Random Access on Narrow Decision Diagrams in External Memory

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SPIN 2024

Adiar

I/O-efficient Decision Diagrams

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github.com/ssoelvsten/adiar

Features

Optimisations

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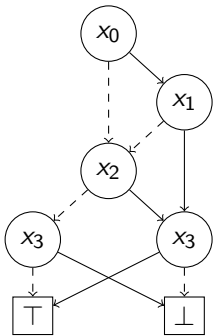
Optimisations



$$[\quad ((0,0),(2,0),(1,0)) \quad ,$$

$$((1,0),(2,0),(3,1)) \quad ,$$

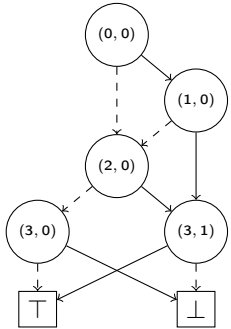
$$((2,0),(3,0),(3,1)) \quad ,$$

$$((3,0),\top,\bot) \quad ,$$

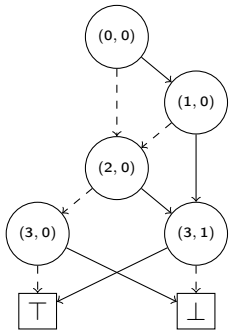
$$((3,1),\bot,\top) \quad]$$

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Node-based representation of prior shown BDDs



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

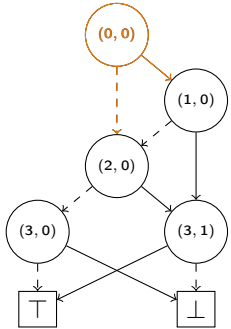


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]

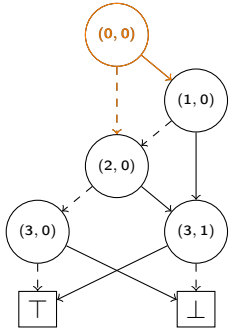


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[

]



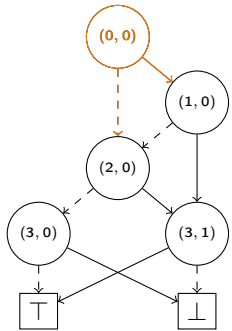
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count}:

[$((0,0) \xrightarrow{\top} (1,0), \quad 1)$,

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

]



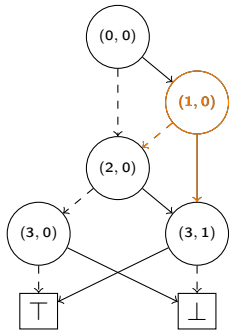
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$$

$]$

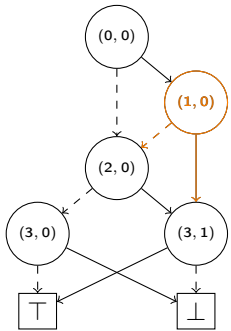


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count}:

$$[\quad ((0,0) \xrightarrow{\top} (1,0), \quad 1) \quad ,$$
$$((0,0) \xrightarrow{\bot} (2,0), \quad 1) \quad ,$$
$$]$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

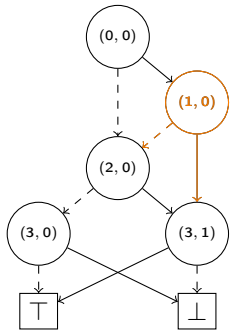
Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

[

$((0, 0) \xrightarrow{\perp} (2, 0), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count}:

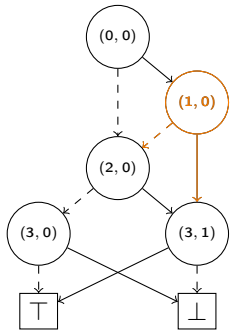
[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue: Q_{count}:

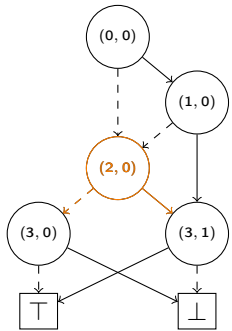
[

$((0,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\perp} (2,0), \quad 1)$,

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

]

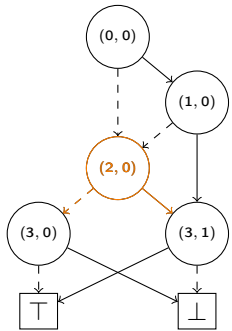


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2,0)$	0	0

Priority Queue: Q_{count}:

$$
\left[
\begin{array}{lll}
((0,0) \xrightarrow{\perp} (2,0), & 1) & , \\
((1,0) \xrightarrow{\perp} (2,0), & 1) & , \\
 & & \\
((1,0) \xrightarrow{\top} (3,1), & 1) & , \\
 & &]
\end{array}
\right.
$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	1	0

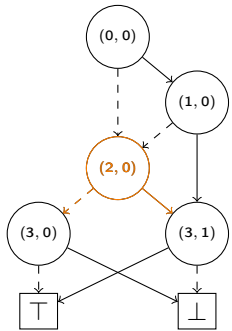
Priority Queue: Q_{count}:

$$[$$

$$((1,0) \xrightarrow{\perp} (2,0), \quad 1) \quad ,$$

$$((1,0) \xrightarrow{\top} (3,1), \quad 1) \quad ,$$

$$]$$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

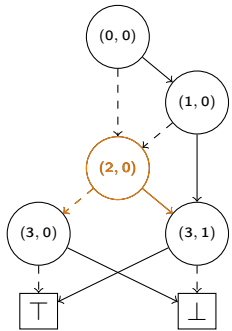
Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$,

]



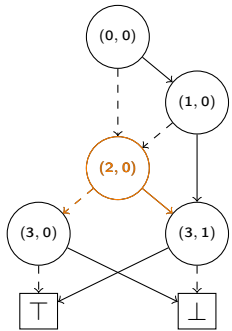
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue: Q_{count}:

[

$$((2,0) \xrightarrow{\perp} (3,0), \quad 2) \quad ,$$
$$((1,0) \xrightarrow{\top} (3,1), \quad 1) \quad ,$$
$$((2,0) \xrightarrow{\top} (3,1), \quad 2) \quad]$$



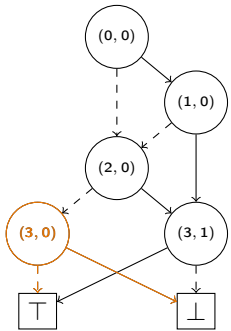
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 0)$	0	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



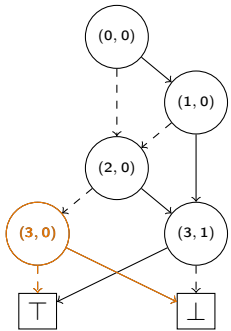
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3,0)$	0	0

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\perp} (3,0), \quad 2)$,
$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,
$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

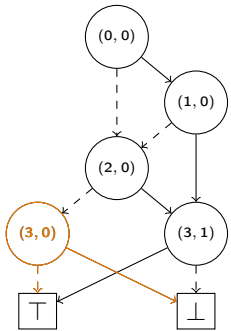
Seek	Sum	Result
$(3, 0)$	2	0

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

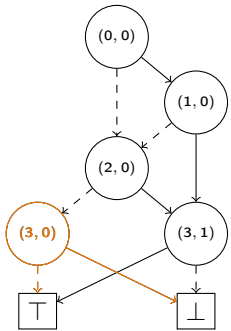
Seek	Sum	Result
$(3,0)$	2	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

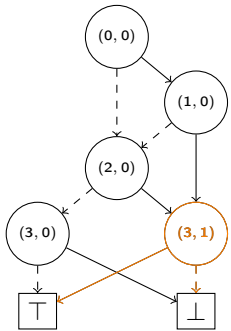
Seek	Sum	Result
$(3,1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

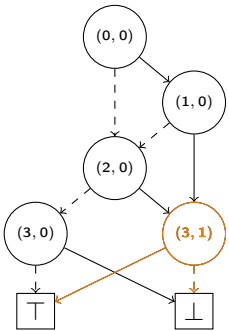
Seek	Sum	Result
$(3,1)$	0	2

Priority Queue: Q_{count}:

[

$((1,0) \xrightarrow{\top} (3,1), \quad 1)$,

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



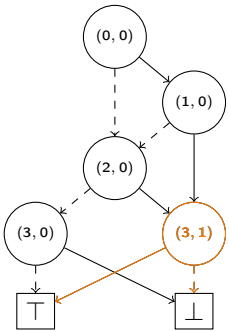
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3,1)$	1	2

Priority Queue: Q_{count}:

[

$((2,0) \xrightarrow{\top} (3,1), \quad 2)$]



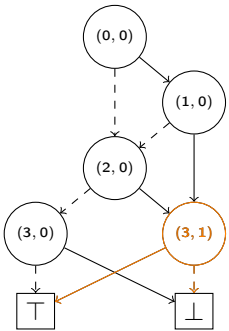
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	2

Priority Queue: Q_{count}:

[

]



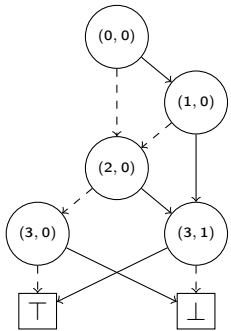
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	3	5

Priority Queue: Q_{count}:

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result

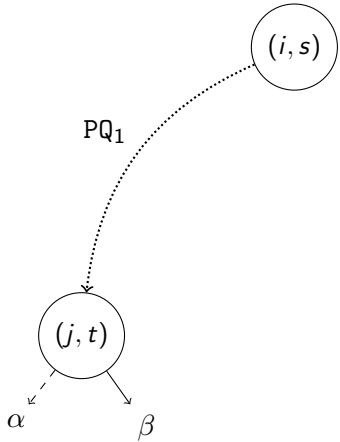
5

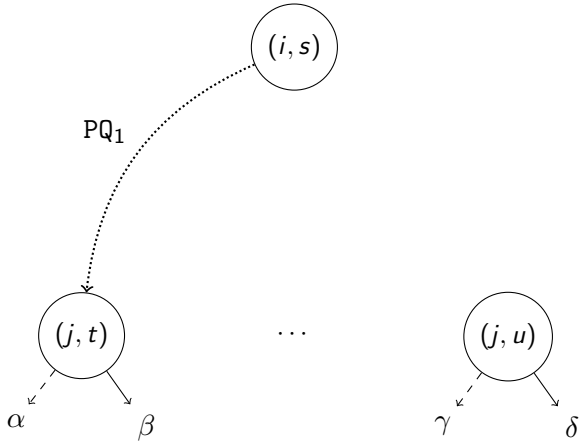
Priority Queue: Q_{count}:

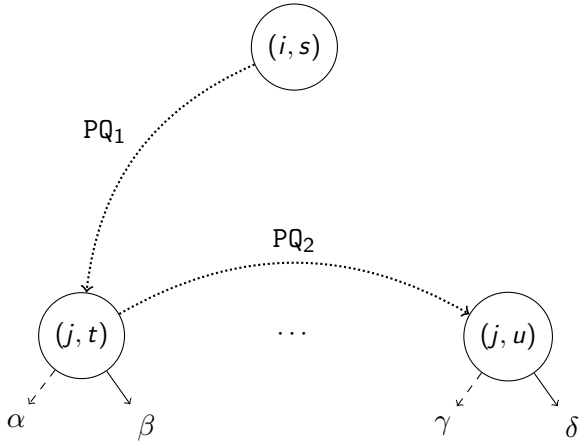
[

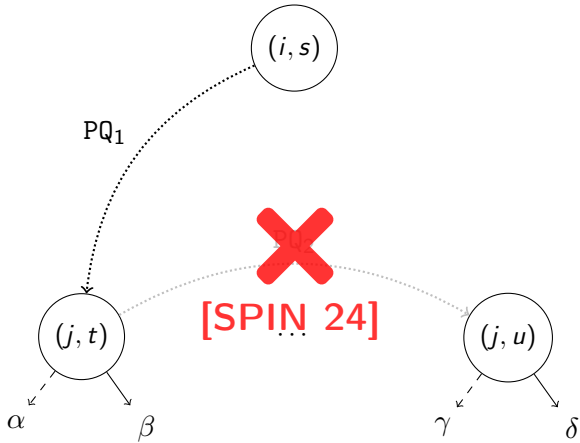
]

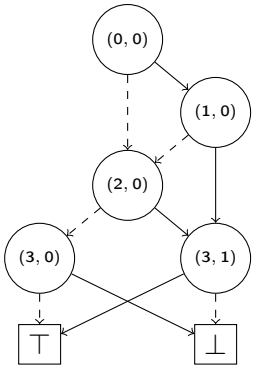
(i, s)

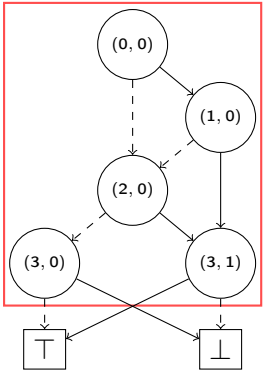

(i, s)
$\mathtt{PQ}_1$
(j, t)
α
β

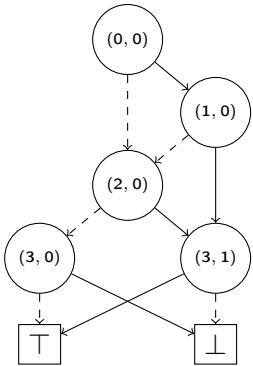

(i, s)
$\mathtt{PQ}_1$
(j, t)
$\cdots$
(j, u)
α
β
γ
δ

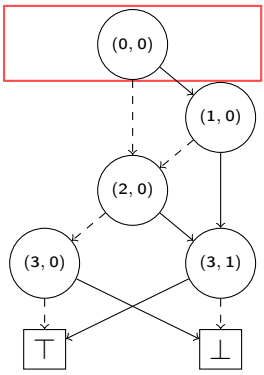

(i, s)
$\mathtt{PQ}_1$
$\mathtt{PQ}_2$
(j, t)
$\cdots$
(j, u)
α
β
γ
δ

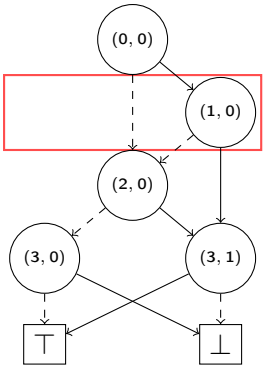

(i, s)
$\mathtt{PQ}_1$
(j, t)
(j, u)
[SPIN 24]
α
β
γ
δ

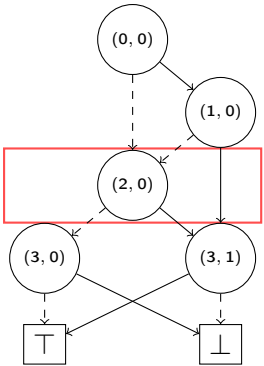

(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥

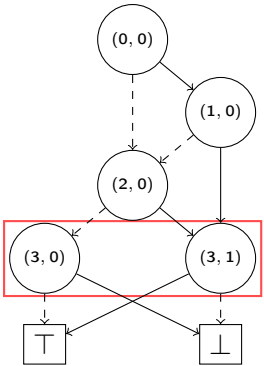

(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥

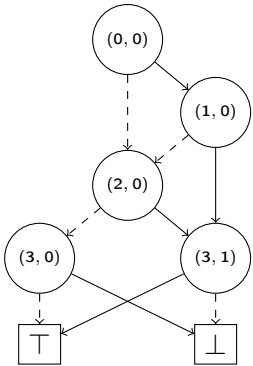

(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥

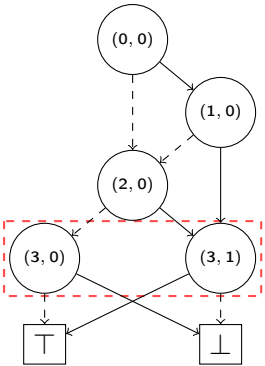

(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥


(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥


(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥

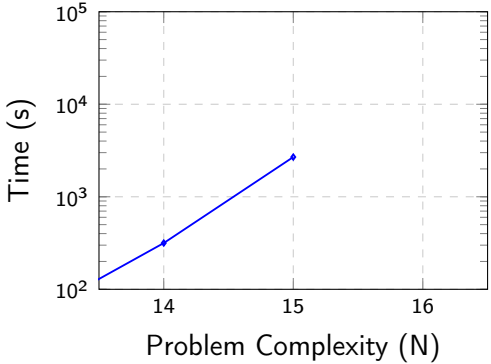

(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥


(0, 0)
(1, 0)
(2, 0)
(3, 0)
(3, 1)
⊤
⊥



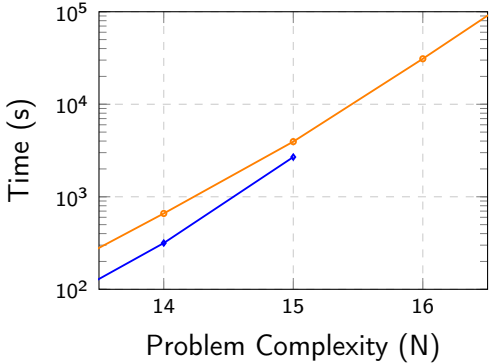
Width

2



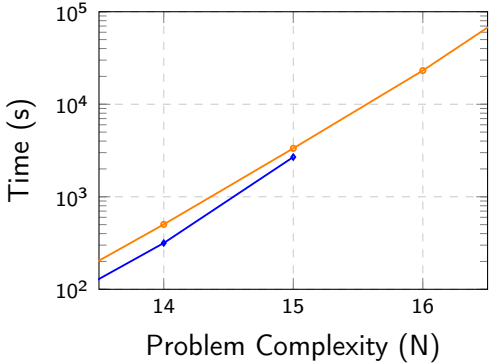
			$N = 15$
◇	CUDD v3.0	:	44.8 min

Queens | 300 GiB of RAM



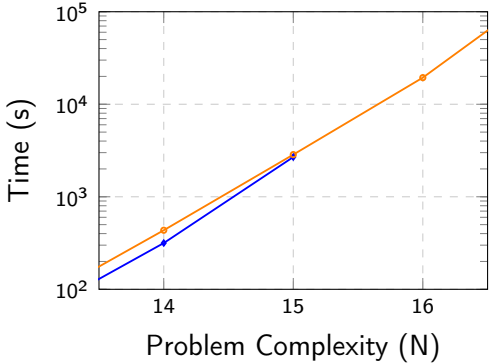
				$N = 15$
◇	CUDD	v3.0	:	44.8 min
○	Adiar	v1.0	:	66.7 min

Queens | 300 GiB of RAM



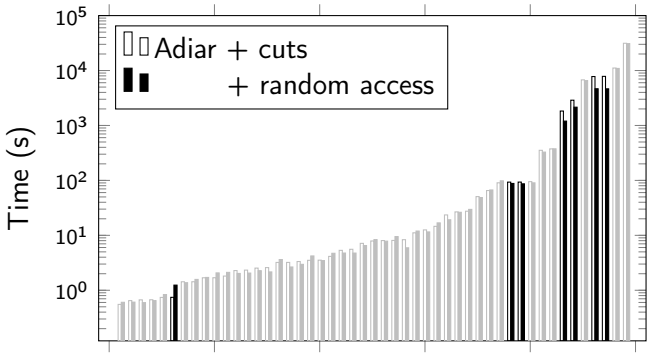
				$N = 15$
◇	CUDD	v3.0	:	44.8 min
○	Adiar	v1.0	:	66.7 min
	+ cuts		:	56.8 min

Queens | 300 GiB of RAM



				$N = 15$
◇	CUDD	v3.0	:	44.8 min
○	Adiar	v1.0	:	66.7 min
	+ cuts		:	56.8 min
	+ random access		:	47.2 min

Queens | 300 GiB of RAM



Speed Ups

6 (+26.9%)

Slowdowns

1 (-66.8%)

EPFL Circuit Verification | 300 GiB of RAM

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ssoelvsten.github.io

Adiar

github.com/ssoelvsten/adiar

ssoelvsten.github.io/adiar

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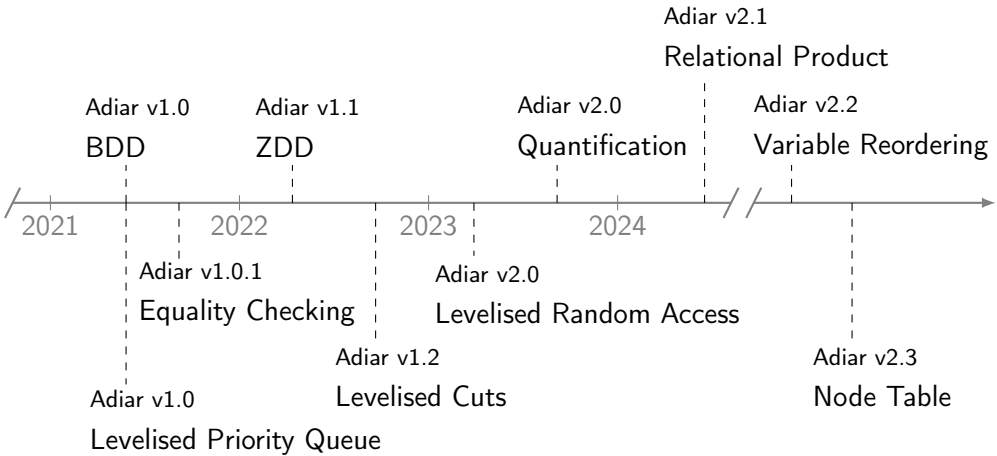
Adiar v2.1
Relational Product
Adiar v1.0
BDD
Adiar v1.1
ZDD
Adiar v2.0
Quantification
Adiar v2.2
Variable Reordering
2021
2022
2023
2024
Adiar v1.0.1
Equality Checking
Adiar v2.0
Levelised Random Access
Adiar v1.2
Levelised Cuts
Adiar v2.3
Node Table
Adiar v1.0
Levelised Priority Queue