

An External Memory Relational Product

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$\exists \vec{x}. \phi(\dots)$ **(Push)**

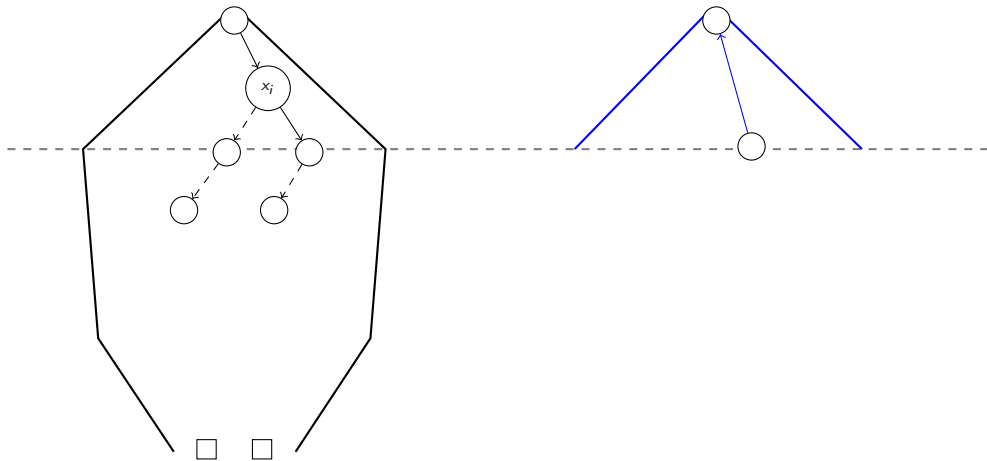


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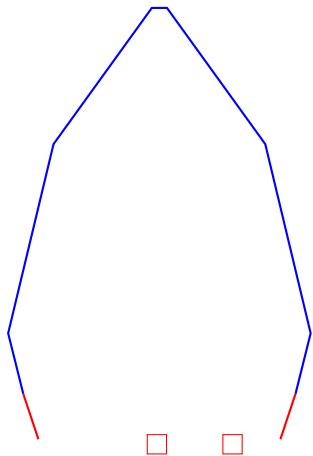
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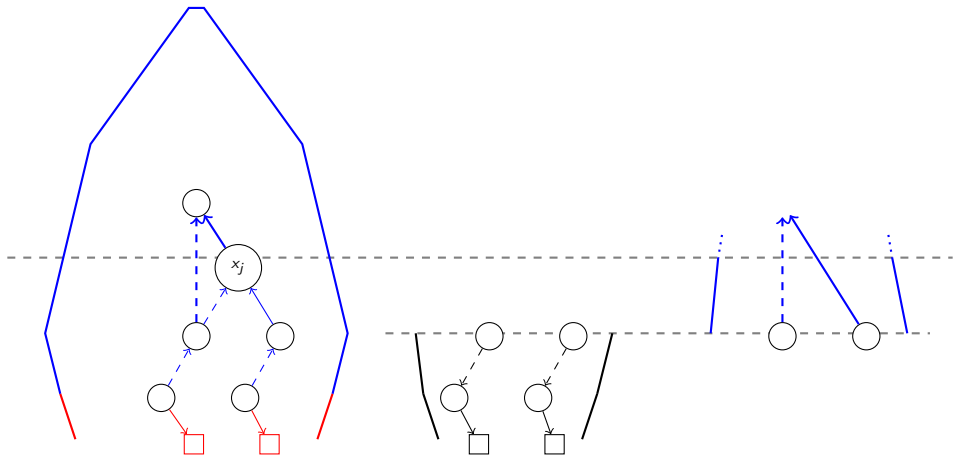


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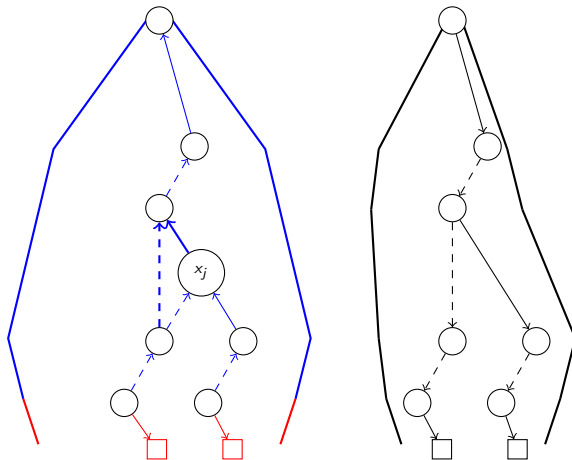
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Definition

A relabelling π is monotonic if $x_i < x_j \implies \pi(x_i) < \pi(x_j)$

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If π is monotonic

- *1-Var / Push:*

Apply π in $O(L_N)$ extra time during the final bottom-up Reduce sweep.

- *Bounce:*

$x'_i < x'_j \implies x_i < x_j$: π can be applied during the outermost Reduce sweep.

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If π is not monotonic

to be continued...