

# An External Memory Relational Product

---

**Steffan Christ Sølvesten**, Jaco van de Pol

September 12, 2023



$$RelProd(S, T) \equiv ( \exists \vec{x}. S(\vec{x}) \wedge T(\vec{x}, \vec{x}') ) [\vec{x}' / \vec{x}]$$

$$RelProd(S, T) \equiv ( \exists \vec{x}. S(\vec{x}) \wedge T(\vec{x}, \vec{x}') )[\vec{x}'/\vec{x}]$$

$$\exists x_i. \phi \equiv \phi[x_i/\top] \vee \phi[x_i/\perp]$$



$$\exists x_i. \phi \equiv \phi[x_i/\top] \vee \phi[x_i/\perp]$$



$$\exists x_i. \phi \equiv \phi[x_i/\top] \vee \phi[x_i/\perp]$$



$$\exists x_i. \phi \equiv \phi[x_i/\top] \vee \phi[x_i/\perp]$$



$$\exists x_i. \phi \equiv \phi[x_i/\top] \vee \phi[x_i/\perp]$$





$$\exists x_i. \phi \equiv \phi[x_i/\top] \vee \phi[x_i/\perp]$$



$$\exists x_i. \phi \equiv \phi[x_i/\top] \vee \phi[x_i/\perp]$$



$$\exists x_i. \phi \equiv \phi[x_i/\top] \vee \phi[x_i/\perp]$$





$\exists \vec{x}. \phi(\dots)$     **(Push)**



$\exists \vec{x}. \phi(\dots)$     **(Push)**



$\exists \vec{x}. \phi(\dots)$  (Push)



$\exists \vec{x}. \phi(\dots)$  (Push)





$\exists \vec{x}. \phi(\dots)$  (Push)



$\exists \vec{x}. \phi(\dots)$  (Push)



$\exists \vec{x}. \phi(\dots)$  (Push)



$\exists \vec{x}. \phi(\dots)$  (Push)



$\exists \vec{x}. \phi(\dots)$  (Push)





$\exists \vec{x}. \phi(\dots)$     **(Bounce)**

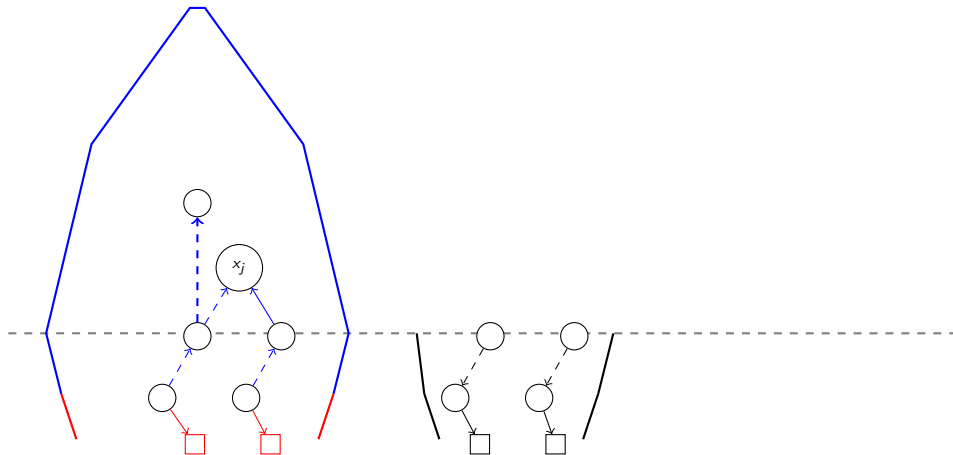


$\exists \vec{x}. \phi(\dots)$     **(Bounce)**





$\exists \vec{x}. \phi(\dots)$     **(Bounce)**



$\exists \vec{x}. \phi(\dots)$     **(Bounce)**



$\exists \vec{x}. \phi(\dots)$     **(Bounce)**



$\exists \vec{x}. \phi(\dots)$     **(Bounce)**



$\exists \vec{x}. \phi(\dots)$     **(Bounce)**



$\exists \vec{x}. \phi(\dots)$     **(Bounce)**



$\exists \vec{x}. \phi(\dots)$     **(Bounce)**



$\exists \vec{x}. \phi(\dots)$     **(Bounce)**



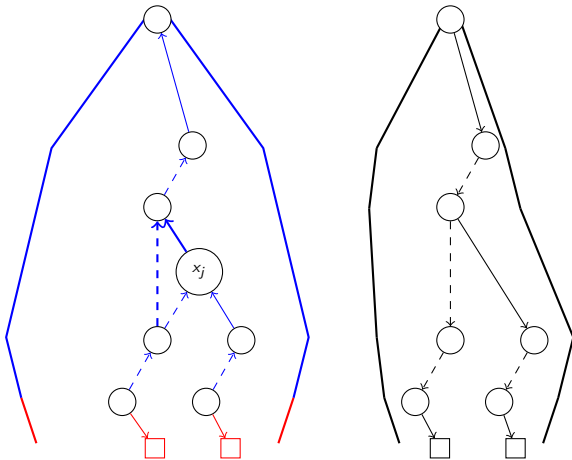


$\exists \vec{x}. \phi(\dots)$     **(Bounce)**



$\exists \vec{x}. \phi(\dots)$     **(Bounce)**



$$\exists \vec{x}. \phi(\dots) \quad (\text{Bounce})$$




$$RelProd(S, T) \equiv ( \exists \vec{x}. S(\vec{x}) \wedge T(\vec{x}, \vec{x}') ) [\vec{x}' / \vec{x}]$$

**Definition**

A relabelling  $\pi$  is monotonic if  $x_i < x_j \implies \pi(x_i) < \pi(x_j)$

## Definition

A relabelling  $\pi$  is monotonic if  $x_i < x_j \implies \pi(x_i) < \pi(x_j)$

If  $\pi$  is monotonic

- *1-Var / Push:*

Apply  $\pi$  in  $O(L_N)$  extra time during the final bottom-up Reduce sweep.

- *Bounce:*

$x'_i < x'_j \implies x_i < x_j$ :  $\pi$  can be applied during the outermost Reduce sweep.

That is, applying  $\pi$  can (essentially) be done for free.

## Definition

A relabelling  $\pi$  is monotonic if  $x_i < x_j \implies \pi(x_i) < \pi(x_j)$

### If $\pi$ is monotonic

- *1-Var / Push:*

Apply  $\pi$  in  $O(L_N)$  extra time during the final bottom-up Reduce sweep.

- *Bounce:*

$x'_i < x'_j \implies x_i < x_j$ :  $\pi$  can be applied during the outermost Reduce sweep.

That is, applying  $\pi$  can (essentially) be done for free.

### If $\pi$ is not monotonic

to be continued...