

Random Access on Narrow Decision Diagrams in External Memory

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Adiar

I/O-efficient Decision Diagrams

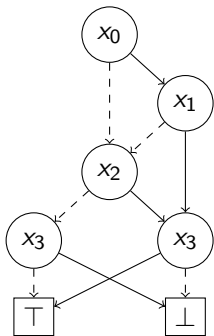
github.com/ssoelvsten/adiar

 Features

 Optimisations

 Features

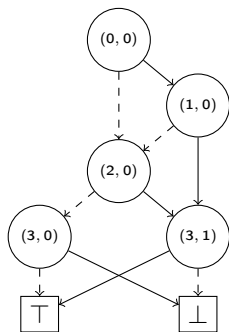
 Optimisations



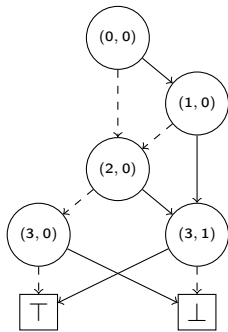
[((0, 0), (2, 0), (1, 0)) ,
 ((1, 0), (2, 0), (3, 1)) ,
 ((2, 0), (3, 0), (3, 1)) ,
 ((3, 0), T, ⊥) ,
 ((3, 1), ⊥, T)]

(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Node-based representation of prior shown BDDs



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[

]

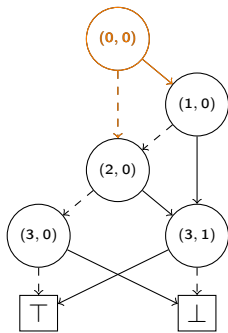


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue: Q_{count} :

[$((0,0) \xrightarrow{\top} (1,0), 1)$,
 $((0,0) \xrightarrow{\perp} (2,0), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\top} (1, 0), 1)$,
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	0	0

Priority Queue: Q_{count} :

[$((0, 0) \xrightarrow{\top} (1, 0), 1)$,
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(1, 0)	1	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), \quad 1) \quad ,$
]

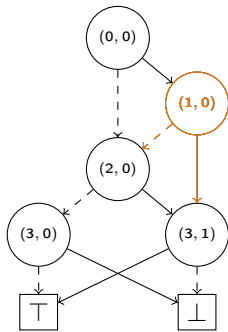


(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(1, 0)$	1	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	0	0

Priority Queue: Q_{count} :

[
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	0	0

Priority Queue: Q_{count} :

[

$((0, 0) \xrightarrow{\perp} (2, 0), 1)$,
$((1, 0) \xrightarrow{\perp} (2, 0), 1)$,
$((1, 0) \xrightarrow{\top} (3, 1), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	1	0

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\perp} (2, 0), 1)$,

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$,
]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(2, 0)	2	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$,
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$,
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	0	0

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\perp} (3, 0),$	2	,
$((1, 0) \xrightarrow{\top} (3, 1),$	1	,
$((2, 0) \xrightarrow{\top} (3, 1),$	2]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	0

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



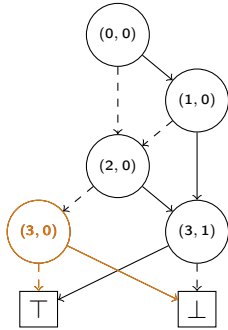
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 0)	2	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	0	2

Priority Queue: Q_{count} :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$,
 $((2, 0) \xrightarrow{T} (3, 1), 2)$]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
$(3, 1)$	1	2

Priority Queue: Q_{count} :

[

$((2, 0) \xrightarrow{\top} (3, 1), \quad 2) \quad]$



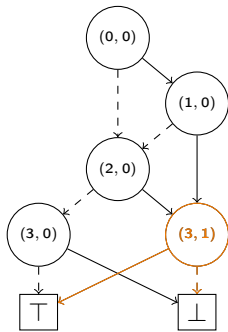
(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek	Sum	Result
(3, 1)	3	2

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek
(3, 1)

Sum
3

Result
5

Priority Queue: Q_{count} :

[

]



(a) $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

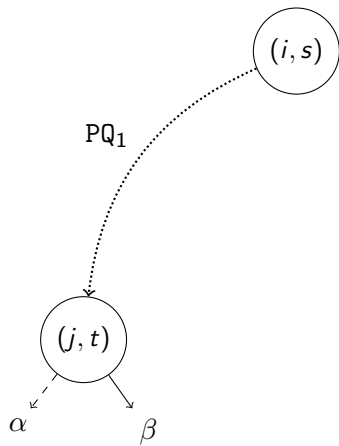
Result
5

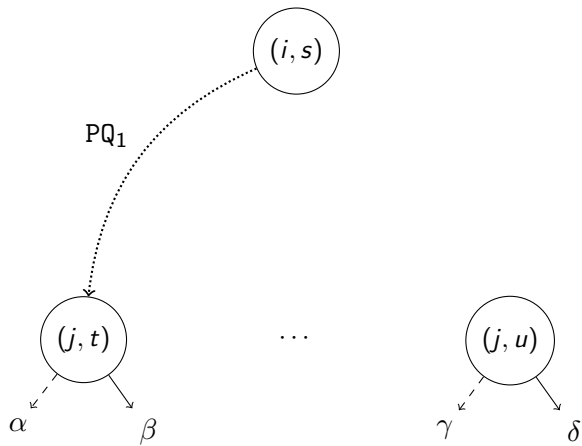
Priority Queue: Q_{count} :

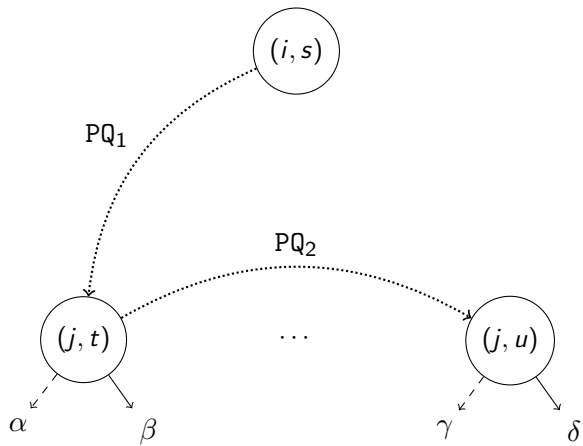
[

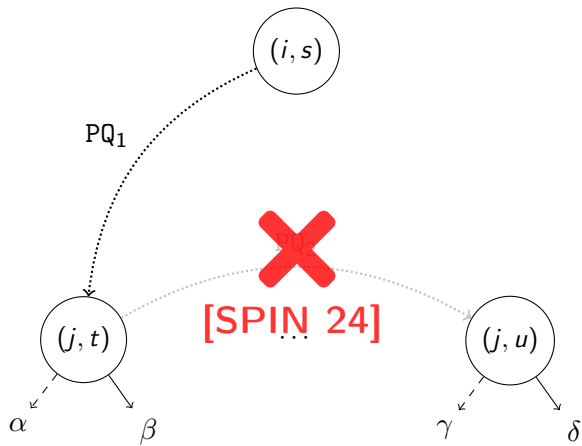
]

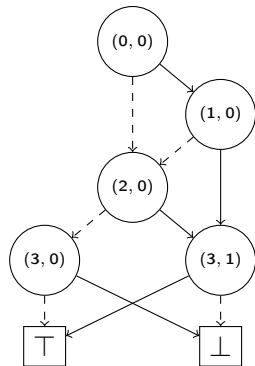
(i, s)

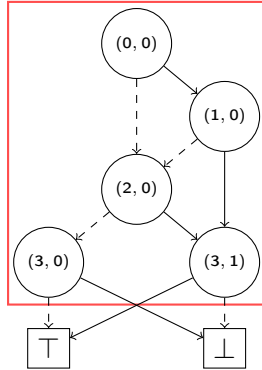


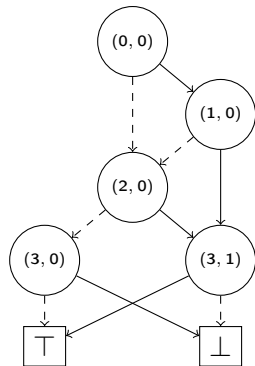


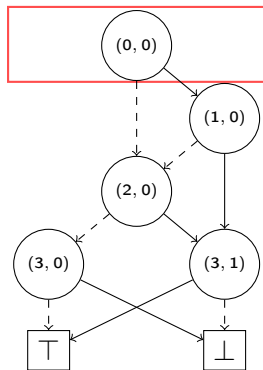


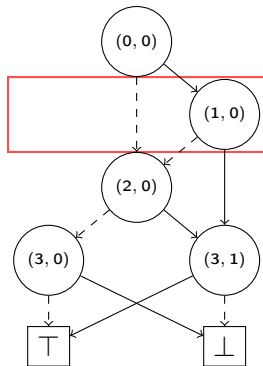


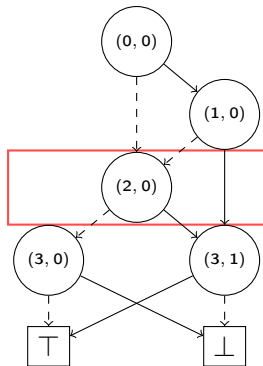


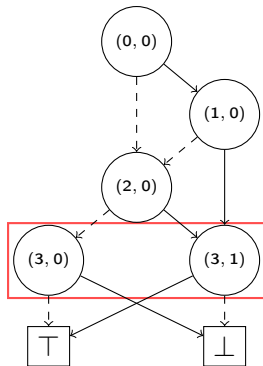


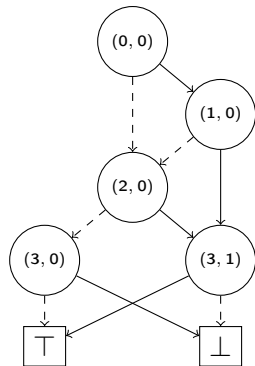


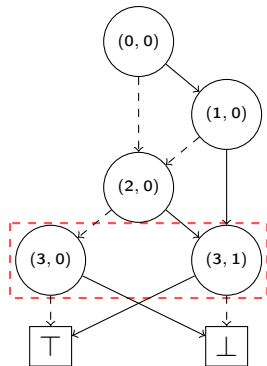






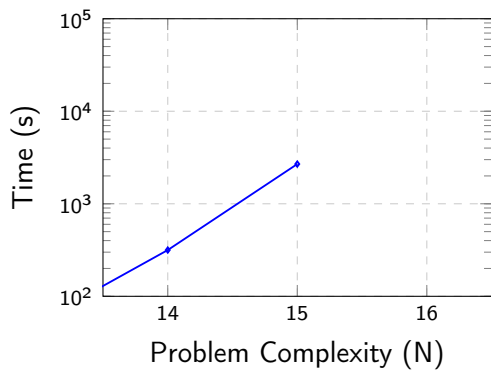







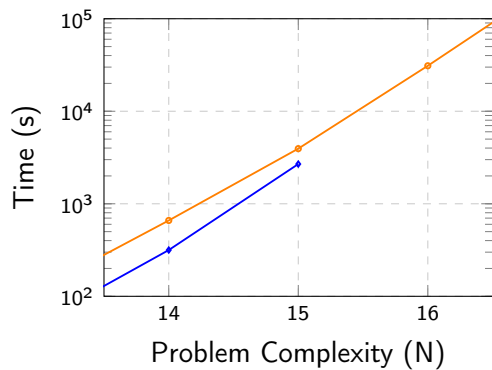
Width

2




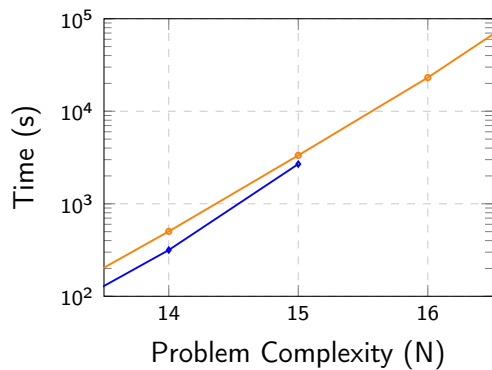
Queens | 300 GiB of RAM


 $N = 15$
◇ CUDD v3.0 : 44.8 min




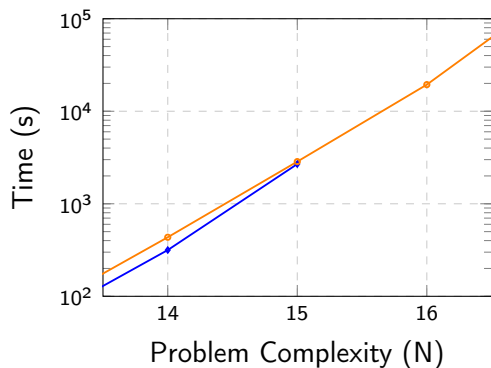
Queens | 300 GiB of RAM

<div>  $N = 15$ </div>			
◇	CUDD	v3.0	: 44.8 min
○	Adiar	v1.0	: 66.7 min



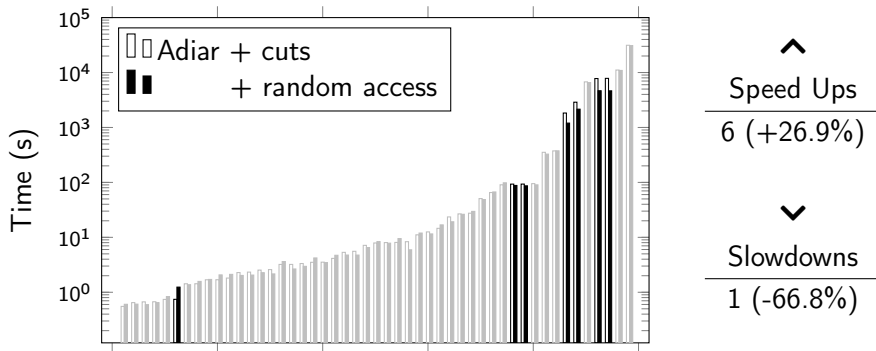
Queens | 300 GiB of RAM

<div>  </div> <div>$N = 15$</div>			
◇	CUDD	v3.0	: 44.8 min
○	Adiar	v1.0	: 66.7 min
	+ cuts		: 56.8 min



Queens | 300 GiB of RAM

<div>🕒</div> <div>$N = 15$</div>			
◇	CUDD	v3.0	: 44.8 min
○	Adiar	v1.0	: 66.7 min
	+ cuts		: 56.8 min
	+ random access		: 47.2 min



EPFL Circuit Verification | 300 GiB of RAM

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Adiar

🔗 github.com/ssoelvsten/adiar

📖 ssoelvsten.github.io/adiar

