Adiar

Binary Decision Diagrams in External Memory

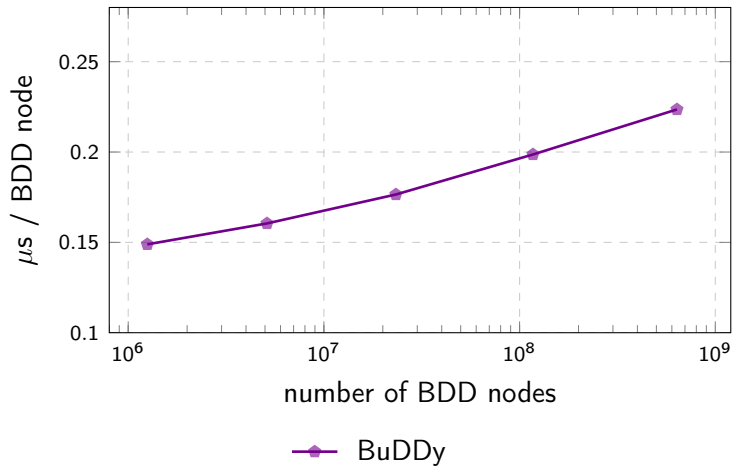---

Steffan Christ Sølvsten, Jaco van de Pol,
Anna Blume Jakobsen, and Mathias Weller Berg Thomasen
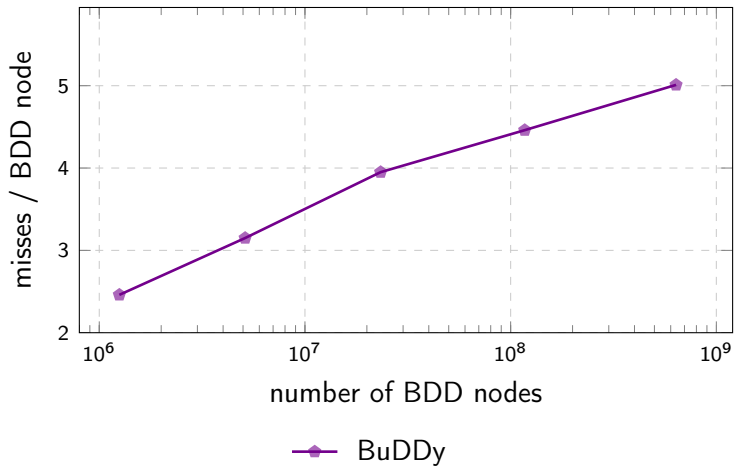TACAS 2022

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Minimal running time for the *Queens* problems.

Cache-misses for the *Queens* problems.

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

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Priority Queue: Q_{count}:
[

]

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$$

$$]$$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$

$\quad ((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$

$]$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$[\quad ((0, 0) \xrightarrow{\top} (1, 0), \quad 1) \quad ,$
$\quad ((0, 0) \xrightarrow{\bot} (2, 0), \quad 1) \quad ,$

$]$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

$[$

$\quad ((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$]$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

[

$((0, 0) \xrightarrow{\perp} (2, 0),$ 1) ,

$((1, 0) \xrightarrow{\perp} (2, 0),$ 1) ,

$((1, 0) \xrightarrow{\top} (3, 1),$ 1) ,

]

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2,0)$	0	0

Priority Queue: Q_{count}:

$[$

$\quad ((0,0) \xrightarrow{\perp} (2,0), \quad 1) \quad ,$

$\quad ((1,0) \xrightarrow{\perp} (2,0), \quad 1) \quad ,$

$\quad ((1,0) \xrightarrow{\top} (3,1), \quad 1) \quad ,$

$]$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

$$[\\
((0,0) \xrightarrow{\perp} (2,0), \quad 1), \\
((1,0) \xrightarrow{\perp} (2,0), \quad 1), \\
\\
((1,0) \xrightarrow{\top} (3,1), \quad 1), \\
]$$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	1	0

Priority Queue: Q_{count}:

$[$

$\quad ((1, 0) \overset{\perp}{\dashrightarrow} (2, 0), \quad 1) \quad ,$

$\quad ((1, 0) \overset{\top}{\rightarrow} (3, 1), \quad 1) \quad ,$

$]$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$$

]

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

$$
\begin{aligned}
[\quad & \\
((2, 0) &\xrightarrow{\perp} (3, 0), \quad 2) \quad, \\
((1, 0) &\xrightarrow{\top} (3, 1), \quad 1) \quad, \\
((2, 0) &\xrightarrow{\top} (3, 1), \quad 2) \quad]
\end{aligned}
$$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3,0)$	0	0

Priority Queue: Q_{count}:

$$[$$

$$((2,0) \xrightarrow{\perp} (3,0), \quad 2) \quad ,$$
$$((1,0) \xrightarrow{\top} (3,1), \quad 1) \quad ,$$
$$((2,0) \xrightarrow{\top} (3,1), \quad 2) \quad]$$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\perp} (3, 0),$ 2) ,
$((1, 0) \xrightarrow{\top} (3, 1),$ 1) ,
$((2, 0) \xrightarrow{\top} (3, 1),$ 2)]

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)\qquad,$
$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)\qquad]$

3

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	2	2

Priority Queue: Q_{count}:

$[$

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$

3

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	1	2

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2 \quad$]

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count}:

[

]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3,1)$	3	5

Priority Queue: Q_{count}:

[

]

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

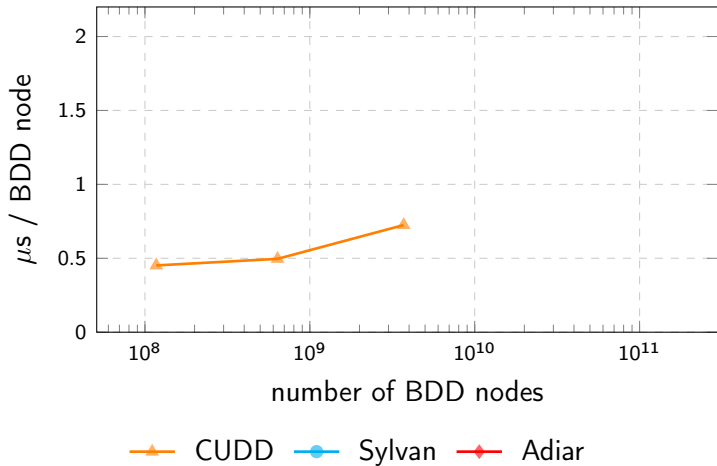Result
5

Priority Queue: Q_{count}:
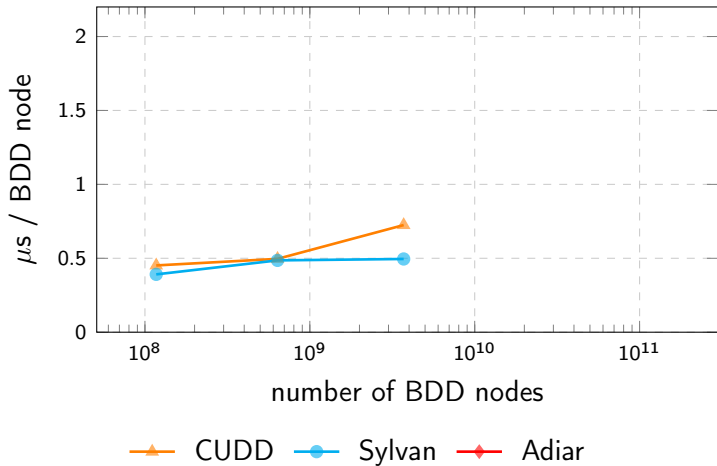[

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Adiar

I/O-efficient Decision Diagrams

github.com/ssoelvsten/adiar

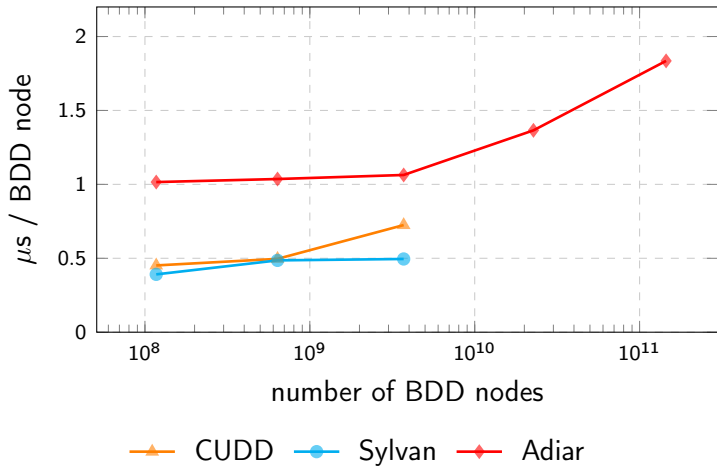Minimal running time for the *Queens* problems.

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Minimal running time for the *Queens* problems.

Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38
$O(N \log N)$	0.058

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38
$O(N \log N)$	0.058
$O(N)$	0.006

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

Steffan Christ Sølvsten

✉ soelvsten@cs.au.dk

🌐 ssoelvsten.github.io

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</> github.com/ssoelvsten/adiar

📕 ssoelvsten.github.io/adiar

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