

# MLightDP

---

**Steffan Christ Sølvesten** and Anna Mie Hansen

2<sup>nd</sup> of January, 2020



### Definition (Differential Privacy)

Given parameters  $\epsilon \geq 0$  then a probabilistic algorithm  $M : A \mapsto B$  is  $\epsilon$ -differentially private if

$$\forall a_1, a_2 \in A \forall O \subset B :$$

$$a_1 \Psi a_2 \implies \Pr [M(a_1) \in O] \leq e^\epsilon \Pr [M(a_2) \in O] \ .$$



## Randomness Alignment

Consider  $q^{(1)}, q^{(2)} \in \mathbb{R}$ , where

$$-1 \leq q^{(1)} - q^{(2)} = \hat{q} \leq 1 .$$

## Randomness Alignment

Consider  $q^{(1)}, q^{(2)} \in \mathbb{R}$ , where

$$-1 \leq q^{(1)} - q^{(2)} = \hat{q} \leq 1 .$$

If  $\eta_1 \sim \text{Lap}(1/\epsilon)$  for  $q^{(1)}$  we'd like a  $\eta_2 \sim \text{Lap}(1/\epsilon)$  for  $q^{(2)}$  such that

$$\eta_2 = f(\eta_1) = \eta_1 + \hat{q} ,$$

because then

$$q^{(1)} + \eta_1 = q^{(1)} + \eta_1 + \hat{q} = q^{(2)} + \eta_2 ,$$

## Randomness Alignment

Consider  $q^{(1)}, q^{(2)} \in \mathbb{R}$ , where

$$-1 \leq q^{(1)} - q^{(2)} = \hat{q} \leq 1 .$$

If  $\eta_1 \sim \text{Lap}(1/\epsilon)$  for  $q^{(1)}$  we'd like a  $\eta_2 \sim \text{Lap}(1/\epsilon)$  for  $q^{(2)}$  such that

$$\eta_2 = f(\eta_1) = \eta_1 + \hat{q} ,$$

because then

$$q^{(1)} + \eta_1 = q^{(1)} + \eta_1 + \hat{q} = q^{(2)} + \eta_2 ,$$

and then

$$\Pr \left[ q^{(1)} + \text{Lap}(1/\epsilon) \in O \right] \leq e^\epsilon \Pr \left[ q^{(2)} + \text{Lap}(1/\epsilon) \in O \right] .$$

# LightDP

Based on the above, Zhang and Kifer '17 propose a type system *LightDP* with dependent types, to prove differential privacy of algorithms.

$$\tau ::= num_{\mathbb{d}} \mid num_* \mid bool_0 \mid list\ \tau \mid \tau_1 \rightarrow \tau_2$$

The number type also contains the numerical *distances* between the two executions.

# LightDP

A program is differentially private, if

---



# LightDP

A program is differentially private, if

- The returned value has distance 0

$$\frac{\Gamma \vdash e : \mathcal{B}_0 \text{ or } \Gamma \vdash e : \textit{list } \mathcal{B}_0 \text{ or } \dots}{\Gamma \vdash \textit{return } e \rightarrow \textit{return } e} \text{(T-Return)}^1$$

---

<sup>1</sup>These rules are variations of the ones by Zhang and Kifer, to remove the disconnect between the rule and their/our implementation.

# LightDP

A program is differentially private, if

- The returned value has distance 0

$$\frac{\Gamma \vdash e : \mathcal{B}_0 \text{ or } \Gamma \vdash e : \text{list } \mathcal{B}_0 \text{ or } \dots}{\Gamma \vdash \text{return } e \rightarrow \text{return } e} \text{ (T-Return)}^1$$

- The privacy budget variable  $v_\epsilon$  is less or equal to  $\epsilon$  in the *transformed* program on all return paths.

$$\frac{\Gamma(\eta) = \text{num}_{\mathsf{d}} \quad e : \text{num}_0}{\Gamma \vdash \eta := \text{Lap}(e) \rightarrow \text{havoc } \eta; \quad v_\epsilon := v_\epsilon + |\mathsf{d}|/e} \text{ (T-Laplace)}^1$$

<sup>1</sup>These rules are variations of the ones by Zhang and Kifer, to remove the disconnect between the rule and their/our implementation.



MLightDP

**Example:** `laplace_mechanism.mldp`

Remember, the randomness alignment example from before where  $-1 \leq q^{(1)} - q^{(2)} = \hat{q} \leq 1$  and adding  $\eta \sim \text{Lap}(1/\epsilon)$  to  $q$  makes them differentially private.

## Example: `laplace_mechanism.mldp`

Remember, the randomness alignment example from before where  $-1 \leq q^{(1)} - q^{(2)} = \hat{q} \leq 1$  and adding  $\eta \sim \text{Lap}(1/\epsilon)$  to  $q$  makes them differentially private.

```
1  
2      LaplaceMechanism(q: float)
```

## Example: `laplace_mechanism.mldp`

Remember, the randomness alignment example from before where  $-1 \leq q^{(1)} - q^{(2)} = \hat{q} \leq 1$  and adding  $\eta \sim \text{Lap}(1/\epsilon)$  to  $q$  makes them differentially private.

```
1
2   LaplaceMechanism(q: float)
3   {
4       let eta : float          = Lap(1 / epsilon);
```

## Example: `laplace_mechanism.mldp`

Remember, the randomness alignment example from before where  $-1 \leq q^{(1)} - q^{(2)} = \hat{q} \leq 1$  and adding  $\eta \sim \text{Lap}(1/\epsilon)$  to  $q$  makes them differentially private.

```
1
2   LaplaceMechanism(q: float)
3   {
4       let eta : float      = Lap(1 / epsilon);
5       return q + eta;
6   }
```



## Example: `laplace_mechanism.mldp`

Remember, the randomness alignment example from before where  $-1 \leq q^{(1)} - q^{(2)} = \hat{q} \leq 1$  and adding  $\eta \sim \text{Lap}(1/\epsilon)$  to  $q$  makes them differentially private.

```
1      precondition: -1.0 <= ^q /\ ^q <= 1.0
2      LaplaceMechanism(q: float[^q], ^q : float[0])
3      {
4          let eta : float          = Lap(1 / epsilon);
5          return q + eta;
6      }
```

## Example: laplace\_mechanism.mldp

Remember, the randomness alignment example from before where  $-1 \leq q^{(1)} - q^{(2)} = \hat{q} \leq 1$  and adding  $\eta \sim \text{Lap}(1/\epsilon)$  to  $q$  makes them differentially private.

```
1      precondition: -1.0 <= ^q /\ ^q <= 1.0
2      LaplaceMechanism(q: float[^q], ^q : float[0])
3      {
4          let eta : float[-^q] = Lap(1 / epsilon);
5          return q + eta;
6      }
```

## Example: `laplace_mechanism.mldp`

Remember, the randomness alignment example from before where  $-1 \leq q^{(1)} - q^{(2)} = \hat{q} \leq 1$  and adding  $\eta \sim \text{Lap}(1/\epsilon)$  to  $q$  makes them differentially private.

```
1      precondition: -1.0 <= ^q /\ ^q <= 1.0
2      LaplaceMechanism(q: float[*])
3      {
4          let eta : float[-^q] = Lap(1 / epsilon);
5          return q + eta;
6      }
```

## Example: laplace\_mechanism.dfy

The transformed *Dafny* program then is:

```
1    method LaplaceMechanism(epsilon : real, q : real, ghost q_hat0 : real)
2      returns (_, real)
3      requires epsilon > 0.0
4      requires -1.0 <= q_hat0 && q_hat0 <= 1.0
5    {
```

## Example: laplace\_mechanism.dfy

The transformed *Dafny* program then is:

```
1    method LaplaceMechanism(epsilon : real, q : real, ghost q_hat0 : real)
2      returns (_, real)
3      requires epsilon > 0.0
4      requires -1.0 <= q_hat0 && q_hat0 <= 1.0
5    {
6      ghost var v_epsilon : real := 0.0;
```

## Example: laplace\_mechanism.dfy

The transformed *Dafny* program then is:

```
1    method LaplaceMechanism(epsilon : real, q : real, ghost q_hat0 : real)
2      returns (_, real)
3      requires epsilon > 0.0
4      requires -1.0 <= q_hat0 && q_hat0 <= 1.0
5    {
6      ghost var v_epsilon : real := 0.0;
7
8      assert 0.0 == 0.0; // T-Laplace
9      assert 0.0 == 0.0; // T-OTimes
10     assert 0.0 == 0.0; // T-OTimes
```

## Example: laplace\_mechanism.dfy

The transformed *Dafny* program then is:

```
1  method LaplaceMechanism(epsilon : real, q : real, ghost q_hat0 : real)
2    returns (_, real)
3    requires epsilon > 0.0
4    requires -1.0 <= q_hat0 && q_hat0 <= 1.0
5  {
6    ghost var v_epsilon : real := 0.0;
7
8    assert 0.0 == 0.0; // T-Laplace
9    assert 0.0 == 0.0; // T-OTimes
10   assert 0.0 == 0.0; // T-OTimes
11   var eta :| 0.0 <= eta;
12   if * { eta := eta; } else { eta := -eta; }
```

## Example: laplace\_mechanism.dfy

The transformed *Dafny* program then is:

```
1  method LaplaceMechanism(epsilon : real, q : real, ghost q_hat0 : real)
2    returns (_, real)
3    requires epsilon > 0.0
4    requires -1.0 <= q_hat0 && q_hat0 <= 1.0
5  {
6    ghost var v_epsilon : real := 0.0;
7
8    assert 0.0 == 0.0; // T-Laplace
9    assert 0.0 == 0.0; // T-OTimes
10   assert 0.0 == 0.0; // T-OTimes
11   var eta :| 0.0 <= eta;
12   if * { eta := eta; } else { eta := -eta; }
13   v_epsilon := v_epsilon + Abs(-q_hat0) / ((1 as real) / epsilon);
```



## Example: laplace\_mechanism.dfy

The transformed *Dafny* program then is:

```
1  method LaplaceMechanism(epsilon : real, q : real, ghost q_hat0 : real)
2    returns (_, real)
3    requires epsilon > 0.0
4    requires -1.0 <= q_hat0 && q_hat0 <= 1.0
5  {
6    ghost var v_epsilon : real := 0.0;
7
8    assert 0.0 == 0.0; // T-Laplace
9    assert 0.0 == 0.0; // T-OTimes
10   assert 0.0 == 0.0; // T-OTimes
11   var eta :| 0.0 <= eta;
12   if * { eta := eta; } else { eta := -eta; }
13   v_epsilon := v_epsilon + Abs(-q_hat0) / ((1 as real) / epsilon);
14
15   assert q_hat0 + -q_hat0 == 0.0; // T-Return
16   assert v_epsilon <= epsilon; // T-Return
```

## Example: laplace\_mechanism.dfy

The transformed *Dafny* program then is:

```
1  method LaplaceMechanism(epsilon : real, q : real, ghost q_hat0 : real)
2    returns (_, real)
3    requires epsilon > 0.0
4    requires -1.0 <= q_hat0 && q_hat0 <= 1.0
5  {
6    ghost var v_epsilon : real := 0.0;
7
8    assert 0.0 == 0.0; // T-Laplace
9    assert 0.0 == 0.0; // T-OTimes
10   assert 0.0 == 0.0; // T-OTimes
11   var eta :| 0.0 <= eta;
12   if * { eta := eta; } else { eta := -eta; }
13   v_epsilon := v_epsilon + Abs(-q_hat0) / ((1 as real) / epsilon);
14
15   assert q_hat0 + -q_hat0 == 0.0; // T-Return
16   assert v_epsilon <= epsilon; // T-Return
17   return q + eta;
18 }
```

# Examples

Verified?	Example Program
✓	<code>laplace_mechanism.mldp</code>
✓	<code>sparse_vector.mldp</code> of Zhang and Kifer '17
✓ <sup>1</sup>	<code>numerical_sparse_vector.mldp</code> of Zhang and Kifer '17
✓ <sup>1</sup>	<code>gap_sparse_vector.mldp</code> of Wang et. al. '19
	<code>partial_sum.mldp</code> of Zhang and Kifer '17
	<code>smart_sum.mldp</code> of Zhang and Kifer '17

---

<sup>1</sup>We had to modify the given invariants to make these work.



# Comparison

<i>LightDP</i>	<i>MLightDP</i>	Feature
✓	✓	DP Type checking
	✓	Output to a Software Verification tool
	✓	Semantic Analysis of distances
		Refinement: Distance inference
	–	Refinement: Mutable Dependency tracking
	–	Free usage of the Laplace
		Automating non-linearity
	–	Other

# MLightDP

The Goal of our project is to create a less prototypical experience than their current prototype.

- 1 Create more intuitive typing annotations.
- 2 Be capable of accommodating the complete pipeline.
- 3 Dispense with assumptions of Zhang and Kifer about the input.
- 4 Make their rules of more fine-grained / less conservative.
- 5 Soundly add new operators, functions, and alternative random distributions to the language.