

# Adiar 1.1 : Zero-suppressed Decision Diagrams in External Memory

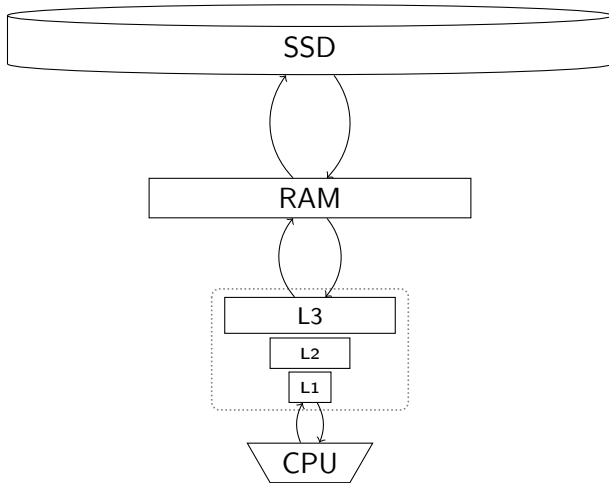
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**Steffan Christ Sølvesten** and Jaco van de Pol

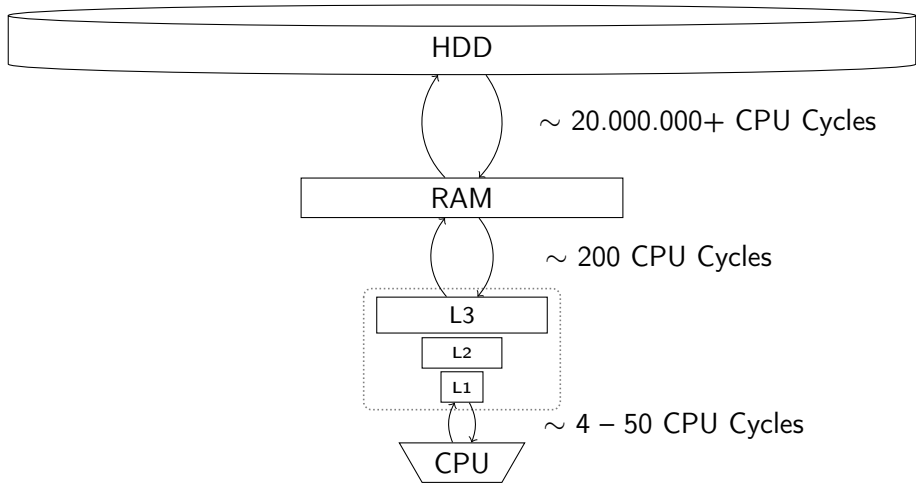
NFM 2023











# Adiar

Binary Decision Diagrams  
in External Memory

`github.com/ssoelvsten/adiar`

# Adiar

**Multi-terminal** Decision Diagrams  
in External Memory

`github.com/ssoelvsten/adiar`

# Adiar

**Quantum Multi-valued** Decision Diagrams  
in External Memory

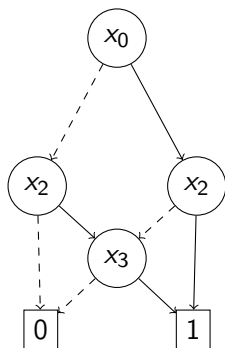
`github.com/ssoelvsten/adiar`



# Adiar

**Zero-suppressed** Decision Diagrams  
in External Memory

`github.com/ssoelvsten/adiar`







**BDD:**  $f : \mathbb{B}^n \rightarrow \mathbb{B}$



**ZDD:**  $A \subseteq \mathbb{B}^n$

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {  
    return prod2<bdd_policy>(f, g, o);  
}
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)  {  
    return prod2<zdd_policy>(A, B, o);  
}
```

```
bdd bdd_apply(bdd f, bdd g, bool_op o)  {  
    return prod2<bdd_policy>(f, g, o);  
}
```

```
zdd zdd_binop(zdd A, zdd B, bool_op o)  {  
    return prod2<zdd_policy>(A, B, o);  
}
```





Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



Running time for *3D Tic-Tac-Toe* with 300 GiB of RAM.



*Done*

BDD   ZDD

*Doable*

MTBDD

LDD

QMDD

*Done*

BDD

ZDD

(K)FDD

Tagged/Chained BDD

*Open*

Clock DD

MDD

*Doable*

MTBDD

LDD

QMDD

*Done*

BDD

ZDD

(K)FDD

Tagged/Chained BDD





# Steffan Christ Sølvsten

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## Adiar

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🔗 [github.com/ssoelvsten/adiar](https://github.com/ssoelvsten/adiar)

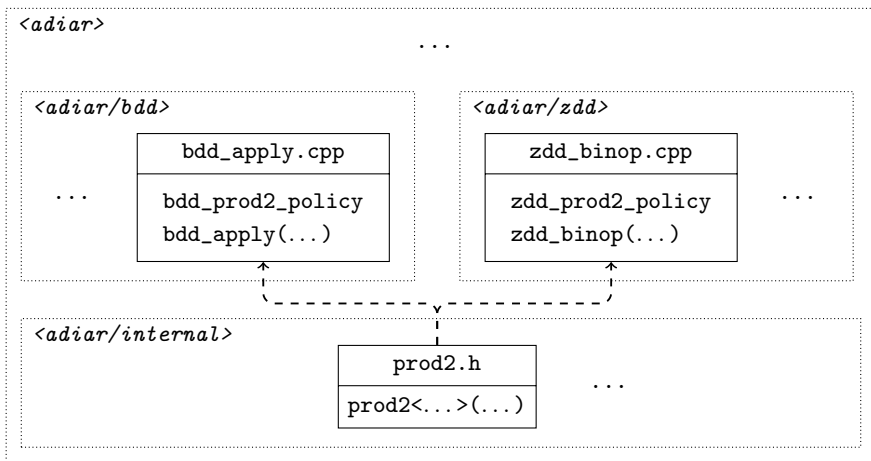
📖 [ssoelvsten.github.io/adiar](https://ssoelvsten.github.io/adiar)



Function	Operation Semantics
<b>ZDD Constructors</b>	
zdd_empty()	$\emptyset$
zdd_null()	$\{\emptyset\}$
zdd_singleton(var)	$\{x_{\text{var}}\}$
zdd_vars(vars)	$\{\bigcup_{i \in \text{vars}} \{x_i\}\}$
zdd_singletons(vars)	$\{\{x_i\} \mid i \in \text{vars}\}$
zdd_powerset(vars)	$\mathcal{P}(\text{vars})$
zdd_sized_set(vars, $k, \odot$ )	$\{s \in \mathcal{P}(\text{vars}) \mid  s  \odot k\}$
<b>ZDD Manipulation</b>	
zdd_binop( $A, B, \otimes$ )	$\{x \mid x \in A \otimes x \in B\}$
zdd_change( $A, \text{vars}$ )	$\{(a \setminus \text{vars}) \cup (\text{vars} \setminus a) \mid a \in A\}$
zdd_complement( $A, \text{dom}$ )	$\mathcal{P}(\text{dom}) \setminus A$
zdd_expand( $A, \text{vars}$ )	$\bigcup_{a \in A} \{a \cup v \mid v \in \mathcal{P}(\text{vars})\}$
zdd_offset( $A, \text{vars}$ )	$\{a \in A \mid \text{vars} \cap a = \emptyset\}$
zdd_onset( $A, \text{vars}$ )	$\{a \in A \mid \text{vars} \subseteq a\}$
zdd_project( $A, \text{vars}$ )	$\bigcup_{a \in A} \{a \cap \text{vars}\}$

Function	Operation Semantics
<b>Counting</b>	
zdd_size( $A$ )	$ A $
zdd_nodecount( $A$ )	# ZDD Nodes in $A$
zdd_varcount( $A$ )	# Non-empty Levels in $A$
<b>Predicates</b>	
zdd_equal( $A, B$ )	$A = B$
zdd_unequal( $A, B$ )	$A \neq B$
zdd_subseteq( $A, B$ )	$A \subseteq B$
zdd_disjoint( $A, B$ )	$A \cap B = \emptyset$
<b>Set elements</b>	
zdd_contains( $A, a$ )	$a \in A$
zdd_minelem( $A$ )	$a \in A \text{ s.t. } \forall a' \in A. a \leq a'$
zdd_maxelem( $A$ )	$a \in A \text{ s.t. } \forall a' \in A. a' \leq a$
<b>Conversion</b>	
zdd_from( $f, \text{dom}$ )	$\{x \in \mathcal{P}(\text{dom}) \mid f(x) = \top\}$
bdd_from( $A, \text{dom}$ )	$\vec{x} : \mathcal{P}(\text{dom}) \mapsto \vec{x} \in A$

Operations provided by Adiar in <adiar/zdd.h>.





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



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Priority Queue:  $Q_{count}$ :

[

]

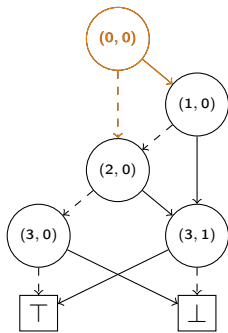


**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[  $((0,0) \xrightarrow{\top} (1,0), 1)$  ,  
 $((0,0) \xrightarrow{\perp} (2,0), 1)$  ,

]





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	0	0

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(2, 0)</b>	0	0

Priority Queue:  $Q_{count}$ :

[

$((0, 0) \xrightarrow{\perp} (2, 0), 1)$	,
$((1, 0) \xrightarrow{\perp} (2, 0), 1)$	,
$((1, 0) \xrightarrow{\top} (3, 1), 1)$	,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(2, 0)</b>	1	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(2, 0)$	2	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
]





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(2, 0)</b>	2	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(3, 0)</b>	0	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2	]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(3, 0)</b>	0	0

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2	]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	0

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$	,
$((2, 0) \xrightarrow{T} (3, 1), 2)$	]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(3, 1)</b>	0	2

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	1	2

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad ]$





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	3	2

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
<b>(3, 1)</b>	3	5

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result  
5

Priority Queue:  $Q_{count}$ :

[

]

# Steffan Christ Sølvsten

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## Adiar

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