Predicting Memory Demands of BDD Operations using Maximum Graph Cuts

Steffan Christ Sølvsten and Jaco van de Pol

ATVA 2023

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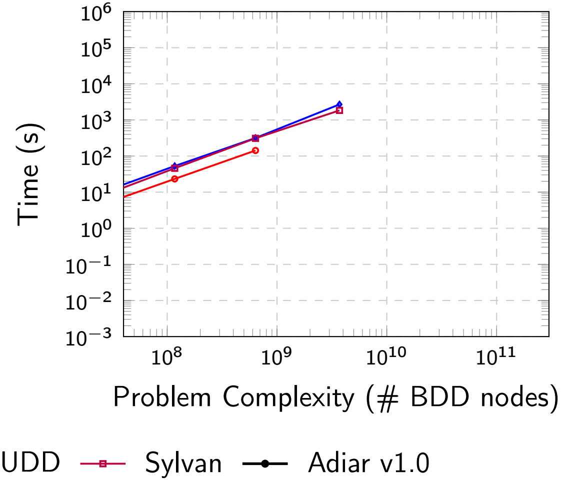
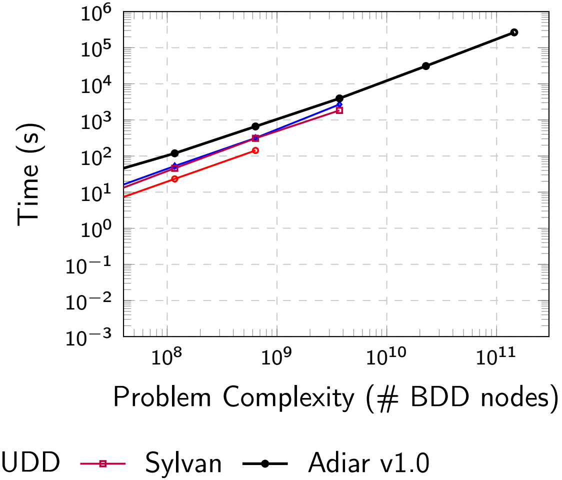


Running Time to solve *N-Queens* problems.



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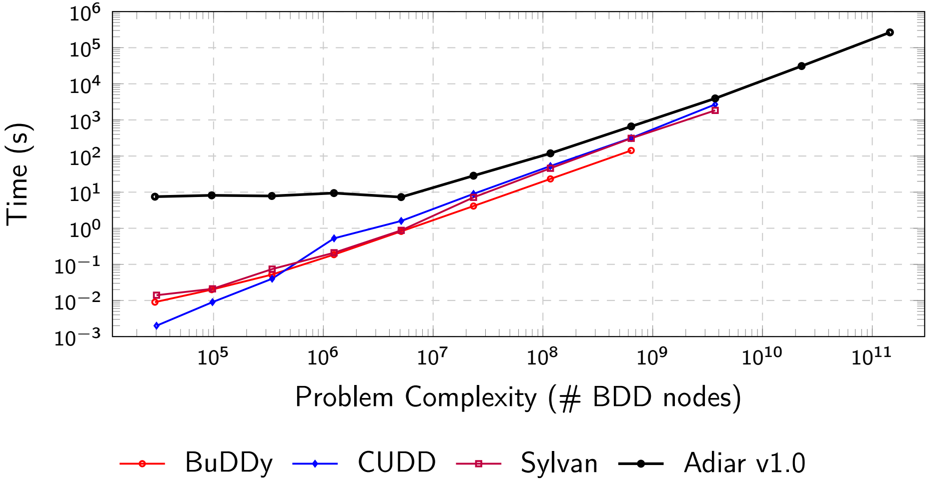

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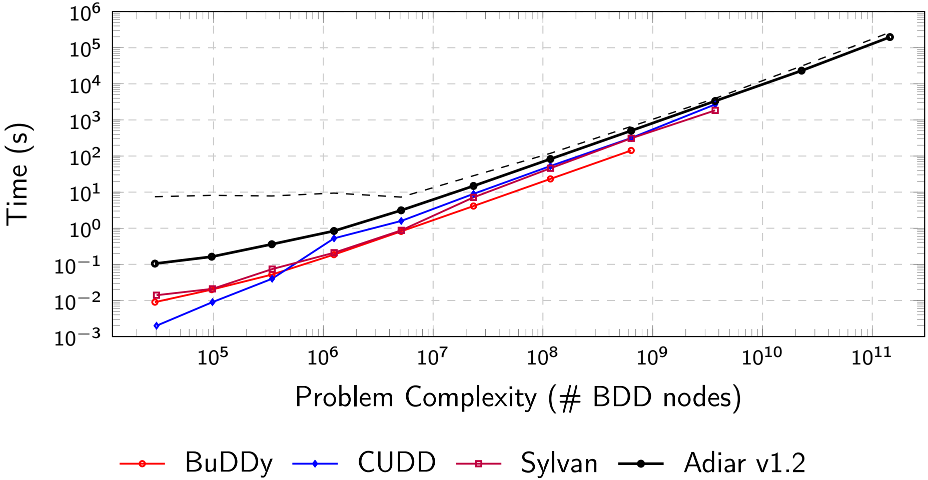

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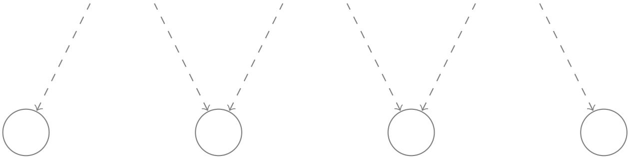
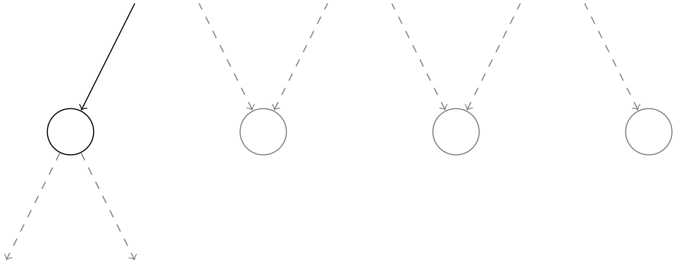
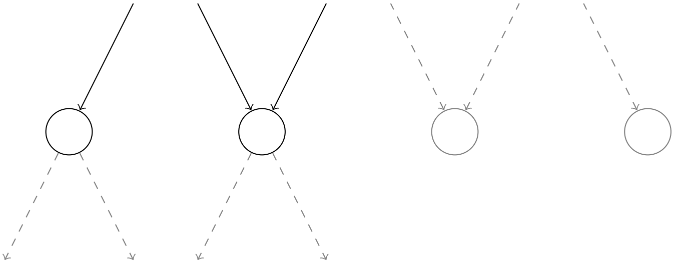
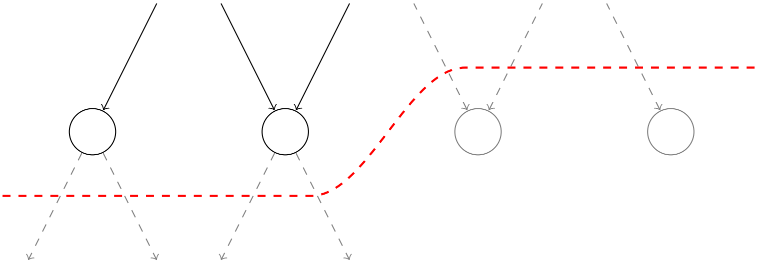
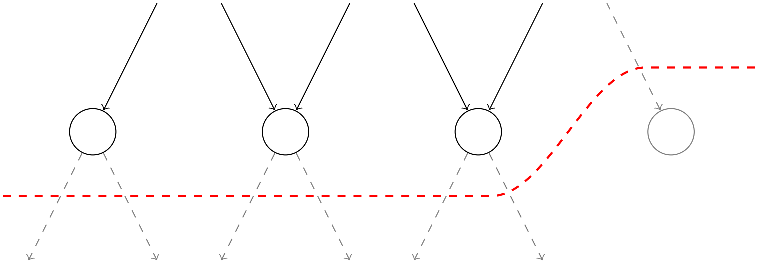
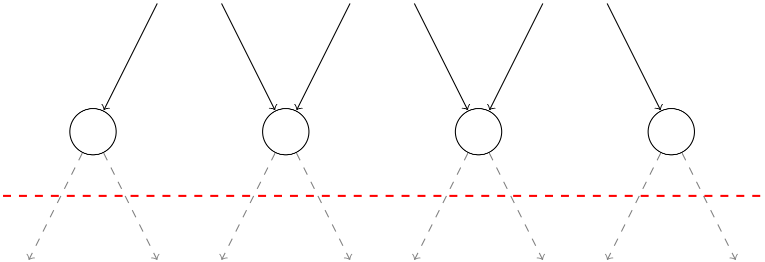




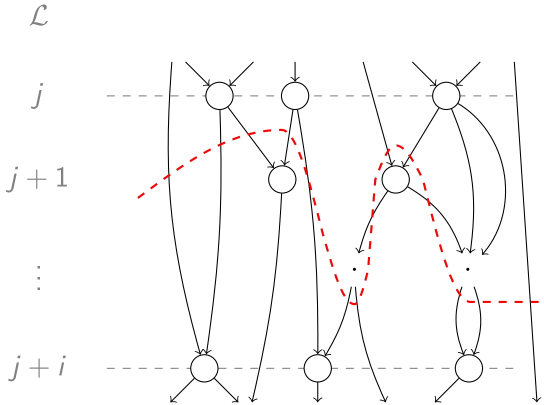




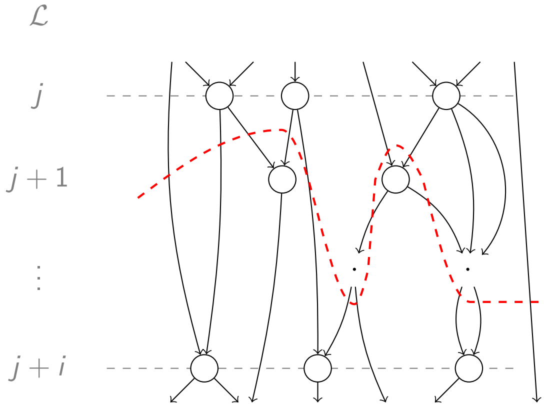




i-level cut



i-level cut

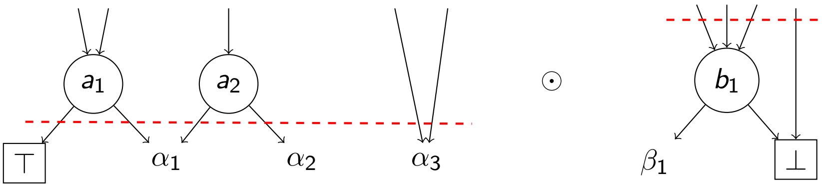


Lemma (**Sølvsten, Van de Pol 2023**)
The maximum i-level cut problem is in P for $i \in \{1, 2\}$.

Theorem (**Lampis, Kaouri, Mitsou 2011**)
The maximum i-level cut problem is NP-complete for $i \geq 4$.

Theorem (Sølvsten, Van de Pol 2023)
Given maximum 2-level cuts size C_f for f and C_g for g, the maximum 2-level cut for $f \odot g$ is less than or equal to $C_f \cdot C_g$.

Proof.



$\square$

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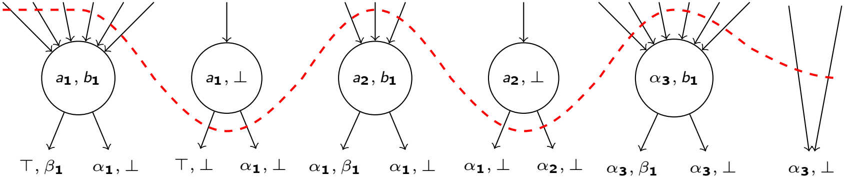

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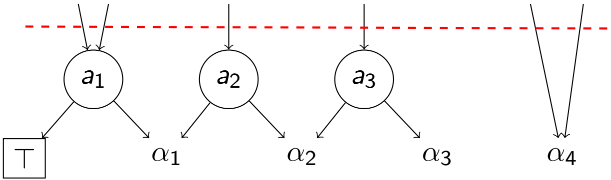
Lemma (Sølvsten, Van de Pol 2023)
The maximum 2-level cut for f is at most $\frac{3}{2}$ larger than its maximum 1-level cut.

Proof.



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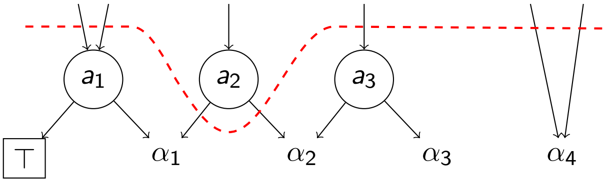

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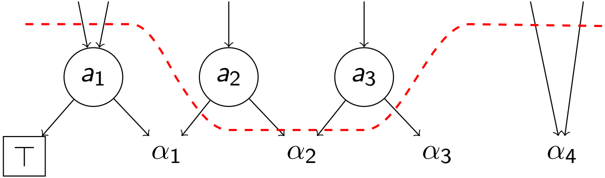

□

		Overhead	Precision
1-level cut	:	1.0%	69.2%
2-level cut	:	3.3%	86.3%

Possible to process a

1.1 GiB BDD

with only

128 MiB Memory

	Running Time
Adiar v1.0 :	56.5 hours

Verification of the 15 smallest EPFL circuits.

		Running Time	
Adiar v1.0	:	56.5 hours	
Adiar v1.2	:	4.0 hours	(−93%)[1]

Verification of the 15 smallest EPFL circuits.

[1] 52.1 of these hours were saved on just verifying the `sin` circuit alone.

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ssoelvsten.github.io

Adiar

github.com/ssoelvsten/adiar

ssoelvsten.github.io/adiar

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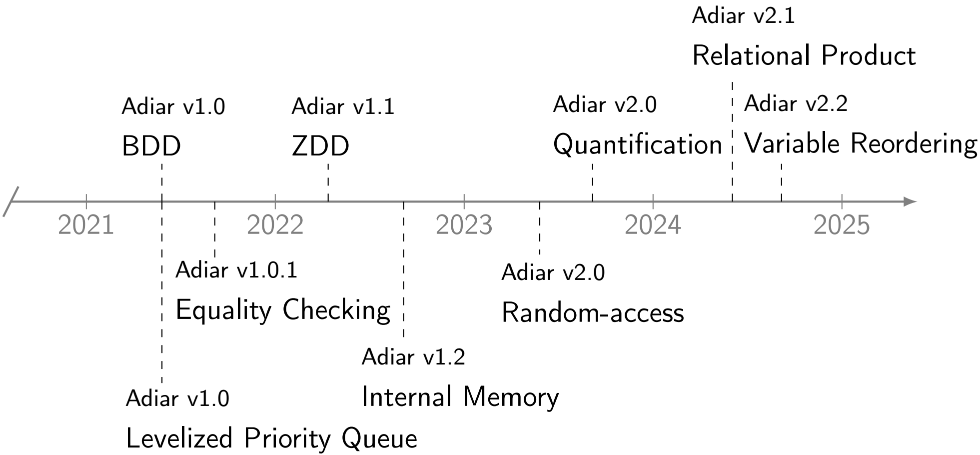
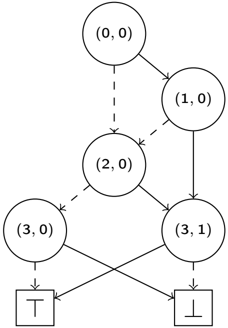

Adiar v2.1
Relational Product
Adiar v1.0
BDD
Adiar v1.1
ZDD
Adiar v2.0
Quantification
Adiar v2.2
Variable Reordering
2021
2022
2023
2024
2025
Adiar v1.0.1
Equality Checking
Adiar v2.0
Random-access
Adiar v1.2
Internal Memory
Adiar v1.0
Levelized Priority Queue

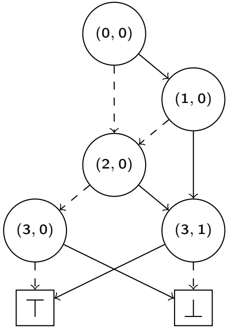
	Sufficient?	Overhead	Memory[2]	Disk R/W	Transition Cost
DF ▸ Adiar (▸)	✗	3×	–	2×	–
DF ‖ Adiar (‖)	✓	–	3×	2×	–
DF ➞ Adiar 1.0	✗ [1]	–	–	–	$\Omega(N \log N)$
State Pattern (➞)	✓ [4]	$\sim$ 20% [3]	2×	–	$\Omega(N)$
i-level cut (/)	✓ [4]	1%	–	–	–

Comparison of possible solutions.

[1] There can be a gap between when depth-first runs out of memory and Adiar 1.0 has no overhead.
[2] Decreasing the memory dedicated to an external memory data structure impacts its performance.
[3] Runtime polymorphism adds a 20% to 30% overhead [Stroustrup].
[4] This solves the gap[1]; a *non-trivial* integration with depth-first algorithms can cover tiny cases.



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

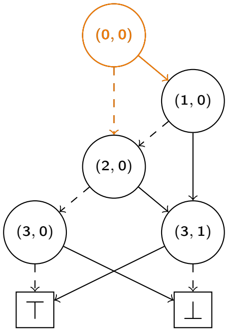


(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]



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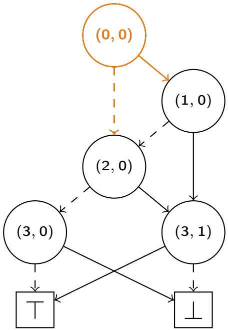
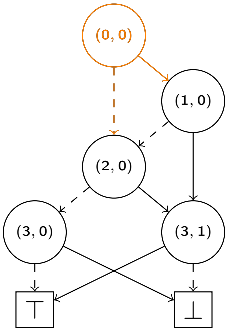


(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

$$\begin{array}{l} [\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad , \\ \quad\;\; ((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad , \\ \\ \\ \\ \\ \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad] \end{array}$$

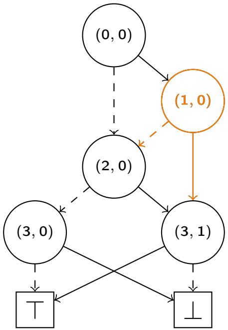


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$$
$$((0,0) \xrightarrow{\perp} (2,0), \quad 1) \quad ,$$
$$]$$

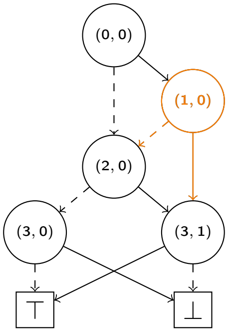


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$$
\begin{array}{llll}
[& ((0,0) \xrightarrow{\top} (1,0), & 1) & , \\
 & ((0,0) \xrightarrow{\bot} (2,0), & 1) & , \\
 & & &]
\end{array}
$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

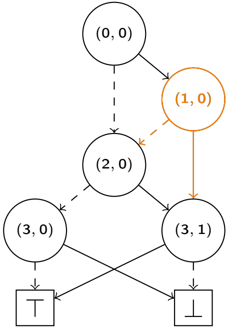
Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

[

$((0,0) \xrightarrow{\perp} (2,0),\quad 1)$,

]



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Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

[
$((0,0) \xrightarrow{\perp} (2,0),\quad 1)$,
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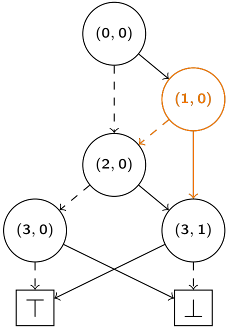
$((1,0) \xrightarrow{\top} (3,1),\quad 1)$,
]

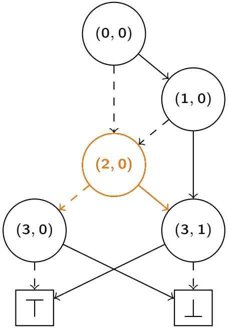


(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	0	0

Priority Queue: Q_{count}:

$$
\begin{array}{l}
[\\
\quad ((0,0) \xrightarrow{\perp} (2,0), \quad 1) \quad , \\
\quad ((1,0) \xrightarrow{\perp} (2,0), \quad 1) \quad , \\
\\
\quad ((1,0) \xrightarrow{\top} (3,1), \quad 1) \quad , \\
\quad]
\end{array}
$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

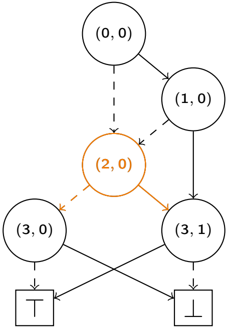
[

$((0, 0) \xrightarrow{\perp} (2, 0), \quad 1)$,

$((1, 0) \xrightarrow{\perp} (2, 0), \quad 1)$,

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2,0)$	1	0

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

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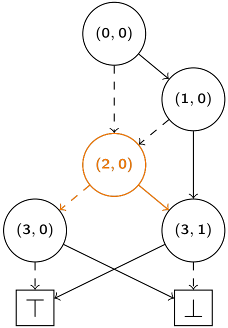


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

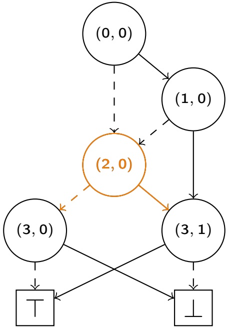
Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]



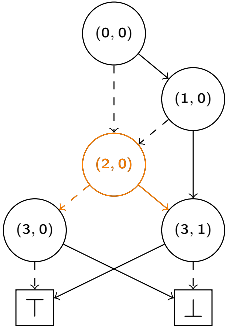
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2,0)$	2	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0),\quad 2)$,
$((1,0) \xrightarrow{\top} (3,1),\quad 1)$,
$((2,0) \xrightarrow{\top} (3,1),\quad 2)$]

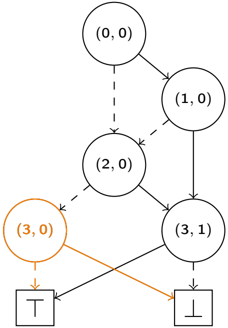


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3,0)$	0	0

Priority Queue: Q_{count}:

$$\begin{array}{lll} [& & \\ & ((2,0) \xrightarrow{\bot} (3,0), \quad 2) & , \\ & ((1,0) \xrightarrow{\top} (3,1), \quad 1) & , \\ & ((2,0) \xrightarrow{\top} (3,1), \quad 2) &] \end{array}$$

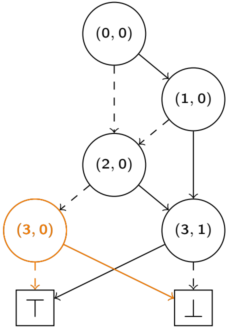


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count}:

$$[\ ((2,0) \xrightarrow{\perp} (3,0),\ 2),\ ((1,0) \xrightarrow{\top} (3,1),\ 1),\ ((2,0) \xrightarrow{\top} (3,1),\ 2)\ ]$$



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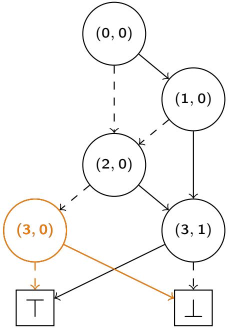
Seek	Sum	Result
$(3,0)$	2	0

Priority Queue: Q_{count}:

$$\begin{array}{l} [\\ ((1,0) \xrightarrow{\top} (3,1),\quad 1)\quad , \\ ((2,0) \xrightarrow{\top} (3,1),\quad 2)\quad] \end{array}$$

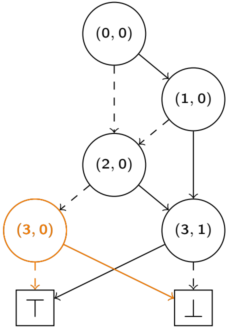


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

Priority Queue: Q_{count}:

$$[\ ((1,0) \xrightarrow{\top} (3,1),\ 1),\ ((2,0) \xrightarrow{\top} (3,1),\ 2)\ ]$$



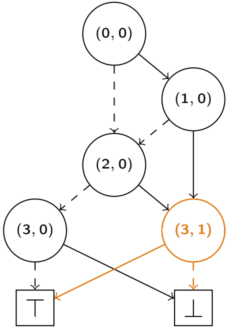
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1),\quad 1)$,
$((2,0) \xrightarrow{\top} (3,1),\quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

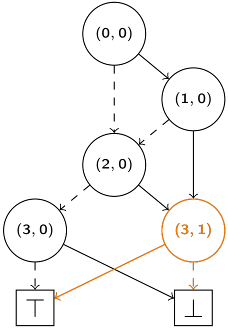
Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]

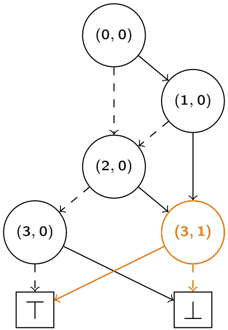


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	1	2

Priority Queue: Q_{count}:

[$((2,0) \xrightarrow{\top} (3,1),\quad 2)$]



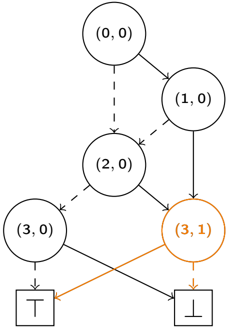
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count}:

[

]



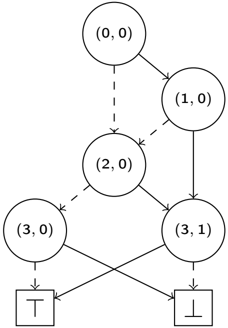
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	3	5

Priority Queue: Q_{count}:

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result

5

Priority Queue: Q_{count}:

[

]

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ssoelvsten.github.io

Adiar

github.com/ssoelvsten/adiar

ssoelvsten.github.io/adiar

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