Predicting Memory Demands of BDD Operations using Maximum Graph Cuts
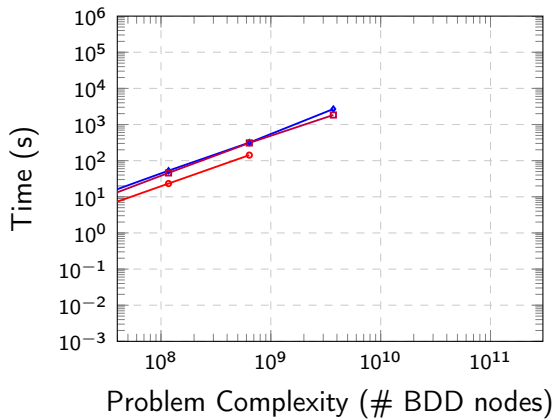
Steffan Christ Sølvsten and Jaco van de Pol
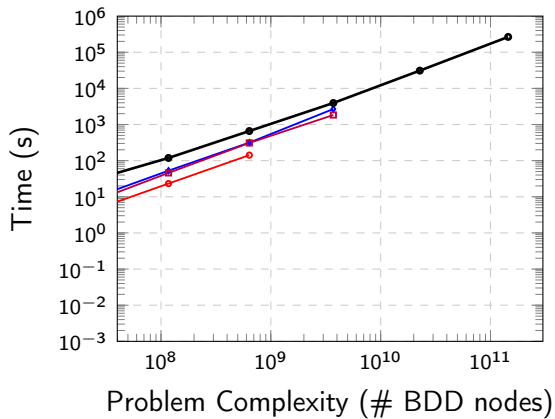
ATVA 2023

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Running Time to solve *N-Queens* problems.

1

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2

i-level cut

i-level cut

Lemma (Sølvsten, Van de Pol 2023)
The maximum i-level cut problem is in
P for $i \in \{1, 2\}$.

Theorem (Lampis, Kaouri, Mitsou 2011)
The maximum i-level cut problem is
NP-complete for $i \geq 4$.

Theorem (Sølvsten, Van de Pol 2023)
Given maximum 2-level cuts size C_f for f and C_g for g, the maximum 2-level cut for $f \odot g$ is less than or equal to $C_f \cdot C_g$.

Proof.

□

Theorem (Sølvsten, Van de Pol 2023)
Given maximum 2-level cuts size C_f for f and C_g for g, the maximum 2-level cut for $f \odot g$ is less than or equal to $C_f \cdot C_g$.
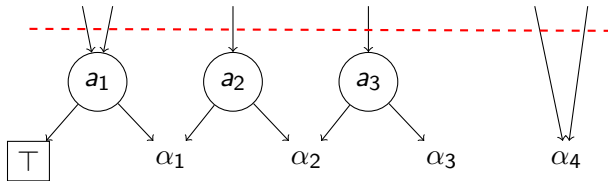
Proof.

4

Lemma (Sølvsten, Van de Pol 2023)
The maximum 2-level cut for f is at most $\frac{3}{2}$ larger than its maximum 1-level cut.
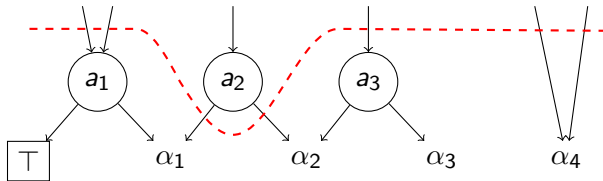
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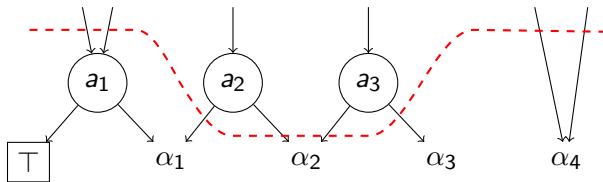
Proof.

		+⏱ Overhead	📏 Precision
1-level cut	:	1.0%	76.5%
2-level cut	:	3.3%	77.4%

Possible to process a

1.1 GiB BDD

with only

128 MiB Memory

⏱

Running Time

Adiar v1.0	:	56.5 hours

Verification of the 15 smallest EPFL circuits.

⏱

Running Time

Adiar v1.0	:	56.5 hours
Adiar v1.2	:	4.0 hours (−93%)[1]

Verification of the 15 smallest EPFL circuits.

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[1] 52.1 of these hours were saved on just verifying the sin circuit alone.
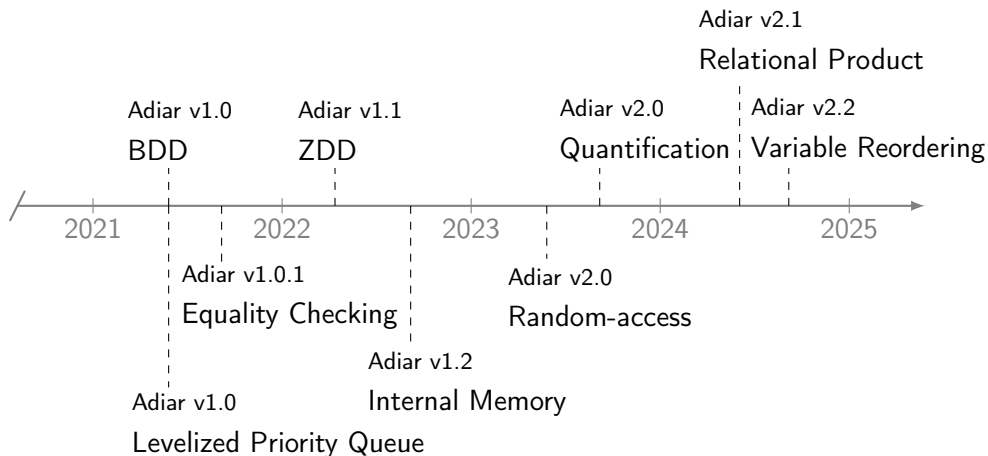
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Adiar

</> github.com/ssoelvsten/adiar

📖 ssoelvsten.github.io/adiar

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Adiar v2.1
Relational Product

Adiar v1.0   Adiar v1.1   Adiar v2.0   Adiar v2.2
BDD   ZDD   Quantification   Variable Reordering

2021   2022   2023   2024   2025

Adiar v1.0.1
Equality Checking

Adiar v2.0
Random-access

Adiar v1.2
Internal Memory

Adiar v1.0
Levelized Priority Queue

9

	🗎 Sufficient?	+⏱ Overhead	🖳 Memory[2]	🗄 Disk R/W	🔄 Transition Cost
DF ▶ Adiar (🖳 ▶ 🗄)	✘	3×	–	2×	–
DF ∥ Adiar (🖳 ∥ 🗄)	✔	–	3×	2×	–
DF → Adiar 1.0	✘ [1]	–	–	–	$\Omega(N \log N)$
State Pattern (🖳 → 🗄)	✔ [4]	∼ 20% [3]	2×	–	$\Omega(N)$
i-level cut (🖳 / 🗄)	✔ [4]	1%	–	–	–

Comparison of possible solutions.

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[1] There can be a gap between when depth-first runs out of memory and Adiar 1.0 has no overhead.
[2] Decreasing the memory dedicated to an external memory data structure impacts its performance.
[3] Runtime polymorphism adds a 20% to 30% overhead [Stroustrup].
[4] This solves the gap[1]; a *non-trivial* integration with depth-first algorithms can cover tiny cases.

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Priority Queue: Q_{count}:
[

]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

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(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$

$\quad ((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$

$]$

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:
$$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$$
$$]$$

11

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$

$\quad ((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$

$]$

11

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

[

$((0, 0) \xrightarrow{\perp} (2, 0), \quad 1$,

]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

$[$

$\quad ((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$\quad ((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$\quad ((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$

$]$

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(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

$$
\begin{aligned}
[\quad & \\
& ((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad , \\
& ((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad , \\
& \\
& ((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad , \\
&]
\end{aligned}
$$

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(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

$[$

$((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$

$]$

11

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(2,0)$	1	0

Priority Queue: Q_{count}:

$$
\begin{aligned}
[\\
((1,0) \xrightarrow{\bot} (2,0), \quad 1) \quad , \\
((1,0) \xrightarrow{\top} (3,1), \quad 1) \quad , \\
]
\end{aligned}
$$

11

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \qquad ,$$
]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

$[$

$((2, 0) \xrightarrow{\perp} (3, 0), \quad 2) \qquad ,$
$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \qquad ,$
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \qquad]$

11

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\perp} (3, 0),\quad 2)\quad ,$
$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)\quad ,$
$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)\quad]$

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

$$[$$

$$((2, 0) \xrightarrow{\perp} (3, 0), \quad 2 \quad ,$$
$$((1, 0) \xrightarrow{\top} (3, 1), \quad 1 \quad ,$$
$$((2, 0) \xrightarrow{\top} (3, 1), \quad 2 \quad]$$

11

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)\quad$,
$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)\quad$]

11

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	2	2

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2)$]

11

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$

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(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3,1)$	0	2

Priority Queue: Q_{count}:

[

$$((1,0) \xrightarrow{\top} (3,1), \quad 1) \quad ,$$
$$((2,0) \xrightarrow{\top} (3,1), \quad 2) \quad]$$
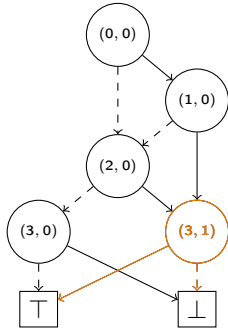
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	1	2

Priority Queue: Q_{count}:

[

$$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count}:

[

]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	5

Priority Queue: Q_{count}:

[

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(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result
5

Priority Queue: Q_{count}:
[

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Adiar

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