

# Adiar 1.1 : Zero-suppressed Decision Diagrams in External Memory

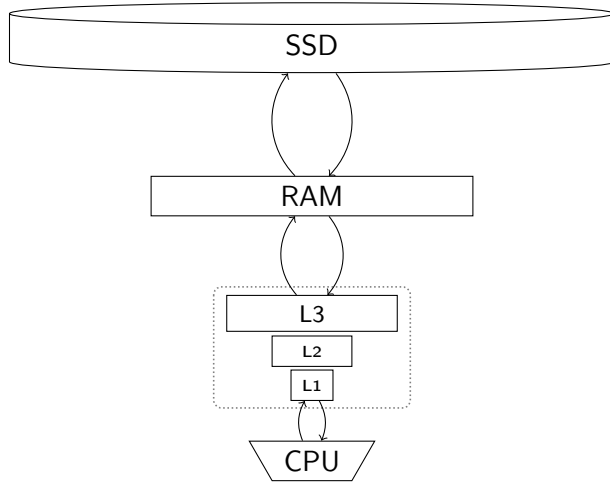
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**Steffan Christ Sølvesten** and Jaco van de Pol

NFM 2023











# Adiar

Binary Decision Diagrams  
in External Memory

`github.com/ssoelvsten/adiar`

# Adiar

**Multi-terminal** Decision Diagrams  
in External Memory

`github.com/ssoelvsten/adiar`

# Adiar

**Quantum Multi-valued** Decision Diagrams  
in External Memory

`github.com/ssoelvsten/adiar`



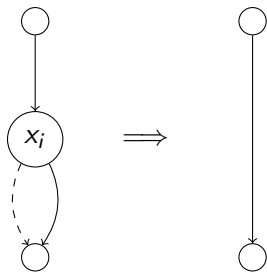
# Adiar

**Zero-suppressed** Decision Diagrams  
in External Memory

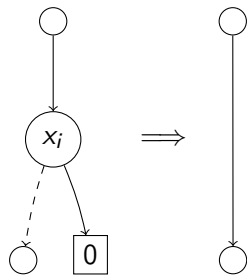
`github.com/ssoelvsten/adiar`







**BDD:**  $f : \mathbb{B}^n \rightarrow \mathbb{B}$



**ZDD:**  $A \subseteq \mathbb{B}^n$

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)
```

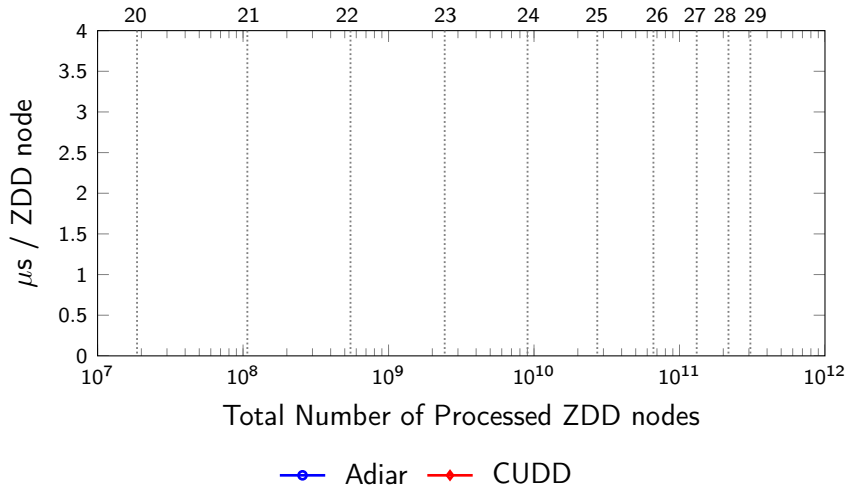
```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {  
    return prod2<bdd_policy>(f, g, o);  
}
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)  {  
    return prod2<zdd_policy>(A, B, o);  
}
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {  
    return prod2<bdd_policy>(f, g, o);  
}
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)  {  
    return prod2<zdd_policy>(A, B, o);  
}
```





Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



*Done*

BDD   ZDD

*Doable*

MTBDD

LDD

QMDD

*Done*

BDD

ZDD

(K)FDD

Tagged/Chained BDD

*Open*

Clock DD

MDD

*Doable*

MTBDD

LDD

QMDD

*Done*

BDD

ZDD

(K)FDD

Tagged/Chained BDD





# Steffan Christ Sølvesten

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🌐 [ssoelvsten.github.io](https://ssoelvsten.github.io)

## Adiar

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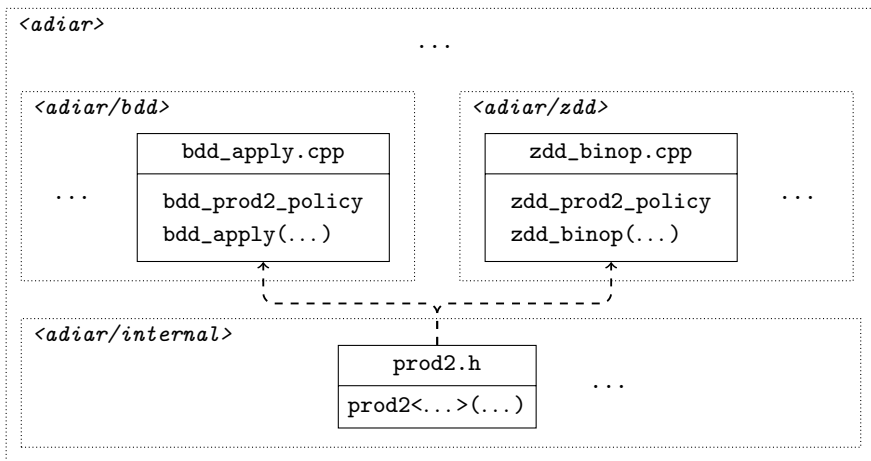
🔗 [github.com/ssoelvsten/adiar](https://github.com/ssoelvsten/adiar)

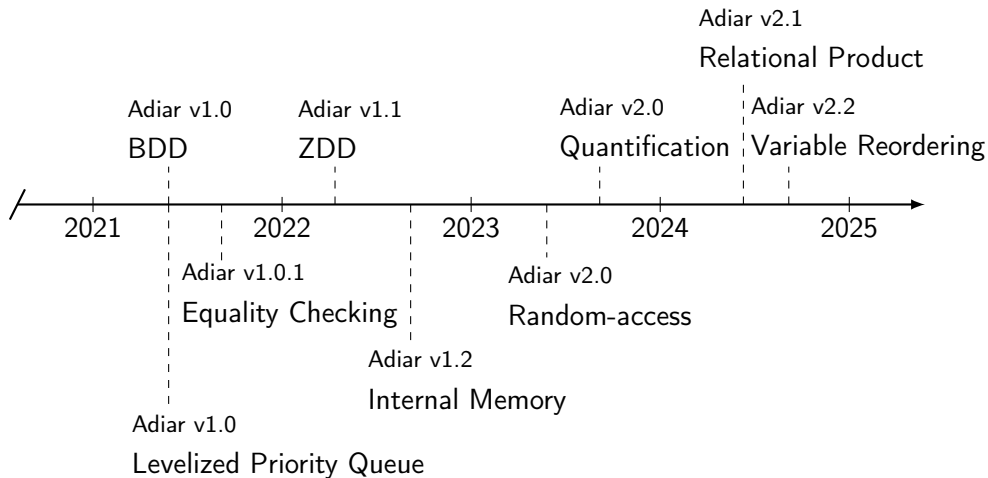
📖 [ssoelvsten.github.io/adiar](https://ssoelvsten.github.io/adiar)

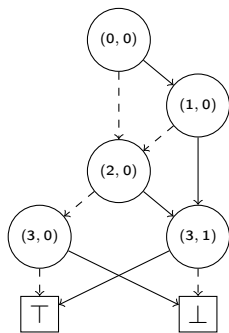
| Function                          | Operation Semantics   |
|-----------------------------------|---|
| <b>ZDD Constructors</b>           |   |
| zdd_empty()                       | $\emptyset$   |
| zdd_null()                        | $\{\emptyset\}$   |
| zdd_singleton(var)                | $\{x_{\text{var}}\}$  |
| zdd_vars(vars)                    | $\{\bigcup_{i \in \text{vars}} \{x_i\}\}$                                   |
| zdd_singletons(vars)              | $\{\{x_i\} \mid i \in \text{vars}\}$  |
| zdd_powerset(vars)                | $\mathcal{P}(\text{vars})$  |
| zdd_sized_set(vars, $k, \odot$ )  | $\{s \in \mathcal{P}(\text{vars}) \mid  s  \odot k\}$                       |
| <b>ZDD Manipulation</b>           |   |
| zdd_binop( $A, B, \otimes$ )      | $\{x \mid x \in A \otimes x \in B\}$  |
| zdd_change( $A, \text{vars}$ )    | $\{(a \setminus \text{vars}) \cup (\text{vars} \setminus a) \mid a \in A\}$ |
| zdd_complement( $A, \text{dom}$ ) | $\mathcal{P}(\text{dom}) \setminus A$                                       |
| zdd_expand( $A, \text{vars}$ )    | $\bigcup_{a \in A} \{a \cup v \mid v \in \mathcal{P}(\text{vars})\}$        |
| zdd_offset( $A, \text{vars}$ )    | $\{a \in A \mid \text{vars} \cap a = \emptyset\}$                           |
| zdd_onset( $A, \text{vars}$ )     | $\{a \in A \mid \text{vars} \subseteq a\}$                                  |
| zdd_project( $A, \text{vars}$ )   | $\bigcup_{a \in A} \{a \cap \text{vars}\}$                                  |

| Function                    | Operation Semantics                                       |
|-----------------------------|---|
| <b>Counting</b>             |   |
| zdd_size( $A$ )             | $ A $   |
| zdd_nodecount( $A$ )        | # ZDD Nodes in $A$  |
| zdd_varcount( $A$ )         | # Non-empty Levels in $A$                                 |
| <b>Predicates</b>           |   |
| zdd_equal( $A, B$ )         | $A = B$   |
| zdd_unequal( $A, B$ )       | $A \neq B$  |
| zdd_subseteq( $A, B$ )      | $A \subseteq B$   |
| zdd_disjoint( $A, B$ )      | $A \cap B = \emptyset$                                    |
| <b>Set elements</b>         |   |
| zdd_contains( $A, a$ )      | $a \in A$   |
| zdd_minelem( $A$ )          | $a \in A \text{ s.t. } \forall a' \in A. a \leq a'$       |
| zdd_maxelem( $A$ )          | $a \in A \text{ s.t. } \forall a' \in A. a' \leq a$       |
| <b>Conversion</b>           |   |
| zdd_from( $f, \text{dom}$ ) | $\{x \in \mathcal{P}(\text{dom}) \mid f(x) = \top\}$      |
| bdd_from( $A, \text{dom}$ ) | $\vec{x} : \mathcal{P}(\text{dom}) \mapsto \vec{x} \in A$ |

Operations provided by Adiar in <adiar/zdd.h>.







**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[  $((0,0) \xrightarrow{\top} (1,0), 1)$  ,  
 $((0,0) \xrightarrow{\perp} (2,0), 1)$  ,

]





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek     | Sum | Result |
|----------|-----|--------|
| $(1, 0)$ | 0   | 0      |

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek     | Sum | Result |
|----------|-----|--------|
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[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(1, 0)</b> | 1   | 0      |

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(1, 0)</b> | 1   | 0      |

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek   | Sum | Result |
|--------|-----|--------|
| (2, 0) | 0   | 0      |

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(2, 0)</b> | 0   | 0      |

Priority Queue:  $Q_{count}$ :

[

|  |   |
|--|---|
| $((0, 0) \xrightarrow{\perp} (2, 0), 1)$ | , |
| $((1, 0) \xrightarrow{\perp} (2, 0), 1)$ | , |
| $((1, 0) \xrightarrow{\top} (3, 1), 1)$  | , |

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek     | Sum | Result |
|----------|-----|--------|
| $(2, 0)$ | 1   | 0      |

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(2, 0)</b> | 2   | 0      |

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
]





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(2, 0)</b> | 2   | 0      |

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), \quad 2) \quad ,$   
 $((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$   
 $((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad ]$



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(3, 0)</b> | 0   | 0      |

Priority Queue:  $Q_{count}$ :

[

|                                       |   |   |
|---------------------------------------|---|---|
| $((2, 0) \xrightarrow{\perp} (3, 0),$ | 2 | , |
| $((1, 0) \xrightarrow{\top} (3, 1),$  | 1 | , |
| $((2, 0) \xrightarrow{\top} (3, 1),$  | 2 | ] |



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek   | Sum | Result |
|--------|-----|--------|
| (3, 0) | 0   | 0      |

Priority Queue:  $Q_{count}$ :

[

|                                       |   |   |
|---------------------------------------|---|---|
| $((2, 0) \xrightarrow{\perp} (3, 0),$ | 2 | , |
| $((1, 0) \xrightarrow{\top} (3, 1),$  | 1 | , |
| $((2, 0) \xrightarrow{\top} (3, 1),$  | 2 | ] |



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek   | Sum | Result |
|--------|-----|--------|
| (3, 0) | 2   | 0      |

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(3, 0)</b> | 2   | 2      |

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek   | Sum | Result |
|--------|-----|--------|
| (3, 1) | 0   | 2      |

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(3, 1)</b> | 0   | 2      |

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(3, 1)</b> | 1   | 2      |

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad ]$





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek   | Sum | Result |
|--------|-----|--------|
| (3, 1) | 3   | 2      |

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(3, 1)</b> | 3   | 5      |

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result  
5

Priority Queue:  $Q_{count}$ :

[

]

# Steffan Christ Sølvsten

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## Adiar

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