

# Adiar

## Binary Decision Diagrams in External Memory

---

**Steffan Christ Sølvesten**, Jaco van de Pol,  
Anna Blume Jakobsen, and Mathias Weller Berg Thomasen

TACAS 2022







Minimal running time for the *Queens* problems.



Cache-misses for the *Queens* problems.





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[

]





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Priority Queue:  $Q_{count}$ :

[  $((0,0) \xrightarrow{\top} (1,0), 1)$  ,  
 $((0,0) \xrightarrow{\perp} (2,0), 1)$  ,

]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek   | Sum | Result |
|--------|-----|--------|
| (1, 0) | 0   | 0      |

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek     | Sum | Result |
|----------|-----|--------|
| $(1, 0)$ | 0   | 0      |

Priority Queue:  $Q_{count}$ :

[  $((0, 0) \xrightarrow{\top} (1, 0), 1)$  ,  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek     | Sum | Result |
|----------|-----|--------|
| $(1, 0)$ | 1   | 0      |

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek     | Sum | Result |
|----------|-----|--------|
| $(1, 0)$ | 1   | 0      |

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek     | Sum | Result |
|----------|-----|--------|
| $(2, 0)$ | 0   | 0      |

Priority Queue:  $Q_{count}$ :

[  
 $((0, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(2, 0)</b> | 0   | 0      |

Priority Queue:  $Q_{count}$ :

[

|  |   |
|--|---|
| $((0, 0) \xrightarrow{\perp} (2, 0), \quad 1)$ | , |
| $((1, 0) \xrightarrow{\perp} (2, 0), \quad 1)$ | , |
| $((1, 0) \xrightarrow{\top} (3, 1), \quad 1)$  | , |

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek     | Sum | Result |
|----------|-----|--------|
| $(2, 0)$ | 1   | 0      |

Priority Queue:  $Q_{count}$ :

[  
 $((1, 0) \xrightarrow{\perp} (2, 0), 1)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 ]





**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(2, 0)</b> | 2   | 0      |

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(2, 0)</b> | 2   | 0      |

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{\perp} (3, 0), 2)$  ,  
 $((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek   | Sum | Result |
|--------|-----|--------|
| (3, 0) | 0   | 0      |

Priority Queue:  $Q_{count}$ :

[

|                                       |   |   |
|---------------------------------------|---|---|
| $((2, 0) \xrightarrow{\perp} (3, 0),$ | 2 | , |
| $((1, 0) \xrightarrow{\top} (3, 1),$  | 1 | , |
| $((2, 0) \xrightarrow{\top} (3, 1),$  | 2 | ] |



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek   | Sum | Result |
|--------|-----|--------|
| (3, 0) | 0   | 0      |

Priority Queue:  $Q_{count}$ :

[

|                                       |   |   |
|---------------------------------------|---|---|
| $((2, 0) \xrightarrow{\perp} (3, 0),$ | 2 | , |
| $((1, 0) \xrightarrow{\top} (3, 1),$  | 1 | , |
| $((2, 0) \xrightarrow{\top} (3, 1),$  | 2 | ] |



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(3, 0)</b> | 2   | 0      |

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(3, 0)</b> | 2   | 2      |

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), \quad 1) \quad ,$   
 $((2, 0) \xrightarrow{T} (3, 1), \quad 2) \quad ]$



(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek   | Sum | Result |
|--------|-----|--------|
| (3, 1) | 0   | 2      |

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{T} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{T} (3, 1), 2)$  ]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(3, 1)</b> | 0   | 2      |

Priority Queue:  $Q_{count}$ :

[

$((1, 0) \xrightarrow{\top} (3, 1), 1)$  ,  
 $((2, 0) \xrightarrow{\top} (3, 1), 2)$  ]





(a)  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek   | Sum | Result |
|--------|-----|--------|
| (3, 1) | 1   | 2      |

Priority Queue:  $Q_{count}$ :

[

$((2, 0) \xrightarrow{T} (3, 1), \quad 2) \quad ]$



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

| Seek          | Sum | Result |
|---------------|-----|--------|
| <b>(3, 1)</b> | 3   | 2      |

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Seek  
**(3, 1)**

Sum  
3

Result  
5

Priority Queue:  $Q_{count}$ :

[

]



**(a)**  $(x_0 \wedge x_1 \wedge x_3) \vee (x_2 \oplus x_3)$

Result  
5

Priority Queue:  $Q_{count}$ :

[

]



# Adiar

[github.com/ssoelvsten/adiar](https://github.com/ssoelvsten/adiar)



—•— CUDD    —■— Sylvan    —●— Adiar

Minimal running time for the *Queens* problems.



—♦— CUDD —■— Sylvan —●— Adiar

Minimal running time for the *Queens* problems.





Minimal running time for the *Queens* problems.



Minimal running time for the *Queens* problems.



| Algorithm                         | Time (s) |
|-----------------------------------|----------|
| $f \leftrightarrow g \equiv \top$ | 0.38     |

Checking the (EPFL Benchmark) *voter* circuit's single output gate ( $|N_f| = |N_g| = 5.76$  MiB).

| Algorithm                         | Time (s) |
|-----------------------------------|----------|
| $f \leftrightarrow g \equiv \top$ | 0.38     |
| $O(N \log N)$                     | 0.058    |

Checking the (EPFL Benchmark) *voter* circuit's single output gate ( $|N_f| = |N_g| = 5.76$  MiB).

| Algorithm                         | Time (s) |
|-----------------------------------|----------|
| $f \leftrightarrow g \equiv \top$ | 0.38     |
| $O(N \log N)$                     | 0.058    |
| $O(N)$                            | 0.006    |

Checking the (EPFL Benchmark) *voter* circuit's single output gate ( $|N_f| = |N_g| = 5.76$  MiB).



# Steffan Christ Sølvsten

---

✉ [soelvsten@cs.au.dk](mailto:soelvsten@cs.au.dk)

🌐 [ssoelvsten.github.io](https://ssoelvsten.github.io)

## Adiar

---

🔗 [github.com/ssoelvsten/adiar](https://github.com/ssoelvsten/adiar)

📖 [ssoelvsten.github.io/adiar](https://ssoelvsten.github.io/adiar)