

2-D DOA estimation Using MUSIC algorithm with Uniform Circular Array

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Abstract— as one key technology in third generation communication, smart antenna can effectively increase system capacity and reduce co-channel interference for deployment and implementation of information and communication technologies in the Smart Cities. This article expounds in detail how to obtain Direction of Arrival (DOA) information using Multiple Signal Classification (MUSIC) algorithm with Uniform Circular Arrays (UCAs) based on theoretical analysis of receiving spatial signal by uniform circular arrays. A number of simulation results were carried using MATLAB and compared with the experimented one. The comparison show that proposed MUSIC algorithm is more accurate and stable.

Key words— MUSIC, DOA, UCA, Smart antenna, Smart Cities

I. INTRODUCTION

Smart antenna has been widely used in many applications such as radar, sonar and wireless communication systems. Considerable research efforts have been made to estimate the direction of arrival (DOA) and various array signal process techniques for DOA estimation have been proposed. The DOA estimation for uniform circular arrays (UCAs) has been developed in these scenarios, which desired all-azimuth angle coverage. By the virtue of their geometry, UCAs are able to provide 360° of coverage in azimuth plane. Moreover, they are known to be isotropic. That is, they can estimate the DOA of incident signal with uniform resolution in the azimuth plane. In addition, direction patterns synthesized with UCAs can be electronically rotated in the plane of the array without significant change of beam shape [1-4].

The smart cities need infrastructural development and housing plan by using smart technologies for the comfortable life of the people. While developing smart cities, there are many issues that need to be considered, such as population, culture, technology and growth. The main issue, however, is the high population of a city. It needs to analyze the impact of traffic and health monitoring systems. GPS has limitations such as selective availability and anti-spoofing. Instead of using GPS, an IRNSS constellation can be used to improve the quality of service wider availability across the regions. [2]

DOA estimation uses antenna arrays. It is known that antenna radiation main lobe beam width is inversely

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proportional to the number of elements in antenna. So, if we consider a single antenna then array pattern will be wider and the resolution cannot be good. Instead of using single antenna, an antenna array system is used in DOA estimation which will improve the resolution of the received signals (Resolution in DOA estimation is the ability to distinguish two signals arriving at different angles). An array system has a multiple elements distributed in space.

There are various methods to estimate the angle of arrival (DOA) of radio signals on the antenna array. DOA estimation techniques can be broadly divided into three different categories namely; conventional methods subspace based methods and maximum likelihood methods. Convolution methods are based on the concepts of beam forming and null steering, but it requires a large number of elements to provide high resolution. Examples of this method are delay and sum and Capon's minimum variance method [3].

One major limitation of this method is poor resolution that is its ability to separate closely spaced signals. Unlike conventional methods, subspace methods exploit the information of the received data resulting in high resolution. Two main subspace based algorithms are Multiple Signal Classification and Estimation of Signal Parameters via Rotational Invariance Techniques.

The DOA algorithms are classified as quadratic (non subspace) type and subspace type. The Bartlett and Capon (Minimum Variance Distortion less Response) [5] are quadratic type algorithms. Both methods are highly dependent on physical size of array aperture, which results in poor resolution and accuracy. Subspace based DOA estimation method is based on the eigen decomposition. The subspace based DOA estimation algorithms MUSIC and ESPRIT provide high resolution; they are more accurate and not limited to physical size of array aperture.

These algorithms give information about number of incident signals and DOA of each signal. Maximum likelihood method is one of the first techniques to be investigated for DOA estimation but has the drawback of intensive computational complexity [4-6].

The DOA estimation technology is focused on high resolution estimation algorithm. In multiple DOA estimation

algorithms, a promising method for smart antenna array is MUSIC algorithm [7, 8, 9], which uses multiple signal classification technique based on exploiting the eigen structure of the input covariance matrix. It is very important that MUSIC algorithm for uniform circular arrays smart antenna is modeled and simulated.

This article expounds in detail how to obtain DOA information using a modified MUSIC algorithm through reconstruction data matrix with conjugate data. The comparison between the proposed method and the experimented one show that the proposed method can solve the problem of coherent signals and estimate clearly the direction of arrival.

II. SYSTEM MODEL AND THE PRINCIPLE OF MUSIC ALGORITHM

We assume that there are N uniform circular array, M narrow band far field signals from different incident direction [10-11]. The radius of the circular array is denoted as r and wavelength of narrow band is denoted as λ . The incident angle of the signals is shown in Fig.1.

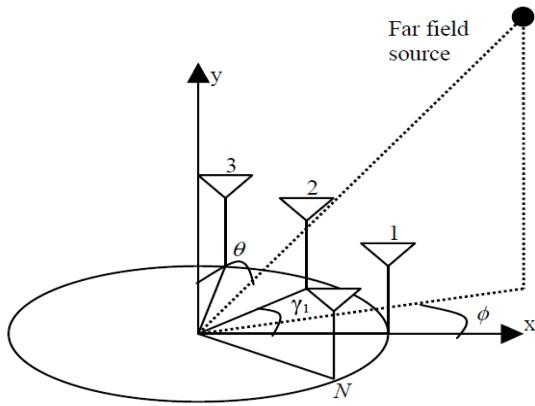


Fig.1. Uniform circular array geometry.

The received array signal can

$$X(t) = AS(t) + N(t) \quad (1)$$

Where $A=[a(\theta_1, \phi_1), \dots, a(\theta_M, \phi_M)]$ is a matrix of the M steering vectors, which represents the possible value set of DOA and $a(\theta_1, \phi_1) \left[e^{j2\pi \frac{r}{\lambda} \sin \Phi_m \cos \theta_m}, \dots, e^{j2\pi \frac{r}{\lambda} \sin \Phi_m \cos \frac{2\pi(N-1)}{N}} \right]$ and $S=[S_1(t), \dots, S_M(t)]^T$ is a signal source vector of size ($M \times 1$). Where the statistical expectation is denoted by $E[\cdot]$ and $[\cdot]^T$ denote transpose of signal vector .

The correlation matrix of received vector can be written as:

$$\begin{aligned} R_X &= E[XX^H] \\ &= E[ASS^H A^H] + E[WW^H] \\ &= AVA^H + \sigma^2 I \\ &= R_S + \sigma^2 I \end{aligned} \quad (2)$$

Where σ^2 is the variance of white Gaussian noise vector (W), V is covariance matrix of signal vector (S) which is a full rank matrix of order $M \times M$ given by,

$$\begin{aligned} V &= E[SS^H] \\ &= \begin{bmatrix} E[|S_1|^2] & \dots & \dots & 0 \\ 0 & E[|S_2|^2] & \dots & 0 \\ \vdots & \ddots & \dots & \vdots \\ 0 & 0 & \dots & E[|S_M|^2] \end{bmatrix} \end{aligned} \quad (3)$$

R_S is a signal covariance matrix of order ($N \times N$) with rank M given by:

$$R_S = \begin{bmatrix} E[|S_1|^2] & \dots & \dots & 0 & \dots & 0 \\ 0 & E[|S_2|^2] & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \dots & \vdots & \dots & 0 \\ 0 & 0 & \dots & E[|S_M|^2] & \dots & 0 \\ 0 & 0 & \dots & 0 & \dots & 0 \end{bmatrix} \quad (4)$$

So R_S , has $N-M$ eigenvectors corresponding to zero eigen values. We know that steering vector $a(\theta_i, \phi_i)$ which is in the signal subspace is orthogonal to noise subspace let Q_n be such an eigenvector.

$$R_S Q_n = AVA^H Q_n = 0 \quad (5)$$

Since V is a positive definite matrix:

$$a^H(\theta_i, \phi_i) Q_n = 0 \quad (6)$$

This implies that signal steering vectors are orthogonal to eigen vector corresponding to noise subspace. So the MUSIC algorithm searches through all angles and plots the spatial spectrum:

$$P_{MUSIC}(\theta, \phi) = \frac{1}{(a^H(\theta, \phi) Q_n Q_n^H a(\theta, \phi))} \quad (7)$$

III. THE PROPOSED ALGORITHM

In the modified algorithm, we will reconstruct the data matrix, let

$$Y = TX^* \quad (8)$$

The '*' represents complex conjugate, T is an N order inverse identity matrix which is called transition matrix. The covariance matrix of the data Y is

$$R_Y = T R_X^* T \quad (9)$$

We introduce a new array covariance matrix, which is the sum of R_Y and R_X

$$\begin{aligned} R &= R_Y + R_X \\ &= AR_S A^* + T[A R_S A]^* T + 2\sigma^2 I \end{aligned} \quad (10)$$

According to matrix theory, if q is an eigenvector corresponding to a zero eigen value of matrix $AR_S A$, then q must also be an eigenvector correspond to the zero eigen value of matrix $T[A R_S A]^* T$. We observe that matrix R_X , R_Y and R have the same noise subspace. By performing eigen value decomposition with R , we get its eigen values and its eigen vectors. According to the estimated number of signal sources,

the noise subspace among the eigen vectors can be distinguished. With the new noise subspace, we can construct MUSIC spatial spectrum

$$P_{\text{MUSIC}}(\theta, \varphi) = \frac{1}{(A(\theta, \varphi)^H q_n q_n^H A(\theta, \varphi))} \quad (11)$$

IV. SIMULATION AND COMPARATIVE RESULTS

In the following step, the simulation results of the proposed MUSIC algorithm investigated at this research work was compared with the experimented one [12, 13, 14]. The comparison was made in the same condition: a uniform circle array (UCA) with five antennas, radius $r=124$ mm. the radiation source is pulse signal and the distance between the radiating antenna and the direction finder receiving antenna approximately is 8 m. The carrier frequency is 6 GHz. And according to the estimated received signal, the receiving data SNR is above 20 dB according to [12], a 4x4 planar antenna array with 0.5λ element spacing for [13] and UCA with 8 antenna element, 2 source and noise = 12 dB, with BPSK modulation , the search step of MUSIC is 0.1° and the noise intensity is -12dB according to [14] .

In the simulation showed in Fig.2, we note that the proposed can resolve clearly the azimuth elevation ($133.6^\circ, 137.8^\circ$) and ($78.6^\circ, 82.4^\circ$) respectively and the peaks are sharp, while the Music only fond one peak around there. To confirm the first simulation result, another simulation showed in Fig.3, using unequal power signal arriving at azimuth and elevation are ($128.4, 116$) and ($78, 84$), so the Fig.3 confirm that the proposed Music algorithm can resolve clearly the angles(θ, φ) and the peaks become Sharp.

From the simulation results shown in Fig.2 and Fig.3, both of the two algorithms can get a correct estimation of the direction angle of independent signals. Because of reconstructing the data covariance matrix in the modified algorithm, which is equivalent to utilize the information of the data one more time, the peak of spectrum becomes sharper and the precision is higher compared to the results shown in [12].

Fig.4 shows that angles have the same values but the proposed method presents a good magnitude for angles ($99.48, 50.13$); ($64.88, 15.1$), the magnitude is 40.15 dB, 38.84dB respectively. We resolved that the proposed method can detect, estimate the DOA clearly and present an efficient magnitude with +0.24% for angles ($99.48, 50.13$) and +0.02% for angles ($64.88, 15.1$) compared to result given in [13].

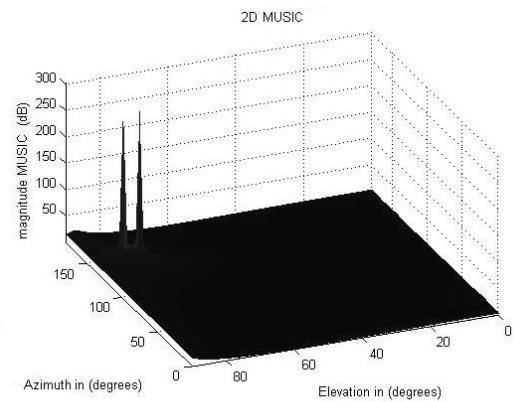


Fig.2. Proposed method simulation for azimuth and elevation ($133.6, 137.8$); ($78.6, 82.4$)

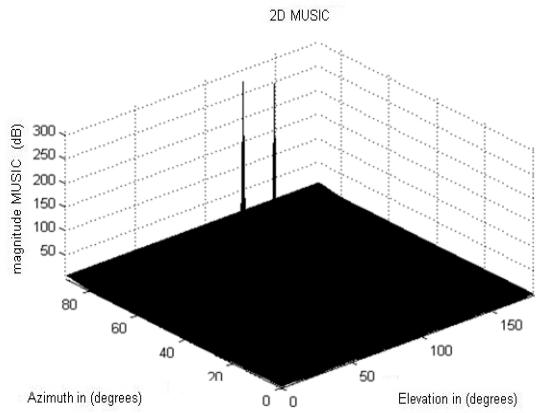


Fig.3. Confirmation simulation for azimuth and elevation ($128.4, 116$); ($78, 84$)

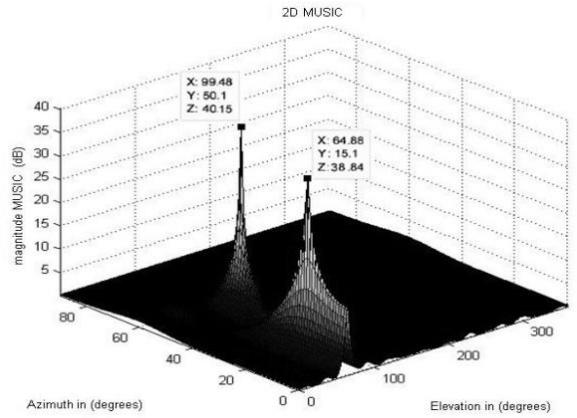


Fig.4. Proposed algorithm for azimuth and elevation ($99.48, 50.13$); ($64.88, 15.1$)

Table1 illustrate the results of the comparison between proposed method and Music method indicate at [12]. It can be seen that the proposed method estimates 3 DOA more accurately while the experimented one cannot detect angles when the Number of signals exceeds 2. The proposed one gives a less error margin to estimate DOA.

TABLE.1.COMPARATIVE RESULT FOR DIFFERENT SIGNALS

Signal		θ_{in}	θ_{out}	$\Delta\theta_{out}$	φ_{in}	φ_{out}	$\Delta\varphi_{out}$
1	[12]	78 84	78.5 82.5	+0.0064 0	128.4 116.0	129.5 117	+0.0082 +0.0086
	Pm	78 84	78 84	0 0	128.4 116.0	128.1 116.2	-0.0023 +0.0017
	[12]	77.0 85.8	76.0 86.5	-0.0065 0	128.2 120	129.5 121.5	77.0 85.8
	Pm	77.0 85.8	77.0 85.8	0 0	128.2 120	128.2 120.1	00 +0.0008
2	[12]	78.6 82.4	0 0	-1 -1	133.6 137.8	0 0	-1 -1
	Pm	78.6 82.4	79.0 85.4	+0.005 +0.036	133.6 137.8	134 137	+0.0029 -0.0028
	[12]	78.6 82.4	0 0	-1 -1	133.6 137.8	0 0	-1 -1
	Pm	78.6 82.4	79.0 85.4	+0.005 +0.036	133.6 137.8	134 137	+0.0029 -0.0028
3	[12]	78.6 82.4	0 0	-1 -1	133.6 137.8	0 0	-1 -1
	Pm	78.6 82.4	79.0 85.4	+0.005 +0.036	133.6 137.8	134 137	+0.0029 -0.0028

TABLE.2.COMPARATIVE RESULT FOR DIFFERENT RADIUS

Radius	Pm	[14]	Error (deg) [14]	Error (deg) Pm
0.1 λ	57.100000	57.100002	+0.10002	+0.10002
0.5 λ	57.100004	57.100004	+1.00004	+1.00004
0.8 λ	56.900000	56.900002	-1.90002	-1
1.0 λ	57.000000	57.000004	+0.00004	0

- Pm: is the proposed method.

Table.2 shows that even if the values of number radius change, the proposed method give a higher precision and value of peaks, contrary to experimented one[14], it cannot detect all angles even if number of signals increases. Results indicate that proposed method MUSIC based on UCA does not have a problem of aperture vagueness.

V. CONCLUSION

Modern society has a firm foundation of science and technology, which plays a key role for economic growth. This study has introduced the 2-D DOA estimation algorithm for Uniform circular array; we have put forward a modified MUSIC algorithm using the method of reconstructing the data

covariance matrix to reduce the algorithm takes full advantage of geometrical norm of UCA, the simulation results and the correlations among the incident signals the proposed comparison have testified the effectiveness of this algorithm. This method provides a new access to estimate DOA.

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