GRAVITATIONAL LENSING

13 - MICROLENSING RATES - MULTIPLE POINT LENSES

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OPTICAL DEPTH (FROM THE LAST LESSON)

$$\tau(D_S) = \frac{1}{\Omega}\int_0^{D_S}[\Omega D_L^2 n(D_L)](\pi\theta_E^2)dD_L$$

$$\begin{aligned}
\tau(D_S) &= \frac{4\pi G}{c^2}\int_0^{D_S}\rho(D_L)D_L^2\frac{D_{LS}}{D_L D_S}dD_L \\
&= \frac{4\pi G}{c^2}\int_0^{D_S}\rho(D_L)D_L\frac{D_S - D_L}{D_S}dD_L \\
&= \frac{4\pi G}{c^2}\int_0^{D_S}\rho(D_L)\frac{D_L}{D_S}\left(1-\frac{D_L}{D_S}\right)D_S dD_L
\end{aligned}$$

with the substitution $x = D_L/D_S,\ dx = dD_L/D_S$

$$\tau(D_S) = \frac{4\pi G}{c^2}D_S^2\int_0^1\rho(x)x(1-x)dx$$

EVENT RATE

The optical depth gives the probability that a source is (micro-)lensed at any instant. The next question is: how many events will we detect by monitoring a certain number of stars during a time interval?

To answer this question, we have to consider the relative motion of sources and lenses, which determines the timescale of events.

It is easier to think in terms of static sources behind moving lenses.

We also assume that the lenses move with the same transverse velocity.

EVENT RATE

$$r_E = D_\mathrm{L}\theta_E$$

$2r_E$

vdt

$$dA = 2r_E v dt = 2r_E^2 \frac{dt}{t_E}$$

Multiplying by the number of lenses and integrating over distance we obtain the area useful for microlensing during the time dt. Dividing by the solid angle, we obtain a probability that a source undergoes a micro lensing event in the time dt:

$$d\tau = \frac{1}{\Omega}\int_0^{D_\mathrm{S}} n(D_\mathrm{L})\Omega dA dD_\mathrm{L} = 2\int_0^{D_\mathrm{S}} n(D_\mathrm{L}) r_E^2 \frac{dt}{t_E} dD_\mathrm{L}$$

EVENT RATE

If we monitor N stars, the number of events expected per unit time will be:

$$\Gamma = \frac{d(N_\star \tau)}{dt} = \frac{2N_\star}{\pi} \int_0^{D_\mathrm{S}} n(D_\mathrm{L}) \frac{\pi r_E^2}{t_E} dD_\mathrm{L}$$

If we assume that all Einstein crossing times are identical:

$$\Gamma = \frac{2N_\star}{\pi t_E} \tau$$

As an order of magnitude:

$$\Gamma \approx 1200 \mathrm{yr}^{-1} \frac{N_\star}{10^8} \frac{\tau}{10^{-6}} \left(\frac{t_E}{19 \mathrm{days}} \right)^{-1}$$

EVENT RATE

For comparison: OGLE IV detected 1500-2000 candidates/year in the 2011-2017 campaigns.

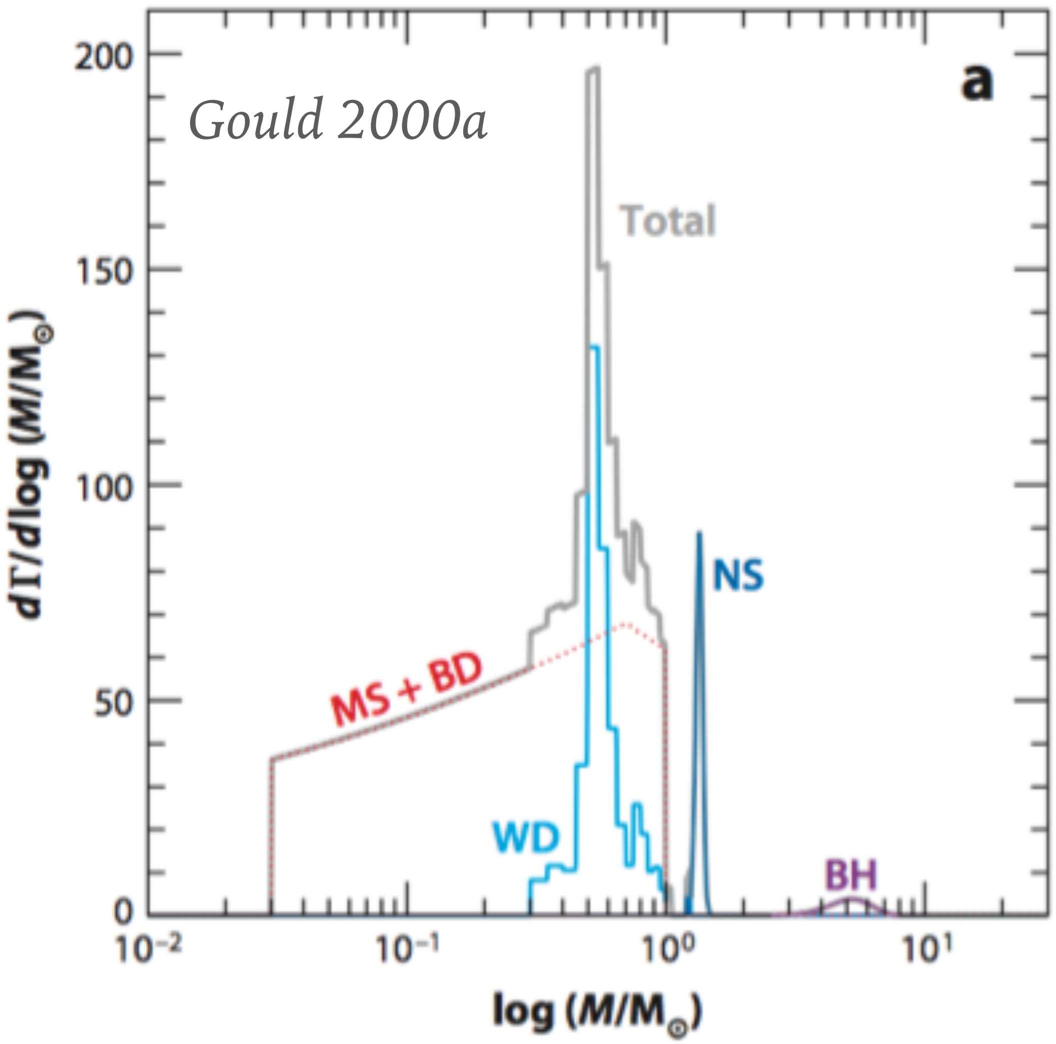
Note that:

$$\Gamma \propto t_E^{-1} \propto M^{-1/2}$$

We can use the distribution of event timescales to probe the kinematics of the Milky Way and the stellar populations in the galaxy.

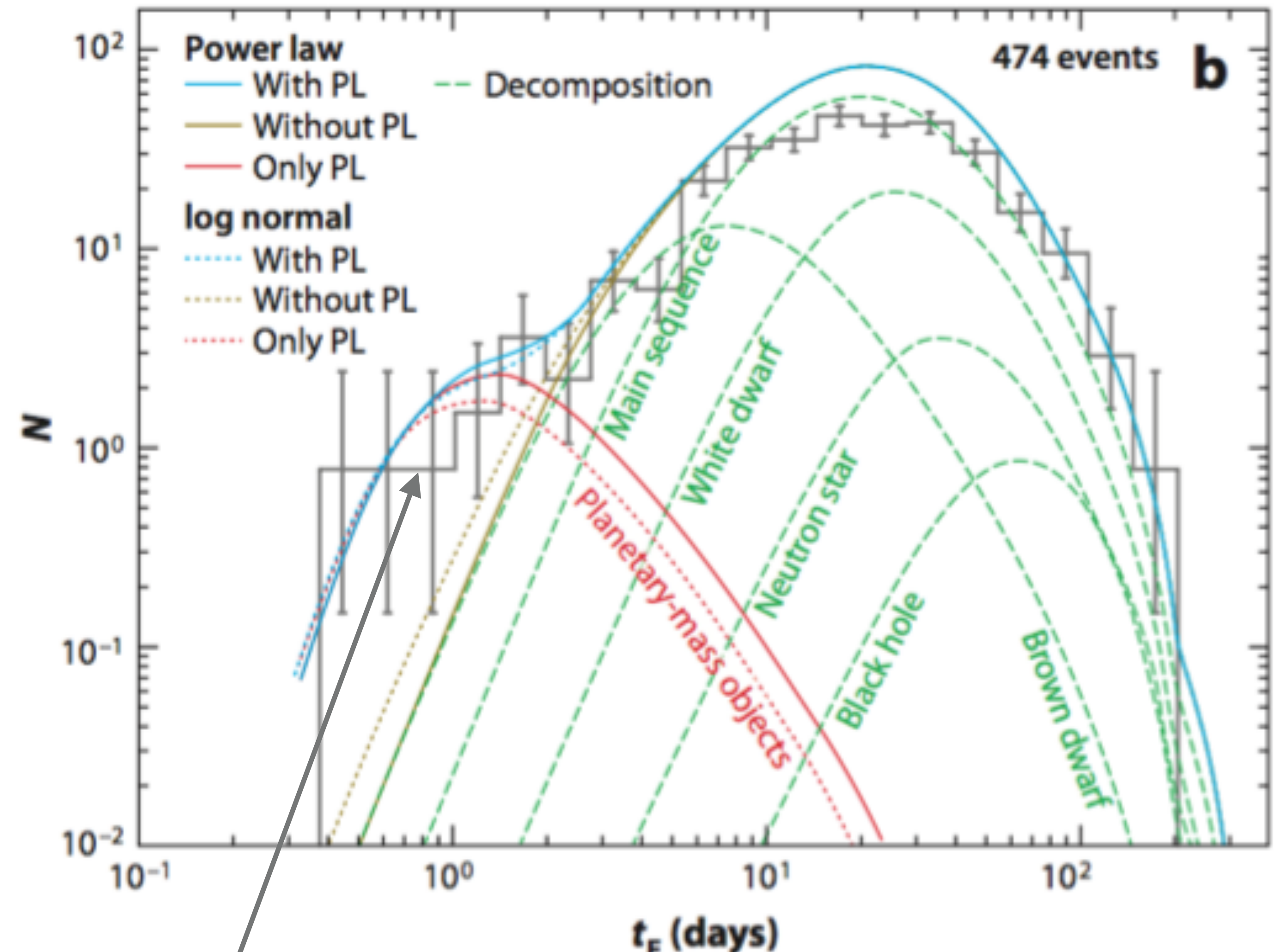
PROBING THE STELLAR POPULATIONS WITH MICROLENSING

Gaudi, 2012, Ann. Rev. Astron. Astrophys. 50, 411



Theoretical estimate of the rate of microlensing events towards the galactic bulge

Sumi et al 2011



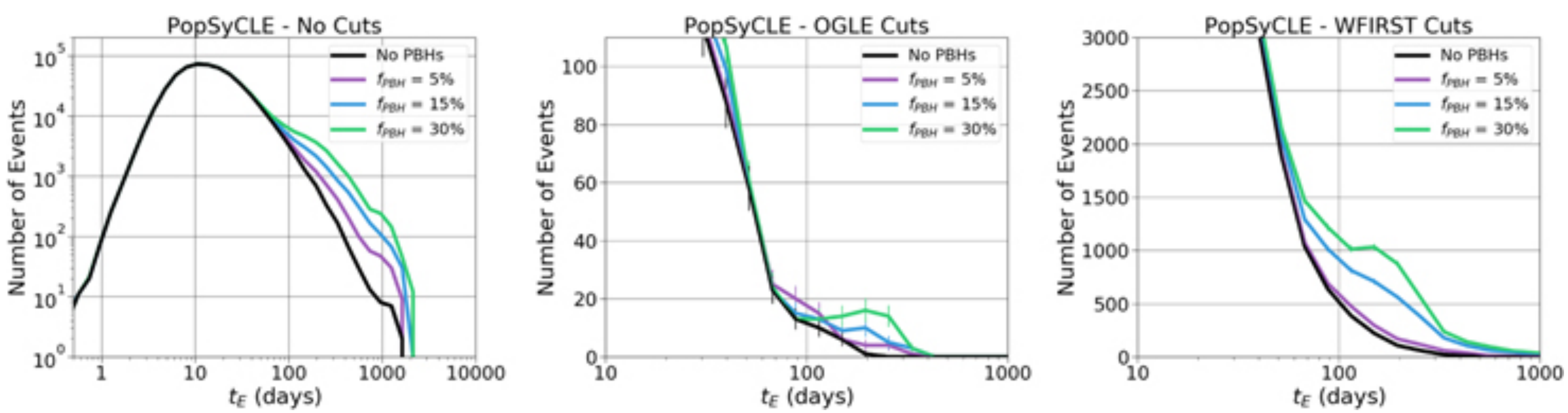
Distribution of microlensing event timescales observed by the MOA collaboration (2006-2007)

SOME IMPORTANT FACTS

- several collaborations have implemented the microlensing idea (proposed by B. Paczynski). These groups have monitored the galactic bulge and the Magellanic Clouds searching for microlensing events
- the relatively high rate of detections favored a barred model of the galaxy
- Towards the Magellanic Clouds, no ‘short’ events (timescales from a few hours up to 20 days) have been seen by any group. This places strong limits on ‘Jupiters’ in the dark halo: specifically, compact objects in the mass range 10^{-6}–0.05 solar masses contribute less than 10% of the dark matter around our Galaxy. This is a very important result, as these objects were previously thought to be the most plausible form of baryonic dark matter, and (for masses below 0.01 solar masses) they would have been virtually impossible to detect directly.

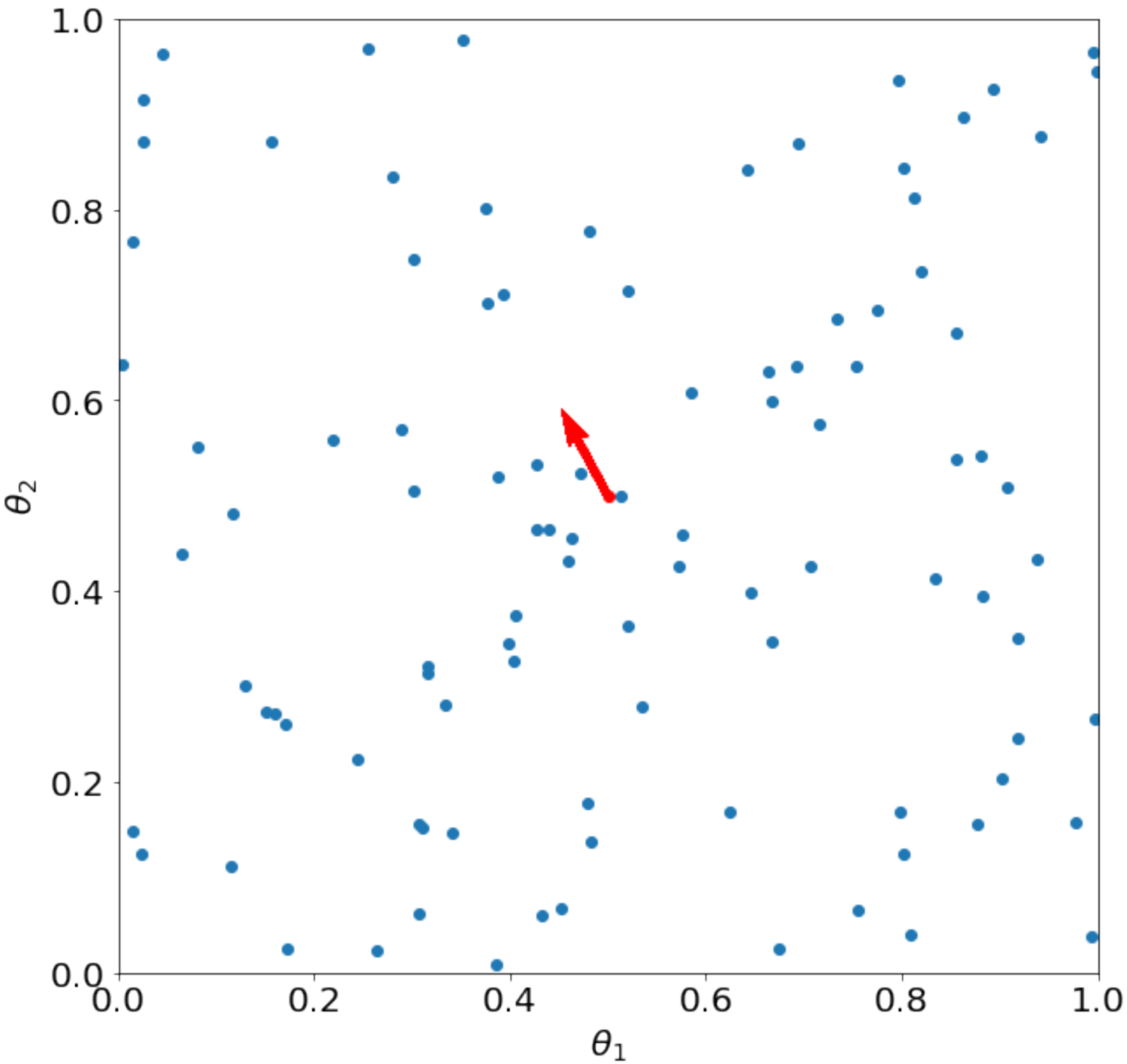
SOME IMPORTANT FACTS

- In general: all detections of microlensing events are most likely caused by known stellar populations. BHs can contribute to 2% of the total mass of the halo.
- The recent detection of GW from merging BHs with intermediate masses has revived the idea of BHs as dark-matter candidates. For such lenses, the time scale of the events would be large so that past microlensing events may not have detected them.



Lu et al. 2019

MULTIPLE POINT MASSES



We conceder a system of N point masses at the same distance D_L.

As seen, a light ray crossing the lens plane at the portion θ will experience the deflection

$$\hat{\alpha}(\vec{\theta}) = \frac{4G}{c^2 D_L} \sum_{i=1}^{N} \frac{M_i}{|\vec{\theta} - \vec{\theta}_i|^2} (\vec{\theta} - \vec{\theta}_i)$$

MULTIPLE POINT MASSES

- compared to an individual point mass, the spatial symmetry is broken
- The mass scale of the system is the total mass=sum of the individual masses
- We may use this mass to define an equivalent Einstein radius and use it to scale all angles

MULTIPLE POINT MASSES

$$M_{tot} = \sum_{i=1}^{N} M_i. \qquad\qquad m_i = M_i / M_{tot}$$

$$\vec{\alpha}(\vec{\theta}) = \sum_{i=1}^{N} \frac{D_{\rm LS}}{D_{\rm L} D_{\rm S}} \frac{4GM_i}{c^2} \frac{(\vec{\theta}-\vec{\theta}_i)}{|\vec{\theta}-\vec{\theta}_i|^2} \frac{M_{tot}}{M_{tot}} = \sum_{i=1}^{N} m_i \frac{\theta_E^2}{|\vec{\theta}-\vec{\theta}_i|^2} (\vec{\theta}-\vec{\theta}_i)$$

dividing by θ_E:

$$\vec{\alpha}(\vec{x}) = \sum_{i=1}^{N} \frac{m_i}{|\vec{x}-\vec{x}_i|^2} (\vec{x}-\vec{x}_i)$$

$$\vec{y} = \vec{x} - \sum_{i=1}^{N} \frac{m_i}{|\vec{x}-\vec{x}_i|^2} (\vec{x}-\vec{x}_i)$$

COMPLEX LENS EQUATION (WITT, 1990)

$$\vec{y} = \vec{x} - \sum_{i=1}^{N} \frac{m_i}{|\vec{x} - \vec{x}_i|^2} (\vec{x} - \vec{x}_i)$$

$$z = x_1 + i x_2 \qquad z_s = y_1 + i y_2$$

$$\alpha(z) = \sum_{i=1}^{N} m_i \frac{(z - z_i)}{(z - z_i)(z^* - z_i^*)} = \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

$$z_s = z - \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

COMPLEX LENS EQUATION (WITT, 1990)

- Thus:

$$z_s = z - \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

- Taking the conjugate:

$$z^* = z_s^* + \sum_{i=1}^{N} \frac{m_i}{z - z_i}$$

- We obtain z* and substitute it back into the original equation, which results in a (N^2+1)th order complex polynomial equation

- This equation can be solved only numerically, even in the case of a binary lens

COMPLEX LENS EQUATION (WITT, 1990)

- Note that the solutions are not necessarily solutions of the lens equations (spurious solutions)
- One has to check if the solutions are solutions of the lens equation
- Rhie (2001,2003): maximum number of images is 5(N-1) for N>2

JACOBIAN DETERMINANT

The Jacobian determinant is (on the real plane):

$$\det A = \frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2} - \left(\frac{\partial y_1}{\partial x_2}\right)^2$$

How do we write it in complex notation?

JACOBIAN DETERMINANT

Note that in lensing these two derivatives are equal!

The complex derivatives (Wirtinger derivatives) of z_s are:

$$
\begin{aligned}
\frac{\partial z_s}{\partial z} &= \frac{1}{2}\left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} + \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} - \frac{\partial y_1}{\partial x_2}\right) \\
\frac{\partial z_s}{\partial z^*} &= \frac{1}{2}\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)(y_1 + iy_2) = \frac{1}{2}\left(\frac{\partial y_1}{\partial x_1} - \frac{\partial y_2}{\partial x_2}\right) + \frac{i}{2}\left(\frac{\partial y_2}{\partial x_1} + \frac{\partial y_1}{\partial x_2}\right)
\end{aligned}
$$

Thus:

$$
\left(\frac{\partial z_s}{\partial z}\right)^2 = \frac{1}{4}\left[\left(\frac{\partial y_1}{\partial x_1}\right)^2 + \left(\frac{\partial y_1}{\partial x_2}\right)^2 + 2\frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2}\right]
$$

$$
\left(\frac{\partial z_s}{\partial z^*}\right)\left(\frac{\partial z_s}{\partial z^*}\right)^* = \frac{1}{4}\left[\left(\frac{\partial y_1}{\partial x_1}\right)^2 + \left(\frac{\partial y_1}{\partial x_2}\right)^2 - 2\frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2}\right] + \left(\frac{\partial y_1}{\partial x_2}\right)^2
$$

By taking the difference of these two equations:

$$
\left(\frac{\partial z_s}{\partial z}\right)^2 - \left(\frac{\partial z_s}{\partial z^*}\right)\left(\frac{\partial z_s}{\partial z^*}\right)^* = \frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2} - \left(\frac{\partial y_1}{\partial x_2}\right)^2 = \det A
$$

JACOBIAN DETERMINANT

Now, we can use the lens equation:

$$z_s = z - \sum_{i=1}^{N} \frac{m_i}{z^* - z_i^*}$$

To obtain:

$$\frac{\partial z_s}{\partial z} = 1 \qquad \frac{\partial z_s}{\partial z^*} = \sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2}$$

so that

$$\left(\frac{\partial z_s}{\partial z}\right)^2 - \left(\frac{\partial z_s}{\partial z^*}\right)\left(\frac{\partial z_s}{\partial z^*}\right)^* = \frac{\partial y_1}{\partial x_1}\frac{\partial y_2}{\partial x_2} - \left(\frac{\partial y_1}{\partial x_2}\right)^2 = \det A$$

$$\Rightarrow \quad \det A = 1 - \left|\sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2}\right|^2$$

CRITICAL LINES

From this equation:

$$\det A = 1 - \left| \sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2} \right|^2$$

We see that on the critical lines (detA=0)

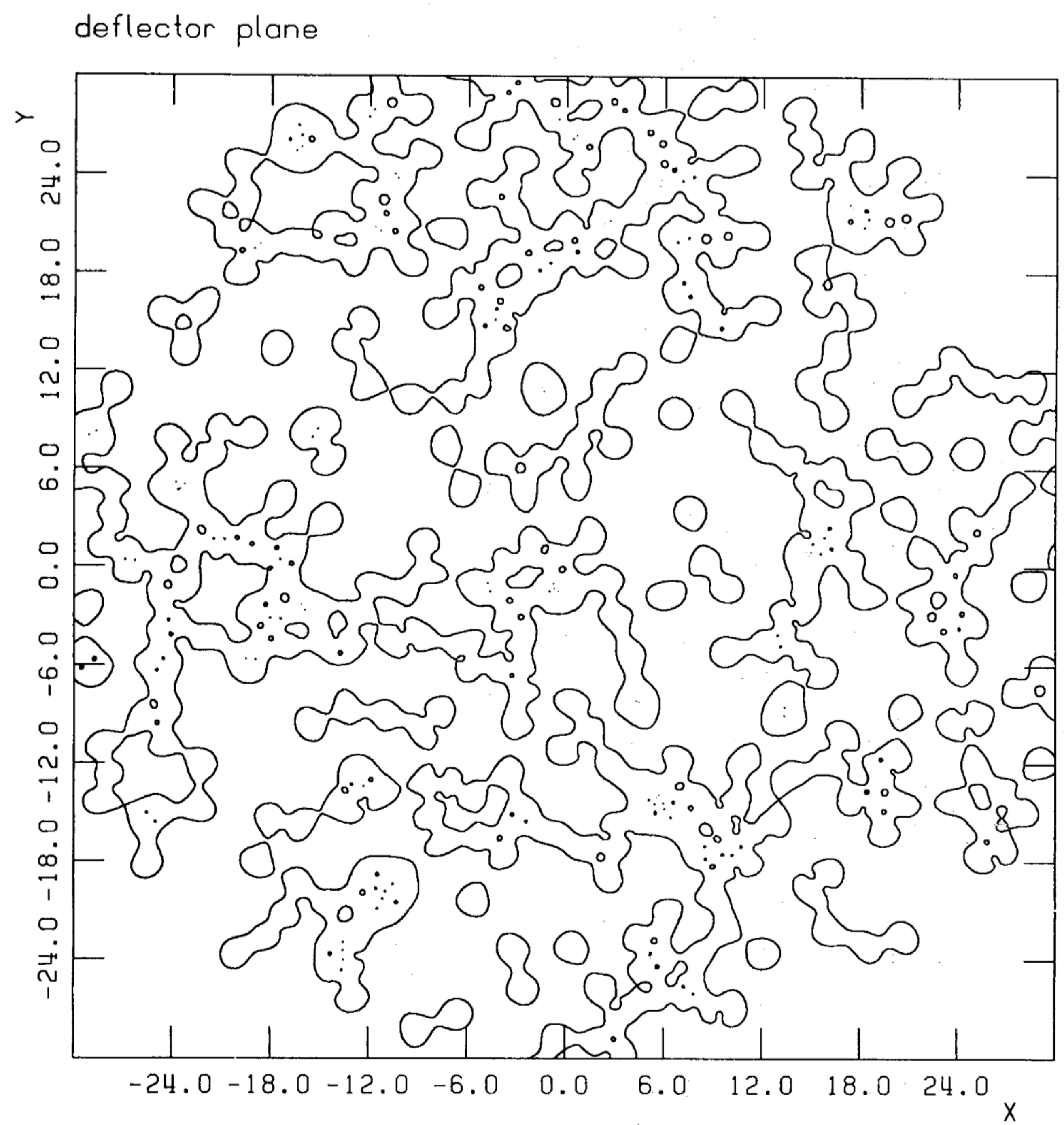
$$\left| \sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2} \right|^2 = 1$$

This sum has to be satisfied on the unit circle:

$$\sum_{i=1}^{N} \frac{m_i}{(z^* - z_i^*)^2} = e^{i\phi} \qquad \phi \in [0, 2\pi)$$

Getting rid of the fraction, this equation can be turned into a polynomial of degree 2N: for each phase, there are <=2N critical points. Solving for all phases, we find up to 2N critical lines.

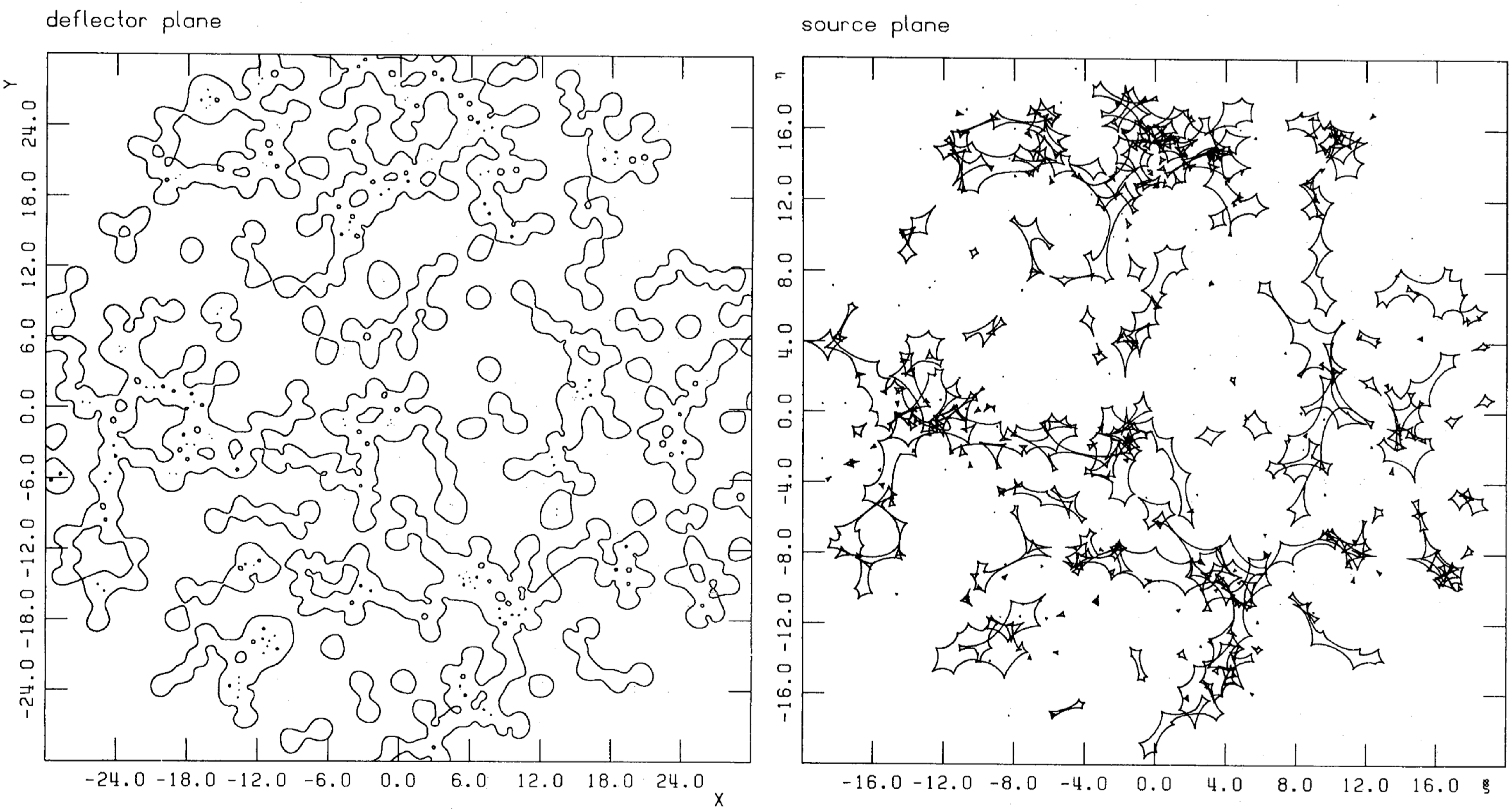
CRITICAL LINES



critical lines originated by 400 stars

Witt, 1990, A&A, 236, 311

CRITICAL LINES AND CAUSTICS



critical lines and caustics originated by 400 stars

Witt, 1990, A&A, 236, 311