

Detection of Collusive Tenders in Infrastructure Projects: Learning from Operation Car Wash

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Abstract: Procurement practices are often characterized by competitive tendering. The overarching purpose of this is to ingrain transparency, probity, and value for money into the processes of acquiring goods and services. When tenderers collude and clients are unable to detect them, bids will become uncompetitive. Yet, there have been a limited number of effective practical tools and methods developed that can be used by procurement authorities, controllers, and public officials to detect collusive tendering. Using data obtained from the Brazilian Federal Police and their ongoing criminal investigation titled Operation Car Wash, a robust and practical probabilistic method is developed. The main findings were that the method was able to accurately identify (81%–96%) the occurrence of collusion during a sealed tendering process. Conclusions are drawn from the lessons learned from the forensic investigations, indicating that the approach presented for detecting collusive behavior during tendering is grounded in reality. This paper presents a new way to utilize statistics and probability to identify the presence of and control collusion in public- and private-sector tendering. DOI: [10.1061/\(ASCE\)CO.1943-7862.0001737](https://doi.org/10.1061/(ASCE)CO.1943-7862.0001737). © 2019 American Society of Civil Engineers.

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Introduction

The corruption of every government begins nearly always with that of principles. —Charles de Montesquieu, 1748

There has been a mandate for governments worldwide to acquire their goods and services through a process of competitive tendering as part of an established governance and procurement strategy that is positioned to ensure competition forces suppliers to compete. In theory, this approach assumes that governments and taxpayers obtain better value for money. This also applies to the procurement of public infrastructure, which provides the foundations to support the social and economic fabric of an economy. The procurement of public infrastructure is a core responsibility of governments, as they have “high and direct implications on a country’s economic capacity, human development, social inclusion, and environmental sustainability” (OECD 2017, p. 180).

Funding for public infrastructure has been traditionally obtained from taxes paid by individuals and businesses. Public sector

agencies responsible for putting together a business case for a new infrastructure asset are also often charged with ensuring its delivery. As a result, so the procurement process is streamlined and efficiencies can be obtained. While the benefits of having such a decentralized procurement approach for infrastructure have been widely espoused (McCue and Pitzer 2000; Vagstad 2000; Bardhan and Mookherjee 2006), there have been calls also for centralized accountability and decision making to exist (OECD 2016, 2017). Irrespective of the procurement approach that is adopted, the process of competitive tendering must ensure accountability, transparency, and fair opportunities in spending taxpayers’ money.

A semicentralized approach, where an administrative unit is responsible for the acquisition of services and goods in its scope, is common (OECD 2016). Large procurement agencies, for example, are responsible for all tendering activities and afford fixed committees. Those of a smaller nature tend to have limited human resources to exclusively manage the procurement process—in Brazil, municipalities need to sporadically establish an *ad hoc* committee to obtain tenders to construct a specific infrastructure asset, which can be observed in other countries as well. Despite public procurement officials being the front line of defense (ACCC 2011, p. 15) as they need to be aware of market structures, behavior, and bidding patterns that may indicate collusion (OECD 2016, p. 6), there is a proclivity for such committees to possess limited knowledge about the practices of procurement, particularly the intricacies of tendering. In this case, the committee that is formed assumes responsibility for managing and controlling the behavior of construction organizations that are providing bids to deliver the required infrastructure.

Construction organizations, however, are comprised of highly skilled people who are attuned and knowledgeable in preparing tender submissions. The mere survival of many of these organizations is dependent upon them securing contracts, making a profit, and being accountable to individual owners or their shareholders. Senior management, however, may be tempted to engage in unlawful activities to ensure that the goals of their organization can be met (e.g., increased profit and market share). Construction

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organizations do not have to operate or conform to the governance, structures, policies, and procedures that governments need to abide by in order to ensure taxpayers are provided with value for money, therefore rendering it possible for them to engage in illegal activities should they choose to do so.

The asymmetry that exists between the public-sector buyers and private contractors provides a breeding ground for the formation of cartels to emerge (Harrington 1989). A cartel exists when contractors agree to act together instead of competing against each other. This agreement is designed to drive up the profits of cartel members while maintaining the illusion of competition. Such anti-competitive conduct may manifest in the form of price fixing; rigging bids (collusive tenders); controlling the output or limiting the supply of materials, equipment, or labor; and sharing markets (OECD 2003; Massimo 2004). The detection of cartels is a challenge (Osborne 1976; Chotibhongs and Arditi 2012), but methods to uncover them can be categorized as follows (Harrington 2005):

- Structural, entailing the identification of markets with traits deemed to be conducive to collusion, and
- Behavioral, which, involves observing how firms coordinate and their results.

Considerable pessimism exists as to the efficacy of structural approaches for unearthing collusion due to the ambiguity that surrounds the presence of cartelization. The upshot is that there has been a preference to adopt a behavioral approach whereby prices, market shares, and other economic data are analyzed (Harrington 2005). Armed with the knowledge and the conditions that engender collusion, the public sector, particularly in Brazil, does not possess robust practical tools to effectively detect the collusive behavior of competitors. Left unattended, the economic loss to an economy and adverse impact on society are insurmountable to digest. The consequences of the corruption scandal that has unfolded in Brazil's public procurement, as a result of Operation Car Wash, has stymied economic growth (Moro 2018). *Investigations are still ongoing; however, findings suggest Brazil's economy has lost a significant amount of resources while the wrongdoings that have materialized from the Operation Car Wash scandal continue to have adverse economic and social impacts.* For example, unemployment has increased by a staggering 6 million from 2014 to the end of 2017 (Watts 2017; IBGE 2018). Put simply, corruption has placed Brazil's economic and social well-being and political stability into a state of disarray. The corruption that exists within public procurement has been suggested as being the primary contributor (Watts 2017).

Drawing on actual data obtained from the Operation Car Wash investigation, this paper develops a probabilistic method for detecting the presence of collusive tendering (also known as bid rigging). While it is acknowledged that an array of theoretical models have been previously developed to detect collusion, they seldom reflect the behaviors that materialize in practice (Gupta 2001; Signor et al. 2019). Other methods are difficult to apply as "they use very expensive measurement tools such as employing independent engineers to evaluate projects" or "rely on a single narrow indicator, which may or may not be the primary vehicle for corruption" (Fazekas and Tóth 2014, p. 2). In addition, developed models and explanations around corruption in procurement have often been grounded on data collated through questionnaire surveys (Shan et al. 2015, 2017; Zhang et al. 2017). Such data have lacked context and meaning, which has rendered their results inconsequential in practice. Alternatively, data derived from secondary sources often reduce acts of corruption in procurement to be simply an abstraction rather a reflection of reality (Chotibhongs and Arditi 2012; Latour and Morselli 2017; Signor et al. 2017).

This paper fills this void by providing a much-needed context for detecting collusive tendering practices that are grounded in practice and real-life events. Studies of this nature have been found rarely, if at all, in the normative literature. The research also fills the main requirements stipulated for being classified as a good indicator of corruption in public procurement (Fazekas and Tóth 2014), as it

- can be used in a real-time basis to identify the presence of collusion;
- is derived solely from objective data at a micro level;
- can be used to generate comparative findings; and
- is open ended, allowing for the continuous extension, improvement, and adjustment to particular contexts.

While findings are presented within the context of Brazil, the method that is used to underpin the identification of collusion can be utilized in other countries if some basic data on pre-tender estimates are available. Procurement authorities and law enforcement agencies that are confronted with the presence of collusive tendering will be able to practically use the proposed approach during the auctioning phase and their investigations. When collusion is detected, with a level of confidence, procurement authorities can adopt varying strategies according to the case at hand. In the case of isolated cases, with low levels of confidence, they can be treated as administrative violations and result in new calls for bids to be undertaken using different competitors. Repeated cases with a high confidence level of collusion can be referred to law enforcement officials so they can instigate a detailed investigation to determine if collusion prevails when juxtaposed with additional evidence. In addition, the proposed tool that is developed can be used to help members of the Organization for Economic Cooperation and Development (OECD) to prevent misconduct, ensure compliance and monitoring, and establish a principle for enhancing the integrity of public procurement (OECD 2009a).

Collusive Tendering

The problems associated with competition avoidance and collusion during tendering have been known for centuries (Smith 1776) as organizations strive to maximize their profits by changing, bending, and breaking rules (Rothkopf and Harstad 1994). This malevolent practice is legally suppressed by governments, and the Sherman Antitrust Act of 1890 (US DoJ 2012) is an example of a law that is used to initiate legal proceedings against those parties that engage in collusion. In the case of a trial, physical proof is produced rarely, if at all, to demonstrate the presence of collusion and thus scientific evidence becomes necessary. In this case, probability and statistics assume a relevant role (National Research Council 2011), embraced with enthusiasm by the courts (Meier 1986). Naturally, this prominence led to the necessity of further discussions about the statisticians' role, as authoritatively exposed by Fisher (1986).

Regarding the application of statistics in procurement, it is necessary to refer to the seminal work of Friedman (1956), which spurred a plethora of studies to be undertaken in an array of industrial sectors. Essentially, Friedman (1956) aimed to determine the optimum bid required to maximize the expected profit of a given competitor based on (1) the cost estimate, (2) the number of competitors, and (3) the past behavior of the competitors. Friedman's (1956) study applied statistical and probabilistic methods to address the viewpoint of an honest competitor and did not mention the possibility of collusion among participants. Similar studies that have extended this work include those of Pelto (1971), McCaffer and Pettitt (1976), Ballesteros-Pérez et al. (2012), Skitmore (2014), and Ballesteros-Pérez and Skitmore (2017). Subsequently,

Friedman's work and its statistical and probabilistic principles were the basis for numerous developments on methods and techniques to detect collusive tendering in construction as this can be found in the works of Zarkada-Fraser and Skitmore (2000), Zarkada-Fraser (2000), Doree (2004), Priemus (2004), Ballesteros-Pérez et al. (2013), Le et al. (2014), and Signor et al. (2017), to name a few. While it is beyond our scope to provide a detailed review of these research articles, it is important to highlight that they aimed to detect collusion by relying substantially on the historic bidding behavior of each competitor, which can be difficult to obtain.

Procurement Policies

The need for monitoring and the difficulty of detecting collusive bid rigging has been widely recognized by many countries such as Australia, the United States (US), and those in the European Union (EU) (ACCC 2011; US DoJ 2012; EU 2014; OECD 2016). Australia recognizes the importance of government purchasing to the country's economy and the likelihood for cartels to be formed. However, the Australian Competition and Consumer Commission (ACCC) only offers a set of generic indicators for the detection of collusive practices, warning that many so-called signs for its presence can be ambiguous. While the ACCC emphasizes the need to study the bidding history of products, past tenders, and price movements over time and briefly cites the probability of the occurrence of certain bids, there is a paucity of available tools to detect the possibility of collusion (ACCC 2011).

In the US, bid rigging was legally prohibited by the Sherman Act enacted in 1890 (US DoJ 2012). According to the US antitrust agencies, bid rigging in government procurement provides the main connection with corruption. Bid rigging training can be provided for public procurement officials and investigators as they are best placed to detect and prevent its occurrence in public contracts (OECD 2014). The sheer volume of case materials and the complexity of the information that is required to be reviewed to determine collusion necessitate the use of computers and advanced technology equipment to ensure its effective detection (US DoJ 2012).

The European Commission, for example, relies on laws and comprehensive manuals to forestall collusion in public procurement (EU 2018). The Directive 2014/24/EU lists collusion as a factor that invalidates tenders but does not detail the methods needed to detect collusive practices, alluding to the existence of legislative differences between the Member States (EU 2014). However, EU competition regulators have begun to consider using algorithms to detect anticompetitive practices (Chee 2018), with the aim of extending their use to public procurement in the future.

The OECD has produced a plethora of publications and comprehensive manuals to address bid rigging in public procurement (OECD 2014, 2016, 2017). The OECD's (2016) manual identifies specific sections on preventing and detecting bid rigging. The OECD has suggested that procurement authorities are best positioned to combat bid rigging, though they have indicated that they are not generally provided with the incentives to do so, particularly as approximately 39% of its members do not recognize procurement as being a professional discipline. Furthermore, in the case of procuring infrastructure, public procurement officials need to have an understanding and knowledge of construction to help them detect bid rigging. According to OECD (2016, p. 15), the detection of bid rigging can be based on behavioral screenings, which

is facilitated by the increasing availability of reliable comprehensive data on public tenders, which allow competition authorities to develop different screening techniques, identify

markers which may show collusion, and test them empirically. In designing screens, authorities have focused on patterns which might indicate collusive bidding, such as submission of identical bids, a high correlation between bids, lack of correlation between the supplier's costs and the bid submitted, and significant differences between the winning and losing bid. Some competition authorities have further developed electronic screening programmes to detect bid rigging through monitoring bids and bidding patterns on a systematic basis. Such programmes are designed to quantify the probability of bid rigging using specific markers such as the rate of successful bids, bid price, number of failed bids, price increases, etc.

While numerous methods to detect collusion have been propagated in the extant literature, there is a consensus amongst studies that the identification of abnormal bidding behaviors only indicates the likelihood for this practice to exist (Lanzillotti 1998; Baker and Rubinfeld 1999; OECD 2009b). In fact, abrupt changes in the patterns of the competitors or bid rotating, which are typical markers of cartels, may be lawful and may only occur in momentary conditions of corporate equilibrium. In reality, then, collusion itself can only be proven by an investigation, which is undertaken typically through law enforcement (OECD 2009b; Lanzillotti 2017).

Detection of Collusive Tendering

In consideration of the absence of a robust tool for detecting collusive tendering, this article presents an automatable probabilistic method, which is based on the analysis of the joint behavior of competitors who act together to succeed in bid rigging. This method is developed based on the tenders of a Brazilian state-owned oil company at the center of Operation Car Wash, which typically prepares pre-tender estimates (PTE) of works to be constructed prior to soliciting tenders. Such pre-tender estimates are supposed to act as a benchmark where expected bid prices fall within a specified range. However, tenders may contain errors (e.g., incorrect quantities). If bidders identify these errors, they are required to report them and modify the pre-tender estimate accordingly. In the case of the probabilistic method presented in this article, it is assumed that the pre-tender estimates prepared by the state-owned oil company are accurate and reliable.

It is assumed that honest competitors prepare their own independent cost estimates and apply a markup that reflects specific behavioral characteristics. For example, when bidders have a desire to win a particular tender, they may choose to decrease their markup, and when they want to make a higher profit, their markup may be increased. The basis of the bid is influenced by several endogenous and exogenous factors such as prevailing economic and general market conditions; current capacity to undertake the work; and short-, medium-, and long-term strategies (e.g., Biruk et al. 2017; Signor et al. 2019). Such factors vary with time and their combination can be impossible to model or forecast. While the behavior of each bid is difficult to predict, in contrast, a set of bids received from competing entities can behave predictably (Signor et al. 2017). Given the randomness of each bid, it is assumed that in a competitive uncapped bidding environment where there are honest competitors, the differences between bids and official pre-tender estimates follow a normal distribution with a mean $\mu = 0$ and an unknown standard deviation (σ). Theoretically, the standard deviation is difficult to predefine. However, it is possible to assume that it is limited, as the vast majority of the bids range from being unrealistic and to being overpriced.

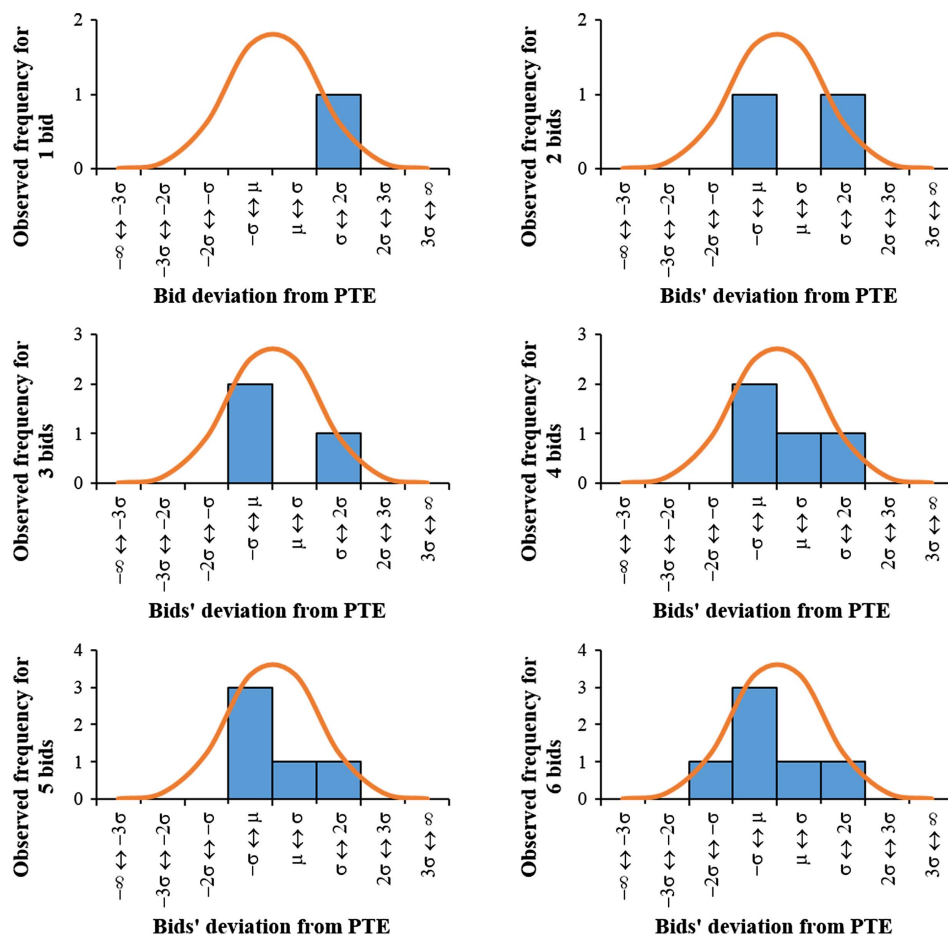


Fig. 1. Random bids for 1–6 honest competitors sequentially added to the set.

For the purpose of the research presented in this article, it is arbitrarily assumed that 90% of the responsible bids fall between $\pm 20\%$ around the pre-tender estimate and that a majority of experienced bidders are in this category under intense competition. This results in a standard deviation of approximately 0.12 ($\sigma \approx 0.12$). Notably, this limit of $\pm 20\%$ is applied on a case-by-case basis and thus does not define the price limits for the projects. It is acknowledged that several other distributions other than a normal distribution may exist (Pelto 1971; Signor et al. 2017; Ballesteros-Pérez and Skitmore 2017). Nevertheless, since their differences are minor in the region of $\pm 20\%$, a normal distribution is selected. Likewise, the means and standard deviations of each tender vary. For example, larger variations are expected in unprecedented projects and smaller variations in repetitive projects. For these reasons, whenever a history of previous honest bidding is available, both the probability distribution and its parameters can be adjusted, and the method proposed here continues to be valid.

Based on the assumption that honest and independent bids vary randomly around the pre-tender estimate (according to a normal distribution), it is, therefore, possible to identify instances of suspicious bidding based on a probabilistic calculation of bidders' global behavior. This approach is based on the premise that full collusion requires all the competitors to agree to partake in bid rigging. It is also expected that most honest and normally distributed bids are unlikely to significantly deviate from the expected mean (either positively or negatively). At this juncture, it is important to emphasize that if only one bid is received (unlikely and even illegal in several countries), then it can be considered to be random; even

if it has a significantly higher value, it is possible that it could have arisen only by chance. In the case of having an additional bid, an initial tendency toward equilibrium is expected, although any imbalances that occur also may be coincidental. However, according to the law of large numbers, the tendency to achieve equilibrium rises with three bids (usually the minimum number of competitors in usual tenders in several countries) and continues to increase along with the number of competitors, as shown in Fig. 1, constructed from hypothetical data randomly generated according to a normal distribution.

Fig. 1 suggests that within an honest competitive bidding environment, a reasonable pre-tender estimate is unlikely to be significantly and simultaneously exceeded by all bids (i.e., in the case of full collusion). A method that can, therefore, determine the joint probability of a set of bids greater than B_1, B_2, \dots, B_n occurring by chance is needed. This can be undertaken by comparing this joint probability to a limit value that is stipulated in accordance with the number of competitors and a desired level of confidence. Thus, if the joint probability exceeds the set's threshold, the result of the tender is considered to have fully colluded.

To determine the presence of collusion, we first need to assess the probabilities $P(i)$ of random bids having smaller values than each observed bid i using the cumulative density function (CDF) of the appropriate distribution, as Fig. 2 exemplifies.

Once all the individual probabilities $P(i)$ are assessed, it is possible to calculate the joint probability $P(x)$ of a random set B_1, B_2, \dots, B_n , being observed by chance, multiplying its constituent probabilities as denoted by the following equation:

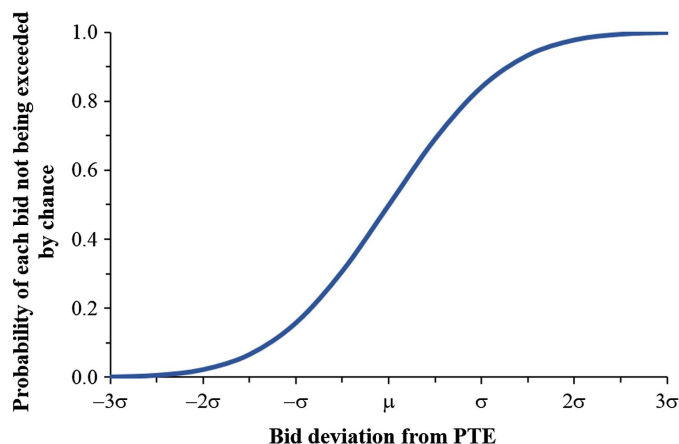


Fig. 2. Probability of each valid bid $P(i)$ not being exceeded by chance in an honest tender.

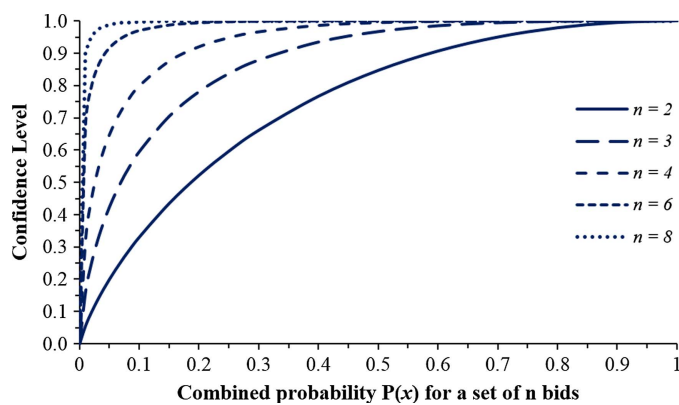


Fig. 3. Examples of CDFs for tenders with different numbers of bids.

$$P(x) = P(1) \cap P(2) \cap \dots \cap P(n) = \prod_{i=1}^n P(i) \quad (1)$$

The probability $P(x)$ of an investigated set of n simultaneous bids being observed by chance can be compared to limit values according to n and a chosen confidence level. These limit values are evaluated using the incomplete Gamma function that describes the CDF represented as follows and exemplified in Fig. 3:

$$\pi_n(x) = \frac{\Gamma(n, -\ln(x))}{(n-1)!} \quad [0 \leq x \leq 1] \quad (2)$$

Table 1 summarizes the limit values with which the inspected sets of bids must be compared. When the set's probability $P(x)$ exceeds any threshold, the tender is considered to have fully colluded at the respective confidence level.

Operation Car Wash: Collusion at a Brazilian State-Owned Oil Company

As was previously highlighted, it is difficult to assess the effectiveness of cartel detection methods as collusive practices are only effectively confirmed through police investigations. In addressing this issue, the results of the proposed method in this article are compared to real data obtained from the findings of Operation Car Wash, which is an ongoing criminal investigation by the Brazilian

Table 1. Limit values for the probabilities $P(x)$, by confidence levels

Bids	Confidence level		
	90%	95%	99%
1	9.00×10^{-1}	9.50×10^{-1}	9.90×10^{-1}
2	5.88×10^{-1}	7.01×10^{-1}	8.62×10^{-1}
3	3.32×10^{-1}	4.41×10^{-1}	6.47×10^{-1}
4	1.75×10^{-1}	2.55×10^{-1}	4.39×10^{-1}
5	8.78×10^{-2}	1.39×10^{-1}	2.78×10^{-1}
6	4.28×10^{-2}	7.33×10^{-2}	1.68×10^{-1}
7	2.03×10^{-2}	3.74×10^{-2}	9.73×10^{-2}
8	9.50×10^{-3}	1.87×10^{-2}	5.47×10^{-2}
9	4.37×10^{-3}	9.14×10^{-3}	3.00×10^{-2}
10	1.99×10^{-3}	4.40×10^{-3}	1.61×10^{-2}
15	3.36×10^{-5}	9.65×10^{-5}	5.66×10^{-4}
20	4.92×10^{-7}	1.75×10^{-6}	1.54×10^{-5}
25	6.55×10^{-9}	2.83×10^{-8}	3.54×10^{-7}
30	8.16×10^{-11}	4.19×10^{-10}	7.25×10^{-9}

Federal Police and other control agencies and has been identified as the world's largest corruption scandal (Watts 2017). To avoid jeopardizing the current investigation only publicly available data are presented in this article.

During the course of the Operation Car Wash investigation, it was discovered that a group of construction companies, the so-called Club of 16, colluded in a process of bid rigging for tenders that had been solicited by a state-owned oil company, using several techniques described by Fazekas et al. (2013). A detailed explanation of this case can be found in the work of Moro (2018). Signor et al. (2017) scientifically proved the presence of collusive bidding using statistical methods based on the assumption that the full colluded tenders could be previously identified using circumstantial evidence.

However, for law enforcement and (principally) for the procurement authorities, the existence of previous evidence is an exception. The rule is that those assigned to control or investigate any particular tender have only limited information, usually restricted to the tender itself. To replicate this situation in this article, data from 101 tenders from the oil company were analyzed without a preemptive bias that collusive bidding had occurred. An extract of the data set containing the 101 tenders (683 bids in total) is shown with brevity in Table 2.

The fourth column of Table 2 lists the differences between the bid and pre-tender estimate, assessed according to the following equation:

$$\text{Difference} \frac{\text{Bid}}{\text{PTE}} = \left(\frac{\text{Bid}}{\text{PTE}} - 1 \right) \times 100\% \quad (3)$$

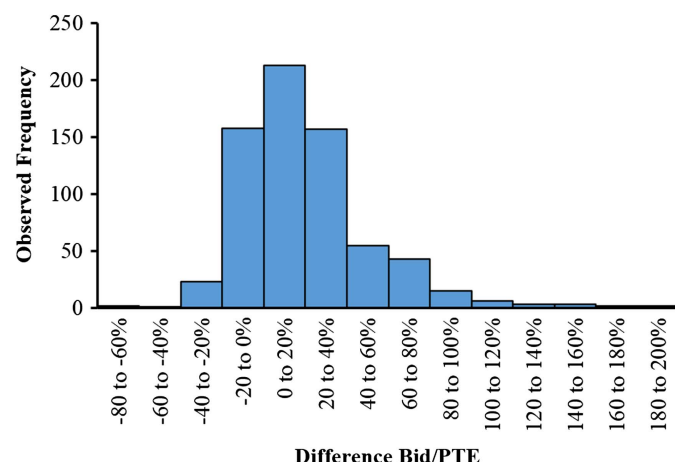
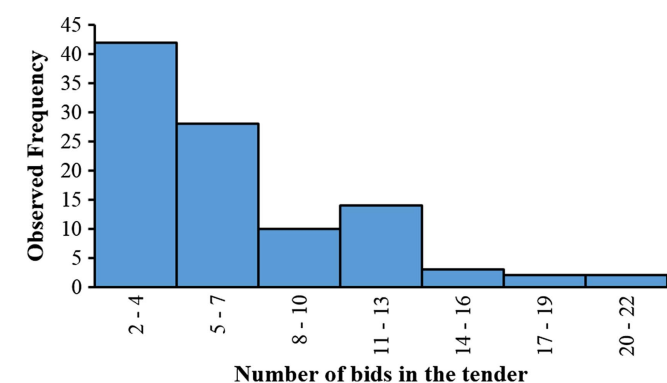
While Fig. 4 displays the histogram of the 683 differences between the bid and pre-tender estimate, Fig. 5 presents the number of bids that ranged from 2 to 21 in each one of the 101 tenders (fifth column in Table 2).

The sixth column in Table 2 contains the probabilities $P(i)$ of a random honest bid being smaller than each observed bid. From these values, the probabilities $P(x)$ for each tender's set of bids were assessed according to Eq. (1) and were recorded in the last column in Table 2. These values were compared to the thresholds for the confidence levels of 90%, 95%, and 99% to decide whether they have fully colluded or not. The results are presented in Fig. 6 and in the Appendix.

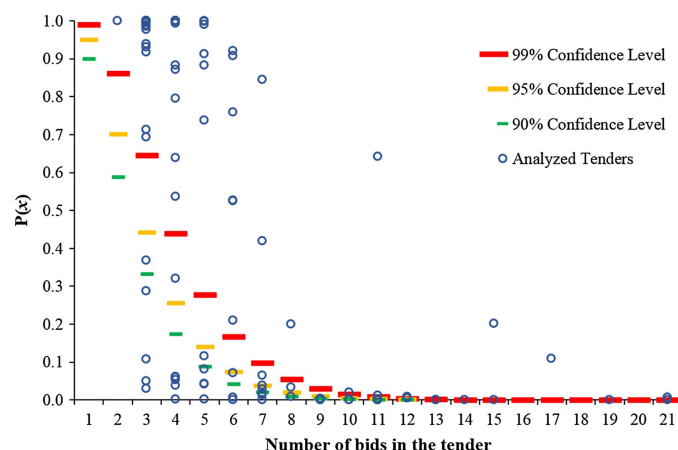
To evaluate the effectiveness of the proposed probabilistic method, the results were compared with the actual data obtained from the Operation Car Wash investigation. At the time of writing

Table 2. Extract of the analyzed data

Tender	Bid	Competitors	Difference bid/PTE (%)	Bids in the tender	Probabilities	
					Single bid $P(i)$	Set $P(x)$
1	1	Consortium contractor LF/Contractor BS	6.9	11	0.72	0.64
	2	Consortium contractor LD/Contractor LL	15.6		0.90	
	3	Contractor LI	28.8		0.99	
	4	Contractor LE	49.1		1.00	
	5	Contractor DI	56.5		1.00	
	6	Consortium contractor LJ/Contractor DD	66.2		1.00	
	7	Contractor DC	72.9		1.00	
	8	Contractor LH	79.8		1.00	
	9	Contractor LK	80.1		1.00	
	10	Contractor LM	93.3		1.00	
	11	Contractor DG	143.1		1.00	
2	12	Contractor AC	−10.8	4	0.18	0.05
	13	Contractor AP	−0.5		0.48	
	14	Contractor AJ	7.4		0.73	
	15	Contractor AK	10.3		0.81	
...
101	682	Consortium contractor LC/Contractor LI/Contractor LJ	42.2	2	1.00	1.00
	683	Consortium contractor LA/Contractor LF	55.4		1.00	

**Fig. 4.** Histogram of the differences bid/PTE for the 683 observed bids.**Fig. 5.** Histogram of the number of bids in each one of the 101 tenders analyzed.

this article, the construction companies that were involved in the collusive tendering have made full confessions relating to 27 of the 101 tenders implemented by the state-owned oil company. The method presented in this article successfully detected 26 (or 96%)

**Fig. 6.** Thresholds for different confidence levels and the results for every tender analyzed.

of these colluded tenders. Tender 69 in the Appendix is the only case when confessed collusion remained undetected by this method. This tender received four bids with differences to the PTE of −8.8%, 0.2%, 2.6%, and 12.9% respectively. The probability $P(x)$ for the set being observed by chance equals 0.059, lower than the 0.175 threshold for the 90% confidence level.

Fig. 7 provides a visualization of the bids within Tender 69 that did not depart significantly from the probabilistic distribution that was adopted to describe the honest tenders. If the oil company's pre-tender estimate was correct, this shows the known fact that the cartel can operate below the expected project costs, thereby resulting in the public experiencing a smaller discount than honest bids could offer.

The fact that Tender 69 has not been flagged does not mean that all tenders whose lowest bid is below the pre-tender estimate are cleared of any wrongdoing. As we can see, the method flags Tender 74 as colluding. In this case, the winning bid was also lower than the pre-tender estimate (the difference bid/PTE equals −0.5%) but, despite the circumstantial evidence, a confession has not yet been made. Fig. 8 illustrates that this flagging is due to the five other bids in the same tender, which were simultaneously above the pre-tender

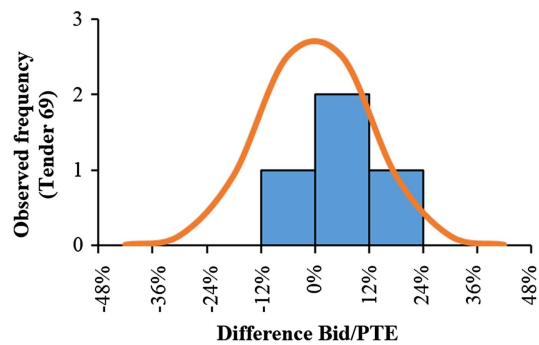


Fig. 7. Tender 69 graphical representation.

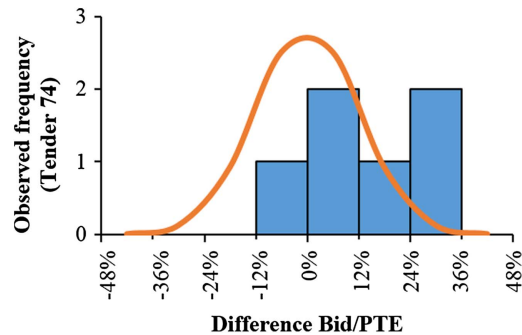


Fig. 8. Tender 74 graphical representation.

estimate in such a way that the imbalance was flagged as full collusion. The five cover bids exceeded the pre-tender estimate by 6.5%, 7.3%, 13.1%, 24.6%, and 32.9% respectively.

Discussion

The Appendix presents all the results of the analysis and shows that the probabilistic method presented in this study was able to correctly detect 96% of tenders where bidders confessed to collusion. Moreover, the findings of the investigation have identified that in six tenders where the Club of 16 contractors had colluded, they were not successful because outsiders' bids were presented (these tenders are referred to as *partly colluded*). Five of the six partly colluded tenders were identified as being cartelized at a 90% confidence level and four at 95% and 99% confidence levels (higher confidence levels naturally tend to rank the tenders as competitive). Additionally, the method flagged between 38% and 49% of the tenders where no confession was made and therefore they should be considered to be honest. At this point, the researchers need to point out that the column identified as Reality should be treated with caution as investigations are still ongoing. It is expected that further developments emerging from Operation Car Wash will lead to more confessions (or assumptions) of collusion, which tend to increase the accuracy of the method presented in this article. Nonetheless, in light of the confessions made by contractors that were involved with the collusive behavior, and the knowledge that has been acquired through objective science, the findings presented in this article can be considered to be reliable and valid.

The method presented here has the practical advantage of allowing any procurement authority to determine whether or a tender is being fully colluded at the time the bids are disclosed, and therefore dispensing with the need to examine the historical

bidding behavior of every competitor provided a but-for scenario is available (not a bid). However, as already emphasized, the accuracy of this probabilistic analysis is limited by two important assumptions that are also required by similar methods: (1) the probabilistic distribution adopted to describe the but-for scenario and (2) the validity of the pre-tender estimates.

It is important to emphasize that the requirement of a noninfringement scenario exists for any kind of tool proposed to detect abnormal bidding; only by knowing the honest bidding behavior can the dishonest be detected. Moreover, the normal distribution is usually adopted to describe this but-for scenario, which does not mean any kind of restriction to other distributions that may prove to be adequate to a particular case. In addition, the validity of the pre-tender estimate is a *conditio sine qua non* since Friedman's (1956) seminal study, which is not expected to change. In fact, an accurate, technically based pre-tender estimate (and not a careless political estimation) is also a condition for responsibly managing public money within a fair and healthy competitive environment as it can enable the completion of the desired works within budget and on time. Therefore, in every analyzed tender, the procurement authority must analyze whether the pre-tender estimate contribute to successful project outcomes. In the case of the state-owned oil company, its pre-tender estimates were not reviewed when competitors presented at least one bid within the -15% to $+20\%$ margin. Unfortunately, it was observed that in a few cases the pre-tender estimate was not reviewed at all even when it was clearly inaccurate, which can lead to punctual misflags of the method presented here.

To observe the model's sensitivity to these assumptions, even though the normal distribution with parameters $\mu = 0$, $\sigma = 0.12$ is deemed to be a robust estimator for ideal bids, the calculation of the probabilities was performed assuming that honest bids are described by a normal distribution with $\bar{x} = 0.08$ and $s = 0.185$ (Signor et al. 2017). Thus, even when it is admitted that the state-owned oil company's pre-tender estimate is generally below the honest bids' average, the probability method presented in this article was able to correctly detect 93% of the confessed cartels at 90% and 95% confidence levels. In addition, considering a greater variability of the bids, this percentage of detection could drop to 81% for the confidence level of 99%. Of the six partially cartelized tenders, a similar detection rate of 50% at the 90% and 95% confidence levels would occur and only one out of six would be flagged at the 99% confidence level. For those tenders without a confession of cartelization, 19%–31% were flagged. It is assumed that if the oil company had corrected its pre-tender estimate in all the analyzed tenders, the rate of adjustment of the method would be even better.

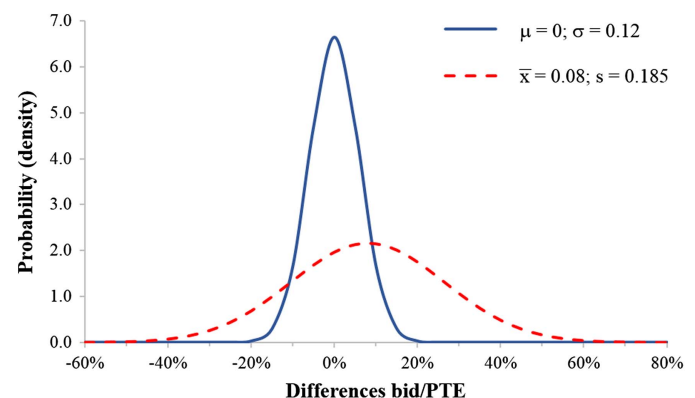


Fig. 9. PDFs of the two tested normal distributions.

When analyzing the changes in the detection rates for these two normal distributions ($\mu = 0$, $\sigma = 0.12$ or $\bar{x} = 0.08$, $s = 0.185$) representing different but-for scenarios, it can be observed that the adoption of larger values to mean and standard deviation lead to smaller detection ratios. This is expected as the increase in these parameters denotes a greater tolerance, reducing the possibility of classifying any tender as being fully colluded. Fig. 9 illustrates the PDF for these two possibilities, allowing us to anticipate that fewer bids would be classified as abnormally high according to the dashed curve.

Conclusions

The robust and practical probabilistic method that has been developed, based on data derived from Operation Car Wash, has the significant potential to be effectively used to detect collusion in sealed, uncapped tenders. The developed probabilistic method presents different probability distributions to describe the but-for scenario for a case being examined. The ability of the developed method to successfully identify the presence of collusion provides procurement authorities with the ability to apply it in practice, as it is straightforward to use and can be automated. Furthermore, the method can be used single-handedly or in conjunction with other existing approaches to overcome the malevolence that afflicts taxpayers worldwide, allowing procurement authorities to impose administrative penalties such as barring construction organizations from bidding for future works or applying other legal measures associated to public procurement. In addition to these practical applications, the method may also be used by law enforcement agencies as a subsidiary source of evidence. While in cases of criminal prosecution there is a consensus that proving collusion should consider detailed investigations, the proposed method can be used to assist the judges to dismiss the reasonable doubt, especially when collusive behavior by the same contractors is repeatedly flagged at high confidence levels.

Appendix. Results of Detecting Collusion during Bidding

Tender	Bids in the tender	Set probability $P(x)$	Confidence level			Reality
			90%	95%	99%	
1	11	6.42×10^{-1}	Collusion	Collusion	Collusion	—
2	4	5.24×10^{-2}	—	—	—	—
3	6	5.26×10^{-1}	Collusion	Collusion	Collusion	—
4	3	4.92×10^{-2}	—	—	—	—
5	7	1.38×10^{-2}	—	—	—	—
6	7	6.35×10^{-2}	Collusion	Collusion	—	—
7	6	9.07×10^{-8}	—	—	—	—
8	8	7.65×10^{-3}	—	—	—	—
9	5	1.16×10^{-1}	Collusion	—	—	—
10	4	1.86×10^{-3}	—	—	—	—
11	3	1.00×10^0	Collusion	Collusion	Collusion	—
12	3	3.68×10^{-1}	Collusion	—	—	—
13	4	9.99×10^{-1}	Collusion	Collusion	Collusion	—
14	5	4.18×10^{-2}	—	—	—	—
15	7	8.44×10^{-1}	Collusion	Collusion	Collusion	—
16	11	1.77×10^{-14}	—	—	—	—
17	4	6.38×10^{-1}	Collusion	Collusion	Collusion	—
18	21	2.10×10^{-8}	—	—	—	—
19	13	2.99×10^{-7}	—	—	—	—
20	11	3.45×10^{-7}	—	—	—	—
21	9	9.32×10^{-4}	—	—	—	—
22	19	2.46×10^{-9}	—	—	—	—

Appendix. (Continued.)

Tender	Bids in the tender	Set probability $P(x)$	Confidence level			Reality
			90%	95%	99%	
23	11	1.90×10^{-4}	—	—	—	—
24	7	1.66×10^{-2}	—	—	—	—
25	10	9.95×10^{-21}	—	—	—	—
26	4	5.36×10^{-1}	Collusion	Collusion	Collusion	—
27	15	2.01×10^{-1}	Collusion	Collusion	Collusion	—
28	5	9.99×10^{-1}	Collusion	Collusion	Collusion	Partly colluded
29	12	4.59×10^{-3}	Collusion	Collusion	Collusion	—
30	11	7.28×10^{-4}	—	—	—	—
31	6	6.36×10^{-3}	—	—	—	—
32	4	8.83×10^{-1}	Collusion	Collusion	Collusion	Collusion
33	12	7.98×10^{-3}	Collusion	Collusion	Collusion	—
34	4	1.00×10^0	Collusion	Collusion	Collusion	—
35	17	1.09×10^{-1}	Collusion	Collusion	Collusion	—
36	11	3.86×10^{-4}	—	—	—	—
37	6	7.07×10^{-2}	Collusion	—	—	Partly colluded
38	5	4.30×10^{-2}	—	—	—	—
39	11	1.21×10^{-2}	Collusion	Collusion	Collusion	—
40	11	2.77×10^{-12}	—	—	—	—
41	12	4.82×10^{-3}	Collusion	Collusion	Collusion	—
42	12	4.78×10^{-3}	Collusion	Collusion	Collusion	—
43	9	3.45×10^{-4}	—	—	—	—
44	8	1.99×10^{-1}	Collusion	Collusion	Collusion	—
45	5	8.12×10^{-2}	—	—	—	—
46	8	3.27×10^{-2}	Collusion	Collusion	—	—
47	21	5.72×10^{-3}	Collusion	Collusion	Collusion	—
48	7	3.86×10^{-2}	Collusion	Collusion	—	—
49	15	5.68×10^{-7}	—	—	—	—
50	14	8.51×10^{-12}	—	—	—	—
51	6	5.24×10^{-1}	Collusion	Collusion	Collusion	Partly colluded
52	3	2.86×10^{-1}	—	—	—	—
53	4	3.20×10^{-1}	Collusion	Collusion	—	—
54	3	6.92×10^{-1}	Collusion	Collusion	Collusion	—
55	10	3.15×10^{-5}	—	—	—	—
56	5	1.35×10^{-3}	—	—	—	—
57	5	7.38×10^{-1}	Collusion	Collusion	Collusion	Partly colluded
58	3	1.07×10^{-1}	—	—	—	—
59	7	1.57×10^{-7}	—	—	—	—
60	13	3.06×10^{-6}	—	—	—	—
61	9	2.93×10^{-4}	—	—	—	—
62	9	2.96×10^{-3}	—	—	—	Partly colluded
63	10	1.94×10^{-2}	Collusion	Collusion	Collusion	Partly colluded
64	4	7.95×10^{-1}	Collusion	Collusion	Collusion	—
65	4	8.72×10^{-1}	Collusion	Collusion	Collusion	—
66	3	3.02×10^{-2}	—	—	—	—
67	3	9.88×10^{-1}	Collusion	Collusion	Collusion	—
68	4	3.78×10^{-2}	—	—	—	—
69	4	5.89×10^{-2}	—	—	—	Collusion
70	4	9.93×10^{-1}	Collusion	Collusion	Collusion	Collusion
71	3	1.00×10^0	Collusion	Collusion	Collusion	Collusion
72	7	4.18×10^{-1}	Collusion	Collusion	Collusion	—
73	5	8.82×10^{-1}	Collusion	Collusion	Collusion	Collusion
74	6	2.09×10^{-1}	Collusion	Collusion	Collusion	—
75	4	9.97×10^{-1}	Collusion	Collusion	Collusion	—
76	7	2.76×10^{-2}	Collusion	—	—	—
77	3	1.00×10^0	Collusion	Collusion	Collusion	Collusion
78	3	9.94×10^{-1}	Collusion	Collusion	Collusion	Collusion
79	3	9.95×10^{-1}	Collusion	Collusion	Collusion	Collusion
80	5	9.90×10^{-1}	Collusion	Collusion	Collusion	—
81	3	9.96×10^{-1}	Collusion	Collusion	Collusion	Collusion

Appendix. (Continued.)

Tender	Bids in the tender	Set probability $P(x)$	Confidence level			Reality
			90%	95%	99%	
82	4	6.10×10^{-2}	—	—	—	—
83	4	9.99×10^{-1}	Collusion	Collusion	Collusion	Collusion
84	5	1.00×10^0	Collusion	Collusion	Collusion	Collusion
85	6	9.21×10^{-1}	Collusion	Collusion	Collusion	Collusion
86	4	1.00×10^0	Collusion	Collusion	Collusion	Collusion
87	3	9.77×10^{-1}	Collusion	Collusion	Collusion	Collusion
88	3	9.39×10^{-1}	Collusion	Collusion	Collusion	Collusion
89	3	9.17×10^{-1}	Collusion	Collusion	Collusion	—
90	3	1.00×10^0	Collusion	Collusion	Collusion	Collusion
91	5	9.12×10^{-1}	Collusion	Collusion	Collusion	Collusion
92	3	1.00×10^0	Collusion	Collusion	Collusion	Collusion
93	3	9.85×10^{-1}	Collusion	Collusion	Collusion	Collusion
94	3	7.13×10^{-1}	Collusion	Collusion	Collusion	Collusion
95	3	9.30×10^{-1}	Collusion	Collusion	Collusion	Collusion
96	4	1.00×10^0	Collusion	Collusion	Collusion	Collusion
97	3	1.00×10^0	Collusion	Collusion	Collusion	Collusion
98	6	7.59×10^{-1}	Collusion	Collusion	Collusion	Collusion
99	4	1.00×10^0	Collusion	Collusion	Collusion	Collusion
100	6	9.08×10^{-1}	Collusion	Collusion	Collusion	Collusion
101	2	1.00×10^0	Collusion	Collusion	Collusion	Collusion
Full collusion detection rate			96%	96%	96%	—
Partly collusion detection rate			83%	67%	67%	—
Still no confession, detected			49%	44%	38%	—
Still no confession, cleared			51%	56%	62%	—

Data Availability Statement

Data generated or analyzed during the study are available from the corresponding author by request. Information about the *Journal's* data-sharing policy can be found here: [https://ascelibrary.org/doi/10.1061/\(ASCE\)CO.1943-7862.0001263](https://ascelibrary.org/doi/10.1061/(ASCE)CO.1943-7862.0001263).

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Supplemental Data

The data set is available online in the ASCE Library (www.ascelibrary.org).

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