GRAVITATIONAL LENSING

10 - MICROLENSING

Massimo Meneghetti

AA 2018-2019

MICROLENSING

- Microlensing is a lensing regime which include effects produced by a broad range of masses: from planets to ensembles of stars
- given the small sizes of the lens, these are (to first-order) assimilated to point masses.
- microlensing effects are mostly detectable and searched within our own galaxy, in particular by monitoring huge amounts of stars in the bulge of the MW or in the Magellanic Clouds
- nevertheless, microlensing effects are important also in extragalactic lenses. Small masses in distant galaxies, for example, introduce perturbations to the lensing signal of their hosts

THE POINT MASS LENS MODEL

- The deflection angle of the point mass lens was derived in the first lecture
- the lensing potential can be readily derived

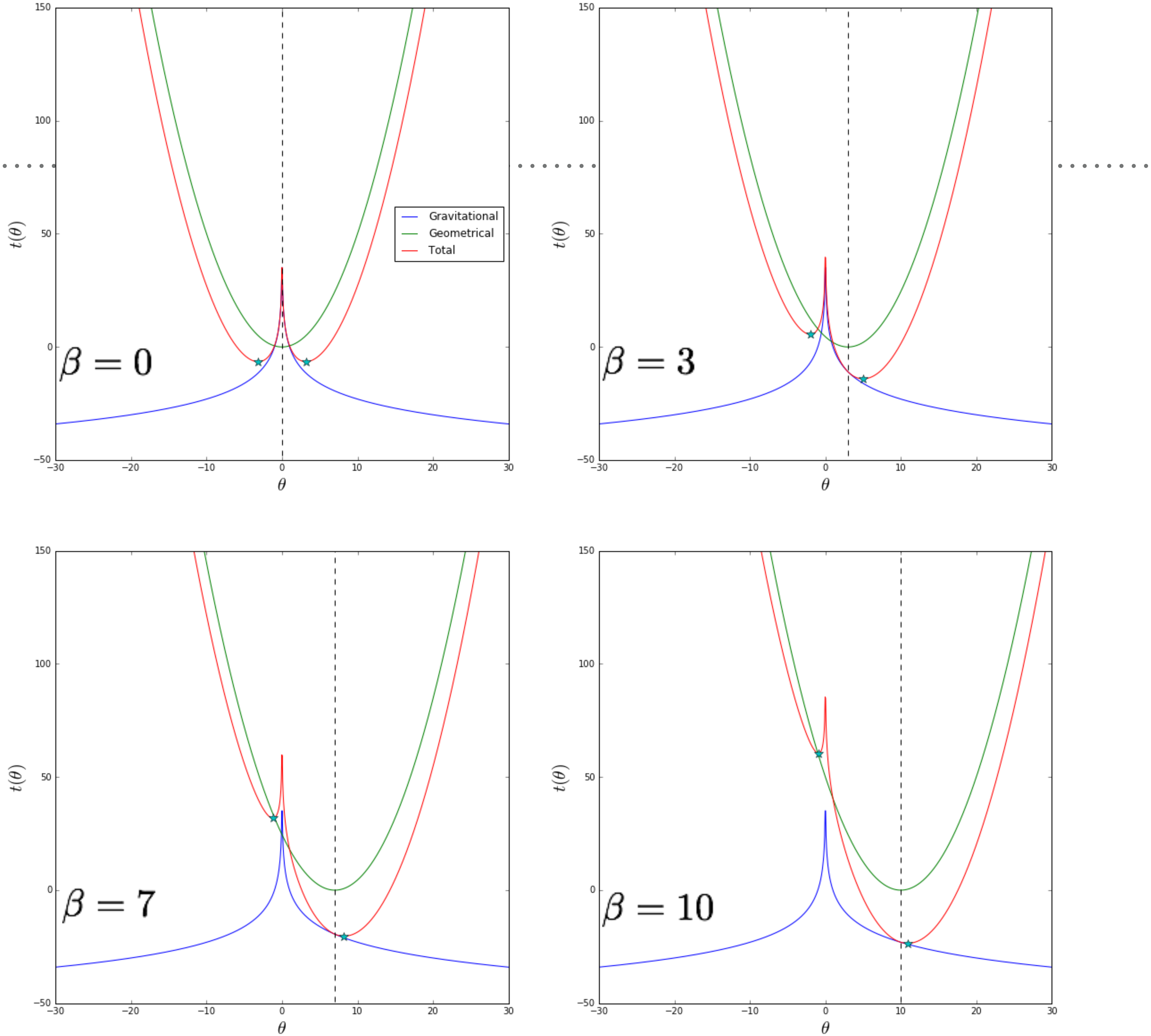
$$\vec{\hat{\alpha}} = \frac{4GM}{c^2}\frac{\vec{\xi}}{|\vec{\xi}|^2} = \frac{4GM}{c^2 D_{\mathrm{L}}}\frac{\vec{\theta}}{|\vec{\theta}|^2}$$

$$\vec{\alpha} = \frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}}\vec{\hat{\alpha}} = \vec{\nabla}\hat{\Psi}$$

$$\nabla \ln|\vec{x}| = \frac{\vec{x}}{|\vec{x}|^2}$$

$$\Downarrow$$

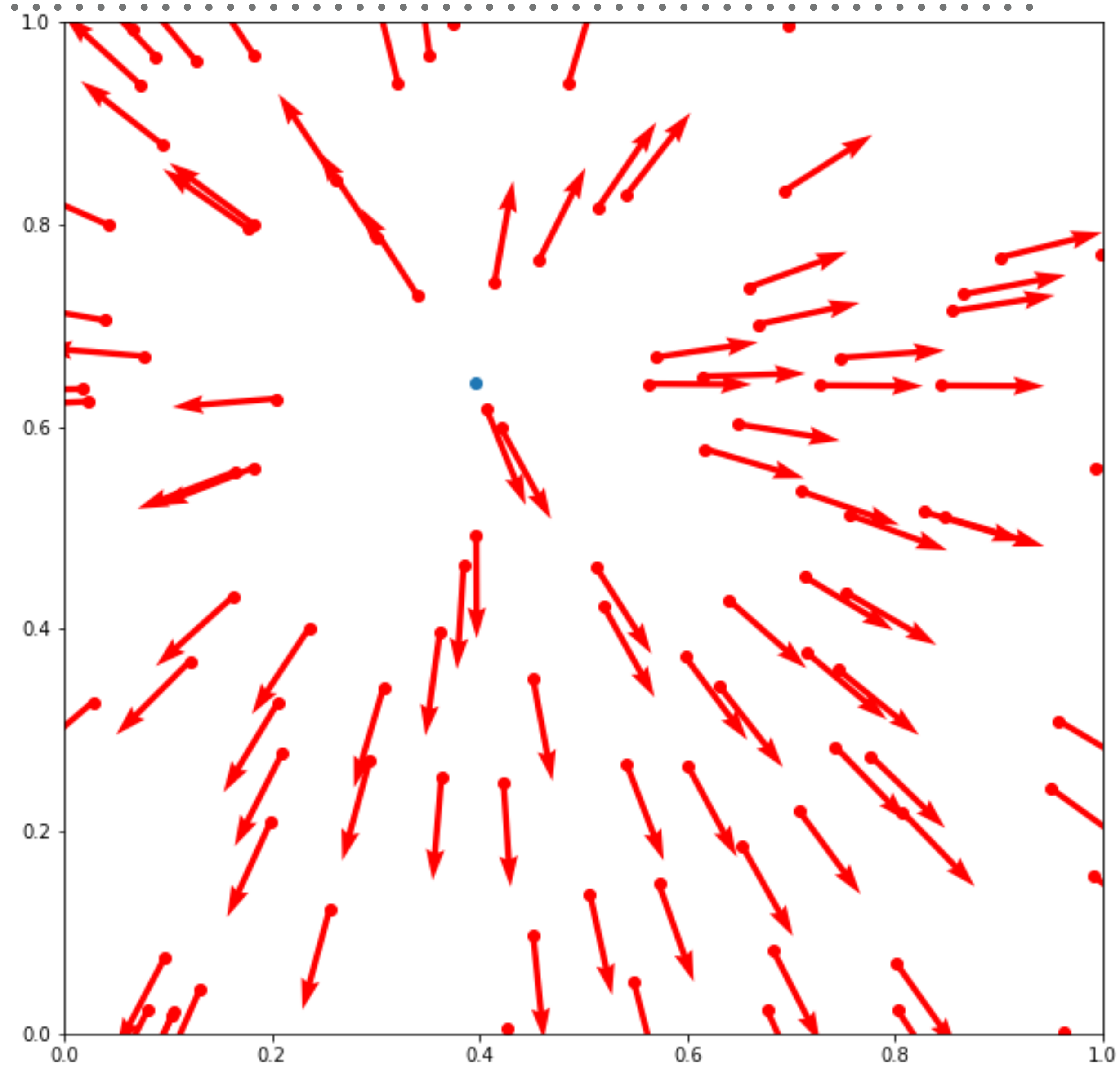
$$\hat{\Psi}(\vec{\theta}) = \frac{4GM}{c^2}\frac{D_{\mathrm{LS}}}{D_{\mathrm{L}}D_{\mathrm{S}}}\ln|\vec{\theta}|$$



$\beta = 0$
$\beta = 3$
$\beta = 7$
$\beta = 10$
Gravitational
Geometrical
Total
$t(\theta)$
θ

LENS EQUATION

- the deflection angle always points away from the lens
- given the symmetry of the lens, we can omit the vector notation in most equations
- the lens equation reads:
- this is clearly quadratic in θ
- so, for each source there are two images, whose positions can be determined by solving the lens equation



$$\hat{\alpha} = \frac{4GM}{c^2 \xi} = \frac{4GM}{c^2 D_L \theta}$$

$$\beta = \theta - \frac{4GM}{c^2 D_L \theta} \frac{D_{LS}}{D_S}$$

SOLUTIONS OF THE LENS EQUATION

- we introduce the Einstein radius:

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}}$$

- by inserting into the lens equation:

$$\beta = \theta - \frac{4GM}{c^2 D_{\rm L}\theta}\frac{D_{\rm LS}}{D_{\rm S}}$$

$$\Downarrow$$

- if we divide by θ_E, we obtain an a-dimensional form of the lens equation

$$\beta = \theta - \frac{\theta_E^2}{\theta}$$

$$y = x - \frac{1}{x}$$

- this is a very convenient way of writing the lens equation, because we get rid of all constants.

SOLUTIONS OF THE LENS EQUATION

$$y = x - \frac{1}{x}$$

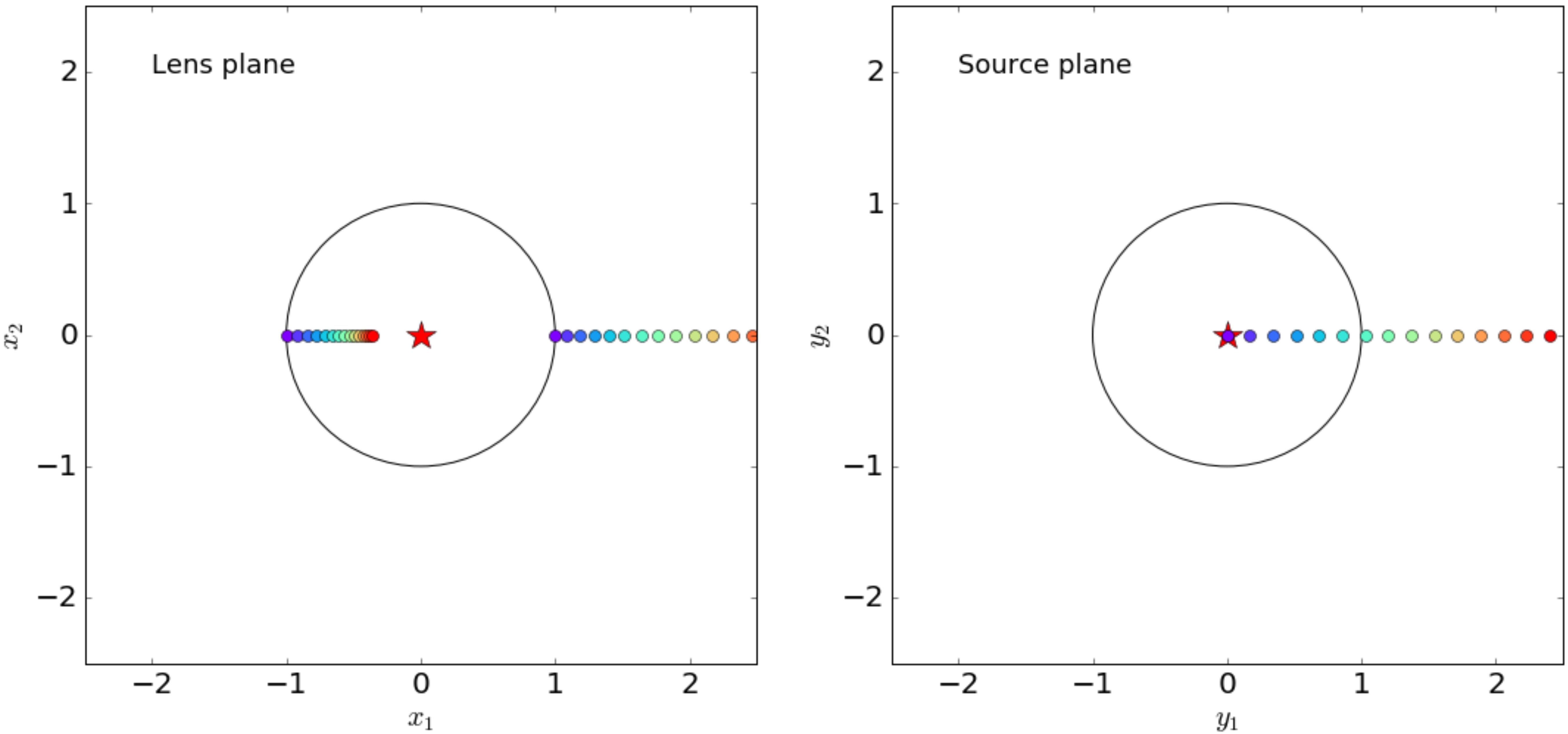
$$\Downarrow$$

$$x^2 - xy - 1 = 0$$

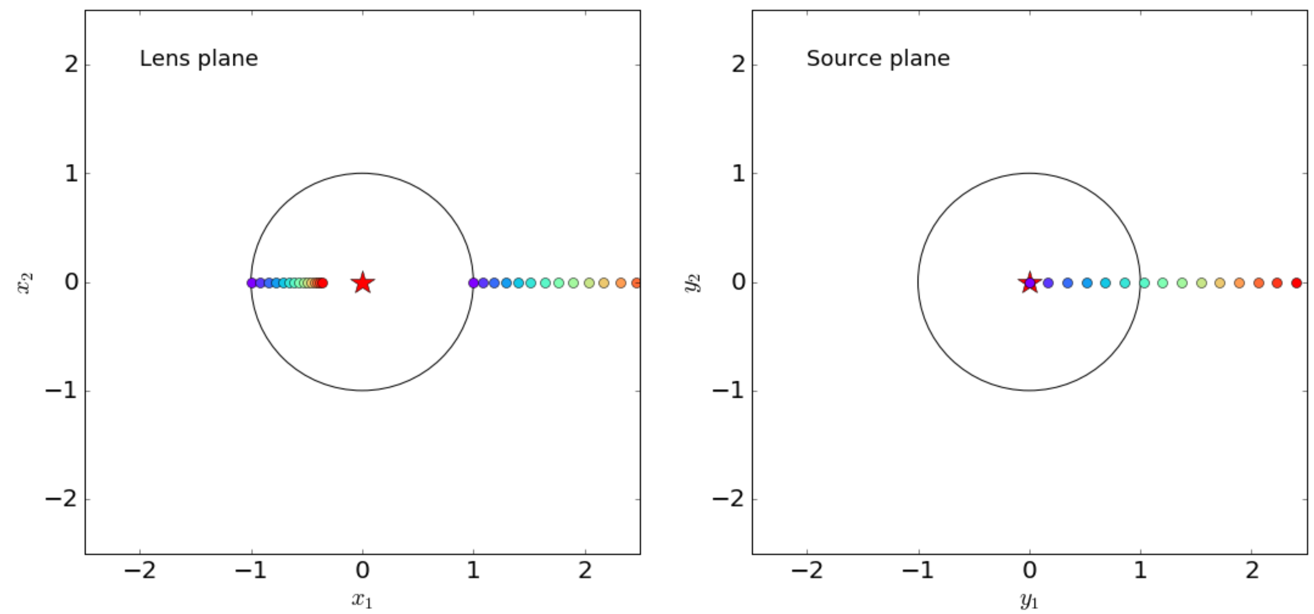
$$\Downarrow$$

$$x_{\pm} = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right]$$

SOLUTIONS OF THE LENS EQUATION



PROPERTIES OF THE IMAGES



$$x_{\pm} = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right]$$

$$x_+ > 1 \qquad\qquad -1 < x_- < 0$$

One of the images is internal to the Einstein radius, the other is external

For $y=0$, the image is a full ring: $x_{\pm} = \pm 1$

This is the ***Einstein ring***

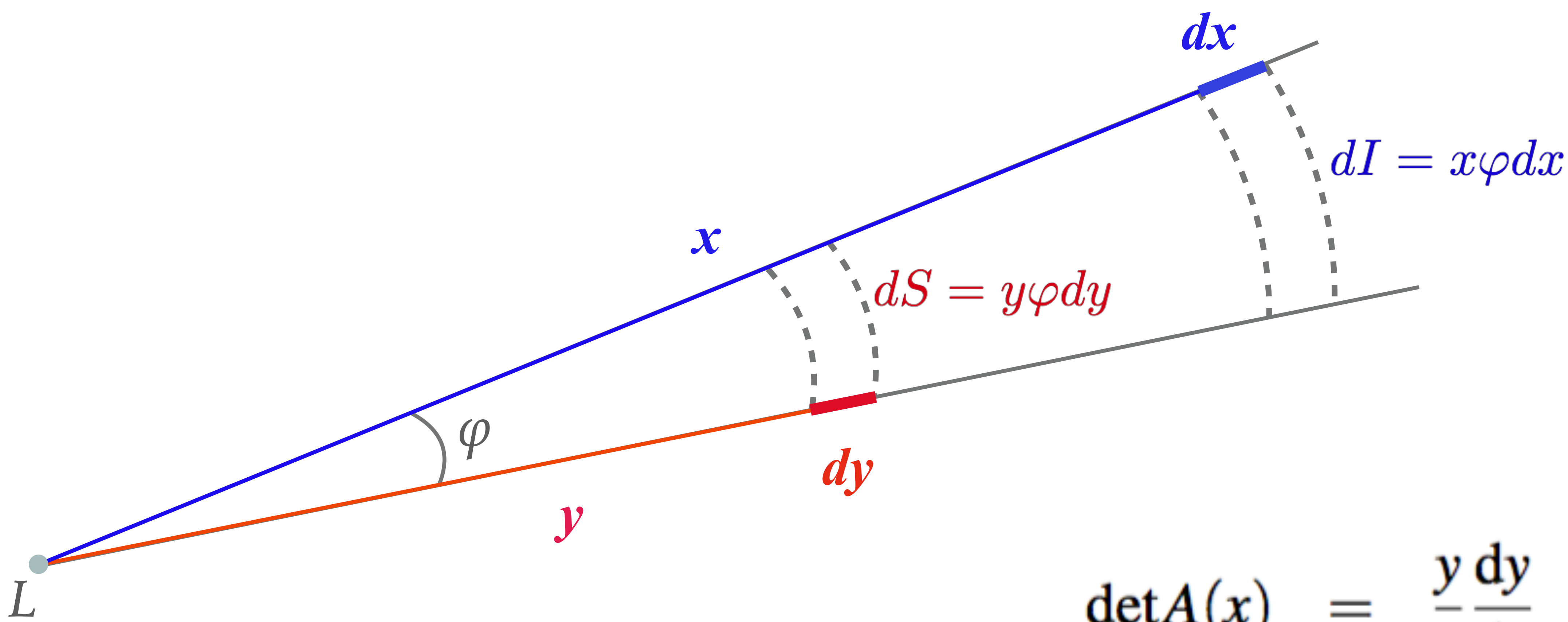
SIZE OF THE EINSTEIN RADIUS

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}} \qquad\qquad D \equiv \frac{D_{\rm L}D_{\rm S}}{D_{\rm LS}}$$

$$\begin{aligned}\theta_E &\approx (10^{-3})''\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D}{10\mathrm{kpc}}\right)^{-1/2},\\ &\approx 1''\left(\frac{M}{10^{12}M_\odot}\right)^{1/2}\left(\frac{D}{\mathrm{Gpc}}\right)^{-1/2},\end{aligned}$$

For a star like the sun within the MW, the Einstein radius is of the order of milli-arcseconds!

MAGNIFICATION



$$\det A(x) = \frac{y}{x}\frac{dy}{dx}$$

CRITICAL LINES AND CAUSTICS

From the lens equation, it follows that:

$$\lambda_t(x) = \frac{y}{x} = \left(1 - \frac{1}{x^2}\right)$$
$$\lambda_r(x) = \frac{\mathrm{d}y}{\mathrm{d}x} = \left(1 + \frac{1}{x^2}\right) .$$

The second eigenvalue is always positive (no critical line). The first is zero on the circle

$$x^2 = 1$$

Thus, the Einstein ring is the tangential critical line! The corresponding caustic is a point at $y=0$

IMAGE MAGNIFICATION

Clearly,

$$\det A(x) = \frac{y}{x}\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\lambda_t(x) = \frac{y}{x} = \left(1 - \frac{1}{x^2}\right)$$

$$\lambda_r(x) = \frac{\mathrm{d}y}{\mathrm{d}x} = \left(1 + \frac{1}{x^2}\right) .$$

$$\Rightarrow \quad \mu(x) = \left(1 - \frac{1}{x^4}\right)^{-1}$$

IMAGE PARITY

Note that:

$$y > 0 \quad \Rightarrow \quad \begin{aligned} x_+ &> 0 \\ x_- &< 0 \end{aligned}$$

$$\mu_t = \frac{x}{y} \quad \Rightarrow \quad \begin{aligned} \mu_t(x_+) &> 0 \\ \mu_t(x_-) &< 0 \end{aligned}$$

$$\mu_r = \frac{dx}{dy} > 0$$

⇒ *Thus the parity of the images is different!*

Not surprising given that the two images are separated by the critical line

SOURCE MAGNIFICATION

Let's compute now the source magnification. This is the sum of the magnifications of the two images

$$x_\pm = \frac{1}{2}\left[y \pm \sqrt{y^2+4}\right]$$

$$\begin{aligned} \frac{x}{y} &= \frac{1}{2}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right) \\ \frac{\mathrm{d}x}{\mathrm{d}y} &= \frac{1}{2}\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) . \end{aligned}$$

Thus the magnifications at the two image positions are

$$\begin{aligned} \mu_\pm(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\ &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\ &= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right) \end{aligned}$$

SOURCE MAGNIFICATION

The total magnification is obtained by summing the magnifications of the images:

$$\begin{aligned}\mu_{\pm}(y) &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y}\right)\left(1 \pm \frac{y}{\sqrt{y^2+4}}\right) \\ &= \frac{1}{4}\left(1 \pm \frac{\sqrt{y^2+4}}{y} \pm \frac{y}{\sqrt{y^2+4}} + 1\right) \\ &= \frac{1}{4}\left(2 \pm \frac{2y^2+4}{y\sqrt{y^2+4}}\right) = \frac{1}{2}\left(1 \pm \frac{y^2+2}{y\sqrt{y^2+4}}\right)\end{aligned}$$

$$\Rightarrow \quad \mu(y) = \mu_{+}(y) + |\mu_{-}(y)| = \frac{y^2+2}{y\sqrt{y^2+4}}$$

The sum of the signed magnification is one!

We can take a series expansion of the magnification to see that $\mu \propto 1 + 2/y^4$ for $y \to \infty$

Thus, the magnification drops quickly as the source moves away from the lens!

SOURCE MAGNIFICATION

In addition:

$$\left|\frac{\mu_+}{\mu_-}\right| = \frac{1+\frac{y^2+2}{y\sqrt{y^2+4}}}{\frac{y^2+2}{y\sqrt{y^2+4}}-1} = \frac{y^2+2+y\sqrt{y^2+4}}{y^2+2-y\sqrt{y^2+4}}$$

$$\frac{1}{2}\left(y+\sqrt{y^2+4}\right)^2 = y^2+2+y\sqrt{y^2+4}$$

$$\frac{1}{2}\left(y-\sqrt{y^2+4}\right)^2 = y^2+2-y\sqrt{y^2+4}$$

$$\left|\frac{\mu_+}{\mu_-}\right| = \left(\frac{y+\sqrt{y^2+4}}{y-\sqrt{y^2+4}}\right)^2 = \left(\frac{x_+}{x_-}\right)^2 .$$

Series expansion at infinity:

$$\left|\frac{\mu_+}{\mu_-}\right| \propto y^4$$

As we move the source away from the lens, the image in x_+ dominates the flux budget very soon.

$$\lim_{y\to\infty}\mu_- = 0$$

$$\lim_{y\to\infty}\mu_+ = 1$$

A SOURCE ON THE EINSTEIN RING

For a source on the Einstein ring:

$$x_\pm = \frac{1 \pm \sqrt{5}}{2} \qquad \mu_\pm = \left[1 - \left(\frac{2}{1 \pm \sqrt{5}}\right)^4\right]^{-1}$$

Therefore: $\mu = |\mu_+| + |\mu_-| = 1.17 + 0.17 = 1.34$

$$\Delta m = -2.5 \log \mu \sim 0.3$$

Given how quickly the magnification drops by moving the source away from the lens, we can assume that only sources within the Einstein radius are magnified in a significant way.

For this reason, the circle within the Einstein radius is assumed to be the cross section for microlensing.

SIZE OF THE EINSTEIN RADIUS

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}} \qquad\qquad D \equiv \frac{D_{\rm L}D_{\rm S}}{D_{\rm LS}}$$

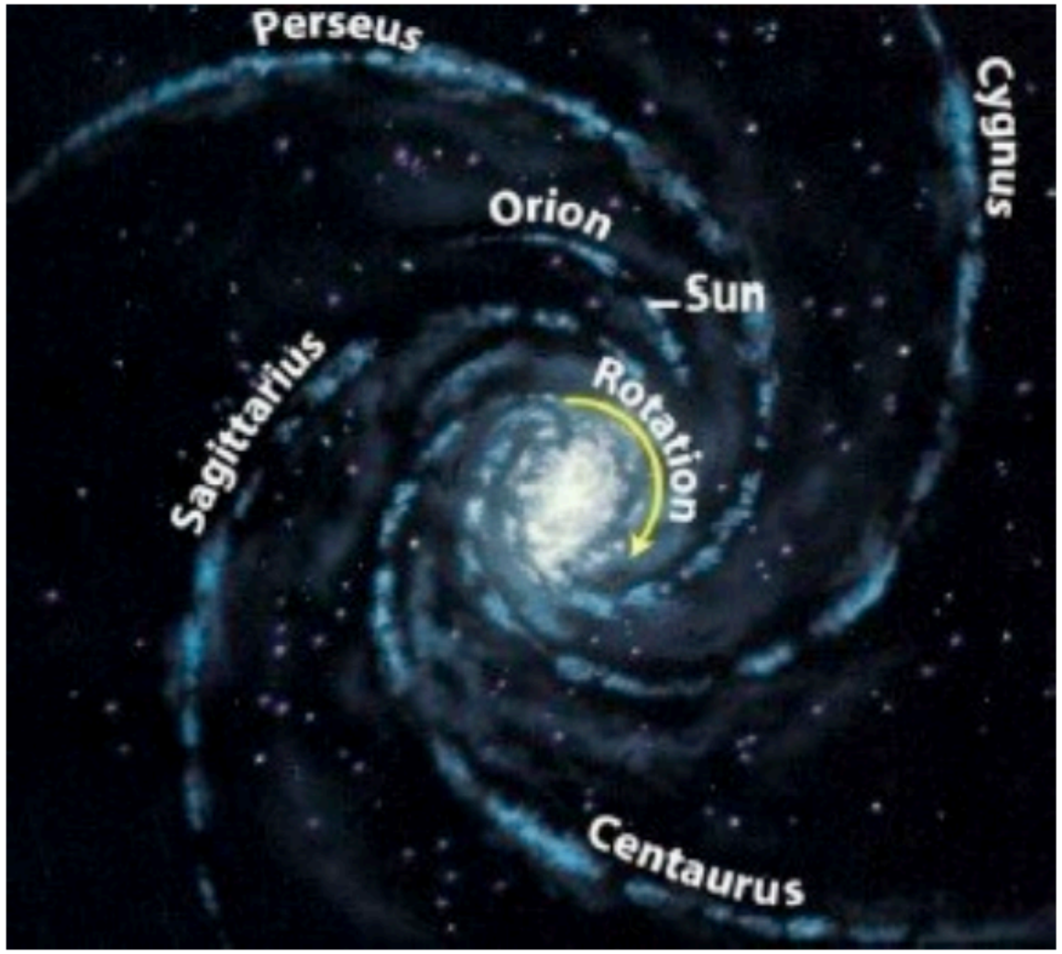
$$\begin{aligned}\theta_E &\approx (10^{-3})'' \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D}{10\mathrm{kpc}}\right)^{-1/2}, \\ &\approx 1'' \left(\frac{M}{10^{12}M_\odot}\right)^{1/2} \left(\frac{D}{\mathrm{Gpc}}\right)^{-1/2},\end{aligned}$$

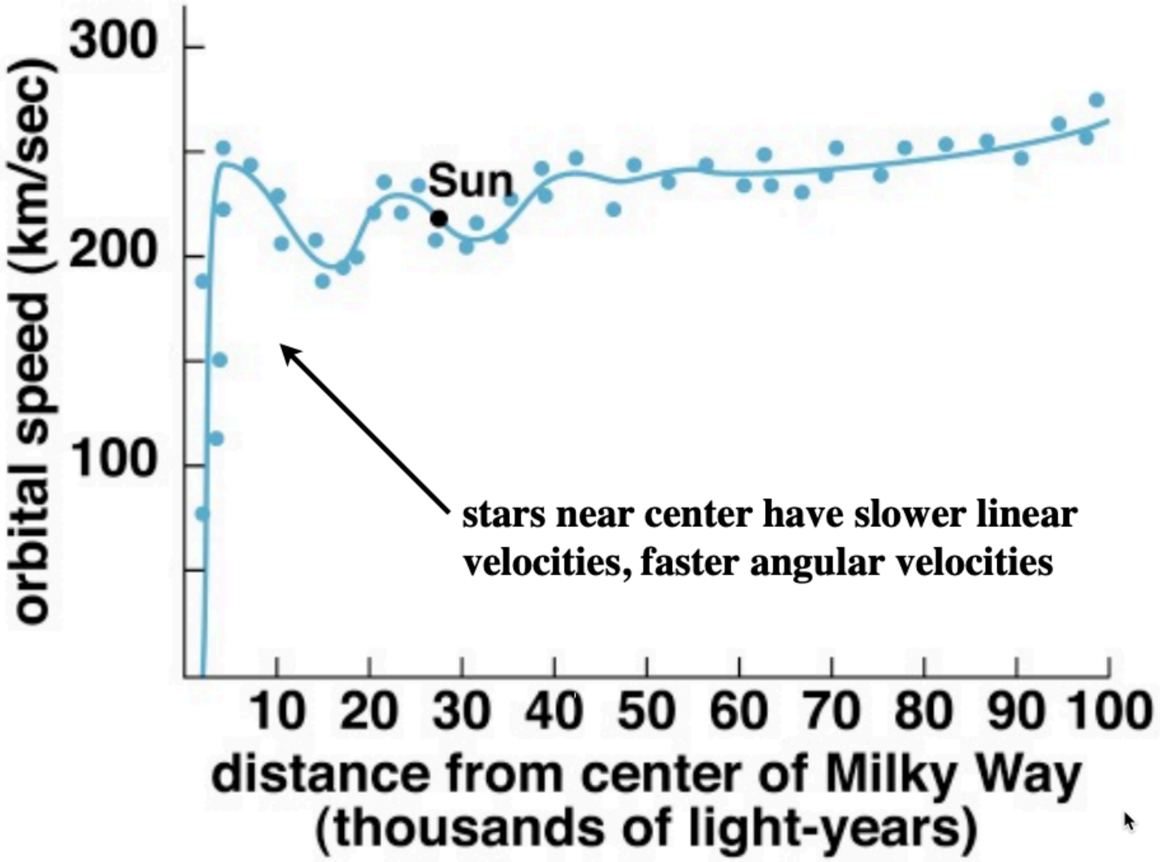
For a star like the sun within the MW, the Einstein radius is of the order of milli-arcseconds!

MICROLENSING OBSERVABLES?

- typical Einstein radii for lenses in the MW are $\sim$1 mas
- thus, the image separation is too small to resolve the images
- magnification is small also for relatively close pairs of lenses and sources
- how to detect a microlensing event?

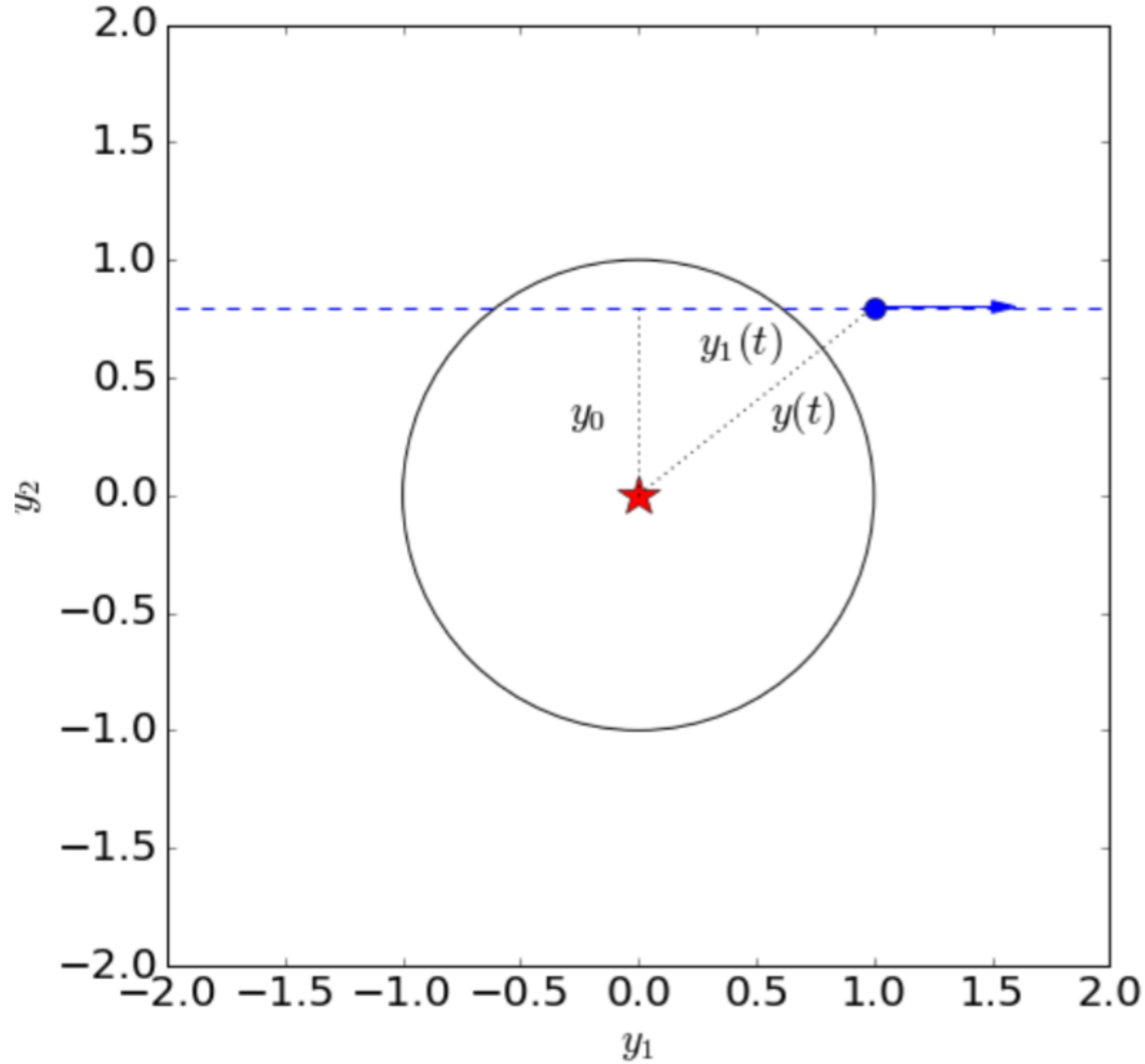
DIFFERENTIAL ROTATION





MICROLENSING LIGHT CURVE

- stars (including the sun) rotate around the galactic center
- rotation is differential (i.e. speed depends on distance)
- this introduces a relative velocity between the lenses and the sources (either in the bulge or in the MCs)
- this causes the relative distance between the sources and the lenses to vary over time…



MICROLENSING LIGHT CURVE

Assume a linear trajectory of the source relative to the lens, with impact parameter y_0

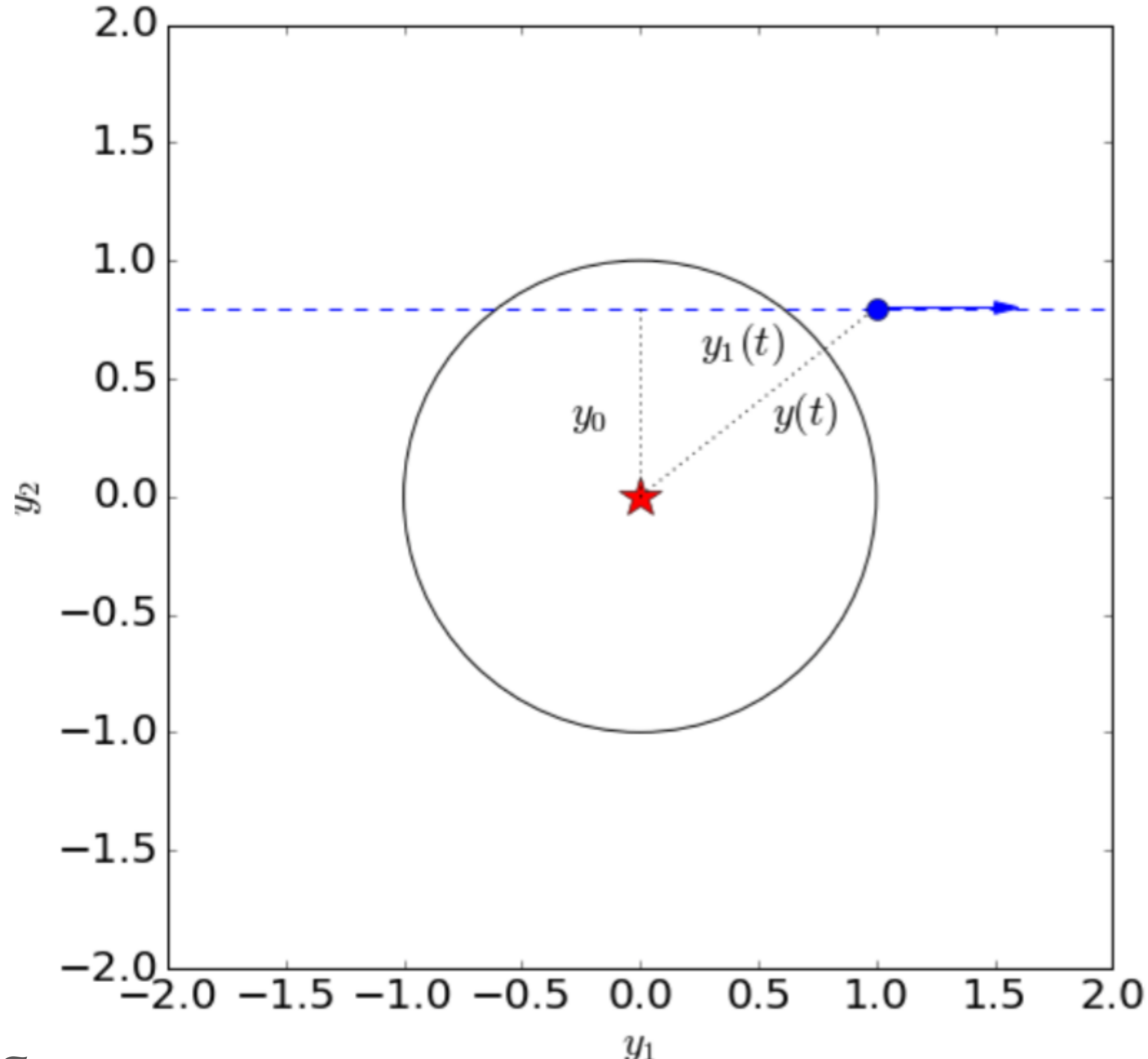
Assume also constant transverse velocity v:

$$y_1(t) = \frac{v(t - t_0)}{D_L \theta_E}$$

We can define a characteristic time of the event:

$$t_E = \frac{D_L \theta_E}{v} = \frac{\theta_E}{\mu_{rel}}$$

*This is the **Einstein radius crossing time***



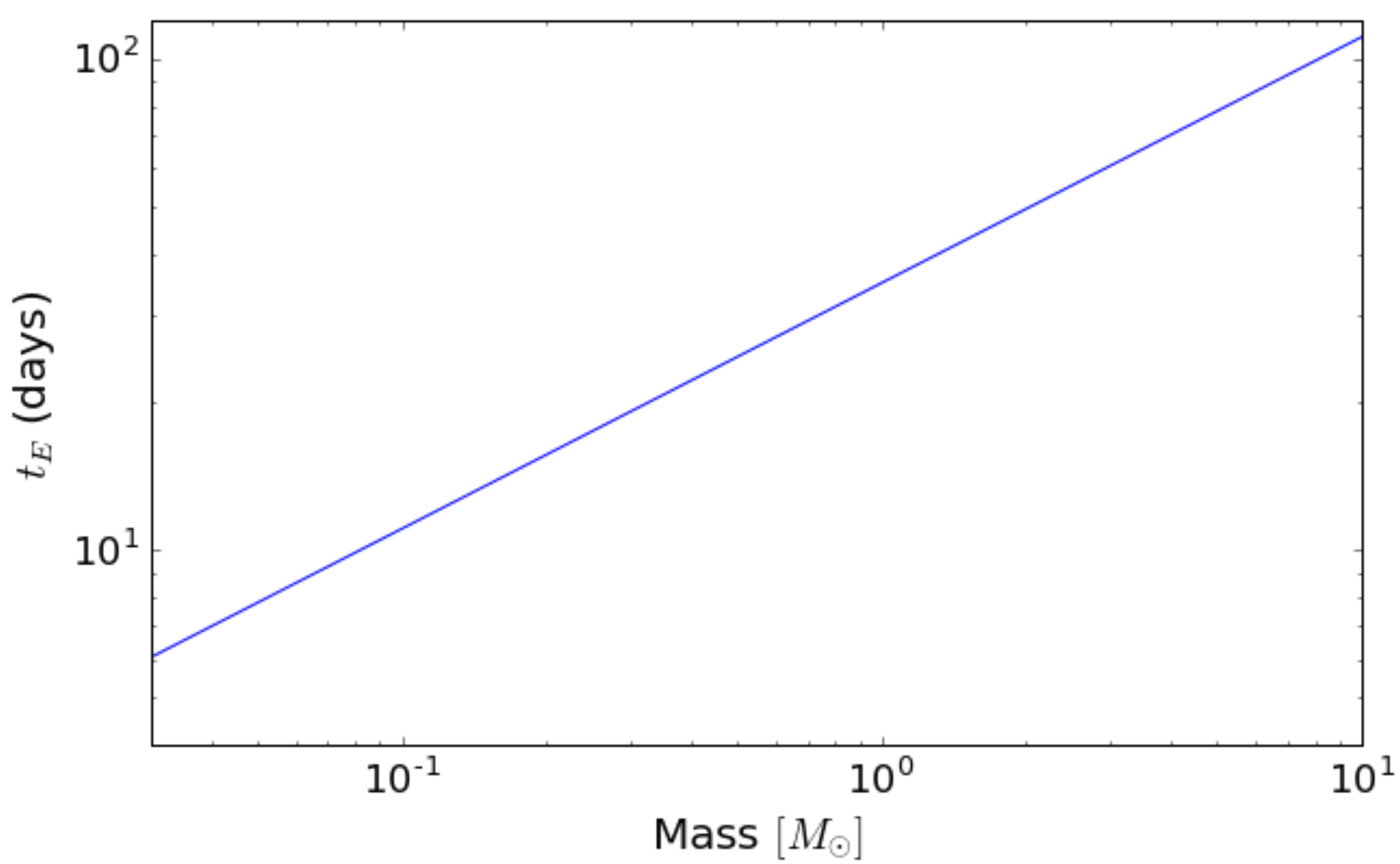
MICROLENSING LIGHT CURVE

Given the definition of Einstein radius

$$\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}}$$

The order of magnitude of the t_E is

$$t_E \approx 19 \text{ days} \sqrt{4\frac{D_{\rm L}}{D_{\rm S}}\left(1-\frac{D_{\rm L}}{D_{\rm S}}\right)}\left(\frac{D_{\rm S}}{8{\rm kpc}}\right)^{1/2}\left(\frac{M}{0.3M_\odot}\right)^{1/2}\left(\frac{v}{200{\rm km}/s}\right)^{-1}$$



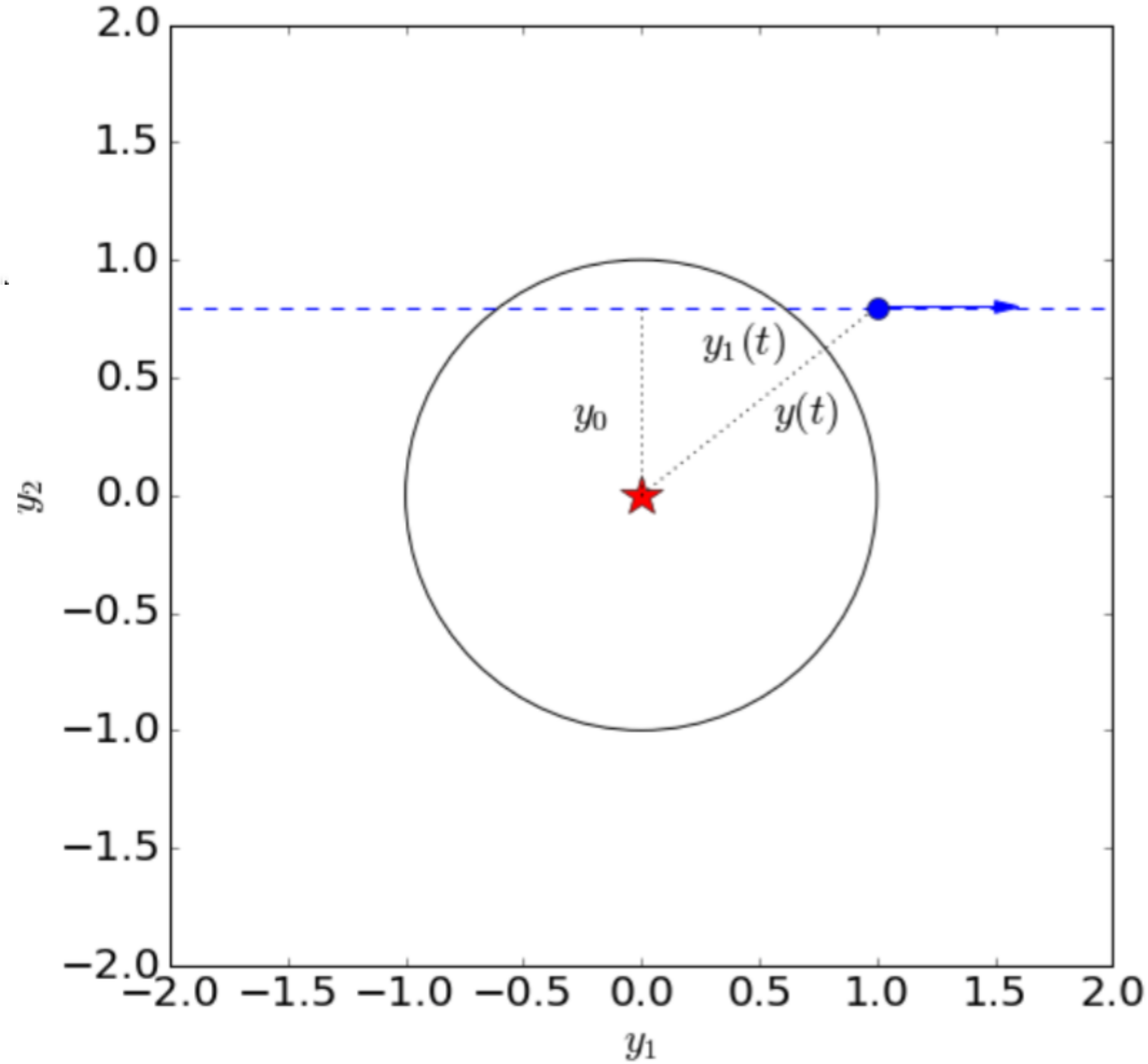
MICROLENSING LIGHT CURVE

We obtain

$$y_1(t) = \frac{v(t-t_0)}{D_L\theta_E} \quad \Rightarrow \quad y_1(t) = \frac{(t-t_0)}{t_E}$$

Thus:

$$y(t) = \sqrt{y_0^2 + y_1^2(t)} = \sqrt{y_0^2 + \frac{(t-t_0)^2}{t_E^2}}$$



EXAMPLE OF STANDARD LIGHT CURVE