-x.s)*(y.t-x.s))>0;
}

vec inter(line x,line y){
    vec a=y.s-x.s,b=x.t-x.s,c=y.t-y.s;
    return y.s+c*((a*b)/(b*c));
}
bool out(line l,vec p){
    return sign((l.t-l.s)*(p-l.s))<0;
}

int n,tot=0;
db ans=inf;
int main(){
    scanf("%d",&n);
    for(int i=1;i<=n;++i)scanf("%lf",&p1[i].x);
    for(int i=1;i<=n;++i)scanf("%lf",&p1[i].y);
    for(int i=1;i<n;++i)a[i]=line(p1[i],p1[i+1]);
    a[n]=line(vec(p1[1].x,inf),vec(p1[1].x,p1[1].y));
    a[n+1]=line(vec(p1[n].x,p1[n].y),vec(p1[n].x,inf));

    sort(a+1,a+n+2,cmp);
    for(int i=1;i<=n;++i){
        if(!sign(ang(a[i])-ang(a[i+1])))continue;
        a[++tot]=a[i];
    }a[++tot]=a[n+1];

    int l=1,r=0;
    q[++r]=a[1],q[++r]=a[2];
    for(int i=3;i<=tot;++i){
        while(l<r&&out(a[i],inter(q[r],q[r-1])))--r;
        while(l<r&&out(a[i],inter(q[l],q[l+1])))++l;
        q[++r]=a[i];
    }
    while(l<r&&out(q[l],inter(q[r],q[r-1])))--r;
    while(l<r&&out(q[r],inter(q[l],q[l+1])))++l;
//......
```

```
}
```

8.5 旋转卡壳

```
if(top==3)return !printf("%d\n",dis(a[sta[1]],a[sta[2]]));
for(int i=1,j=2;i<top;++i){
    while(area(a[sta[i]],a[sta[i+1]],a[sta[j]])>=area(a[sta[i]],a[sta[i+1]],a[
        sta[j%top+1]]))j=j%top+1;
    ans=max(ans,max(dis(a[sta[i]],a[sta[j]]),dis(a[sta[i+1]],a[sta[j]])));
}printf("%d\n",ans);
```

8.6 多项式复合 (yurzhang)

$O(n \log n\sqrt{n \log n})$，奇慢无比，慎用

```
#pragma GCC optimize("Ofast,inline")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,sse4.1,sse4.2,popcnt,abm,mmx,avx,
    avx2,tune=native")
#include <cstdio>
#include <cstring>
#include <cmath>
#include <algorithm>

#define MOD 998244353
#define G 332748118
#define N 262210
#define re register
#define gc pa==pb&&(pb=(pa=buf)+fread(buf,1,100000,stdin),pa==pb)?EOF:*pa++
typedef long long ll;
static char buf[100000],*pa(buf),*pb(buf);
static char pbuf[3000000],*pp(pbuf),st[15];
int read() {
    re int x(0);re char c(gc);
    while(c<'0'||c>'9')c=gc;
    while(c>='0'&&c<='9')
        x=x*10+c-48,c=gc;
    return x;
}
void write(re int v) {
    if(v==0)
        *pp++=48;
    else {
        re int tp(0);
        while(v)
            st[++tp]=v%10+48,v/=10;
        while(tp)
            *pp++=st[tp--];
    }
    *pp++=32;
}

int pow(re int a,re int b) {
    re int ans(1);
    while(b)
        ans=b&1?(ll)ans*a%MOD:ans,a=(ll)a*a%MOD,b>>=1;
```

```cpp
        return ans;
}

int inv[N],ifac[N];
void pre(re int n) {
    inv[1]=ifac[0]=1;
    for(re int i(2);i<=n;++i)
        inv[i]=(ll)(MOD-MOD/i)*inv[MOD%i]%MOD;
    for(re int i(1);i<=n;++i)
        ifac[i]=(ll)ifac[i-1]*inv[i]%MOD;
}

int getLen(re int t) {
    return 1<<(32-__builtin_clz(t));
}

int lmt(1),r[N],w[N];
void init(re int n) {
    re int l(0);
    while(lmt<=n)
        lmt<<=1,++l;
    for(re int i(1);i<lmt;++i)
        r[i]=(r[i>>1]>>1)|((i&1)<<(l-1));
    re int wn(pow(3,(MOD-1)/lmt));
    w[lmt>>1]=1;
    for(re int i((lmt>>1)+1);i<lmt;++i)
        w[i]=(ll)w[i-1]*wn%MOD;
    for(re int i((lmt>>1)-1);i;--i)
        w[i]=w[i<<1];
}

void DFT(int*a,re int l) {
    static unsigned long long tmp[N];
    re int u(__builtin_ctz(lmt)-__builtin_ctz(l)),t;
    for(re int i(0);i<l;++i)
        tmp[i]=(a[r[i]>>u])%MOD;
    for(re int i(1);i<l;i<<=1)
        for(re int j(0),step(i<<1);j<l;j+=step)
            for(re int k(0);k<i;++k)
                t=(ll)w[i+k]*tmp[i+j+k]%MOD,
                tmp[i+j+k]=tmp[j+k]+MOD-t,
                tmp[j+k]+=t;
    for(re int i(0);i<l;++i)
        a[i]=tmp[i]%MOD;
}

void IDFT(int*a,re int l) {
    std::reverse(a+1,a+l);DFT(a,l);
    re int bk(MOD-(MOD-1)/l);
    for(re int i(0);i<l;++i)
        a[i]=(ll)a[i]*bk%MOD;
}

int n,m;
int a[N],b[N],c[N];

void getInv(int*a,int*b,int deg) {
```

```
    if(deg==1)
        b[0]=pow(a[0],MOD-2);
    else {
        static int tmp[N];
        getInv(a,b,(deg+1)>>1);
        re int l(getLen(deg<<1));
        for(re int i(0);i<l;++i)
            tmp[i]=i<deg?a[i]:0;
        DFT(tmp,l),DFT(b,l);
        for(re int i(0);i<l;++i)
            b[i]=(2ll-(ll)tmp[i]*b[i]%MOD+MOD)%MOD*b[i]%MOD;
        IDFT(b,l);
        for(re int i(deg);i<l;++i)
            b[i]=0;
    }
}

void getDer(int*a,int*b,int deg) {
    for(re int i(0);i+1<deg;++i)
        b[i]=(ll)a[i+1]*(i+1)%MOD;
    b[deg-1]=0;
}

void getComp(int*a,int*b,int k,int m,int&n,int*c,int*d) {
    if(k==1) {
        for(re int i(0);i<m;++i)
            c[i]=0,d[i]=b[i];
        n=m,c[0]=a[0];
    } else {
        static int t1[N],t2[N];
        int nl(n),nr(n),*cl,*cr,*dl,*dr;
        getComp(a,b,k>>1,m,nl,cl=c,dl=d);
        getComp(a+(k>>1),b,(k+1)>>1,m,nr,cr=c+nl,dr=d+nl);
        n=std::min(n,nl+nr-1);
        re int _l(getLen(nl+nr));
        for(re int i(0);i<_l;++i)
            t1[i]=i<nl?dl[i]:0;
        for(re int i(0);i<_l;++i)
            t2[i]=i<nr?cr[i]:0;
        DFT(t1,_l),DFT(t2,_l);
        for(re int i(0);i<_l;++i)
            t2[i]=(ll)t1[i]*t2[i]%MOD;
        IDFT(t2,_l);
        for(re int i(0);i<n;++i)
            c[i]=((i<nl?cl[i]:0)+t2[i])%MOD;
        for(re int i(0);i<_l;++i)
            t2[i]=i<nr?dr[i]:0;
        DFT(t2,_l);
        for(re int i(0);i<_l;++i)
            t2[i]=(ll)t1[i]*t2[i]%MOD;
        IDFT(t2,_l);
        for(re int i(0);i<n;++i)
            d[i]=t2[i];
    }
}

void getComp(int*a,int*b,int*c,int deg) {
```

```
        static int ts[N],ps[N],c0[N],_t1[N],idM[N];
        int M(std::max((int)ceil(sqrt(deg/log2(deg))*2.5),2)),_n(deg+deg/M);
        getComp(a,b,deg,M,_n,c0,_t1);
        re int _l(getLen(_n+deg));
        for(re int i(_n);i<_l;++i)
            c0[i]=0;
        for(re int i(0);i<_l;++i)
            ps[i]=i==0;
        for(re int i(0);i<_l;++i)
            ts[i]=M<=i&&i<deg?b[i]:0;
        getDer(b,_t1,M);
        for(re int i(M-1);i<deg;++i)
            _t1[i]=0; /// Important!!!
        getInv(_t1,idM,deg);
        for(int i=deg;i<_l;++i)
            idM[i]=0;
        DFT(ts,_l),DFT(idM,_l);
        for(re int t(0);t*M<deg;++t) {
            for(re int i(0);i<_l;++i)
                _t1[i]=i<deg?c0[i]:0;
            DFT(ps,_l),DFT(_t1,_l);
            for(re int i(0);i<_l;++i)
                _t1[i]=(ll)_t1[i]*ps[i]%MOD,
                ps[i]=(ll)ps[i]*ts[i]%MOD;
            IDFT(ps,_l),IDFT(_t1,_l);
            for(re int i(deg);i<_l;++i)
                ps[i]=0;
            for(re int i(0);i<deg;++i)
                c[i]=((ll)_t1[i]*ifac[t]+c[i])%MOD;
            getDer(c0,c0,_n);
            for(re int i(_n-1);i<_l;++i)
                c0[i]=0;
            DFT(c0,_l);
            for(re int i(0);i<_l;++i)
                c0[i]=(ll)c0[i]*idM[i]%MOD;
            IDFT(c0,_l);
            for(re int i(_n-1);i<_l;++i)
                c0[i]=0;
        }
}

int main() {
    n=read(),m=read();
    for(re int i(0);i<=n;++i)
        a[i]=read();
    for(re int i(0);i<=m;++i)
        b[i]=read();

    m=(n>m?n:m)+1;
    pre(m);init(m*5);
    getComp(a,b,c,m);

    for(re int i(0);i<=n;++i)
        write(c[i]);
    fwrite(pbuf,1,pp-pbuf,stdout);
    return 0;
}
```

8.7 下降幂多项式乘法

$O(n \log n)$。

```cpp
#include<cstdio>
#include<algorithm>
const int N=524288,md=998244353,g3=(md+1)/3;
typedef long long LL;
int n,m,A[N],B[N],fac[N],iv[N],rev[N],C[N],g[20][N],lim,M;
int pow(int a,int b){
    int ret=1;
    for(;b;b>>=1,a=(LL)a*a%md)if(b&1)ret=(LL)ret*a%md;
    return ret;
}
void upd(int&a){a+=a>>31&md;}
void init(int n){
    int l=-1;
    for(lim=1;lim<n;lim<<=1)++l;M=l+1;
    for(int i=1;i<lim;++i)
    rev[i]=((rev[i>>1])>>1)|((i&1)<<l);
}
void NTT(int*a,int f){
    for(int i=1;i<lim;++i)if(i<rev[i])std::swap(a[i],a[rev[i]]);
    for(int i=0;i<M;++i){
        const int*G=g[i],c=1<<i;
        for(int j=0;j<lim;j+=c<<1)
        for(int k=0;k<c;++k){
            const int x=a[j+k],y=a[j+k+c]*(LL)G[k]%md;
            upd(a[j+k]+=y-md),upd(a[j+k+c]=x-y);
        }
    }
    if(!f){
        const int iv=pow(lim,md-2);
        for(int i=0;i<lim;++i)a[i]=(LL)a[i]*iv%md;
        std::reverse(a+1,a+lim);
    }
}
int main(){
    scanf("%d%d",&n,&m);++n,++m;
    for(int i=0;i<20;++i){
        int*G=g[i];
        G[0]=1;
        const int gi=G[1]=pow(3,(md-1)/(1<<i+1));
        for(int j=2;j<1<<i;++j)G[j]=(LL)G[j-1]*gi%md;
    }
    for(int i=0;i<n;++i)scanf("%d",A+i);
    for(int i=0;i<m;++i)scanf("%d",B+i);
    for(int i=*fac=1;i<N;++i)
    fac[i]=fac[i-1]*(LL)i%md;
    iv[N-1]=pow(fac[N-1],md-2);
    for(int i=N-2;~i;--i)iv[i]=(i+1LL)*iv[i+1]%md;
    init(n+m<<1);
    for(int i=0;i<n+m-1;++i)C[i]=iv[i];
    NTT(A,1),NTT(B,1),NTT(C,1);
    for(int i=0;i<lim;++i)A[i]=(LL)A[i]*C[i]%md,B[i]=(LL)B[i]*C[i]%md;
    NTT(A,0),NTT(B,0);
    for(int i=0;i<lim;++i)C[i]=0;
    for(int i=0;i<n+m-1;++i)
```

```
    C[i]=(i&1)?md-iv[i]:iv[i];
    for(int i=0;i<lim;++i)A[i]=(LL)A[i]*B[i]%md*fac[i]%md;
    for(int i=n+m-1;i<lim;++i)A[i]=0;
    NTT(A,1),NTT(C,1);
    for(int i=0;i<lim;++i)A[i]=(LL)A[i]*C[i]%md;
    NTT(A,0);
    for(int i=0;i<n+m-1;++i)printf("%d%c",A[i]," \n"[i==n+m-2]);
    return 0;
}
```

8.8 平面欧几里得距离最小生成树

10^5，400ms。

By Claris.

```
#include<cstdio>
#include<algorithm>
#include<cmath>
using namespace std;
typedef long long ll;
const int N=100010;
const ll inf=2000000000000000001LL;
const double eps=1e-9;
inline int sgn(double x){
  if(x>eps)return 1;
  if(x<-eps)return -1;
  return 0;
}
struct P{
  double x,y;
  P(){}
  P(double _x,double _y){x=_x,y=_y;}
  bool operator<(const P&a)const{return sgn(x-a.x)<0||sgn(x-a.x)==0&&sgn(y-a.y)
      <0;}
  P operator-(const P&a)const{return P(x-a.x,y-a.y);}
  double operator&(const P&a)const{return x*a.y-y*a.x;}
  double operator|(const P&a)const{return x*a.x+y*a.y;}
}p[N];
struct PI{
  ll x,y;
  PI(){}
  PI(ll _x,ll _y){x=_x,y=_y;}
}loc[N],pool[N];
inline double check(const P&a,const P&b,const P&c){return (b-a)&(c-a);}
inline double dis2(const P&a){return a.x*a.x+a.y*a.y;}
inline bool cross(int a,int b,int c,int d){
  return sgn(check(p[a],p[c],p[d])*check(p[b],p[c],p[d]))<0&&sgn(check(p[c],p[a
      ],p[b])*check(p[d],p[a],p[b]))<0;
}
inline ll dis(const PI&a,const PI&b){return (a.x-b.x)*(a.x-b.x)+(a.y-b.y)*(a.y-
    b.y);}
inline bool cmpx(const PI&a,const PI&b){return a.x<b.x;}
inline bool cmpy(int a,int b){return pool[a].y<pool[b].y;}
struct P3{
  double x,y,z;
  P3(){}
```

```
  P3(double _x,double _y,double _z){x=_x,y=_y,z=_z;}
  bool operator<(const P3&a)const{return sgn(x-a.x)<0||sgn(x-a.x)==0&&sgn(y-a.y
      )<0;}
  P3 operator-(const P3&a)const{return P3(x-a.x,y-a.y,z-a.z);}
  double operator|(const P3&a)const{return x*a.x+y*a.y+z*a.z;}
  P3 operator&(const P3&a)const{return P3(y*a.z-z*a.y,z*a.x-x*a.z,x*a.y-y*a.x)
      ;}
}ori[N];
inline P3 check(const P3&a,const P3&b,const P3&c){return (b-a)&(c-a);}
inline P3 gp3(const P&a){return P3(a.x,a.y,a.x*a.x+a.y*a.y);}
inline int cal(double x){
  int y=x;
  for(int i=y-2;i<=y+2;i++)if(!sgn(x-i))return i;
}
bool incir(int a,int b,int c,int d){
  P3 aa=gp3(p[a]),bb=gp3(p[b]),cc=gp3(p[c]),dd=gp3(p[d]);
  if(sgn(check(p[a],p[b],p[c]))<0)swap(bb,cc);
  return sgn(check(aa,bb,cc)|(dd-aa))<0;
}
int n,i,j,et,la[N],tot,l,r,q[N<<2];
struct E{
  int to,l,r;
  E(){}
  E(int _to,int _l,int _r=0){to=_to,l=_l,r=_r;}
}e[N<<5];
inline void add(int x,int y){
  e[++et]=E(y,la[x]),e[la[x]].r=et,la[x]=et;
  e[++et]=E(x,la[y]),e[la[y]].r=et,la[y]=et;
}
inline void del(int x){
  e[e[x].r].l=e[x].l;
  e[e[x].l].r=e[x].r;
  la[e[x^1].to]==x?la[e[x^1].to]=e[x].l:1;
}
void delaunay(int l,int r){
  if(r-l<=2){
    for(int i=l;i<r;i++)for(int j=i+1;j<=r;j++)add(i,j);
    return;
  }
  int i,j,mid=(l+r)>>1,ld=0,rd=0,id,op;
  delaunay(l,mid),delaunay(mid+1,r);
  for(tot=0,i=l;i<=r;q[++tot]=i++)
    while(tot>1&&sgn(check(p[q[tot-1]],p[q[tot]],p[i]))<0)tot--;
  for(i=1;i<tot&&!ld;i++)if(q[i]<=mid&&mid<q[i+1])ld=q[i],rd=q[i+1];
  for(;add(ld,rd),1;){
    id=op=0;
    for(i=la[ld];i;i=e[i].l)
      if(sgn(check(p[ld],p[rd],p[e[i].to]))>0)
        if(!id||incir(ld,rd,id,e[i].to))op=-1,id=e[i].to;
    for(i=la[rd];i;i=e[i].l)
      if(sgn(check(p[rd],p[ld],p[e[i].to]))<0)
        if(!id||incir(ld,rd,id,e[i].to))op=1,id=e[i].to;
    if(op==0)break;
    if(op==-1){
      for(i=la[ld];i;i=e[i].l)
      if(cross(rd,id,ld,e[i].to))del(i),del(i^1),i=e[i].r;
      ld=id;
```

```
    }else{
      for(i=la[rd];i;i=e[i].l)
      if(cross(ld,id,rd,e[i].to))del(i),del(i^1),i=e[i].r;
      rd=id;
    }
  }
}
namespace DS{
int m,tot,a[N],f[N],g[N],v[N<<1],nxt[N<<1],ed,col[N];ll w[N<<1];
double ans;
struct E{int x,y;ll w;E(){}E(int _x,int _y,ll _w){x=_x,y=_y,w=_w;}}e[N<<3];
inline bool cmp(const E&a,const E&b){return a.w<b.w;}
inline void newedge(int x,int y,ll z){e[++tot]=E(x,y,z);}
int F(int x){return f[x]==x?x:f[x]=F(f[x]);}
inline void merge(int x,int y,ll z){
  if(F(x)==F(y))return;
  f[f[x]]=f[y];
  v[++ed]=y;w[ed]=z;nxt[ed]=g[x];g[x]=ed;
  v[++ed]=x;w[ed]=z;nxt[ed]=g[y];g[y]=ed;
  ans+=sqrt(z);
}
inline void work(){
  sort(e+1,e+tot+1,cmp);
  for(ed=0,i=1;i<=n;i++)f[i]=i,g[i]=0;
  for(i=1;i<=tot;i++)merge(e[i].x,e[i].y,e[i].w);
  printf("%.15f\n",ans);
}
}
int main(){
  while(~scanf("%d",&n)){
    for(i=0;i<=n+1;i++)la[i]=0;
    et=1;
    DS::tot=0;
    for(i=1;i<=n;i++){
      ll x,y;
      scanf("%lld%lld",&x,&y);
      p[i]=P(x,y);
      loc[i]=PI(x,y);
      ori[i]=P3(x,y,i);
    }
    sort(p+1,p+n+1);
    sort(ori+1,ori+n+1);
    delaunay(1,n);
    for(i=1;i<=n;i++)for(j=la[i];j;j=e[j].l){
      int x=cal(ori[i].z),y=cal(ori[e[j].to].z);
      DS::newedge(x,y,dis(loc[x],loc[y]));
    }
    DS::work();
  }
}
```

8.9 弦图找错

```
#include <bits/stdc++.h>
using namespace std;
const int MAXN = 200005;
```

```
using lint = long long;
using pi = pair<int, int>;

// the algorithm may be wrong. if you have any ideas for proving / disproving
    this, please contact me.

vector<int> gph[MAXN];
int n, m, cnt[MAXN], idx[MAXN];
int mark[MAXN], vis[MAXN], par[MAXN];

void report(int x, int y){
    gph[x].erase(find(gph[x].begin(), gph[x].end(), y));
    gph[y].erase(find(gph[y].begin(), gph[y].end(), x));
    for(int i=1; i<=n; i++){
        if(binary_search(gph[i].begin(), gph[i].end(), x) &&
            binary_search(gph[i].begin(), gph[i].end(), y)){
            mark[i] = 1;
        }
    }
    queue<int> que;
    vis[x] = 1;
    que.push(x);
    while(!que.empty()){
        int x = que.front(); que.pop();
        for(auto &i : gph[x]){
            if(!mark[i] && !vis[i]){
                par[i] = x;
                vis[i] = 1;
                que.push(i);
            }
        }
    }
    assert(vis[y]);
    vector<int> v;
    while(y){
        v.push_back(y);
        y = par[y];
    }
    printf("NO\n%d\n", v.size());
    for(auto &i : v) printf("%d ", i-1);
}

int main(){
    scanf("%d %d",&n,&m);
    for(int i=0; i<m; i++){
        int s, e; scanf("%d %d",&s,&e);
        s++, e++;
        gph[s].push_back(e);
        gph[e].push_back(s);
    }
    for(int i=1; i<=n; i++) sort(gph[i].begin(), gph[i].end());
    priority_queue<pi> pq;
    for(int i=1; i<=n; i++) pq.emplace(cnt[i], i);
    vector<int> ord;
    while(!pq.empty()){
        int x = pq.top().second, y = pq.top().first;
        pq.pop();
```

```
            if(cnt[x] != y || idx[x]) continue;
            ord.push_back(x);
            idx[x] = n + 1 - ord.size();
            for(auto &i : gph[x]){
                if(!idx[i]){
                    cnt[i]++;
                    pq.emplace(cnt[i], i);
                }
            }
        }
    }
    reverse(ord.begin(), ord.end());
    for(auto &i : ord){
        int minBef = 1e9;
        for(auto &j : gph[i]){
            if(idx[j] > idx[i]) minBef = min(minBef, idx[j]);
        }
        minBef--;
        if(minBef < n){
            minBef = ord[minBef];
            for(auto &j : gph[i]){
                if(idx[j] > idx[minBef] && !binary_search(gph[minBef].begin(),
                    gph[minBef].end(), j)){
                    report(minBef, i);
                    return 0;
                }
            }
        }
    }
    puts("YES");
    for(auto &i : ord) printf("%d␣", i-1);
}
```

8.10 $O(\frac{nm}{\omega})$ **LCS**

```
/*
 * Author : _Wallace_
 * Source : https://www.cnblogs.com/-Wallace-/
 * Problem : LOJ #6564. 最长公共子序列
 * Standard : GNU C++ 03
 * Optimal : -Ofast
 */
#include <algorithm>
#include <cstddef>
#include <cstdio>
#include <cstring>

typedef unsigned long long ULL;

const int N = 7e4 + 5;
int n, m, u;

struct bitset {
  ULL t[N / 64 + 5];

  bitset() {
    memset(t, 0, sizeof(t));
```

```cpp
    }
    bitset(const bitset &rhs) {
      memcpy(t, rhs.t, sizeof(t));
    }

    bitset& set(int p) {
      t[p >> 6] |= 1llu << (p & 63);
      return *this;
    }
    bitset& shift() {
      ULL last = 0llu;
      for (int i = 0; i < u; i++) {
        ULL cur = t[i] >> 63;
        (t[i] <<= 1) |= last, last = cur;
      }
      return *this;
    }
    int count() {
      int ret = 0;
      for (int i = 0; i < u; i++)
        ret += __builtin_popcountll(t[i]);
      return ret;
    }

    bitset& operator = (const bitset &rhs) {
      memcpy(t, rhs.t, sizeof(t));
      return *this;
    }
    bitset& operator &= (const bitset &rhs) {
      for (int i = 0; i < u; i++) t[i] &= rhs.t[i];
      return *this;
    }
    bitset& operator |= (const bitset &rhs) {
      for (int i = 0; i < u; i++) t[i] |= rhs.t[i];
      return *this;
    }
    bitset& operator ^= (const bitset &rhs) {
      for (int i = 0; i < u; i++) t[i] ^= rhs.t[i];
      return *this;
    }

    friend bitset operator - (const bitset &lhs, const bitset &rhs) {
      ULL last = 0llu; bitset ret;
      for (int i = 0; i < u; i++){
        ULL cur = (lhs.t[i] < rhs.t[i] + last);
        ret.t[i] = lhs.t[i] - rhs.t[i] - last;
        last = cur;
      }
      return ret;
    }
} p[N], f, g;

signed main() {
  scanf("%d%d", &n, &m), u = n / 64 + 1;
  for (int i = 1, c; i <= n; i++)
    scanf("%d", &c), p[c].set(i);
  for (int i = 1, c; i <= m; i++) {
```

```
    scanf("%d", &c), (g = f) |= p[c];
    f.shift(), f.set(0);
    ((f = g - f) ^= g) &= g;
  }
  printf("%d\n", f.count());
  return 0;
}
```

另一个实现

```cpp
#include <bits/stdc++.h>
#pragma GCC target("popcnt,bmi")

using namespace std;
using ull = uint64_t;

const int N = 70005, M = 1136;

int n, m;
ull g[N][M], f[M];

int read() {
    const int M = 1e6;
    static streambuf *in = cin.rdbuf();
#define gc (p1 == p2 && (p2 = (p1 = buf) + in -> sgetn(buf, M), p1 == p2) ? -1 \
    : *p1++)
    static char buf[M], *p1, *p2;
    int c = gc, r = 0;

    while (c < 48)
        c = gc;

    while (c > 47)
        r = r * 10 + (c & 15), c = gc;

    return r;
}
int main() {
    cin.tie(0)->sync_with_stdio(0);
    cin >> n >> m;

    for (int i = 0; i < n; i++)
        g[read()][i / 62] |= 1ULL << (i % 62);

    int lim = (n - 1) / 62;

    for (int i = 0; i < m; i++) {
        int c = 1;
        auto can = g[read()];

        for (int j = 0; j <= lim; j++) {
            ull x = f[j], y = x | can[j];
            x += x + c + (~y & (1ULL << 62) - 1);
            f[j] = x & y, c = x >> 62;
        }
    }
```

```
    int ans = 0;

    for (int i = 0; i <= lim; i++)
        ans += __builtin_popcountll(f[i]);

    cout << ans;
}
```

8.11 区间 LIS（排列）

```
#include<bits/stdc++.h>
using namespace std;
//dengyaotriangle!

const int maxn=100005;

int pool[(int)5e7];int ps;
inline int *aloc(int x){
    ps+=x;return pool+ps-x;
}
void unit_monge_mult(int *a,int *b,int *r,int n){
    if(n==2){
        if(a[0]==0&&b[0]==0)r[0]=0,r[1]=1;
        else r[0]=1,r[1]=0;
        return;
    }
    if(n==1){r[0]=0;return;}
    int lps=ps;
    int d=n/2;
    int *a1=aloc(d),*a2=aloc(n-d),*b1=aloc(d),*b2=aloc(n-d);
    int *mpa1=aloc(d),*mpa2=aloc(n-d),*mpb1=aloc(d),*mpb2=aloc(n-d);
    int p[2]={0,0};
    for(int i=0;i<n;i++){
        if(a[i]<d)a1[p[0]]=a[i],mpa1[p[0]]=i,p[0]++;
        else a2[p[1]]=a[i]-d,mpa2[p[1]]=i,p[1]++;
    }
    p[0]=p[1]=0;
    for(int i=0;i<n;i++){
        if(b[i]<d)b1[p[0]]=b[i],mpb1[p[0]]=i,p[0]++;
        else b2[p[1]]=b[i]-d,mpb2[p[1]]=i,p[1]++;
    }
    int *c1=aloc(d),*c2=aloc(n-d);
    unit_monge_mult(a1,b1,c1,d),unit_monge_mult(a2,b2,c2,n-d);
    int *cpx=aloc(n),*cpy=aloc(n),*cqx=aloc(n),*cqy=aloc(n);
    for(int i=0;i<d;i++)cpx[mpa1[i]]=mpb1[c1[i]],cpy[mpa1[i]]=0;
    for(int i=0;i<n-d;i++)cpx[mpa2[i]]=mpb2[c2[i]],cpy[mpa2[i]]=1;
    for(int i=0;i<n;i++)r[i]=cpx[i];
    for(int i=0;i<n;i++)cqx[cpx[i]]=i,cqy[cpx[i]]=cpy[i];
    int hi=n,lo=n,his=0,los=0;
    for(int i=0;i<n;i++){
        if(cqy[i]^(cqx[i]>=hi))his--;
        while(hi>0&&his<0){
            hi--;
            if(cpy[hi]^(cpx[hi]>i))his++;
```

```
        }
        while(lo>0&&los<=0){
            lo--;
            if(cpy[lo]^(cpx[lo]>=i))los++;
        }
        if(los>0&&hi==lo)r[lo]=i;
        if(cqy[i]^(cqx[i]>=lo))los--;
    }
    ps=lps;
}
void subunit_monge_mult(int*a,int*b,int*c,int n){
    int lps=ps;
    int *za=aloc(n),*zb=aloc(n),*res=aloc(n),*vis=aloc(n),*mpa=aloc(n),*mpb=aloc
        (n),*rb=aloc(n);
    memset(vis,0,sizeof(int)*n);
    memset(mpa,-1,sizeof(int)*n);
    memset(mpb,-1,sizeof(int)*n);
    memset(rb,-1,sizeof(int)*n);
    int ca=n;
    for(int i=n-1;i>=0;i--)if(a[i]!=-1){
        vis[a[i]]=1;ca--;za[ca]=a[i];mpa[ca]=i;
    }
    for(int i=n-1;i>=0;i--)if(!vis[i])za[--ca]=i;
    memset(vis,-1,sizeof(int)*n);
    for(int i=0;i<n;i++)if(b[i]!=-1)vis[b[i]]=i;
    ca=0;
    for(int i=0;i<n;i++)if(vis[i]!=-1){
        mpb[ca]=i;rb[vis[i]]=ca++;
    }
    for(int i=0;i<n;i++)if(rb[i]==-1)rb[i]=ca++;
    for(int i=0;i<n;i++)zb[rb[i]]=i;
    unit_monge_mult(za,zb,res,n);
    memset(c,-1,sizeof(int)*n);
    for(int i=0;i<n;i++)if(mpa[i]!=-1&&mpb[res[i]]!=-1)c[mpa[i]]=mpb[res[i]];
    ps=lps;
}


void solve(int *p,int *ret,int n){
    if(n==1){ret[0]=-1;return;}
    int lps=ps,d=n/2;
    int *pl=aloc(d),*pr=aloc(n-d);
    for(int i=0;i<d;i++)pl[i]=p[i];
    for(int i=0;i<n-d;i++)pr[i]=p[i+d];
    int *vis=aloc(n);memset(vis,-1,sizeof(int)*n);
    for(int i=0;i<d;i++)vis[pl[i]]=i;
    int *tl=aloc(d),*tr=aloc(n-d),*mpl=aloc(d),*mpr=aloc(n-d);
    int ca=0;
    for(int i=0;i<n;i++)if(vis[i]!=-1)mpl[ca]=i,tl[vis[i]]=ca++;
    ca=0;memset(vis,-1,sizeof(int)*n);
    for(int i=0;i<n-d;i++)vis[pr[i]]=i;
    for(int i=0;i<n;i++)if(vis[i]!=-1)mpr[ca]=i,tr[vis[i]]=ca++;
    int *vl=aloc(d),*vr=aloc(n-d);
    solve(tl,vl,d),solve(tr,vr,n-d);
    int *sl=aloc(n),*sr=aloc(n);
    iota(sl,sl+n,0);iota(sr,sr+n,0);
    for(int i=0;i<d;i++)sl[mpl[i]]=(vl[i]==-1?-1:mpl[vl[i]]);
```

```
    for(int i=0;i<n-d;i++)sr[mpr[i]]=(vr[i]==-1?-1:mpr[vr[i]]);
    subunit_monge_mult(sl,sr,ret,n);
    ps=lps;
}
int invp[maxn],res_monge[maxn];
int main(){
    ios::sync_with_stdio(0);cin.tie(0);
    int n,q;
    cin>>n>>q;
    vector<int> a(n);
    for(int i=0;i<n;i++)cin>>a[i],invp[a[i]]=i;
    solve(invp,res_monge,n);
    vector<int> fwk(n+1),ans(q);
    vector<vector<pair<int,int> > > qry(n+1);
    for(int i=0;i<q;i++){
        int l,r;
        cin>>l>>r;
        qry[l].push_back({r,i});
        ans[i]=r-l;
    }
    for(int i=n-1;i>=0;i--){
        if(res_monge[i]!=-1){
            for(int p=res_monge[i]+1;p<=n;p+=p&-p)fwk[p]++;
        }
        for(auto& z:qry[i]){
            int id,c;tie(id,c)=z;
            for(int p=id;p;p-=p&-p)ans[c]-=fwk[p];
        }
    }
    for(int i=0;i<q;i++)cout<<ans[i]<<'\n';
    return 0;
}
```

8.12 区间 LCS

$s_{[0,a)}$ 和 $t_{[b,c)}$ 的 LCS

```
#include<bits/stdc++.h>
using namespace std;
//dengyaotriangle!

const int maxn=1005;
const int maxq=500005;
int n,m,q;
char a[maxn],b[maxn];
struct qryt{
    int x,nxt;
}z[maxq];
int qry[maxn][maxn];
int ans[maxq];
int r[maxn];
int bit[maxn];

int main(){
    ios::sync_with_stdio(0);cin.tie(0);
```

```
    cin>>q>>b>>a;n=strlen(a);m=strlen(b);
    //q,s,t
    for(int i=1;i<=q;i++){
        int a,b,c;
        cin>>a>>b>>c;
        if(a){
            ans[i]=c-b;
            z[i].x=b;z[i].nxt=qry[a][c];
            qry[a][c]=i;
        }
    }
    for(int i=0;i<n;i++)r[i]=i;
    for(int i=0;i<m;i++){
        int lp=-1;
        for(int j=0;j<n;j++)if(a[j]==b[i]){lp=j;break;}
        if(lp!=-1){
            for(int j=lp+1;j<n;j++){
                if(a[j]!=b[i]){
                    if(r[j-1]<r[j])swap(r[j-1],r[j]);
                }
            }
            for(int i=n-1;i>lp;i--)r[i]=r[i-1];
            r[lp]=-1;
        }
        for(int i=0;i<=n;i++)bit[i]=0;
        for(int j=0;j<n;j++){
            if(r[j]!=-1){
                for(int p=n-r[j];p<=n;p+=p&-p)bit[p]++;
            }
            for(int y=qry[i+1][j+1];y;y=z[y].nxt){
                for(int p=n-z[y].x;p;p-=p&-p)ans[y]-=bit[p];
            }
        }
    }
    for(int i=1;i<=q;i++)cout<<ans[i]<<'\n';
    return 0;
}
```

8.13 毛毛虫剖分

　　毛毛虫剖分，一种由轻重链剖分（HLD）推广而成的树上结点重标号方法，支持修改／查询一只毛毛虫的信息，并且可以对毛毛虫的身体和足分别修改／查询不同信息.

　　严格强于树剖，而且复杂度和树剖一样哦！

　　一些定义（默认在一棵树上）：

　　毛毛虫：一条链和与这条链邻接的所有结点构成的集合. 虫身（身体）：毛毛虫的链部分. 虫足（足）：毛毛虫除虫身的部分. 重标号方法首先重剖求出重链. DFS，若现在处理到结点 u：若 u 还未被标号，则为其标号. 若 u 是重链头，遍历这条重链，将邻接这条链的结点依次标号. 先递归重儿子，再递归轻儿子. 重标号性质对于重链，除链头外的结点标号连续. 对于任意结点，其轻儿子标号连续. 对于以重链头为根的子树，与这条重链邻接的所有结点标号连续. 这样就可以随便维护毛毛虫信息了，顺便还能维护链信息，子树信息等.

　　时间复杂度同轻重链剖分.

　　以 SAM 为例，若我们只保留所有的转移边 (u, v)，满足到达 u 的路径数目大于到达 v 的路径数目一半，且从 v 出发的路径数目大于从 u 出发的路径数目一半，这样剩余的子图显然会形成若干条链，且每个点恰好在一条链上。这样，我们容易证明，从根结点出发的任何一条路径，至多经过 $O(\log n)$ 条不在链上的转移边（也意味着至多经过 $O(\log n)$ 条链）。