

# Adiar:

## Binary Decision Diagrams in External Memory

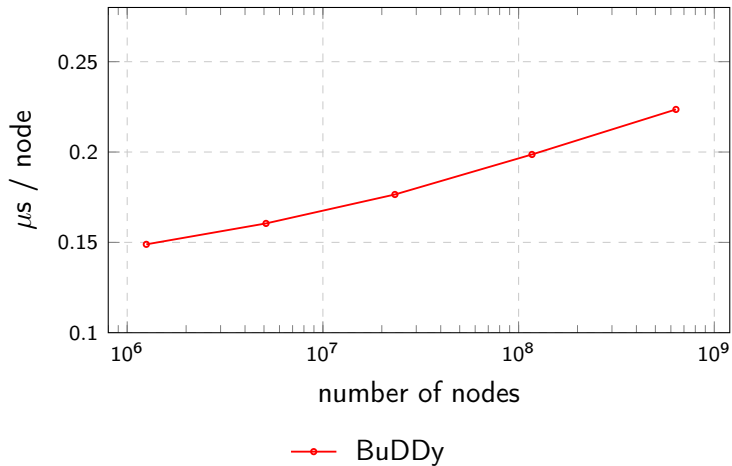
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**Steffan Christ Sølvesten**, Jaco van de Pol,  
Anna Blume Jakobsen, and Mathias Weller Berg Thomasen

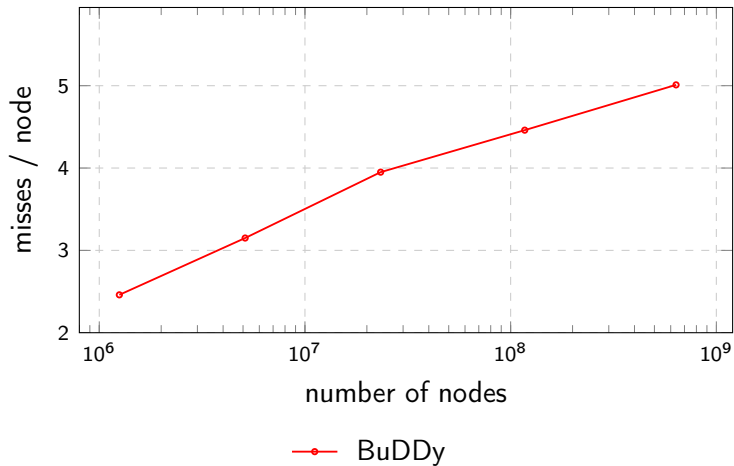
TACAS 2022





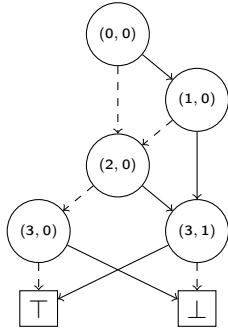


**Figure 1:** Minimal running time for the *Queens* problems.



**Figure 2:** Cache-misses for the *Queens* problems.





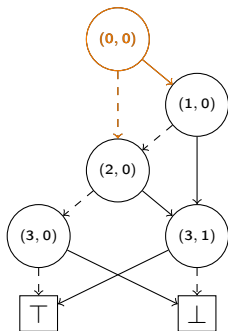
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[

]

**Figure 3:** In-order traversal of BDD



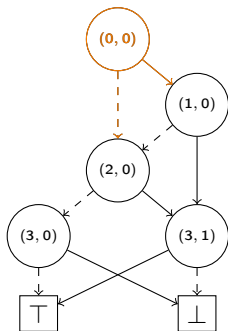
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

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[

]

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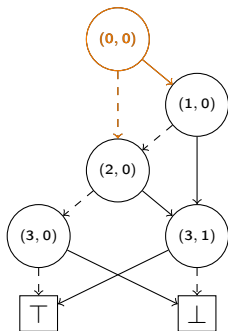
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[	$((0, 0) \xrightarrow{T} (1, 0), \quad 1)$	,
	$((0, 0) \xrightarrow{F} (2, 0), \quad 1)$	,
]		

**Figure 3:** In-order traversal of BDD





(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

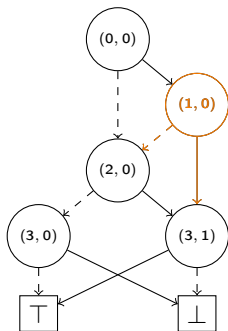
Seek	Sum	Result
(1, 0)	0	0

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{T} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]

**Figure 3:** In-order traversal of BDD



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

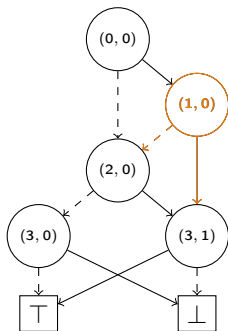
Seek	Sum	Result
(1, 0)	0	0

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{T} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]

**Figure 3:** In-order traversal of BDD



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

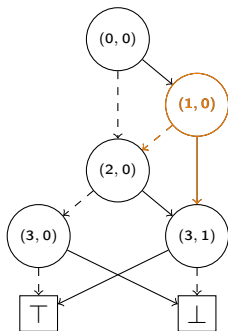
Seek	Sum	Result
(1, 0)	1	0

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]

**Figure 3:** In-order traversal of BDD



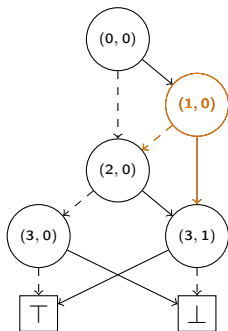
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	1	0

Priority Queue:  $Q_{count}$ :

[	
$((0, 0) \xrightarrow{\perp} (2, 0), 1)$	,
$((1, 0) \xrightarrow{\perp} (2, 0), 1)$	,
$((1, 0) \xrightarrow{\top} (3, 1), 1)$	,
]	

Figure 3: In-order traversal of BDD



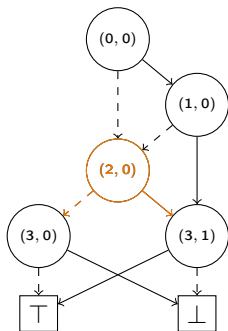
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	0	0

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]

**Figure 3:** In-order traversal of BDD



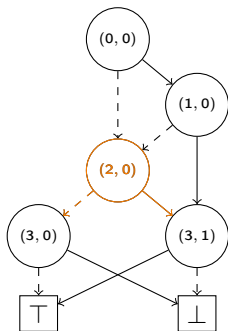
(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	0	0

Priority Queue:  $Q_{count}$ :

[	
$((0, 0) \xrightarrow{\perp} (2, 0), 1)$	,
$((1, 0) \xrightarrow{\perp} (2, 0), 1)$	,
$((1, 0) \xrightarrow{\top} (3, 1), 1)$	,
]	

**Figure 3:** In-order traversal of BDD



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	1	0

Priority Queue:  $Q_{count}$ :

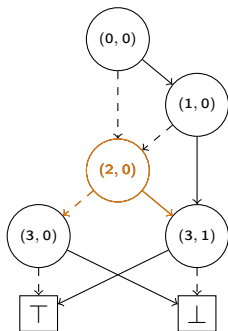
[

$((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,

]

**Figure 3:** In-order traversal of BDD



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

**Figure 3:** In-order traversal of BDD

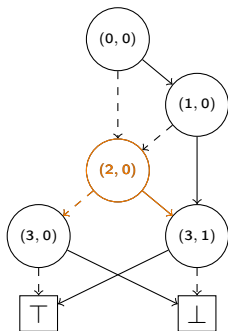
Seek	Sum	Result
(2, 0)	2	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
]





(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

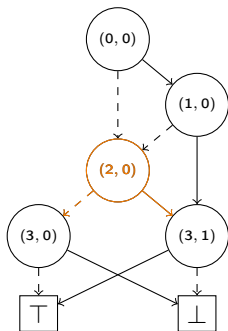
Seek	Sum	Result
(2, 0)	2	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]

**Figure 3:** In-order traversal of BDD



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

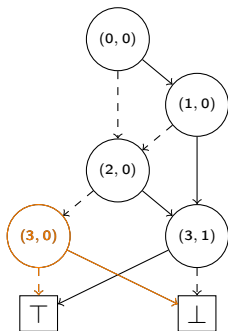
Seek	Sum	Result
(3, 0)	0	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]

**Figure 3:** In-order traversal of BDD



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

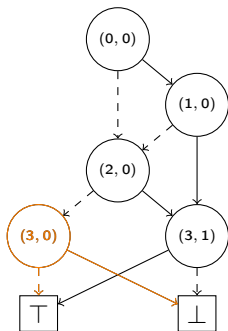
Seek	Sum	Result
<b>(3, 0)</b>	0	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]

**Figure 3:** In-order traversal of BDD



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

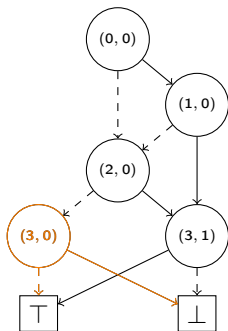
Seek	Sum	Result
(3, 0)	2	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]

**Figure 3:** In-order traversal of BDD



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

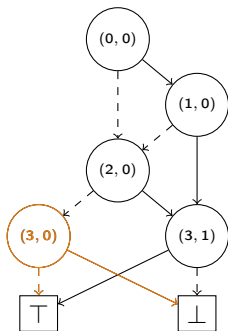
Seek	Sum	Result
(3, 0)	2	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]

**Figure 3:** In-order traversal of BDD



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

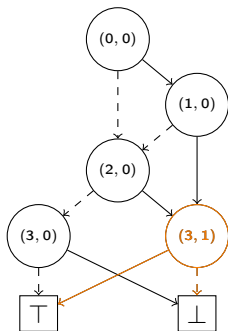
Figure 3: In-order traversal of BDD

Seek	Sum	Result
(3, 1)	0	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

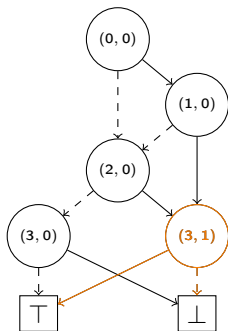
Seek	Sum	Result
<b>(3, 1)</b>	0	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), \quad 1) \quad ,$   
 $((2, 0) \xrightarrow{T} (3, 1), \quad 2) \quad ]$

**Figure 3:** In-order traversal of BDD



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

**Figure 3:** In-order traversal of BDD

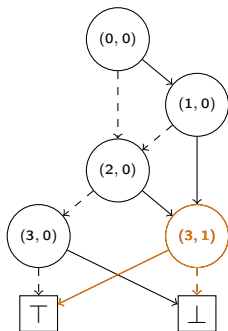
Seek	Sum	Result
<b>(3, 1)</b>	1	2

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad ]$





(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

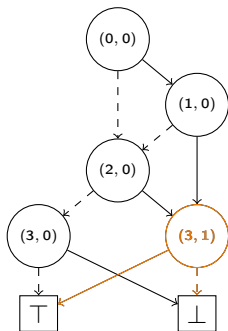
Seek	Sum	Result
(3, 1)	3	2

Priority Queue:  $Q_{count}$ :

[

]

**Figure 3:** In-order traversal of BDD



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	3	5

Priority Queue:  $Q_{count}$ :

[

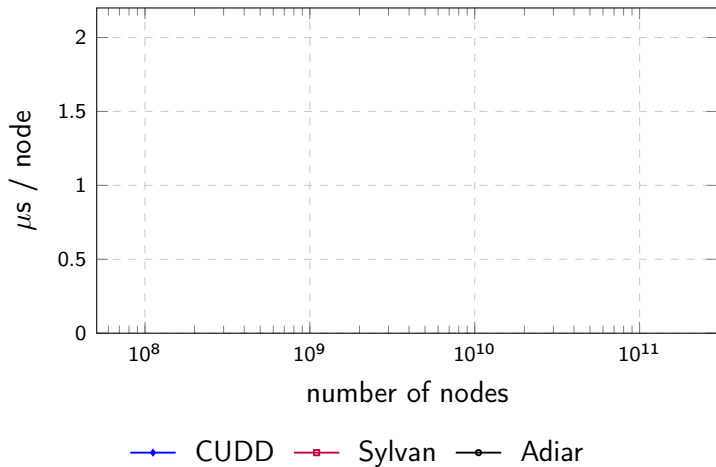
]

**Figure 3:** In-order traversal of BDD

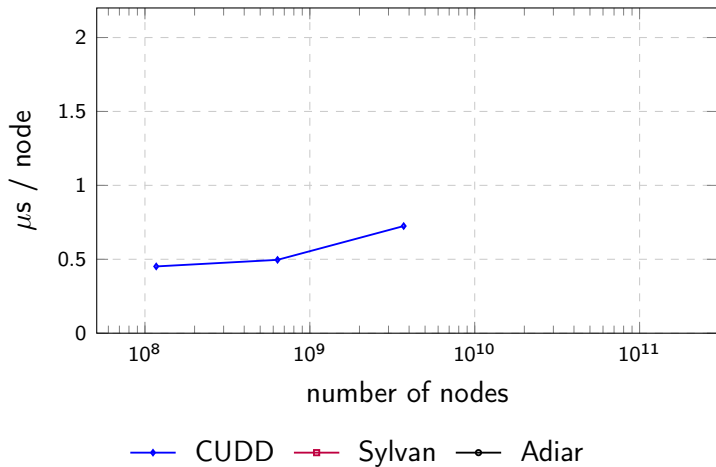


# Adiar

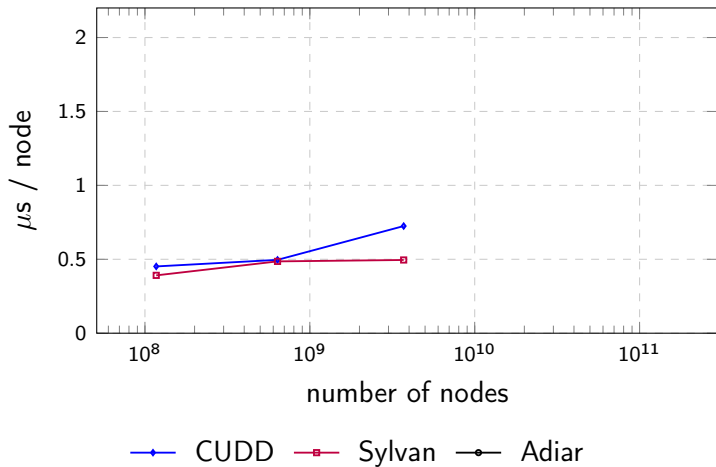
[github.com/ssoelvsten/adiar](https://github.com/ssoelvsten/adiar)



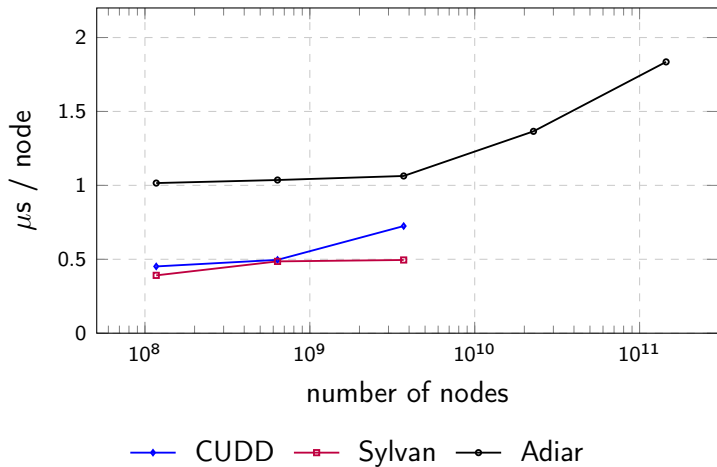
**Figure 4:** Minimal running time for the *Queens* problems.



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# Steffan Christ Sølvsten

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🐦 [@ssoelvsten](https://twitter.com/ssoelvsten)

## Adiar

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🔗 [github.com/ssoelvsten/adiar](https://github.com/ssoelvsten/adiar)

📄 [ssoelvsten.github.io/adiar](https://ssoelvsten.github.io/adiar)

