

Many Body Physics

① Lecture 25 :→

a vector with n elements:

$$|u\rangle = \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{pmatrix}$$

Python
 # C like notation
 starts from 0

$$\langle u | = \left[x_0^* \ x_1^* \ \dots \ x_{n-1}^* \right]$$

$$= |u\rangle^\dagger$$

Orthonormal Basis :→

$$|y_i/n\rangle = \sum_{i=0}^{n-1} y_i^* x_i'$$

Outer Product :→

$$|u\rangle \langle u| =$$

$$\begin{pmatrix} u_0 u_0^* & u_1 u_1^* & \dots & u_{n-1} u_{n-1}^* \\ \vdots & & & \\ u_{n-1} u_0^* & \dots & \dots & u_{n-1} u_{n-1}^* \end{pmatrix}$$

Tensor Product : \Rightarrow

$$|x\rangle \otimes |y\rangle = |xy\rangle$$

$$|x\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (10)$$

$$|y\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (11)$$

Computational Basis : \Rightarrow

$$\langle y|x\rangle = 0$$

$$\langle n|x\rangle = 1$$

$$\langle y|y\rangle = 1$$

$$|x\rangle \otimes |n\rangle = \begin{pmatrix} 1 & \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ 0 & \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{pmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= |xn\rangle$$

Single particle Wavefunction \Rightarrow

$$\psi_{\alpha}(\vec{r}) \otimes \epsilon_{m_s} = |\alpha m_s\rangle$$

Radial
part

spin
part

compact notation.

\Downarrow

no. of nodes (hydrogen atom).
 m_l line

Total wavefunction: Tensor product of spin part & radial part

$$|m\rangle \otimes |n\rangle = |0\rangle \otimes |0\rangle = |00\rangle = |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$|0 \otimes 1\rangle = \begin{pmatrix} 0 & [1 \\ 0] \\ 1 & [1 \\ 0] \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = |01\rangle$$

$$|1 \otimes 0\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = |10\rangle$$

$$|1 \otimes 1\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = |11\rangle$$

$$\langle 11|00 \rangle = 0$$

$$\langle 00|00 \rangle = 1$$

$$\langle 11|11 \rangle = 1$$

$$\langle 01|01 \rangle = 1$$

$$\langle 10|10 \rangle = 1$$

$$|0\rangle\langle 0| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$|1\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I = \sum_{i=0}^{n-1} |i\rangle\langle i|$$

↓
Sum upto
all computational basis

$$i:0 ; |0\rangle$$

$$i:1 ; |1\rangle$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{P} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\hat{Q} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|0\rangle\langle 0|$$

$$|1\rangle\langle 1|$$

$$\hat{P}^2 = \hat{P}$$

Idempotent operator

$$\hat{Q}^2 = \hat{Q}$$

Projection operation

$$\hat{P}\hat{Q} = 0$$

Orthogonal operation

→ We try to reduce the dimensionality

$$I = \hat{P} + \hat{Q} = \underbrace{\sum_{i=0}^{d-1} |i\rangle\langle i|}_{P} + \underbrace{\sum_{i=d+1}^{\infty} |i\rangle\langle i|}_{Q}$$

of our computational basis is $|0\rangle$ & $|1\rangle$ so we can define a new state

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$|0\rangle$ & $|1\rangle$ are may not be necessarily eigenbasis of full problem

$$|\Psi\rangle = \sum_{i=0}^{\infty} C_i |i\rangle$$

$$\hat{P}|\Psi\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \left[\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right]$$

$$= \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |\alpha\rangle$$

$$\hat{Q}|\Psi\rangle = P|1\rangle$$

Density Matrix : \rightarrow

$$f = |\Psi\rangle \langle \Psi| = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \begin{bmatrix} \alpha^* & \beta^* \end{bmatrix}$$

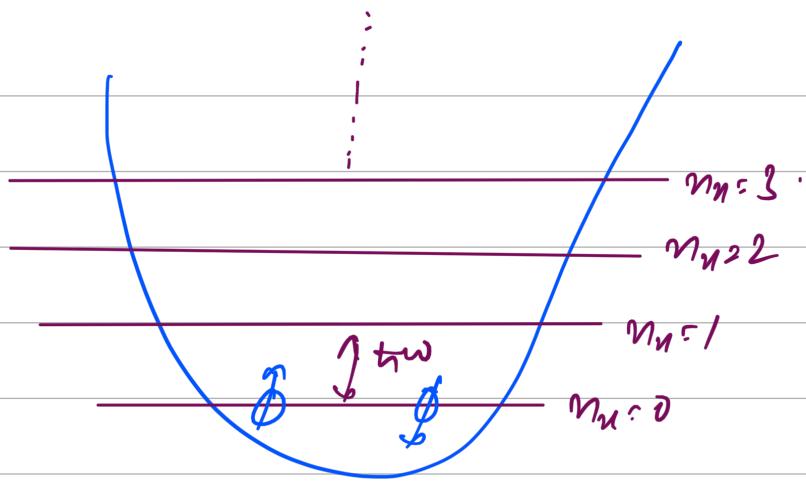
$$= \begin{bmatrix} \alpha\alpha^* & \alpha\beta^* \\ \beta\alpha^* & \beta\beta^* \end{bmatrix}$$

$$\text{Tr}(f)_2 \quad |\alpha|^2 + |\beta|^2 = 1.$$

= Sum of all probabilities

Computational Basis : \rightarrow Harmonic Oscillator (quantum state)

Bose Einstein Condensate, Nuclear physics,



$$\rightarrow E_{n_n} = \left(n_n + \frac{1}{2} \right) \hbar \omega$$

Hamiltonian (1 particle)

$$H_0 = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial n^2} + \frac{1}{2} k n^2$$

Computational Basis

$$\psi_{n_n}(n) = \langle n | n_n \rangle$$

$$n_n \rightarrow i$$

Truncate. Here,

$$\text{Basis} \rightarrow i = 0, 1, 2, \dots, -1 \nearrow \infty$$

{ first truncation in the computational basis in the level of single particle

$$\phi_i(n)$$

$$\hat{h}_0 \phi_i(n) = \sum_j \phi_j(n).$$

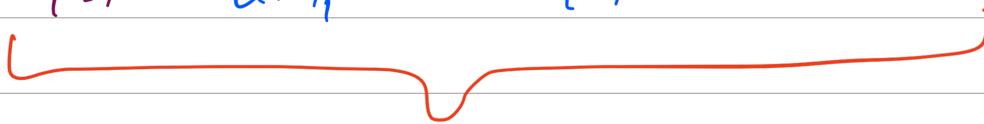
$$\hat{h}_0 |i\rangle = E_i |i\rangle$$



One Body operator because it acts at one particle at a time

$$N = \# \text{ of particles}$$

$$\hat{H} = \sum_{i=1}^N -\frac{\hbar^2 \nabla_i^2}{2m_i} + \sum_{i=1}^N V_{\text{ext}}(\vec{r}_i)$$



one Body part of the Hamiltonian.

$$\underline{H_0}.$$

$$+ \sum_{i < j}^N v(|\vec{r}_i - \vec{r}_j|) \approx \underbrace{\left\{ v(r_i, r_j) \right.}_{r_{ij}}$$

$H_I \equiv$ interaction part.

Construct many body ansatz $|\Psi_i\rangle$ based on the single particle computational basis.

$$|\Psi_i\rangle = \prod_{j=0}^d \phi_j(x_j)$$

for fermions:

so we say

$$H_0 |\Psi_i\rangle = \prod_{j=0}^d \phi_j(n_j)$$

$$= H_0$$

$$= E_i (|\Psi_i\rangle)$$

$$h_0 (\Psi_i) = E_i (\Psi_i)$$

Exact state function

$$H |\Psi_k\rangle = E_k |\Psi_k\rangle.$$

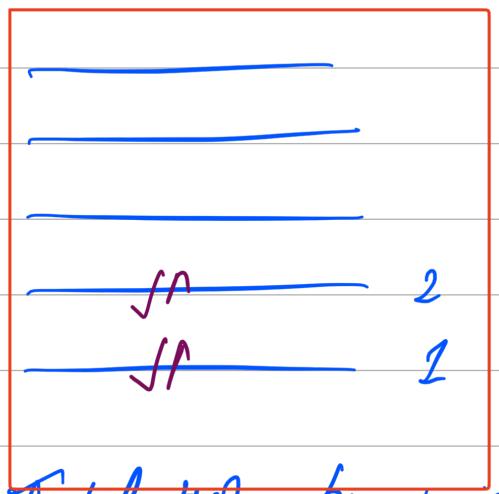
$$|\Psi_k\rangle = \sum_{j=0}^d c_{kj} |\Psi_j\rangle$$

(1) → Another truncation for many body

Computational Many Body Basis

Suppose we have single particle state \rightarrow

n single particle states with
double degeneracy



new Hilbert space where we sum
our calculations - 1 additional
truncation.

in single particle states:

Total # of fermionic states = Configuration

$$= \binom{2n}{N}$$

$$= \frac{(2n)!}{(2n-N)! N!}$$