

Adiar 1.1 : Zero-suppressed Decision Diagrams in External Memory

Steffan Christ Sølvesten and Jaco van de Pol

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Adiar

Binary Decision Diagrams
in External Memory

`github.com/ssoelvsten/adiar`

Adiar

Multi-terminal Decision Diagrams
in External Memory

`github.com/ssoelvsten/adiar`

Adiar

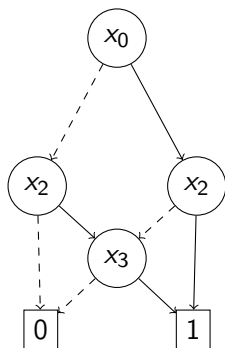
Quantum Multi-valued Decision Diagrams
in External Memory

`github.com/ssoelvsten/adiar`

Adiar

Zero-suppressed Decision Diagrams
in External Memory

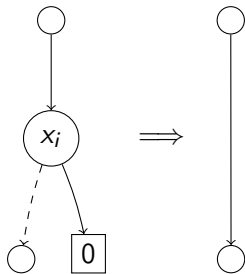
`github.com/ssoelvsten/adiar`







BDD: $f : \mathbb{B}^n \rightarrow \mathbb{B}$



ZDD: $A \subseteq \mathbb{B}^n$

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
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```

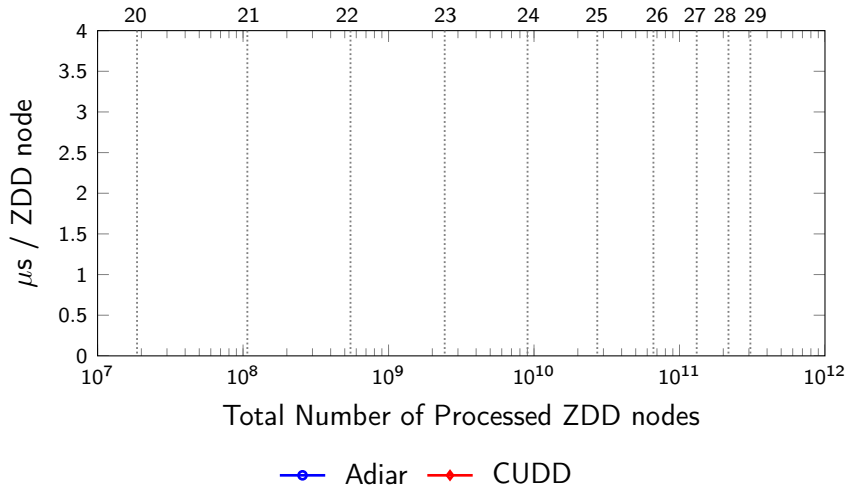
```
zdd zdd_binop(zdd A, zdd B, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {  
    return prod2<bdd_policy>(f, g, o);  
}
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)  {  
    return prod2<zdd_policy>(A, B, o);  
}
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {  
    return prod2<bdd_policy>(f, g, o);  
}
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)  {  
    return prod2<zdd_policy>(A, B, o);  
}
```

Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.

Done

BDD ZDD

Doable

MTBDD

LDD

QMDD

Done

BDD

ZDD

(K)FDD

Tagged/Chained BDD

Open

Clock DD

MDD

Doable

MTBDD

LDD

QMDD

Done

BDD

ZDD

(K)FDD

Tagged/Chained BDD

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Adiar

🔗 github.com/ssoelvsten/adiar

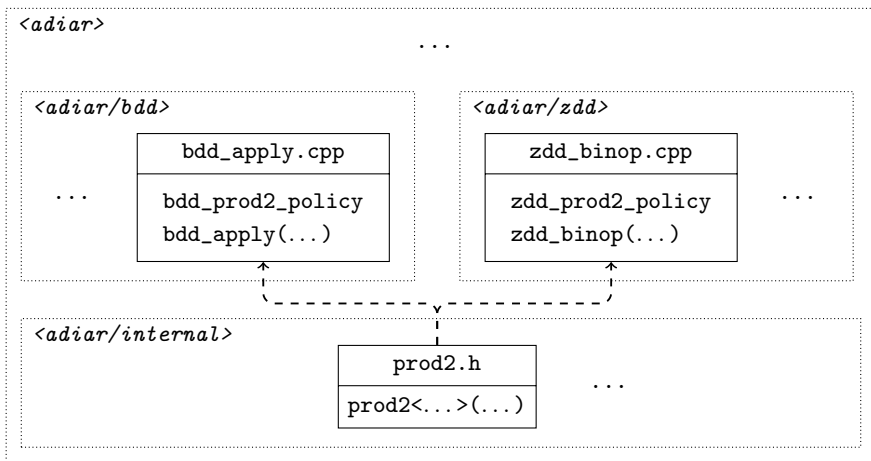
📖 ssoelvsten.github.io/adiar



Function	Operation Semantics
ZDD Constructors	
<code>zdd_empty()</code>	\emptyset
<code>zdd_null()</code>	$\{\emptyset\}$
<code>zdd_singleton(var)</code>	$\{x_{\text{var}}\}$
<code>zdd_vars(vars)</code>	$\{\bigcup_{i \in \text{vars}} \{x_i\}\}$
<code>zdd_singletons(vars)</code>	$\{\{x_i\} \mid i \in \text{vars}\}$
<code>zdd_powerset(vars)</code>	$\mathcal{P}(\text{vars})$
<code>zdd_sized_set(vars, k, \odot)</code>	$\{s \in \mathcal{P}(\text{vars}) \mid s \odot k\}$
ZDD Manipulation	
<code>zdd_binop(A, B, \otimes)</code>	$\{x \mid x \in A \otimes x \in B\}$
<code>zdd_change(A, vars)</code>	$\{(a \setminus \text{vars}) \cup (\text{vars} \setminus a) \mid a \in A\}$
<code>zdd_complement(A, dom)</code>	$\mathcal{P}(\text{dom}) \setminus A$
<code>zdd_expand(A, vars)</code>	$\bigcup_{a \in A} \{a \cup v \mid v \in \mathcal{P}(\text{vars})\}$
<code>zdd_offset(A, vars)</code>	$\{a \in A \mid \text{vars} \cap a = \emptyset\}$
<code>zdd_onset(A, vars)</code>	$\{a \in A \mid \text{vars} \subseteq a\}$
<code>zdd_project(A, vars)</code>	$\bigcup_{a \in A} \{a \cap \text{vars}\}$

Function	Operation Semantics
Counting	
<code>zdd_size(A)</code>	$ A $
<code>zdd_nodecount(A)</code>	# ZDD Nodes in A
<code>zdd_varcount(A)</code>	# Non-empty Levels in A
Predicates	
<code>zdd_equal(A, B)</code>	$A = B$
<code>zdd_unequal(A, B)</code>	$A \neq B$
<code>zdd_subseteq(A, B)</code>	$A \subseteq B$
<code>zdd_disjoint(A, B)</code>	$A \cap B = \emptyset$
Set elements	
<code>zdd_contains(A, a)</code>	$a \in A$
<code>zdd_minelem(A)</code>	$a \in A \text{ s.t. } \forall a' \in A. a \leq a'$
<code>zdd_maxelem(A)</code>	$a \in A \text{ s.t. } \forall a' \in A. a' \leq a$
Conversion	
<code>zdd_from(f, dom)</code>	$\{x \in \mathcal{P}(\text{dom}) \mid f(x) = \top\}$
<code>bdd_from(A, dom)</code>	$\vec{x} : \mathcal{P}(\text{dom}) \mapsto \vec{x} \in A$

Operations provided by Adiar in `<adiar/zdd.h>`.





(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[

]



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(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[$((0,0) \xrightarrow{\top} (1,0), 1)$,
 $((0,0) \xrightarrow{\perp} (2,0), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\top} (1, 0), 1)$,
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\top} (1, 0), 1)$,
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	1	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	0	0

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$((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
$((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
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]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	1	0

Priority Queue: Q_{count} :

[
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0), \quad 2) \quad ,$
 $((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
 $((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	0

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1),$	1)	,
$((2, 0) \xrightarrow{T} (3, 1),$	2)]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	1	2

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{T} (3, 1), \quad 2) \quad]$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	3	2

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek
(3, 1)

Sum
3

Result
5

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result
5

Priority Queue: Q_{count} :

[

]

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