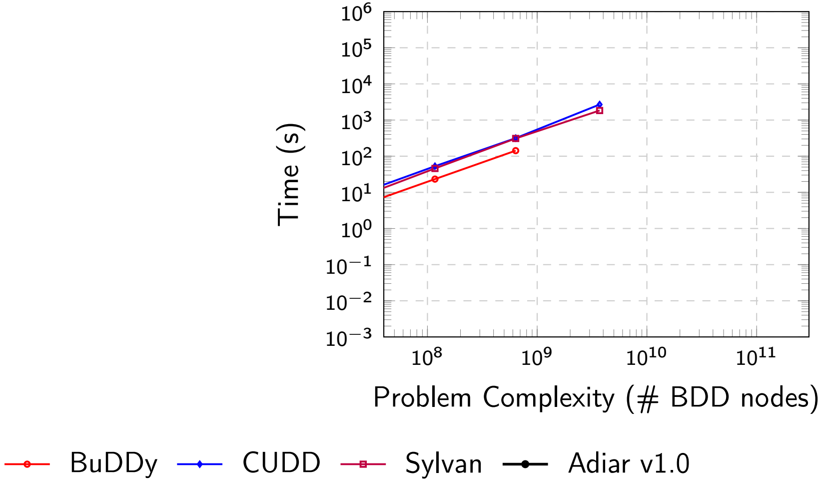
Predicting Memory Demands of BDD Operations using Maximum Graph Cuts

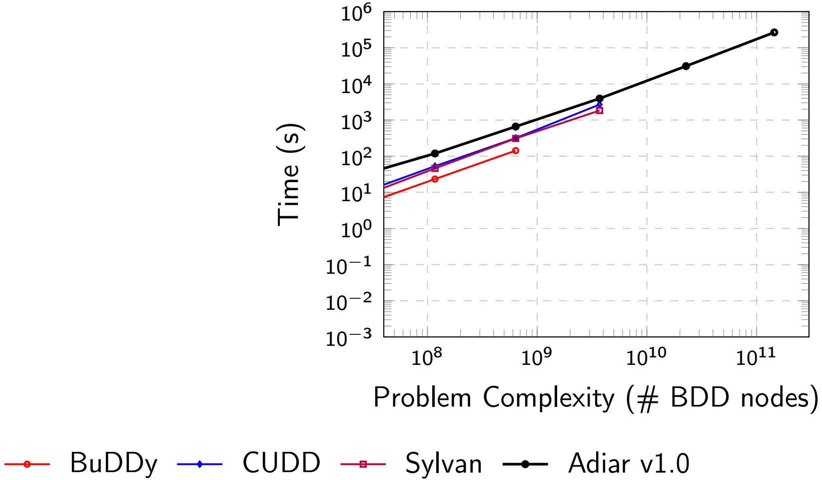
Steffan Christ Sølvsten and Jaco van de Pol

ATVA 2023

AARHUS UNIVERSITY



Running Time to solve *N-Queens* problems.



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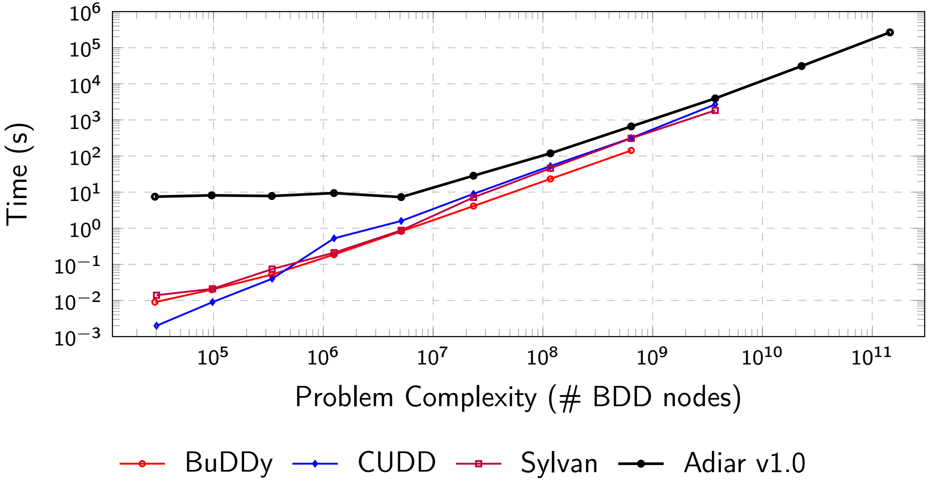

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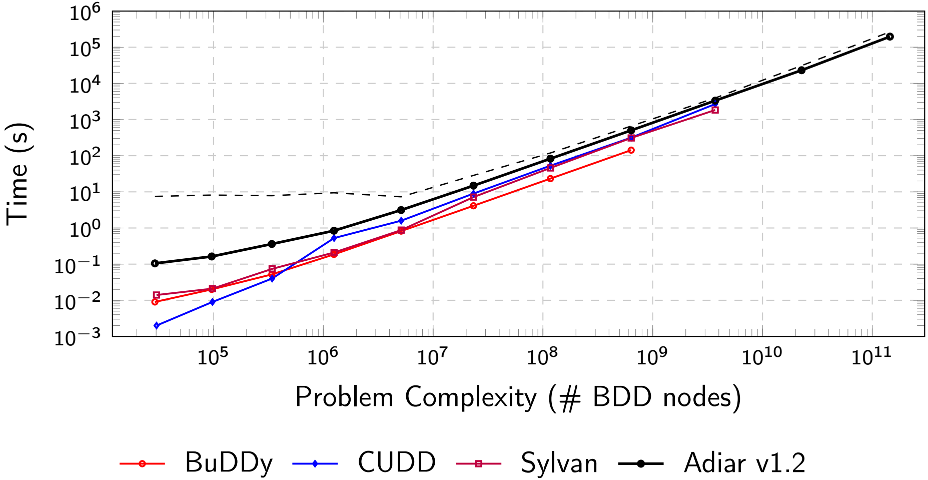
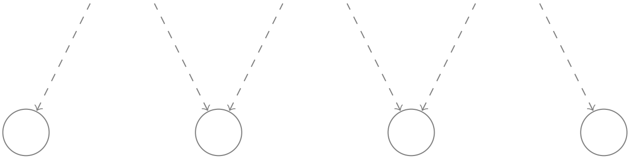
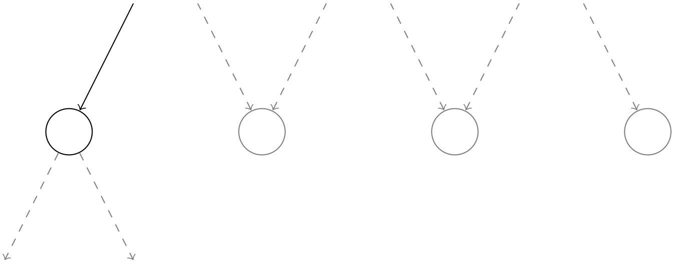
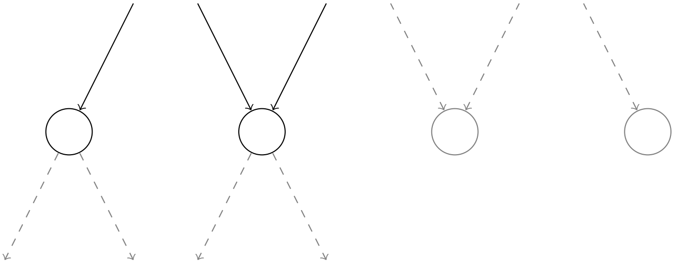
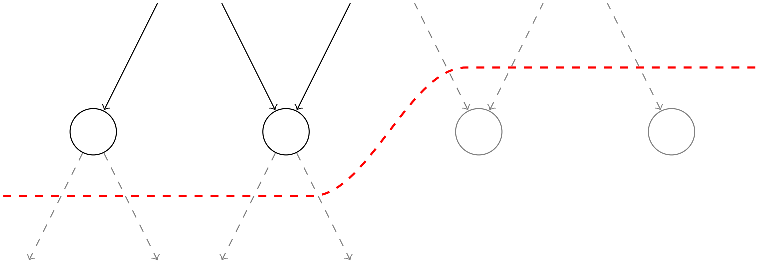


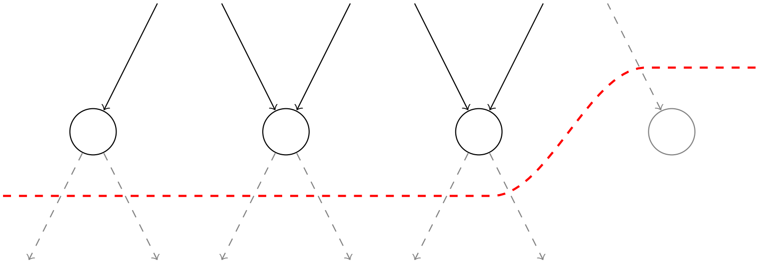
Running Time to solve *N-Queens* problems.

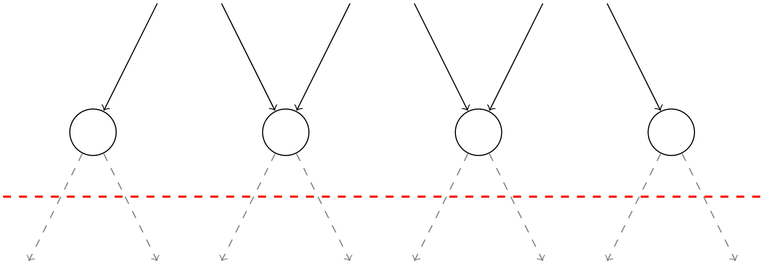




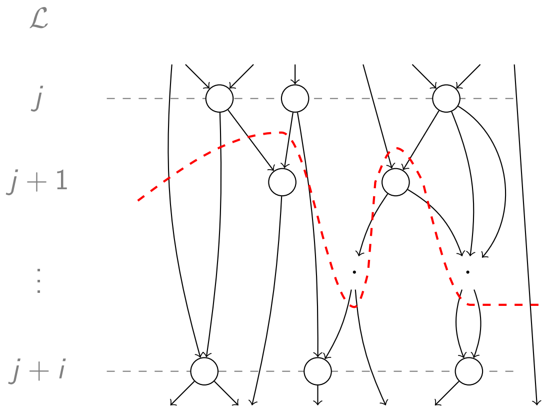




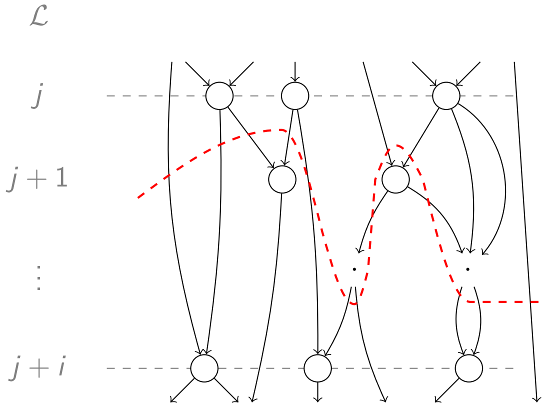




i-level cut



i-level cut



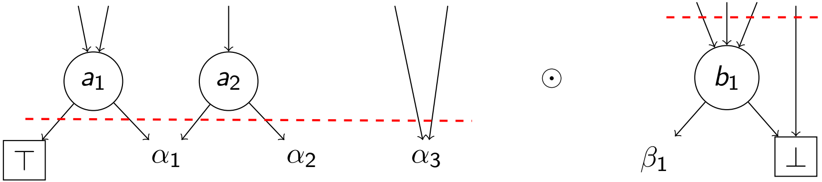
Lemma (**Sølvsten, Van de Pol 2023**)
The maximum i-level cut problem is in P for $i \in \{1, 2\}$.

Theorem (**Lampis, Kaouri, Mitsou 2011**)
The maximum i-level cut problem is NP-complete for $i \geq 4$.

Theorem (**Sølvsten, Van de Pol 2023**)

Given maximum 2-level cuts size C_f for f and C_g for g, the maximum 2-level cut for $f \odot g$ is less than or equal to $C_f \cdot C_g$.

Proof.



$\square$

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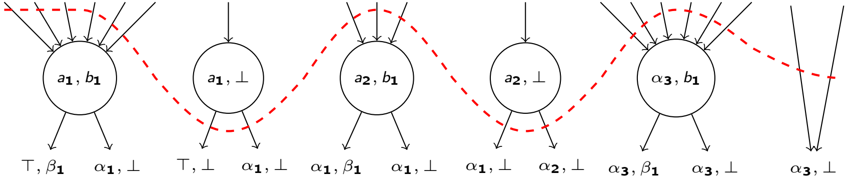
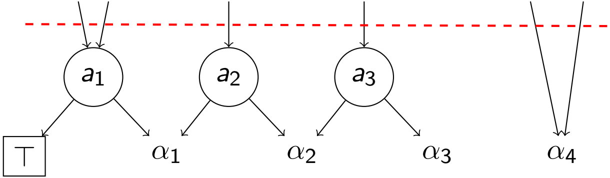


□

Lemma (**Sølvsten, Van de Pol 2023**)
The maximum 2-level cut for f is at most $\frac{3}{2}$ larger than its maximum 1-level cut.

Proof.



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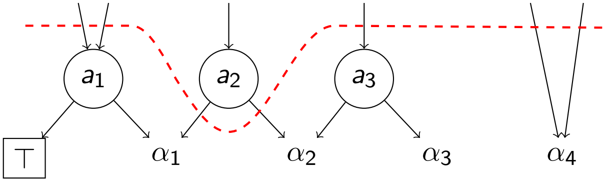

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Lemma (Sølvsten, Van de Pol 2023)
The maximum 2-level cut for f is at most $\frac{3}{2}$ larger than its maximum 1-level cut.

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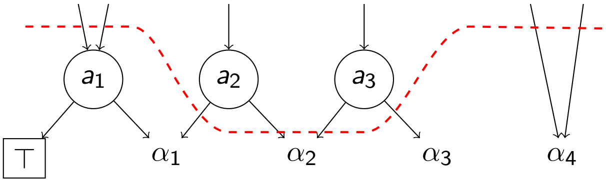

□

		Overhead	Precision
1-level cut	:	1.0%	69.2%
2-level cut	:	3.3%	86.3%

Possible to process a

1.1 GiB BDD

with only

128 MiB Memory

	Running Time
Adiar v1.0 :	56.5 hours

Verification of the 15 smallest EPFL circuits.

		Running Time	
Adiar v1.0	:	56.5 hours	
Adiar v1.2	:	4.0 hours	(-93%)[1]

Verification of the 15 smallest EPFL circuits.

[1] 52.1 of these hours were saved on just verifying the `sin` circuit alone.

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ssoelvsten.github.io

Adiar

github.com/ssoelvsten/adiar

ssoelvsten.github.io/adiar

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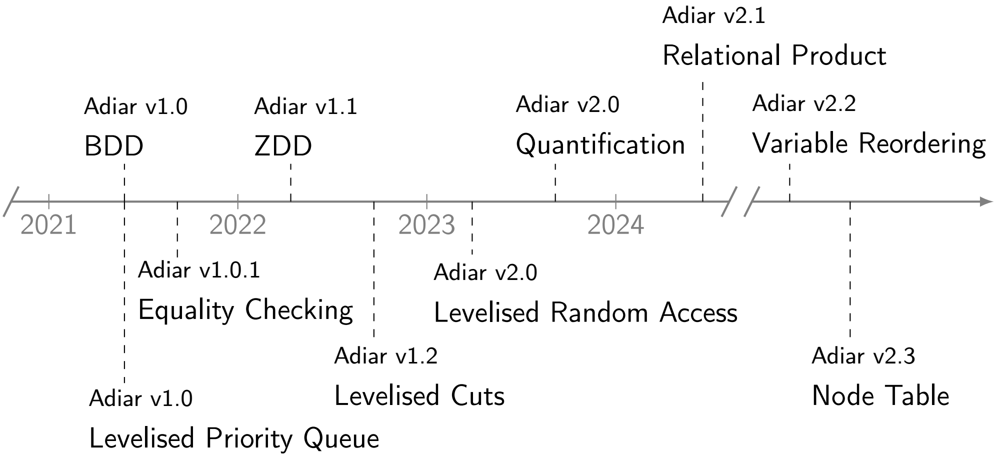
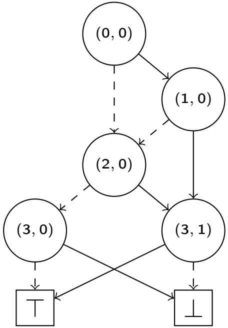


Adiar v2.1
Relational Product
Adiar v1.0
BDD
Adiar v1.1
ZDD
Adiar v2.0
Quantification
Adiar v2.2
Variable Reordering
2021
2022
2023
2024
Adiar v1.0.1
Equality Checking
Adiar v2.0
Levelised Random Access
Adiar v1.2
Levelised Cuts
Adiar v2.3
Node Table
Adiar v1.0
Levelised Priority Queue

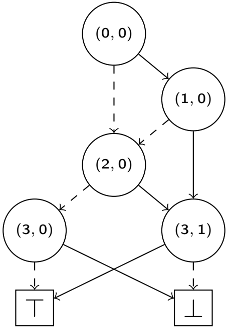
	Sufficient?	Overhead	Memory[2]	Disk R/W	Transition Cost
DF ▸ Adiar (▸)	✘	$3\times$	–	$2\times$	–
DF ‖ Adiar (‖)	✔	–	$3\times$	$2\times$	–
DF ➞ Adiar 1.0	✘ [1]	–	–	–	$\Omega(N \log N)$
State Pattern (➞)	✔ [4]	$\sim 20\%$ [3]	$2\times$	–	$\Omega(N)$
i-level cut (/)	✔ [4]	1%	–	–	–

Comparison of possible solutions.

[1] There can be a gap between when depth-first runs out of memory and Adiar 1.0 has no overhead.
[2] Decreasing the memory dedicated to an external memory data structure impacts its performance.
[3] Runtime polymorphism adds a 20% to 30% overhead [Stroustrup].
[4] This solves the gap[1]; a *non-trivial* integration with depth-first algorithms can cover tiny cases.



(a) $\left(x_0 \wedge x_1 \wedge x_3\right) \vee \left(x_2 \oplus x_3\right)$

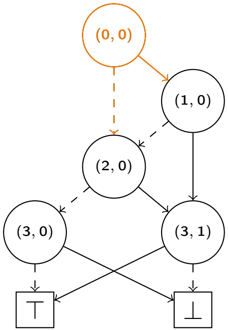


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

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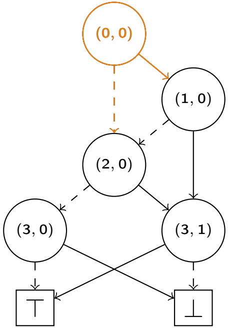
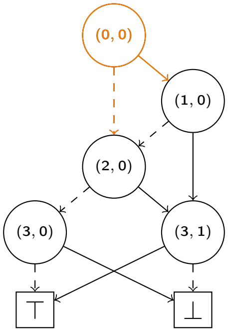


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0),\quad 1)\quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0),\quad 1)\quad ,$$
$$]$$

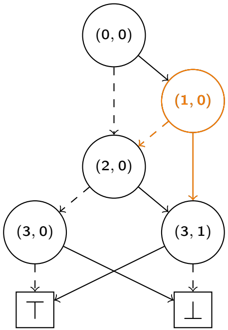


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$$
$$((0,0) \xrightarrow{\perp} (2,0), \quad 1) \quad ,$$
$$]$$

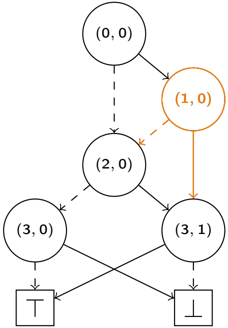


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$$\left[\begin{array}{ll} ((0,0) \xrightarrow{\top} (1,0), & 1), \\ ((0,0) \xrightarrow{\bot} (2,0), & 1), \end{array} \right]$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

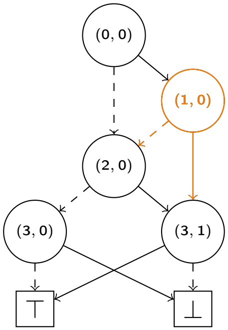
Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

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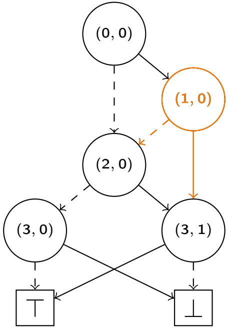
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]



(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

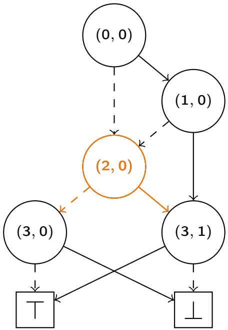
[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]

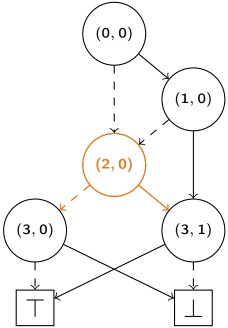


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

$$
\begin{array}{l}
[\\
\quad ((0,0) \xrightarrow{\perp} (2,0), \quad 1) \quad , \\
\quad ((1,0) \xrightarrow{\perp} (2,0), \quad 1) \quad , \\
\\
\quad ((1,0) \xrightarrow{\top} (3,1), \quad 1) \quad , \\
\quad]
\end{array}
$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	1	0

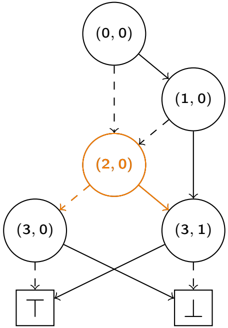
Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

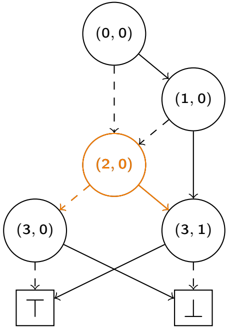
Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$

]

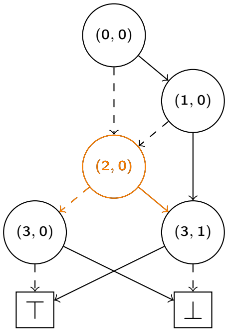


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2,0)$	2	0

Priority Queue: Q_{count}:

$$[\;((2,0) \xrightarrow{\bot} (3,0),\ 2),\quad ((1,0) \xrightarrow{\top} (3,1),\ 1),\quad ((2,0) \xrightarrow{\top} (3,1),\ 2)\;]$$



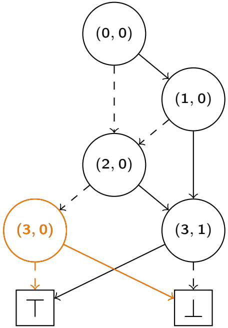
(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\bot} (3,0),\quad 2)$,
$((1,0) \xrightarrow{\top} (3,1),\quad 1)$,
$((2,0) \xrightarrow{\top} (3,1),\quad 2)$]



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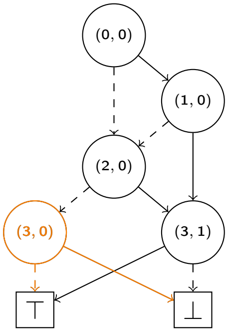
Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

$$
\left[\begin{array}{lll}
((2,0) \xrightarrow{\perp} (3,0), & 2) & , \\
((1,0) \xrightarrow{\top} (3,1), & 1) & , \\
((2,0) \xrightarrow{\top} (3,1), & 2) &
\end{array}\right]
$$



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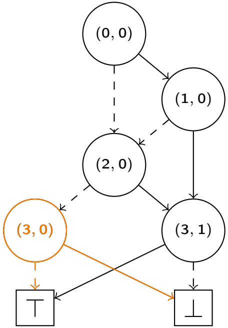
Seek	Sum	Result
$(3,0)$	2	0

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1),\quad 1)$,

$((2,0) \xrightarrow{\top} (3,1),\quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

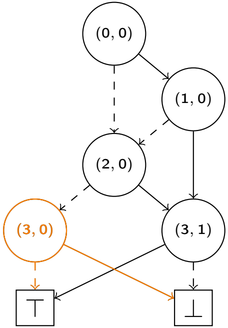
Seek	Sum	Result
(3, 0)	2	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1),\quad 1)$,

$((2,0) \xrightarrow{\top} (3,1),\quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

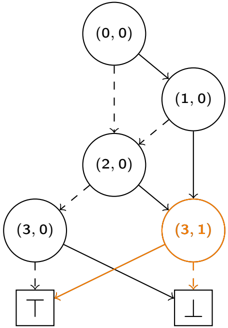
Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)$,

$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)$]



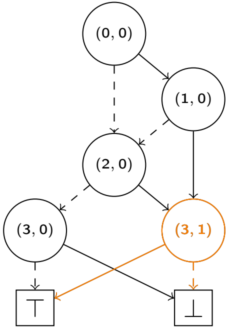
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1),\quad 1)$,
$((2,0) \xrightarrow{\top} (3,1),\quad 2)$]



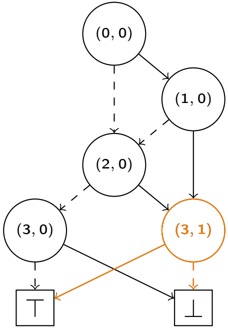
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	1	2

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



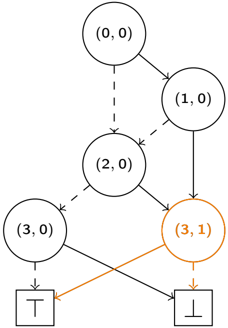
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count}:

[

]



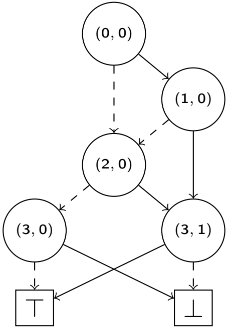
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	5

Priority Queue: Q_{count}:

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result

5

Priority Queue: Q_{count}:

[

]

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ssoelvsten.github.io

Adiar

github.com/ssoelvsten/adiar

ssoelvsten.github.io/adiar

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