An External Memory Relational Product

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$$RelProd(S, T) \equiv (\ \exists \vec{x}.\ S(\vec{x}) \wedge T(\vec{x}, \vec{x'})\ )[\vec{x'}/\vec{x}]$$

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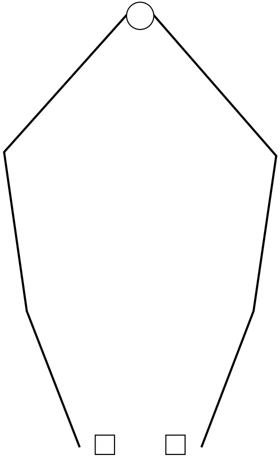

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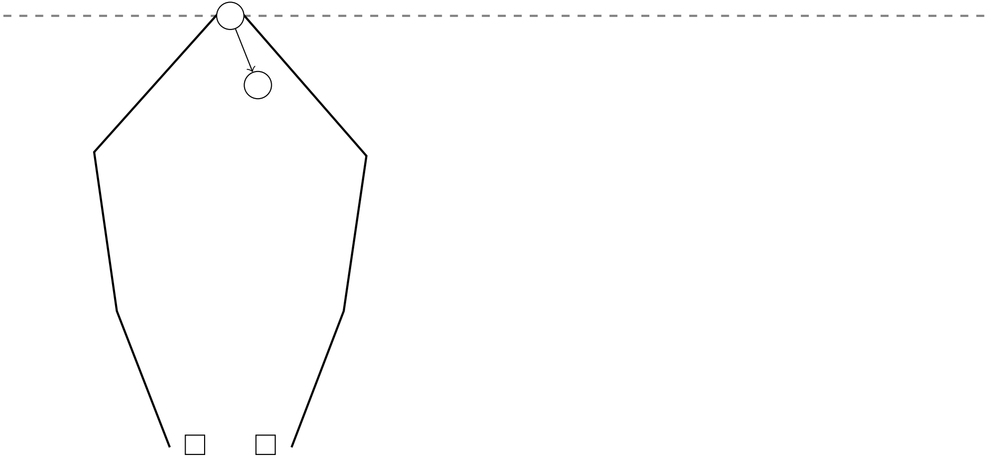

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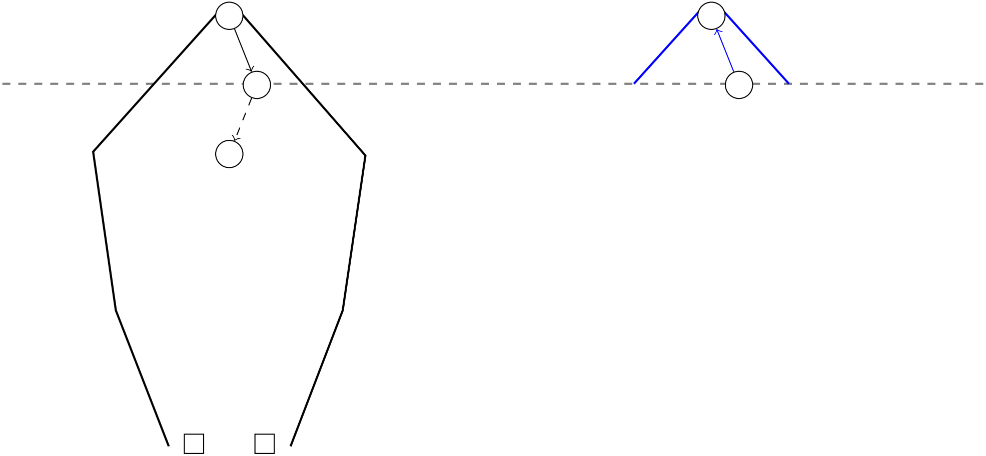

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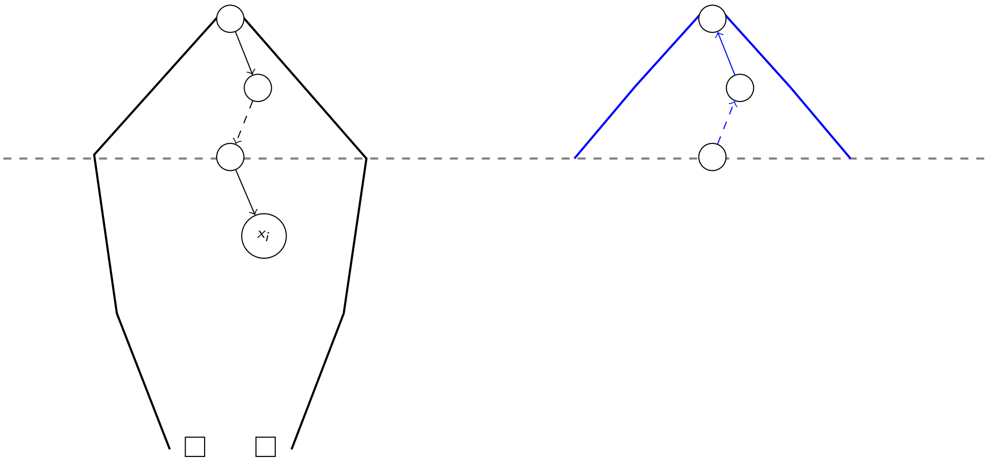

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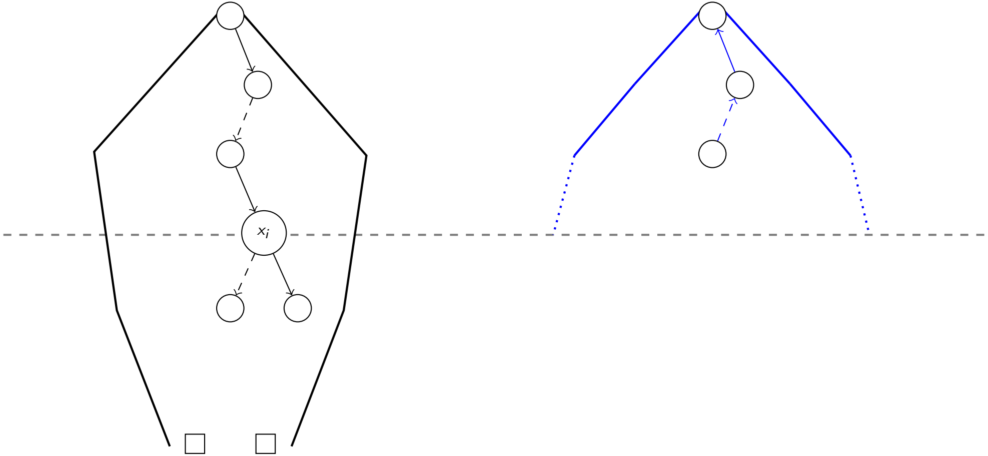

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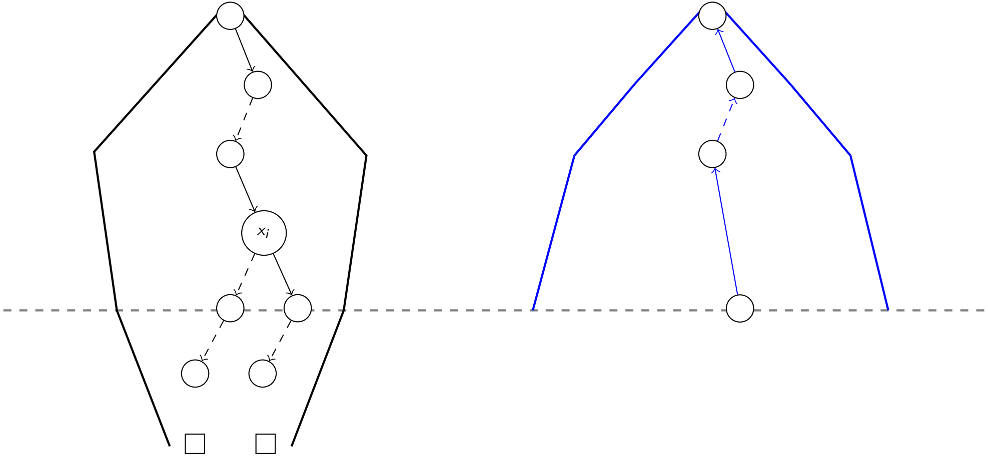

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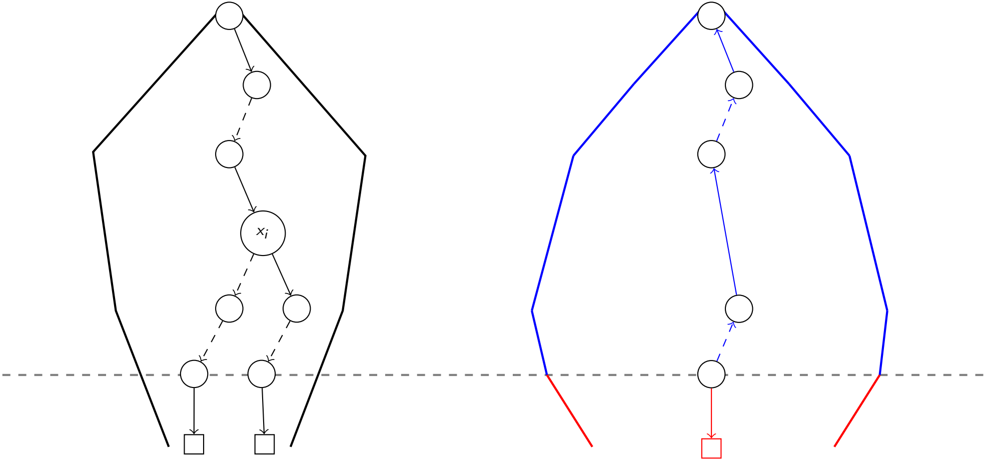

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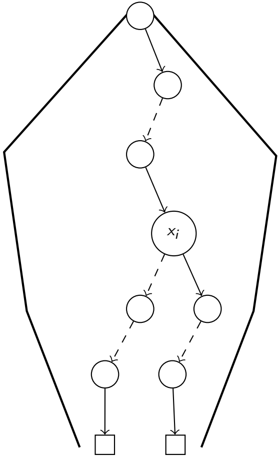

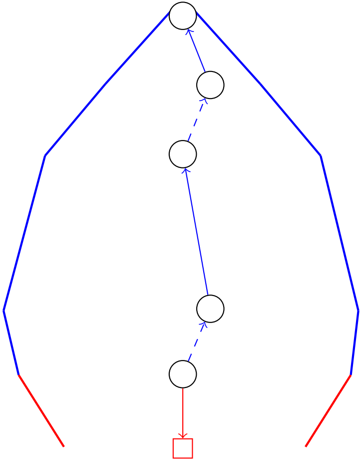

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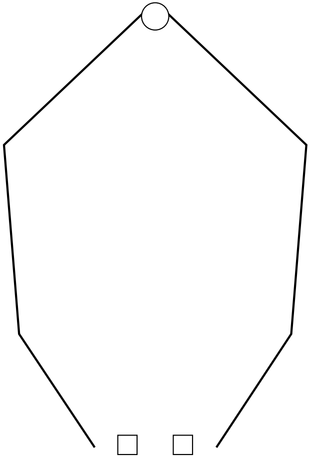

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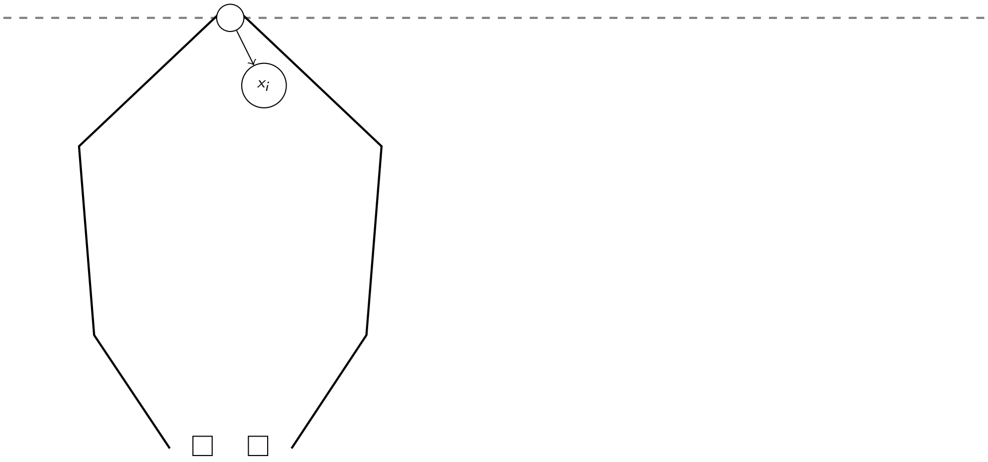

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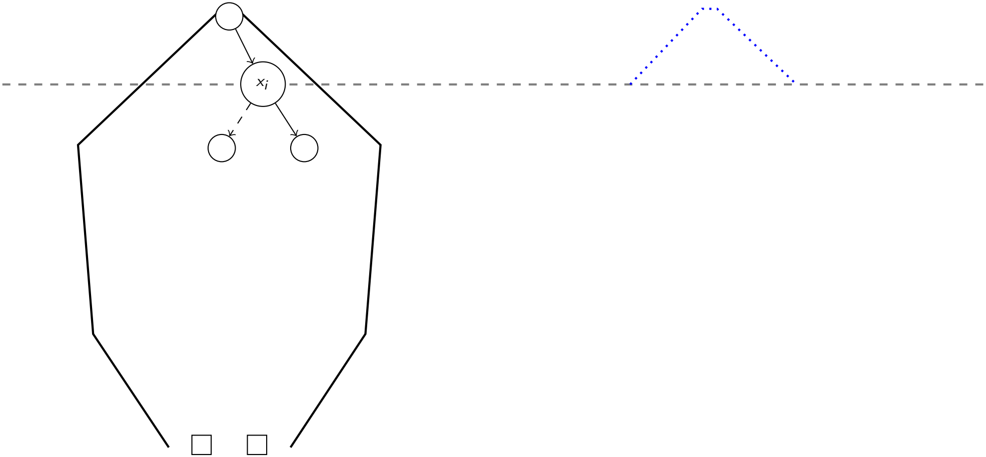

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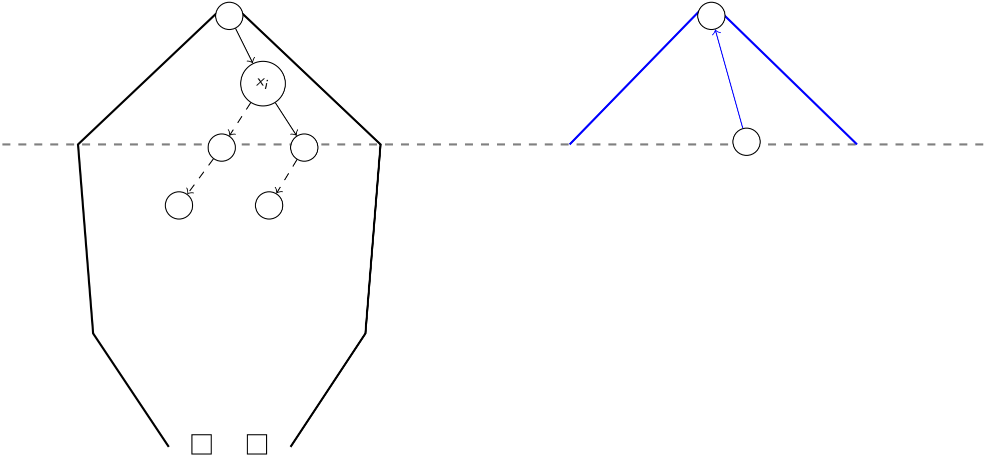

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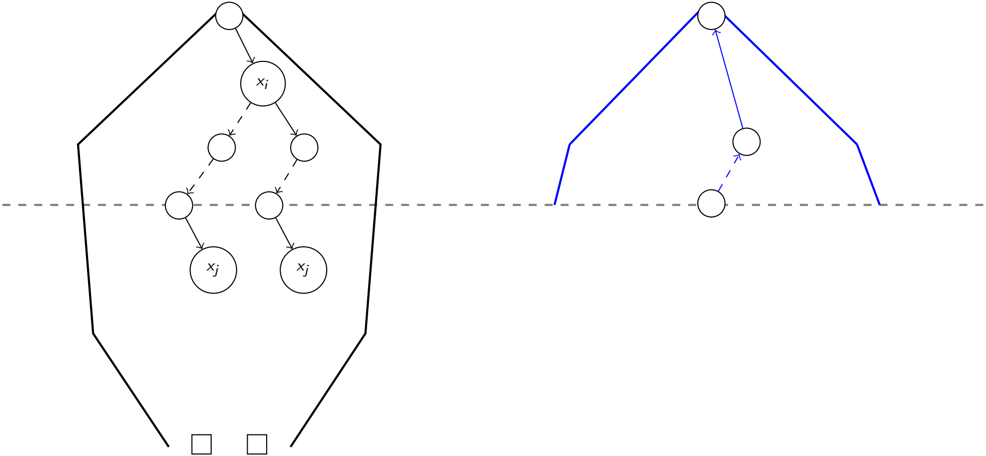

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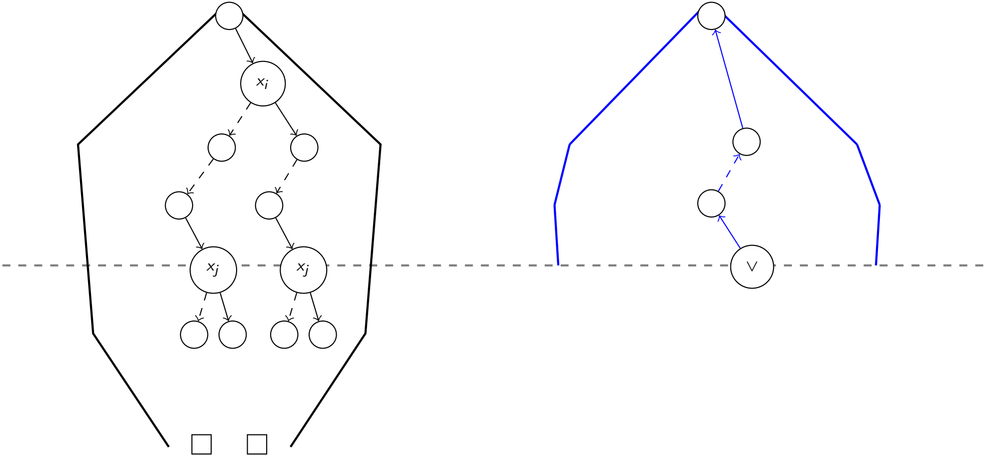

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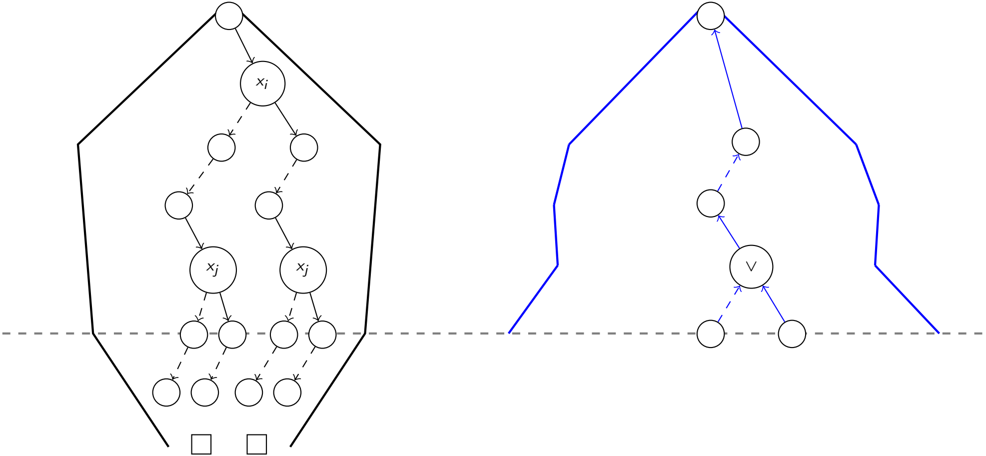

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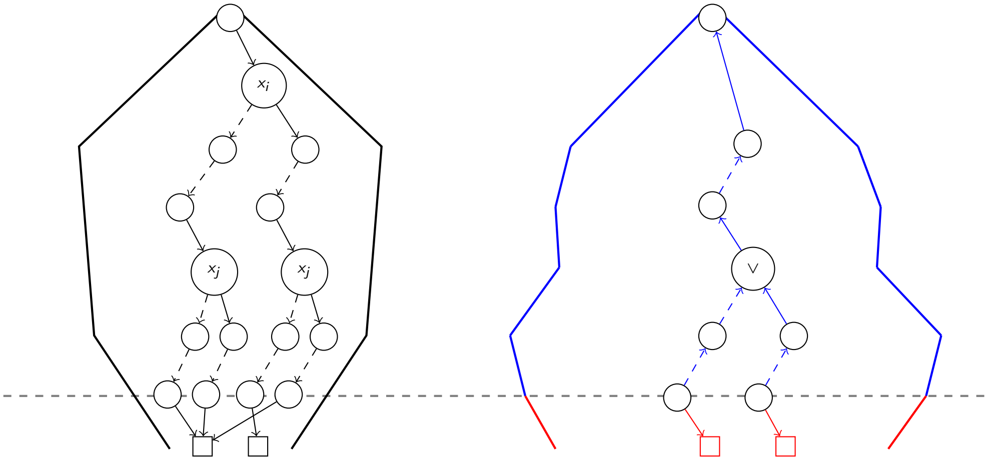

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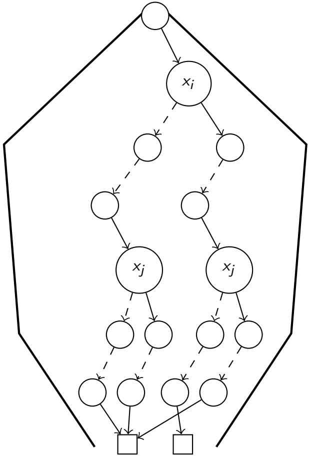

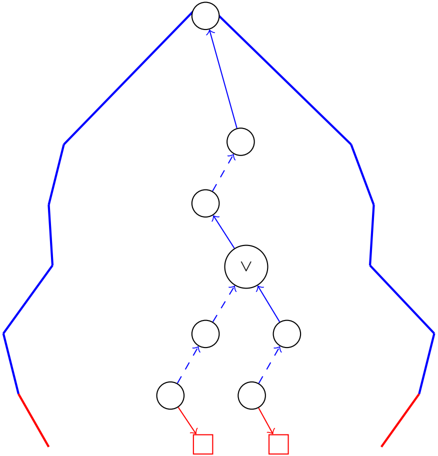

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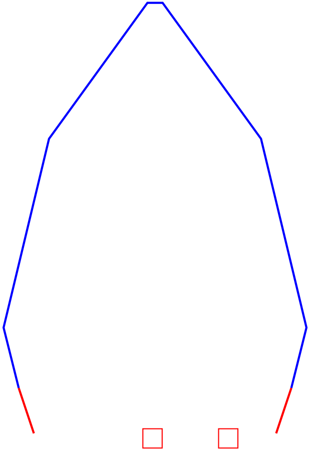

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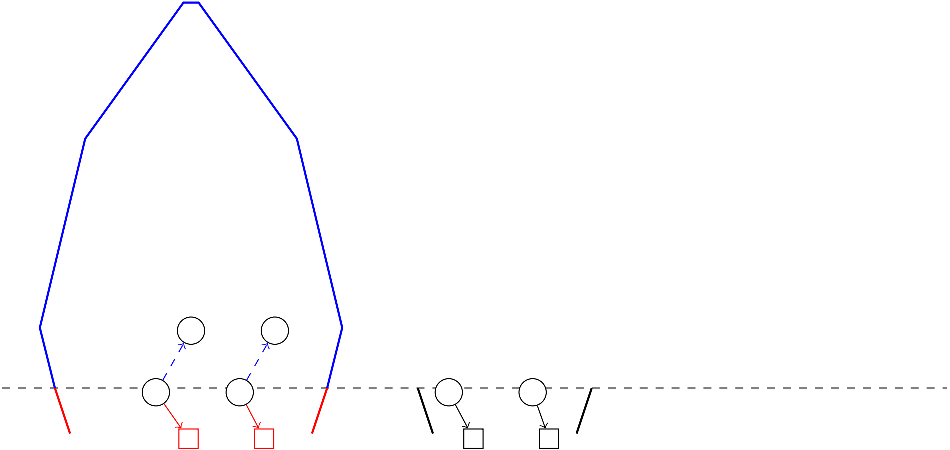

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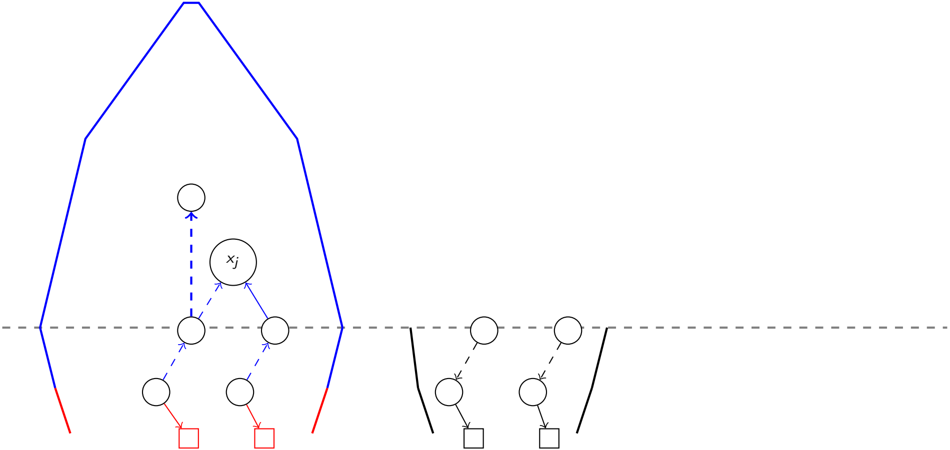

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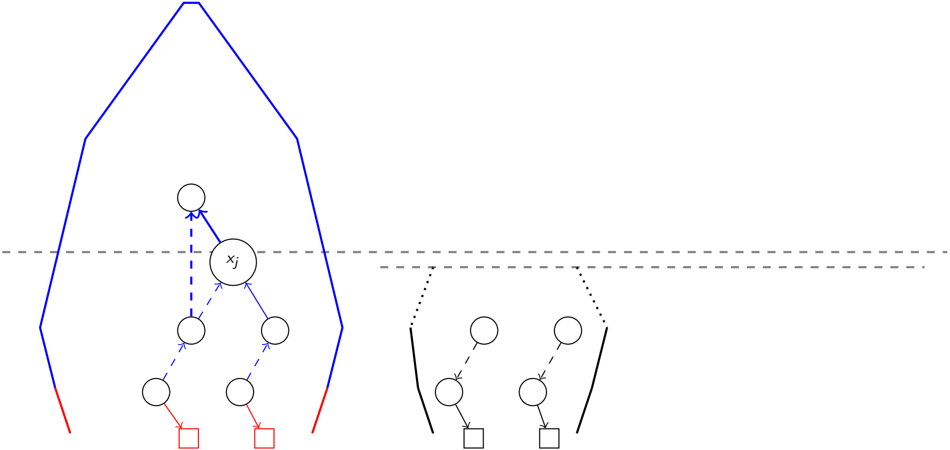

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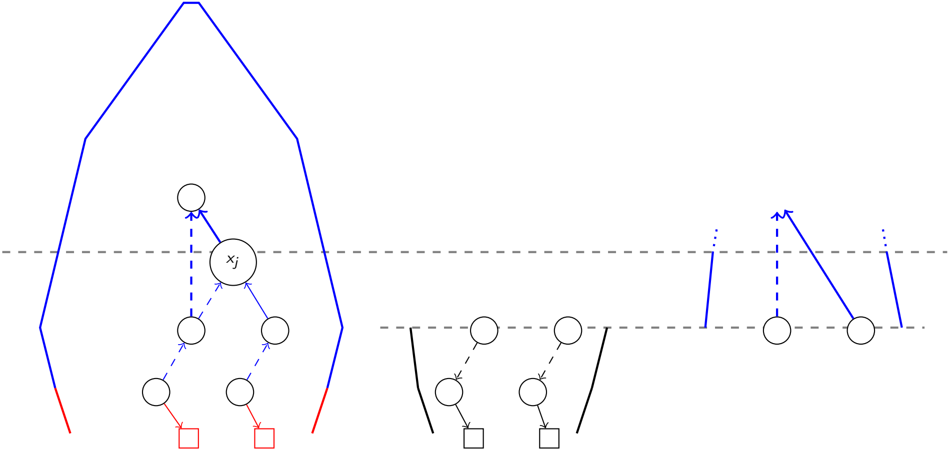

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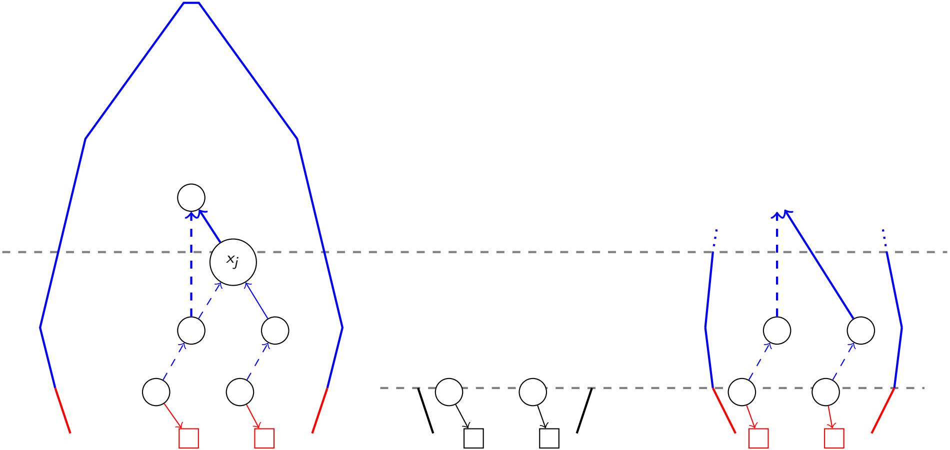

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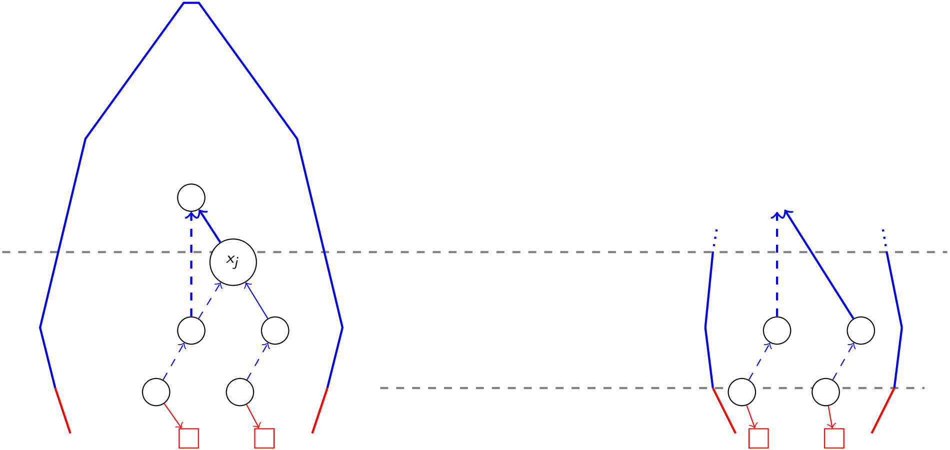

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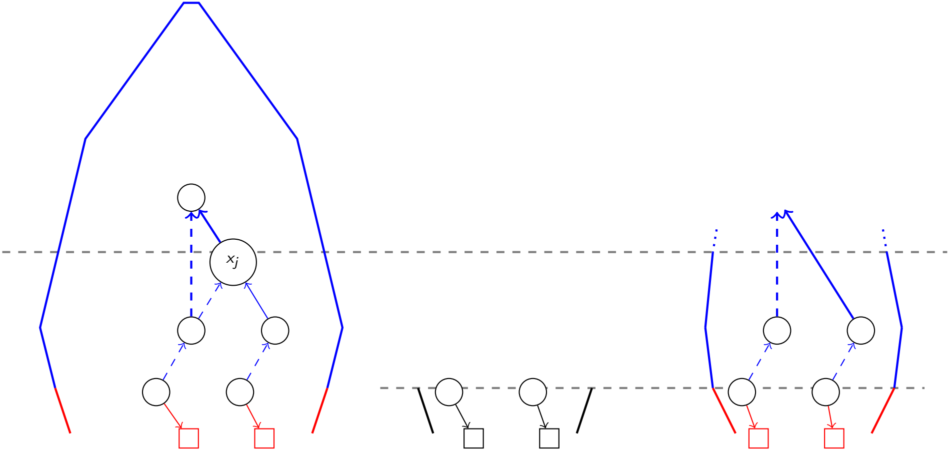

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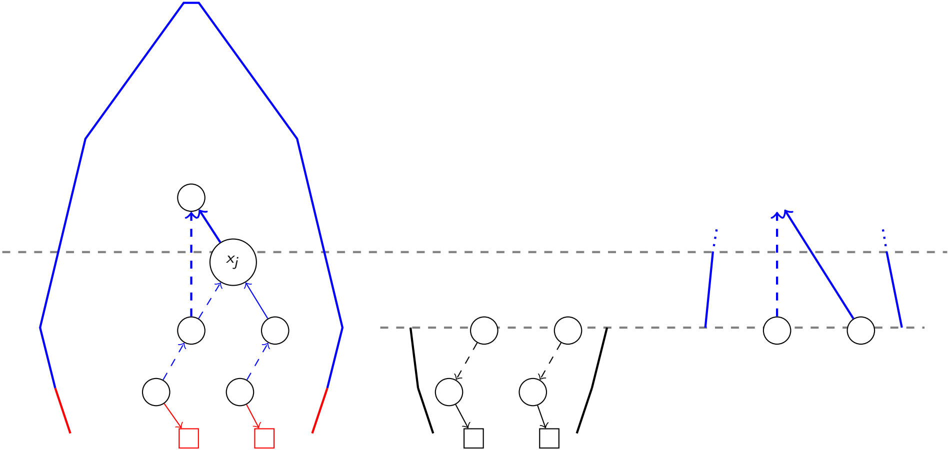

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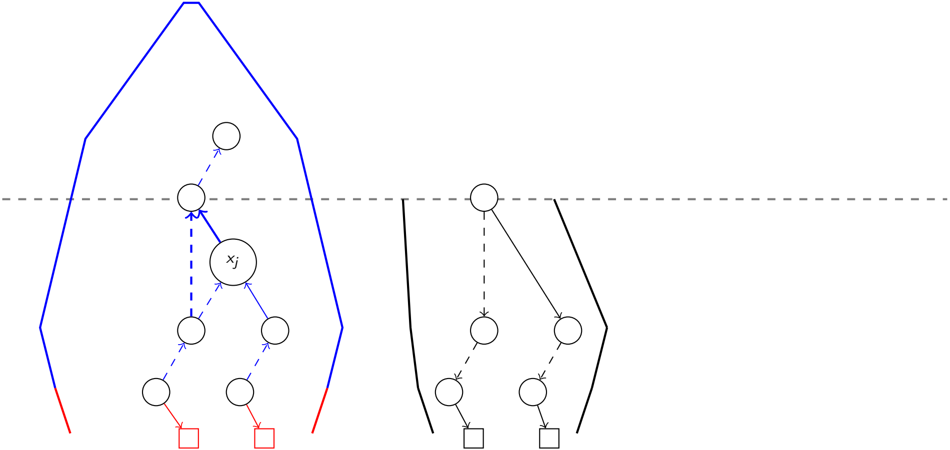

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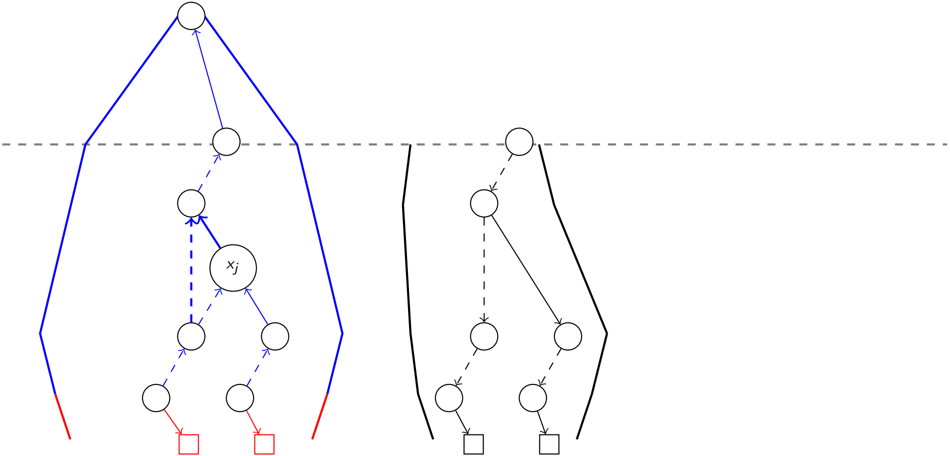

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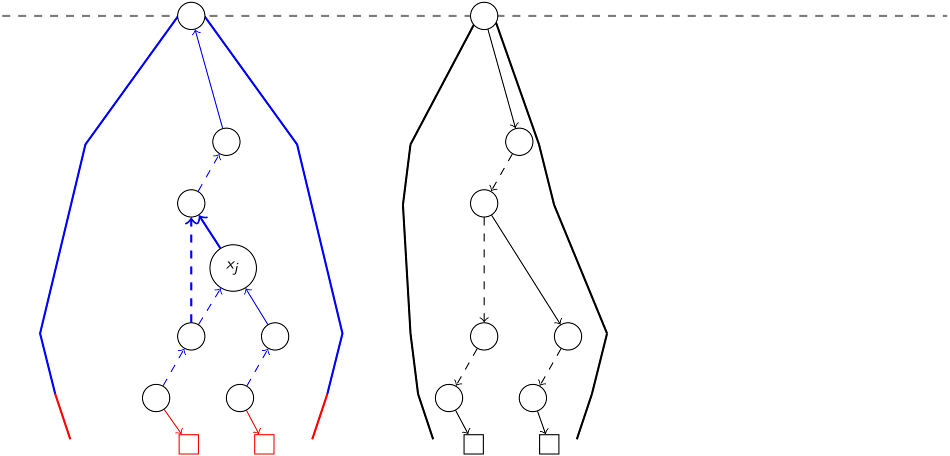

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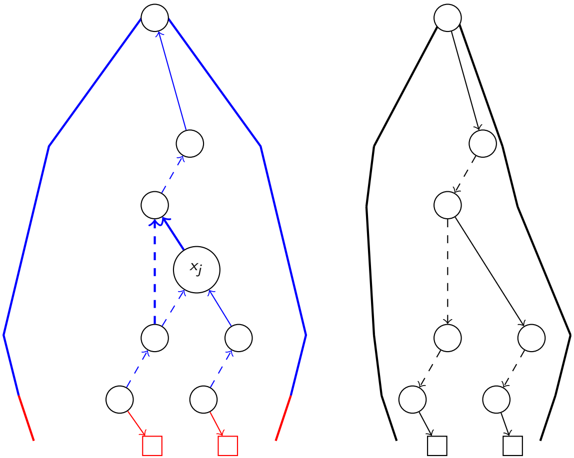

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  Apply π in $O(L_N)$ extra time during the final bottom-up Reduce sweep.
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If π is not monotonic

to be continued...