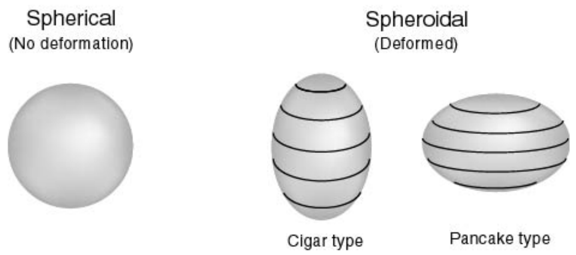
Sunday, October 29, 2023 10:41 AM

Rotational - anisotropy in the nucleus

The deformation reflects the highly anisotropic mass distribution.



The Coriolis and centrifugal forces that act in such coordinate frames perturb the structure of a rotating object.

The condition that the fluctuations in shape be small compared with the average deformation, ΔR << R, is therefore equivalent to the adiabatic condition ω(rot) << ω(vib).

SYMMETRIES OF DEFORMATION: ROTATIONAL DEGREES OF FREEDOM

A separation of the motion into intrinsic and rotational components :

$H = H_{intr}(q,p) + H_{rot,\alpha}(P_w)$

The intrinsic motion is described by the coordinate q and conjugate momenta p, which are measured relative to body-fixed coordinate frame and are therefore scalers w.r.t. Rotations of the external coordinate system. The rotational Hamiltonian does not depend on the orientations ω and is a function of conjugate angular momenta P_w, which may depend on quantum number α.

The orientation of a body in three-dimensional space involves three angular variables, such as the Euler angles

$\omega = \phi, \theta, \psi.$

The eigenstates of the Hamiltonian are of the product form :

$\Psi_{\alpha I \Sigma} = \Phi_\alpha(q)\, \Phi_{\alpha, I}(\omega)$

and three quantum numbers are needed in order to specify the state of motion.

The total angular momentum I and its component M= I, on a space-fixed axis provide two of these quantum numbers; the third may be obtained by considering the component of I with respect to an intrinsic coordinate system with orientation ω.

For each instrinsic state α the spectrum involves a sequence of rotational levels, specified by a set of angular momentum quantum number denoted by I in above equation.

Degrees of Freedom Associated with Spatial Rotations

As a commuting set of angular momentum variables, we may thus choose I^2, Iz,, and I3. The eigenvalues of I3, are denoted by K and have the same range of values as does M.

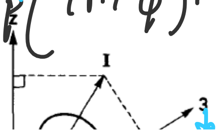
The orientation is characterzed by azimuthal angle ϕ

$K = I, I-1, \ldots, -I$

The associated rotational wave function is

$$\Phi_M(\phi) = \frac{1}{\sqrt{2\pi}} \exp(iM\phi).$$

Coordinate system fixed in lab



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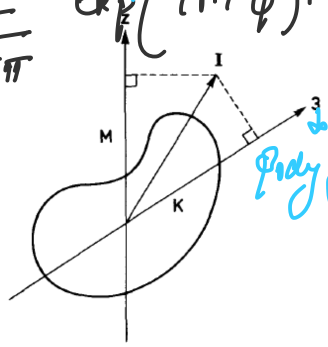

Body fixed coordinate system

Figure 4-1 Angular momentum quantum numbers describing rotational motion in three dimensions. The z axis belongs to a coordinate system fixed in the laboratory, while the 3 axis is part of a body-fixed coordinate system (compare the $\mathscr{K}$ and $\mathscr{K}'$ systems defined in Fig. 1A-1, Vol. I, p. 76).

The rotational wave function can be written as

$$\Phi_{IKM}(\omega) = \left(\frac{2I+1}{8\pi^2}\right)^{1/2} \mathscr{D}^I_{MK}(\omega)$$

Rotational Matrix.

for K=0 $\mathscr{D}$ reduces to spherical harmonics.

amplitudes $C_{\alpha I}(K)$ depends on the relative mag.

$$\Phi_{\alpha, I, K=0, M}(\omega) = \frac{1}{\sqrt{2\pi}} Y_{IM}(\theta, \phi)$$

Consequences of Axial Symmetry :

For K= 0, the rotational wave function is the same as for the angular motion of a point particle with no spin.

If system possesses axial symmetry two consequences are :

For finite K, the rotational motion corresponds to the angular motion of a particle with helicity h = K

1. The projection I3, on the symmetry axis is a constant of the motion.

invariance of Ham w.r.t rot^n about symmetric axis.

2. There are no collective rotations about the symmetry axis

While I^2 and Iz are constants of the motion for any rotationally invariant Hamiltonian, the commutator of I, with the Hamiltonian depends on intrinsic properties of the system. In general, therefore, the stationary states involve a superposition of components with different values of K,

I_3 is conservation if the 3 axis is the symmetric axis for the favour of rotation

$$\Phi_{\alpha I M} = \left(\frac{2I+1}{8\pi^2}\right)^{1/2} \sum_K C_{\alpha I}(K)\, \mathscr{D}^I_{MK}(\omega)$$

There's no collective rotation for a spherical nuclei.

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$$\Phi_{\tau IM}=\left(\frac{2I+1}{8\pi^2}\right)^{1/2}\sum_{K} C_{\tau I}(K)\,\mathcal{D}^{I}_{MK}(\omega)$$

Third rotational quantum #

It follows that the quantum number K represents the angular momentum of the intrinsic motion and has a fixed value for the rotational band based on a given intrinsic state. (In diatomic molecules, the angular momentum of the collective rotational motion is perpendicular to the symmetry axis because the nuclei can be treated as mass points and because the electrons do not rotate collectively in the axially symmetric binding field.

The restriction on the rotational degrees of freedom resulting from axial symmetry corresponds to the constraint

$$I_3 = J_3 \quad \rightarrow (4.10)$$

where J_3 is the operator representing the component of intrinsic angular momentum

The constraint (4.10) ensures that the total nuclear wave function, which is a product of intrinsic and rotational wave functions (see Eq. (4-4)), is independent of the value of psi

R

Invariance

A further reduction in the rotational degrees of freedom follows if the intrinsic Hamiltonian is invariant with respect to a rotation of 180" about an axis perpendicular to the symmetry axis.

R invariance is the only possible invariance; Infact variance w.r.t. any other rotation would imply an ∞ of symmetry axes & Hence spherical symmetry

$R_e = R_i$

invariance under infinitesimal rotations about symmetry axis.

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