GRAVITATIONAL LENSING

LECTURE 13

Docente: Massimo Meneghetti

AA 2016-2017

CONTENTS

- microlensing: optical depth and event rates

WHAT IS THE PROBABILITY TO OBSERVE A MICROLENSING EVENT?

- microlensing event: a variation of the source flux which follows the law we derived in the last lecture:

$$\mu(y) = |\mu_+(y)| + |\mu_-(y)| = \frac{y^2+2}{y\sqrt{y^2+4}}$$

- two important things to remember:
  - the total magnification drops as $\mu \propto 1 + 2/y^4$ for $y \to \infty$
  - the ratio of image magnification grows as $\left|\frac{\mu_+}{\mu_-}\right| \propto y^4$
- for these reasons, we anticipated that the microlensing cross section is

$$\sigma = \pi\theta_E^2$$

- more generally, **the cross section is the area (or the solid angle) within which a source has to be located in order to undergo a microlensing event.** In our case this $\mu \gtrsim 1.34$

OPTICAL DEPTH

The number of lenses in the spherical shell centered on the observer and with radius D_L to D_L+dD_L is

$$dN_L = 4\pi D_{\mathrm{L}}^2 n(D_{\mathrm{L}}) dD_{\mathrm{L}}$$

where $n(D_L)$ is the number density of lenses.

OPTICAL DEPTH

The number of lenses in the spherical shell centered on the observer and with radius D_L to D_L+dD_L is

$$dN_L = 4\pi D_{\mathrm{L}}^2 n(D_{\mathrm{L}}) dD_{\mathrm{L}}$$

where $n(D_L)$ is the number density of lenses.

Each of these lenses has a cross section $\sigma = \pi\theta_E^2$

OPTICAL DEPTH

The number of lenses in the spherical shell centered on the observer and with radius D_L to D_L+dD_L is

$$dN_L = 4\pi D_{\mathrm{L}}^2 n(D_{\mathrm{L}}) dD_{\mathrm{L}}$$

where $n(D_L)$ is the number density of lenses.

Each of these lenses has a cross section $\sigma = \pi\theta_E^2$

Summing all the cross sections and dividing by the total area of the sky, we measure the probability that a source at distance D_S is lensed by a lens at distance D_L

$$\frac{1}{4\pi}[4\pi D_{\mathrm{L}}^2 n(D_{\mathrm{L}})](\pi\theta_E^2) dD_{\mathrm{L}}$$

OPTICAL DEPTH

The number of lenses in the spherical shell centered on the observer and with radius D_L to D_L+dD_L is

$$dN_L = 4\pi D_{\rm L}^2 n(D_{\rm L}) dD_{\rm L}$$

where $n(D_L)$ is the number density of lenses.

Each of these lenses has a cross section $\sigma = \pi\theta_E^2$

Summing all the cross sections and dividing by the total area of the sky, we measure the probability that a source at distance D_S is lensed by a lens at distance D_L

$$\frac{1}{4\pi}[4\pi D_{\rm L}^2 n(D_{\rm L})](\pi\theta_E^2) dD_{\rm L}$$

Integrating over D_L, we obtain the ***optical depth****:*

$$\tau(D_{\rm S}) = \frac{1}{4\pi}\int_0^{D_{\rm S}} [4\pi D_{\rm L}^2 n(D_{\rm L})](\pi\theta_E^2) dD_{\rm L}$$

OPTICAL DEPTH

If all lenses have the same mass: $n(D_{\rm L}) = \rho(D_{\rm L})/M$

On the other hand, $\theta_E \equiv \sqrt{\frac{4GM}{c^2}\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}}$

Therefore, the optical depth does not depend on the mass of the lenses, but only on the mass density!

OPTICAL DEPTH

$$\tau(D_{\rm S}) = \frac{1}{4\pi}\int_0^{D_{\rm S}} [4\pi D_{\rm L}^2 n(D_{\rm L})](\pi\theta_E^2)\, dD_{\rm L}$$

OPTICAL DEPTH

$$\tau(D_{\rm S}) = \frac{1}{4\pi}\int_0^{D_{\rm S}} [4\pi D_{\rm L}^2 n(D_{\rm L})](\pi\theta_E^2)dD_{\rm L}$$

$$\Downarrow$$

$$\tau(D_{\rm S}) \;=\; \frac{4\pi G}{c^2}\int_0^{D_{\rm S}} \rho(D_{\rm L})D_{\rm L}^2\frac{D_{\rm LS}}{D_{\rm L}D_{\rm S}}dD_{\rm L}$$

OPTICAL DEPTH

$$\tau(D_S) = \frac{1}{4\pi}\int_0^{D_S}[4\pi D_L^2 n(D_L)](\pi\theta_E^2)dD_L$$

$$\Downarrow$$

$$\begin{aligned}\tau(D_S) &= \frac{4\pi G}{c^2}\int_0^{D_S}\rho(D_L)D_L^2\frac{D_{LS}}{D_L D_S}dD_L \\ &= \frac{4\pi G}{c^2}\int_0^{D_S}\rho(D_L)D_L\frac{D_S - D_L}{D_S}dD_L\end{aligned}$$

OPTICAL DEPTH

$$\tau(D_S) = \frac{1}{4\pi}\int_0^{D_S}[4\pi D_L^2 n(D_L)](\pi\theta_E^2)dD_L$$

$$\Downarrow$$

$$\begin{aligned}
\tau(D_S) &= \frac{4\pi G}{c^2}\int_0^{D_S}\rho(D_L)D_L^2\frac{D_{LS}}{D_L D_S}dD_L \\
&= \frac{4\pi G}{c^2}\int_0^{D_S}\rho(D_L)D_L\frac{D_S - D_L}{D_S}dD_L \\
&= \frac{4\pi G}{c^2}\int_0^{D_S}\rho(D_L)\frac{D_L}{D_S}\left(1 - \frac{D_L}{D_S}\right)D_S dD_L
\end{aligned}$$

OPTICAL DEPTH

$$\tau(D_S) = \frac{1}{4\pi}\int_0^{D_S}[4\pi D_L^2 n(D_L)](\pi\theta_E^2)dD_L$$

$$\begin{aligned}
\tau(D_S) &= \frac{4\pi G}{c^2}\int_0^{D_S}\rho(D_L)D_L^2\frac{D_{LS}}{D_L D_S}dD_L \\
&= \frac{4\pi G}{c^2}\int_0^{D_S}\rho(D_L)D_L\frac{D_S - D_L}{D_S}dD_L \\
&= \frac{4\pi G}{c^2}\int_0^{D_S}\rho(D_L)\frac{D_L}{D_S}\left(1-\frac{D_L}{D_S}\right)D_S dD_L
\end{aligned}$$

with the substitution $x = D_L/D_S,\ dx = dD_L/D_S$

$$\tau(D_S) = \frac{4\pi G}{c^2}D_S^2\int_0^1\rho(x)x(1-x)dx$$

OPTICAL DEPTH DENSITY

$$\frac{d\tau}{dx} \propto \rho(x)x(1-x)$$

The function which modulates the lens contribution to the optical depth has a peak at x=0.5 (half way between the observer and the source).

Then, whether most of the optical depth is accumulated near the observer or near the source, depends on the mass density profile…

OPTICAL DEPTH (SIMPLEST CASE)

Uniform distribution of the lenses

$$\rho(x) = \rho_0 = \text{const.}$$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 \int_0^1 x(1-x)dx = \frac{2}{3}\frac{\pi G}{c^2} D_S^2 \rho_0$$

OPTICAL DEPTH (SIMPLEST CASE)

Uniform distribution of the lenses

$$\rho(x) = \rho_0 = \text{const.}$$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 \int_0^1 x(1-x)dx = \frac{2}{3}\frac{\pi G}{c^2} D_S^2 \rho_0$$

Note that:

$$M_{gal} = \tfrac{4}{3}\pi D_S^3 \rho_0$$

OPTICAL DEPTH (SIMPLEST CASE)

Uniform distribution of the lenses

$$\rho(x) = \rho_0 = \text{const.}$$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 \int_0^1 x(1-x)dx = \frac{2}{3}\frac{\pi G}{c^2} D_S^2 \rho_0$$

Note that:

$$M_{gal} = \tfrac{4}{3}\pi D_S^3 \rho_0$$

$$\tau(D_S) = \frac{GM_{gal}}{2c^2 D_S} = \frac{V_{circ}^2}{2c^2}$$

OPTICAL DEPTH (SIMPLEST CASE)

Uniform distribution of the lenses

$$\rho(x) = \rho_0 = \text{const.}$$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 \int_0^1 x(1-x)dx = \frac{2}{3}\frac{\pi G}{c^2} D_S^2 \rho_0$$

Note that:

$$M_{gal} = \tfrac{4}{3}\pi D_S^3 \rho_0$$

$$\tau(D_S) = \frac{GM_{gal}}{2c^2 D_S} = \frac{V_{circ}^2}{2c^2}$$

$$\tau \approx 2.6 \times 10^{-7}$$

OPTICAL DEPTH (JUST A BIT MORE REALISTIC)

Lenses in the galactic disk:

scale of the disk

$$\rho(R) = \rho_0 \exp\left(-(R - R_0)/R_D\right)$$

density in the sun neighborhood

distance of earth from the galactic center

distance of lenses from the galactic center

OPTICAL DEPTH (JUST A BIT MORE REALISTIC)

Lenses in the galactic disk:

scale of the disk

$$\rho(R) = \rho_0 \exp\left(-(R - R_0)/R_D\right)$$

density in the sun neighborhood

distance of earth from the galactic center

distance of lenses from the galactic center

$$R = D_{LS}$$

OPTICAL DEPTH (JUST A BIT MORE REALISTIC)

Lenses in the galactic disk:

scale of the disk

$$\rho(R) = \rho_0 \exp\left(-(R - R_0)/R_D\right)$$

density in the sun neighborhood

distance of earth from the galactic center

distance of lenses from the galactic center

$$R = D_{LS} \qquad\qquad R_0 = D_S$$

OPTICAL DEPTH (JUST A BIT MORE REALISTIC)

Lenses in the galactic disk:

scale of the disk

$$\rho(R) = \rho_0 \exp\left(-(R - R_0)/R_D\right)$$

density in the sun neighborhood

distance of earth from the galactic center

distance of lenses from the galactic center

$$R = D_{LS} \qquad\qquad R_0 = D_S$$

$$R - R_0 = D_{LS} - D_S = D_S - D_L - D_S = -D_L$$

OPTICAL DEPTH (JUST A BIT MORE REALISTIC)

Lenses in the galactic disk:

scale of the disk

$$\rho(R) = \rho_0 \exp\left(-(R - R_0)/R_D\right)$$

density in the sun neighborhood

distance of earth from the galactic center

distance of lenses from the galactic center

$$R = D_{LS} \qquad\qquad R_0 = D_S$$

$$R - R_0 = D_{LS} - D_S = D_S - D_L - D_S = -D_L$$

$$\rho(D_L) = \rho_0 \exp\left(D_L/R_D\right)$$

OPTICAL DEPTH (JUST A BIT MORE REALISTIC)

Making the substitutions $\mathbf{x = D_L/D_S}$ $x' = R_D/D_S$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 \int_0^1 \exp{(x/x')} x(1-x)dx$$

$$\tau(D_S) = \frac{4\pi G}{c^2}\rho_0 D_S^2 x'^2[2x' - 1 + \exp{(1/x')}(2x' - 1)]$$

$$D_S = 8 \text{ kpc} \qquad R_D = 3 \text{ kpc} \qquad \rho_0 = 0.1 M_\odot \text{ pc}^{-3}$$

$$\tau \approx 2.9 \times 10^{-6}$$

EVENT RATE

The optical depth gives the probability that a source is (micro-)lensed at any instant. The next question is: how many events will we detect by monitoring a certain number of stars during a time interval?

To answer this question, we have to consider the relative motion of sources and lenses, which determines the timescale of events.

It is easier to think in terms of static sources behind moving lenses.

We also assume that the lenses move with the same transverse velocity.

EVENT RATE

Multiplying by the number density of lenses and integrating over distance we obtain the area useful for microlensing during the time dt:

$$d\tau = \int_0^{D_\mathrm{S}} n(D_\mathrm{L})\, dA\, dD_\mathrm{L} = 2\int_0^{D_\mathrm{S}} n(D_\mathrm{L})\, r_E^2 \frac{dt}{t_E}\, dD_\mathrm{L}$$

EVENT RATE

Multiplying by the number density of lenses and integrating over distance we obtain the area useful for microlensing during the time dt:

$$d\tau = \int_0^{D_\mathrm{S}} n(D_\mathrm{L})\, dA\, dD_\mathrm{L} = 2\int_0^{D_\mathrm{S}} n(D_\mathrm{L})\, r_E^2 \frac{dt}{t_E}\, dD_\mathrm{L}$$

EVENT RATE

Multiplying by the number density of lenses and integrating over distance we obtain the area useful for microlensing during the time dt:

$$d\tau = \int_0^{D_S} n(D_L) dA dD_L = 2 \int_0^{D_S} n(D_L) r_E^2 \frac{dt}{t_E} dD_L$$

EVENT RATE

$$dA = 2r_E v dt = 2r_E^2 \frac{dt}{t_E}$$

Multiplying by the number density of lenses and integrating over distance we obtain the area useful for microlensing during the time dt:

$$d\tau = \int_0^{D_S} n(D_L) dA dD_L = 2\int_0^{D_S} n(D_L) r_E^2 \frac{dt}{t_E} dD_L$$

EVENT RATE

If we monitor N stars, the number of events expected per unit time will be:

$$\Gamma = \frac{d(N_\star \tau)}{dt} = \frac{2N_\star}{\pi} \int_0^{D_\mathrm{S}} n(D_\mathrm{L}) \frac{\pi r_E^2}{t_E} dD_\mathrm{L}$$

If we assume that all Einstein crossing times are identical:

$$\Gamma = \frac{2N_\star}{\pi t_E} \tau$$

As an order of magnitude:

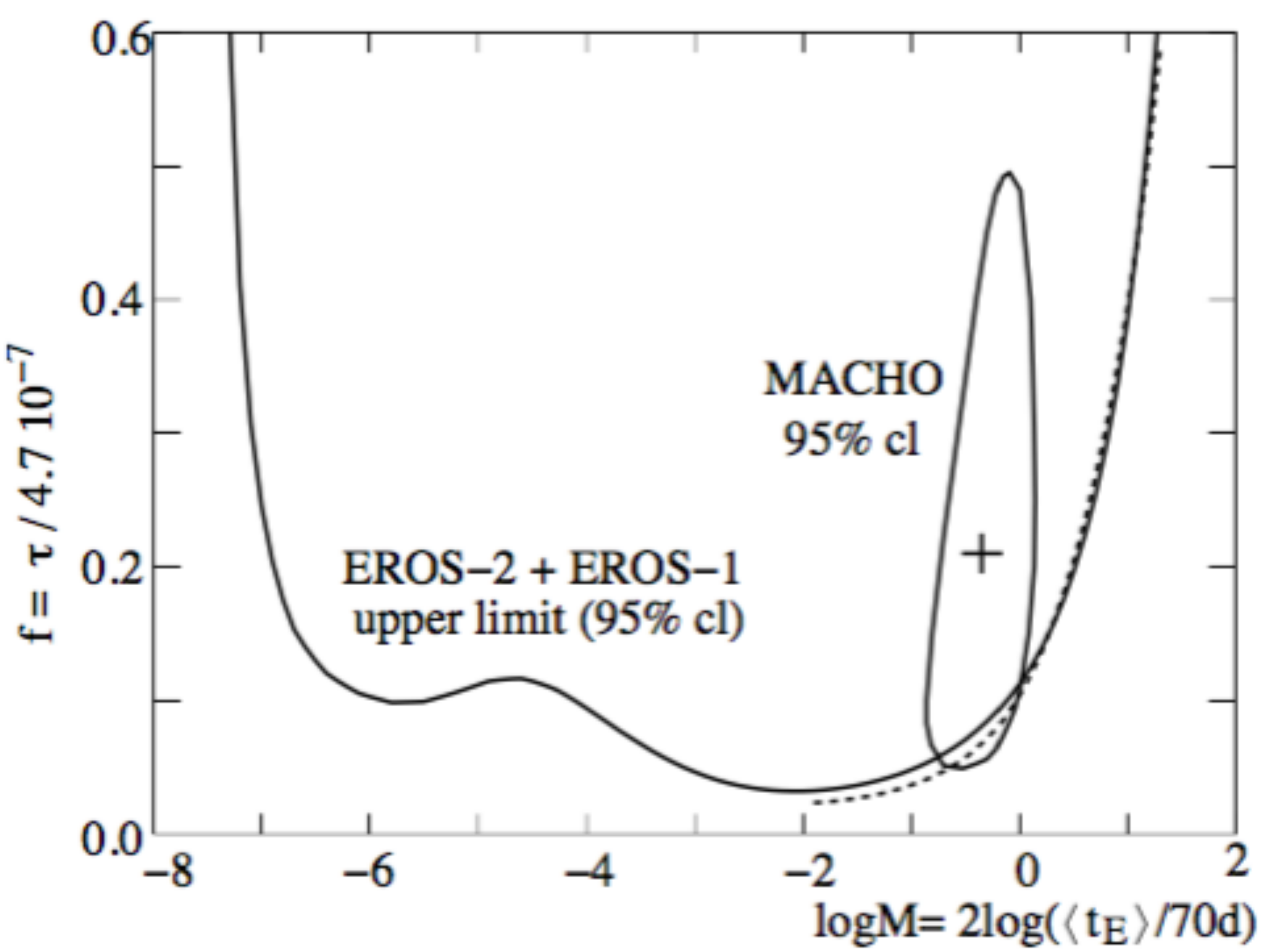
$$\Gamma \approx 1200 \mathrm{yr}^{-1} \frac{N_\star}{10^8} \frac{\tau}{10^{-6}} \left(\frac{t_E}{19 \mathrm{days}} \right)^{-1}$$

SOME IMPORTANT FACTS

- several collaborations have implemented the microlensing idea (proposed by B. Paczynski). These groups have monitored the galactic bulge and the Magellanic Clouds searching for microlensing events
- for example, the OGLE-III collaboration, by monitoring 2x10^8 stars in the galactic bulge detected ~600 events/yr.
- this is about ~25% of the expected rate
- note that the event rate $\Gamma \sim t_E^{-1} \sim M^{-1/2}$. Thus, while the optical depth does not depend on mass, the event rate does. The timescale distribution can thus be used to probe the kinematics and the mass function of lenses in the Milky Way.

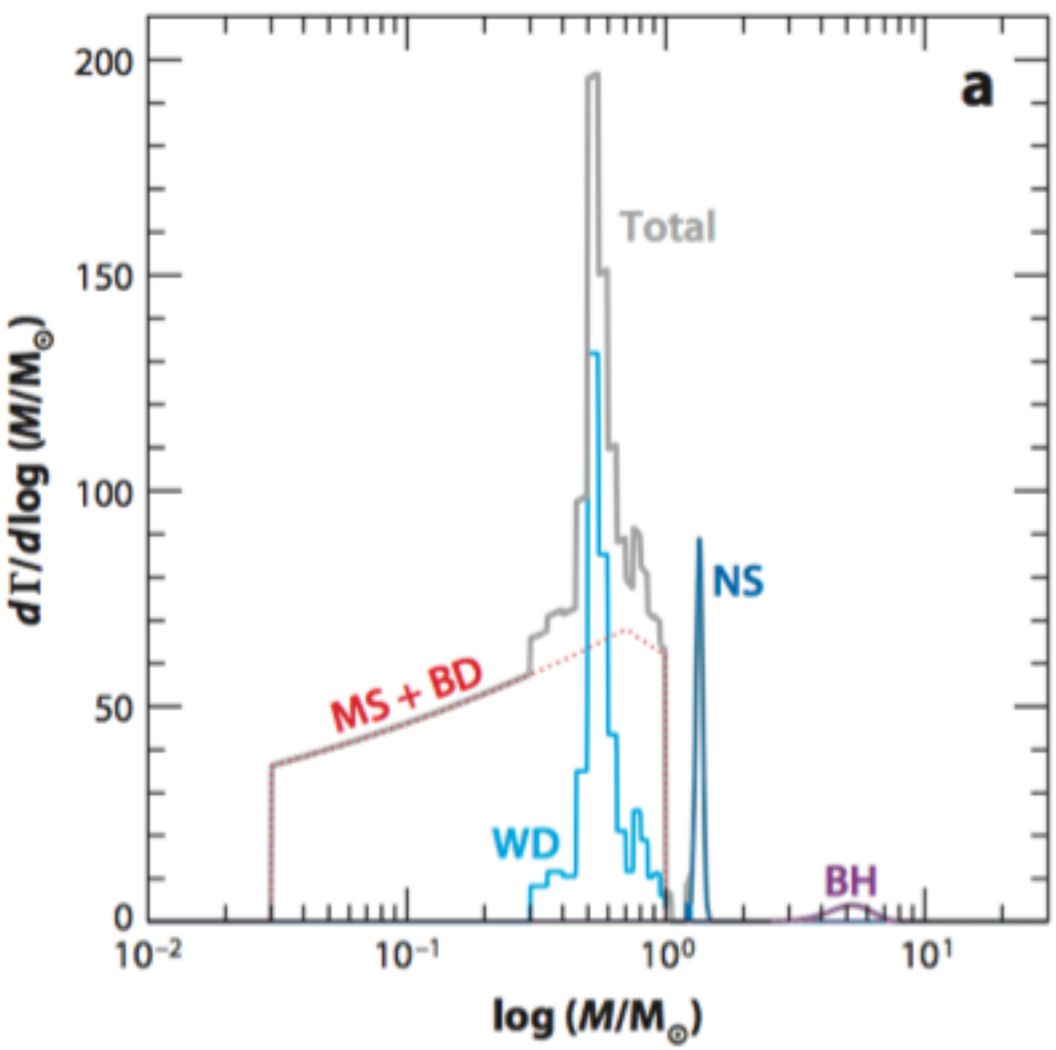
PAST RESULTS IN MICROLENSING RESEARCH

- searches for MACHOs (<20% of the halo)
- galactic structure (essentially, the known stellar populations in the galaxy and in the LMC/SMC can explain all the microlensing signal)

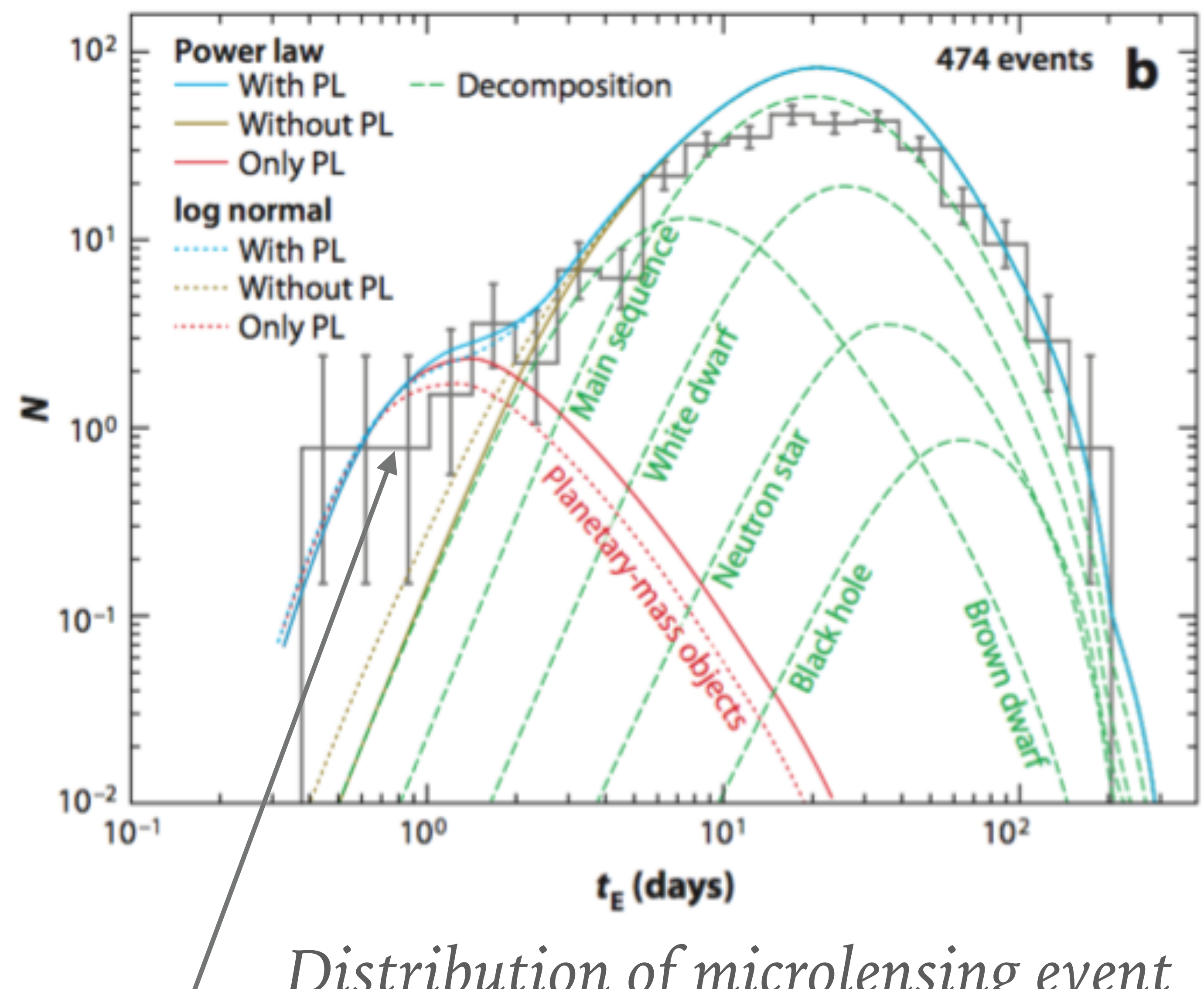


PROBING THE STELLAR POPULATIONS WITH MICROLENSING

Gaudi, 2012, Ann. Rev. Astron. Astrophys. 50, 411



Theoretical estimate of the rate of microlensing events towards the galactic bulge (Gould, 2000)

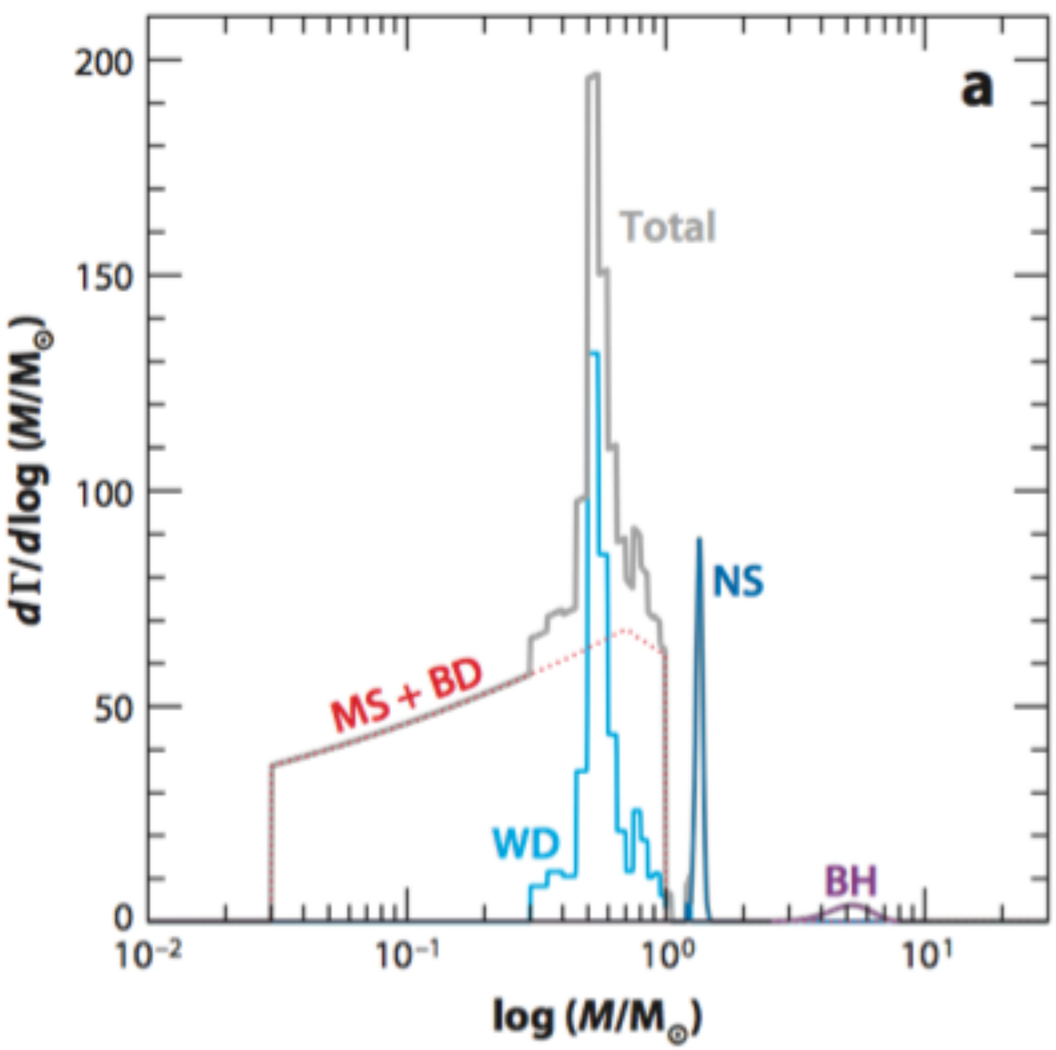


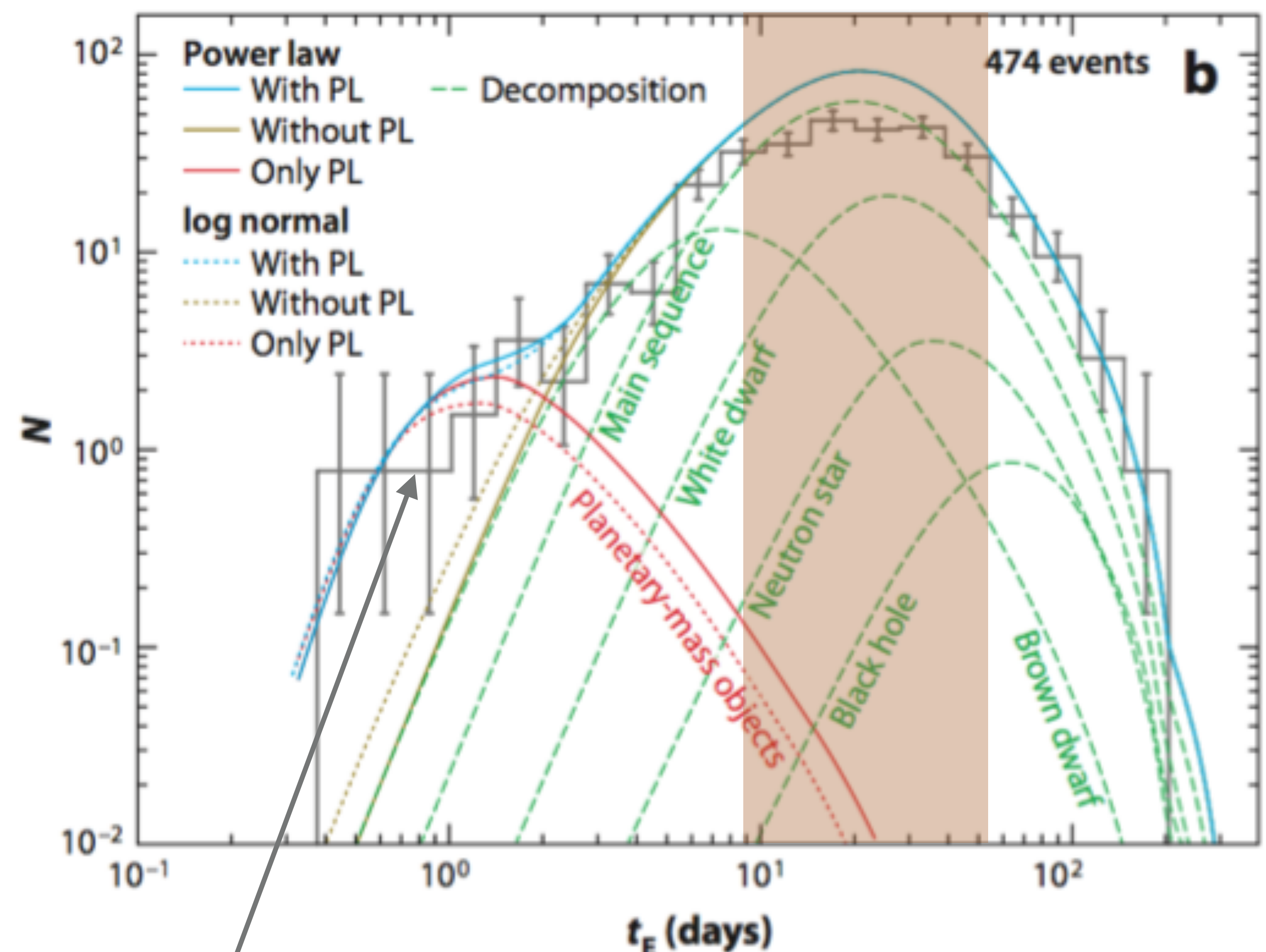
Distribution of microlensing event timescales observed by the MOA collaboration (2006-2007) [Sumi et al. 2011]

PROBING THE STELLAR POPULATIONS WITH MICROLENSING

Gaudi, 2012, Ann. Rev. Astron. Astrophys. 50, 411

wide range of masses at fixed t_E!





Theoretical estimate of the rate of microlensing events towards the galactic bulge (Gould, 2000)

Distribution of microlensing event timescales observed by the MOA collaboration (2006-2007) [Sumi et al. 2011]