GRAVITATIONAL LENSING

7 - TIME DELAYS

Massimo Meneghetti

AA 2018-2019

GRAVITATIONAL TIME DELAY

- In a lensing phenomenon, light travels with an effective velocity *c'<c*. As seen, this implies an effective refractive index *n>1*
- The effective refractive index is expressed in terms of the Newtonian potential
- If we compare the travel times of two photons, one traveling at velocity c and the other at velocity c', we notice that the second accumulates a time delay t_{grav}
- This time delay is called *gravitational* time delay, or *Shapiro* time delay (Shapiro, 1964)

$$n = 1 - \frac{2\Phi}{c^2}$$

$$\begin{aligned} t_{grav} &= \int \frac{dz}{c'} - \int \frac{dz}{c} \\ &= \frac{1}{c} \int (n-1) dz \\ &= -\frac{2}{c^3} \int \Phi dz \end{aligned}$$

GRAVITATIONAL TIME DELAY

- In a lensing phenomenon, light travels with an effective velocity $c' < c$. As seen, this implies an effective refractive index $n > 1$
- The effective refractive index is expressed in terms of the Newtonian potential
- If we compare the travel times of two photons, one traveling at velocity c and the other at velocity c', we notice that the second accumulates a time delay t_{grav}
- This time delay is called *gravitational* time delay, or *Shapiro* time delay (Shapiro, 1964)

$$\begin{aligned} t_{grav} &= \int \frac{dz}{c'} - \int \frac{dz}{c} \\ &= \frac{1}{c} \int (n-1) dz \\ &= -\frac{2}{c^3} \int \Phi dz \end{aligned}$$

$$\hat{\Psi}(\vec{\theta}) = \frac{D_{LS}}{D_L D_S} \frac{2}{c^2} \int \Phi(\vec{\theta}, z) dz$$

$$t_{grav} = -\frac{1}{c} \frac{D_S D_L}{D_{LS}} \hat{\Psi}$$

GRAVITATIONAL TIME DELAY

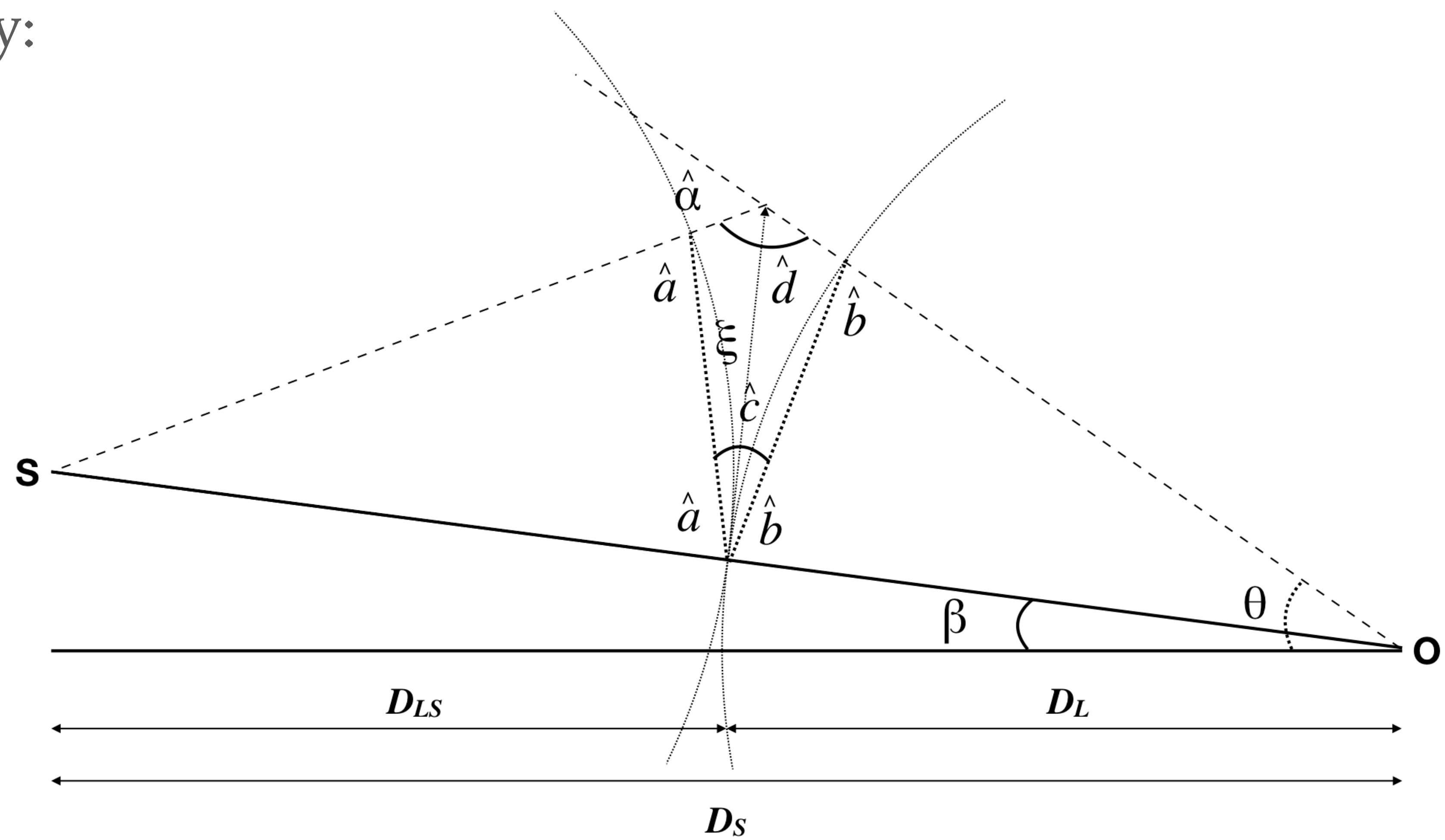
- In a lensing phenomenon, light travels with an effective velocity *c'<c*. As seen, this implies an effective refractive index *n>1*
- The effective refractive index is expressed in terms of the Newtonian potential
- If we compare the travel times of two photons, one traveling at velocity c and the other at velocity c', we notice that the second accumulates a time delay t_{grav}
- This time delay is called *gravitational* time delay, or *Shapiro* time delay (Shapiro, 1964)

$$t_{grav} = -\frac{1}{c}\frac{D_S D_L}{D_{LS}}\hat{\Psi}$$

This time delay does not account yet for the different path of photons!

GEOMETRICAL TIME DELAY

- We need to combine the gravitational time delay to the so called *geometrical* time delay:

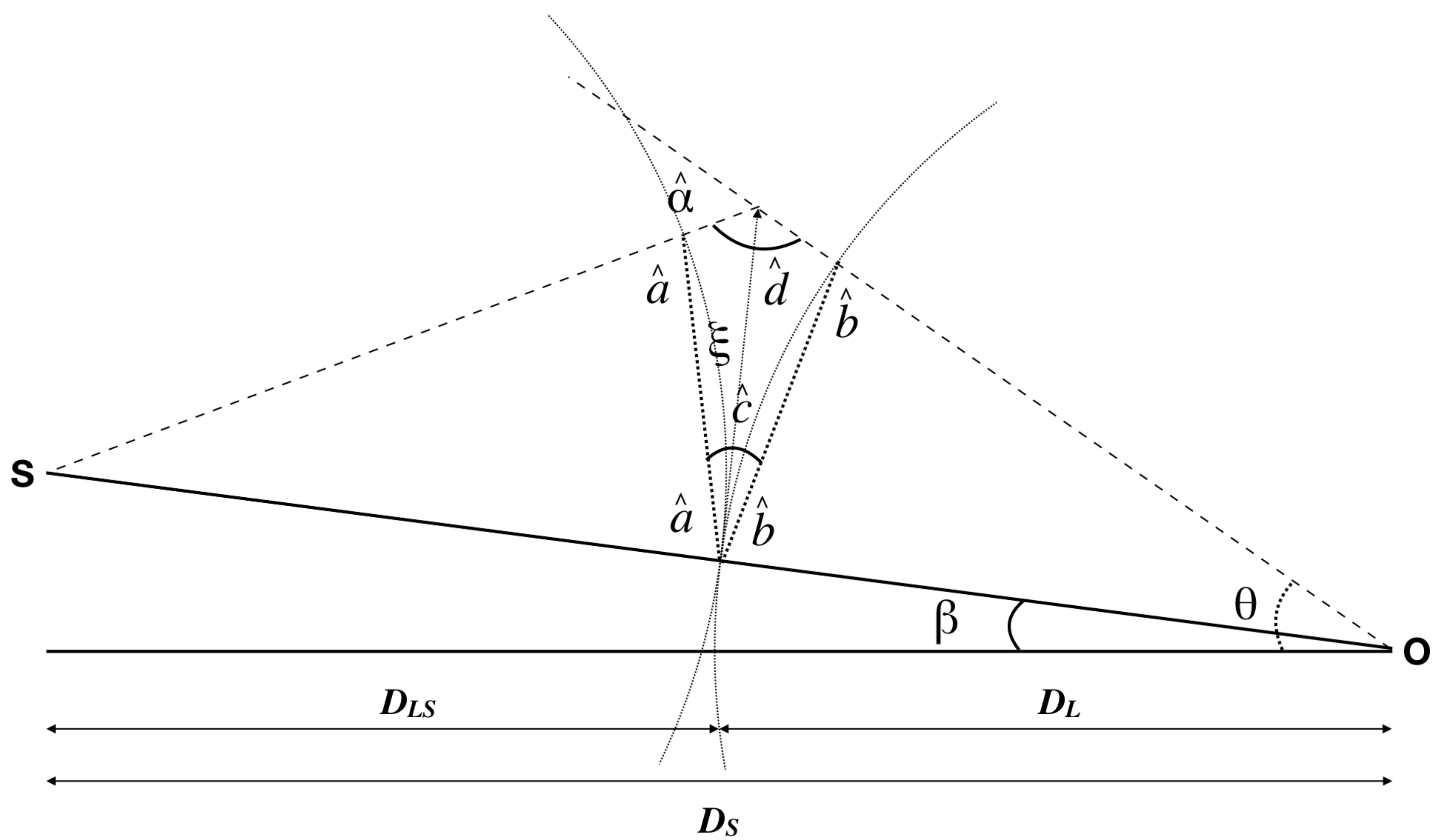


$$(\pi - \hat{a}) + (\pi - \hat{b}) + \hat{c} + \hat{d} = 2\pi \Rightarrow \hat{a} + \hat{b} = \hat{c} + \hat{d}$$

$$\hat{a} + \hat{b} + \hat{c} = \pi \Rightarrow 2\hat{c} = \pi - \hat{d}$$

$$\hat{\alpha} + \hat{d} = \pi \Rightarrow \hat{d} = \pi - \hat{\alpha}$$

GEOMETRICAL TIME DELAY



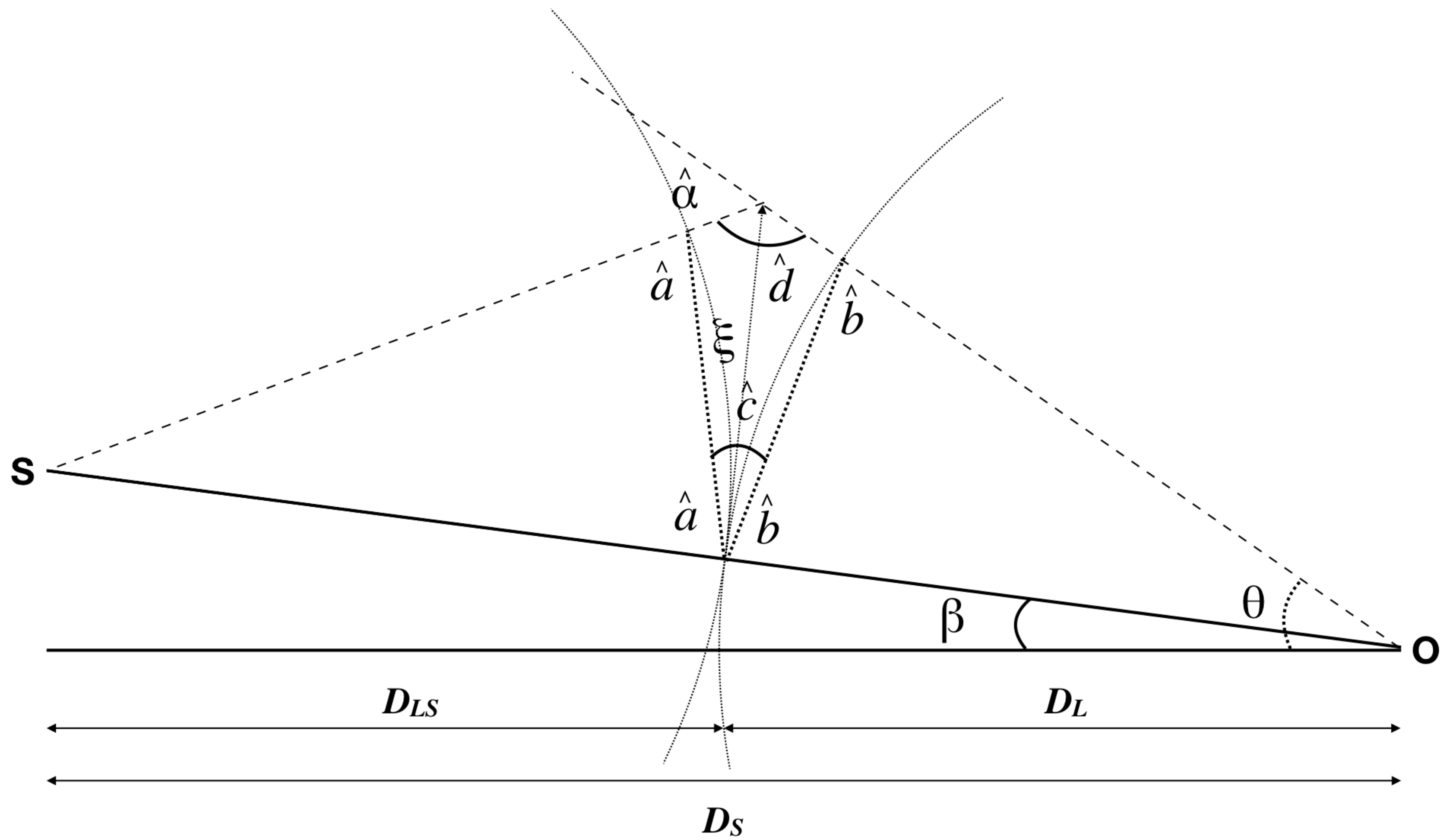
$$(\pi - \hat{a}) + (\pi - \hat{b}) + \hat{c} + \hat{d} = 2\pi \Rightarrow \hat{a} + \hat{b} = \hat{c} + \hat{d}$$

$$\hat{a} + \hat{b} + \hat{c} = \pi \Rightarrow 2\hat{c} = \pi - \hat{d}$$

$$\hat{\alpha} + \hat{d} = \pi \Rightarrow \hat{d} = \pi - \hat{\alpha}$$

$$\begin{aligned} \Rightarrow \hat{c} &= \frac{\hat{\alpha}}{2} \\ &= \frac{1}{2}\frac{D_S}{D_{LS}}\alpha \\ &= \frac{1}{2}\frac{D_S}{D_{LS}}(\theta - \beta) \end{aligned}$$

GEOMETRICAL TIME DELAY



$$
\begin{aligned}
\Rightarrow \hat{c} &= \frac{\hat{\alpha}}{2} \\
&= \frac{1}{2}\frac{D_S}{D_{LS}}\alpha \\
&= \frac{1}{2}\frac{D_S}{D_{LS}}(\theta-\beta)
\end{aligned}
$$

$$
\begin{aligned}
\xi &= D_L(\theta-\beta) \\
t_{geom} &= \frac{1}{c}\xi\hat{c} \\
&= \frac{1}{2c}\frac{D_S D_L}{D_{LS}}(\theta-\beta)^2
\end{aligned}
$$

TOTAL TIME DELAY

$$t_{geom} = \frac{1}{2c}\frac{D_S D_L}{D_{LS}}(\theta - \beta)^2$$

$$t_{grav} = -\frac{1}{c}\frac{D_S D_L}{D_{LS}}\hat{\Psi}$$

$$\begin{aligned} t_{tot} &= t_{geom} + t_{grav} \\ &= \frac{1}{2c}\frac{D_S D_L}{D_{LS}}(\theta - \beta)^2 - \frac{1}{c}\frac{D_S D_L}{D_{LS}}\hat{\Psi}(\theta) \\ &= \frac{1}{c}\frac{D_S D_L}{D_{LS}}\left[\frac{1}{2}(\theta - \beta)^2 - \hat{\Psi}(\theta)\right] \end{aligned}$$

TOTAL TIME DELAY

$$\begin{aligned} t_{tot} &= t_{geom} + t_{grav} \\ &= \frac{1}{2c}\frac{D_S D_L}{D_{LS}}(\theta-\beta)^2 - \frac{1}{c}\frac{D_S D_L}{D_{LS}}\hat{\Psi}(\theta) \\ &= \frac{1}{c}\frac{D_S D_L}{D_{LS}}\left[\frac{1}{2}(\theta-\beta)^2 - \hat{\Psi}(\theta)\right] \end{aligned}$$

Accounting for the expansion of the universe and for the fact that this is a surface:

$$t_{tot}(\vec{\theta}) = \frac{1+z_L}{c}\frac{D_S D_L}{D_{LS}}\left[\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \hat{\Psi}(\vec{\theta})\right]$$

TOTAL TIME DELAY

$$t_{tot}(\vec{\theta}) = \frac{1+z_L}{c}\frac{D_S D_L}{D_{LS}}\left[\frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \hat{\Psi}(\vec{\theta})\right]$$

$$\tau(\vec{\theta}) = \frac{1}{2}(\vec{\theta}-\vec{\beta})^2 - \hat{\Psi}(\vec{\theta})$$

Fermat potential

$$D_{\Delta t} = (1+z_L)\frac{D_S D_L}{D_{LS}}$$

Time delay distance

TIME DELAY SURFACE

$$t(\vec{\theta}) = t_{geom} + t_{grav} \propto \left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \hat{\Psi} \right)$$

$$\vec{\nabla} t(\vec{\theta}) \propto \left(\vec{\theta} - \vec{\beta} - \vec{\nabla} \hat{\Psi} \right)$$

TIME DELAY SURFACE

$$t(\vec{\theta}) = t_{geom} + t_{grav} \propto \left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \hat{\Psi} \right)$$

$$\vec{\nabla} t(\vec{\theta}) \propto \left(\vec{\theta} - \vec{\beta} - \vec{\nabla}\hat{\Psi} \right)$$

Lens equation!

TIME DELAY SURFACE

$$t(\vec{\theta}) = t_{geom} + t_{grav} \propto \left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \hat{\Psi} \right)$$

$$\vec{\nabla} t(\vec{\theta}) \propto \left(\vec{\theta} - \vec{\beta} - \vec{\nabla}\hat{\Psi} \right)$$

Lens equation!

Images form at the stationary points of t!

TIME DELAY SURFACE

$$t(\vec{\theta}) = t_{geom} + t_{grav} \propto \left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \hat{\Psi} \right)$$

$$\vec{\nabla} t(\vec{\theta}) \propto \left(\vec{\theta} - \vec{\beta} - \vec{\nabla}\hat{\Psi} \right)$$

Lens equation!

Images form at the stationary points of t!

$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial\theta_i \partial\theta_j} \propto (\delta_{ij} - \Psi_{ij})$$

TIME DELAY SURFACE

$$t(\vec{\theta}) = t_{geom} + t_{grav} \propto \left(\frac{1}{2}(\vec{\theta} - \vec{\beta})^2 - \hat{\Psi} \right)$$

$$\vec{\nabla} t(\vec{\theta}) \propto \left(\vec{\theta} - \vec{\beta} - \vec{\nabla}\hat{\Psi} \right)$$

Lens equation!

Images form at the stationary points of t!

$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial\theta_i \partial\theta_j} \propto (\delta_{ij} - \Psi_{ij})$$ ***This is the Jacobian!***

TYPES OF IMAGES

- minima (eigenvalues of A are both positive, hence $\mathrm{det}A>0$ and $\mathrm{Tr}\ A>0$; positive magnification)
- saddle (eigenvalues have opposite signs, thus $\mathrm{det}A<0$; negative magnification)
- maxima (eigenvalues are both negative, hence $\mathrm{det}A>0$ and $\mathrm{Tr}\ A<0$; positive magnification)
- Let see some examples…