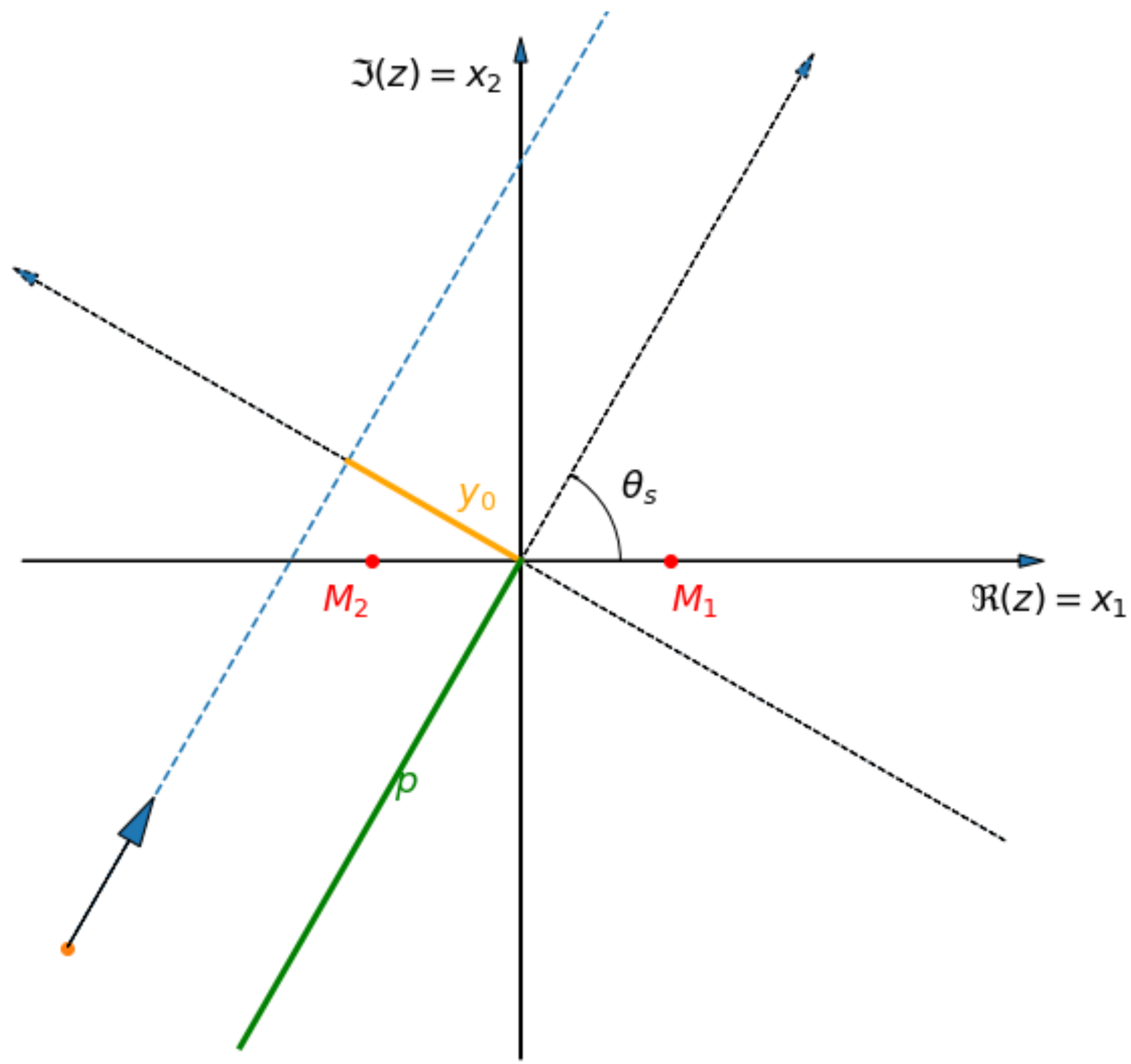
GRAVITATIONAL LENSING

12 - BINARY LENSES

Massimo Meneghetti

AA 2019-2020

GEOMETRY OF A BINARY LENS



- Two point masses $M_1$and M_2
- Origin is chosen to coincide with the midpoint between the two masses Real axis passes through the two lenses.
- θ_S inclination of the source trajectory relative to the real axis
- y_0: impact parameter with respect to the origin
- t_0: time of minimum distance from the origin
- $p = \dfrac{t - t_0}{t_E}$: time from t_0 in units of t_E
- Position of the source in the complex plane:

$$\Re(z) = p \cos \theta_S - y_0 \sin \theta_S$$

$$\Im(z) = p \sin \theta_S + y_0 \cos \theta_S$$

LENS EQUATION FOR THE BINARY LENS

$$z_s = z - \sum_{i=1}^{2} \frac{m_i}{z^* - z_i^*} = z - \frac{m_1}{z^* - z_1^*} - \frac{m_2}{z^* - z_2^*}$$

Take the complex conjugate to compute z^* and insert it back… we obtain a polynomial equation of degree $(N^2+1) = 5$:

$$p_5(z) = \sum_{i=0}^{5} c_i z^i = 0 \qquad \Delta m = \frac{m_1 - m_2}{2} \quad m = \frac{m_1 + m_2}{2} \quad z_2 = -z_1 \quad z_1 = z_1^*$$

$$c_0 = z_1^2[4(\Delta m)^2 z_s + 4m\Delta m z_1 + 4\Delta m z_s z_s^* z_1 + 2m z_s^* z_1^2 + z_s z_s^{*2} z_1^2 - 2\Delta m z_1^3 - z_s z_1^4]$$

$$c_1 = -8m\Delta m z_s z_1 - 4(\Delta m)^2 z_1^2 - 4m^2 z_1^2 - 4m z_s z_s^* z_1^2 - 4\Delta m z_s^* z_1^3 - z_s^{*2} z_1^4 + z_1^6$$

$$c_2 = 4m^2 z_s + 4m\Delta m z_1 - 4\Delta m z_s z_s^* z_1 - 2z_s z_s^{*2} z_1^2 + 4\Delta m z_1^3 + 2z_s z_1^4$$

$$c_3 = 4m z_s z_s^* + 4\Delta m z_s^* z_1 + 2z_s^{*2} z_1^2 - 2z_1^4$$

$$c_4 = -2m z_s^* + z_s z_s^{*2} - 2\Delta m z_1 - z_s z_1^2$$

$$c_5 = z_1^2 - z_s^{*2}$$

Up to 5 images

CRITICAL LINES AND CAUSTICS

$$\det A = 1 - \left| \sum_{i=1}^{2} \frac{m_i}{(z^* - z_i^*)^2} \right|^2 = 1 - \left| \frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2} \right|^2 = 0$$

Or

$$\left| \frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2} \right|^2 = 1 \Rightarrow \frac{m_1}{(z^* - z_1^*)^2} + \frac{m_2}{(z^* - z_2^*)^2} = e^{i\phi} \quad \forall \;\; \phi \in [0,2\pi)$$

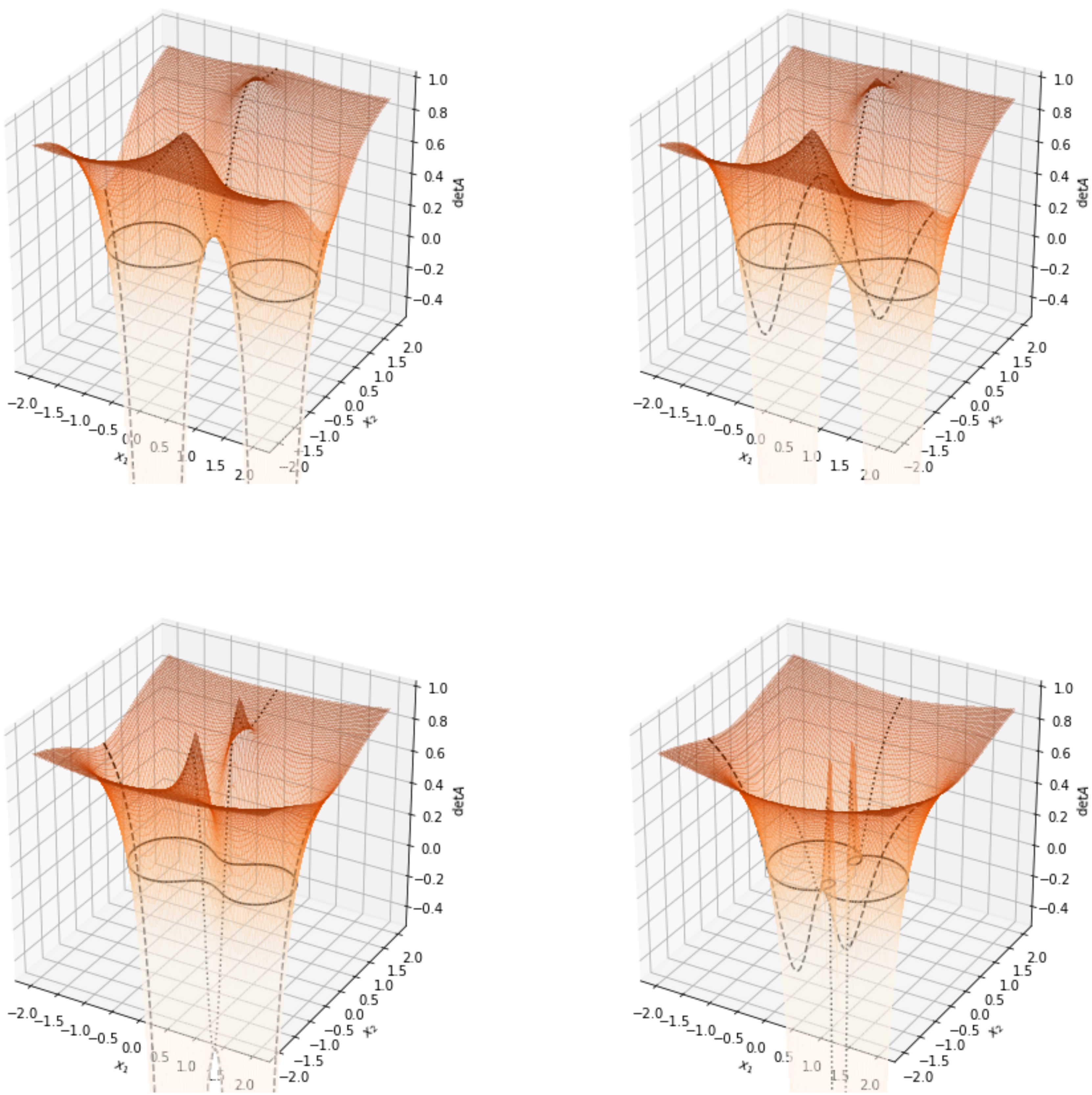
Which can be reduced to

$$p_4(z) = z^4 - z^2(2z_1^{*2} + e^{i\phi}) - zz_1^* 2(m_1 - m_2)e^{i\phi} + z_1^{*2}(z_1^{*2} - e^{i\phi}) = 0$$

Once the roots are found (critical points), the caustics can be obtained using the lens equation:

$$z_{cau} = z_{crit} - \frac{m_1}{z_{crit}^* - z_1^*} - \frac{m_2}{z_{crit}^* - z_2^*}$$

$\det A$ SURFACES



Depending on the distance between the two lenses

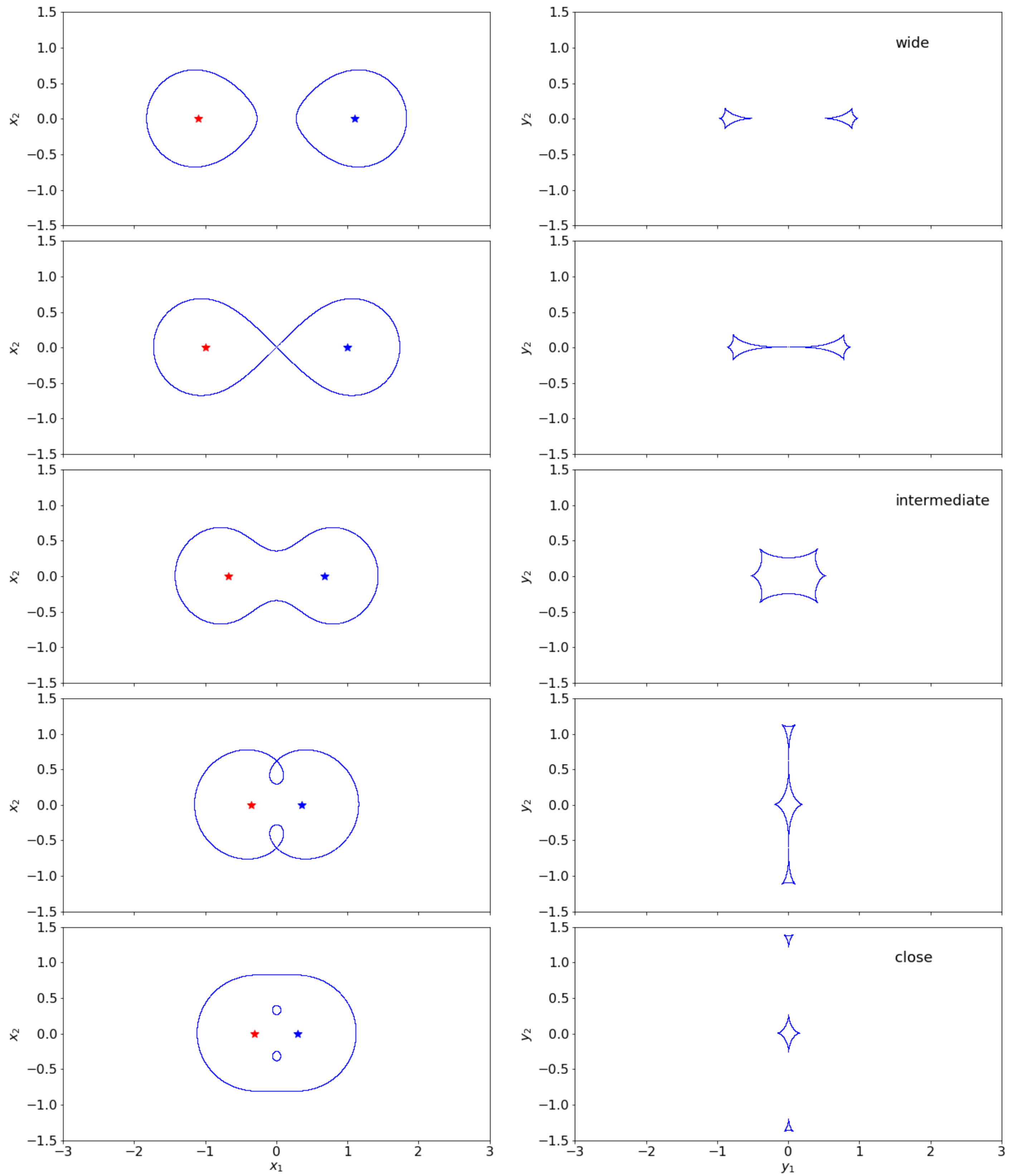
$$d = z_1 - z_2 = 2z_1$$

we can have one, two, or three critical lines. Note that the transitions between these three regimes happen when critical lines touch.

This happens at saddle points of the $\det A$ *surface!*

$$\frac{\partial \det A}{\partial z^*} = 0$$

CAUSTIC (AND CRITICAL LINE) TOPOLOGIES



$$d > d_{WI}$$

$$d = d_{WI} = (m_1^{1/3} + m_2^{1/3})^{3/2}$$

$$d_{IC} < d < d_{WI}$$

$$d = d_{IC} = (m_1^{1/3} + m_2^{1/3})^{-3/4}$$

$$d < d_{IC}$$

MAGNIFICATION

On the lens plane, the magnification is

$$\det A(z) = 1 - \left| \sum_{i=1}^{2} \frac{m_i}{(z^* - z_i^*)^2} \right| \qquad \mu(z) = \det A(z)^{-1}$$

Remember that even microlensing by binary lenses will be revealed through magnification effects.

No single images will be observed! Thus, what matters is the total magnification of all images of a given source:

$$\mu(z_s) = \sum_{images} |\mu_{ima}(z_{ima})|$$

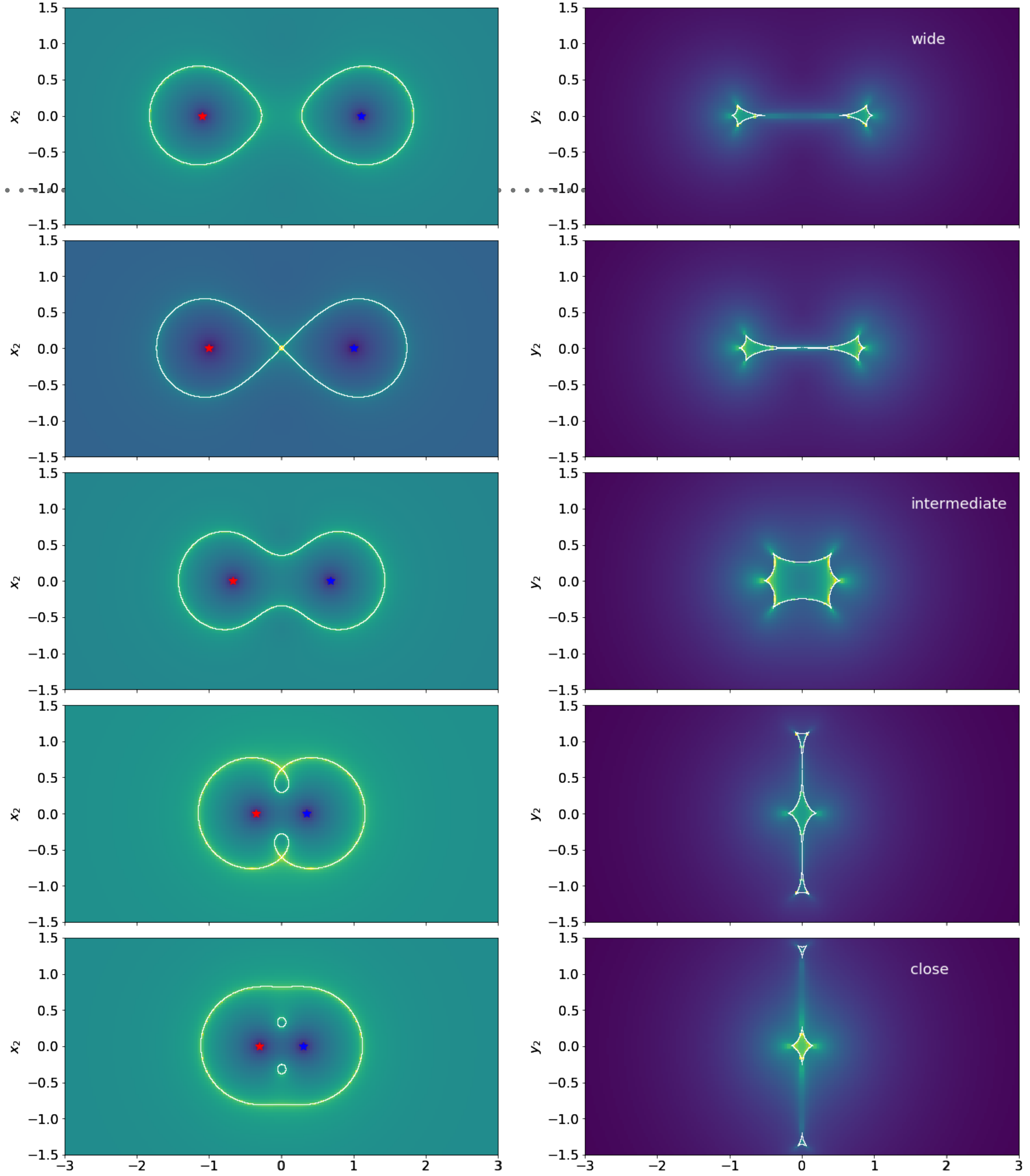
where z_{ima} are now the positions of the images of the source at z_s

EXAMPLES

Shown are maps of the magnification on the lens (left) and on the source plane (right)

We can recognise the critical lines and the caustics (where magnification diverges)

These maps are very important: depending on the trajectory on the source relative to the lens, several of these features will be impressed in the light-curves!



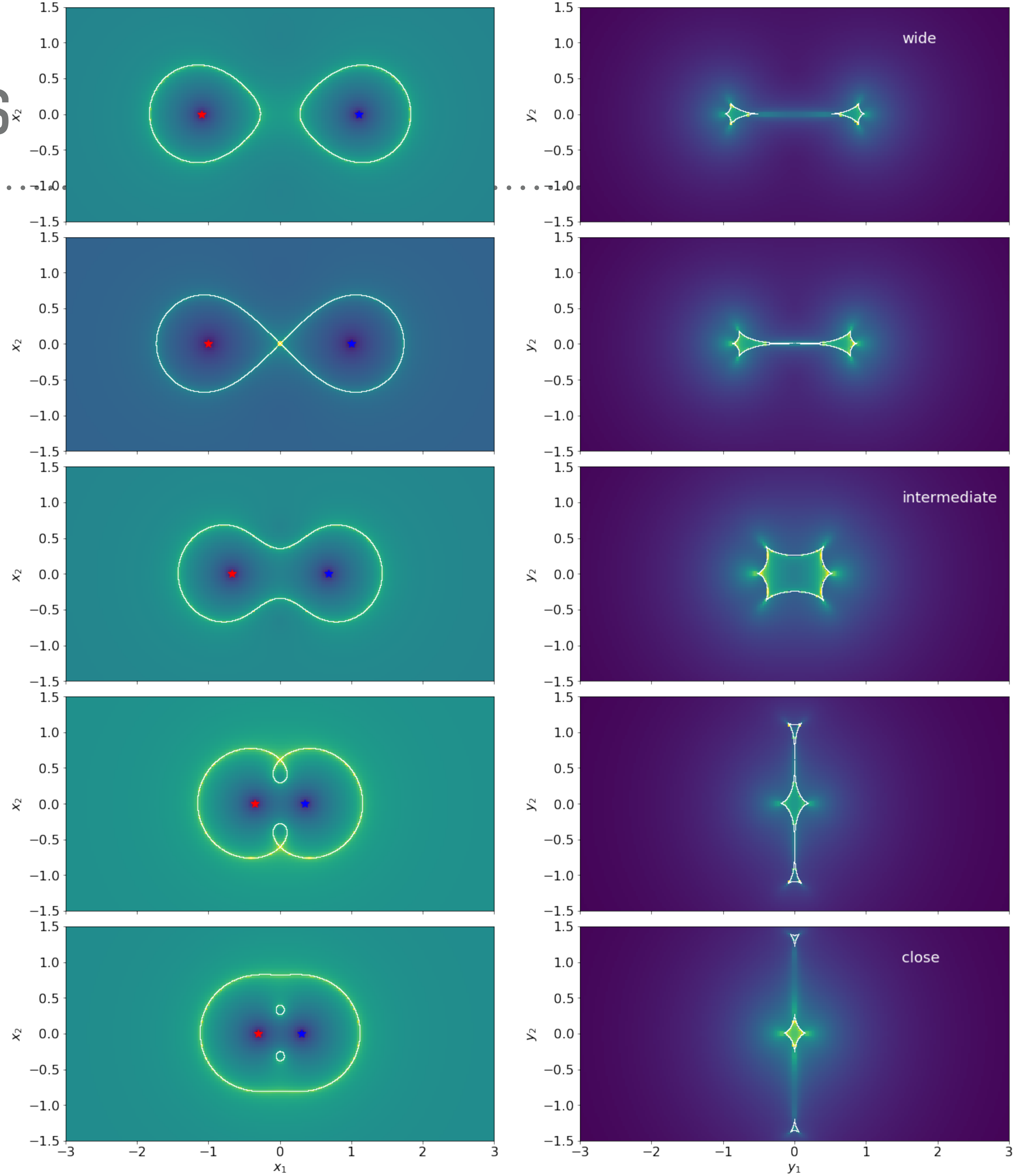
INTERESTING PROPERTIES

Lobes of high magnification near the ***cusps***

Sharp magnification changes on the ***folds***

Inside the caustics: *moderately high magnification* $\mu > 3$ *(Witt & Mao 1995)*

Extended regions of high magnification in ***between the caustics*** *in wide and close systems*



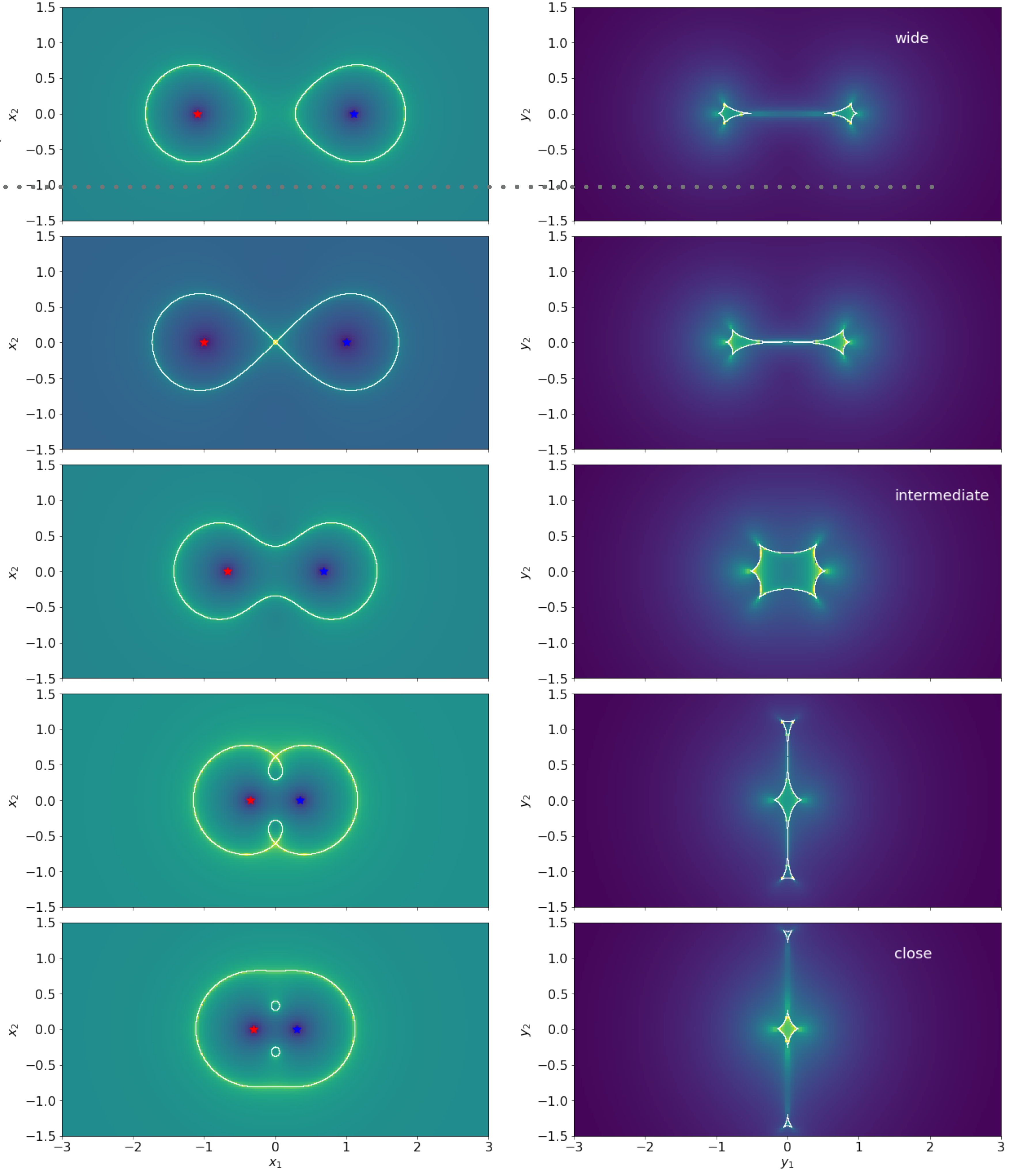
DEPENDENCE ON $q = M_1/M_2$

As q changes, the morphologies of the magnification maps, critical lines and caustics change

Critical lines: larger around the primary lens, smaller around the secondary

Wide systems: smaller caustic for the primary, larger for the secondary

Close systems: secondary caustics move to the back of the primary



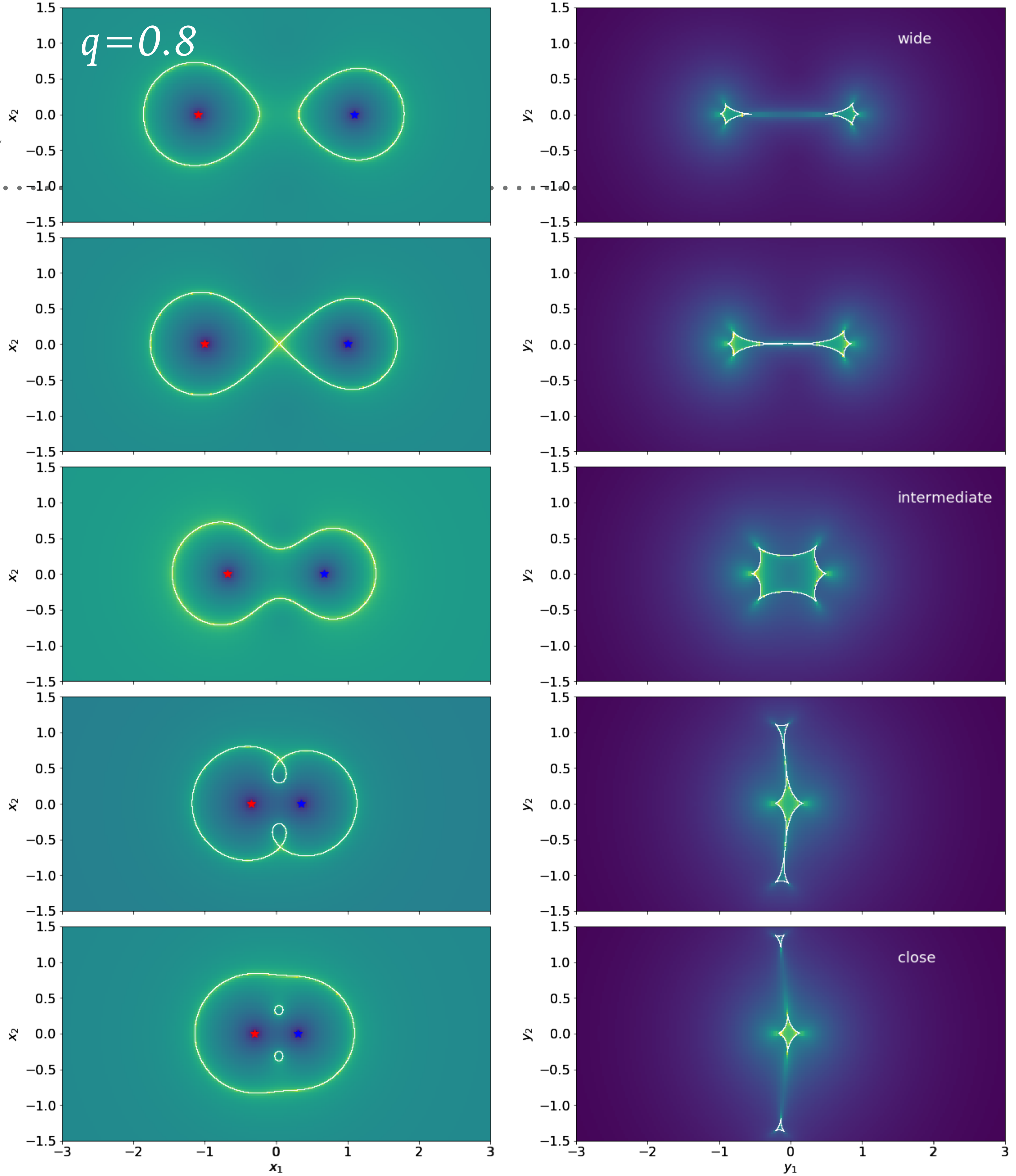
DEPENDENCE ON $q = M_1/M_2$

As q changes, the morphologies of the magnification maps, critical lines and caustics change

Critical lines: larger around the primary lens, smaller around the secondary

Wide systems: smaller caustic for the primary, larger for the secondary

Close systems: secondary caustics move to the back of the primary



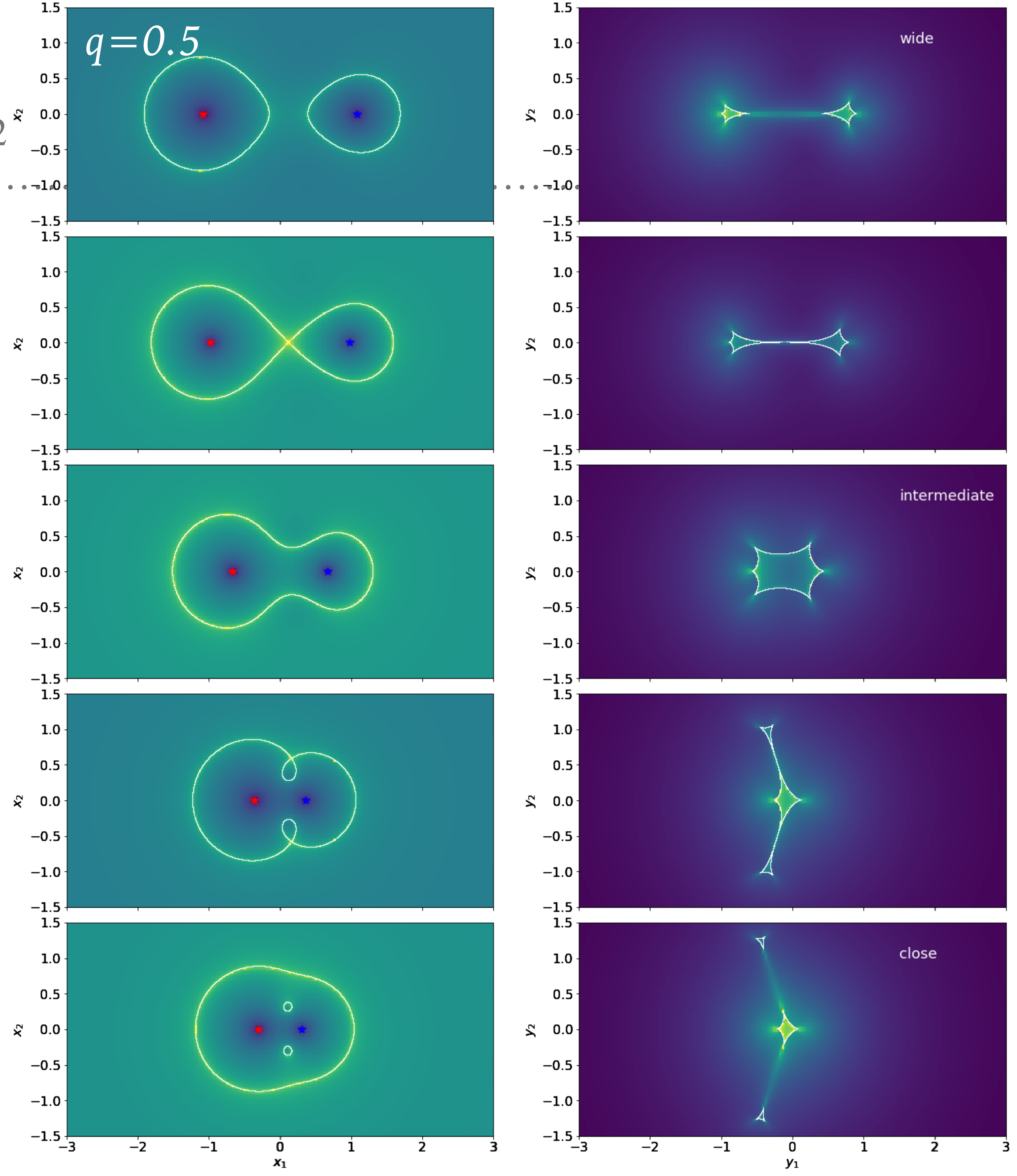
DEPENDENCE ON $q = M_1/M_2$

As q changes, the morphologies of the magnification maps, critical lines and caustics change

Critical lines: larger around the primary lens, smaller around the secondary

Wide systems: smaller caustic for the primary, larger for the secondary

Close systems: secondary caustics move to the back of the primary



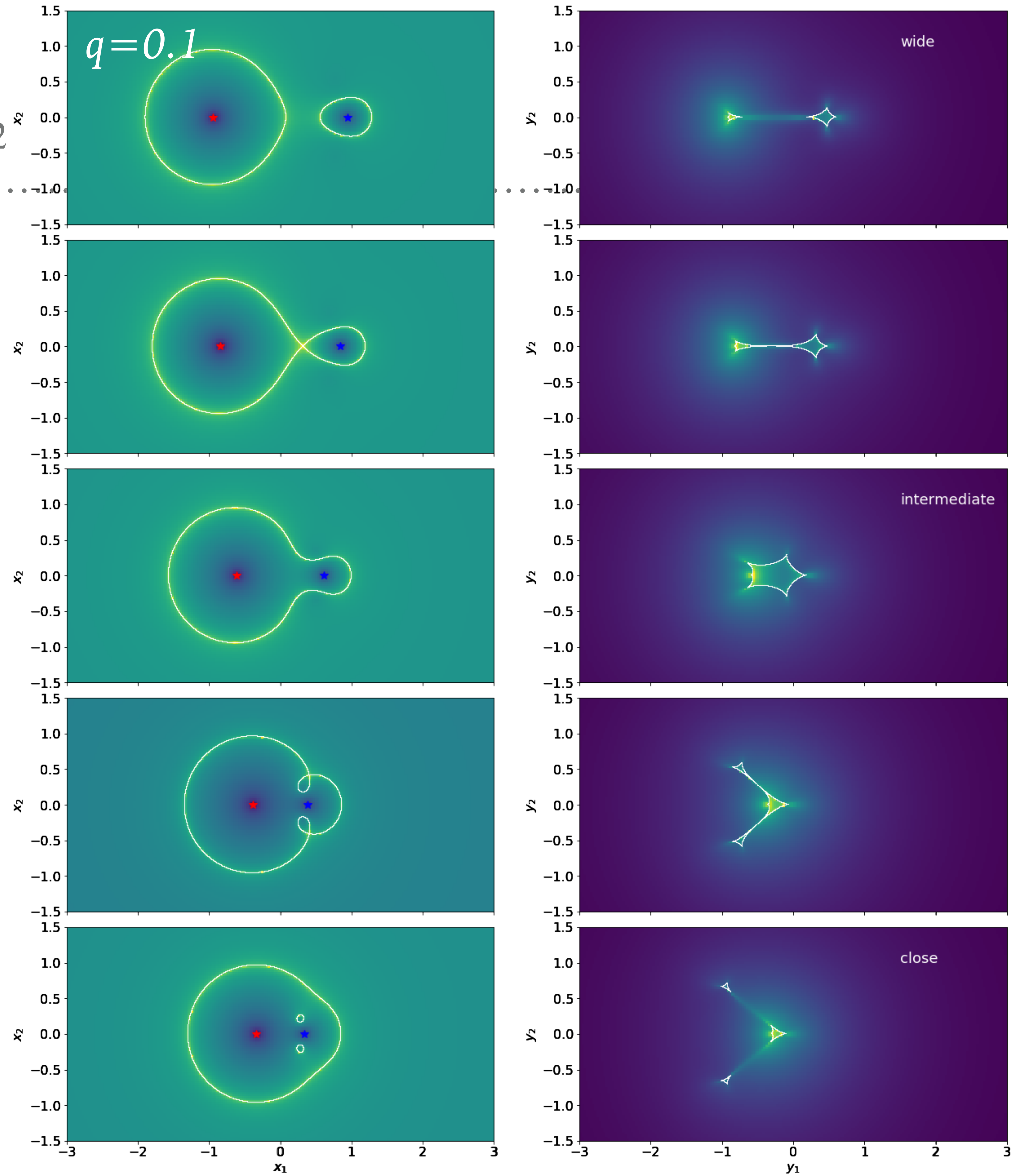
DEPENDENCE ON $q = M_1/M_2$

As q changes, the morphologies of the magnification maps, critical lines and caustics change

Critical lines: larger around the primary lens, smaller around the secondary

Wide systems: smaller caustic for the primary, larger for the secondary

Close systems: secondary caustics move to the back of the primary



SOME OBSERVED LIGHT CURVES