An External Memory Relational Product

Steffan Christ Sølvsten, Jaco van de Pol

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AARHUS
UNIVERSITY

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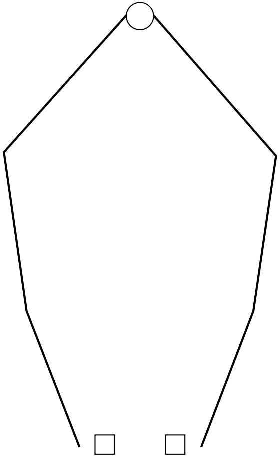

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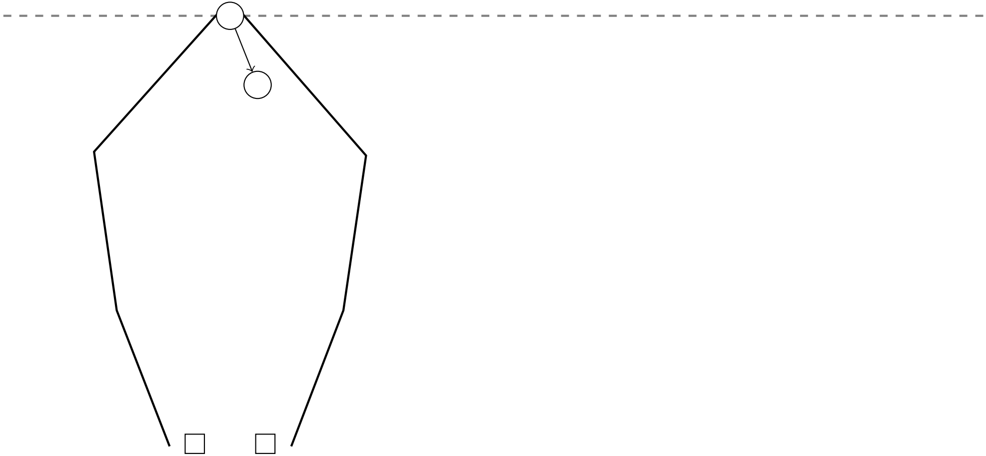

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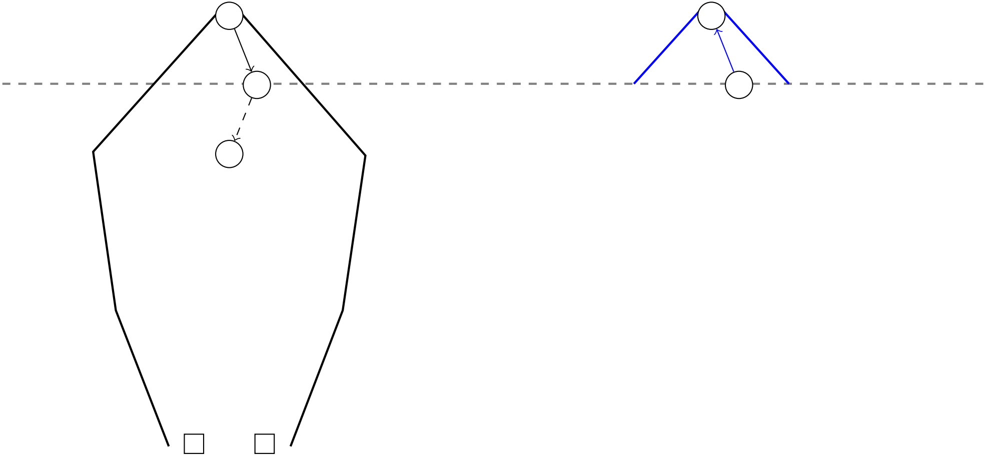

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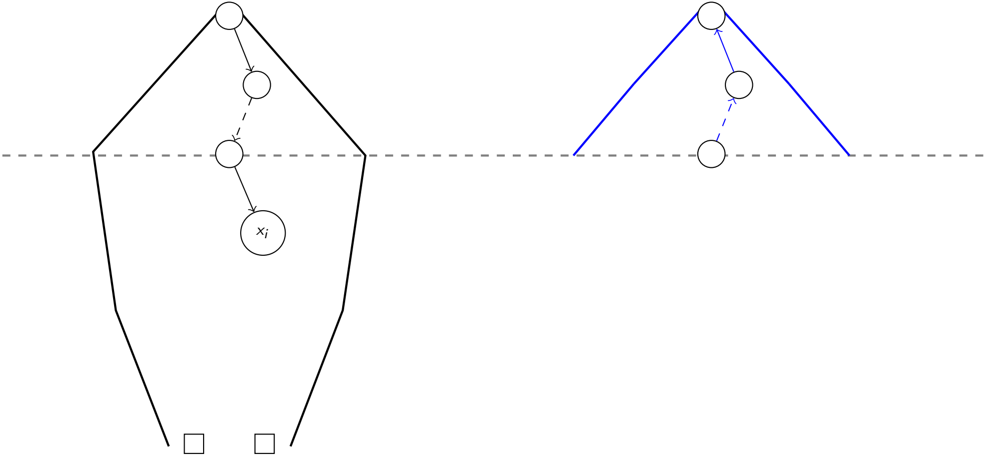

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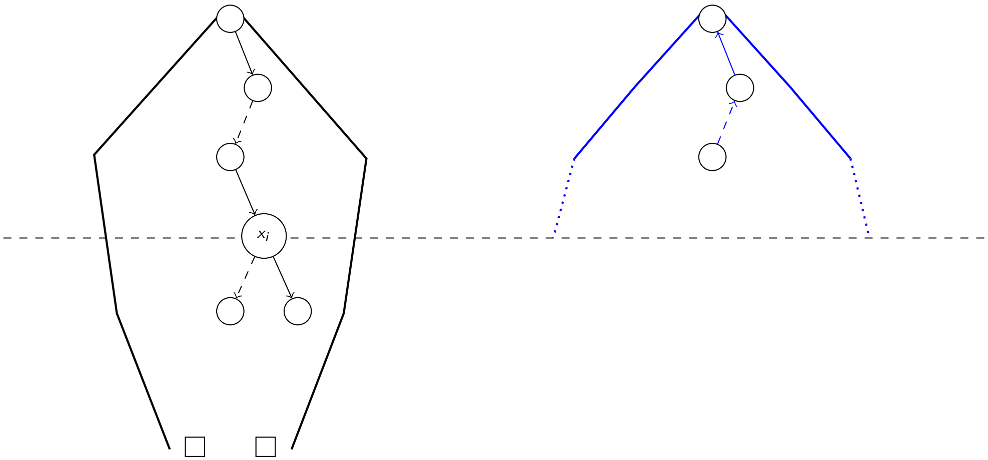

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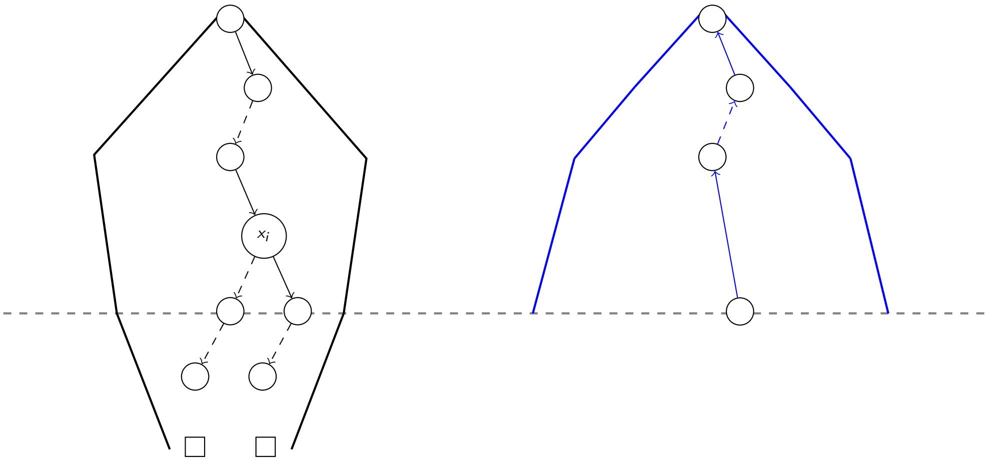

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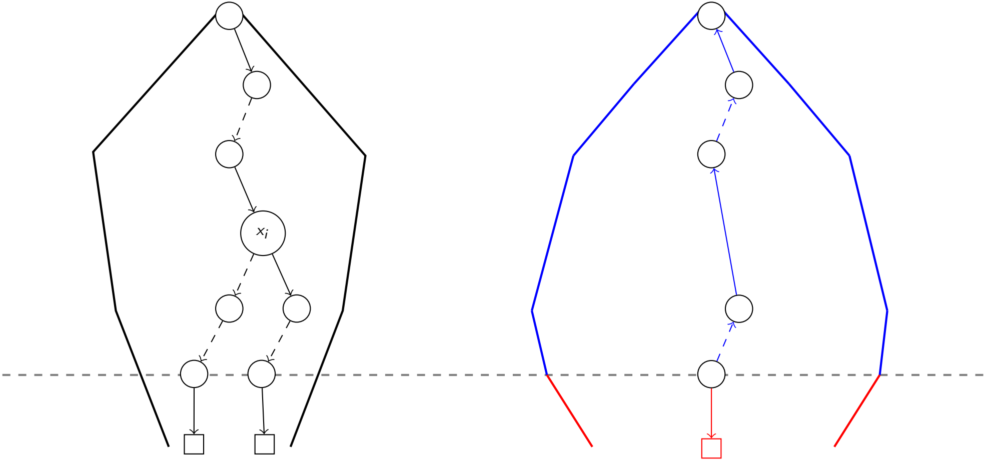

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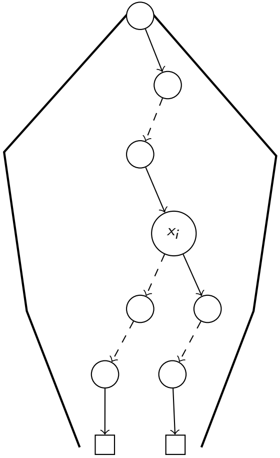

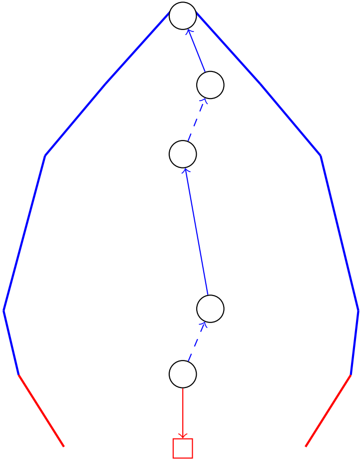

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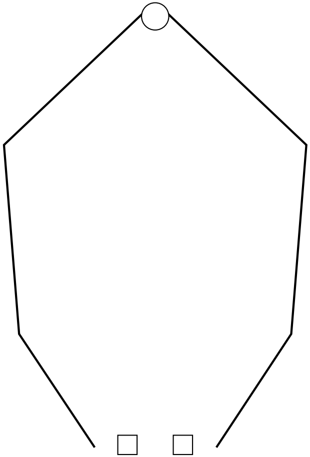

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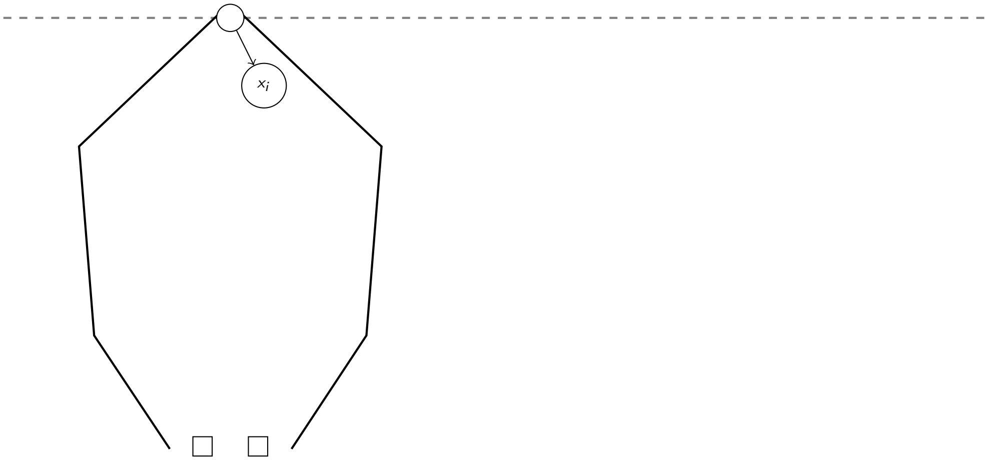

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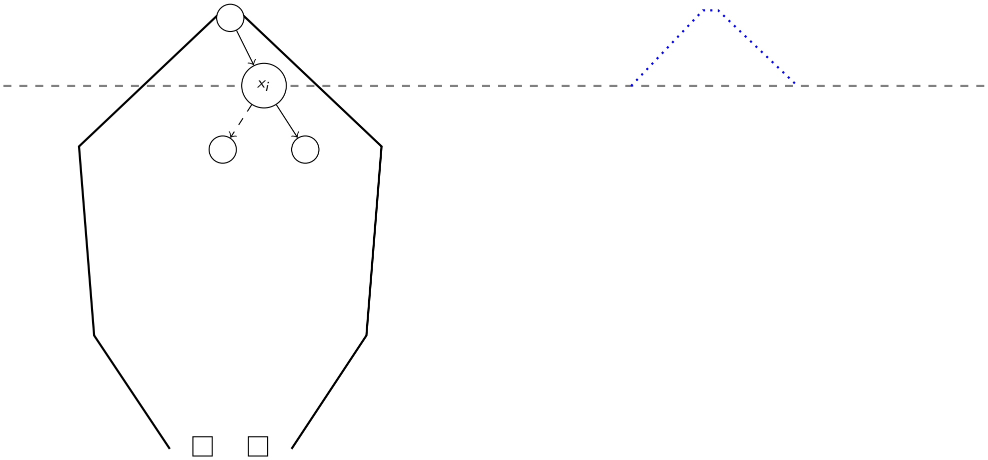

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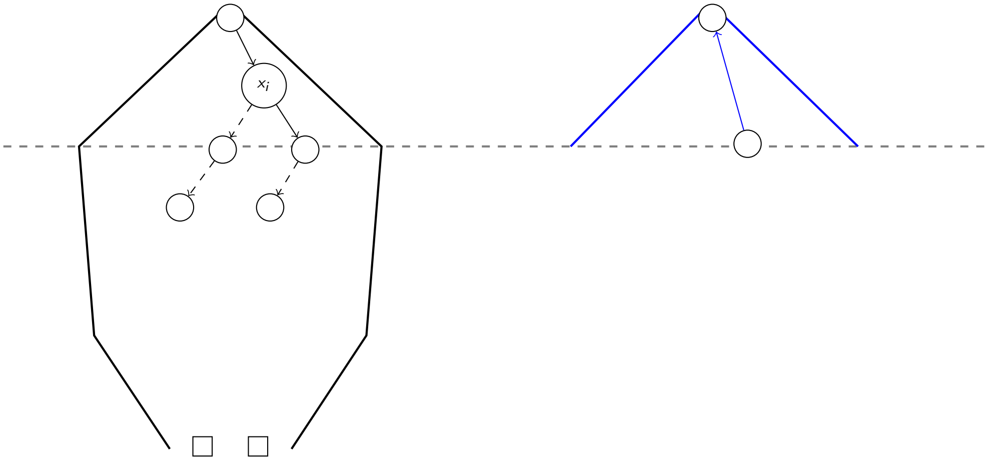

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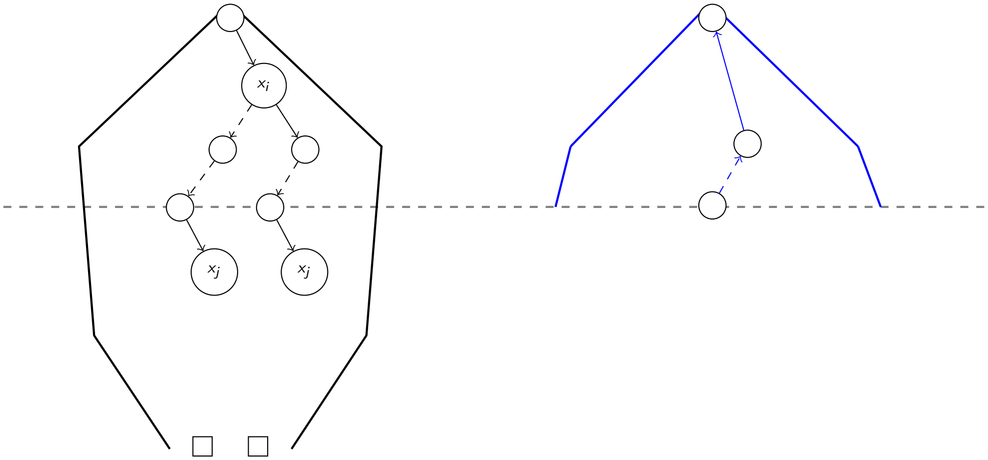

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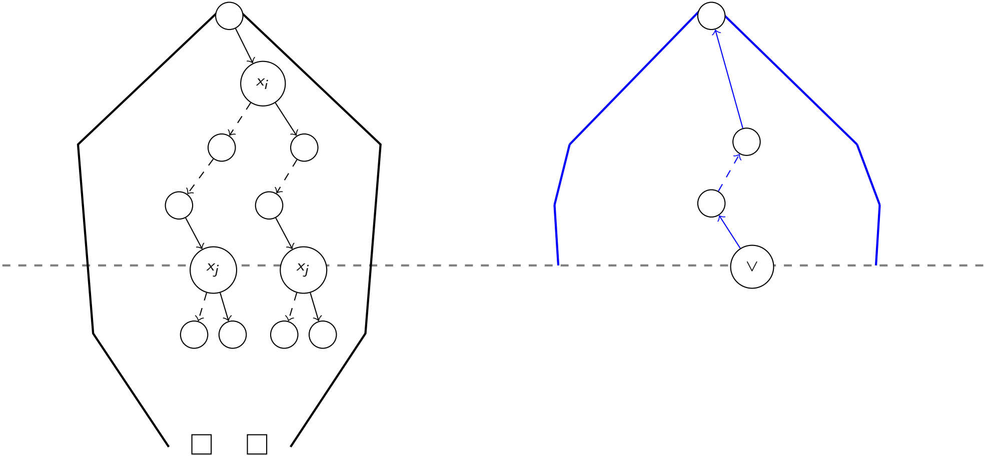

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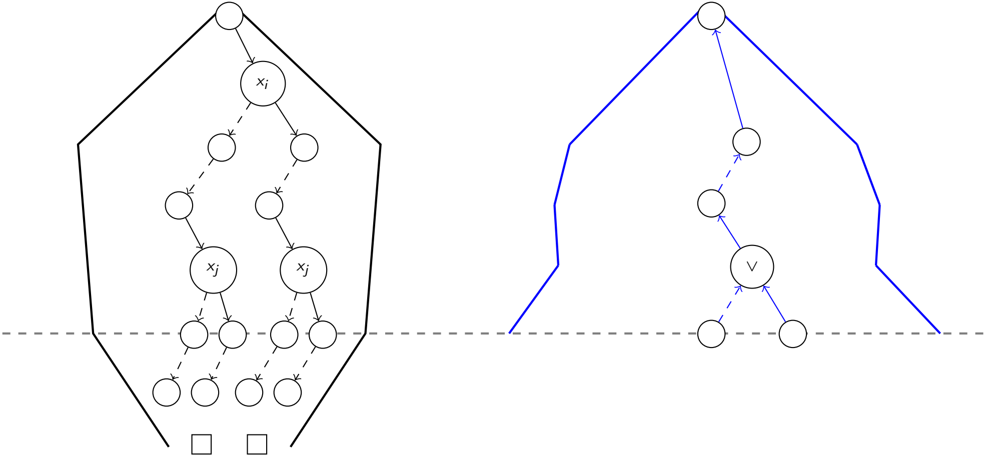

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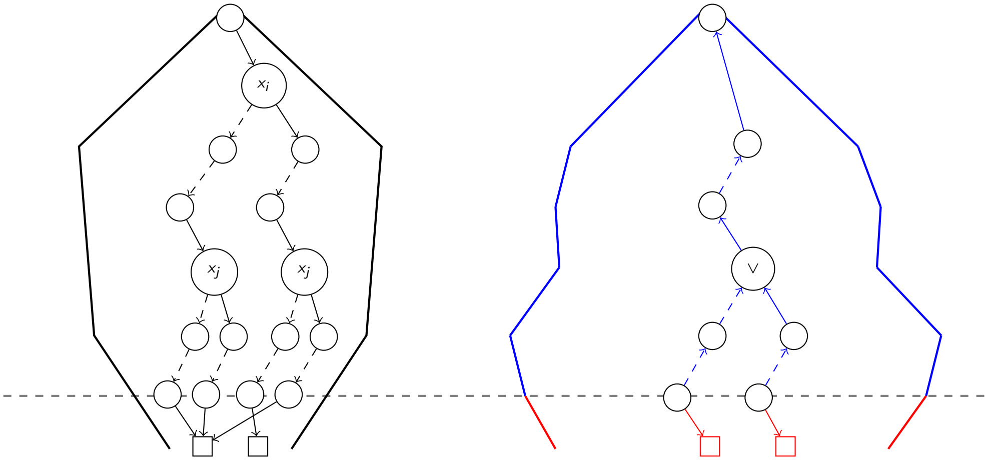

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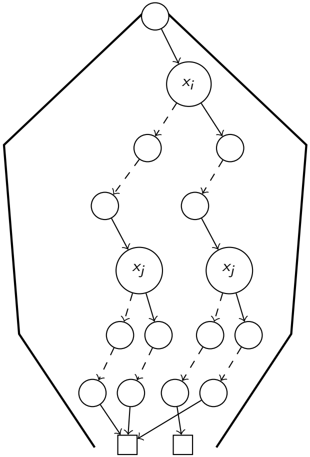

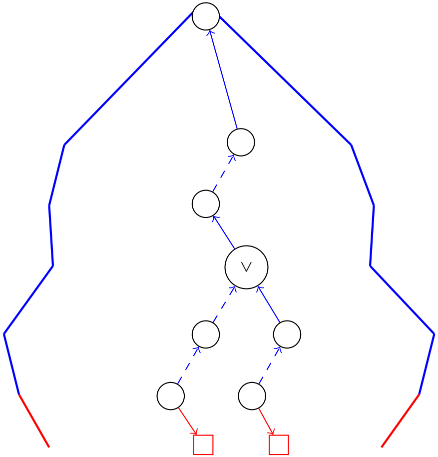

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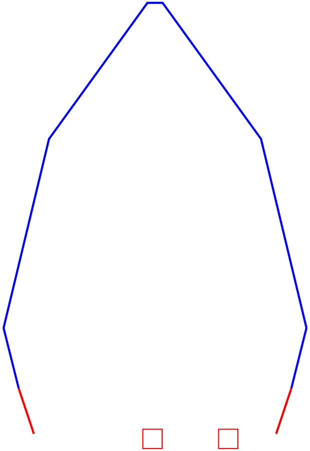

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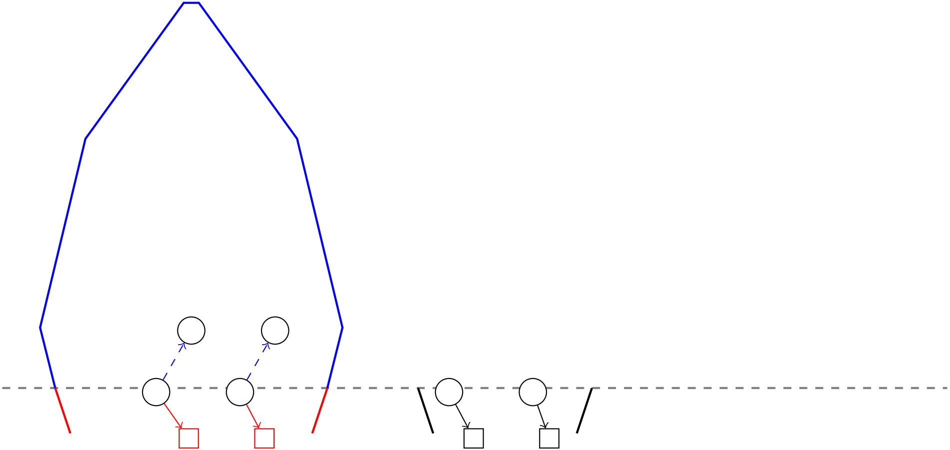

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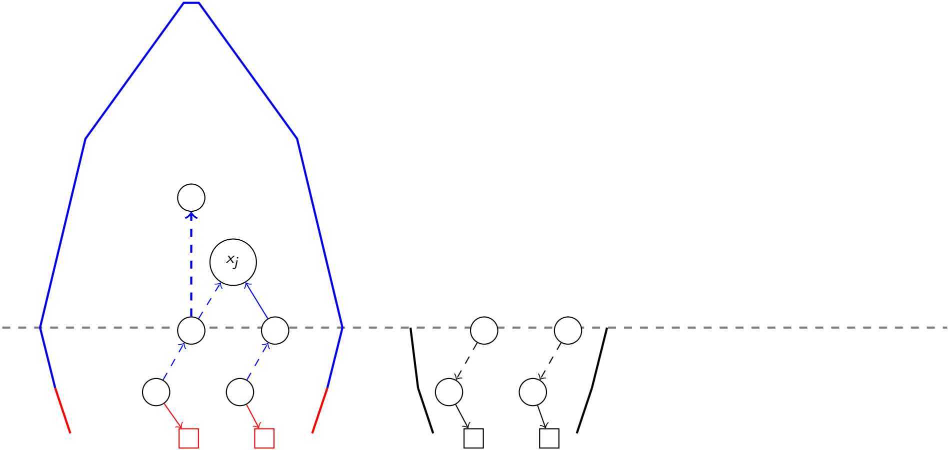

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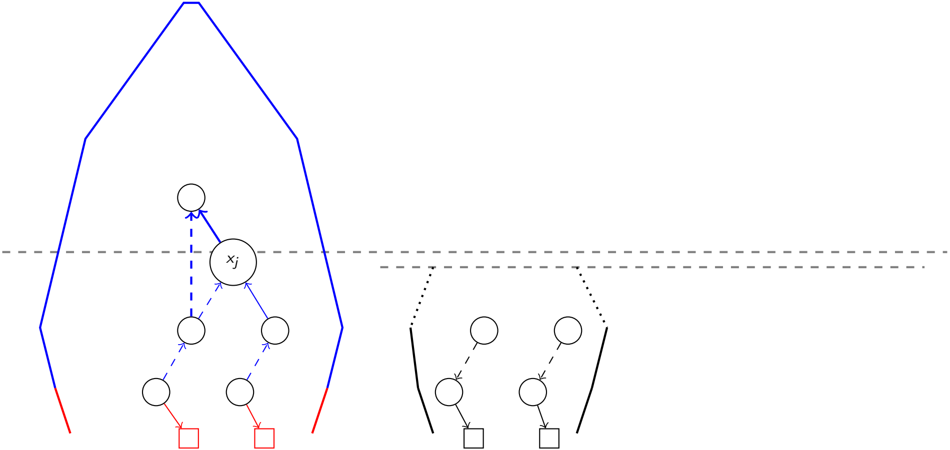

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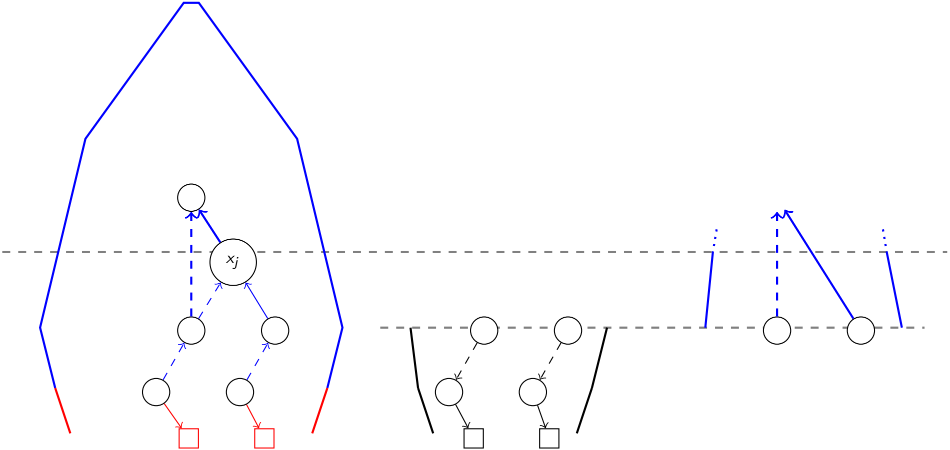

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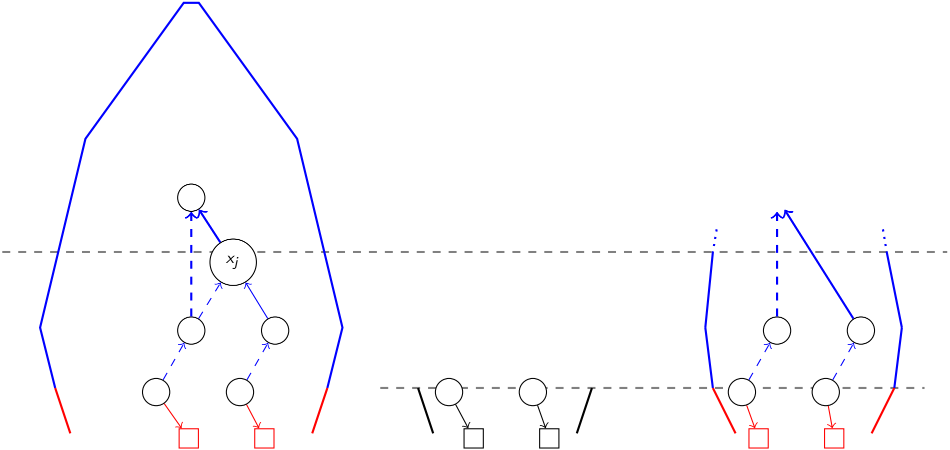

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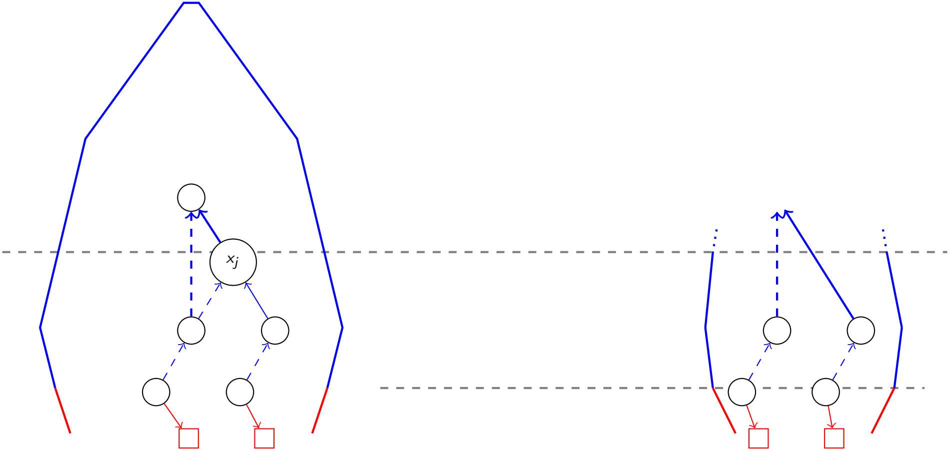

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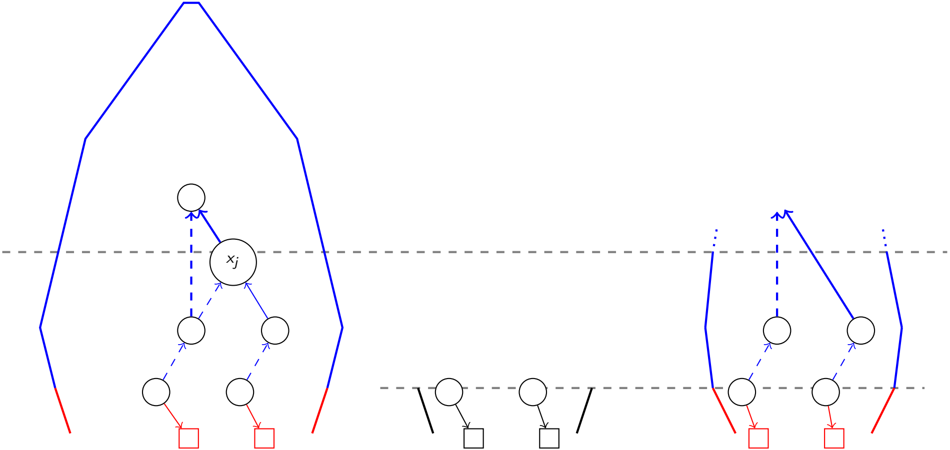

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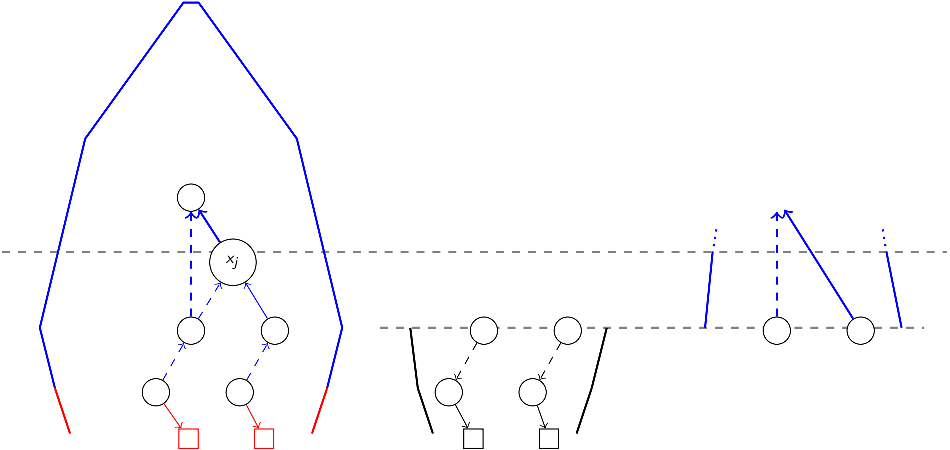

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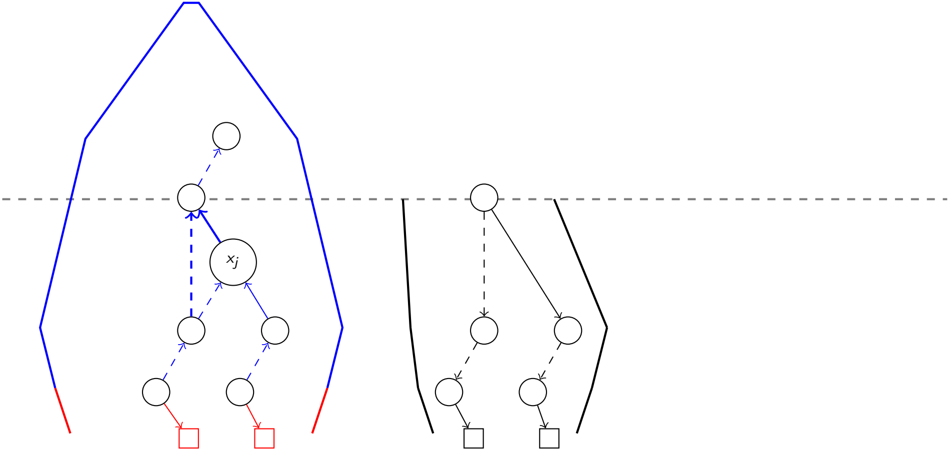

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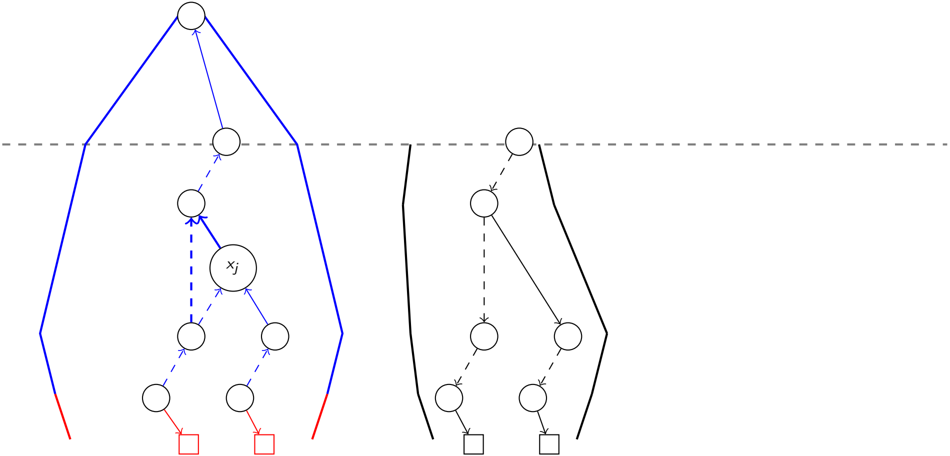

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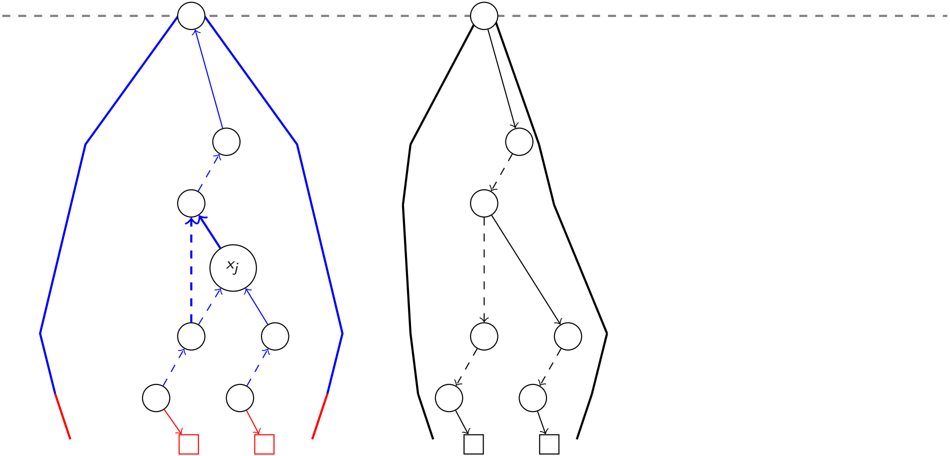

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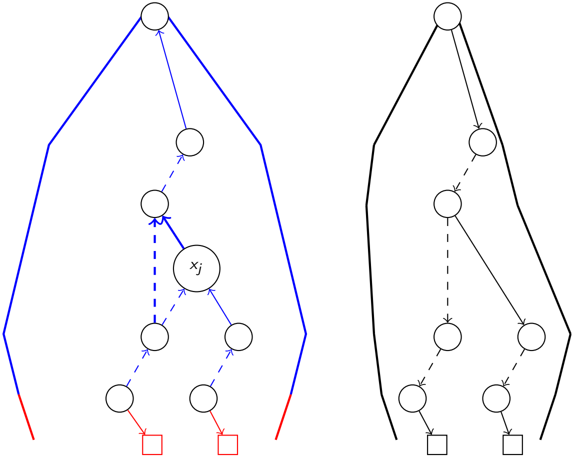

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  Apply π in $O(L_N)$ extra time during the final bottom-up Reduce sweep.
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If π is not monotonic

to be continued...