# Adiar 1.1 : Zero-suppressed Decision Diagrams in External Memory

**Steffan Christ Sølvsten** and Jaco van de Pol

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AARHUS UNIVERSITY

SSD

RAM

L3

L2

L1

CPU

SSD

$\sim 200.000+$ CPU Cycles

RAM

$\sim 200$ CPU Cycles

L3

L2

L1

$\sim 4 - 50$ CPU Cycles

CPU

HDD

$\sim 20.000.000+$ CPU Cycles

RAM

$\sim 200$ CPU Cycles

L3

L2

L1

$\sim 4 - 50$ CPU Cycles

CPU

# Adiar

Binary Decision Diagrams
in External Memory

github.com/ssoelvsten/adiar

# Adiar

**Multi-terminal** Decision Diagrams
in External Memory

github.com/ssoelvsten/adiar

# Adiar

## **Quantum Multi-valued** Decision Diagrams in External Memory

github.com/ssoelvsten/adiar

# Adiar

**Zero-suppressed** Decision Diagrams
in External Memory

github.com/ssoelvsten/adiar

**BDD:** $f : \mathbb{B}^n \to \mathbb{B}$        **ZDD:** $A \subseteq \mathbb{B}^n$

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)




zdd zdd_binop(zdd A, zdd B, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {
    return prod2<bdd_policy>(f, g, o);
}


zdd zdd_binop(zdd A, zdd B, bool_op o)  {
    return prod2<zdd_policy>(A, B, o);
}
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {

    return prod2<bdd_policy>(f, g, o);

}


zdd zdd_binop(zdd A, zdd B, bool_op o)  {

    return prod2<zdd_policy>(A, B, o);

}
```

Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.

Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.

Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.

*Done*

BDD   ZDD

*Open*

Clock DD

*Doable*    MTBDD  LDD          QMDD

*Done*

MDD          BDD  ZDD    (K)FDD

Tagged/Chained BDD

# Steffan Christ Sølvsten

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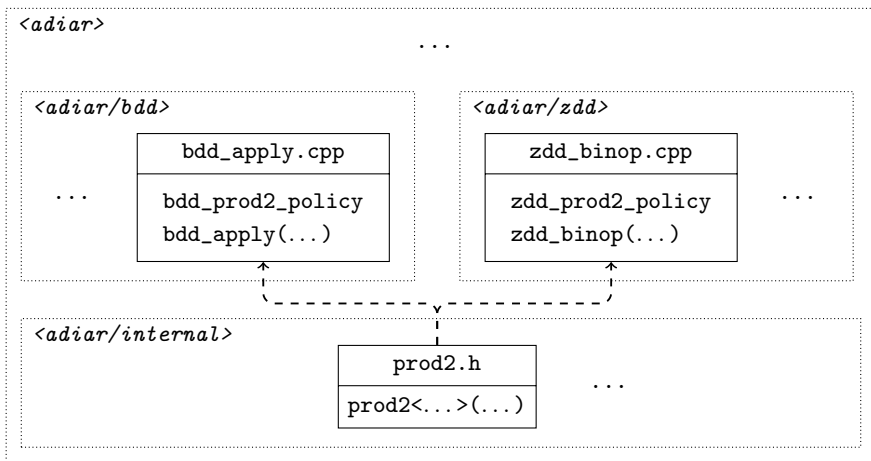🌐 ssoelvsten.github.io

## Adiar

</> github.com/ssoelvsten/adiar

📘 ssoelvsten.github.io/adiar

**AARHUS UNIVERSITY**
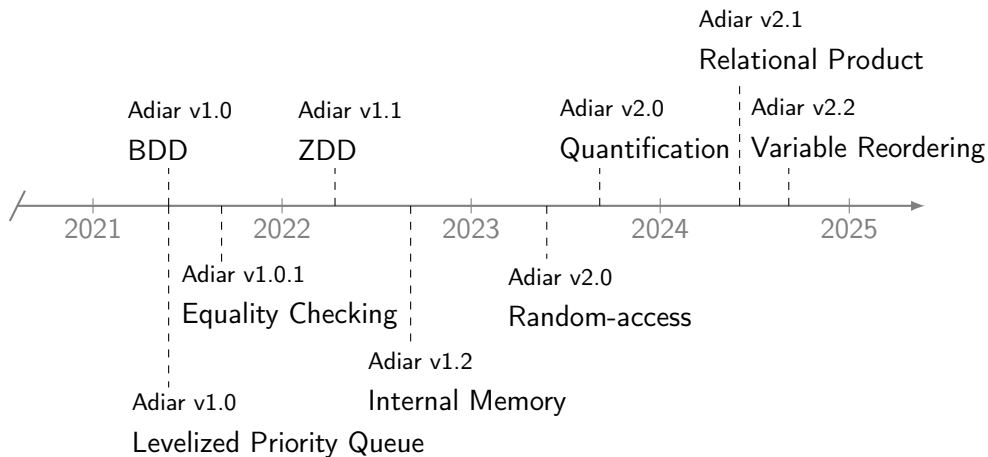
| Function | Operation Semantics |
|---|---|
| **ZDD Constructors** ||
| `zdd_empty()` | $\emptyset$ |
| `zdd_null()` | $\{\emptyset\}$ |
| `zdd_singleton(var)` | $\{x_{\textbf{var}}\}$ |
| `zdd_vars(vars)` | $\{\bigcup_{i \in \textbf{vars}} \{x_i\}\}$ |
| `zdd_singletons(vars)` | $\{\{x_i\} \mid i \in \textbf{vars}\}$ |
| `zdd_powerset(vars)` | $\mathcal{P}(\textbf{vars})$ |
| `zdd_sized_set(vars,k,⊙)` | $\{s \in \mathcal{P}(\textbf{vars}) \mid \lvert s \rvert \odot k\}$ |
| **ZDD Manipulation** ||
| `zdd_binop(A,B,⊗)` | $\{x \mid x \in A \otimes x \in B\}$ |
| `zdd_change(A, vars)` | $\{(a \setminus \textbf{vars}) \cup (\textbf{vars} \setminus a) \mid a \in A\}$ |
| `zdd_complement(A, dom)` | $\mathcal{P}(\textbf{dom}) \setminus A$ |
| `zdd_expand(A, vars)` | $\bigcup_{a \in A}\{a \cup v \mid v \in \mathcal{P}(\textbf{vars})\}$ |
| `zdd_offset(A, vars)` | $\{a \in A \mid \textbf{vars} \cap a = \emptyset\}$ |
| `zdd_onset(A, vars)` | $\{a \in A \mid \textbf{vars} \subseteq a\}$ |
| `zdd_project(A, vars)` | $\bigcup_{a \in A}\{a \cap \textbf{vars}\}$ |

| Function | Operation Semantics |
|---|---|
| **Counting** ||
| `zdd_size(A)` | $\lvert A \rvert$ |
| `zdd_nodecount(A)` | # ZDD Nodes in $A$ |
| `zdd_varcount(A)` | # Non-empty Levels in $A$ |
| **Predicates** ||
| `zdd_equal(A, B)` | $A = B$ |
| `zdd_unequal(A, B)` | $A \neq B$ |
| `zdd_subseteq(A, B)` | $A \subseteq B$ |
| `zdd_disjoint(A, B)` | $A \cap B = \emptyset$ |
| **Set elements** ||
| `zdd_contains(A, a)` | $a \in A$ |
| `zdd_minelem(A)` | $a \in A$ s.t. $\forall a' \in A \,.\, a \leq a'$ |
| `zdd_maxelem(A)` | $a \in A$ s.t. $\forall a' \in A \,.\, a' \leq a$ |
| **Conversion** ||
| `zdd_from(f, dom)` | $\{x \in \mathcal{P}(\textbf{dom}) \mid f(x) = \top\}$ |
| `bdd_from(A, dom)` | $\vec{x} : \mathcal{P}(\textbf{dom}) \mapsto \vec{x} \in A$ |

Operations provided by Adiar in `<adiar/zdd.h>`.

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

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**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: $Q_{count}$:

[

]

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: $Q_{count}$:

[

]

**(a)** $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Priority Queue: $Q_{count}$:

$[ \quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$

$\quad ((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$

$]$

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(1, 0)$ | 0 | 0 |

Priority Queue: $Q_{count}$:

$\big[ \quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$

$\quad \quad ((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$

$\big]$

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(1, 0)$ | 0 | 0 |

Priority Queue: $Q_{count}$:

$$\left[ \begin{array}{ll} ((0,0) \xrightarrow{\top} (1,0), & 1) \quad , \\ ((0,0) \xrightarrow{\bot} (2,0), & 1) \quad , \end{array} \right.$$

$\left. \vphantom{\begin{array}{l} 1 \\ 1 \\ 1 \end{array}} \right]$

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(1, 0)$ | 1 | 0 |

Priority Queue: $Q_{count}$:

[

$((0, 0) \xrightarrow{\perp} (2, 0), \quad 1)$ ,

]

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(1, 0)$ | 1 | 0 |

Priority Queue: $Q_{count}$:

$[$

$\quad ((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$\quad ((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$\quad ((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$

$]$

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(2, 0)$ | 0 | 0 |

Priority Queue: $Q_{count}$:

[

$((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$

]

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(2, 0)$ | 0 | 0 |

Priority Queue: $Q_{count}$:

[

    $((0, 0) \xrightarrow{\perp} (2, 0),$    1)     ,

    $((1, 0) \xrightarrow{\perp} (2, 0),$    1)     ,

    $((1, 0) \xrightarrow{\top} (3, 1),$    1)     ,

    ]

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(2, 0)$ | 1 | 0 |

Priority Queue: $Q_{count}$:

$[$

$\quad ((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$\quad ((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$

$]$

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(2, 0)$ | 2 | 0 |

Priority Queue: $Q_{count}$:

[

$$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \qquad ,$$

]

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(2, 0)$ | 2 | 0 |

Priority Queue: $Q_{count}$:

$[$

$((2, 0) \xrightarrow{\perp} (3, 0), \quad 2) \quad ,$
$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad ]$

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(3, 0)$ | 0 | 0 |

Priority Queue: $Q_{count}$:

$$
\begin{aligned}
[ \\
((2, 0) &\xrightarrow{\perp} (3, 0), \quad 2) \quad , \\
((1, 0) &\xrightarrow{\top} (3, 1), \quad 1) \quad , \\
((2, 0) &\xrightarrow{\top} (3, 1), \quad 2) \quad ]
\end{aligned}
$$

**(a)** $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

Seek      Sum      Result
$(3, 0)$      0      0

Priority Queue: $Q_{count}$:

$$[$$

$$((2, 0) \xrightarrow{\bot} (3, 0), \quad 2) \quad ,$$
$$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$$
$$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad ]$$

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(3, 0)$ | 2 | 0 |

Priority Queue: $Q_{count}$:

$[$

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)\quad ,$
$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)\quad ]$

10

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(3, 0)$ | 2 | 2 |

Priority Queue: $Q_{count}$:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad ]$

**(a)** $(x_0 \land x_1 \land x_3) \lor (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(3, 1)$ | 0 | 2 |

Priority Queue: $Q_{count}$:

[

$((1, 0) \xrightarrow{\top} (3, 1),\quad 1)\quad,$
$((2, 0) \xrightarrow{\top} (3, 1),\quad 2)\quad]$

10

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(3, 1)$ | 0 | 2 |

Priority Queue: $Q_{count}$:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$ ,
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2)$ ]

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(3,1)$ | 1 | 2 |

Priority Queue: $Q_{count}$:

$[$

$((2,0) \xrightarrow{\top} (3,1), \quad 2) \quad ]$

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(3, 1)$ | 3 | 2 |

Priority Queue: $Q_{count}$:

[

]

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|------|-----|--------|
| $(3, 1)$ | 3 | 5 |

Priority Queue: $Q_{count}$:

[

]

**(a)** $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result
5

Priority Queue: $Q_{count}$:
[

]

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## Adiar

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