An External Memory Relational Product

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AARHUS
UNIVERSITY

$$RelProd(S,T) \equiv (\ \exists\vec{x}.\ S(\vec{x}) \wedge T(\vec{x},\vec{x'})\ )[\vec{x'}/\vec{x}]$$

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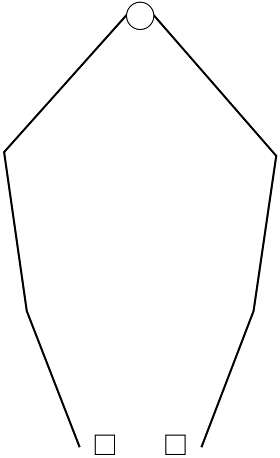
$\exists x_i.\ \phi \equiv \phi[x_i/\top] \lor \phi[x_i/\bot]$



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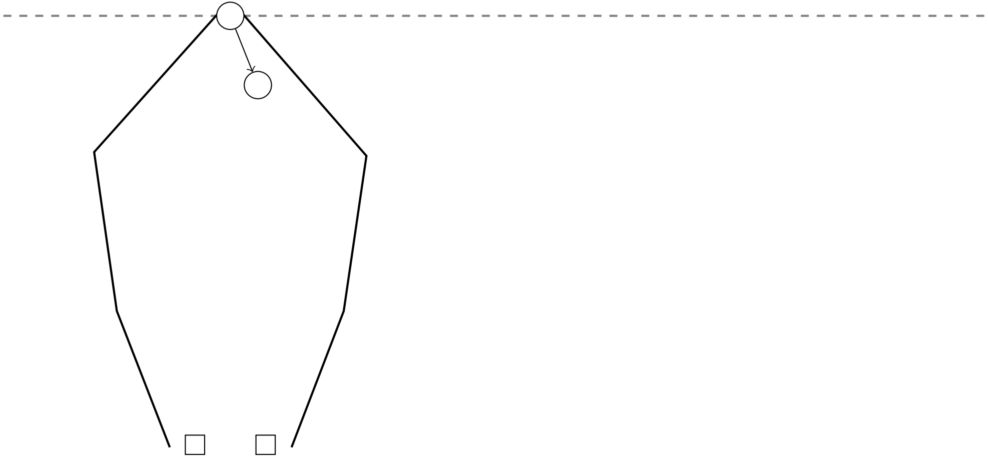

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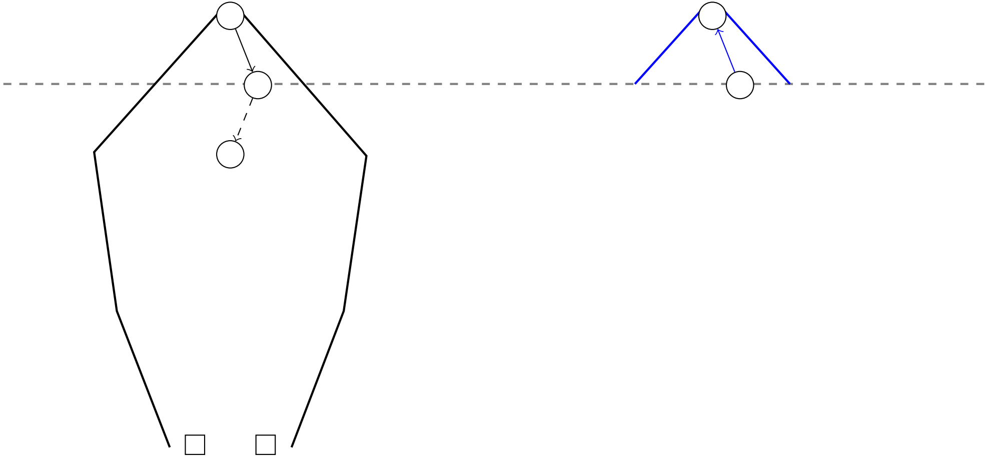

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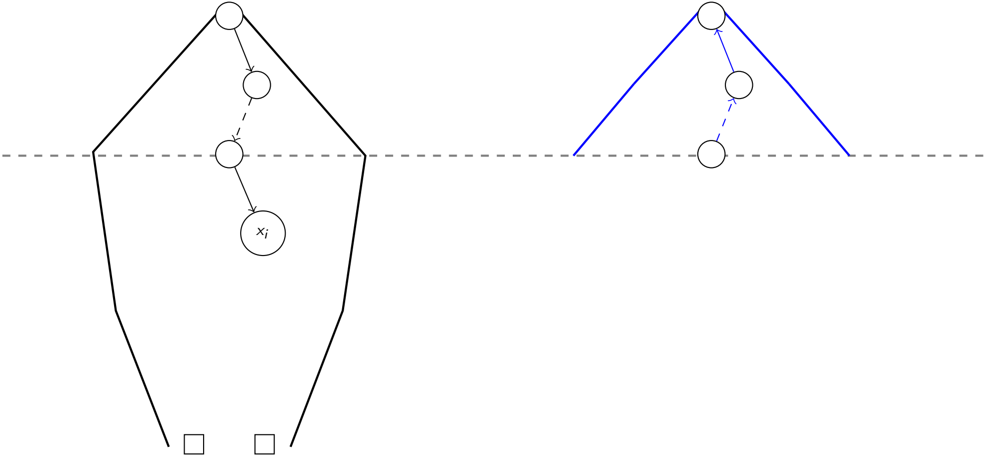

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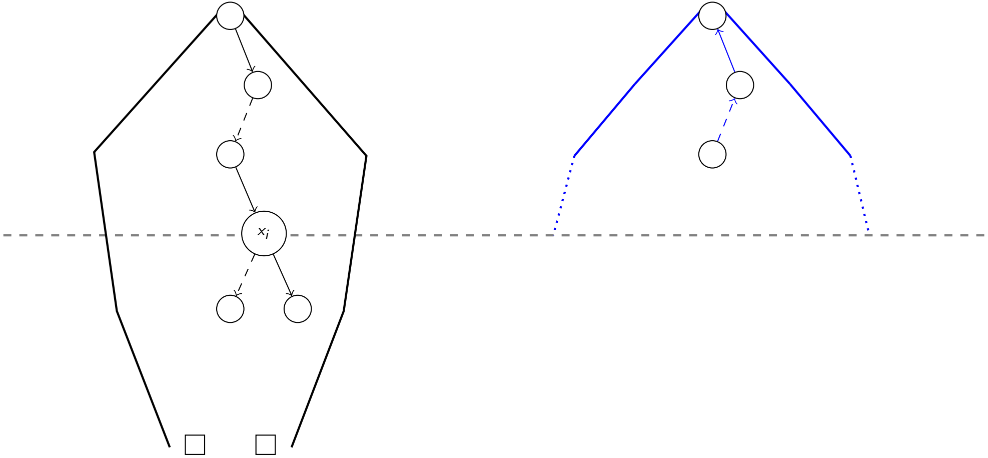

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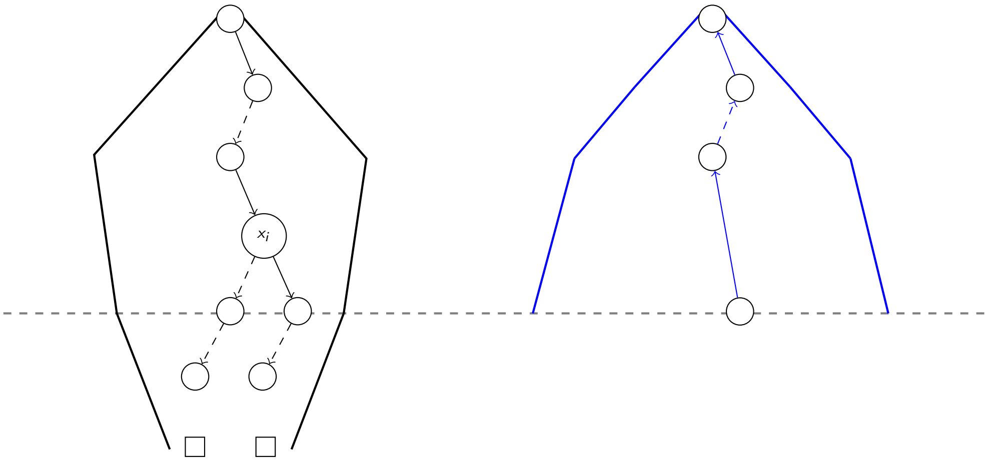

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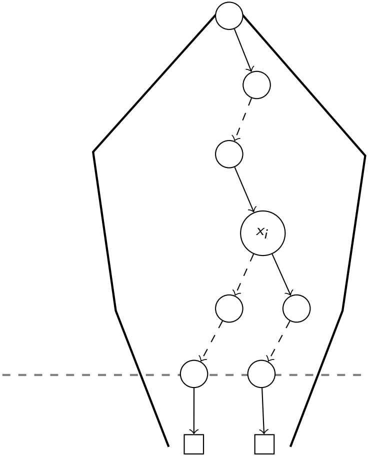

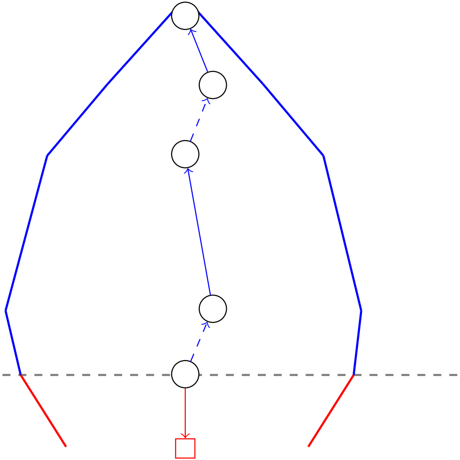

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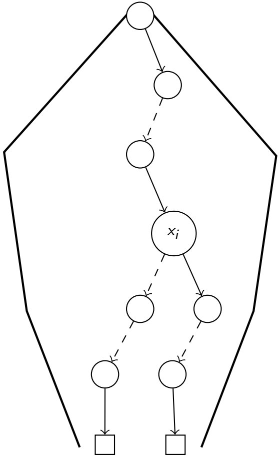

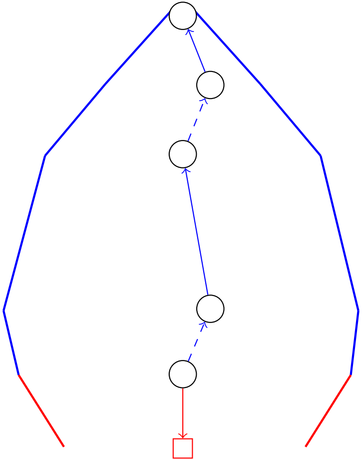

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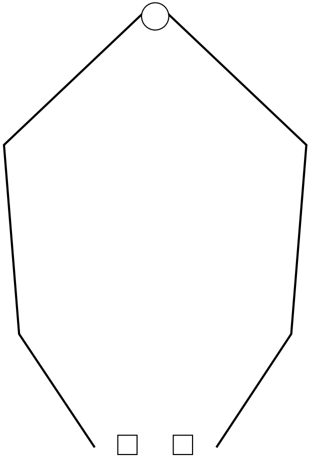

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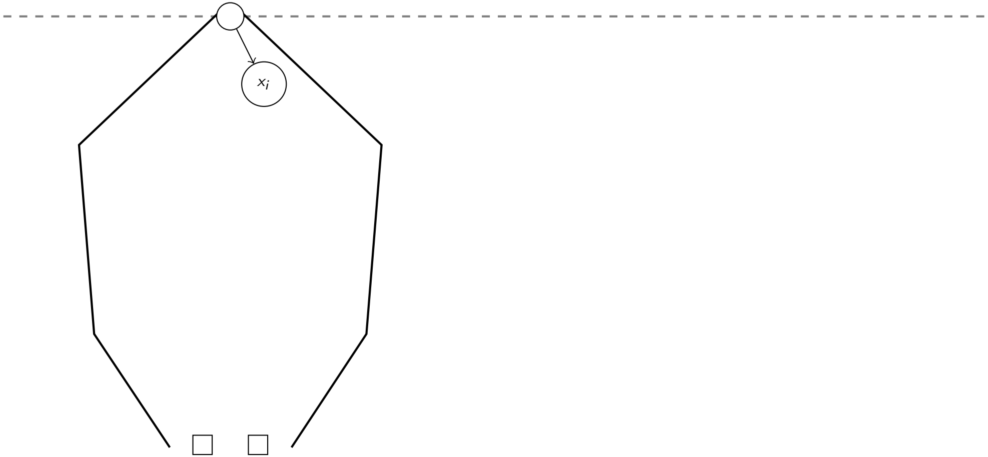

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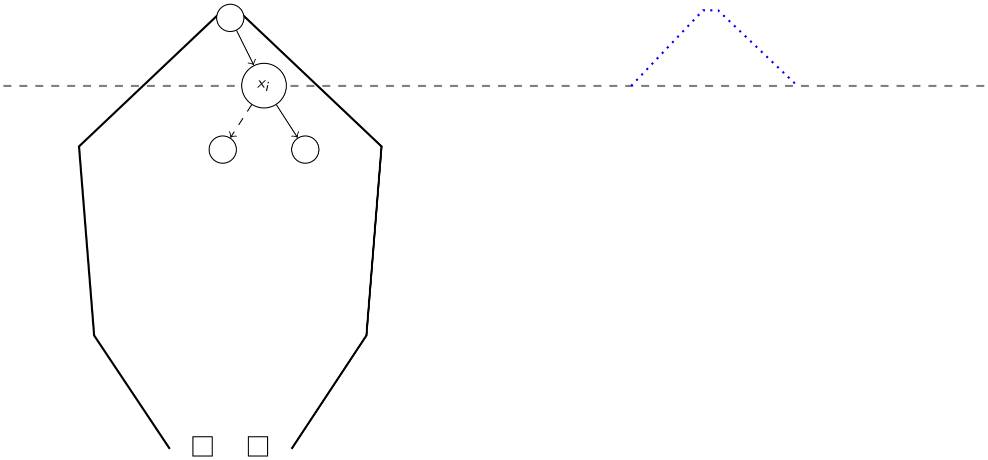

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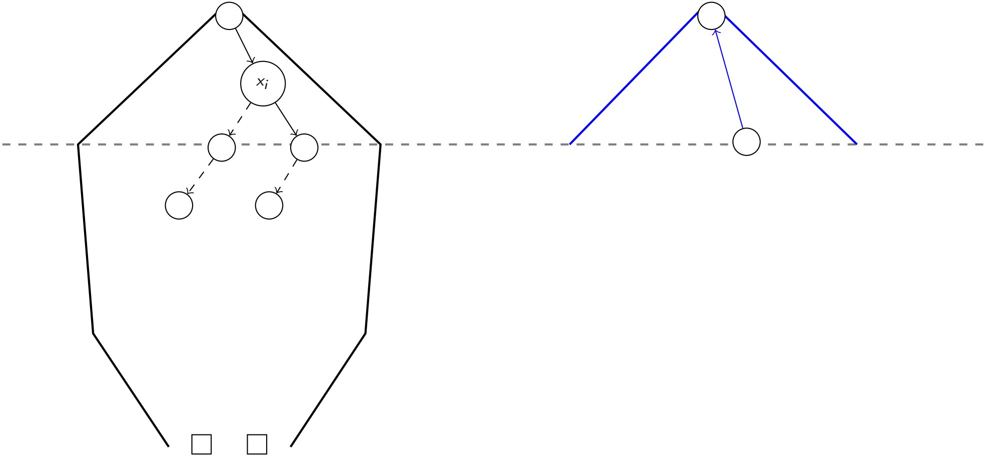

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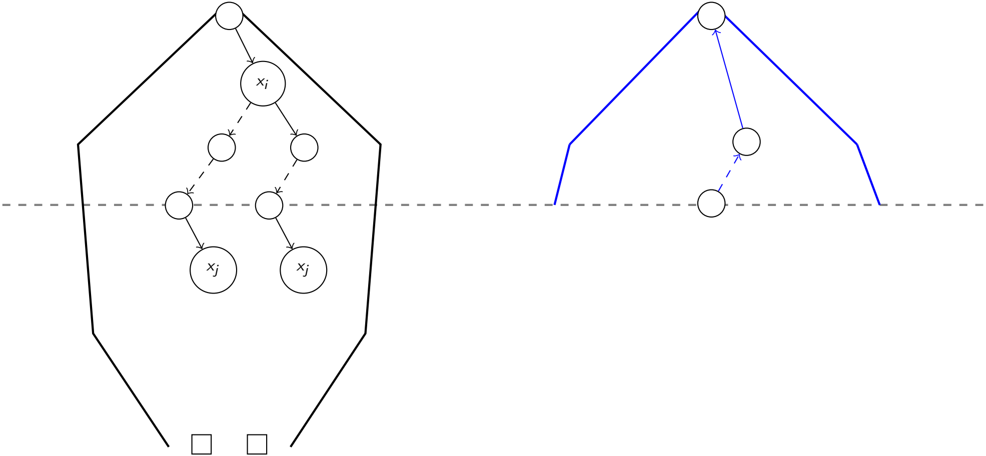

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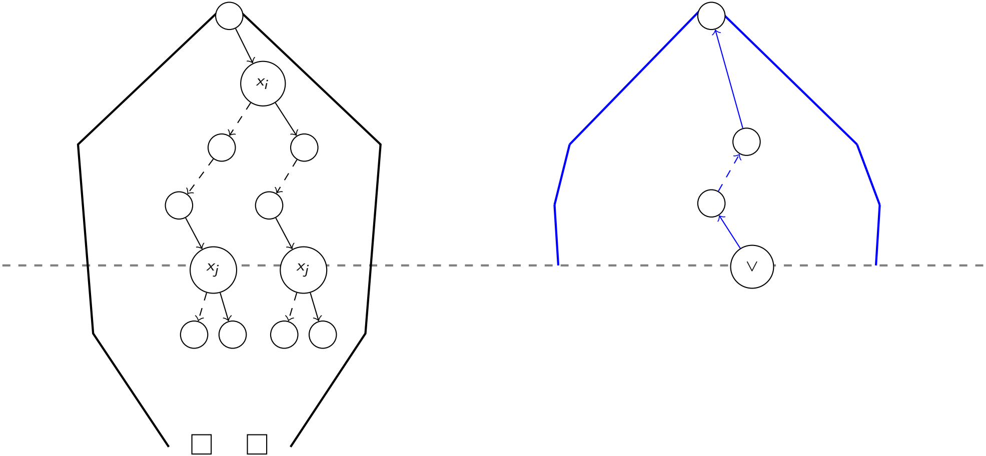

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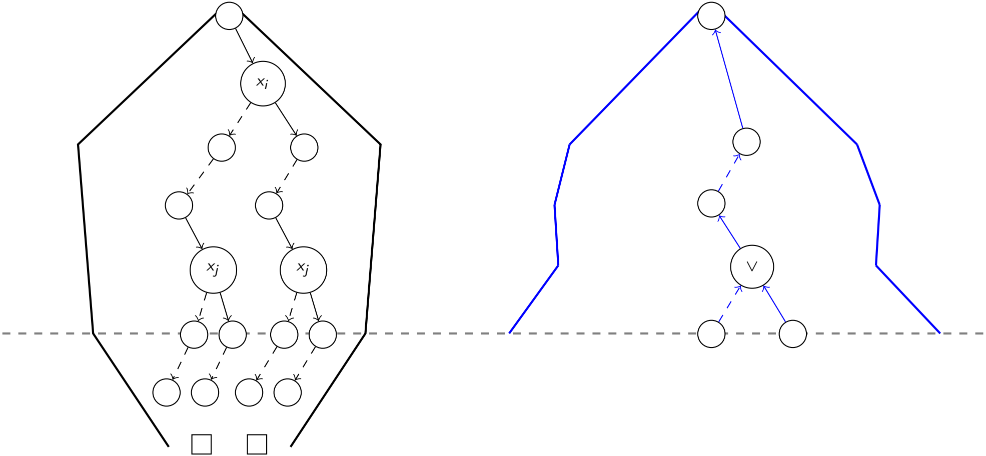

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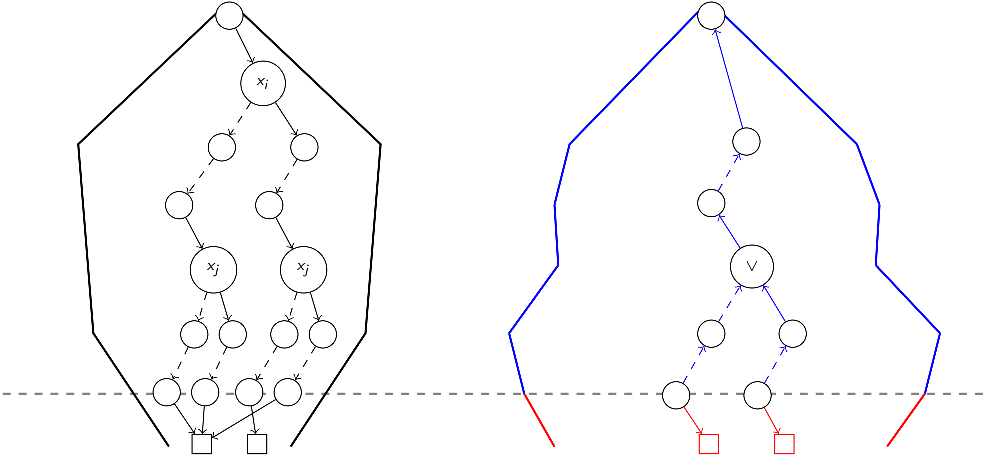

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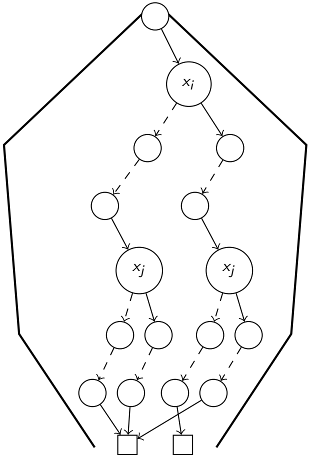

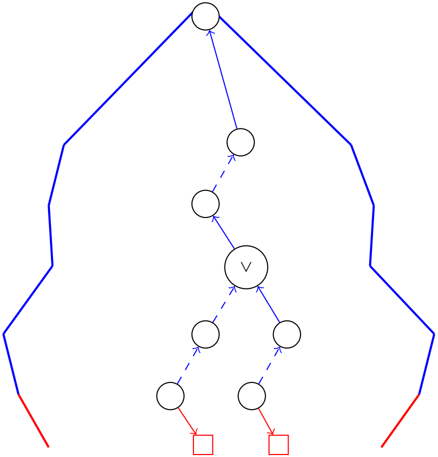

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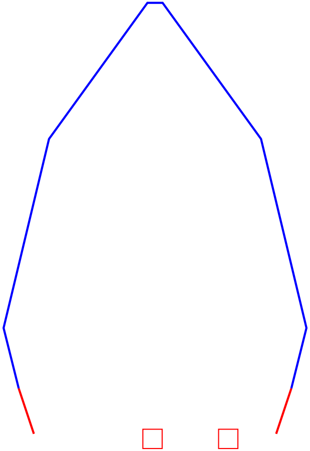

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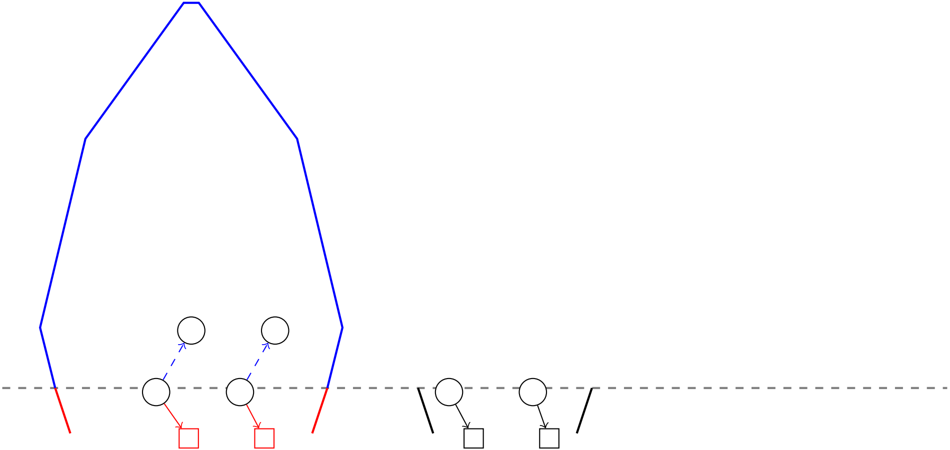

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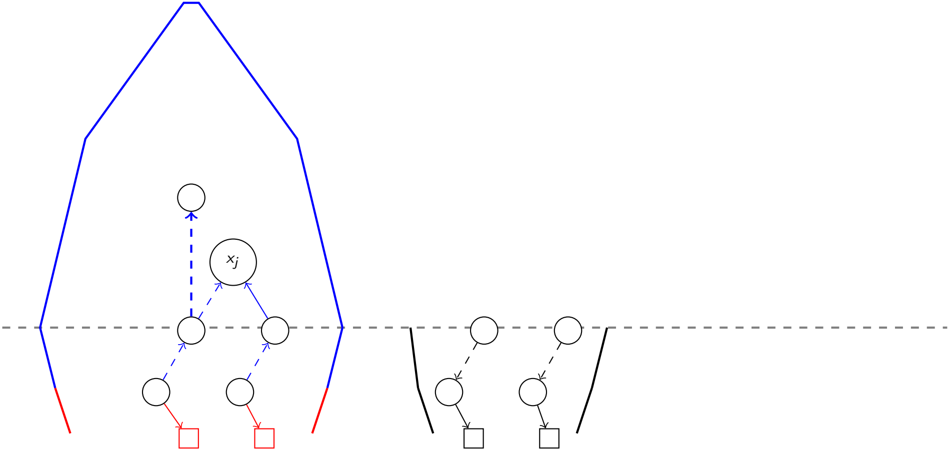

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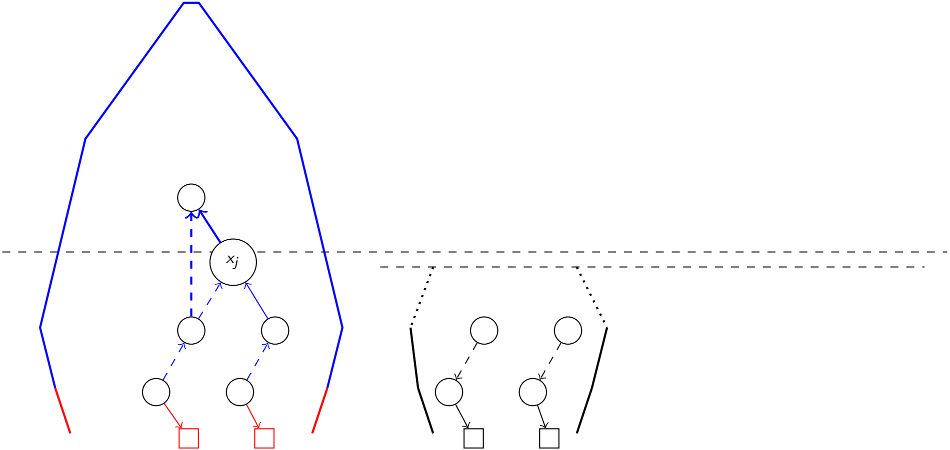

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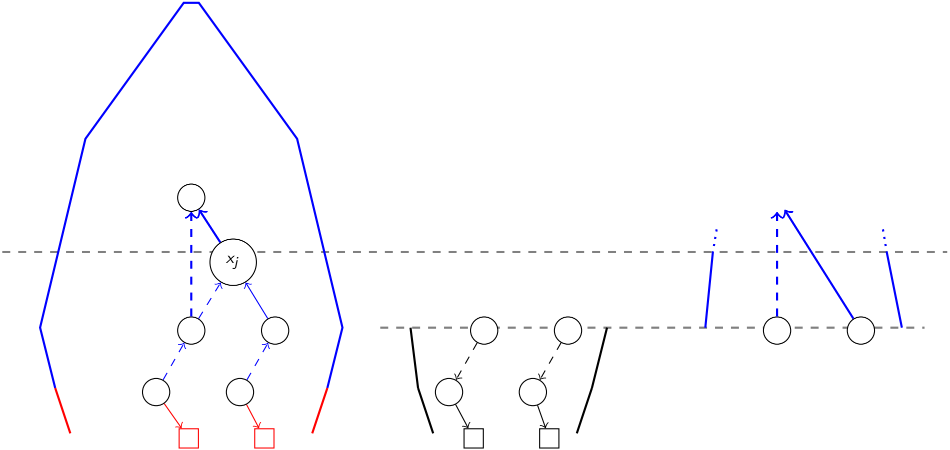

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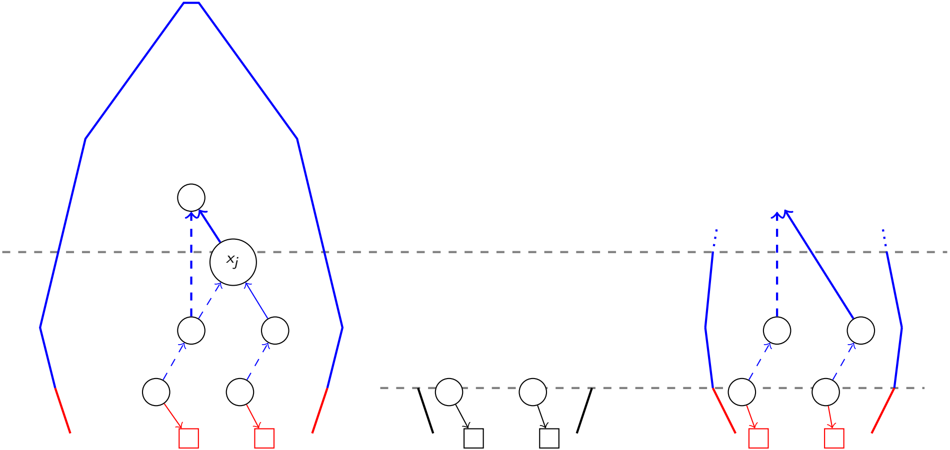

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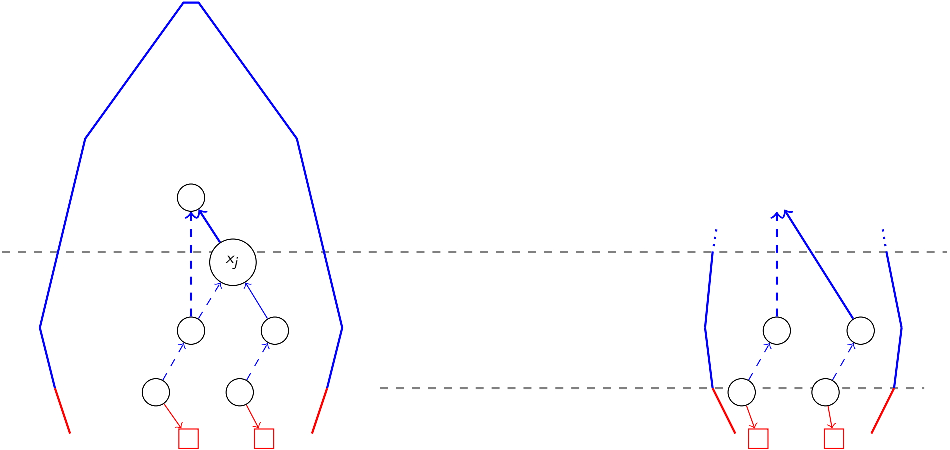

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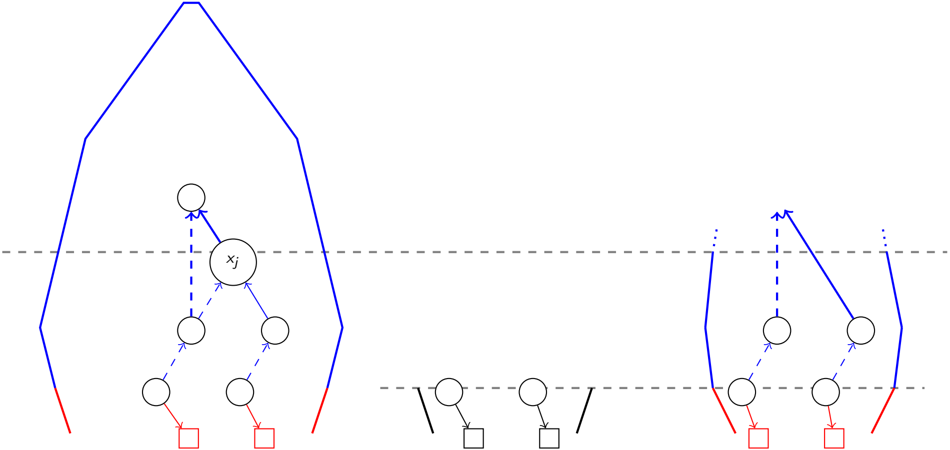

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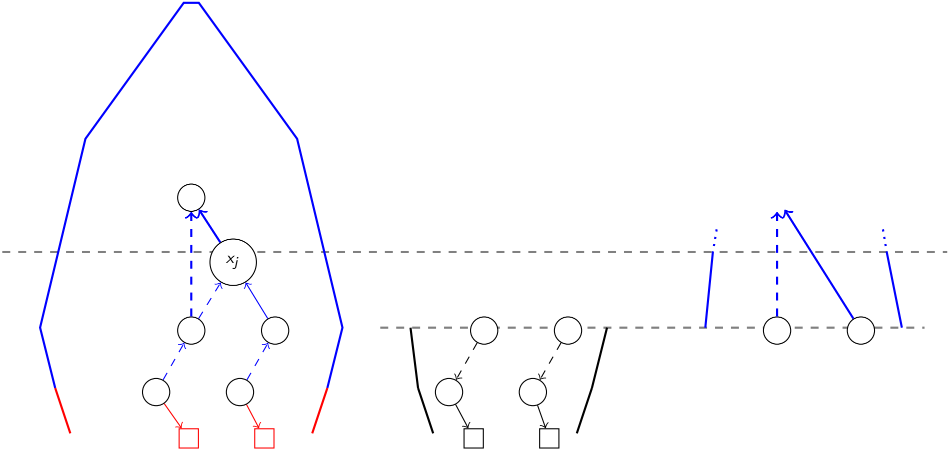

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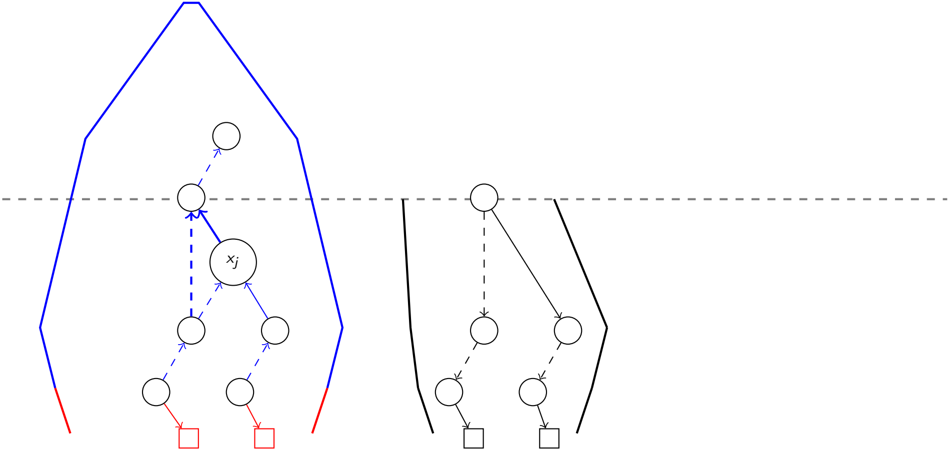

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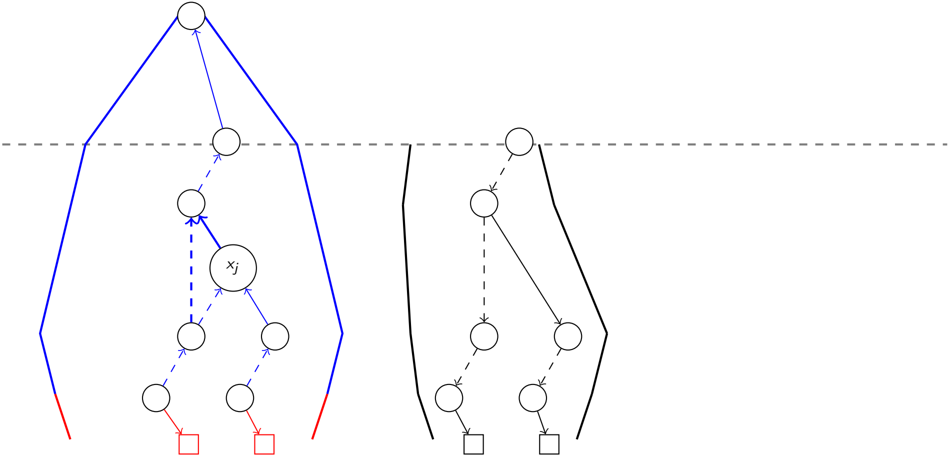

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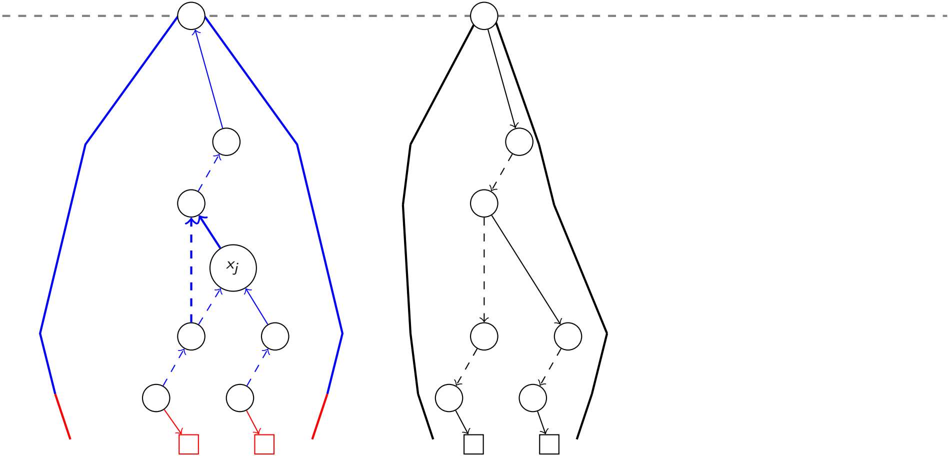

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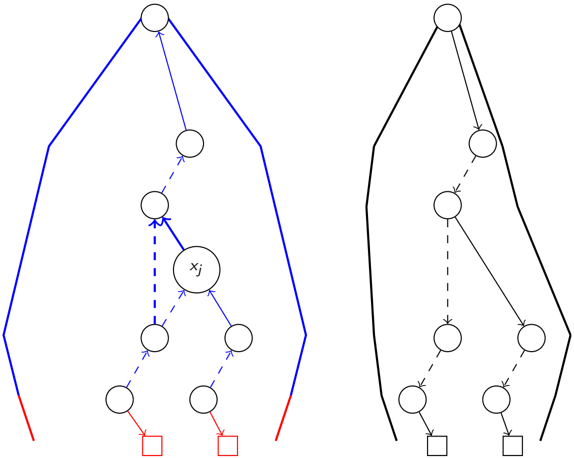

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If π is monotonic

- *1-Var / Push*:
  Apply π in $O(L_N)$ extra time during the final bottom-up Reduce sweep.
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  $x_i' < x_j' \implies x_i < x_j$: π can be applied during the outermost Reduce sweep.

That is, applying π can (essentially) be done for free.

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If π is not monotonic

to be continued...