GRAVITATIONAL LENSING

3 - DEFLECTION ANGLE (CONTINUATION) - LENS EQUATION

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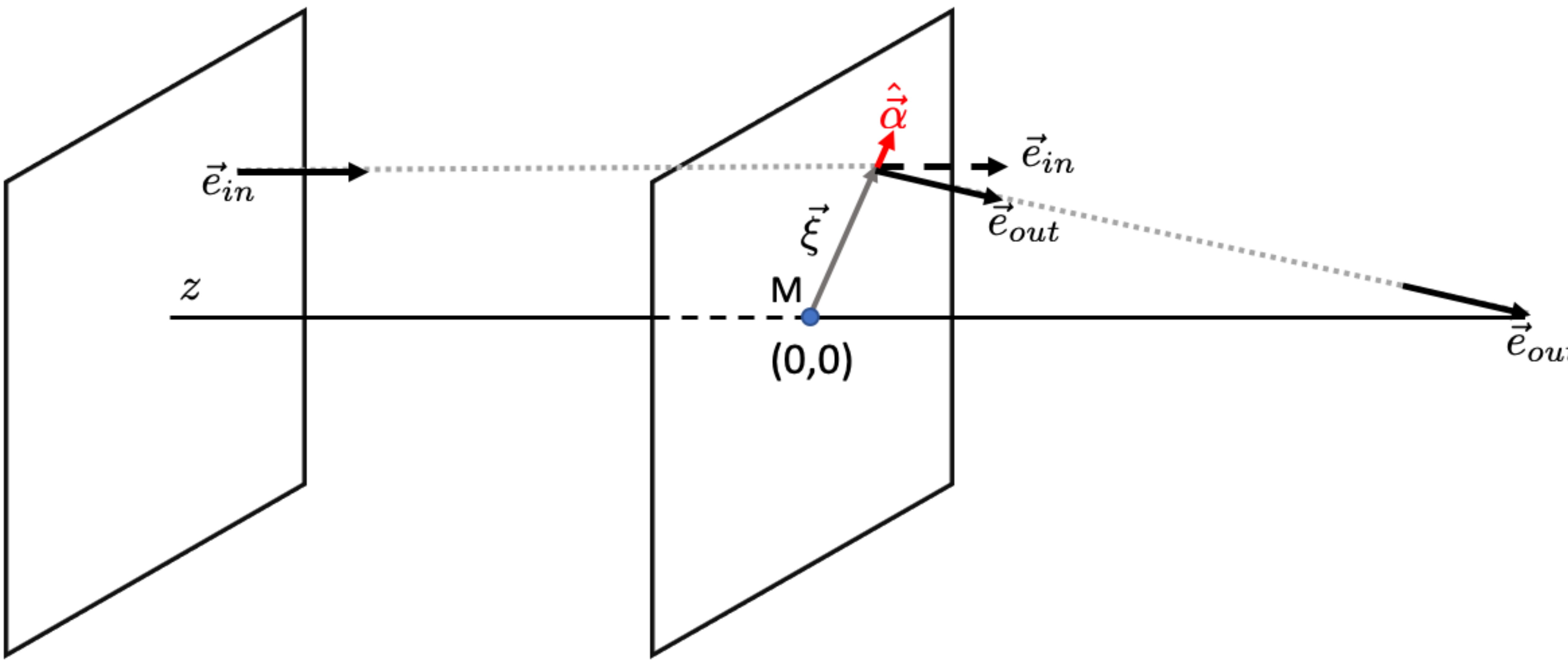
AA 2018-2019

GENERALISATION OF THE DEFLECTION ANGLE FORMULA

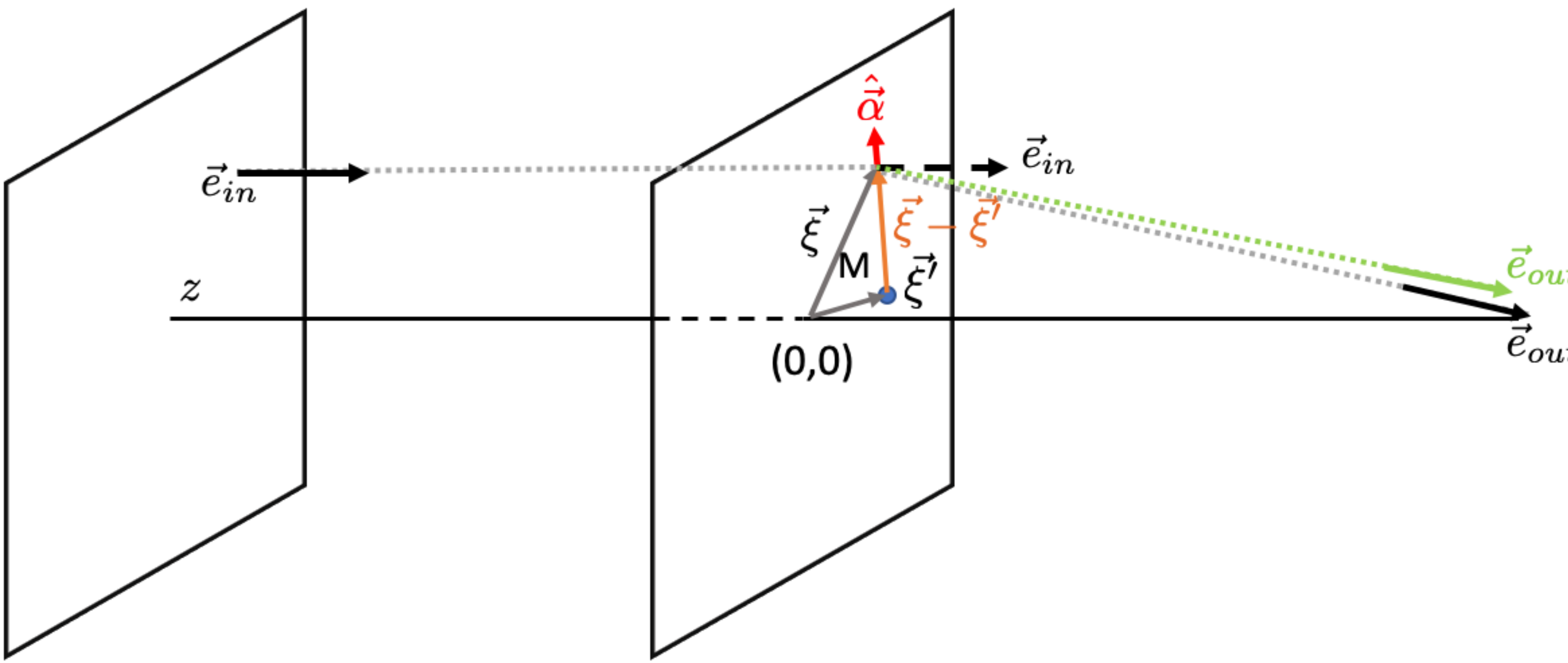
$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4GM}{c^2 \xi} \vec{e}_{\xi} = \frac{4GM}{c^2} \frac{\vec{\xi}}{|\xi|^2}$$



Using “Thin screen approximation”

GENERALISATION OF THE DEFLECTION ANGLE FORMULA

$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4GM}{c^2} \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2}$$



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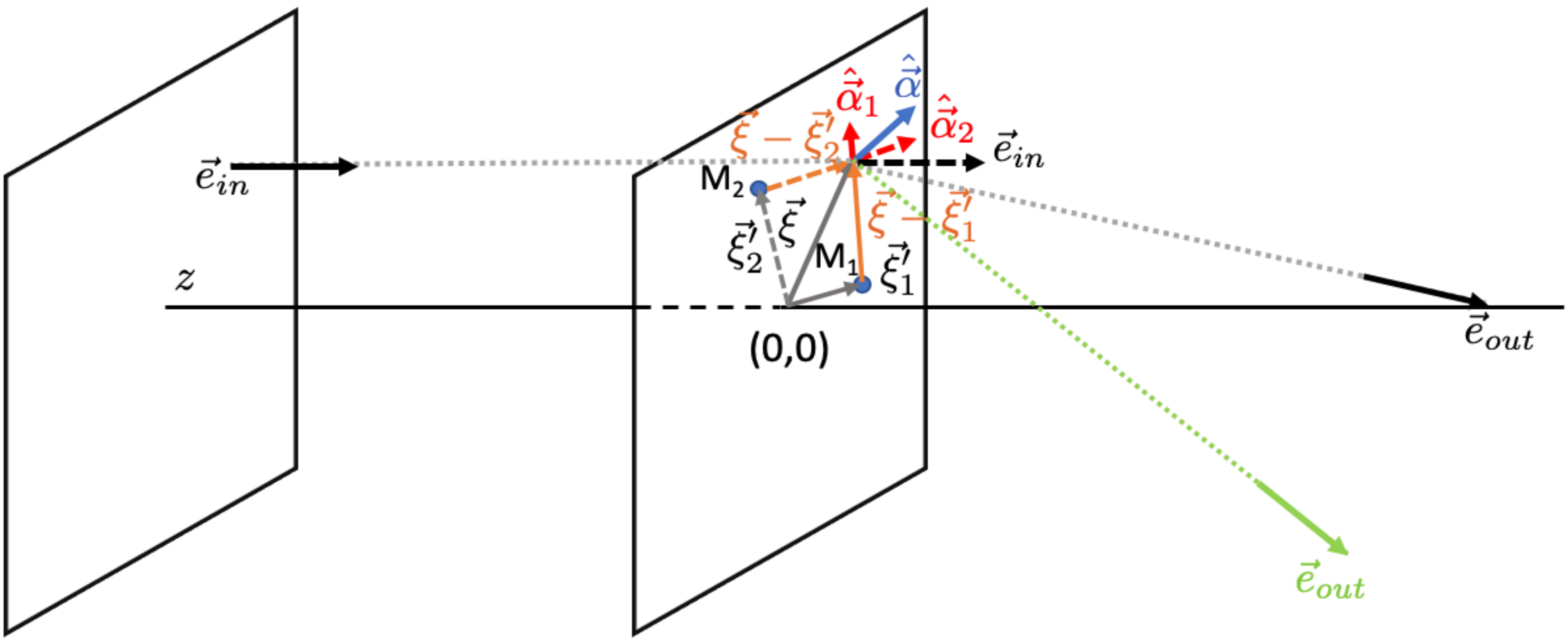
GENERALISATION OF THE DEFLECTION ANGLE FORMULA

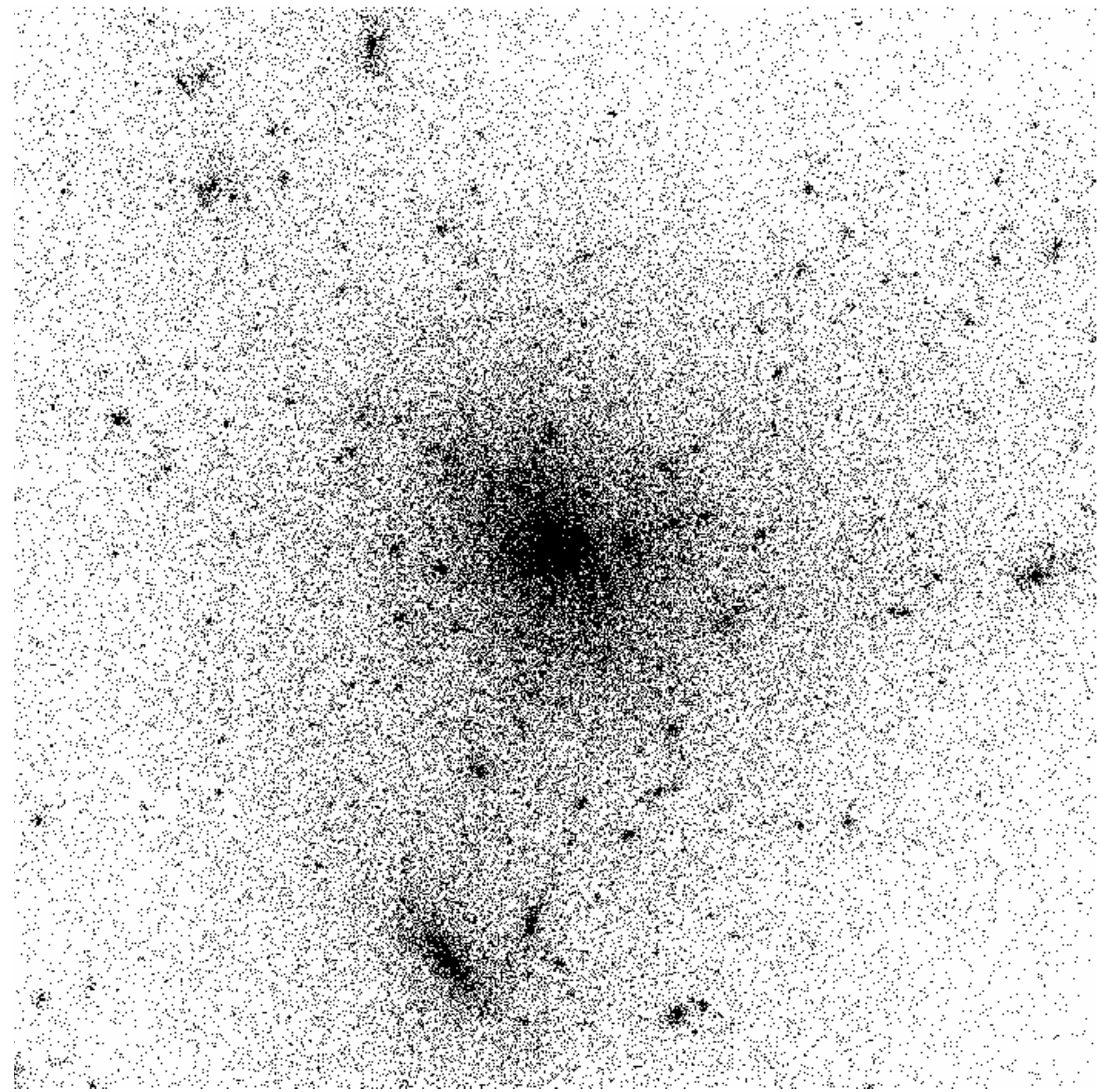
$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \sum_i M_i \frac{\vec{\xi} - \vec{\xi}\,'}{|\vec{\xi} - \vec{\xi}\,'|^2}$$



Using "Thin screen approximation"

DEFLECTION BY AN ENSEMBLE OF POINT MASSES

- Structure formation is often studied using numerical simulations
- Galaxies, galaxy clusters, etc. are described by ensembles of particles
- The calculation of the deflection angle by direct summation of all contributions from each particle has a computational cost $O(N^2)$



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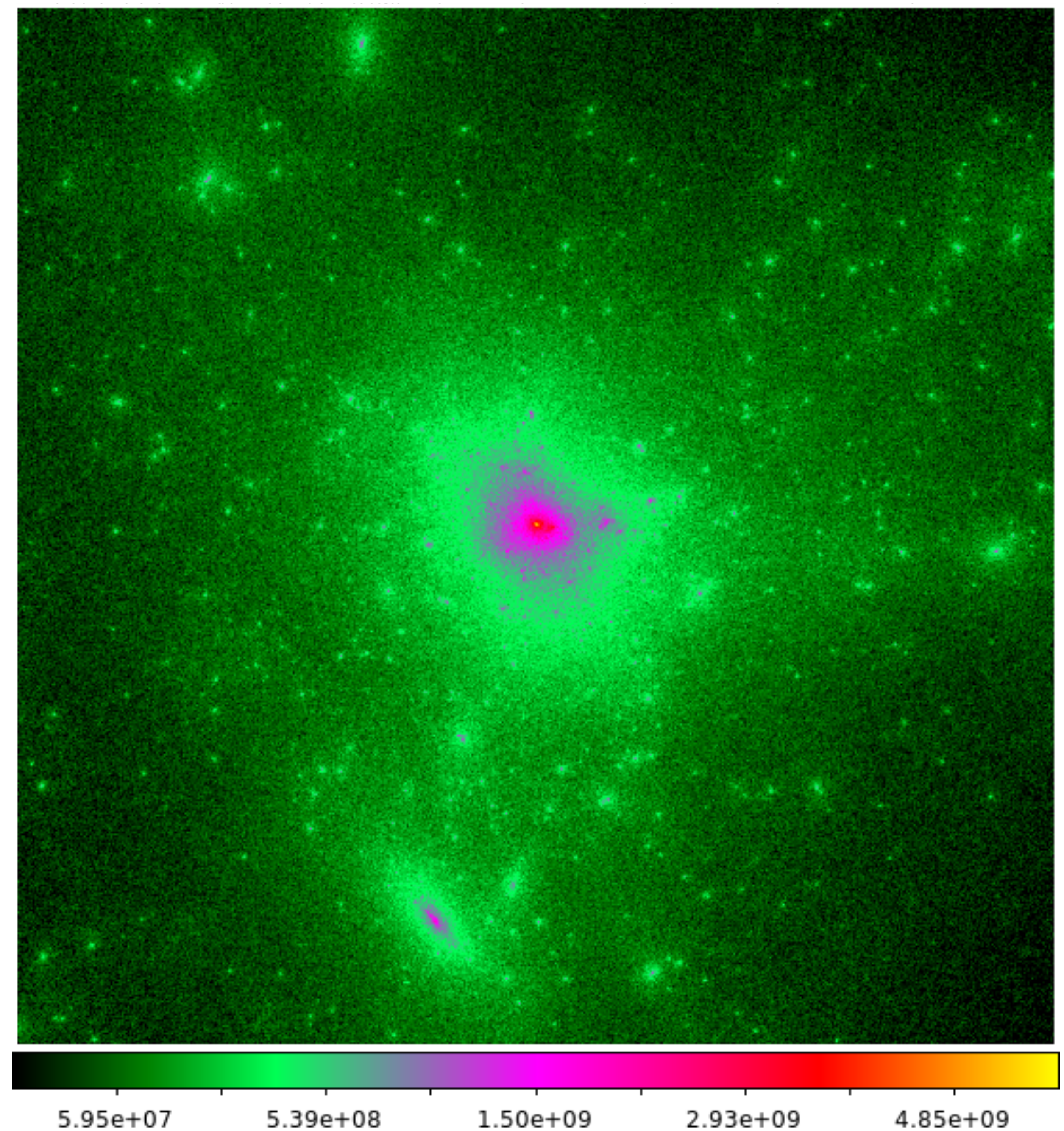


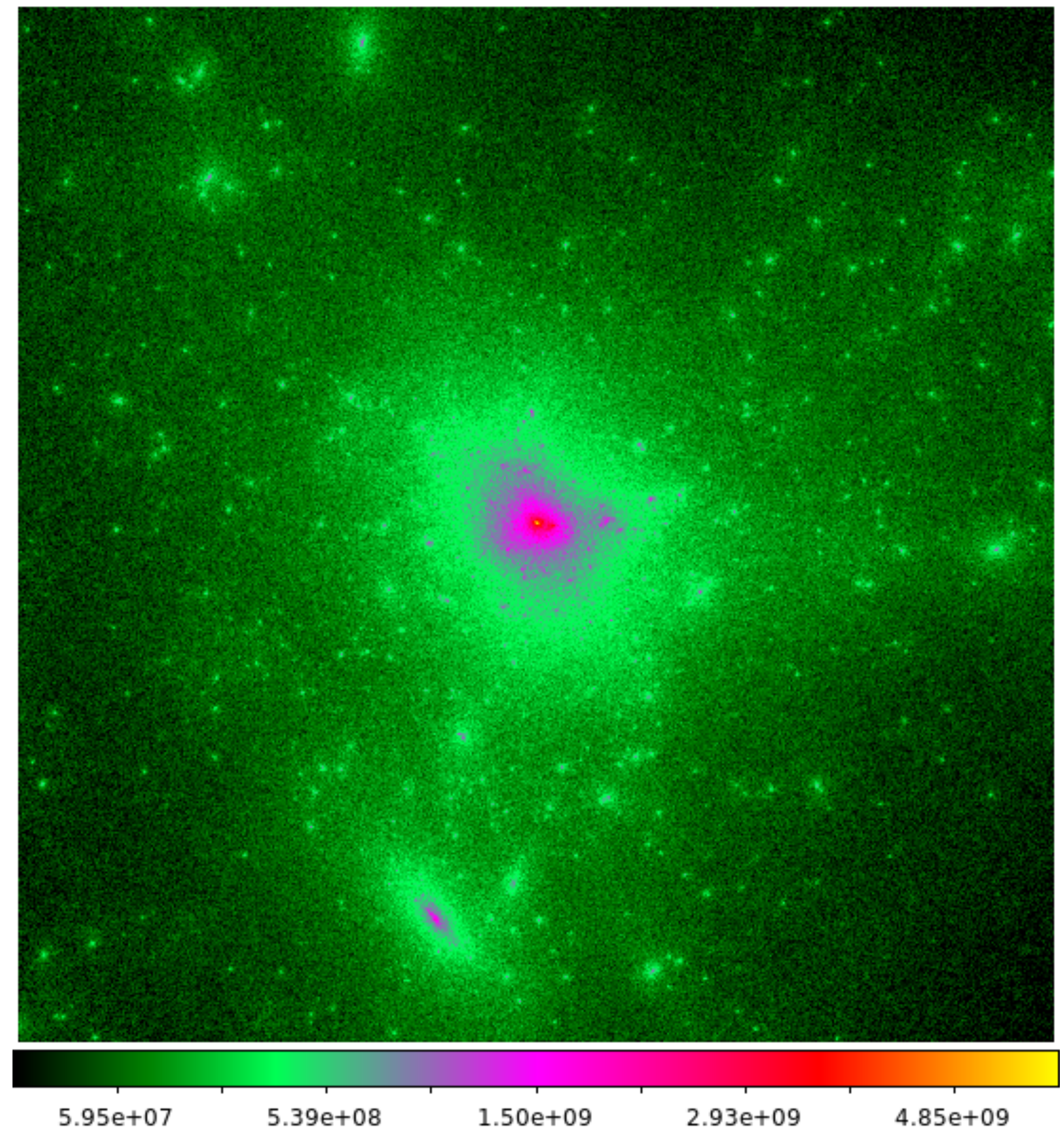
DEFLECTION BY AN EXTENDED MASS DISTRIBUTION

- This can be easily generalized to the case of a continuum distribution of mass

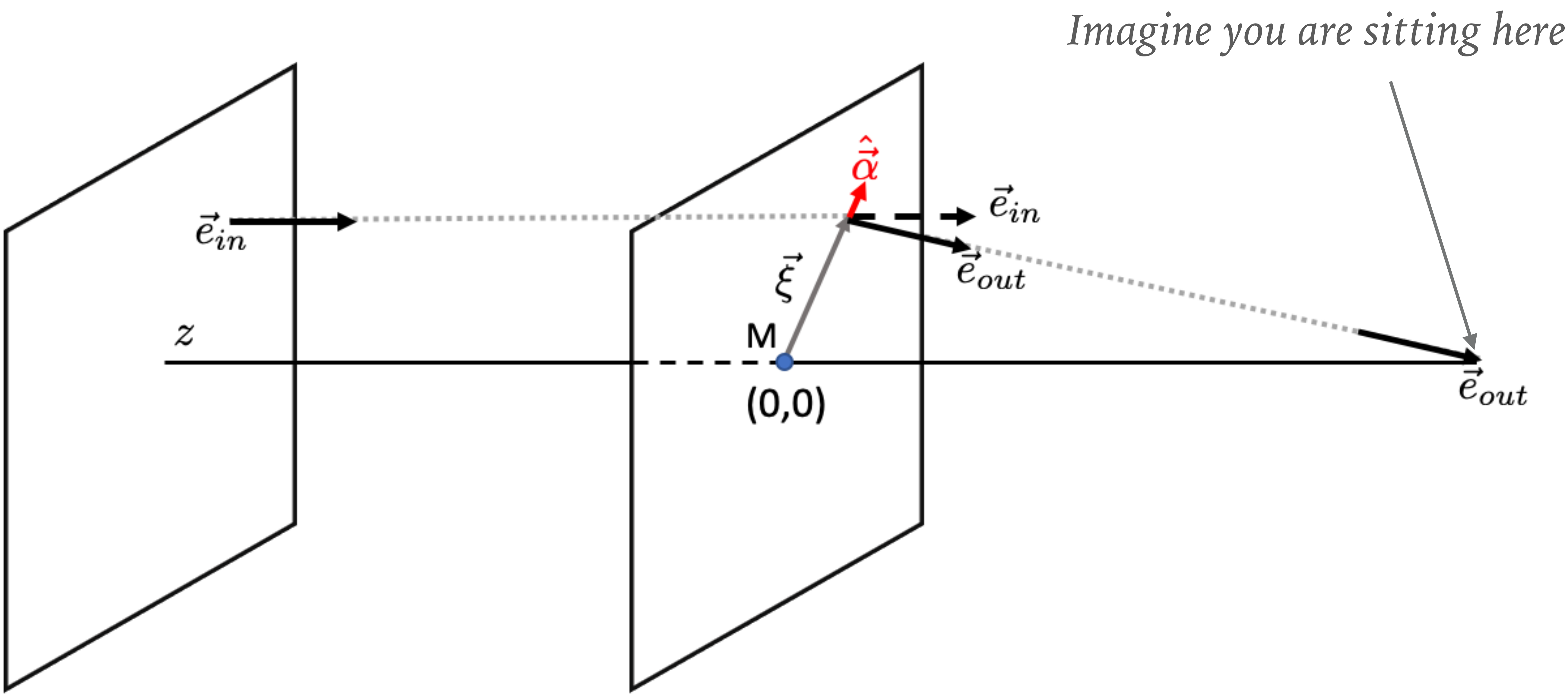
$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z) dz$$

$$d\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2} \Sigma(\vec{\xi}') d\xi'^2$$

$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{\vec{\xi} - \vec{\xi}'}{|\vec{\xi} - \vec{\xi}'|^2} \Sigma(\vec{\xi}') d\xi'^2$$



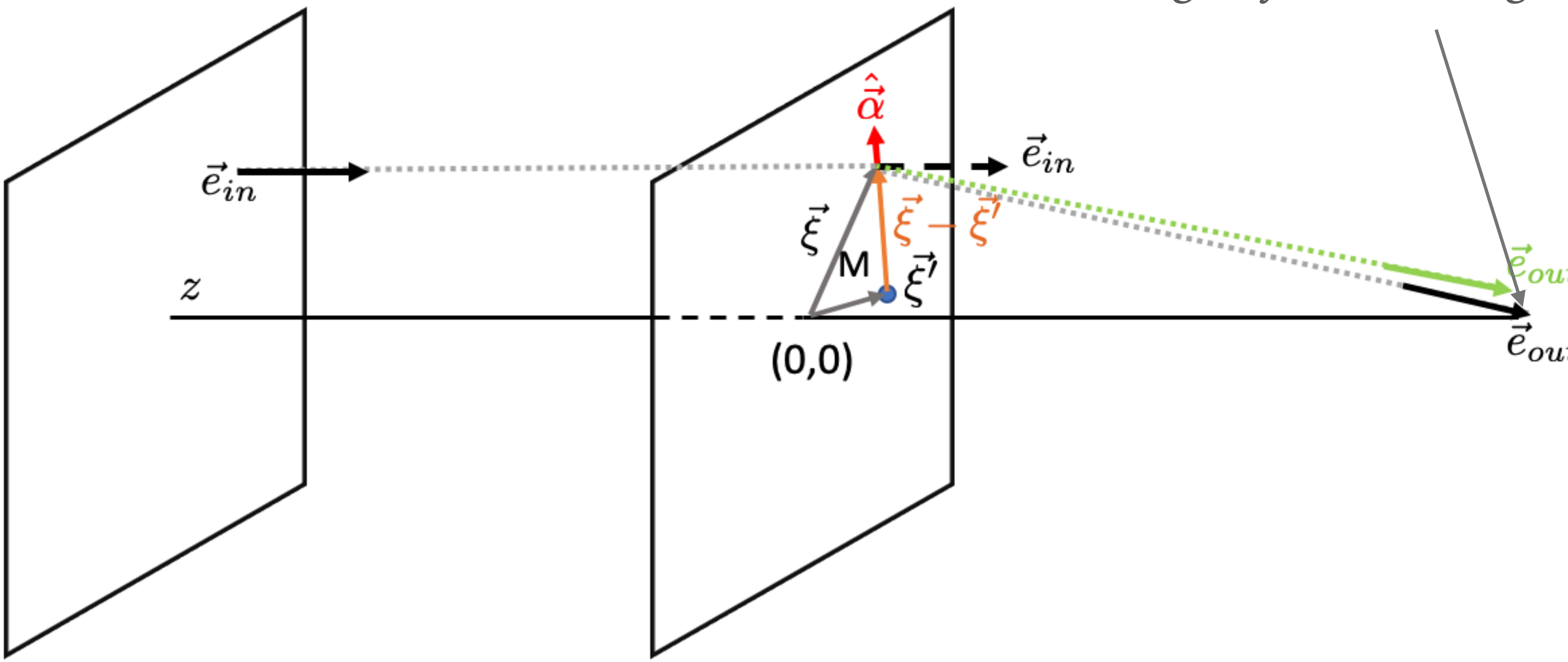
IS ANY DEFLECTION RIGHT?



Some rays are deflected in the right way, others are not!

IS ANY DEFLECTION RIGHT?

Imagine you are sitting here



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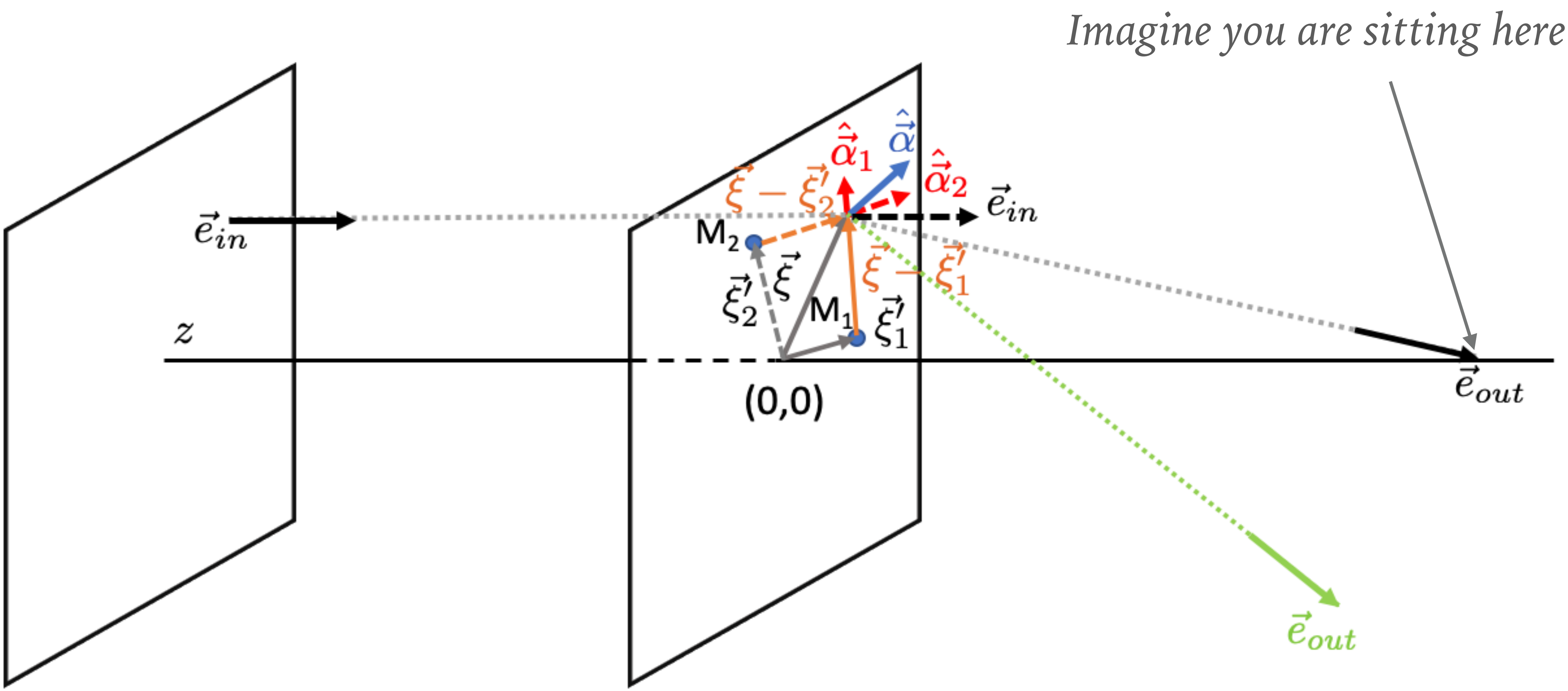

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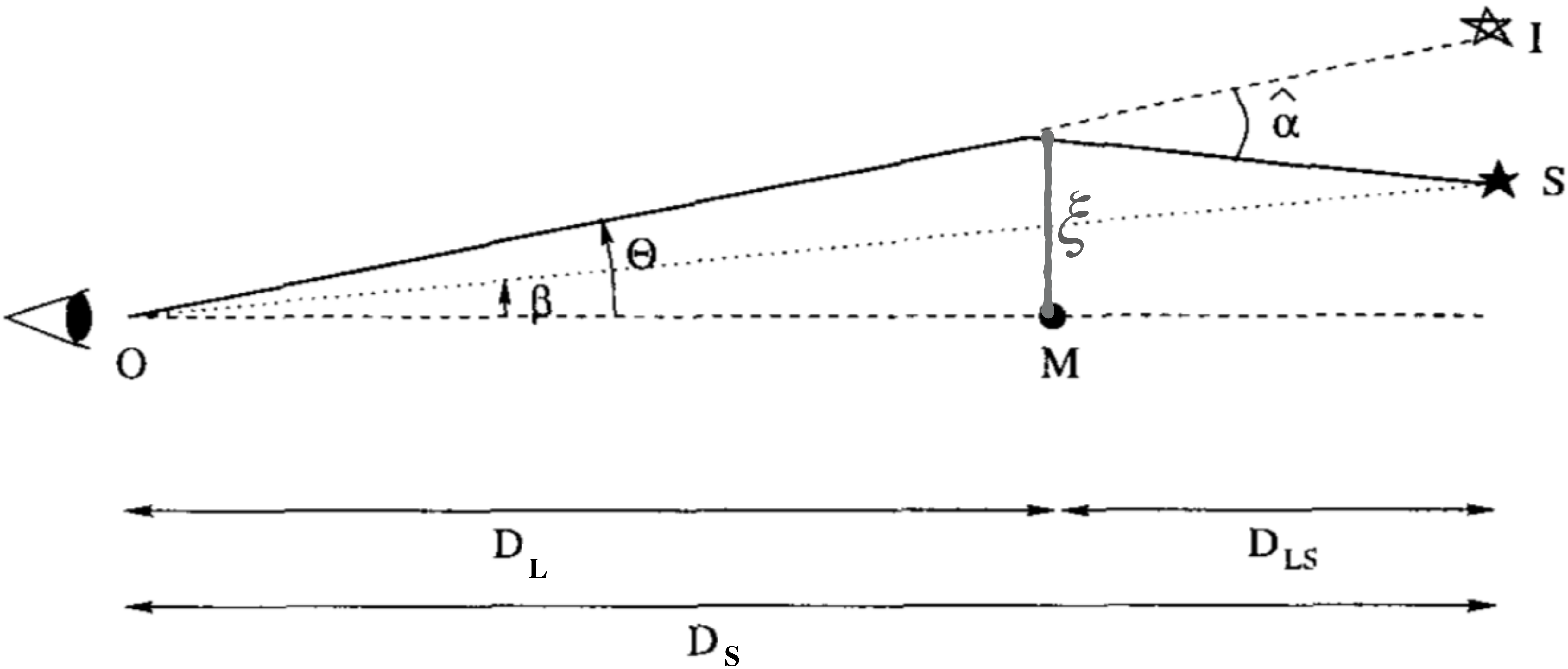
LENS EQUATION



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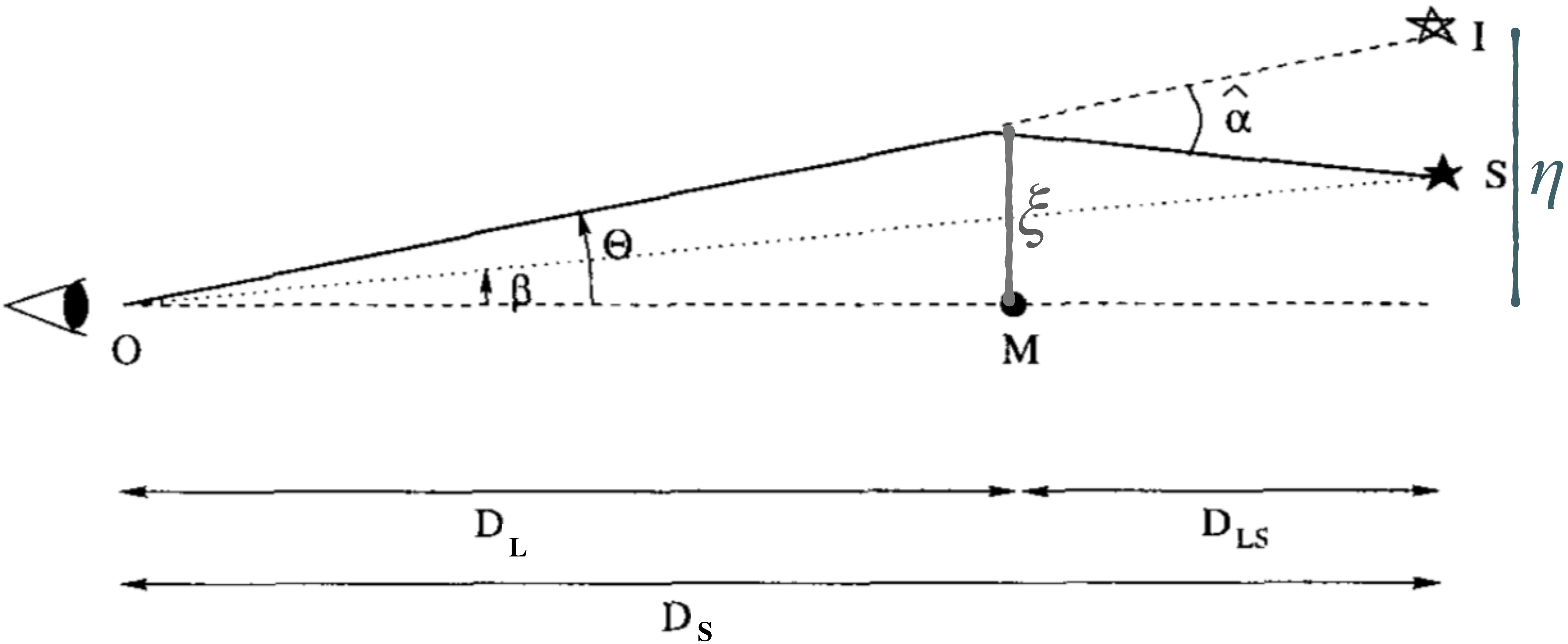

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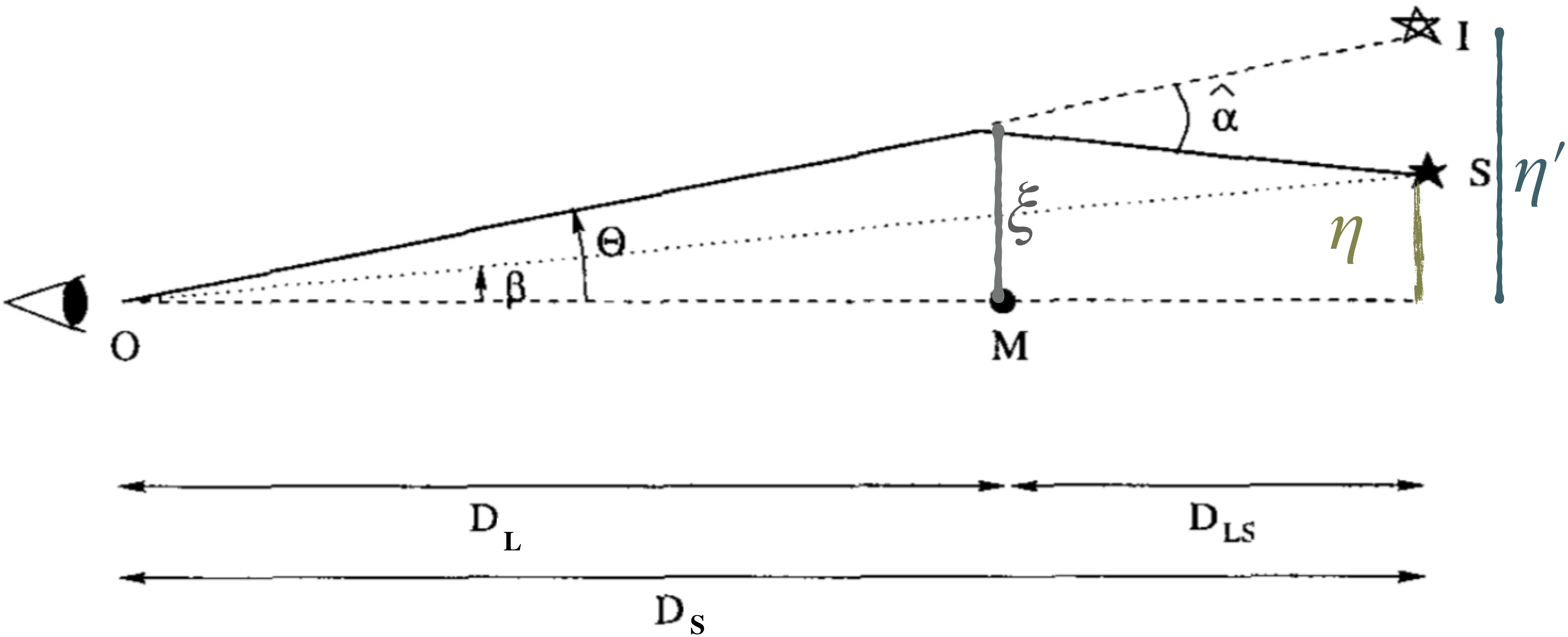
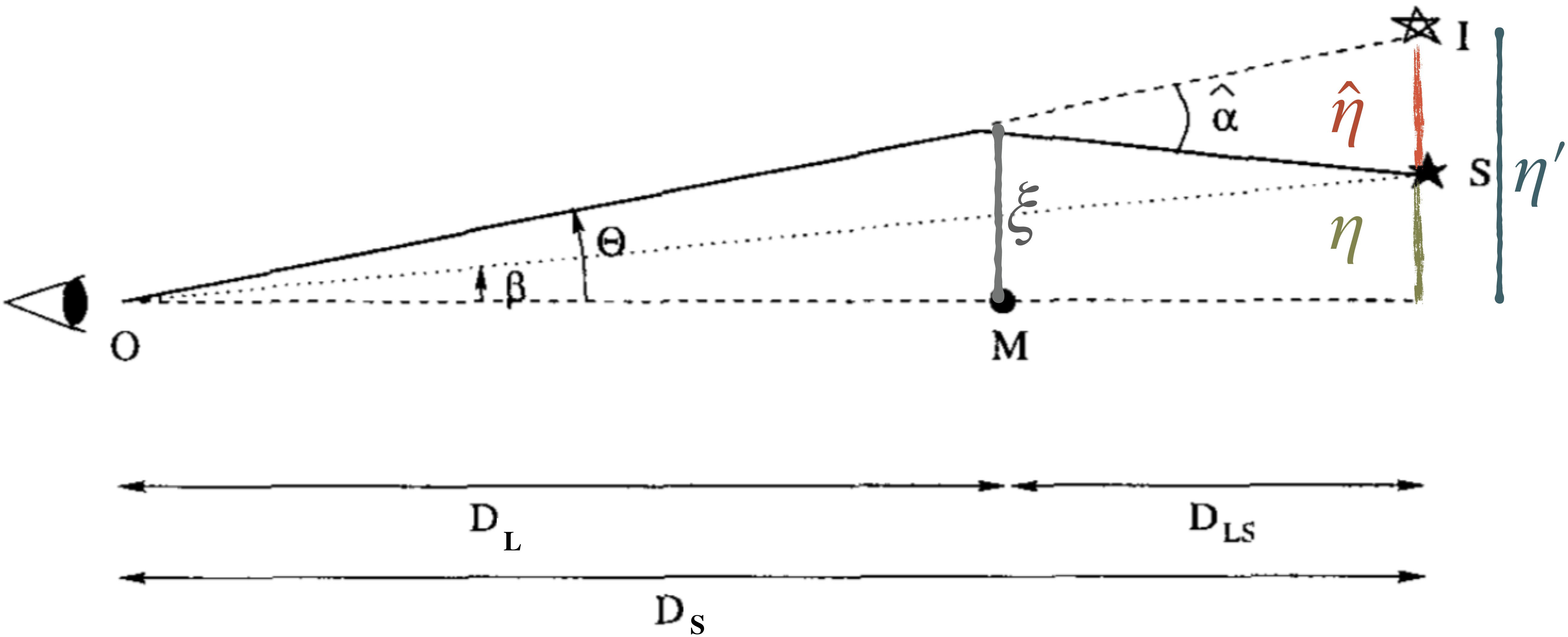


LENS EQUATION

$$\eta' = \eta + \hat{\eta}$$



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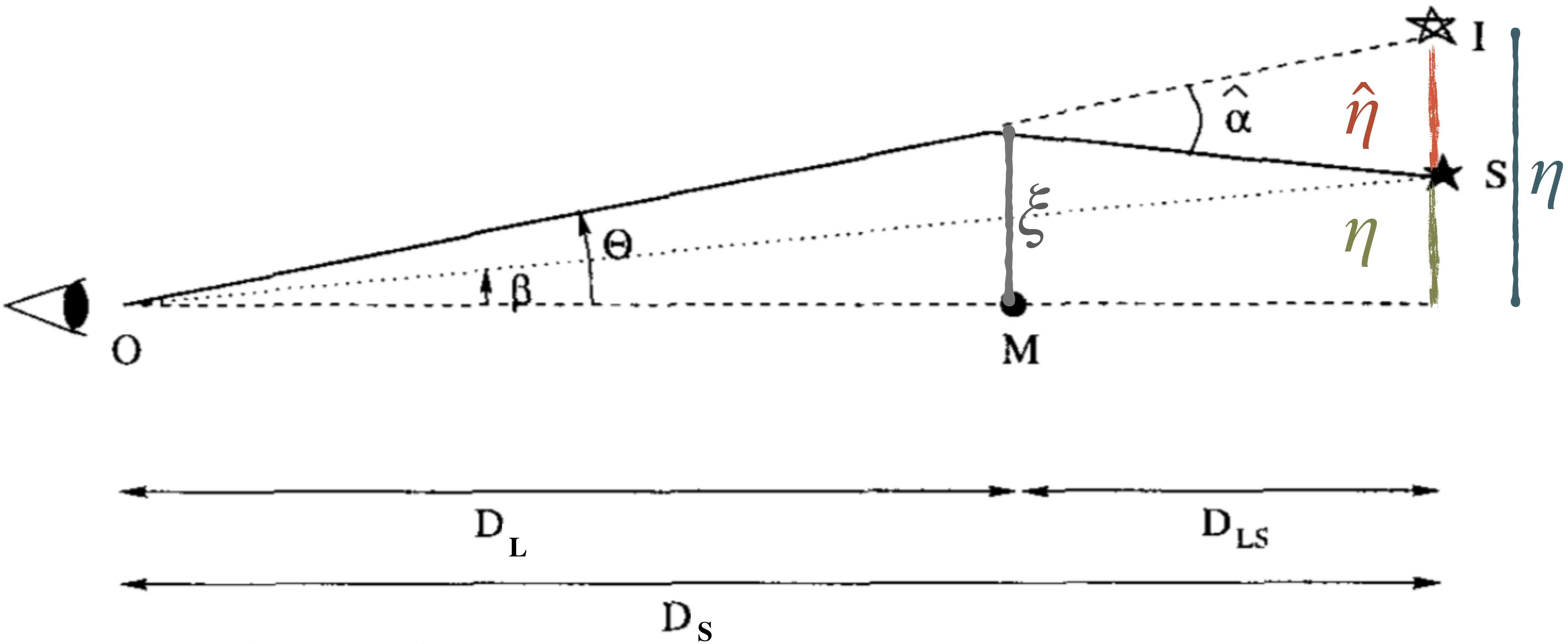

$$\eta' = D_S \theta$$

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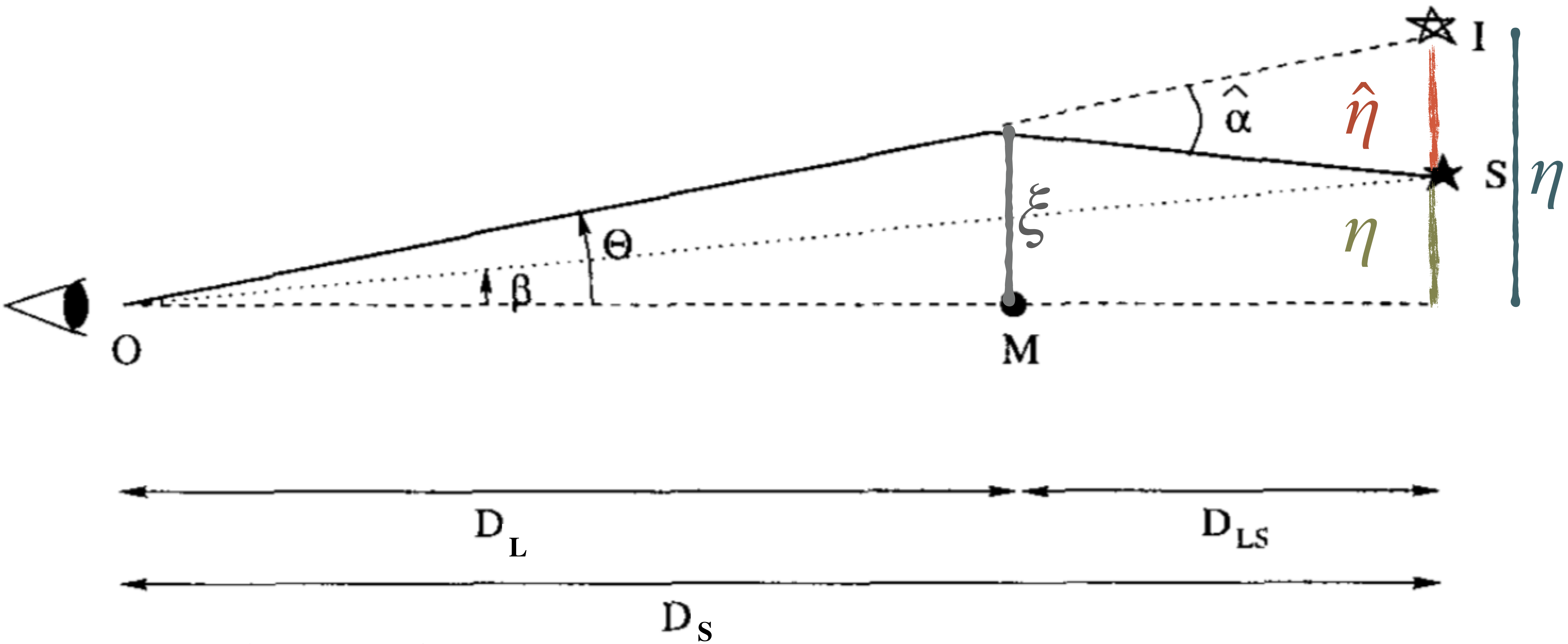

$$\eta' = D_S\theta \qquad \eta = D_S\beta$$

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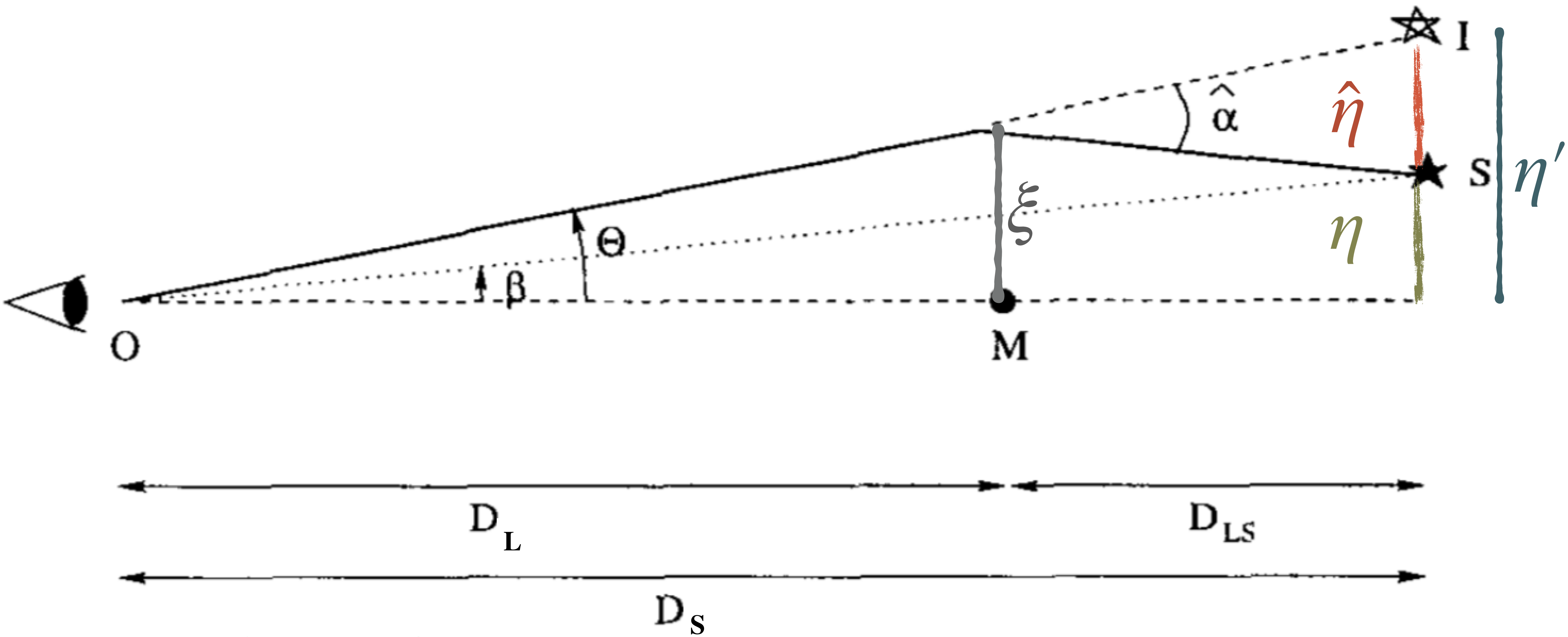
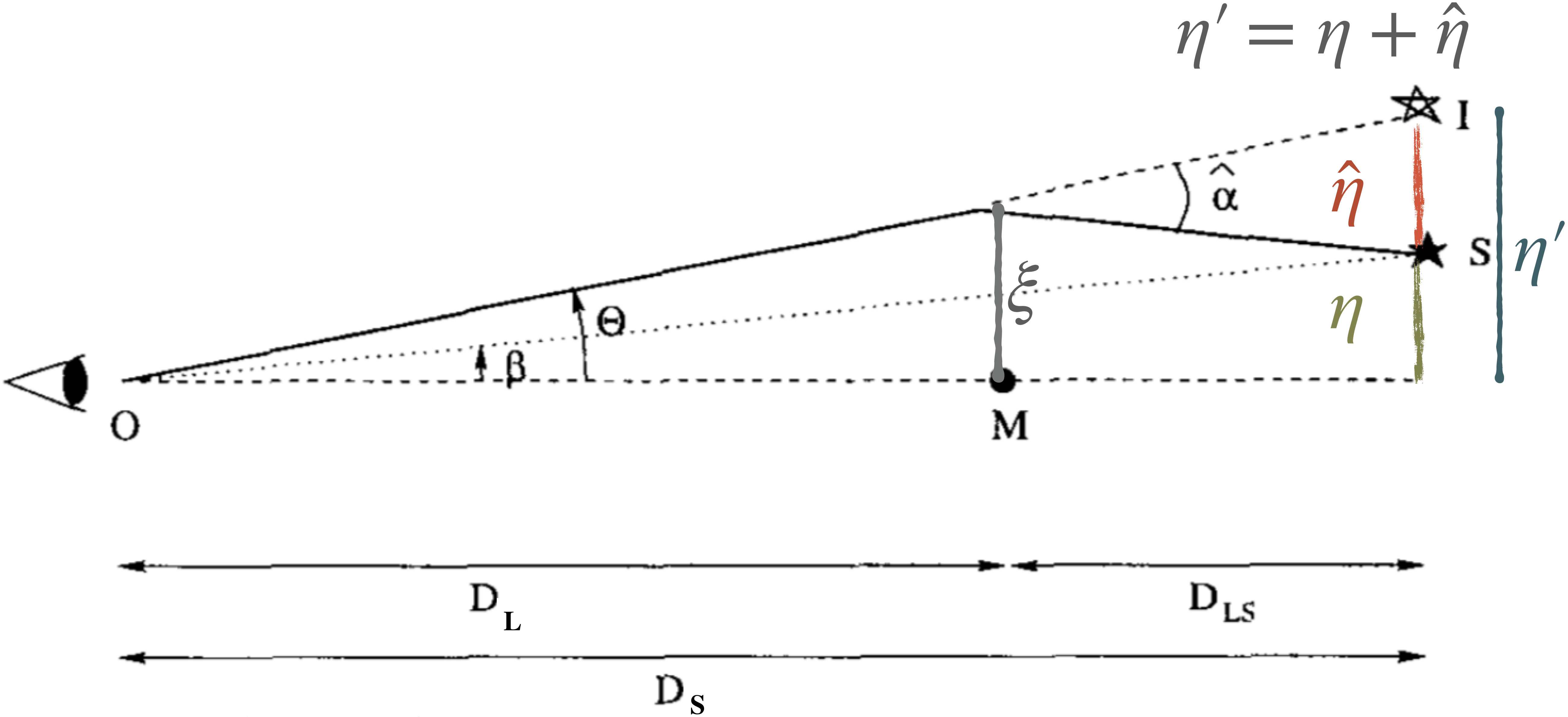


$$\eta' = D_S\theta \qquad \eta = D_S\beta \qquad \hat{\eta} = D_{LS}\hat{\alpha}(\theta)$$

LENS EQUATION



$\eta' = D_S\theta$ $\eta = D_S\beta$ $\hat{\eta} = D_{LS}\hat{\alpha}(\theta)$

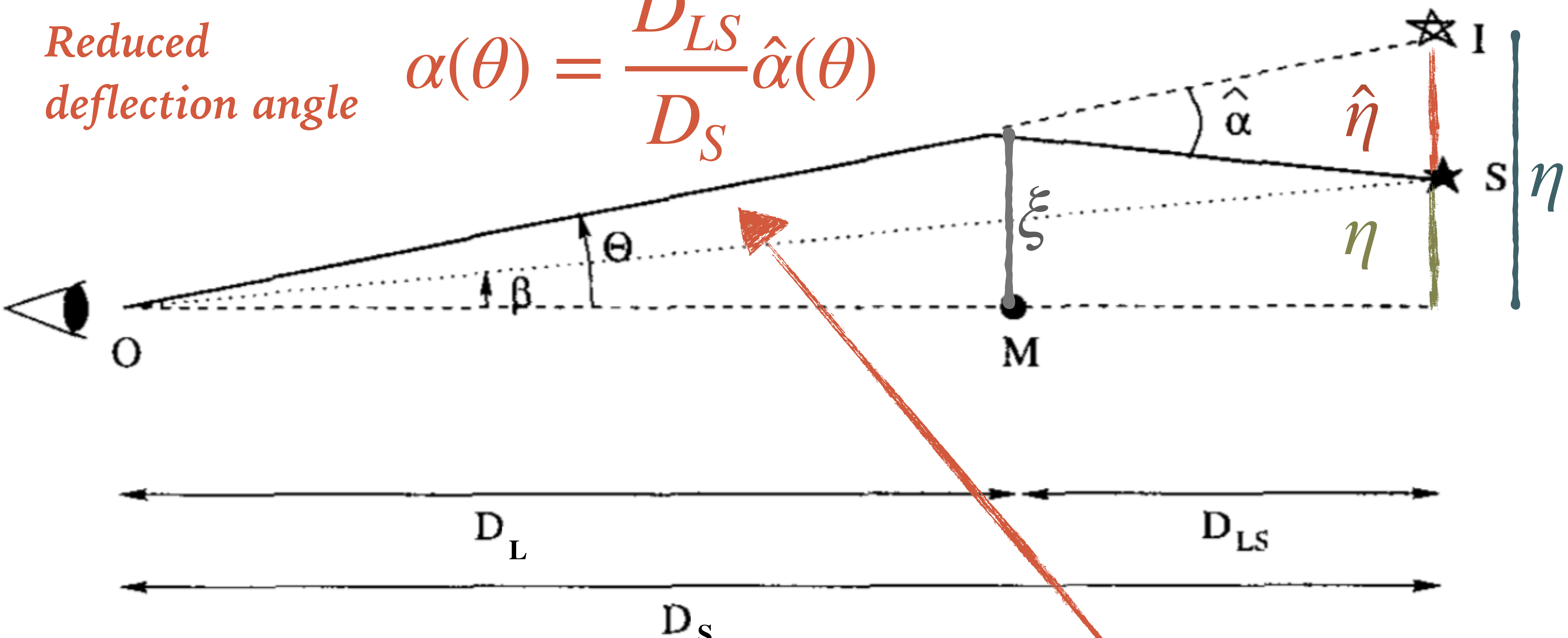
$$D_S\theta = D_S\beta + D_{LS}\hat{\alpha}(\theta) \Rightarrow \beta = \theta - \frac{D_{LS}}{D_S}\hat{\alpha}(\theta)$$

LENS EQUATION

$$\eta' = \eta + \hat{\eta}$$

Reduced deflection angle

$$\alpha(\theta) = \frac{D_{LS}}{D_S}\hat{\alpha}(\theta)$$



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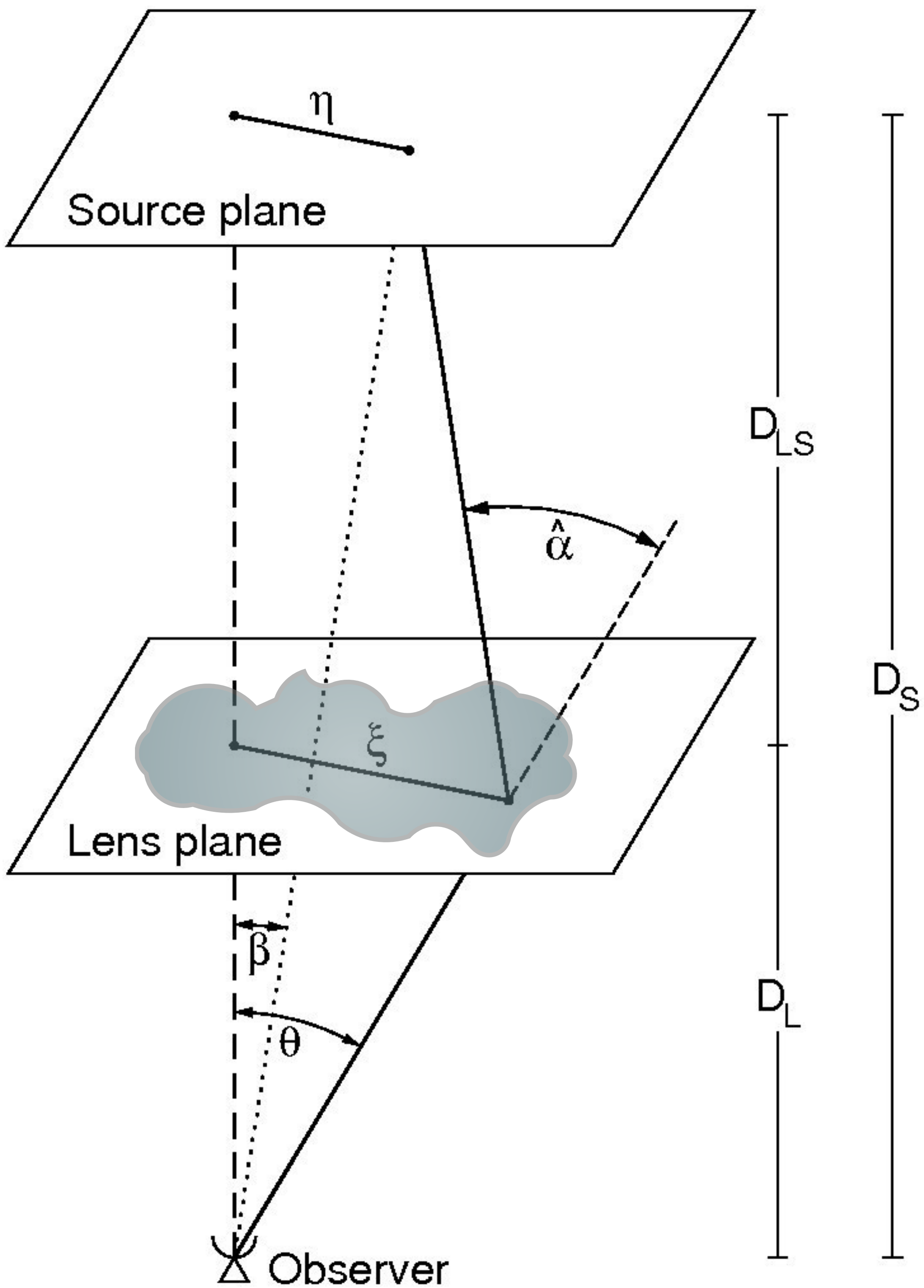

Remember that:

1) *positions on the lens and source planes are defined by vectors*
2) *the deflection angle itself is a vector*

$$\vec{\theta} = \frac{\vec{\xi}}{D_L} \qquad \vec{\beta} = \frac{\vec{\eta}}{D_S}$$

$$\vec{\beta} = \vec{\theta} - \frac{D_{LS}}{D_S}\vec{\hat{\alpha}}(\vec{\theta}) = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

DIMENSIONLESS NOTATION

*Quite often, an alternative way is chosen to write the lens equation: the so called "**dimension-less**" notation.*

*This implies the choice of a **reference angle (or length)** to scale the source and image positions and the deflection angle:*

$$\vec{\theta} = \frac{\vec{\xi}}{D_L} \quad \vec{\beta} = \frac{\vec{\eta}}{D_S} \quad \vec{\alpha}(\vec{\theta}) = \frac{D_{LS}}{D_S}\hat{\vec{\alpha}}(\vec{\theta}) \quad \vec{\beta} = \vec{\theta} - \frac{D_{LS}}{D_S}\hat{\vec{\alpha}}(\vec{\theta}) = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

$$\theta_0 = \frac{\xi_0}{D_L} = \frac{\eta_0}{D_S}$$

the reference angle subtends the reference scales on the lens and on the source planes

$$\frac{\vec{\theta}}{\theta_0} = \frac{\vec{\beta}}{\theta_0} - \frac{\vec{\alpha}(\vec{\theta})}{\theta_0}$$

dividing both members of the lens equation by the reference angle…

$$\vec{y} = \vec{x} - \vec{\alpha}(\vec{x})$$

$$\vec{\alpha}(\vec{x}) = \frac{\vec{\alpha}(\vec{\theta})}{\theta_0} = \frac{D_L}{\xi_0}\vec{\alpha}(\vec{\theta})$$

LENSING POTENTIAL

$$\hat{\vec{\alpha}} = \frac{2}{c^2}\int_{-\infty}^{+\infty} \vec{\nabla}_{\perp}\Phi dz$$

This formula tells us that the deflection is caused by the projection of the Newtonian gravitational potential on the lens plane.

$$\hat{\Psi}(\vec{\theta}) = \frac{D_{\mathsf{LS}}}{D_{\mathsf{L}}D_{\mathsf{S}}}\frac{2}{c^2}\int \Phi(D_{\mathsf{L}}\vec{\theta}, z)\mathrm{d}z$$

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We introduce the ***effective lensing potential***

1 *the lensing potential is the projection of the 3D potential*

2 *the lensing potential scales with distances*

OTHER PROPERTIES OF THE LENSING POTENTIAL

$$\vec{\nabla}_\theta \hat{\Psi}(\vec{\theta}) = \vec{\alpha}(\vec{\theta})$$

The reduced deflection angle is the gradient of the lensing potential

$$\begin{aligned}
\vec{\nabla}_\theta \hat{\Psi}(\vec{\theta}) &= D_{\mathrm{L}} \vec{\nabla}_\perp \hat{\Psi} = \vec{\nabla}_\perp \left(\frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}} \frac{2}{c^2} \int \hat{\Phi}(\vec{\theta}, z) \mathrm{d}z \right) \\
&= \frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}} \frac{2}{c^2} \int \vec{\nabla}_\perp \Phi(\vec{\theta}, z) \mathrm{d}z \\
&= \vec{\alpha}(\vec{\theta})
\end{aligned}$$

NOTE THAT…

… the same result holds if we use the dimension-less notation:

$$\vec{\nabla}_x = \frac{\xi_0}{D_\mathrm{L}} \vec{\nabla}_\theta$$

$$\vec{\nabla}_x \hat{\Psi} = \frac{\xi_0}{D_\mathrm{L}} \vec{\nabla}_\theta \hat{\Psi} = \frac{\xi_0}{D_\mathrm{L}} \vec{\alpha}$$

By multiplying both sides of the equation by D_L^2/ξ_0^2 *we obtain:*

$$\frac{D_\mathrm{L}^2}{\xi_0^2} \vec{\nabla}_x \hat{\Psi} = \frac{D_\mathrm{L}}{\xi_0} \vec{\alpha} \quad \Rightarrow \quad \Psi = \frac{D_\mathrm{L}^2}{\xi_0^2} \hat{\Psi} \quad \Rightarrow \quad \vec{\nabla}_x \Psi(\vec{x}) = \vec{\alpha}(\vec{x})$$

We have introduced the ***dimensionless counter-part of the lensing potential!***

OTHER PROPERTIES OF THE LENSING POTENTIAL

$$\triangle_\theta \hat{\Psi}(\vec{\theta}) = 2\kappa(\vec{\theta})$$

The laplacian of the lensing potential is twice the ***convergence:***

$$\kappa(\vec{\theta}) \equiv \frac{\Sigma(\vec{\theta})}{\Sigma_{\rm cr}} \quad \text{with} \quad \Sigma_{\rm cr} = \frac{c^2}{4\pi G}\frac{D_{\rm S}}{D_{\rm L}D_{\rm LS}}$$

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$[G]=L^3/M/T^2$

$[c^2]=L^2/T^2$

$[D_X]=L$

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$[G] = L^3/M/T^2$

$[c^2] = L^2/T^2$

$[D_X] = L$

The critical surface density is a characteristic density to distinguish between strong and weak gravitational lenses!

OTHER PROPERTIES OF THE LENSING POTENTIAL

$$\triangle_\theta \hat{\Psi}(\vec{\theta}) = 2\kappa(\vec{\theta})$$

The laplacian of the lensing potential is twice the ***convergence:***

We start from the poisson equation

$$\triangle\Phi = 4\pi G\rho$$

The surface mass density is then:

$$\Sigma(\vec{\theta}) = \frac{1}{4\pi G}\int_{-\infty}^{+\infty} \triangle\Phi dz$$

$$\kappa(\vec{\theta}) = \frac{1}{c^2}\frac{D_{\mathrm{L}}D_{\mathrm{LS}}}{D_{\mathrm{S}}}\int_{-\infty}^{+\infty} \triangle\Phi dz$$

Let's introduce the Laplacian operator on the lens plane:

$$\triangle_\theta = \frac{\partial^2}{\partial\theta_1^2} + \frac{\partial^2}{\partial\theta_2^2} = D_{\mathrm{L}}^2\left(\frac{\partial^2}{\partial\xi_1^2} + \frac{\partial^2}{\partial\xi_2^2}\right) = D_{\mathrm{L}}^2\left(\triangle - \frac{\partial^2}{\partial z^2}\right)$$

Then:

$$\triangle\Phi = \frac{1}{D_{\mathrm{L}}^2}\triangle_\theta\Phi + \frac{\partial^2\Phi}{\partial z^2}$$

OTHER PROPERTIES OF THE LENSING POTENTIAL

With this substitution:

$$\kappa(\vec{\theta}) = \frac{1}{c^2}\frac{D_{\rm LS}}{D_{\rm S}D_{\rm L}}\left[\triangle_\theta \int_{-\infty}^{+\infty} \Phi \mathrm{d}z + D_{\rm L}^2 \int_{-\infty}^{+\infty} \frac{\partial^2 \Phi}{\partial z^2} \mathrm{d}z\right]$$

where the second term in the sum is zero, if the lens if gravitationally bound!

Given the definition of lensing potential:

$$\kappa(\theta) = \frac{1}{2}\triangle_\theta \hat{\Psi}$$

Note that:

$$\triangle_\theta = D_{\rm L}^2 \triangle_\xi = \frac{D_{\rm L}^2}{\xi_0^2}\triangle_x \qquad \kappa(\theta) = \frac{1}{2}\triangle_\theta \hat{\Psi} = \frac{1}{2}\frac{\xi_0^2}{D_{\rm L}^2}\triangle_\theta \Psi$$

$$\Rightarrow \quad \kappa(\vec{x}) = \frac{1}{2}\triangle_x \Psi(\vec{x})$$

DIMENSIONLESS NOTATION

From

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi'})\Sigma(\vec{\xi'})}{|\vec{\xi} - \vec{\xi'}|^2} \, \mathrm{d}^2\xi'$$

we obtain

$$\vec{\alpha}(\vec{x}) = \frac{1}{\pi} \int_{\mathbf{R}^2} \mathrm{d}^2x' \kappa(\vec{x}') \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|}$$

Using

$$\vec{\nabla}_x \Psi(\vec{x}) = \vec{\alpha}(\vec{x})$$

$$\Psi(\vec{x}) = \frac{1}{\pi} \int_{\mathbf{R}^2} \kappa(\vec{x}') \ln |\vec{x} - \vec{x}'| \mathrm{d}^2x'$$

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we obtain

$$\vec{\alpha}(\vec{x}) = \frac{1}{\pi} \int_{\mathbf{R}^2} \mathrm{d}^2 x' \kappa(\vec{x}') \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|}$$

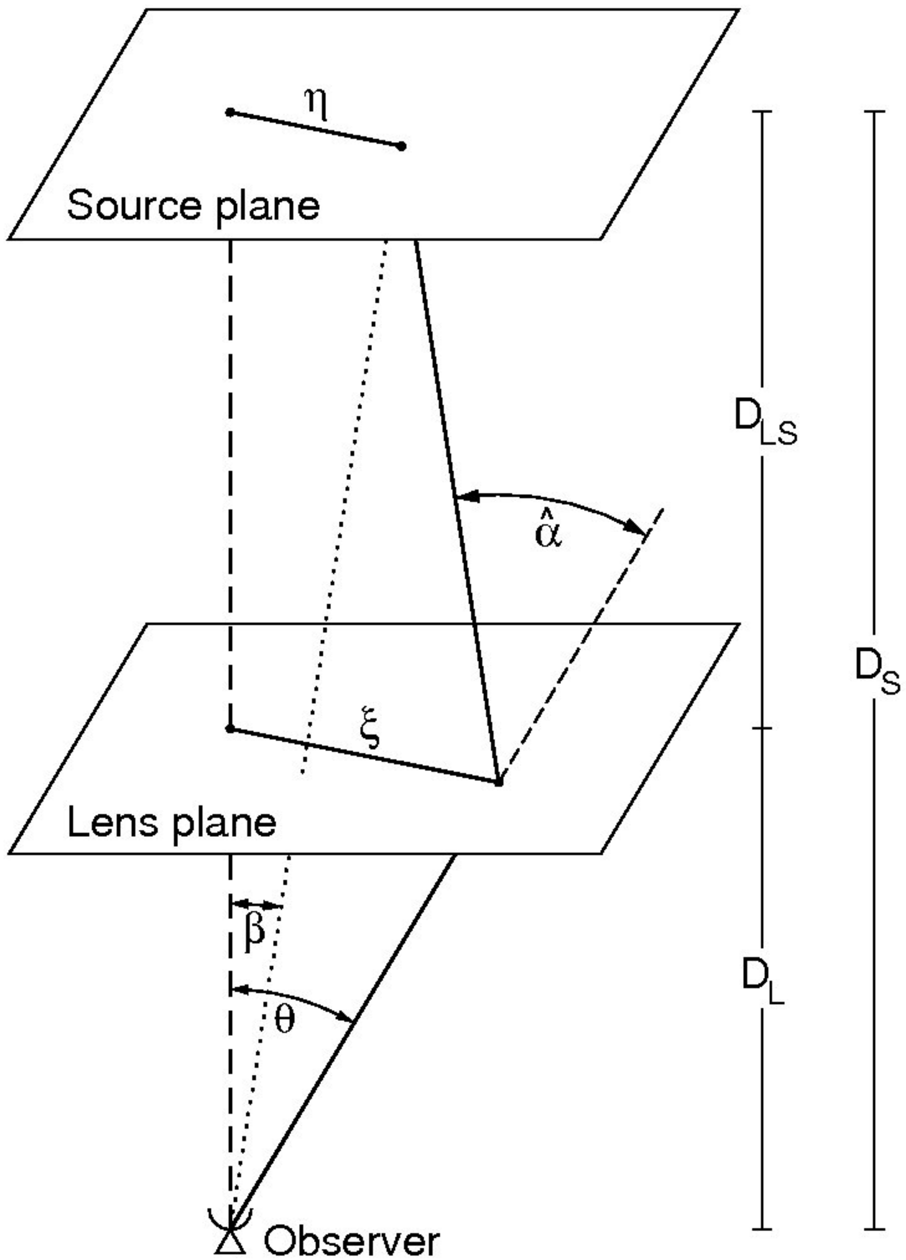
Using

$$\vec{\nabla}_x \Psi(\vec{x}) = \vec{\alpha}(\vec{x})$$

Convolution kernels

$$\Psi(\vec{x}) = \frac{1}{\pi} \int_{\mathbf{R}^2} \kappa(\vec{x}') \ln|\vec{x} - \vec{x}'| \mathrm{d}^2 x'$$

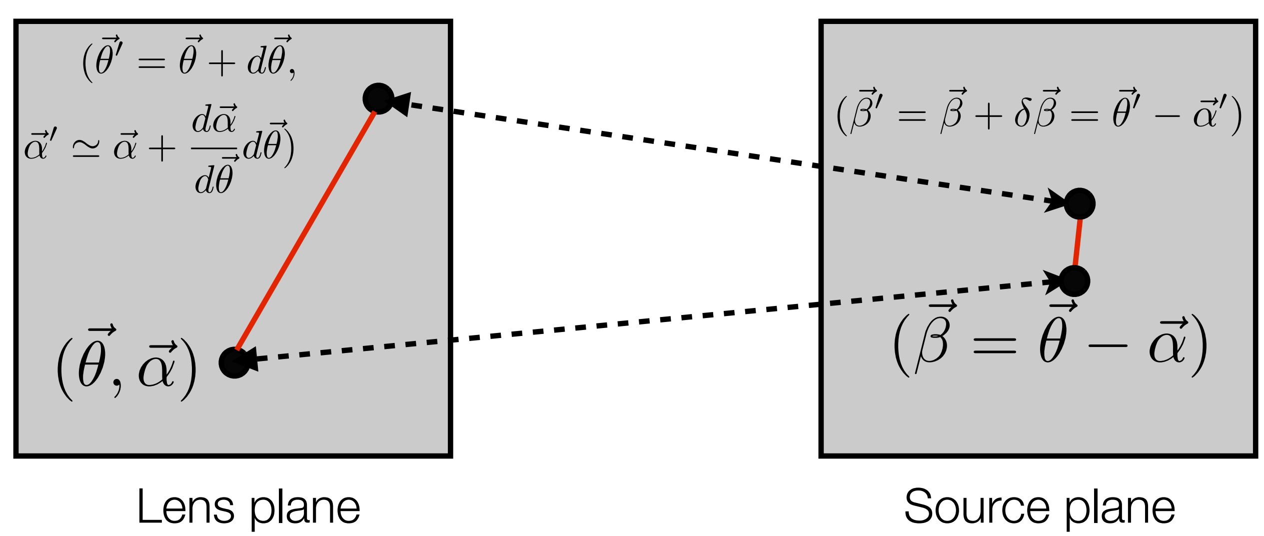
LENS MAPPING (FIRST ORDER)



- *we derived the lens equation*

$$\vec{\beta} = \vec{\theta} - \frac{D_{LS}}{D_S}\hat{\vec{\alpha}}(\vec{\theta}) = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

- *Assuming that the d.a. does not vary significantly over the scale dΘ:*



$$(\vec{\beta}' - \vec{\beta}) = \left(I - \frac{d\vec{\alpha}}{d\vec{\theta}}\right)(\vec{\theta}' - \vec{\theta}) = A(\vec{\theta}' - \vec{\theta})$$

LENS MAPPING (FIRST ORDER)

$$A \equiv \frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \left(\delta_{ij} - \frac{\partial \alpha_i(\vec{\theta})}{\partial \theta_j} \right) = \left(\delta_{ij} - \frac{\partial^2 \hat{\Psi}(\vec{\theta})}{\partial \theta_i \partial \theta_j} \right)$$

*A is called "**the lensing Jacobian**": it is a symmetric second rank tensor describing the first order mapping between lens and source planes.*

This tensor can be written as the sum of an isotropic part, proportional to its trace, and an anisotropic, traceless part.

$$A_{iso,i,j} = \frac{1}{2}\mathrm{Tr}A\delta_{i,j}$$

$$A_{aniso,i,j} = A_{i,j} - \frac{1}{2}\mathrm{Tr}A\delta_{i,j}$$

ANISOTROPIC PART

$$\left(A-\frac{1}{2}\mathrm{tr}A\cdot I\right)_{ij} = \delta_{ij}-\hat{\Psi}_{ij}-\frac{1}{2}(1-\hat{\Psi}_{11}+1-\hat{\Psi}_{22})\delta_{ij}$$

$$= -\hat{\Psi}_{ij}+\frac{1}{2}(\hat{\Psi}_{11}+\hat{\Psi}_{22})\delta_{ij}$$

$$= \begin{pmatrix} -\frac{1}{2}(\hat{\Psi}_{11}-\hat{\Psi}_{22}) & -\hat{\Psi}_{12} \\ -\hat{\Psi}_{12} & \frac{1}{2}(\hat{\Psi}_{11}-\hat{\Psi}_{22}) \end{pmatrix}$$

$$\boxed{\frac{\partial^2\hat{\Psi}(\vec{\theta})}{\partial\theta_i\partial\theta_j}\equiv\hat{\Psi}_{ij}}$$

Introducing the ***shear****:*

$$\gamma_1 = \frac{1}{2}(\hat{\Psi}_{11}-\hat{\Psi}_{22})$$

$$\gamma_2 = \hat{\Psi}_{12}=\hat{\Psi}_{21}\,,$$

$$\Gamma = \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix}$$

Symmetric, trace-less tensor with eigenvalues:

$$\pm\sqrt{\gamma_1^2+\gamma_2^2}=\pm\gamma$$

ISOTROPIC PART

$$\begin{aligned}\frac{1}{2}\mathrm{tr}A\cdot I &= \left[1-\frac{1}{2}(\hat{\Psi}_{11}+\hat{\Psi}_{22})\right]\delta_{ij} \\ &= \left(1-\frac{1}{2}\triangle\hat{\Psi}\right)\delta_{ij} = (1-\kappa)\delta_{ij}\end{aligned}$$

Remember: $\triangle_\theta \Psi(\vec{\theta}) = 2\kappa(\vec{\theta})$

THE SHEAR IS NOT A VECTOR!

$$\det A = (1-\kappa-\gamma)(1-\kappa+\gamma)$$
$$\gamma = \sqrt{\gamma_1^2+\gamma_2^2}$$

There is thus an orthogonal coordinate transformation $R(\varphi)$, a rotation by an angle φ, which brings the Jacobian matrix into diagonal form.

The Jacobian matrix transforms as

$$A \rightarrow A' = R(\varphi)^T A R(\varphi)$$

This shows that the shear components transform under coordinate rotations as

$$\gamma_1 \rightarrow \gamma_1' = \gamma_1 \cos(2\varphi) + \gamma_2 \sin(2\varphi)$$
$$\gamma_2 \rightarrow \gamma_2' = -\gamma_1 \sin(2\varphi) + \gamma_2 \cos(2\varphi)$$

i.e. unlike a vector! Since the shear components are mapped onto each other after rotations of $\varphi = \pi$ rather than $\varphi = 2\pi$, they form a so-called spin-2 field.

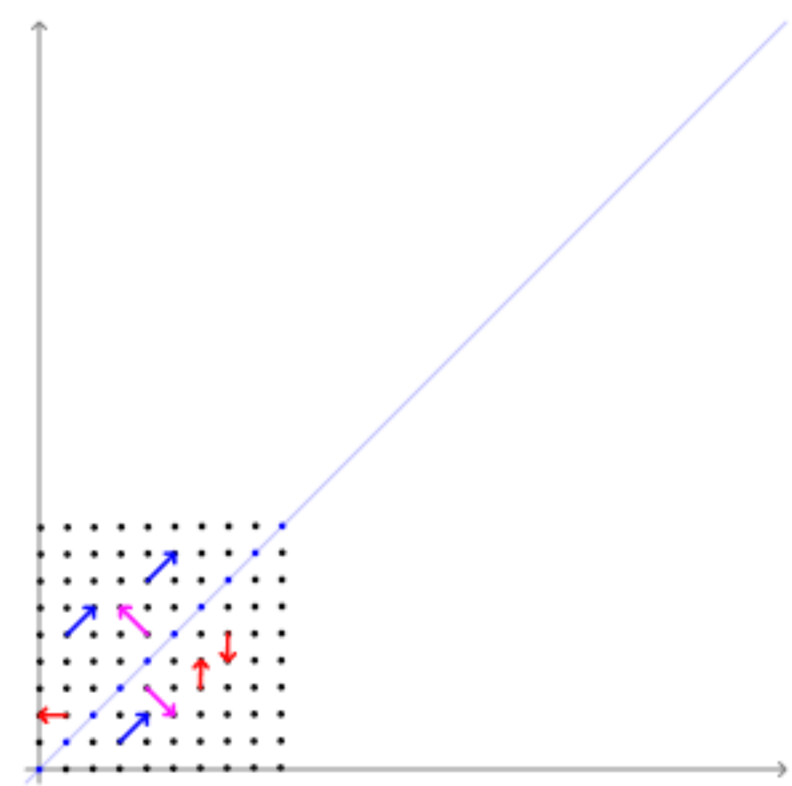
LENSING JACOBIAN

$$
\begin{aligned}
A &= \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} \\
&= (1-\kappa)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \gamma \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix}
\end{aligned}
$$

Lens mapping at first order is a linear application, distorting areas.

Distortion directions are given by the ***eigenvectors*** *of A.*

Distortion amplitudes in these directions are given by the ***eigenvalues****.*



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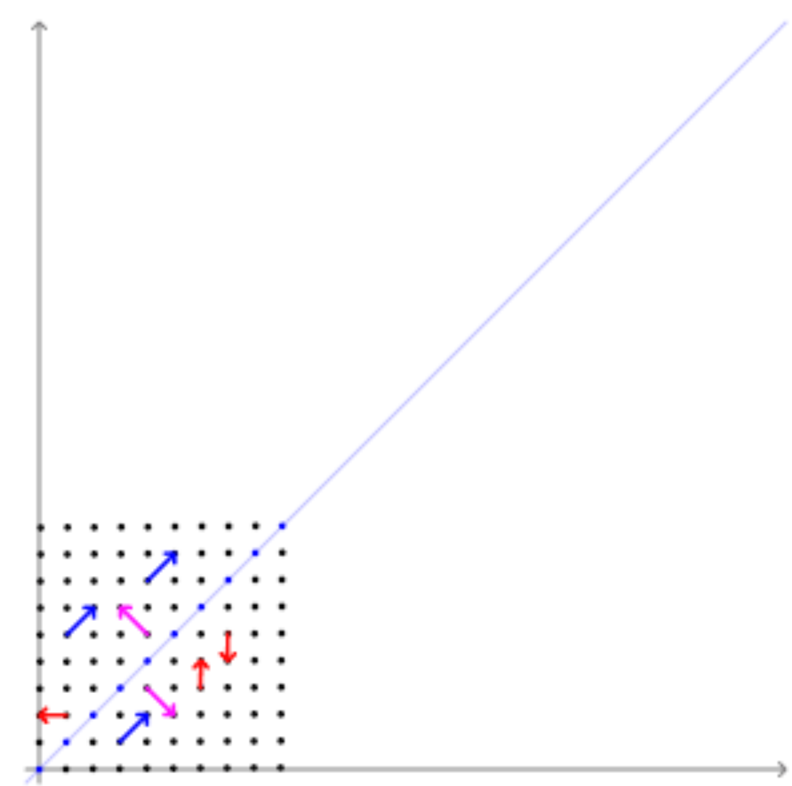


EXAMPLE: FIRST ORDER DISTORTION OF A CIRCULAR SOURCE

$$\beta_1^2 + \beta_2^2 = \beta^2$$

In the reference frame where A is diagonal:

$$\left(\begin{array}{c} \beta_1 \\ \beta_2 \end{array}\right) = \left(\begin{array}{cc} 1-\kappa-\gamma & 0 \\ 0 & 1-\kappa+\gamma \end{array}\right)\left(\begin{array}{c} \theta_1 \\ \theta_2 \end{array}\right)$$

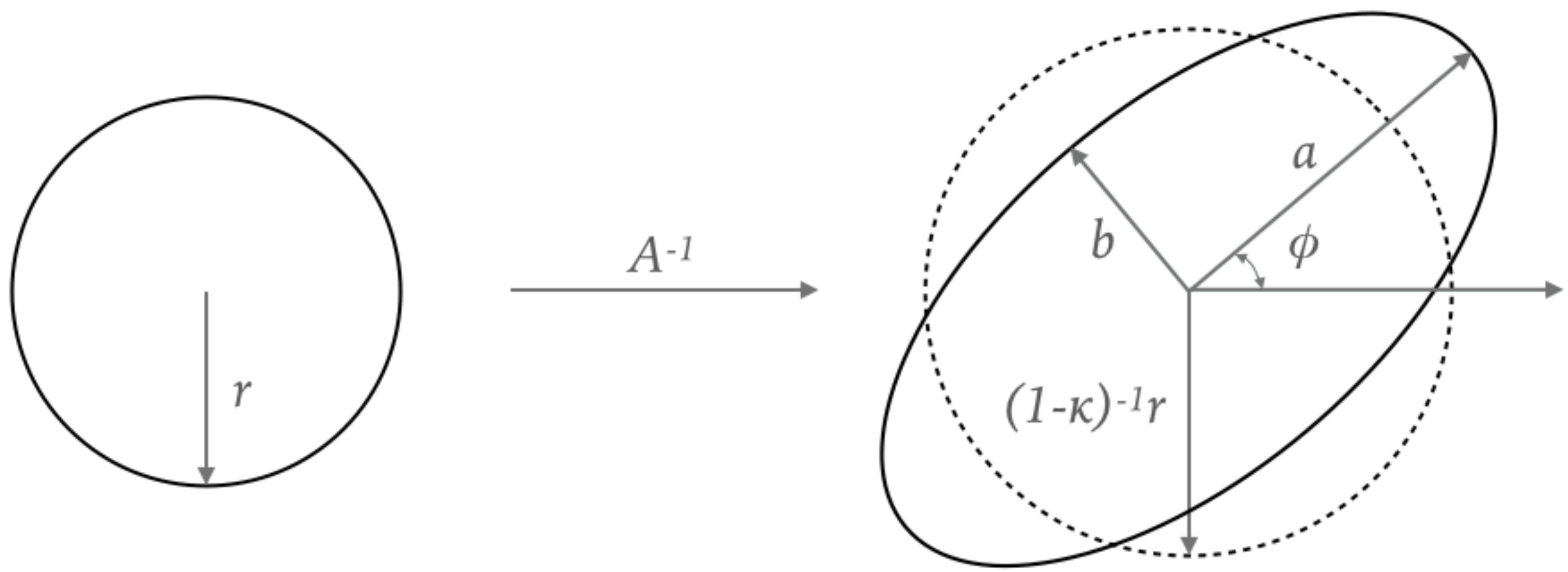
$$\begin{aligned} \beta_1 &= (1-\kappa-\gamma)\theta_1 \\ \beta_2 &= (1-\kappa+\gamma)\theta_2 \end{aligned}$$

$$\beta^2 = \beta_1^2 + \beta_2^2 = (1-\kappa-\gamma)^2\theta_1^2 + (1-\kappa+\gamma)^2\theta_2^2$$

This is the equation of an ellipse with semi-axes:

$$a = \frac{\beta}{1-\kappa-\gamma} \qquad b = \frac{\beta}{1-\kappa+\gamma}$$

EXAMPLE: FIRST ORDER DISTORTION OF A CIRCULAR SOURCE



convergence: *responsible for isotropic expansion or contraction*

shear: *responsible for anisotropic distortion*

Ellipticity: $$e = \frac{a-b}{a+b} = \frac{\gamma}{1-\kappa} = g$$

ON THE SPIN-2 NATURE OF SHEAR: QUIZ

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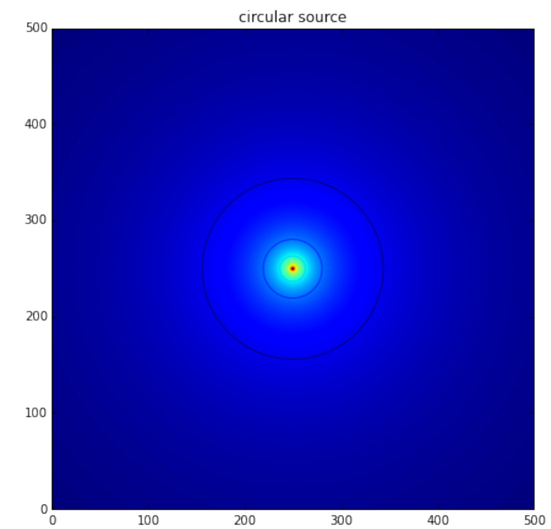
- Let's consider a circular source



ON THE SPIN-2 NATURE OF SHEAR: QUIZ

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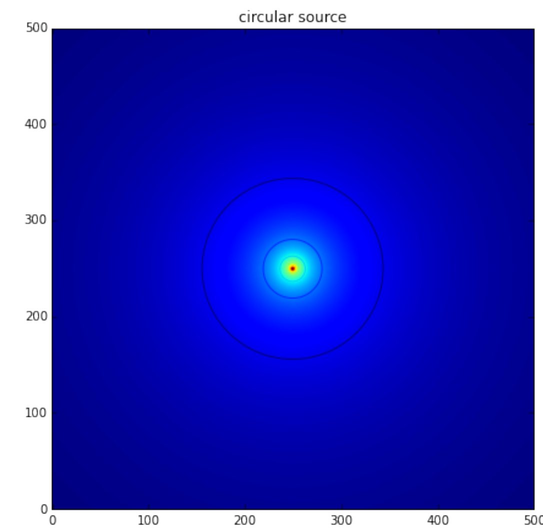
- How is it distorted if we apply a pure convergence trasformation?



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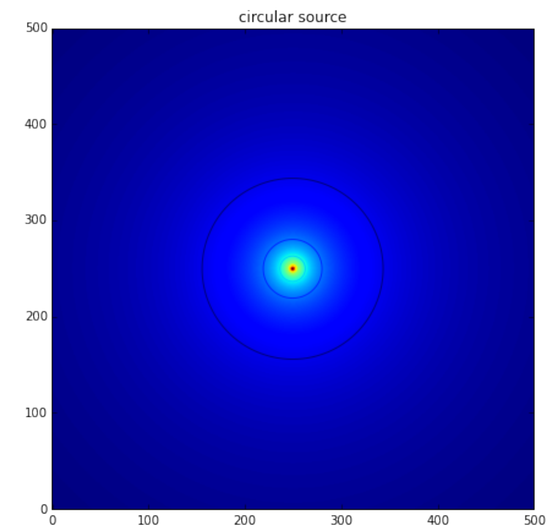

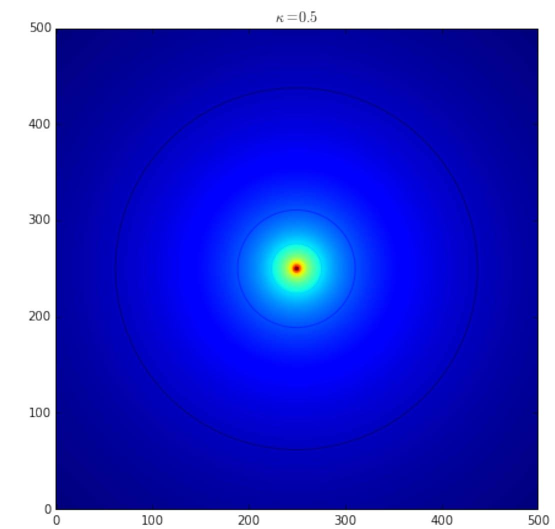

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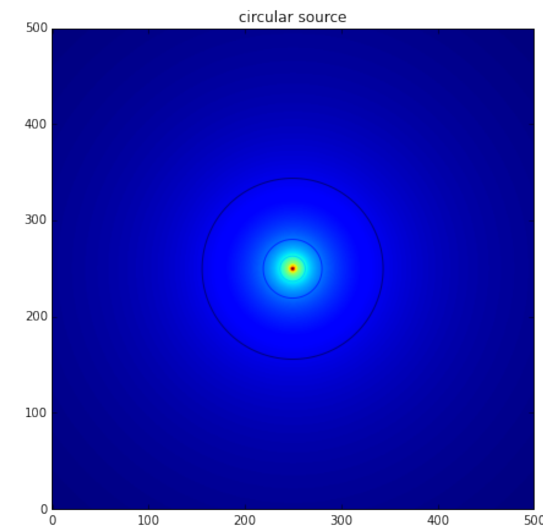

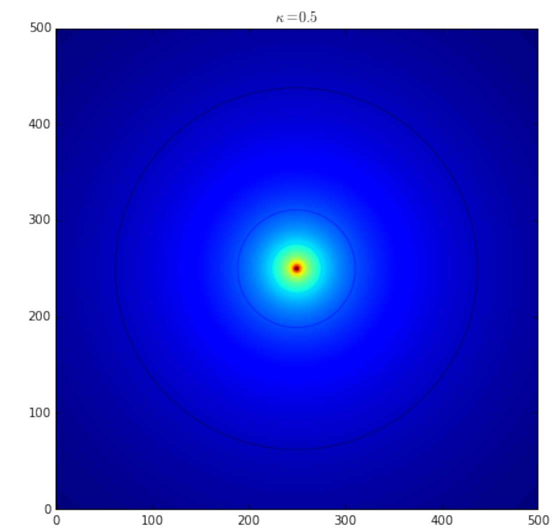

- Now, assume that $\gamma_1 > 0$ and $\gamma_2 = 0$. How is the image distorted?

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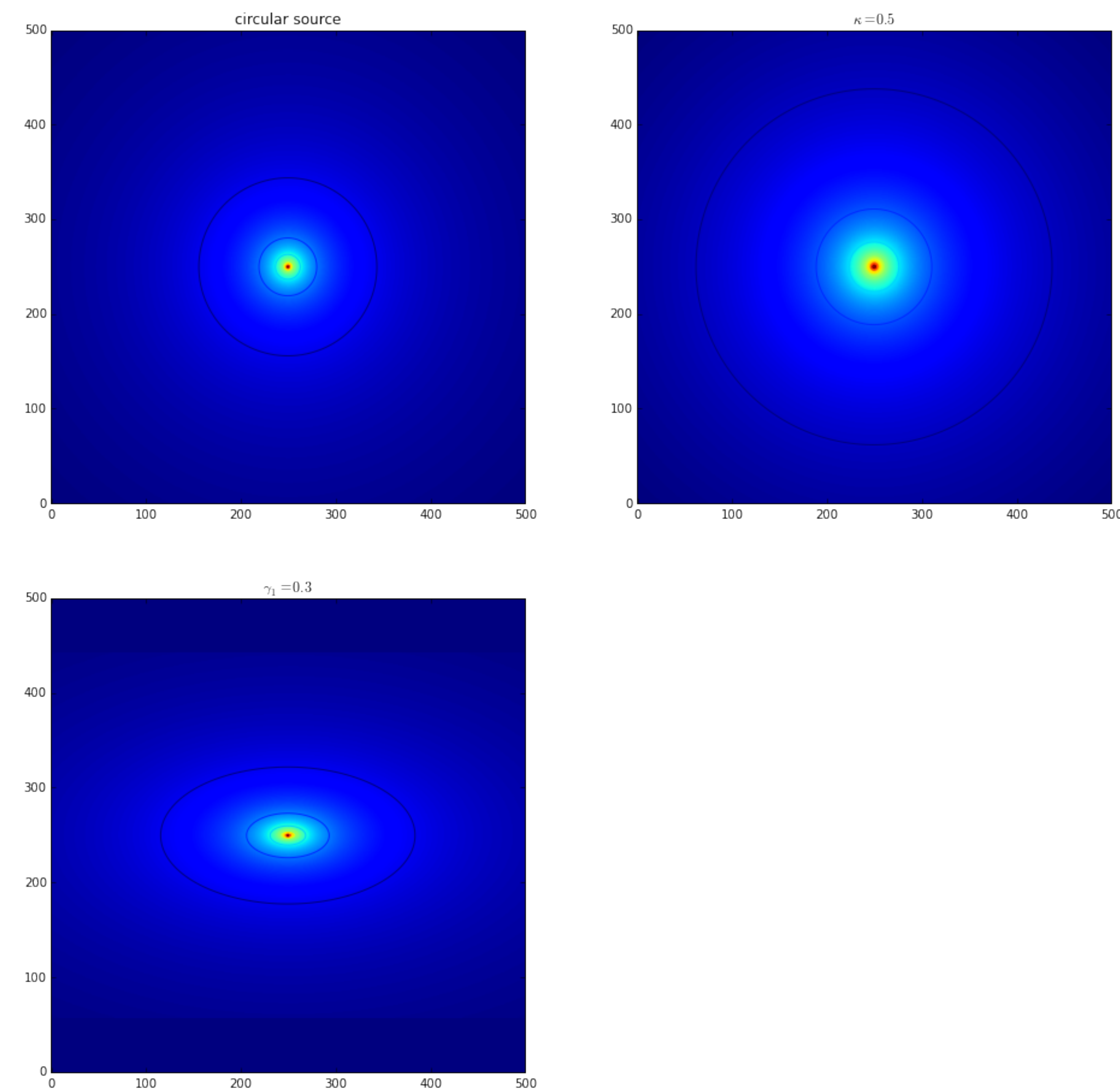

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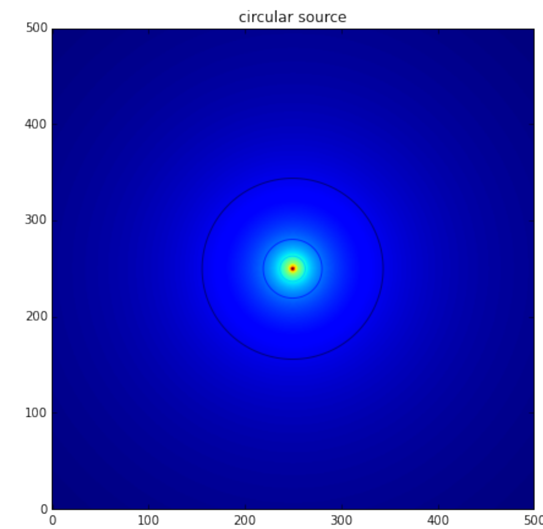

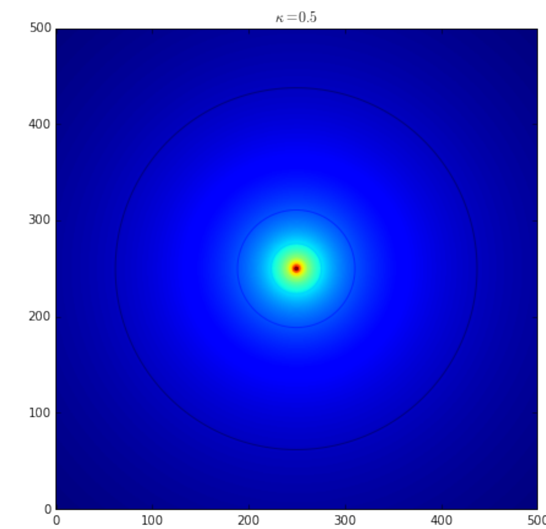

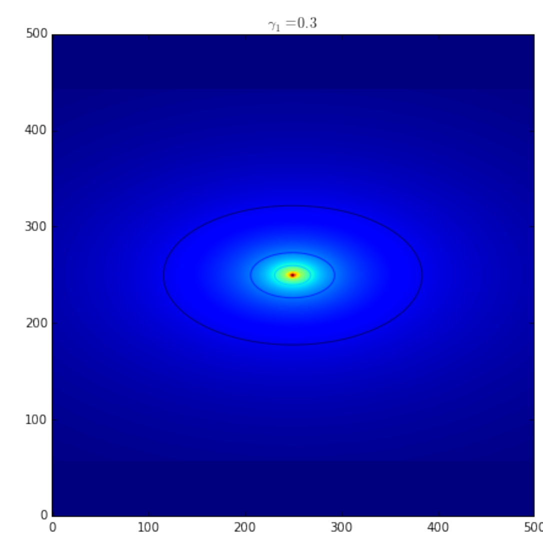

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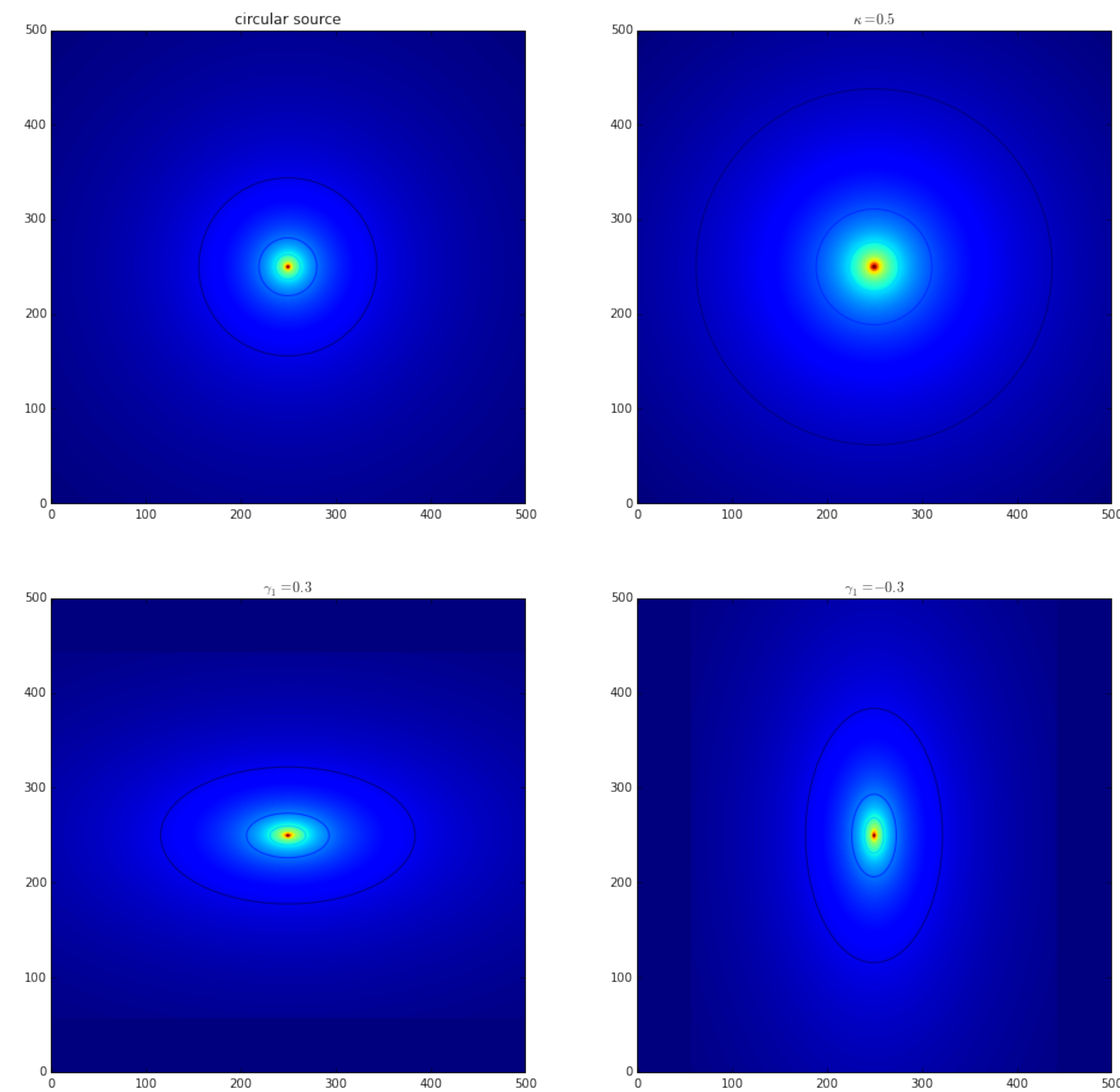

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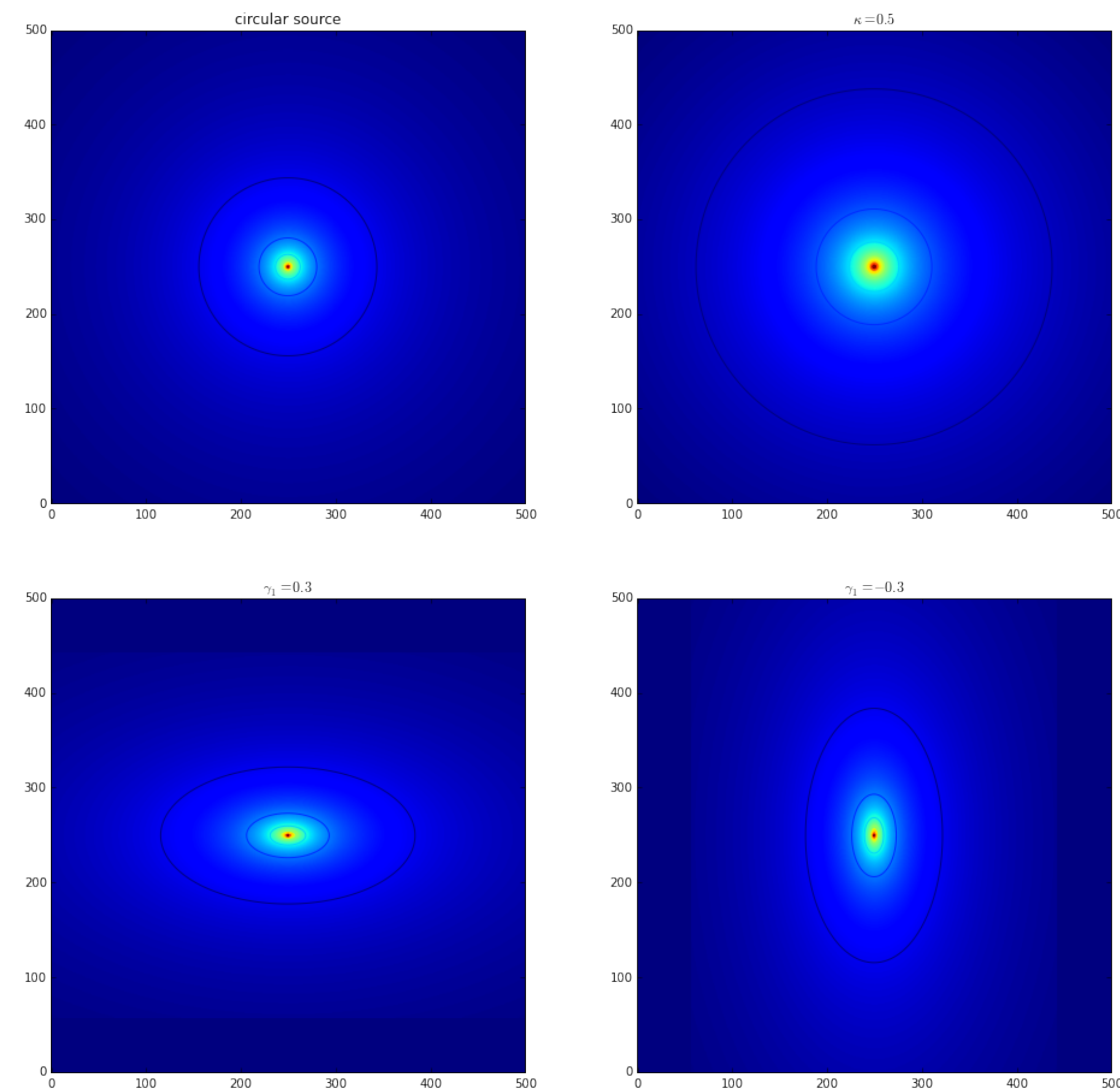

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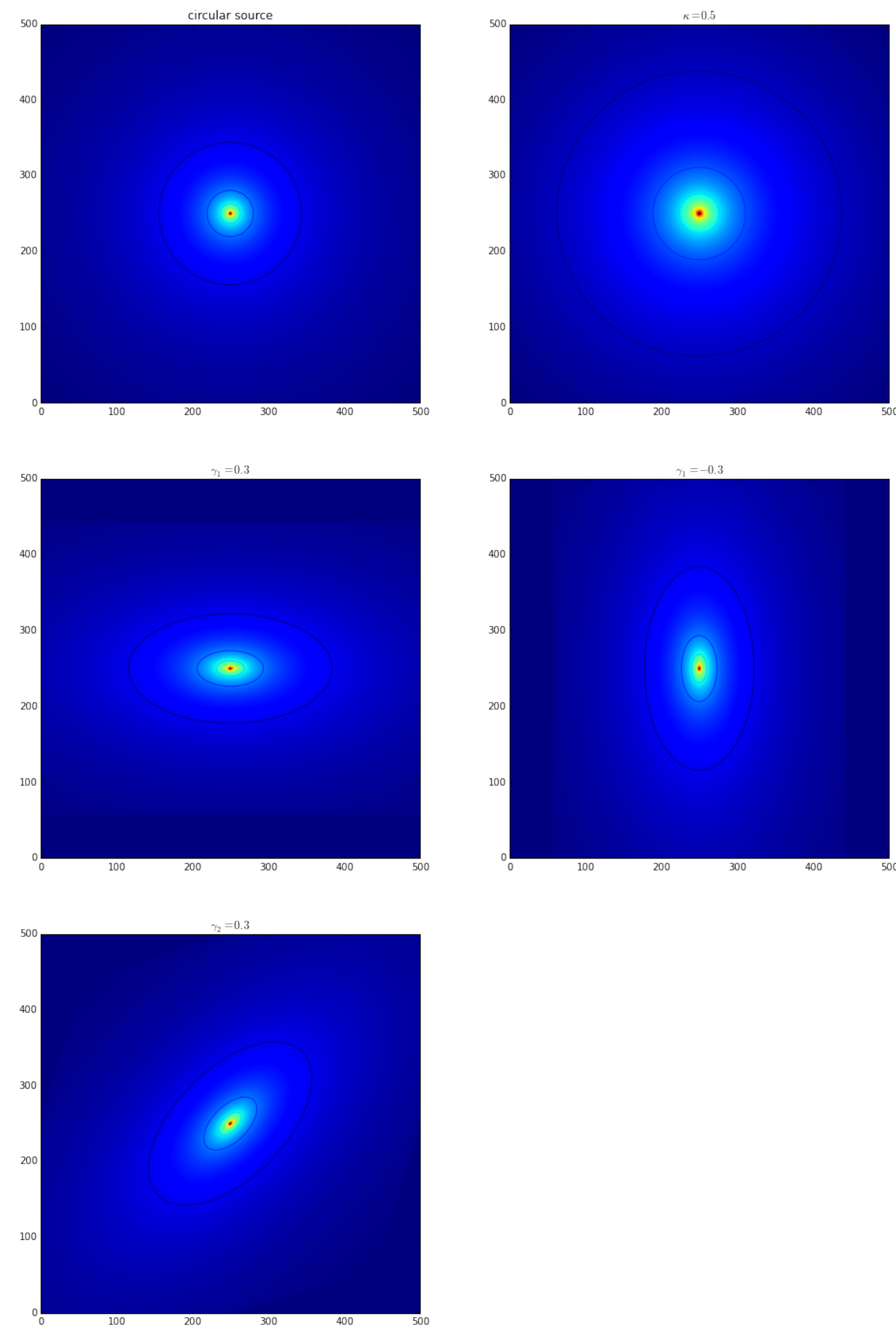

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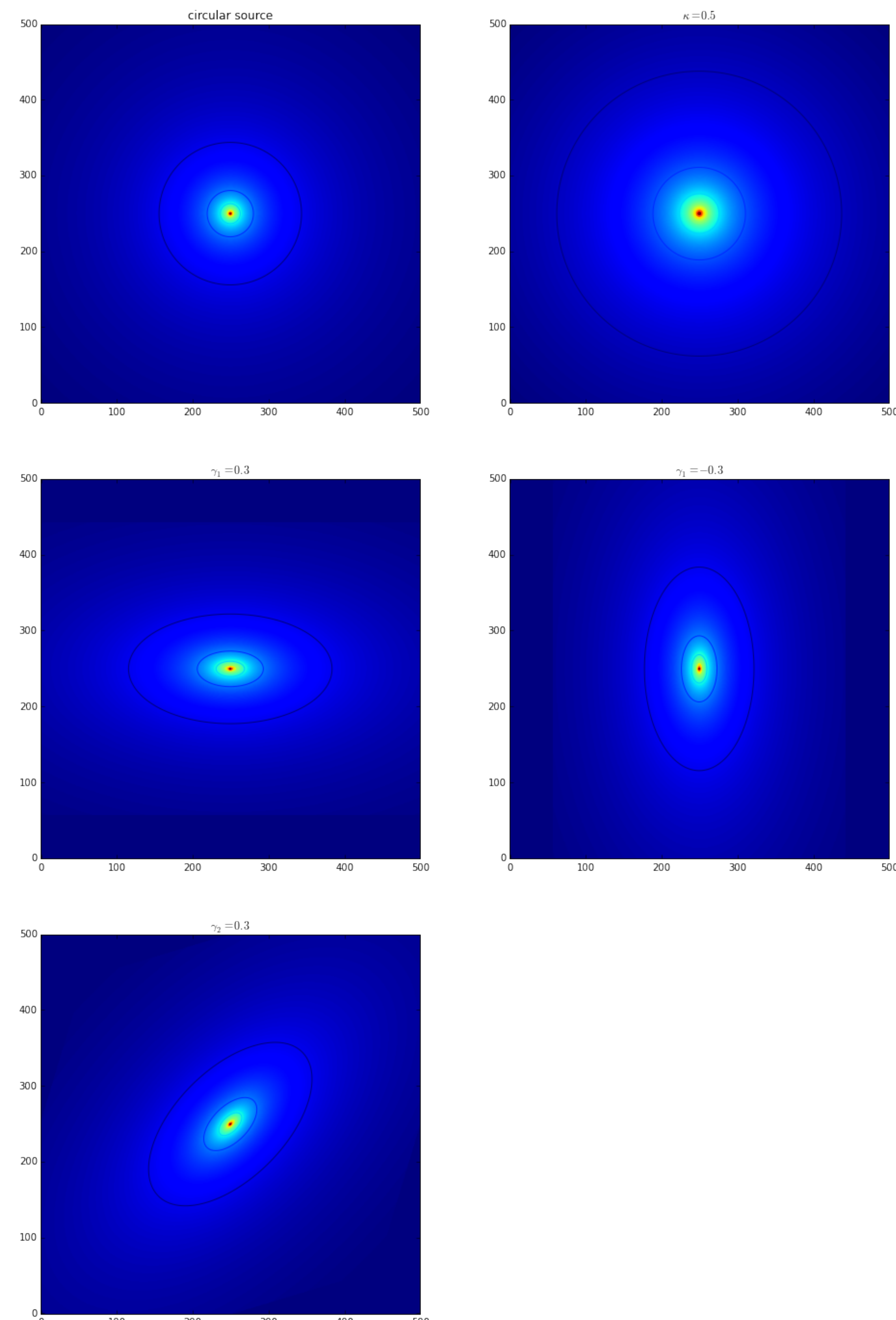

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- Let’s set $\gamma_1 = 0$. How is the image distorted if $\gamma_2 > 0$?
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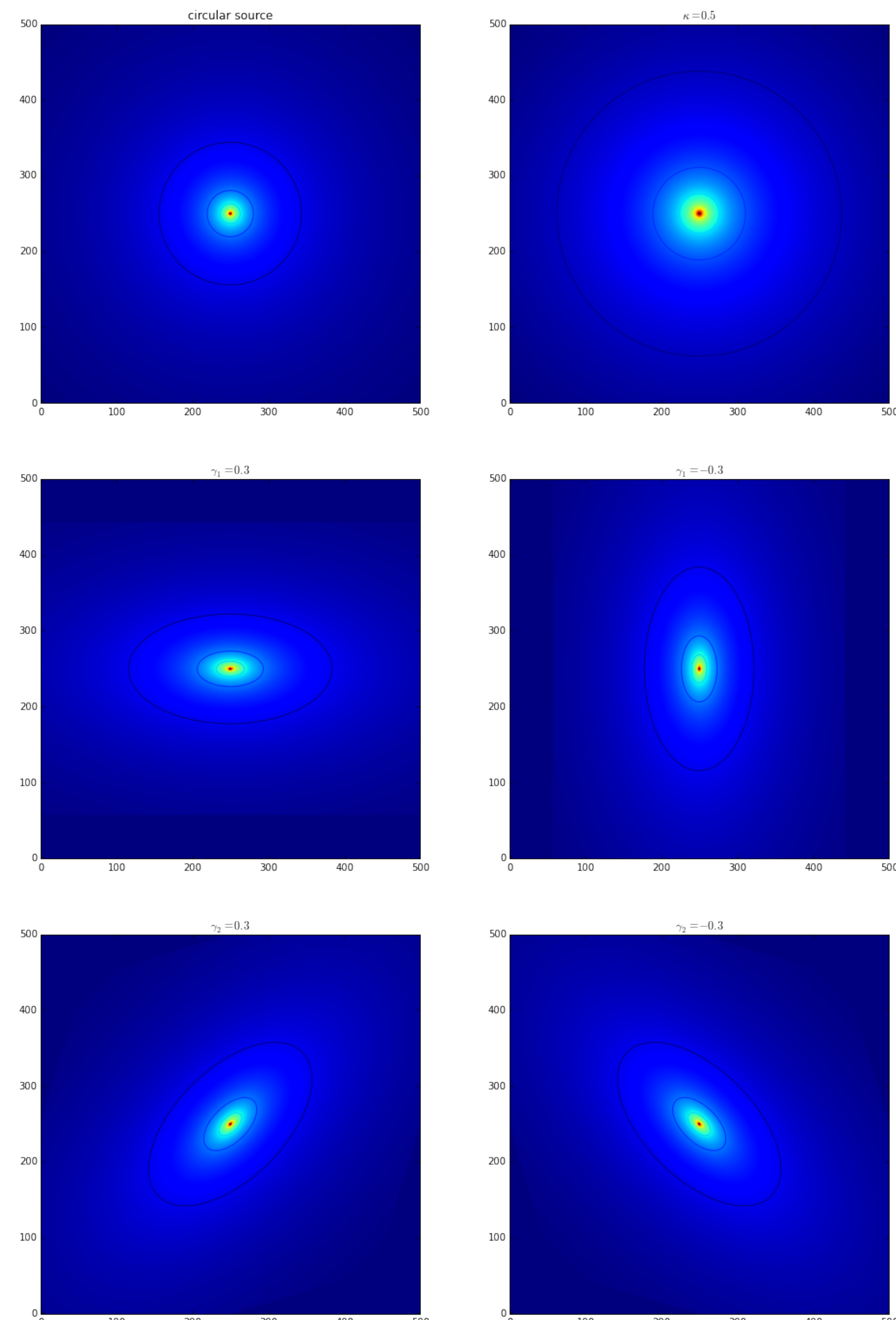

SHEAR DISTORTIONS