An External Memory Relational Product

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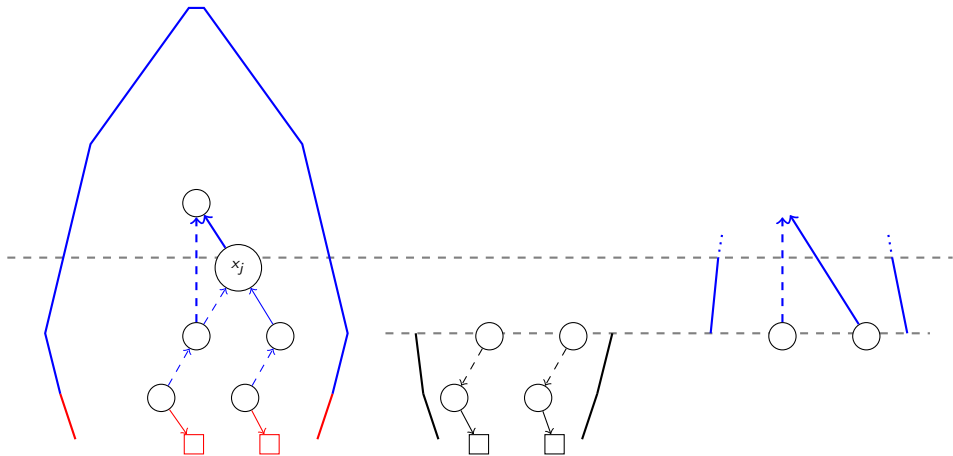
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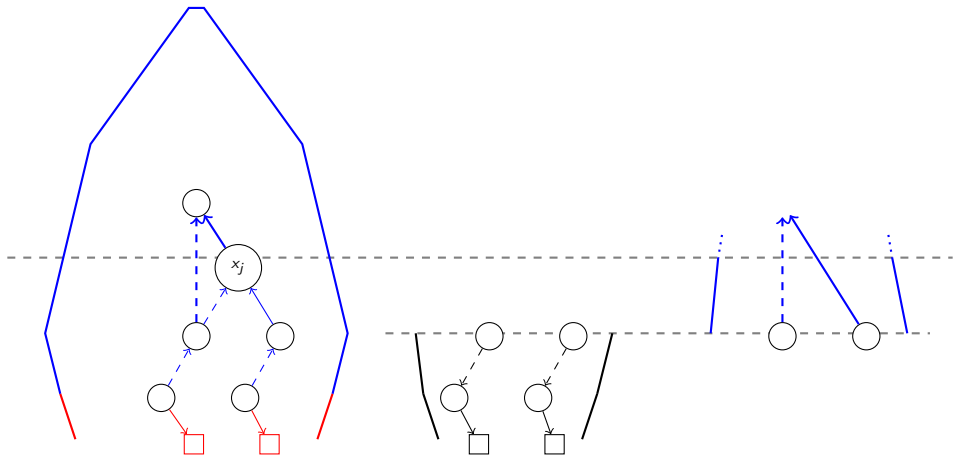$\exists \vec{x}. \phi(\dots)$ **(Bounce)**

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If π is monotonic

- *1-Var / Push*:
  Apply π in $O(L_N)$ extra time during the final bottom-up Reduce sweep.
- *Bounce*:
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If π is not monotonic

  to be continued...