

Adiar 1.1 : Zero-suppressed Decision Diagrams in External Memory

Steffan Christ Sølvesten and Jaco van de Pol

18th of May, 2023









Adiar

Binary Decision Diagrams
in External Memory

`github.com/ssoelvsten/adiar`

Adiar

Multi-terminal Decision Diagrams
in External Memory

`github.com/ssoelvsten/adiar`

Adiar

Quantum Multi-valued Decision Diagrams
in External Memory

`github.com/ssoelvsten/adiar`

Adiar

Zero-suppressed Decision Diagrams
in External Memory

`github.com/ssoelvsten/adiar`







BDD: $f : \mathbb{B}^n \rightarrow \mathbb{B}$



ZDD: $A \subseteq \mathbb{B}^n$

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {  
    return prod2<bdd_policy>(f, g, o)  
}
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)  {  
    return prod2<zdd_policy>(A, B, o)  
}
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {  
    return prod2<bdd_policy>(f, g, o)  
}
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)  {  
    return prod2<zdd_policy>(A, B, o)  
}
```

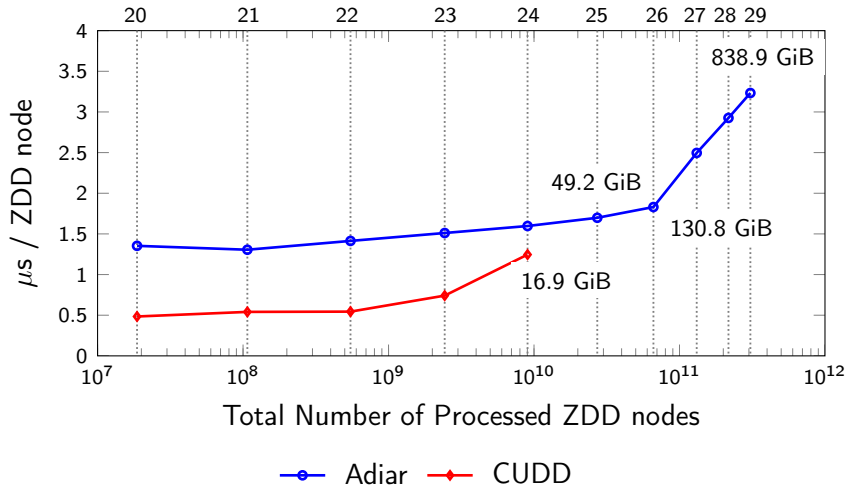



—●— Adiar —◆— CUDD

Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.

Done

BDD ZDD

Doable

MTBDD

LDD

QMDD

Done

BDD

ZDD

(K)FDD

Tagged/Chained BDD

Open

Clock DD

MDD

Doable

MTBDD

LDD

QMDD

Done

BDD

ZDD

(K)FDD

Tagged/Chained BDD

Steffan Christ Sølvsten

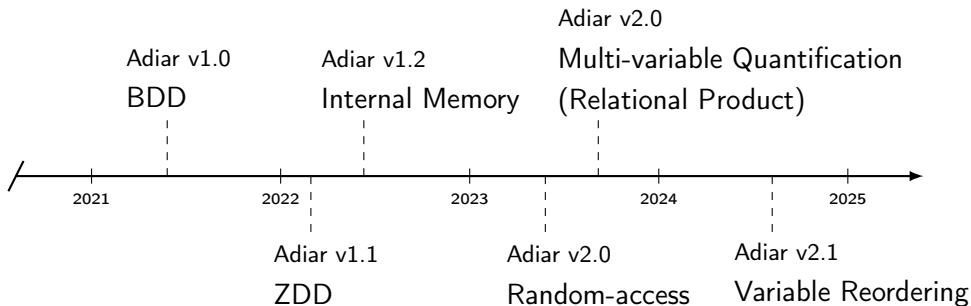
✉ soelvsten@cs.au.dk

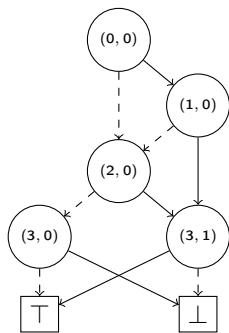
🐦 [@ssoelvsten](https://twitter.com/ssoelvsten)

Adiar

🔗 github.com/ssoelvsten/adiar

📖 ssoelvsten.github.io/adiar





(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

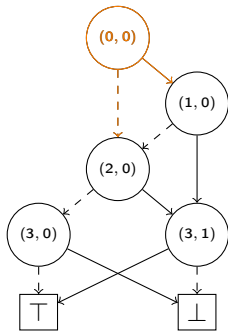


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[

]

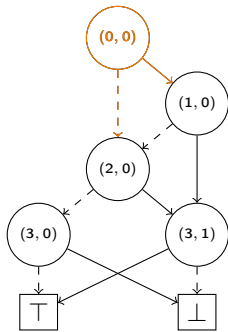


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[

]



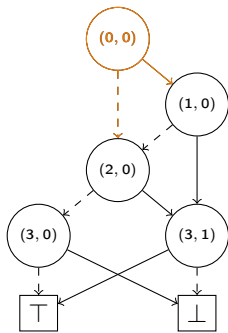
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[$((0,0) \xrightarrow{\top} (1,0), 1)$,

$((0,0) \xrightarrow{\perp} (2,0), 1)$,

]



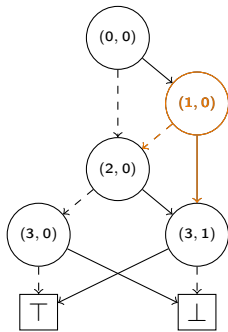
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\top} (1, 0), 1)$,
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]



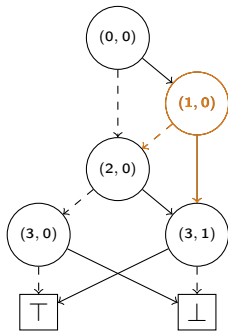
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\top} (1, 0), 1)$,
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	1	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

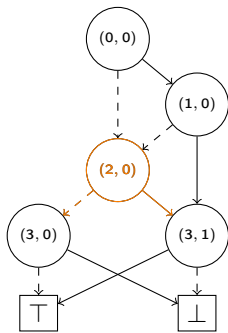
Seek	Sum	Result
(2, 0)	0	0

Priority Queue: Q_{count} :

[

$((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
$((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
$((1, 0) \xrightarrow{\top} (3, 1), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	1	0

Priority Queue: Q_{count} :

[
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]



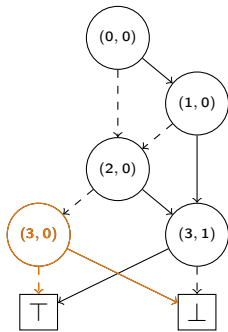
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2]



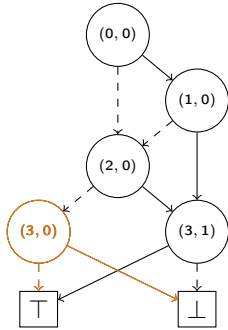
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	0

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



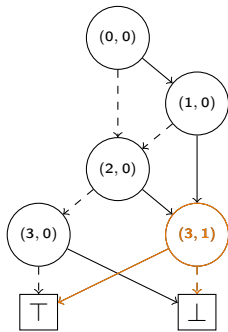
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



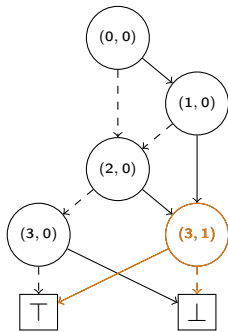
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$
 $((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	1	2

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	3	2

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek
(3, 1)

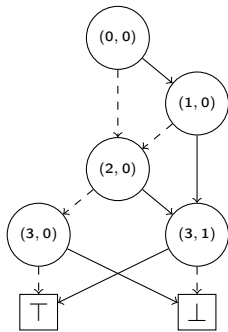
Sum
3

Result
5

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result
5

Priority Queue: Q_{count} :

[

]

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Adiar

🔗 github.com/ssoelvsten/adiar

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