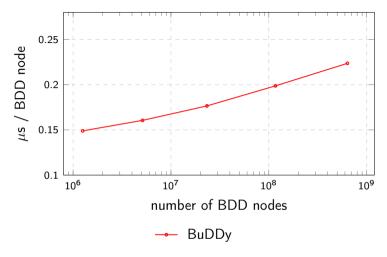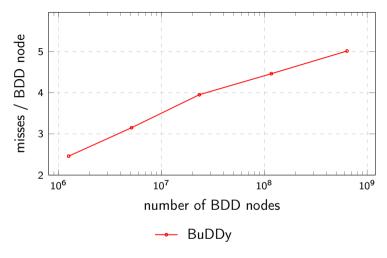# Adiar

Binary Decision Diagrams in External Memory

Steffan Christ Sølvsten, Jaco van de Pol,
Anna Blume Jakobsen, and Mathias Weller Berg Thomasen
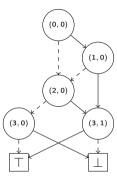TACAS 2022

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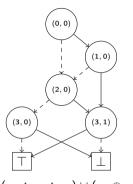Minimal running time for the *Queens* problems.

Cache-misses for the *Queens* problems.

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:
[

]

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:
$$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$$
$$]$$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$[\quad ((0, 0) \xrightarrow{\top} (1, 0), \quad 1) \quad ,$

$\quad ((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$]$

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$[\quad ((0, 0) \xrightarrow{\top} (1, 0), \quad 1) \quad ,$
$\quad ((0, 0) \xrightarrow{\bot} (2, 0), \quad 1) \quad ,$

$]$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

$$[\quad ((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad , $$

$$]$$

(a) $\left(x_0 \wedge x_1 \wedge x_3\right) \vee \left(x_2 \oplus x_3\right)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

[

$((0, 0) \xrightarrow{\perp} (2, 0), \quad 1)$,

$((1, 0) \xrightarrow{\perp} (2, 0), \quad 1)$,

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,

]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

[

$((0, 0) \xrightarrow{\perp} (2, 0),\quad 1)$,

$((1, 0) \xrightarrow{\perp} (2, 0),\quad 1)$,

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)$,

]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

$$\big[\\ ((0,0) \xrightarrow{\perp} (2,0), \quad 1) \quad , \\ ((1,0) \xrightarrow{\perp} (2,0), \quad 1) \quad , \\ \\ ((1,0) \xrightarrow{\top} (3,1), \quad 1) \quad , \\ \big]$$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	1	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\perp} (2, 0),\quad 1)\quad ,$

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)\quad ,$

]

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,

]

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\perp} (3, 0),\quad 2)\quad ,$

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)\quad ,$

$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)\quad]$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\perp} (3, 0),\quad 2)$,
$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)$,
$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)$]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\perp} (3, 0), \quad 2) \quad ,$
$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	2	0

Priority Queue: Q_{count}:

$$[$$

$$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$$
$$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$$

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2)$]

3

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)$,
$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)$]

3

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	0	2

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)$,
$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)$]

3

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	1	2

Priority Queue: Q_{count}:

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad$]

3

(a) $\left(x_0 \wedge x_1 \wedge x_3\right) \vee \left(x_2 \oplus x_3\right)$

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	5

Priority Queue: Q_{count}:

[

]

3

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$
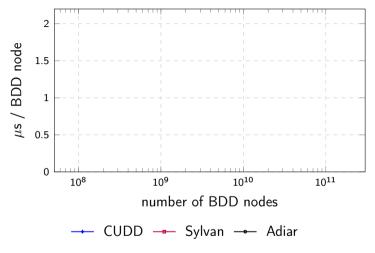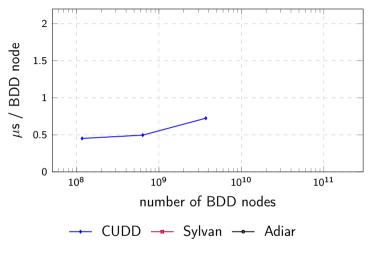
Result
5

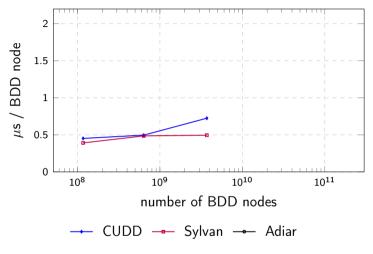Priority Queue: Q_{count}:
[

]
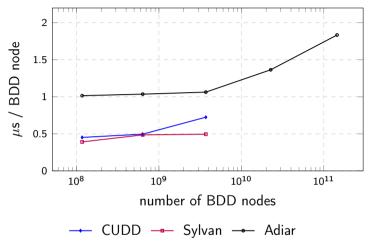
Adiar

github.com/ssoelvsten/adiar

Minimal running time for the *Queens* problems.

4

Minimal running time for the *Queens* problems.

Minimal running time for the *Queens* problems.

Minimal running time for the *Queens* problems.

Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38
$O(N \log N)$	0.058

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38
$O(N \log N)$	0.058
$O(N)$	0.006

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

Steffan Christ Sølvsten

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🐦 @ssoelvsten

Adiar

</> github.com/ssoelvsten/adiar

📕 ssoelvsten.github.io/adiar

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