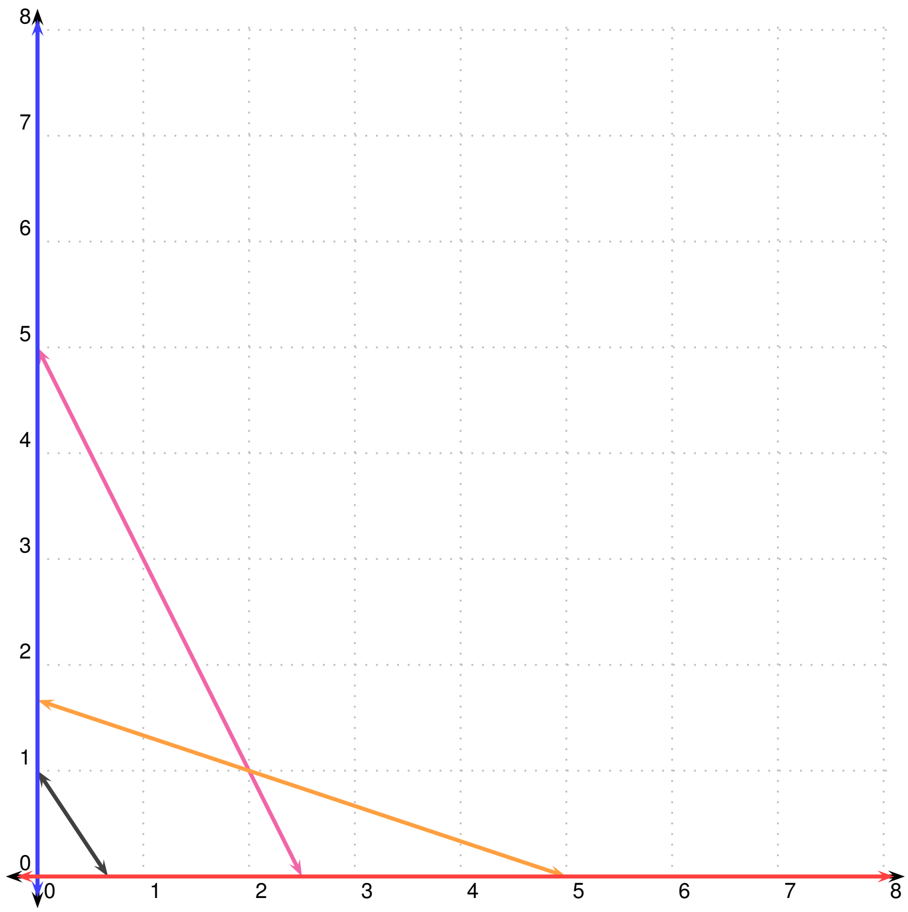
1. Consider the following linear programming problem

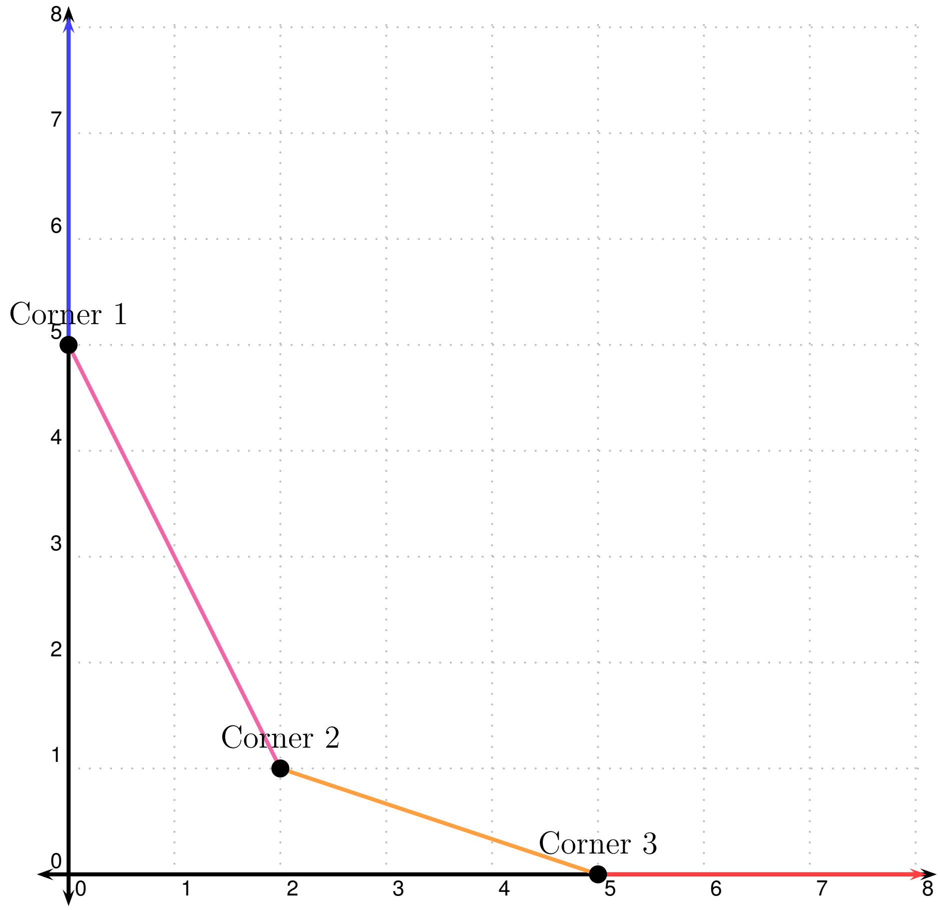
$$\begin{aligned} \text{minimize} \quad & 4y_1 + 7y_2 \\ \text{subject to} \quad & 2y_1 + y_2 \geq 5 \\ & 3y_1 + 2y_2 \geq 2 \\ & y_1 + 3y_2 \geq 5 \\ & y_1, y_2 \geq 0 \end{aligned}$$

Solve this problem using the graphical method. (Hint: determine the feasible region, and evaluate the corner points using algebra. Notice this is a minimization problem)

We can find our feasible region by graphing the equalities portion of the inequalities.



Since everything is greater than or equal to, we know the feasablie region is above all the lines, so our solution is inside the following region:



From here we can test the corner points of our feasible region, which occur at: $(0,5)$, $(2,1)$, and $(5,0)$. Substituting: $4(5)+7(0) = 20$, $4(2)+7(1) = 15$, and $4(0)+7(5) = 35$, $min(20,15,35) = 15$ thus the minimized solution is $\boxed{15}$

2. Consider the following vectors and matrix:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ 5 \\ 3 \\ 7 \\ 5 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 & -2 & 4 & 5 \\ 3 & 1 & 1 & 2 & 4 \\ 0 & 1 & -1 & 2 & 2 \end{bmatrix}$$

Perform the following operations:

(a) $\mathbf{c}^T\mathbf{x}$

$$\mathbf{c}^T = \begin{bmatrix} 1 & 5 & 3 & 7 & 5 \end{bmatrix}$$

$$\mathbf{c}^T\mathbf{x} = \begin{bmatrix} 1 & 5 & 3 & 7 & 5 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \boxed{\begin{bmatrix} x_1 & 5x_2 & 3x_3 & 7x_4 & 5x_5 \end{bmatrix}}$$

(b) $A\mathbf{c}$

$$\begin{bmatrix} 1 & 0 & -2 & 4 & 5 \\ 3 & 1 & 1 & 2 & 4 \\ 0 & 1 & -1 & 2 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \\ 3 \\ 7 \\ 5 \end{bmatrix} = \begin{bmatrix} 1+0-6+28+25 \\ 3+5+3+14+20 \\ 0+5-3+14+10 \end{bmatrix} = \boxed{\begin{bmatrix} 48 \\ 45 \\ 26 \end{bmatrix}}$$

3. Consider the following optimization problem:

$$\begin{aligned} \text{maximize} \quad & 5x_1 + 2x_2 + 5x_3 \\ \text{subject to} \quad & 2x_1 + 3x_2 + x_3 \leq 4 \\ & x_1 + 2x_2 + 3x_3 \leq 7 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Write the problem above in matrix-vector notation.

First we must rewrite the conditions of the problem, for all feasible solutions: x_1, x_2, x_3, the value on the LHS is at most the value on the RHS, so we have that:

$$2x_1 + 3x_2 + x_3 + s_1 = 4$$

$$x_1 + 2x_2 + 3x_3 + s_2 = 7$$

Since we are maximizing, the solution function becomes:

$$5x_1 - 2x_2 - 5x_3 + z = 0$$

Thus, the final matrix is:

$$\boxed{\left[\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & z & C \\ \hline 2 & 3 & 1 & 1 & 0 & 0 & 4 \\ 1 & 2 & 3 & 0 & 1 & 0 & 7 \\ -5 & -2 & -5 & 0 & 0 & 1 & 0 \end{array}\right]}$$