

# Adiar 1.1 : Zero-suppressed Decision Diagrams in External Memory

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**Steffan Christ Sølvesten** and Jaco van de Pol

NFM 2023











# Adiar

Binary Decision Diagrams  
in External Memory

`github.com/ssoelvsten/adiar`

# Adiar

**Multi-terminal** Decision Diagrams  
in External Memory

`github.com/ssoelvsten/adiar`

# Adiar

**Quantum Multi-valued** Decision Diagrams  
in External Memory

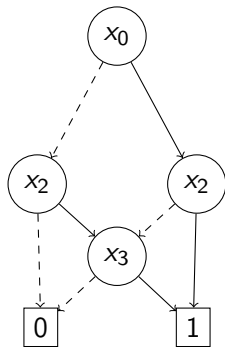
`github.com/ssoelvsten/adiar`



# Adiar

**Zero-suppressed** Decision Diagrams  
in External Memory

`github.com/ssoelvsten/adiar`







**BDD:**  $f : \mathbb{B}^n \rightarrow \mathbb{B}$



**ZDD:**  $A \subseteq \mathbb{B}^n$

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {  
    return prod2<bdd_policy>(f, g, o);  
}
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)  {  
    return prod2<zdd_policy>(A, B, o);  
}
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {  
    return prod2<bdd_policy>(f, g, o);  
}
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)  {  
    return prod2<zdd_policy>(A, B, o);  
}
```

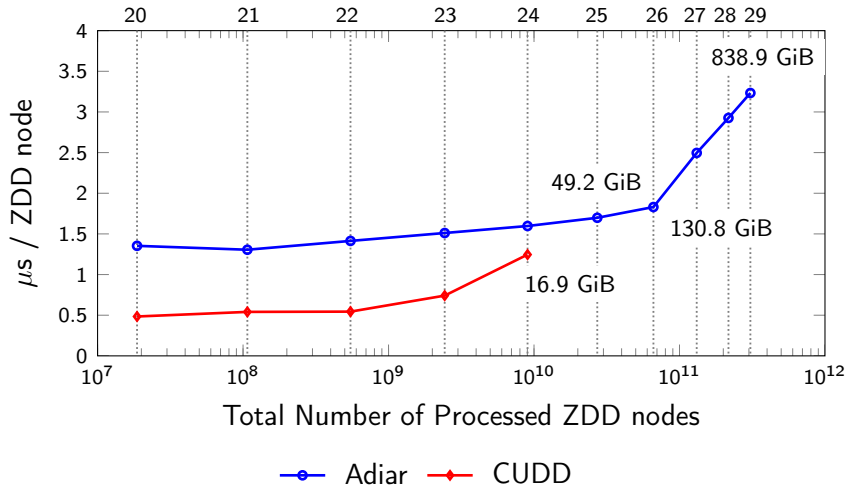




Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



*Done*

BDD   ZDD

*Doable*

MTBDD

LDD

QMDD

*Done*

BDD

ZDD

(K)FDD

Tagged/Chained BDD

*Open*

Clock DD

MDD

*Doable*

MTBDD

LDD

QMDD

*Done*

BDD

ZDD

(K)FDD

Tagged/Chained BDD





# Steffan Christ Sølvsten

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🌐 [ssoelvsten.github.io](https://ssoelvsten.github.io)

## Adiar

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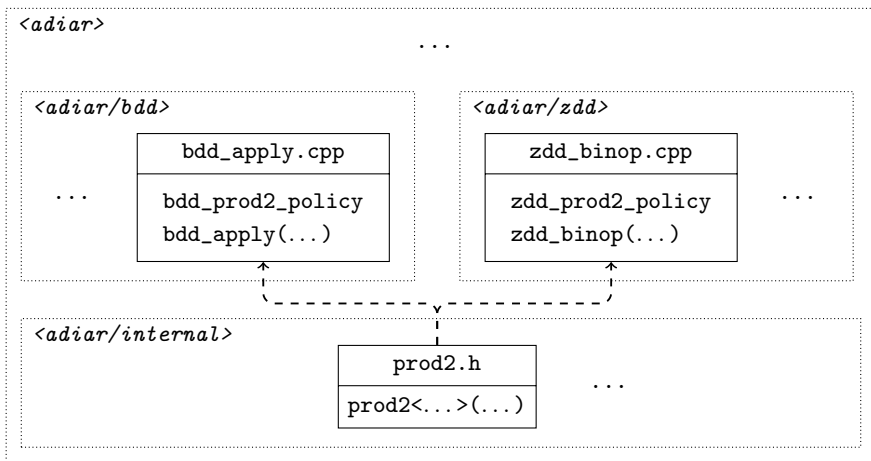
📖 [ssoelvsten.github.io/adiar](https://ssoelvsten.github.io/adiar)



Function	Operation Semantics
<b>ZDD Constructors</b>	
<code>zdd_empty()</code>	$\emptyset$
<code>zdd_null()</code>	$\{\emptyset\}$
<code>zdd_singleton(var)</code>	$\{x_{var}\}$
<code>zdd_vars(vars)</code>	$\{\bigcup_{i \in vars} \{x_i\}\}$
<code>zdd_singletons(vars)</code>	$\{\{x_i\} \mid i \in vars\}$
<code>zdd_powerset(vars)</code>	$\mathcal{P}(vars)$
<code>zdd_sized_set(vars, k, <math>\odot</math>)</code>	$\{s \in \mathcal{P}(vars) \mid  s  \odot k\}$
<b>ZDD Manipulation</b>	
<code>zdd_binop(A, B, <math>\otimes</math>)</code>	$\{x \mid x \in A \otimes x \in B\}$
<code>zdd_change(A, vars)</code>	$\{(a \setminus vars) \cup (vars \setminus a) \mid a \in A\}$
<code>zdd_complement(A, dom)</code>	$\mathcal{P}(dom) \setminus A$
<code>zdd_expand(A, vars)</code>	$\bigcup_{a \in A} \{a \cup v \mid v \in \mathcal{P}(vars)\}$
<code>zdd_offset(A, vars)</code>	$\{a \in A \mid vars \cap a = \emptyset\}$
<code>zdd_onset(A, vars)</code>	$\{a \in A \mid vars \subseteq a\}$
<code>zdd_project(A, vars)</code>	$\bigcup_{a \in A} \{a \cap vars\}$

Function	Operation Semantics
<b>Counting</b>	
<code>zdd_size(A)</code>	$ A $
<code>zdd_nodecount(A)</code>	# ZDD Nodes in A
<code>zdd_varcount(A)</code>	# Non-empty Levels in A
<b>Predicates</b>	
<code>zdd_equal(A, B)</code>	$A = B$
<code>zdd_unequal(A, B)</code>	$A \neq B$
<code>zdd_subseteq(A, B)</code>	$A \subseteq B$
<code>zdd_disjoint(A, B)</code>	$A \cap B = \emptyset$
<b>Set elements</b>	
<code>zdd_contains(A, a)</code>	$a \in A$
<code>zdd_minelem(A)</code>	$a \in A \text{ s.t. } \forall a' \in A. a \leq a'$
<code>zdd_maxelem(A)</code>	$a \in A \text{ s.t. } \forall a' \in A. a' \leq a$
<b>Conversion</b>	
<code>zdd_from(f, dom)</code>	$\{x \in \mathcal{P}(dom) \mid f(x) = \top\}$
<code>bdd_from(A, dom)</code>	$\vec{x} : \mathcal{P}(dom) \mapsto \vec{x} \in A$

Operations provided by Adiar in `<adiar/zdd.h>`.





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



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Priority Queue:  $Q_{count}$ :

[

]



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[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[  $((0,0) \xrightarrow{\top} (1,0), 1)$  ,  
 $((0,0) \xrightarrow{\perp} (2,0), 1)$  ,

]





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
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Priority Queue:  $Q_{count}$ :

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 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 ]

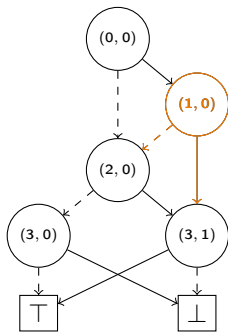


(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	1	0

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(2, 0)</b>	0	0

Priority Queue:  $Q_{count}$ :

[

$((0, 0) \xrightarrow{\perp} (2, 0), 1)$	,
$((1, 0) \xrightarrow{\perp} (2, 0), 1)$	,
$((1, 0) \xrightarrow{\top} (3, 1), 1)$	,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	1	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1) \quad ,$

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
]





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(2, 0)</b>	2	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(3, 0)</b>	0	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2	]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(3, 0)</b>	0	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2	]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$	,
$((2, 0) \xrightarrow{T} (3, 1), 2)$	]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(3, 1)</b>	0	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(3, 1)</b>	1	2

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad ]$





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek  
**(3, 1)**

Sum  
3

Result  
2

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek  
**(3, 1)**

Sum  
3

Result  
5

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result  
5

Priority Queue:  $Q_{count}$ :

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]

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