# GRAVITATIONAL LENSING

## 9 - MICROLENSING LIGHT CURVES

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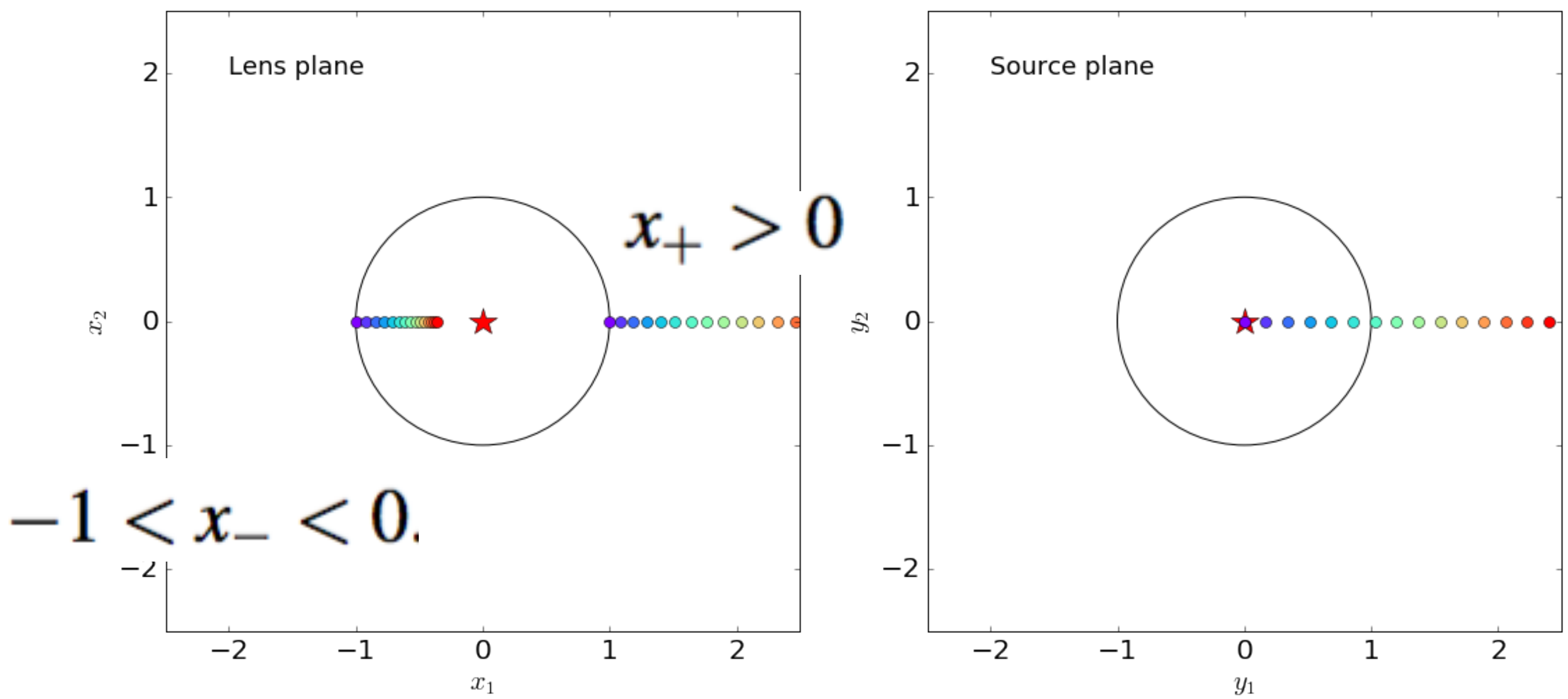
# PROPERTIES OF THE IMAGES

*From the last lesson*



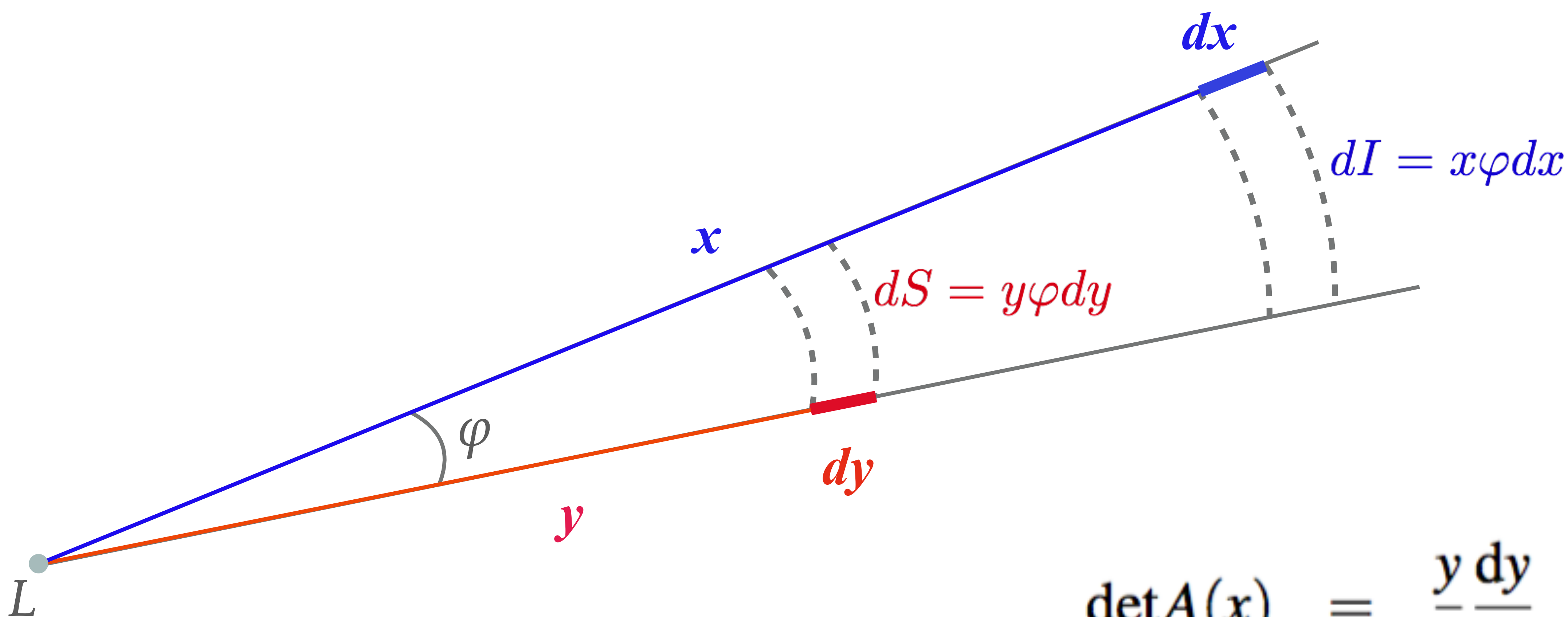

$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right]$$

*One of the images is internal to the Einstein radius, the other is external*

*For $y=0$, the image is a full ring:* $x_\pm = \pm 1$

*This is the* ***Einstein ring***

# MAGNIFICATION




$$\det A(x) = \frac{y}{x}\frac{\mathrm{d}y}{\mathrm{d}x}$$

# CRITICAL LINES AND CAUSTICS

*From the lens equation, it follows that:*

$$\lambda_t(x) = \frac{y}{x} = \left(1 - \frac{1}{x^2}\right)$$
$$\lambda_r(x) = \frac{\mathrm{d}y}{\mathrm{d}x} = \left(1 + \frac{1}{x^2}\right) .$$

*The second eigenvalue is always positive (no critical line). The first is zero on the circle*

$$x^2 = 1$$

*Thus, the Einstein ring is the tangential critical line! The corresponding caustic is a point at y=0*

# IMAGE MAGNIFICATION

*Clearly,*

$$\begin{aligned} \det A(x) &= \frac{y}{x}\frac{\mathrm{d}y}{\mathrm{d}x} \\ \lambda_t(x) &= \frac{y}{x} = \left(1 - \frac{1}{x^2}\right) \\ \lambda_r(x) &= \frac{\mathrm{d}y}{\mathrm{d}x} = \left(1 + \frac{1}{x^2}\right). \end{aligned}$$

$$\Rightarrow \quad \mu(x) = \left(1 - \frac{1}{x^4}\right)^{-1}$$

# IMAGE PARITY

*Note that:*

$$y > 0 \Rightarrow \begin{array}{l} x_+ > 0 \\ x_- < 0 \end{array}$$

$$\mu_t = \frac{x}{y} \Rightarrow \begin{array}{l} \mu_t(x_+) > 0 \\ \mu_t(x_-) < 0 \end{array}$$

$$\mu_r = \frac{dx}{dy} > 0$$

$\Rightarrow$ *Thus the parity of the images is different!*

# SOURCE MAGNIFICATION

*Let's compute now the source magnification. This is the sum of the magnifications of the two images*

$$x_{\pm} = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right]$$

$$\begin{aligned} \frac{x}{y} &= \frac{1}{2}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right) \\ \frac{\mathrm{d}x}{\mathrm{d}y} &= \frac{1}{2}\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) . \end{aligned}$$

*Thus the magnifications at the two image positions are*

$$\begin{aligned} \mu_{\pm}(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\ &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\ &= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right) \end{aligned}$$

# SOURCE MAGNIFICATION

*The total magnification is obtained by summing the magnifications of the images:*

$$\begin{aligned}\mu_{\pm}(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right)\\ &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right)\\ &= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right)\end{aligned}$$

$$\Rightarrow \quad \mu(y) = \mu_+(y) + |\mu_-(y)| = \frac{y^2+2}{y\sqrt{y^2+4}}$$

*The sum of the signed magnification is one!*

*We can take a power series of the magnification to see that* $\mu \propto 1 + 2/y^4$ for $y \to \infty$.

*Thus, the magnification drops quickly as the source moves away from the lens!*

# SOURCE MAGNIFICATION

*In addition:*

$$\left|\frac{\mu_+}{\mu_-}\right| = \frac{1+\frac{y^2+2}{y\sqrt{y^2+4}}}{\frac{y^2+2}{y\sqrt{y^2+4}}-1} = \frac{y^2+2+y\sqrt{y^2+4}}{y^2+2-y\sqrt{y^2+4}}$$

$$\frac{1}{2}\left(y+\sqrt{y^2+4}\right)^2 = y^2+2+y\sqrt{y^2+4}$$

$$\frac{1}{2}\left(y-\sqrt{y^2+4}\right)^2 = y^2+2-y\sqrt{y^2+4}$$

$$\left|\frac{\mu_+}{\mu_-}\right| = \left(\frac{y+\sqrt{y^2+4}}{y-\sqrt{y^2+4}}\right)^2 = \left(\frac{x_+}{x_-}\right)^2.$$

*Power series at infinity:*

$$\left|\frac{\mu_+}{\mu_-}\right| \propto y^4$$

*As we move the source away from the lens, the image in $x_+$ dominates the flux budget very soon.*

$$\lim_{y\to\infty}\mu_- = 0$$

$$\lim_{y\to\infty}\mu_+ = 1$$

# A SOURCE ON THE EINSTEIN RING

*For a source on the Einstein ring:*

$$x_{\pm} = \frac{1 \pm \sqrt{5}}{2} \qquad \mu_{\pm} = \left[1 - \left(\frac{2}{1 \pm \sqrt{5}}\right)^4\right]^{-1}$$

*Therefore:* $$\mu = |\mu_+| + |\mu_-| = 1.17 + 0.17 = 1.34$$

$$\Delta m = -2.5 \log \mu \sim 0.3$$

*Given how quickly the magnification drops by moving the source away from the lens, we can assume that only sources within the Einstein radius are magnified in a significant way.*

*For this reason, the circle within the Einstein radius is assumed to be the cross section for microlensing.*

# SIZE OF THE EINSTEIN RADIUS

From the last lesson

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{\mathrm{LS}}}{D_{\mathrm{L}}D_{\mathrm{S}}}} \qquad D \equiv \frac{D_{\mathrm{L}}D_{\mathrm{S}}}{D_{\mathrm{LS}}}$$

$$\begin{aligned}\theta_E &\approx (10^{-3})''\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D}{10\mathrm{kpc}}\right)^{-1/2},\\ &\approx 1''\left(\frac{M}{10^{12}M_\odot}\right)^{1/2}\left(\frac{D}{\mathrm{Gpc}}\right)^{-1/2},\end{aligned}$$

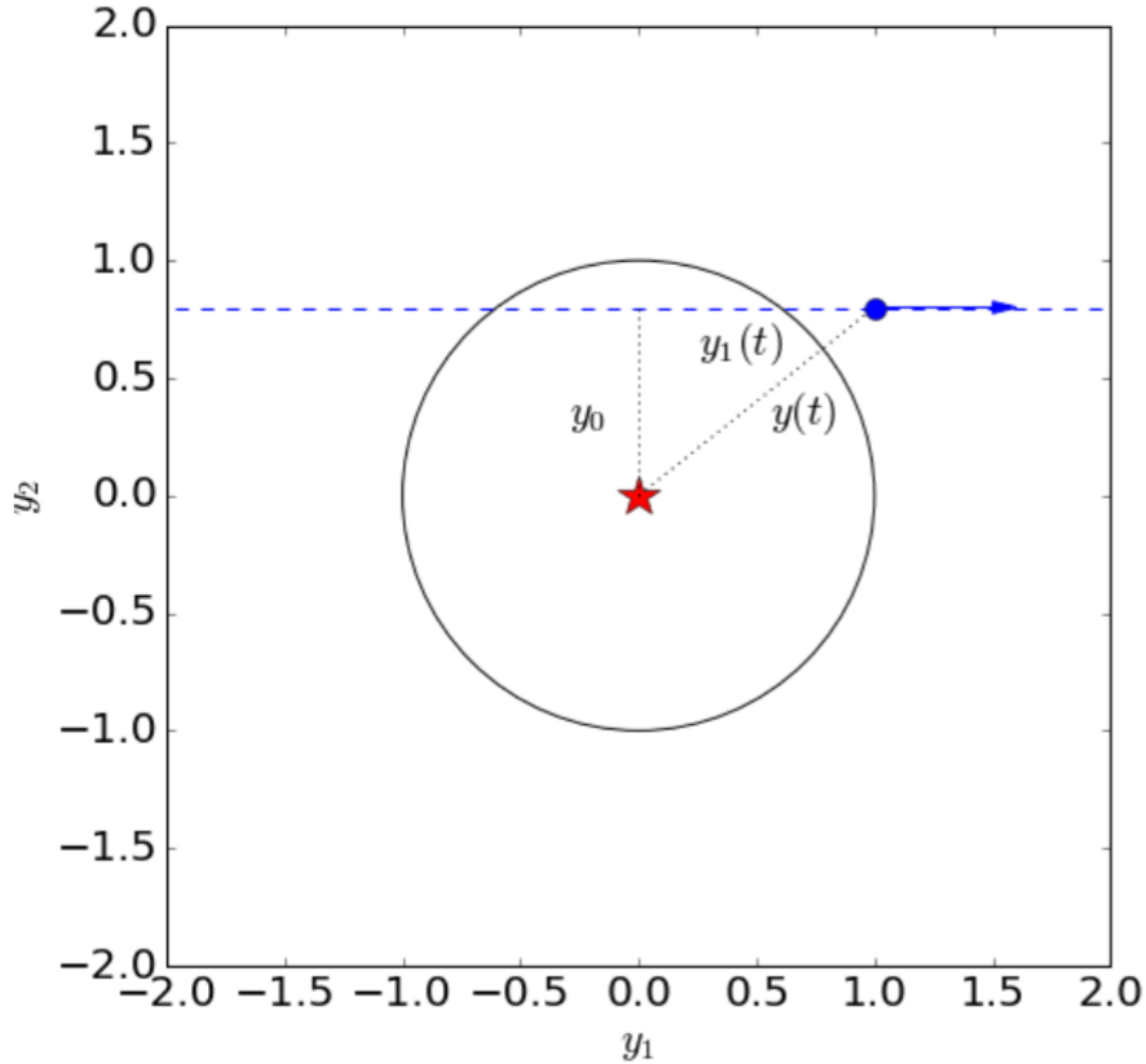
*For a star like the sun within the MW, the Einstein radius is of the order of milli-arcseconds!*

# MICROLENSING LIGHT CURVE

- typical Einstein radii for lenses in the MW are $\sim$1 mas
- thus, the image separation is too small to resolve the images
- magnification is small also for relatively close pairs of lenses and sources
- how to detect a microlensing event?

# MICROLENSING LIGHT CURVE

- stars (including the sun) rotate around the galactic center
- rotation is differential (i.e. speed depends on distance)
- this introduces a relative velocity between the lenses and the sources (either in the bulge or in the MCs)
- this causes the relative distance between the sources and the lenses to vary over time…



# MICROLENSING LIGHT CURVE

*Assume a linear trajectory of the source relative to the lens, with impact parameter $y_0$*

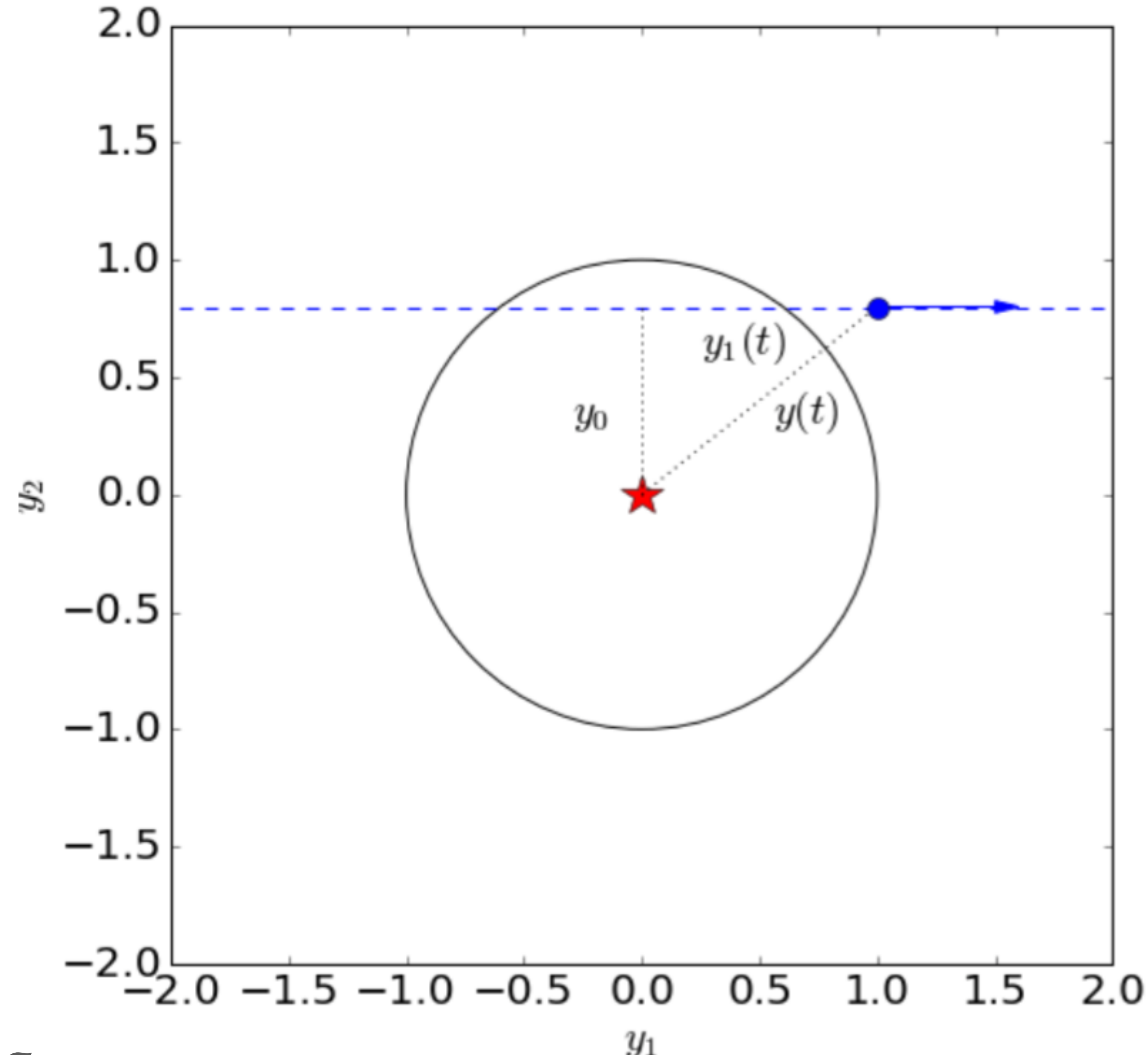
*Assume also constant transverse velocity $v$:*

$$y_1(t) = \frac{v(t-t_0)}{D_L\theta_E}$$

*We can define a characteristic time of the event:*

$$t_E = \frac{D_L\theta_E}{v} = \frac{\theta_E}{\mu_{rel}}$$

*This is the* ***Einstein radius crossing time***



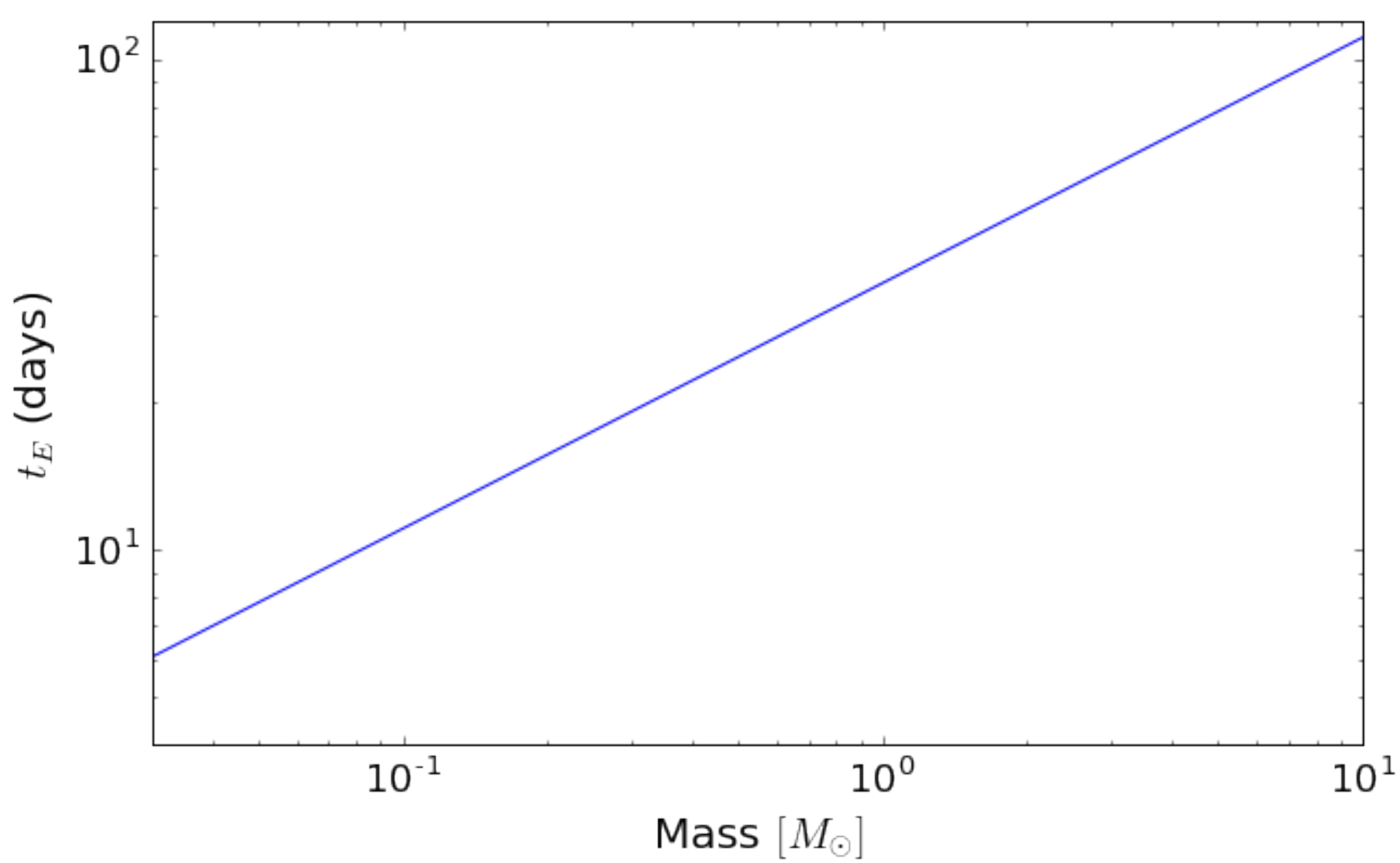
# MICROLENSING LIGHT CURVE

*Given the definition of Einstein radius*

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}}$$

*The order of magnitude of the $t_E$ is*

$$t_E \approx 19\text{ days}\sqrt{4\frac{D_{\rm L}}{D_{\rm S}}\left(1-\frac{D_{\rm L}}{D_{\rm S}}\right)}\left(\frac{D_{\rm S}}{8\text{kpc}}\right)^{1/2}\left(\frac{M}{0.3M_\odot}\right)^{1/2}\left(\frac{v}{200\text{km}/s}\right)^{-1}$$



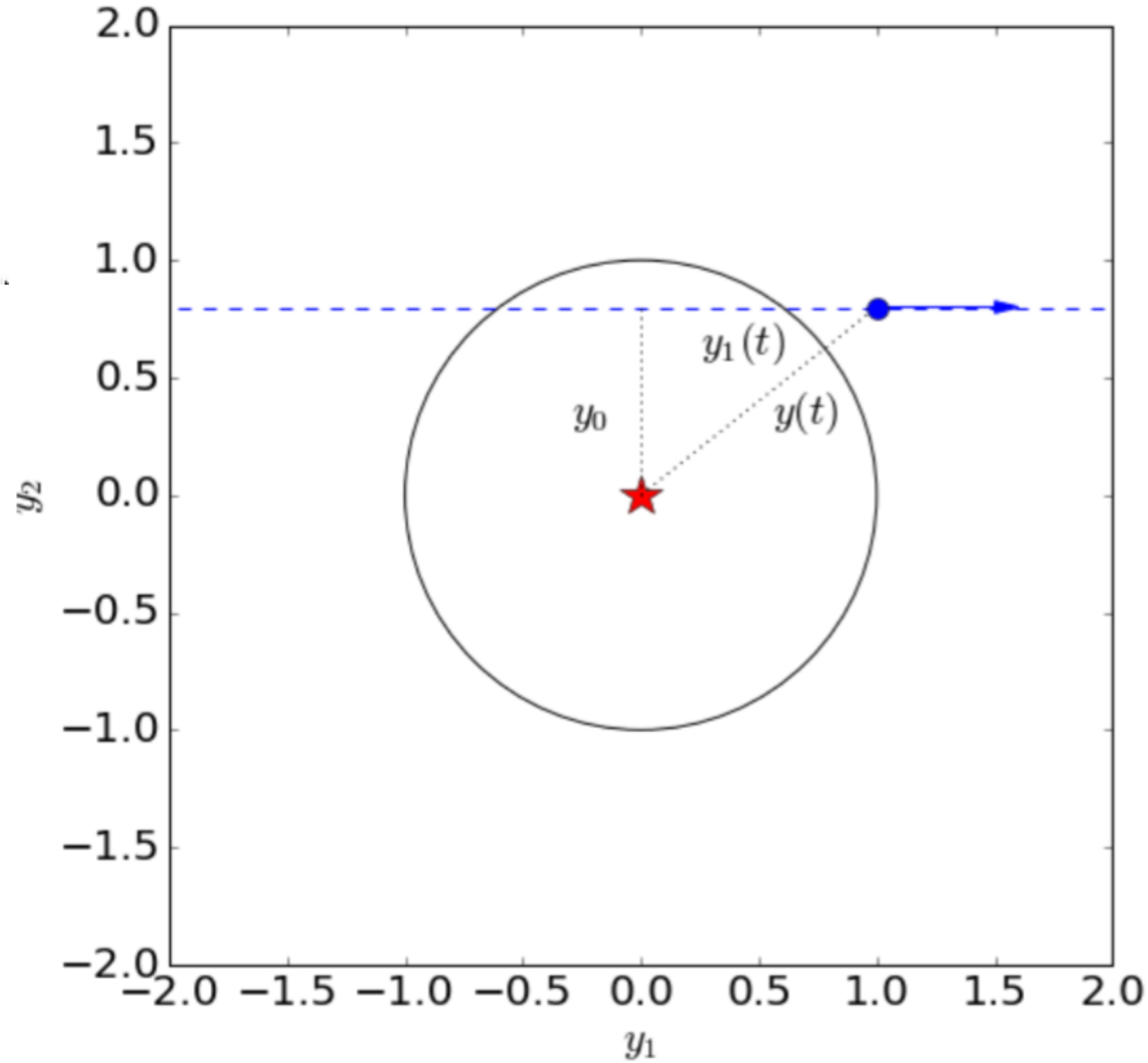
# MICROLENSING LIGHT CURVE

*We obtain*

$$y_1(t) = \frac{v(t-t_0)}{D_L\theta_E} \quad \Rightarrow \quad y_1(t) = \frac{(t-t_0)}{t_E}$$

*Thus:*

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t-t_0)^2}{t_E^2}}$$



# EXAMPLE OF STANDARD LIGHT CURVE