# Efficient Equality Checking for Non-Shared Binary Decision Diagrams

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 $f\leftrightarrow g\equiv \top$ 

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$$\underbrace{O(\mathsf{sort}(\mathit{N}^2))}_{\mathsf{Apply}} + \underbrace{O(\mathsf{sort}(\mathit{N}^2))}_{\mathsf{Reduce}} + \underbrace{O(1))}_{\mathsf{check is} \ \top} = O(\mathsf{sort}(\mathit{N}^2))$$

# Theorem (Bryant '86)

Let  $\pi$  be a variable order and  $f: \mathbb{B}^n \to \mathbb{B}$  then there exists a unique (up to isomorphism) Reduced Ordered Binary Decision Diagram representing f with ordering  $\pi$ .

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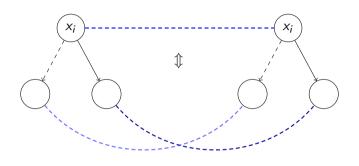
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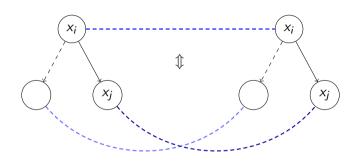
## Trivial cases: $f \not\equiv g$ if there is a mismatch in

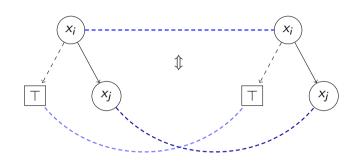
•	$N_f \neq N_g$	Number of nodes	O(1) I/Os
•	$L_f  eq L_g$	Number of levels	O(1) I/Os

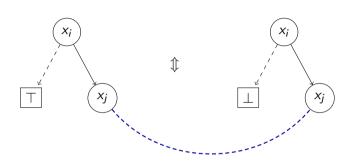
$$lacksquare$$
  $N_{f,i} 
eq N_{g,i}$  Number of nodes on a level  $O(L/B)$  I/Os

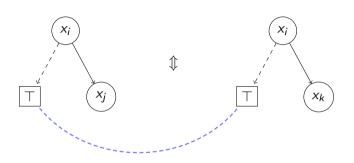
■ 
$$L_{f,i} \neq L_{g,i}$$
 Label of an *i*th level  $O(L/B)$  I/Os

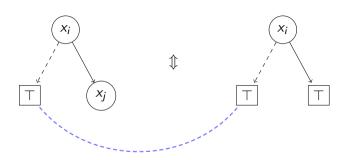












## IsIsomorphic(f, g)

- Check whether root  $v_f$  of f and root  $v_g$  of g have a local violation.
- Check  $low(v_f) \sim low(v_g)$  and  $high(v_f) \sim high(v_g)$  "recursively".

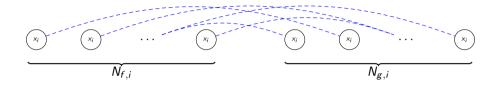
Return false on first violation. If there are no violations then return true.

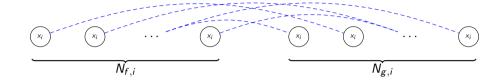
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Return false if more than  $N_{f,i} = N_{g,i}$  pairs of nodes  $(v_f, v_g)$  are checked on level i.

$$O(\operatorname{sort}(N))$$

#### Observation

The output of Reduce has the following properties

■ Nodes on level i have their identifiers consecutively numbered

$$MAX - N_{f,i} + 1, \dots, MAX - 1, MAX$$
.

■ Nodes on level *i* are output sorted by their children

$$((i_1, id_1), low_1, high_1) <_{lex(i, low, high)} ((i_2, id_2), low_2, high_2)$$
,

where

$$\forall (i, id) : (i, id) < \bot < \top^{-1}$$
.

<sup>&</sup>lt;sup>1</sup>Assuming the BDD is not negated. If that is the case then  $(i, id) < \top < \bot$ .

#### **Theorem**

If  $G_f$  and  $G_g$  are outputs of Reduce.

 $G_f \sim G_g \iff For \ all \ i \in [0; N) \ the \ node \ G_f[i] \ matches \ G_g[i] \ numerically.$ 

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 $\Rightarrow$  : Strong induction on BDD levels bottom-up  $\dots$ 

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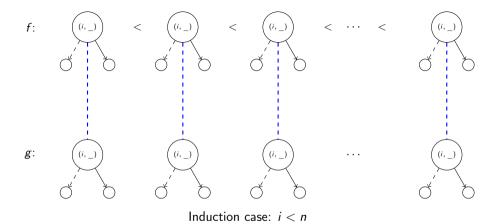
 $\Rightarrow$ : Strong induction on BDD levels bottom-up . . .

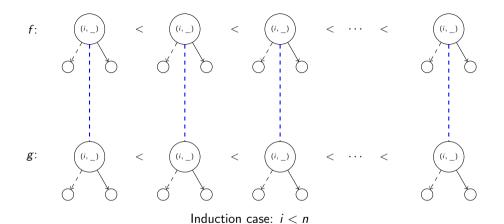
## Corollary

If  $G_f$  and  $G_g$  are outputs of Reduce then  $f \equiv g$  is computable using  $2 \cdot N/B$  I/Os.<sup>2</sup>

<sup>&</sup>lt;sup>2</sup>Assuming they are both unnegated (or both negated).







Algorithm	Time (s)
$f\leftrightarrow g\equiv \top$	0.38
O(sort(N))	0.058
$2 \cdot N/B$	0.006

Checking the (EPFL Benchmark) *voter* circuit's single output gate ( $|N_f| = |N_g| = 5.76$  MiB).