

GRAVITATIONAL LENSING

LECTURE 9

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CONTENTS

- time delay surface

TIME DELAY SURFACE

$$t(\vec{\theta}) = t_{geom} + t_{grav} \propto \left(\frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \hat{\Psi} \right)$$

$$\vec{\nabla} t(\vec{\theta}) \propto \left(\vec{\theta} - \vec{\beta} - \vec{\nabla} \hat{\Psi} \right)$$

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Lens equation!

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Images form at the stationary points of t !

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$$T_{ij} = \frac{\partial^2 t(\vec{\theta})}{\partial \theta_i \partial \theta_j} \propto (\delta_{ij} - \Psi_{ij}) \text{ *This is the Jacobian!*}$$

TYPES OF IMAGES

- minima (eigenvalues of A are both positive, hence $\det A > 0$ and $\text{Tr } A > 0$; positive magnification)
- saddle (eigenvalues have opposite signs, thus $\det A < 0$; negative magnification)
- maxima (eigenvalues are both negative, hence $\det A > 0$ and $\text{Tr } A < 0$; positive magnification)
- Let see some examples...

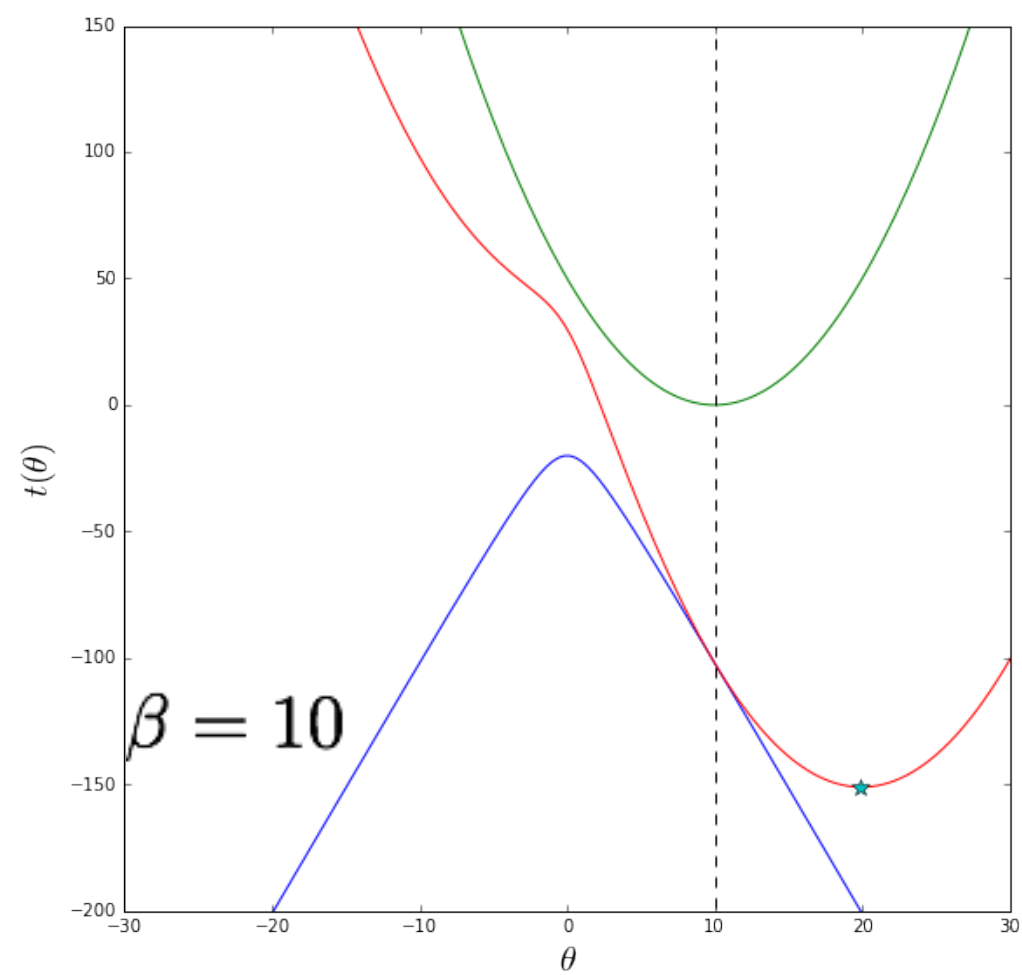
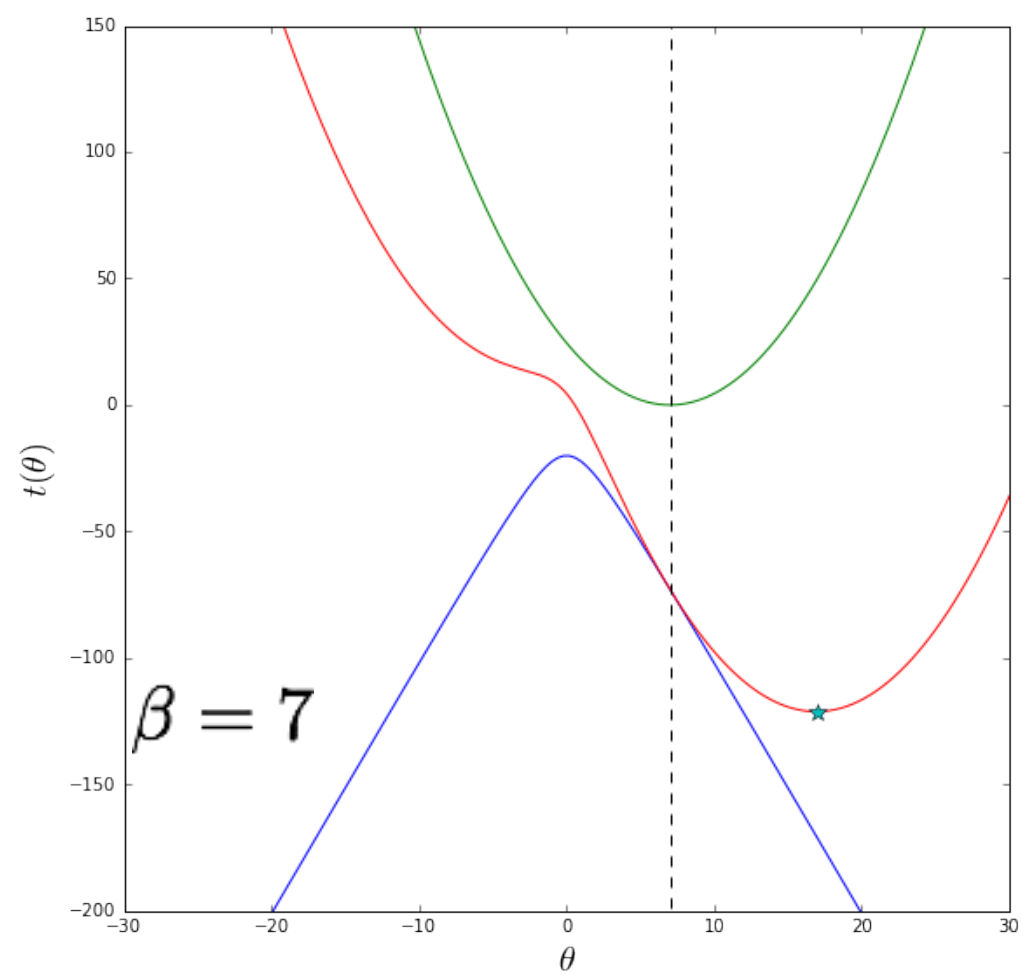
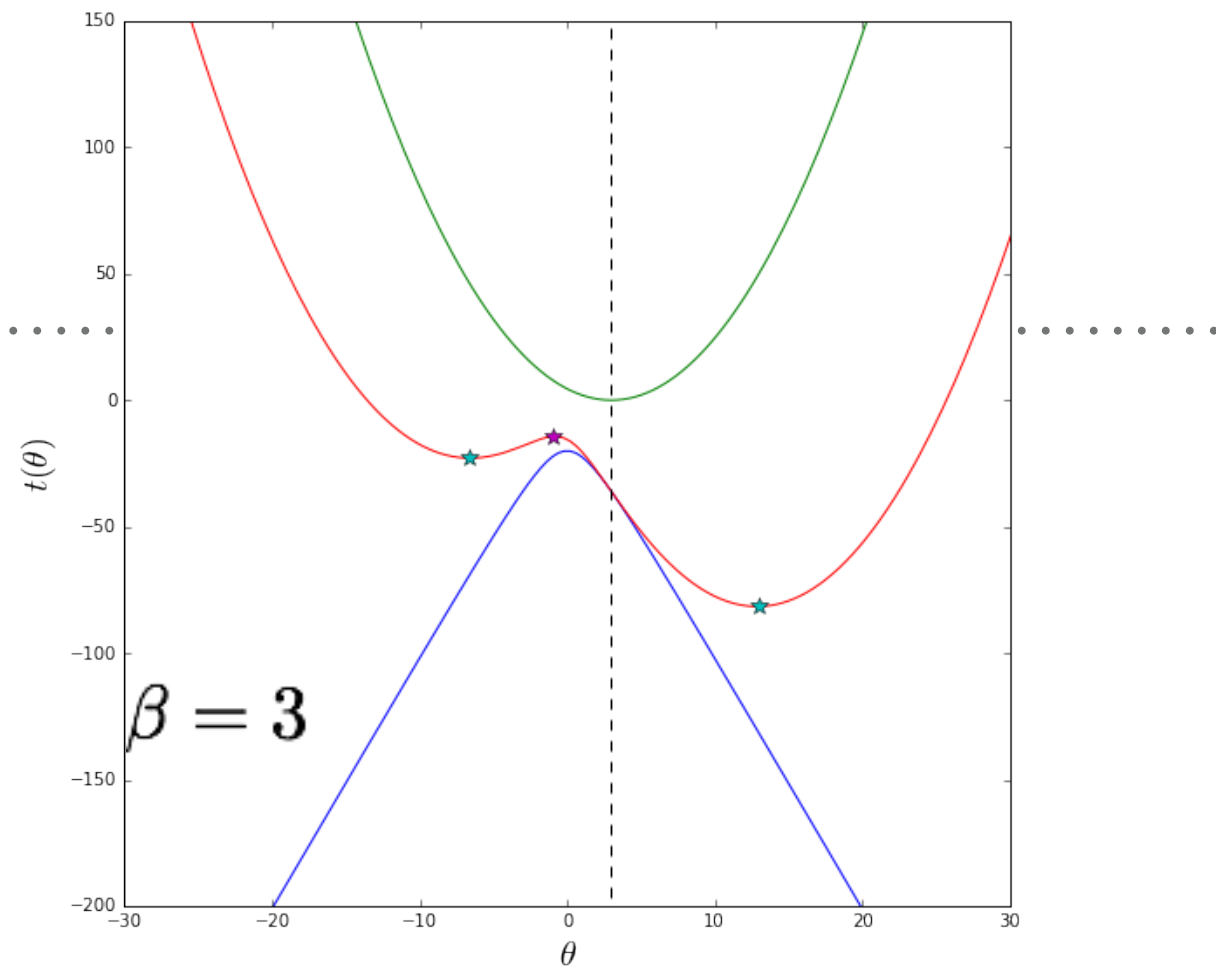
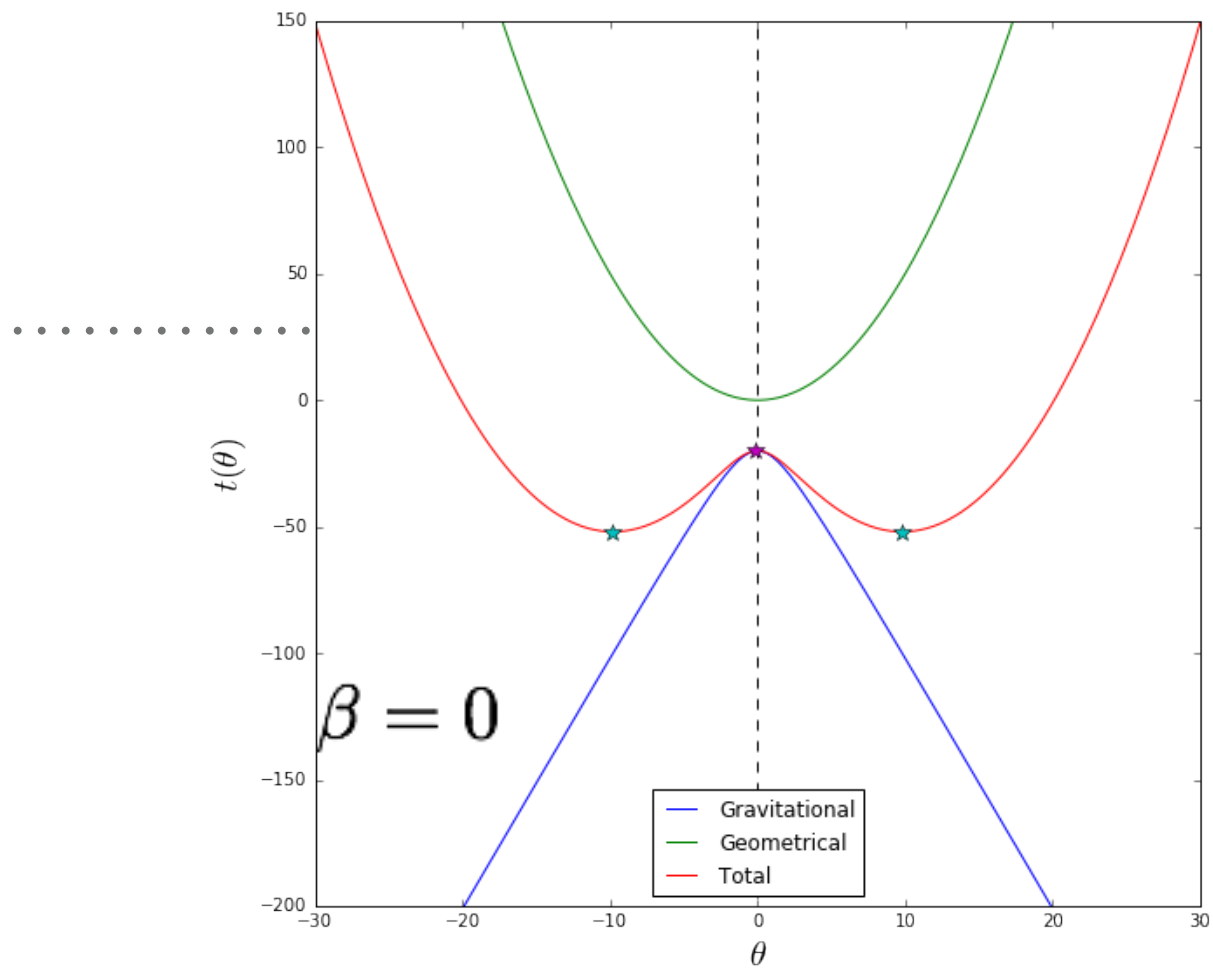
EXAMPLE OF TIME DELAY SURFACE

Toy potential:

$$\psi(\theta) \propto \frac{1}{\sqrt{\theta^2 + \theta_c^2}}$$

Assuming axial-symmetry, we can discuss the time-delay function instead of the time delay surface.

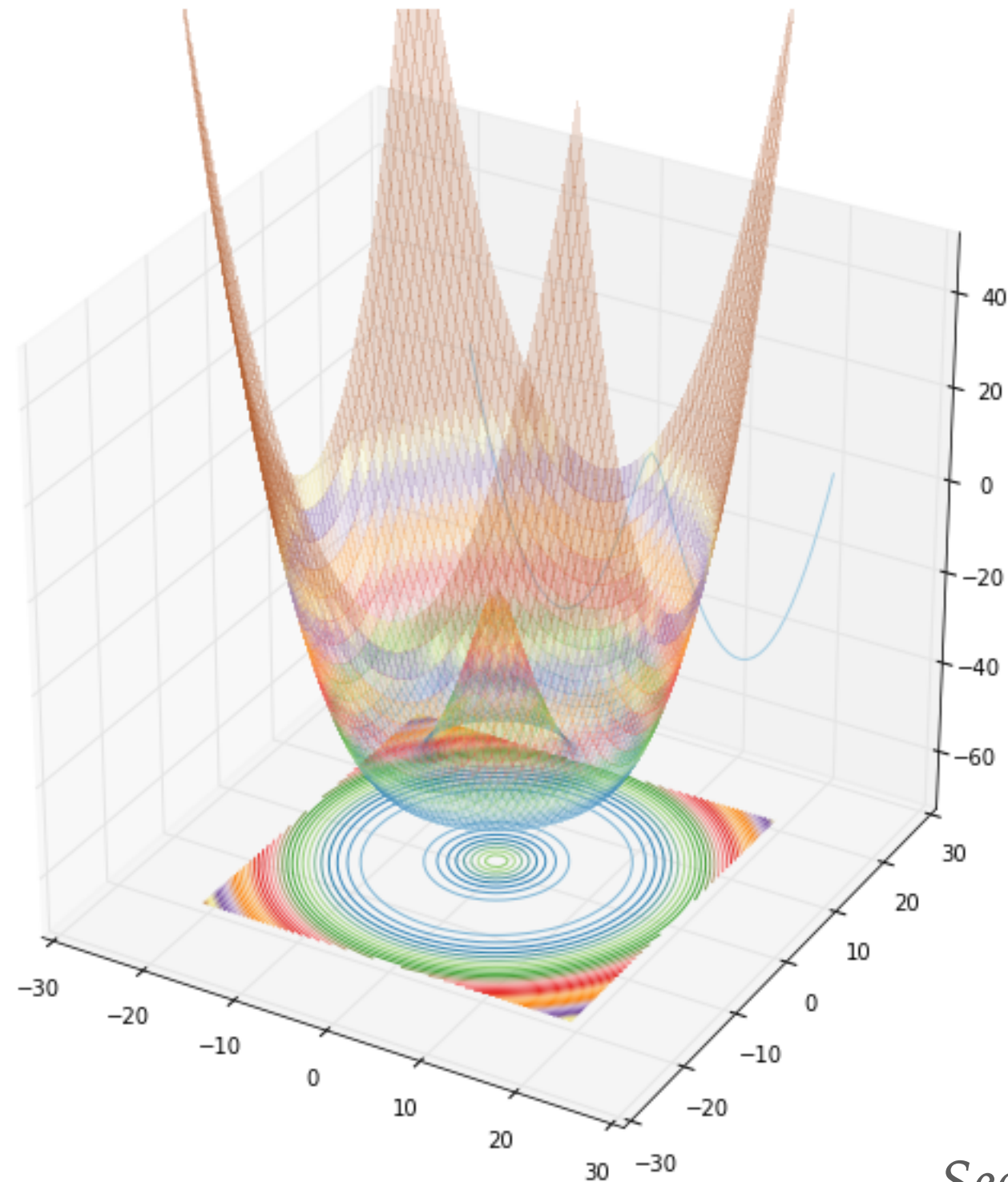
$$t(\theta) \propto \left[\frac{1}{2}(\theta - \beta)^2 - \psi(\theta) \right]$$



SOME INTERESTING PROPERTIES

- image multiplicity depends on the relative position of lens and source
- couples of images disappear after approaching each other
- the time-delay function is flat when this happens!
- $\det A=0$ means infinite magnification: the images disappear on the critical lines!
- this happens every time a source crosses a caustic!

THE TIME DELAY SURFACE OF AN AXIALLY SYMMETRIC LENS



See the jupyter notebook!