# Adiar

Binary Decision Diagrams in External Memory

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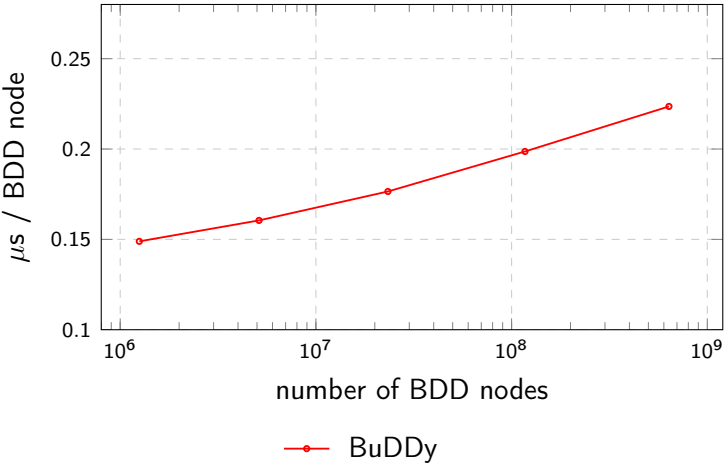
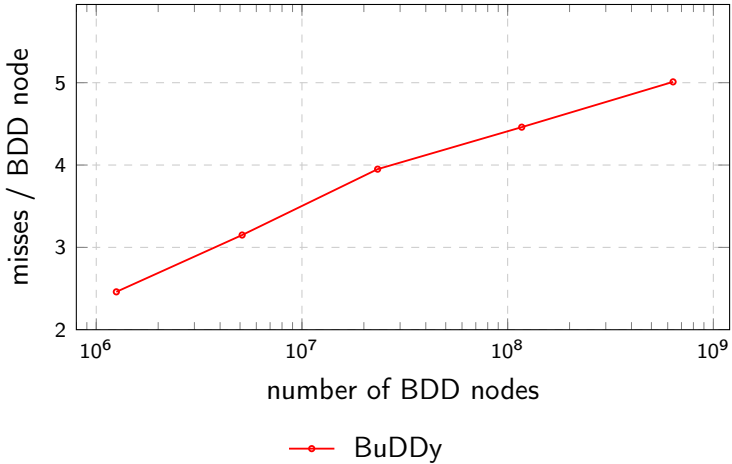
Anna Blume Jakobsen, and Mathias Weller Berg Thomasen

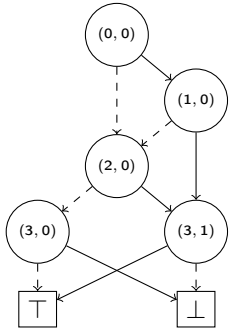
TACAS 2022



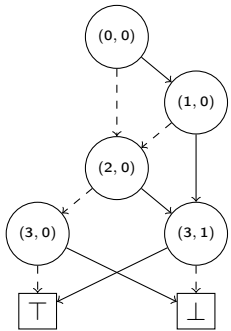

Minimal running time for the *Queens* problems.




Cache-misses for the *Queens* problems.




**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

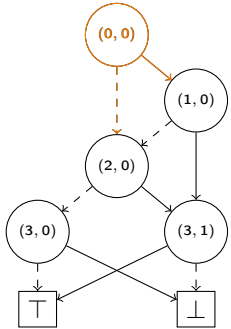



**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: $Q_{count}$:

[

]

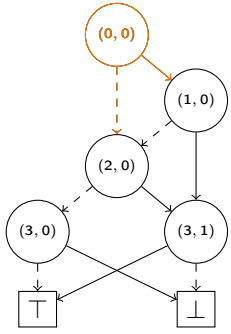



**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: $Q_{count}$:

[

]

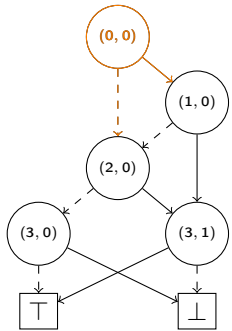



**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: $Q_{count}$:

$$\left[ \begin{array}{ll} ((0,0) \xrightarrow{\top} (1,0), & 1) \\ ((0,0) \xrightarrow{\bot} (2,0), & 1) \end{array} \begin{array}{l} , \\ , \end{array} \right.$$

]



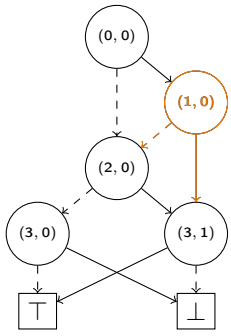

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(1, 0)$ | 0 | 0 |

Priority Queue: $Q_{count}$:

$$[\quad ((0,0) \xrightarrow{\top} (1,0),\quad 1) \quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0),\quad 1) \quad ,$$

$]$



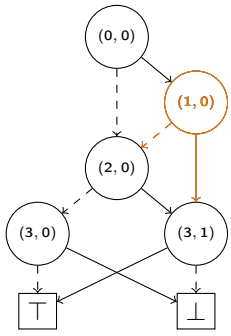

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(1,0)$ | 0 | 0 |

Priority Queue: $Q_{count}$:

$$[\quad ((0,0) \xrightarrow{\top} (1,0),\quad 1)\quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0),\quad 1)\quad ,$$

]




**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

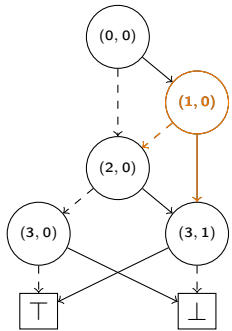
| Seek | Sum | Result |
|---|---|---|
| $(1, 0)$ | 1 | 0 |

Priority Queue: $Q_{count}$:

[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$ ,

]




**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(1,0)$ | 1 | 0 |

Priority Queue: $Q_{count}$:

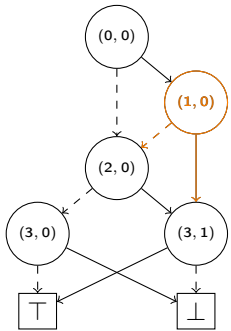
[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$ ,

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$ ,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$ ,

]

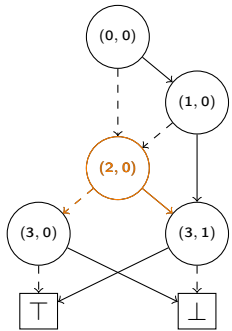



**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(2, 0)$ | 0 | 0 |

Priority Queue: $Q_{count}$:

$$
\begin{bmatrix}
((0,0) \xrightarrow{\perp} (2,0), & 1) & , \\
((1,0) \xrightarrow{\perp} (2,0), & 1) & , \\
 & & \\
((1,0) \xrightarrow{\top} (3,1), & 1) & , \\
 & & ]
\end{bmatrix}
$$

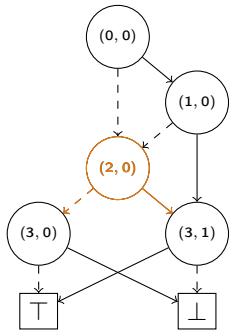



**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(2, 0)$ | 0 | 0 |

Priority Queue: $Q_{count}$:

$$
\begin{bmatrix}
((0,0) \xrightarrow{\perp} (2,0), & 1) & , \\
((1,0) \xrightarrow{\perp} (2,0), & 1) & , \\
 & & \\
((1,0) \xrightarrow{\top} (3,1), & 1) & , \\
 & & ]
\end{bmatrix}
$$




**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(2, 0)$ | 1 | 0 |

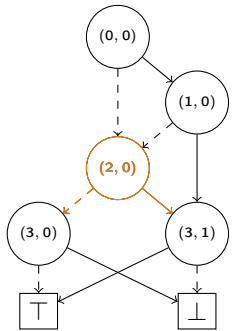
Priority Queue: $Q_{count}$:

[

$((1, 0) \xrightarrow{\perp} (2, 0), \quad 1)$ ,

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$ ,

]




**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

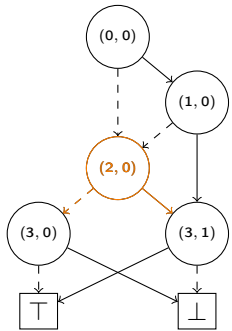
| Seek | Sum | Result |
|---|---|---|
| $(2, 0)$ | 2 | 0 |

Priority Queue: $Q_{count}$:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$ ,

]



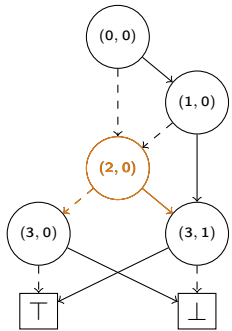

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(2, 0)$ | 2 | 0 |

Priority Queue: $Q_{count}$:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$ ,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$ ,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$ ]



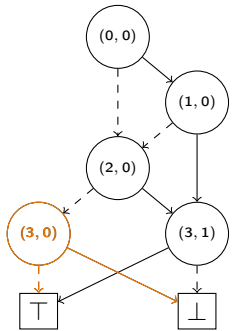

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(3, 0)$ | 0 | 0 |

Priority Queue: $Q_{count}$:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$ ,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$ ,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$ ]



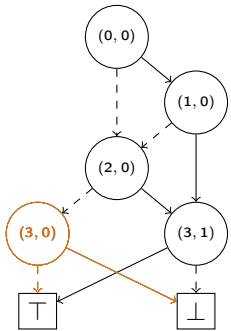

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(3, 0)$ | 0 | 0 |

Priority Queue: $Q_{count}$:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$ ,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$ ,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$ ]




**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

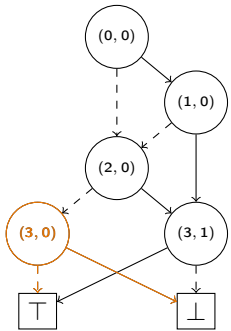
| Seek | Sum | Result |
|---|---|---|
| $(3, 0)$ | 2 | 0 |

Priority Queue: $Q_{count}$:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$ ,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$ ]




**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

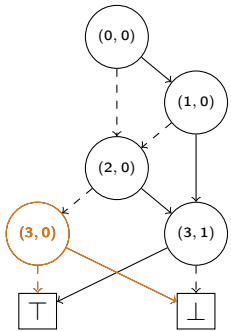
| Seek | Sum | Result |
|---|---|---|
| $(3, 0)$ | 2 | 2 |

Priority Queue: $Q_{count}$:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$ ,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$ ]




**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

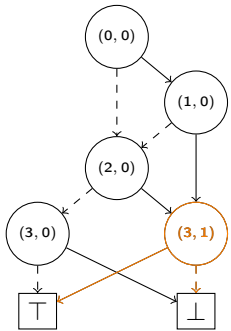
| Seek | Sum | Result |
|---|---|---|
| $(3, 1)$ | 0 | 2 |

Priority Queue: $Q_{count}$:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$ ,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$ ]




**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

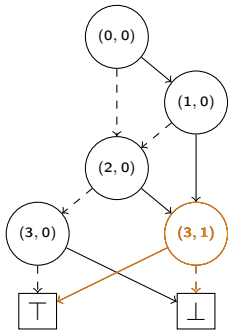
| Seek | Sum | Result |
|---|---|---|
| $(3, 1)$ | 0 | 2 |

Priority Queue: $Q_{count}$:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$ ,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$ ]



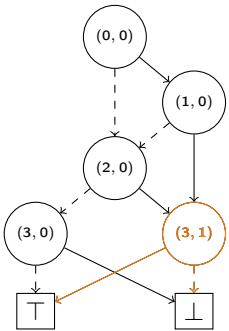

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(3,1)$ | 1 | 2 |

Priority Queue: $Q_{count}$:

[

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$ ]



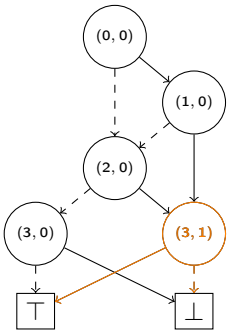

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(3, 1)$ | 3 | 2 |

Priority Queue: $Q_{count}$:

[

]



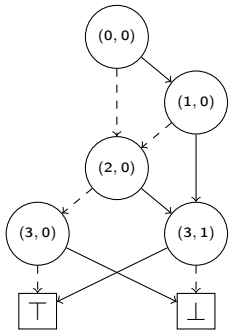

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---|---|---|
| $(3, 1)$ | 3 | 5 |

Priority Queue: $Q_{count}$:

[

]




**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result

5

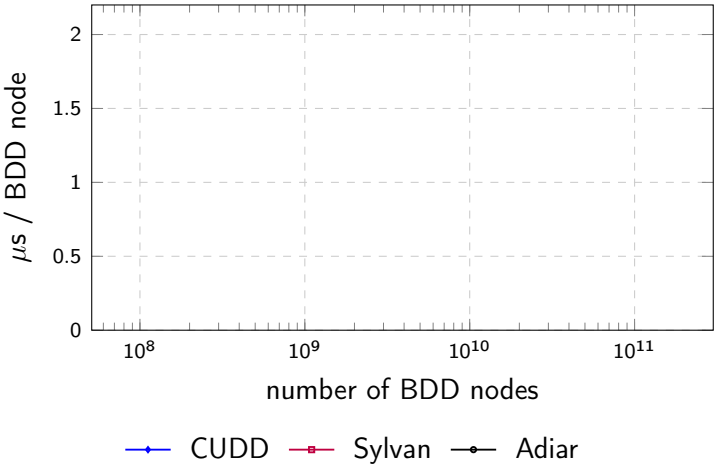
Priority Queue: $Q_{count}$:

[

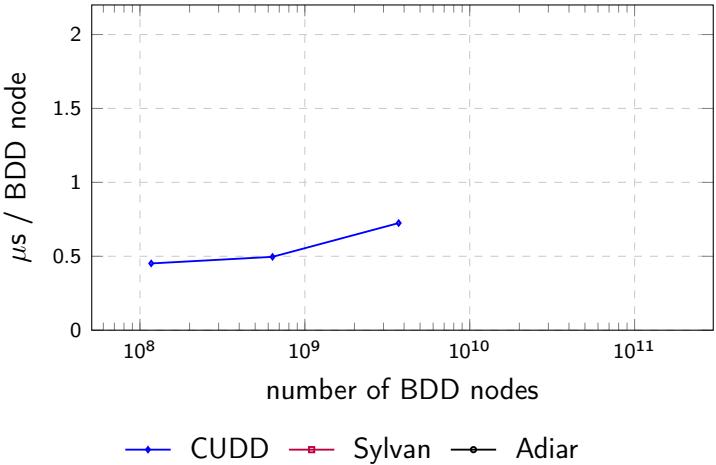
]

# Adiar

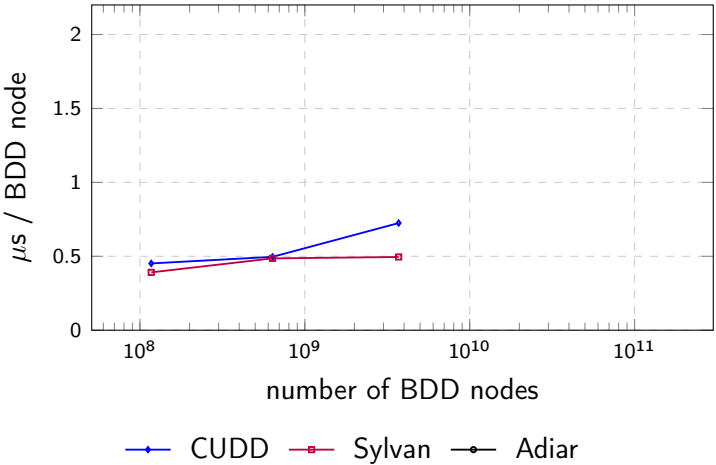
github.com/ssoelvsten/adiar



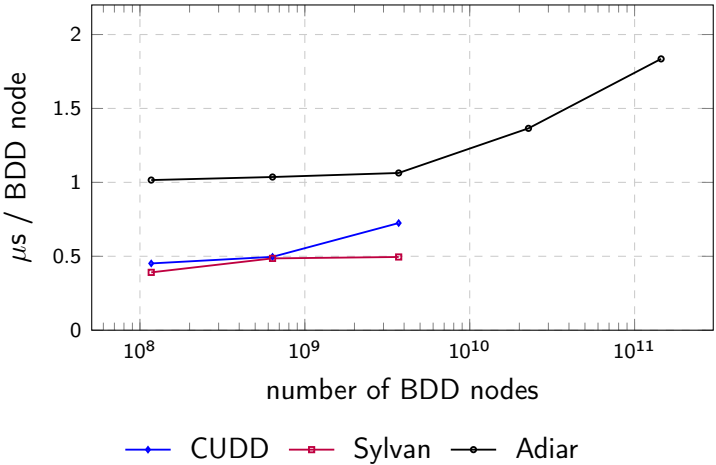

Minimal running time for the *Queens* problems.




Minimal running time for the *Queens* problems.




Minimal running time for the *Queens* problems.




Minimal running time for the *Queens* problems.

| Algorithm | Time (s) |
|---|---|
| $f \leftrightarrow g \equiv \top$ | 0.38 |

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

| Algorithm | Time (s) |
|---|---|
| $f \leftrightarrow g \equiv \top$ | 0.38 |
| $O(N \log N)$ | 0.058 |

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

| Algorithm | Time (s) |
|---|---|
| $f \leftrightarrow g \equiv \top$ | 0.38 |
| $O(N \log N)$ | 0.058 |
| $O(N)$ | 0.006 |

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

# Steffan Christ Sølvsten

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ssoelvsten.github.io


## Adiar

github.com/ssoelvsten/adiar

ssoelvsten.github.io/adiar


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