An External Memory Relational Product

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AARHUS
UNIVERSITY

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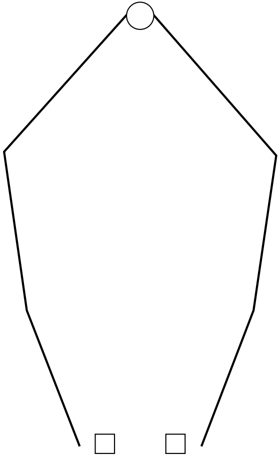

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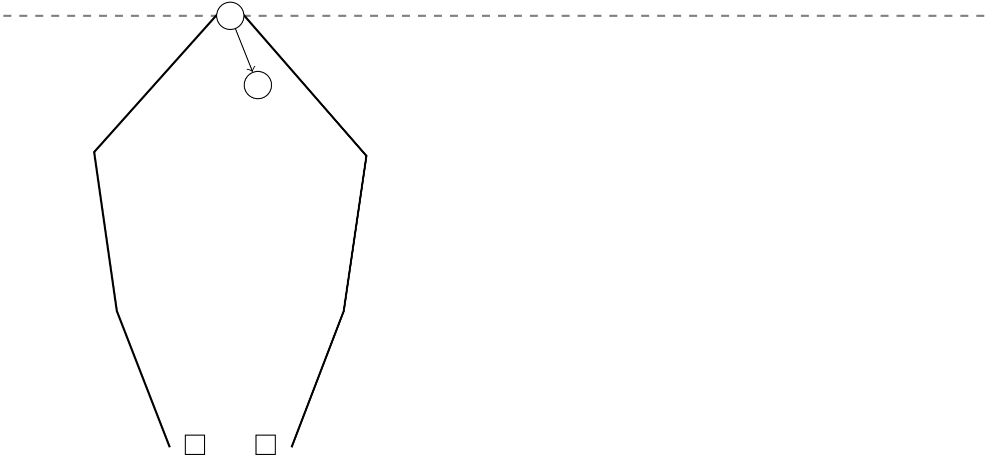

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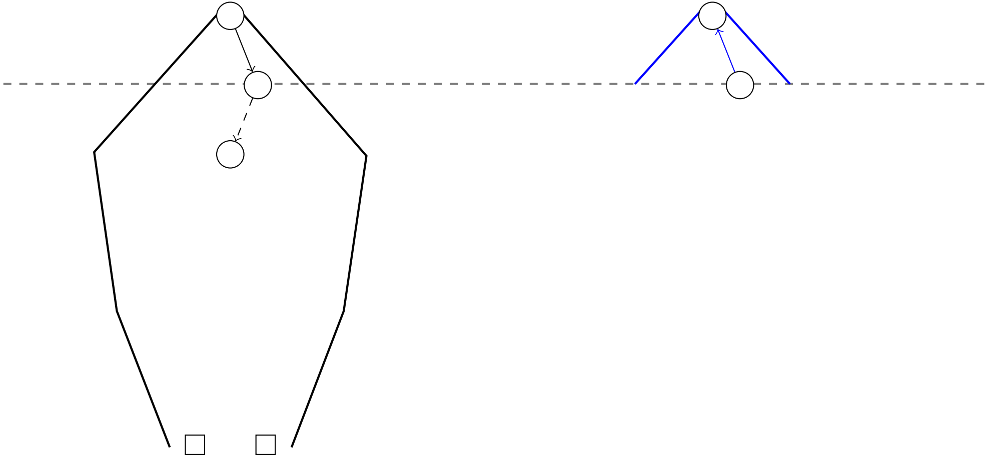

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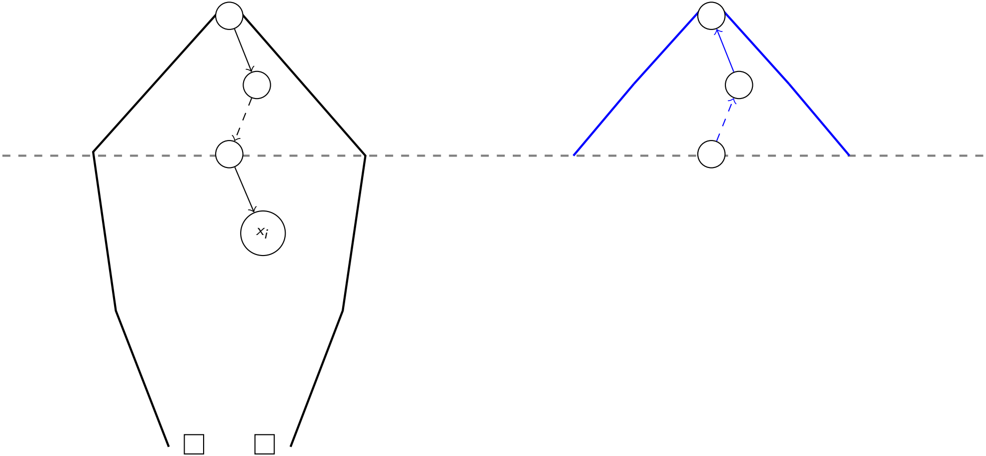

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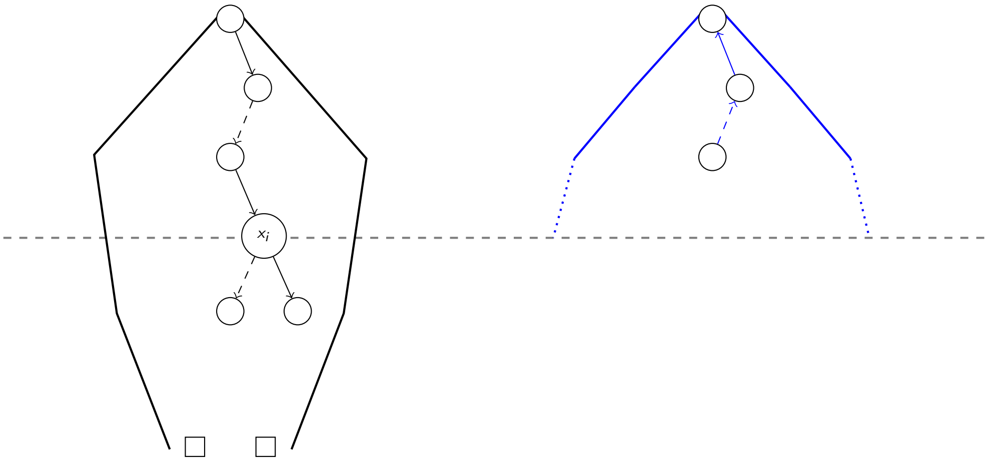

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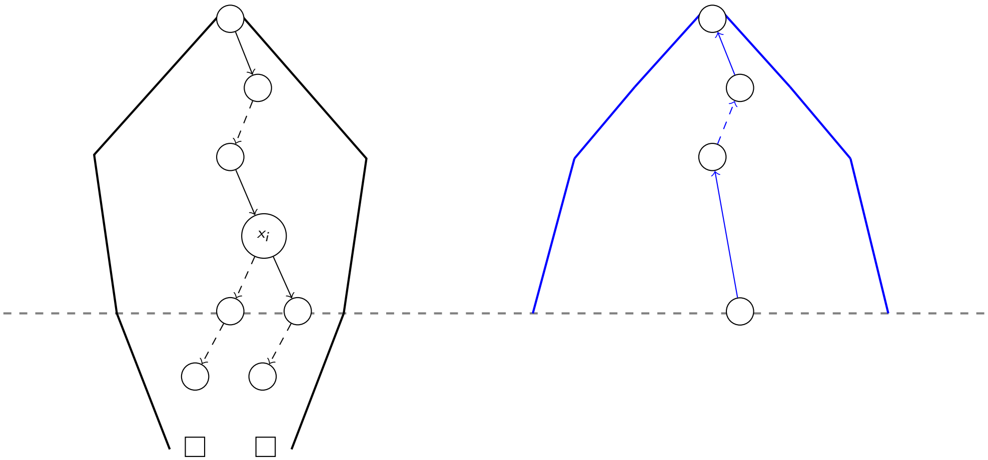

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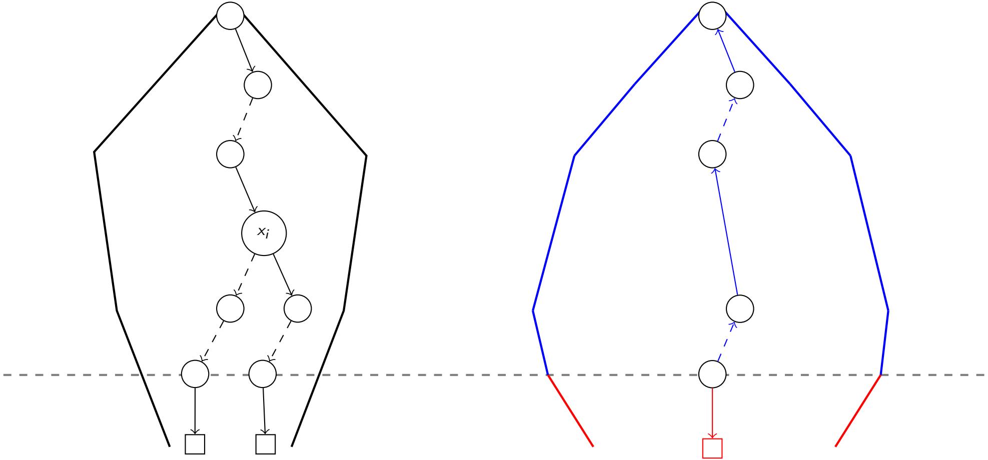

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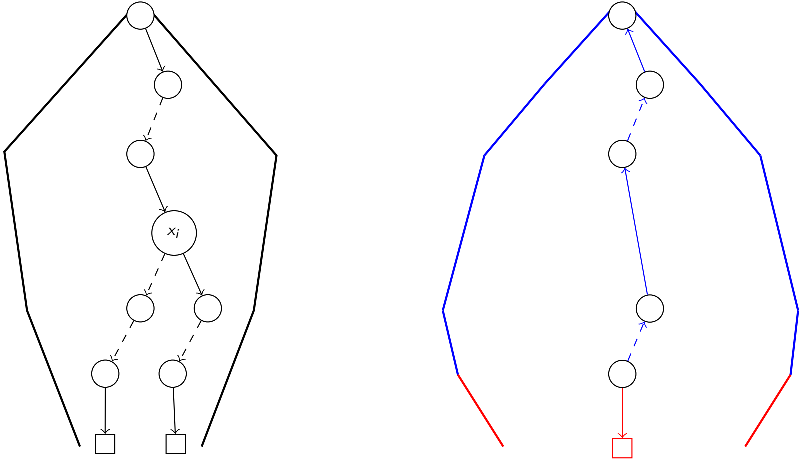

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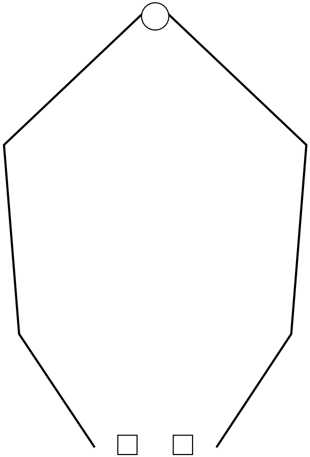

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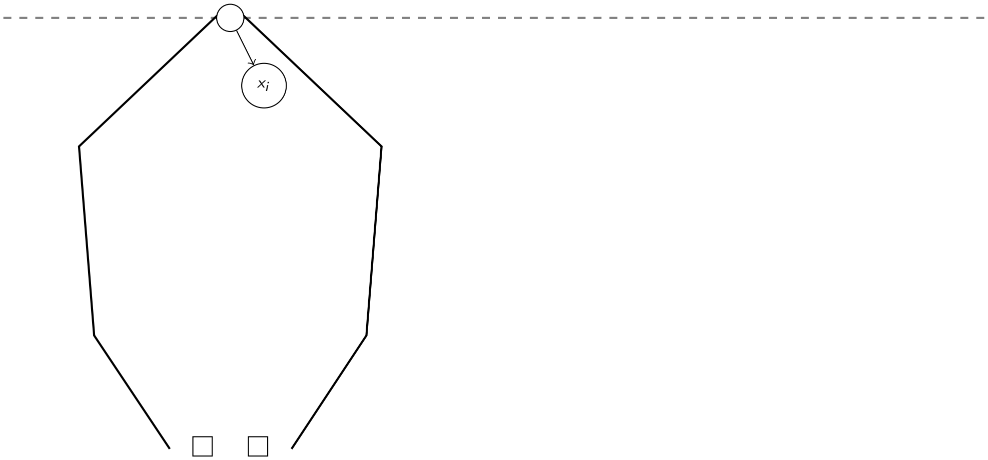

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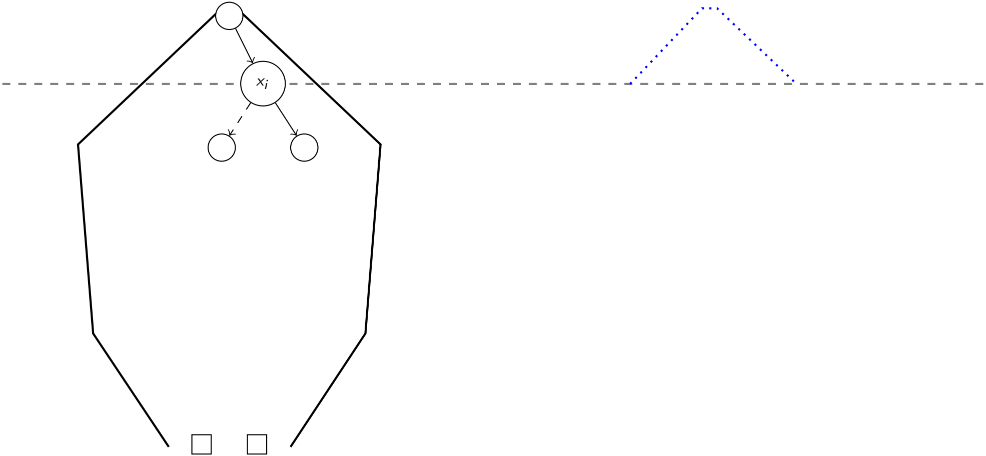

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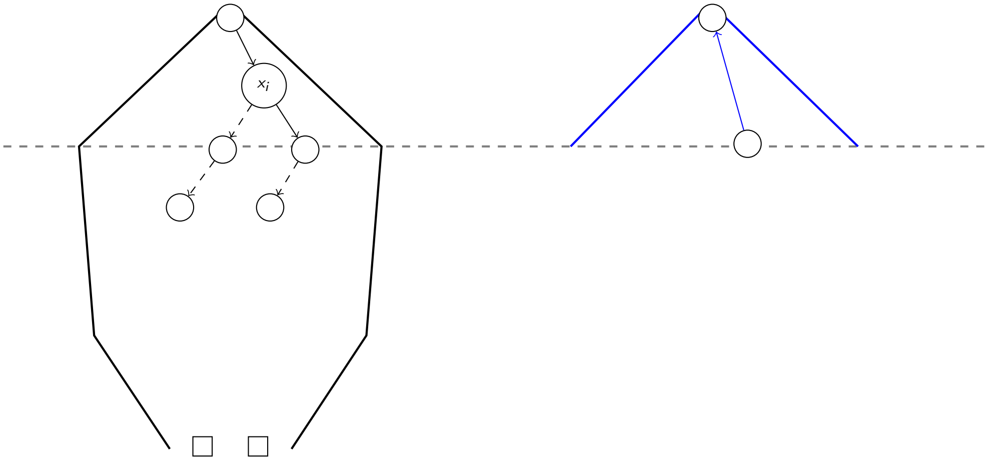

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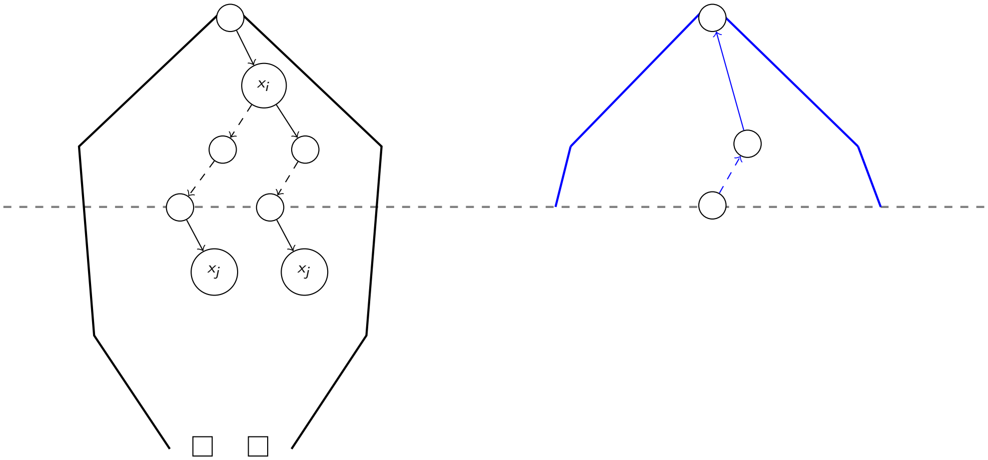

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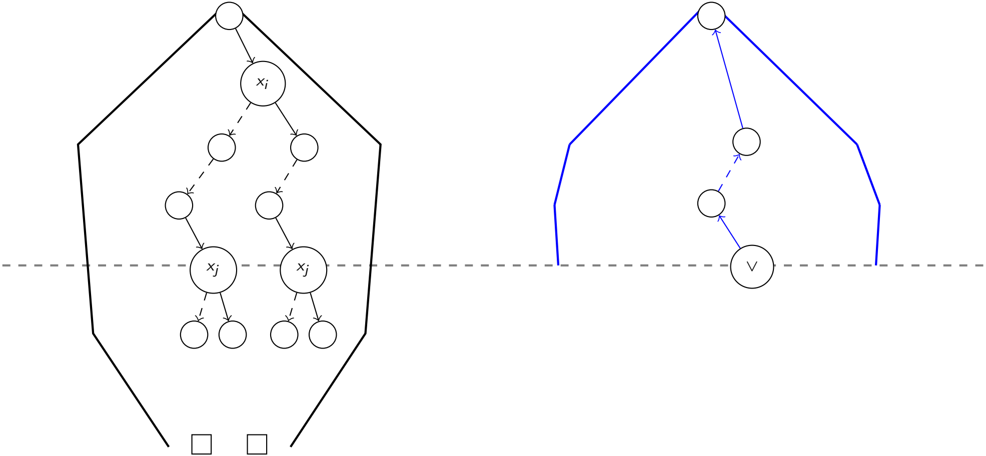

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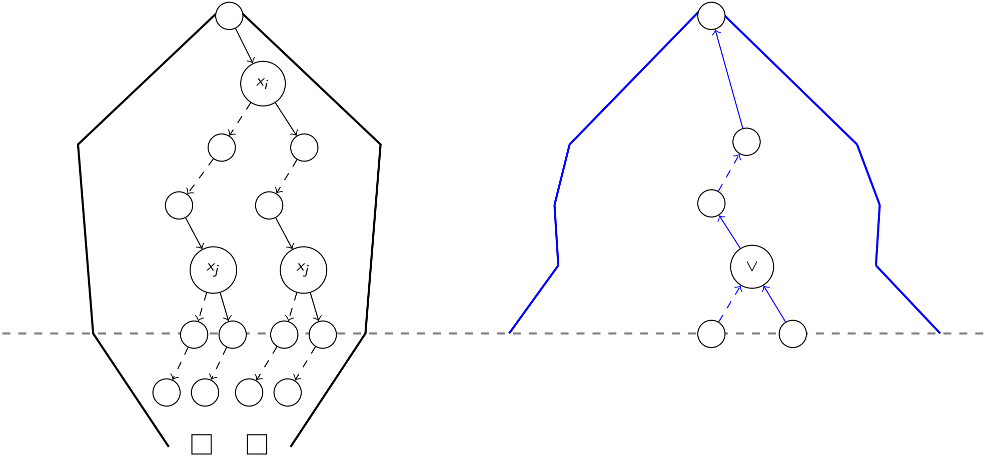

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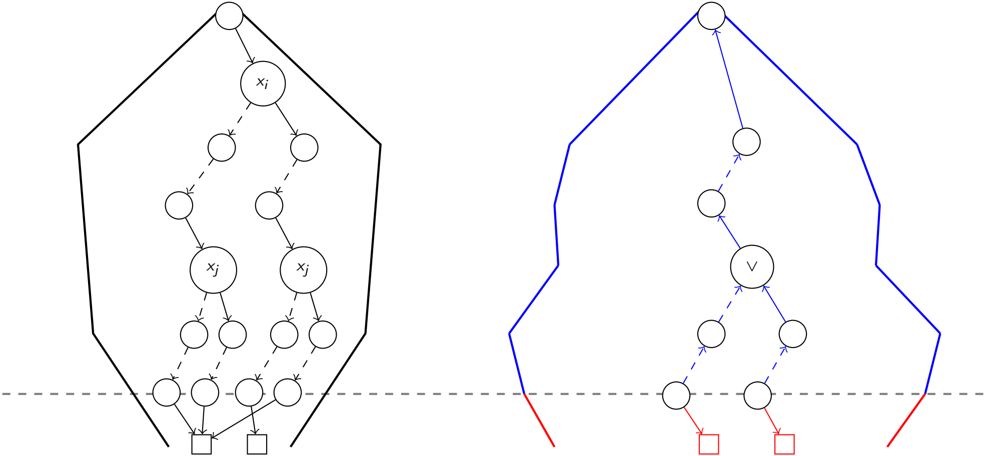

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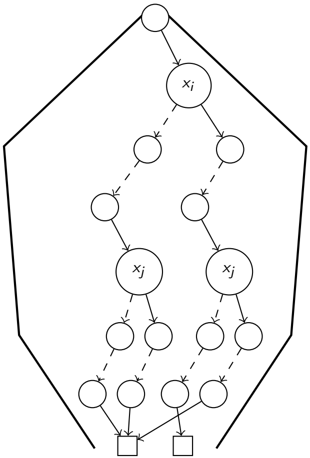

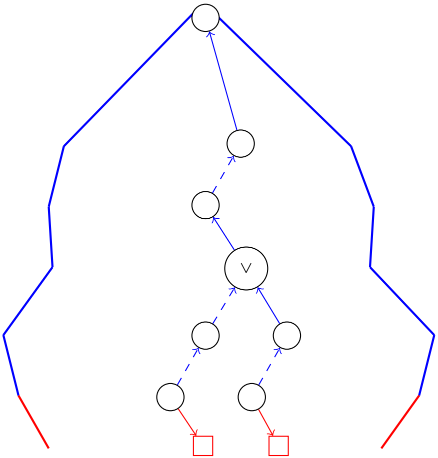

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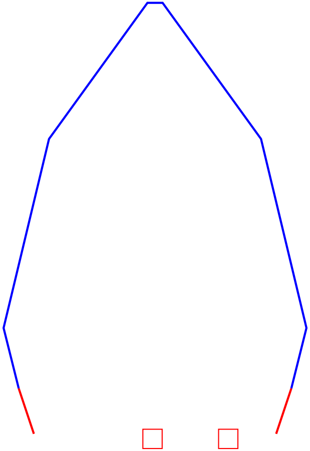

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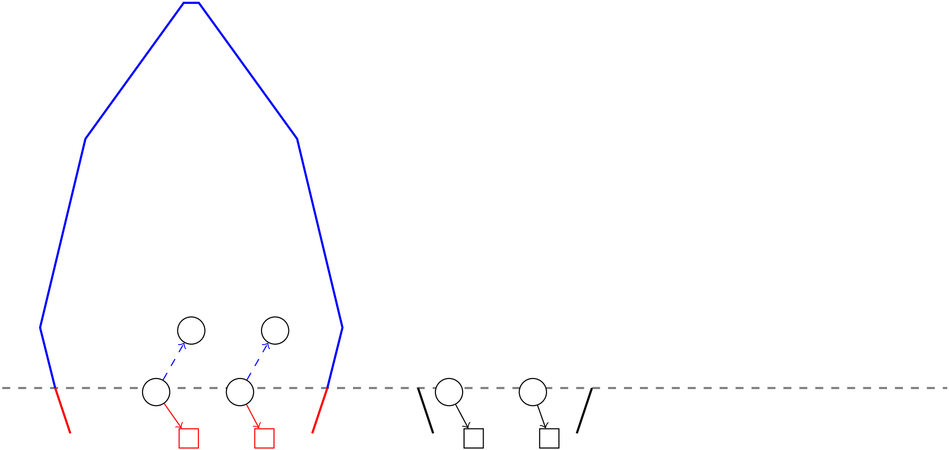

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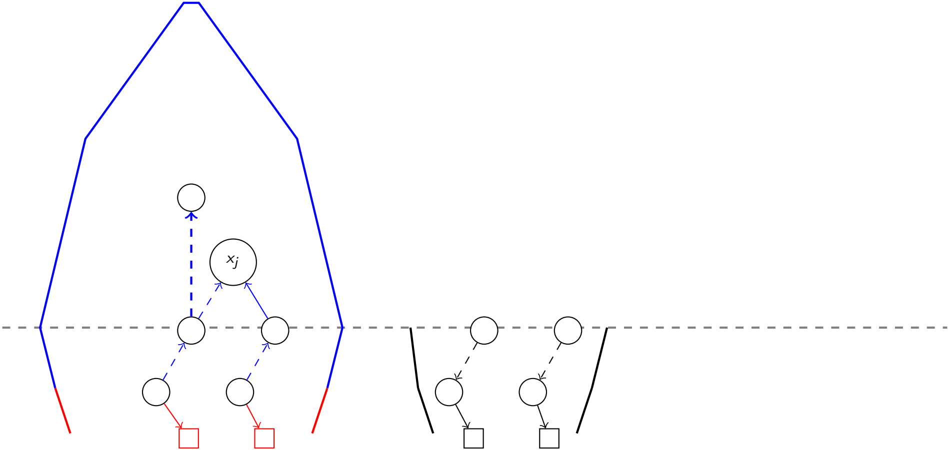

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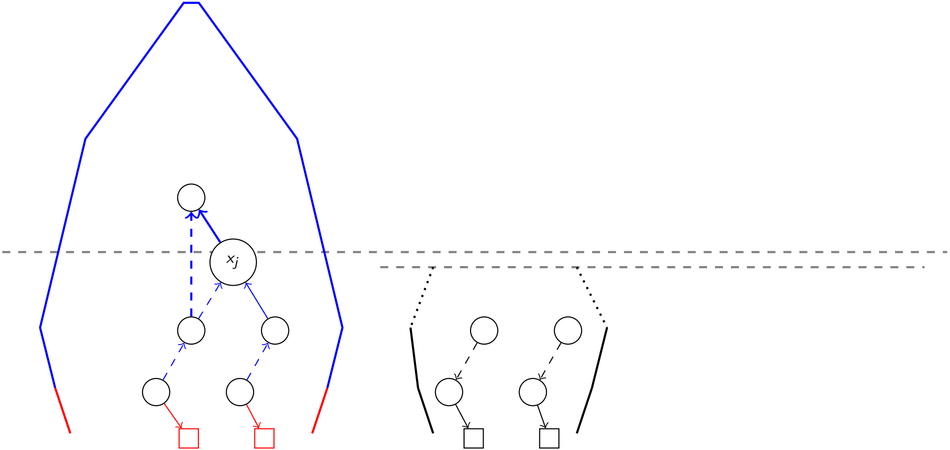

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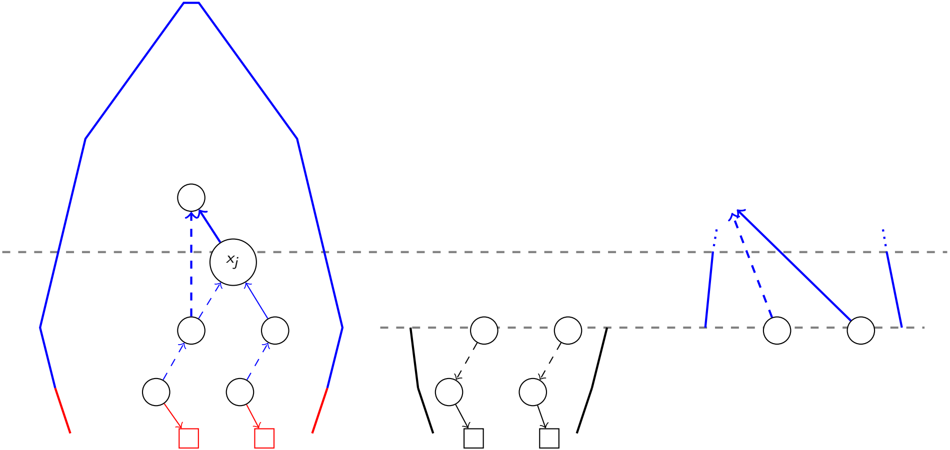

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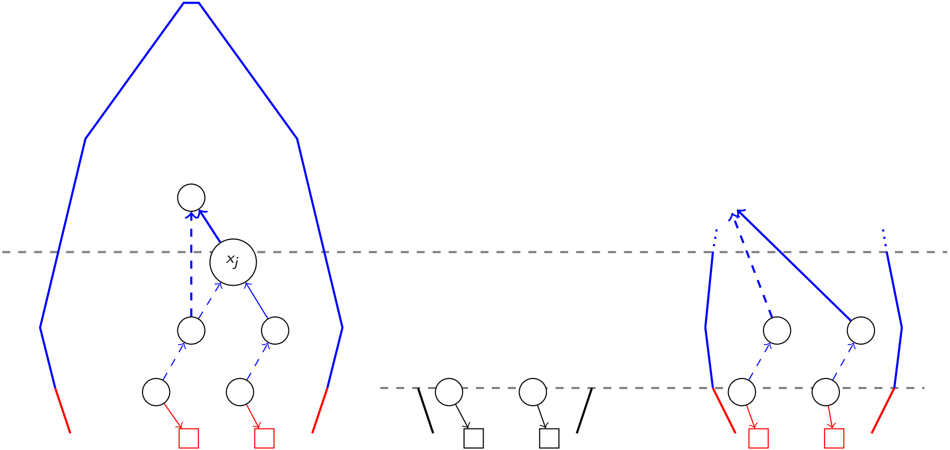

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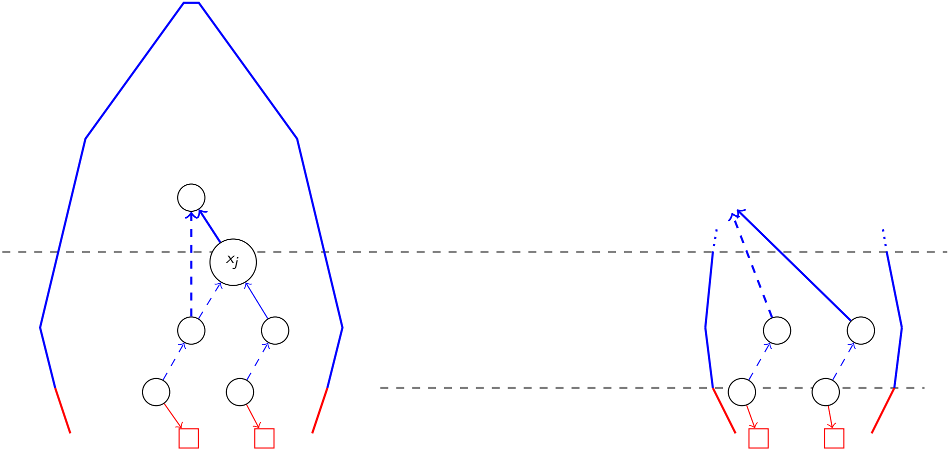

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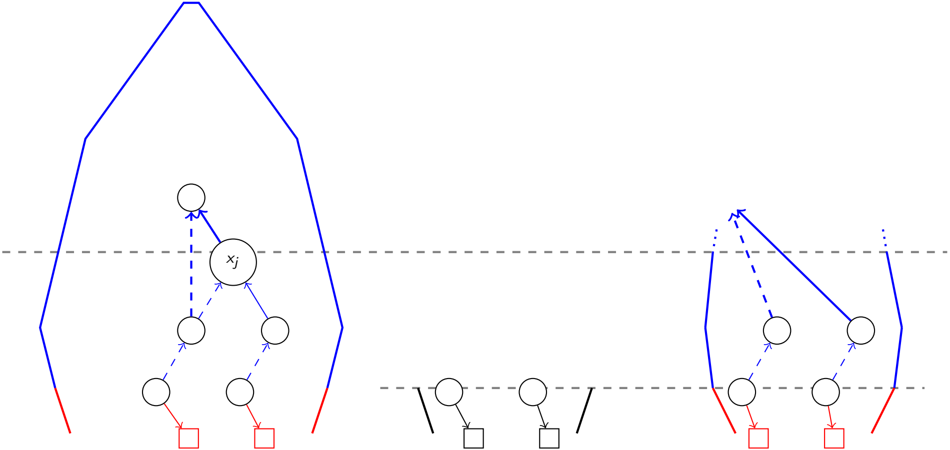

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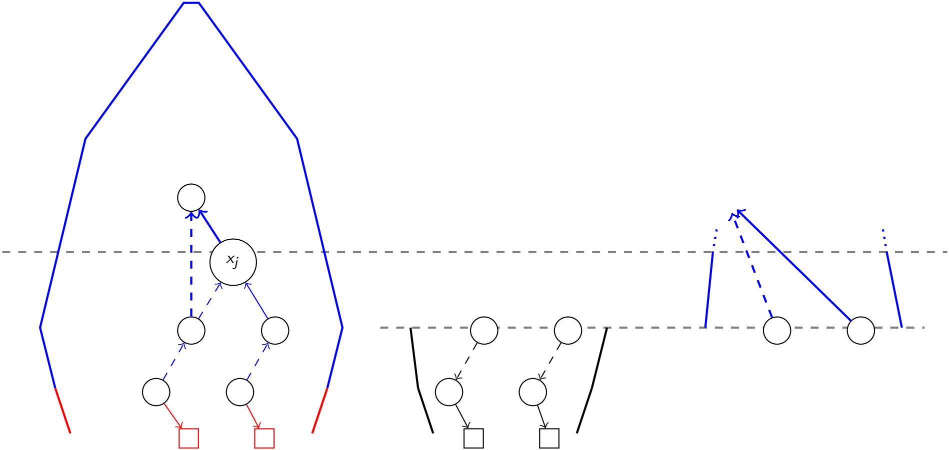

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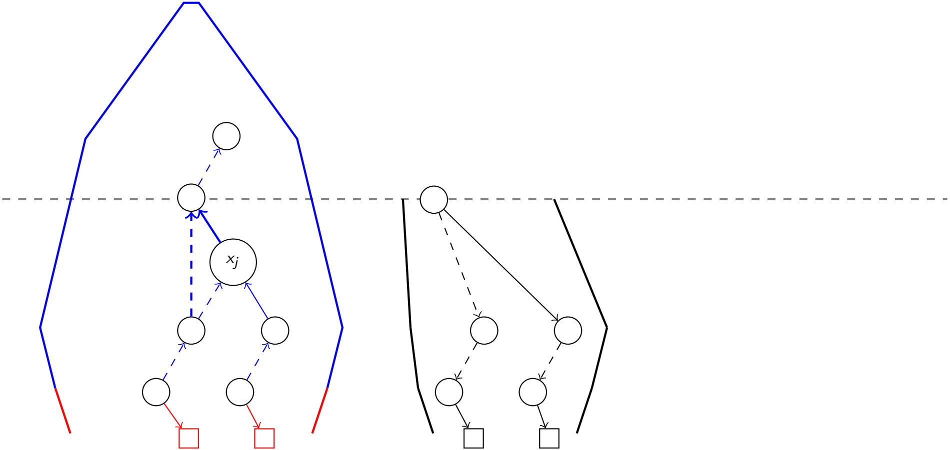

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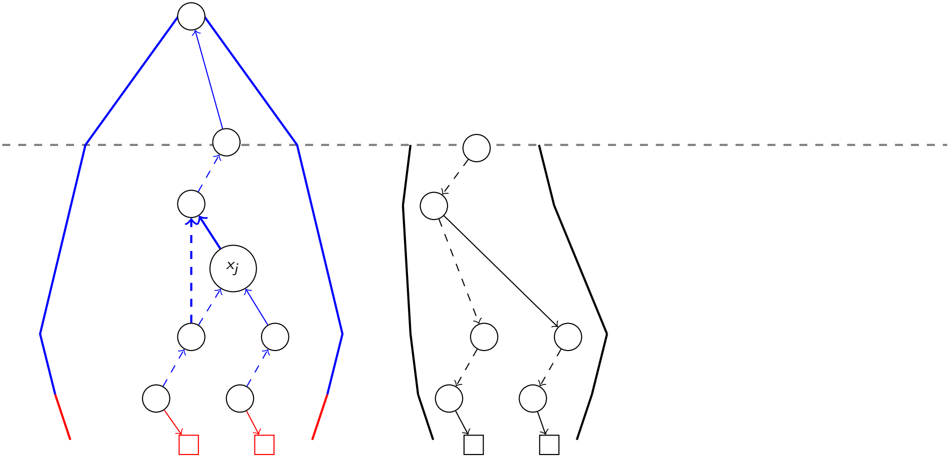

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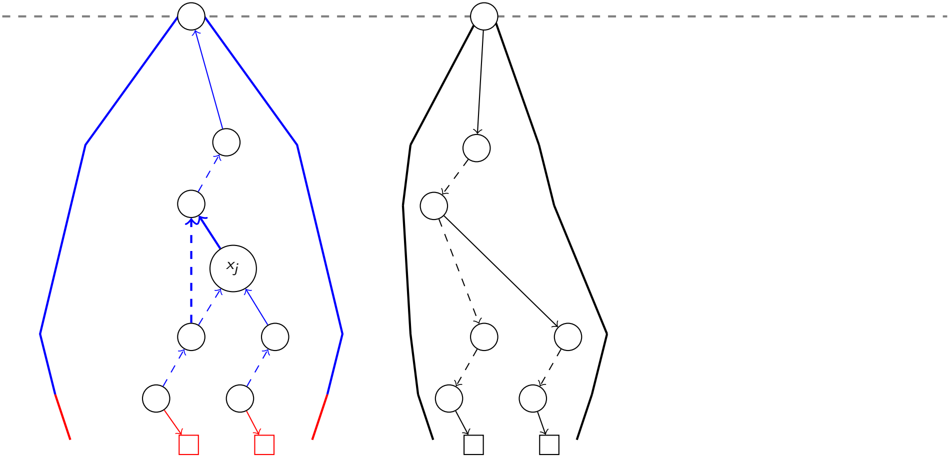

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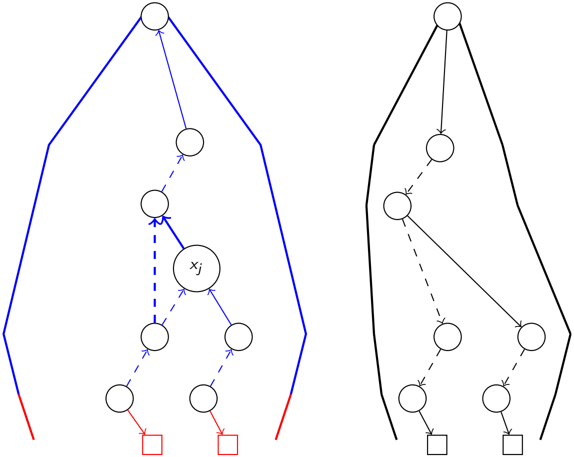

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If π is not monotonic

to be continued...