

# An External Memory Relational Product

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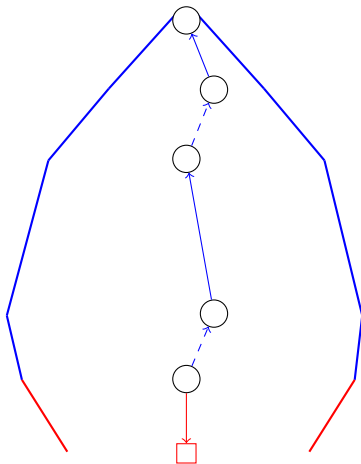
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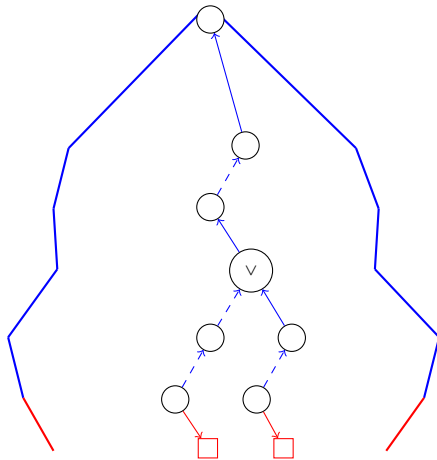


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Apply  $\pi$  in  $O(L_N)$  extra time during the final bottom-up Reduce sweep.

- *Bounce:*

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### If $\pi$ is not monotonic

to be continued...