

Adiar

Binary Decision Diagrams in External Memory

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Minimal running time for the *Queens* problems.



Cache-misses for the *Queens* problems.



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



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Priority Queue: Q_{count} :

[

]



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(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\top} (1, 0), 1)$,

$((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|----------|-----|--------|
| $(1, 0)$ | 0 | 0 |

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\top} (1, 0), 1)$,
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|----------|-----|--------|
| $(1, 0)$ | 0 | 0 |

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{T} (1, 0), 1)$,
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|--------|-----|--------|
| (1, 0) | 1 | 0 |

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|--------|-----|--------|
| (1, 0) | 1 | 0 |

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|--------|-----|--------|
| (2, 0) | 0 | 0 |

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|----------|-----|--------|
| $(2, 0)$ | 0 | 0 |

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|----------|-----|--------|
| $(2, 0)$ | 1 | 0 |

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\perp} (2, 0), 1)$,

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---------------|-----|--------|
| (2, 0) | 2 | 0 |

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|----------|-----|--------|
| $(2, 0)$ | 2 | 0 |

Priority Queue: Q_{count} :

[

| | |
|--|---|
| $((2, 0) \xrightarrow{\perp} (3, 0), \quad 2)$ | , |
| $((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$ | , |
| $((2, 0) \xrightarrow{\top} (3, 1), \quad 2)$ |] |



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|----------|-----|--------|
| $(3, 0)$ | 0 | 0 |

Priority Queue: Q_{count} :

[

| | |
|--|---|
| $((2, 0) \xrightarrow{\perp} (3, 0), \quad 2)$ | , |
| $((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$ | , |
| $((2, 0) \xrightarrow{\top} (3, 1), \quad 2)$ |] |



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---------------|-----|--------|
| (3, 0) | 0 | 0 |

Priority Queue: Q_{count} :

[

| | |
|--|---|
| $((2, 0) \xrightarrow{\perp} (3, 0), \quad 2)$ | , |
| $((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$ | , |
| $((2, 0) \xrightarrow{\top} (3, 1), \quad 2)$ |] |



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|--------|-----|--------|
| (3, 0) | 2 | 0 |

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---------------|-----|--------|
| (3, 0) | 2 | 2 |

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---------------|-----|--------|
| (3, 1) | 0 | 2 |

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek | Sum | Result |
|---------------|-----|--------|
| (3, 1) | 0 | 2 |

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek
(3, 1)

Sum
1

Result
2

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{T} (3, 1), \quad 2) \quad]$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek
(3, 1)

Sum
3

Result
2

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek
(3, 1)

Sum
3

Result
5

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result
5

Priority Queue: Q_{count} :

[

]

Adiar

github.com/ssoelvsten/adiar



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Minimal running time for the *Queens* problems.



—♦— CUDD —□— Sylvan —○— Adiar

Minimal running time for the *Queens* problems.



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Minimal running time for the *Queens* problems.

| Algorithm | Time (s) |
|-----------------------------------|----------|
| $f \leftrightarrow g \equiv \top$ | 0.38 |

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

| Algorithm | Time (s) |
|-----------------------------------|----------|
| $f \leftrightarrow g \equiv \top$ | 0.38 |
| $O(N \log N)$ | 0.058 |

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

| Algorithm | Time (s) |
|-----------------------------------|----------|
| $f \leftrightarrow g \equiv \top$ | 0.38 |
| $O(N \log N)$ | 0.058 |
| $O(N)$ | 0.006 |

Checking the (EPFL Benchmark) *voter* circuit's single output gate ($|N_f| = |N_g| = 5.76$ MiB).

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Adiar

🔗 github.com/ssoelvsten/adiar

📄 ssoelvsten.github.io/adiar

