# GRAVITATIONAL LENSING

## 2 - DEFLECTION OF LIGHT

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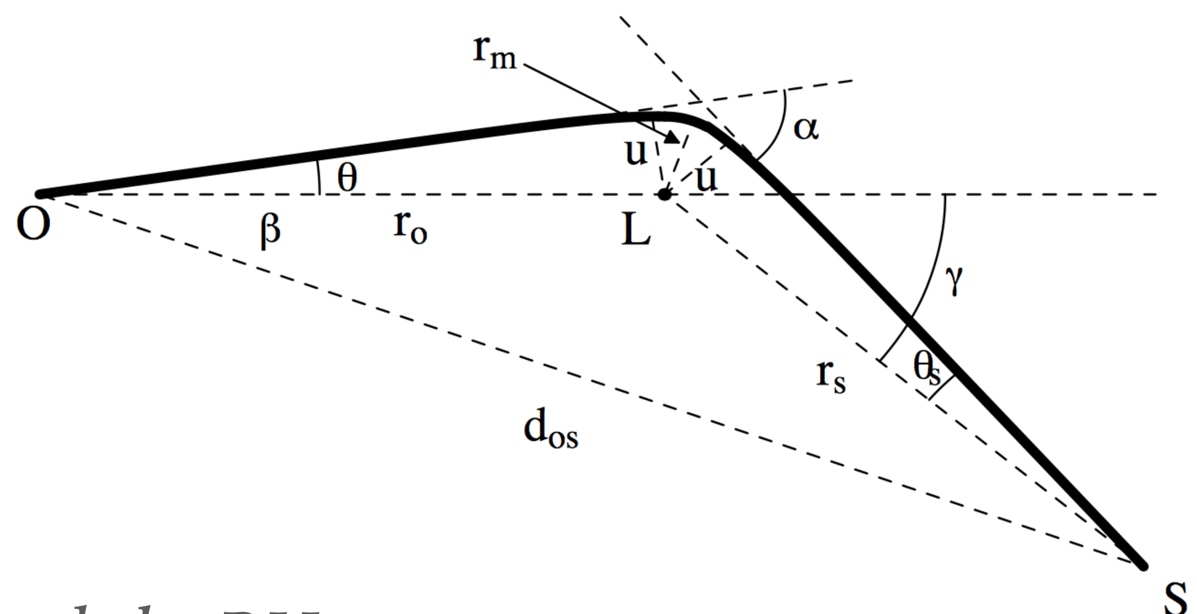
# DEFLECTION OF LIGHT BY A BLACK HOLE

*suggested reading: [http://arxiv.org/pdf/0911.2187v2.pdf](http://arxiv.org/pdf/0911.2187v2.pdf)*

*Generic static spherically symmetric metric:*

$$ds^2 = A(R)dt^2 - B(R)dR^2 - C(R)(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\hat{\alpha} = -\pi + \frac{2G}{c^2}\int_{R_m}^{\infty} u\sqrt{\frac{B(R)}{C(R)[C(R)/A(R) - u^2]}}dR$$




*u=impact parameter*

*$R_m$=minimum distance between the photon and the BH*

$$u^2 = \frac{C(R_m)}{A(R_m)}$$

# DEFLECTION OF LIGHT BY A BLACK HOLE

*For the Schwarzschild metric:*

$$A(R) = 1 - 2GM/Rc^2 \qquad B(R) = A(R)^{-1} \qquad C(R) = R^2$$

*The weak-field limit holds for* $R_m \gg 2GM/c^2$

*The exact solution of the integral in the previous slide was found by Darwin (1959):*

$$\hat{\alpha} = -\pi + 4\frac{G}{c^2}\sqrt{R_m/s}F(\varphi, m)$$

$$\begin{aligned} s &= \sqrt{(R_m - 2M)(R_m + 6M)} \\ m &= (s - R_m + 6M)/2s \\ \varphi &= \arcsin\sqrt{2s/(3R_m - 6M + s)} \end{aligned}$$

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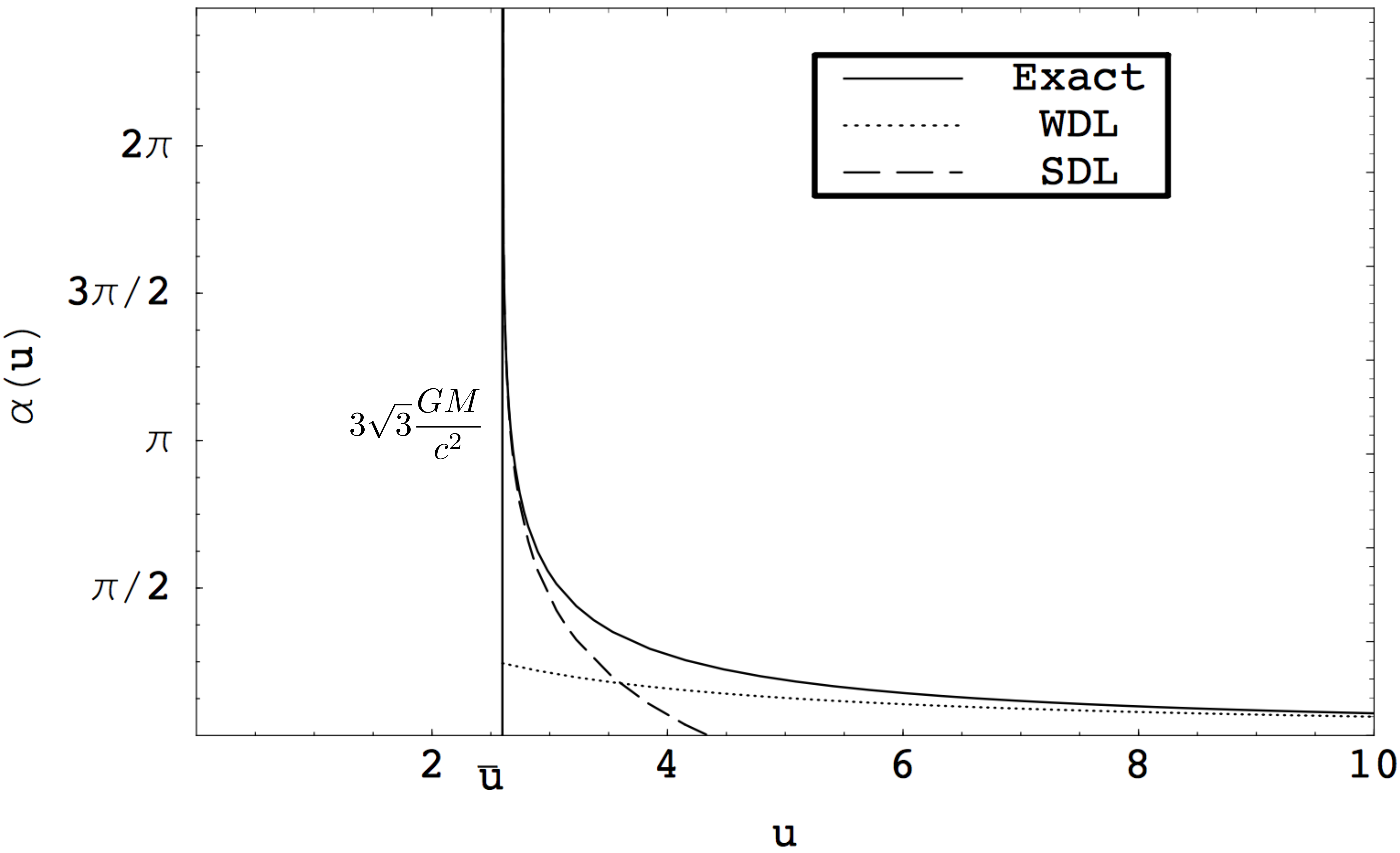
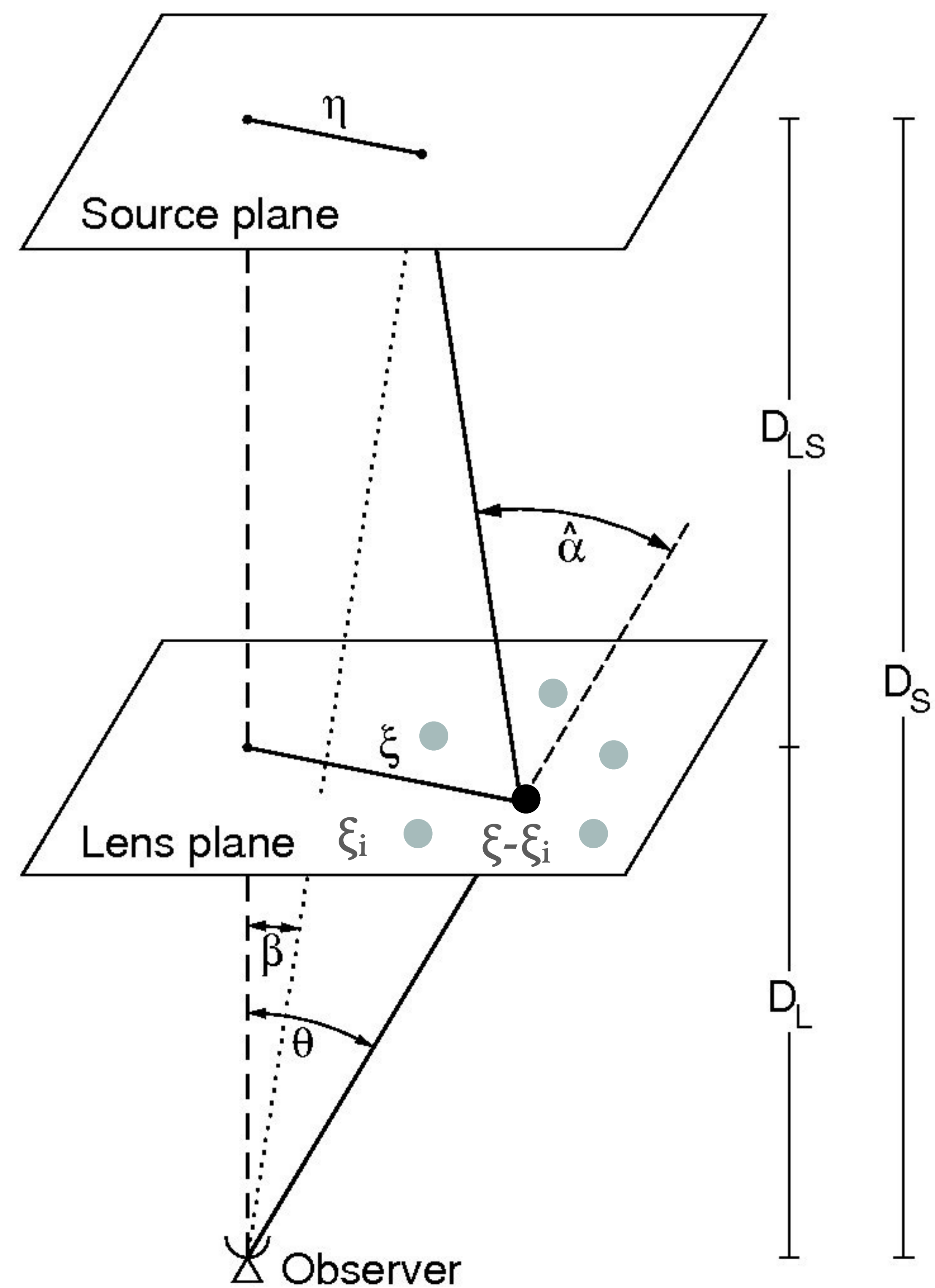
*for the Schwarzschild metric*



# DEFLECTION BY AN ENSEMBLE OF POINT MASSES

- Thin screen approximation
- Remaining in the weak field limit, one can use the superposition principle
- The deflection angle by a system of point masses is the vectorial sum of the deflection angles of the single lenses
- Example: studying the deflection by mass distributions obtained from N-body/hydrodynamical simulations



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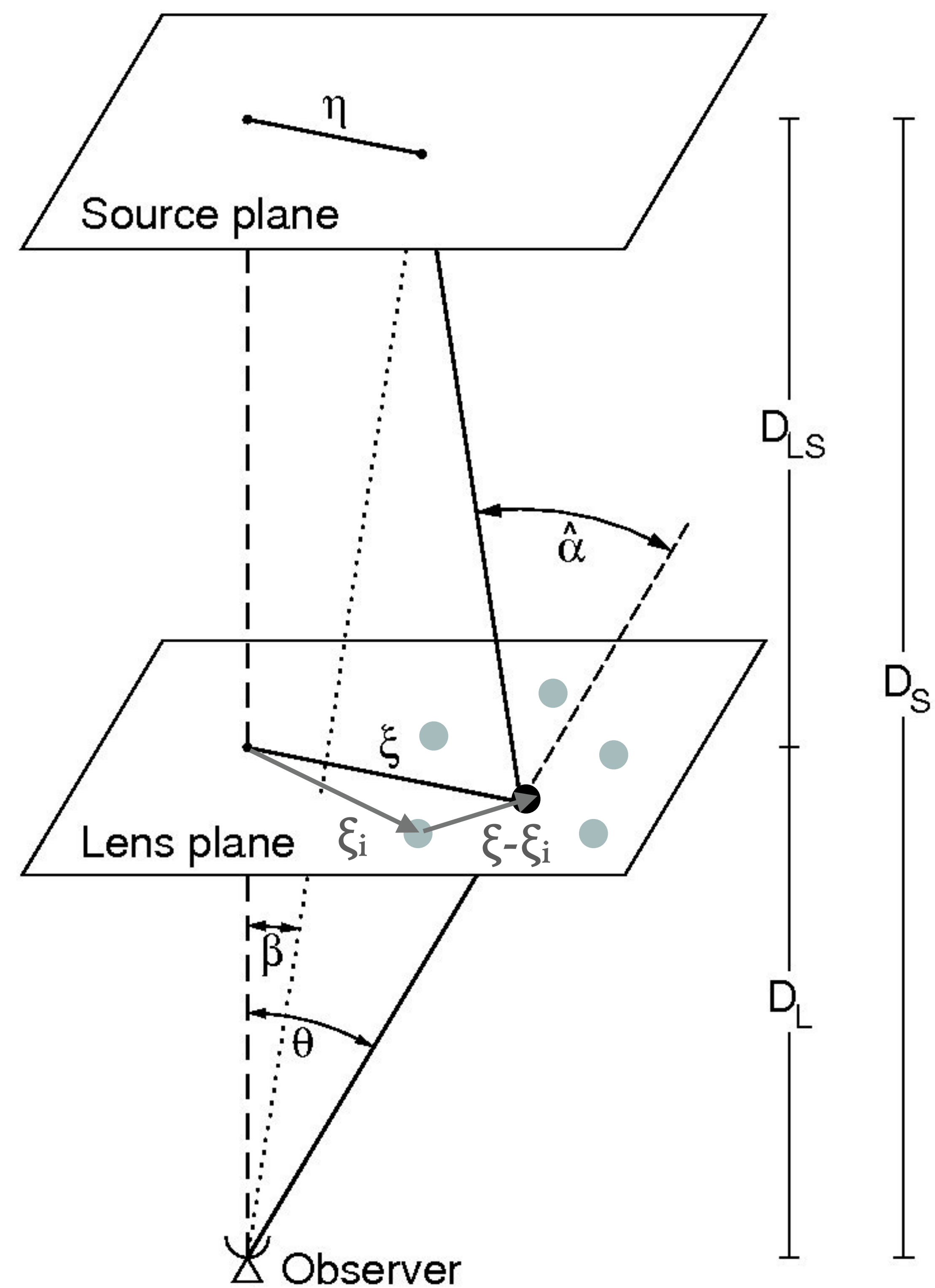
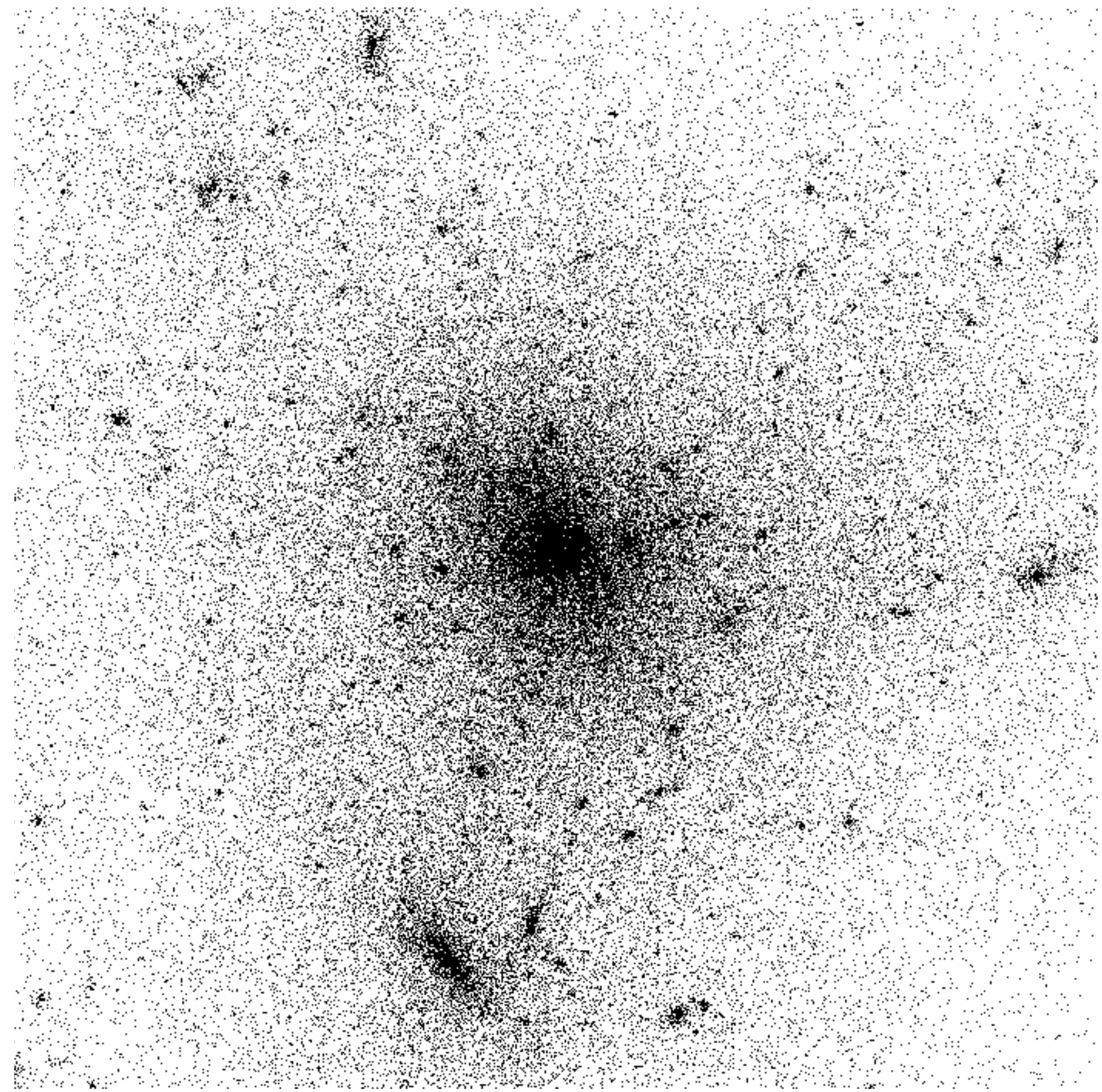
$$\hat{\vec{\alpha}}(\vec{\xi}) = \sum_i \hat{\vec{\alpha}}_i(\vec{\xi} - \vec{\xi}_i) = \frac{4G}{c^2} \sum_i M_i \frac{\vec{\xi} - \vec{\xi}_i}{|\vec{\xi} - \vec{\xi}_i|^2}$$



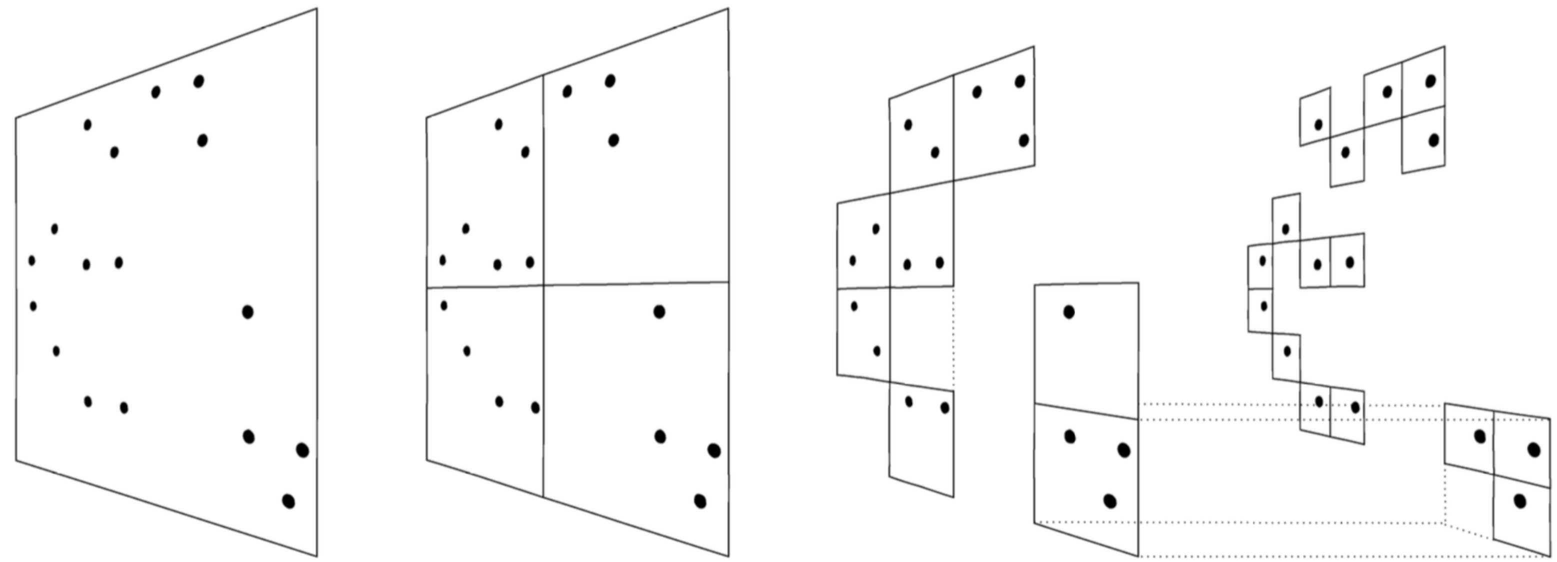
# DEFLECTION BY AN ENSEMBLE OF POINT MASSES

- Structure formation is often studied using numerical simulations
- Galaxies, galaxy clusters, etc. are described by ensembles of particles
- The calculation of the deflection angle by direct summation of all contributions from each particle has a computational cost $O(N^2)$



# POSSIBLE SOLUTION: TREE ALGORITHM (BARNES & HUT, 1986)

*Barnes & Hut "oct-tree"*



*Short-range contributions (direct summation): particles in cells subtending large angles*

*Long-range contributions (grouped, Taylor expansion of the deflection potential,…): particles in cells subtending angles smaller than a chosen threshold*

*Cost of calculations scales as O(NLog(N))*

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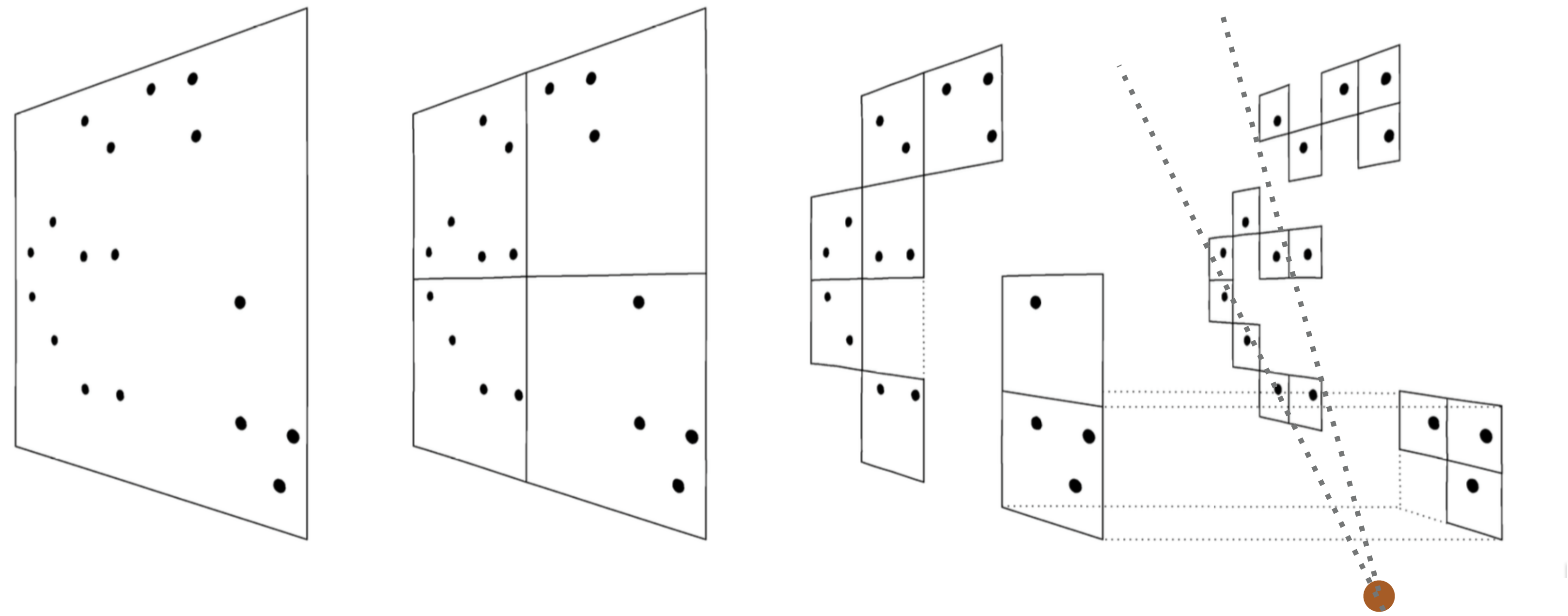

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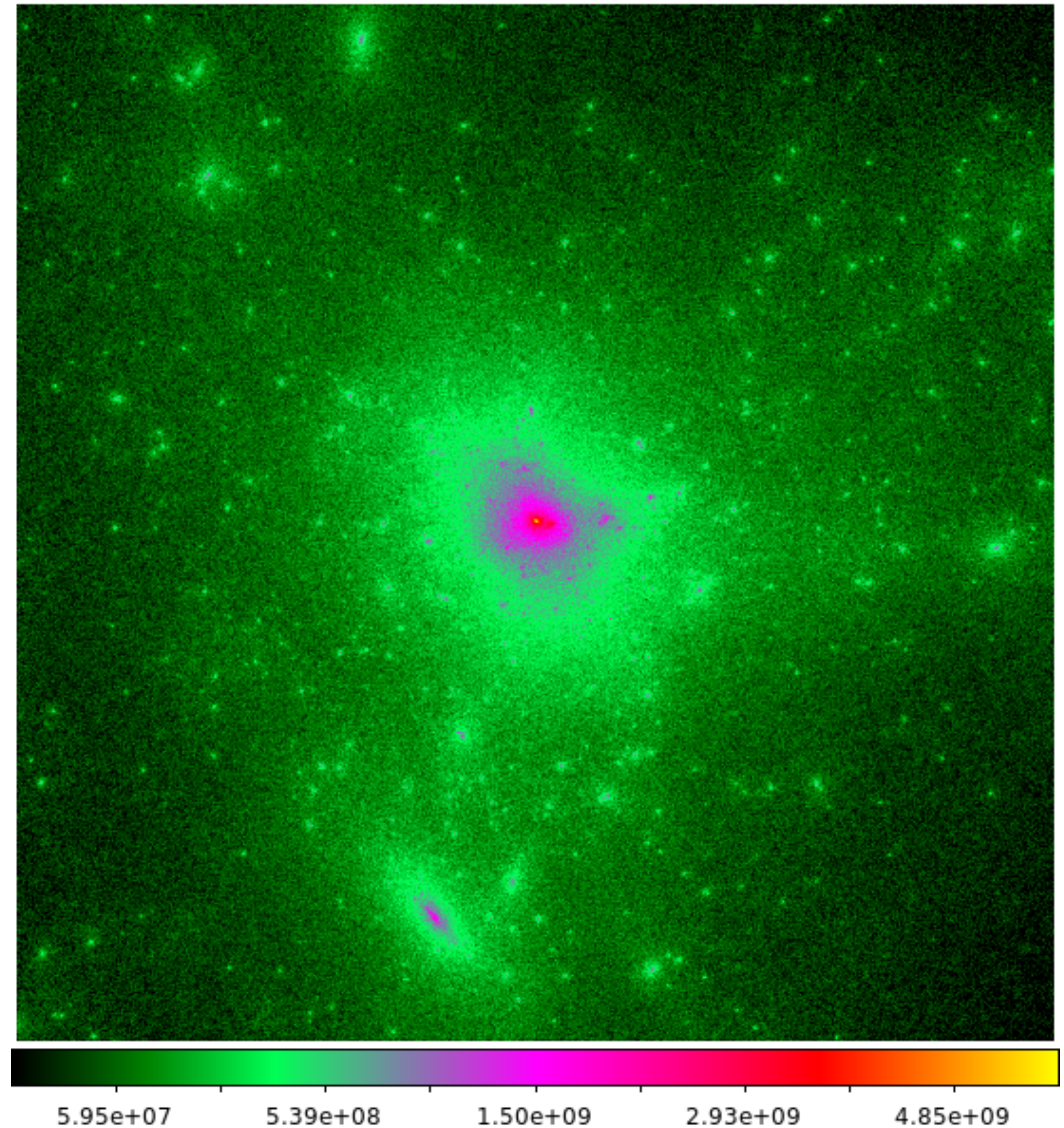
# DEFLECTION BY AN EXTENDED MASS DISTRIBUTION

- This can be easily generalized to the case of a continuum distribution of mass

$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z)\ \mathrm{d}z$$

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}')\Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2}\ \mathrm{d}^2\xi'$$



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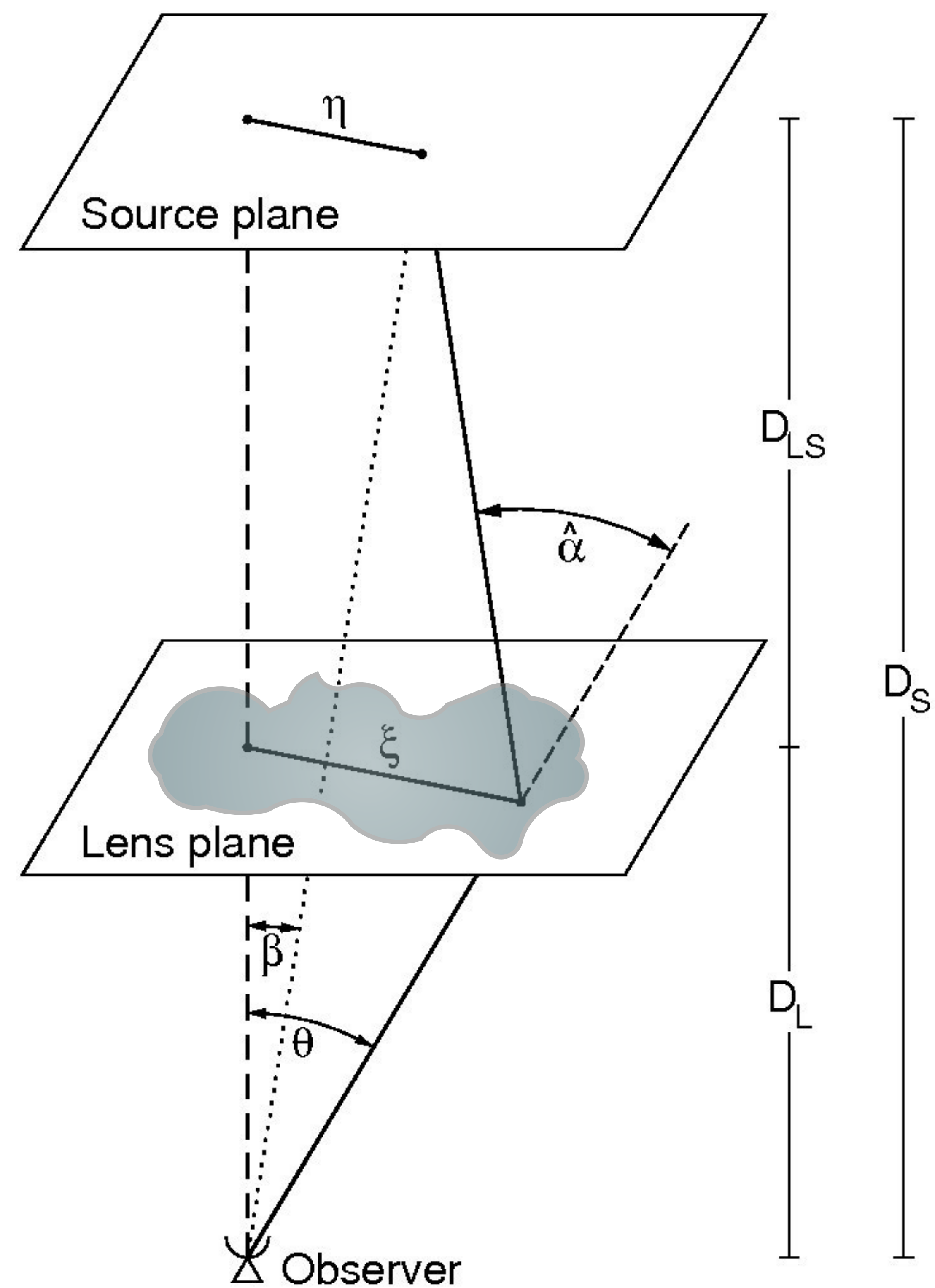

# HOW TO COMPUTE THIS DEFLECTION ANGLE?

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi}')\Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2} \, \mathrm{d}^2\xi'$$

*This is a convolution!*

*Kernel function:* $$\vec{K}(\vec{\xi}) \propto \frac{\vec{\xi}}{|\vec{\xi}|^2}$$

$$\tilde{\hat{\alpha}}_i(\vec{k}) \propto \tilde{\Sigma}(\vec{k})\tilde{K}_i(\vec{k})$$

*This is the typical problem to be solved using FFT (Cooley and Tukey, 1965)*