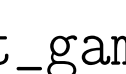
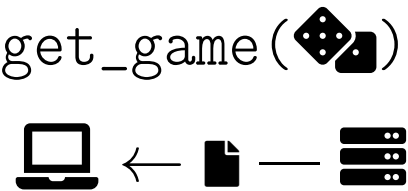
Anagram Trees

Steffan Christ Sølvsten

Wordrow

wordrow.io

get_game();



Reason: (*1.*) Simple backend (*2.*) Small file size

Contents

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Anagrams

Consider an *alphabet* $\Sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_k\}$ and words x, y, ... from a *language* $L \subseteq \Sigma^*$.

Definition (Rohit Parikh, 1961)
The *Parikh vector* of a word $x \in \Sigma^*$ is $\Psi(x) \triangleq \langle |\sigma_1|, |\sigma_2|, \ldots, |\sigma_k| \rangle$.

Example
For $\Sigma = \{a, b, c\}$, $\Psi(abb) = \langle 1, 2, 0 \rangle$ and $\Psi(abab) = \Psi(abba) = \langle 2, 2, 0 \rangle$.

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Example
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Theorem (Rohit Parikh, 1961)
Given a Context-Free Language, $L \subseteq \Sigma^*$, one can efficiently construct the set of all Parikh vectors. One can use this to identify that $x \in \Sigma^*$ **cannot** *be in the language.*

More Details: cs.umu.se/kurser/TDBC92/VT06/final/3.pdf

Anagrams

Consider an *alphabet* $\Sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_k\}$ and words x, y, ... from a *language* $L \subseteq \Sigma^*$.

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Example
For $\Sigma = \{a, b, c\}$, $\Psi(abb) = \langle 1, 2, 0 \rangle$ and $\Psi(abab) = \Psi(abba) = \langle 2, 2, 0 \rangle$.

Definition (Anagram)
$x, y \in \Sigma^*$ are *anagrams* if $\Psi(x) = \Psi(y)$.

Definition (Subanagram)
$x \in \Sigma^*$ is a *subanagram* of $y \in \Sigma^*$ if $\Psi(x) \leq \Psi(y)$.

Anagrams

Lemma
Given $x, y \in \Sigma^n$, one can compute whether $\Psi(x) = \Psi(y)$ in $\mathcal{O}(n + |\Sigma|)$ time.

Lemma
Given $x, y \in \Sigma^*$, one can compute whether $\Psi(x) \leq \Psi(y)$ in $\mathcal{O}(|x| + |y| + |\Sigma|)$ time.

Proof.

□

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Proof.

Compute the Parikh vectors similar to the first half of *Counting Sort*. □

Example
Counting the number of *a*'s, *b*'s, and *c*'s in *aba* and *aab* both yield $\langle 2, 1, 0 \rangle$.

Anagrams

Lemma
Given $x, y \in \Sigma^n$, computing whether $\Psi(x) = \Psi(y)$ takes $\mathcal{O}(sort(n))$ time.

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Anagrams

Lemma
Given $x, y \in \Sigma^n$, computing whether $\Psi(x) = \Psi(y)$ takes $\mathcal{O}(sort(n))$ time.

Proof.

Sort words x and y in $\mathcal{O}(\text{sort}(n))$ time. Then, check whether they now are the very same word in $O(n)$ time. □

Example

x =	b	a	a		x =	c	a	b
y =	a	b	a		y =	a	b	a

Anagrams

Lemma
Given $x, y \in \Sigma^n$, computing whether $\Psi(x) = \Psi(y)$ takes $\mathcal{O}(sort(n))$ time.

Proof.

Sort words x and y in $\mathcal{O}(\text{sort}(n))$ time. Then, check whether they now are the very same word in $O(n)$ time. □

Example

x =	a	a	b		x =	c	a	b
y =	a	a	b		y =	a	b	a

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Given $x, y \in \Sigma^n$, computing whether $\Psi(x) = \Psi(y)$ takes $\mathcal{O}(sort(n))$ time.

Proof.

Sort words x and y in $\mathcal{O}(\text{sort}(n))$ time. Then, check whether they now are the very same word in $O(n)$ time. □

Example

x =	**a**	a	b		x =	c	a	b
y =	**a**	a	b		y =	a	b	a

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Lemma
Given $x, y \in \Sigma^n$, computing whether $\Psi(x) = \Psi(y)$ takes $\mathcal{O}(sort(n))$ time.

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Sort words x and y in $\mathcal{O}(\text{sort}(n))$ time. Then, check whether they now are the very same word in $O(n)$ time. □

Example

x =	a	**a**	b		x =	c	a	b
y =	a	**a**	b		y =	a	b	a

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Given $x, y \in \Sigma^n$, computing whether $\Psi(x) = \Psi(y)$ takes $\mathcal{O}(sort(n))$ time.

Proof.

Sort words x and y in $\mathcal{O}(\text{sort}(n))$ time. Then, check whether they now are the very same word in $O(n)$ time. □

Example

x =	a	a	**b**		x =	c	a	b
y =	a	a	**b**		y =	a	b	a

Anagrams

Lemma
Given $x, y \in \Sigma^n$, computing whether $\Psi(x) = \Psi(y)$ takes $\mathcal{O}(sort(n))$ time.

Proof.

Sort words x and y in $\mathcal{O}(\text{sort}(n))$ time. Then, check whether they now are the very same word in $O(n)$ time. □

Example

x =	a	a	b	✔	x =	c	a	b
y =	a	a	b		y =	a	b	a

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Example

x =	a	a	b	✔	x =	a	b	c
y =	a	a	b		y =	a	a	b

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Example

x =	a	a	b	✔	x =	a	**b**	c
y =	a	a	b		y =	a	**a**	b

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Given $x, y \in \Sigma^n$, computing whether $\Psi(x) = \Psi(y)$ takes $\mathcal{O}(sort(n))$ time.

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Sort words x and y in $\mathcal{O}(\text{sort}(n))$ time. Then, check whether they now are the very same word in $O(n)$ time. □

Example

x = a a b ✔ y = a a b

x = a b c ! y = a a b

Anagrams

Lemma
Given $x, y \in \Sigma^*$, checking $\Psi(x) \leq \Psi(y)$ takes $\mathcal{O}(sort(|x|) + sort(|y|))$ time.

Proof.

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Given $x, y \in \Sigma^*$, checking $\Psi(x) \leq \Psi(y)$ takes $\mathcal{O}(sort(|x|) + sort(|y|))$ time.

Proof.

Again, sort words x and y. Now, match each symbol of x with ones in y; skip symbols of y if x is "ahead". □

Example

x =	b	a		x =	c	a	
y =	a	b	a	y =	a	b	a

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Lemma
Given $x, y \in \Sigma^*$, checking $\Psi(x) \leq \Psi(y)$ takes $\mathcal{O}(sort(|x|) + sort(|y|))$ time.

Proof.

Again, sort words x and y. Now, match each symbol of x with ones in y; skip symbols of y if x is "ahead". □

Example

x =	a	b			x =	c	a
y =	a	a	b		y =	a	b	a

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y =	**a**	a	b		y =	a	b	a

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Example

x =	a	**b**			x =	c	a	
y =	a	**a**	b		y =	a	b	a

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Given $x, y \in \Sigma^*$, checking $\Psi(x) \leq \Psi(y)$ takes $\mathcal{O}(sort(|x|) + sort(|y|))$ time.

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x =	a	**b**			x =	c	a	
y =	a	a	**b**		y =	a	b	a

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Example

x =	a	b		✔	x =	a	c
y =	a	a	b		y =	a	a	b

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Proof.

Again, sort words x and y. Now, match each symbol of x with ones in y; skip symbols of y if x is "ahead". □

Example

x =	a	b		✔	x =	a	**c**	
y =	a	a	b		y =	a	a	**b**

Anagrams

Lemma
Given $x, y \in \Sigma^*$, checking $\Psi(x) \leq \Psi(y)$ takes $\mathcal{O}(sort(|x|) + sort(|y|))$ time.

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Again, sort words x and y. Now, match each symbol of x with ones in y; skip symbols of y if x is "ahead". □

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Proof.

Again, sort words x and y. Now, match each symbol of x with ones in y; skip symbols of y if x is "ahead". □

Example

Contents

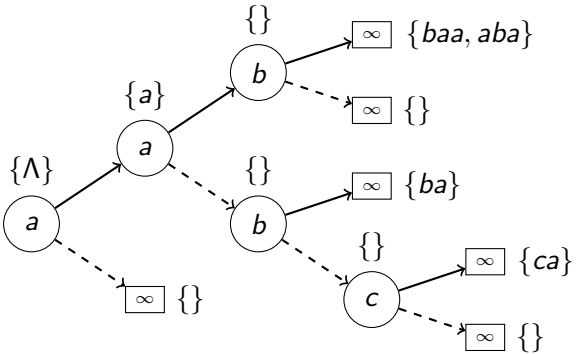
Anatree

Given an alphabet, Σ, and an ordering on its symbols, $< : \Sigma \times \Sigma \to \{\top, \bot\}$, the *Anatree* data structure manages a set of words $L \subseteq \Sigma^*$ on which one can do

Operation	
`insert(x)` `delete(x)`	$\mathcal{O}(\text{sort}(\lvert x \rvert) + \lvert \Sigma \rvert)$
`contains(x)`	$\mathcal{O}(\text{sort}(\lvert x \rvert) + \lvert \Sigma \rvert)$
`anagrams(x)`	$\mathcal{O}(\text{sort}(\lvert x \rvert) + \lvert \Sigma \rvert + T)$
`subanagrams(x)`	$\mathcal{O}(\text{sort}(\lvert x \rvert) + \min(N_{\text{Tree}}, 2^{\lvert x \rvert} \cdot \lvert \Sigma \rvert) + T)$

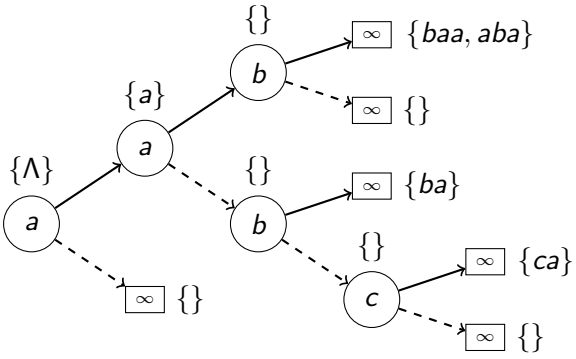
where N_{Tree} is the size of the Anagram tree and T is the output size.

Anatree



$L = \{\Lambda, a, ba, ca, aba, baa\}$

Anatree.`contains(ba)`

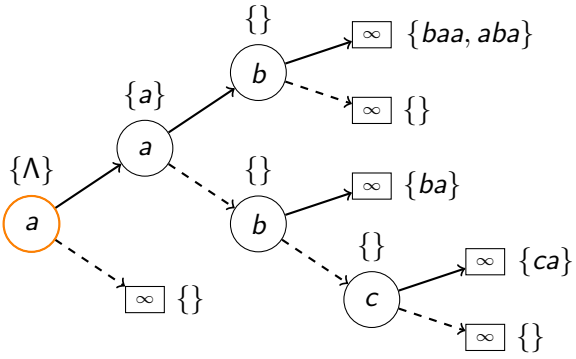


$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
contains(x):
  n := find(root, sort(x), 0)
  return n ≠ NIL & n.contains(x)

find(n, x', i):
```

Anatree.contains(ba)



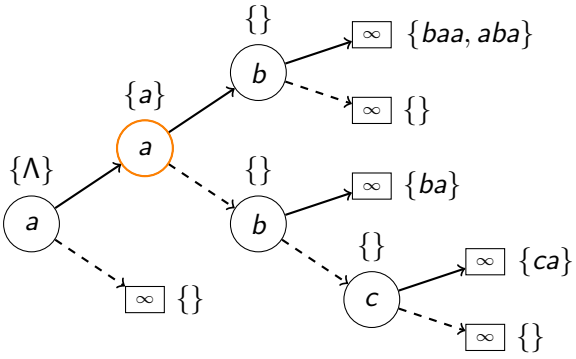
$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
contains(x):
  n := find(root, sort(x), 0)
  return n ≠ NIL & n.contains(x)

find(n, x', i):
```

```
  if x'[i] = n.char
    return find(n.true , x', i+1)
```

Anatree.contains(ba)

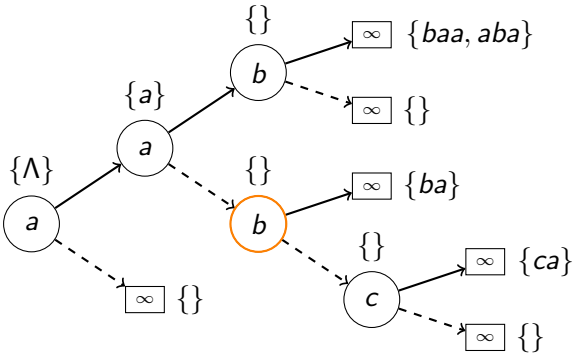


$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
contains(x):
  n := find(root, sort(x), 0)
  return n ≠ NIL & n.contains(x)

find(n, x', i):
  if x'[i] > n.char
    return find(n.false, x', i)
  if x'[i] = n.char
    return find(n.true , x', i+1)
```

Anatree.contains(ba)



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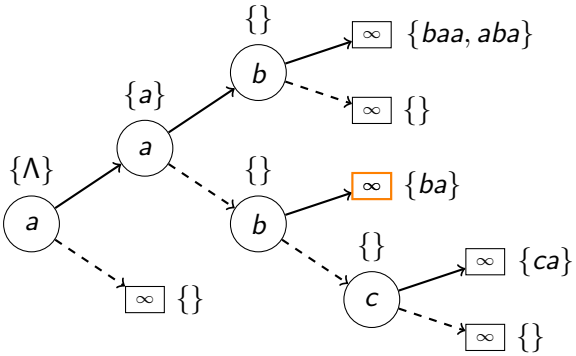
```
contains(x):
  n := find(root, sort(x), 0)
  return n ≠ NIL & n.contains(x)

find(n, x', i):
```

```
  if x'[i] > n.char
    return find(n.false, x', i)
  if x'[i] = n.char
    return find(n.true , x', i+1)
```

Anatree.contains(ba)



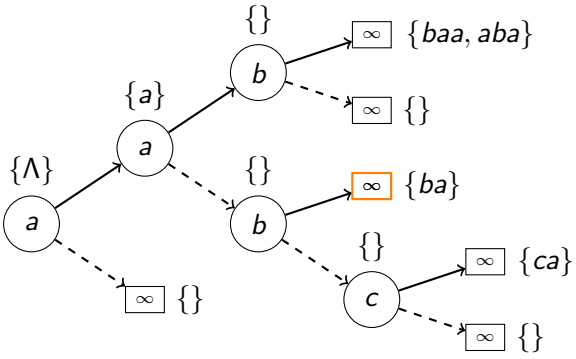
$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
contains(x):
  n := find(root, sort(x), 0)
  return n ≠ NIL & n.contains(x)

find(n, x', i):
  if i = x'.length
    return n


  if x'[i] > n.char
    return find(n.false, x', i)
  if x'[i] = n.char
    return find(n.true , x', i+1)
```

Anatree.contains(ba) = Yes



$L = \{\Lambda, a, ba, ca, aba, baa\}$

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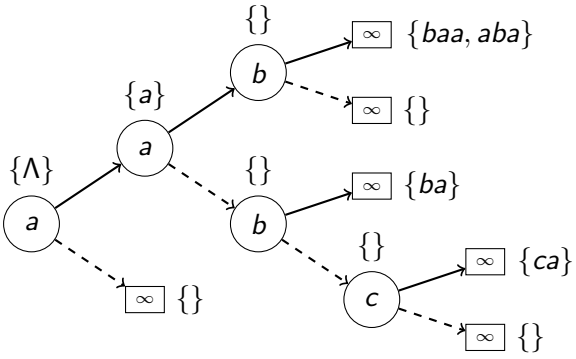
  if x'[i] = n.char
    return find(n.true , x', i+1)
```

Anatree.`contains(aca)`



$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
contains(x):
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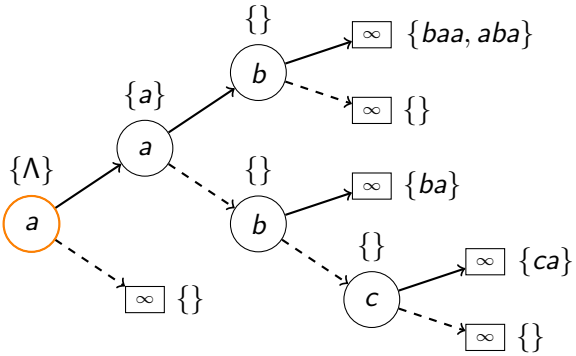
  if x'[i] > n.char
    return find(n.false, x', i)
  if x'[i] = n.char
    return find(n.true , x', i+1)
```

Anatree.contains(aca)



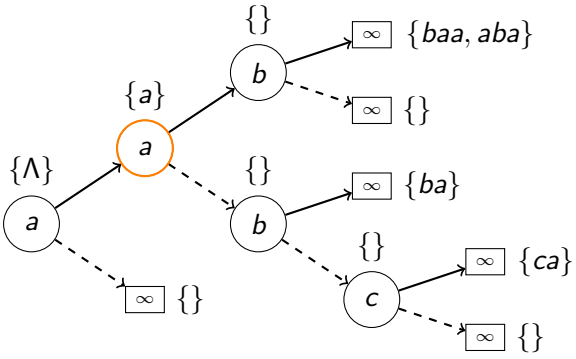
$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
contains(x):
  n := find(root, sort(x), 0)
  return n ≠ NIL & n.contains(x)

find(n, x', i):
  if i = x'.length
    return n


  if x'[i] > n.char
    return find(n.false, x', i)
  if x'[i] = n.char
    return find(n.true , x', i+1)
```

Anatree.contains(aca)



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    return find(n.false, x', i)
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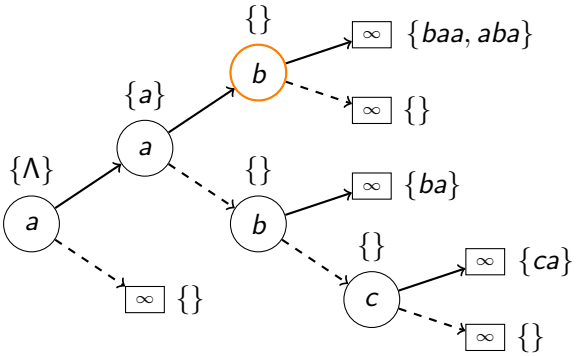
    return find(n.true , x', i+1)
```

Anatree.contains(aca)



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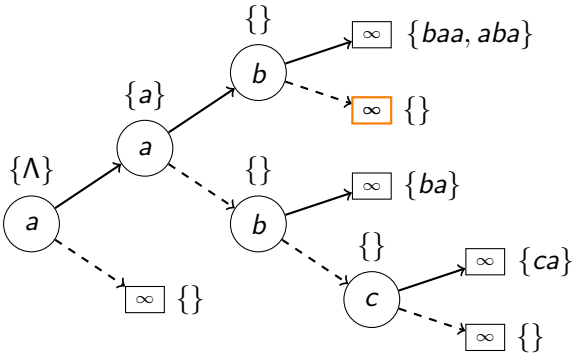
Anatree.contains(aca)

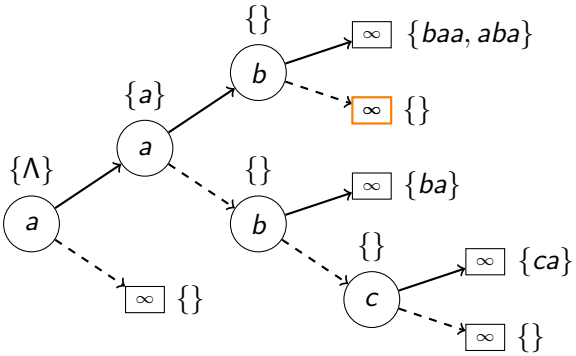


$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
contains(x):
  n := find(root, sort(x), 0)
  return n ≠ NIL & n.contains(x)

find(n, x', i):
  if i = x'.length
    return n
  if x'[i] < n.char
    return NIL
  if x'[i] > n.char
    return find(n.false, x', i)
  if x'[i] = n.char
    return find(n.true , x', i+1)
```

Anatree.contains(aca) = No



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find(n, x', i):
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    return n
  if x'[i] < n.char
    return NIL
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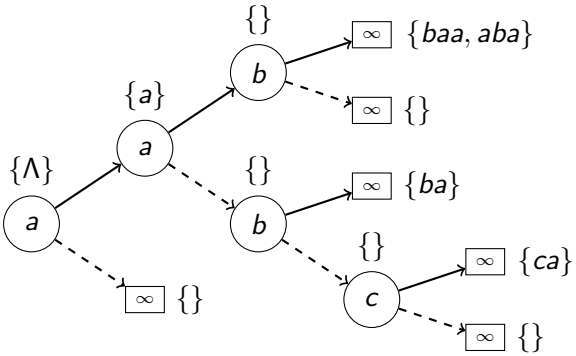
    return find(n.false, x', i)
  if x'[i] = n.char
    return find(n.true , x', i+1)
```

Anatree.`contains(...)`



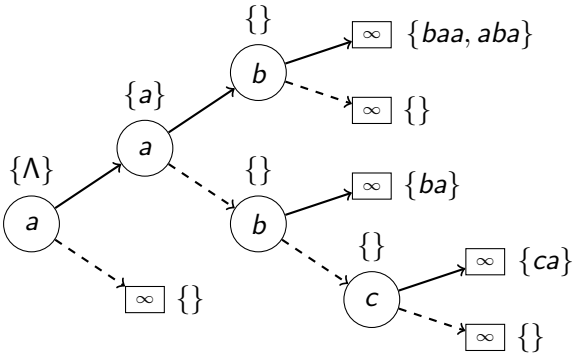
$L = \{\Lambda, a, ba, ca, aba, baa\}$

Lemma
`find(n, sort(x), i)` runs in $\mathcal{O}(sort(|x|) + |\Sigma|)$ *time.*

Proof.
$\mathcal{O}(1)$ time is spent per node. At most $|x|$ *high* edges and $|\Sigma|$ *low* edges are traversed, meaning at most $|x| + |\Sigma|$ nodes are visited.

On top of this, add the $\mathcal{O}(\text{sort}(|x|))$ time to sort x into x'. □

Anatree.`contains(...)`



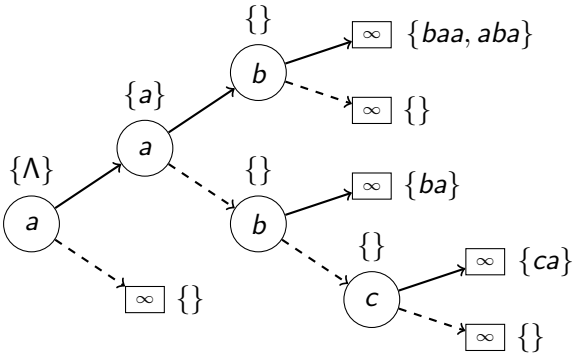
$L = \{\Lambda, a, ba, ca, aba, baa\}$

Lemma
`find(n, sort(x), i)` runs in
$\mathcal{O}(sort(|x|) + |\Sigma|)$ *time.*

Proof.
$\mathcal{O}(1)$ time is spent per node... □

Corollary
`contains(x)` runs in
$\mathcal{O}(sort(|x|) + |\Sigma|)$ *time.*

Anatree.`anagrams(...)`



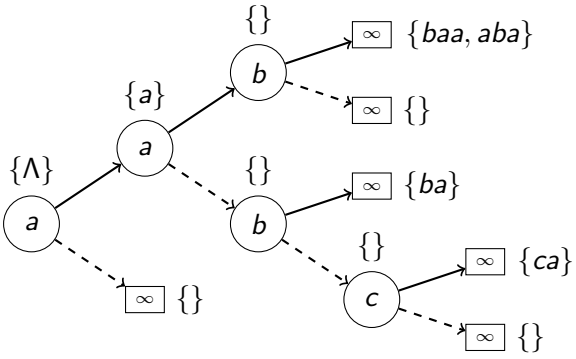
$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
anagrams(x):
  n := find(root, sort(x), 0)
  if n ≠ NIL
    output words in n
```

Corollary
`anagrams(x)` runs in $\mathcal{O}(sort(|x|) + |\Sigma| + T)$ *time.*

Proof.
It takes $\mathcal{O}(\texttt{sort}(|x|) + |\Sigma|)$ time to find n and then another $\mathcal{O}(T)$ time to output its content. □

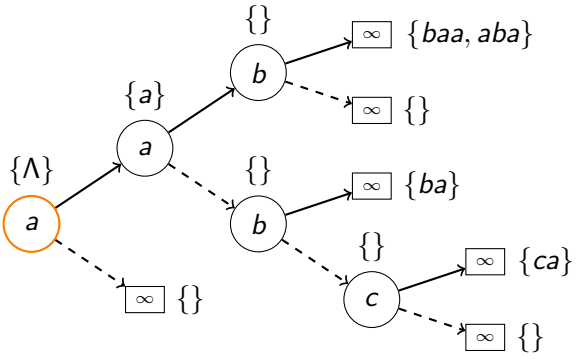
Anatree.`subanagrams(a) =`



$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
subanagrams(x):
  subanagrams'(root, sort(x), 0)
```

Anatree.subanagrams(a) = Λ

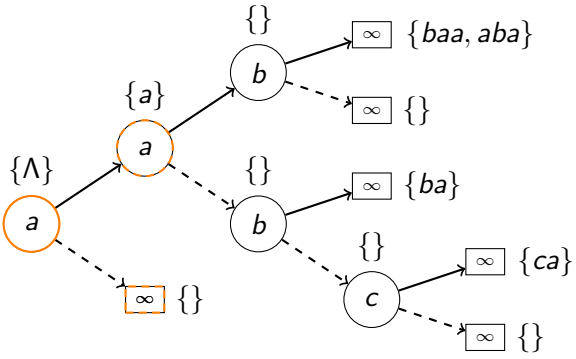


$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
subanagrams(x):
  subanagrams'(root, sort(x), 0)

subanagrams'(n, x', i):
  output words in n
```

Anatree.subanagrams(a) = Λ



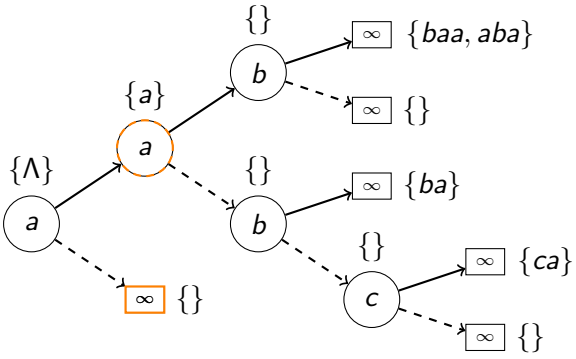
$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
subanagrams(x):
  subanagrams'(root, sort(x), 0)

subanagrams'(n, x', i):
  output words in n
```

```
if x'[i] = n.char:
  subanagrams'(n.false, x', i+1)
  subanagrams'(n.true,  x', i+1)
```

Anatree.subanagrams(a) = Λ



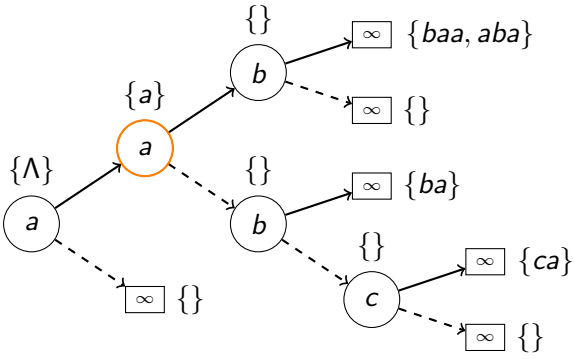
$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
subanagrams(x):
  subanagrams'(root, sort(x), 0)

subanagrams'(n, x', i):
  output words in n
  if n.char = ∞:
    return

  if x'[i] = n.char:
    subanagrams'(n.false, x', i+1)
    subanagrams'(n.true,  x', i+1)
```

Anatree.subanagrams(a) = Λ, a



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```
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  subanagrams'(root, sort(x), 0)

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  if i = x'.length:
    return


  if x'[i] = n.char:
    subanagrams'(n.false, x', i+1)
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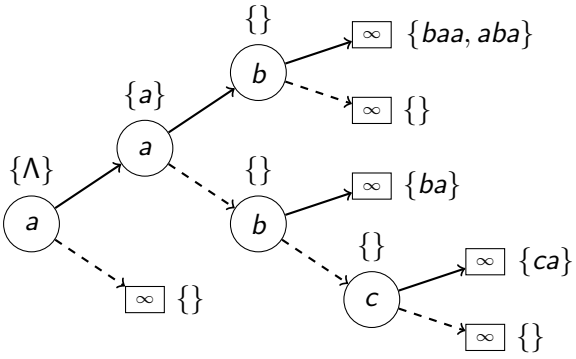
Anatree.subanagrams(a) = Λ, a



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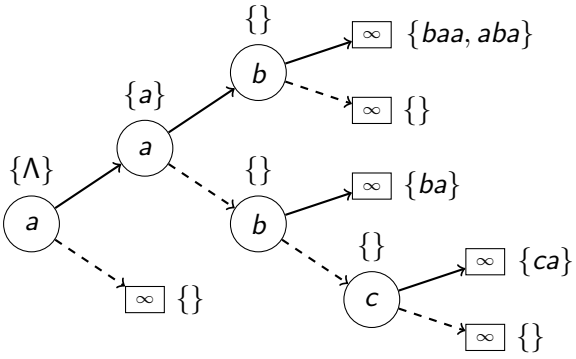
Anatree.subanagrams(abb) =



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```
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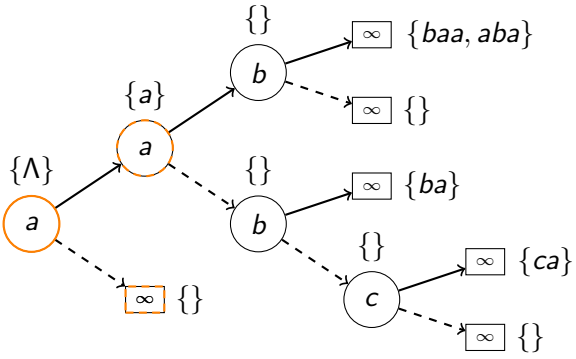
  subanagrams'(root, sort(x), 0)

subanagrams'(n, x', i):
  output words in n
  if n.char = ∞:
    return


  if i = x'.length:
    return


  if x'[i] = n.char:
    subanagrams'(n.false, x', i+1)
    subanagrams'(n.true,  x', i+1)
```

Anatree.subanagrams(abb) = Λ



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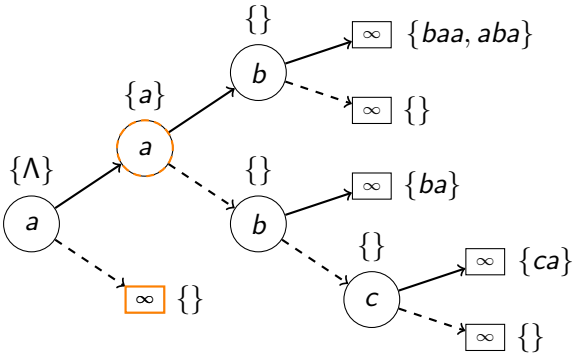

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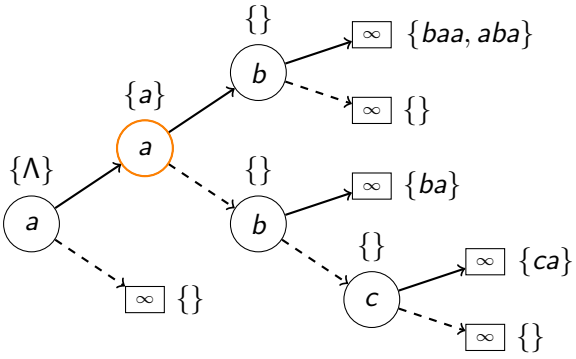

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```
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  subanagrams'(root, sort(x), 0)

subanagrams'(n, x', i):
  output words in n
  if n.char = ∞:
    return


  if i = x'.length:
    return
  if x'[i] > n.char:
    subanagrams'(n.false, x', i)
  if x'[i] = n.char:
    subanagrams'(n.false, x', i+1)
    subanagrams'(n.true,  x', i+1)
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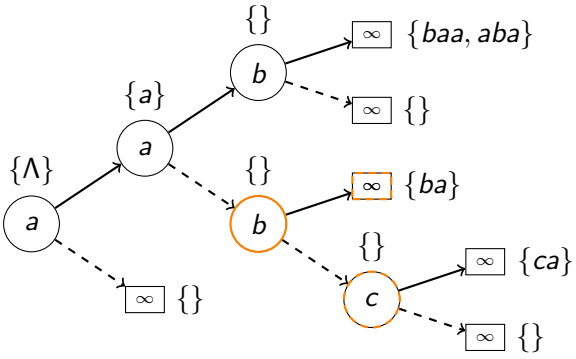
Anatree.subanagrams(abb) = Λ, a



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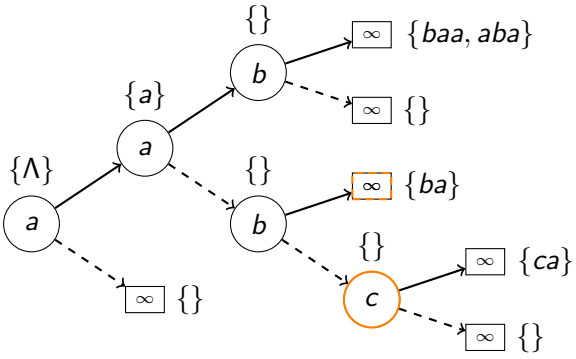
Anatree.subanagrams(abb) = Λ, a



$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
subanagrams(x):
  subanagrams'(root, sort(x), 0)

subanagrams'(n, x', i):
  output words in n
  if n.char = ∞:
    return
  while x'[i] < n.char:
    i++
  if i = x'.length:
    return
  if x'[i] > n.char:
    subanagrams'(n.false, x', i)
  if x'[i] = n.char:
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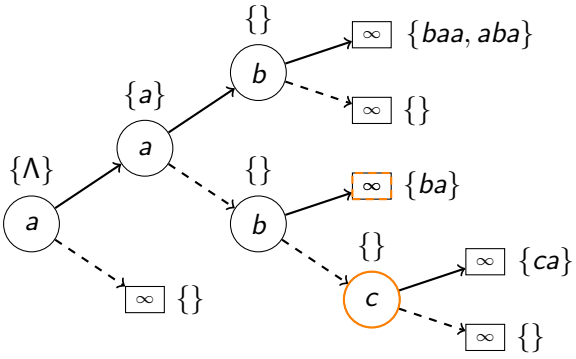
Anatree.subanagrams(abb) = Λ, a



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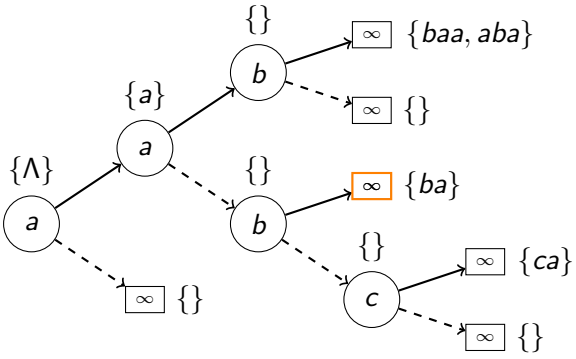
```

Anatree.subanagrams(abb) = Λ, a, ab



$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
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  subanagrams'(root, sort(x), 0)

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  while x'[i] < n.char:
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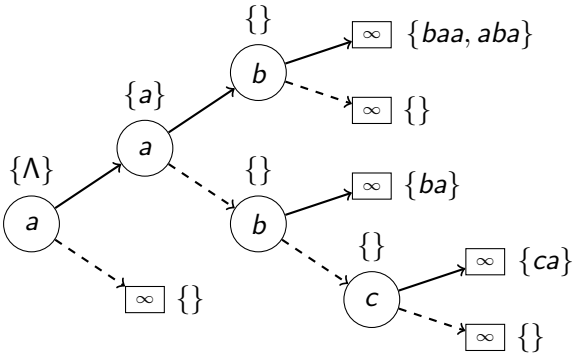
```

Anatree.subanagrams(abb) = Λ, a, ab



$L = \{\Lambda, a, ba, ca, aba, baa\}$

```
subanagrams(x):
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subanagrams'(n, x', i):
  output words in n
  if n.char = ∞:
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  while x'[i] < n.char:
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  if i = x'.length:
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  if x'[i] > n.char:
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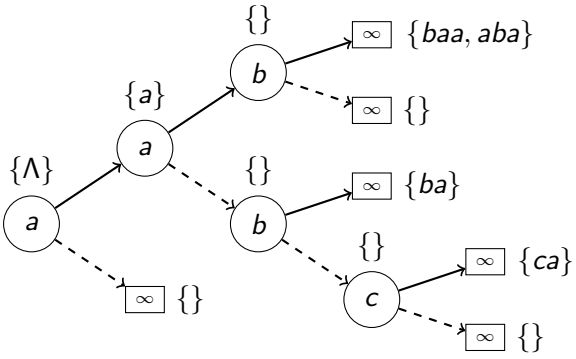
    subanagrams'(n.false, x', i+1)
    subanagrams'(n.true,  x', i+1)
```

Anatree.`subanagrams(...)`

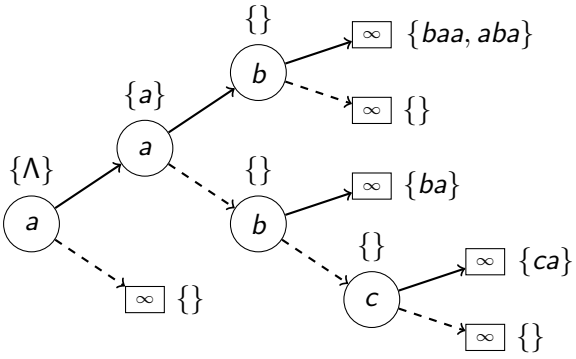


$L = \{\Lambda, a, ba, ca, aba, baa\}$

Lemma

For $N = \sum_{i=1}^{k} |x_i|$*, the anatree has size,* N_{tree}*, at most* N*.*

Anatree.`subanagrams(...)`



$L = \{\Lambda, a, ba, ca, aba, baa\}$

Lemma

For $N = \sum_{i=1}^{k} |x_i|$, the anatree has size, N_{tree}, at most N.

Theorem

`subanagrams(x)` runs in $\mathcal{O}(sort(|x|) + \min(N_{tree}, 2^{|x|} \cdot |\Sigma|) + T)$ time.

Proof.

It takes $\mathcal{O}(\text{sort}(|x|))$ time to sort x and another $\mathcal{O}(T)$ to write the output.

For every match, the recursion splits in two. Each of these $2^{|x|}$ matches have $|\Sigma|$ or fewer mismatches. □

Anatree.`keys(...)`

Definition
The subset L' of $L \subseteq \Sigma^*$ is a set of keys w.r.t. Ψ if for all $x, y \in L'$ then $\Psi(x) \neq \Psi(y)$.

Anatree.`keys(...)`

Definition
The subset L' of $L \subseteq \Sigma^*$ is a set of keys w.r.t. Ψ if for all $x, y \in L'$ then $\Psi(x) \neq \Psi(y)$.

Theorem
`keys(length)` runs in $\mathcal{O}(\min(N_{tree}, 2^{length} \cdot |\Sigma|) + T)$ *time.*

Proof.
Left as an exercise to the reader... □

Anatree.`insert`(Λ)

```
insert(x):
  root = insert'(root, sort(x), 0, x)

insert'(n, x', i, x):
```

$\{\}$

$\boxed{\infty}$

Anatree.insert(Λ)

```
insert(x):
  root = insert'(root, sort(x), 0, x)

insert'(n, x', i, x):
  if i = x'.length:
    n.insert(x)
  return n
```

{Λ}

∞

Anatree.`insert`(*ba*)

```
insert(x):
  root = insert'(root, sort(x), 0, x)

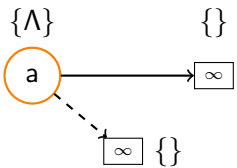
insert'(n, x', i, x):
  if i = x'.length:
    n.insert(x)
```

$\{\Lambda\}$

$\boxed{\infty}$

```
  return n
```

Anatree.insert(*ba*)

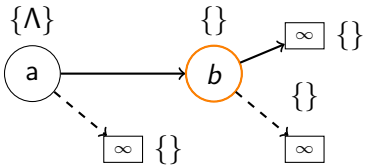



```
insert(x):
  root = insert'(root, sort(x), 0, x)

insert'(n, x', i, x):
  if i = x'.length:
    n.insert(x)
  else if n.char = ∞:
    n = node{ char: x'[i], false: ∞, true: ∞ }
    n.true = insert'(n.true, x', i+1, x)

  return n
```

Anatree.insert(*ba*)

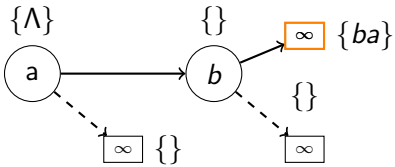



```
insert(x):
  root = insert'(root, sort(x), 0, x)

insert'(n, x', i, x):
  if i = x'.length:
    n.insert(x)
  else if n.char = ∞:
    n = node{ char: x'[i], false: ∞, true: ∞ }
    n.true = insert'(n.true, x', i+1, x)

  return n
```

Anatree.insert(*ba*)

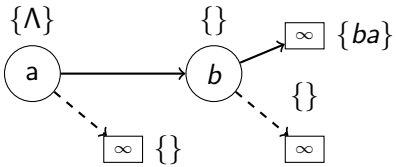



```
insert(x):
  root = insert'(root, sort(x), 0, x)

insert'(n, x', i, x):
  if i = x'.length:
    n.insert(x)
  else if n.char = ∞:
    n = node{ char: x'[i], false: ∞, true: ∞ }
    n.true = insert'(n.true, x', i+1, x)

  return n
```

Anatree.`insert(`a`)`

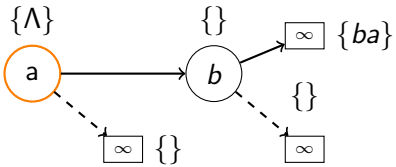



```
insert(x):
  root = insert'(root, sort(x), 0, x)

insert'(n, x', i, x):
  if i = x'.length:
    n.insert(x)
  else if n.char = ∞:
    n = node{ char: x'[i], false: ∞, true: ∞ }
    n.true = insert'(n.true, x', i+1, x)

  return n
```

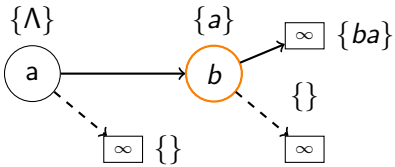
Anatree.insert(a)




```
insert(x):
  root = insert'(root, sort(x), 0, x)

insert'(n, x', i, x):
  if i = x'.length:
    n.insert(x)
  else if n.char = ∞:
    n = node{ char: x'[i], false: ∞, true: ∞ }
    n.true = insert'(n.true, x', i+1, x)
  else if x'[i] == m.char:
    n.true = insert'(n.true, x', i+1, x)
  return n
```

Anatree.insert(a)

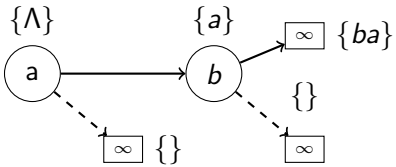



```
insert(x):
  root = insert'(root, sort(x), 0, x)

insert'(n, x', i, x):
  if i = x'.length:
    n.insert(x)
  else if n.char = ∞:
    n = node{ char: x'[i], false: ∞, true: ∞ }
    n.true = insert'(n.true, x', i+1, x)

  else if x'[i] == m.char:
    n.true = insert'(n.true, x', i+1, x)
  return n
```

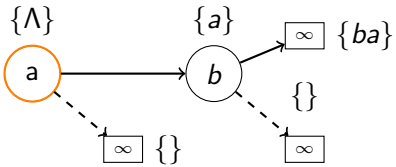
Anatree.`insert(`*baa*`)`




```
insert(x):
  root = insert'(root, sort(x), 0, x)

insert'(n, x', i, x):
  if i = x'.length:
    n.insert(x)
  else if n.char = ∞:
    n = node{ char: x'[i], false: ∞, true: ∞ }
    n.true = insert'(n.true, x', i+1, x)
  else if x'[i] == m.char:
    n.true = insert'(n.true, x', i+1, x)
  return n
```

Anatree.insert(*baa*)




```
insert(x):
  root = insert'(root, sort(x), 0, x)

insert'(n, x', i, x):
  if i = x'.length:
    n.insert(x)
  else if n.char = ∞:
    n = node{ char: x'[i], false: ∞, true: ∞ }
    n.true = insert'(n.true, x', i+1, x)
  else if x'[i] == m.char:
    n.true = insert'(n.true, x', i+1, x)
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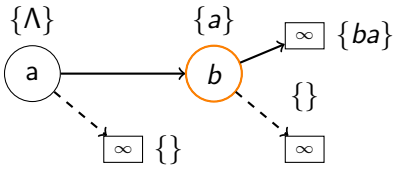
Anatree.insert(*baa*)

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  else if x'[i] == m.char:
    n.true = insert'(n.true, x', i+1, x)
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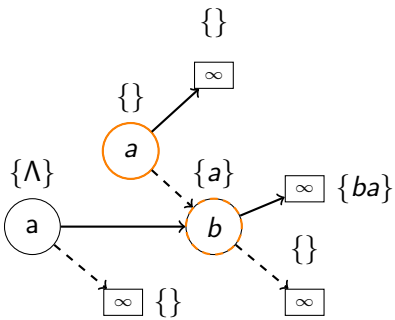
```

Anatree.`insert(`*baa*`)`

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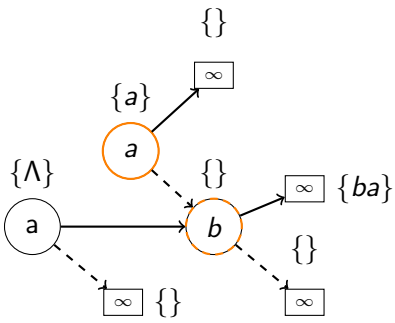
Anatree.insert(*baa*)

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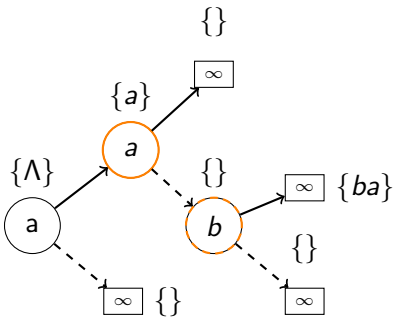


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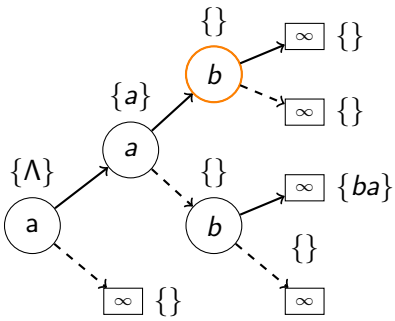
Anatree.insert(*baa*)

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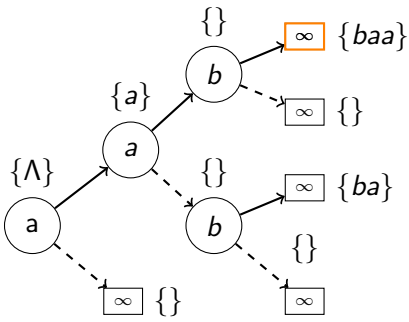
```

Anatree.insert(*baa*)

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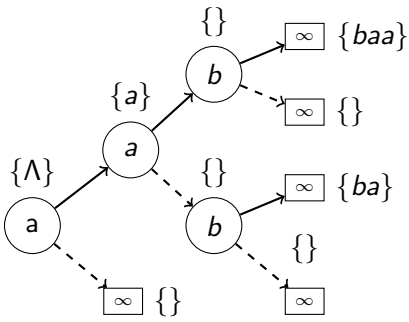
    return n'


  else if x'[i] == m.char:
    n.true = insert'(n.true, x', i+1, x)
  return n
```

Anatree.`insert(`*aba*`)`

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    n.true = insert'(n.true, x', i+1, x)
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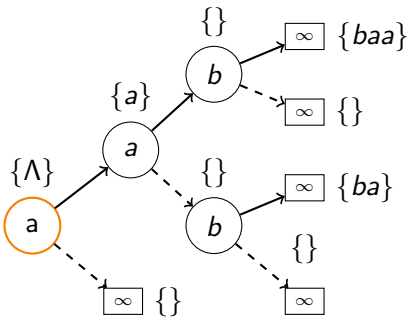
Anatree.insert(*aba*)

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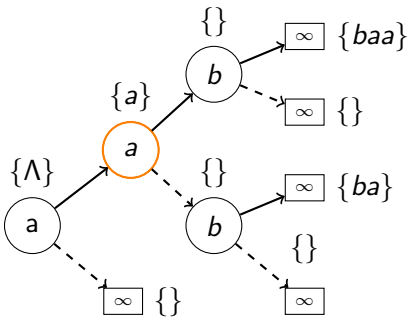
Anatree.insert(*aba*)

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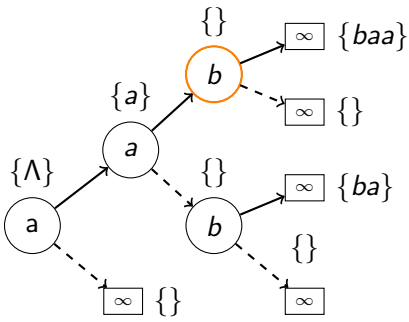
Anatree.insert(*aba*)

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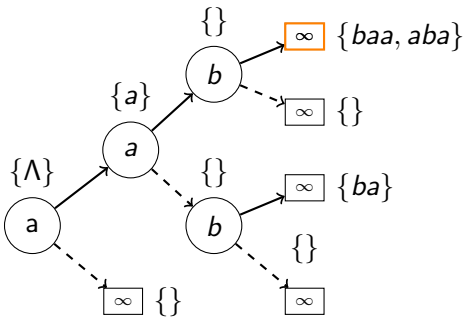
Anatree.insert(*aba*)

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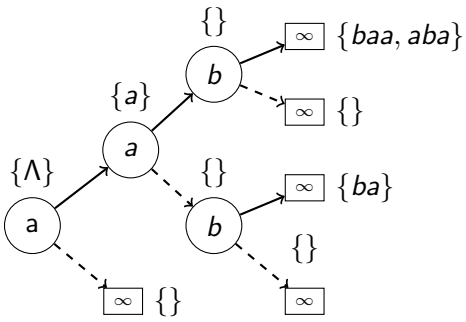
```

Anatree.insert(*ca*)

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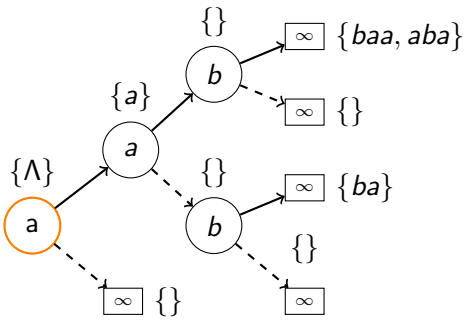
Anatree.insert(*ca*)

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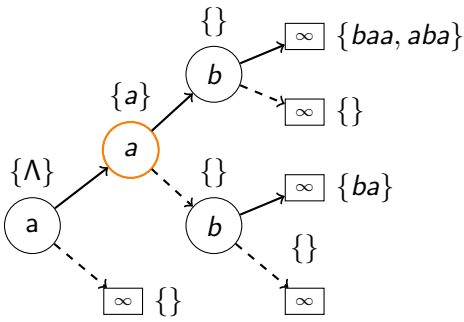


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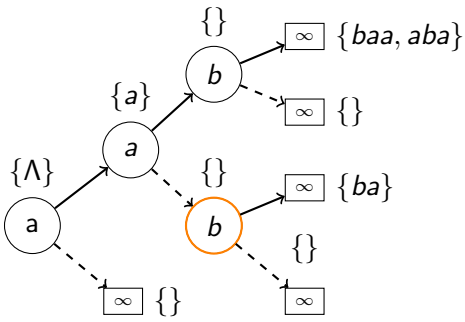


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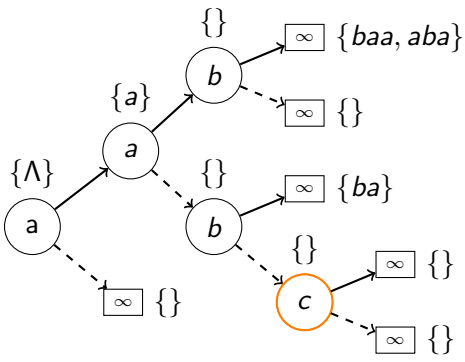


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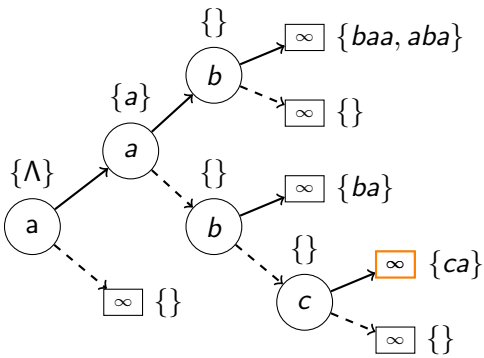


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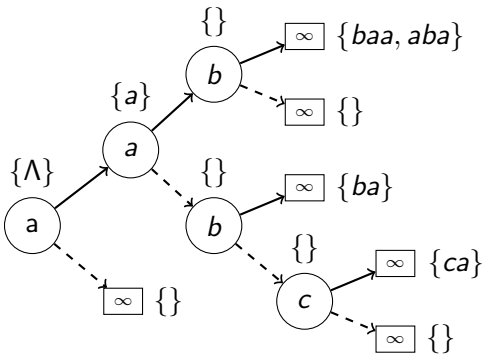
Anatree.`insert(...)`

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```

Anatree.`insert(...)`

Theorem
`insert(x)` runs in $\mathcal{O}(sort(|x|) + \Sigma)$ time.

Proof.
Similar argument as for `find(`n`, `x'`, `i`)`. □

Anatree.`insert(...)`

Theorem
`insert(x)` runs in $\mathcal{O}(sort(|x|) + \Sigma)$ time.

Proof.
Similar argument as for `find(`n`, `x'`, `i`)`. □

Corollary
For $N = \sum_{i=1}^{k} |x_i|$, `insert(`x_1`, `x_2`, ..., `x_k`)` requires $\mathcal{O}(sort(N) + k \cdot |\Sigma|)$ time.

Proof.
Follows from complexity of `insert(`x_i`)` and `sort` distributes over $+$ in $\mathcal{O}$-notation:

$$\mathcal{O}(\text{sort}(N_1) + \text{sort}(N_2)) = \mathcal{O}(\text{sort}(N_1 + N_2))$$

□

Anatree.`delete(...)`

Theorem
`delete(x)` runs in $\mathcal{O}(\mathit{sort}(|x|) + |\Sigma|)$ *time.*

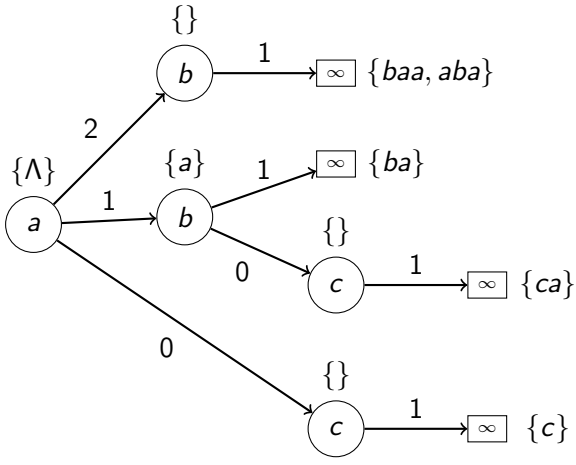
Proof.
Left as an exercise to the reader... □

Anatree

		Dictionary		Anatree			
		# Words	# Symbols	Size	#Keys	`insert` (s)	`subanagrams` (s)
	DK	32863	177308	62687	8513	12.62	1.05
	DE	23587	127562	55047	8201	9.46	0.88
	EN	40804	218342	75697	11741	10.62	1.43
	ES	39650	219776	56103	7502	8.45	0.89

Contents

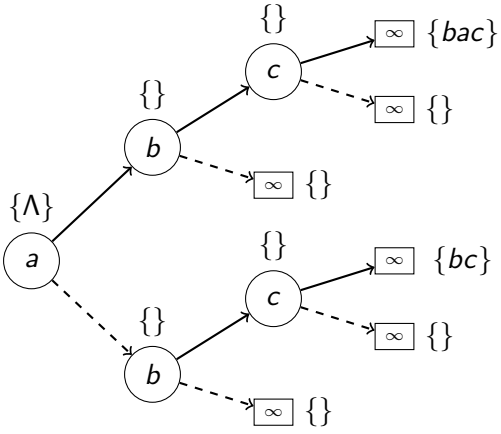
Multi-valued Anatree

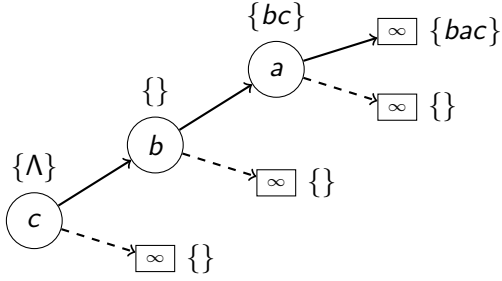


$L = \{\Lambda, a, c, ba, ca, aba, baa\}$

Contents

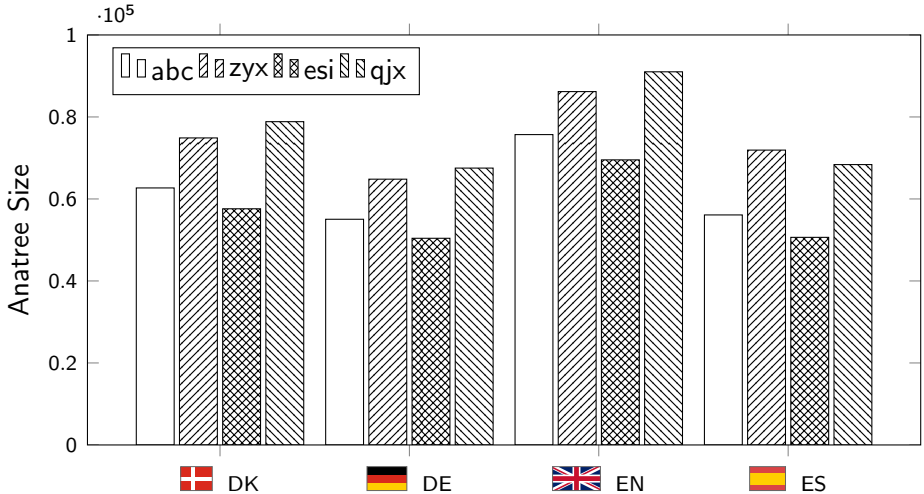
Letter Ordering



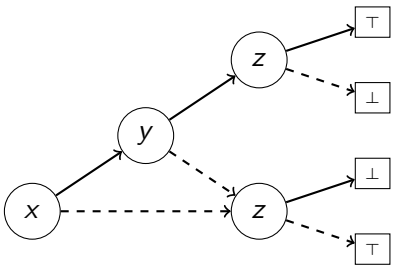


$$L = \{\Lambda, bc, bac\}$$

Letter Ordering

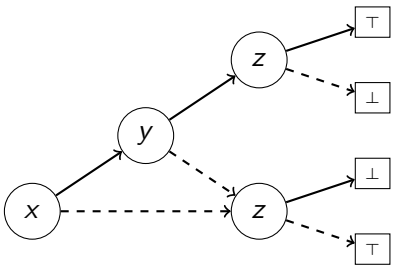


Binary Decision Diagrams



$f(x, y, z) \equiv \neg((x \wedge y) \oplus z)$

Binary Decision Diagrams

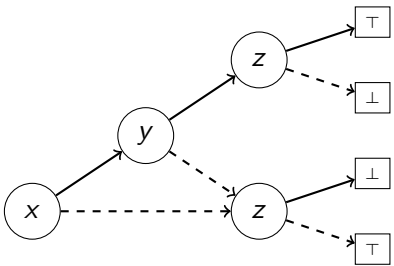


$f(x, y, z) \equiv \neg((x \wedge y) \oplus z)$

Used in the context of:

- Model Checking
- Compilers
- Game Solving

Binary Decision Diagrams



$f(x, y, z) \equiv \neg((x \wedge y) \oplus z)$

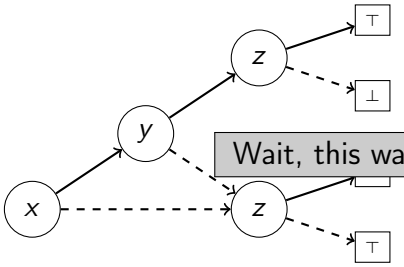
Used in the context of:

- Model Checking
- Compilers
- Game Solving

Features of BDDs:

- (Often) Smaller than Formula/Set
- Operation Complexity depends on BDD Size
- Size depends on Variable Ordering

Binary Decision Diagrams



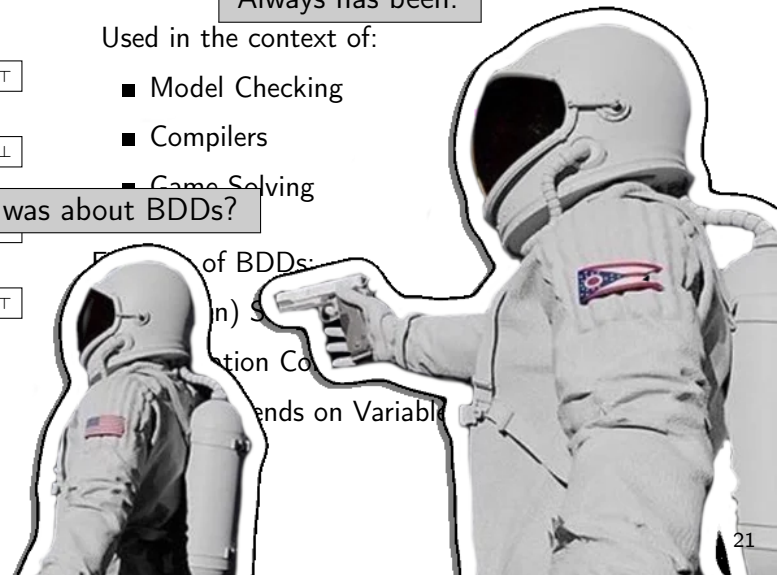
$f(x, y, z) \equiv \neg((x \wedge y) \oplus z)$

Always has been.

Used in the context of:

- Model Checking
- Compilers
- Game Solving

Wait, this was about BDDs?



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Wordrow

wordrow.io

github.com/ssoelvsten/wordrow

Anatree

github.com/ssoelvsten/anatree

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