Efficient External Memory Algorithms for Binary Decision Diagram Manipulation

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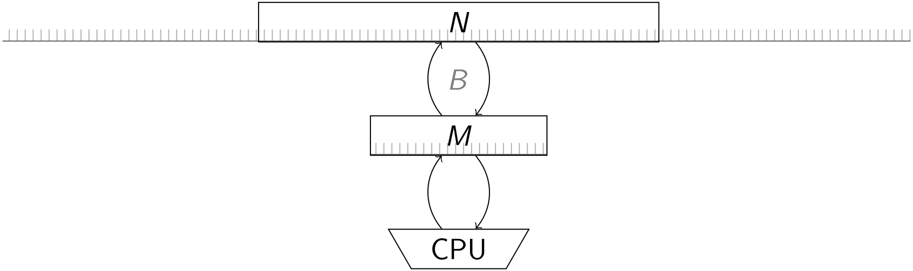
AARHUS UNIVERSITY



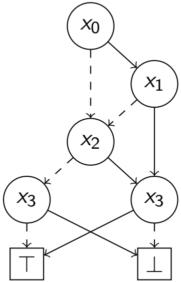
The I/O model by Aggarwal and Vitter '87

For any realistic values of N, M, and B we have that

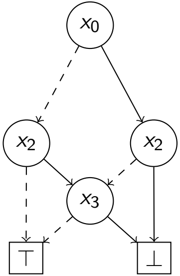
$$N/B \quad < \quad \mathrm{sort}(N) \triangleq N/B \cdot \log_{M/B} N/B \quad \ll \quad N \ ,$$

Theorem (Aggarwal and Vitter '87)
N elements can be sorted in $\Theta(sort(N))$ I/Os.

Theorem (Arge '95)
N elements can be inserted in and extracted from a Priority Queue in $\Theta(sort(N))$ I/Os.



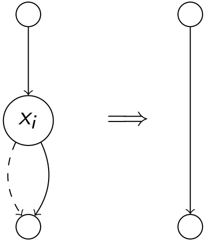
(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$



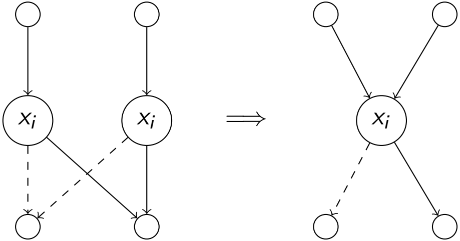
(b) $\neg(x_0 \ ? \ x_2 \lor x_3 \ : \ x_2 \land x_3)$

Examples of (Reduced Ordered) Binary Decision Diagrams.

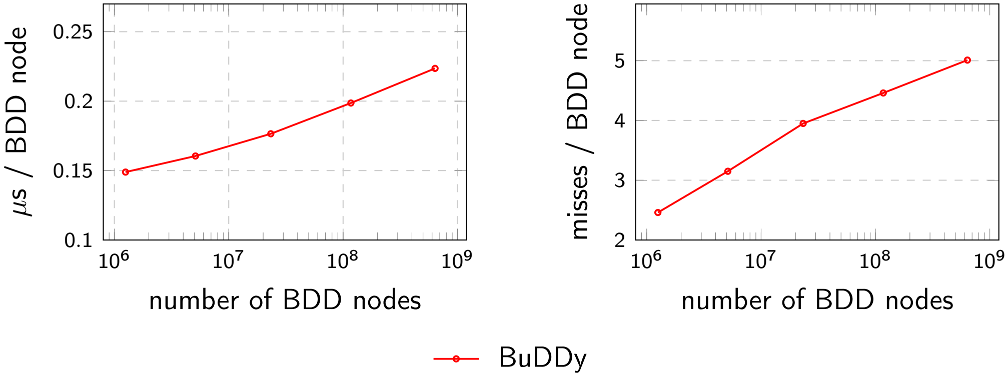
Theorem (Bryant '86)
For a fixed variable order, if one exhaustively applies the two rules below, then one obtains the Reduced OBDD, which is a unique canonical form of the function.



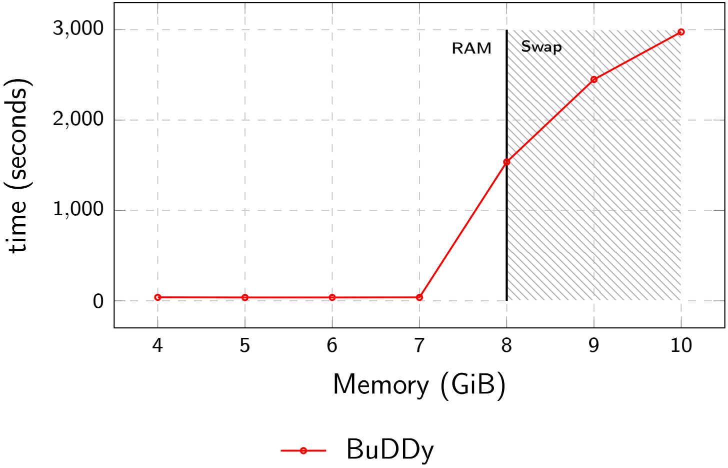
(1) Remove redundant nodes



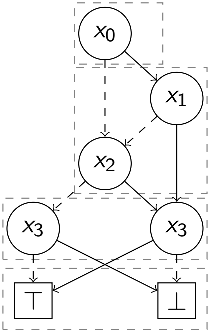
(2) Merge duplicate nodes



Cache behaviour for the N-Queens problem.



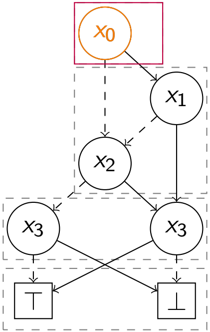
Running time for *Tic-Tac-Toe* with $N = 21$.



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

$$M = 4,\ B = 2$$

node I/Os	cache lookups
0	0



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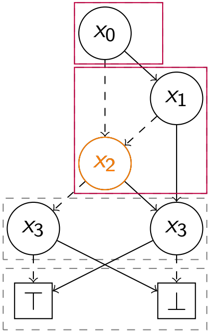
node I/Os	cache lookups
1	1



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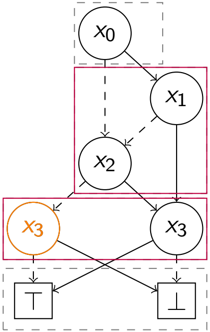
$$M = 4,\ B = 2$$

node I/Os	cache lookups
2	2



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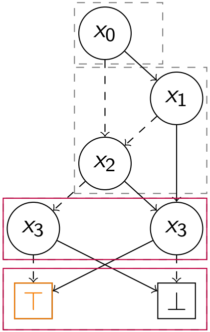
node I/Os	cache lookups
3	3



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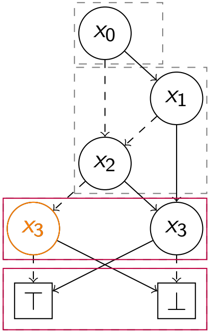
node I/Os	cache lookups
4	3



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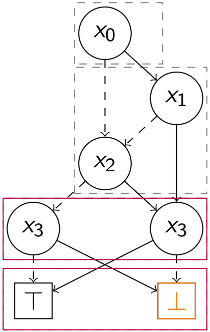

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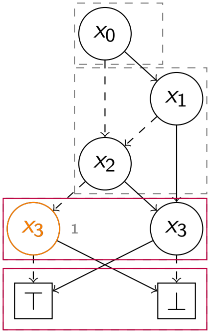

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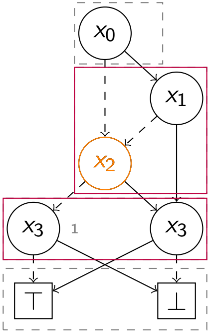

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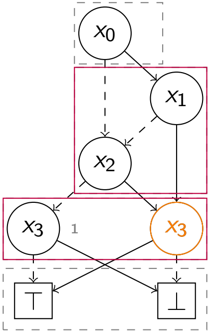
node I/Os	cache lookups
5	3



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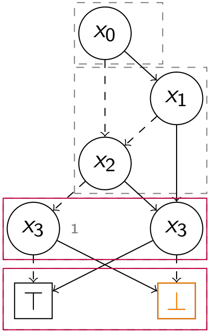
$M = 4,\ B = 2$

node I/Os	cache lookups
5	4



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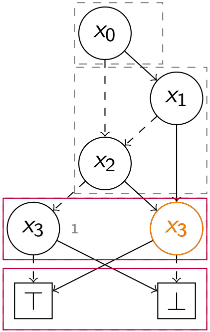
$$M = 4,\ B = 2$$

node I/Os	cache lookups
6	4



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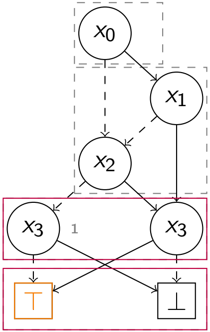

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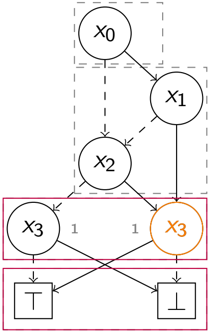

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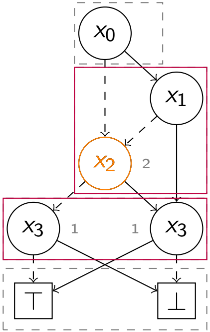

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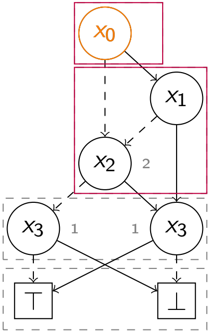
node I/Os	cache lookups
7	4



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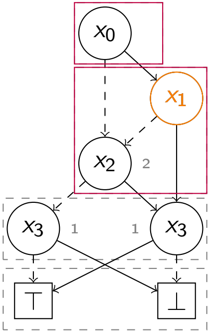
$$M = 4,\ B = 2$$

node I/Os	cache lookups
8	4



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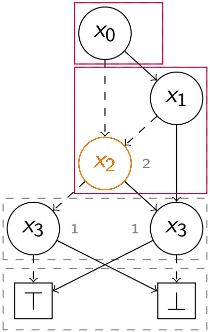
$$M = 4,\ B = 2$$

node I/Os	cache lookups
8	5



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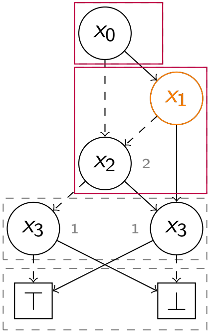
node I/Os	cache lookups
8	6



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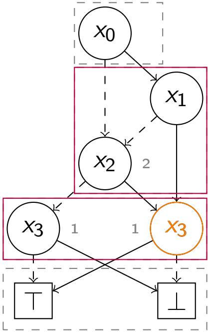

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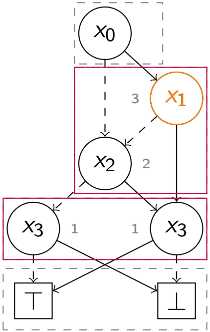
$$M = 4,\ B = 2$$

node I/Os	cache lookups
9	7



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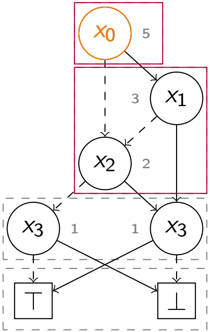

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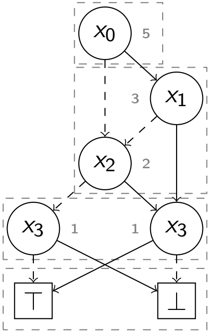
node I/Os	cache lookups
10	7



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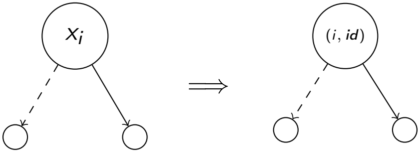
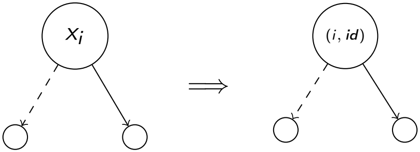
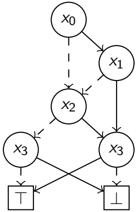
10	7


x_i
(i, id)

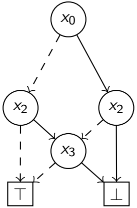


$$(i_1, id_1) < (i_2, id_2) \equiv i_1 < i_2 \vee (i_1 = i_2 \wedge id_i < id_j)$$



$$[\quad ((0,0),(2,0),(1,0)),\quad ((1,0),(2,0),(3,1)),\quad ((2,0),(3,0),(3,1)),\quad ((3,0),\bot,\top),\quad ((3,1),\top,\bot)\quad]$$

(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

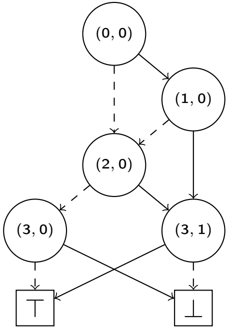


$$[\quad ((0,0),(2,0),(2,1)),\quad ((2,0),\top,(3,0)),\quad ((2,1),(3,0),\bot),\quad ((3,0),\top,\bot)\quad]$$

(b) $\neg(x_0 \;?\; x_2 \lor x_3 \;:\; x_2 \land x_3)$

Node-based representation of prior shown BDDs

CountPaths Example



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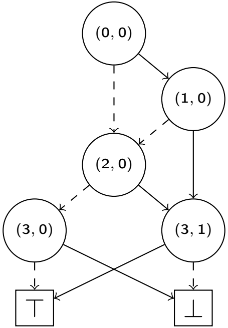
CountPaths Example



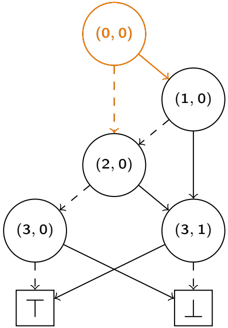
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: $Q_{\boldsymbol{count}}$:

[

]

CountPaths Example



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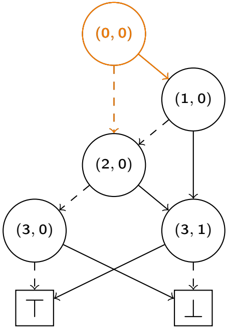
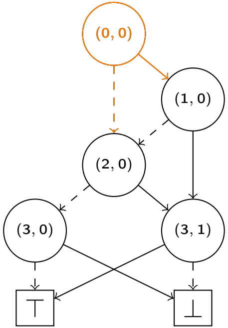


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: $Q_{\boldsymbol{count}}$:

[$((0,0) \xrightarrow{\top} (1,0),\quad 1)$,
$((0,0) \xrightarrow{\perp} (2,0),\quad 1)$,
]

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

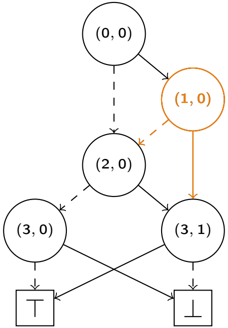
Priority Queue: $Q_{\textit{count}}$:

$[\quad ((0,0) \xrightarrow{\top} (1,0),\ 1),$

$((0,0) \xrightarrow{\bot} (2,0),\ 1),$

$]$

CountPaths Example



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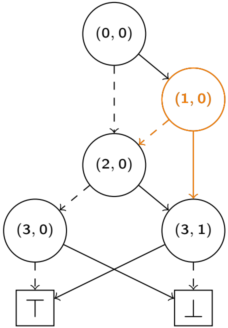
$$((0,0) \xrightarrow{\perp} (2,0), \quad 1) \quad ,$$
$$]$$

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

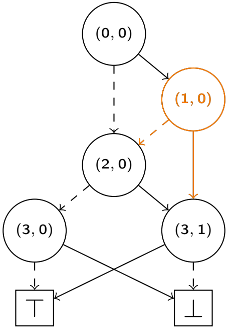
Priority Queue: $Q_{\mathbf{count}}$:

[

$((0,0) \xrightarrow{\perp} (2,0),\quad 1)$,

]

CountPaths Example



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Seek	Sum	Result
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Priority Queue: Q_{count}:

[

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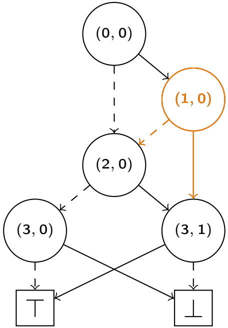
$((1,0) \xrightarrow{\top} (3,1),\quad 1)$,

]

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	0	0

Priority Queue: Q_{count}:

[
$((0,0) \xrightarrow{\perp} (2,0),\quad 1)$,
$((1,0) \xrightarrow{\perp} (2,0),\quad 1)$,
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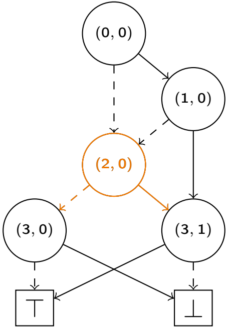

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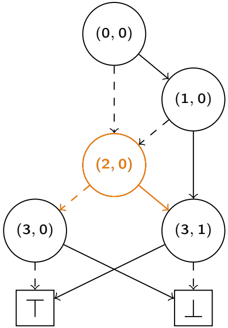

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Seek	Sum	Result
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Priority Queue: Q_{count}:

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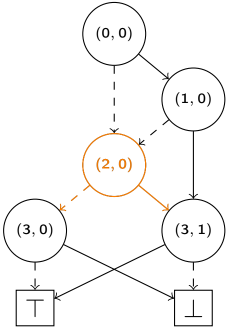


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

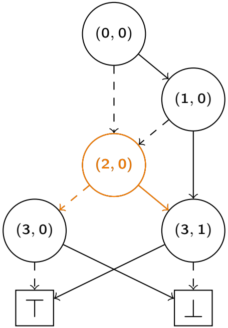
Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,

]

CountPaths Example



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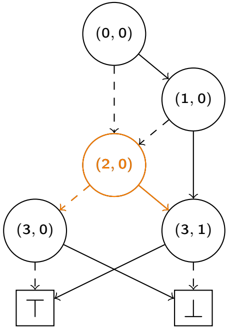
Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: $Q_{\boldsymbol{count}}$:

[
$((2,0) \xrightarrow{\perp} (3,0),\ \ 2)$,
$((1,0) \xrightarrow{\top} (3,1),\ \ 1)$,
$((2,0) \xrightarrow{\top} (3,1),\ \ 2)$]

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: $Q_{\boldsymbol{count}}$:

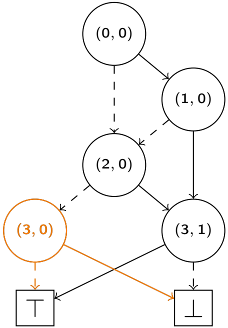
[

$((2, 0) \xrightarrow{\bot} (3, 0), \quad 2)$,

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2)$]

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

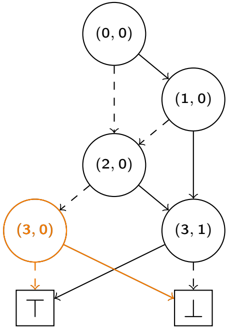
Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: $Q_{\boldsymbol{count}}$:

[

$((2, 0) \xrightarrow{\perp} (3, 0), \quad 2)$,
$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2)$]

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	0

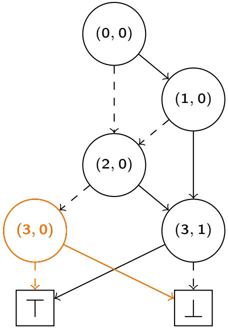
Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

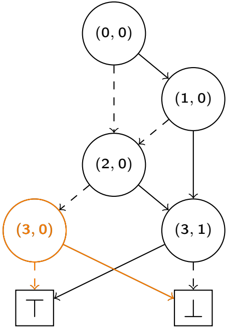
Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1),\quad 1)$,

$((2,0) \xrightarrow{\top} (3,1),\quad 2)$]

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	0	2

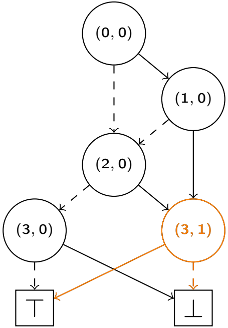
Priority Queue: $Q_{\boldsymbol{count}}$:

[

$((1,0) \xrightarrow{\top} (3,1),\quad 1)$,

$((2,0) \xrightarrow{\top} (3,1),\quad 2)$]

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

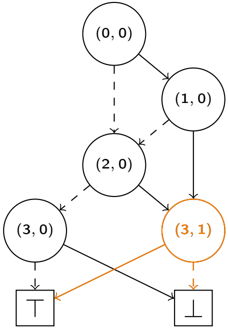
Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]

CountPaths Example



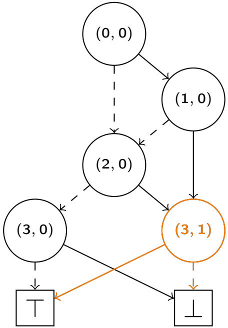
(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	1	2

Priority Queue: Q_{count}:

[$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)$]

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

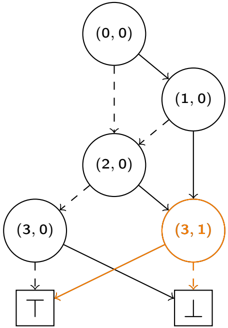
Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count}:

[

]

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

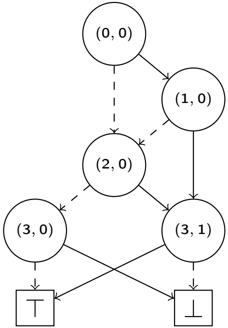
Seek	Sum	Result
$(3, 1)$	3	5

Priority Queue: Q_{count}:

[

]

CountPaths Example



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

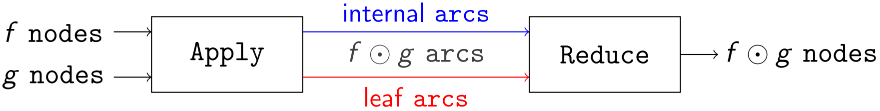
Result

5

Priority Queue: $Q_{\boldsymbol{count}}$:

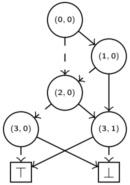
[

]

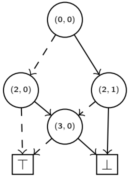


f nodes
g nodes
Apply
internal arcs
$f \odot g$ arcs
leaf arcs
Reduce
$f \odot g$ nodes

Apply Example (∧)



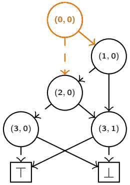
(a) $(x_{\mathbf{0}} \wedge x_{\mathbf{1}} \wedge x_{\mathbf{3}}) \vee (x_{\mathbf{2}} \oplus x_{\mathbf{3}})$



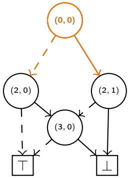
(b) $\neg(x_{\mathbf{0}}\ ?\ x_{\mathbf{2}} \vee x_{\mathbf{3}}\ :\ x_{\mathbf{2}} \wedge x_{\mathbf{3}})$

(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



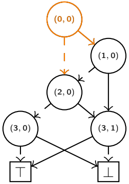
(a) $(x_{\mathbf{0}} \wedge x_{\mathbf{1}} \wedge x_{\mathbf{3}}) \vee (x_{\mathbf{2}} \oplus x_{\mathbf{3}})$



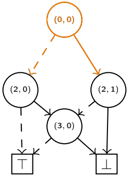
(b) $\neg(x_{\mathbf{0}}\ ?\ x_{\mathbf{2}} \vee x_{\mathbf{3}}\ :\ x_{\mathbf{2}} \wedge x_{\mathbf{3}})$

(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



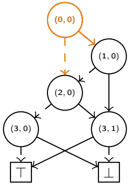
(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Priority Queue: $Q_{\mathbf{app}:1}$:

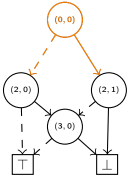
[$(0,0) \xrightarrow{\top} ((1,0),(2,1))$,
$(0,0) \xrightarrow{\bot} ((2,0),(2,0))$,
]

(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

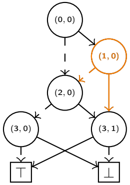
Seek:
$\min((1,0),(2,1))$

Priority Queue: $Q_{\mathit{app}:1}$:

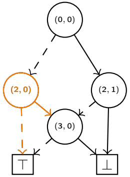
$$[\quad (0,0) \xrightarrow{\top} ((1,0),(2,1)) \quad , \quad (0,0) \xrightarrow{\bot} ((2,0),(2,0)) \quad , \quad]$$

(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0\ ?\ x_2 \vee x_3\ :\ x_2 \wedge x_3)$

Seek:

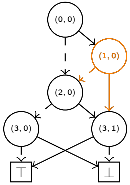
$\min((1,0),(2,1))$

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{1}}$:

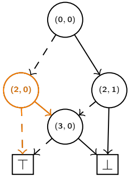
$$[\quad (0,0) \xrightarrow{\top} ((1,0),(2,1)) \quad , \quad (0,0) \xrightarrow{\bot} ((2,0),(2,0)) \quad , \quad]$$

(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

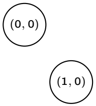


(b) $\neg(x_0 \;?\; x_2 \vee x_3 \;:\; x_2 \wedge x_3)$

Seek:
$\min((1,0),(2,1))$

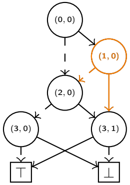
Priority Queue: $Q_{\boldsymbol{app}:\mathbf{1}}$:

$$
\begin{array}{llll}
[& (0,0) \xrightarrow{\top} ((1,0),(2,1)) & , \\
 & (0,0) \xrightarrow{\bot} ((2,0),(2,0)) & , \\
 & (1,0) \xrightarrow{\bot} ((2,0),(2,1)) & , \\
 & (1,0) \xrightarrow{\top} ((3,1),(2,1)) & , \\
 & &]
\end{array}
$$

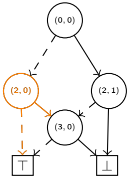


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



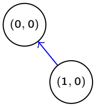
(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Seek:
$\min((1,0),(2,1))$

Priority Queue: $Q_{app:1}$:

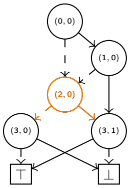
$$[\ (0,0) \xrightarrow{\bot} ((2,0),(2,0)),\quad (1,0) \xrightarrow{\bot} ((2,0),(2,1)),\quad (1,0) \xrightarrow{\top} ((3,1),(2,1)),\ ]$$

Output:
$(0,0) \xrightarrow{\top} (1,0)$

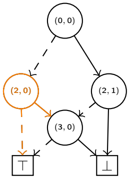


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Seek:
$\min((2,0),(2,0))$

Priority Queue: $Q_{app:1}$:

[

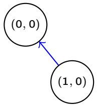
$(0,0) \xrightarrow{\perp} ((2,0),(2,0))$,

$(1,0) \xrightarrow{\perp} ((2,0),(2,1))$,

$(1,0) \xrightarrow{\top} ((3,1),(2,1))$,

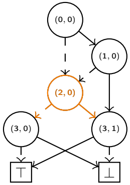
]

Output:

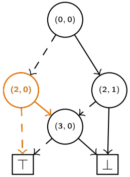


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



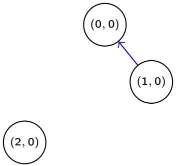
(b) $\neg(x_0 \; ? \; x_2 \vee x_3 \; : \; x_2 \wedge x_3)$

Seek:
$\min((2,0),(2,0))$

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{1}}$:

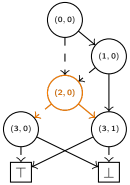
$$
\begin{aligned}
[\; & (0,0) \xrightarrow{\bot} ((2,0),(2,0)) \;, \\
& (1,0) \xrightarrow{\bot} ((2,0),(2,1)) \;, \\
& (1,0) \xrightarrow{\top} ((3,1),(2,1)) \;, \\
& (2,0) \xrightarrow{\top} ((3,1),(3,0)) \;, \\
& (2,0) \xrightarrow{\bot} ((3,0),\top) \;]
\end{aligned}
$$

Output:

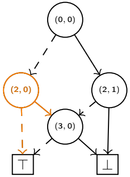


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ?\ x_2 \vee x_3 \ :\ x_2 \wedge x_3)$

Seek:

$\min((2,0),(2,0))$

Priority Queue: $Q_{app:1}$:

[

$(1,0) \xrightarrow{\perp} ((2,0),(2,1))$,

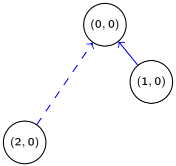
$(1,0) \xrightarrow{\top} ((3,1),(2,1))$,

$(2,0) \xrightarrow{\top} ((3,1),(3,0))$,

$(2,0) \xrightarrow{\perp} ((3,0),\top)$]

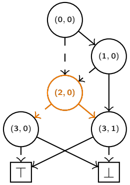
Output:

$(0,0) \xrightarrow{\perp} (2,0)$

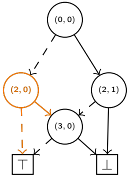


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



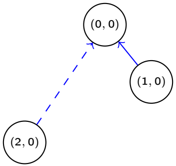
(b) $\neg(x_0 \ ?\ x_2 \vee x_3 \ :\ x_2 \wedge x_3)$

Seek:
min((2, 0), (2, 1))

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{1}}$:

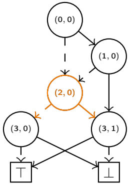
$$\left[\begin{array}{ll} (1,0) \xrightarrow{\bot} ((2,0),(2,1)) & , \\ (1,0) \xrightarrow{\top} ((3,1),(2,1)) & , \\ (2,0) \xrightarrow{\top} ((3,1),(3,0)) & , \\ (2,0) \xrightarrow{\bot} ((3,0),\top) & \end{array}\right]$$

Output:

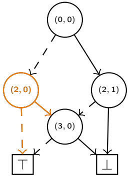


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Seek:
$\min((2,0),(2,1))$

Priority Queue: $Q_{app:1}$:

[

$(1,0) \xrightarrow{\top} ((3,1),(2,1))$,

$(2,0) \xrightarrow{\top} ((3,1),(3,0))$,

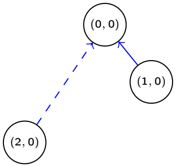
$(2,0) \xrightarrow{\bot} ((3,0),\top)$]

Priority Queue: $Q_{app:2}$:

[$(1,0) \xrightarrow{\bot} ((2,0),(2,1))$ $((3,0),(3,1))$,

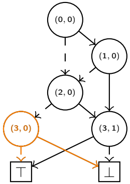
]

Output:

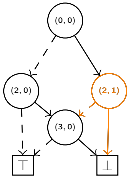


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Seek:
$\max((2,0),(2,1))$

Priority Queue: $Q_{app:1}$:

$[$

$(1,0) \xrightarrow{\top} ((3,1),(2,1))$,

$(2,0) \xrightarrow{\top} ((3,1),(3,0))$,

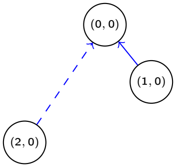
$(2,0) \xrightarrow{\bot} ((3,0),\top)$ $]$

Priority Queue: $Q_{app:2}$:

$[$ $(1,0) \xrightarrow{\bot} ((2,0),(2,1))$ $((3,0),(3,1))$,

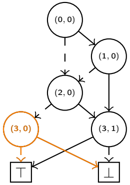
$]$

Output:

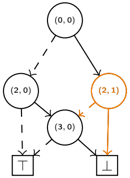


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \; ? \; x_2 \vee x_3 \; : \; x_2 \wedge x_3)$

Seek:
$\max((2,0),(2,1))$

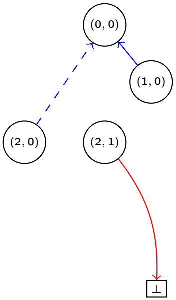
Priority Queue: $Q_{app:1}$:

$$
\begin{aligned}
[\quad & \\
& (1,0) \xrightarrow{\top} ((3,1),(2,1)) \quad , \\
& (2,1) \xrightarrow{\bot} ((3,0),(3,0)) \quad , \\
& (2,0) \xrightarrow{\top} ((3,1),(3,0)) \quad , \\
& (2,0) \xrightarrow{\bot} ((3,0),\top) \quad]
\end{aligned}
$$

Priority Queue: $Q_{app:2}$:

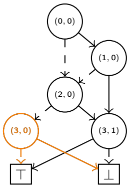
$$
[\quad (1,0) \xrightarrow{\bot} ((2,0),(2,1)) \quad ((3,0),(3,1)) \quad , \quad]
$$

Output:
$(2,1) \xrightarrow{\top} \bot$

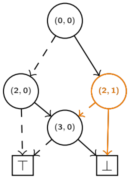


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \;?\; x_2 \vee x_3 \;:\; x_2 \wedge x_3)$

Seek:
$\max((2,0),(2,1))$

Priority Queue: $Q_{app:1}$:

[

$(1,0) \xrightarrow{\top} ((3,1),(2,1))$,
$(2,1) \xrightarrow{\bot} ((3,0),(3,0))$,
$(2,0) \xrightarrow{\top} ((3,1),(3,0))$,

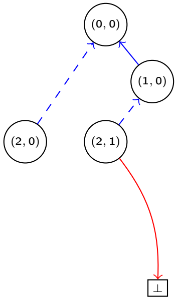
$(2,0) \xrightarrow{\bot} ((3,0),\top)$]

Priority Queue: $Q_{app:2}$:

[

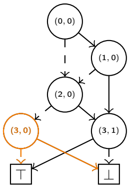
]

Output:
$(1,0) \xrightarrow{\bot} (2,1)$

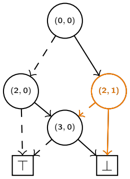


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \;?\; x_2 \vee x_3 \;:\; x_2 \wedge x_3)$

Seek:
$\min((3,1),(2,1))$

Priority Queue: $Q_{app:1}$:

[

$(1,0) \xrightarrow{\top} ((3,1),(2,1))$,
$(2,1) \xrightarrow{\bot} ((3,0),(3,0))$,
$(2,0) \xrightarrow{\top} ((3,1),(3,0))$,

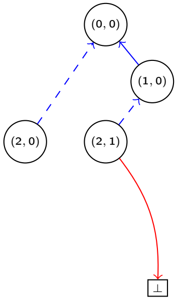
$(2,0) \xrightarrow{\bot} ((3,0),\top)$]

Priority Queue: $Q_{app:2}$:

[

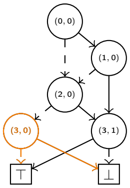
]

Output:

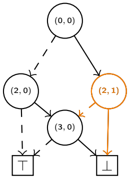


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \;?\; x_2 \vee x_3 \;:\; x_2 \wedge x_3)$

Seek:
$\min((3,1),(2,1))$

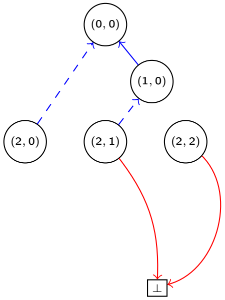
Priority Queue: $Q_{app:1}$:

$$
\begin{aligned}
[\; & (1,0) \xrightarrow{\top} ((3,1),(2,1)) ,\\
& (2,1) \xrightarrow{\perp} ((3,0),(3,0)) ,\\
& (2,0) \xrightarrow{\top} ((3,1),(3,0)) ,\\
& (2,2) \xrightarrow{\perp} ((3,1),(3,0)) ,\\
& (2,0) \xrightarrow{\perp} ((3,0),\top) \;]
\end{aligned}
$$

Priority Queue: $Q_{app:2}$:

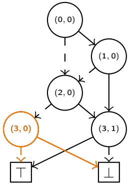
[ ]

Output:
$(2,2) \xrightarrow{\top} \perp$

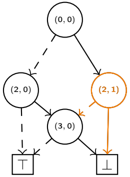


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Seek:
min((3, 1), (2, 1))

Priority Queue: $Q_{\mathit{app}:1}$:

[

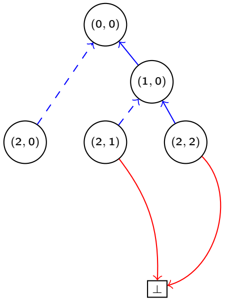
$(2,1) \xrightarrow{\perp} ((3,0),(3,0))$,
$(2,0) \xrightarrow{\top} ((3,1),(3,0))$,
$(2,2) \xrightarrow{\perp} ((3,1),(3,0))$,
$(2,0) \xrightarrow{\perp} ((3,0),\top)$]

Priority Queue: $Q_{\mathit{app}:2}$:

[

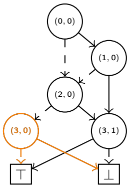
]

Output:
$(1,0) \xrightarrow{\top} (2,2)$

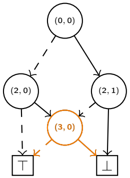


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Seek:
$\min((3,0),(3,0))$

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{1}}$:

[

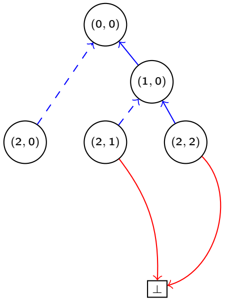
$(2,1) \xrightarrow{\perp} ((3,0),(3,0))$,
$(2,0) \xrightarrow{\top} ((3,1),(3,0))$,
$(2,2) \xrightarrow{\perp} ((3,1),(3,0))$,
$(2,0) \xrightarrow{\perp} ((3,0),\top)$]

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{2}}$:

[

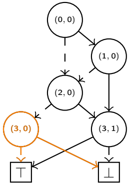
]

Output:

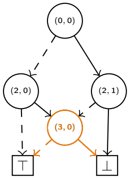


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Seek:
$\min((3,0),(3,0))$

Priority Queue: $Q_{app:1}$:

[

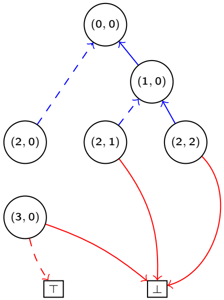
$(2,1) \xrightarrow{\perp} ((3,0),(3,0))$,
$(2,0) \xrightarrow{\top} ((3,1),(3,0))$,
$(2,2) \xrightarrow{\perp} ((3,1),(3,0))$,
$(2,0) \xrightarrow{\perp} ((3,0),\top)$]

Priority Queue: $Q_{app:2}$:

[

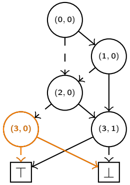
]

Output:
$(3,0) \xrightarrow{\perp} \top, (3,0) \xrightarrow{\top} \perp$

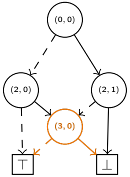


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \; ? \; x_2 \vee x_3 \; : \; x_2 \wedge x_3)$

Seek:
$\min((3,0),(3,0))$

Priority Queue: $Q_{app:1}$:

[

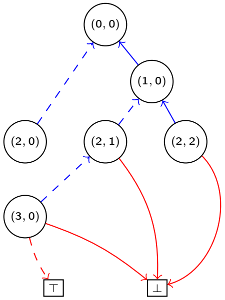
$(2,0) \xrightarrow{\top} ((3,1),(3,0))$,
$(2,2) \xrightarrow{\perp} ((3,1),(3,0))$,
$(2,0) \xrightarrow{\perp} ((3,0),\top)$]

Priority Queue: $Q_{app:2}$:

[

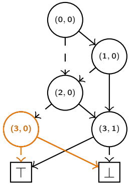
]

Output:
$(2,1) \xrightarrow{\perp} (3,0)$

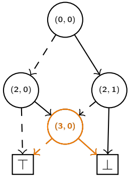


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \;?\; x_2 \vee x_3 \;:\; x_2 \wedge x_3)$

Seek:
$\min((3,1),(3,0))$

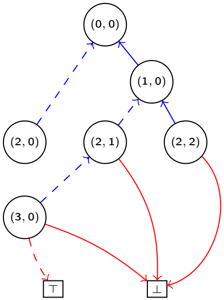
Priority Queue: $Q_{\boldsymbol{app}:\mathbf{1}}$:

$$
\begin{aligned}
[\;\; & (2,0) \xrightarrow{\top} ((3,1),(3,0)) \quad , \\
& (2,2) \xrightarrow{\bot} ((3,1),(3,0)) \quad , \\
& (2,0) \xrightarrow{\bot} ((3,0),\top) \quad]
\end{aligned}
$$

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{2}}$:

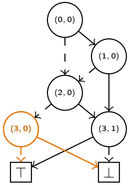
[ ]

Output:

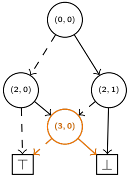


(c) $(a) \wedge (b)$

Apply Example ($\land$)



(a) $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ?\ x_2 \lor x_3 \ :\ x_2 \land x_3)$

Seek:

min((3, 1), (3, 0))

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{1}}$:

[

$(2, 0) \xrightarrow{\bot} ((3, 0), \top)$]

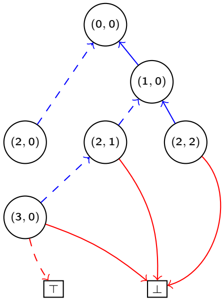
Priority Queue: $Q_{\boldsymbol{app}:\mathbf{2}}$:

[

$(2, 0) \xrightarrow{\top} ((3, 1), (3, 0)) \quad (\top, \bot)$,

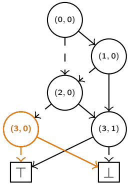
$(2, 2) \xrightarrow{\bot} ((3, 1), (3, 0)) \quad (\top, \bot)$]

Output:

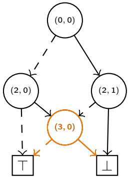


(c) $(a) \land (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \; ? \; x_2 \vee x_3 \; : \; x_2 \wedge x_3)$

Seek:

$\min((3,0), \top)$

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{1}}$:

[

$(2,0) \xrightarrow{\perp} ((3,0), \top)$]

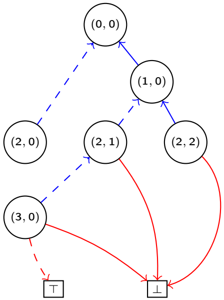
Priority Queue: $Q_{\boldsymbol{app}:\mathbf{2}}$:

[

$(2,0) \xrightarrow{\top} ((3,1),(3,0)) \quad (\top, \perp)$,

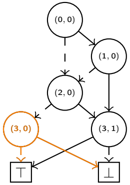
$(2,2) \xrightarrow{\perp} ((3,1),(3,0)) \quad (\top, \perp)$]

Output:

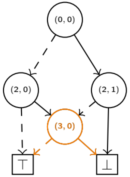


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Seek:

$\min((3,0), \top)$

Priority Queue: $Q_{app:1}$:

[

$(2,0) \xrightarrow{\perp} ((3,0), \top)$]

Priority Queue: $Q_{app:2}$:

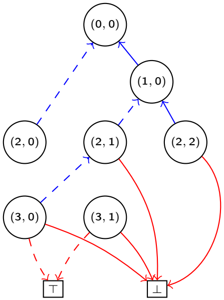
[

$(2,0) \xrightarrow{\top} ((3,1),(3,0)) \quad (\top, \perp)$,

$(2,2) \xrightarrow{\perp} ((3,1),(3,0)) \quad (\top, \perp)$]

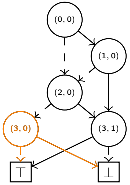
Output:

$(3,1) \xrightarrow{\perp} \top, (3,1) \xrightarrow{\top} \perp$

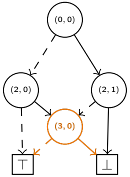


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Seek:
$\min((3,0), \top)$

Priority Queue: $Q_{app:1}$:

[

]

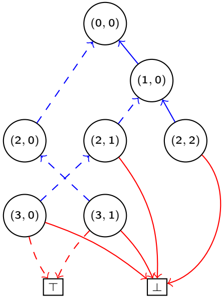
Priority Queue: $Q_{app:2}$:

[

$(2,0) \xrightarrow{\top} ((3,1),(3,0)) \quad (\top, \bot)$,

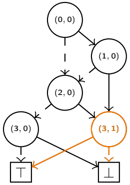
$(2,2) \xrightarrow{\bot} ((3,1),(3,0)) \quad (\top, \bot)$]

Output:
$(2,0) \xrightarrow{\bot} (3,1)$

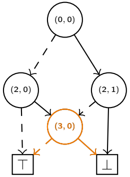


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Seek:
$\max((3,1),(3,0))$

Priority Queue: $Q_{app:1}$:

[

]

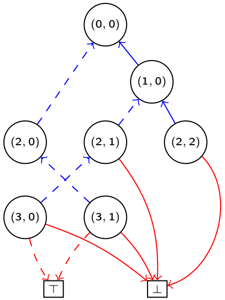
Priority Queue: $Q_{app:2}$:

[

$(2,0) \xrightarrow{\top} ((3,1),(3,0)) \quad (\top,\bot)$,

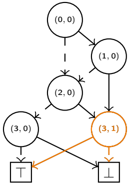
$(2,2) \xrightarrow{\bot} ((3,1),(3,0)) \quad (\top,\bot)$]

Output:

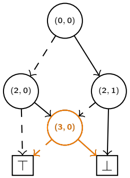


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Seek:
$\max((3,1),(3,0))$

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{1}}$:

[

]

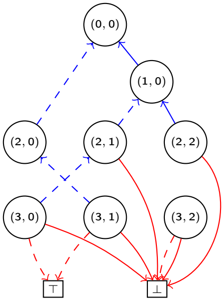
Priority Queue: $Q_{\boldsymbol{app}:\mathbf{2}}$:

[

$(2,0) \xrightarrow{\top} ((3,1),(3,0)) \quad (\top,\bot)$,

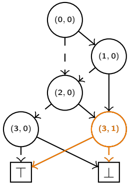
$(2,2) \xrightarrow{\bot} ((3,1),(3,0)) \quad (\top,\bot)$]

Output:
$(3,2) \xrightarrow{\bot} \bot, (3,2) \xrightarrow{\top} \bot$

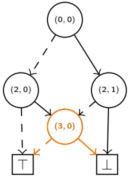


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \; ? \; x_2 \vee x_3 \;\; : \;\; x_2 \wedge x_3)$

Seek:
$\max((3,1),(3,0))$

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{1}}$:

[

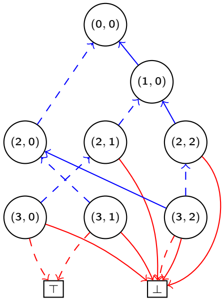
]

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{2}}$:

[

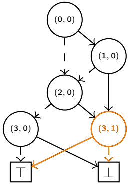
]

Output:
$(2,0) \xrightarrow{\top} (3,2), (2,2) \xrightarrow{\perp} (3,2)$

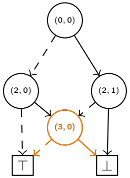


(c) $(a) \wedge (b)$

Apply Example ($\wedge$)



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(b) $\neg(x_0 \ ? \ x_2 \vee x_3 \ : \ x_2 \wedge x_3)$

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{1}}$:

[

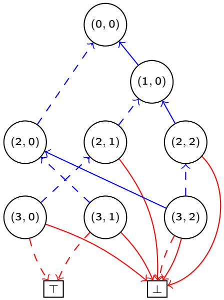
]

Priority Queue: $Q_{\boldsymbol{app}:\mathbf{2}}$:

[

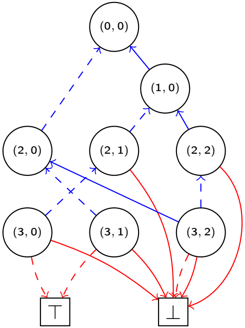
]

Output:



(c) $(a) \wedge (b)$

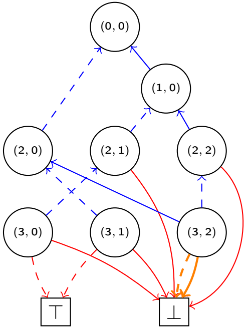
Reduce Example



(c) $(a) \wedge (b)$

(d) $(a) \wedge (b)$ **reduced**

Reduce Example



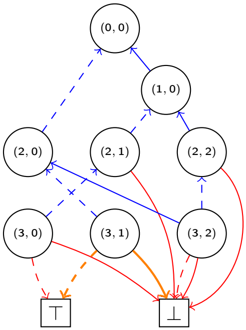
(c) $(a) \wedge (b)$

Level: 3

[$[(3,2) \mapsto \bot]$]

(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

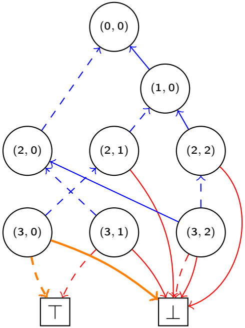
Level: 3

[$[(3,2) \mapsto \bot]$]

[$((3,1), \top, \bot)$,
]

(d) $(a) \wedge (b)$ **reduced**

Reduce Example



(c) $(a) \wedge (b)$

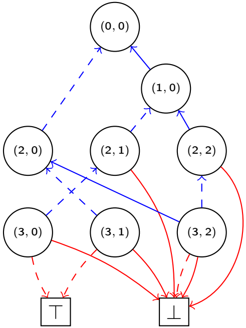
Level: 3

[$[(3,2) \mapsto \bot]$ **]**

[$((3,1), \top, \bot)$,
$((3,0), \top, \bot)$ **]**

(d) $(a) \wedge (b)$ **reduced**

Reduce Example



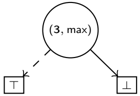
(c) $(a) \wedge (b)$

Level: 3

[$[(3,2) \mapsto \bot]$]

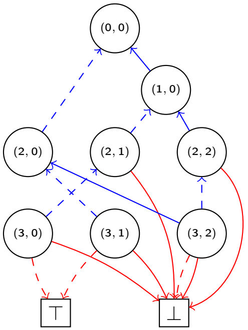
[$[(3,1) \mapsto (3, \text{max})]$,
$((3,0), \top, \bot)$]

Output:
$((3, \text{max}), \top, \bot)$



(d) $(a) \wedge (b)$ **reduced**

Reduce Example



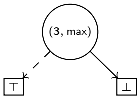
(c) $(a) \wedge (b)$

Level: 3

[$[(3,2) \mapsto \bot]$]

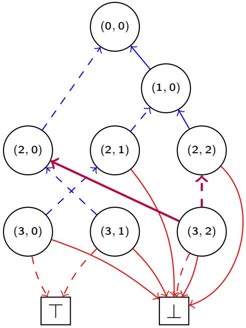
[$[(3,1) \mapsto (3, \max)]$,
$[(3,0) \mapsto (3, \max)]$]

Output:



(d) $(a) \wedge (b)$ **reduced**

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\boldsymbol{red}}$:

[$(2,2) \xrightarrow{\perp} \perp$,

$(2,0) \xrightarrow{\top} \perp$,

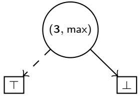
]

Level: 3

[ ]

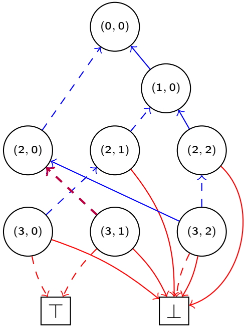
[$[(3,1) \mapsto (3,\text{max})]$,
$[(3,0) \mapsto (3,\text{max})]$]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: Q_{red}:

[$(2,2) \xrightarrow{\bot} \bot$,

$(2,0) \xrightarrow{\top} \bot$,

$(2,0) \xrightarrow{\bot} (3, \max)$,

]

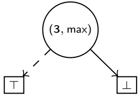
Level: 3

[ ]

[,

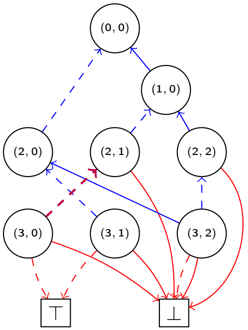
$[(3,0) \mapsto (3, \max)]$]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\boldsymbol{red}}$:

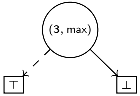
[$(2,2) \xrightarrow{\perp} \perp$,
$(2,1) \xrightarrow{\perp} (3,\text{max})$,
$(2,0) \xrightarrow{\top} \perp$,
$(2,0) \xrightarrow{\perp} (3,\text{max})$,
]

Level: 3

[ ]

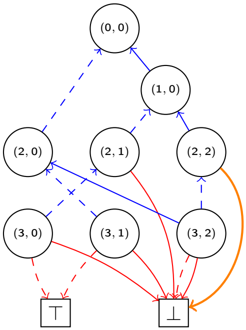
[,
]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: Q_{red}:

[

$(2,1) \xrightarrow{\perp} (3, \max)$,

$(2,0) \xrightarrow{\top} \perp$,

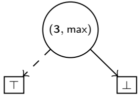
$(2,0) \xrightarrow{\perp} (3, \max)$,

]

Level: 2

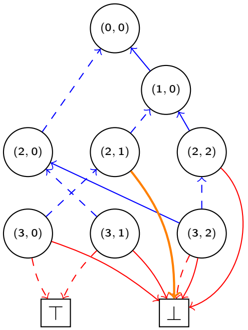
[$[(2,2) \mapsto \perp]$]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\boldsymbol{red}}$:

[

$(2,0) \xrightarrow{\top} \bot$,

$(2,0) \xrightarrow{\bot} (3, \mathrm{max})$,

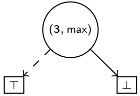
]

Level: 2

[$[(2,2) \mapsto \bot]$]

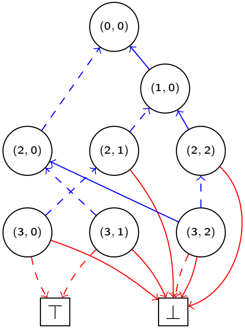
[$((2,1), (3, \mathrm{max}), \bot)$,
]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\boldsymbol{red}}$:

[

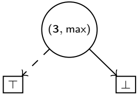
]

Level: 2

[$[(2,2) \mapsto \bot]$]

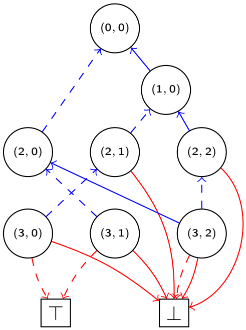
[$((2,1),(3,\text{max}),\bot)$,
$((2,0),(3,\text{max}),\bot)$]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\boldsymbol{red}}$:

[

]

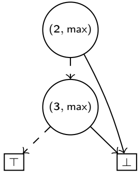
Level: 2

[$[(2,2) \mapsto \bot]$]

[$[(2,1) \mapsto (2,\mathsf{max})]$,
$((2,0),(3,\mathsf{max}),\bot)$]

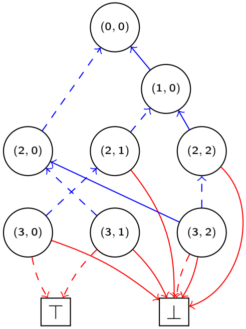
Output:

$((2,\mathsf{max}),(3,\mathsf{max}),\bot)$



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: Q_{red}:

[

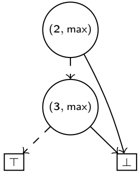
]

Level: 2

[$[(2,2) \mapsto \bot]$]

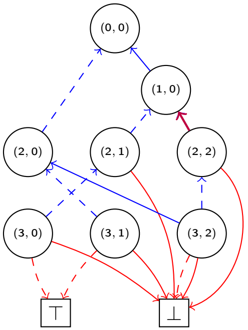
[$[(2,1) \mapsto (2, \text{max})]$,
$[(2,0) \mapsto (2, \text{max})]$]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\textbf{\textit{red}}}$:

[

$(1, 0) \xrightarrow{\top} \bot$,

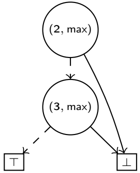
]

Level: 2

[ ]

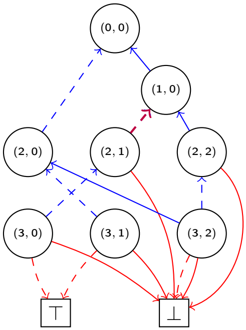
[$[(2, 1) \mapsto (2, \text{max})]$,
$[(2, 0) \mapsto (2, \text{max})]$]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\textbf{\textit{red}}}$:

[

$(1,0) \xrightarrow{\top} \bot$,

$(1,0) \xrightarrow{\bot} (2, \mathrm{max})$,

]

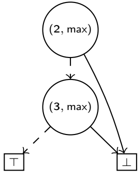
Level: 2

[ ]

[,

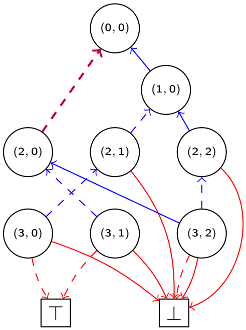
$[(2,0) \mapsto (2, \mathrm{max})]$]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\boldsymbol{red}}$:

[

$(1,0) \xrightarrow{\top} \bot$,

$(1,0) \xrightarrow{\bot} (2, \text{max})$,

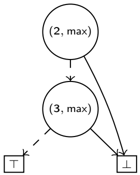
$(0,0) \xrightarrow{\bot} (2, \text{max})$]

Level: 2

[ ]

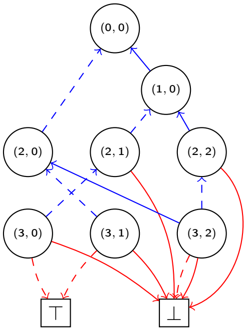
[,
]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\boldsymbol{red}}$:

[

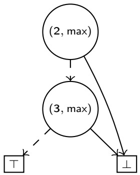
$(0, 0) \xrightarrow{\perp} (2, \text{max})$]

Level: 1

[ ]

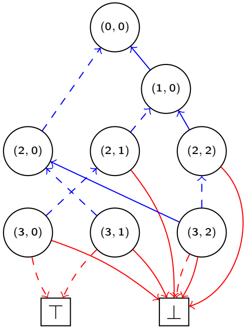
[$((1, 0), (2, \text{max}), \perp)$]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\boldsymbol{red}}$:

[

$(0,0) \xrightarrow{\perp} (2, \max)$]

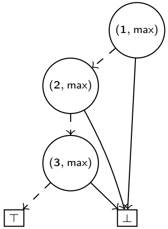
Level: 1

[ ]

[$[(1,0) \mapsto (1, \max)]$]

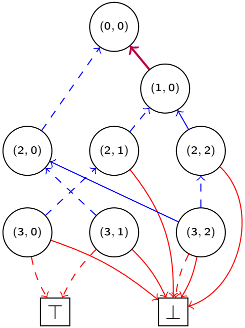
Output:

$((1, \max), (2, \max), \perp)$



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \land (b)$

Priority Queue: Q_{red}:

[

$(0,0) \xrightarrow{\top} (1, \max)$,

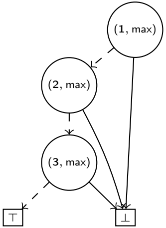
$(0,0) \xrightarrow{\perp} (2, \max)$]

Level: 1

[ ]

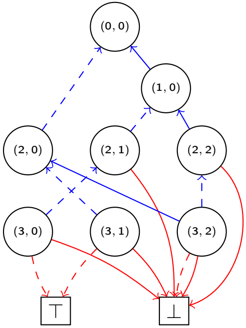
[ ]

Output:



(d) $(a) \land (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\boldsymbol{red}}$:

[

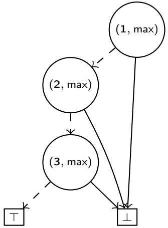
]

Level: 0

[ ]

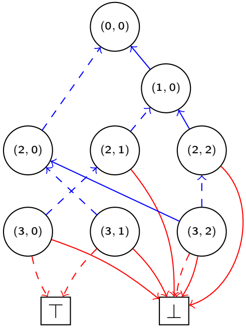
[((0, 0), (2, max), (1, max))]

Output:



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\boldsymbol{red}}$:

[

]

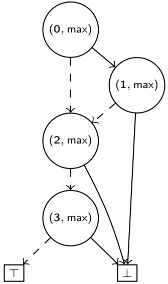
Level: 0

[ ]

[$[(0,0) \mapsto (0, \text{max})]$]

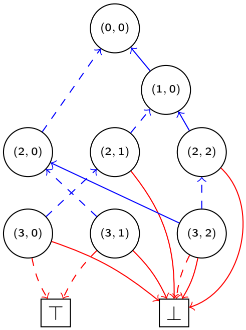
Output:

$((0, \text{max}), (2, \text{max}), (1, \text{max}))$



(d) $(a) \wedge (b)$ reduced

Reduce Example



(c) $(a) \wedge (b)$

Priority Queue: $Q_{\boldsymbol{red}}$:

[

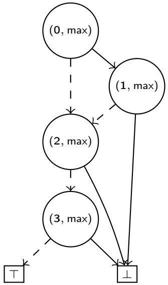
]

Level: 0

[ ]

[ ]

Output:

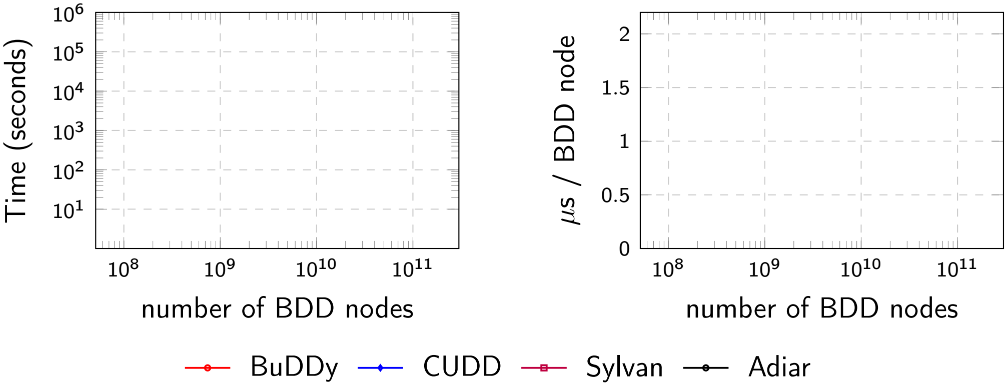


(d) $(a) \wedge (b)$ reduced

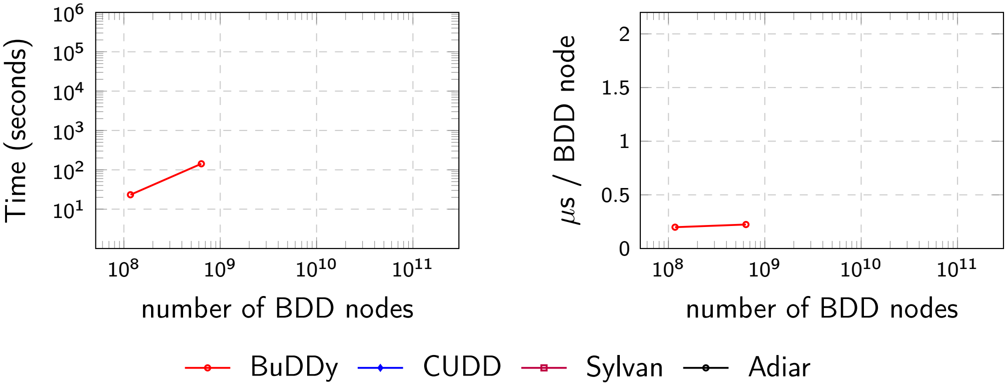
Algorithm		Depth-first	Time-forwarded
Reduce		$O(N)$	$O(\text{sort}(N))$
BDD Manipulation			
Apply	$f \odot g$	$O(N_f \cdot N_g)$	$O(\text{sort}(N_f \cdot N_g))$
If-Then-Else	$f \; ? \; g \; : \; h$	$O(N_f \cdot N_g \cdot N_h)$	$O(\text{sort}(N_f \cdot N_g \cdot N_h))$
Restrict	$f\|_{x_i=v}$	$O(N)$	$O(\text{sort}(N))$
Negation	$\neg f$	$O(1)$	$O(1)$
Quantification	$\exists/\forall v : f\|_{x_i=v}$	$O(N^2)$	$O(\text{sort}(N^2))$
Counting			
Count Paths	$\#$paths in f to $\top$	$O(N)$	$O(\text{sort}(N))$
Count SAT	$\#x : f(x)$	$O(N)$	$O(\text{sort}(N))$
Other			
Equality	$f \equiv g$	$O(1)$	$O(\text{sort}(N))$
Evaluate	$f(x)$	$O(L)$	$O(N/B)$
Min/Max SAT	$\min / \max\{x \mid f(x)\}$	$O(L)$	$O(N/B)$

Adiar

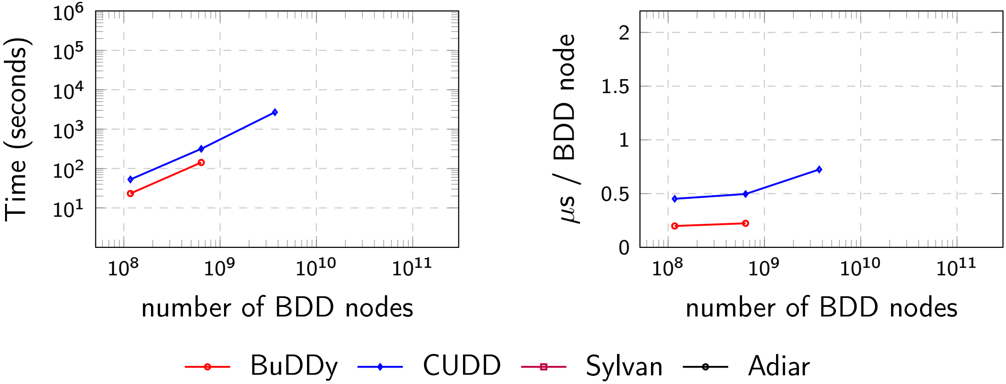
github.com/ssoelvsten/adiar



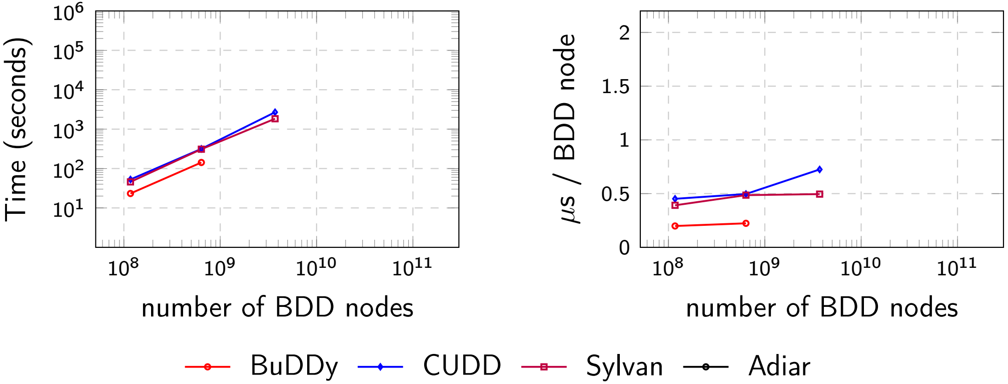
Minimum running times for the N-Queens problem.



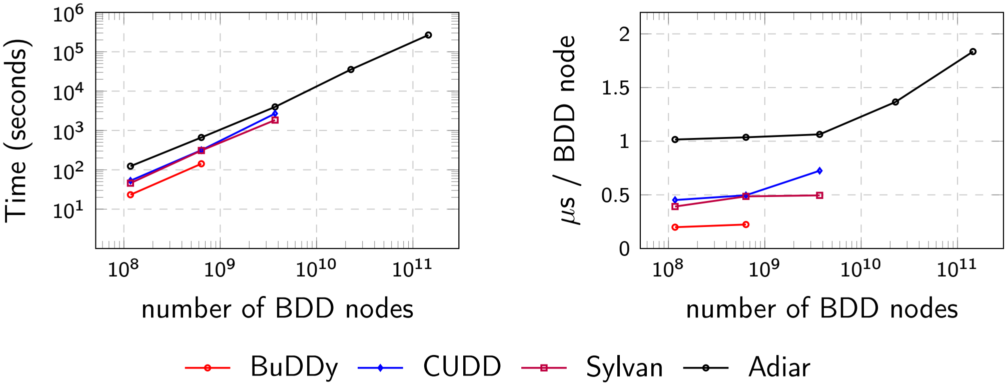
Minimum running times for the N-Queens problem.



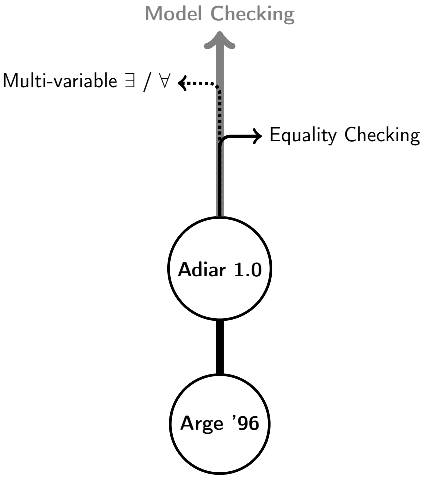
Minimum running times for the N-Queens problem.

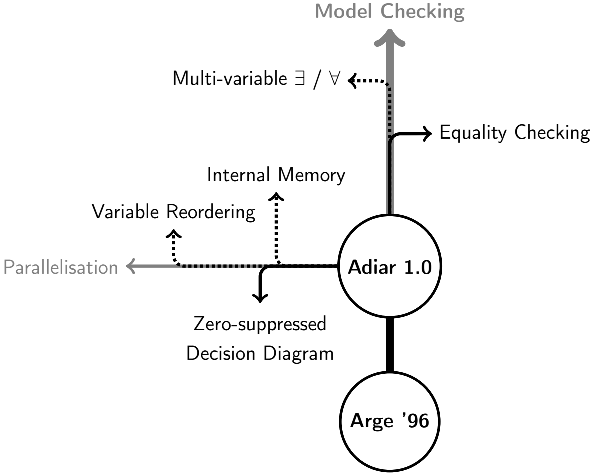


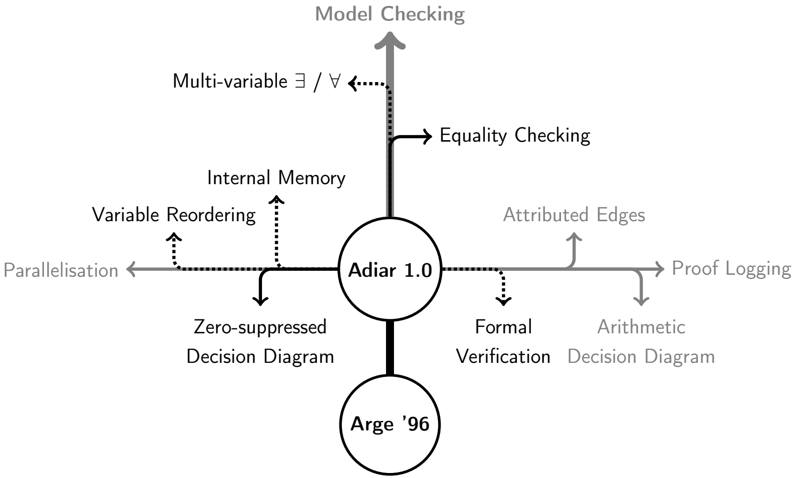
Minimum running times for the *N*-Queens problem.



Minimum running times for the N-Queens problem.


Model Checking
Multi-variable ∃ / ∀
Equality Checking
Adiar 1.0
Arge '96


Model Checking
Multi-variable ∃ / ∀
Equality Checking
Internal Memory
Variable Reordering
Parallelisation
Adiar 1.0
Zero-suppressed
Decision Diagram
Arge '96


Model Checking
Multi-variable ∃ / ∀
Equality Checking
Internal Memory
Variable Reordering
Attributed Edges
Parallelisation
Adiar 1.0
Proof Logging
Zero-suppressed
Decision Diagram
Formal
Verification
Arithmetic
Decision Diagram
Arge '96

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Adiar

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