

# MLightDP

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### Definition (Differential Privacy)

Given parameters  $\epsilon \geq 0$  then a probabilistic algorithm  $M : A \mapsto B$  is  $\epsilon$ -differentially private if

$$\forall a_1, a_2 \in A \forall O \subset B :$$

$$a_1 \Psi a_2 \implies \Pr [M(a_1) \in O] \leq e^\epsilon \Pr [M(a_2) \in O] \ .$$



## Randomness Alignment

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$$\eta_2 = f(\eta_1) = \eta_1 + \hat{q} ,$$

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and then

$$\Pr \left[ q^{(1)} + \text{Lap}(1/\epsilon) \in O \right] \leq e^\epsilon \Pr \left[ q^{(2)} + \text{Lap}(1/\epsilon) \in O \right] .$$

# LightDP

Based on the above, Zhang and Kifer '17 propose a type system *LightDP* with dependent types, to prove differential privacy of algorithms.

$$\tau ::= num_{\mathbb{d}} \mid num_* \mid bool_0 \mid list \tau \mid \tau_1 \rightarrow \tau_2$$

The number type also contains the numerical *distances* between the two executions.

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- The privacy budget variable  $v_\epsilon$  is less or equal to  $\epsilon$  in the *transformed* program on all return paths.

$$\frac{\Gamma(\eta) = \text{num}_{\mathsf{d}} \quad e : \text{num}_0}{\Gamma \vdash \eta := \text{Lap}(e) \rightarrow \text{havoc } \eta; \quad v_\epsilon := v_\epsilon + |\mathsf{d}|/e} \text{ (T-Laplace)}^1$$

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MLightDP

**Example:** `laplace_mechanism.mldp`

Remember, the randomness alignment example from before where  $-1 \leq q^{(1)} - q^{(2)} = \hat{q} \leq 1$  and adding  $\eta \sim \text{Lap}(1/\epsilon)$  to  $q$  makes them differentially private.

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4       let eta : float          = Lap(1 / epsilon);
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1      precondition: -1.0 <= ^q /\ ^q <= 1.0
2      LaplaceMechanism(q: float[^q], ^q : float[0])
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## Example: laplace\_mechanism.dfy

The transformed *Dafny* program then is:

```
1    method LaplaceMechanism(epsilon : real, q : real, ghost q_hat0 : real)
2      returns (_, real)
3      requires epsilon > 0.0
4      requires -1.0 <= q_hat0 && q_hat0 <= 1.0
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17   return q + eta;
18 }
```

# Examples

Verified?	Example Program
✓	<code>laplace_mechanism.mldp</code>
✓	<code>sparse_vector.mldp</code> of Zhang and Kifer '17
✓ <sup>1</sup>	<code>numerical_sparse_vector.mldp</code> of Zhang and Kifer '17
✓ <sup>1</sup>	<code>gap_sparse_vector.mldp</code> of Wang et. al. '19
	<code>partial_sum.mldp</code> of Zhang and Kifer '17
	<code>smart_sum.mldp</code> of Zhang and Kifer '17

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<sup>1</sup>We had to modify the given invariants to make these work.



# Comparison

<i>LightDP</i>	<i>MLightDP</i>	Feature
✓	✓	DP Type checking
	✓	Output to a Software Verification tool
	✓	Semantic Analysis of distances
		Refinement: Distance inference
	–	Refinement: Mutable Dependency tracking
	–	Free usage of the Laplace
		Automating non-linearity
	–	Other

# MLightDP

The Goal of our project is to create a less prototypical experience than their current prototype.

- 1 Create more intuitive typing annotations.
- 2 Be capable of accommodating the complete pipeline.
- 3 Dispense with assumptions of Zhang and Kifer about the input.
- 4 Make their rules of more fine-grained / less conservative.
- 5 Soundly add new operators, functions, and alternative random distributions to the language.