Binary Search Problems

Q: https://codeforces.com/contest/1486/problem/C1

There is an array a of n **different** numbers. In one query you can ask the position of the second maximum element in a subsegment $a[l..r]$. Find the position of the maximum element in the array in no more than **40** queries.

A subsegment $a[l..r]$ is all the elements $al,al+1,...,ar$. After asking this subsegment you will be given the position of the second maximum from this subsegment **in the whole** array.

a= [5,1,4,2,3]
l=1,r=5 -> 3
l=3,r=5 -> 5

In interactive problems:

Judge	**You**
(Knows the array)	(can ask queries and get answer)
int x;	cout<<"? "<<l<<" "<<r<<endl;
cin>>x;	

a= [5,1,4,2,3]
int n;
cin>>n;

int l=1,r=5;
cout<<"? "<<l<<" "<<r<<endl;
int x;
cin>>x; // x=3
cout<<"? "<<4<<" "<<5<<endl;
cin>>x; // x=4
int ans;
cout<<"! "<<ans<<endl;

1…….n
(1,n) -> z
int mid = (n+1)/2;
(1,mid) and (mid+1,n)
a1,a2,a3,a4 -> (a1,a2) (a3,a4) -> =z
(a2,a4) (a1,a3) ->!=z
(a2,a3) (a1,a4) -> !=z
a2 is in first half(1...mid)
(1,mid) -> x

log(n) -> (15-17)*2 <=40

Sol:

```
map<pair<int,int>,int> m;
int query(int l,int r)
{
    if(m.find(mp(l,r))!=m.end()){
        return m[mp(l,r)];
    }
    cout<<"? "<<l<<" "<<r<<endl;
```

```
    int z;
    cin>>z;
    m[mp(l,r)]=z;
    return z;
}
void solve()
{
    int n;
    cin>>n; //5,1,4,2,3
    int l=1,r=n; //l=1,r=3
    while(r-l>1){
        int mid = (l+r)/2;
        int z = query(l,r);
        if(z<=mid){    //a2 is present in the first half
            int x = query(l,mid);
            if(x==z){   //a1 and a2 present in l...mid -> go
left
                r=mid;
            }else{      // a1 and a2 present in different
halfs....go right
                l=mid;
            }
        }else{    //a2 is present in 2nd half(mid+1...r)
            int x = query(mid,r);
            if(x==z){  // a1 and a2 present in same
half...mid to r
                l=mid;
            }else{     // a1 and a2 present in diff
half...go l...mid
```

```
                r=mid;
            }
        }
    }
    int z = query(l,r);
    if(z==l){
        cout<<"! "<<r<<endl;
    }else{
        cout<<"! "<<l<<endl;
    }
}
```

Q: https://atcoder.jp/contests/abc174/tasks/abc174_e

```
#include<bits/stdc++.h>
using namespace std;

int32_t main(){

    int n,k;
    scanf("%d%d",&n,&k);
    vector<int> a(n);
    for(int i=0;i<n;i++) cin>>a[i];
    int low=1,high=1e9;
    // search space-> length of max stick;
    int ans=-1;
    while(low<=high){
        int mid=low+(high-low)/2;
        int req_number_of_cuts=0;
```

```
        for(int i=0;i<n;i++){
            if(a[i]%mid!=0)
req_number_of_cuts+=(a[i]/mid);
            else
req_number_of_cuts+=((a[i]/mid)-1);
        }
        if(req_number_of_cuts<=k){
            ans=mid;
            high=mid-1;
        }
        else low=mid+1;
    }
    cout<<ans<<endl;
}
```

Some useful built-in functions for Binary Search in C++:

1. If you are given a sorted vector (a[i] <= a[i+1]) and a number x, and you need to find the index of the first element >=x, this is called **lower_bound of x**.
In C++, there is a builtin function to find this in O(log N) using binary search.
Eg:

```
int ind = lower_bound(a.begin(), a.end(), x) - a.begin();
```

2. If you are given a sorted vector (a[i] <= a[i+1]) and a number x,

and you need to find the index of the first element >x, this is called **upper_bound of x**.
In C++, there is a builtin function to find this in O(log N) using binary search.

```
int ind = upper_bound(a.begin(), a.end(), x) - a.begin();
```

Suppose, there is no such element, then ind = size of the array.

Q.
https://codeforces.com/edu/course/2/lesson/6/1/practice/contest/283911/problem/D

```
#include <bits/stdc++.h>

using namespace std;

int32_t main() {
 int n;
 cin >> n;

 vector<int> vec(n);

 for (int i = 0; i < n; i++) {
   cin >> vec[i];
 }

 sort(vec.begin(), vec.end());

 int k;
 cin >> k;
```

```
 int l, r;
 while (k--) {
   cin >> l >> r;
   int y = upper_bound(vec.begin(), vec.end(), r) -
vec.begin();
   int x = lower_bound(vec.begin(), vec.end(), l) -
vec.begin();

   cout << y - x << ' ';
 }

 return 0;
}
```

Q.
https://codeforces.com/edu/course/2/lesson/6/2/practice/contest/283932/problem/E

```
#include <bits/stdc++.h>

using namespace std;

typedef long double ld;

const ld eps = 1e-6;  // 10^(-6)

int32_t main() {
 //  fastio;
```

```
 ld c;
 cin >> c;

 long double lo = 1, hi = 1e10, mid;

 while (hi - lo >= eps) {
   mid = lo + (hi - lo) / 2;

   ld val = mid * mid + sqrt(mid);

   if (val <= c) {
     lo = mid;
   } else {
     hi = mid;
   }
 }

 cout << fixed << setprecision(7) << mid << endl;

 return 0;
}
```

Q.
https://codeforces.com/edu/course/2/lesson/6/2/practice/contest/283932/problem/D

D. Children Holiday

time limit per test: 2 seconds
memory limit per test: 512 megabytes
input: standard input
output: standard output

The organizers of the children's holiday are planning to inflate m balloons for it. They invited n assistants, the i-th assistant inflates a balloon in t_i minutes, but every time after z_i balloons are inflated he gets tired and rests for y_i minutes. Now the organizers of the holiday want to know after what time all the balloons will be inflated with the most optimal work of the assistants, and how many balloons each of them will inflate. (If the assistant has inflated the balloon and needs to rest, but he will not have to inflate more balloons, then it is considered that he finished the work immediately after the end of the last balloon inflation, and not after the rest).

Input

The first line of the input contains integers m and n $(0 \le m \le 15000, 1 \le n \le 1000)$. The next n lines contain three integers each, t_i, z_i, and y_i, respectively $(1 \le t_i, y_i \le 100, 1 \le z_i \le 1000)$.

Output

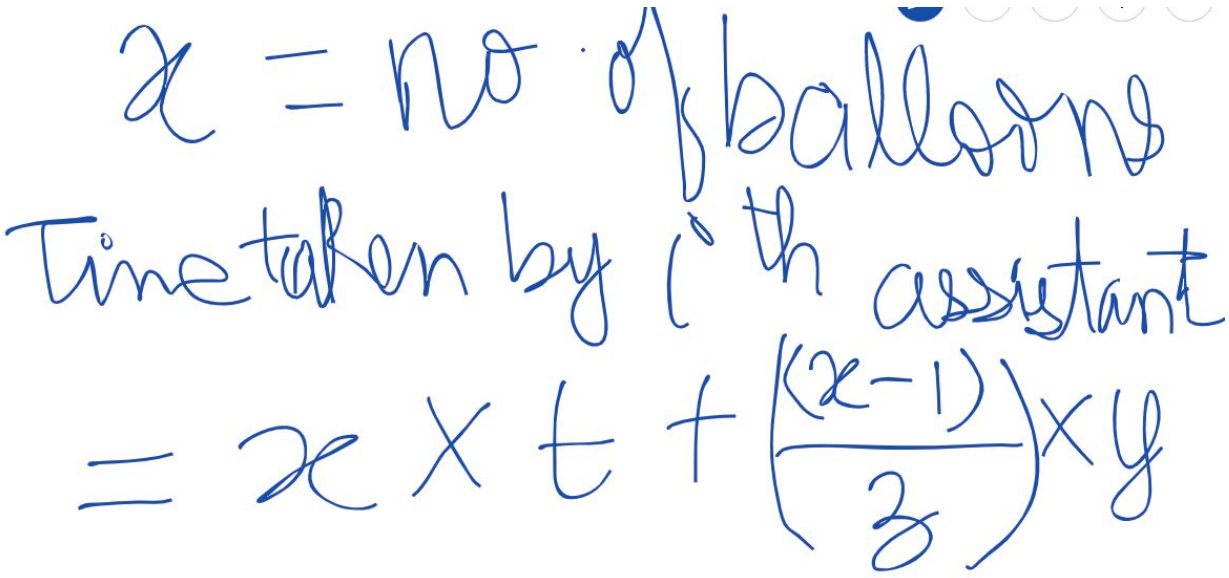
In the first line print the number T, the time it takes for all the balloons to be inflated. On the second line print n numbers, the number of balloons inflated by each of the invited assistants. If there are several optimal answers, output any of them.

Let f(x) = No. of balloons that can be inflated in x minutes

It is a monotonic non-decreasing function.

Let g(x) = Time taken by one person to inflate x balloons.

It is a monotonic non-decreasing function.



https://codeforces.com/edu/course/2/lesson/6/2/practice/contest/283932/problem/D

```cpp
#include <bits/stdc++.h>
#define int long long

using namespace std;

int get_balloons(int p, int ti, int zi, int yi) {
 // p: time provided
 // return the maximum number of balloons that
 // this person can fill in t=x minutes

 int lo = 0;
 int hi = 1e8;
 int mid;
 int ans = 0;
```

```
 while (lo <= hi) {
   mid = lo + (hi - lo) / 2;
   int gx = mid * ti + ((mid - 1) / zi) * yi;
   // time taken to fill x=mid balloons
   if (gx <= p) {
     ans = mid;
     lo = mid + 1;
   } else {
     hi = mid - 1;
   }
 }
 return ans;
}

int32_t main() {
 //  fastio;

 int m, n;
 cin >> m >> n;

 vector<int> t(n), z(n), y(n);

 for (int i = 0; i < n; i++) {
   cin >> t[i] >> z[i] >> y[i];
 }

 int lo = 0;
 int hi = 1e8;
 int mid;
```

```
int ans;

vector<int> balloons;

while (lo <= hi) {
  mid = lo + (hi - lo) / 2;

  // Find the number of balloons that can be
  // inflated in x=mid minutes

  int num = 0;  // no. of balloons filled
  vector<int> temp;
  for (int i = 0; i < n; i++) {
    if (num == m) {
      temp.push_back(0);
      continue;
    }
    int cnt = get_balloons(mid, t[i], z[i], y[i]);
    num += cnt;
    if (num >= m) {
      int diff = num - m;
      cnt -= diff;
      num = m;
    }
    temp.push_back(cnt);
  }

  if (num >= m) {
```

```
      balloons = temp;
      ans = mid;
      hi = mid - 1;
    } else {
      lo = mid + 1;
    }
  }

  cout << ans << '\n';

  for (int i = 0; i < balloons.size(); i++) {
    cout << balloons[i] << ' ';
  }
  return 0;
}
```

Time complexity: O (log (10^8) * n * log (10^8))