

I/O-Efficient Algorithms and Data Structures

Steffan Christ Sølvesten

8th of September, 2023



$0, 1, 2, \dots, i, i+1, \dots, 2i, 2i+1, \dots, N-1$















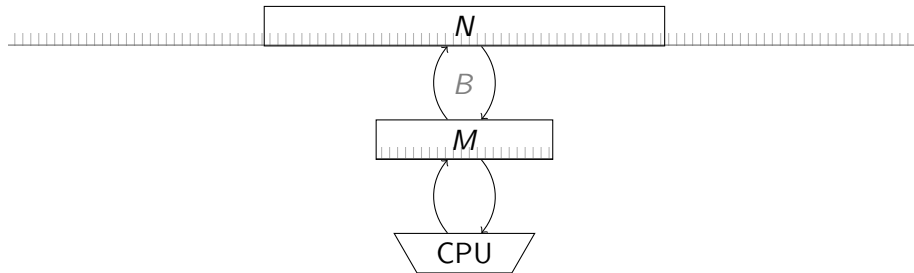






I/O Model

Aggarwal and Vitter '87



I/O Model : Sequential Access

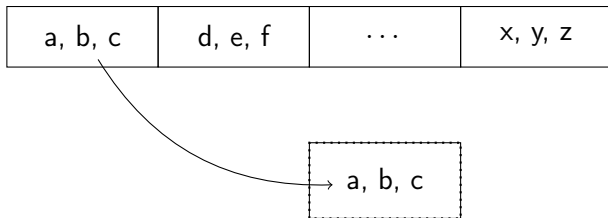
Aggarwal and Vitter '87

a, b, c	d, e, f	...	x, y, z
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æ, ø

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æ, ø, å

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Aggarwal and Vitter '87

a, b, c	d, e, f	...	x, y, z	æ, ø, å
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Time : N
I/O : N/B
Memory : B

I/O Model : Stack



I/O Model : Stack



I/O Model : Stack



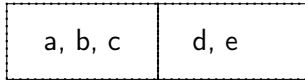
I/O Model : Stack



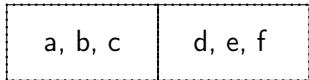
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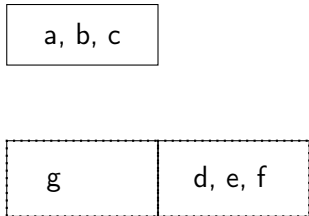
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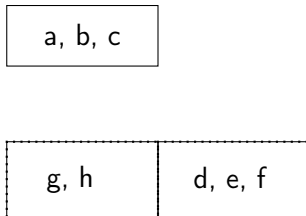
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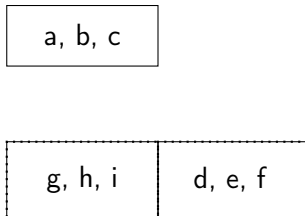
I/O Model : Stack



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I/O Model : Stack



I/O Model : Stack



I/O Model : Stack

a, b, c	d, e, f
---------	---------

g, h, i	j
---------	---

I/O Model : Stack



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I/O Model : Stack



I/O Model : Stack



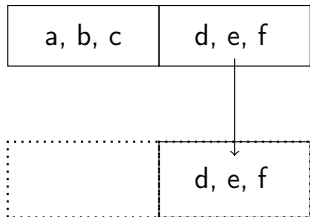
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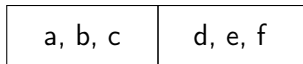
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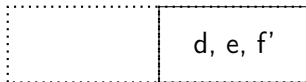
I/O Model : Stack



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I/O Model : Stack

a, b, c	d, e, f
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g', h'	d, e, f'
--------	----------

I/O Model : Stack



I/O Model : Stack



I/O Model : Stack

a, b, c	d, e, f'
---------	----------

g', h', i'	j'
------------	----

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g', h', i'	j'
------------	----

Time : $O(N)$
I/O : $O(N/B)$
Memory : $2B$

I/O Model : M/B-way Mergesort

Aggarwal and Vitter '87



I/O Model : M/B-way Mergesort

Aggarwal and Vitter '87



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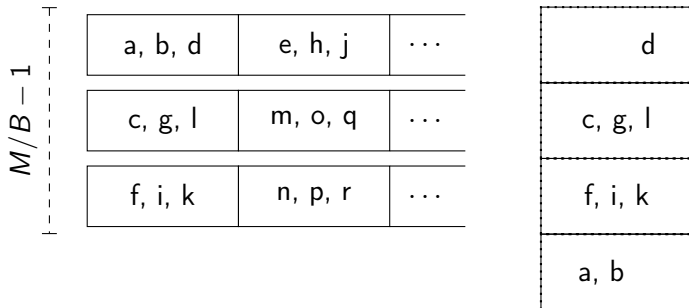
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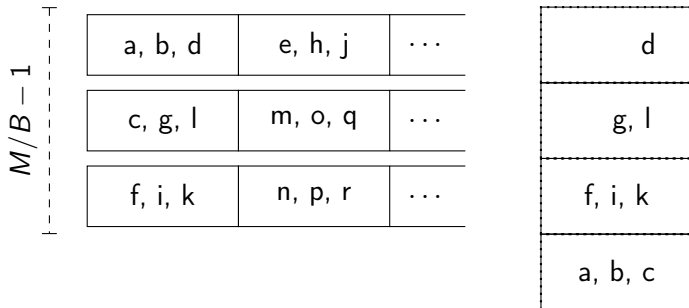
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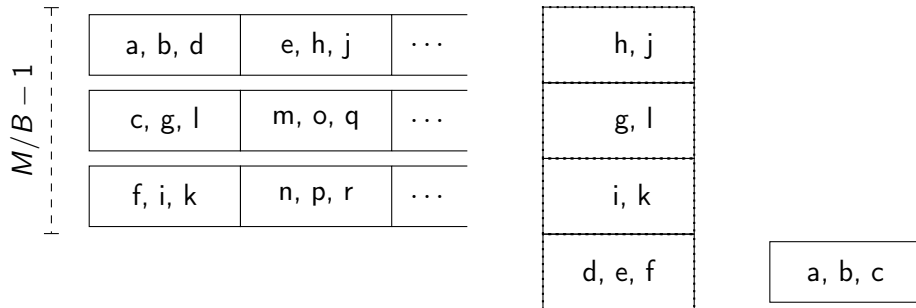
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I/O Model : M/B-way Mergesort

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B

I/O Model : M/B-way Mergesort

Aggarwal and Vitter '87



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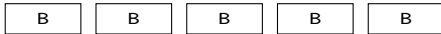
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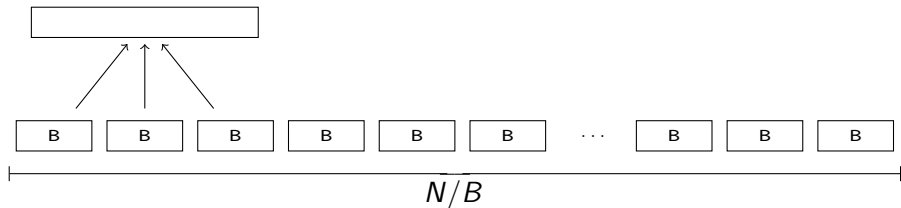
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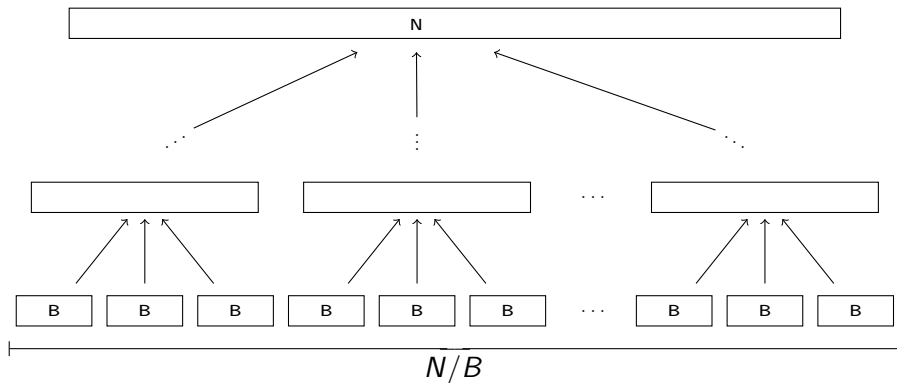
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I/O Model : M/B-way Mergesort

Aggarwal and Vitter '87

Theorem

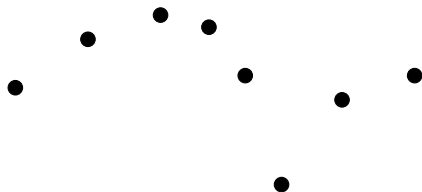
N elements can be sorted in $\Theta(N/B \cdot \log_{M/B}(N/B))$ I/Os.

Convex Hull : Graham Scan

Graham '72

Convex Hull

Compute the *convex hull* for N points in the plane.



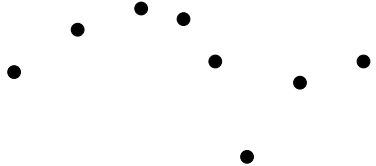
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Convex Hull can be computed in $O(N/B \cdot \log_{M/B}(N/B))$ I/Os.

Convex Hull : Graham Scan

Graham '72

Upper Hull:



Convex Hull : Graham Scan

Graham '72

Upper Hull:

- Sort input points by x -axis

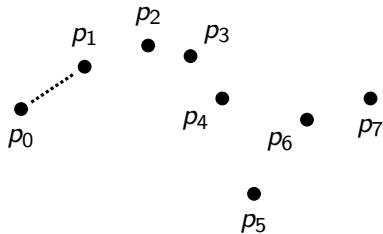


Convex Hull : Graham Scan

Graham '72

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- Initialize stack $S = [p_0, p_1]$



Convex Hull : Graham Scan

Graham '72

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 - 1 Let p_s, p_t be the two top-most points of S
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 - Pop p_t and go-to 1
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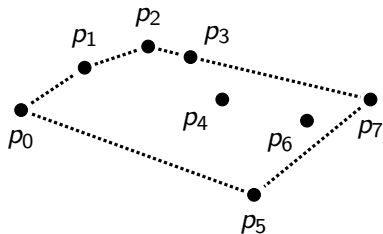
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Lower Hull:

- Symmetric...



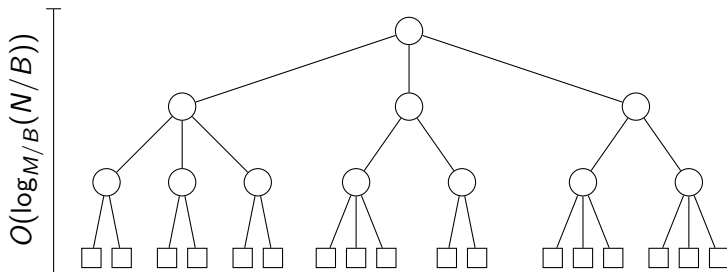
a-b Tree

Huddleston and Mehlhorn '82



Buffer Tree

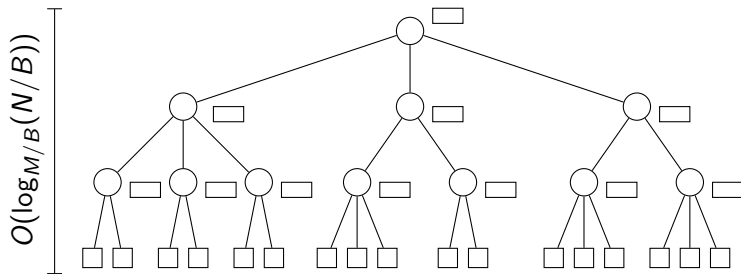
Arge '95



$$a = \frac{1}{4}M/B, \quad b = M/B, \quad \text{Leaf Size} = B$$

Buffer Tree

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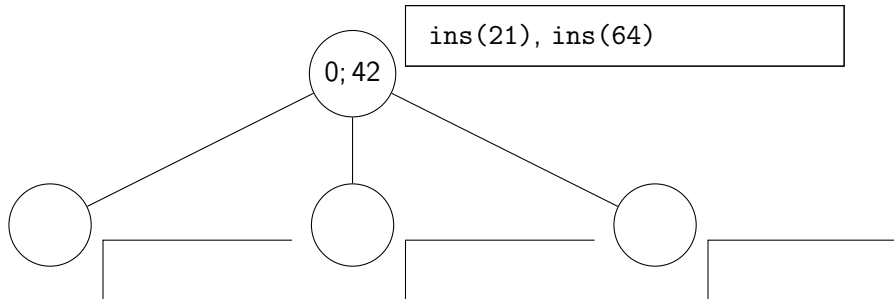
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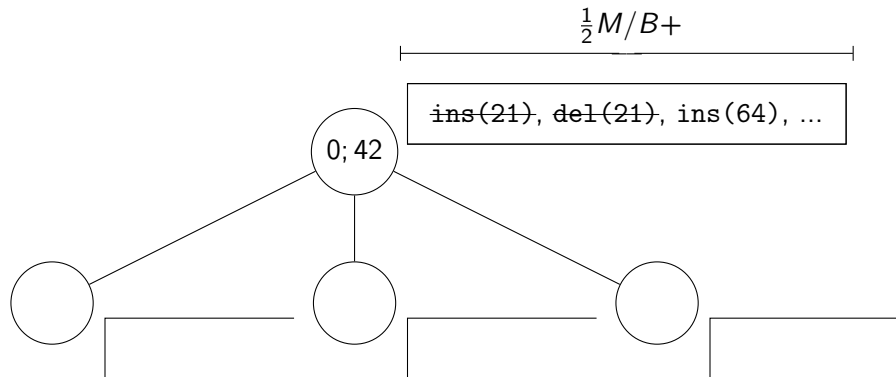
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Theorem

A Buffer Tree can resolve N inserts and deletes in $\Theta(N/B \cdot \log_{M/B}(N/B))$ I/Os.

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A Buffer Tree with N requests can empty all its buffers, and output all remaining sorted elements, in $\Theta(N/B)$ I/Os.

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A Buffer Tree with N requests can empty all its buffers, and output all remaining sorted elements, in $\Theta(N/B)$ I/Os.

Corollary

An I/O-efficient Priority Queue can resolve N push and delete min operations in $\Theta(N/B \cdot \log_{M/B}(N/B))$ I/Os.

Proof.

Use an $M/2$ sized internal memory priority queue, pq. If pq overflows, move $M/4$ the largest elements to a Buffer Tree, t. If pq underflows, obtain the $M/4$ smallest elements from t. \square

Binary Decision Diagrams

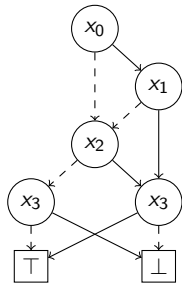
Arge '96, Sølvesten '22

#Paths

Given a Binary Decision Diagram of N nodes, compute the number of paths from the root to the \top terminal.

Theorem

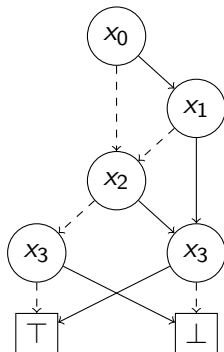
#Paths can be computed in
 $O(N/B \cdot \log_{M/B}(N/B))$ I/Os.



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Binary Decision Diagrams

Arge '96, Sølvsten '22



Decision Diagram

[((0, 0), (2, 0), (1, 0)) ,
((1, 0), (2, 0), (3, 1)) ,
((2, 0), (3, 0), (3, 1)) ,
((3, 0), \top , \perp) ,
((3, 1), \perp , \top)]

On-Disk Format

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Binary Decision Diagrams

Arge '96, Sølvsten '22

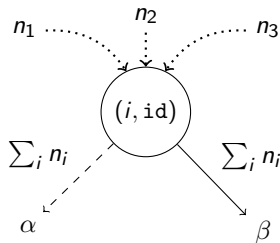


Idea

Count the number of in-going paths to each node.

Binary Decision Diagrams

Arge '96, Sølvsten '22



Time-Forward Processing

Defer work with $Q_{\text{count}} : \text{PriorityQueue}(\langle s \rightarrow t, \mathbb{N} \rangle)$ sorted on t in ascending order.

$$((i, \text{id}) \xrightarrow{\perp} \alpha, \quad \sum_i n_i), \quad ((i, \text{id}) \xrightarrow{\top} \beta, \quad \sum_i n_i)$$

Binary Decision Diagrams

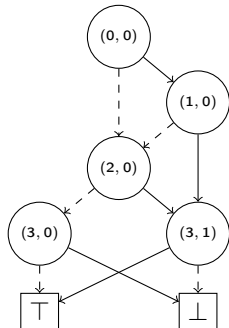
Arge '96, Sølvsten '22



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Arge '96, Sølvsten '22



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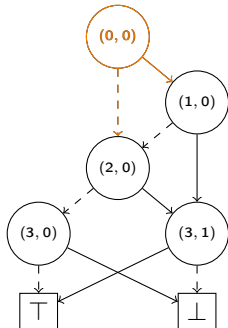
Priority Queue: Q_{count} :

[

]

Binary Decision Diagrams

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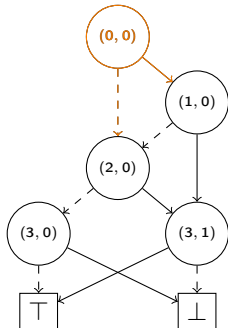
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]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count} :

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Binary Decision Diagrams

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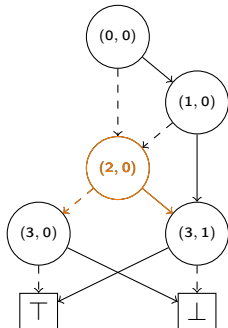
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Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

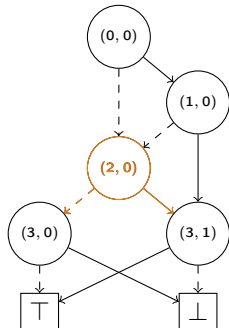
Seek	Sum	Result
(2, 0)	1	0

Priority Queue: Q_{count} :

[
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

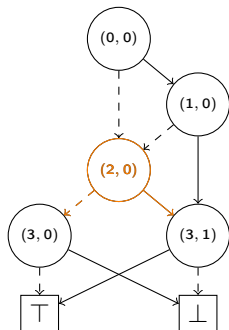
Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

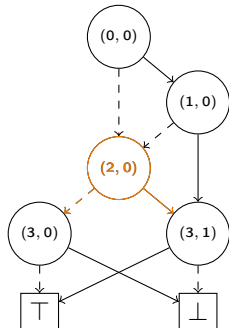
Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$,
$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
$((2, 0) \xrightarrow{\top} (3, 1), 2)$]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	0

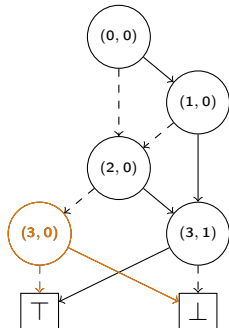
Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	2	2

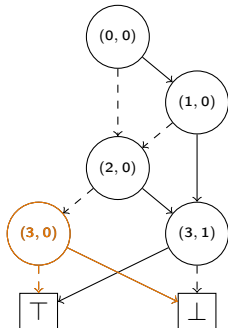
Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

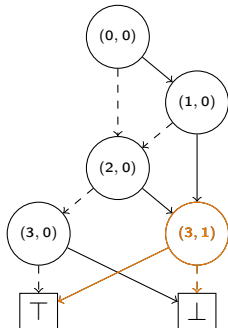
Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	1	2

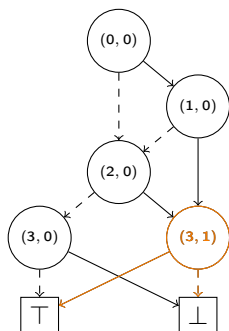
Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count} :

[

]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek

(3, 1)

Sum

3

Result

5

Priority Queue: Q_{count} :

[

]

Binary Decision Diagrams

Arge '96, Sølvsten '22



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result

5

Priority Queue: Q_{count} :

[

]

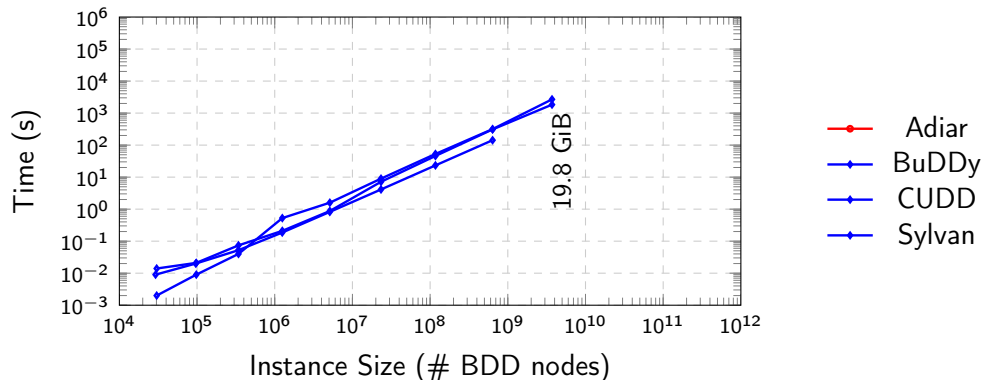
Adiar

I/O-efficient Decision Diagrams

github.com/ssoelvsten/adiar

Binary Decision Diagrams

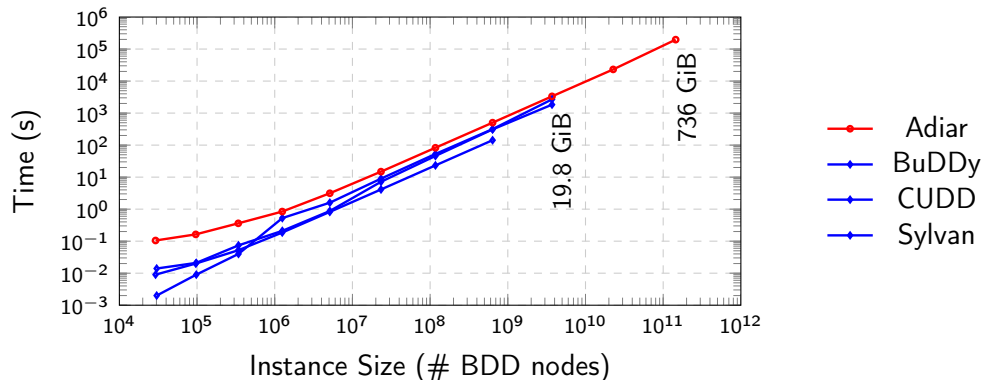
Arge '96, Sølvsten '22



Running time for the *N-Queens* problems.

Binary Decision Diagrams

Arge '96, Sølvsten '22



Running time for the N -Queens problems.

Further Reading : Foundations

- **Aggarwal and Vitter (1987)**

“The Input/Output Complexity of Sorting and Related Problems”

The I/O-model, Sorting, Permutation, FFT, and Matrix transposition.

- **Arge, Goodrich, Nelson, and Sitchinava (2008)**

“Fundamental Parallel Algorithms for Private-cache Chip Multiprocessors.”

The I/O-model for Multi-Threading.

Further Reading : Data Structures

- **Arge (1995)**

"The Buffer Tree: A new technique for Optimal I/O-algorithms"

An I/O-efficient Tree, Priority Queue, and Range Tree.

- **Sanders (2002)**

"Fast Priority Queues for Cached Memory"

A much faster I/O-efficient Priority Queue.

- **Agarwal, Arge and Yi (2006)**

"I/O-Efficient Batched Union-Find and Its Applications to Terrain Analysis"

An I/O-efficient (Lazy) Union-Find.

Further Reading : Algorithms

- **Goodrich, Tsay, Vengroff, and Vitter (1993)**

“External-Memory Computational Geometry”

Distribution Sweeping and other algorithms.

- **Chiang, Goodrich, Grove, Tamassia, Vengroff, and Vitter (1995)**

“External-memory Graph Algorithms”

Time-forward Processing and other algorithms.

- **Arge, Toma, Vitter (2001)**

“I/O-Efficient Algorithms for Problems on Grid-Based Terrains”

The TERRAFLOW algorithm.

Further Reading : Libraries (C++)

- **TPIE : Templated Portable I/O Environment**

github.com/thomasmoelhave/tpie

Duke University and Aarhus University

- **STXXL : Standard Template library for XXL data sets**

github.com/stxxl/stxxl

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Adiar

🔗 github.com/ssoelvsten/adiar

📖 ssoelvsten.github.io/adiar

Distribution Sweeping

Goodrich, Tsay, Vengroff, and Vitter '93

Batched Range Searching

Given N axis-parallel rectangles and N points in the plane, compute for each point p all rectangles containing p .



Theorem

Batched Range Searching can be solved in $O(\text{sort}(N) + \text{scan}(T))$ I/Os.

Distribution Sweeping

Goodrich, Tsay, Vengroff, and Vitter '93

Preprocessing:

Algorithm:



Distribution Sweeping

Goodrich, Tsay, Vengroff, and Vitter '93

Preprocessing:

- Split each rectangle into two vertical lines.

Algorithm:



Distribution Sweeping

Goodrich, Tsay, Vengroff, and Vitter '93

Preprocessing:

- Split each rectangle into two vertical lines.
- Sort all lines and points by their x -value.

Algorithm:



Distribution Sweeping

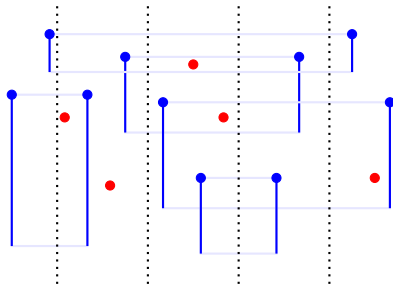
Goodrich, Tsay, Vengroff, and Vitter '93

Preprocessing:

- Split each rectangle into two vertical lines.
- Sort all lines and points by their x -value.

Algorithm:

- Split all data into $\Theta(\sqrt{M/B})$ slabs. Solve these recursively; output is given sorted by y -value.



Distribution Sweeping

Goodrich, Tsay, Vengroff, and Vitter '93

Preprocessing:

- Split each rectangle into two vertical lines.
- Sort all lines and points by their x -value.

Algorithm:

- Split all data into $\Theta(\sqrt{M/B})$ *slabs*. Solve these recursively; output is given sorted by y -value.
- Merge slabs together, report points between line segments outside its slab.
 - Use $\Theta(\sqrt{M/B^2}) = \Theta(\sqrt{M/B})$ multi-slabs to maintain each *active* rectangle.
 - Output points and un-matched line segments.

