Predicting Memory Demands of BDD Operations using Maximum Graph Cuts

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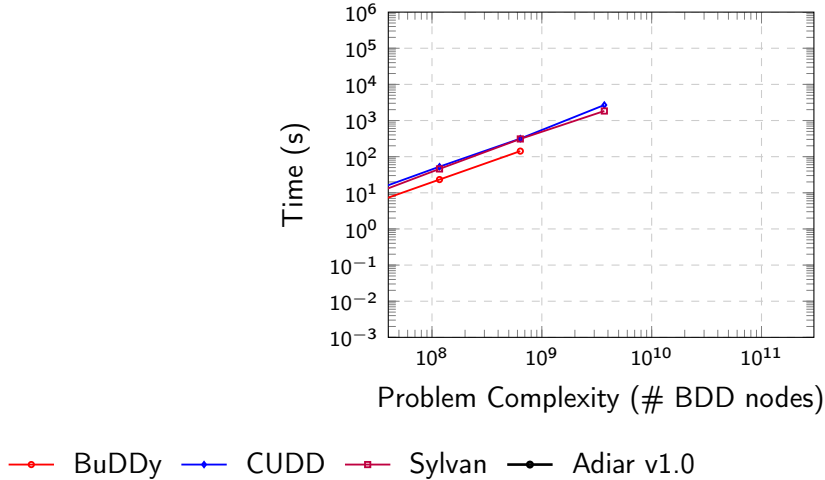
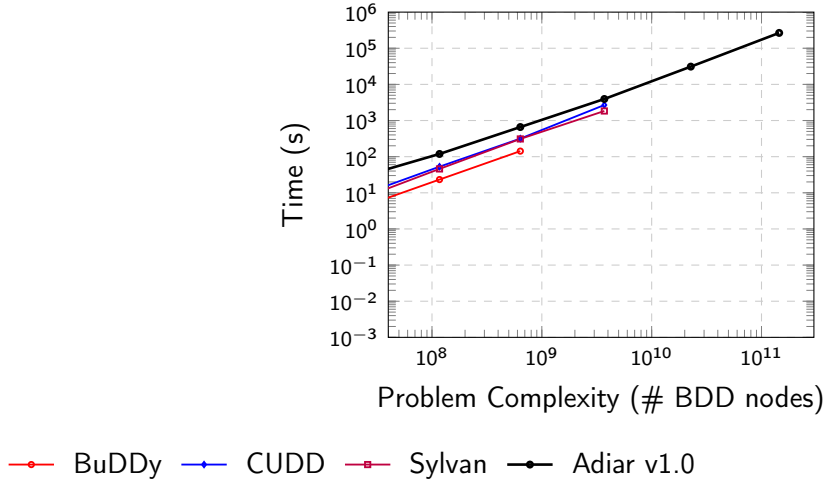
Steffan Christ Sølvsten and Jaco van de Pol

ATVA 2023



Running Time to solve *N-Queens* problems.



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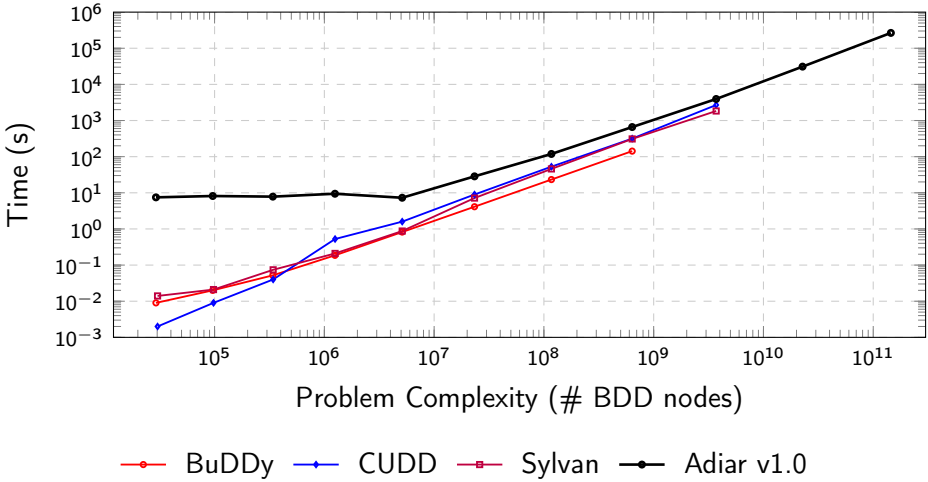

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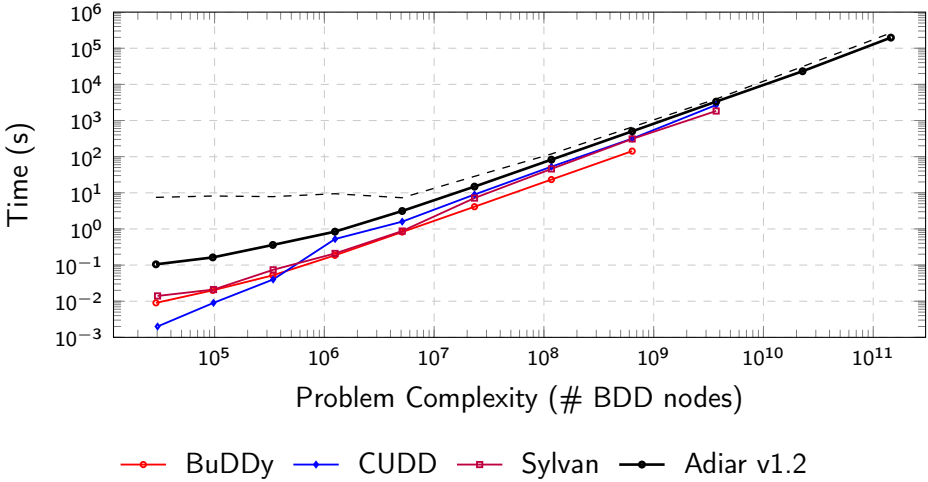

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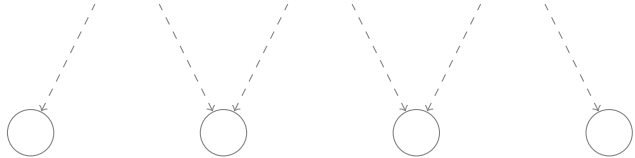
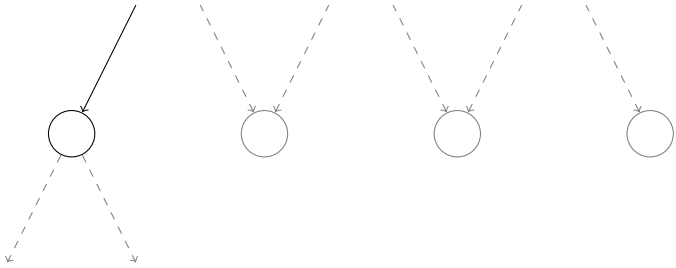
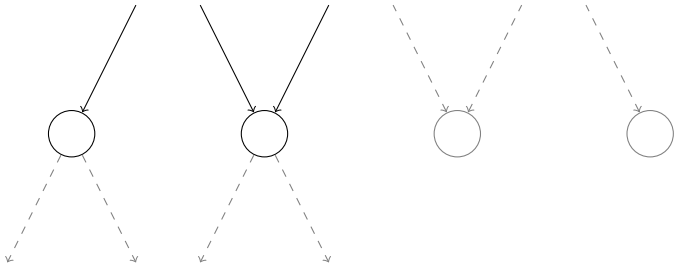
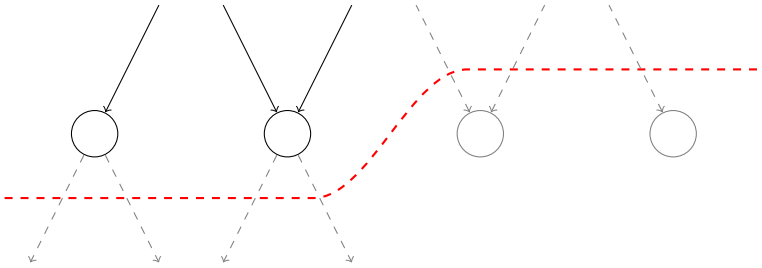
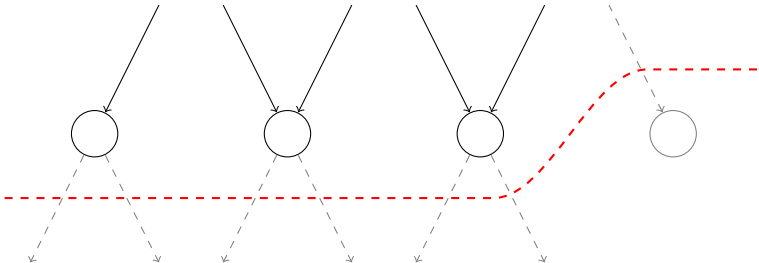
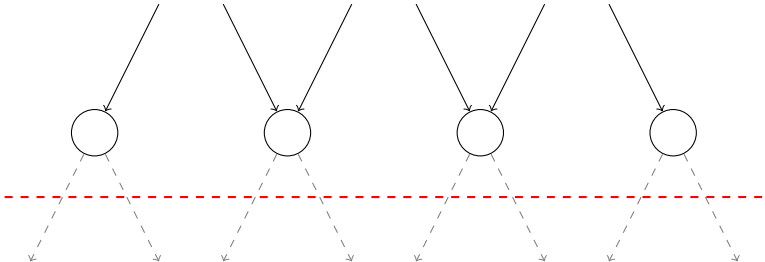




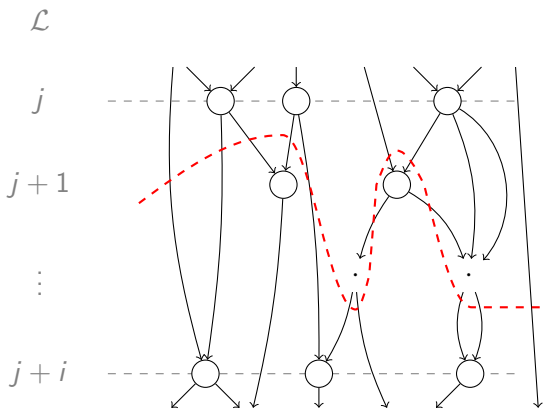








i-level cut



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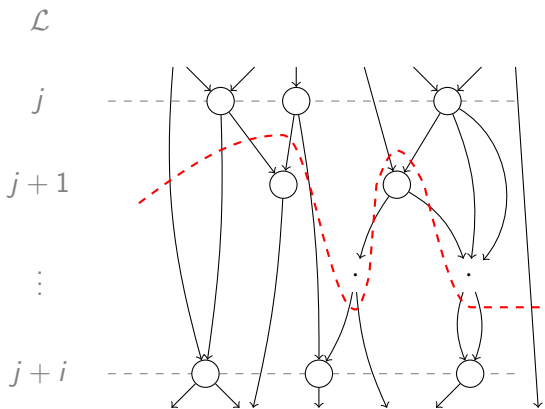
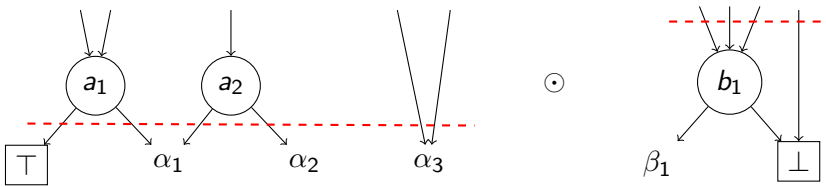


Lemma (**Sølvsten, Van de Pol 2023**)
The maximum i-level cut problem is in P for $i \in \{1, 2\}$.

Theorem (**Lampis, Kaouri, Mitsou 2011**)
The maximum i-level cut problem is NP-complete for $i \geq 4$.

Theorem (Sølvsten, Van de Pol 2023)
Given maximum 2-level cuts size C_f for f and C_g for g, the maximum 2-level cut for $f \odot g$ is less than or equal to $C_f \cdot C_g$.

Proof.



□

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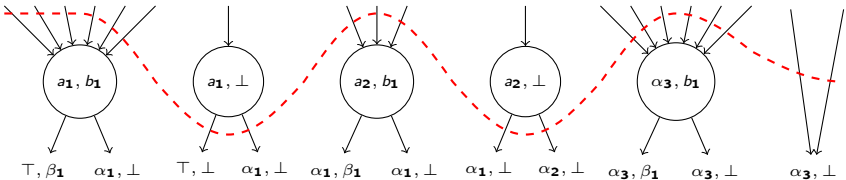
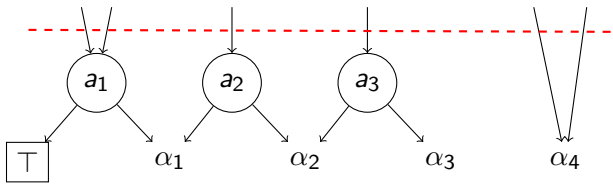


□

Lemma (Sølvsten, Van de Pol 2023)
The maximum 2-level cut for f is at most $\frac{3}{2}$ larger than its maximum 1-level cut.

Proof.



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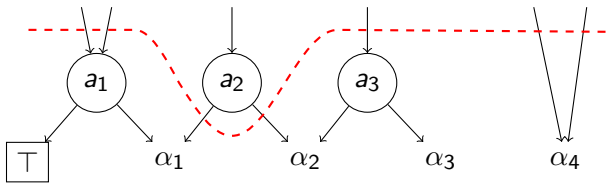

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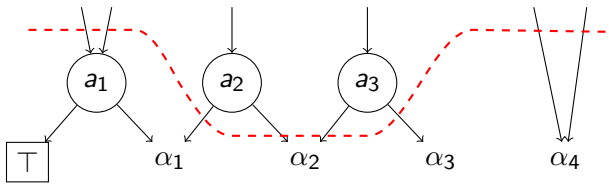

□

1-level cut	:	1.0%
2-level cut	:	3.3%

Computational overhead of piggybacking i-level cut within Adiar.

Possible to process a

1.1 GiB BDD

with only

128 MiB Memory

Adiar v1.0 : 56.5 hours

Running time to verify the 15 smallest EPFL instances.

Adiar v1.0 : 56.5 hours

Adiar v1.2 : 4.0 hours (−93%)[1]

Running time to verify the 15 smallest EPFL instances.

[1] 52.1 of these hours were saved on just verifying the `sin` circuit alone.

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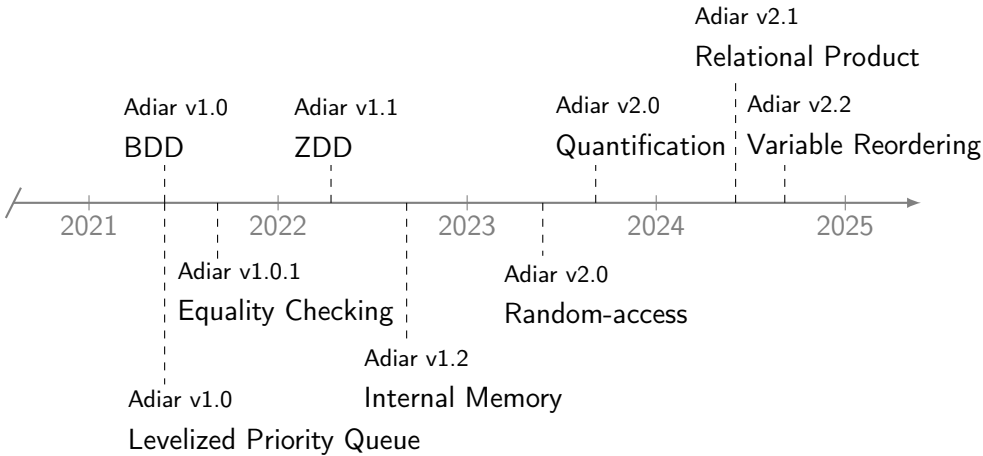
ssoelvsten.github.io

Adiar

github.com/ssoelvsten/adiar

ssoelvsten.github.io/adiar

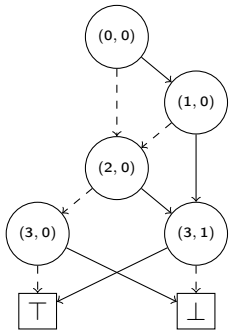
AARHUS
UNIVERSITY


Adiar v2.1
Relational Product
Adiar v1.0
BDD
Adiar v1.1
ZDD
Adiar v2.0
Quantification
Adiar v2.2
Variable Reordering
2021
2022
2023
2024
2025
Adiar v1.0.1
Equality Checking
Adiar v2.0
Random-access
Adiar v1.2
Internal Memory
Adiar v1.0
Levelized Priority Queue

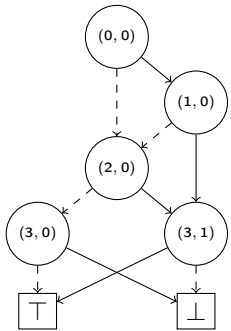
	Sufficient?	Overhead	Memory[2]	Disk R/W	Transition Cost
DF ▸ Adiar (RAM ▸ Disk)	✗	$3\times$	–	$2N$	–
DF ‖ Adiar (RAM ‖ Disk)	✓	–	$M/3$	$2N$	–
DF → Adiar 1.0	✗ [1]	–	–	–	$\Omega(N \log N)$
State Pattern (RAM → Disk)	✓ [4]	$\sim 20\%$ [3]	$M/2$	–	$\Omega(N)$
i-level cut (RAM / Disk)	✓ [4]	1%	–	–	–

Comparison of possible solutions.

[1] There can be a gap between when depth-first runs out of memory and Adiar 1.0 has no overhead.
[2] Decreasing the memory dedicated to an external memory data structure impacts its performance.
[3] Runtime polymorphism adds a 20% to 30% overhead [Stroustrup].
[4] This solves the gap[1]; a *non-trivial* integration with depth-first algorithms can cover tiny cases.



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

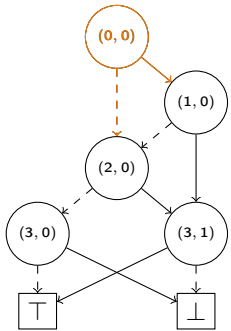


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]



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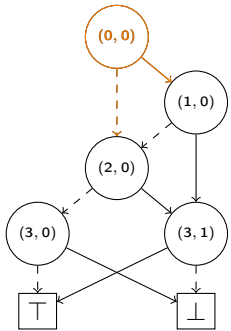
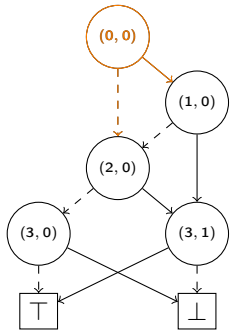


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

$$\left[\begin{array}{ll} ((0,0) \xrightarrow{\top} (1,0), & 1) \\ ((0,0) \xrightarrow{\perp} (2,0), & 1) \end{array} \right]$$

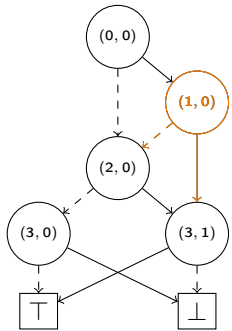


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0),\quad 1)\quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0),\quad 1)\quad ,$$
$$]$$



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Seek	Sum	Result
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Priority Queue: Q_{count}:

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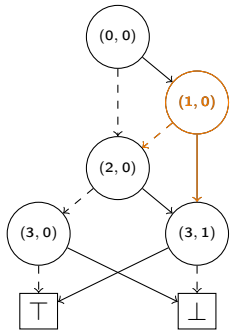
$]$



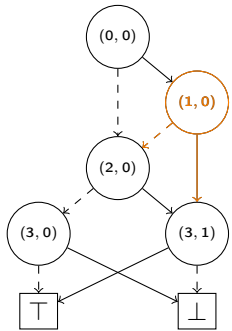
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\perp} (2,0), \quad 1) \quad ,$$

$$]$$



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Seek	Sum	Result
$(1,0)$	1	0

Priority Queue: Q_{count}:

[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

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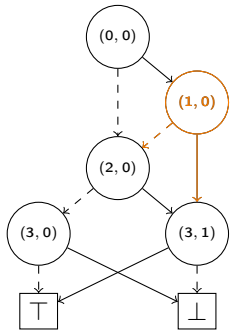
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

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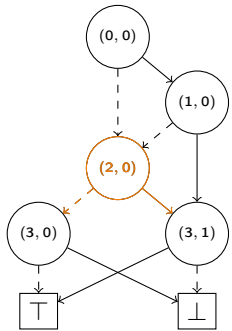
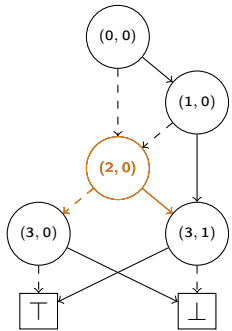


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

$$
\begin{bmatrix}
((0,0) \xrightarrow{\perp} (2,0), & 1) & , \\
((1,0) \xrightarrow{\perp} (2,0), & 1) & , \\
 & & \\
((1,0) \xrightarrow{\top} (3,1), & 1) & , \\
 & &]
\end{bmatrix}
$$



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Priority Queue: Q_{count}:

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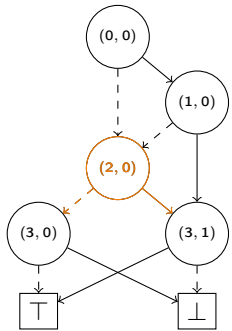


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

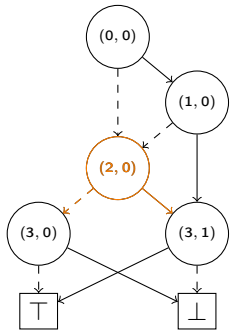
Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,

]



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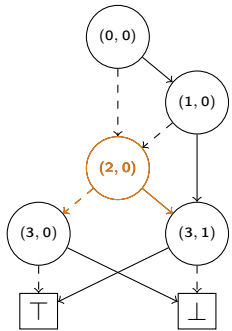

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,
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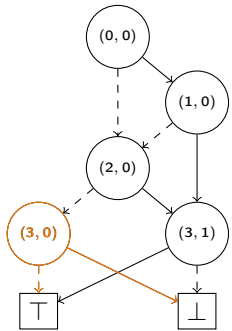

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Priority Queue: Q_{count}:

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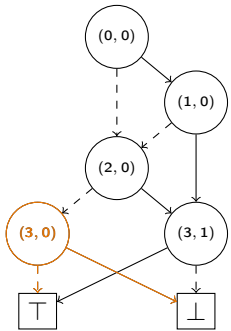

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Seek	Sum	Result
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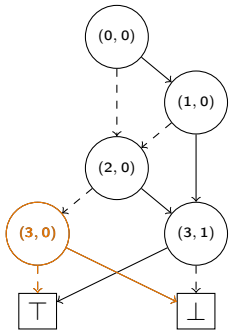
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

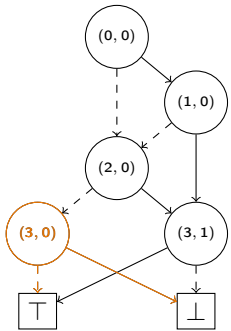
Seek	Sum	Result
$(3, 0)$	2	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

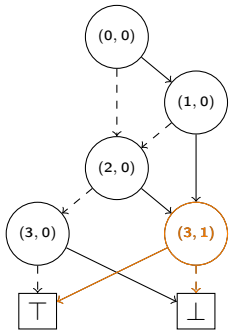
Seek	Sum	Result
$(3,1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

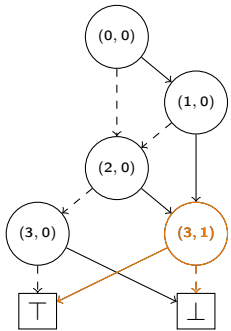
Seek	Sum	Result
$(3,1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



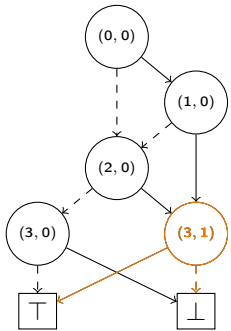
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3,1)$	1	2

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



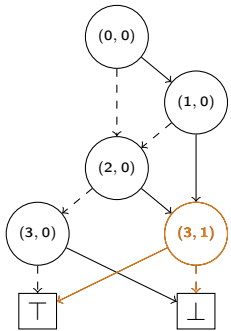
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count}:

[

]



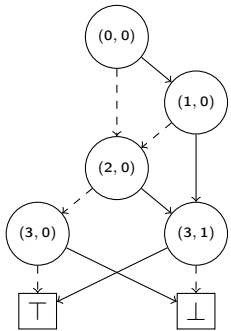
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	5

Priority Queue: Q_{count}:

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result

5

Priority Queue: Q_{count}:

[

]

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