

Adiar:

Binary Decision Diagrams in External Memory

Steffan Christ Sølvesten, Jaco van de Pol,
Anna Blume Jakobsen, and Mathias Weller Berg Thomasen

TACAS 2022



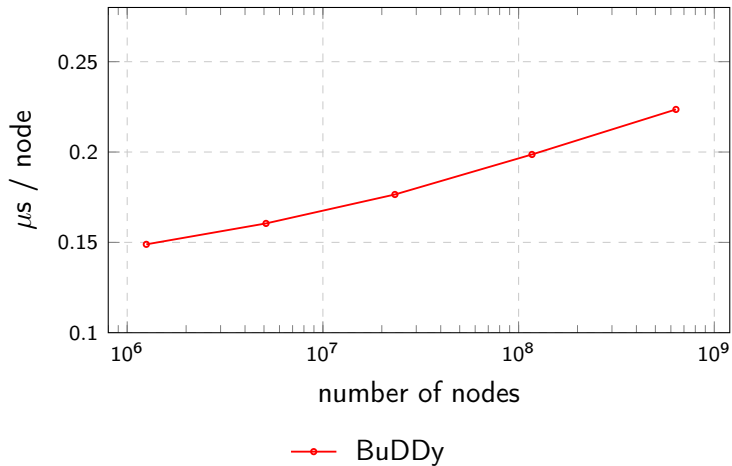


Figure 1: Minimal running time for the *Queens* problems.

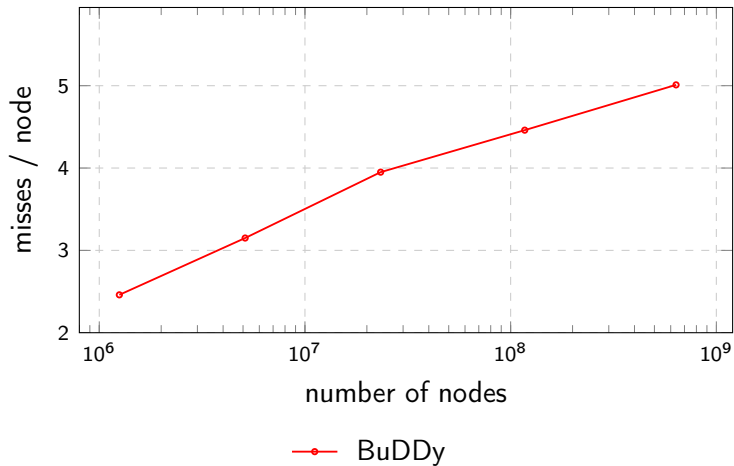
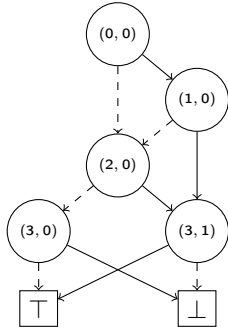


Figure 2: Cache-misses for the *Queens* problems.



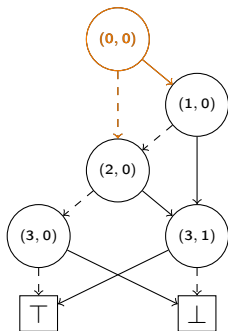
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[

]

Figure 3: In-order traversal of BDD



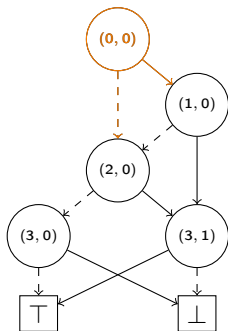
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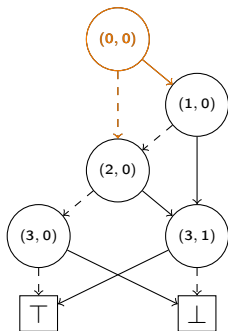


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

| | | |
|---|--|---|
| [| $((0, 0) \xrightarrow{T} (1, 0), \quad 1)$ | , |
| | $((0, 0) \xrightarrow{F} (2, 0), \quad 1)$ | , |
|] | | |

Figure 3: In-order traversal of BDD



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

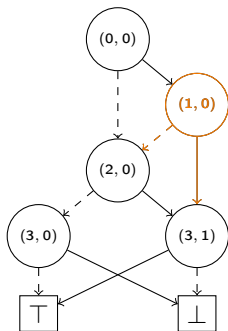
| Seek | Sum | Result |
|--------|-----|--------|
| (1, 0) | 0 | 0 |

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{T} (1, 0), 1)$,
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]

Figure 3: In-order traversal of BDD



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

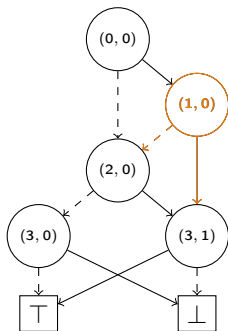
| Seek | Sum | Result |
|--------|-----|--------|
| (1, 0) | 0 | 0 |

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{T} (1, 0), 1)$,
 $((0, 0) \xrightarrow{F} (2, 0), 1)$,

]

Figure 3: In-order traversal of BDD



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

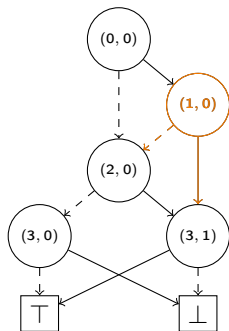
| Seek | Sum | Result |
|--------|-----|--------|
| (1, 0) | 1 | 0 |

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]

Figure 3: In-order traversal of BDD



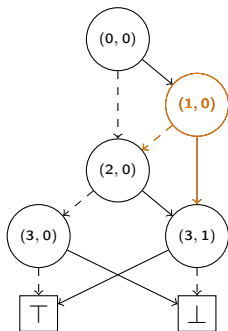
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|--------|-----|--------|
| (1, 0) | 1 | 0 |

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]

Figure 3: In-order traversal of BDD



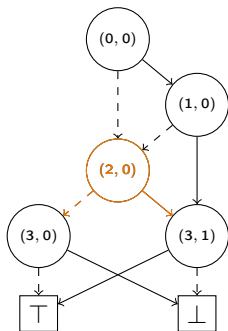
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|--------|-----|--------|
| (2, 0) | 0 | 0 |

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]

Figure 3: In-order traversal of BDD



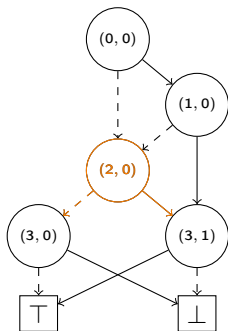
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Figure 3: In-order traversal of BDD

| Seek | Sum | Result |
|--------|-----|--------|
| (2, 0) | 0 | 0 |

Priority Queue: Q_{count} :

| | |
|--|---|
| [| |
| $((0, 0) \xrightarrow{\perp} (2, 0), 1)$ | , |
| $((1, 0) \xrightarrow{\perp} (2, 0), 1)$ | , |
| $((1, 0) \xrightarrow{\top} (3, 1), 1)$ | , |
|] | |



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|--------|-----|--------|
| (2, 0) | 1 | 0 |

Priority Queue: Q_{count} :

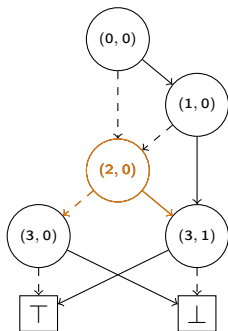
[

$((1, 0) \xrightarrow{\perp} (2, 0), 1)$,

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,

]

Figure 3: In-order traversal of BDD



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

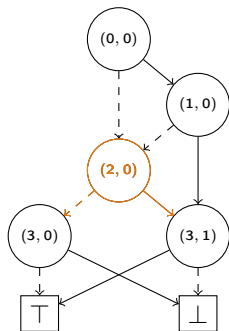
Figure 3: In-order traversal of BDD

| Seek | Sum | Result |
|--------|-----|--------|
| (2, 0) | 2 | 0 |

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

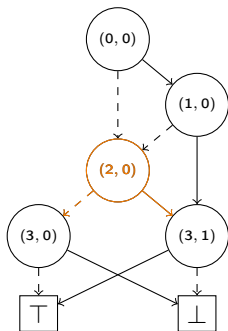
| Seek | Sum | Result |
|--------|-----|--------|
| (2, 0) | 2 | 0 |

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]

Figure 3: In-order traversal of BDD



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

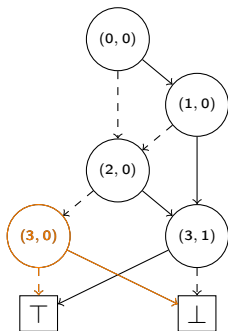
| Seek | Sum | Result |
|--------|-----|--------|
| (3, 0) | 0 | 0 |

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{1} (3, 0), 2)$,
 $((1, 0) \xrightarrow{1} (3, 1), 1)$,
 $((2, 0) \xrightarrow{0} (3, 1), 2)$]

Figure 3: In-order traversal of BDD



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

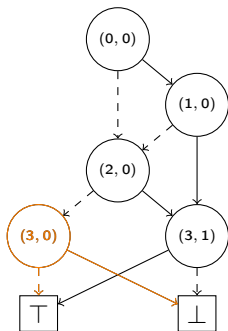
| Seek | Sum | Result |
|--------|-----|--------|
| (3, 0) | 0 | 0 |

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]

Figure 3: In-order traversal of BDD



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

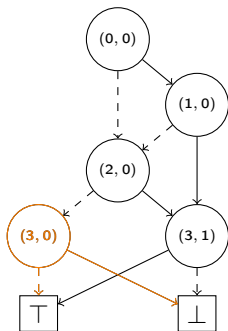
Figure 3: In-order traversal of BDD

| Seek | Sum | Result |
|--------|-----|--------|
| (3, 0) | 2 | 0 |

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

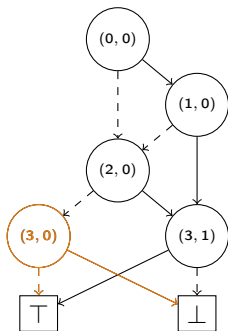
| Seek | Sum | Result |
|--------|-----|--------|
| (3, 0) | 2 | 2 |

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]

Figure 3: In-order traversal of BDD



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

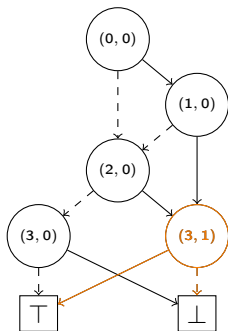
Figure 3: In-order traversal of BDD

| Seek | Sum | Result |
|--------|-----|--------|
| (3, 1) | 0 | 2 |

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

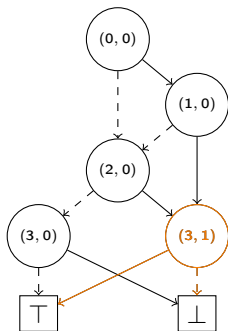
| Seek | Sum | Result |
|---------------|-----|--------|
| (3, 1) | 0 | 2 |

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]

Figure 3: In-order traversal of BDD



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

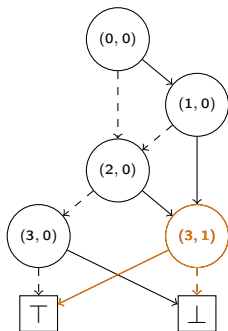
| Seek | Sum | Result |
|---------------|-----|--------|
| (3, 1) | 1 | 2 |

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$

Figure 3: In-order traversal of BDD



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

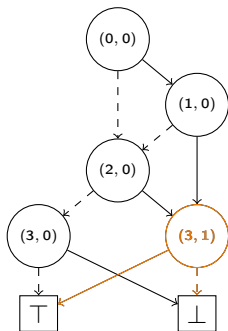
| Seek | Sum | Result |
|--------|-----|--------|
| (3, 1) | 3 | 2 |

Priority Queue: Q_{count} :

[

]

Figure 3: In-order traversal of BDD



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|--------|-----|--------|
| (3, 1) | 3 | 5 |

Priority Queue: Q_{count} :

[

]

Figure 3: In-order traversal of BDD

Adiar

github.com/ssoelvsten/adiar

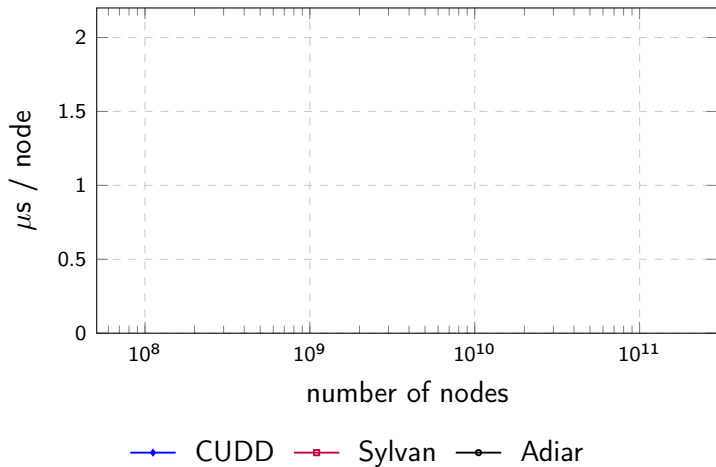


Figure 4: Minimal running time for the *Queens* problems.

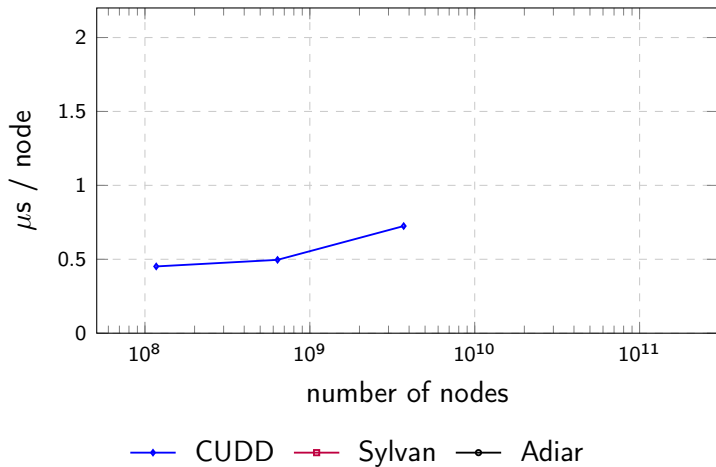


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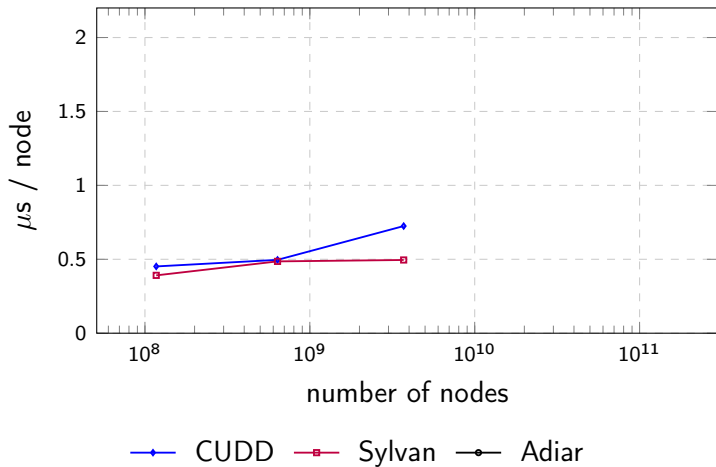


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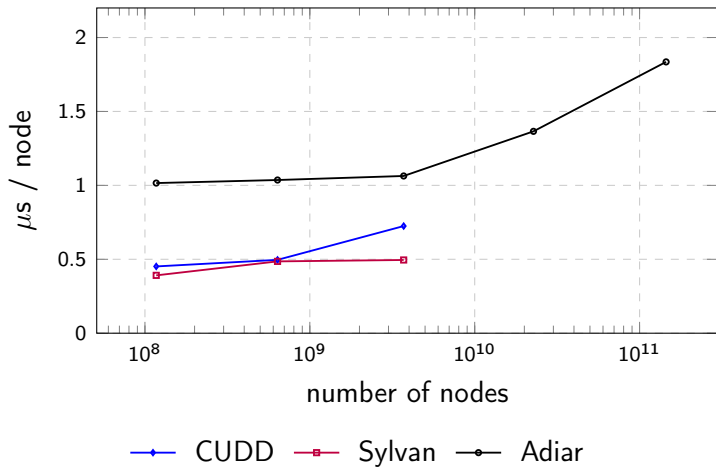


Figure 4: Minimal running time for the *Queens* problems.

Steffan Christ Sølvsten

✉ soelvsten@cs.au.dk

🐦 [@ssoelvsten](https://twitter.com/ssoelvsten)

Adiar

🔗 github.com/ssoelvsten/adiar

📄 ssoelvsten.github.io/adiar

