Adiar 1.1 : Zero-suppressed Decision Diagrams in External Memory

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Steffan Christ Sølvsten and Jaco van de Pol

18th of May, 2023

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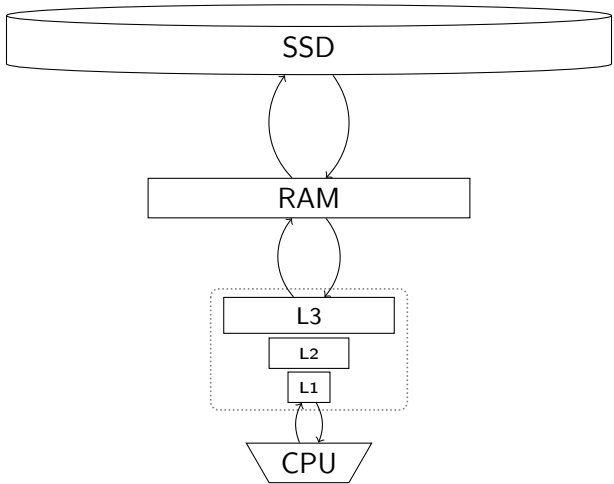
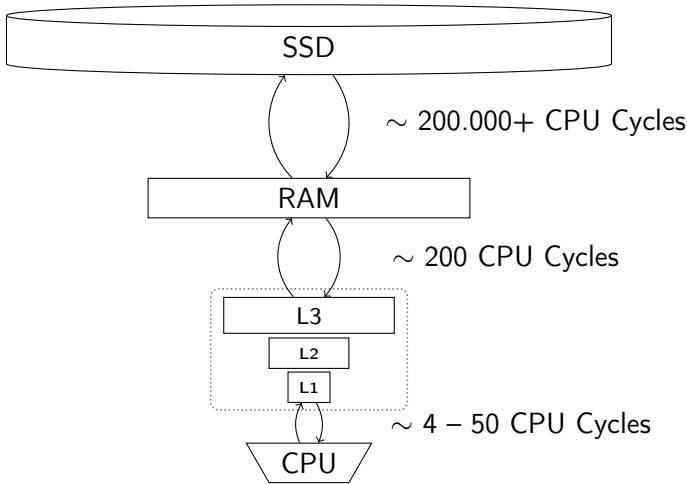
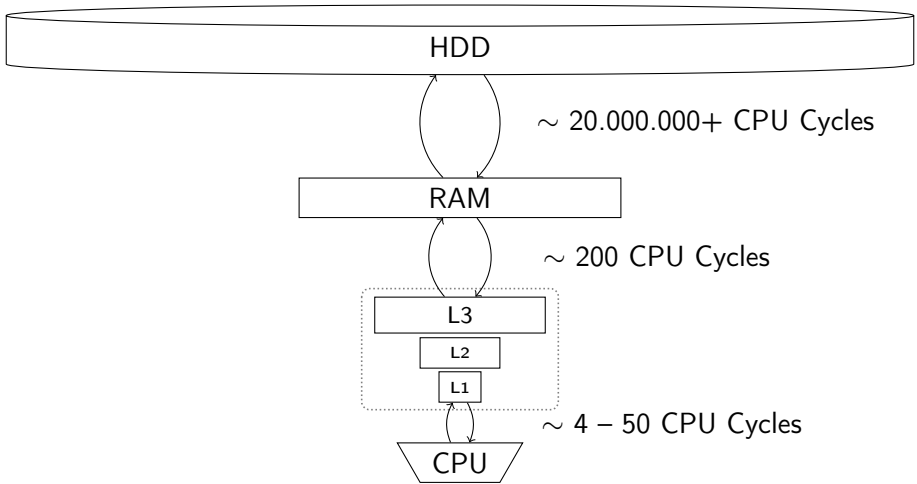

SSD
RAM
L3
L2
L1
CPU


SSD
~ 200.000+ CPU Cycles
RAM
~ 200 CPU Cycles
L3
L2
L1
~ 4 – 50 CPU Cycles
CPU


HDD
~ 20.000.000+ CPU Cycles
RAM
~ 200 CPU Cycles
L3
L2
L1
~ 4 – 50 CPU Cycles
CPU

Adiar

Binary Decision Diagrams
in External Memory

github.com/ssoelvsten/adiar

Adiar

Multi-terminal Decision Diagrams
in External Memory

`github.com/ssoelvsten/adiar`

Adiar

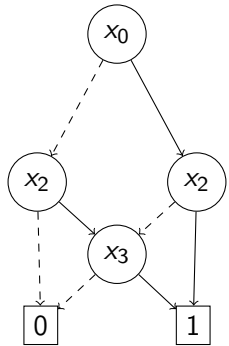
Quantum Multi-valued Decision Diagrams
in External Memory

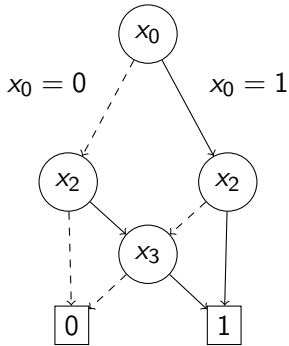
github.com/ssoelvsten/adiar

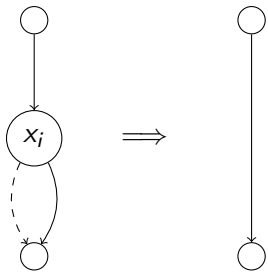
Adiar

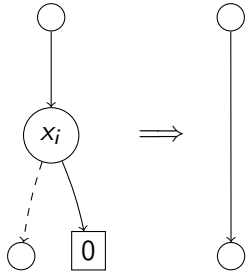
Zero-suppressed Decision Diagrams
in External Memory

`github.com/ssoelvsten/adiar`


x_0
x_2
x_2
x_3
0
1


x_0
$x_0 = 0$
$x_0 = 1$
x_2
x_2
x_3
0
1





BDD: $f : \mathbb{B}^n \to \mathbb{B}$

ZDD: $A \subseteq \mathbb{B}^n$

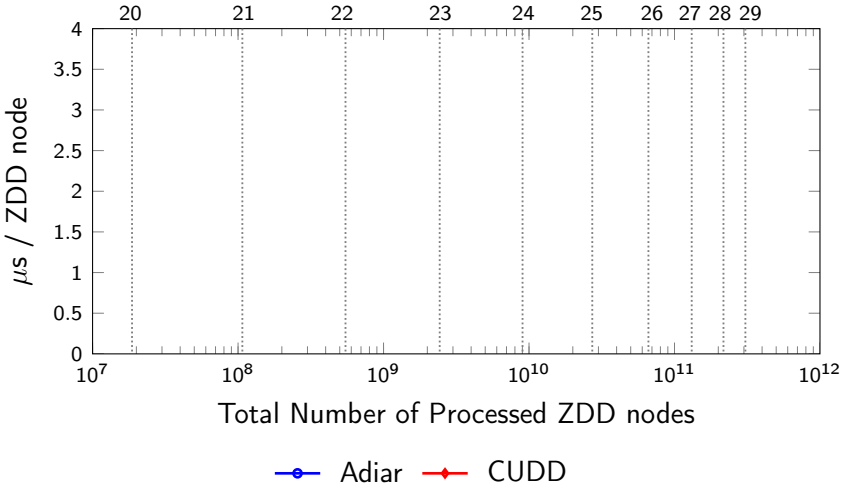
```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

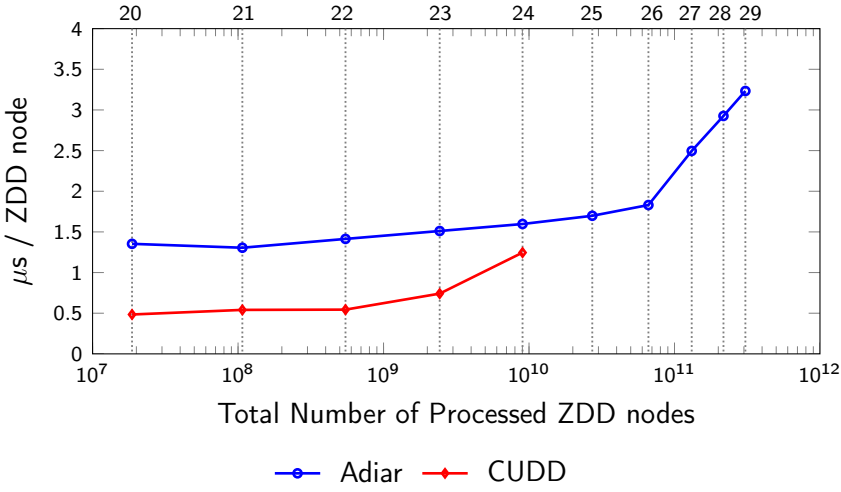
```
zdd zdd_binop(zdd A, zdd B, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {
    return prod2<bdd_policy>(f, g, o);
}

zdd zdd_binop(zdd A, zdd B, bool_op o)  {
    return prod2<zdd_policy>(A, B, o);
}
```

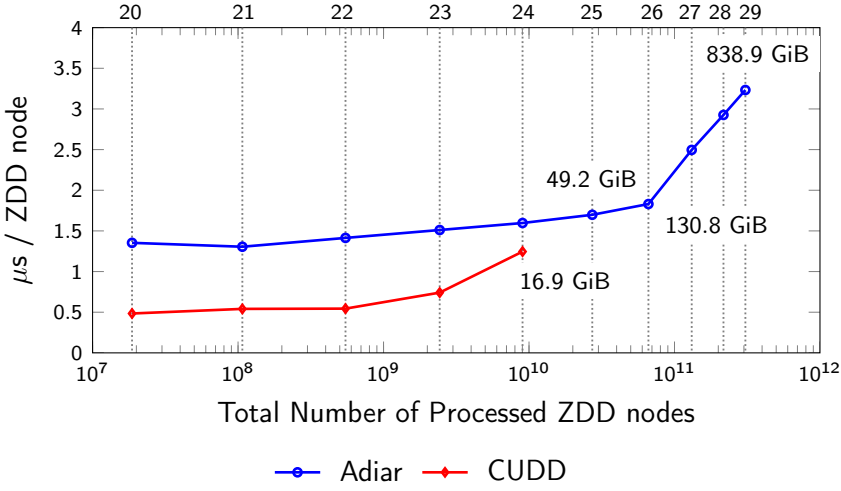
```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {
    return prod2<bdd_policy>(f, g, o);
}

zdd zdd_binop(zdd A, zdd B, bool_op o)  {
    return prod2<zdd_policy>(A, B, o);
}
```



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



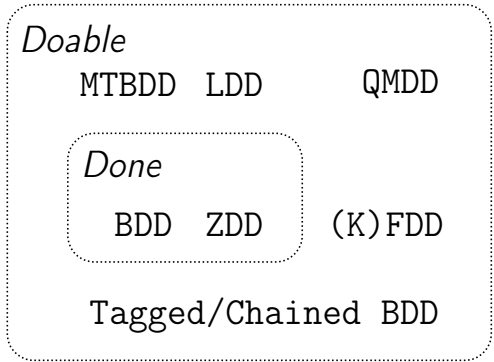
Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.

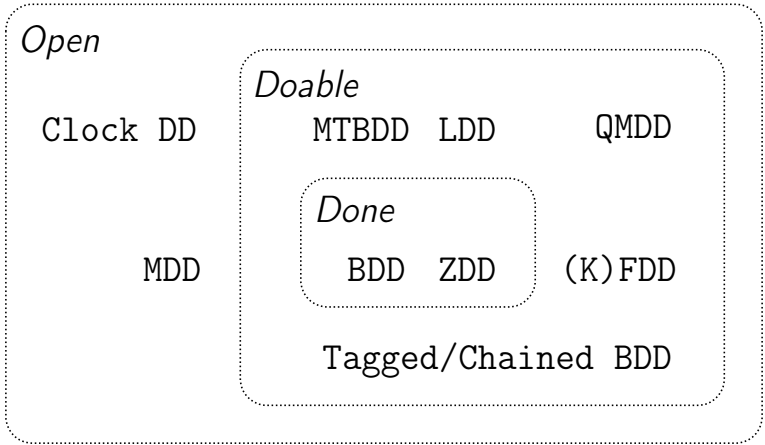


Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.

Done

BDD ZDD


Doable
MTBDD
LDD
QMDD
Done
BDD
ZDD
(K)FDD
Tagged/Chained BDD


Open
Clock DD
MDD
Doable
MTBDD LDD
QMDD
Done
BDD ZDD
(K)FDD
Tagged/Chained BDD

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Adiar

github.com/ssoelvsten/adiar

ssoelvsten.github.io/adiar

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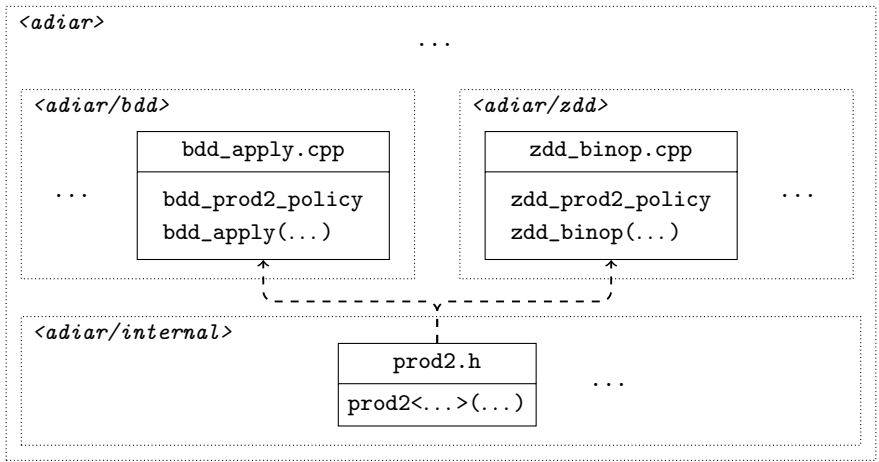
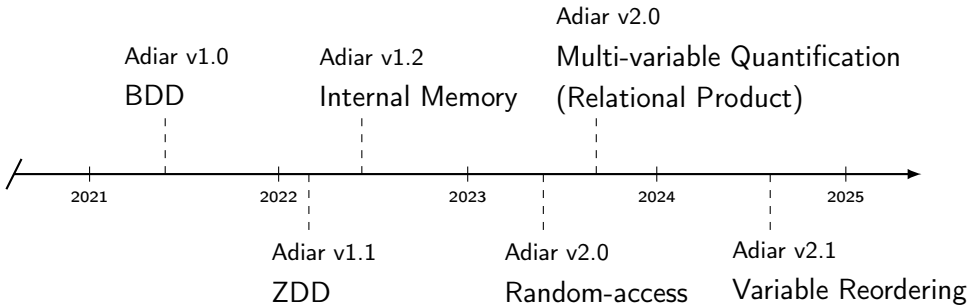
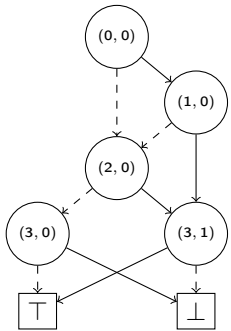
Function	Operation Semantics
ZDD Constructors	
`zdd_empty()`	$\emptyset$
`zdd_null()`	$\{\emptyset\}$
`zdd_singleton(var)`	$\{x_{\textbf{\textit{var}}}\}$
`zdd_vars(vars)`	$\{\bigcup_{i \in \textbf{\textit{vars}}} \{x_i\}\}$
`zdd_singletons(vars)`	$\{\{x_i\} \mid i \in \textbf{\textit{vars}}\}$
`zdd_powerset(vars)`	$\mathcal{P}(\textbf{\textit{vars}})$
`zdd_sized_set(vars,`k`,`$\odot$`)`	$\{s \in \mathcal{P}(\textbf{\textit{vars}}) \mid \lvert s\rvert \odot k\}$
ZDD Manipulation	
`zdd_binop(`A`,`B`,`$\otimes$`)`	$\{x \mid x \in A \otimes x \in B\}$
`zdd_change(`A`, vars)`	$\{(a \setminus \textbf{\textit{vars}}) \cup (\textbf{\textit{vars}} \setminus a) \mid a \in A\}$
`zdd_complement(`A`, dom)`	$\mathcal{P}(\textbf{\textit{dom}}) \setminus A$
`zdd_expand(`A`, vars)`	$\bigcup_{a \in A} \{a \cup v \mid v \in \mathcal{P}(\textbf{\textit{vars}})\}$
`zdd_offset(`A`, vars)`	$\{a \in A \mid \textbf{\textit{vars}} \cap a = \emptyset\}$
`zdd_onset(`A`, vars)`	$\{a \in A \mid \textbf{\textit{vars}} \subseteq a\}$
`zdd_project(`A`, vars)`	$\bigcup_{a \in A} \{a \cap \textbf{\textit{vars}}\}$

Function	Operation Semantics
Counting	
`zdd_size(`A`)`	$\lvert A\rvert$
`zdd_nodecount(`A`)`	# ZDD Nodes in A
`zdd_varcount(`A`)`	# Non-empty Levels in A
Predicates	
`zdd_equal(`A`, `B`)`	$A = B$
`zdd_unequal(`A`, `B`)`	$A \neq B$
`zdd_subseteq(`A`, `B`)`	$A \subseteq B$
`zdd_disjoint(`A`, `B`)`	$A \cap B = \emptyset$
Set elements	
`zdd_contains(`A`, `a`)`	$a \in A$
`zdd_minelem(`A`)`	$a \in A$ s.t. $\forall a' \in A \,.\, a \leq a'$
`zdd_maxelem(`A`)`	$a \in A$ s.t. $\forall a' \in A \,.\, a' \leq a$
Conversion	
`zdd_from(`f`, dom)`	$\{x \in \mathcal{P}(\textbf{\textit{dom}}) \mid f(x) = \top\}$
`bdd_from(`A`, dom)`	$\vec{x} : \mathcal{P}(\textbf{\textit{dom}}) \mapsto \vec{x} \in A$

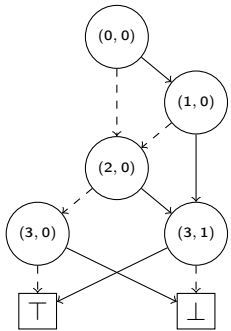
Operations provided by Adiar in `<adiar/zdd.h>`.


<adiar>
...
<adiar/bdd>
bdd_apply.cpp
bdd_prod2_policy
bdd_apply(...)
...
<adiar/zdd>
zdd_binop.cpp
zdd_prod2_policy
zdd_binop(...)
...
<adiar/internal>
prod2.h
prod2<...>(...)
...


Adiar v2.0
Adiar v1.0
Adiar v1.2
Multi-variable Quantification
BDD
Internal Memory
(Relational Product)
2021
2022
2023
2024
2025
Adiar v1.1
Adiar v2.0
Adiar v2.1
ZDD
Random-access
Variable Reordering



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

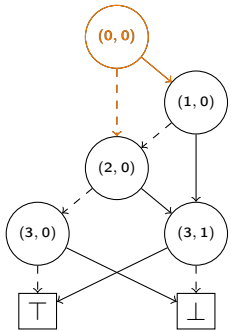


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]

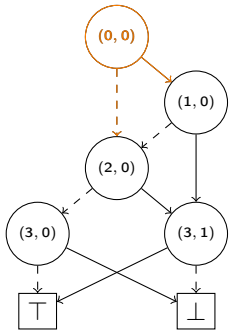


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

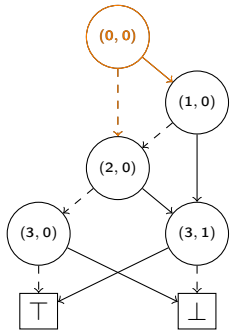
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

$$\left[\begin{array}{ll} ((0,0) \xrightarrow{\top} (1,0), & 1) \\ ((0,0) \xrightarrow{\perp} (2,0), & 1) \end{array} \right]$$

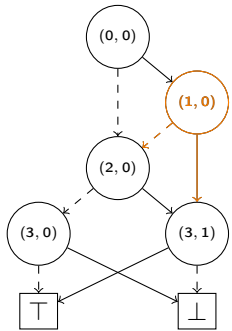


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0),\quad 1)\quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0),\quad 1)\quad ,$$
$$]$$



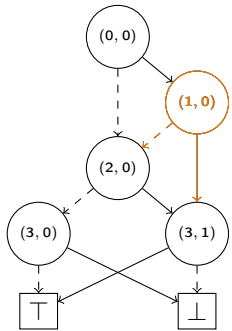
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$$

$]$



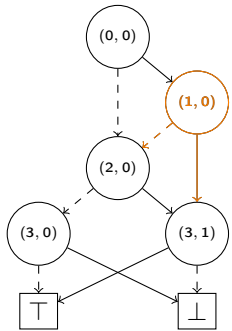
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\perp} (2,0), \quad 1) \quad ,$$

$$]$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1,0)$	1	0

Priority Queue: Q_{count}:

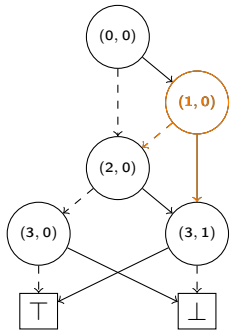
[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

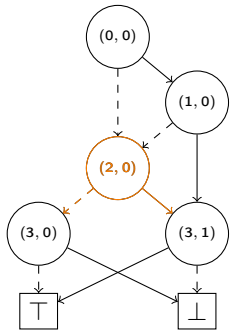
[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]

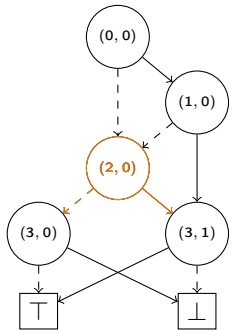


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

$$
\begin{bmatrix}
((0,0) \xrightarrow{\perp} (2,0), & 1) & , \\
((1,0) \xrightarrow{\perp} (2,0), & 1) & , \\
 & & \\
((1,0) \xrightarrow{\top} (3,1), & 1) & , \\
 & &]
\end{bmatrix}
$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2,0)$	1	0

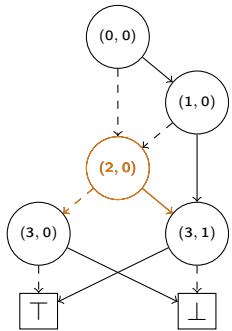
Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

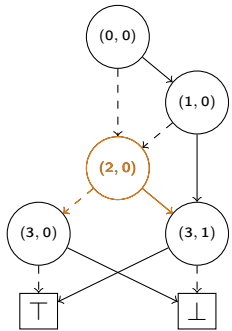
Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,

]



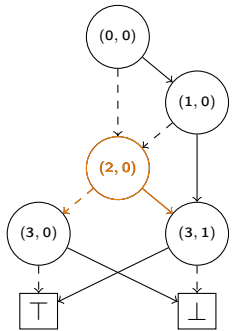
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$$((2,0) \xrightarrow{\perp} (3,0), \quad 2) \quad ,$$
$$((1,0) \xrightarrow{\top} (3,1), \quad 1) \quad ,$$
$$((2,0) \xrightarrow{\top} (3,1), \quad 2) \quad]$$



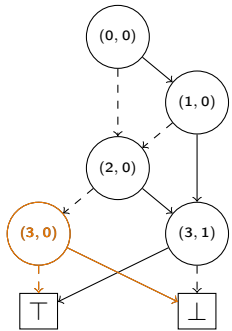
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



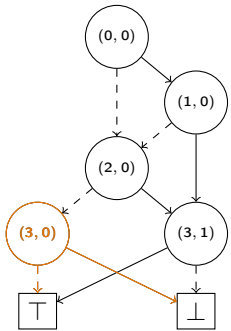
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

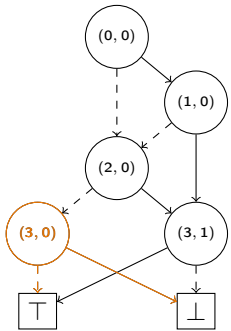
Seek	Sum	Result
$(3, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

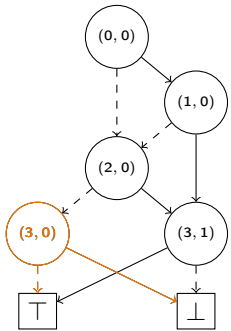
Seek	Sum	Result
$(3, 0)$	2	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

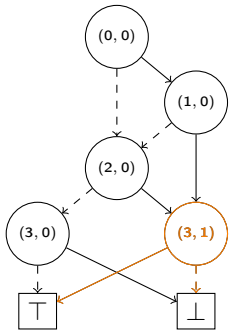
Seek	Sum	Result
$(3,1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

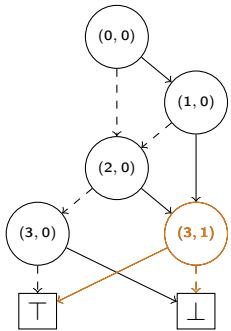
Seek	Sum	Result
$(3,1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



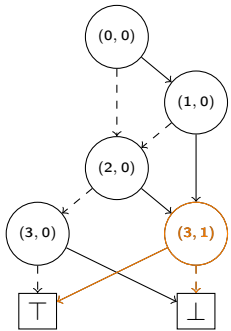
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3,1)$	1	2

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



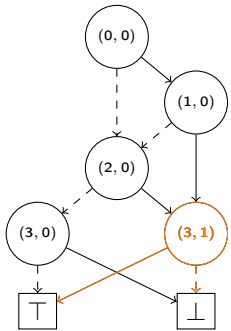
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count}:

[

]



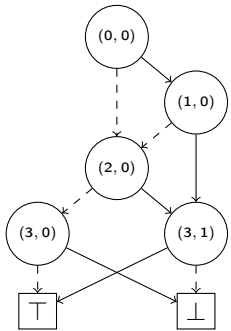
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	5

Priority Queue: Q_{count}:

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result

5

Priority Queue: Q_{count}:

[

]

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