

# Adiar

## Binary Decision Diagrams in External Memory

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Minimal running time for the *Queens* problems.



Cache-misses for the *Queens* problems.





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



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Priority Queue:  $Q_{count}$ :

[

]



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**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	1	0

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	0	0

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
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**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

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[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	1	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
]





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), \quad 2)$	,
$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$	,
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2)$	]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), \quad 2)$	,
$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$	,
$((2, 0) \xrightarrow{\top} (3, 1), \quad 2)$	]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek  
(3, 0)

Sum  
0

Result  
0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(3, 0)</b>	2	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(3, 1)</b>	0	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(3, 1)</b>	0	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek  
**(3, 1)**

Sum  
1

Result  
2

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad ]$



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek  
**(3, 1)**

Sum  
3

Result  
2

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek  
**(3,1)**

Sum  
3

Result  
5

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result  
5

Priority Queue:  $Q_{count}$ :

[

]



# Adiar

[github.com/ssoelvsten/adiar](https://github.com/ssoelvsten/adiar)



—♦— CUDD —■— Sylvan —●— Adiar

Minimal running time for the *Queens* problems.



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Minimal running time for the *Queens* problems.





Minimal running time for the *Queens* problems.



Minimal running time for the *Queens* problems.



Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38

Checking the (EPFL Benchmark) *voter* circuit's single output gate ( $|N_f| = |N_g| = 5.76$  MiB).

Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38
$O(N \log N)$	0.058

Checking the (EPFL Benchmark) *voter* circuit's single output gate ( $|N_f| = |N_g| = 5.76$  MiB).

Algorithm	Time (s)
$f \leftrightarrow g \equiv \top$	0.38
$O(N \log N)$	0.058
$O(N)$	0.006

Checking the (EPFL Benchmark) *voter* circuit's single output gate ( $|N_f| = |N_g| = 5.76$  MiB).



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🐦 [@ssoelvsten](https://twitter.com/ssoelvsten)

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🔗 [github.com/ssoelvsten/adiar](https://github.com/ssoelvsten/adiar)

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