GRAVITATIONAL LENSING

10 – MICROLENSING OPTICAL DEPTH

Massimo Meneghetti

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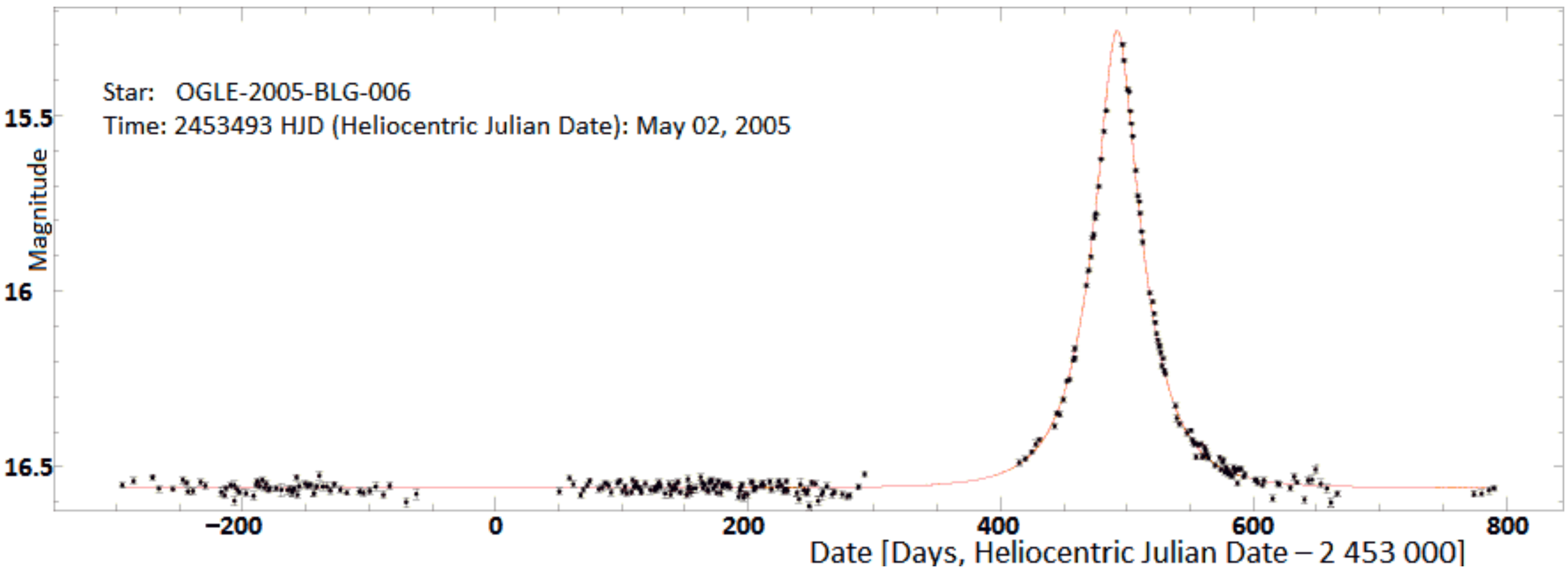
EXAMPLE OF STANDARD LIGHT CURVE



About 90% of the microlensing events are well described by this standard model of the light curve.

NON-STANDARD LIGHT CURVES

Lee et al 2009

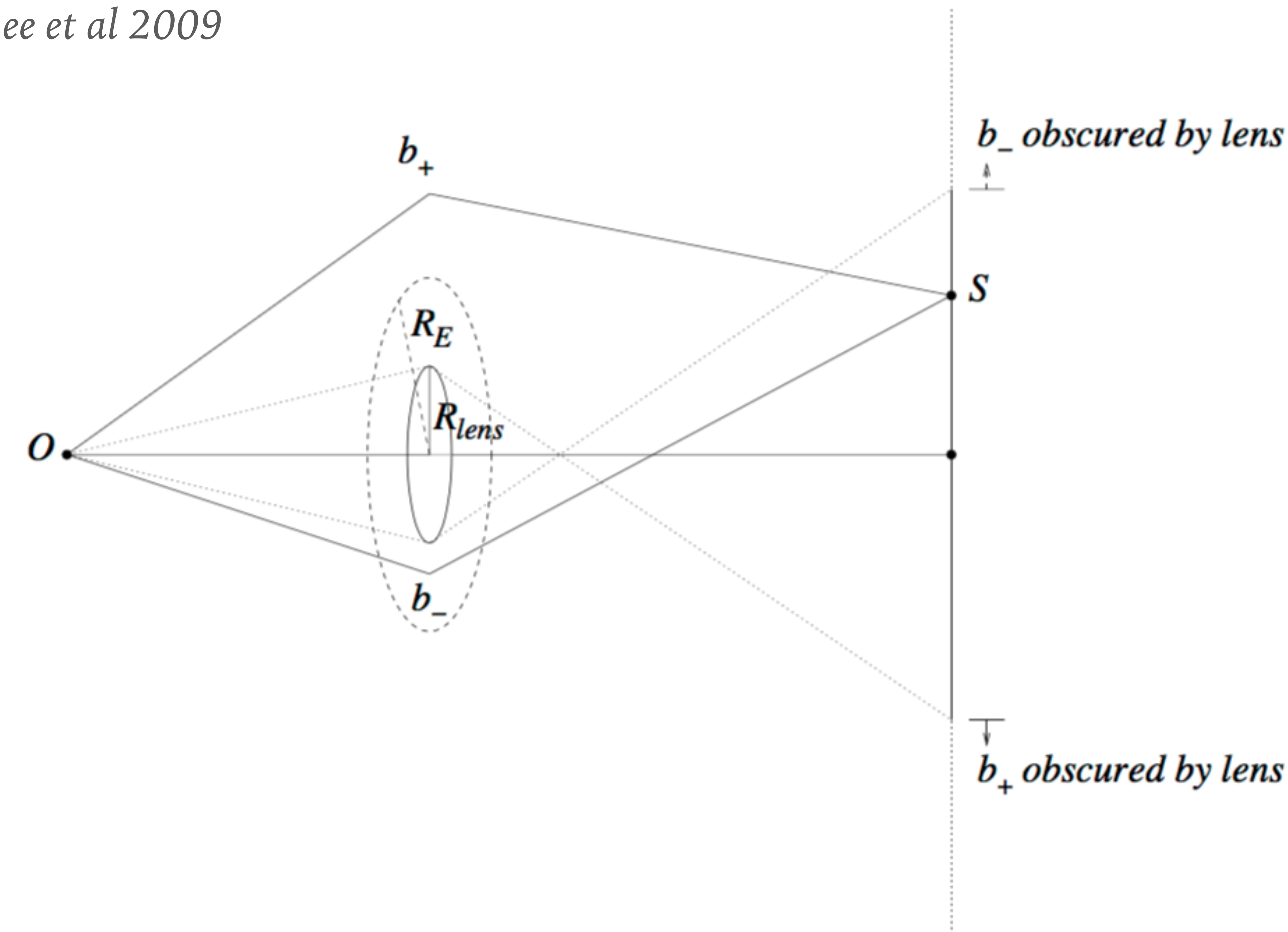
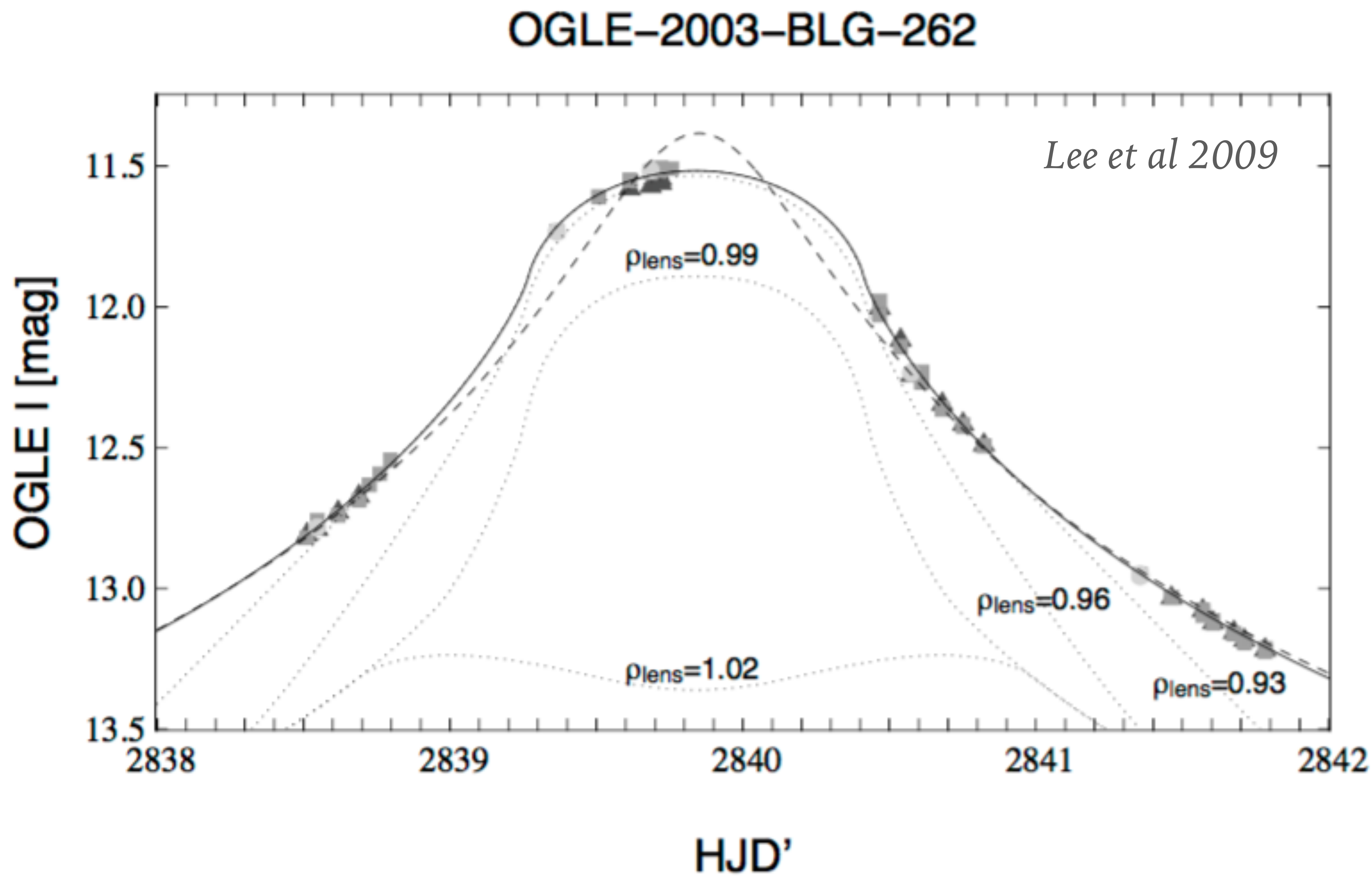


illustration of the finite lens size effect

NON-STANDARD LIGHT CURVES



Gould, 1994: $\mu'(y) \simeq \mu(y)\dfrac{4y}{\pi\rho}E(\vartheta_{max}, y/\rho)$ $\qquad \vartheta_{\max} = \begin{cases} \frac{\pi}{2} & y \le \rho \\ \arcsin(\rho/y) & y > \rho \end{cases}$

WHAT IS THE PROBABILITY TO OBSERVE A MICROLENSING EVENT?

- microlensing event: a variation of the source flux which follows the law we derived in the last lecture:

$$\mu(y) = \mu_+(y) + |\mu_-(y)| = \frac{y^2+2}{y\sqrt{y^2+4}}$$

- two important things to remember:
  - the total magnification drops as $\mu \propto 1 + 2/y^4$ for $v \to \infty$,
  - the ratio of image magnification grows as $\left|\frac{\mu_+}{\mu_-}\right| \propto y^4$
- for these reasons, we anticipated that the microlensing cross section is

$$\sigma = \pi\theta_E^2$$

- more generally, **the cross section is the area (or the solid angle) within which a source has to be located in order to undergo a microlensing event.** In our case this $\mu \gtrsim 1.34$

OPTICAL DEPTH

The number of lenses in the solid angle Ω between D_L to D_L+dD_L is

$$dN_L = \Omega D_\mathrm{L}^2 n(D_\mathrm{L}) dD_\mathrm{L}$$

where $n(D_L)$ is the number density of lenses.

Each of these lenses has a cross section $\sigma = \pi\theta_E^2$

Summing all the cross sections and dividing by the solid angle, we measure the probability that a source at distance D_S is lensed by a lens at distance D_L

*Integrating over D_L, we obtain the **optical depth**:*

$$\tau(D_\mathrm{S}) = \frac{1}{\Omega}\int_0^{D_\mathrm{S}} [\Omega D_\mathrm{L}^2 n(D_\mathrm{L})](\pi\theta_E^2) dD_\mathrm{L}$$

OPTICAL DEPTH

If all lenses have the same mass: $n(D_\mathrm{L}) = \rho(D_\mathrm{L})/M$

On the other hand, $\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_\mathrm{LS}}{D_\mathrm{L}D_\mathrm{S}}}$

Therefore, the optical depth does not depend on the mass of the lenses, but only on the mass density!

Note that if not all masses are equal we should integrate over M

OPTICAL DEPTH

$$\tau(D_\mathrm{S}) = \frac{1}{\Omega}\int_0^{D_\mathrm{S}} [\Omega D_\mathrm{L}^2 n(D_\mathrm{L})](\pi\theta_E^2) dD_\mathrm{L}$$

$$\begin{aligned}\tau(D_\mathrm{S}) &= \frac{4\pi G}{c^2}\int_0^{D_\mathrm{S}} \rho(D_\mathrm{L}) D_\mathrm{L}^2 \frac{D_\mathrm{LS}}{D_\mathrm{L} D_\mathrm{S}} dD_\mathrm{L} \\ &= \frac{4\pi G}{c^2}\int_0^{D_\mathrm{S}} \rho(D_\mathrm{L}) D_\mathrm{L} \frac{D_\mathrm{S} - D_\mathrm{L}}{D_\mathrm{S}} dD_\mathrm{L} \\ &= \frac{4\pi G}{c^2}\int_0^{D_\mathrm{S}} \rho(D_\mathrm{L}) \frac{D_\mathrm{L}}{D_\mathrm{S}} \left(1 - \frac{D_\mathrm{L}}{D_\mathrm{S}}\right) D_\mathrm{S} dD_\mathrm{L}\end{aligned}$$

with the substitution $x = D_\mathrm{L}/D_\mathrm{S},\ dx = dD_\mathrm{L}/D_\mathrm{S}$

$$\tau(D_\mathrm{S}) = \frac{4\pi G}{c^2} D_\mathrm{S}^2 \int_0^1 \rho(x) x(1-x) dx$$

OPTICAL DEPTH DENSITY

$$\frac{d\tau}{dx} \propto \rho(x)x(1-x)$$

The function which modulates the lens contribution to the optical depth has a peak at x=0.5 (half way between the observer and the source).

Then, whether most of the optical depth is accumulated near the observer or near the source, depends on the mass density profile…

OPTICAL DEPTH (SIMPLEST CASE)

Uniform distribution of the lenses

$$\rho(x) = \rho_0 = \text{const.}$$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 \int_0^1 x(1-x)dx = \frac{2}{3}\frac{\pi G}{c^2} D_S^2 \rho_0$$

Note that:

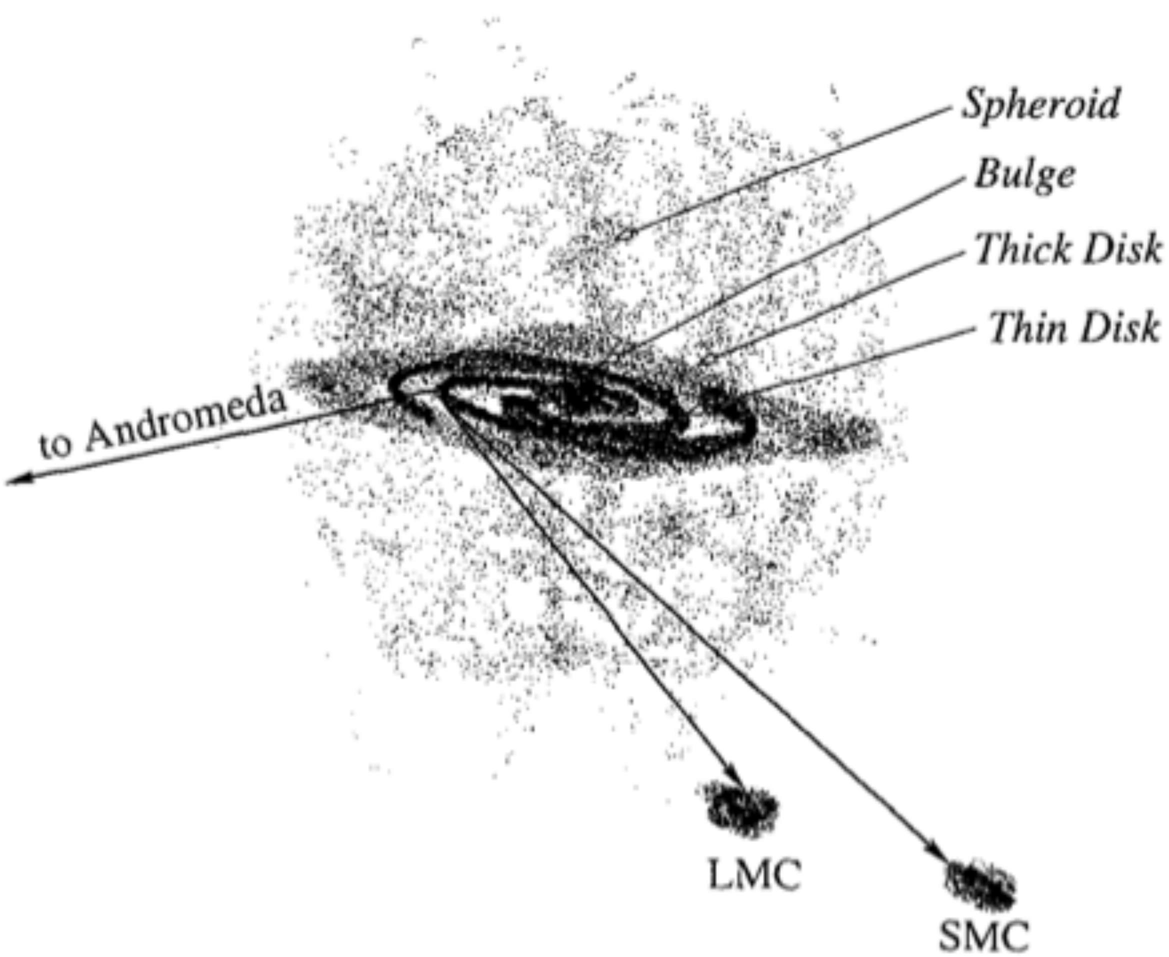
$$M_{gal} = \frac{4}{3}\pi D_S^3 \rho_0$$

$$\tau(D_S) = \frac{GM_{gal}}{2c^2 D_S} = \frac{V_{circ}^2}{2c^2}$$

$$\tau \approx 2.6 \times 10^{-7}$$

The true optical depth towards the galactic center is 3-10 times larger…

A MODEL FOR OUR GALAXY



All these components have their own optical depths...

I) thin & thick disk (young stars & gas)

$$\rho^D(R,z) = \rho_0^D \exp\left(-\frac{R-R_0}{h_R} - \frac{|z|}{h_z}\right)$$

$$\sigma^D \simeq 20 \text{ km/s} \qquad v_{rot}^D \simeq 220 \text{ km/s}$$

$$\sigma^{TD} \simeq 40 \text{ km/s} \qquad v_{rot}^{TD} \simeq 180 \text{ km/s}$$

II) Spheroid (old star halo)

$$\rho^S \propto r^{-3.5} \qquad \sigma^S \simeq 120 \text{ km/s}$$

III) Bulge (contains a bar)

$$\rho^B(s) = \frac{M_0}{8\pi abc} \exp\left[-\frac{s^2}{2}\right]$$

$$s^4 \equiv [(x'/a)^2 + (y'/b)^2]^2 + (z'/c)^4$$

OPTICAL DEPTH (JUST A BIT MORE REALISTIC)

Lenses in the galactic disk:

scale of the disk

$$\rho(R) = \rho_0 \exp\left(-(R - R_0)/R_D\right)$$

density in the sun neighborhood

distance of earth from the galactic center

distance of lenses from the galactic center

$$R = D_{LS} \qquad\qquad R_0 = D_S$$

$$R - R_0 = D_{LS} - D_S = D_S - D_L - D_S = -D_L$$

$$\rho(D_L) = \rho_0 \exp\left(D_L/R_D\right)$$

OPTICAL DEPTH (JUST A BIT MORE REALISTIC)

Making the substitutions $\mathbf{x} = \mathbf{D}_{\mathrm{L}}/\mathbf{D}_{\mathrm{S}}$ $\quad x' = R_D/D_S$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 \int_0^1 \exp\left(x/x'\right) x(1-x)dx$$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 x'^2[2x' - 1 + \exp\left(1/x'\right)(2x' - 1)]$$

$$D_S = 8 \text{ kpc} \qquad R_D = 3 \text{ kpc} \qquad \rho_0 = 0.1 M_\odot \text{ pc}^{-3}$$

$$\tau \approx 2.9 \times 10^{-6}$$

EVENT RATE

The optical depth gives the probability that a source is (micro-)lensed at any instant. The next question is: how many events will we detect by monitoring a certain number of stars during a time interval?

To answer this question, we have to consider the relative motion of sources and lenses, which determines the timescale of events.

It is easier to think in terms of static sources behind moving lenses.

We also assume that the lenses move with the same transverse velocity.

EVENT RATE

$$r_E = D_{\mathrm{L}} \theta_E$$

$$dA = 2r_E v dt = 2r_E^2 \frac{dt}{t_E}$$

Multiplying by the number of lenses and integrating over distance we obtain the area useful for microlensing during the time dt. Dividing by the solid angle, we obtain a probability that a source undergoes a micro lensing event in the time dt:

$$d\tau = \frac{1}{\Omega} \int_0^{D_{\mathrm{S}}} n(D_{\mathrm{L}}) \Omega dA dD_{\mathrm{L}} = 2 \int_0^{D_{\mathrm{S}}} n(D_{\mathrm{L}}) r_E^2 \frac{dt}{t_E} dD_{\mathrm{L}}$$

EVENT RATE

If we monitor N stars, the number of events expected per unit time will be:

$$\Gamma = \frac{d(N_\star \tau)}{dt} = \frac{2N_\star}{\pi} \int_0^{D_\mathrm{S}} n(D_\mathrm{L}) \frac{\pi r_E^2}{t_E} dD_\mathrm{L}$$

If we assume that all Einstein crossing times are identical:

$$\Gamma = \frac{2N_\star}{\pi t_E} \tau$$

As an order of magnitude:

$$\Gamma \approx 1200\mathrm{yr}^{-1} \frac{N_\star}{10^8} \frac{\tau}{10^{-6}} \left(\frac{t_E}{19\mathrm{days}} \right)^{-1}$$

EVENT RATE

For comparison: OGLE IV detected 1500-2000 candidates/year in the 2011-2017 campaigns.

Note that:

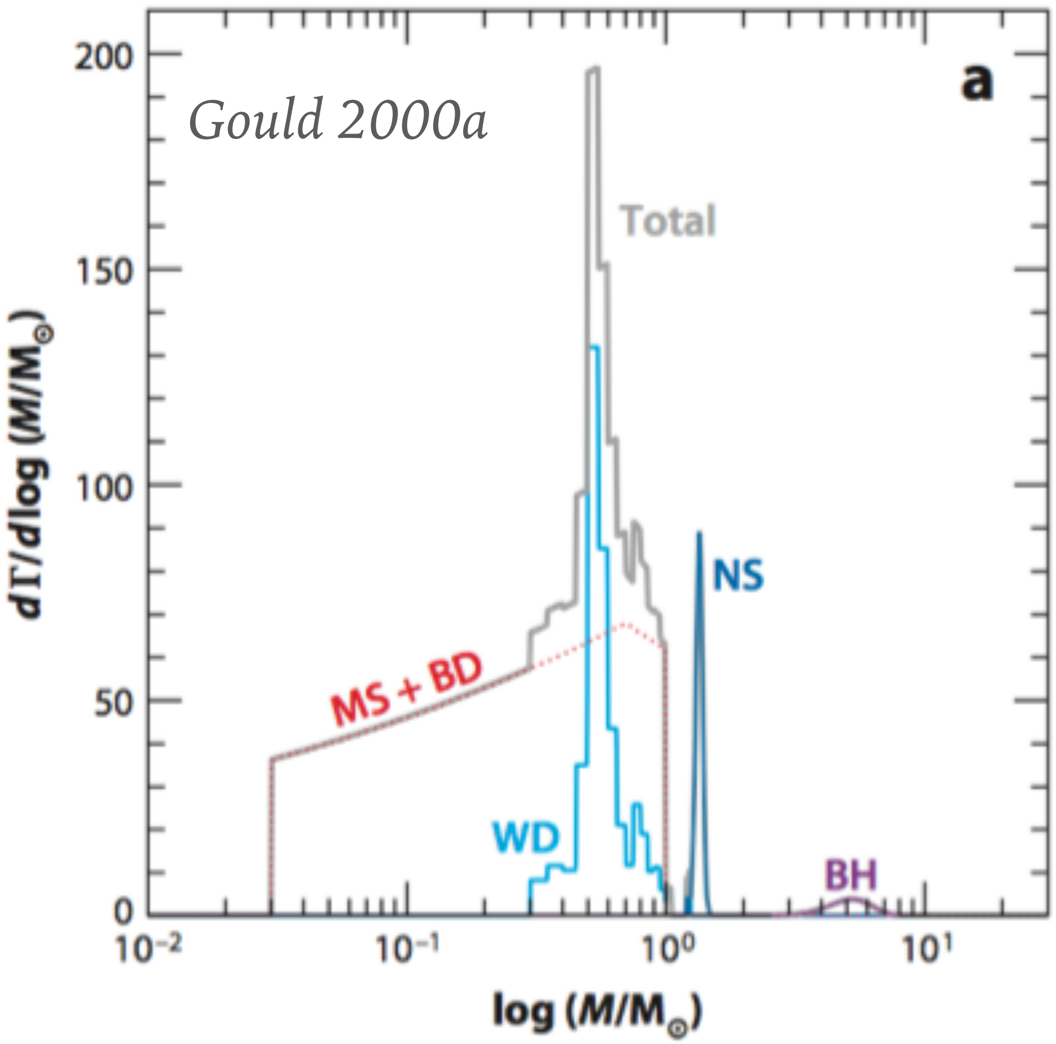
$$\Gamma \propto t_E^{-1} \propto M^{-1/2}$$

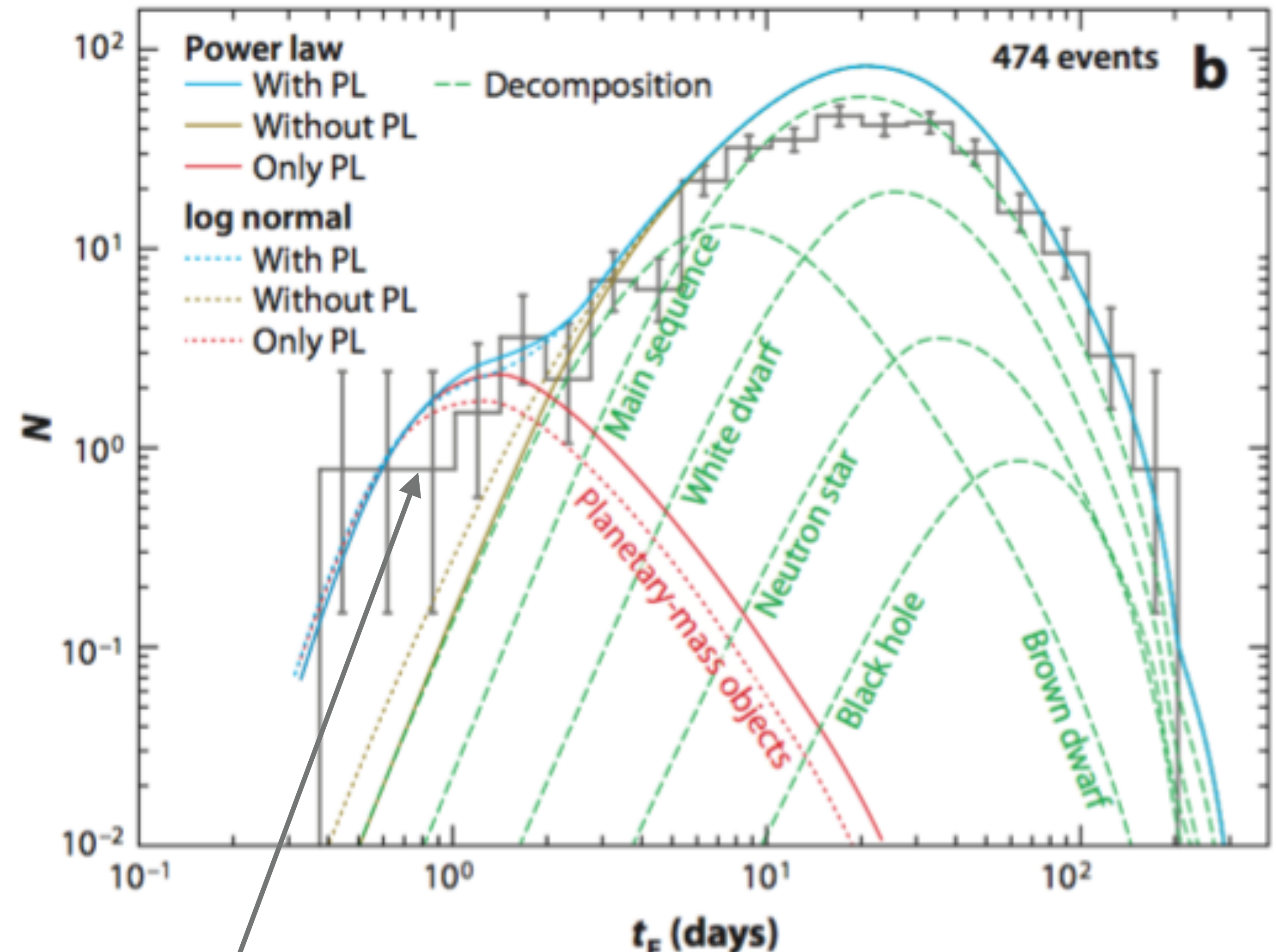
We can use the distribution of event timescales to probe the kinematics of the Milky Way and the stellar populations in the galaxy.

PROBING THE STELLAR POPULATIONS WITH MICROLENSING

Gaudi, 2012, Ann. Rev. Astron. Astrophys. 50, 411

Sumi et al 2011





Theoretical estimate of the rate of microlensing events towards the galactic bulge

Distribution of microlensing event timescales observed by the MOA collaboration (2006-2007)

SOME IMPORTANT FACTS

- several collaborations have implemented the microlensing idea (proposed by B. Paczynski). These groups have monitored the galactic bulge and the Magellanic Clouds searching for microlensing events
- the relatively high rate of detections favored a barred model of the galaxy
- Towards the Magellanic Clouds, no ‘short’ events (timescales from a few hours up to 20 days) have been seen by any group. This places strong limits on ‘Jupiters’ in the dark halo: specifically, compact objects in the mass range 10^{-6}–0.05 solar masses contribute less than 10% of the dark matter around our Galaxy. This is a very important result, as these objects were previously thought to be the most plausible form of baryonic dark matter, and (for masses below 0.01 solar masses) they would have been virtually impossible to detect directly.

SOME IMPORTANT FACTS

- In general: all detections of microlensing events are most likely caused by known stellar populations. BHs can contribute to 2% of the total mass of the halo.
- The recent detection of GW from merging BHs with intermediate masses has revived the idea of BHs as dark-matter candidates. For such lenses, the time scale of the events would be large so that past microlensing events may not have detected them.