GRAVITATIONAL LENSING

LECTURE 14

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CONTENTS

- astrometric microlensing

WHAT IS ASTROMETRIC MICROLENSING?

- during a microlensing event, the two images of the source cannot be resolved ($\theta_E \sim 1$mas)
- their positions and the magnifications change as a function of time
- in particular, the image forming outside the Einstein ring dominates, in terms of flux for most of the time
- what an observer will see is one source at the light centroid, which will move as a function of time depending on where the two images form and on how much flux they emit

THE EQUATIONS

$$
\begin{aligned}
x_{\pm,\parallel} &= \frac{1}{2}(1 \pm Q) y_{\parallel} \\
x_{\pm,\perp} &= \frac{1}{2}(1 \pm Q) y_{\perp}
\end{aligned}
$$

$$
Q = \frac{\sqrt{y^2+4}}{y}
$$

$$
\begin{aligned}
\mu_{\pm}(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\
&= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\
&= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right)
\end{aligned}
$$

$$
\vec{x}_c = \frac{\vec{x}_+ |\mu_+| + \vec{x}_- |\mu_-|}{|\mu_+| + |\mu_-|}
$$

$$
\delta \vec{x}_c = \vec{x}_c - \vec{y}
$$

LIGHT CENTROID SHIFT AMPLITUDE

$$\delta x_c = \frac{\frac{1}{4}\left[(y+\sqrt{y^2+4})\left(1+\frac{y^2+2}{y\sqrt{y^2+4}}\right)-(y-\sqrt{y^2+4})\left(1-\frac{y^2+2}{y\sqrt{y^2+4}}\right)\right]}{\frac{y^2+2}{y\sqrt{y^2+4}}} - y$$

$$= \frac{\frac{1}{4}\left(y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}+\frac{y^2+2}{y}-y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}-\frac{y^2+2}{y}\right)}{\frac{y^2+2}{y\sqrt{y^2+4}}} - y$$

$$= \frac{y}{y^2+2}.$$

Given the sign, the shift points in the same direction of y.

Note that $y \gg \sqrt{2}$, $\delta x_c \approx \frac{1}{y}$

Thus, the shift decreases relatively slow with y… remember the scaling of μ?

LIGHT CENTROID SHIFT AMPLITUDE

$$\begin{aligned}\delta x_c &= \frac{\frac{1}{4}\left[(y+\sqrt{y^2+4})\left(1+\frac{y^2+2}{y\sqrt{y^2+4}}\right)-(y-\sqrt{y^2+4})\left(1-\frac{y^2+2}{y\sqrt{y^2+4}}\right)\right]}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\ &= \frac{\frac{1}{4}\left(y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}+\frac{y^2+2}{y}-y+\sqrt{y^2+4}+\frac{y^2+2}{\sqrt{y^2+4}}-\frac{y^2+2}{y}\right)}{\frac{y^2+2}{y\sqrt{y^2+4}}}-y \\ &= \frac{y}{y^2+2}.\end{aligned}$$

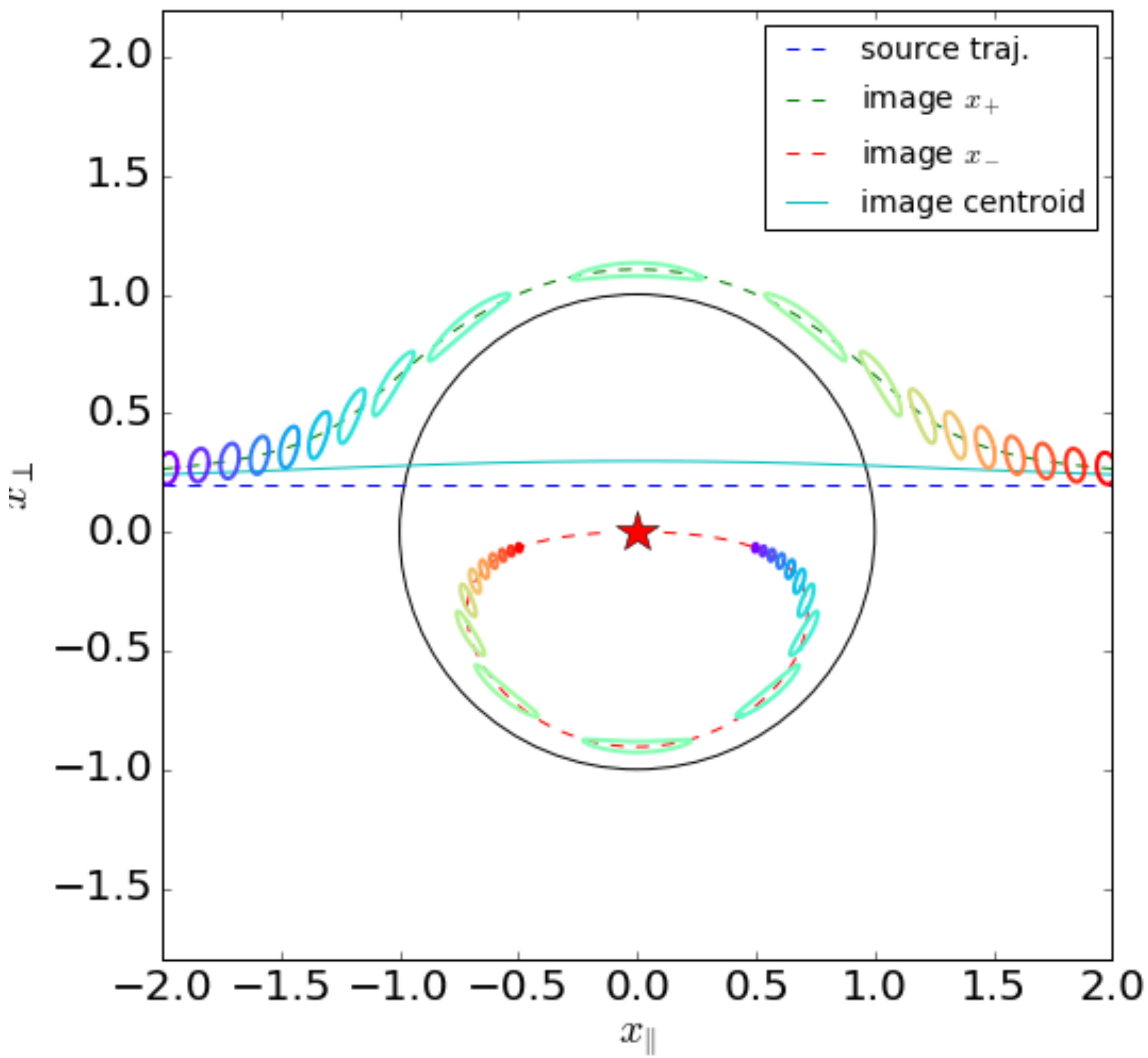
In addition

$$\frac{d(\delta x_c)}{dy} = \frac{2-y^2}{(y^2+2)^2}$$

the shift is maximum at $y=\sqrt{2}$, $\delta x_c = \delta x_{c,max} = (2\sqrt{2})^{-1}$

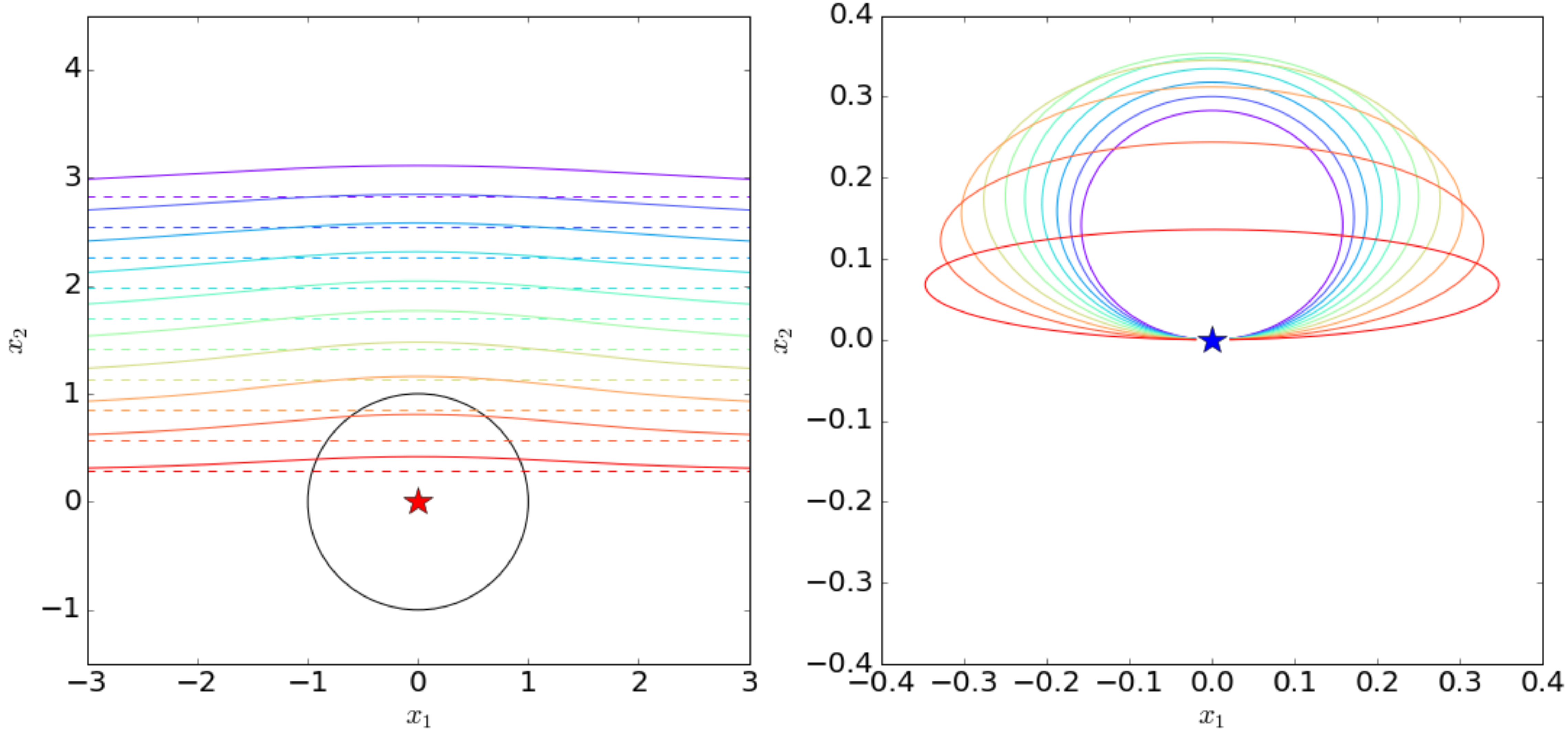
This corresponds to $\sim 0.354\theta_E$ *which is above the accuracy of GAIA*

EXAMPLE



source traj.
image x_+
image x_-
image centroid
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
$x_\perp$
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
$x_\parallel$

WHAT IS THE PATH OF THE CENTROID SHIFT WITH RESPECT TO THE UNPERTURBED SOURCE?



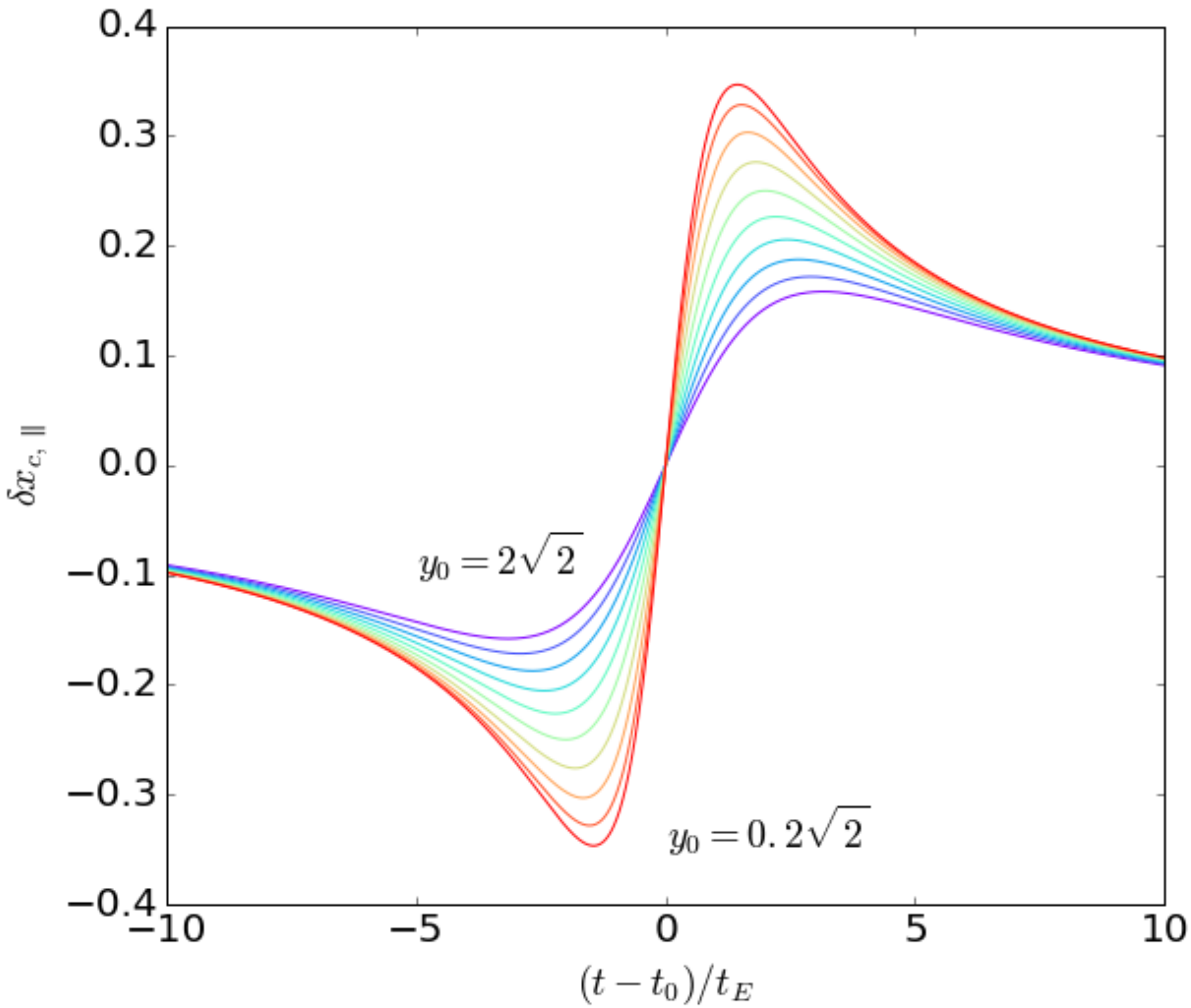
HOW DO WE EXPLAIN THIS PATH?

We can decompose the shift into the components parallel and perpendicular to the motion of the source:

$$\delta x_{c,\parallel} = \frac{y_\parallel}{y^2+2} = \frac{(t-t_0)/t_E}{[(t-t_0)/t_E]^2+y_0^2+2}$$

$$\delta x_{c,\perp} = \frac{y_\perp}{y^2+2} = \frac{y_0}{[(t-t_0)/t_E]^2+y_0^2+2}$$

RESULTS



Antisymmetric!

Taking the derivative:

$$\frac{d(\delta x_{c,\parallel})}{dt} = \frac{y_0^2 + 2 - [(t - t_0)/t_E]^2}{\{[(t - t_0)/t_E]^2 + y_0^2 + 2\}^2}$$

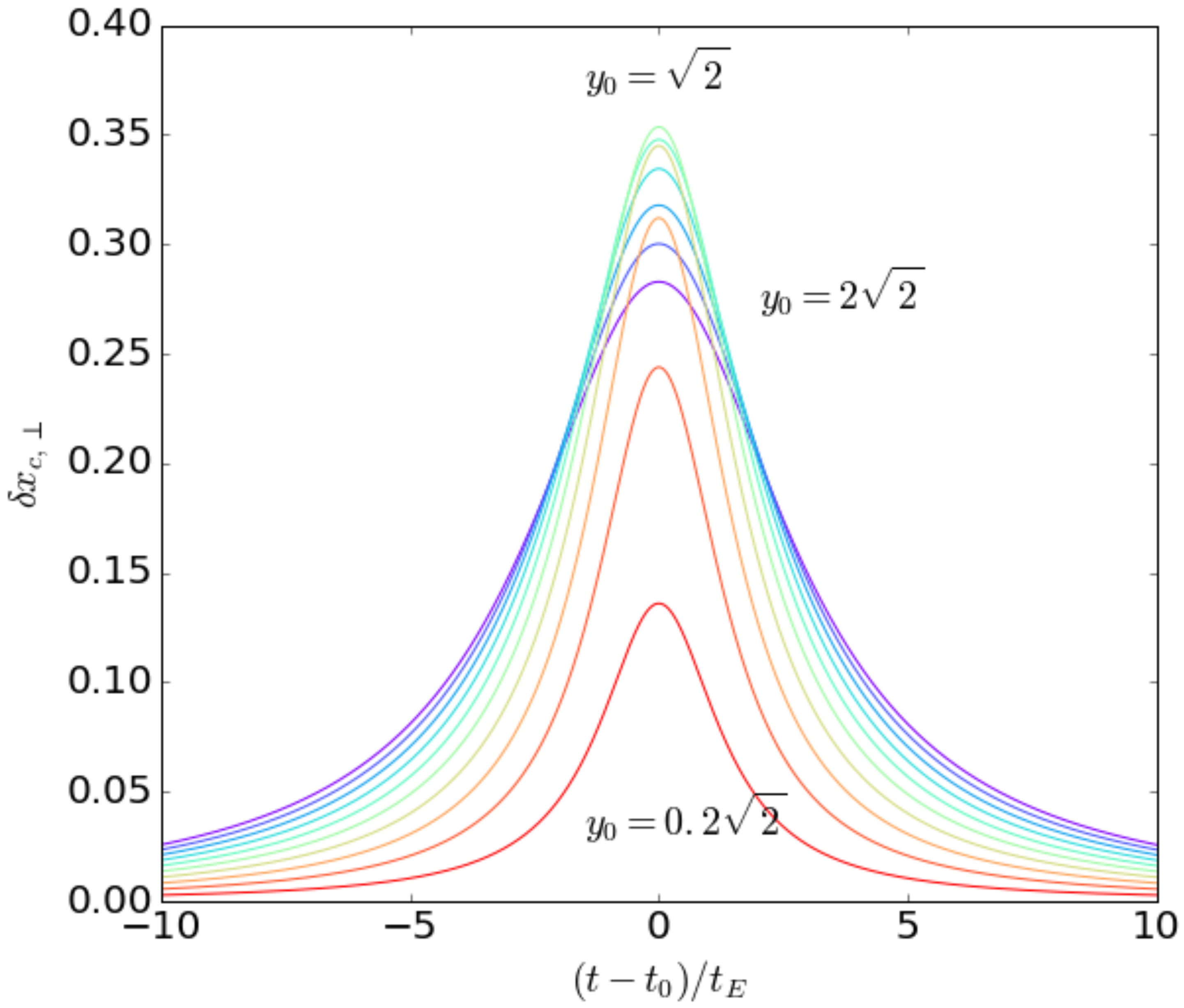
$$(t - t_0)/t_E = \pm\sqrt{y_0^2 + 2}$$

$$\delta x_{c,\parallel} = \pm\frac{1}{2\sqrt{y_0^2 + 2}}$$

For small y0:

$(t - t_0)/t_E \approx \pm\sqrt{2}$ and $\delta x_{c,\parallel} \approx \delta x_{c,max}$.

RESULTS



One maximum in t=t0

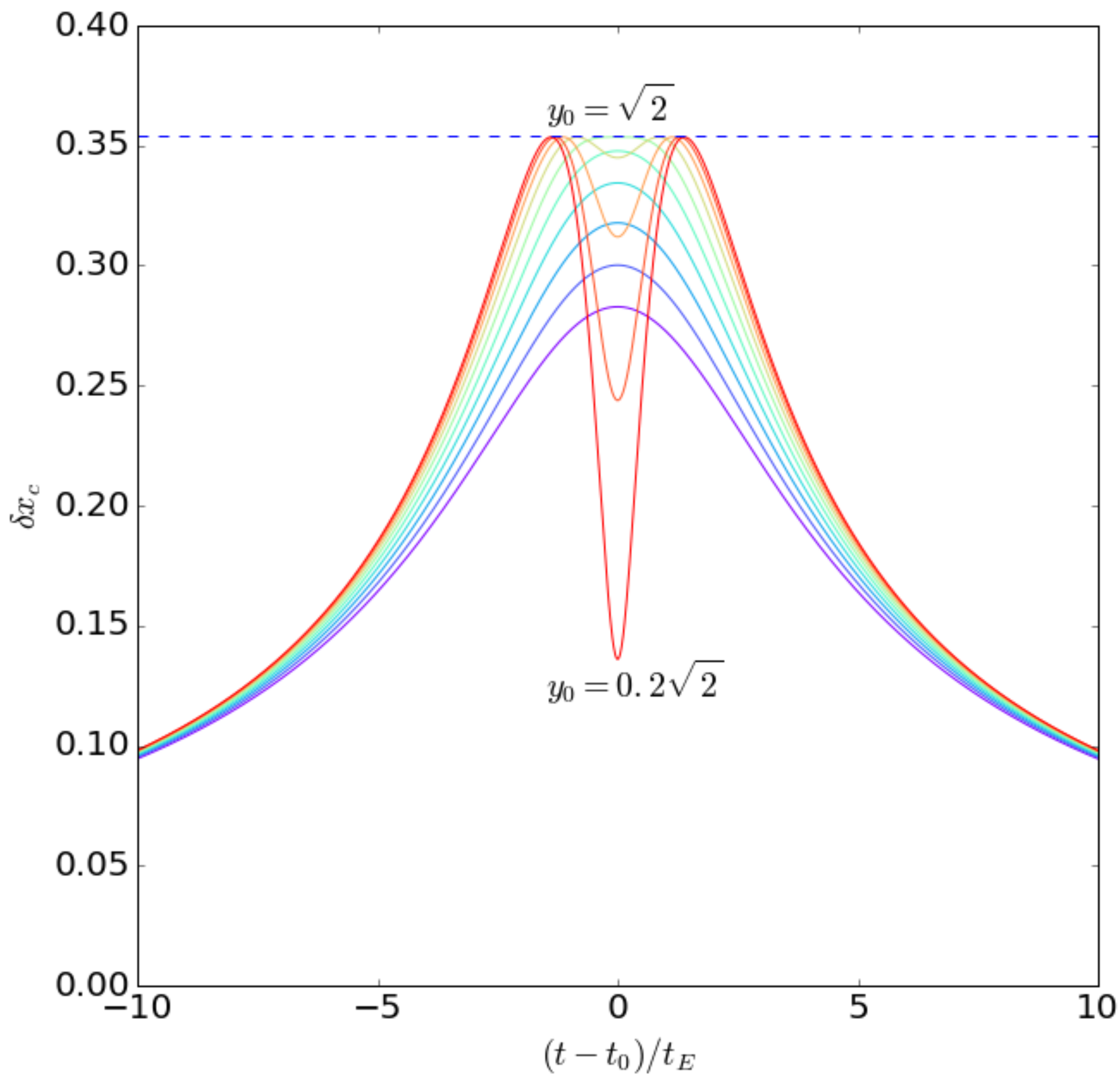
$$\delta x_{c,\perp,max} = \frac{y_0}{y_0^2 + 2}$$

the peak is the highest for

$$y_0 = \sqrt{2},$$

$$\delta x_{c,max}$$

RESULTS



$$\frac{d(\delta x_c)}{dp} = 2p\frac{2-y_0^2-p^2}{2\sqrt{y_0^2+p^2}(y_0^2+p^2+2)^2}$$

For small y0: two maxima and one minimum

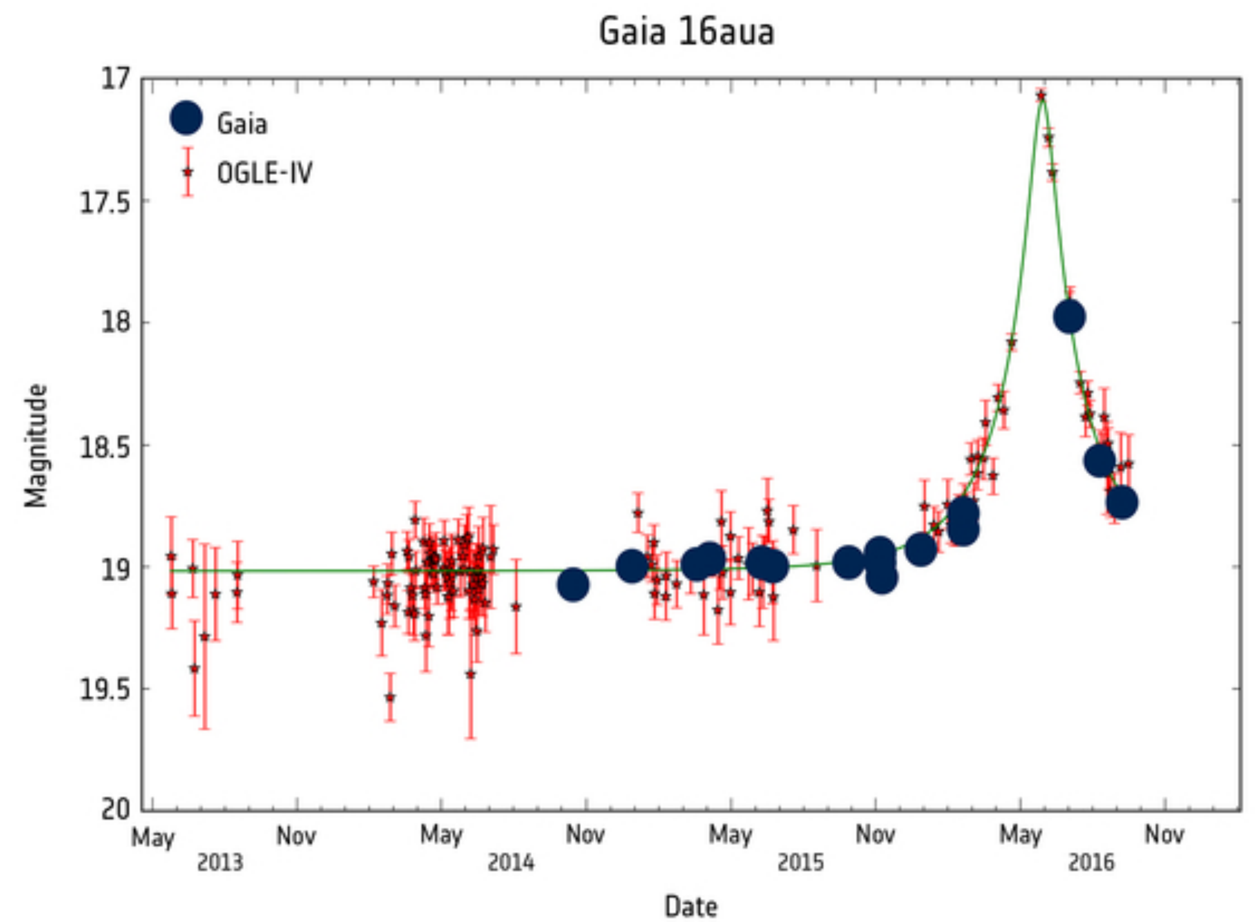
In this case, the shift is mainly parallel to the motion of the source

For large y0: one maximum

In this case, the shift is mainly perpendicular to the motion of the source

GAIA AND MICROLENSING

- GAIA has made the first photometric microlensing detection recently…
- Will it be able to detect the astrometric effect too?



GAIA + OGLE IV