GRAVITATIONAL LENSING

LECTURE 7

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AA 2016-2017

CONTENTS

- convergence and shear
- distance dependence of lensing effects
- magnification
- critical lines and caustics

ON THE SPIN-2 NATURE OF SHEAR: QUIZ

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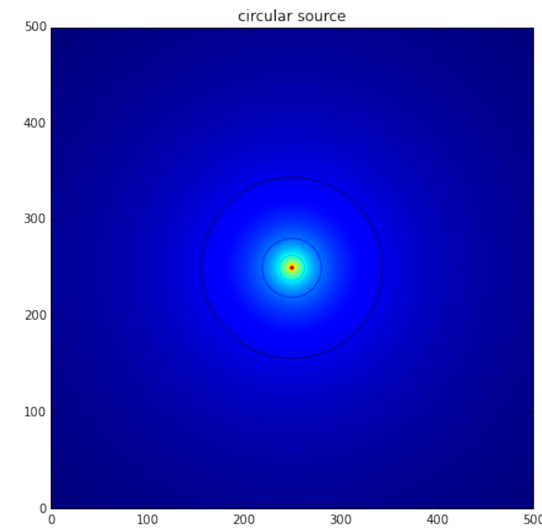

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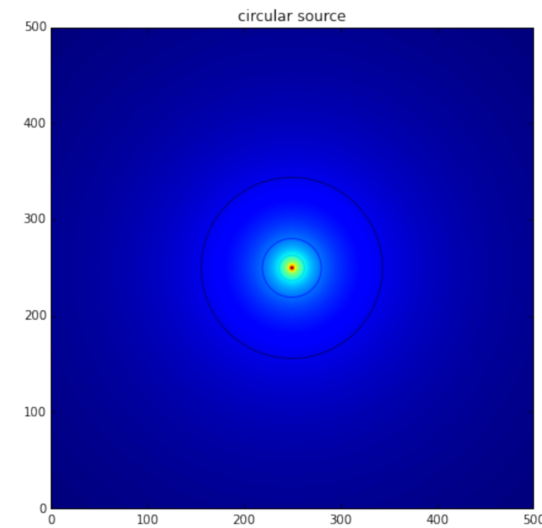

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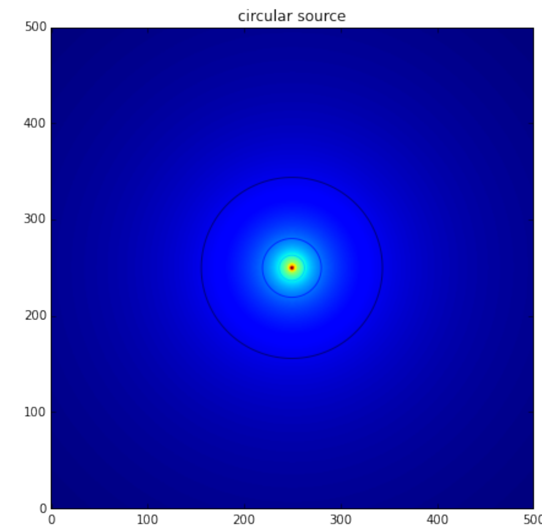

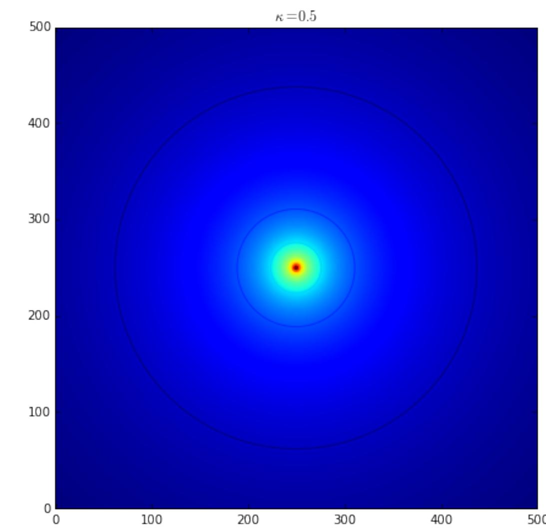

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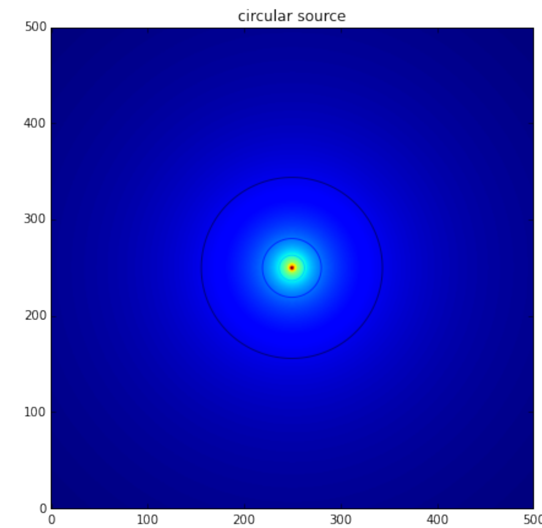

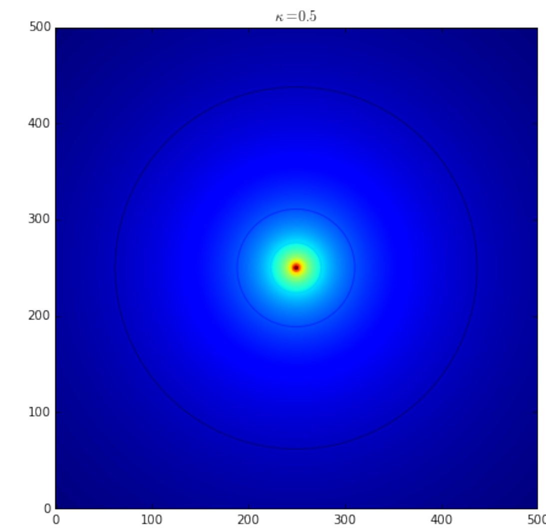

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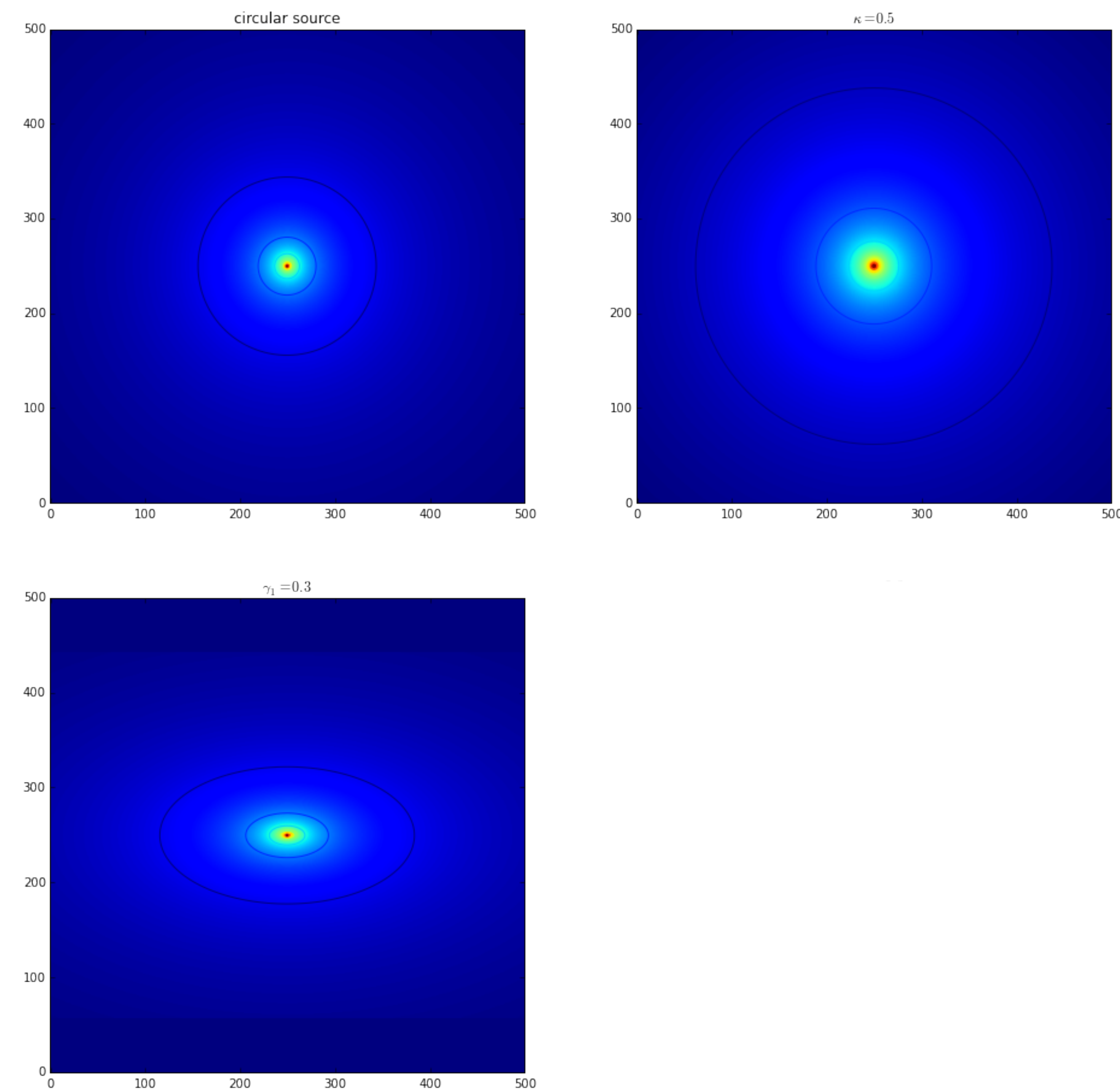

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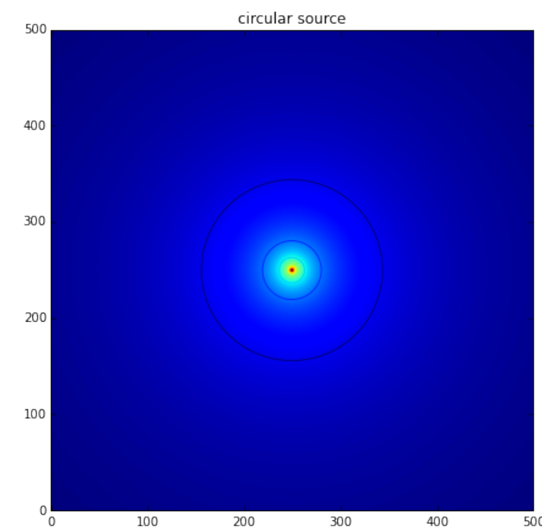

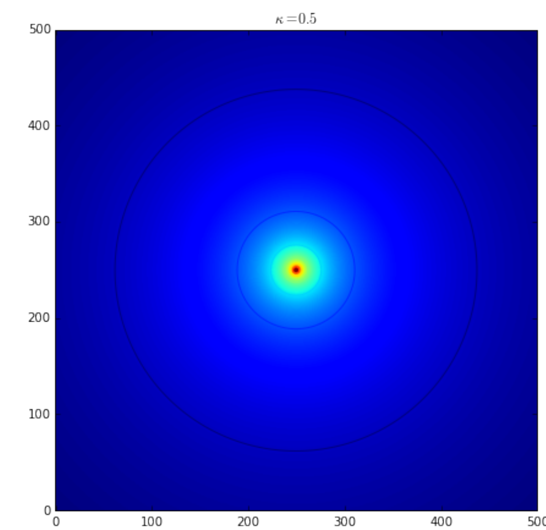

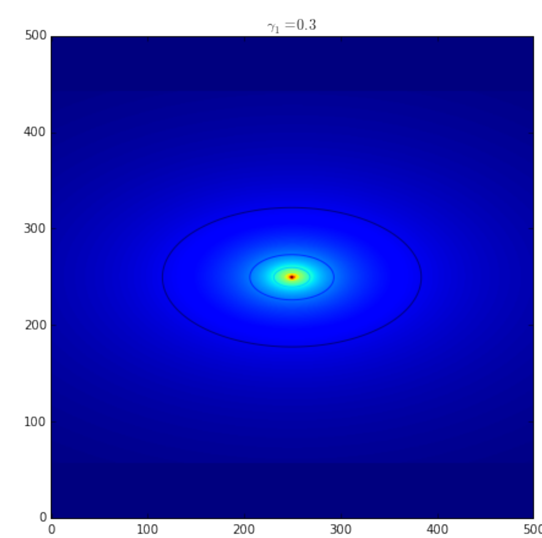

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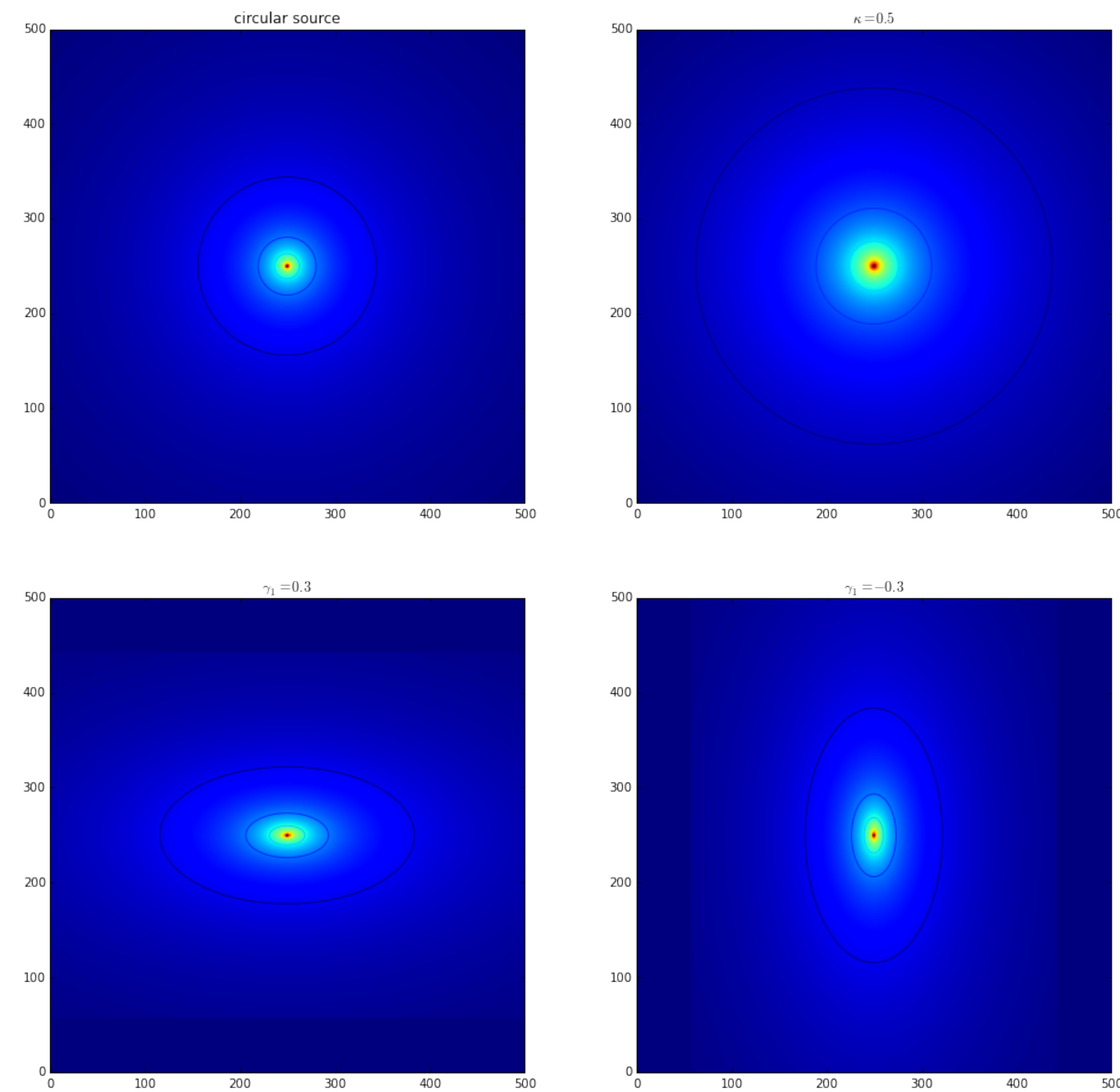

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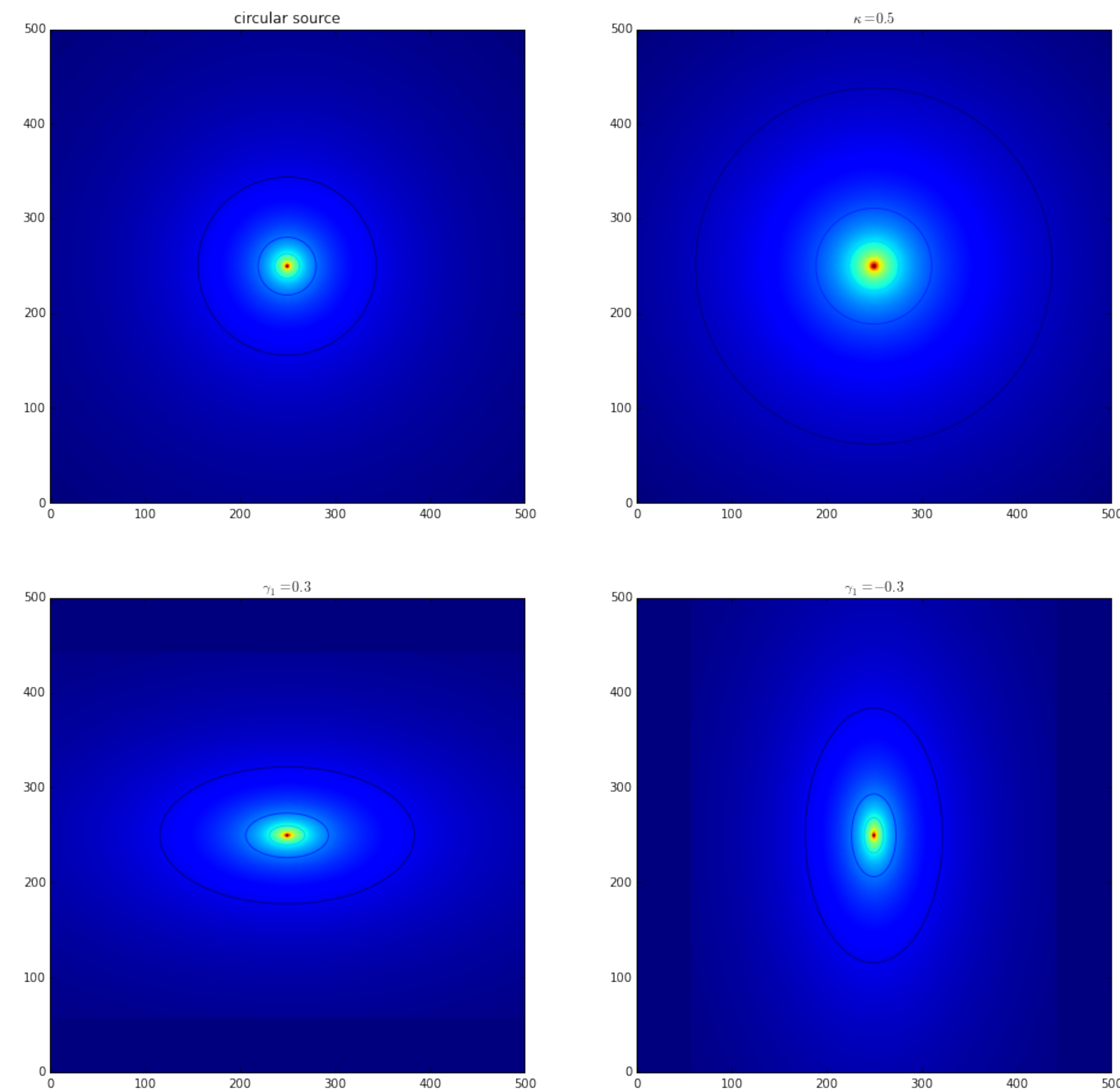

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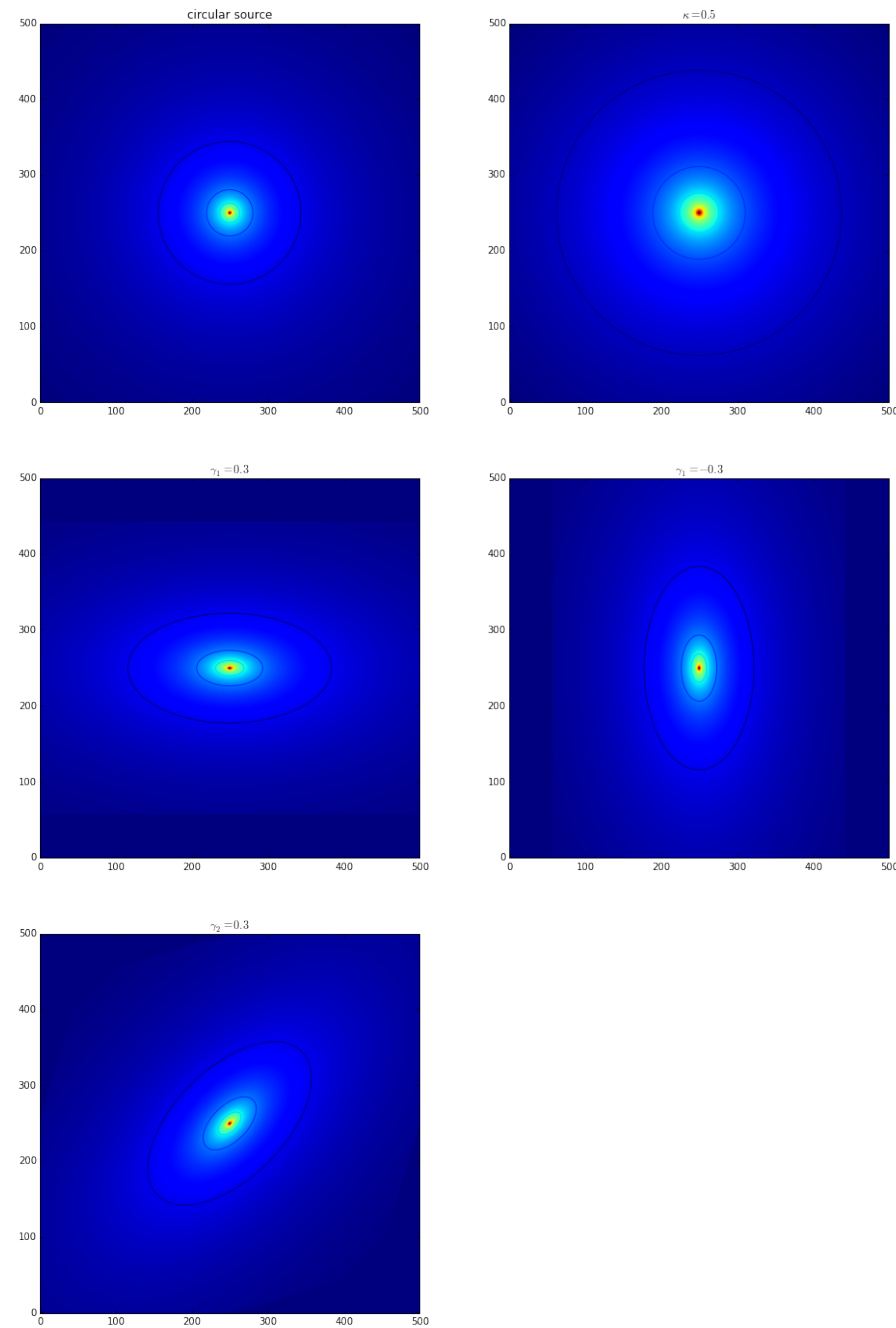

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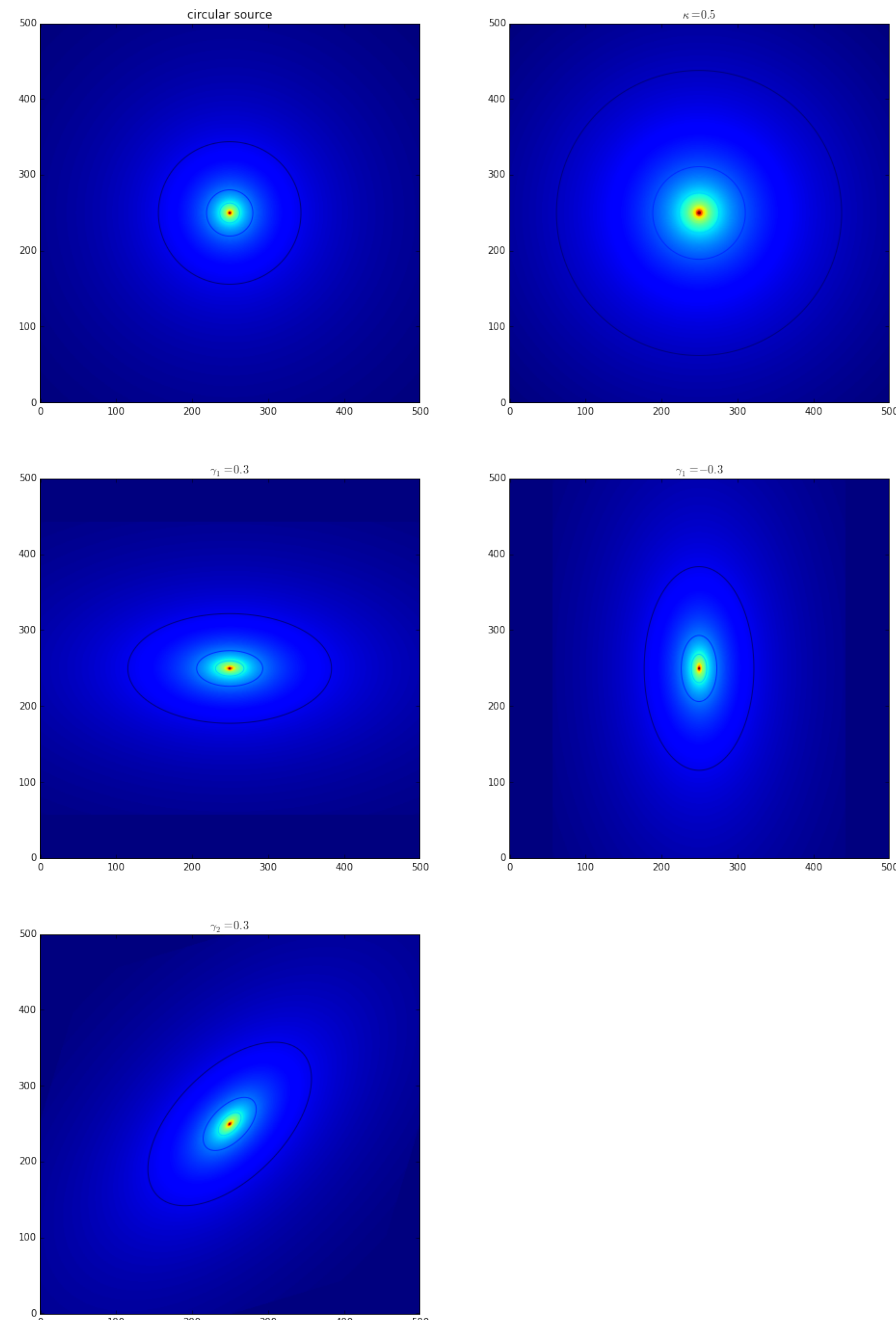

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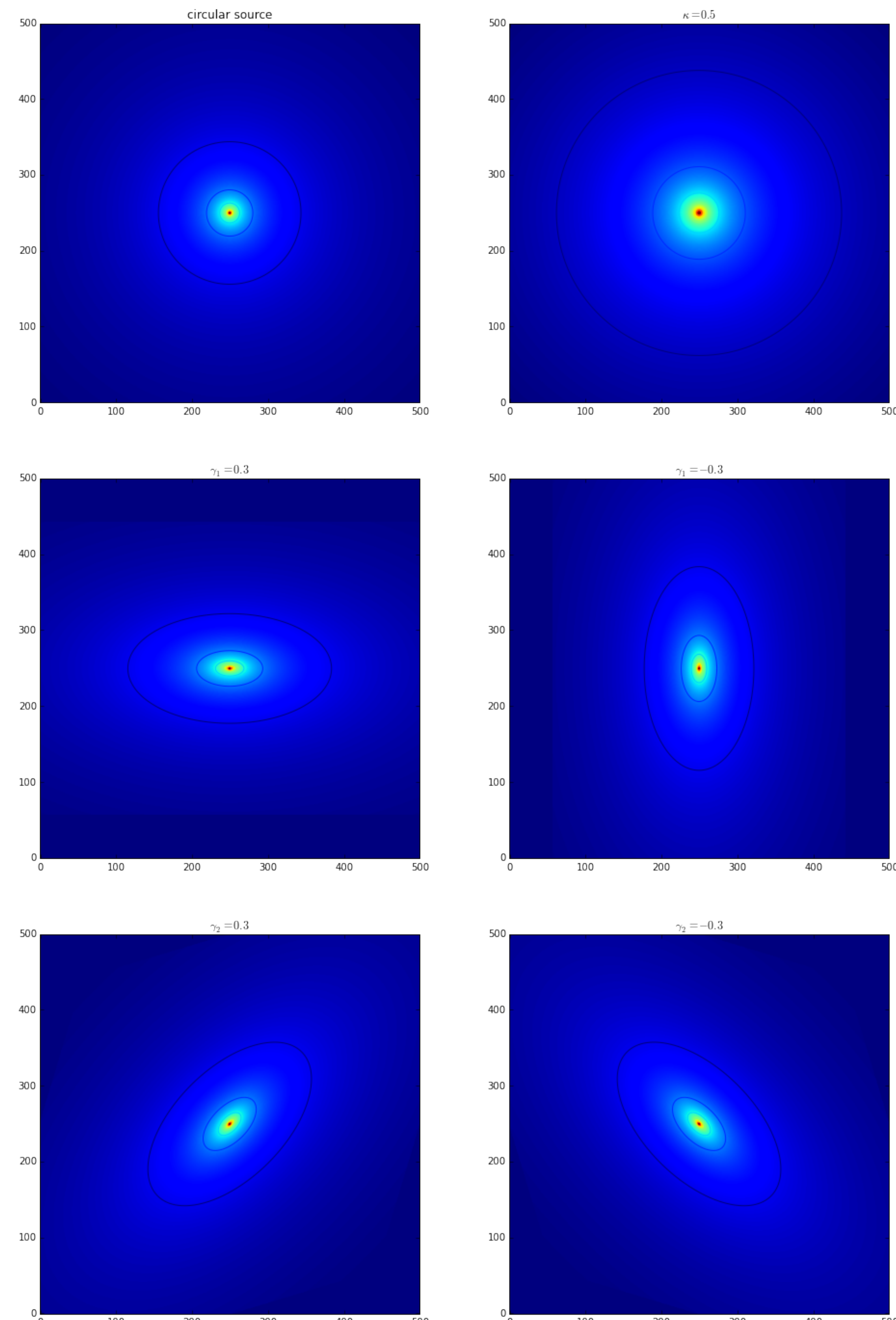
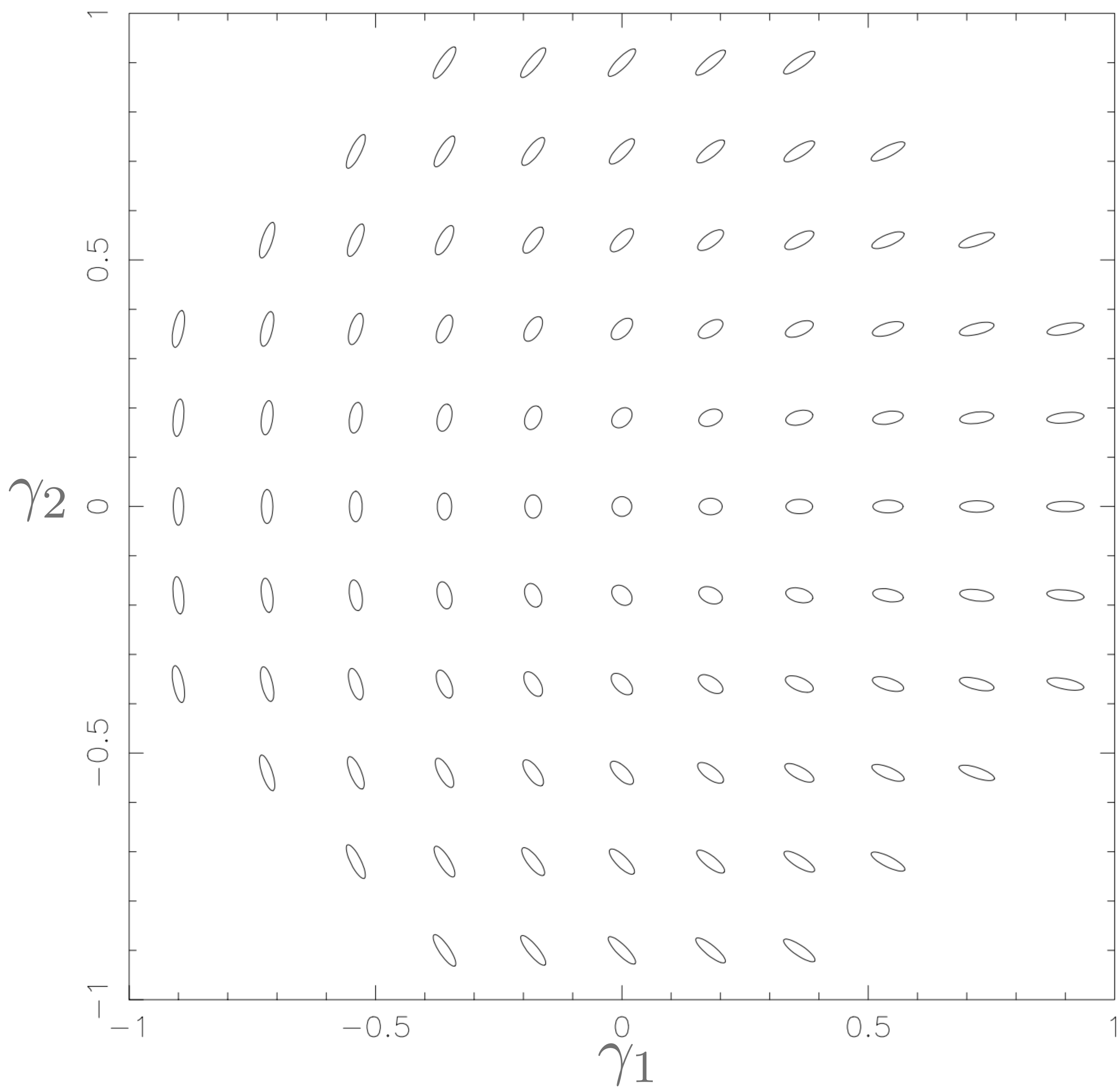


SHEAR DISTORTIONS



DEPENDENCE ON REDSHIFT

We have seen that the lensing potential, the deflection angle, the convergence, the shear… depend on a combination of distances.

For example, the lensing potential is:

$$\hat{\Psi}(\vec{\theta}) = \frac{D_{\mathrm{LS}}}{D_{\mathrm{L}} D_{\mathrm{S}}} \frac{2}{c^2} \int \Phi(D_{\mathrm{L}}\vec{\theta}, z) \mathrm{d}z$$

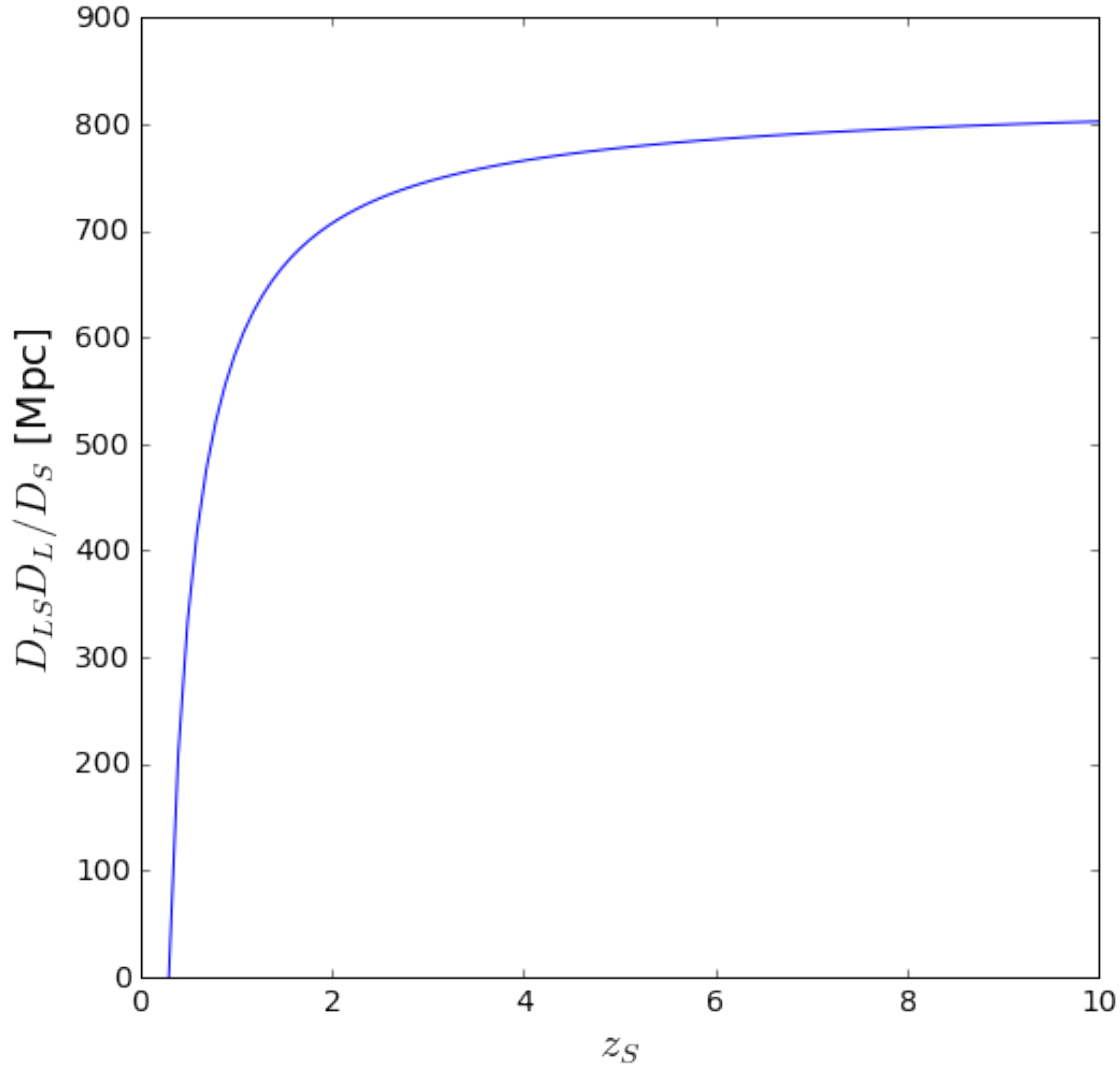
Every spatial derivative of Ψ *introduces a factor* D_L.

The distance ratio $D_{LS}D_L/D_S$ *is called "lensing distance".*

Both the shear and the convergence, being second derivatives of the lensing potential, scale as the lensing distance

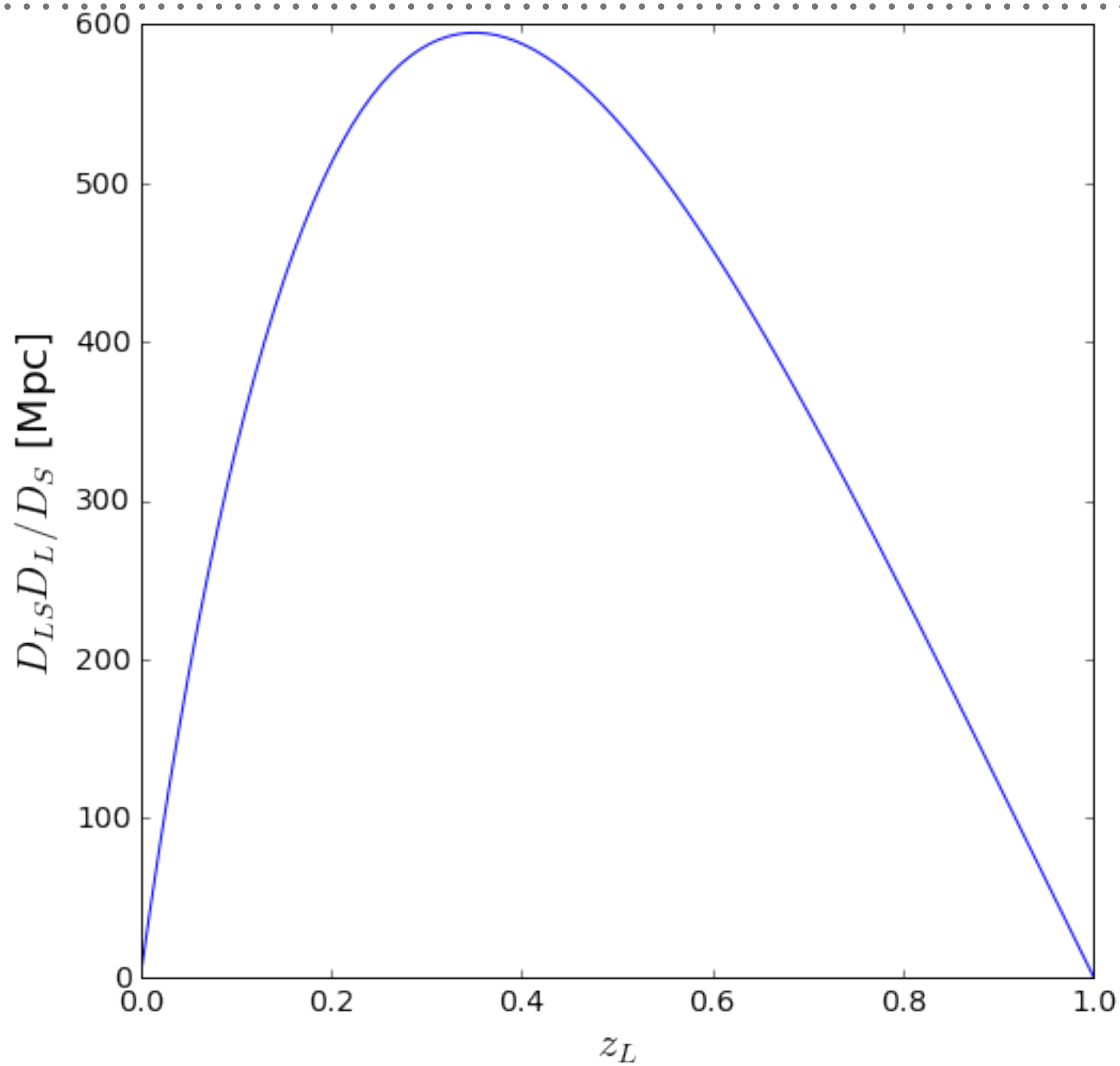
$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_S}{D_{LS} D_L}$$

HOW DOES THE LENSING DISTANCE SCALE WITH SOURCE REDSHIFT?



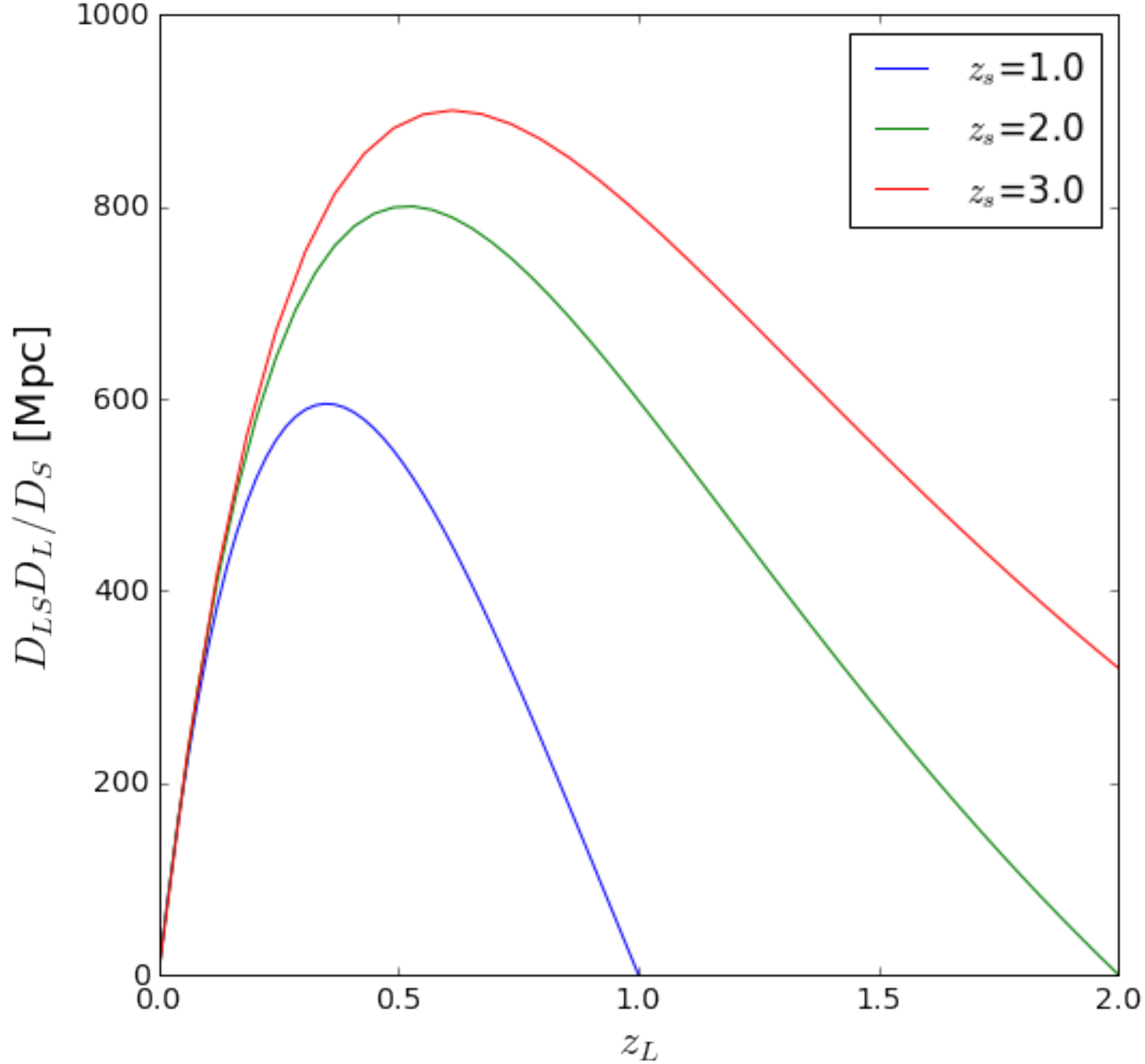
Note that if the lensing distance grows, the critical surface density decreases, the convergence and the shear grow!

HOW DOES THE LENSING DISTANCE SCALE WITH LENS REDSHIFT?



The lensing distance peaks at ~half way between the source and the observer, meaning that there is an optimal distance where the lens produces its largest effects.

HOW DOES THE LENSING DISTANCE SCALE WITH LENS REDSHIFT?



Of course, the peak moves to larger distances as the distance to the source increases.

CONSERVATION OF SURFACE BRIGHTNESS

*The source **surface brightness** is*

$$I_\nu = \frac{dE}{dt dA d\Omega d\nu}$$

In phase space, the radiation emitted is characterized by the density

$$f(\vec{x}, \vec{p}, t) = \frac{dN}{d^3x d^3p}$$

In absence of photon creations or absorptions, f is conserved (Liouville theorem)

$$dN = \frac{dE}{h\nu} = \frac{dE}{cp}$$

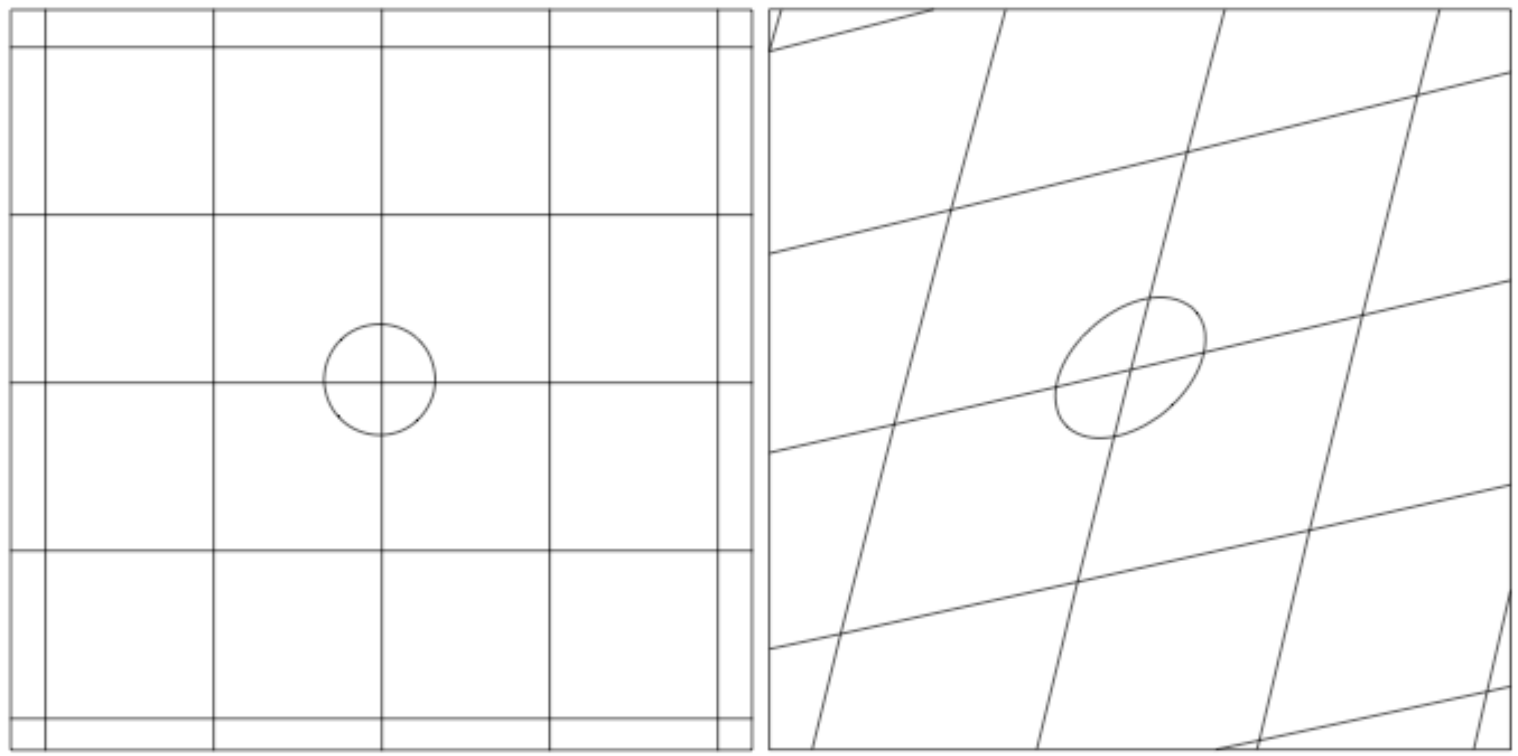
$$d^3x = c dt dA$$

$$d^3\vec{p} = p^2 dp d\Omega$$

$$f(\vec{x}, \vec{p}, t) = \frac{dN}{d^3x d^3p} = \frac{dE}{hcp^3 dA dt d\nu d\Omega} = \frac{I_\nu}{hcp^3}$$

Since GL does not involve creation or absorption of photons, neither it changes the photon momenta (achromatic!), surface brightness is conserved!

MAGNIFICATION



Kneib & Natarajan (2012)

$$F_\nu = \int_I I_\nu(\vec{\theta}) d^2\theta = \int_S I_\nu^S[\vec{\beta}(\vec{\theta})]\mu d^2\beta$$

Lensing changes the amount of photons (flux) we receive from the source by changing the solid angle the source subtends

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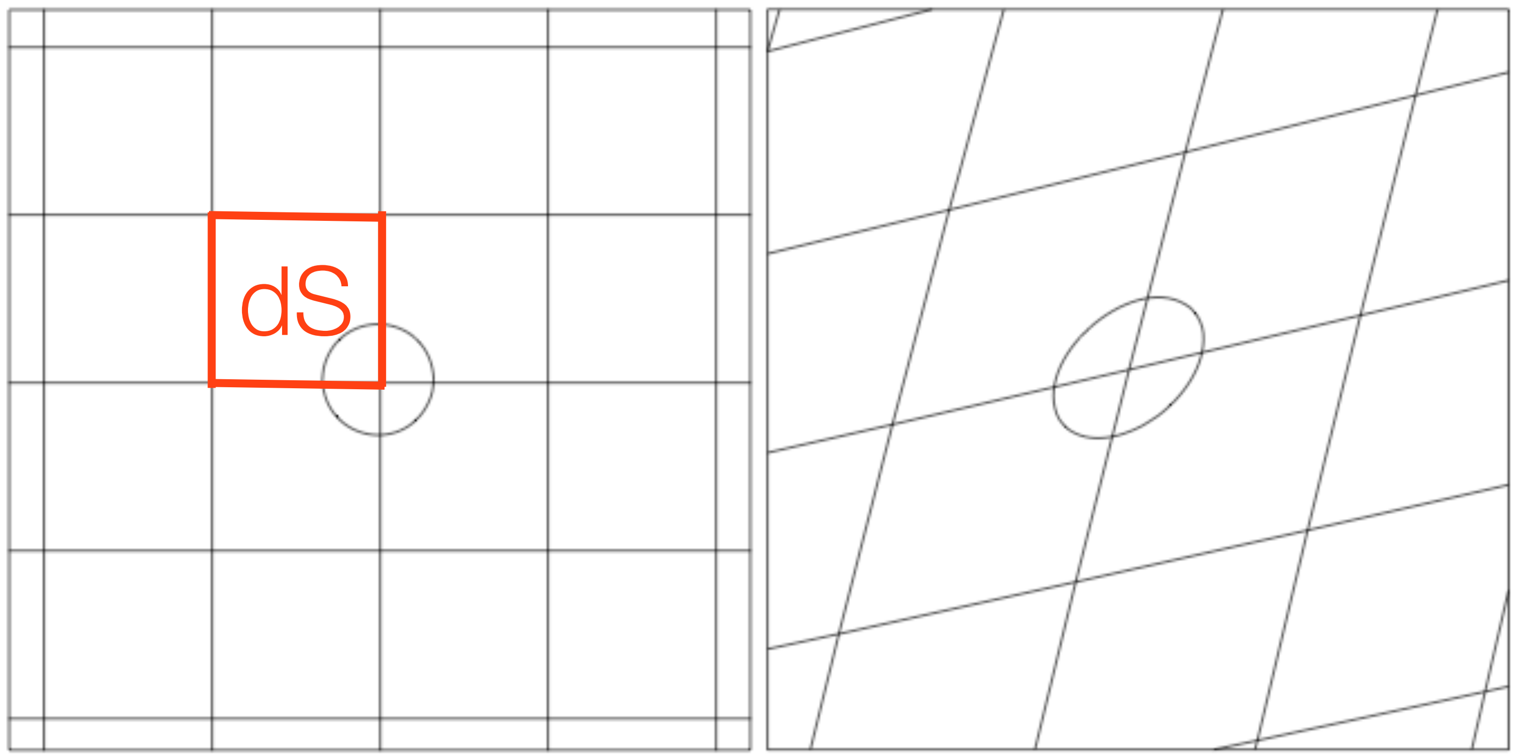

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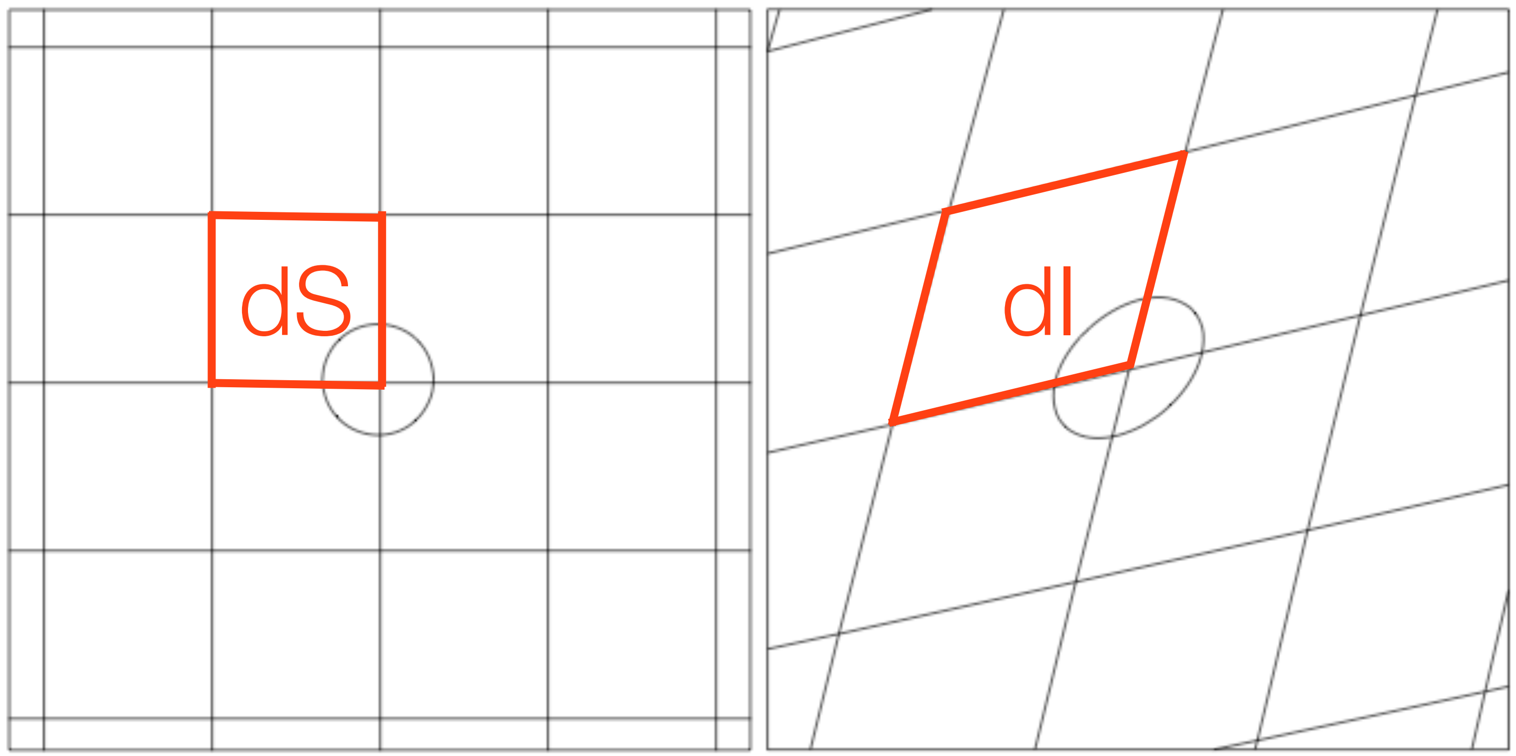

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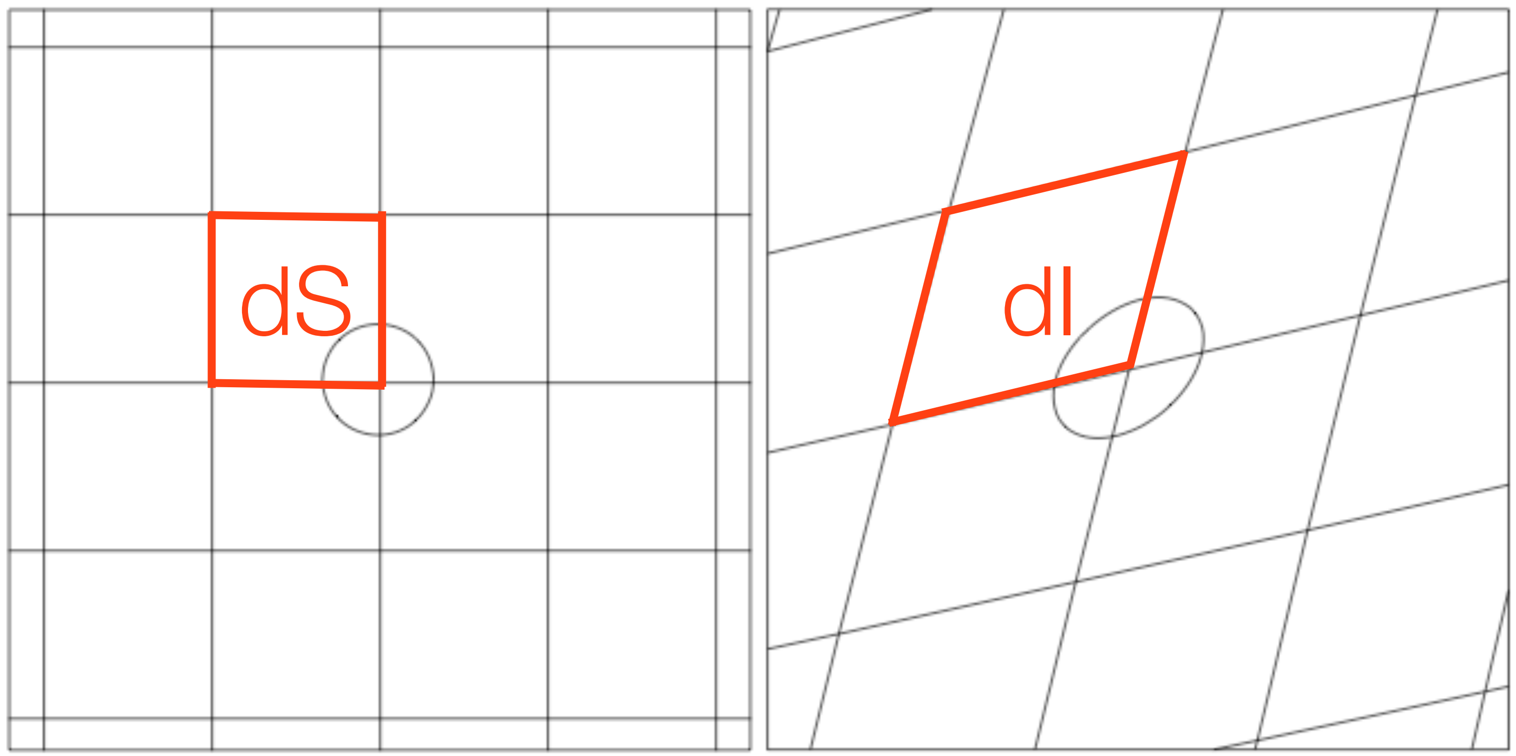

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$$\boxed{\mu = \frac{dI}{dS} = \frac{\delta\theta^2}{\delta\beta^2} = \det A^{-1}}$$

$$F_\nu = \int_I I_\nu(\vec{\theta})d^2\theta = \int_S I_\nu^S[\vec{\beta}(\vec{\theta})]\mu d^2\beta$$

Lensing changes the amount of photons (flux) we receive from the source by changing the solid angle the source subtends

CRITICAL LINES AND CAUSTICS

Both convergence and shear are functions of position on the lens plane:

$\kappa = \kappa(\vec{\theta})$

$\gamma = \gamma(\vec{\theta})$

The determinant of the lensing Jacobian is

$$\det A = (1 - \kappa - \gamma)(1 - \kappa + \gamma) = \mu^{-1}$$

*The **critical lines** are the lines where the eigenvalues of the Jacobian are zero:*

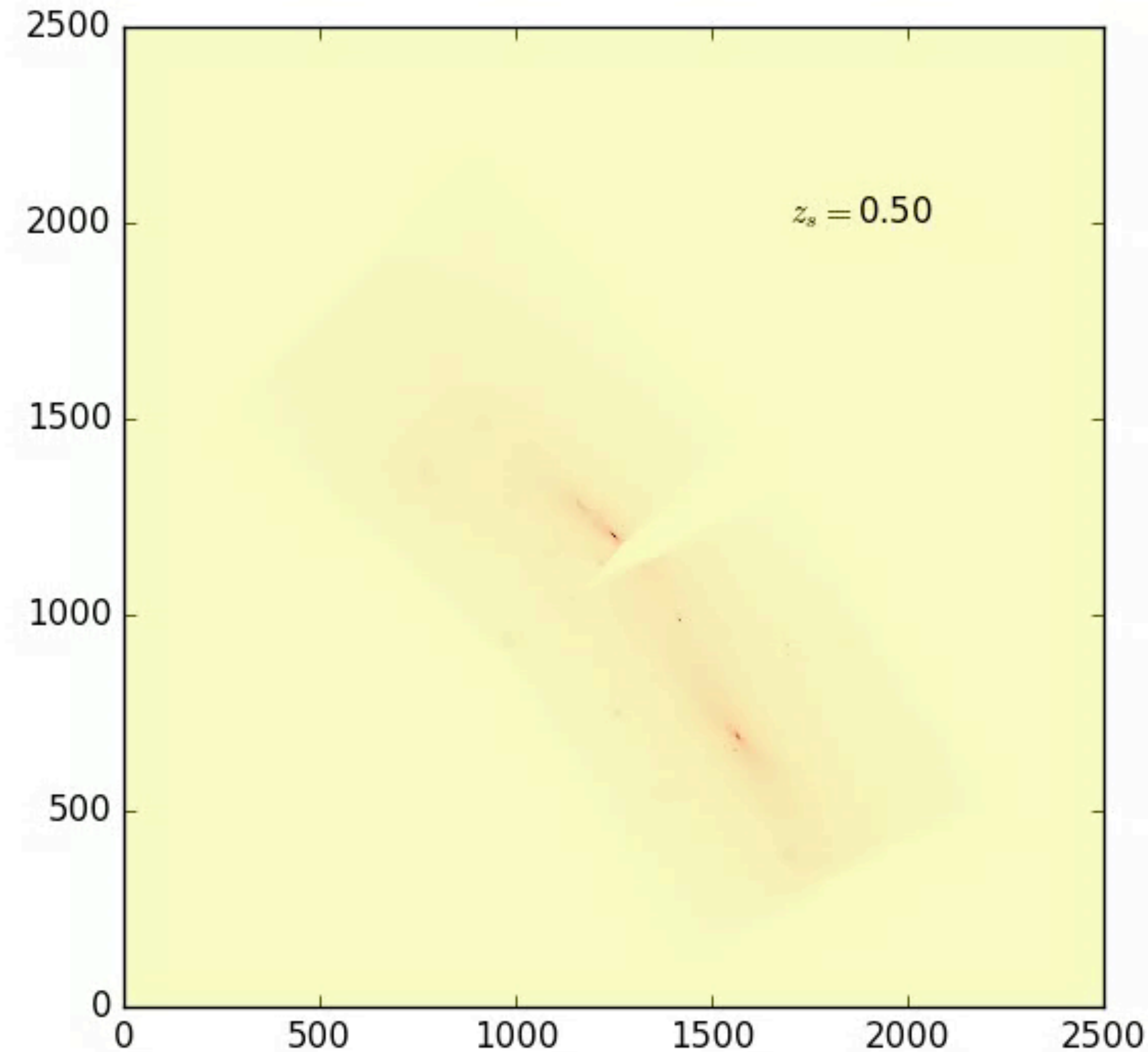
$(1 - \kappa - \gamma) = 0$ *tangential critical line*

$(1 - \kappa + \gamma) = 0$ *radial critical line*

Along these lines the magnification diverges!

*Via the lens equations, they are mapped into the **caustics**…*

VISUALIZING THE CAUSTICS



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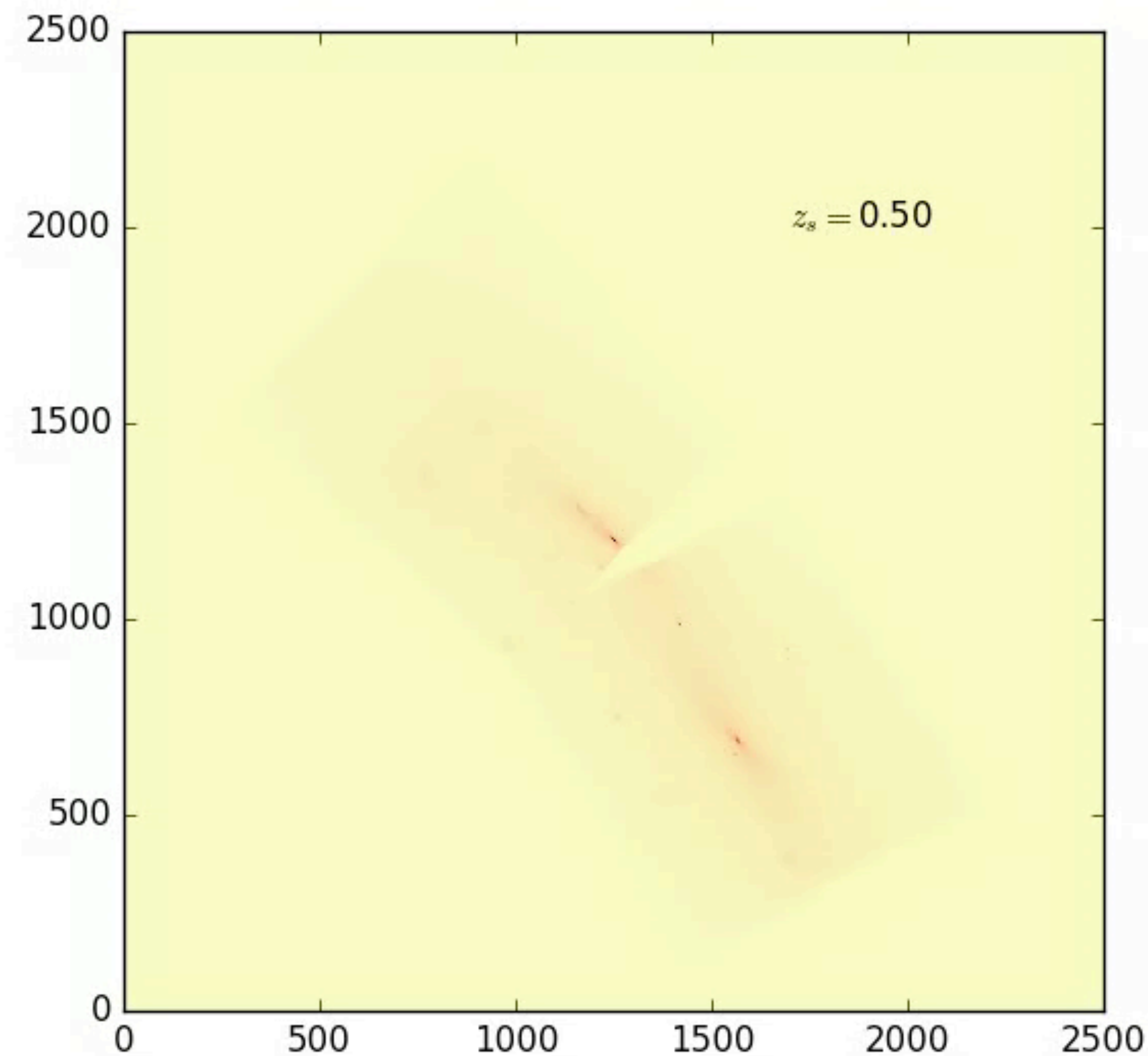

SAMPLED VOLUME