GRAVITATIONAL LENSING

LECTURE 11

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CONTENTS

- microlensing: point mass lenses

MICROLENSING

- Microlensing is a lensing regime which include effects produced by a broad range of masses: from planets to ensembles of stars
- given the small sizes of the lens, these are (to first-order) assimilated to point masses.
- microlensing effects are mostly detectable and searched within our own galaxy, in particular by monitoring huge amounts of stars in the bulge of the MW or in the Magellanic Clouds
- nevertheless, microlensing effects are important also in extragalactic lenses. Small masses in distant galaxies, for example, introduce perturbations to the lensing signal of their hosts

THE POINT MASS LENS MODEL

$$\hat{\vec{\alpha}} = \frac{4GM}{c^2}\frac{\vec{\xi}}{|\vec{\xi}|^2} = \frac{4GM}{c^2 D_{\rm L}}\frac{\vec{\theta}}{|\vec{\theta}|^2}$$

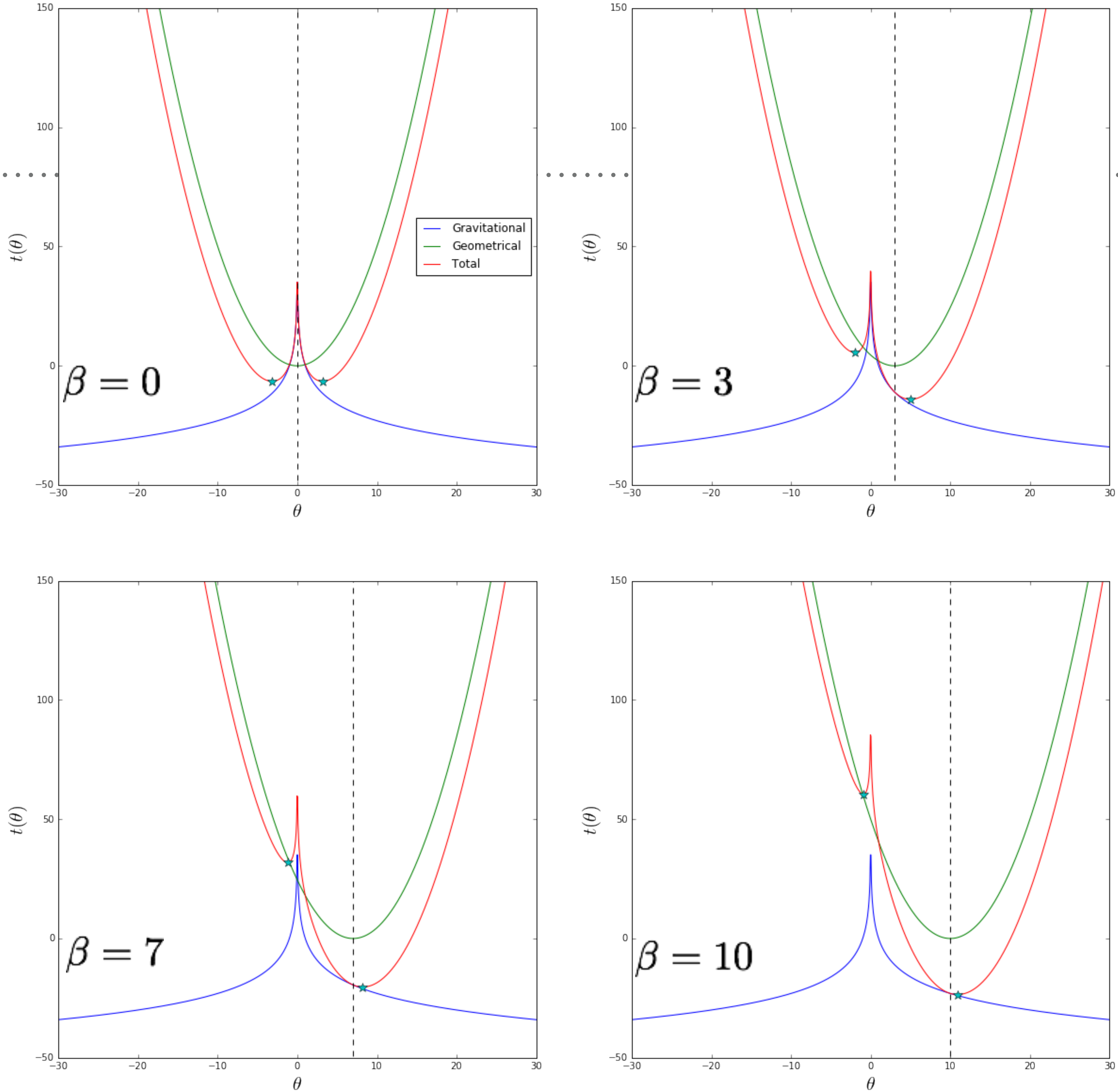
- The deflection angle of the point mass lens was derived in the first lecture
- the lensing potential can be readily derived

$$\vec{\alpha} = \frac{D_{\rm LS}}{D_{\rm S}}\hat{\vec{\alpha}} = \vec{\nabla}\hat{\Psi}$$

$$\nabla \ln|\vec{x}| = \frac{\vec{x}}{|\vec{x}|^2}$$

$$\Downarrow$$

$$\hat{\Psi}(\vec{\theta}) = \frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}\ln|\vec{\theta}|$$



Gravitational
Geometrical
Total
$\beta = 0$
$\beta = 3$
$\beta = 7$
$\beta = 10$
$t(\theta)$
θ

LENS EQUATION

- the deflection angle always points away from the lens
- given the symmetry of the lens, we can omit the vector notation in most equations
- the lens equation reads:
- this is clearly quadratic in θ
- so, for each source there are two images, whose positions can be determined by solving the lens equation

$$\hat{\alpha} = \frac{4GM}{c^2\xi} = \frac{4GM}{c^2 D_\mathrm{L}\theta}$$

$$\beta = \theta - \frac{4GM}{c^2 D_\mathrm{L}\theta}\frac{D_\mathrm{LS}}{D_\mathrm{S}}$$

SOLUTIONS OF THE LENS EQUATION

- we introduce the Einstein radius:

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{\mathrm{LS}}}{D_{\mathrm{L}}D_{\mathrm{S}}}}$$

- by inserting into the lens equation:

$$\beta = \theta - \frac{4GM}{c^2 D_{\mathrm{L}}\theta}\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}}$$

$$\Downarrow$$

$$\beta = \theta - \frac{\theta_E^2}{\theta}$$

- if we divide by θ_E, we obtain an a-dimensional form of the lens equation

$$y = x - \frac{1}{x}$$

- this is a very convenient way of writing the lens equation, because we get rid of all constants.

SOLUTIONS OF THE LENS EQUATION

$$y = x - \frac{1}{x}$$

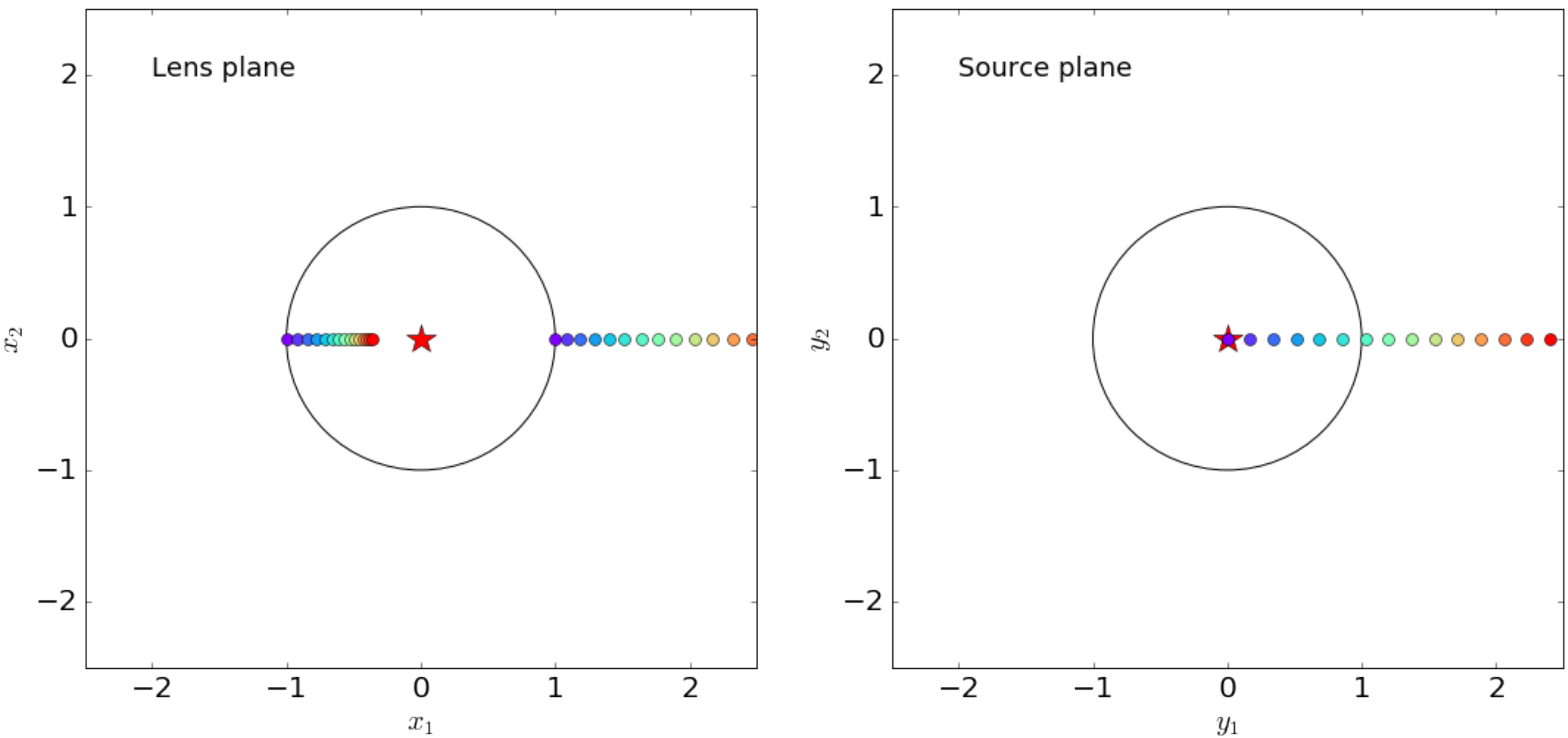
$$\Downarrow$$

$$x^2 - xy - 1 = 0$$

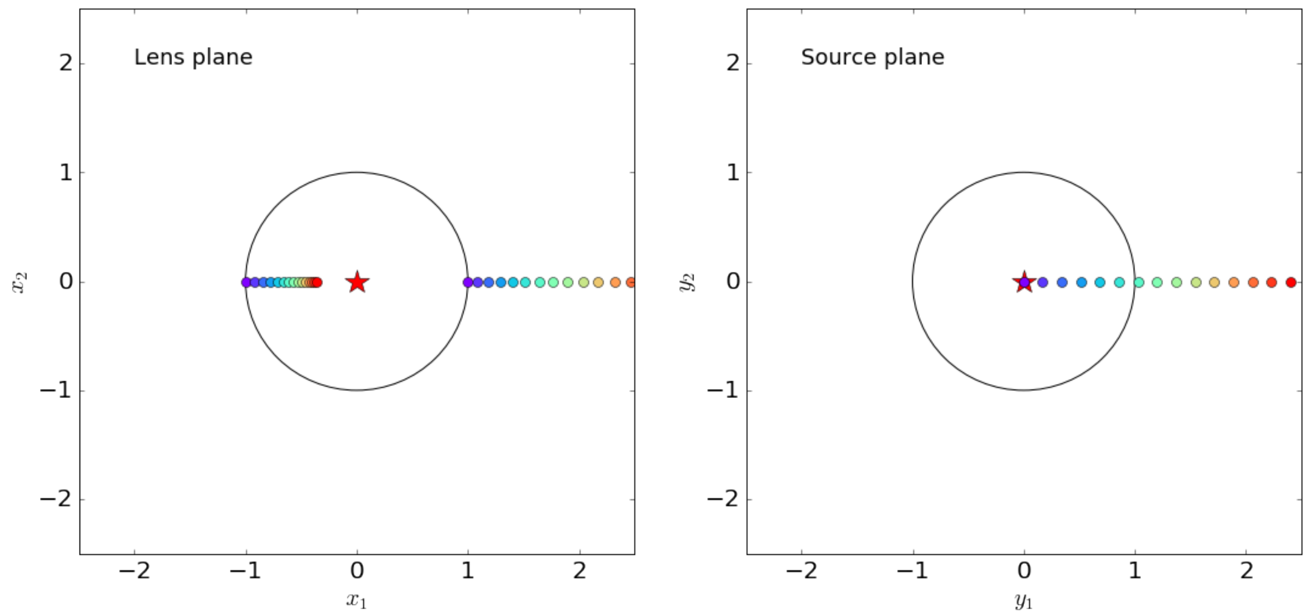
$$\Downarrow$$

$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right]$$

SOLUTIONS OF THE LENS EQUATION



PROPERTIES OF THE IMAGES



$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right]$$

$$x_+ > 0 \qquad\qquad -1 < x_- < 0.$$

One of the images is internal to the Einstein radius, the other is external

For $y=0$, the image is a full ring: $\quad x_\pm = \pm 1$

This is the ***Einstein ring***

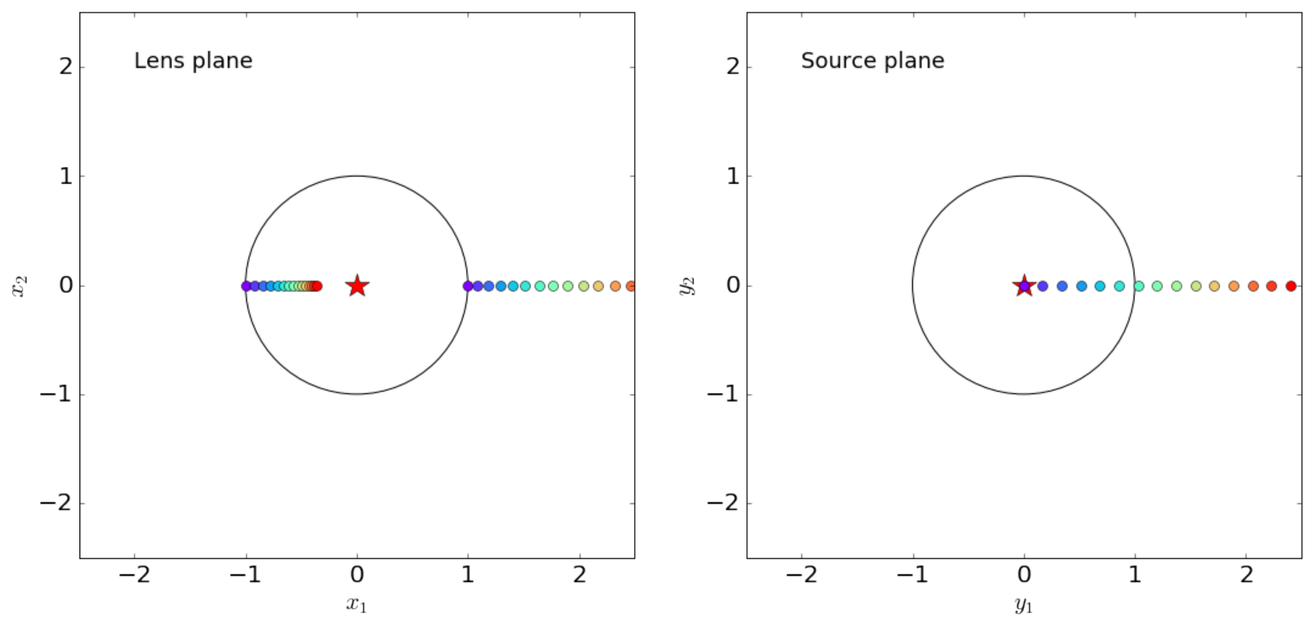
SIZE OF THE EINSTEIN RADIUS

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}} \qquad D \equiv \frac{D_{\rm L}D_{\rm S}}{D_{\rm LS}}$$

$$\begin{aligned}\theta_E &\approx (10^{-3})''\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D}{10\mathrm{kpc}}\right)^{-1/2}, \\ &\approx 1''\left(\frac{M}{10^{12}M_\odot}\right)^{1/2}\left(\frac{D}{\mathrm{Gpc}}\right)^{-1/2},\end{aligned}$$

For a star like the sun within the MW, the Einstein radius is of the order of micro-arcseconds!

PROPERTIES OF THE IMAGES



$$\beta \rightarrow \infty \qquad \theta_- = x_- \theta_E \rightarrow 0 \qquad \theta_+ = x_+ \theta_E \rightarrow \beta$$

For large angular distances between the lens and the source, one image approaches the lens, while the other follows the source.

CRITICAL LINES AND CAUSTICS

Since the lens is axially-symmetric:

$$\det A(x) = \frac{y}{x}\frac{\mathrm{d}y}{\mathrm{d}x}$$

From the lens equation, it follows that:

$$\lambda_t(x) = \frac{y}{x} = \left(1 - \frac{1}{x^2}\right)$$
$$\lambda_r(x) = \frac{\mathrm{d}y}{\mathrm{d}x} = \left(1 + \frac{1}{x^2}\right).$$

The second eigenvalue is always positive (no critical line). The first is zero on the circle

$$x^2 = 1$$

Thus, the Einstein ring is the tangential critical line! The corresponding caustic is a point at $y=0$

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