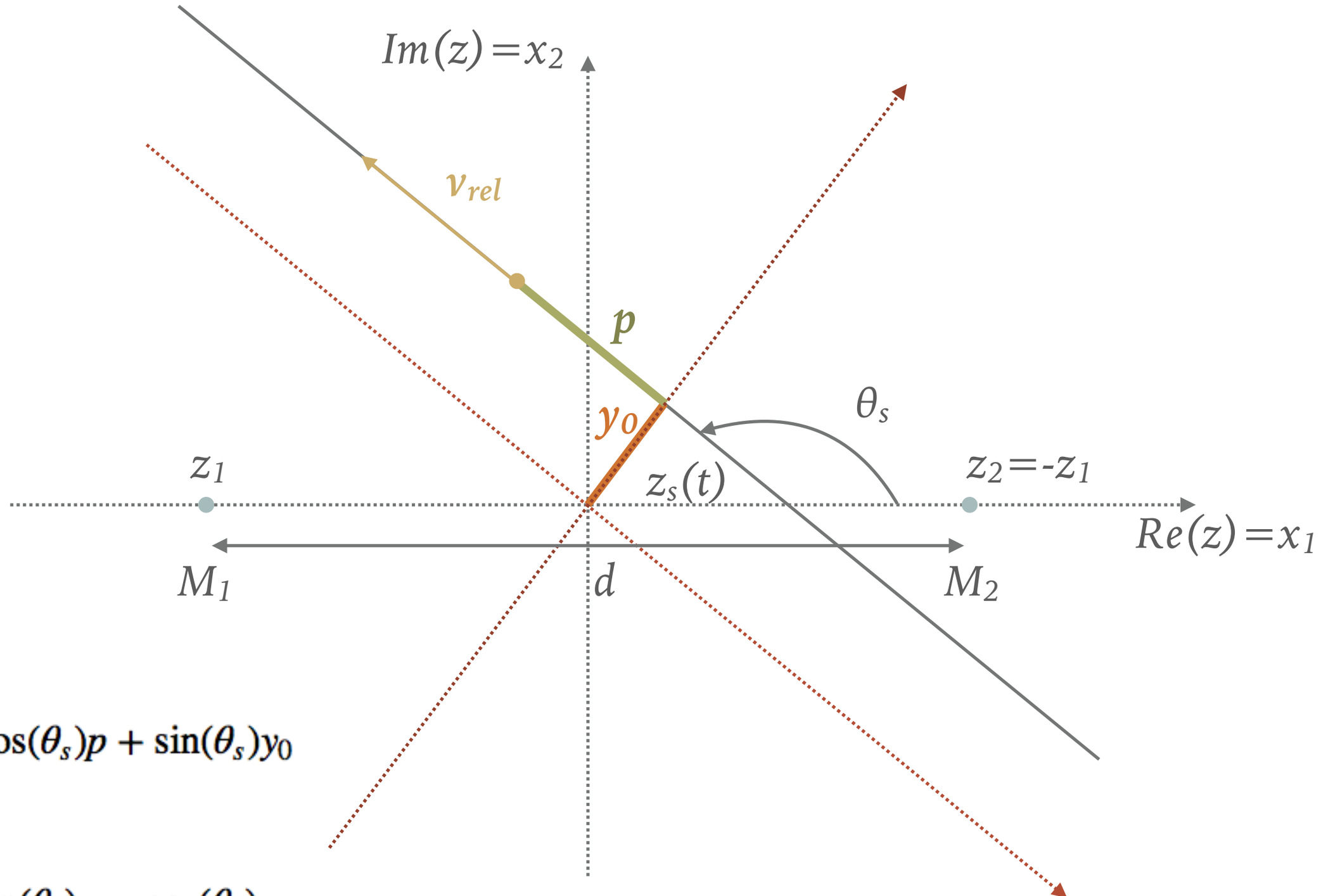
GRAVITATIONAL LENSING

13 – BINARY LENSES

Massimo Meneghetti
AA 2017-2018

BINARY LENSES



$$\Re(z_s) = \cos(\theta_s)p + \sin(\theta_s)y_0$$

$$\Im(z_s) = \sin(\theta_s)p - \cos(\theta_s)y_0$$

BINARY LENSES (SEE NOTEBOOK)

- Lens equation:

$$z_s = z - \frac{m_1}{z^* - z_1^*} - \frac{m_2}{z^* - z_2^*}$$

- determinant of the Jacobian:

$$\det A = 1 - \left|\frac{\partial z_s}{\partial z^*}\right|^2$$

$$\frac{\partial z_s}{\partial z^*} = \frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2}$$

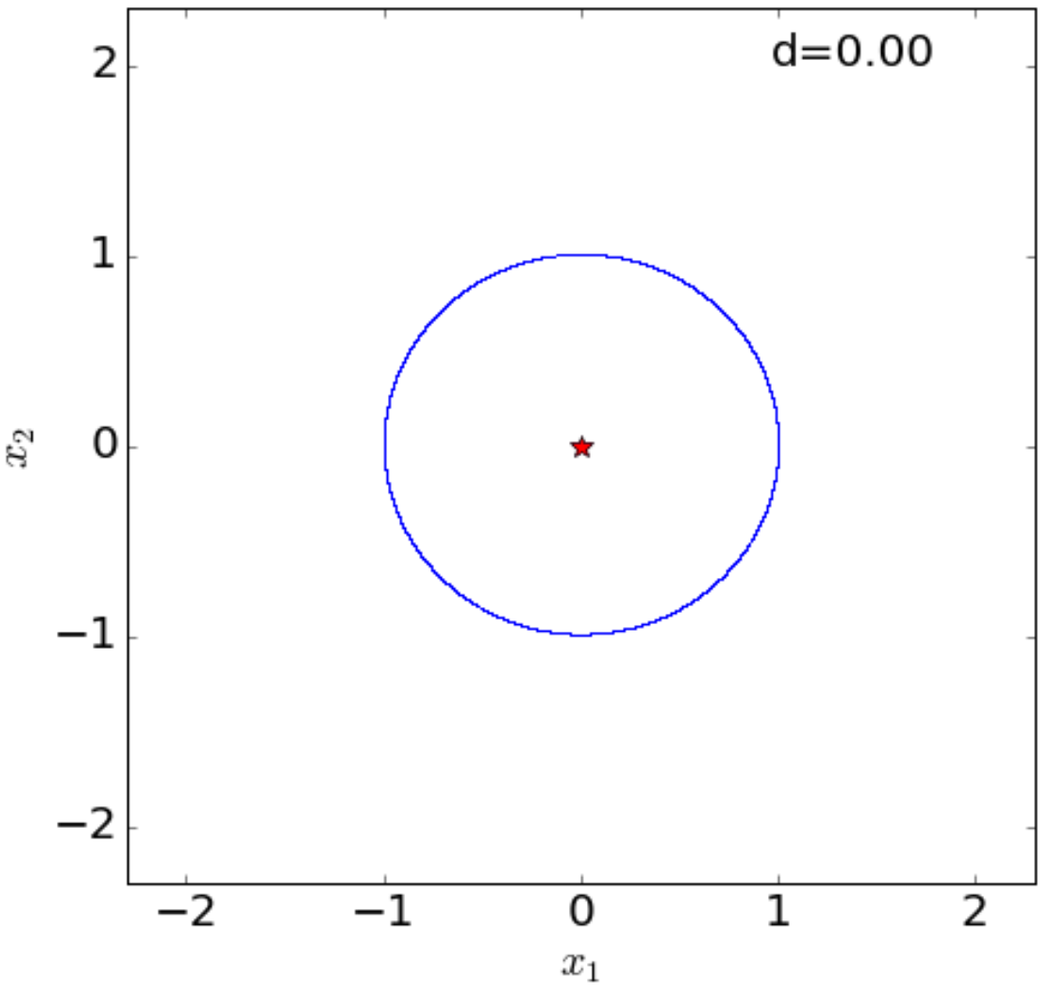
- condition for critical points:

$$\frac{\partial z_s}{\partial z^*} = e^{i\phi}$$

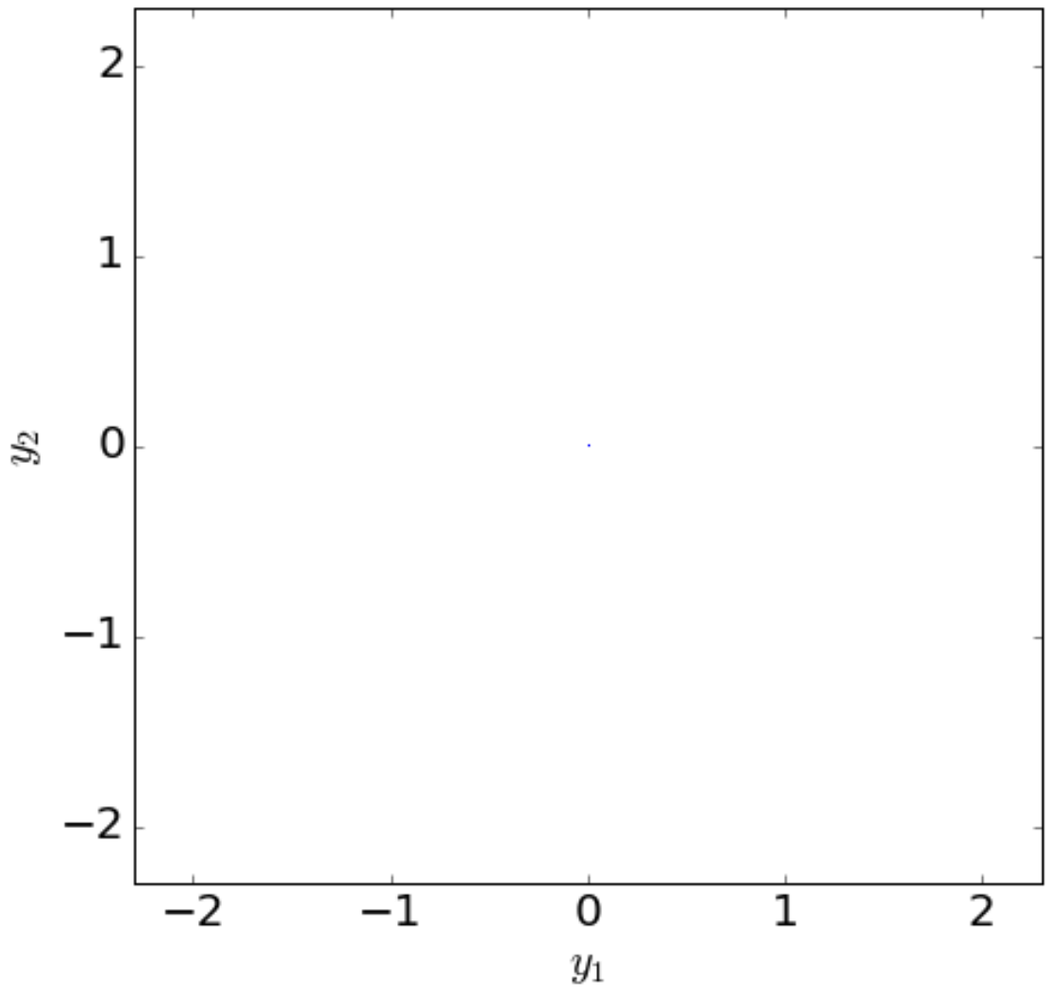
- resulting fourth grade polynomial ($z_2 = -z_1$):

$$z^4 - z^2(2z_1^{*2} + e^{i\phi}) - zz_1^* 2(m_1 - m_2)e^{i\phi} + z_1^{*2}(z_1^{*2} - e^{i\phi}) = 0$$

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

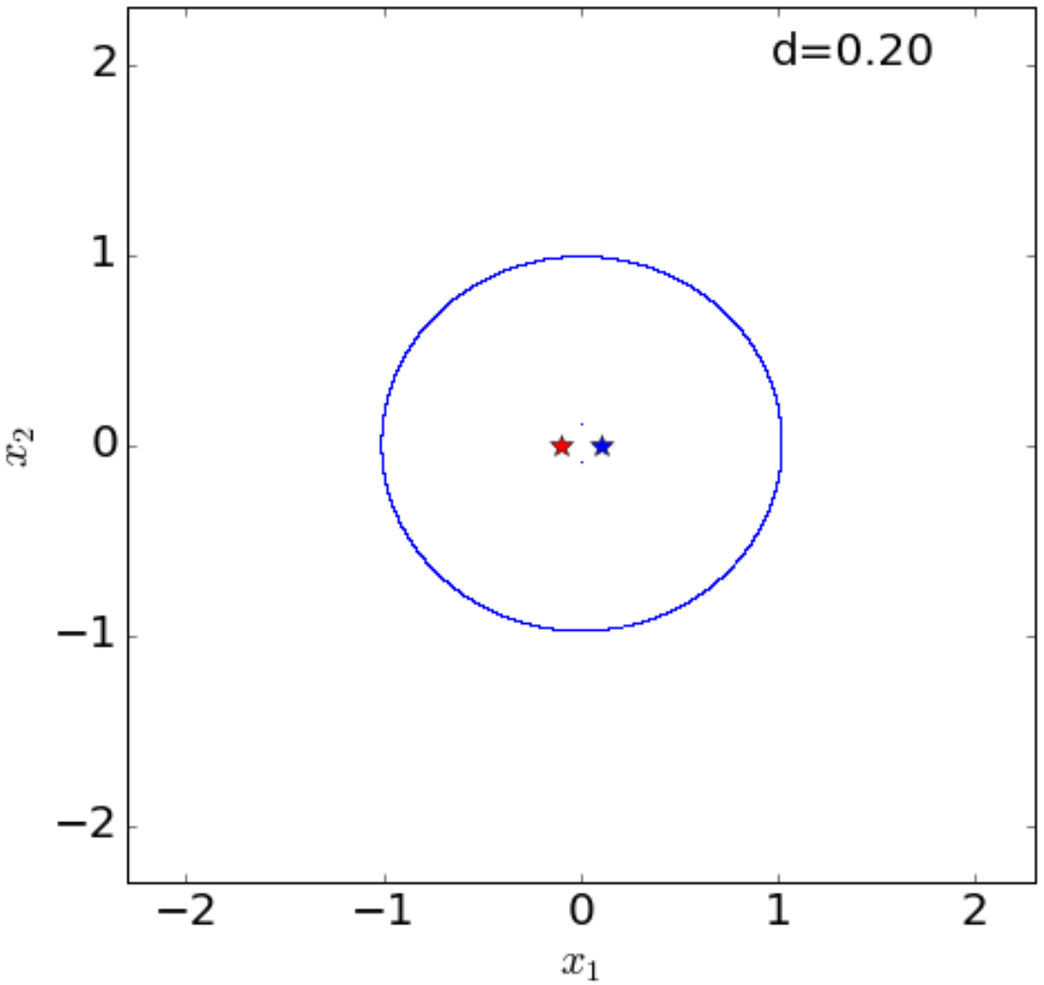


critical lines

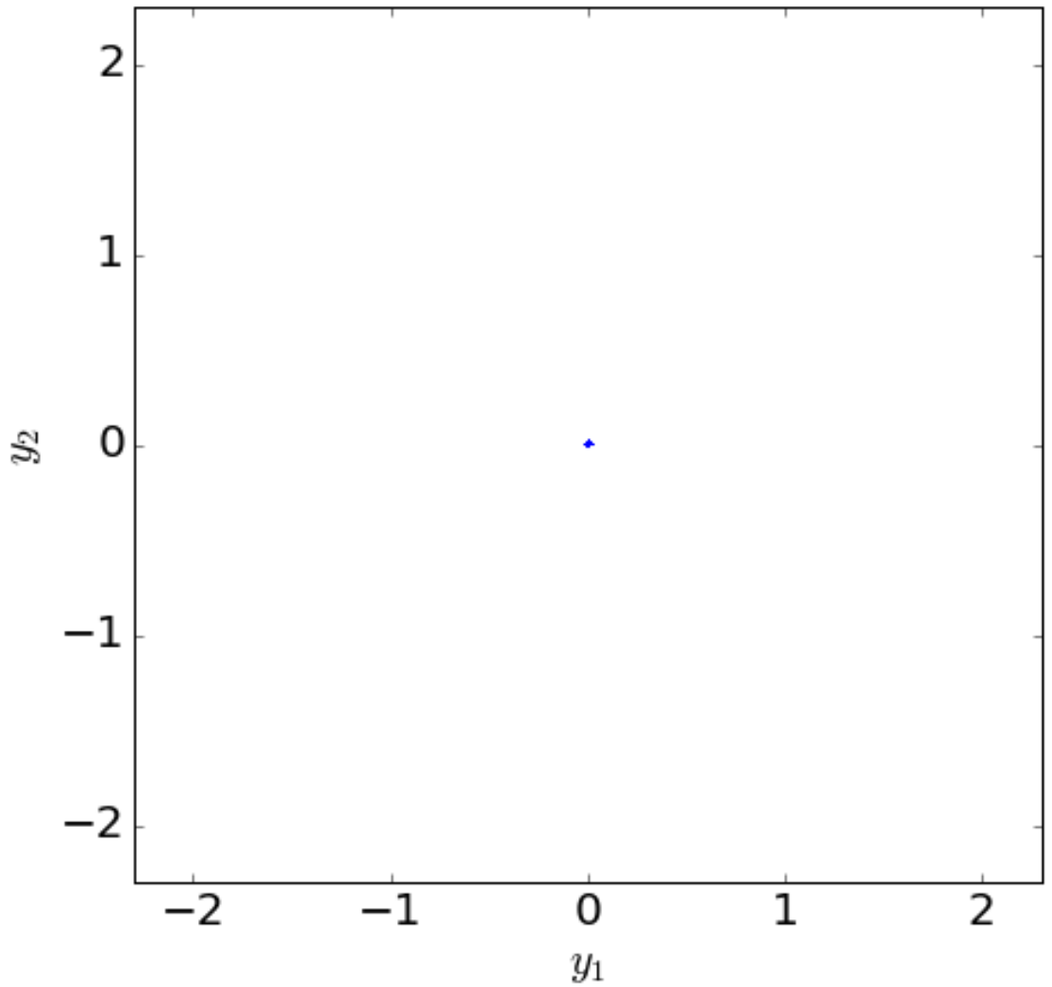


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

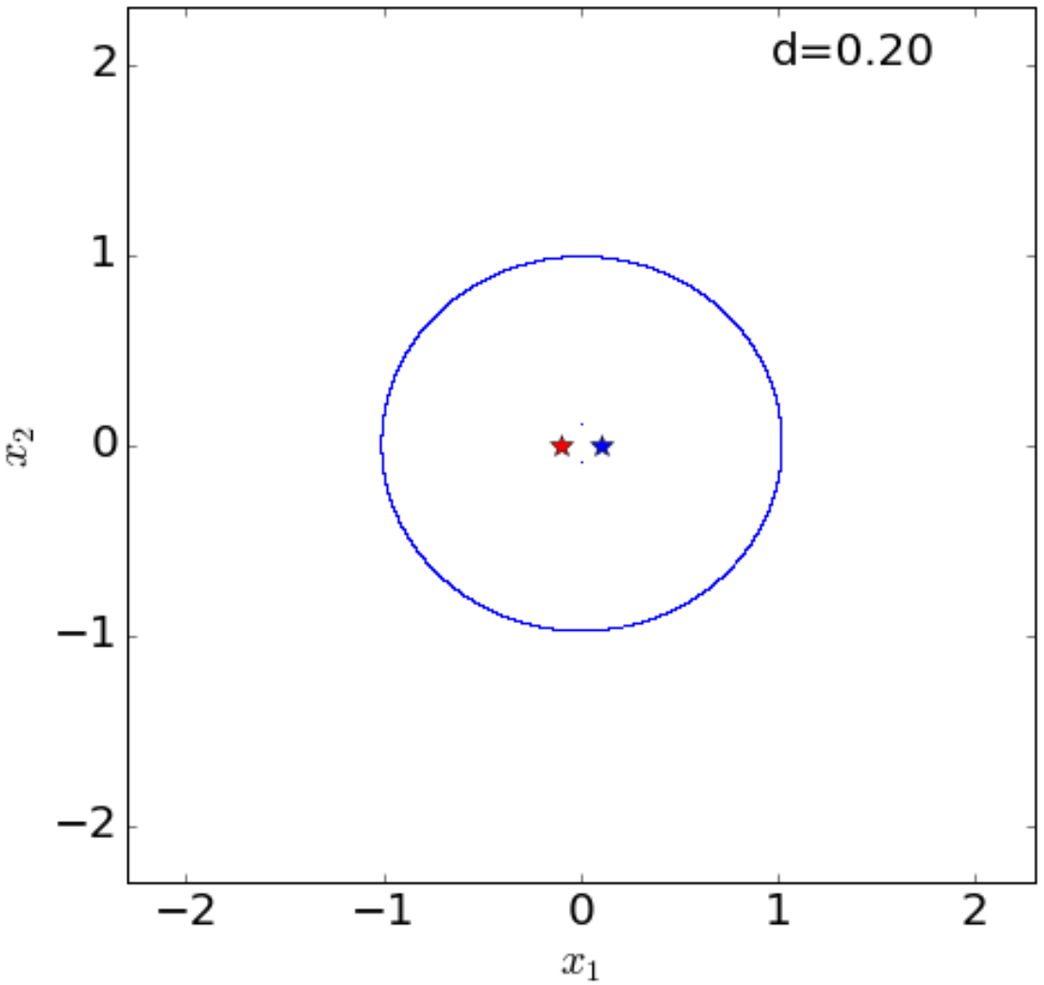


critical lines

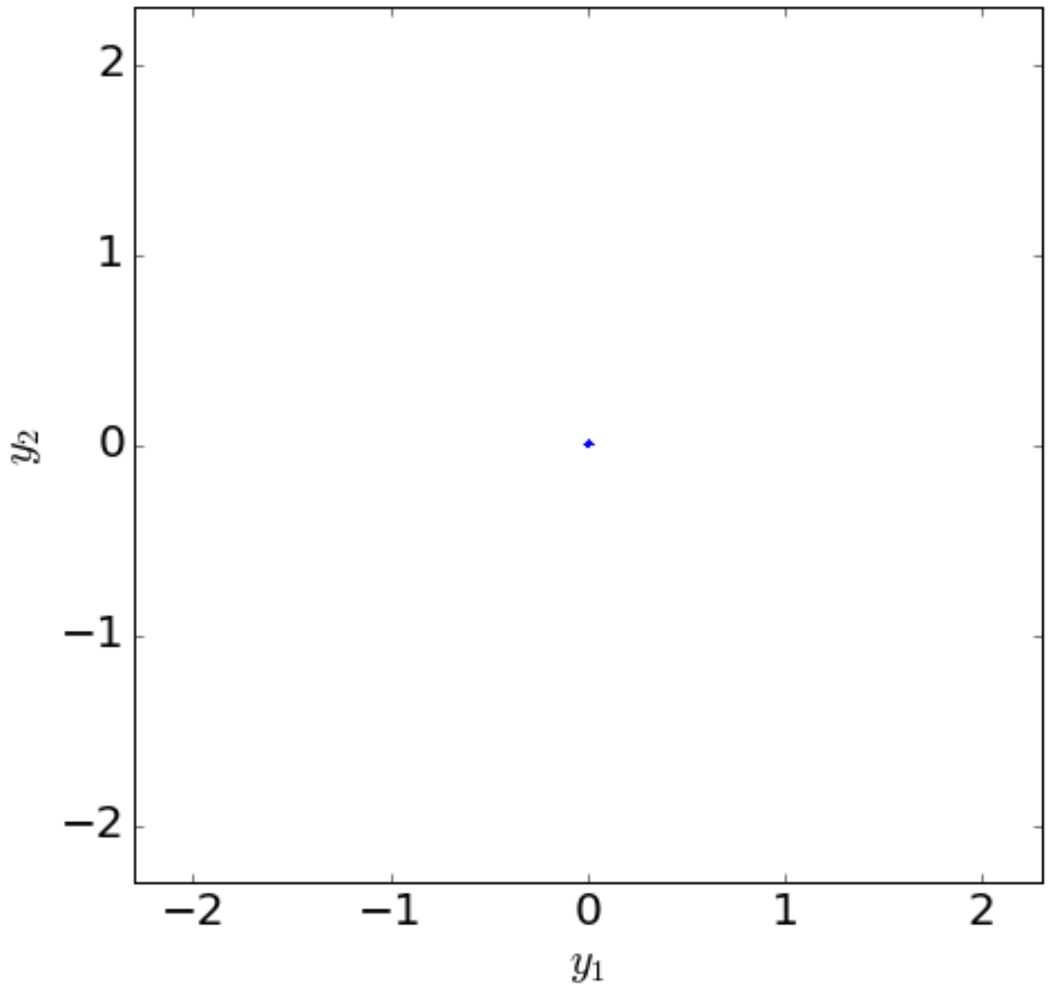


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

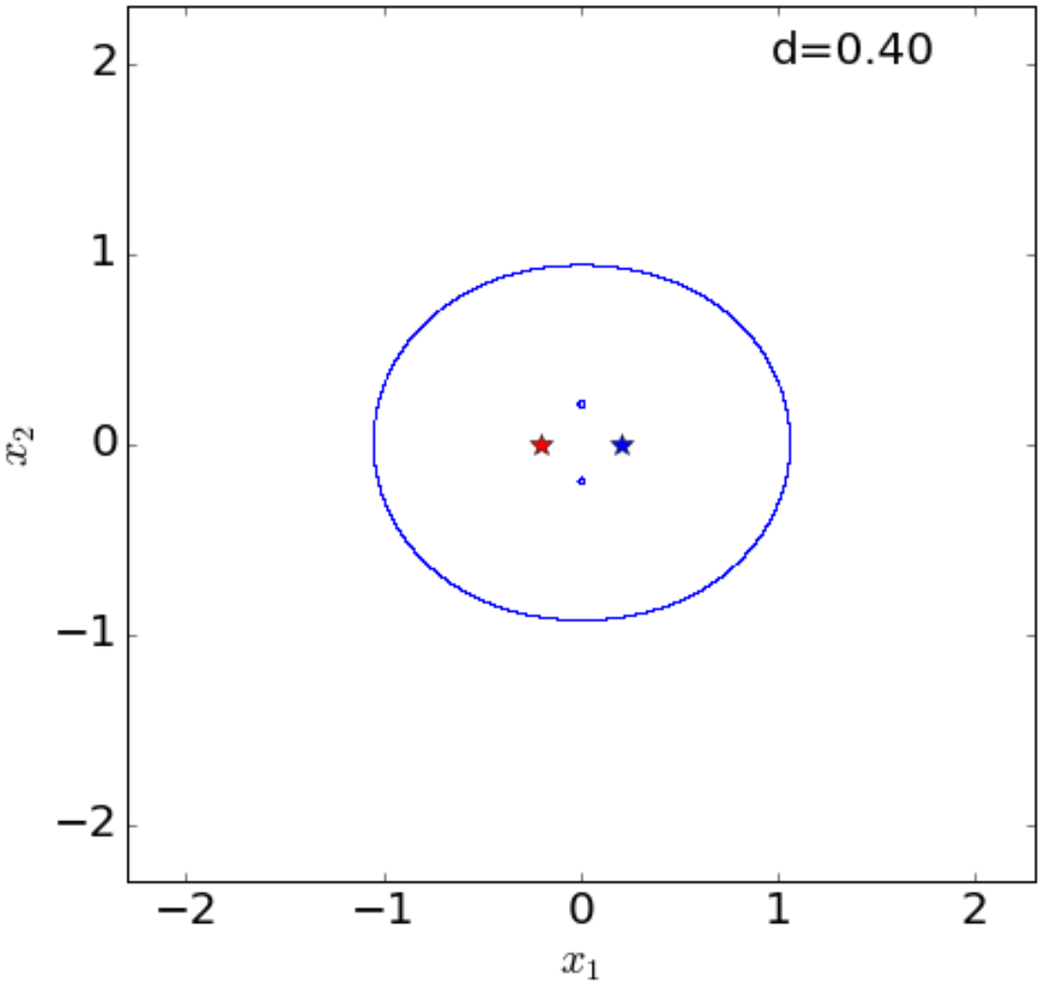


critical lines

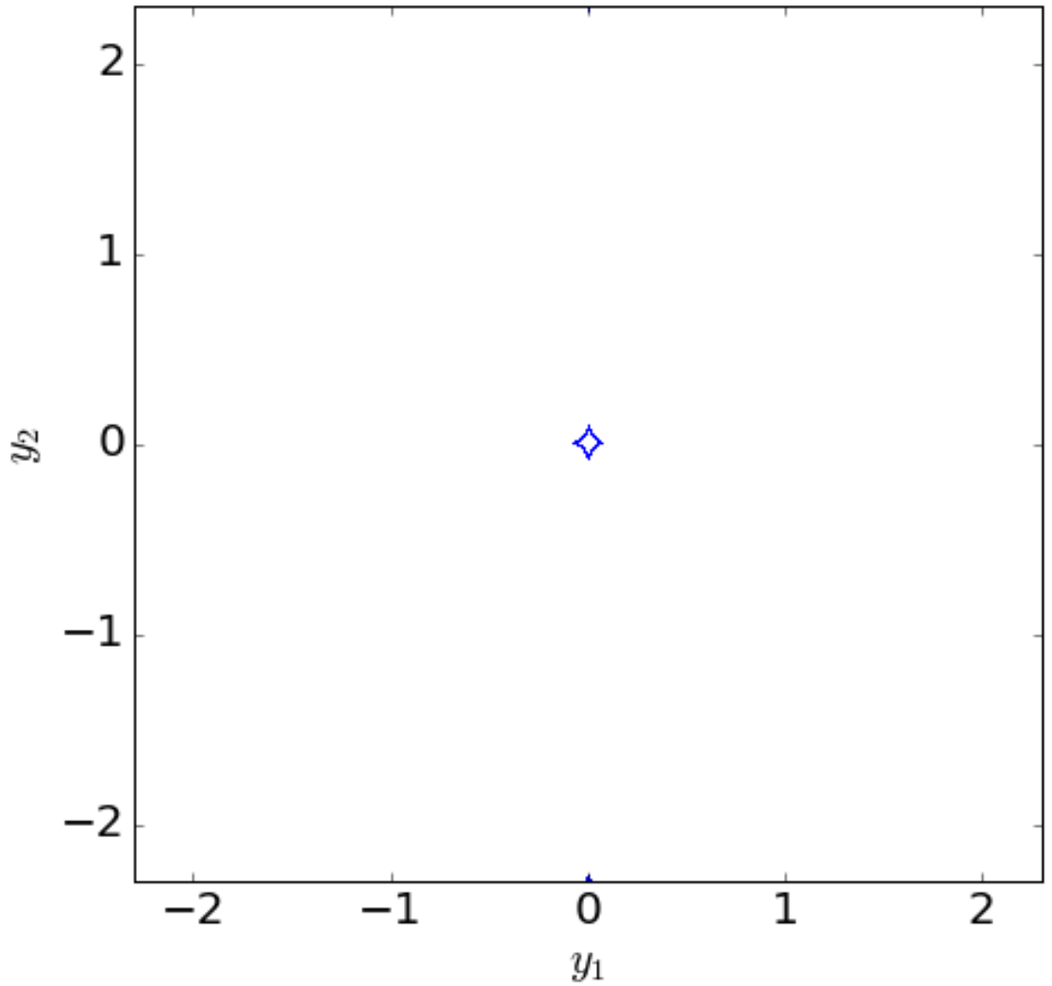


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

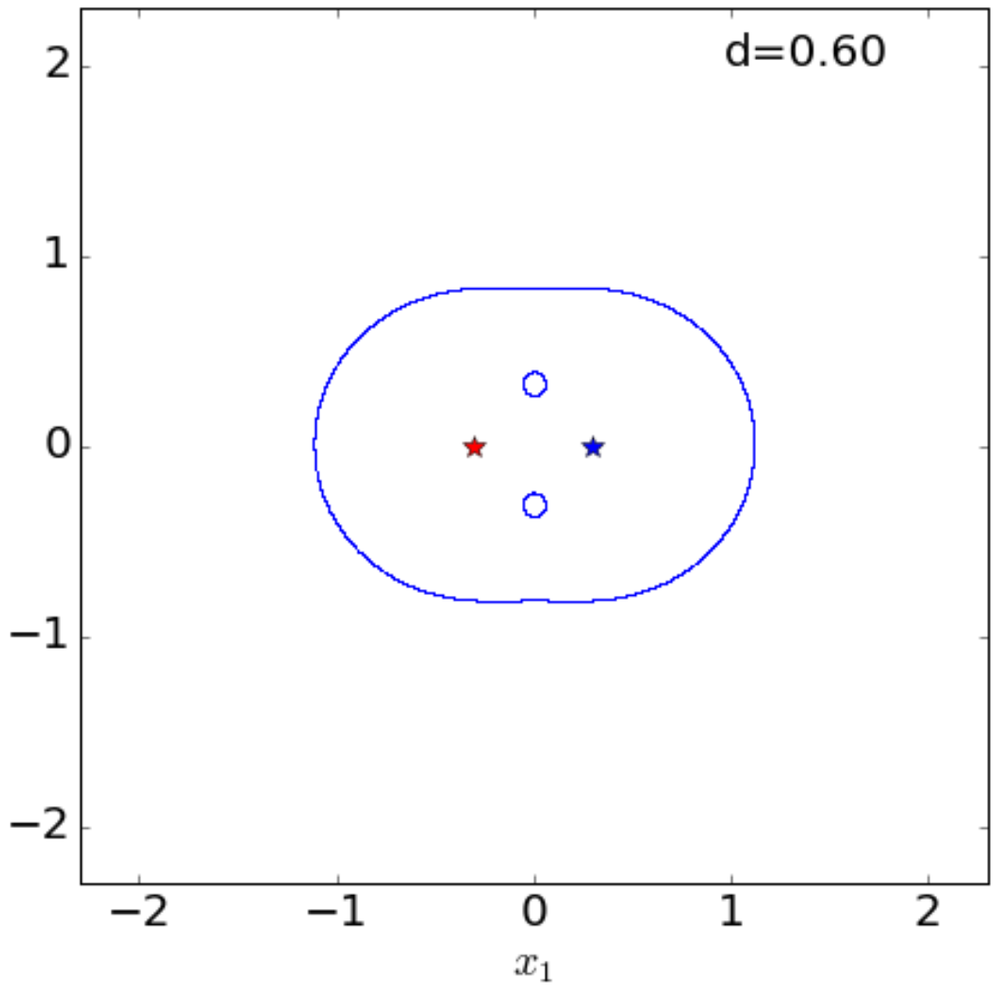


critical lines

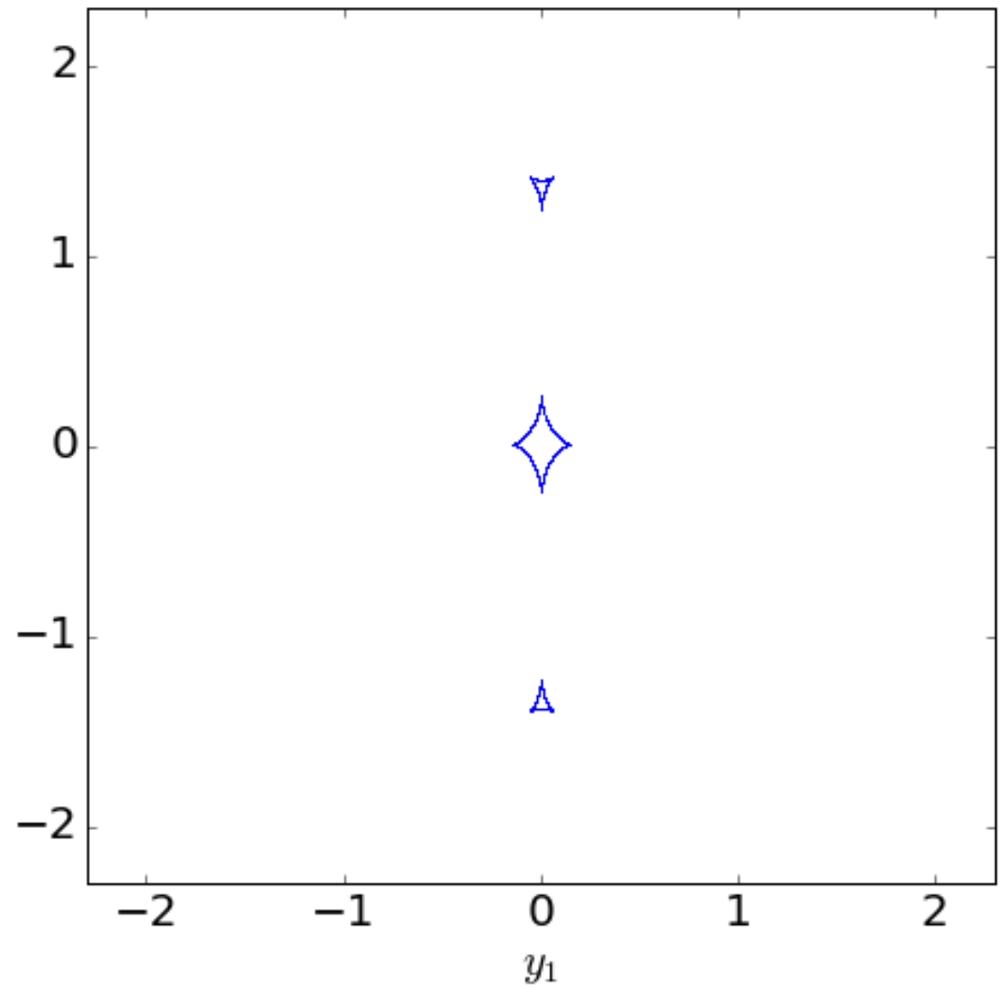


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

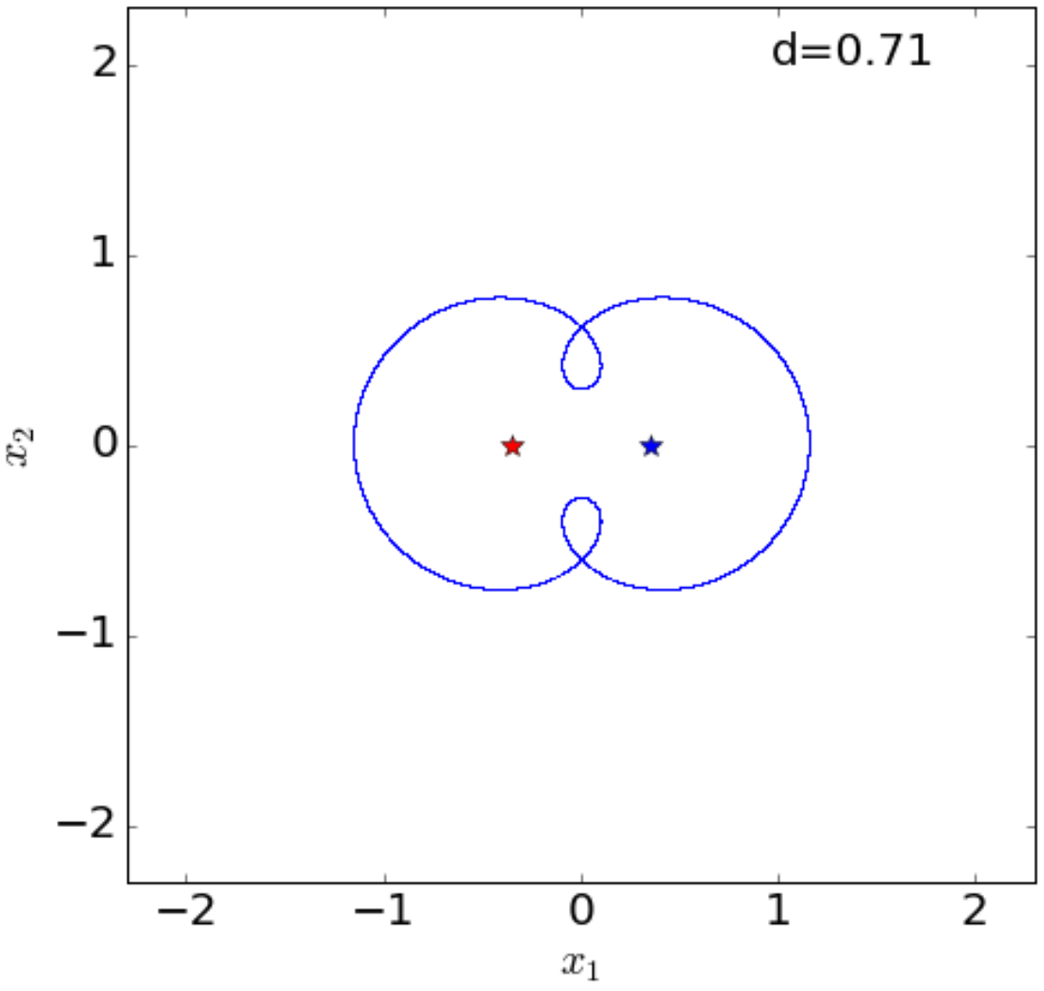


critical lines

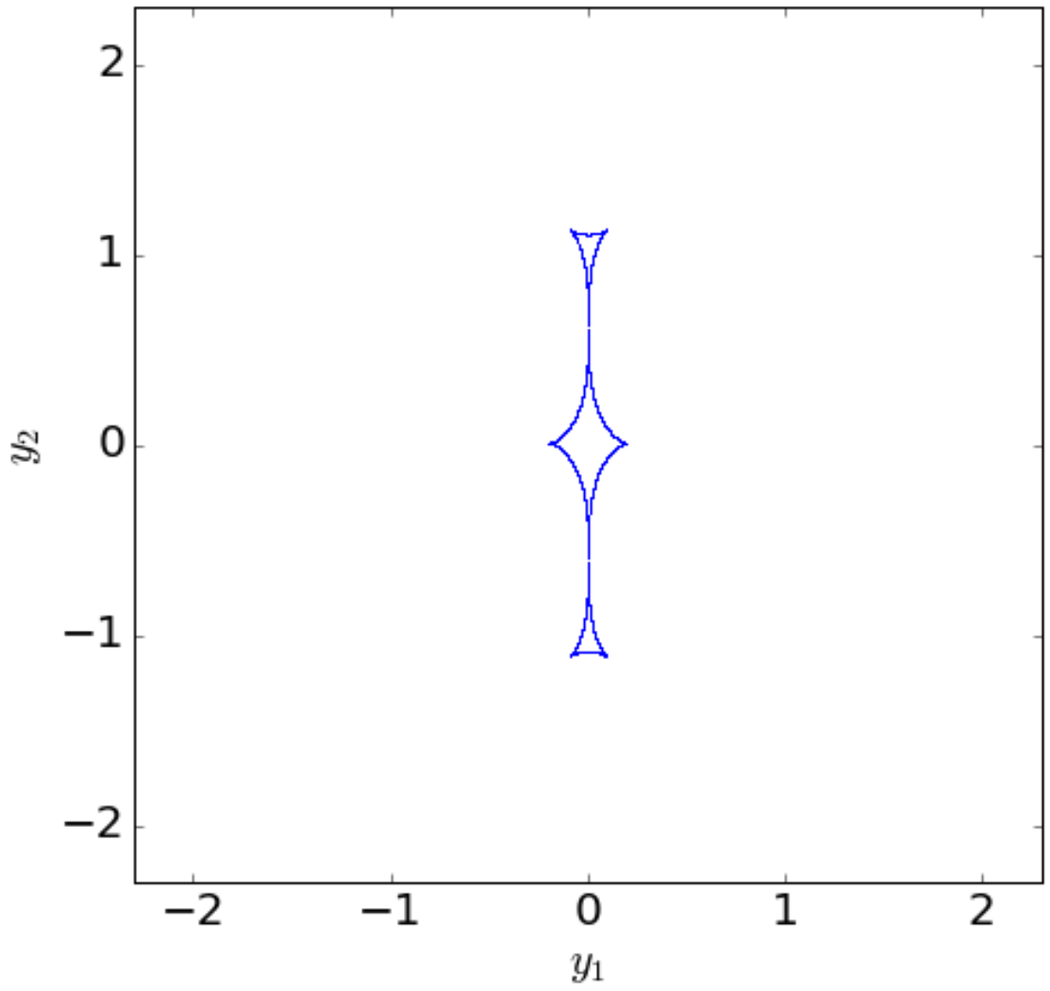


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

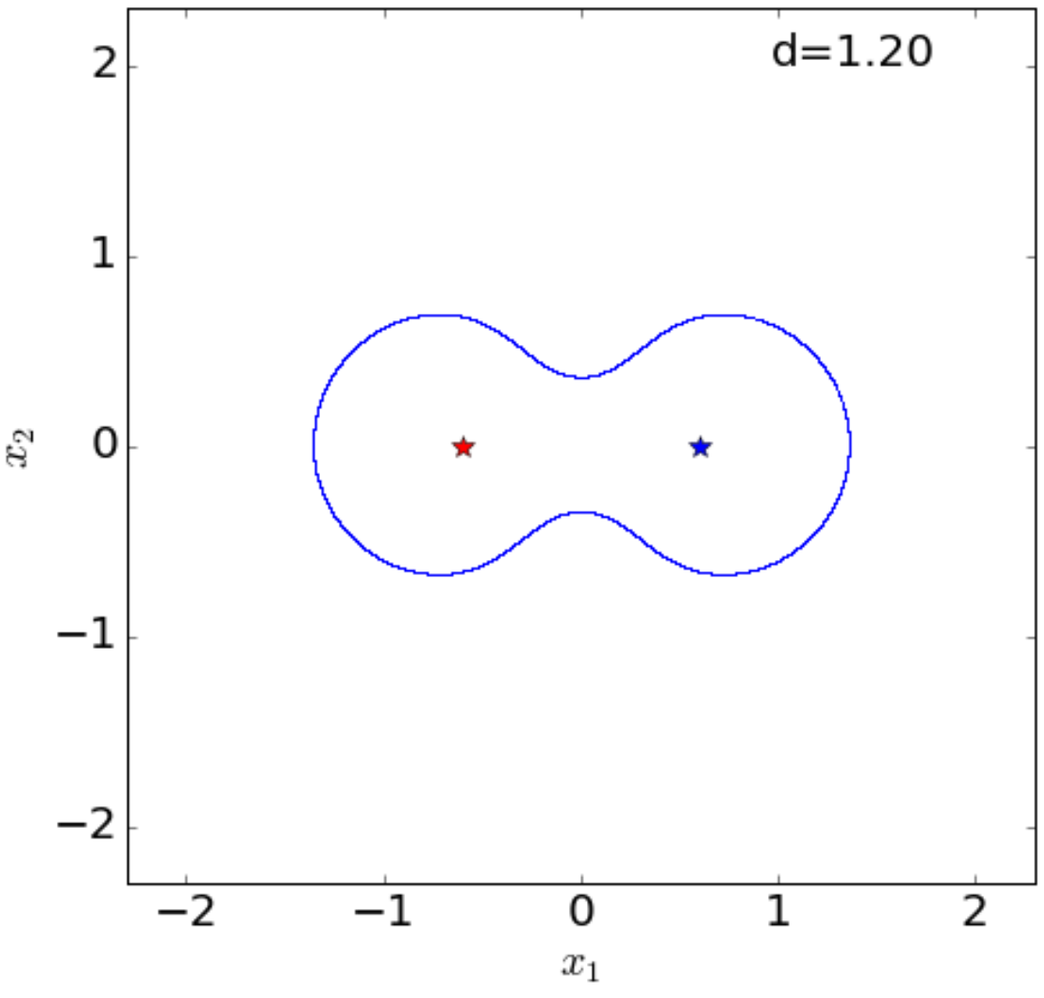


critical lines

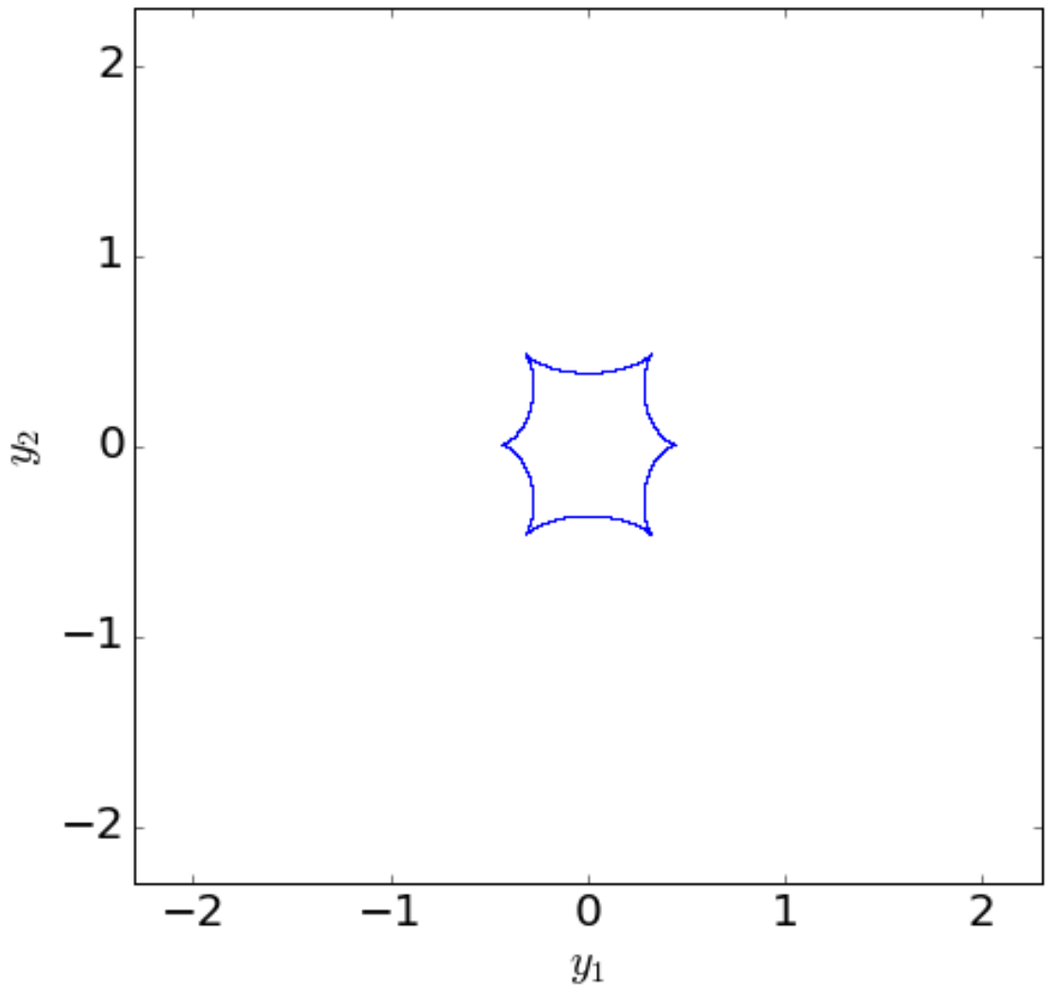


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

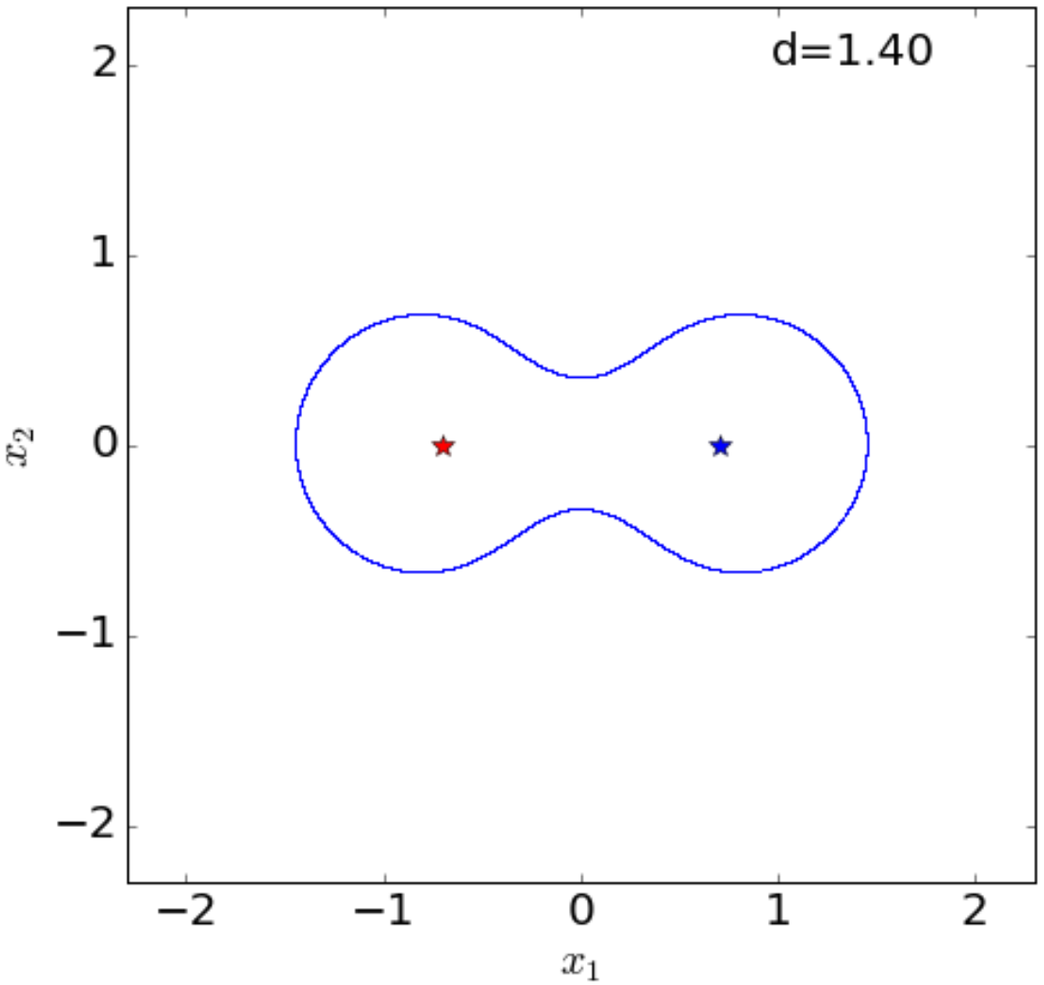


critical lines

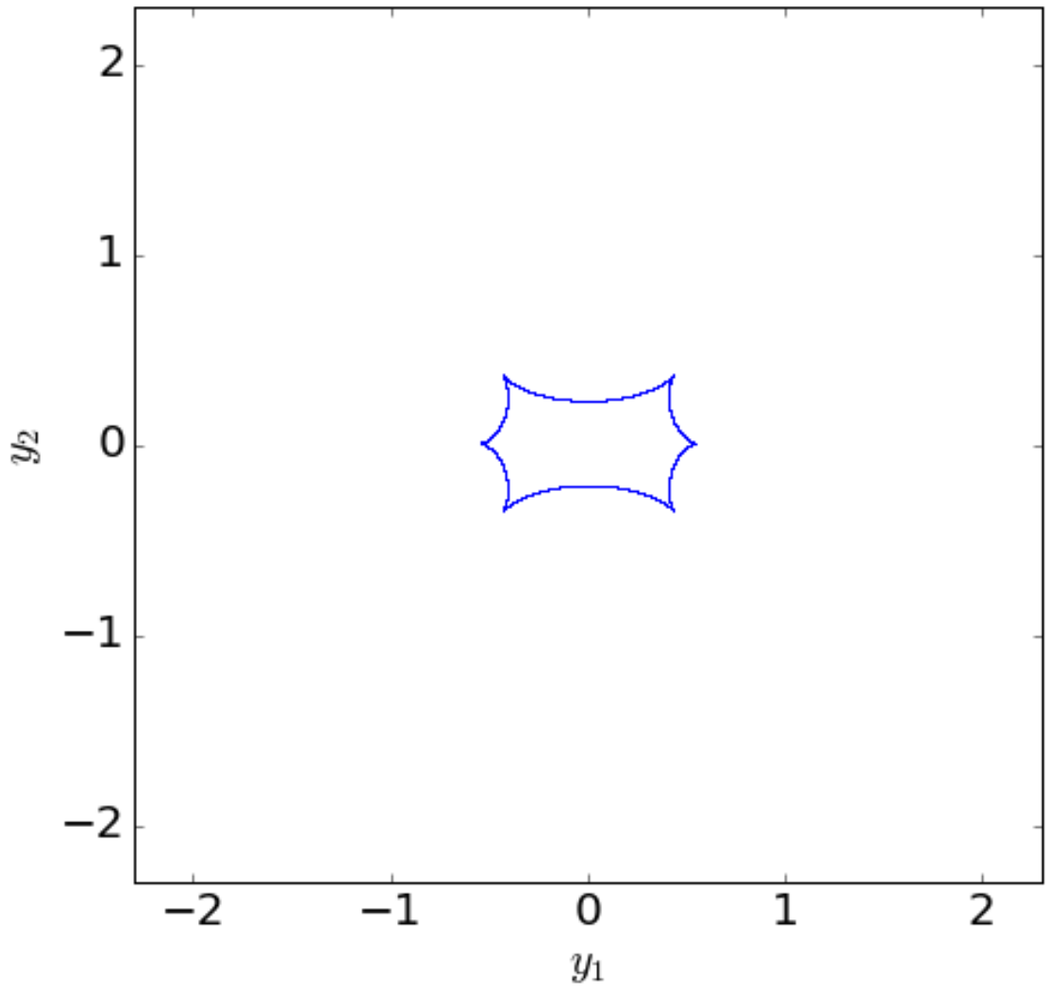


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

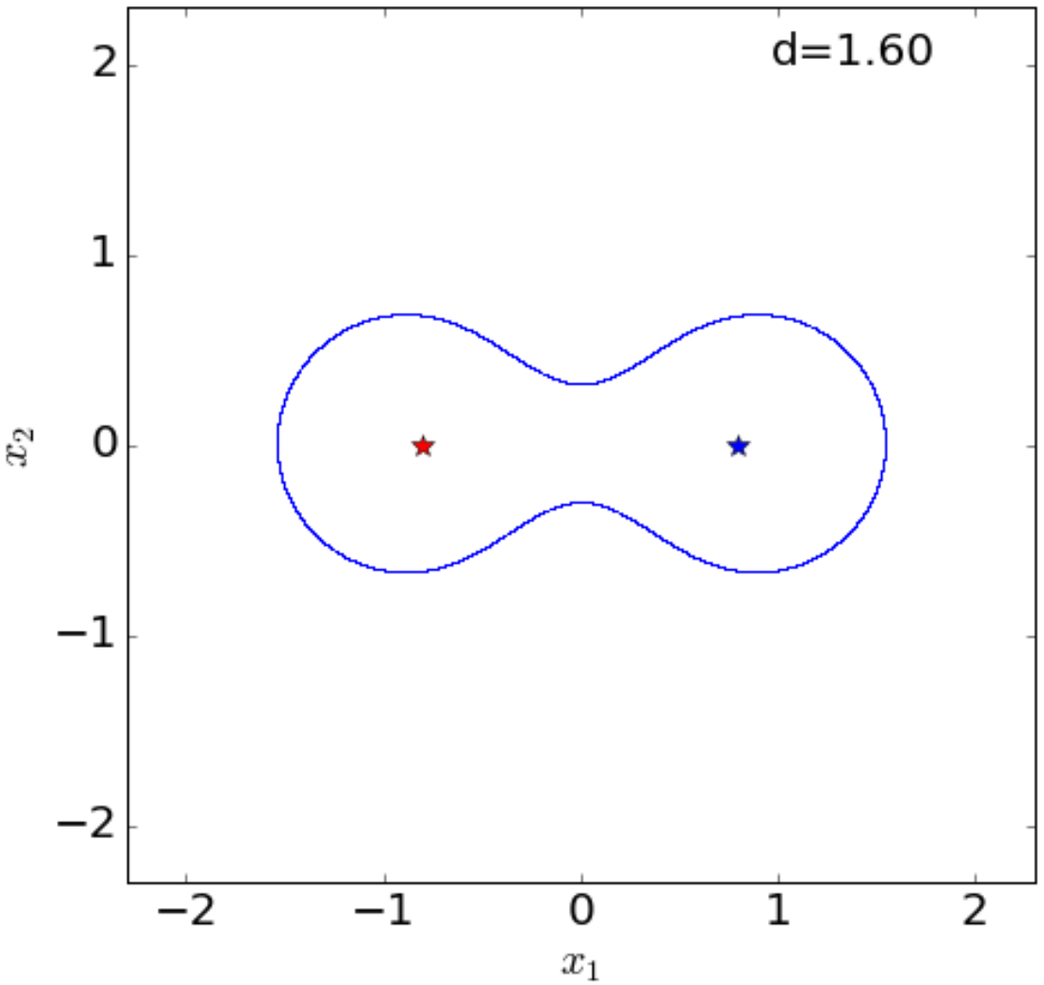


critical lines

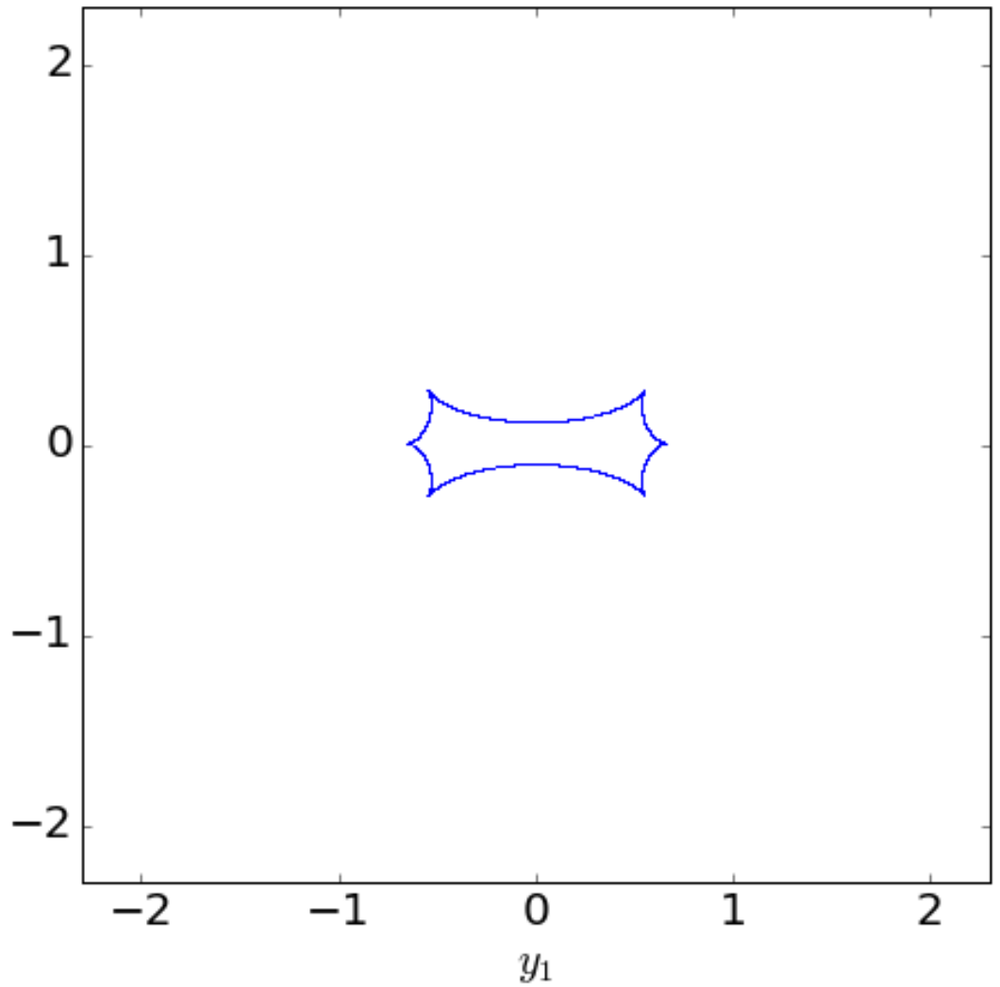


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

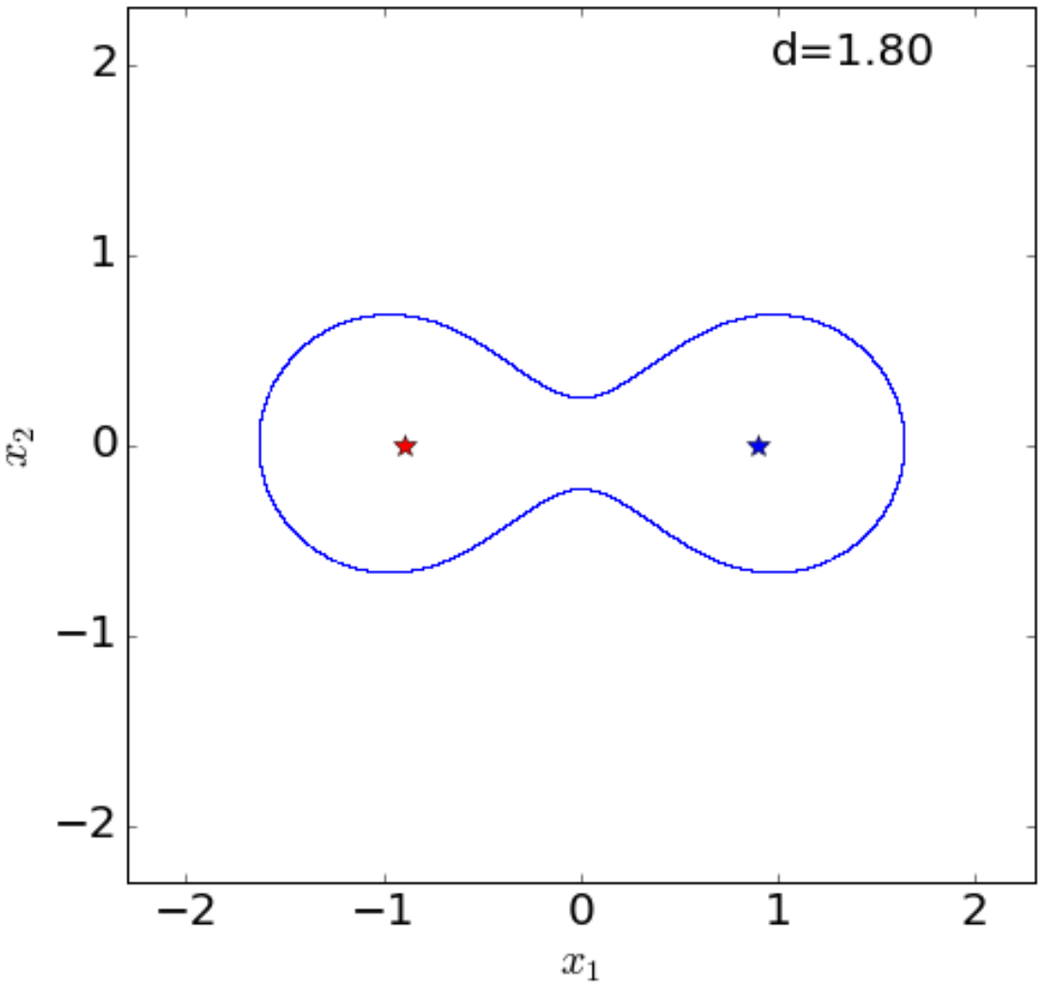


critical lines

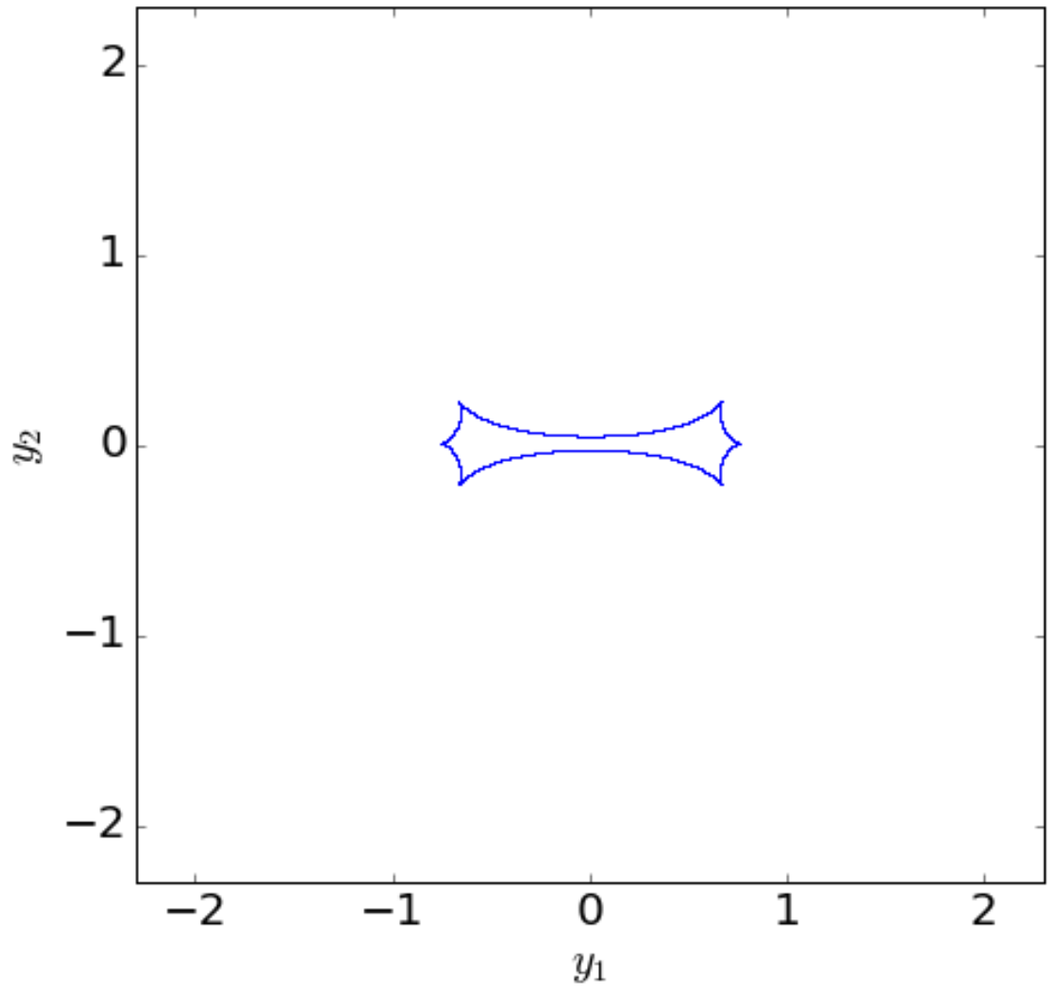


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

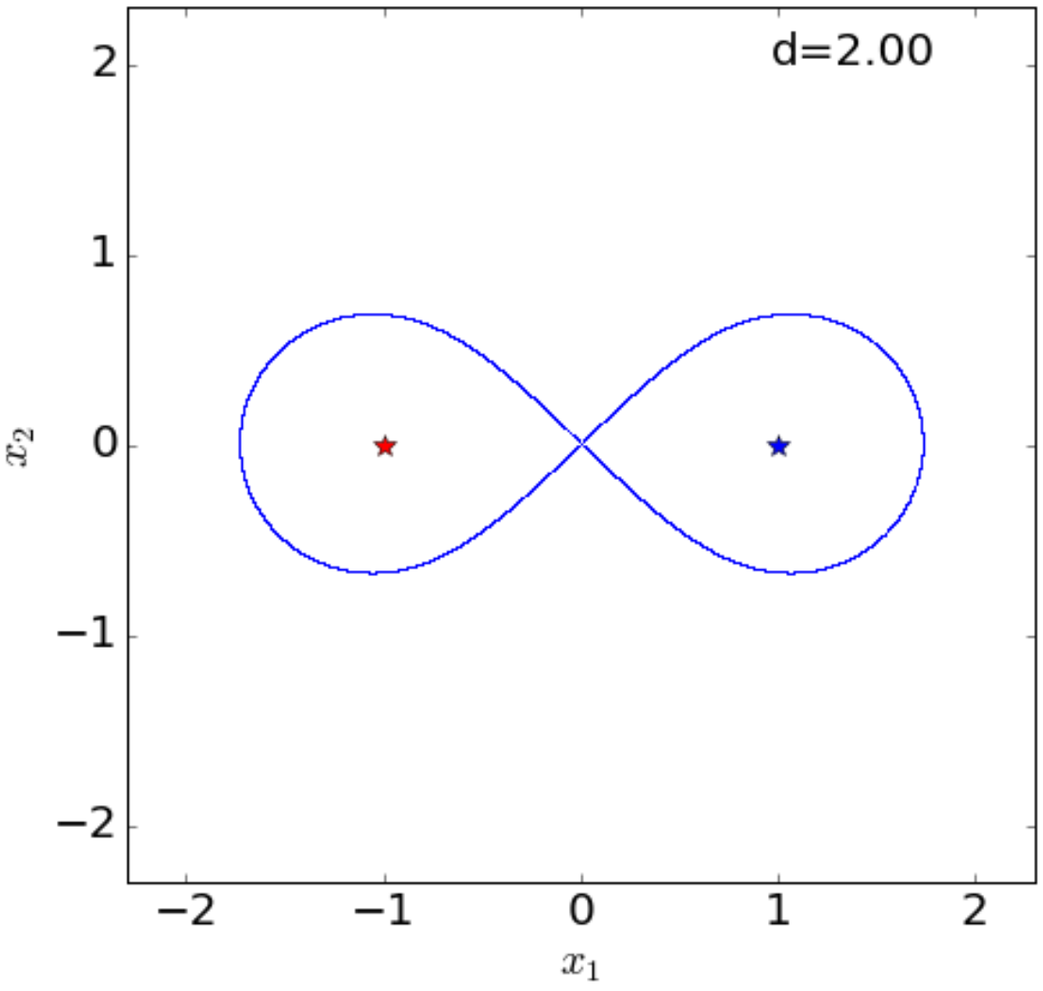


critical lines

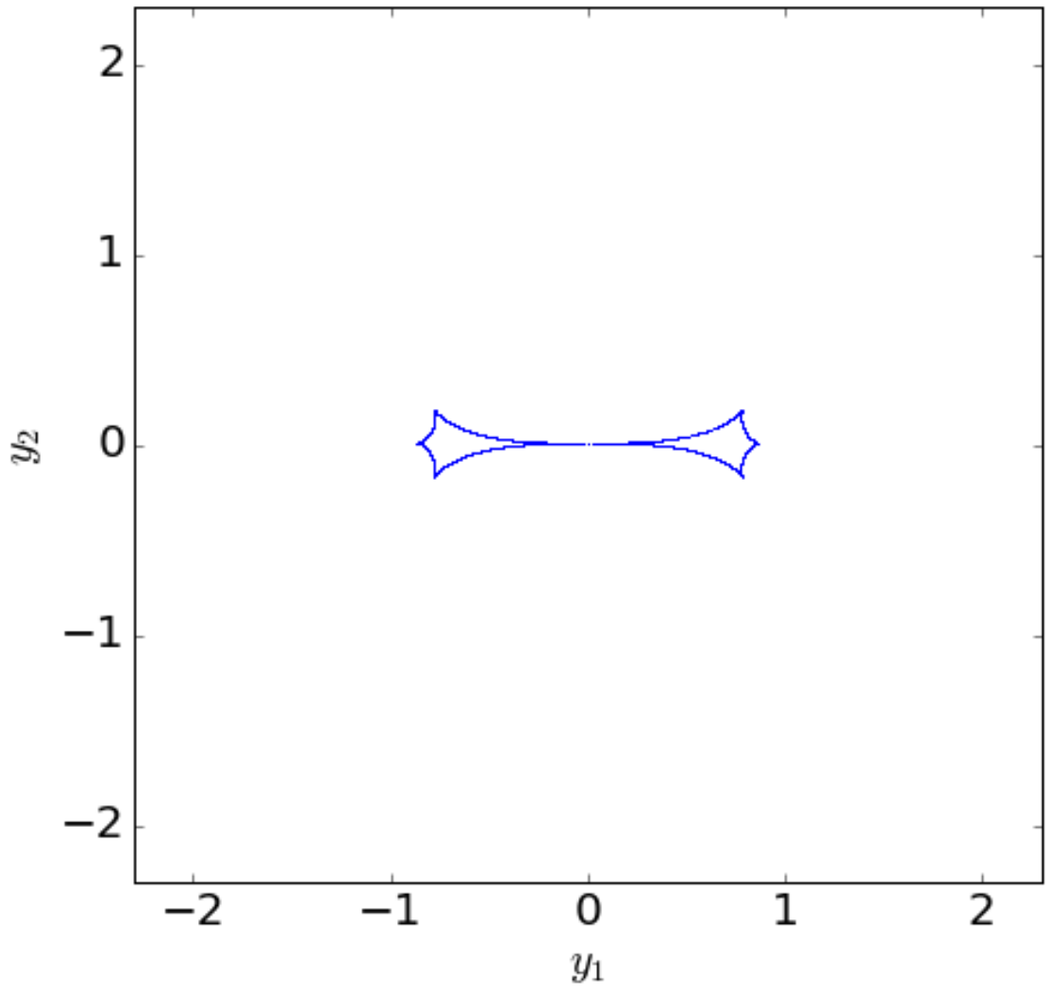


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

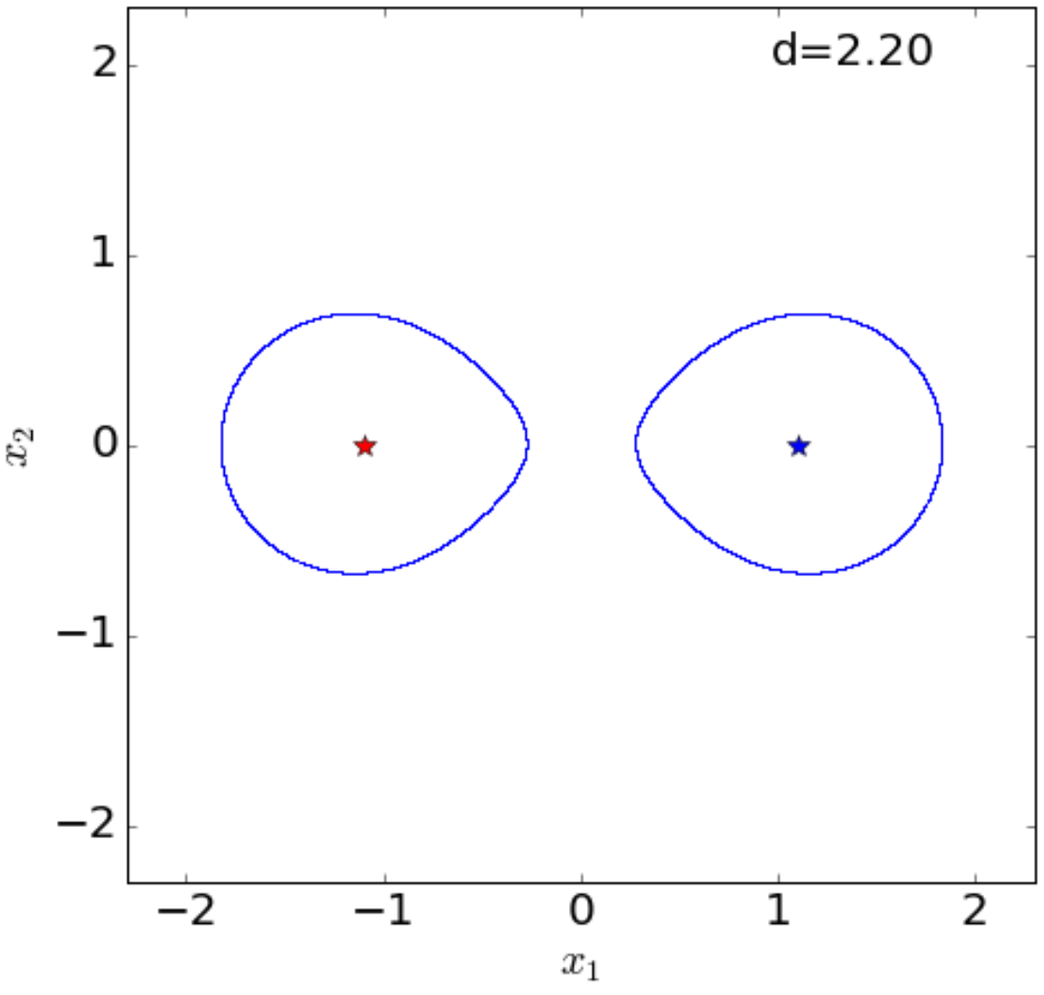


critical lines

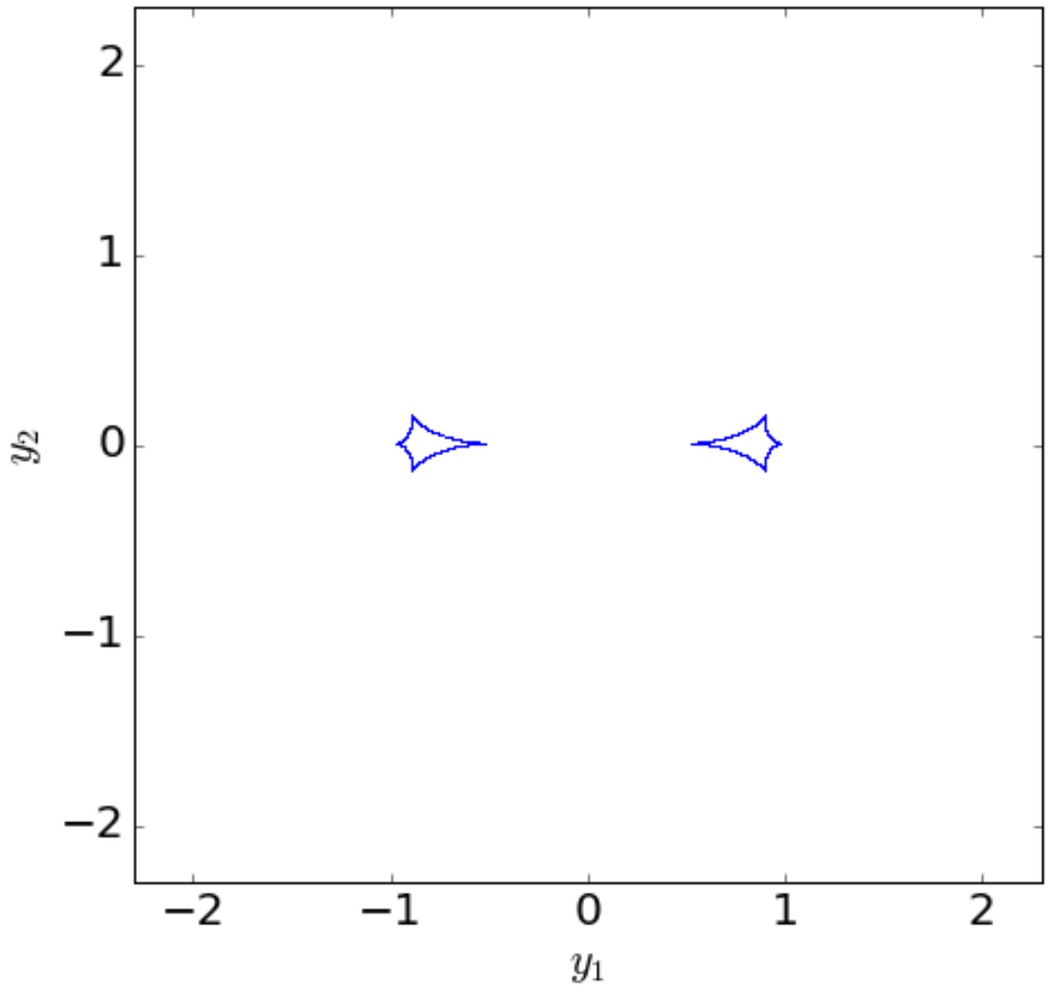


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

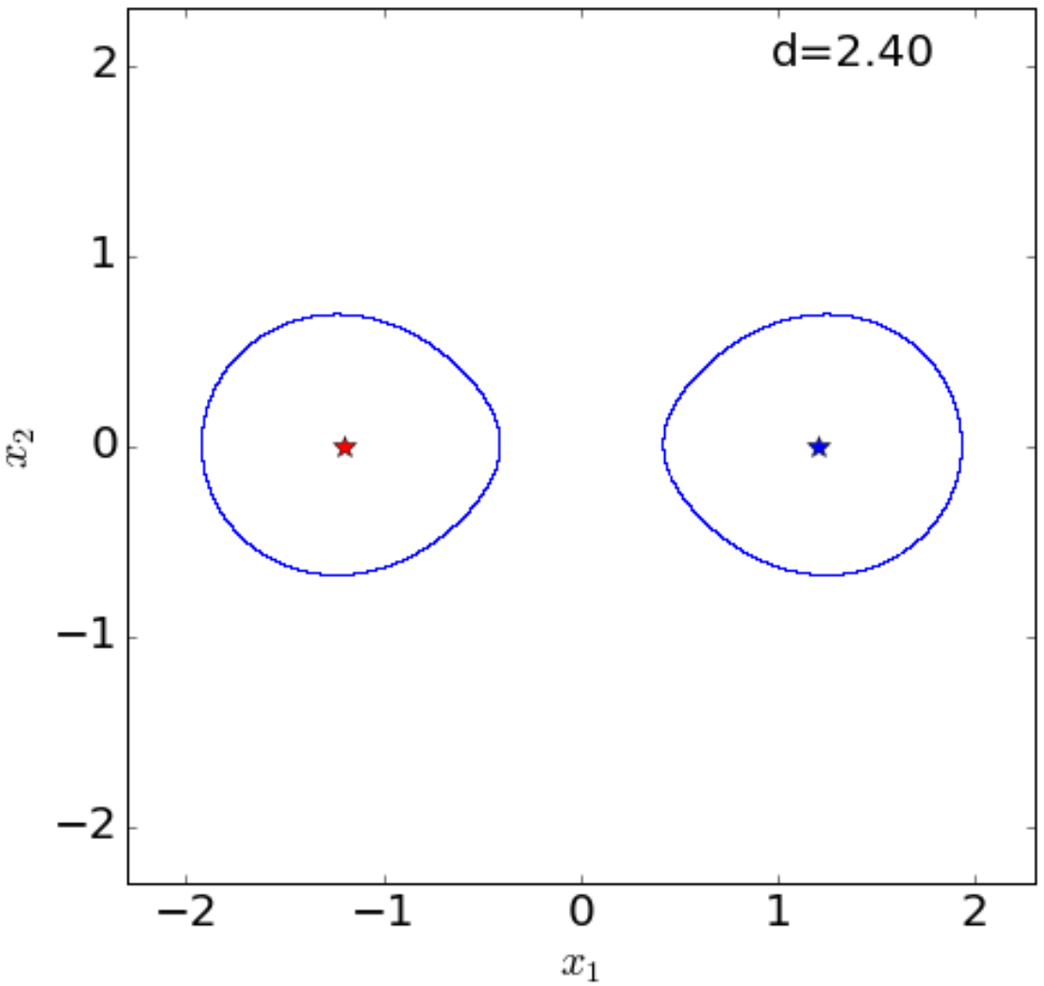


critical lines

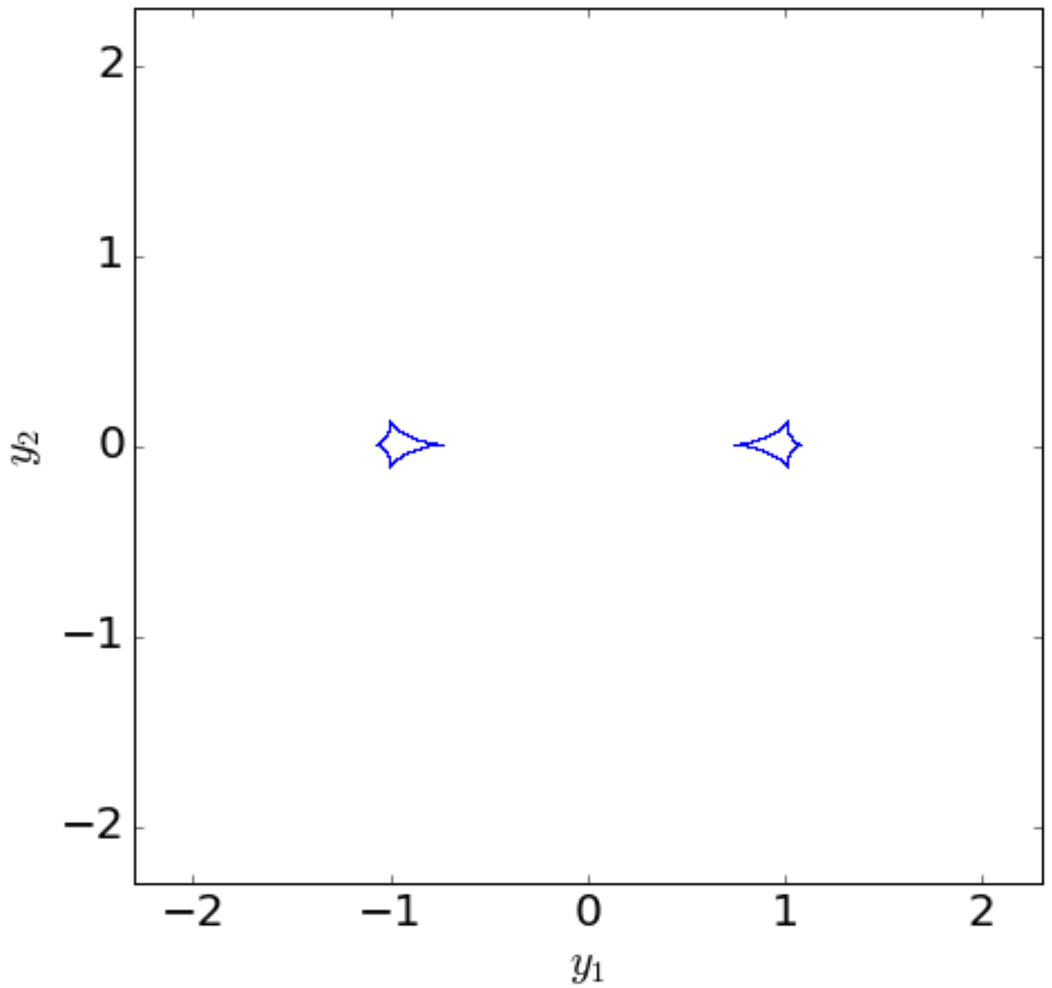


caustics

BINARY LENSES:
TWO LENSES WITH THE SAME MASS (Q=1) AND VARYING DISTANCE

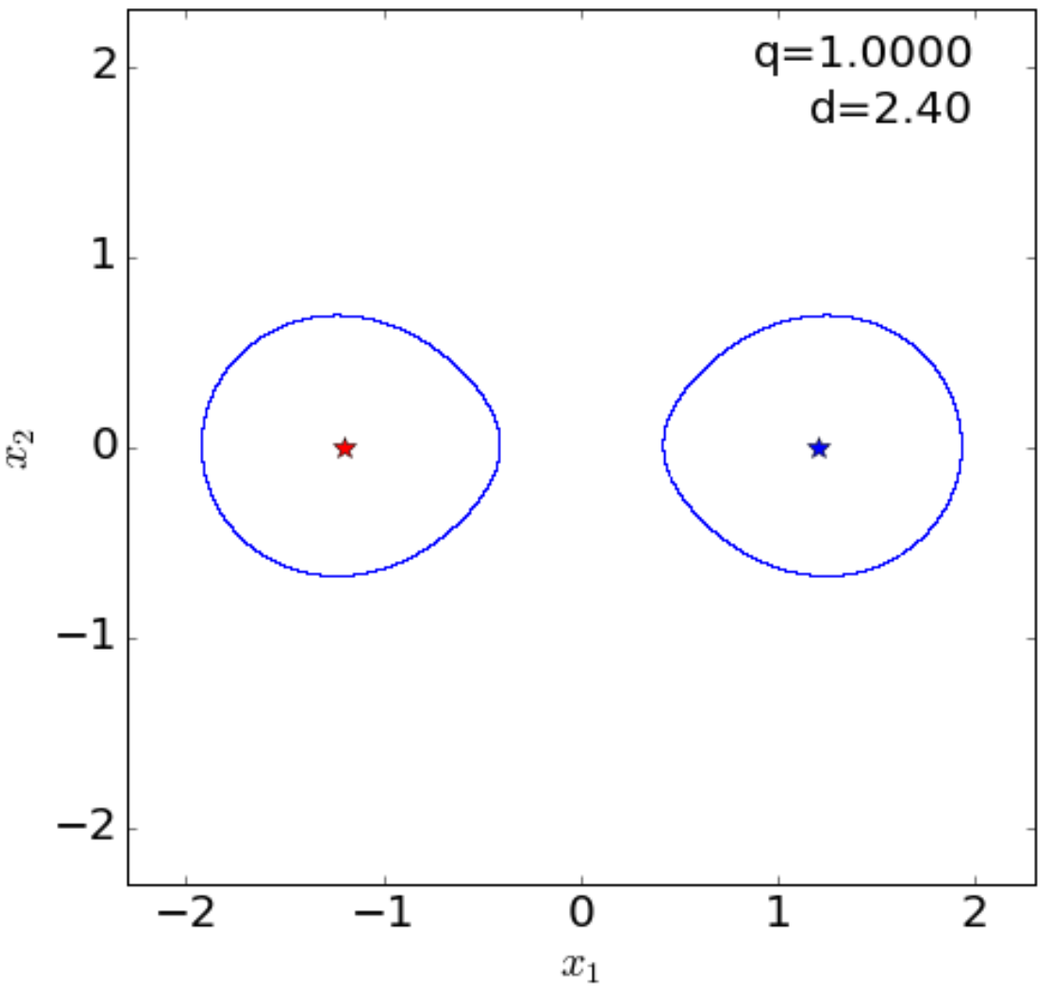


critical lines

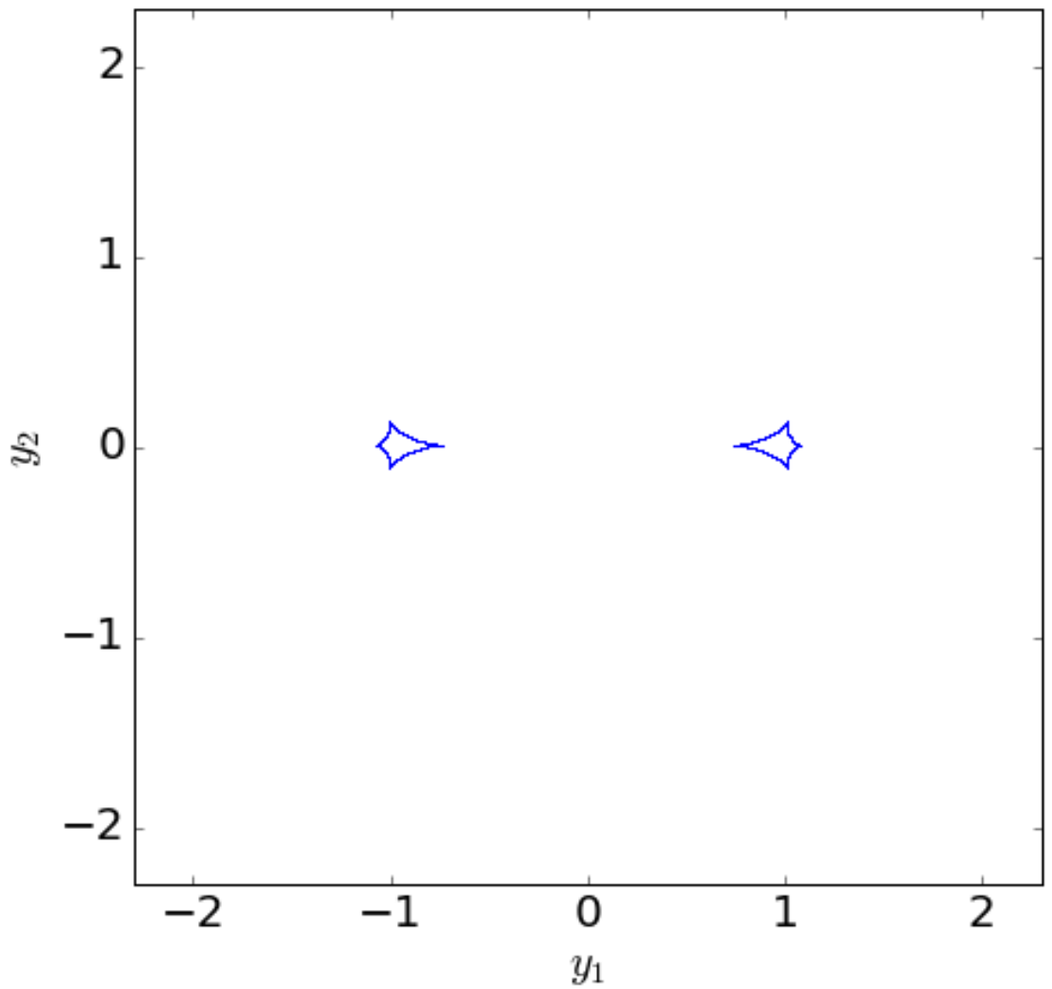


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

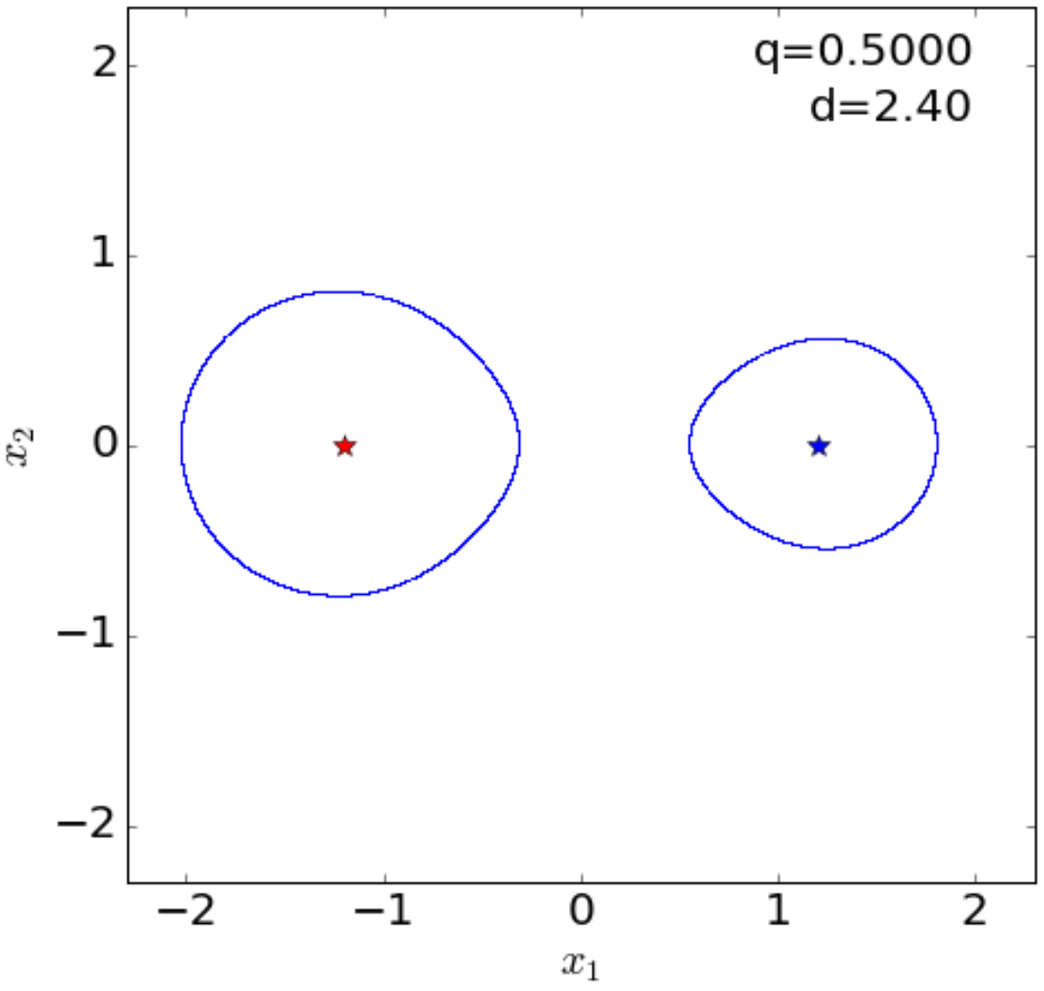


critical lines

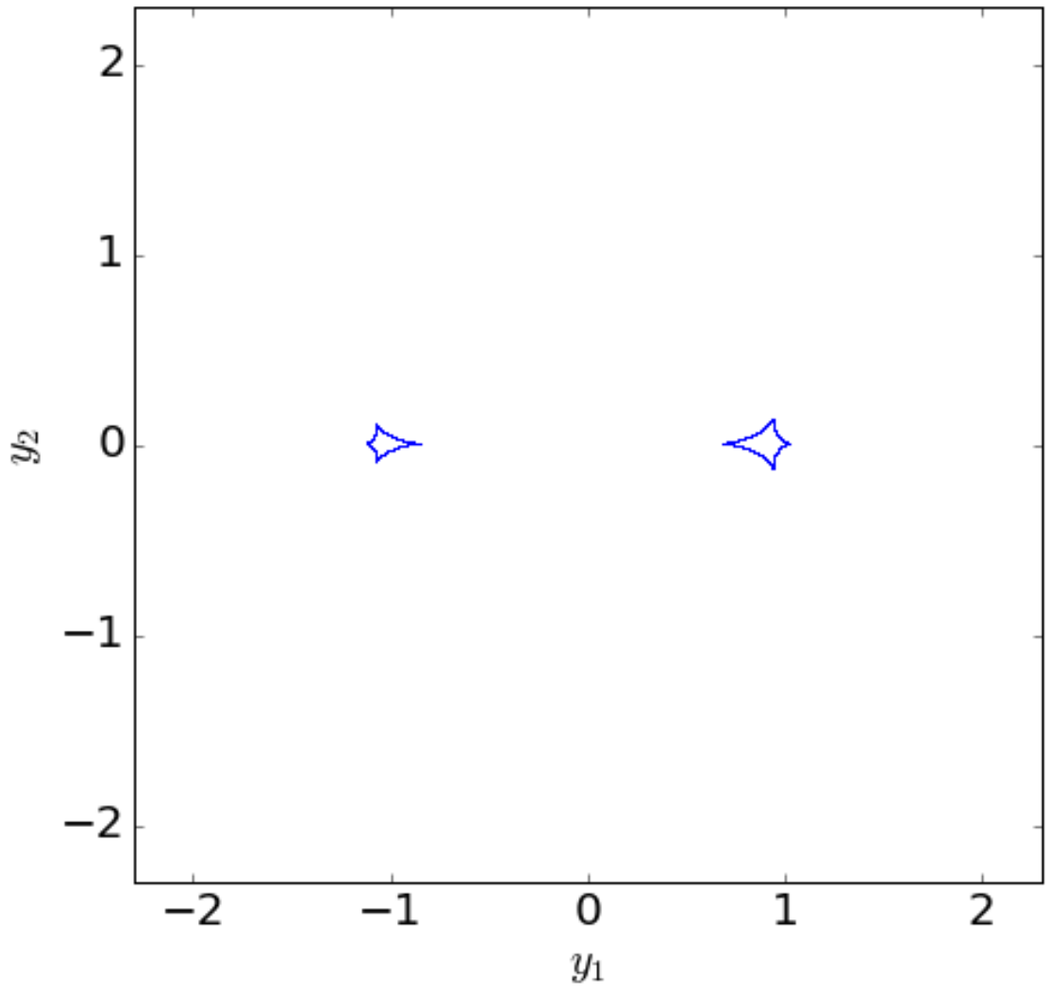


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

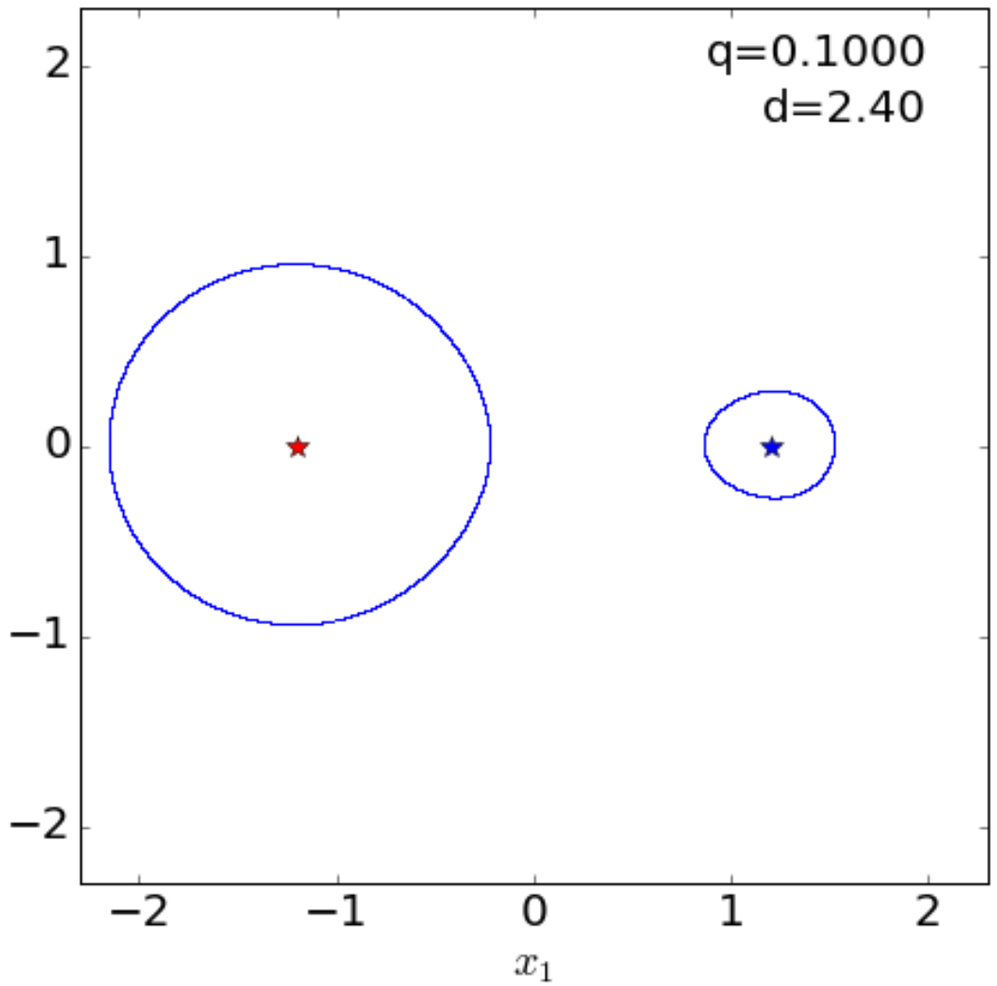


critical lines

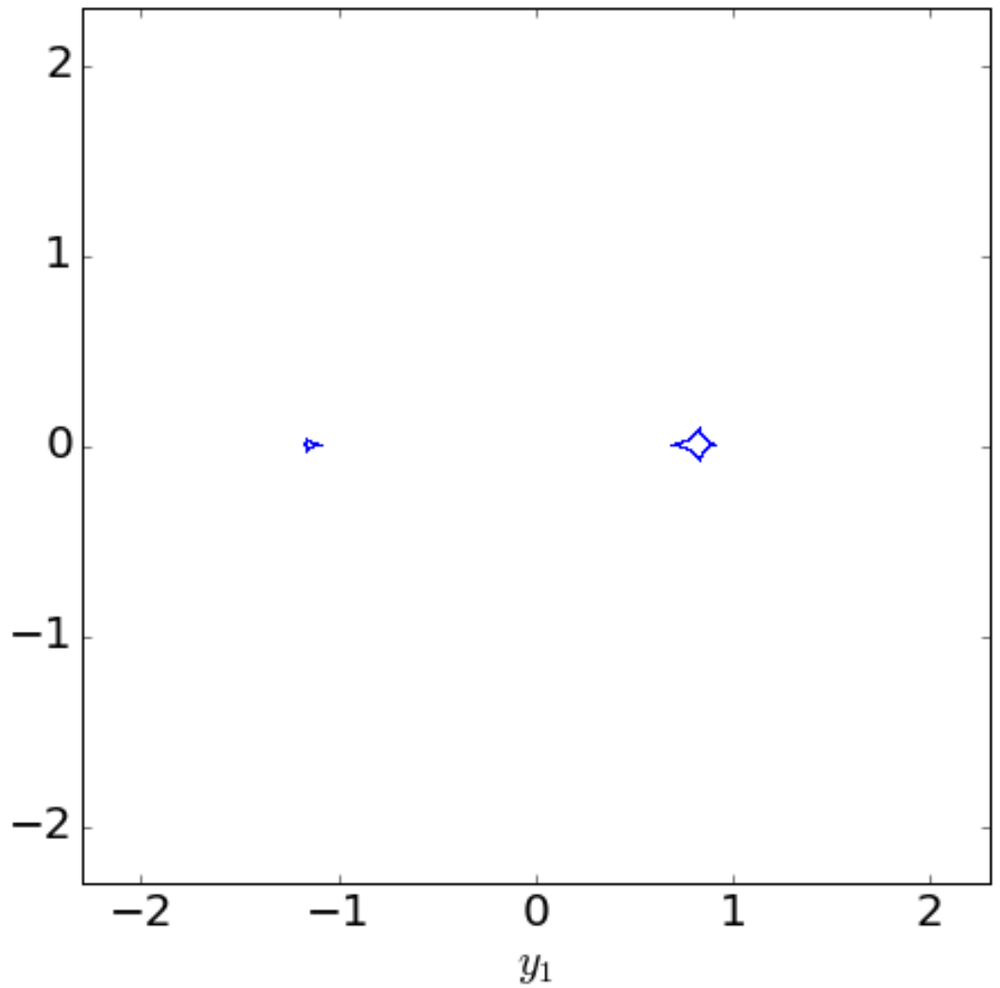


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

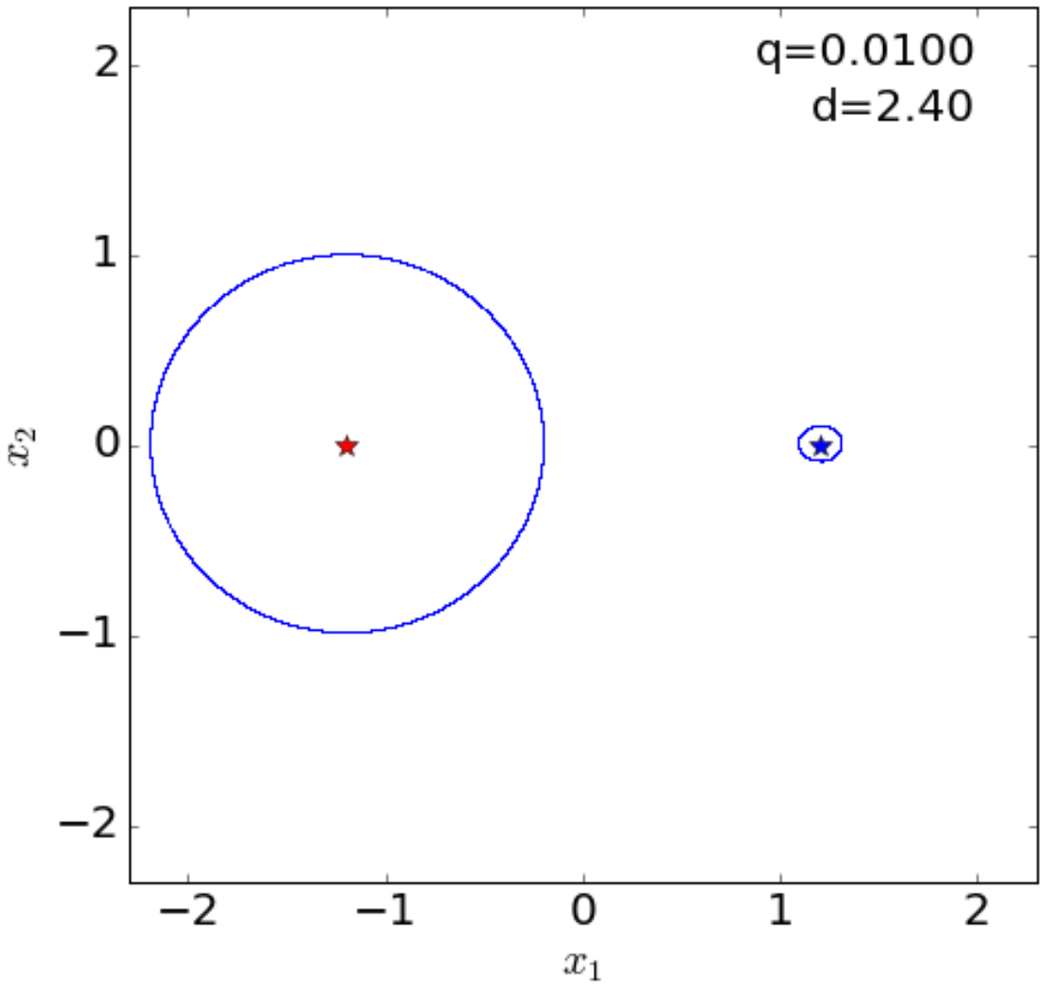


critical lines

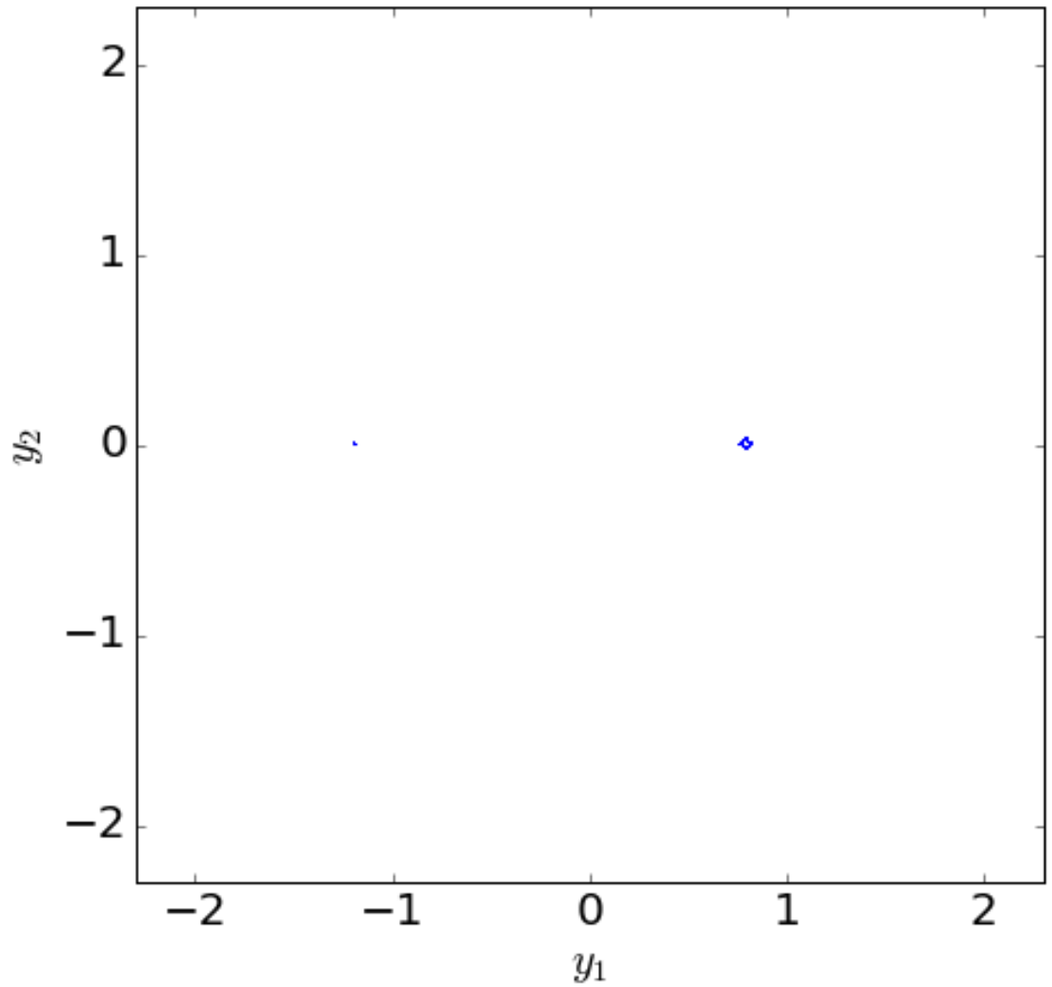


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

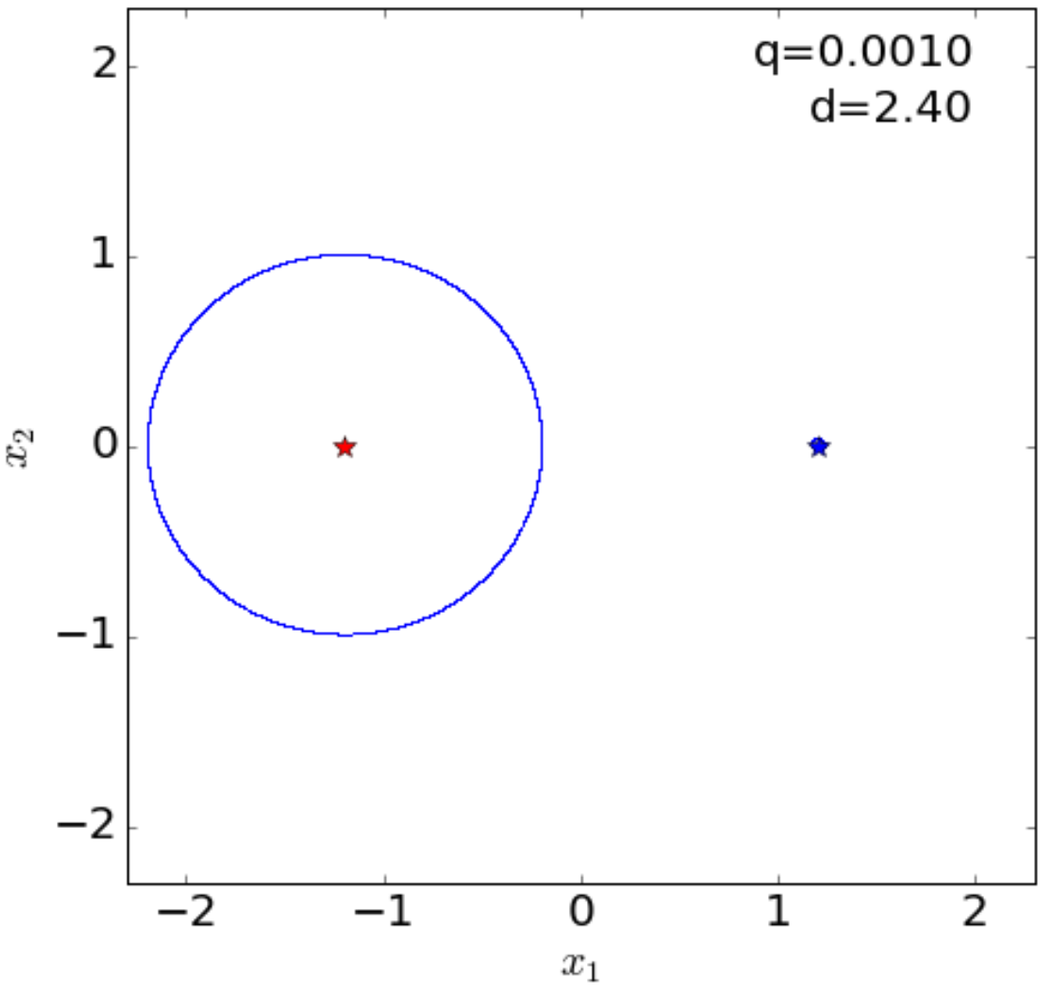


critical lines

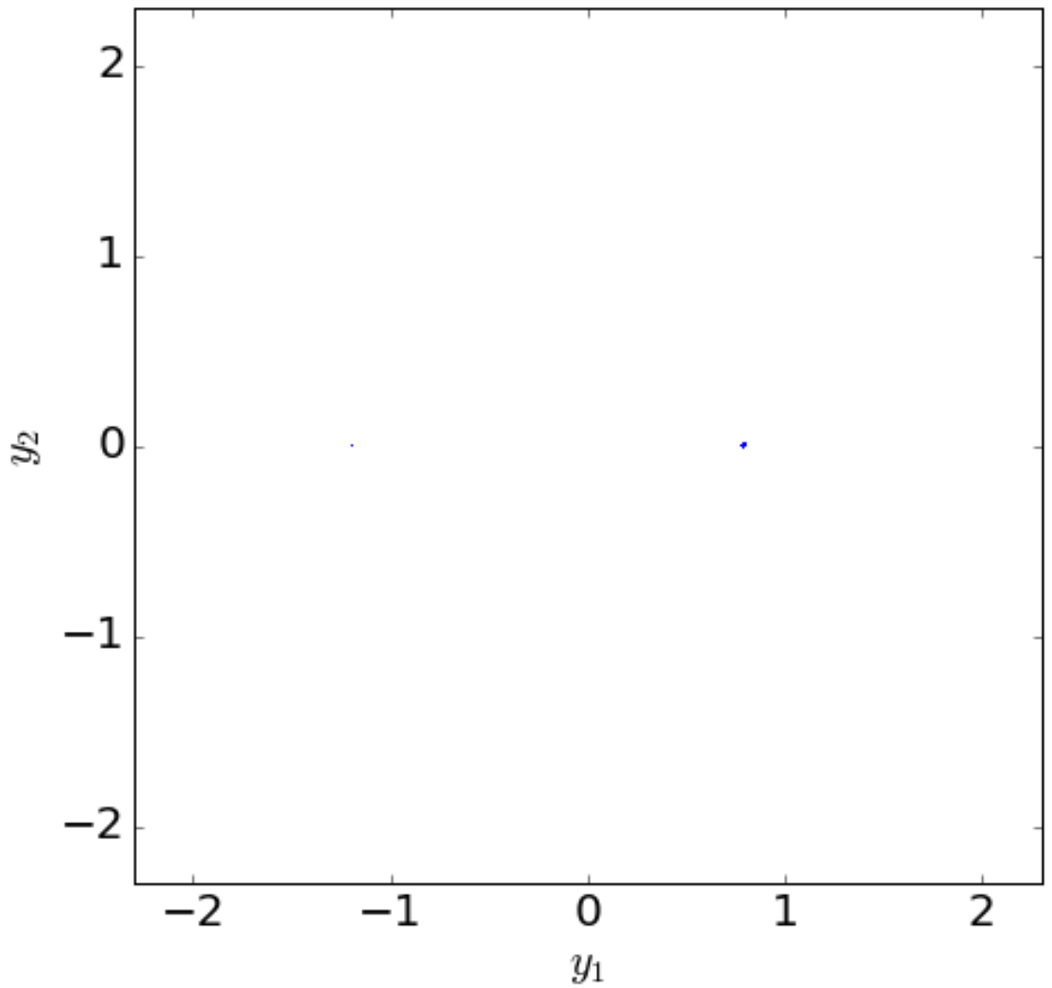


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

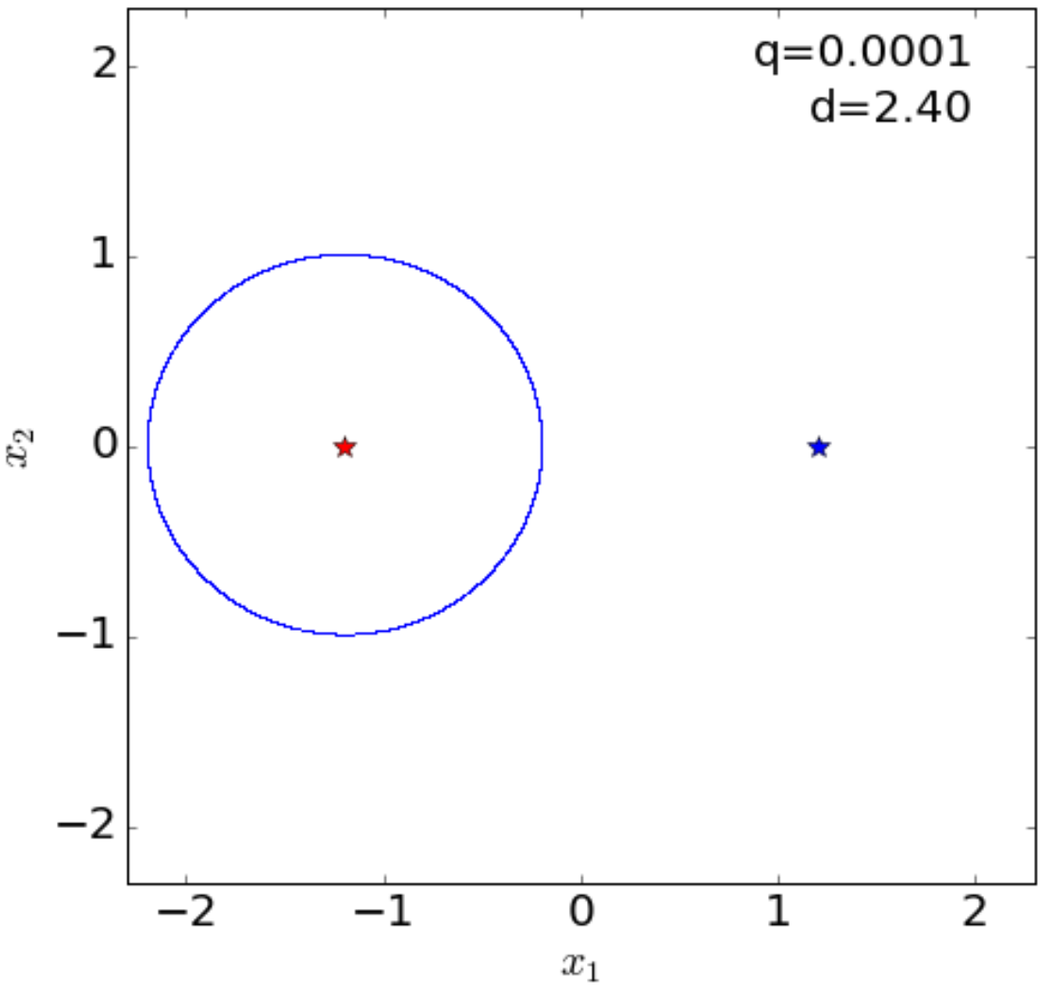


critical lines

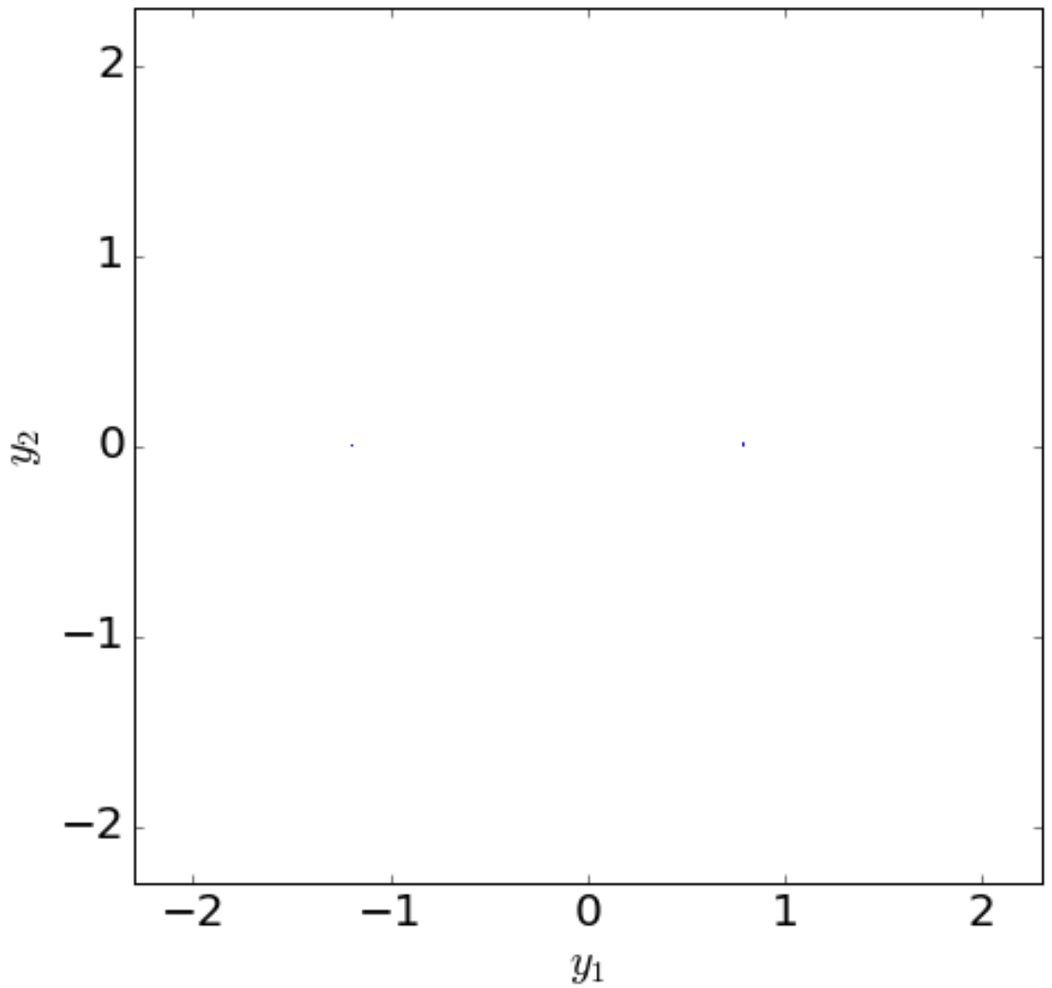


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

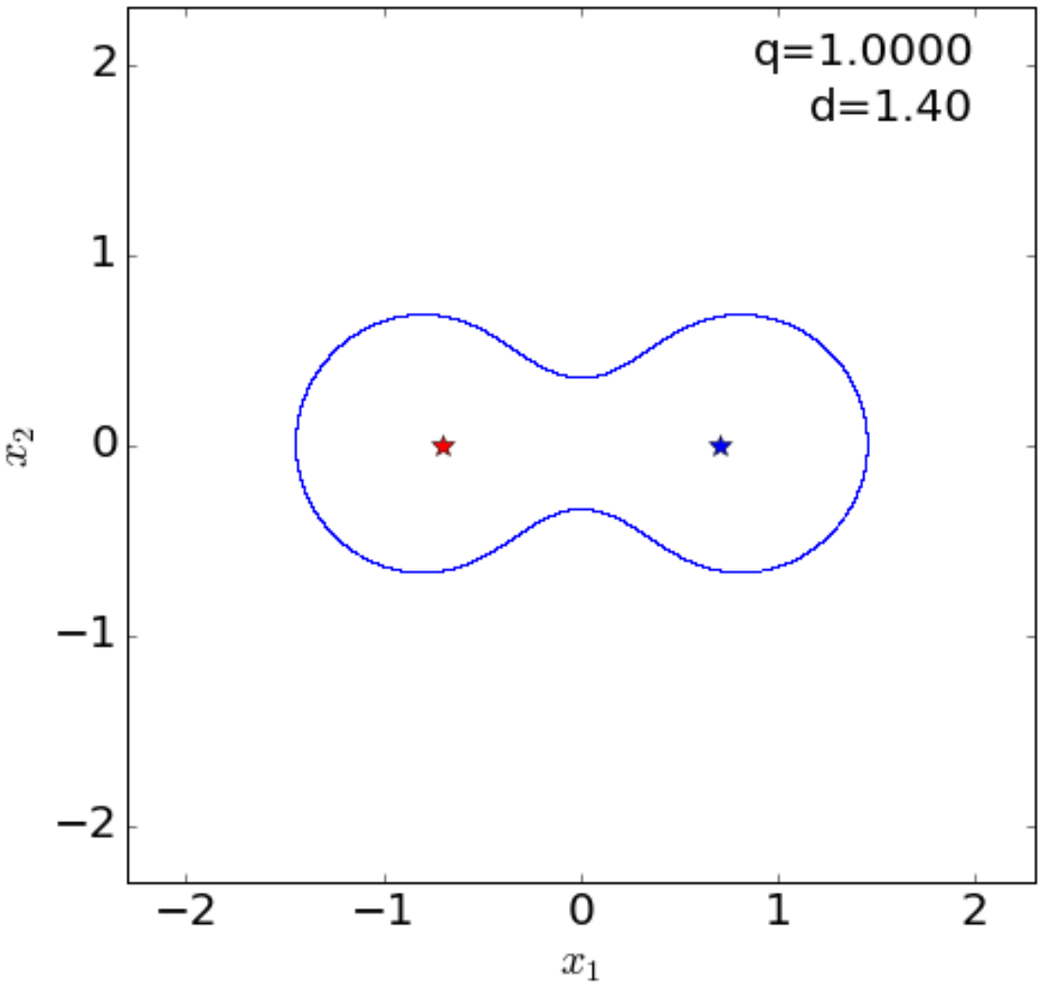


critical lines

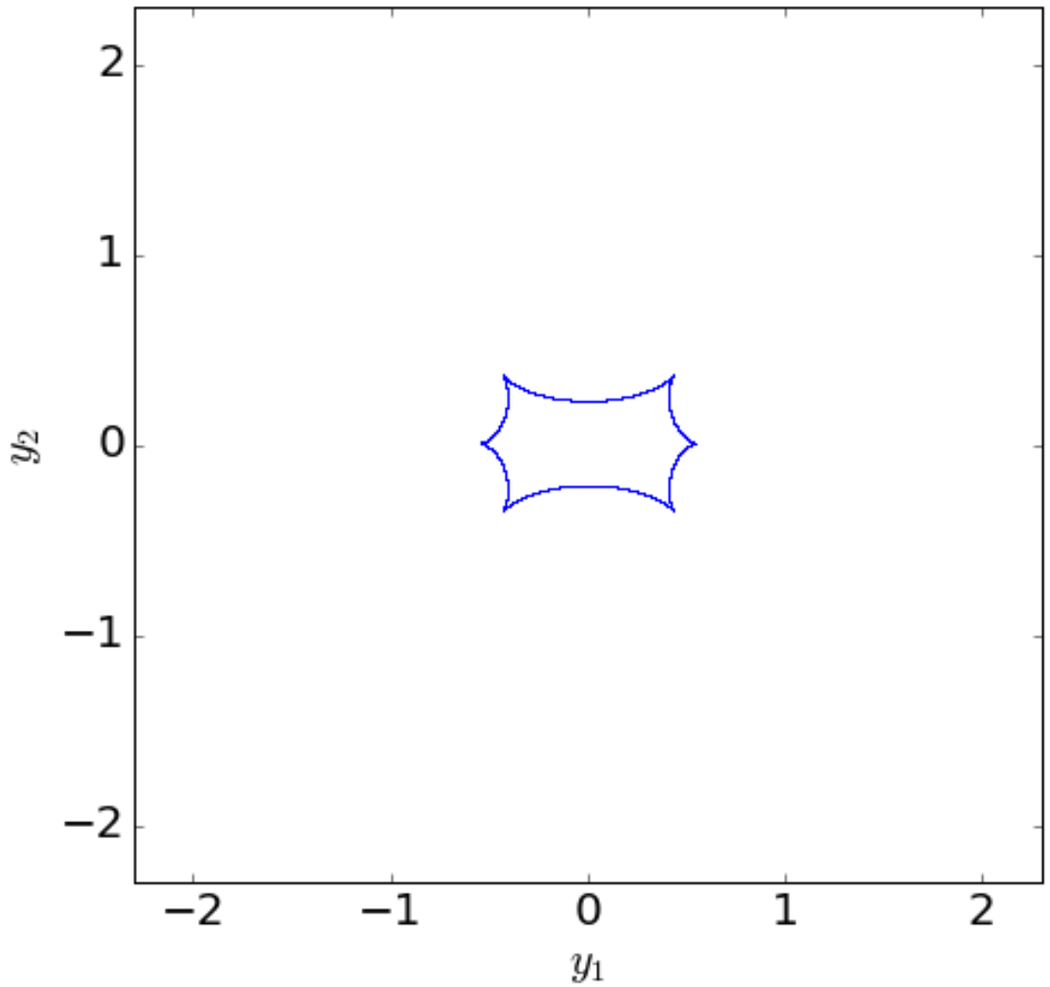


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

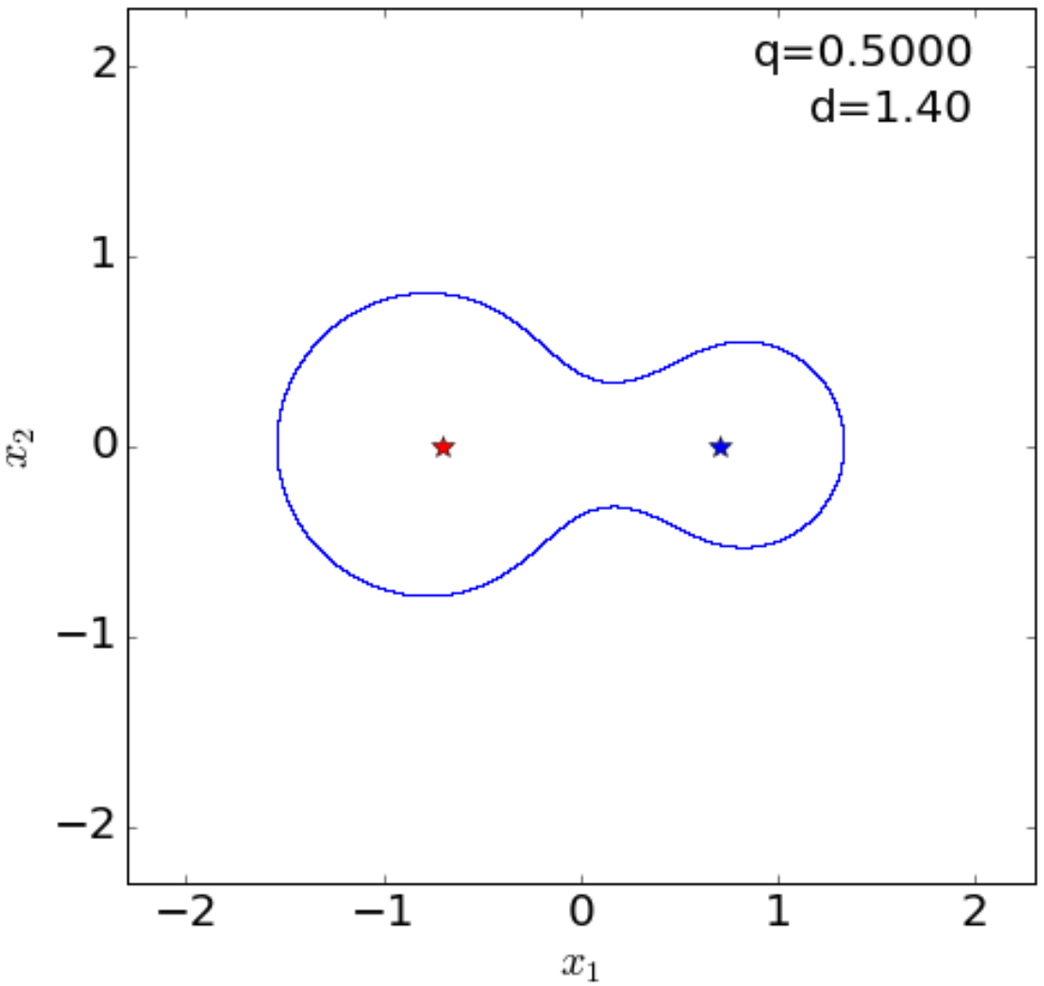


critical lines

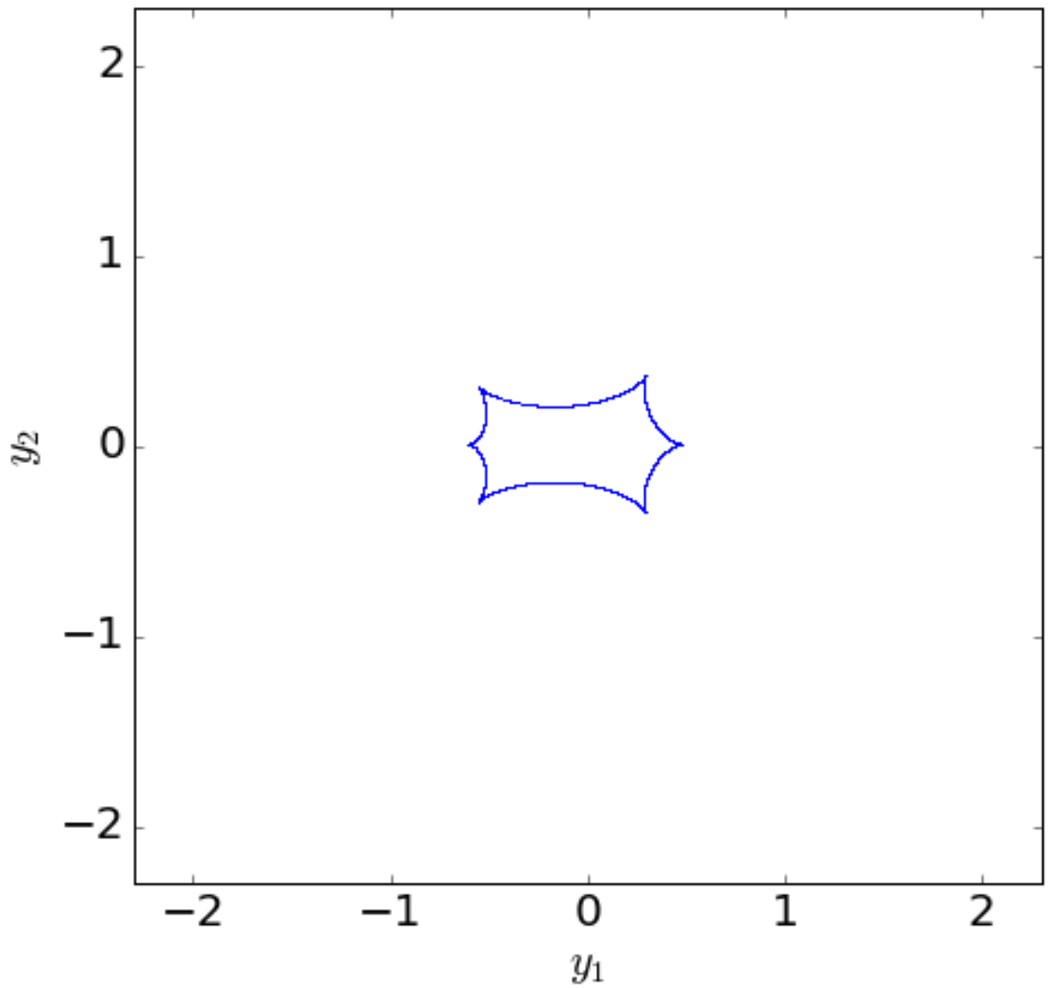


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

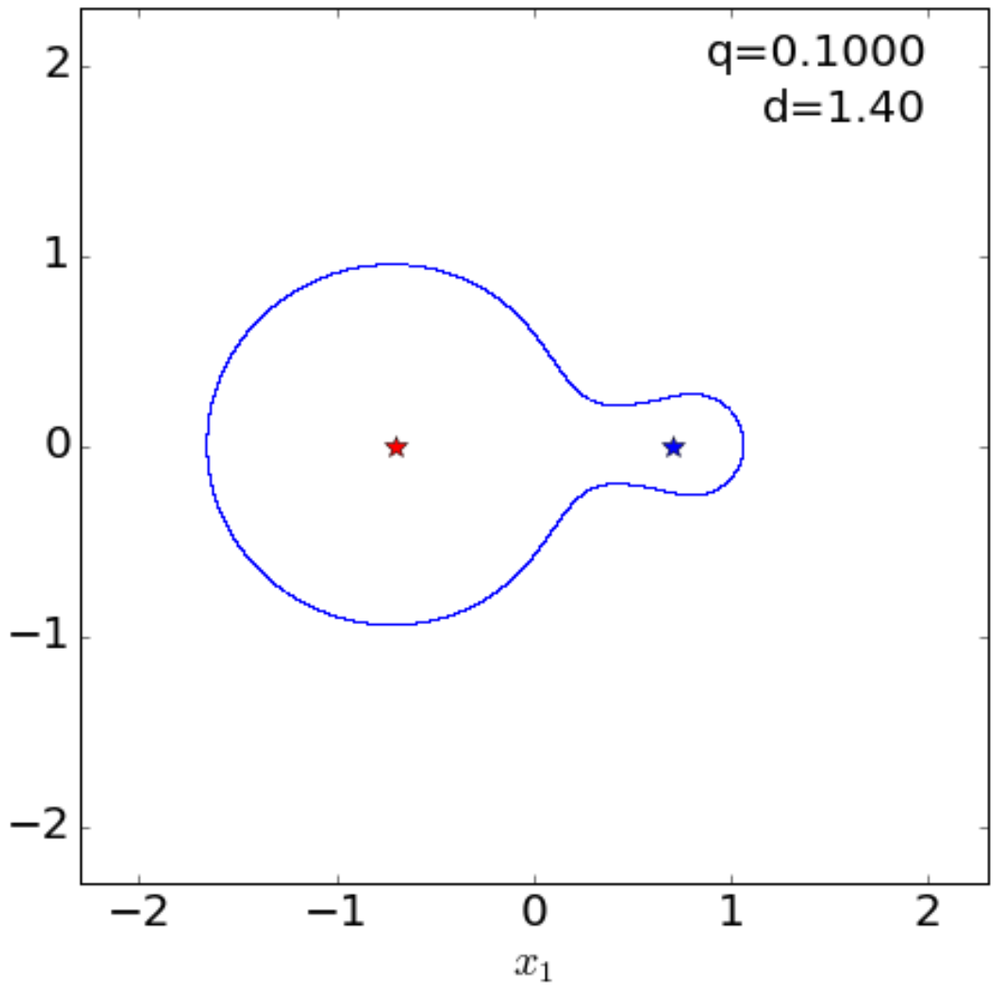


critical lines

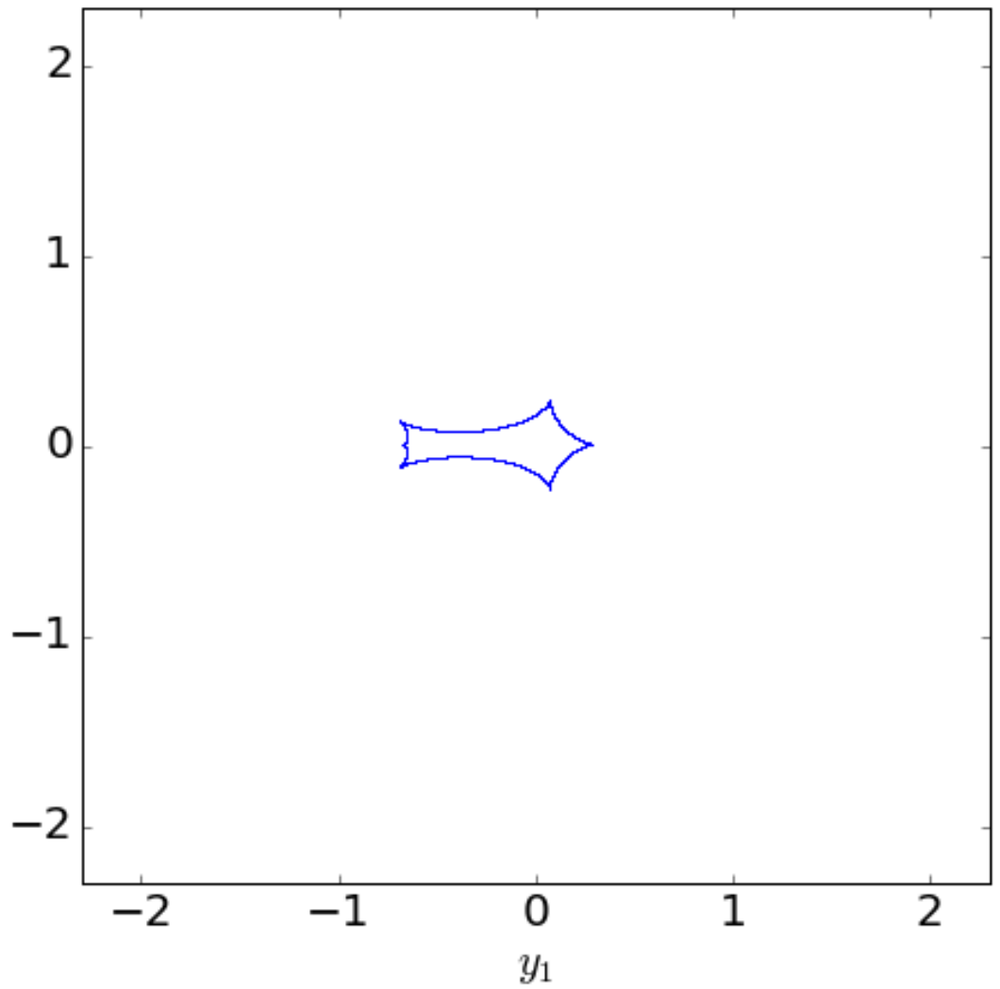


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

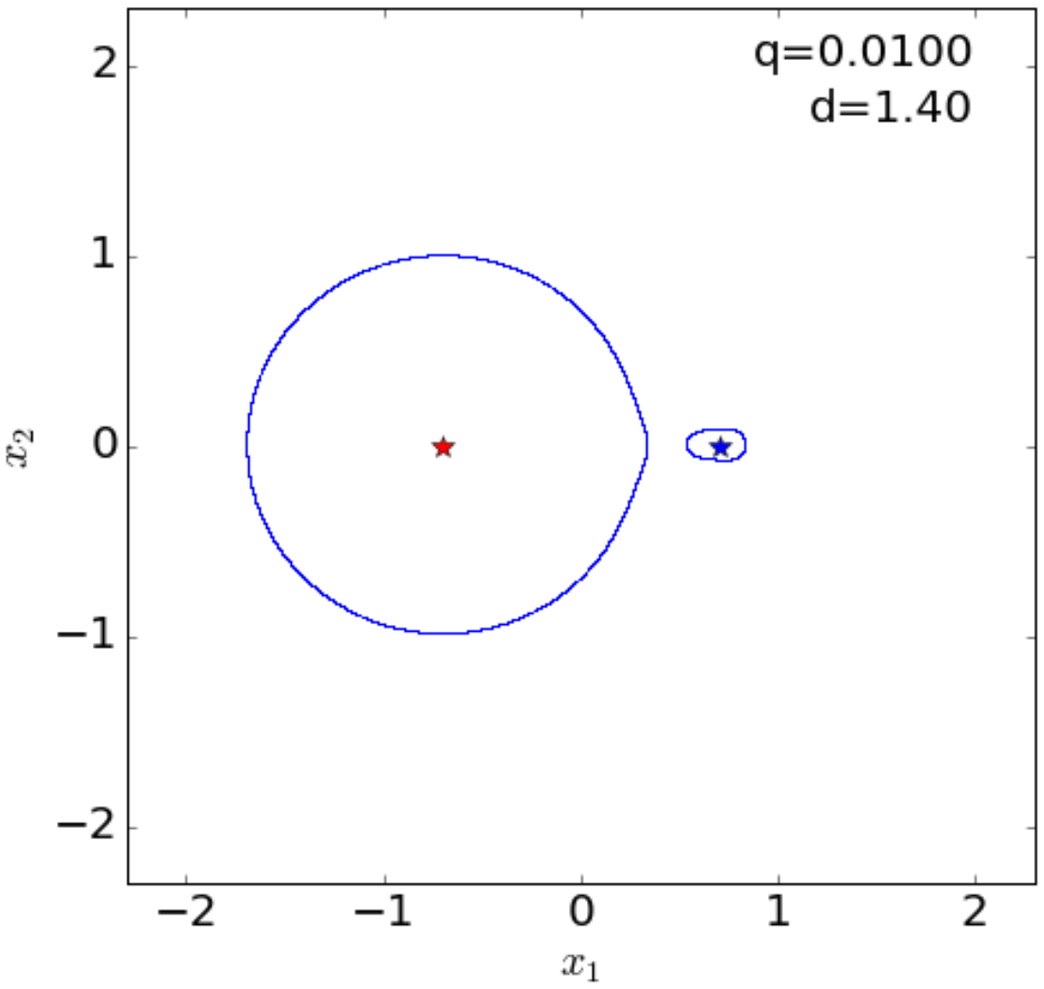


critical lines

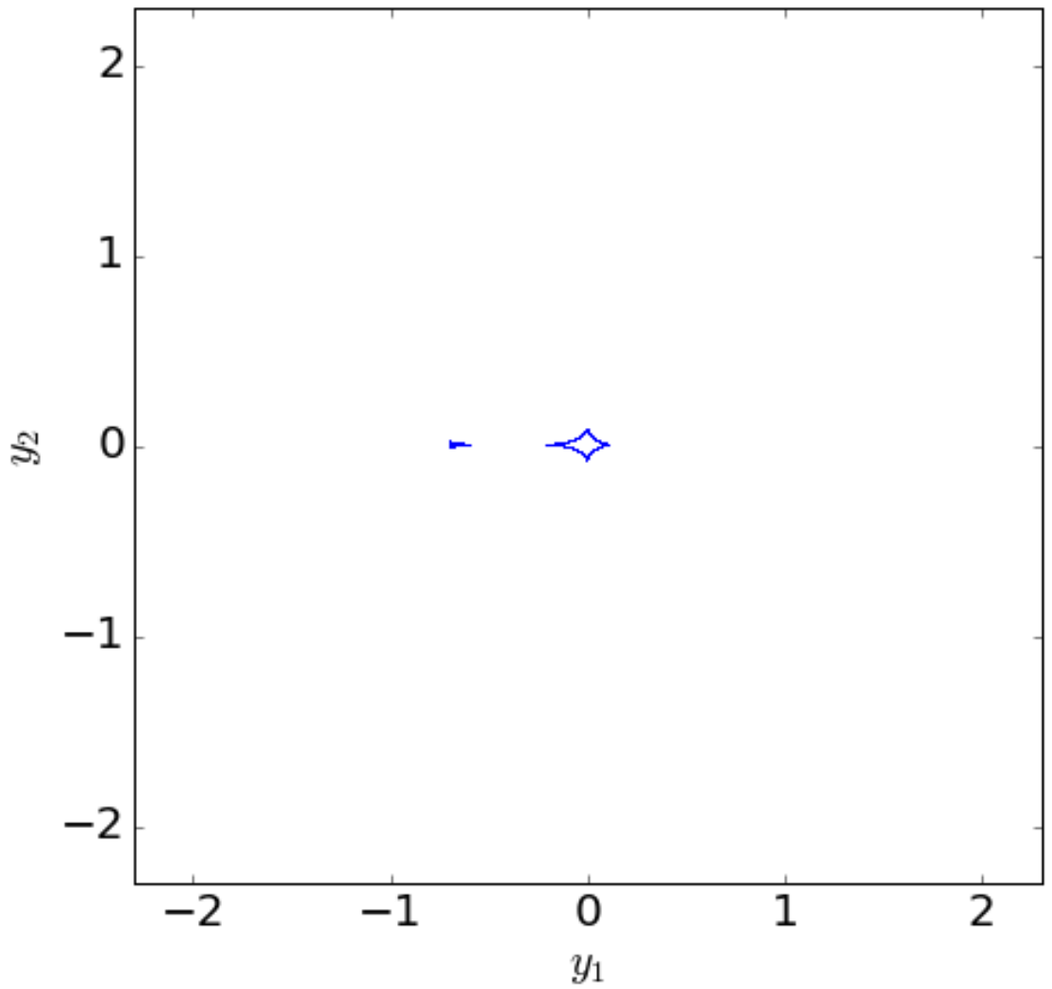


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

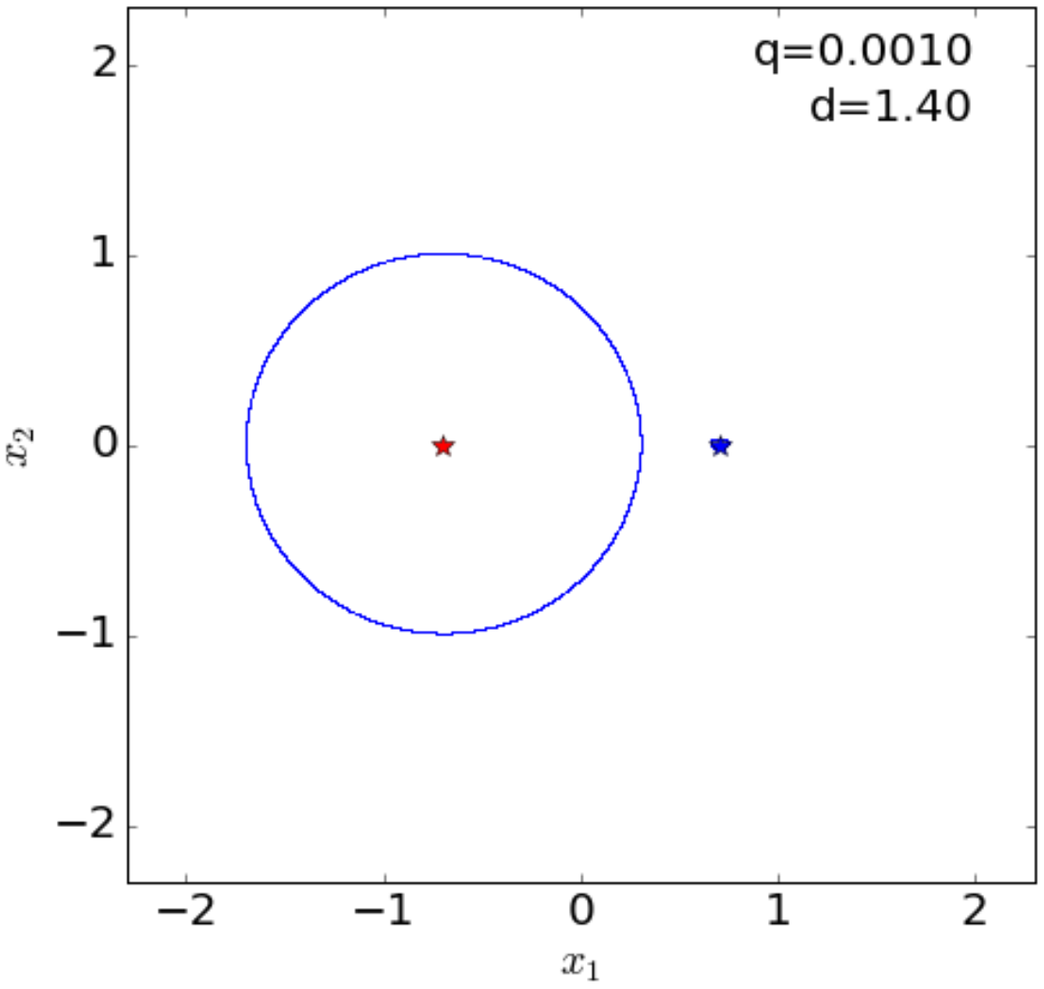


critical lines

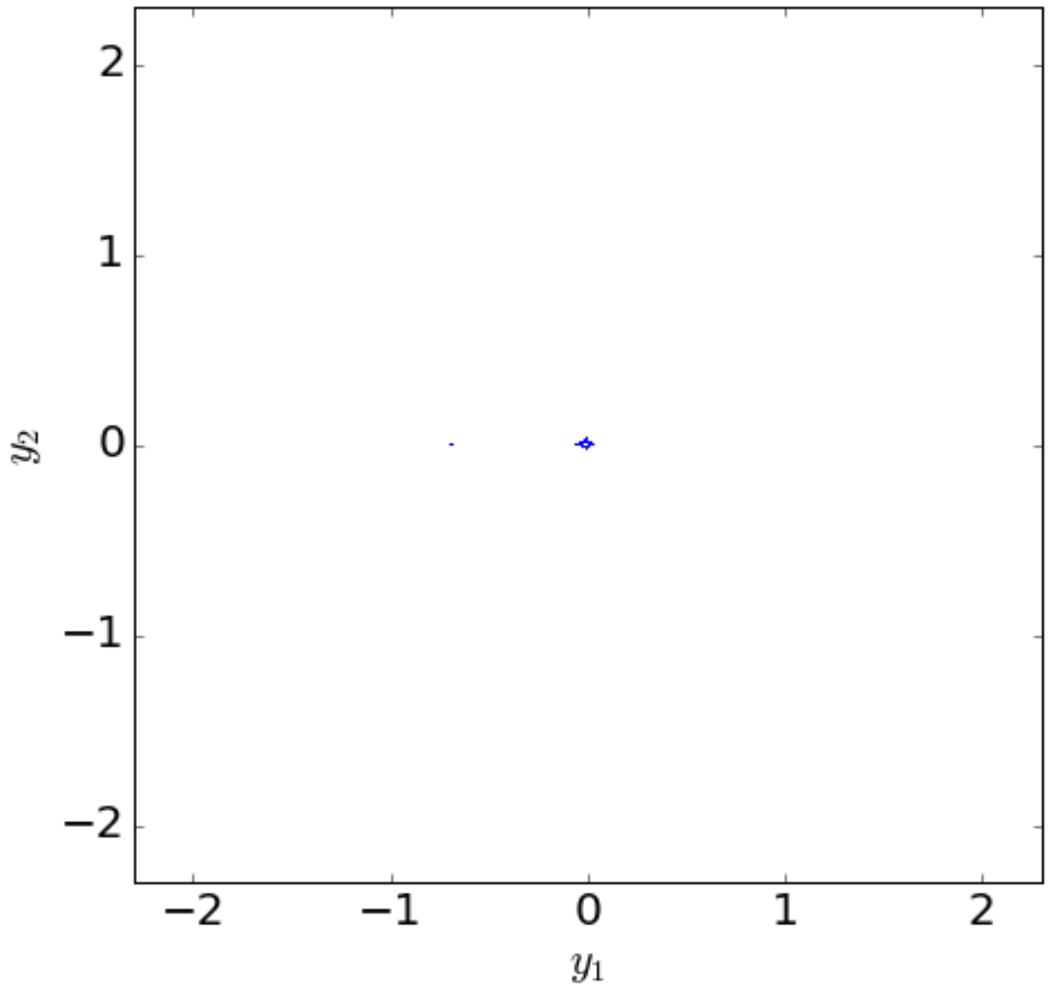


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

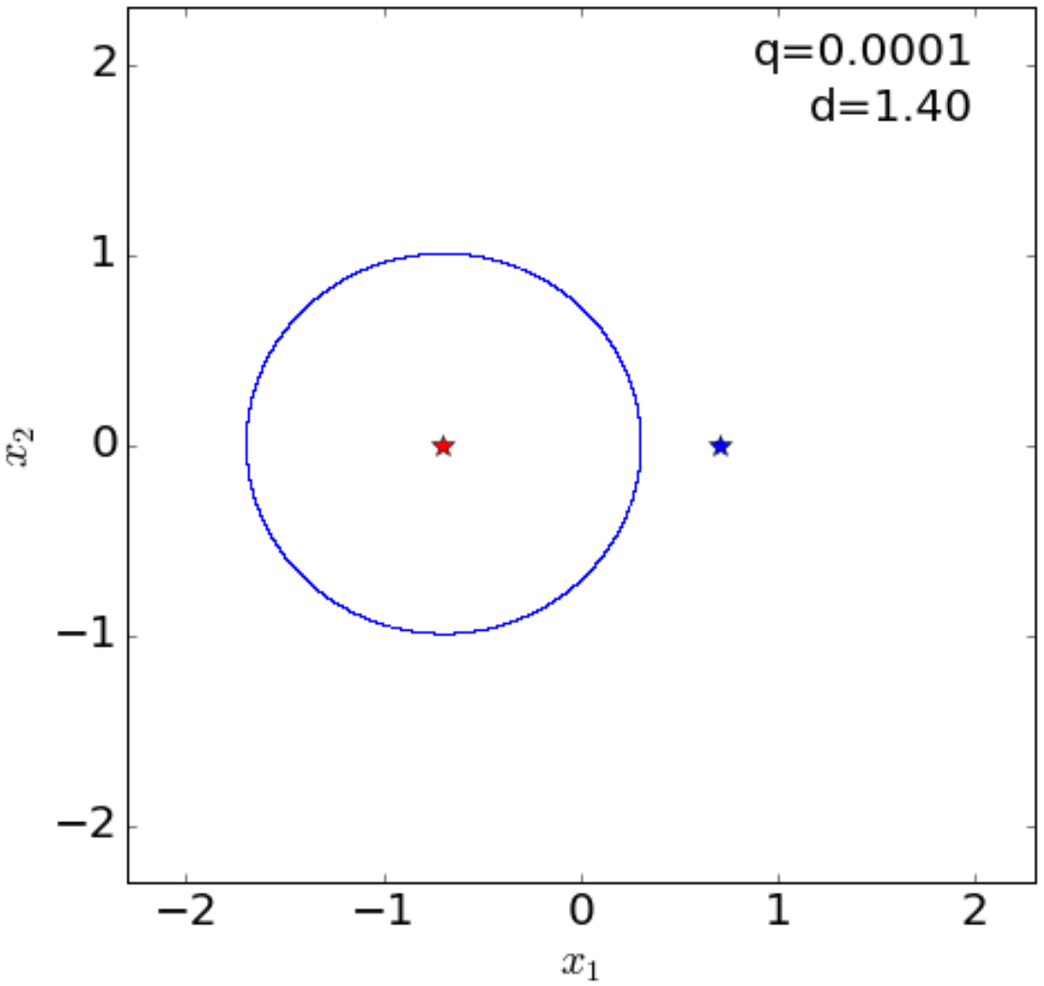


critical lines

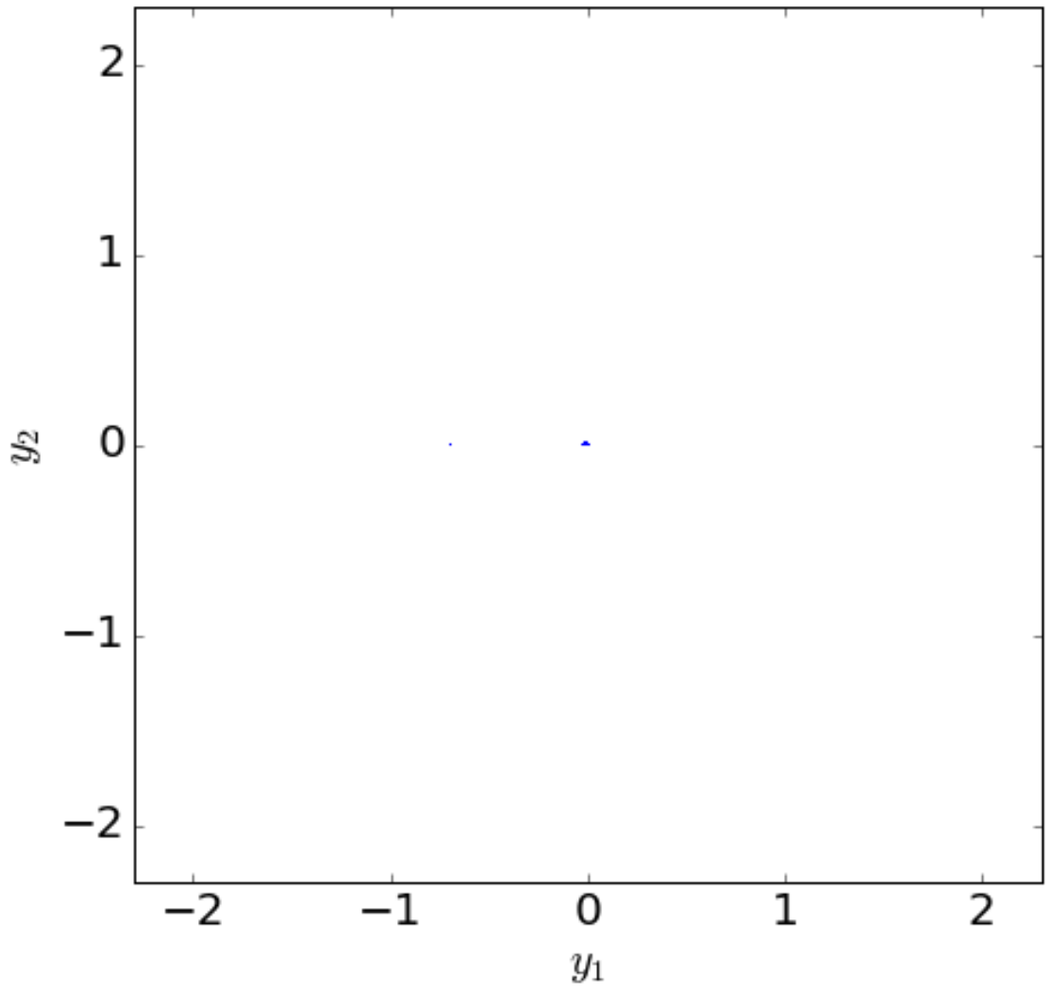


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

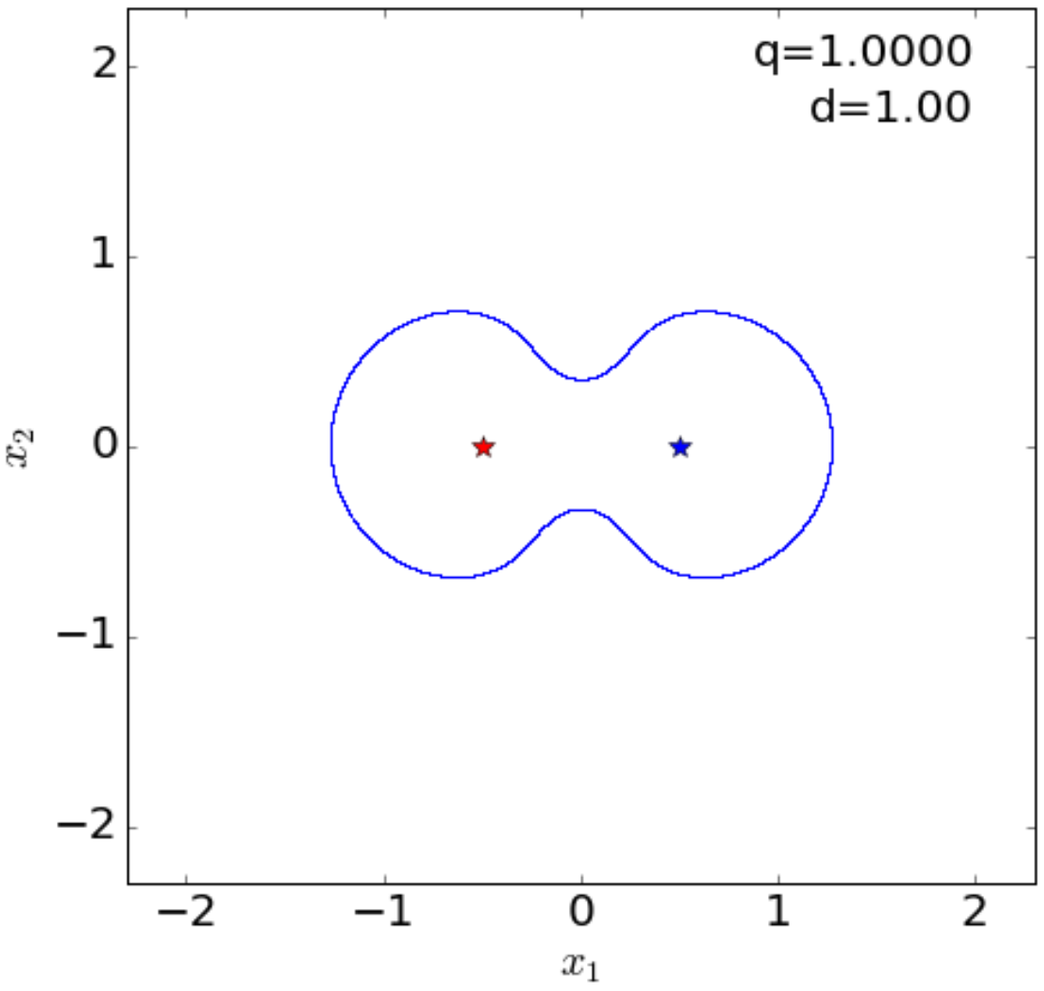


critical lines

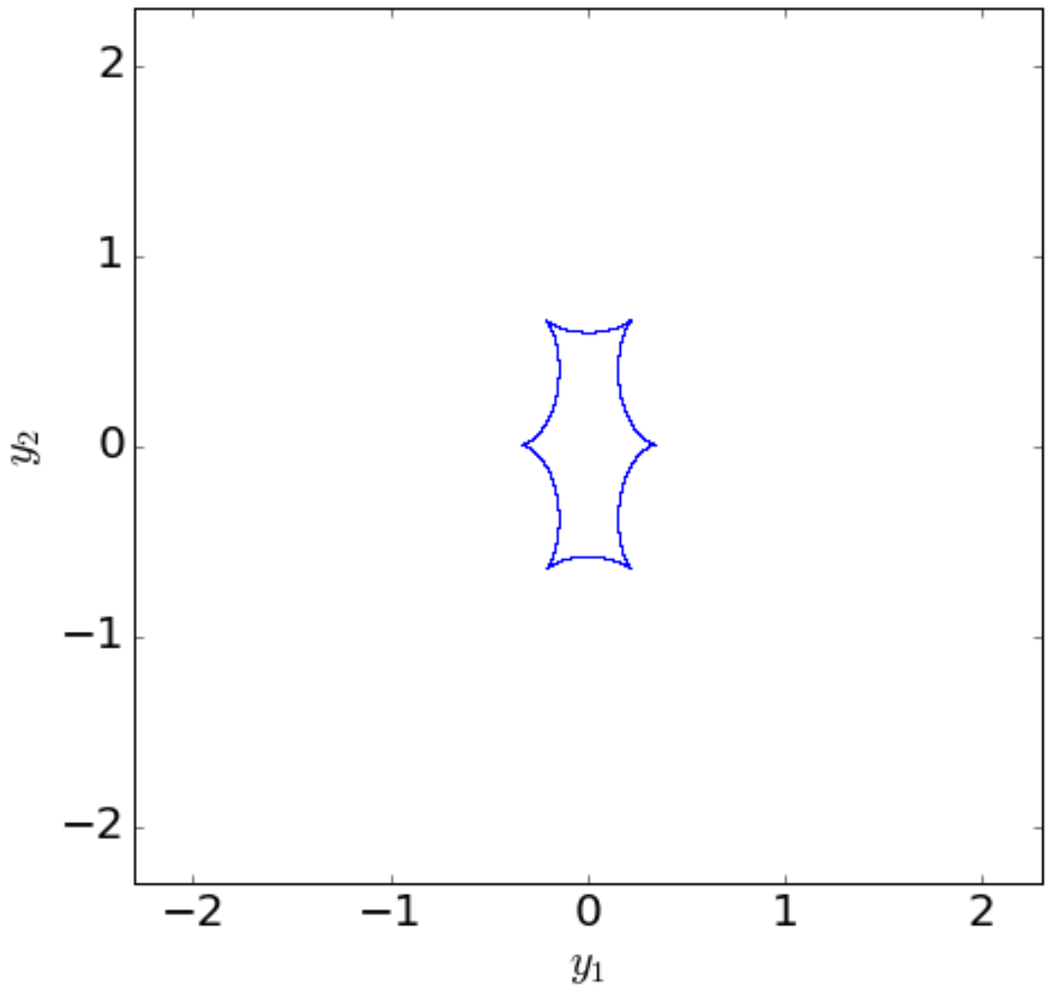


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

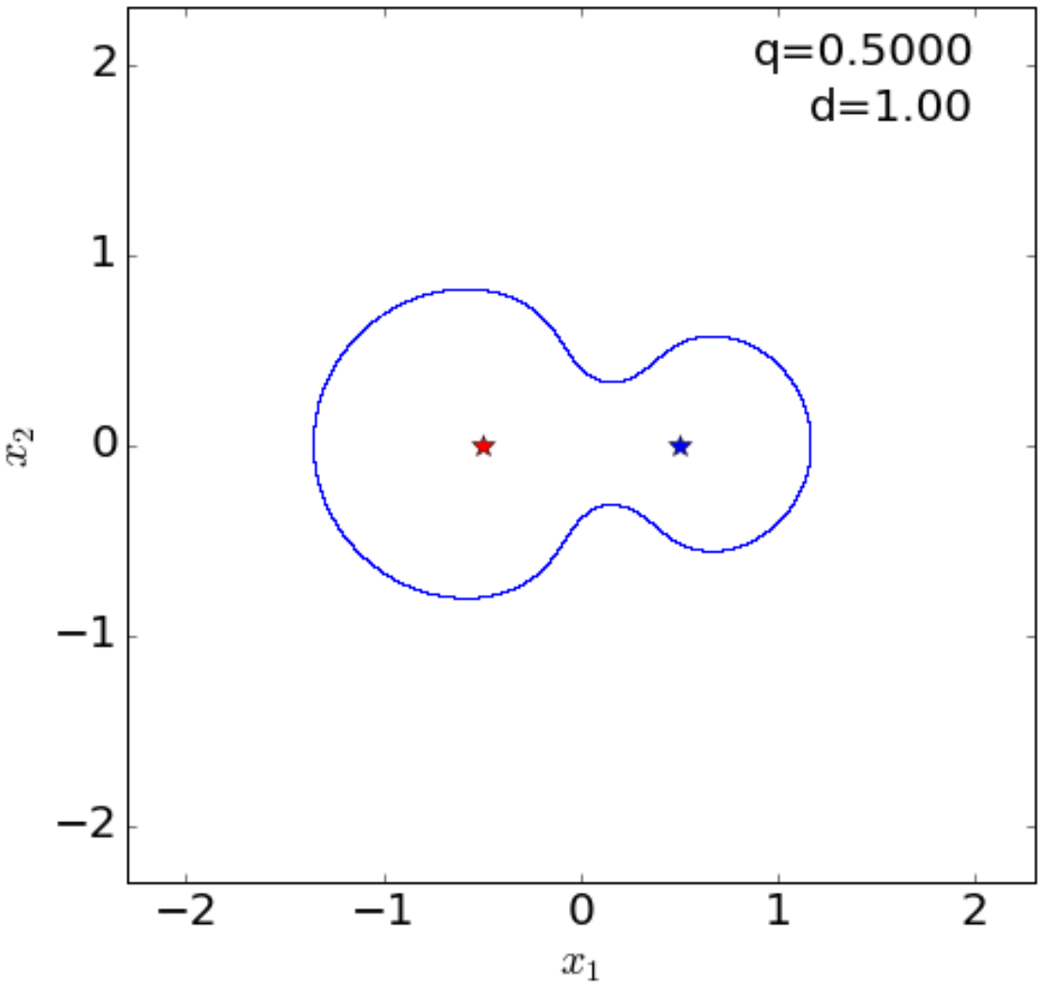


critical lines

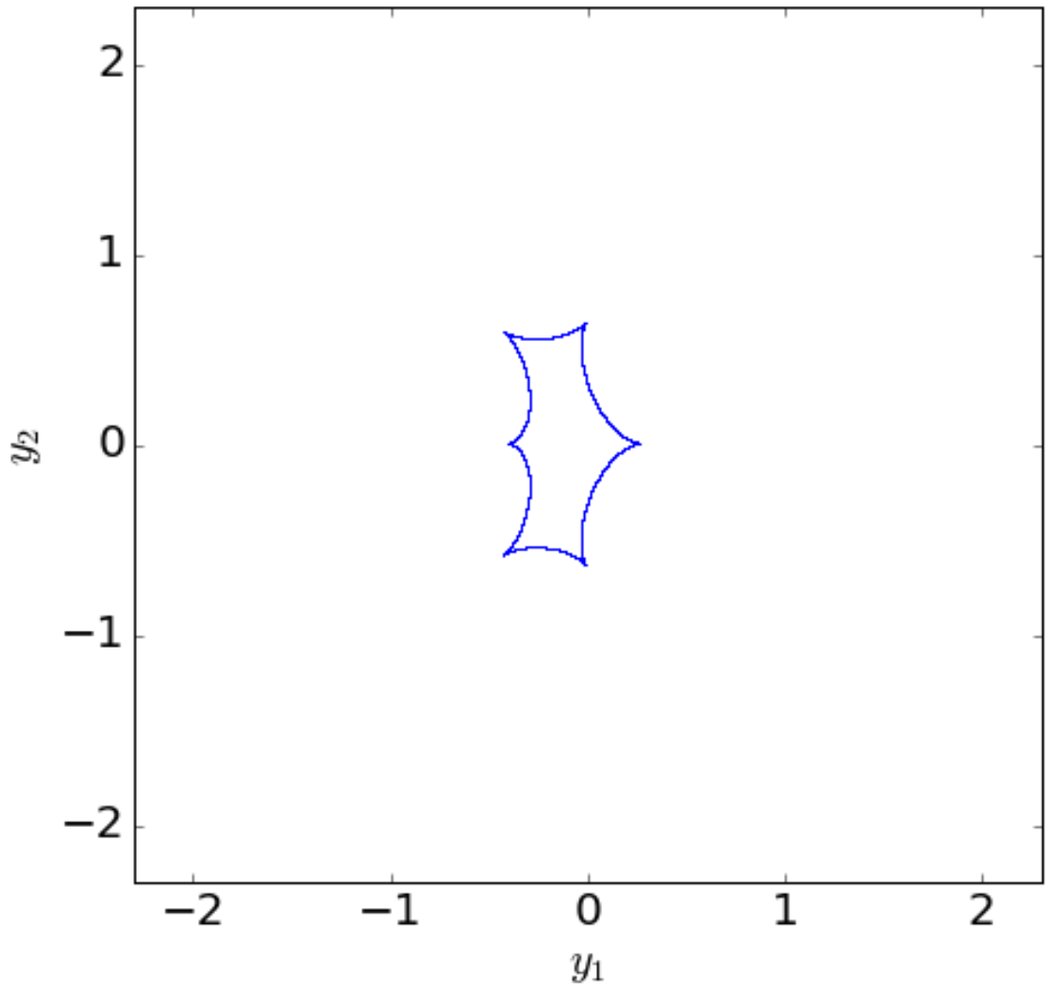


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

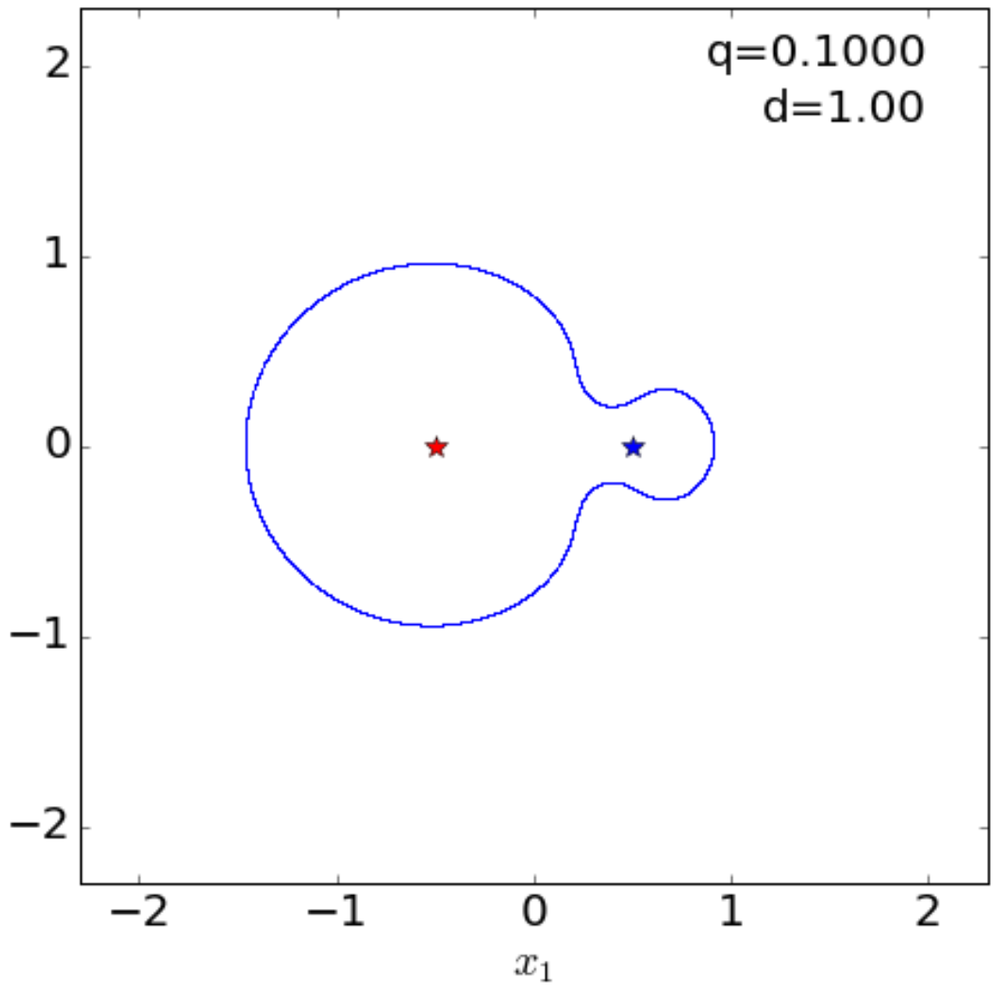


critical lines

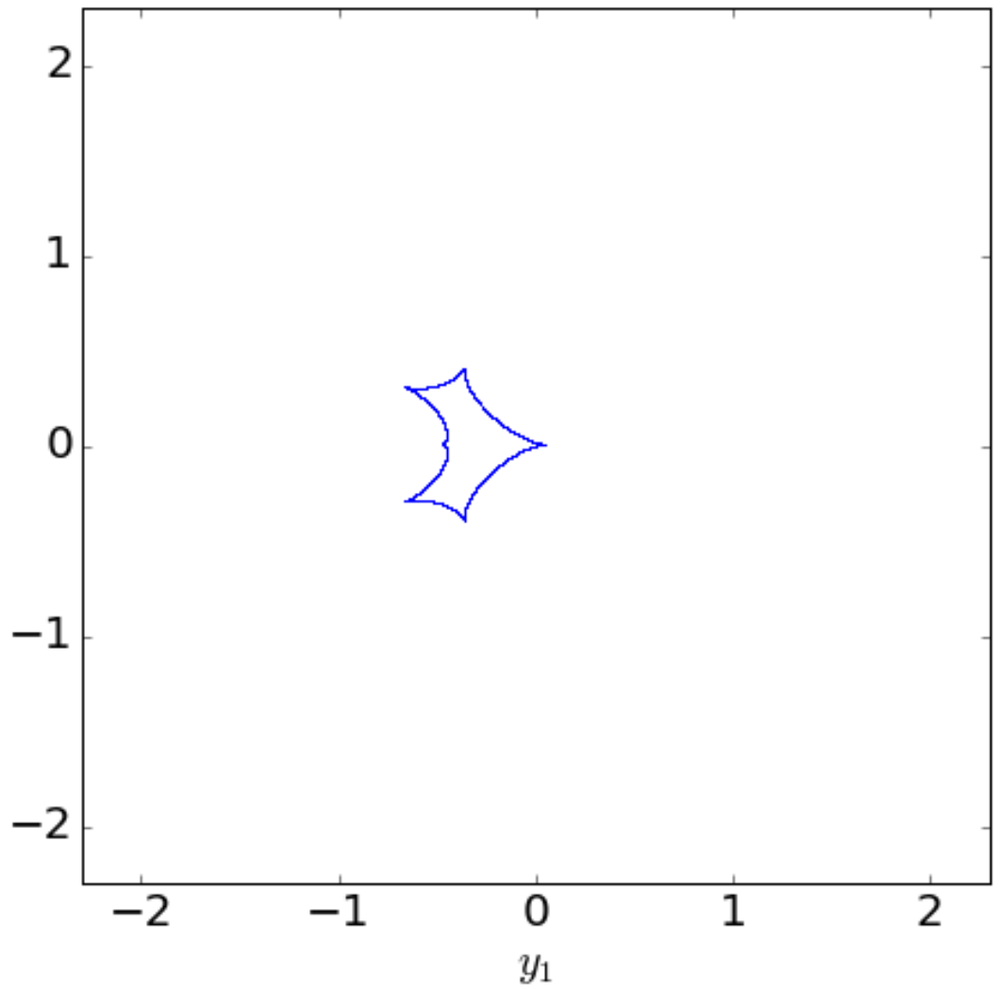


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

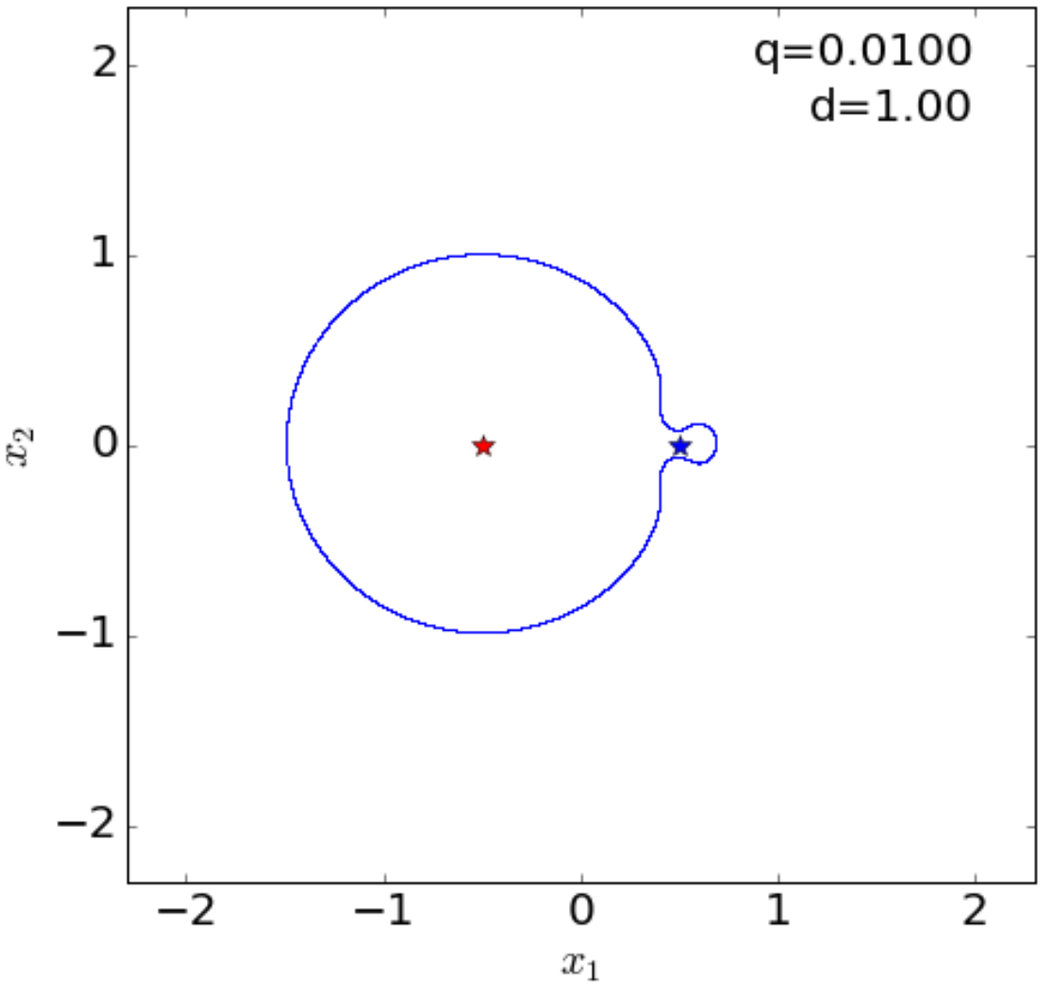


critical lines

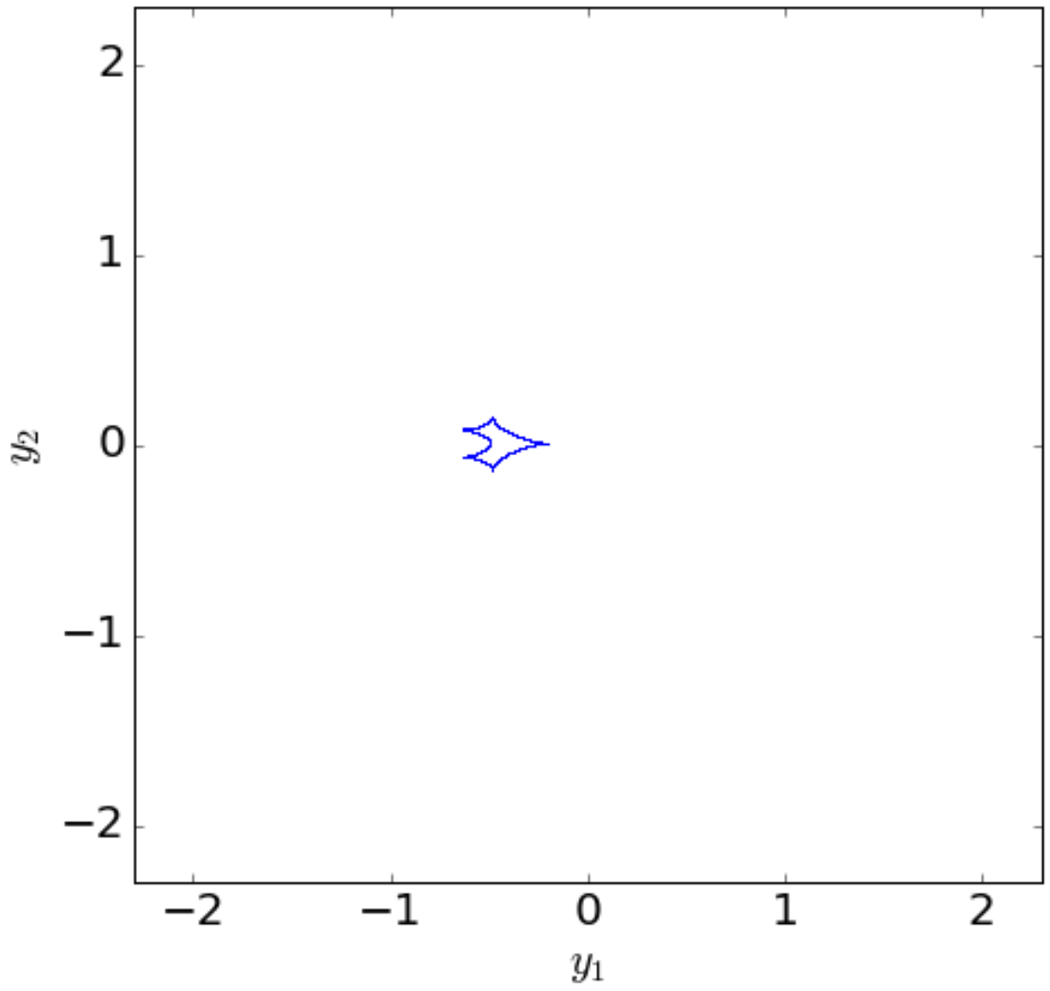


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

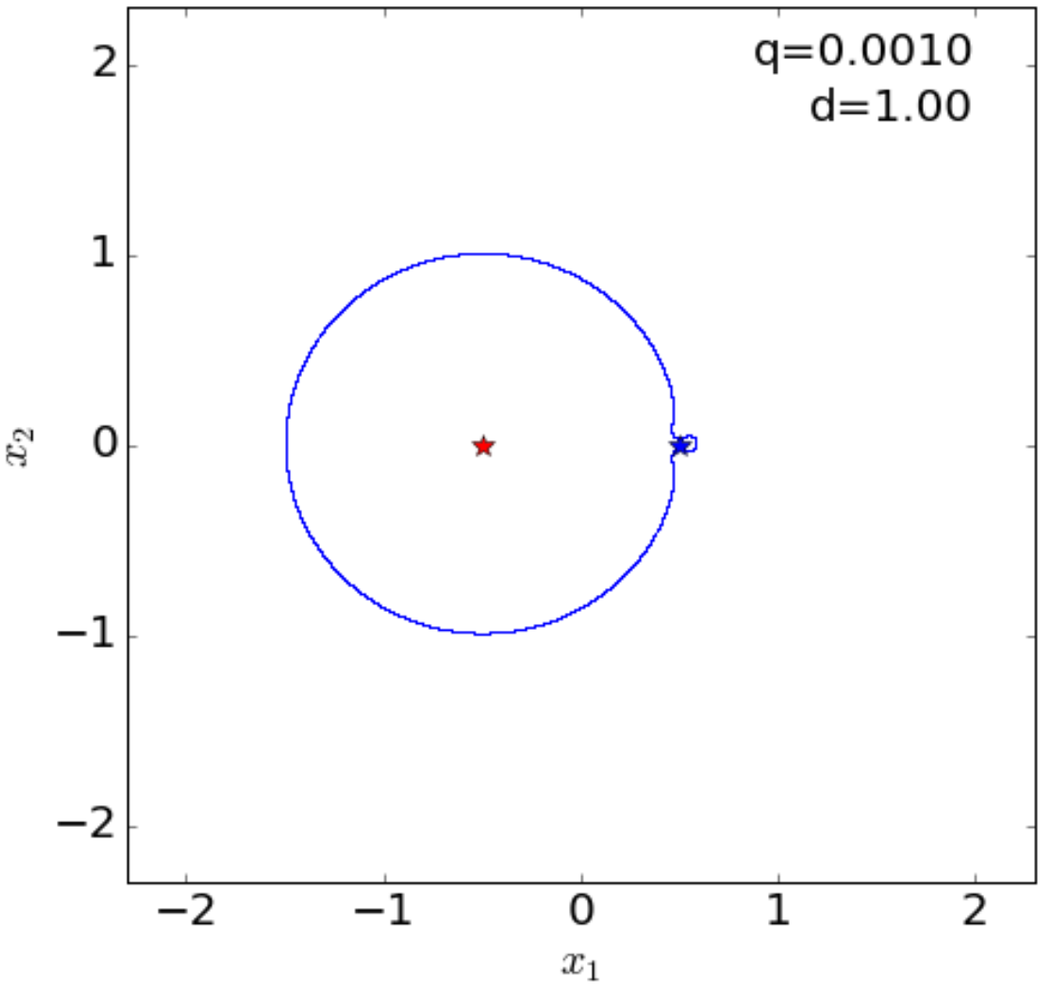


critical lines

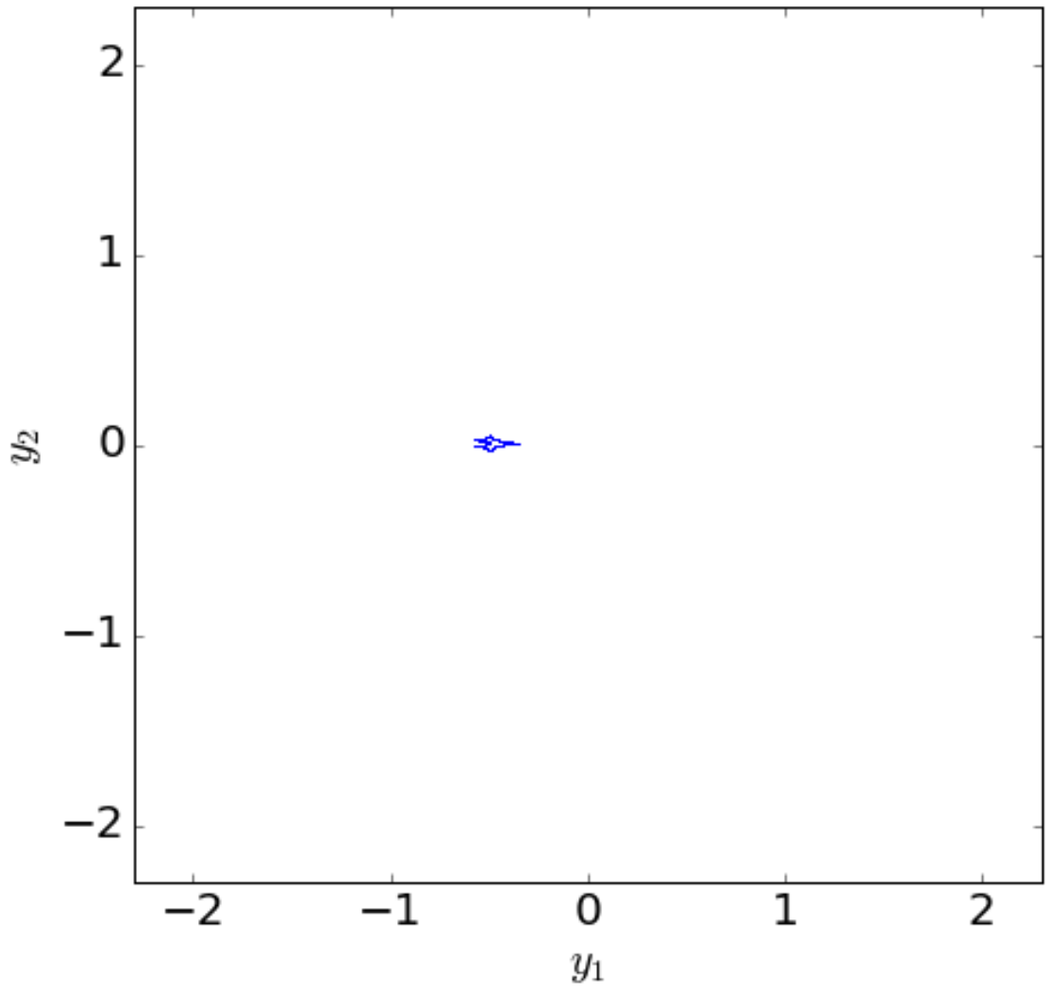


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

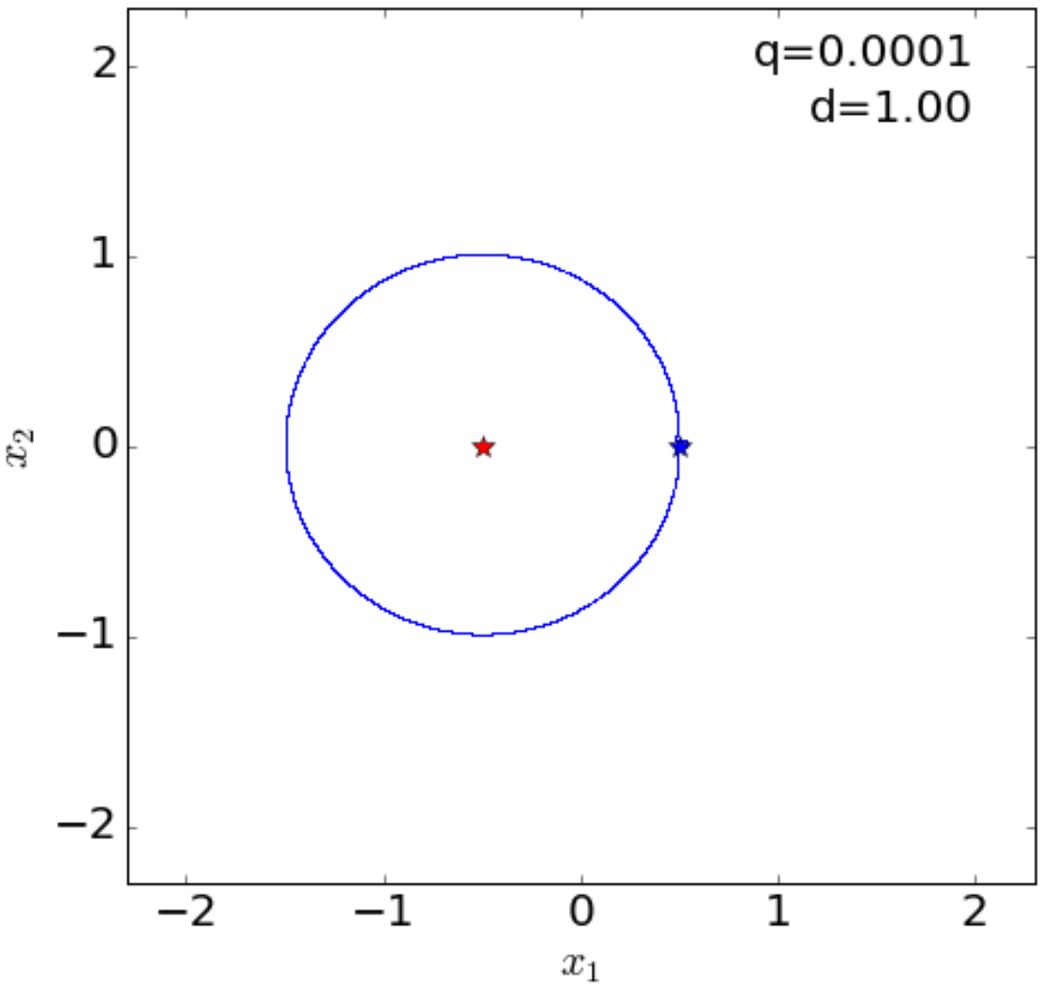


critical lines

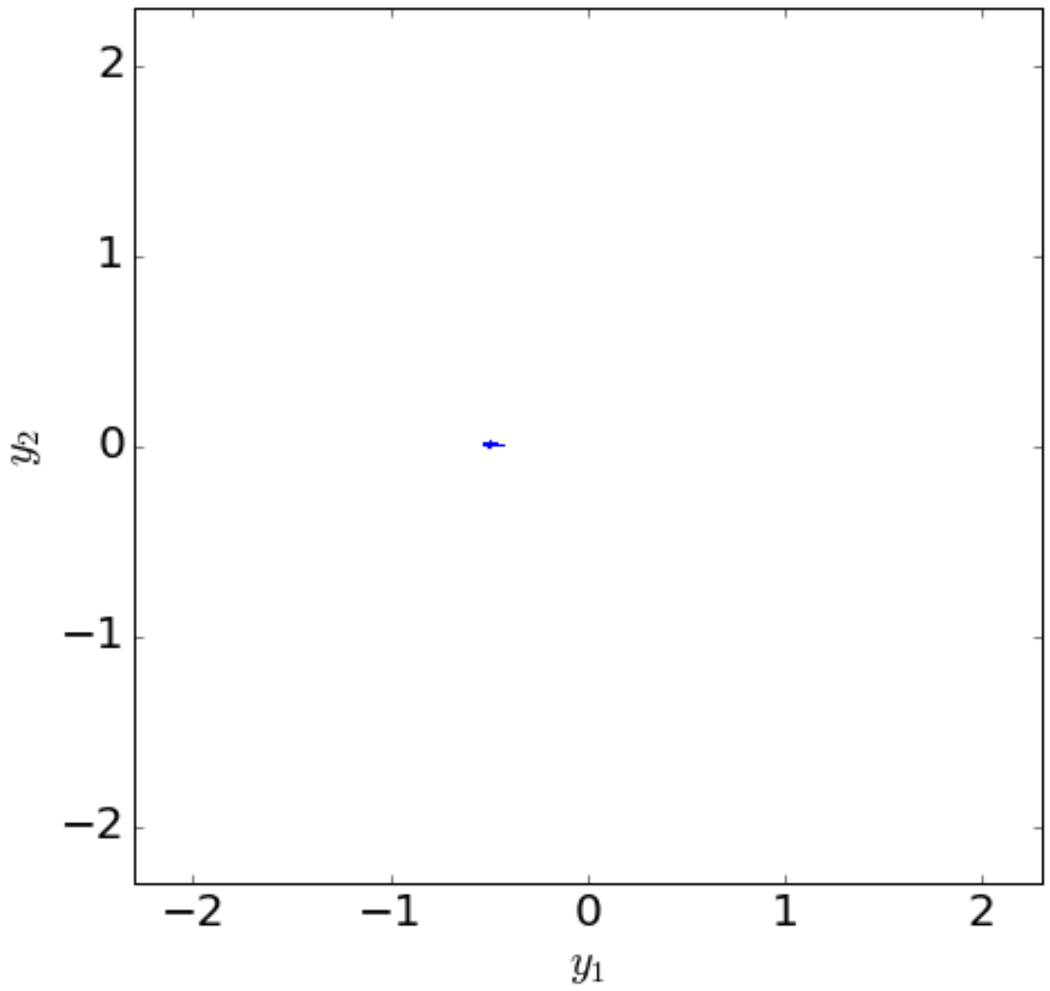


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

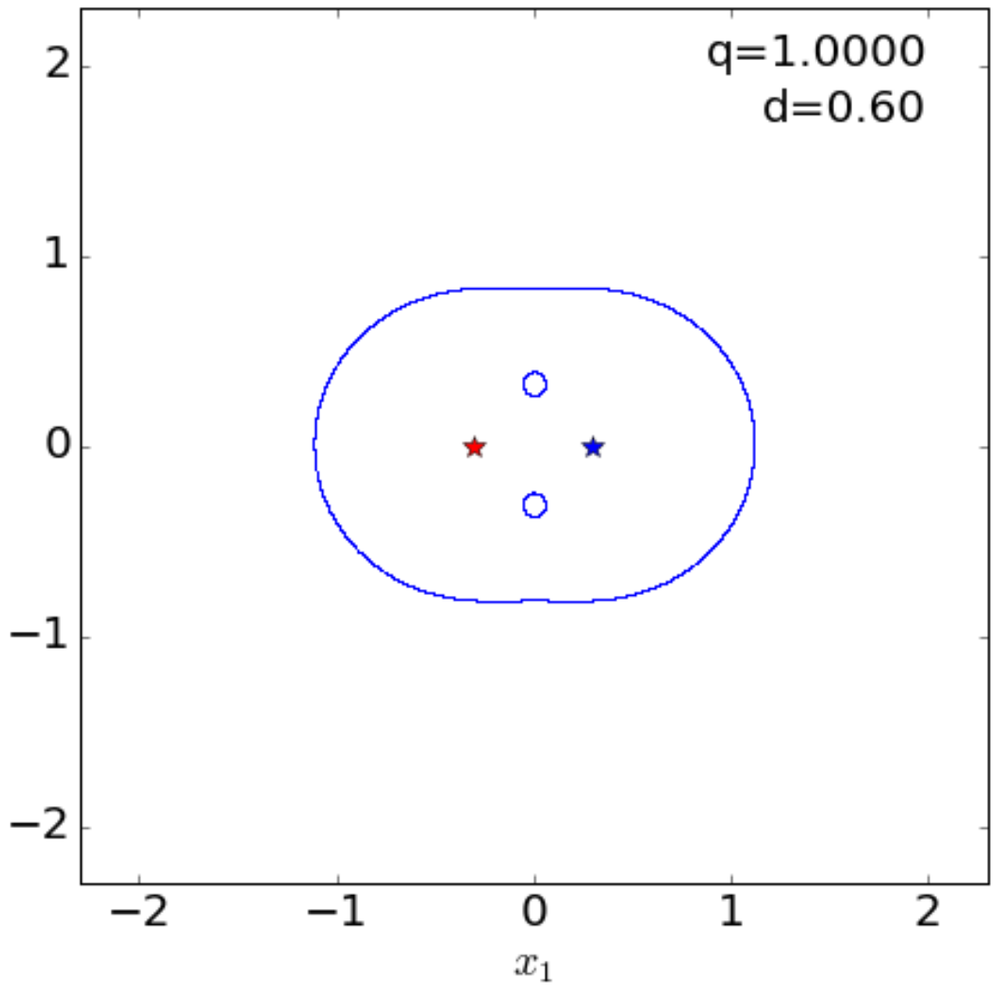


critical lines

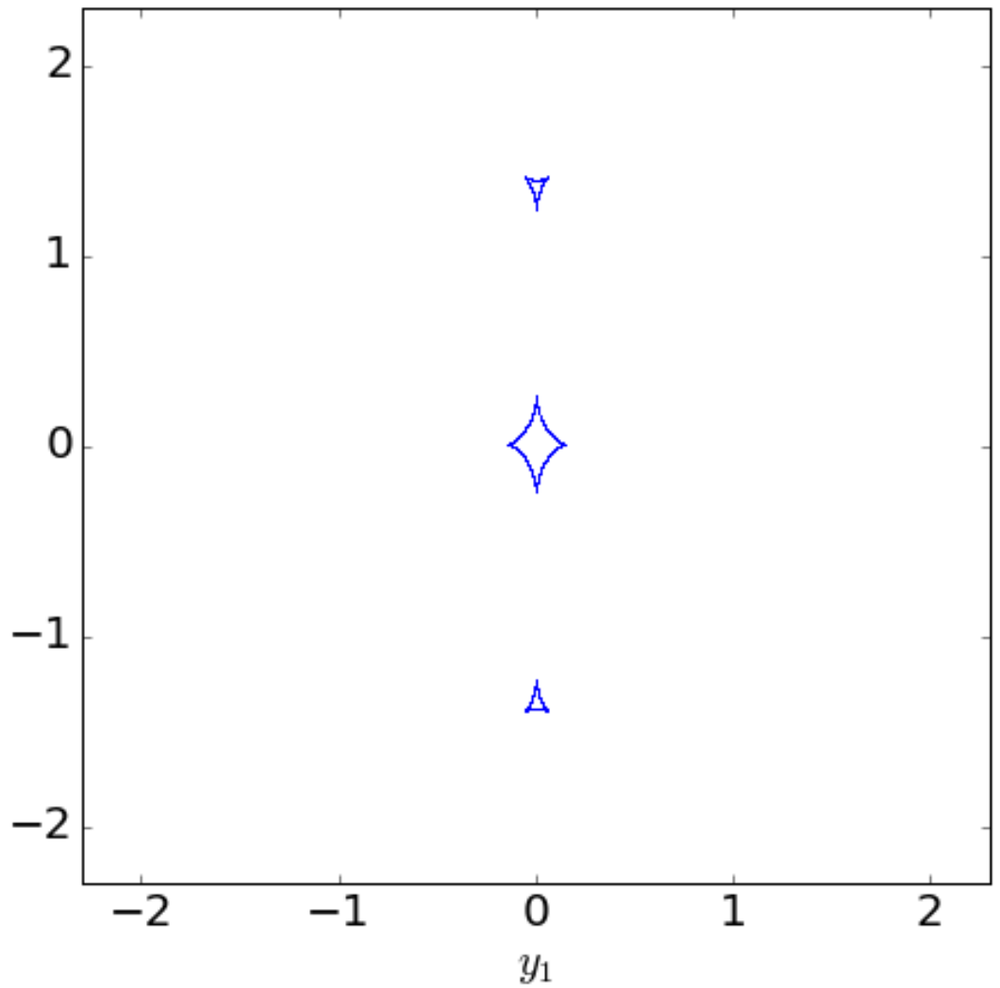


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

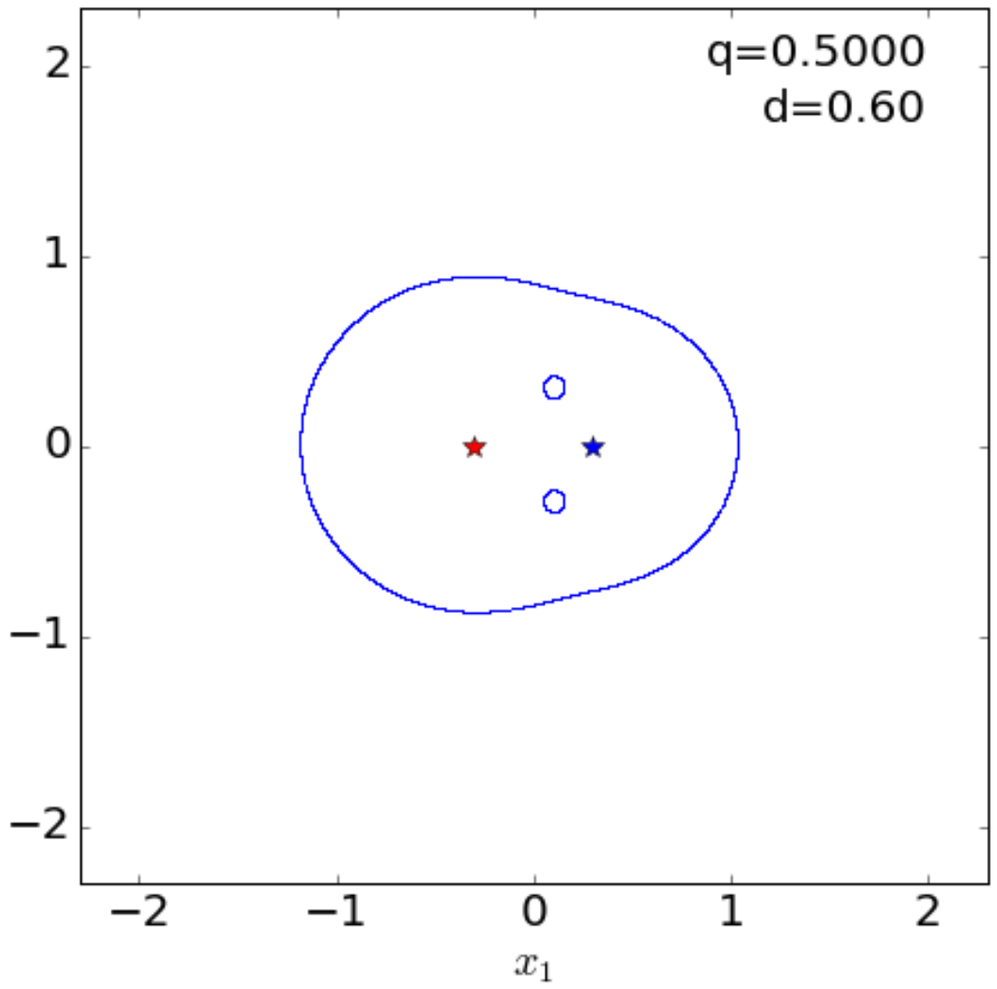


critical lines

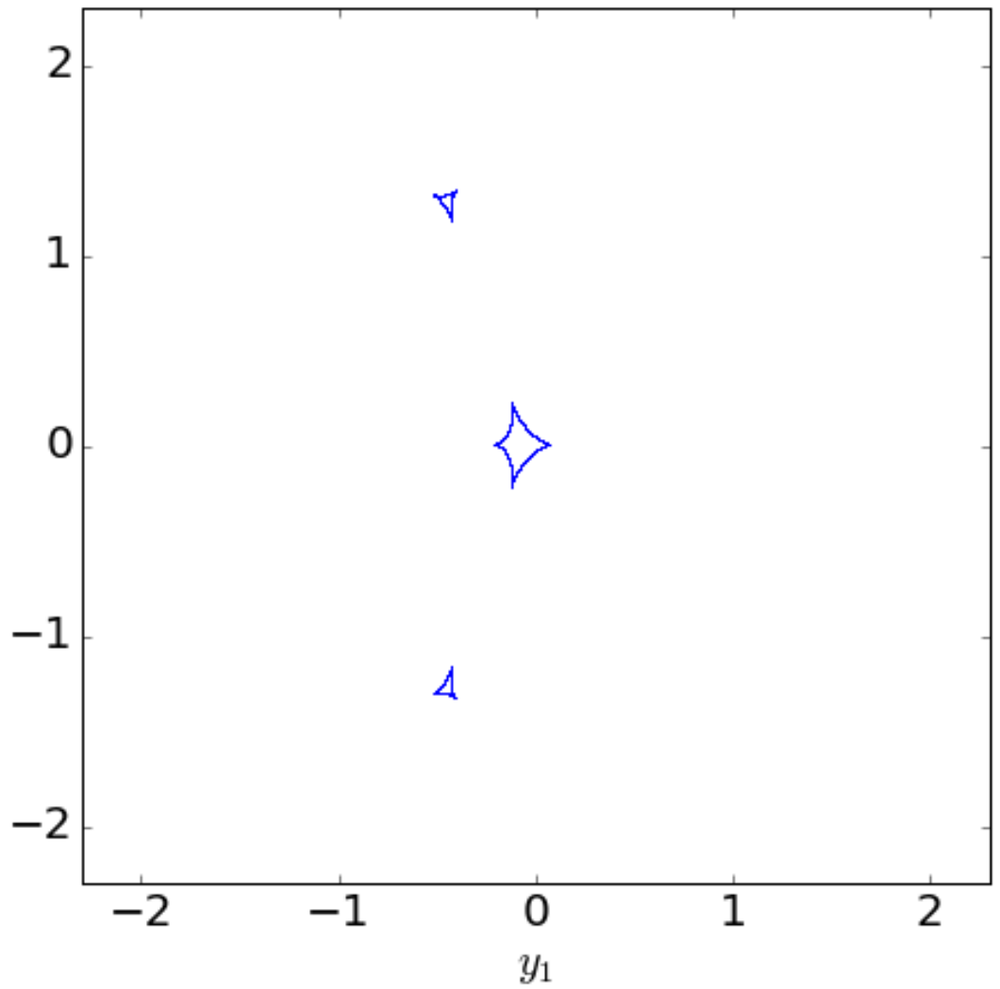


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

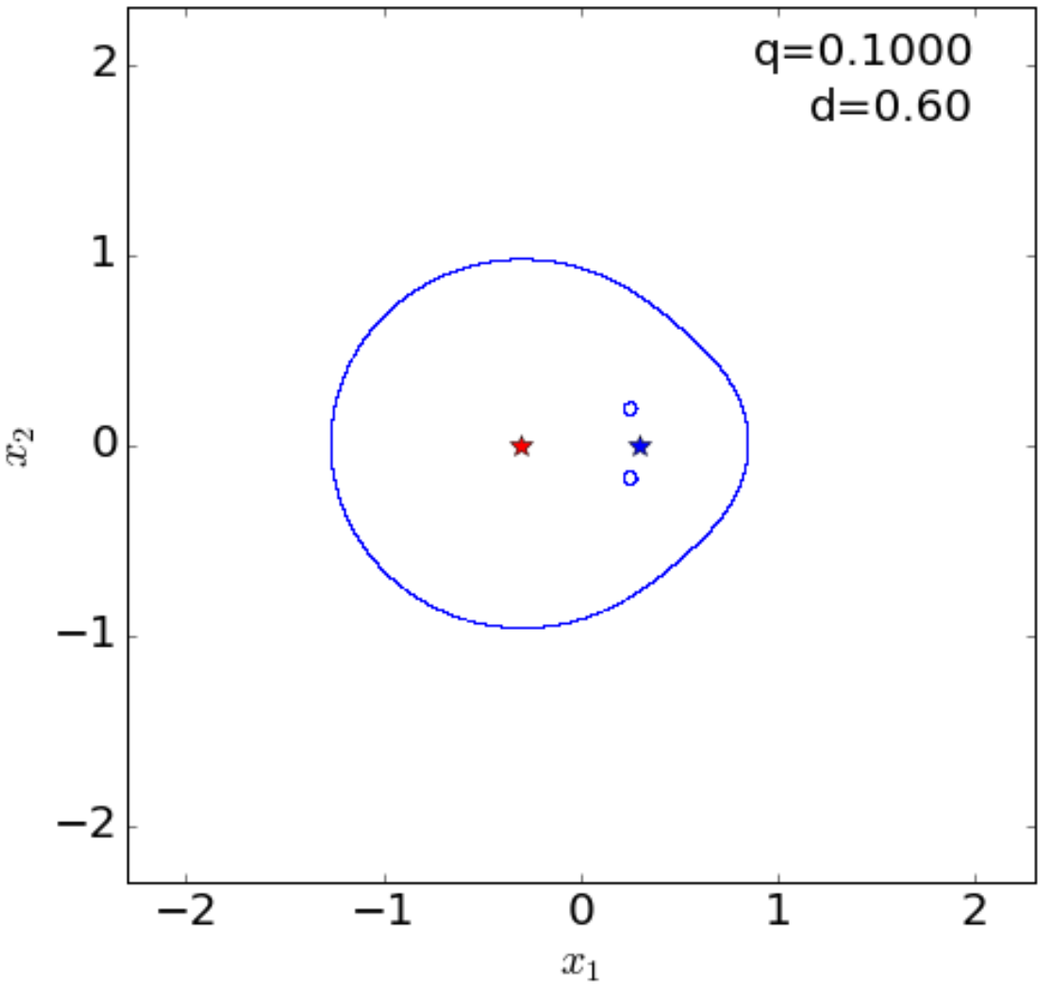


critical lines

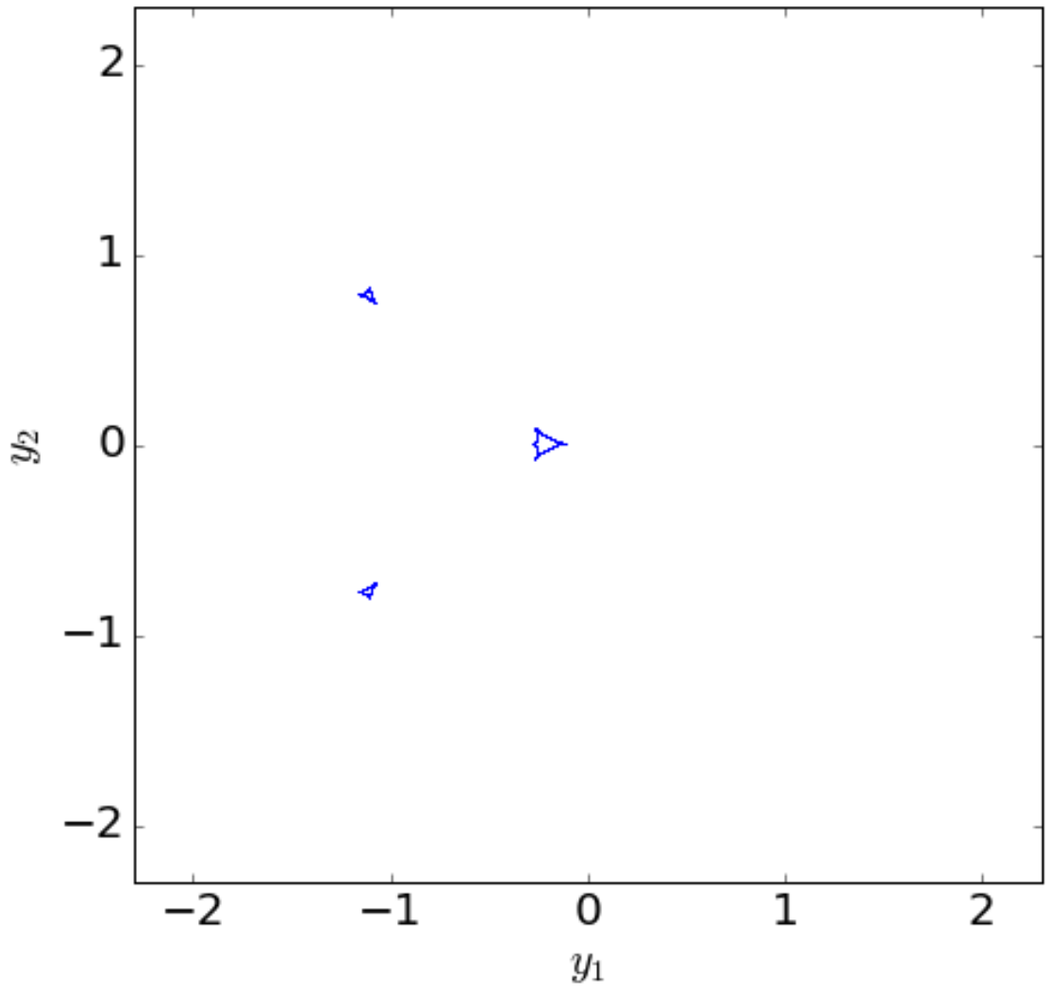


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

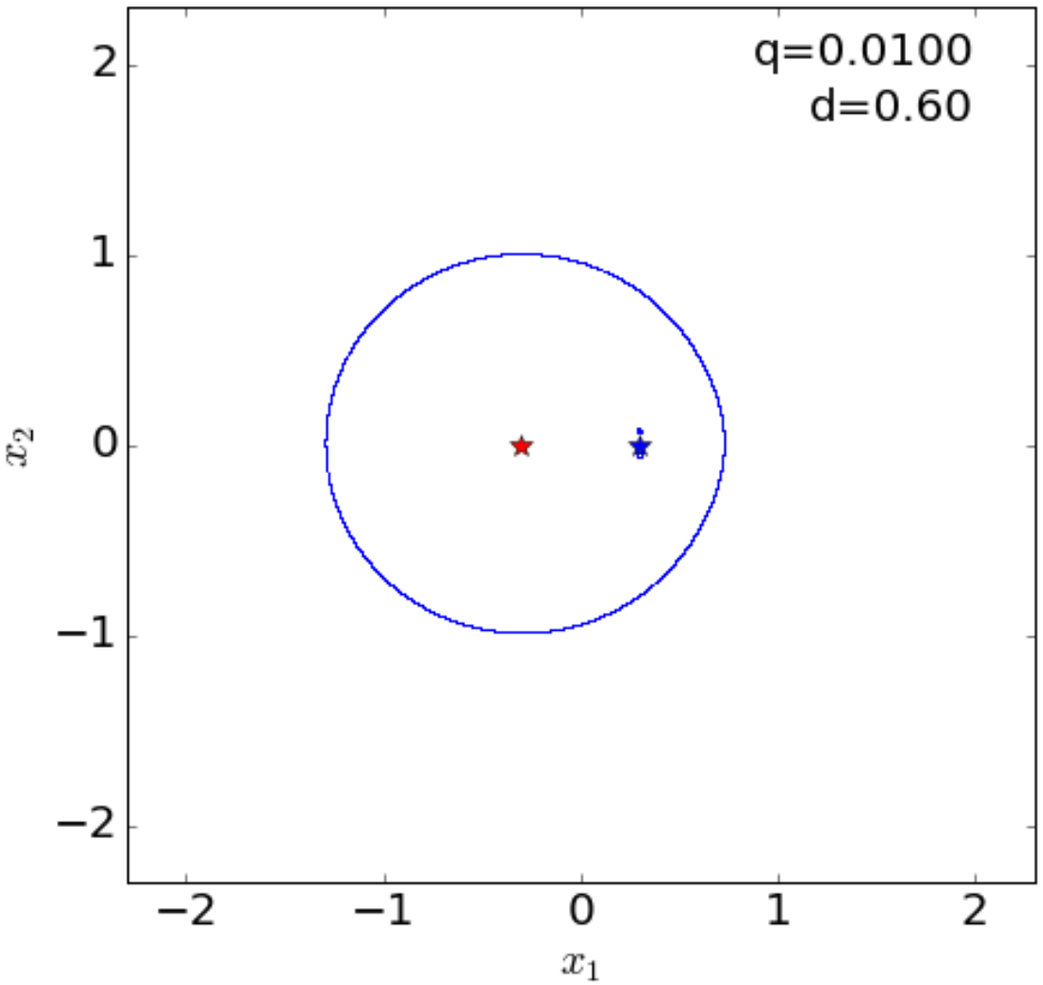


critical lines

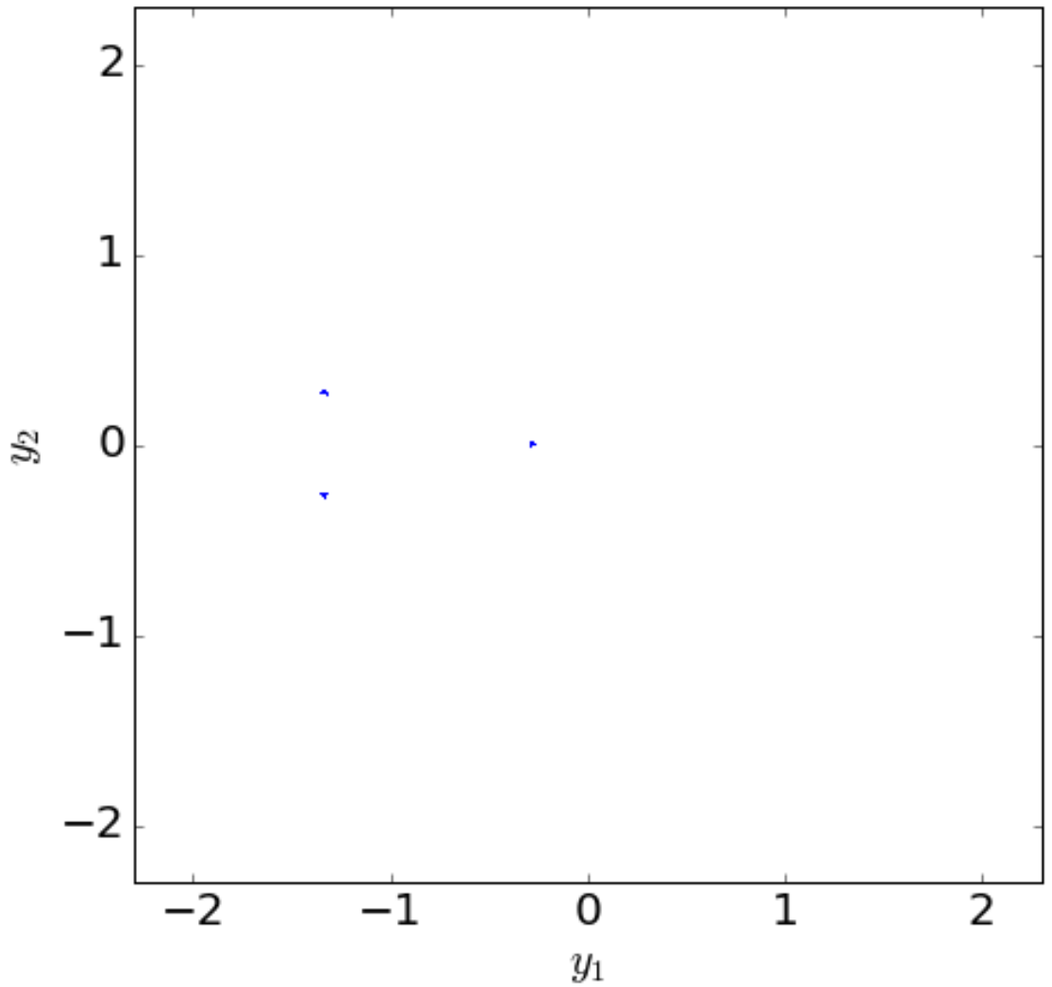


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

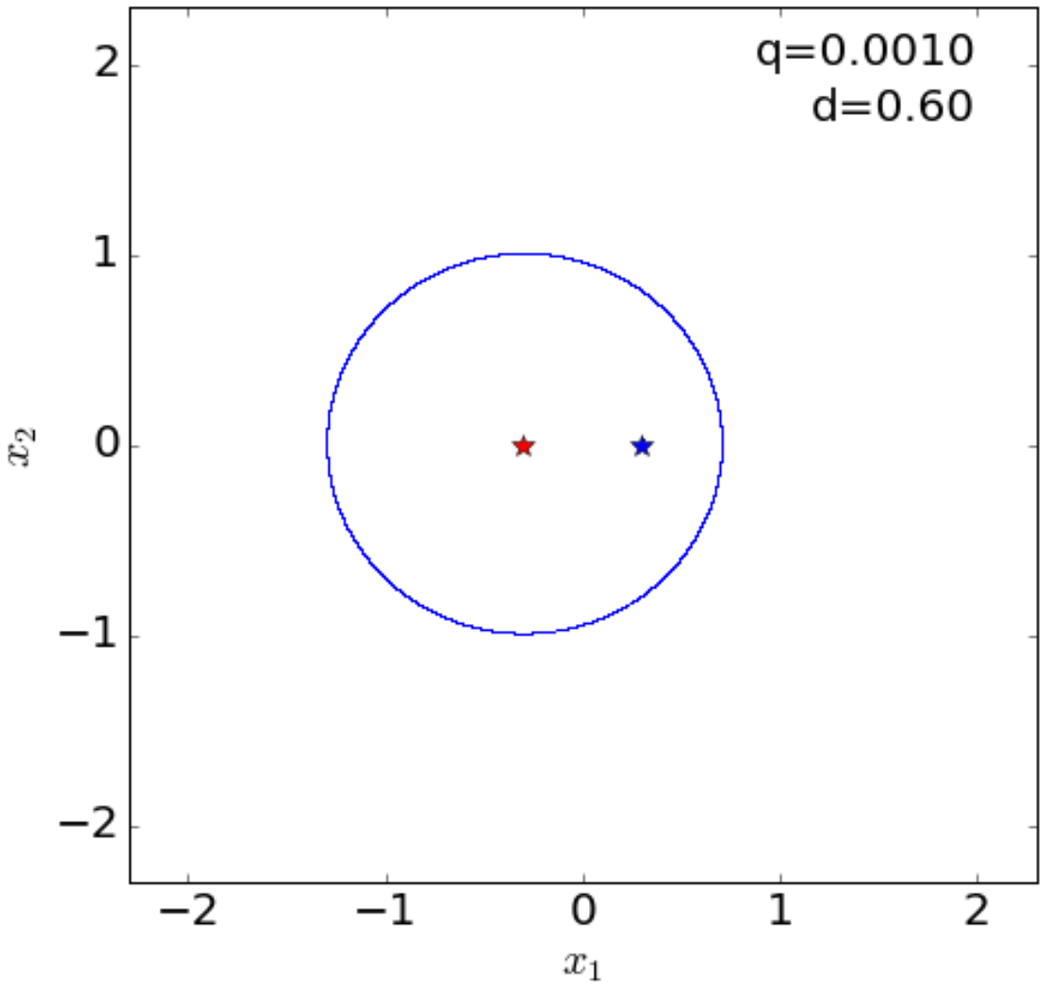


critical lines

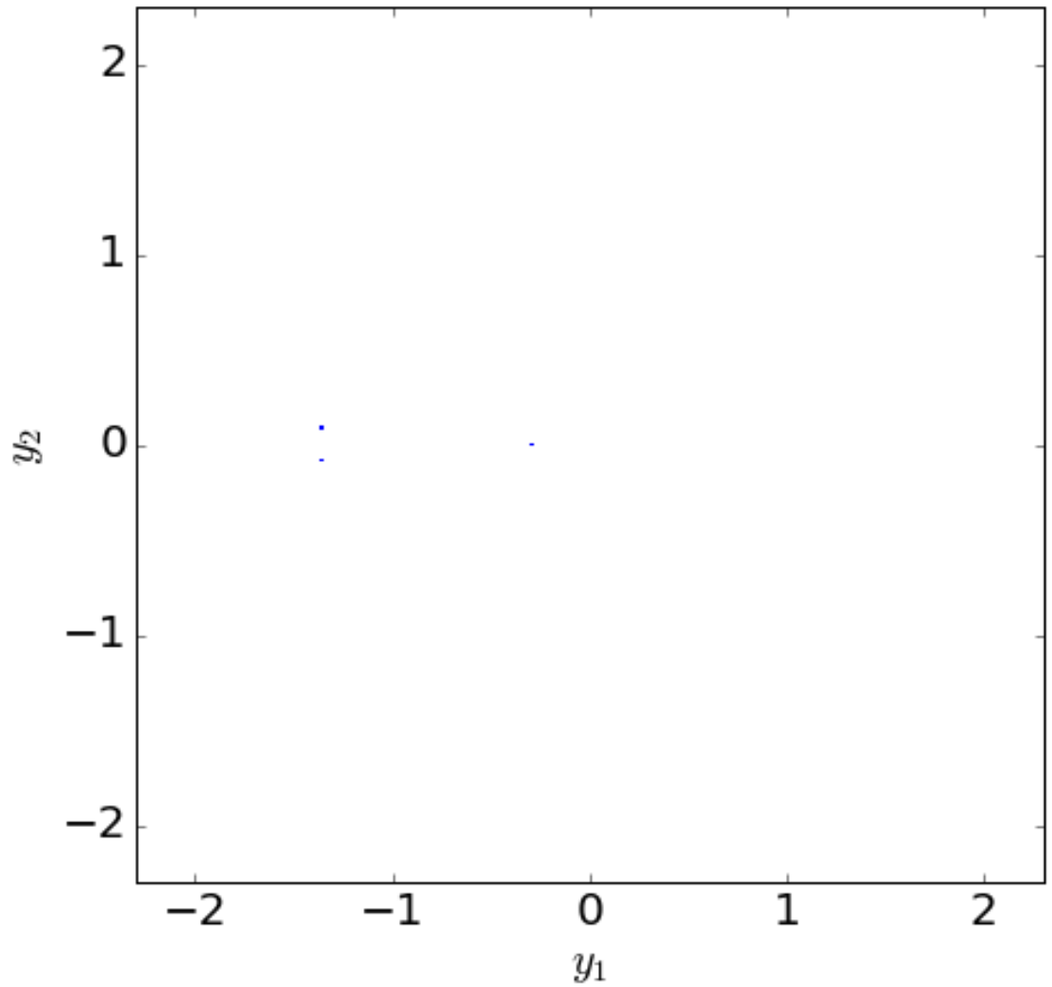


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

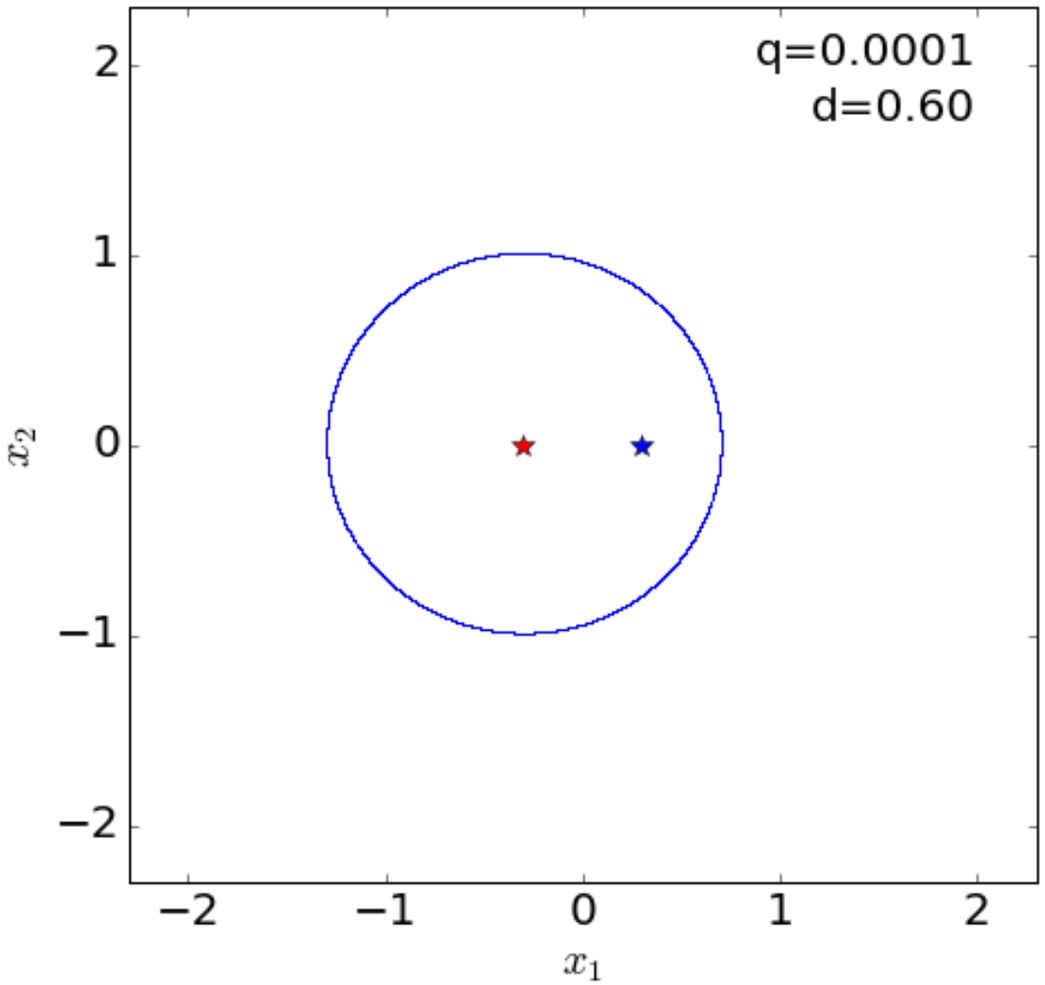


critical lines

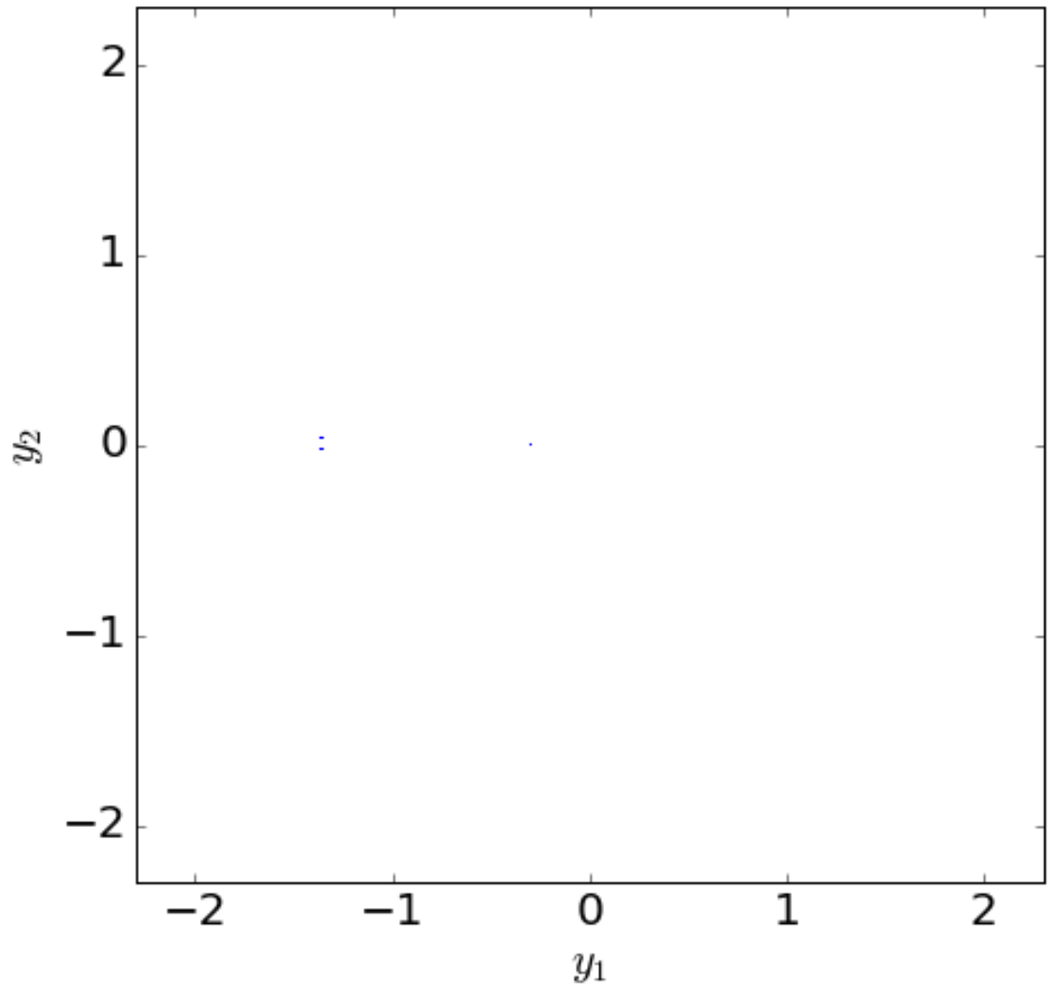


caustics

BINARY LENSES:
TWO LENSES WITH THE VARYING MASS AND FIXED DISTANCE

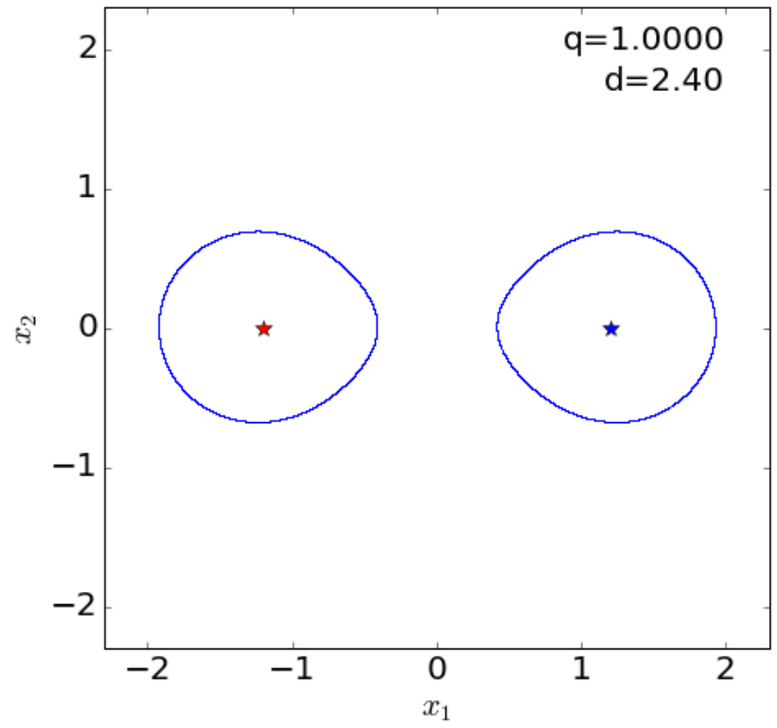


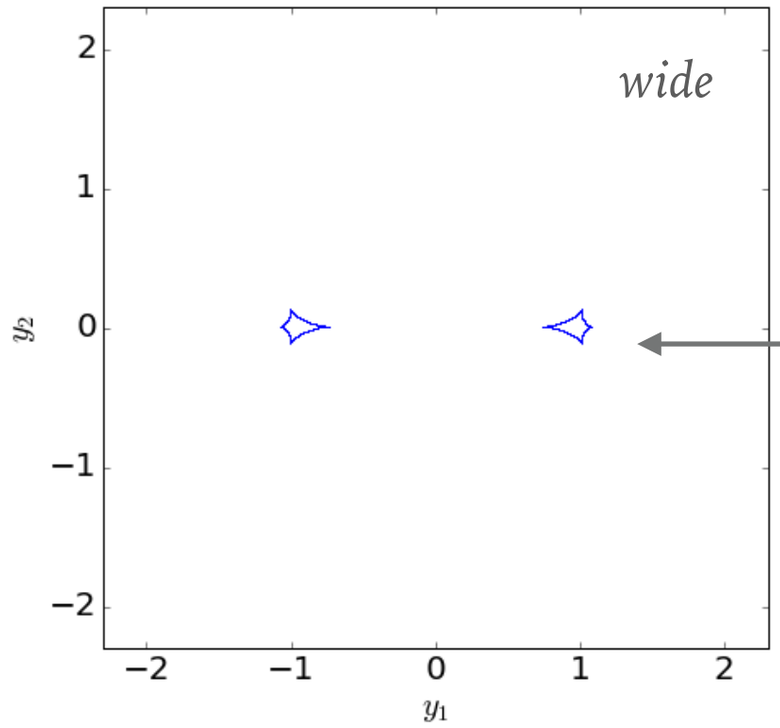
critical lines



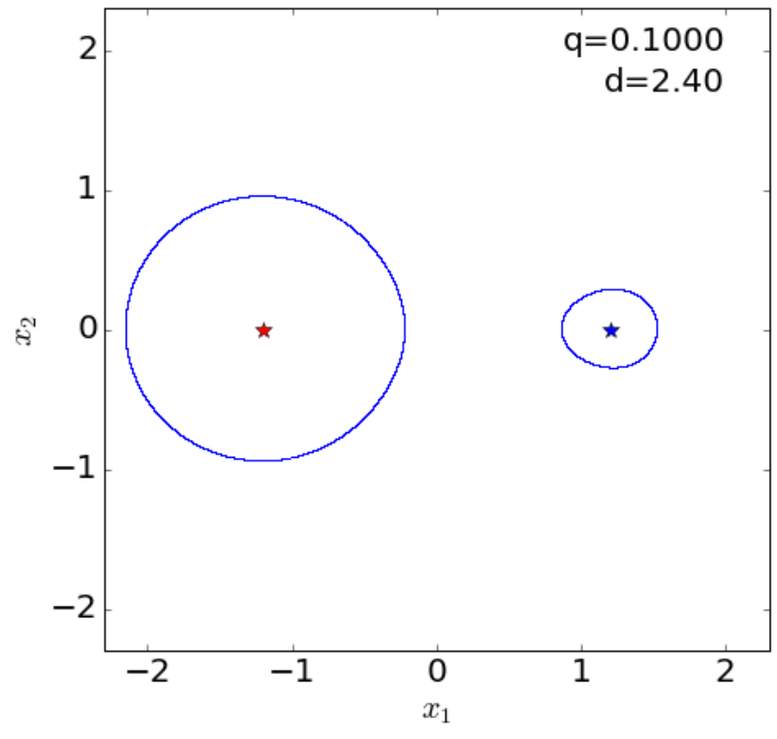
caustics

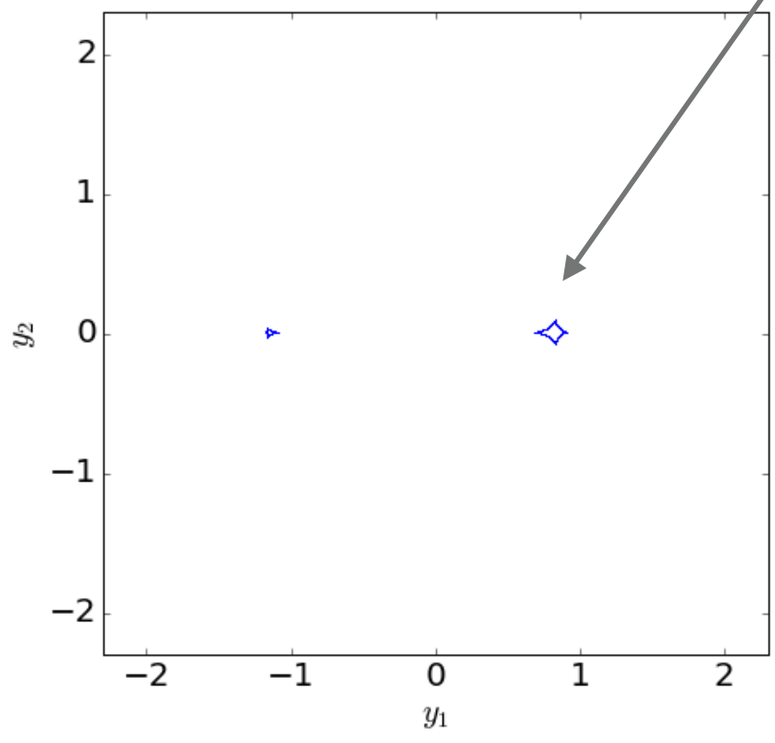
BINARY LENSES: TOPOLOGY CLASSIFICATION



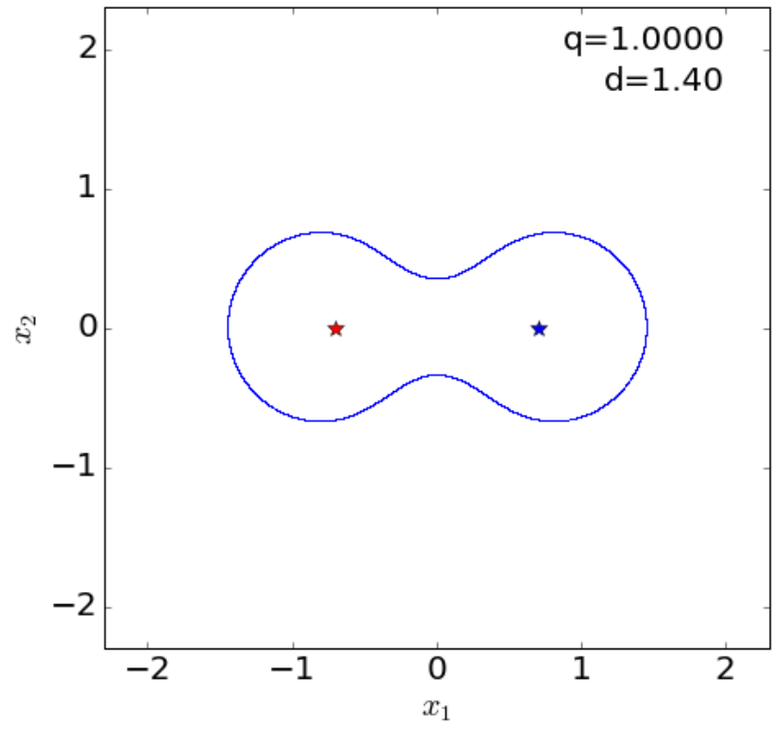


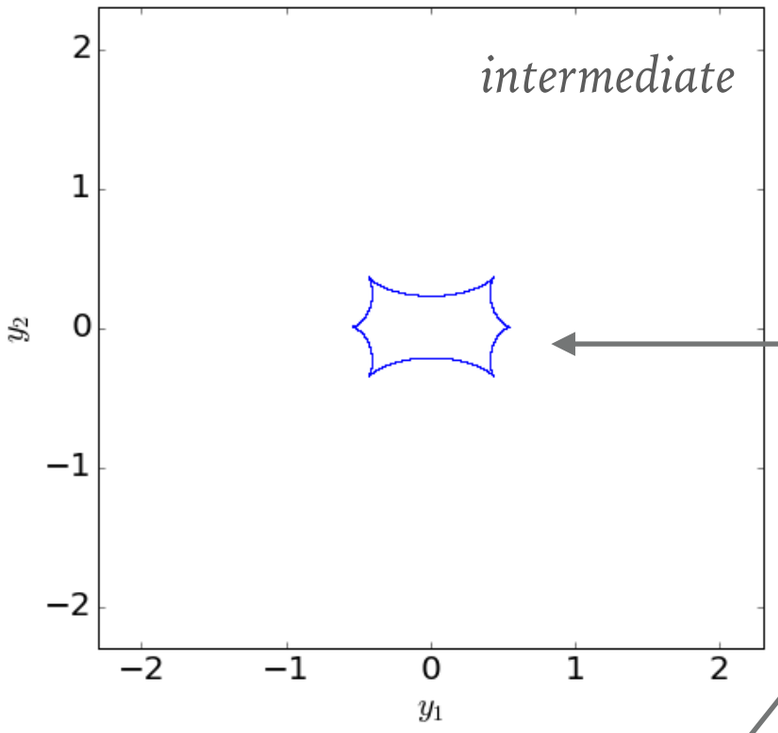
separate 4-cusp caustics



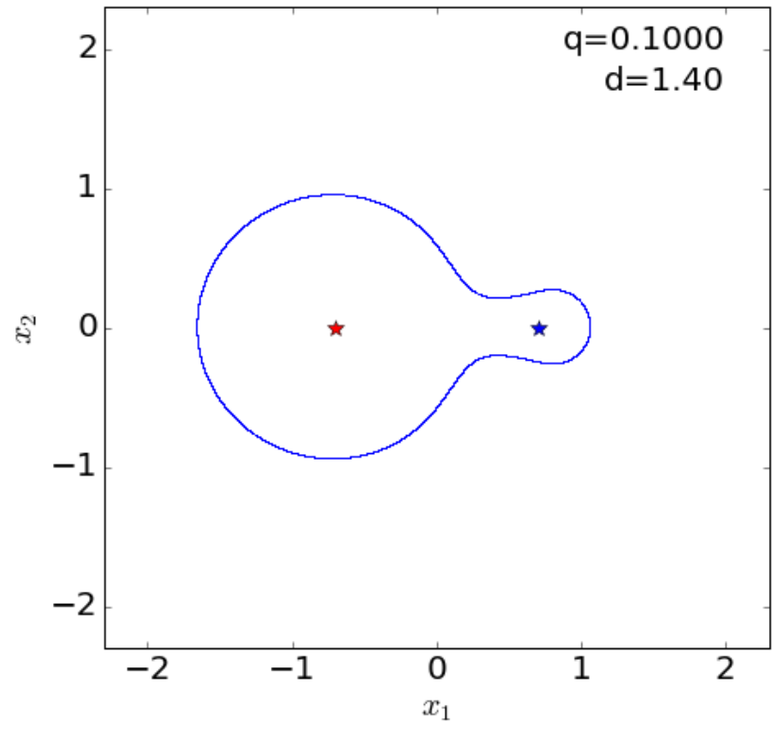


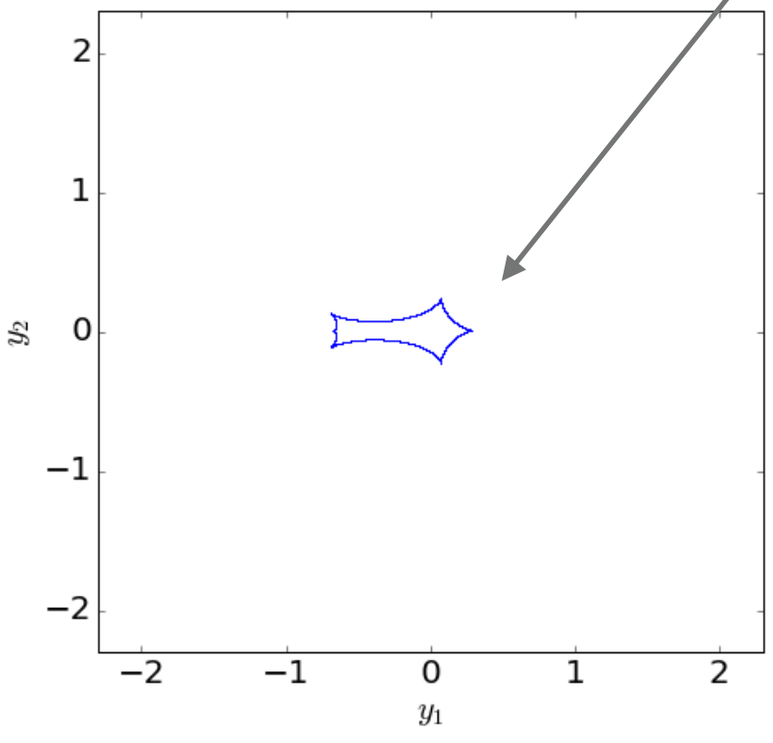
BINARY LENSES: TOPOLOGY CLASSIFICATION



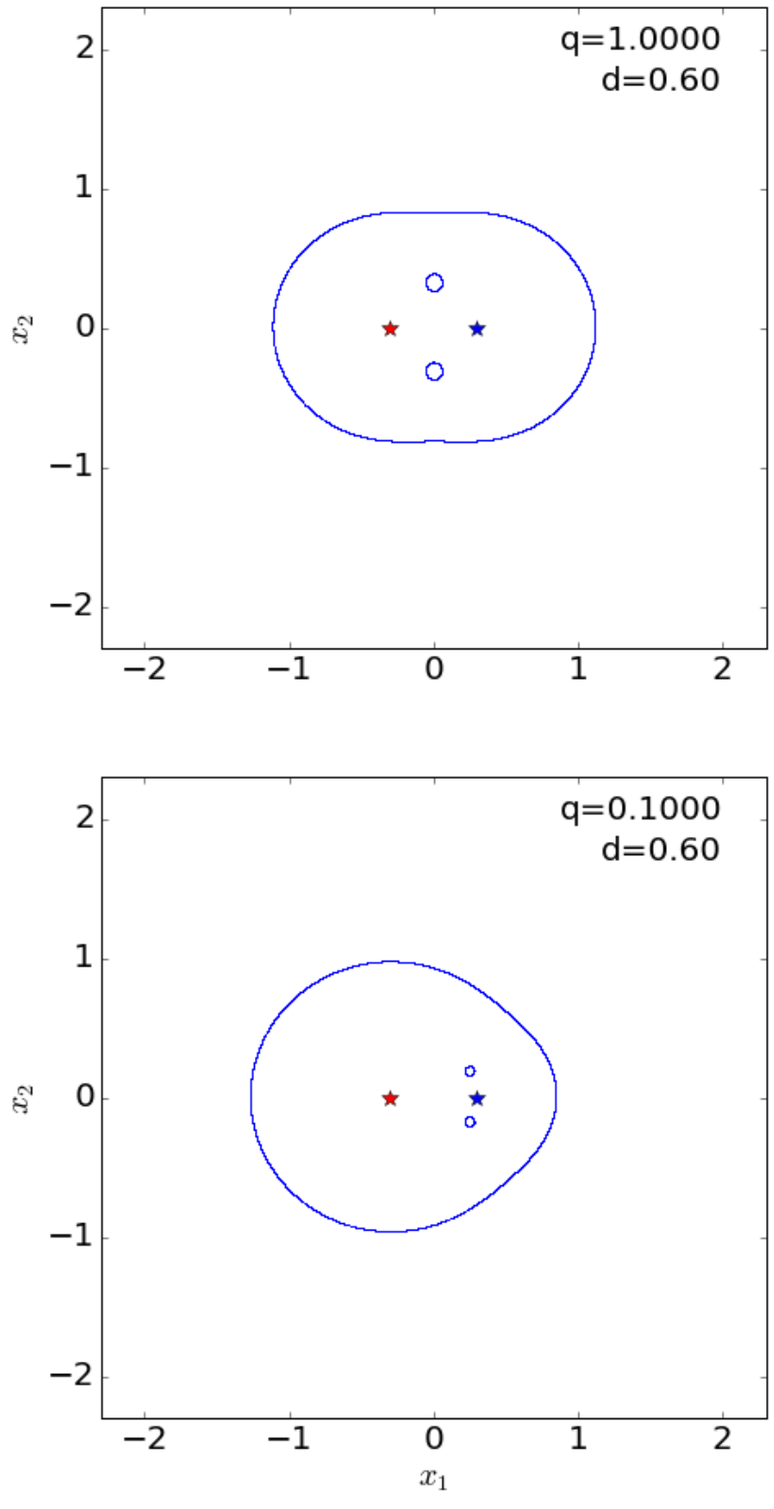


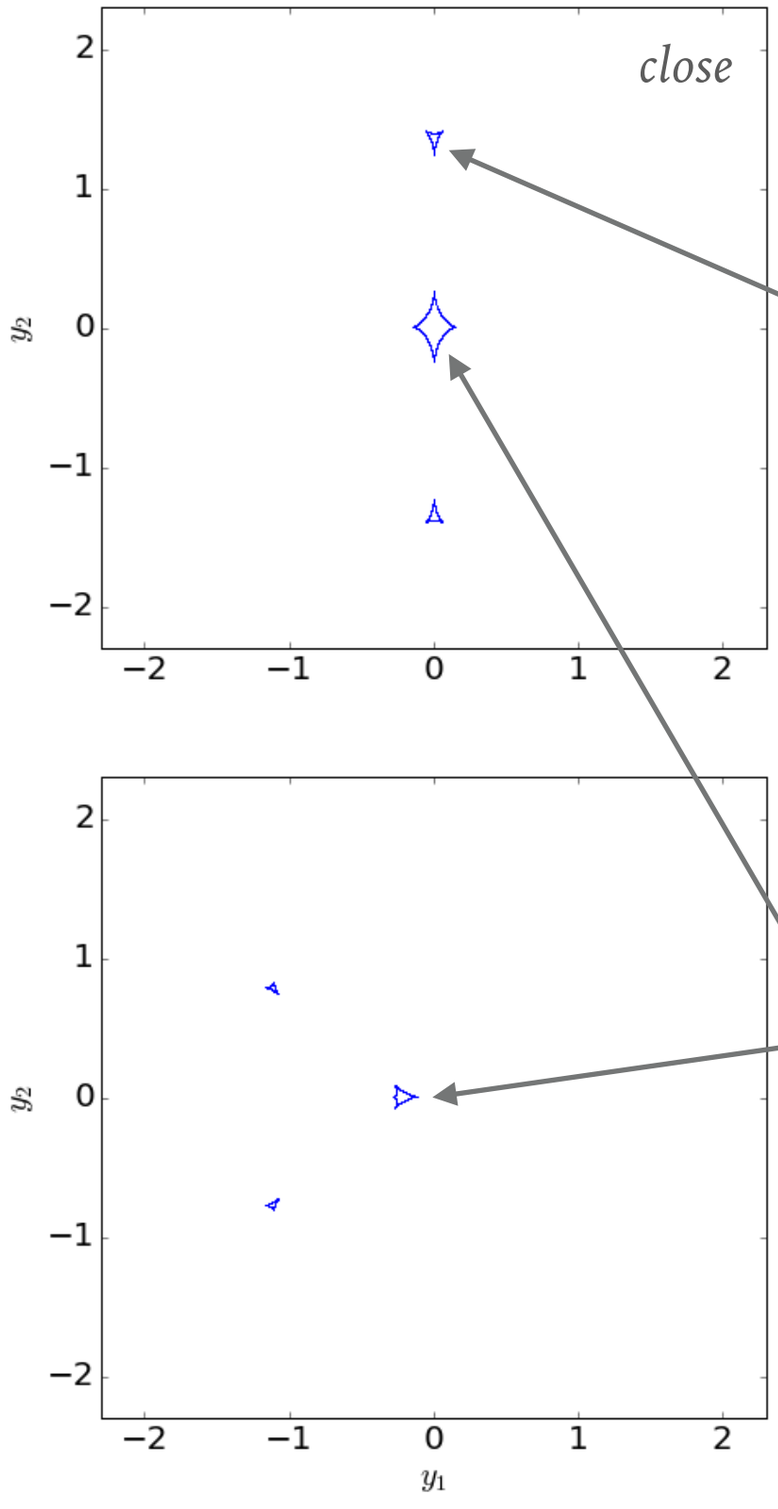
single 6-cusp caustic





BINARY LENSES: TOPOLOGY CLASSIFICATION

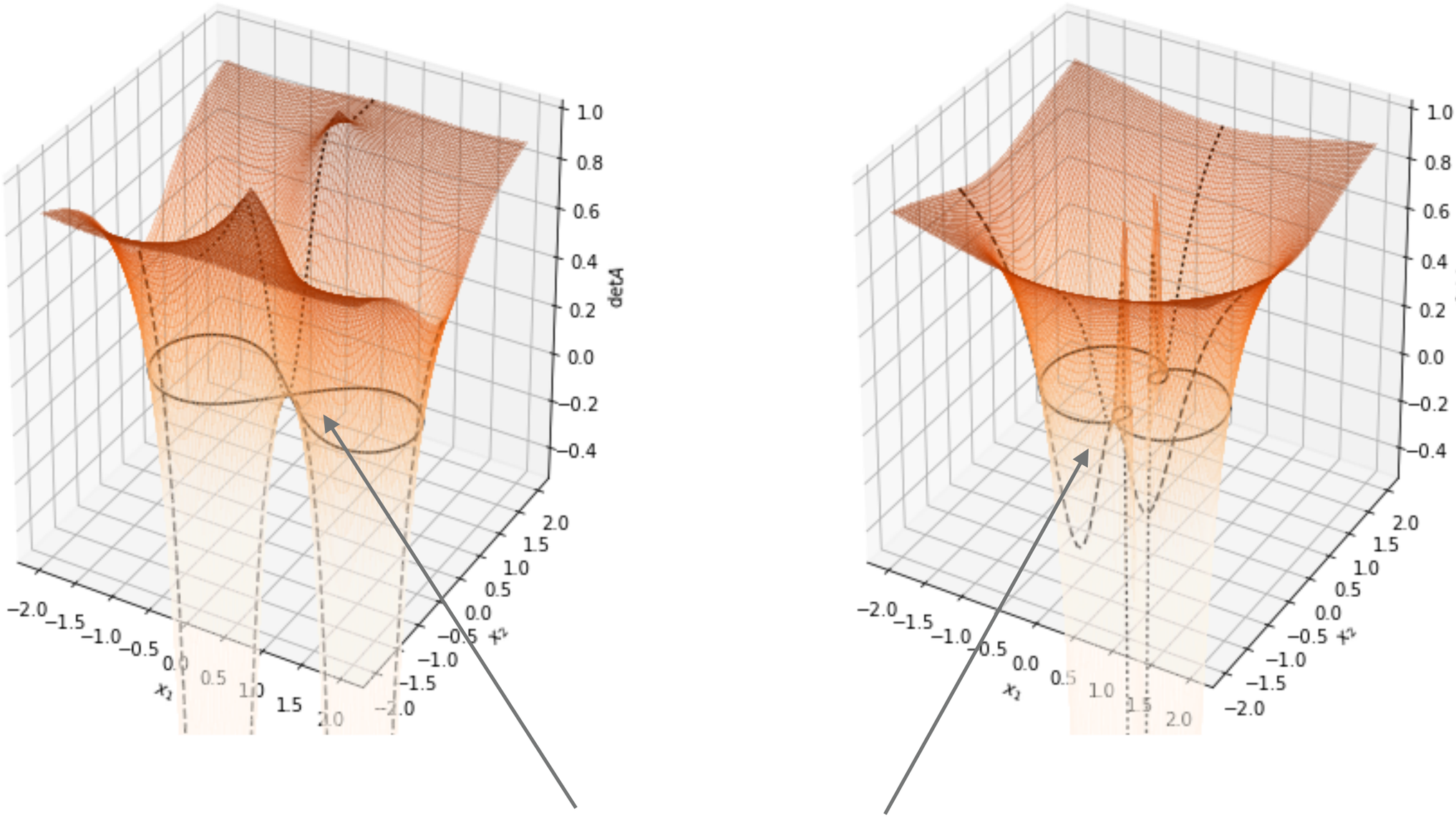




two triangular caustics

single 4-cusp caustic

TRANSITIONS



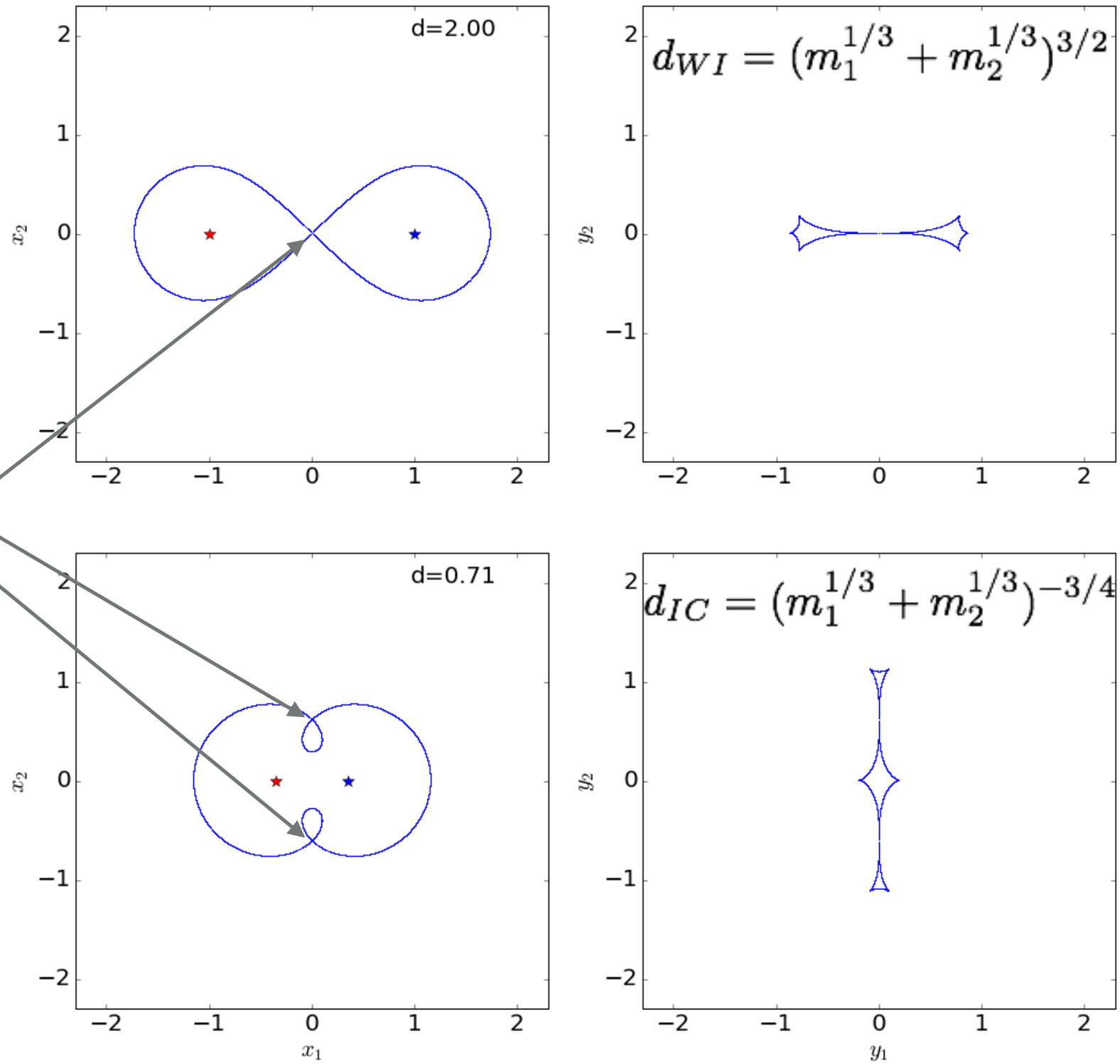
Touching critical lines = saddle points of detA

TRANSITIONS

Touching critical lines

$$\det A = 0$$

$$\frac{\partial \det A}{\partial z^*} = 0$$



MULTIPLE IMAGES

➤ Lens equation:

$$z_s = z - \frac{m_1}{z^* - z_1^*} - \frac{m_2}{z^* - z_2^*}$$

➤ complex polynomial:

$$p_5(z) = \sum_{i=0}^{5} c_i z^i$$

$$\Delta m = \frac{m_1 - m_2}{2} \qquad m = \frac{m_1 + m_2}{2} \qquad z_2 = -z_1 \qquad z_1 = z_1^*$$

$$c_0 = z_1^2[4(\Delta m)^2 z_s + 4m\Delta m z_1 + 4\Delta m z_s z_s^* z_1 + 2m z_s^* z_1^2 + z_s z_s^{*2} z_1^2 - 2\Delta m z_1^3 - z_s z_1^4]$$

$$c_1 = -8m\Delta m z_s z_1 - 4(\Delta m)^2 z_1^2 - 4m^2 z_1^2 - 4m z_s z_s^* z_1^2 - 4\Delta m z_s^* z_1^3 - z_s^{*2} z_1^4 + z_1^6$$

$$c_2 = 4m^2 z_s + 4m\Delta m z_1 - 4\Delta m z_s z_s^* z_1 - 2z_s z_s^{*2} z_1^2 + 4\Delta m z_1^3 + 2z_s z_1^4$$

$$c_3 = 4m z_s z_s^* + 4\Delta m z_s^* z_1 + 2z_s^{*2} z_1^2 - 2z_1^4$$

$$c_4 = -2m z_s^* + z_s z_s^{*2} - 2\Delta m z_1 - z_s z_1^2$$

$$c_5 = z_1^2 - z_s^{*2}$$

Witt & Mao, 1995,
ApJ, 447, L105

➤ 3 or 5 images

IMAGE MAGNIFICATION

- magnification at the image position:

$$\mu = \det A^{-1} = \left[1 - \left|\frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2}\right|\right]^{-1}$$

- total magnification:

$$\mu_{tot} = \sum_{i=1}^{n_i} |\mu_i|$$

- *of course, the magnification varies as a function of z and consequently as a function of t, if the source moves relative to the lens…*