GRAVITATIONAL LENSING

LECTURE 12

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AA 2016-2017

CONTENTS

- microlensing magnification
- single microlensing light-curve

IMAGE MAGNIFICATION

Since the lens is axially-symmetric:

$$\det A(x) = \frac{y}{x}\frac{dy}{dx}$$

From the lens equation, it follows that:

$$\lambda_t(x) = \frac{y}{x} = \left(1 - \frac{1}{x^2}\right)$$

$$\lambda_r(x) = \frac{dy}{dx} = \left(1 + \frac{1}{x^2}\right) .$$

The second eigenvalue is always positive (no critical line). The first is zero on the circle

$$x^2 = 1$$

Note that y/x is the ratio of the source to image sizes in the tangential direction, thus the first eigenvalue is the tangential eigenvalue.

Thus, the Einstein ring is the tangential critical line! The corresponding caustic is a point at y=0

IMAGE MAGNIFICATION

Clearly,

$$
\begin{aligned}
\det A(x) &= \frac{y}{x}\frac{\mathrm{d}y}{\mathrm{d}x} \\
\lambda_t(x) &= \frac{y}{x} = \left(1 - \frac{1}{x^2}\right) \\
\lambda_r(x) &= \frac{\mathrm{d}y}{\mathrm{d}x} = \left(1 + \frac{1}{x^2}\right) .
\end{aligned}
\quad \Rightarrow \quad
\mu(x) = 1 - \frac{1}{x^4}
$$

SOURCE MAGNIFICATION

Let's compute now the source magnification. This is the sum of the magnifications of the two images

$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right] \Rightarrow$$

$$\begin{aligned} \frac{x}{y} &= \frac{1}{2}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right) \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{2}\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \end{aligned}$$

Thus the magnifications at the two image positions are

$$\begin{aligned} \mu_\pm(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\ &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\ &= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right) \end{aligned}$$

SOURCE MAGNIFICATION

Note that:

$$
\begin{aligned}
\mu_\pm(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\
&= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\
&= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right) \\
&= \pm\frac{(y^2+2 \mp y\sqrt{y^2+4})}{2y\sqrt{y^2+4}} \\
&= \pm\frac{(2y^2+4 \mp 2y\sqrt{y^2+4})}{4y\sqrt{y^2+4}} \\
&= \pm\frac{(y-\sqrt{y^2+4})^2}{4y\sqrt{y^2+4}}\,.
\end{aligned}
$$

Thus the parity of the images is different!

SOURCE MAGNIFICATION

The total magnification is obtained by summing the magnifications of the images:

$$\begin{aligned}\mu_\pm(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\ &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\ &= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right)\end{aligned}$$

$$\Rightarrow \quad \mu(y) = |\mu_+(y)| + |\mu_-(y)| = \frac{y^2+2}{y\sqrt{y^2+4}}$$

The sum of the signed magnification is one!

We can take a power series of the magnification to see that $\mu \propto 1 + 2/y^4$ for $y \to \infty$.

Thus, the magnification drops quickly as the source moves away from the lens!

SOURCE MAGNIFICATION

In addition:

$$\left|\frac{\mu_+}{\mu_-}\right| = \left(\frac{y+\sqrt{y^2+4}}{y-\sqrt{y^2+4}}\right)^2 = \left(\frac{x_+}{x_-}\right)^2.$$

$$\lim_{y\to\infty}\mu_- = 0$$

$$\lim_{y\to\infty}\mu_+ = 1$$

$$\left|\frac{\mu_+}{\mu_-}\right| \propto y^4$$

As we move the source away from the lens, the image in x_+ dominates the flux budget very soon.

A SOURCE ON THE EINSTEIN RING

For a source on the Einstein ring:

$$x_\pm = \frac{1 \pm \sqrt{5}}{2} \qquad\qquad \mu_\pm = \left[1 - \left(\frac{2}{1 \pm \sqrt{5}}\right)^4\right]^{-1}$$

Therefore: $$\mu = |\mu_+| + |\mu_-| = 1.17 + 0.17 = 1.34$$

$$\Delta m = -2.5 \log \mu \sim 0.3$$

Given how quickly the magnification drops by moving the source away from the lens, we can assume that only sources within the Einstein radius are magnified in a significant way.

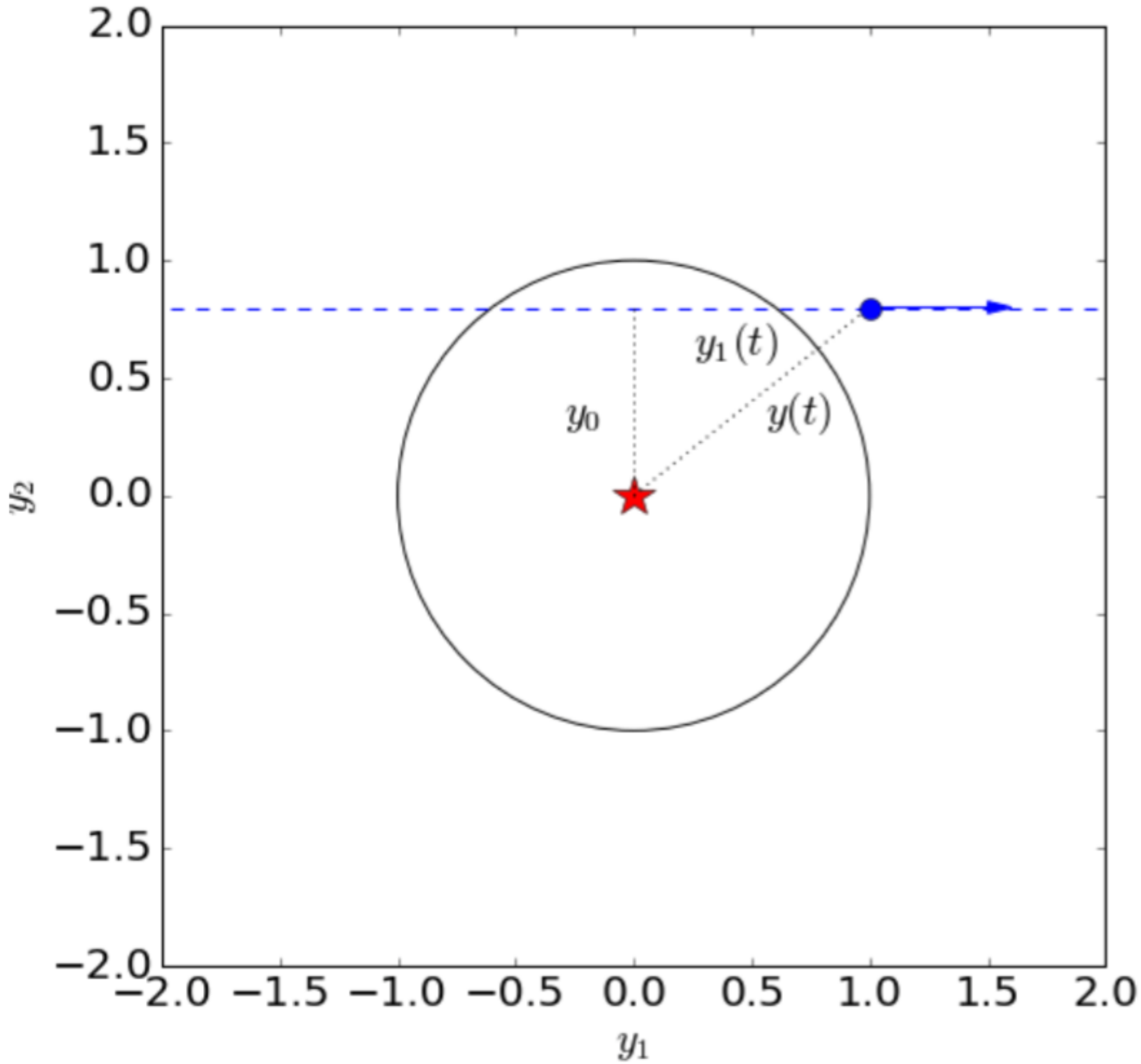
For this reason, the circle within the Einstein radius is assumed to be the cross section for microlensing.

MICROLENSING LIGHT CURVE

- typical Einstein radii for lenses in the MW are $\sim$1 mas
- thus, the image separation is too small to resolve the images
- how to detect a microlensing event?

MICROLENSING LIGHT CURVE

- stars (including the sun) rotate around the galactic center
- rotation is differential (i.e. speed depends on distance)
- this introduces a relative velocity between the lenses and the sources (either in the bulge or in the MCs)
- this causes the relative distance between the sources and the lenses to vary over time…



MICROLENSING LIGHT CURVE

Assume a linear trajectory of the source relative to the lens, with impact parameter y_0

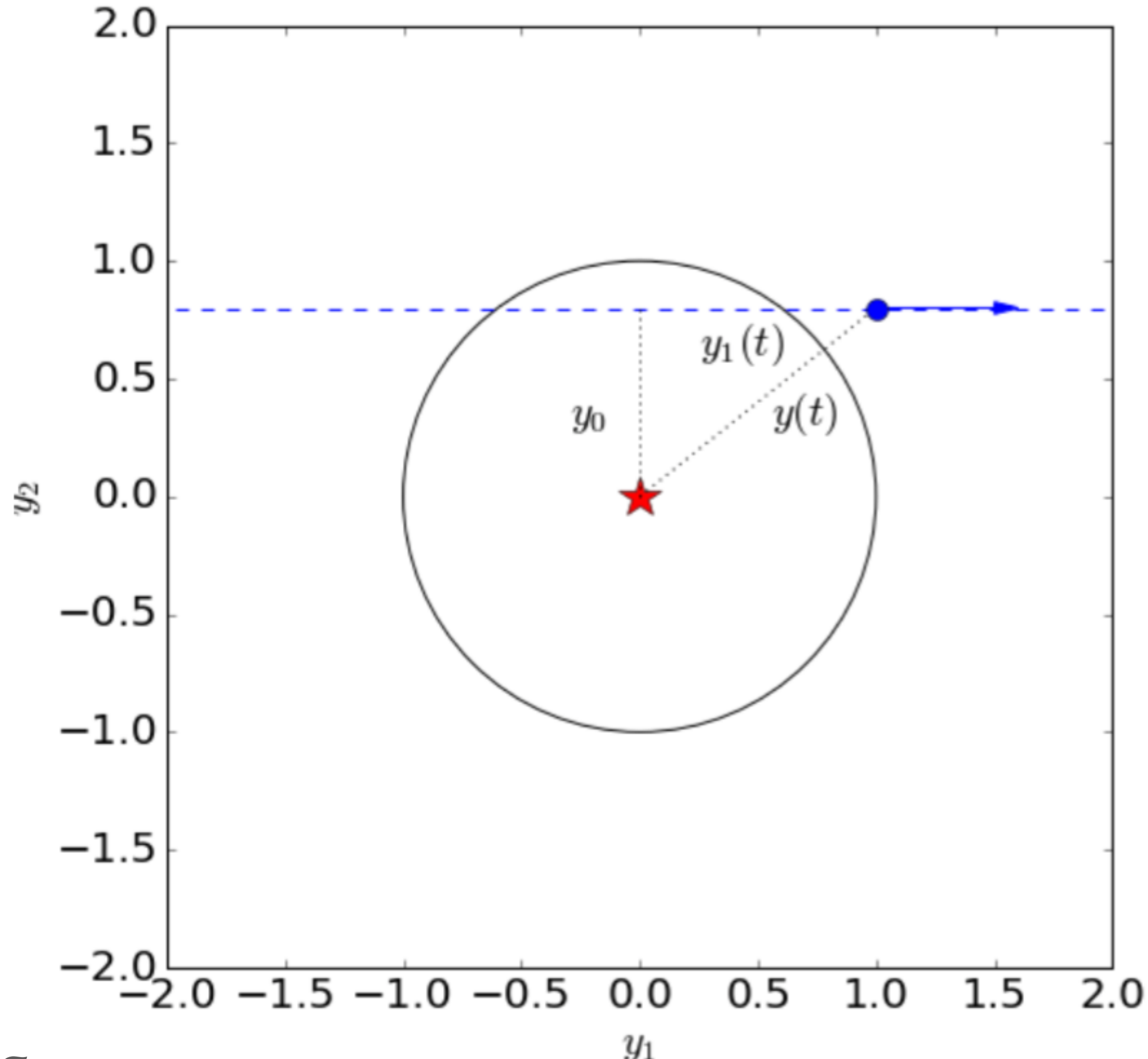
Assume also constant transverse velocity v:

$$y_1(t) = \frac{v(t-t_0)}{D_L\theta_E}$$

We can define a characteristic time of the event:

$$t_E = \frac{D_L\theta_E}{v} = \frac{\theta_E}{\mu_{rel}}$$

This is the ***Einstein radius crossing time***

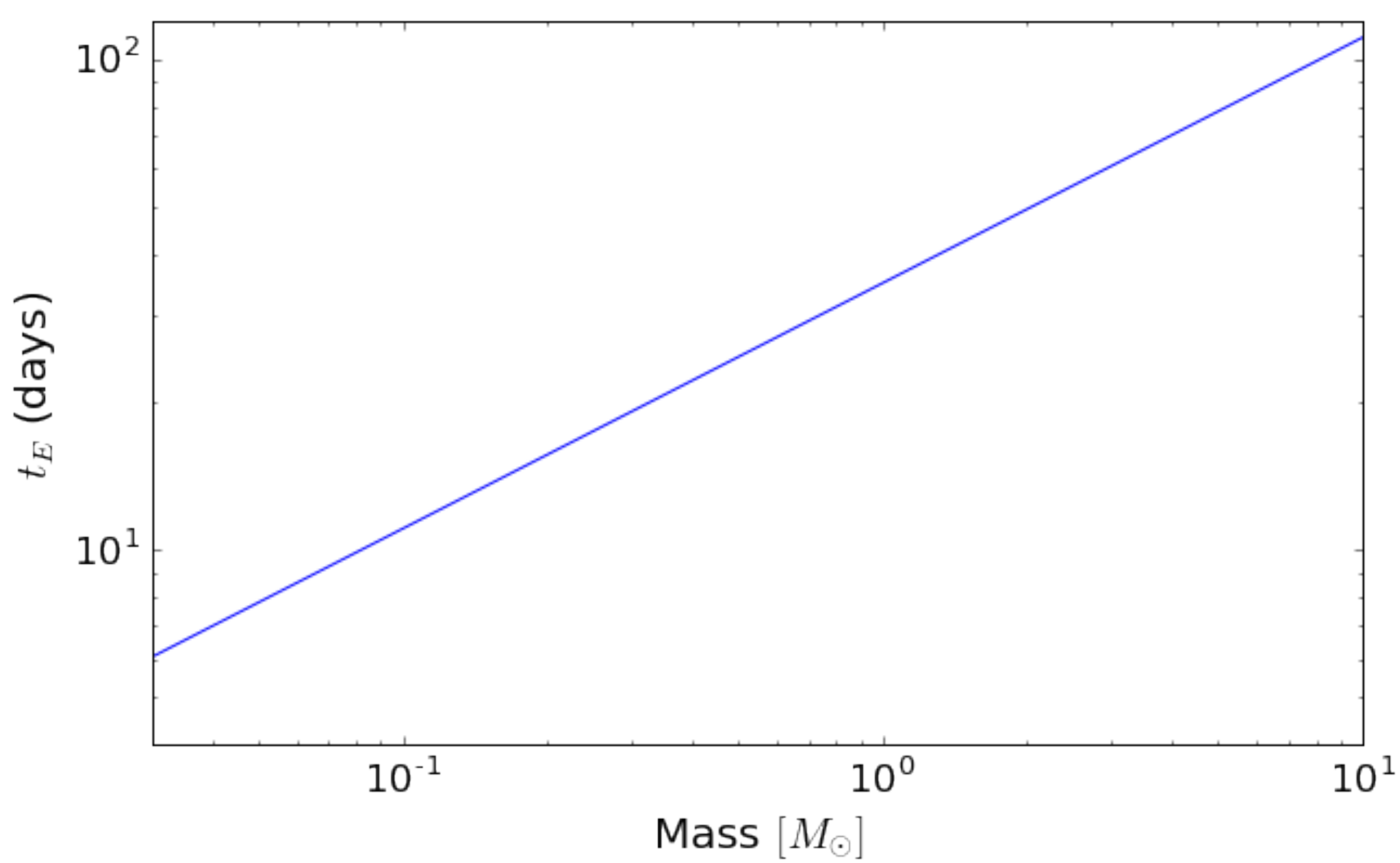


MICROLENSING LIGHT CURVE

Given the definition of Einstein radius $$\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}}$$

The order of magnitude of the t_E *is*

$$t_E \approx 19\text{ days}\sqrt{4\frac{D_{\rm L}}{D_{\rm S}}\left(1-\frac{D_{\rm L}}{D_{\rm S}}\right)}\left(\frac{D_{\rm S}}{8\text{kpc}}\right)^{1/2}\left(\frac{M}{0.3M_\odot}\right)^{1/2}\left(\frac{v}{200\text{km}/s}\right)^{-1}$$



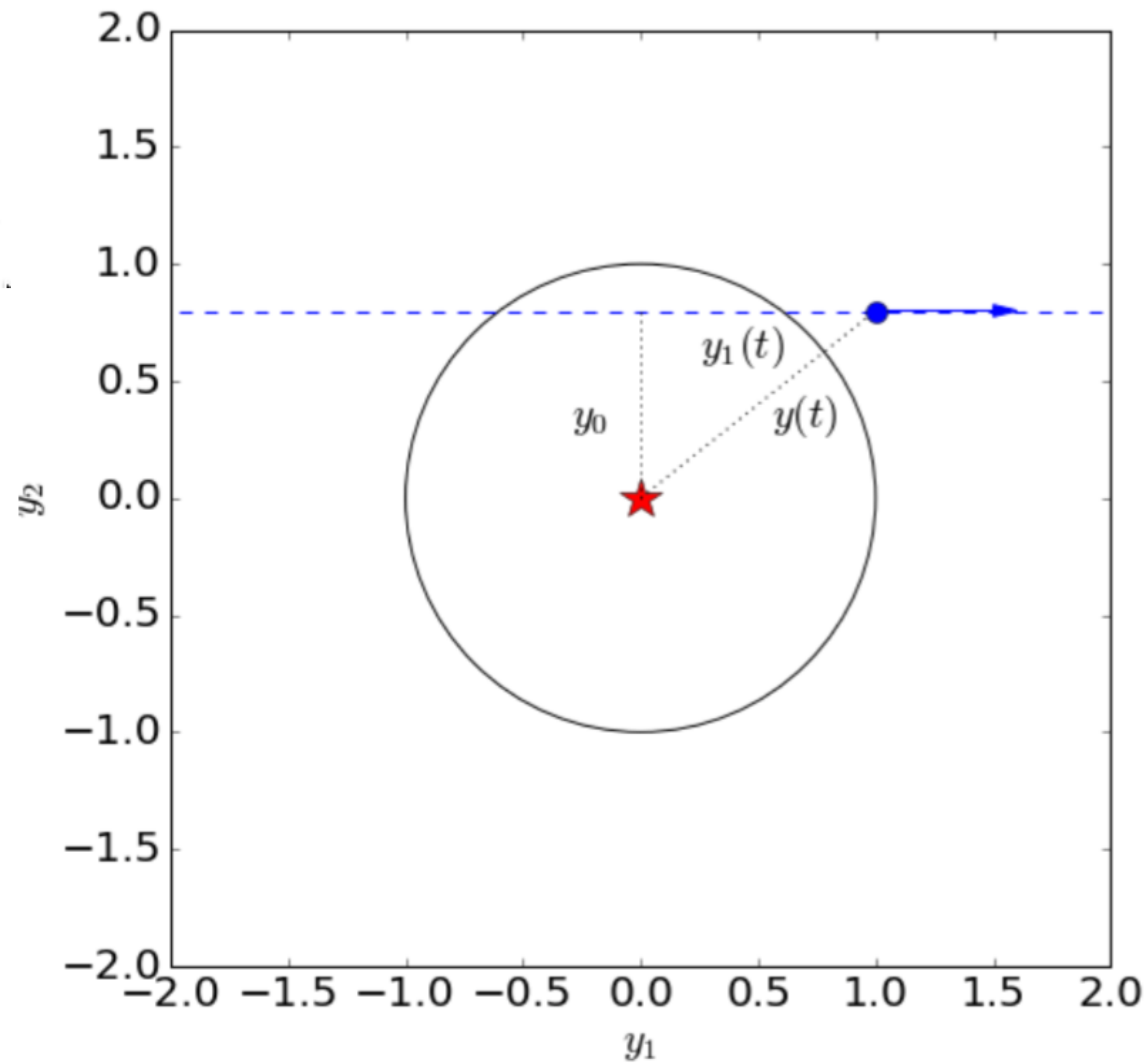
MICROLENSING LIGHT CURVE

We obtain

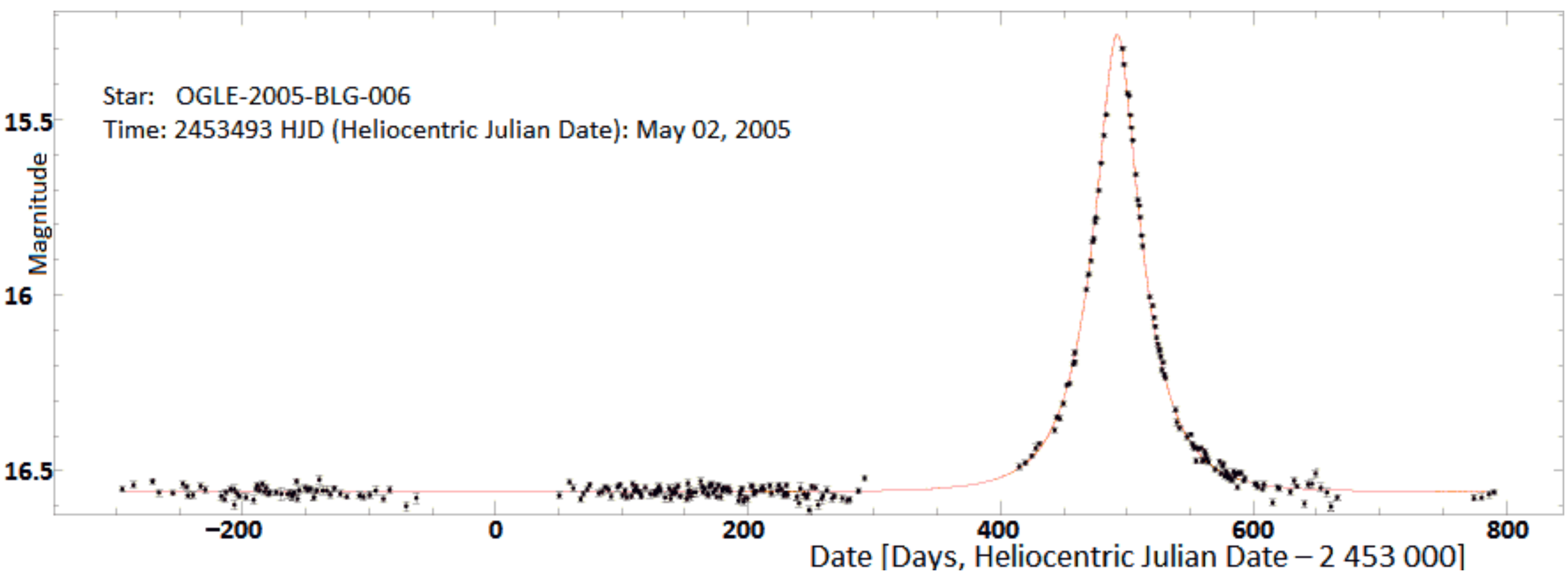
$$y_1(t) = \frac{v(t-t_0)}{D_L\theta_E} \quad \Rightarrow \quad y_1(t) = \frac{(t-t_0)}{t_E}$$

Thus:

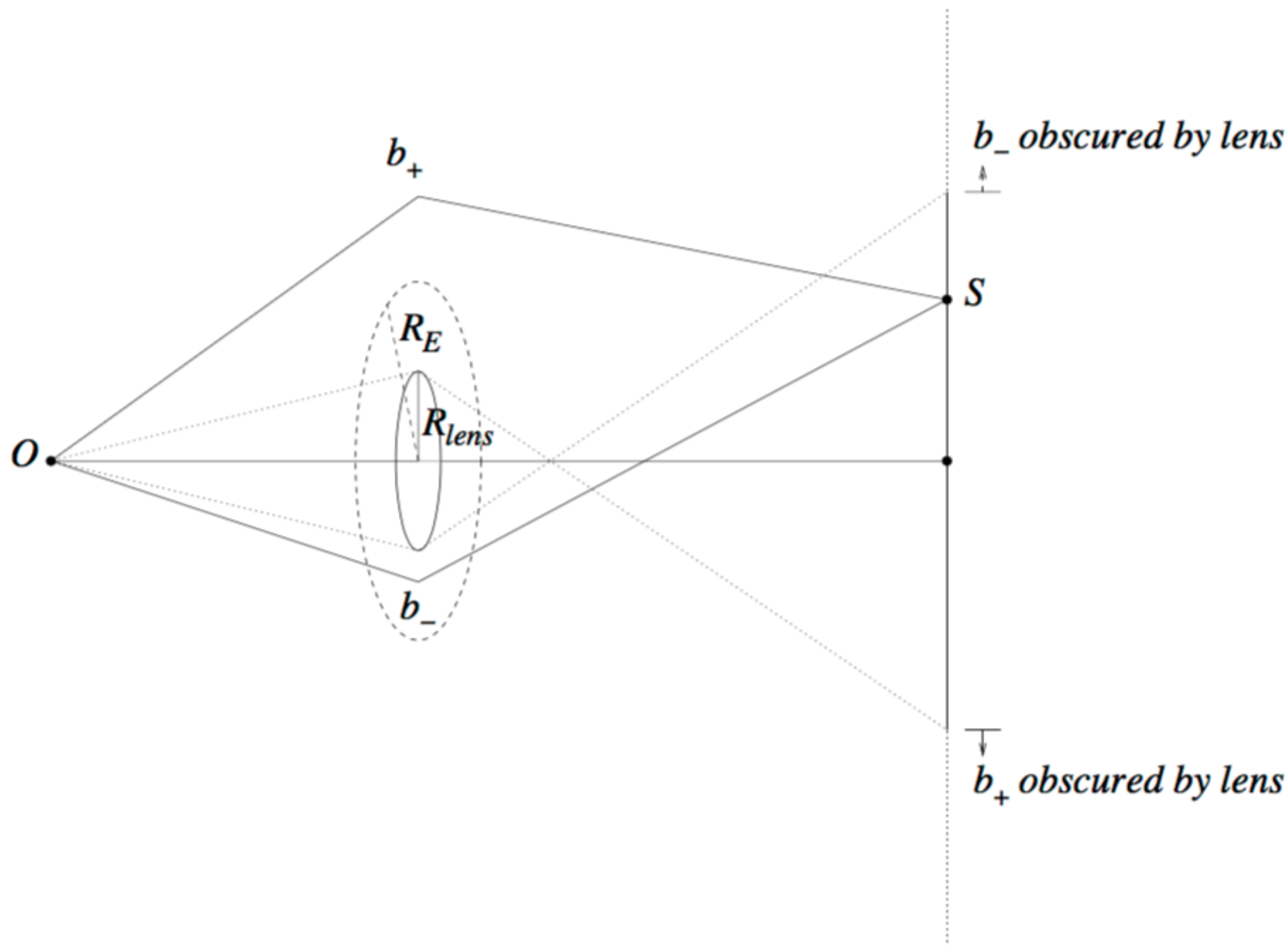
$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t-t_0)^2}{t_E^2}}$$



EXAMPLE OF STANDARD LIGHT CURVE



NON-STANDARD LIGHT CURVES



finite lens size effect

NON-STANDARD LIGHT CURVES