GRAVITATIONAL LENSING

12 - ASTROMETRIC MICROLENSING

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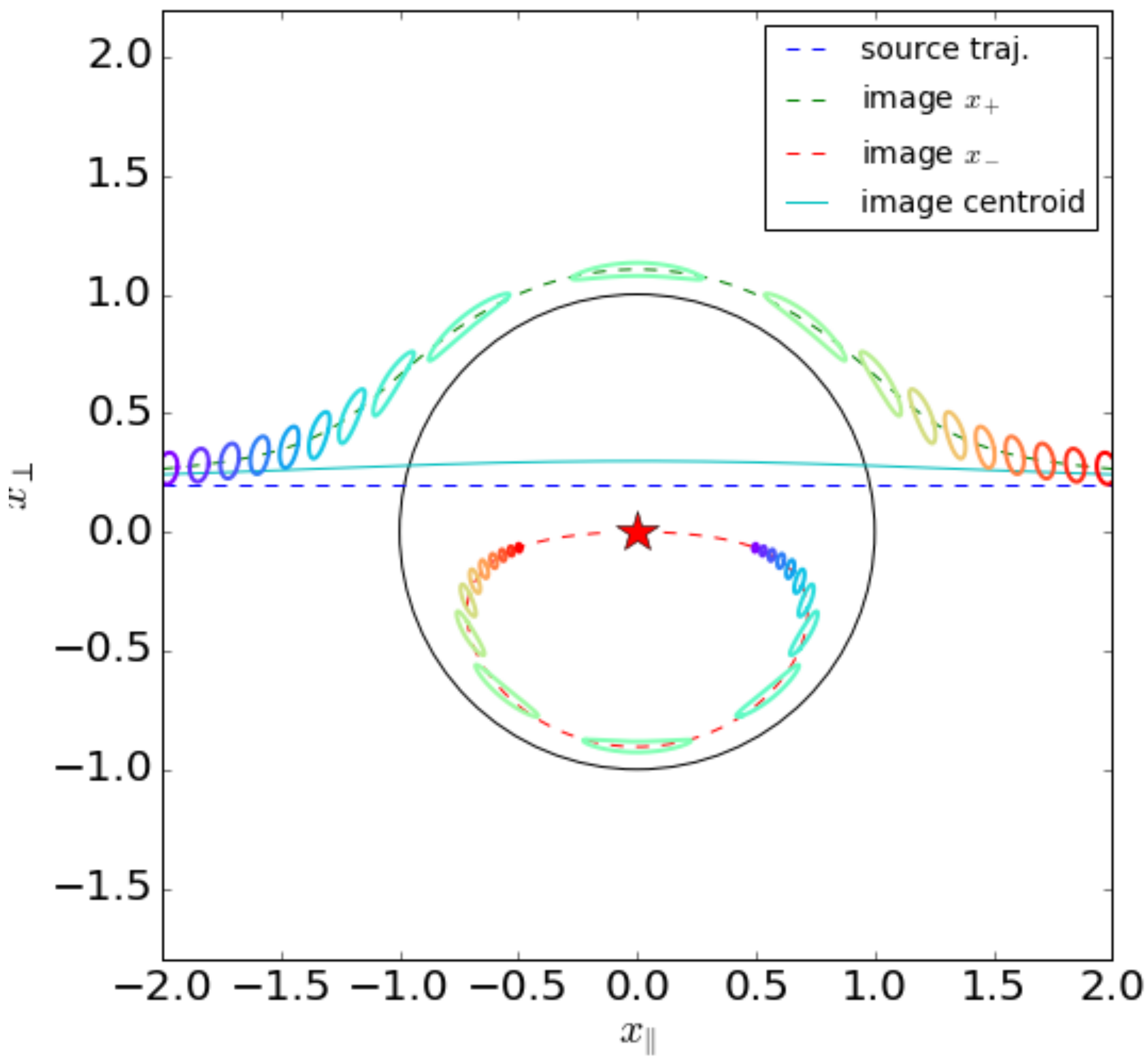
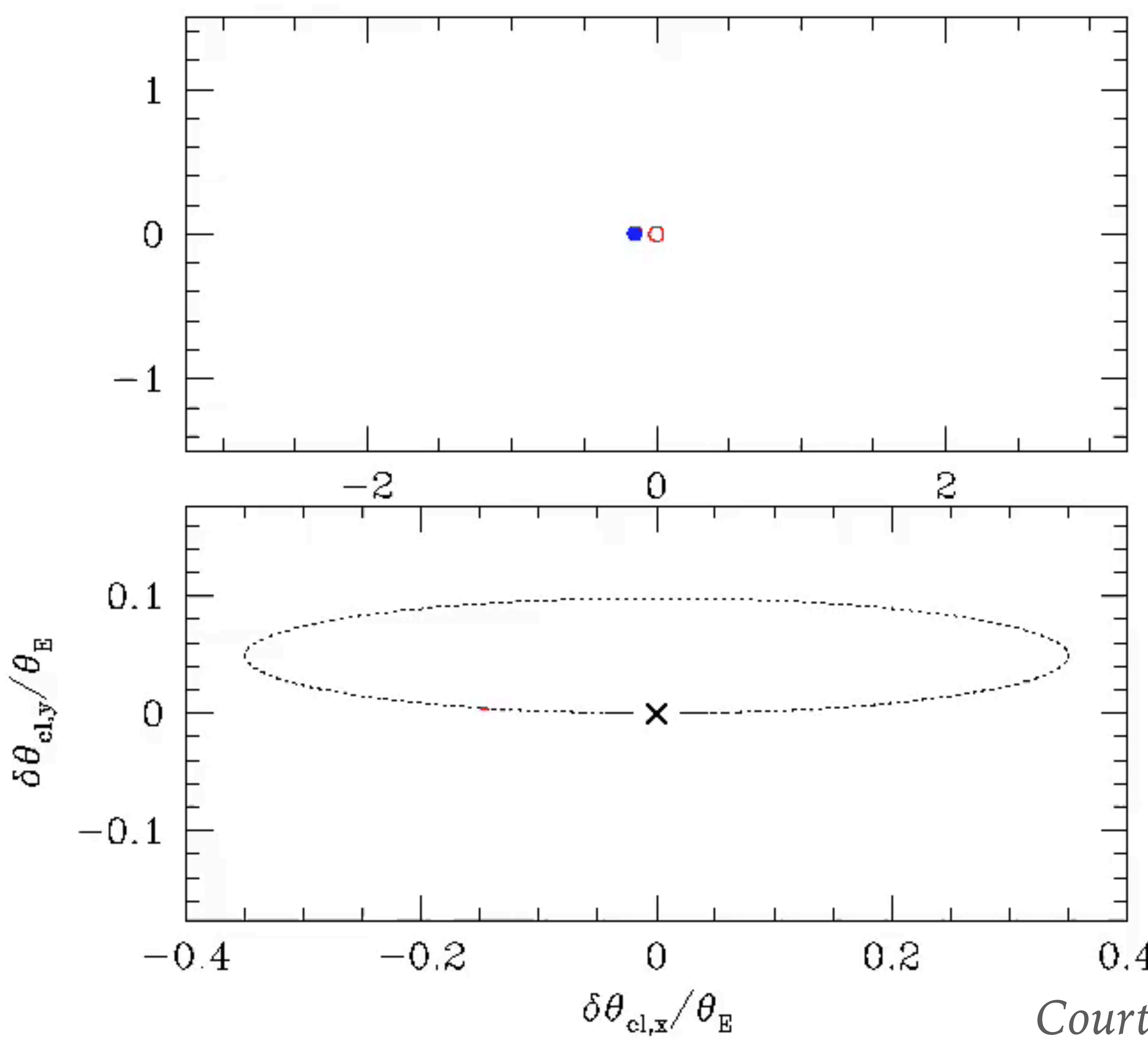
AA 2017-2018

EXAMPLE



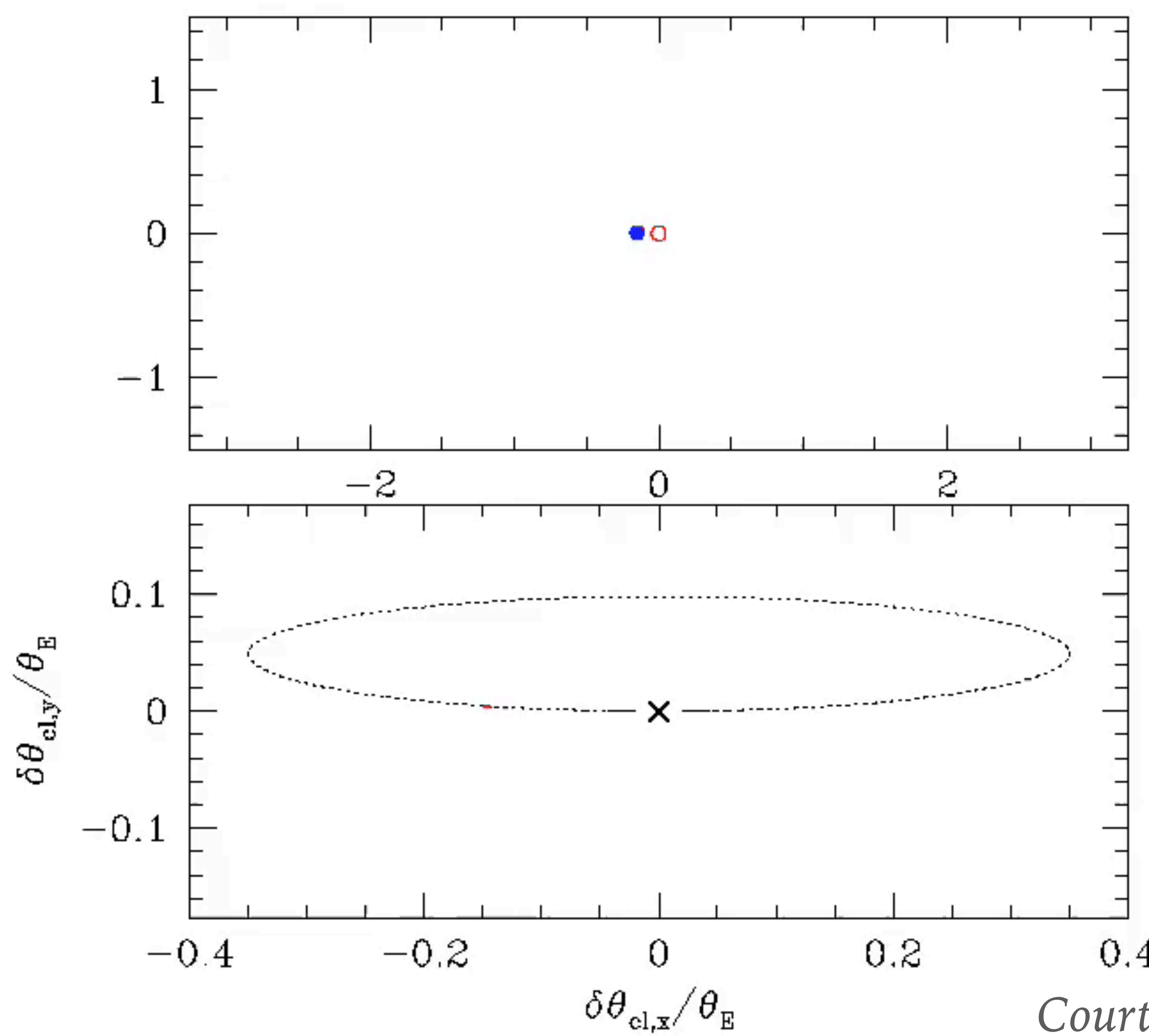
source traj.
image x_+
image x_-
image centroid
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
$x_\perp$
$x_\parallel$

ASTROMETRIC MICROLENSING (ANIMATION)



Courtesy of S. Gaudi

ASTROMETRIC MICROLENSING (ANIMATION)

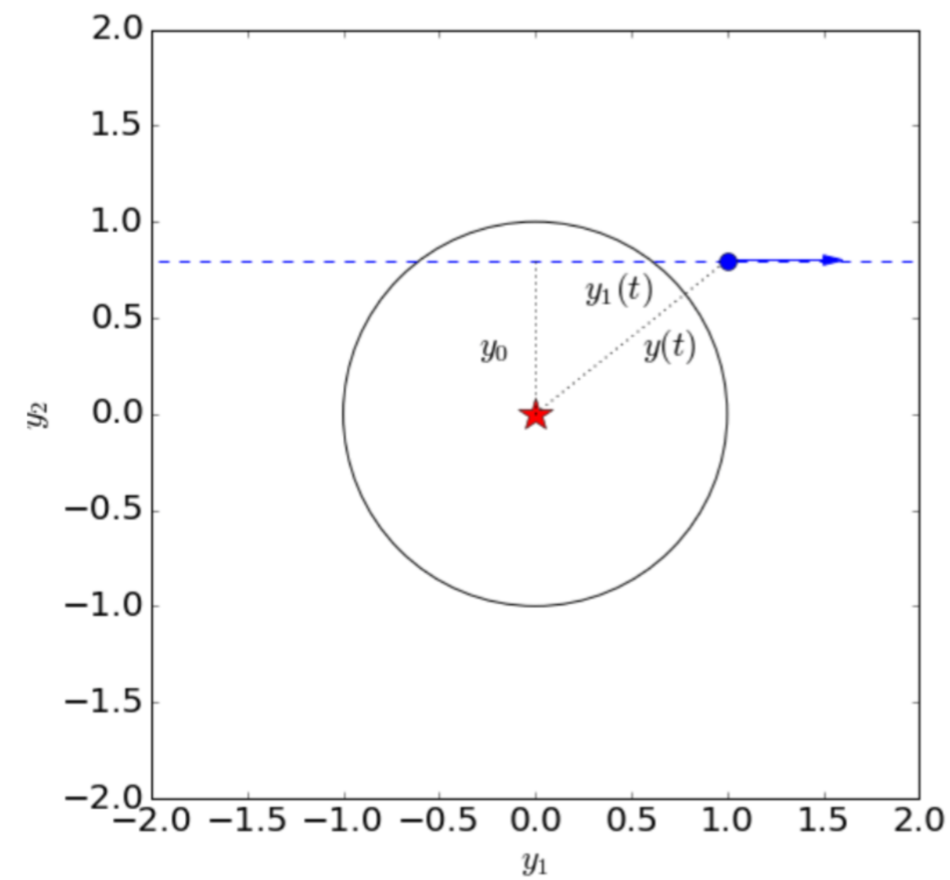


Courtesy of S. Gaudi

HOW DO WE EXPLAIN THIS PATH?

We can decompose the shift into the components parallel and perpendicular to the motion of the source:

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t-t_0)^2}{t_E^2}}$$

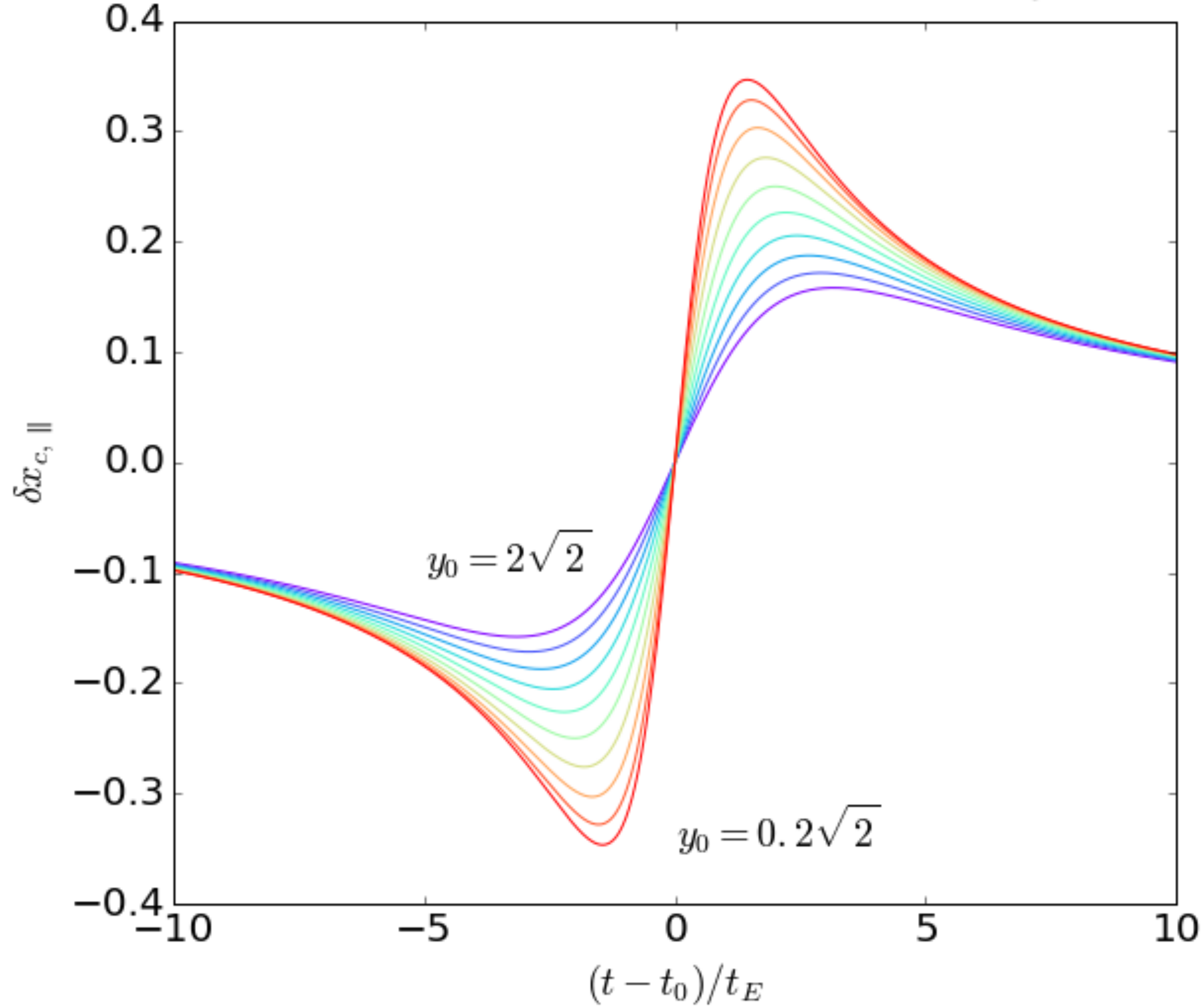


$$\delta x_c = \frac{y}{y^2+2}$$

$$\begin{aligned}
\delta x_{c,\parallel} &= \frac{y_\parallel}{y^2+2} = \frac{(t-t_0)/t_E}{[(t-t_0)/t_E]^2 + y_0^2 + 2} \\
\delta x_{c,\perp} &= \frac{y_\perp}{y^2+2} = \frac{y_0}{[(t-t_0)/t_E]^2 + y_0^2 + 2}
\end{aligned}$$

RESULTS: PARALLEL COMPONENT

$$\delta x_{c,\parallel} = \frac{y_\parallel}{y^2+2} = \frac{(t-t_0)/t_E}{[(t-t_0)/t_E]^2 + y_0^2 + 2}$$



Antisymmetric!

Taking the derivative:

$$\frac{d(\delta x_{c,\parallel})}{dp} = \frac{y_0^2+2-p^2}{(p^2+y_0^2+2)^2}$$

$$(t-t_0)/t_E = \pm\sqrt{y_0^2+2}$$

$$\delta x_{c,\parallel} = \pm\frac{1}{2\sqrt{y_0^2+2}}$$

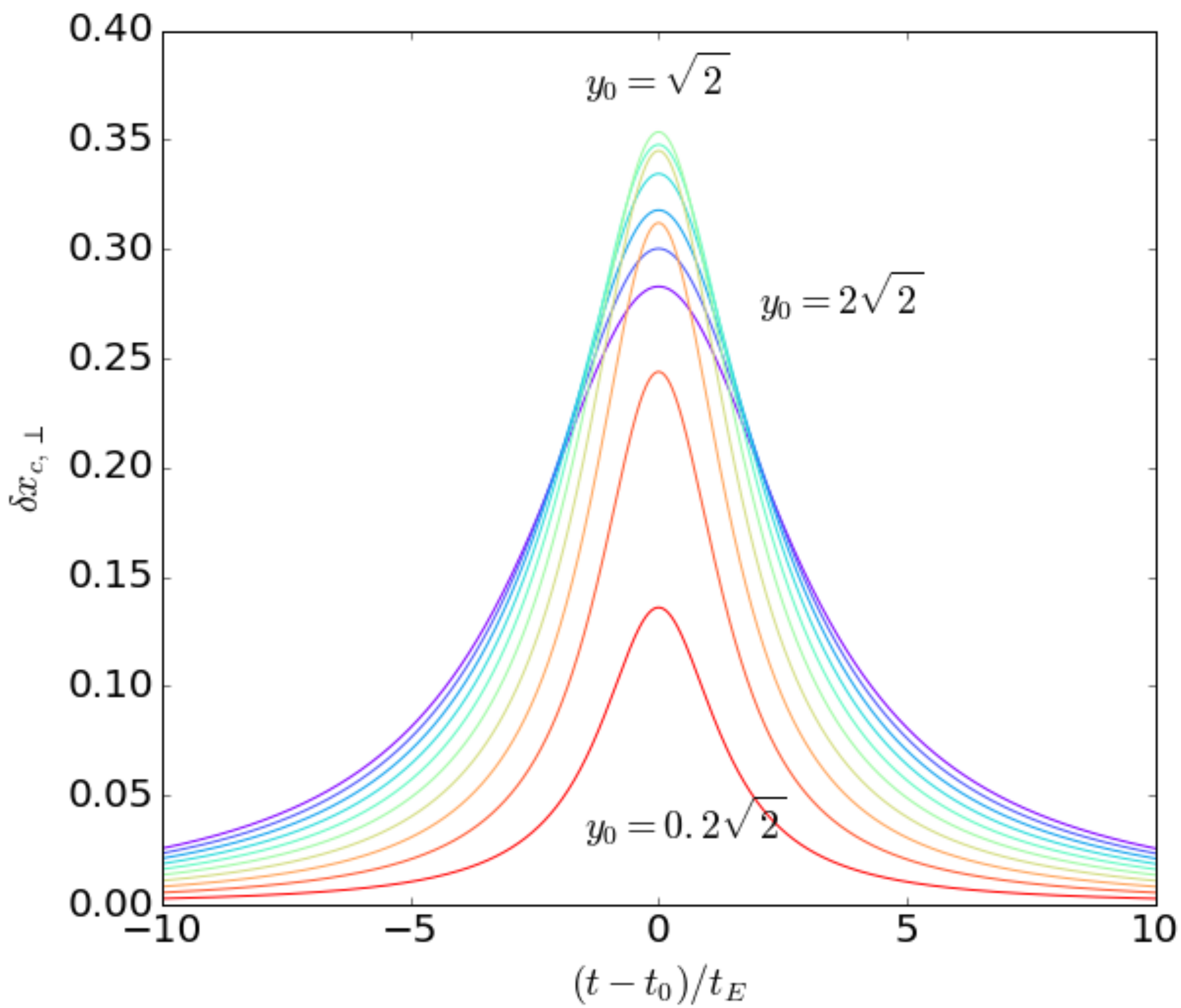
For small y_0:

$(t-t_0)/t_E \approx \pm\sqrt{2}$ and $\delta x_{c,\parallel} \approx \delta x_{c,max}$.

$$y = \sqrt{2}$$

RESULTS: PERPENDICULAR COMPONENT

$$\delta x_{c,\perp} = \frac{y_\perp}{y^2+2} = \frac{y_0}{[(t-t_0)/t_E]^2+y_0^2+2}$$



One maximum in $t = t_0$

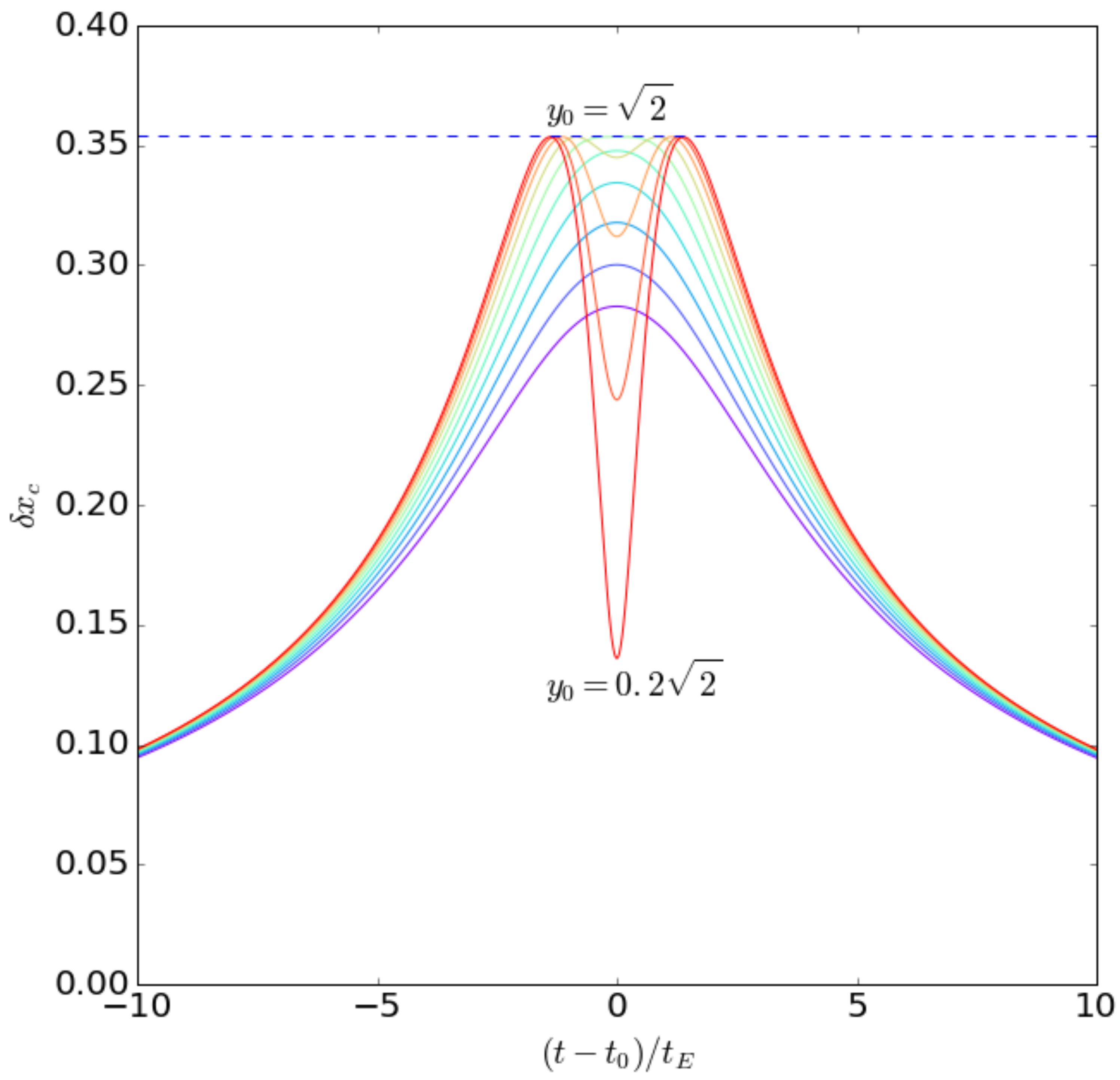
$$\delta x_{c,\perp,max} = \frac{y_0}{y_0^2+2}$$

the peak is the highest for

$$y_0 = \sqrt{2},$$

$$\delta x_{c,max}$$

RESULTS



$$\frac{d(\delta x_c)}{dp} = p\frac{2-y_0^2-p^2}{\sqrt{y_0^2+p^2}(y_0^2+p^2+2)^2}$$

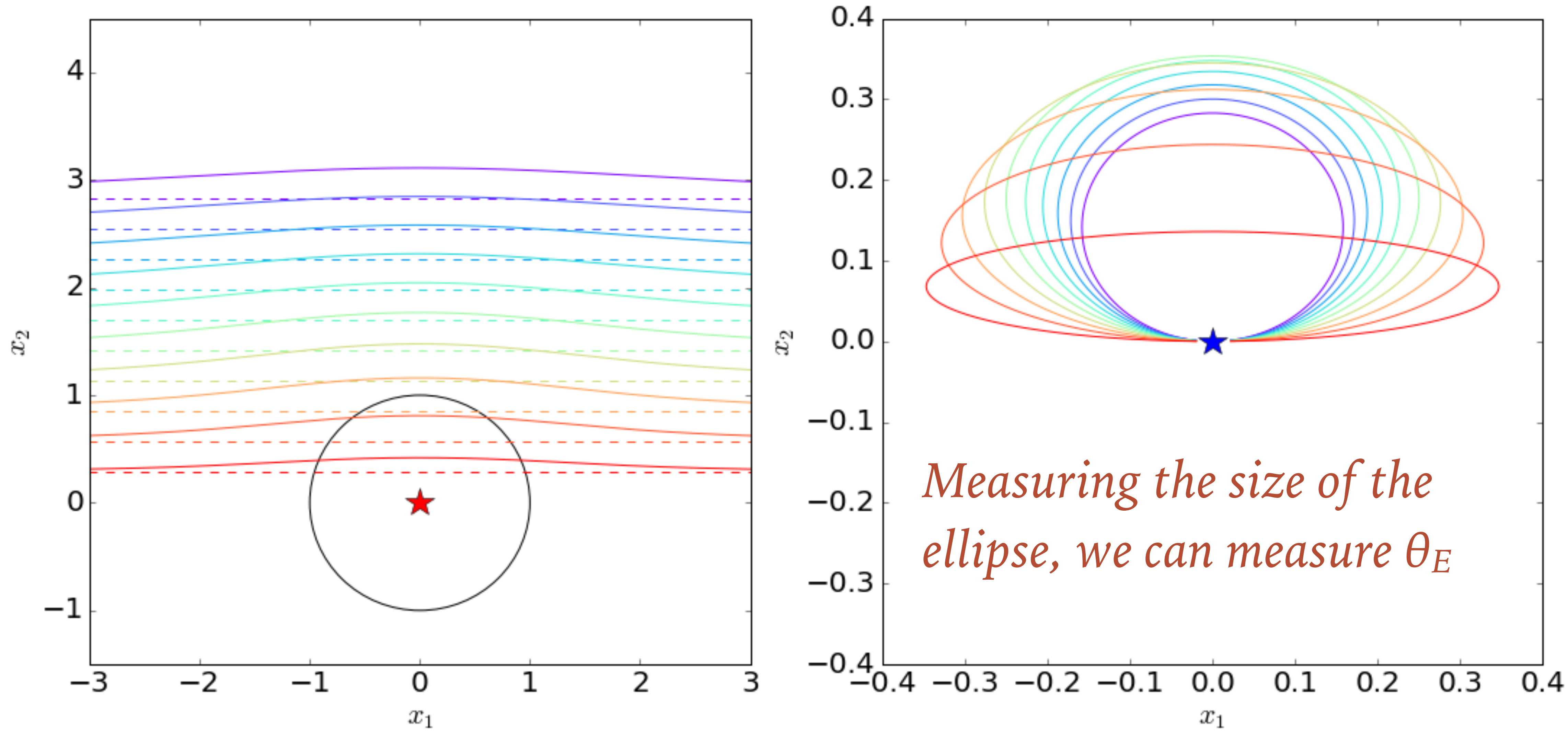
For small y_0: two maxima and one minimum

In this case, the shift is mainly parallel to the motion of the source

For large y_0: one maximum

In this case, the shift is mainly perpendicular to the motion of the source

WHAT IS THE PATH OF THE CENTROID SHIFT WITH RESPECT TO THE UNPERTURBED SOURCE?



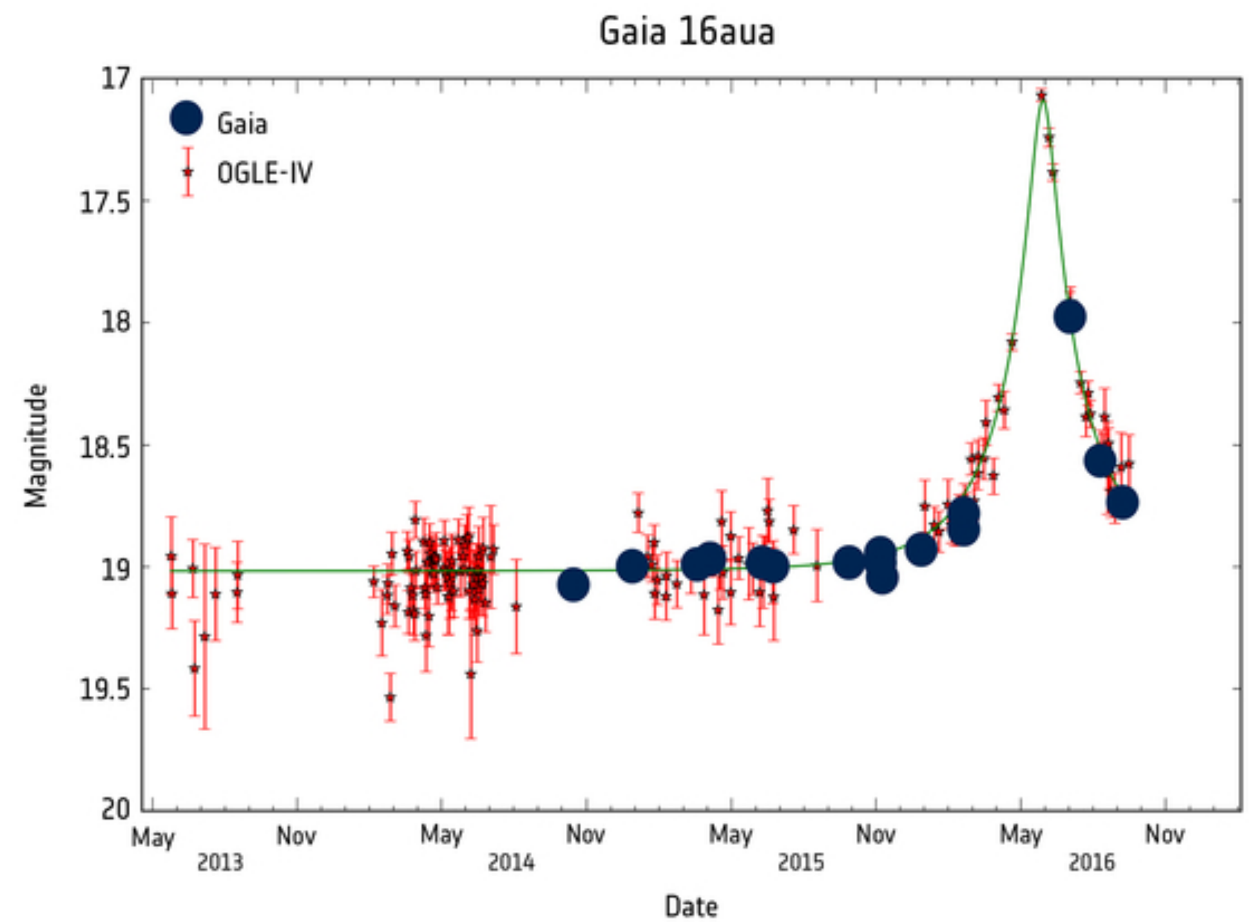
$$
\begin{aligned}
a &= \frac{1}{2}\frac{1}{\sqrt{y_0^2+2}} \\
b &= \frac{1}{2}\frac{y_0}{y_0^2+2} \, .
\end{aligned}
$$

For large impact parameters, the ellipse becomes a circle.

For small impact parameters, it becomes a straight line of length 0.5

GAIA AND MICROLENSING

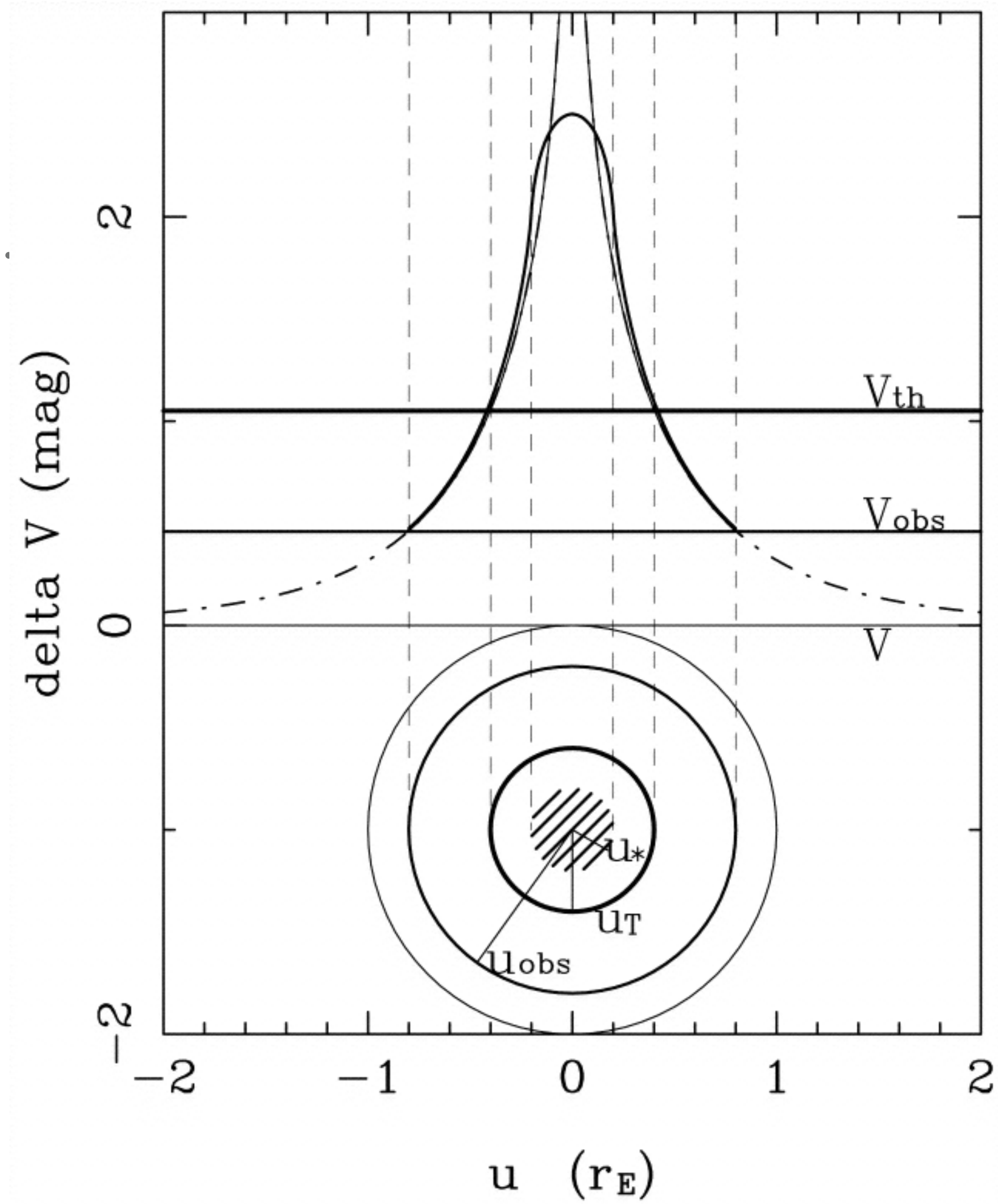
- GAIA has made the first photometric microlensing detection recently…
- Will it be able to detect the astrometric effect too?



GAIA + OGLE IV

FINITE SOURCE SIZE

- microlensing events are detectable when the source passes close or onto the caustics of the lens
- if the source is not point-like, the effect of magnification will be smeared out
- this effect can be used to infer the angular size of the source in units of the Einstein ring radius
- it is often possible to measure the size of the source via its intrinsic color and magnitude using empirical color-surface brightness relations (Kervella et al. 2004)
- in these cases, it is possible to measure the Einstein radius!
- combining with the Einstein cr. time we can measure the proper motion



$$\rho = \frac{\theta_\star}{\theta_E} \qquad \theta_E = \theta_\star \frac{t_E}{t_\star}$$

$$log(2\theta_*) = 0.0755(V - K) + 0.5170 - 0.2K$$

$$\frac{M}{D_{rel}} = \frac{c^2}{4G}\theta_E^2 \qquad \mu_{rel} = \frac{v}{D_L} = \frac{\theta_E}{t_E}$$

MULTIPLE POINT MASSES

- In the case of multiple point masses, we can use the superposition principle to compute the total deflection angle: total deflection angle = sum of individual deflections
- compared to an individual point mass, the spatial symmetry is broken
- The mass scale of the system is the total mass=sum of the individual masses
- We may use this mass to define an equivalent Einstein radius and use it to scale all angles

MULTIPLE POINT MASSES

$$M_{tot} = \sum_{i=1}^{N} M_i. \qquad\qquad m_i = M_i / M_{tot}$$

$$\vec{\alpha}(\vec{\theta}) = \sum_{i=1}^{N} \frac{D_{\rm LS}}{D_{\rm L} D_{\rm S}} \frac{4GM_i}{c^2} \frac{(\vec{\theta} - \vec{\theta}_i)}{|\vec{\theta} - \vec{\theta}_i|^2} \frac{M_{tot}}{M_{tot}} = \sum_{i=1}^{N} m_i \frac{\theta_E^2}{|\vec{\theta} - \vec{\theta}_i|^2} (\vec{\theta} - \vec{\theta}_i)$$

dividing by θ_E:

$$\vec{\alpha}(\vec{x}) = \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{x}_i|^2} (\vec{x} - \vec{x}_i)$$

$$\vec{y} = \vec{x} - \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{x}_i|^2} (\vec{x} - \vec{x}_i)$$

COMPLEX LENS EQUATION (WITT, 1990)

$$\vec{y} = \vec{x} - \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{x}_i|^2} (\vec{x} - \vec{x}_i)$$

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$$z = x_1 + ix_2 \qquad z_s = y_1 + iy_2$$

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$$z = x_1 + ix_2 \qquad\qquad z_s = y_1 + iy_2$$

$$\alpha(z) = \sum_{i=1}^{N} m_i \frac{(z - z_i)}{(z - z_i)(z^* - z_i^*)} = \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

COMPLEX LENS EQUATION (WITT, 1990)

$$\vec{y} = \vec{x} - \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{x}_i|^2} (\vec{x} - \vec{x}_i)$$

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$$z_s = z - \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

COMPLEX LENS EQUATION (WITT, 1990)

- Thus:

$$z_s = z - \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

- Taking the conjugate:

$$z^* = z_s^* + \sum_{i=1}^{N} \frac{m_i}{z - z_i}$$

- We obtain z* and substitute it back into the original equation, which results in a (N^2+1)th order complex polynomial equation

- This equation can be solved only numerically, even in the case of a binary lens

COMPLEX LENS EQUATION (WITT, 1990)

- Note that the solutions are not necessarily solutions of the lens equations (spurious solutions)
- One has to check if the solutions are solutions of the lens equation
- Rhie (2001,2003): maximum number of images is $5(N-1)$ for $N>2$

JACOBIAN DETERMINANT

The Jacobian determinant is (on the real plane):

$$\det A = \frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2} - \left(\frac{\partial y_1}{\partial x_2}\right)^2$$

How do we write it in complex notation?

JACOBIAN DETERMINANT

The complex derivatives of z_s are:

$$
\begin{aligned}
\frac{\partial z_s}{\partial z} &= \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} + \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} - \frac{\partial y_1}{\partial x_2}\right) \\
\frac{\partial z_s}{\partial z^*} &= \frac{1}{2}\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} - \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} + \frac{\partial y_1}{\partial x_2}\right)
\end{aligned}
$$

JACOBIAN DETERMINANT

Note that in lensing these two derivatives are equal!

The complex derivatives of z_s are:

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\end{aligned}
$$

Thus:

$$
\left(\frac{\partial z_s}{\partial z}\right)^2 = \frac{1}{4}\left[\left(\frac{\partial y_1}{\partial x_1}\right)^2 + \left(\frac{\partial y_1}{\partial x_2}\right)^2 + 2\frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2}\right]
$$

$$
\left(\frac{\partial z_s}{\partial z^*}\right)\left(\frac{\partial z_s}{\partial z^*}\right)^* = \frac{1}{4}\left[\left(\frac{\partial y_1}{\partial x_1}\right)^2 + \left(\frac{\partial y_1}{\partial x_2}\right)^2 - 2\frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2}\right] + \left(\frac{\partial y_1}{\partial x_2}\right)^2
$$

JACOBIAN DETERMINANT

Note that in lensing these two derivatives are equal!

The complex derivatives of z_s are:

$$\frac{\partial z_s}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} + \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} - \frac{\partial y_1}{\partial x_2}\right)$$

$$\frac{\partial z_s}{\partial z^*} = \frac{1}{2}\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} - \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} + \frac{\partial y_1}{\partial x_2}\right)$$

Thus:

$$\left(\frac{\partial z_s}{\partial z}\right)^2 = \frac{1}{4}\left[\left(\frac{\partial y_1}{\partial x_1}\right)^2 + \left(\frac{\partial y_1}{\partial x_2}\right)^2 + 2\frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2}\right]$$

$$\left(\frac{\partial z_s}{\partial z^*}\right)\left(\frac{\partial z_s}{\partial z^*}\right)^* = \frac{1}{4}\left[\left(\frac{\partial y_1}{\partial x_1}\right)^2 + \left(\frac{\partial y_1}{\partial x_2}\right)^2 - 2\frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2}\right] + \left(\frac{\partial y_1}{\partial x_2}\right)^2$$

By taking the difference of these two equations:

$$\left(\frac{\partial z_s}{\partial z}\right)^2 - \left(\frac{\partial z_s}{\partial z^*}\right)\left(\frac{\partial z_s}{\partial z^*}\right)^* = \frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2} - \left(\frac{\partial y_1}{\partial x_2}\right)^2 = \det A$$

JACOBIAN DETERMINANT

Now, we can use the lens equation:

$$z_s = z - \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

To obtain:

$$\frac{\partial z_s}{\partial z} = 1 \qquad \frac{\partial z_s}{\partial z^*} = \sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2}$$

so that

$$\det A = 1 - \left| \sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2} \right|^2$$

CRITICAL LINES

From this equation:

$$\det A = 1 - \left| \sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2} \right|^2$$

We see that on the critical lines (detA=0)

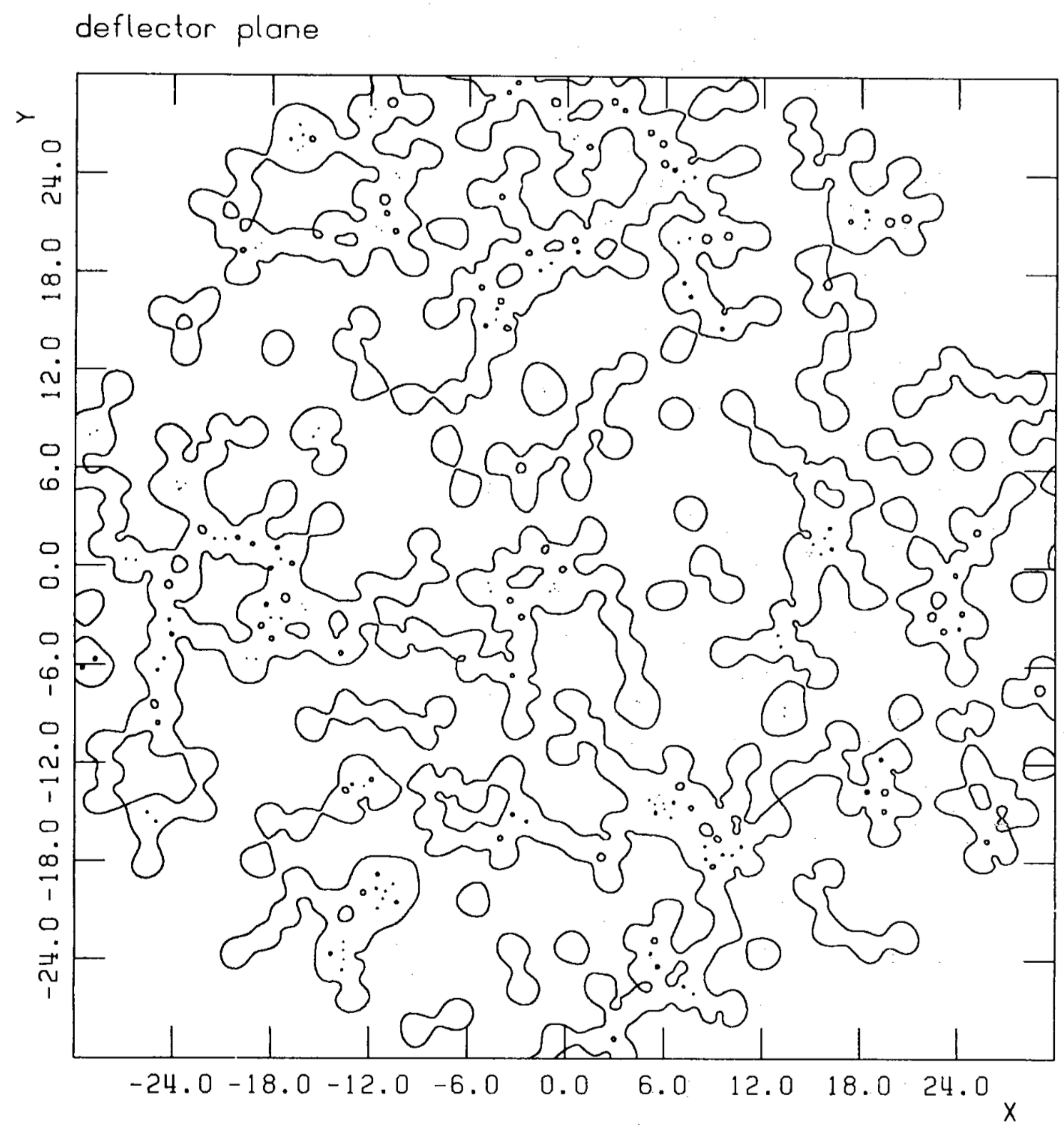
$$\left| \sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2} \right|^2 = 1$$

This sum has to be satisfied on the unit circle:

$$\sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2} = e^{i\phi} \qquad \phi \in [0, 2\pi)$$

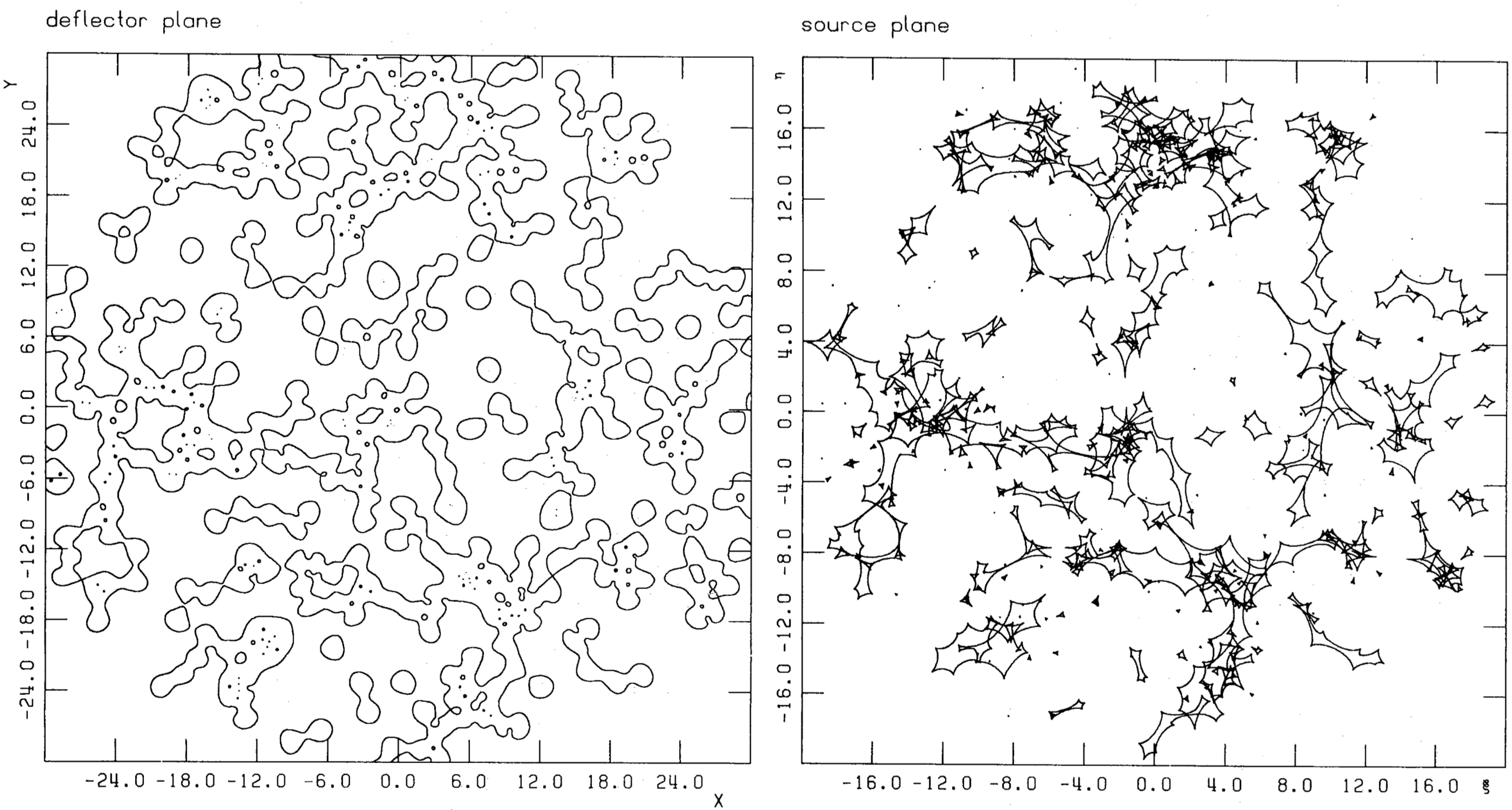
Getting rid of the fraction, this equation can be turned into a polynomial of degree 2N: for each phase, there are <=2N critical points. Solving for all phases, we find up to 2N critical lines.

CRITICAL LINES



critical lines originated by 400 stars *Witt, 1990, A&A, 236, 311*

CRITICAL LINES AND CAUSTICS



critical lines and caustics originated by 400 stars

Witt, 1990, A&A, 236, 311