

GRAVITATIONAL LENSING

8 – MICROLENSING

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MICROLENSING

- Microlensing is a lensing regime which include effects produced by a broad range of masses: from planets to ensembles of stars
- given the small sizes of the lens, these are (to first-order) assimilated to point masses.
- microlensing effects are mostly detectable and searched within our own galaxy, in particular by monitoring huge amounts of stars in the bulge of the MW or in the Magellanic Clouds
- nevertheless, microlensing effects are important also in extragalactic lenses. Small masses in distant galaxies, for example, introduce perturbations to the lensing signal of their hosts

THE POINT MASS LENS MODEL

- The deflection angle of the point mass lens was derived in the first lecture
- the lensing potential can be readily derived

$$\hat{\vec{\alpha}} = \frac{4GM}{c^2} \frac{\vec{\xi}}{|\vec{\xi}|^2} = \frac{4GM}{c^2 D_L} \frac{\vec{\theta}}{|\vec{\theta}|^2}$$

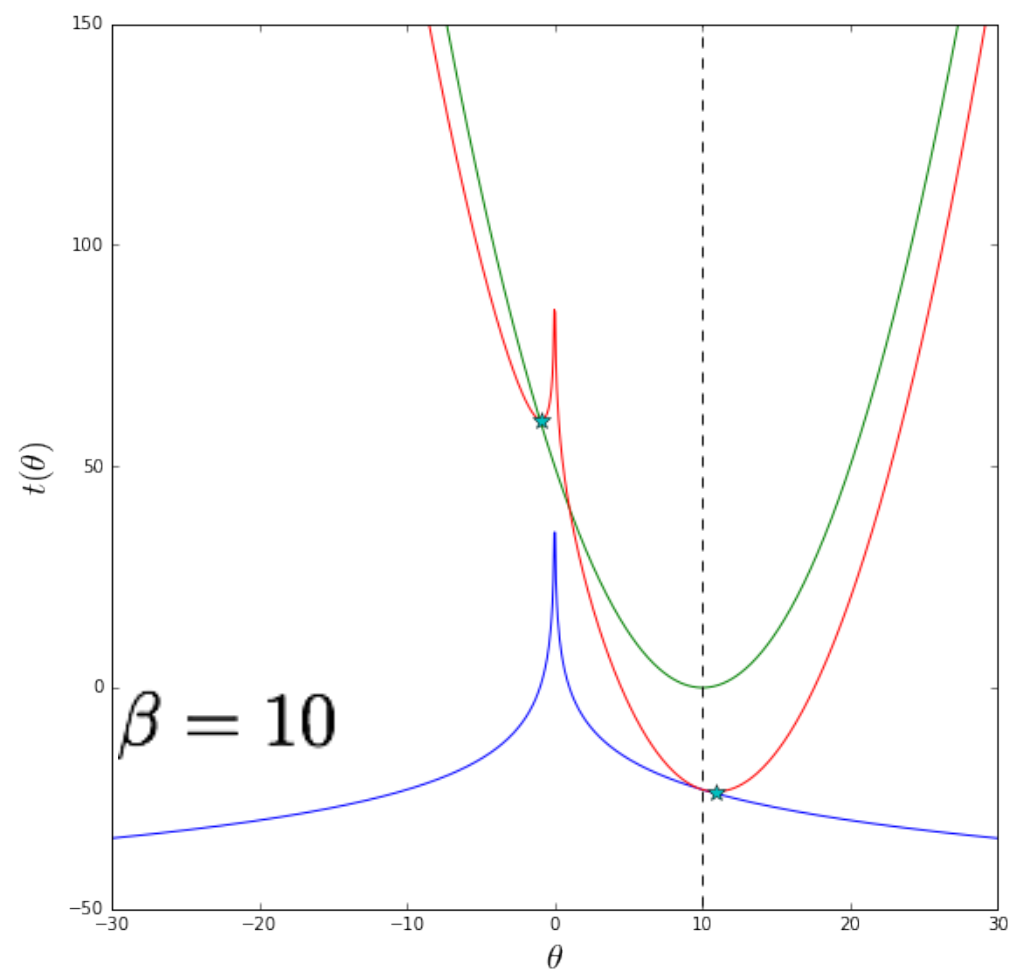
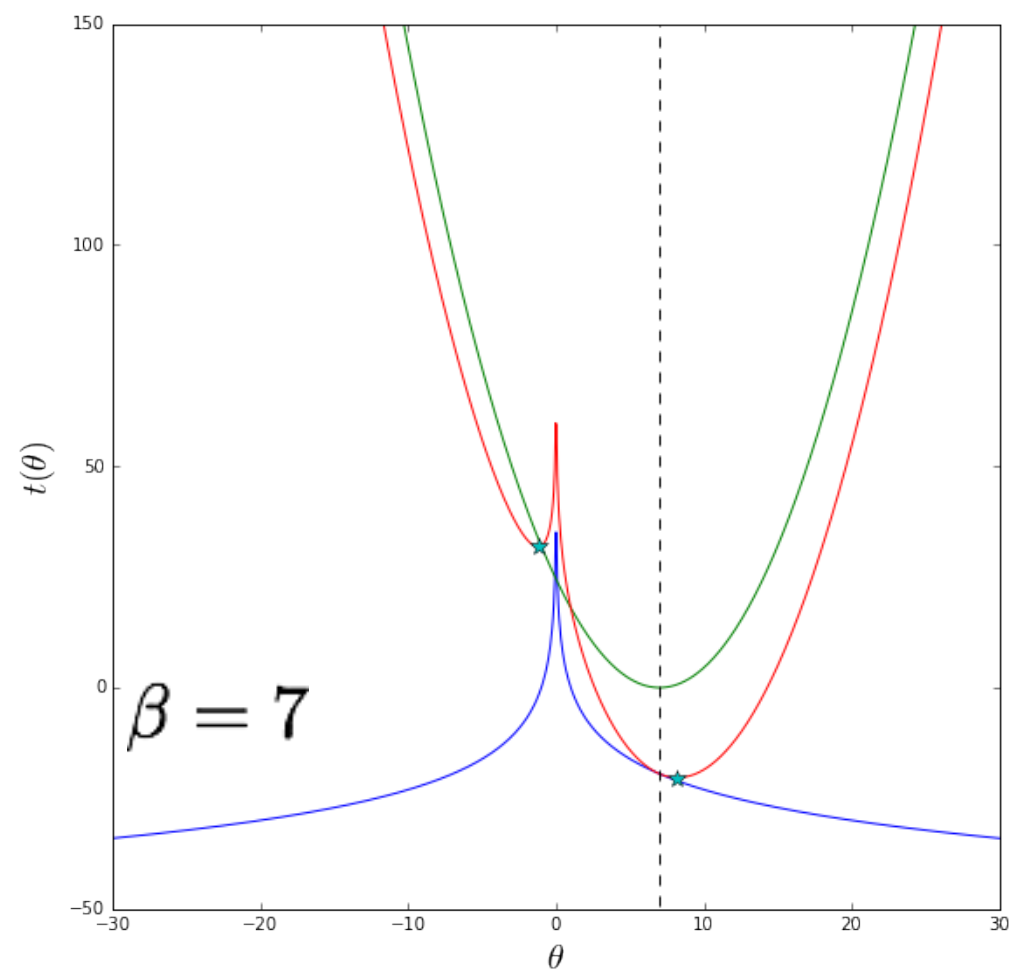
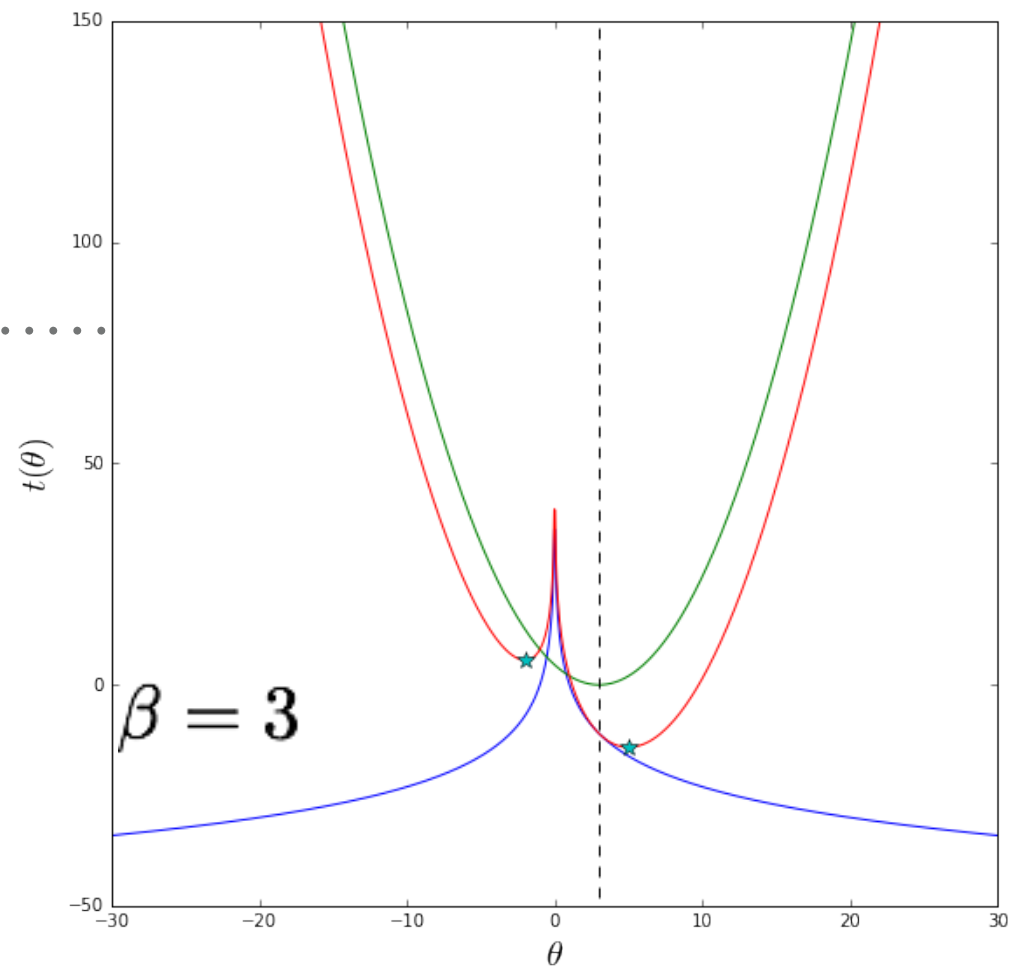
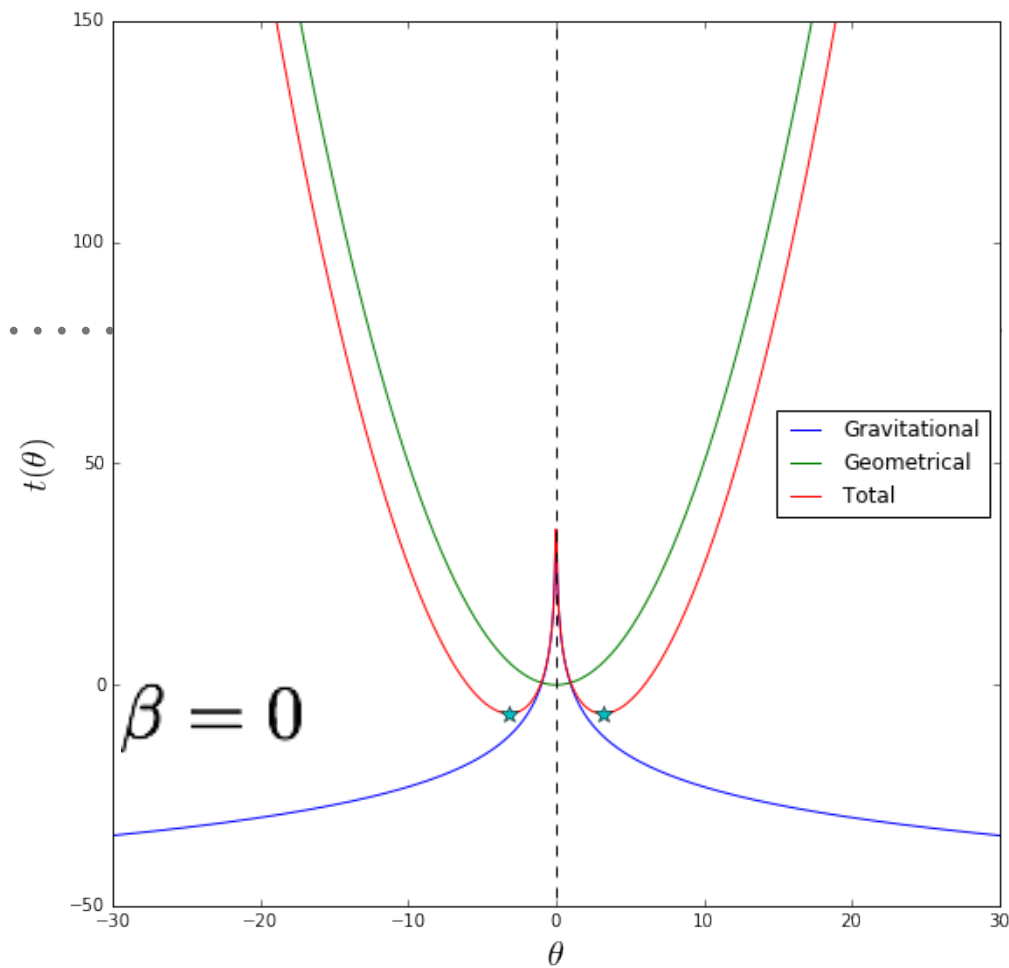
$$\vec{\alpha} = \frac{D_{LS}}{D_S} \hat{\vec{\alpha}} = \vec{\nabla} \hat{\Psi}$$

$$\nabla \ln |\vec{x}| = \frac{\vec{x}}{|\vec{x}|^2}$$



$$\hat{\Psi}(\vec{\theta}) = \frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S} \ln |\vec{\theta}|$$

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LENS EQUATION

- the deflection angle always points away from the lens
- given the symmetry of the lens, we can omit the vector notation in most equations
- the lens equation reads:
- this is clearly quadratic in θ
- so, for each source there are two images, whose positions can be determined by solving the lens equation

$$\hat{\alpha} = \frac{4GM}{c^2\xi} = \frac{4GM}{c^2D_L\theta}$$

$$\beta = \theta - \frac{4GM}{c^2D_L\theta} \frac{D_{LS}}{D_S}$$

SOLUTIONS OF THE LENS EQUATION

- we introduce the Einstein radius:
- by inserting into the lens equation:
- if we divide by θ_E , we obtain an a-dimensional form of the lens equation
- this is a very convenient way of writing the lens equation, because we get rid of all constants.

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}}$$

$$\beta = \theta - \frac{4GM}{c^2 D_L \theta} \frac{D_{LS}}{D_S}$$



$$\beta = \theta - \frac{\theta_E^2}{\theta}$$

$$y = x - \frac{1}{x}$$

SOLUTIONS OF THE LENS EQUATION

$$y = x - \frac{1}{x}$$

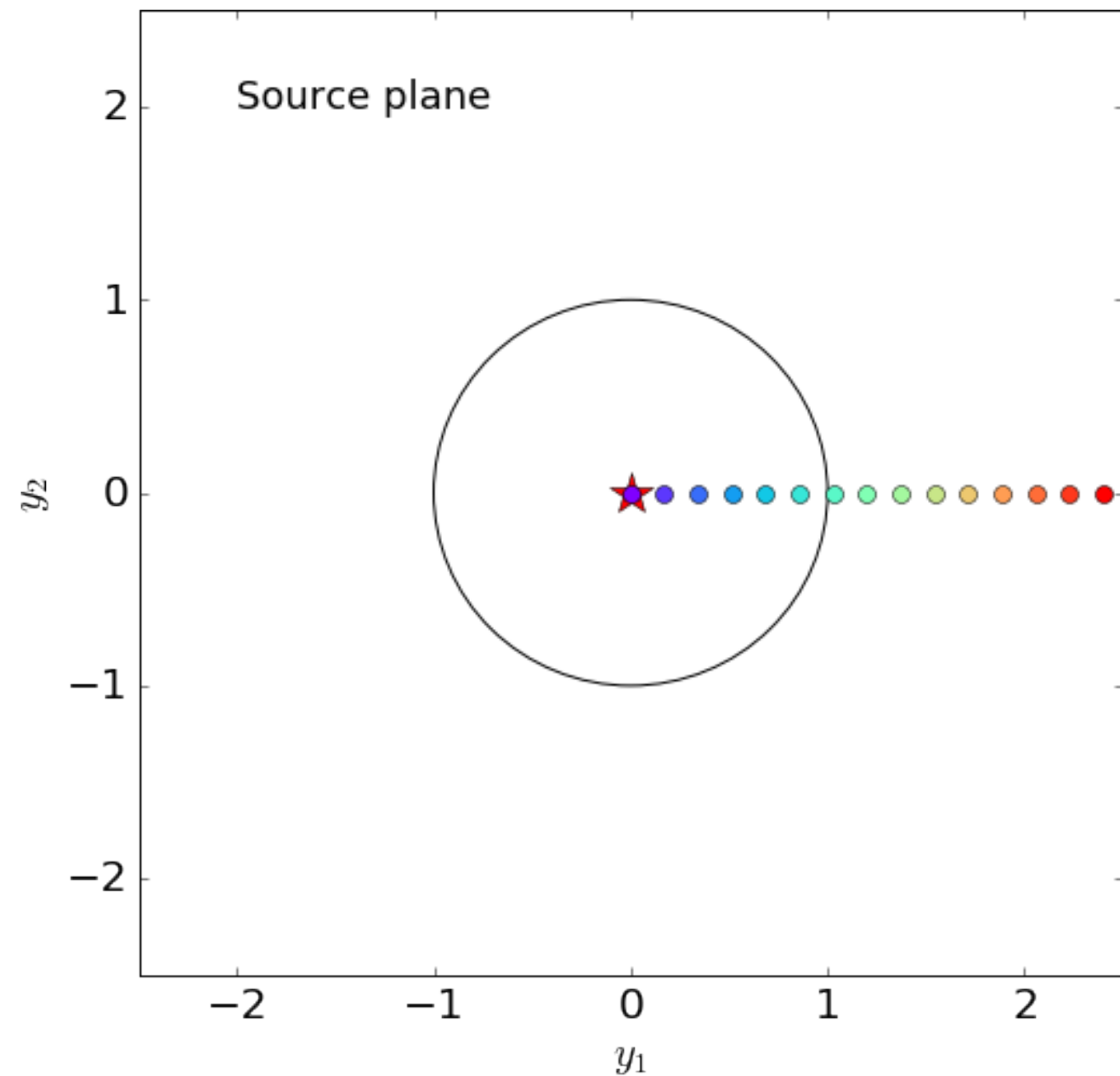
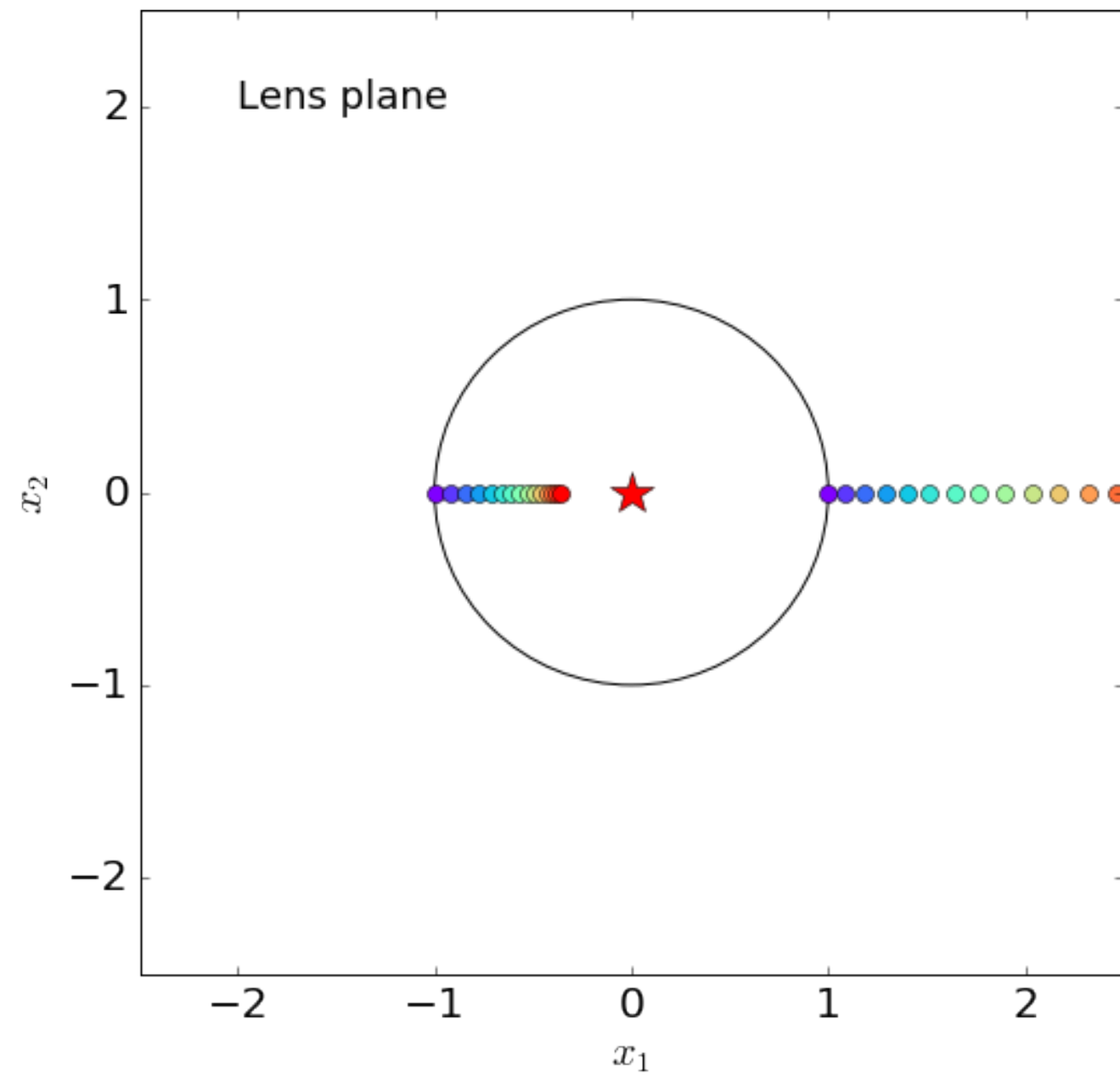


$$x^2 - xy - 1 = 0$$

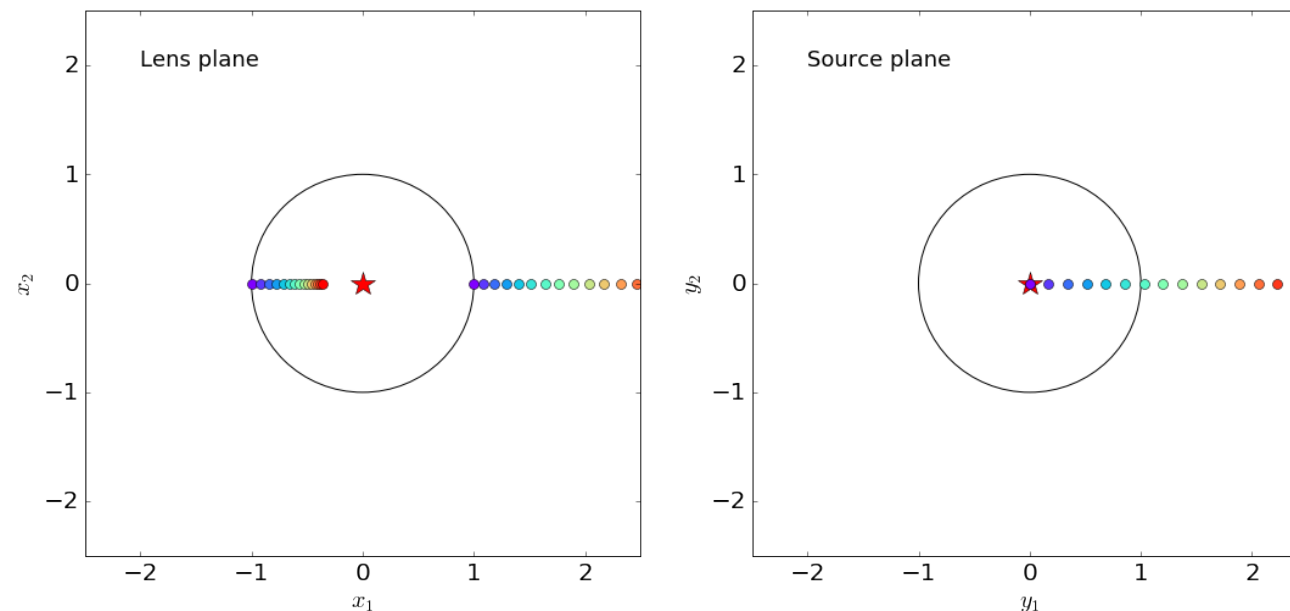


$$x_{\pm} = \frac{1}{2} \left[y \pm \sqrt{y^2 + 4} \right]$$

SOLUTIONS OF THE LENS EQUATION



PROPERTIES OF THE IMAGES



$$x_{\pm} = \frac{1}{2} \left[y \pm \sqrt{y^2 + 4} \right]$$

$$x_{+} > 0$$

$$-1 < x_{-} < 0.$$

One of the images is internal to the Einstein radius, the other is external

For $y=0$, the image is a full ring: $x_{\pm} = \pm 1$

*This is the **Einstein ring***

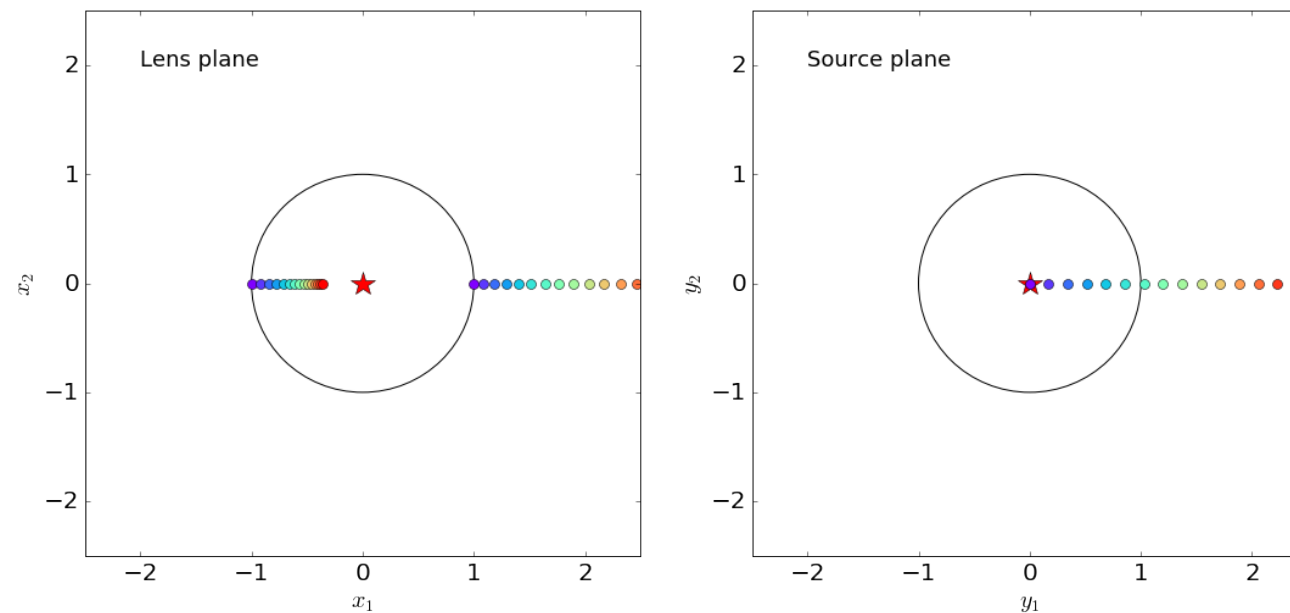
SIZE OF THE EINSTEIN RADIUS

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}} \qquad D \equiv \frac{D_L D_S}{D_{LS}}$$

$$\begin{aligned} \theta_E &\approx (10^{-3})'' \left(\frac{M}{M_\odot} \right)^{1/2} \left(\frac{D}{10 \text{kpc}} \right)^{-1/2}, \\ &\approx 1'' \left(\frac{M}{10^{12} M_\odot} \right)^{1/2} \left(\frac{D}{\text{Gpc}} \right)^{-1/2}, \end{aligned}$$

For a star like the sun within the MW, the Einstein radius is of the order of milli-arcseconds!

PROPERTIES OF THE IMAGES



$$\beta \rightarrow \infty, \quad \theta_- = x_- \theta_E \rightarrow 0, \quad \theta_+ = x_+ \theta_E \rightarrow \beta$$

For large angular distances between the lens and the source, one image approaches the lens, while the other follows the source.