

# An External Memory Relational Product

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$\exists \vec{x}. \phi(\dots)$     **(Bounce)**

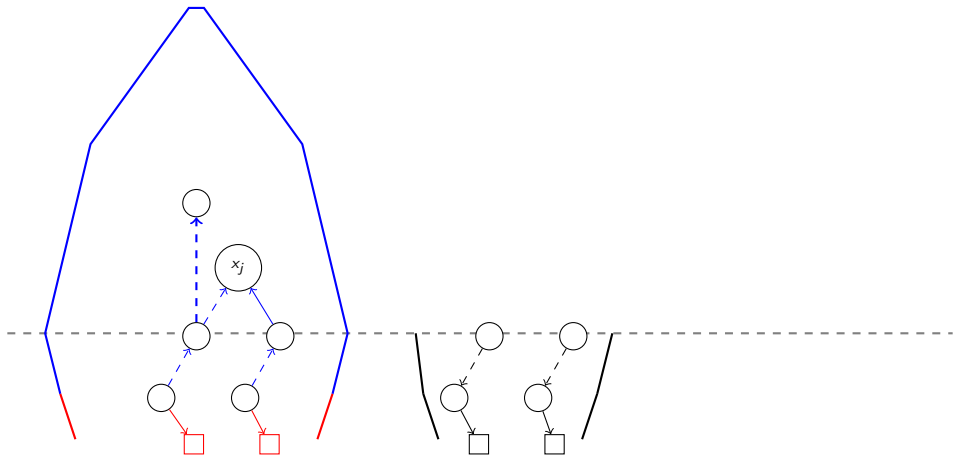


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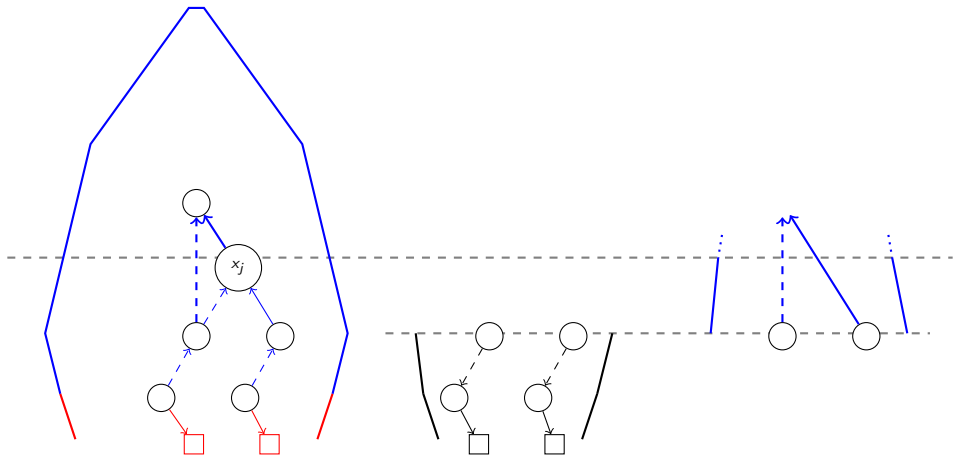


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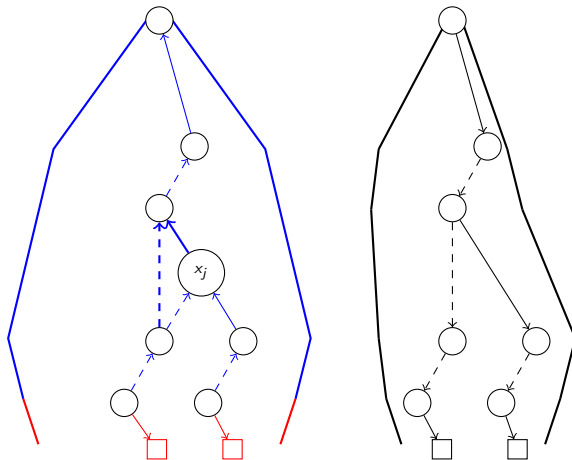
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If  $\pi$  is monotonic

- *1-Var / Push:*

Apply  $\pi$  in  $O(L_N)$  extra time during the final bottom-up Reduce sweep.

- *Bounce:*

$x'_i < x'_j \implies x_i < x_j$ :  $\pi$  can be applied during the outermost Reduce sweep.

That is, applying  $\pi$  can (essentially) be done for free.

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### If $\pi$ is not monotonic

to be continued...