

Persistent Segment Trees

Q) Given an array answer queries of the form $l\ r\ k \rightarrow$ find the k th largest element in $a[l\dots r]$.

Segment trees with each node storing an ordered set $\rightarrow (\log n)^3$ per query

Node 1 $\rightarrow 1, 2, \dots, n \rightarrow$ array(1, ..., n) in sorted form

Node $i \rightarrow l, \dots, r \rightarrow$ array(l, \dots, r) in sorted form

$$T(n) = 2 * T(n/2) + O(n)$$

We can build this seg tree in $O(n \log n)$

k th element is x

We'll find rank of x in this range l, r

Split into m ranges

Node $_i$, node $_{i+1}$, node $_{i+2}$, ..., node $_m$

$$\text{rank}_i(x) + \dots + \text{rank}_m(x) \geq k$$

$$O(\log n * \log n * \log n)$$

Cnt[1, ..., n] \rightarrow cnt[i] = count of value i in the array

$V_0 \rightarrow$ cnt[i] = 0 for all i

$V_i \rightarrow$ cnt[j] = count of value j in $a[1..i]$

Cnt[3] in $a[l..r]$

Cnt[3] in $V_r - \text{cnt}[3]$ in V_{l-1}

//Range update, range max queries.

struct Vertex

{

Vertex *l, *r;

int sum, val;

bool is;

```

Vertex(int va) : l(nullptr), r(nullptr), sum(va), val(0), is(false) {}
Vertex(Vertex *L, Vertex *R) : l(L), r(R), sum(0), val(0), is(0)
{
    if (l) sum = max(sum, l->sum);
    if (r) sum = max(sum, r->sum);
}
};

```

```

Vertex* newlazykid(Vertex *v, int delta, int L, int R)
{
    Vertex *ret = new Vertex(0);
    ret->l = v->l;
    ret->r = v->r;
    ret->val = v->val;
    ret->is = 1;
    ret->val += delta;
    ret->sum = v->sum + delta;

    return ret;
}

```

```

void propagate(Vertex *v, int L, int R)
{
    if(v->is && v->val)
    {
        if(L!=R)
        {
            v->l = newlazykid(v->l, v->val, L, (L+R)/2);
            v->r = newlazykid(v->r, v->val, (L+R)/2+1, R);
        }
        v->is=0;
        v->val=0;
    }
}

```

```

Vertex* build(int tl, int tr)

```

```

{
    if (tl == tr)
        return new Vertex(0);
    int tm = (tl + tr) / 2;
    return new Vertex(build(tl, tm), build(tm+1, tr));
}

```

```

int get_max(Vertex* v, int tl, int tr, int l, int r)
{
    if (l > r)
        return 0;
    if (l <= tl && tr <= r)
        return v->sum;
    propagate(v, tl, tr);
    int tm = (tl + tr) / 2;
    return max(get_max(v->l, tl, tm, l, min(r,tm))
        ,get_max(v->r, tm+1, tr, max(l,tm+1), r));
}

```

```

Vertex* update(Vertex* v, int tl, int tr, int L, int R, int delta)
{
    if(tr < L || R < tl) return v;
    if (L<=tl && tr<=R)
        return newlazykid(v,delta, tl, tr);
    propagate(v, tl, tr);
    int tm = (tl + tr) / 2;
    return new Vertex(update(v->l, tl, tm, L, min(R,tm), delta),
        update(v->r, tm+1, tr, max(L,tm+1), R, delta));
}

```