

Adiar 1.1 : Zero-suppressed Decision Diagrams in External Memory

Steffan Christ Sølvesten and Jaco van de Pol

18th of May, 2023



Adiar

Binary Decision Diagrams
in External Memory

github.com/ssoelvsten/adiar

Adiar

Multi-terminal Decision Diagrams
in External Memory

github.com/ssoelvsten/adiar

Adiar

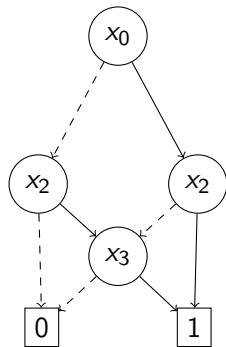
Quantum Multi-valued Decision Diagrams
in External Memory

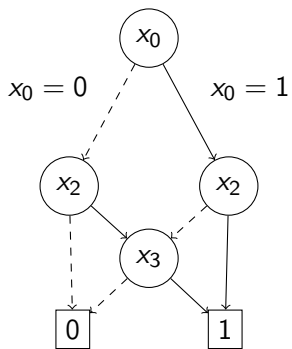
github.com/ssoelvsten/adiar

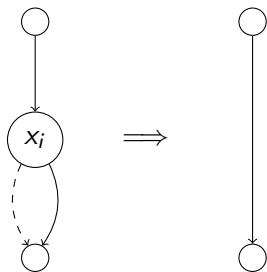
Adiar

Zero-suppressed Decision Diagrams
in External Memory

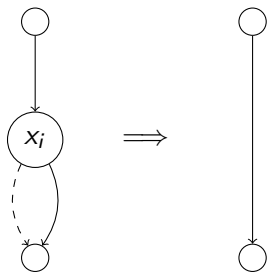
github.com/ssoelvsten/adiar



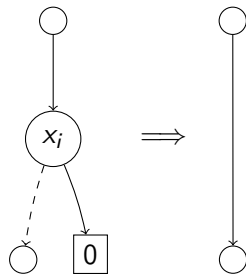




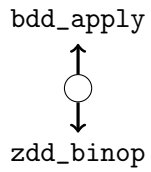
BDD: $f : \mathbb{B}^n \rightarrow \mathbb{B}$



BDD: $f : \mathbb{B}^n \rightarrow \mathbb{B}$



ZDD: $A \subseteq \mathcal{P}(\mathbb{B}^n)$



bdd_pathcount



zdd_size

bdd_apply



zdd_binop

zdd_offset



bdd_restrict

zdd_onset

zdd_disjoint



zdd_equal

bdd_equal

zdd_subseteq

bdd_pathcount



zdd_size

bdd_apply



zdd_binop

zdd_offset



bdd_restrict

zdd_onset

zdd_disjoint



zdd_equal



bdd_equal



zdd_subseteq



bdd_from_zdd

zdd_change

zdd_from_bdd

zdd_expand

zdd_complement

bdd_pathcount



zdd_size

bdd_apply



zdd_binop

zdd_offset



bdd_restrict

zdd_onset

zdd_disjoint



zdd_equal



bdd_equal

zdd_subseteq

bdd_from_zdd

zdd_from_bdd



zdd_change



zdd_expand

zdd_complement

`zdd_binop(A , B , \otimes) = prod2(A , B , \otimes , zdd_strategy)`

$$\text{zdd_binop}(A, B, \otimes) = \text{prod2}<\text{zdd_policy}>(A, B, \otimes)$$

<adiar>

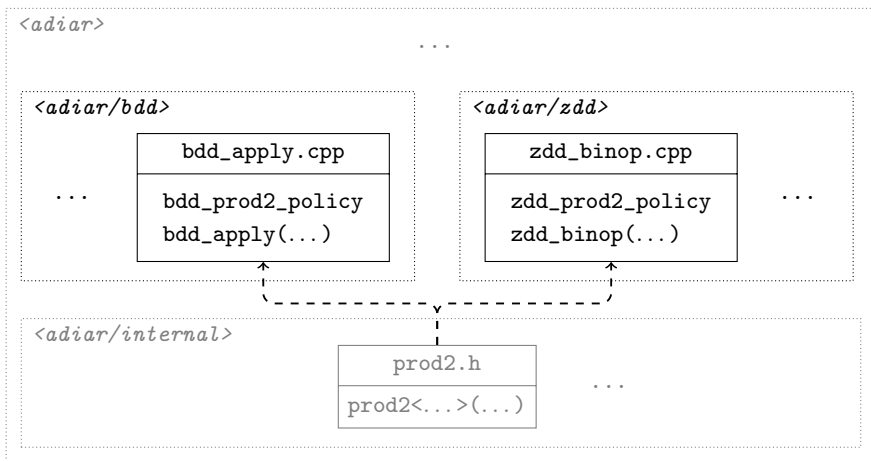
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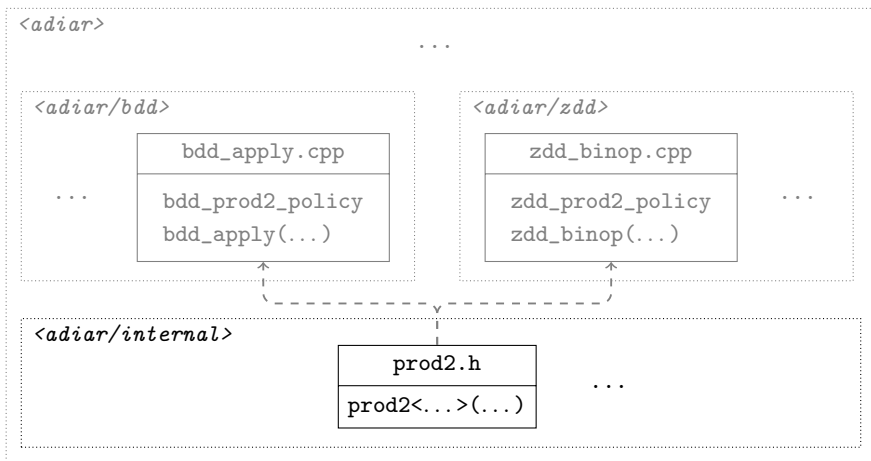
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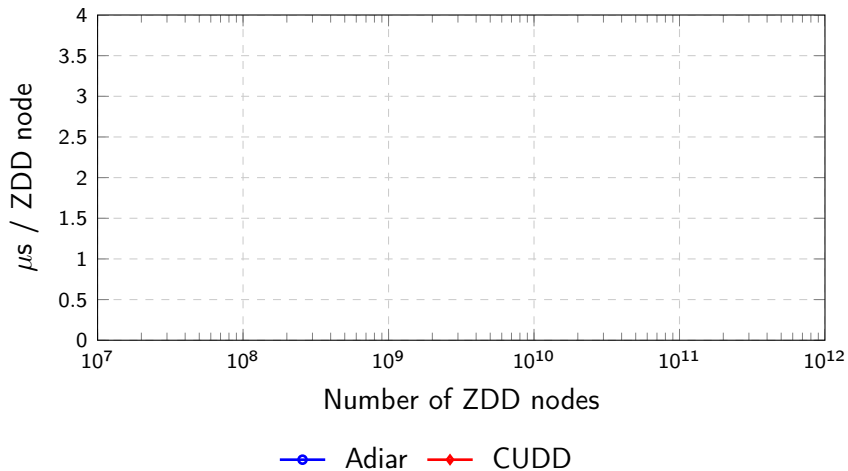
prod2.h

prod2<...>(...)

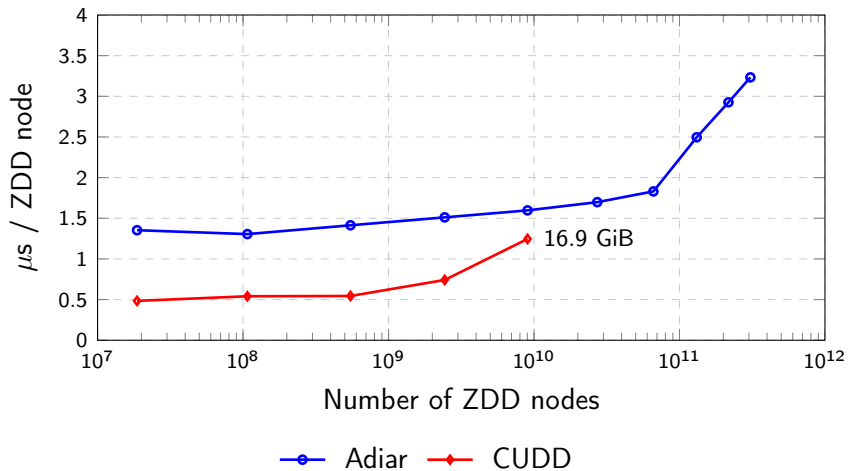
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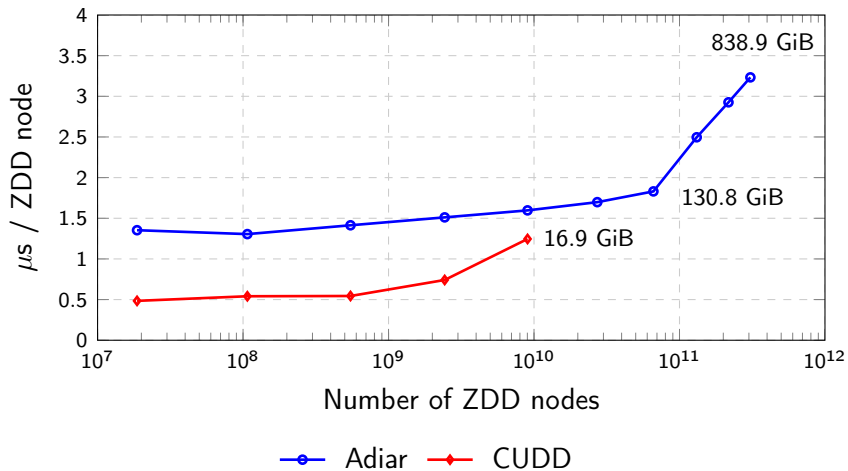




Running time for *Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *Tic-Tac-Toe* with 300 GiB of RAM.

BDD

ZDD

FDD

BDD

ZDD

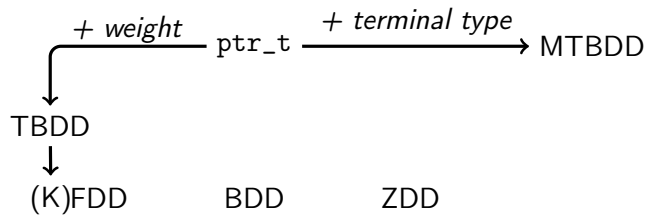
$\text{ptr_t} \xrightarrow{+ \text{terminal type}} \text{MTBDD}$

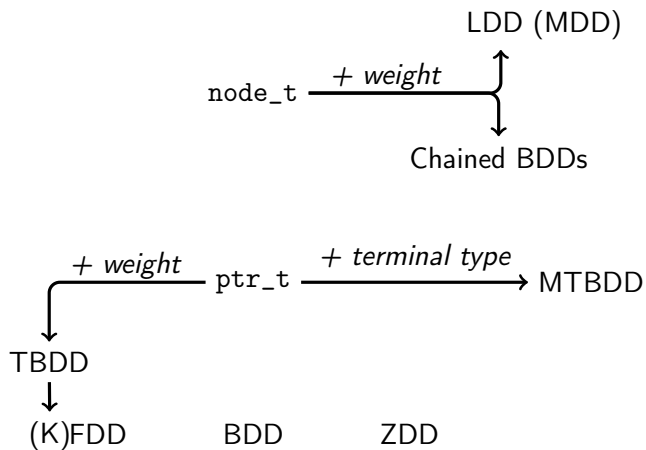
FDD

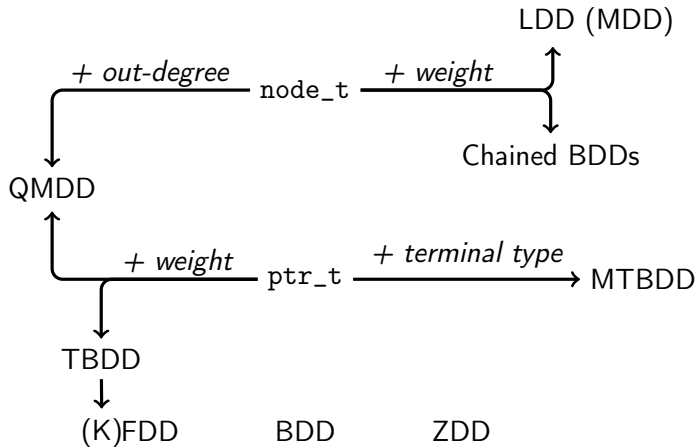
BDD

ZDD









Steffan Christ Sølvsten

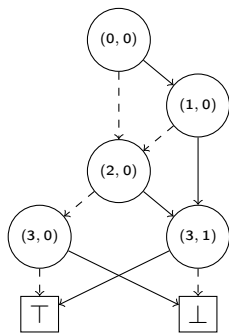
✉ soelvsten@cs.au.dk

🐦 [@ssoelvsten](https://twitter.com/ssoelvsten)

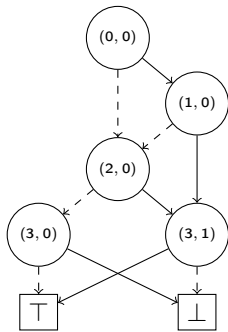
Adiar

📄 github.com/ssoelvsten/adiar

📖 ssoelvsten.github.io/adiar



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

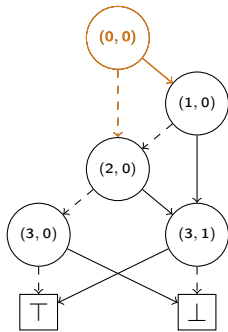


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[

]

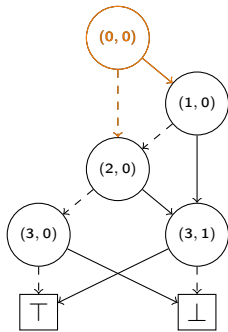


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[

]

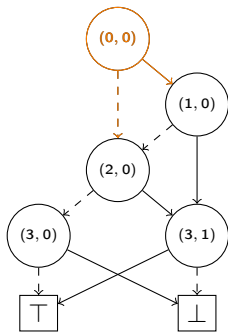


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[$((0,0) \xrightarrow{\top} (1,0), 1)$,
 $((0,0) \xrightarrow{\perp} (2,0), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\top} (1, 0), 1)$,
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]



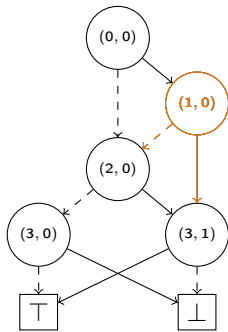
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\top} (1, 0), 1)$,
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]

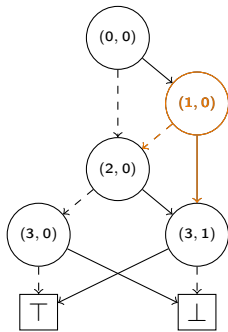


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	1	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
]

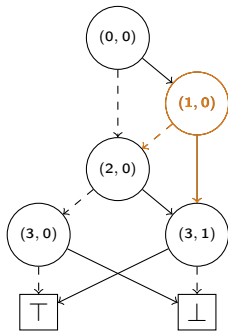


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	1	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]

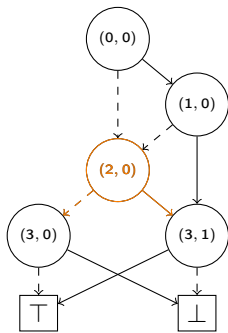


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	0	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

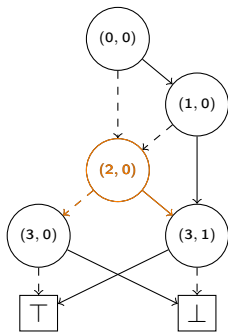
Seek	Sum	Result
(2, 0)	0	0

Priority Queue: Q_{count} :

[

$((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
$((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
$((1, 0) \xrightarrow{\top} (3, 1), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	1	0

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$

]



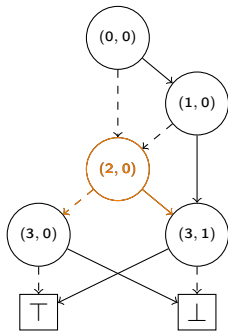
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



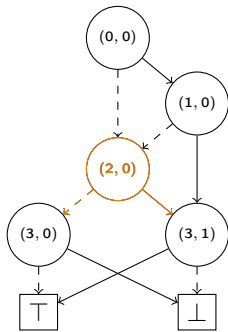
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2]



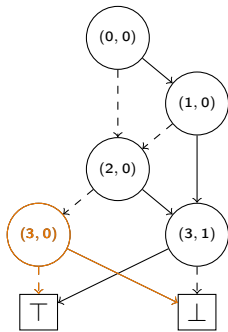
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2]



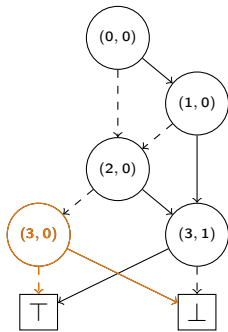
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	0

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



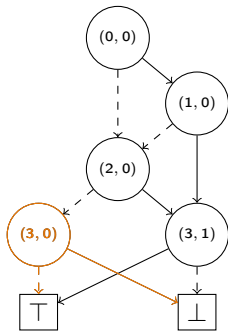
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



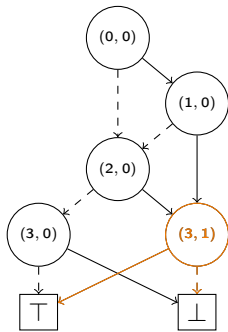
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



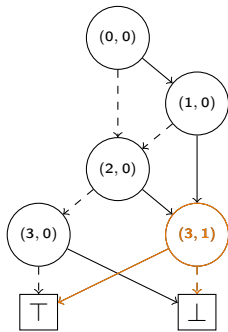
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	1	2

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$



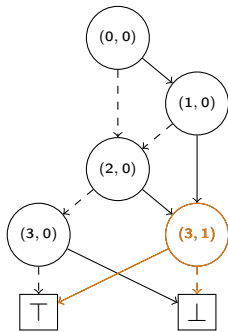
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	3	2

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek
(3, 1)

Sum
3

Result
5

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result
5

Priority Queue: Q_{count} :

[

]

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🐦 [@ssoelvsten](https://twitter.com/ssoelvsten)

Adiar

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📖 ssoelvsten.github.io/adiar