Adiar 1.1 : Zero-suppressed Decision Diagrams in External Memory

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Steffan Christ Sølvsten and Jaco van de Pol

18th of May, 2023

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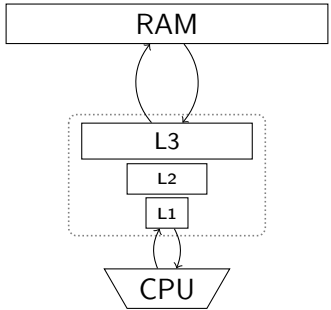
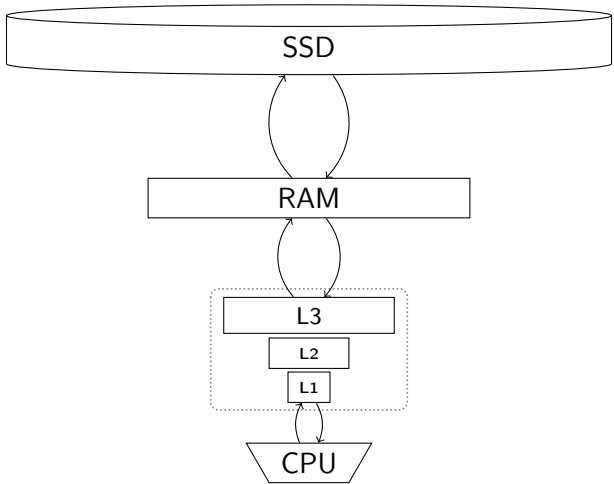
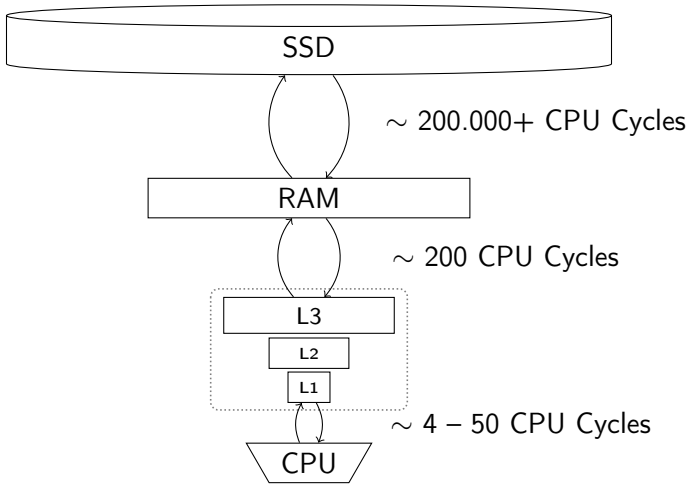
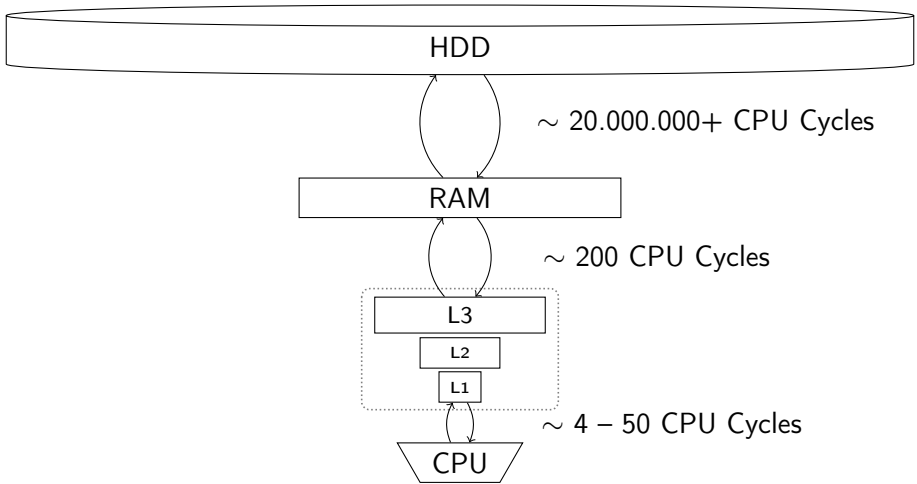

RAM
L3
L2
L1
CPU


SSD
RAM
L3
L2
L1
CPU


SSD
~ 200.000+ CPU Cycles
RAM
~ 200 CPU Cycles
L3
L2
L1
~ 4 – 50 CPU Cycles
CPU

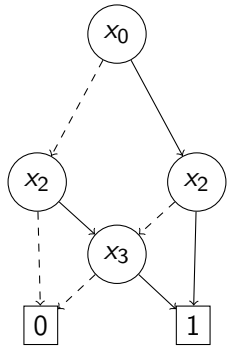

HDD
~ 20.000.000+ CPU Cycles
RAM
~ 200 CPU Cycles
L3
L2
L1
~ 4 – 50 CPU Cycles
CPU

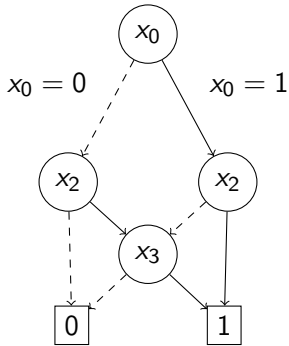
Binary
Decision Diagrams

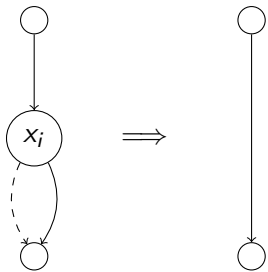
Multi-terminal
Decision Diagrams

Quantum Multi-valued Decision Diagrams

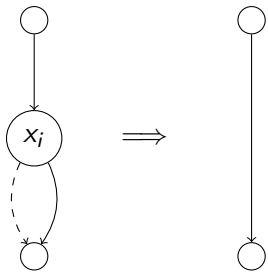
Zero-suppressed
Decision Diagrams

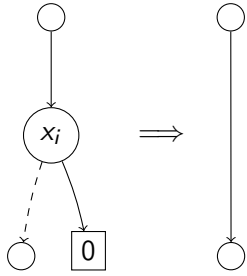

x_0
x_2
x_2
x_3
0
1


x_0
$x_0 = 0$
$x_0 = 1$
x_2
x_2
x_3
0
1



BDD: $f : \mathbb{B}^n \to \mathbb{B}$





BDD: $f : \mathbb{B}^n \to \mathbb{B}$

ZDD: $A \subseteq \mathbb{B}^n$

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

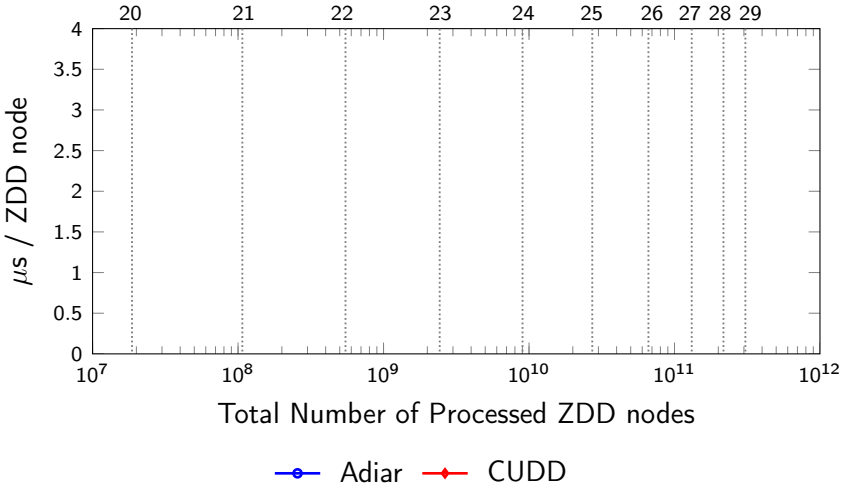
```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)
```

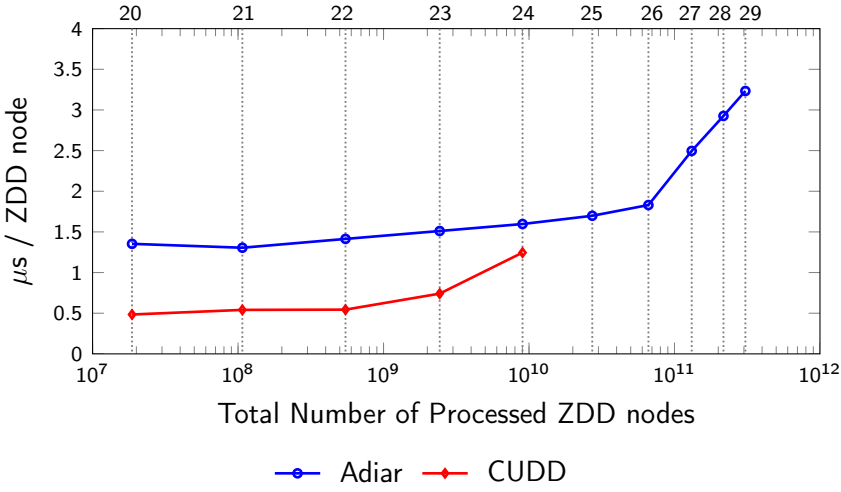
```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {
    return prod2(f, g, o, bdd_strategy)
}

zdd zdd_binop(zdd A, zdd B, bool_op o)  {
    return prod2(A, B, o, zdd_strategy)
}
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {
    return prod2<bdd_policy>(f, g, o)
}

zdd zdd_binop(zdd A, zdd B, bool_op o)  {
    return prod2<zdd_policy>(A, B, o)
}
```

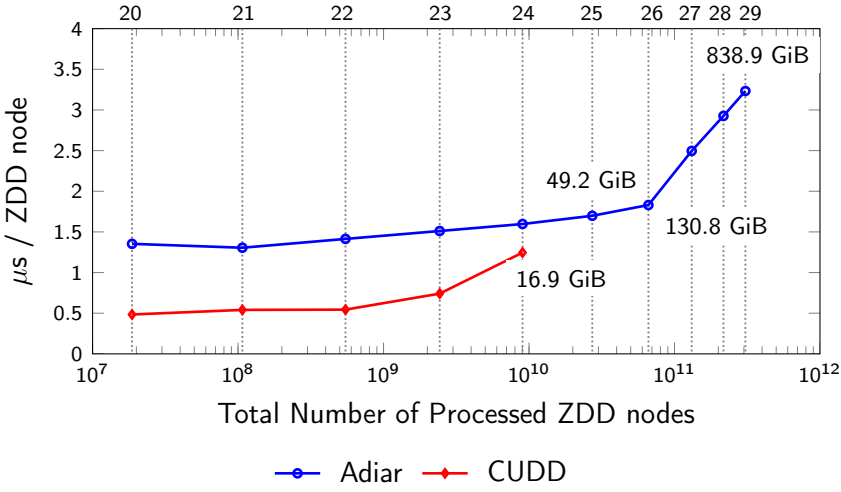
```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {
    return prod2<bdd_policy>(f, g, o)
}

zdd zdd_binop(zdd A, zdd B, bool_op o)  {
    return prod2<zdd_policy>(A, B, o)
}
```



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



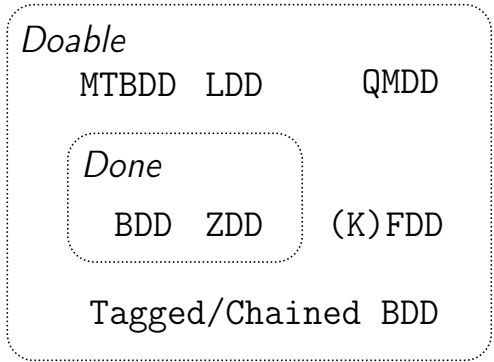
Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.

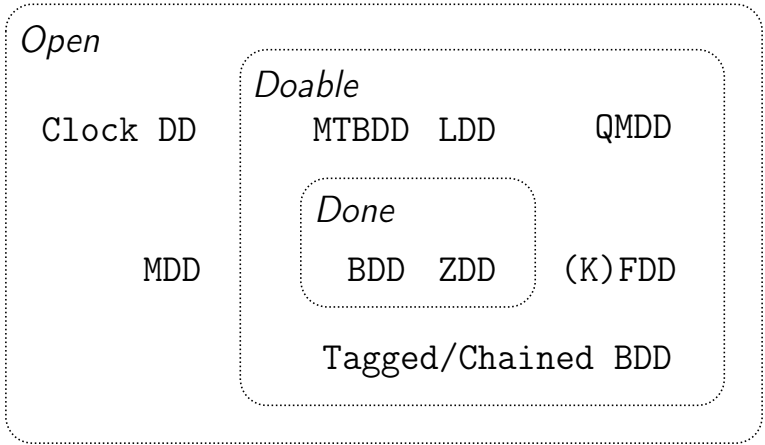


Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.

Done

BDD ZDD


Doable
MTBDD
LDD
QMDD
Done
BDD
ZDD
(K)FDD
Tagged/Chained BDD


Open
Clock DD
MDD
Doable
MTBDD LDD
QMDD
Done
BDD ZDD
(K)FDD
Tagged/Chained BDD

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Adiar

github.com/ssoelvsten/adiar

ssoelvsten.github.io/adiar

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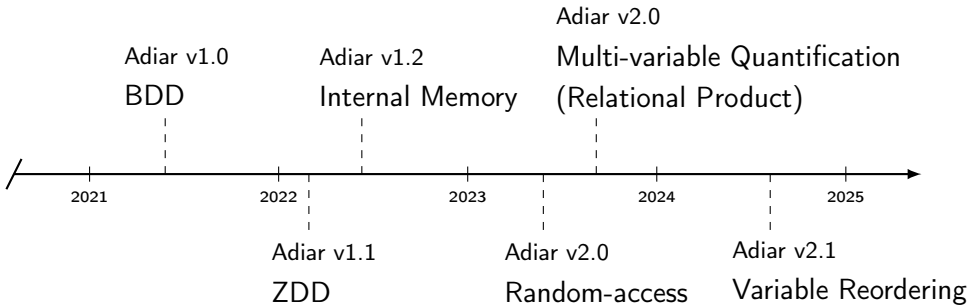
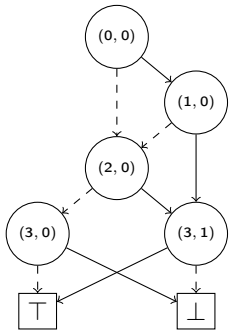
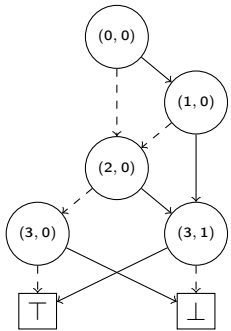

Adiar v2.0
Adiar v1.0
Adiar v1.2
Multi-variable Quantification
BDD
Internal Memory
(Relational Product)
2021
2022
2023
2024
2025
Adiar v1.1
Adiar v2.0
Adiar v2.1
ZDD
Random-access
Variable Reordering



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

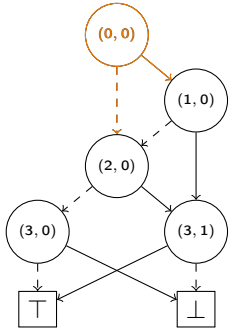


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]

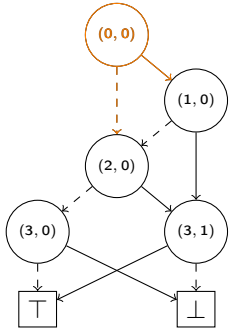


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

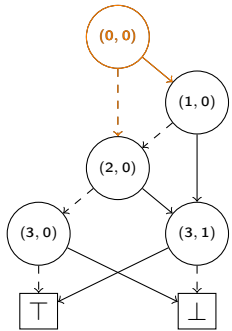
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

$$\left[\begin{array}{ll} ((0,0) \xrightarrow{\top} (1,0), & 1) \\ ((0,0) \xrightarrow{\bot} (2,0), & 1) \end{array} \right]$$



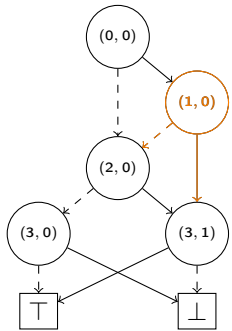
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0),\quad 1)\quad ,$$
$$\quad\ ((0,0) \xrightarrow{\bot} (2,0),\quad 1)\quad ,$$

]



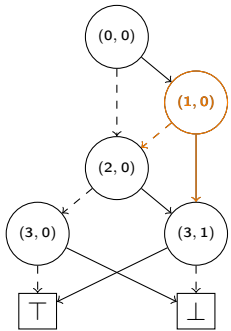
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1,0)$	0	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0),\quad 1)\quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0),\quad 1)\quad ,$$

$]$



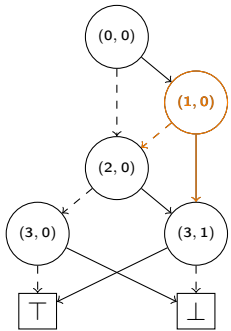
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\perp} (2,0), \quad 1) \quad ,$$

$$]$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

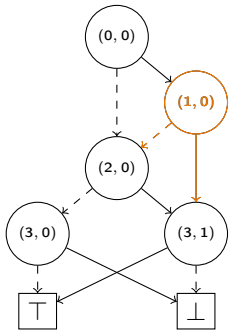
[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]

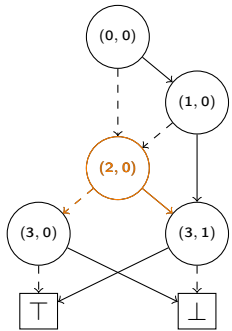


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

$$
\begin{bmatrix}
((0,0) \xrightarrow{\perp} (2,0), & 1) & , \\
((1,0) \xrightarrow{\perp} (2,0), & 1) & , \\
 & & \\
((1,0) \xrightarrow{\top} (3,1), & 1) & , \\
 & &]
\end{bmatrix}
$$

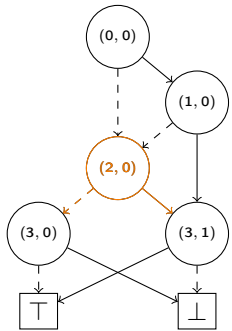


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

$$
\begin{bmatrix}
((0,0) \xrightarrow{\perp} (2,0), & 1) & , \\
((1,0) \xrightarrow{\perp} (2,0), & 1) & , \\
 & & \\
((1,0) \xrightarrow{\top} (3,1), & 1) & , \\
 & &]
\end{bmatrix}
$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2,0)$	1	0

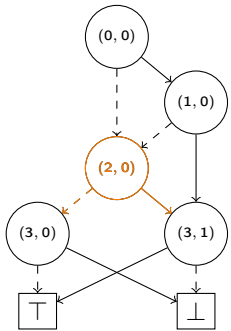
Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

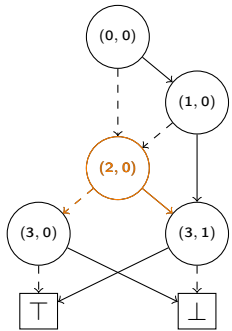
Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,

]



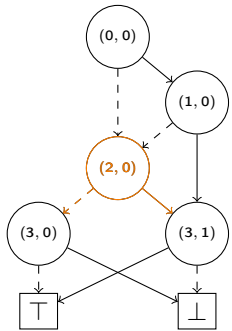
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



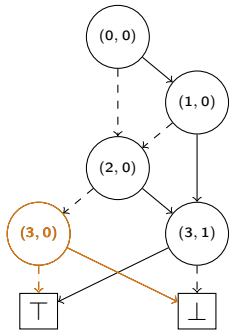
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



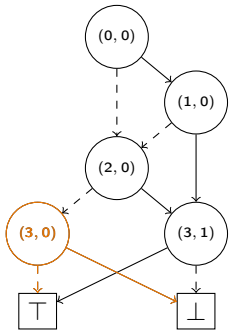
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3,0)$	0	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

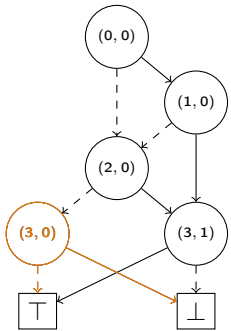
Seek	Sum	Result
$(3, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

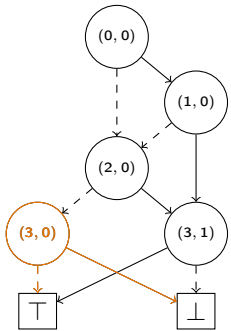
Seek	Sum	Result
$(3,0)$	2	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

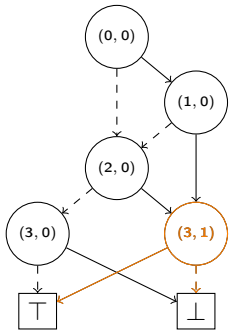
Seek	Sum	Result
$(3,1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

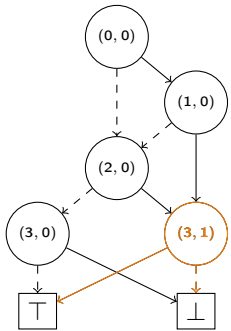
Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



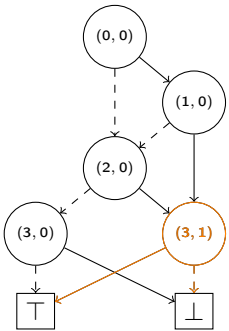
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	1	2

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2)$]



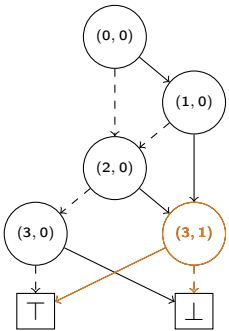
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count}:

[

]



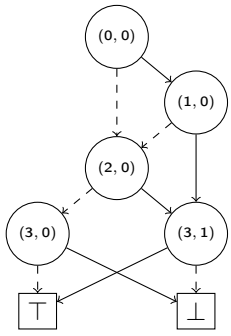
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	5

Priority Queue: Q_{count}:

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result

5

Priority Queue: Q_{count}:

[

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Adiar

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