Neutrino Oscillation

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Abstract

1 Introduction

Why do we believe that Neutrino's oscillate or that they have mass? There are four reasons for this

1. Dark Matter - The observed motion of matter within galaxies cannot be explained by luminous matter seen. This points to 10 times more matter than it actually seen.Is this extra dark matter made up of non-zero mass relic neutrinos from Big Bang?

2. LSND Experiment - An experiment at Los Alamos, LSND, using a $\bar{\nu_\mu}$ beam measures a number of $\bar{\nu_e}$ as well of the expected background oscillations.

3. Solar Neutrino Problem - As by Eddington paper sun produce its energy by nuclear process inside the sun. So neutrino are also produced in this reaction but the measured rate on the earth is 25% - 50% of what is predicted by Standard Solar Model. Could they be oscillating into another neutrino flavor between production and observation?

4. Atmospheric Neutrinos - Produced by the decay of pions and kaons themselves originating from cosmic ray interaction in the Upper Atmosphere. The muon neutrino to electron neutrino ratio is 60% what is expected. Is this due to change in the flavor mix due to oscillations?

   So there are enough hints to justify studying oscillations in great detail.

2 Assumptions

• Neutrinos have masses.
• Neutrinos mix.
• Mixing described by a unitary matrix, U_{fk}, similar to the Cabibo-Kobayashi-Maskawa matrix describing quark mixing.

Then the neutrino weak eigen states: ν_e, ν_μ, ν_τ are each a superposition of the three Mass states: ν_1, ν_2 and ν_3 :

$$|\nu_f> = \sum_{i=1}^{3} U_{fk}|v_k> \ \ ; \text{where k = 1,2,3}$$

At time t = 0, produce a beam of given flavor α
At time t = t we will have:
The different $|v_k>$'s
will evolve differently with time because of the different m_i's in the exponent(that is why we need masses to have oscillations)

Conclusion
At t = 0 we had the EXACT mix to get flavor α.
At t = t we will have ALL flavors present in the beam

3 Two Neutrino Mixing

Now just for a case limit ourselves to two-neutrinos the mixing can be described by a simple rotation matrix

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

θ = Strength of oscillation
Starting with a beam of flavor α at time t =t;

$$|\nu_t = \exp(ip.r)(\cos\theta e^{-iE_1 t}|\nu_1> + \sin\theta e^{-iE_2 t}|\nu_2>)$$

Probability to find the flavor β in the initially pure α beam:

$$P_{\alpha\beta}(\text{t}) = |<\nu_\beta|\nu_t>|^2$$

with

$$\nu_\beta = -\sin\theta|\nu_1> + \cos\theta|\nu_2>$$

which gives:

$$P_{\alpha\beta}(t) = \sin^2 2\theta \sin^2(\frac{E_1 - E_2}{2})t$$

with

$$E_{1,2} = \sqrt{p^2 + m_{1,2}^2} \simeq p(1 + \frac{m_{1,2}^2}{2p^2})$$

$$E_1 - E_2 = \frac{1}{2p}\Delta m^2 \text{ where } \Delta m^2 = (m_1^2 - m_2^2)$$

Setting t = $\frac{L}{c}$, L is distance between production and observation points:

$$P_{\alpha\beta}(t) = \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4pc}$$

Switching from natural to real units this could be written as:

$$P_{\alpha\beta}(t) = \sin^2 2\theta \sin^2 1.27 \frac{L(m)}{E(MeV)} \Delta m^2 (eV^2)$$

with $\lambda = \frac{2.48E}{\Delta m^2}$ = Oscillation length = Distance between maxima.

For small difference between m_{ν_1} and m_{ν_2}, original mix of mass states will not change quickly.
Neutrino must travel far before they have an appreciable probability to oscillate to another flavor. Must compensate smallness of Δm^2 by long distance.

For large mass difference the mix changes very quickly i.e. Fast Oscillations

$$\text{Large } \Delta m^2 \longrightarrow \text{small } \lambda$$

$$\text{For } \lambda << \text{Detector or source size}$$

$$P = \frac{1}{2} \sin^2 2\theta$$

4 Terminology

Disappearance experiment: Start with a beam of given flavor; observe lower rate of same flavor after distance L.

Appearance experiment: Start with a beam of given flavor; observe appearance of a different flavor after a distance L.

Baseline: Distance(L) between production and observation of neutrinos.

Short Base-Line: Typically length = $O(100m)$

Long Base-Line: Typically length = $O(100Km)$

5 A cosmological Neutrino?

Rotational velocity curves of galaxies. The discrepancy between the two curves can be accounted for by adding a dark matter halo surrounding the galaxy.
Expectation at large R:

$$v_R = \frac{1}{\sqrt{R}}; \text{ But instead we find} \Longrightarrow \text{FLAT.}$$

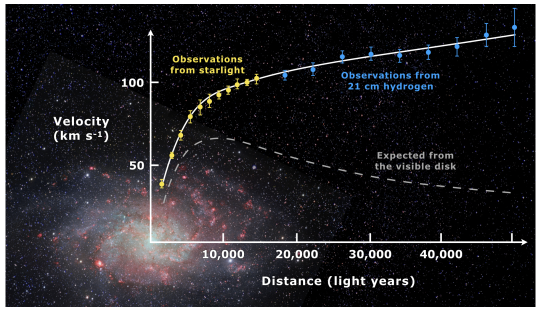

Figure 1: Rotation curve of spiral galaxy Messier 33 (yellow and blue points with error bars), and a predicted one from distribution of the visible matter (gray line).

This points to 10 times more matter than as actually seen. So that could be dark matter as relic neutrinos. Their abundance is nearly as Cosmic Background Radiation $\approx 115/cm^3/species$ and their density is about:

$$\rho = \frac{3}{11} = \sum_1^3 m_i \text{ with } m_i = \text{neutrino mass}$$

If they are distributed in a halo around the galaxies they could explain these rotational velocities and their average mass would be : 30 eV/c^2
For mixed hot and cold Dark matter number reduced to a few eV/c^2

6 Direct limits on neutrino mass

Limits from Direct Measurements			
Neutrino flavor	ν_e	ν_μ	ν_τ
Limit	$2.5\ eV - 10.0\ eV/c^2$	$0.16\ MeV/c^2$	$18.2\ MeV/c^2$
Confidence Level	95%	90%	95%
Method	Tritium end point	Muon momentum from pion decay	$>$ 5 pion decay of the τ at LEP

Limits of precision essentially reached. Thus the only way to go below these limits is through oscillations. But note that oscillations only give differences in masses. Some might have prejudice that ν_τ is the heaviest and other have negligible mass.
Look for oscillation involving the ν_τ in order to have large mass difference and have sizeable effects for reasonable distances.

$$P_{\alpha\beta}(t) = \sin^2 2\theta \sin^2 1.27 \frac{L(m)}{E(MeV)} \Delta m^2 (eV^2)$$

7 Numerology

Solar neutrino experiments were suggesting

$$\Delta m^2 = 10^{-5} eV^2$$

1. Assume the solar effect is due to $\nu_e \rightarrow \nu_\mu$ oscillation
2. The ν_e is vanishingly small.
3. The ν_μ mass is them $\sqrt{10^{-5}} \sim 3{\times}10^{-3} eV^2$
4. The smallness of neutrino, neutrino masses, m_{ν_i} due to a Very Large Majorana Mass, M;

   $$m_{\nu_i} \;=\; \frac{(m_i^F)^2}{M}$$

   where m_i^F is the quark mass.
   Then

   $$\frac{m_{\nu_\mu}}{m_{\nu_\tau}} = \frac{m_\mu^2}{m_\tau^2}$$

   or

   some formula

   Important to look for $\nu_\mu \rightarrow \nu_\tau$ oscillations at high Δm^2 ($\sim$ $1000 eV^2$)

8 Search for $\nu_\mu \rightarrow \nu_\tau$ oscillations

Exclusion panorama in

Appearance experiments is detected by CC interaction

$$\nu_\tau + N \rightarrow \tau^- + X$$