MATHEMATICAL PRINCIPLES FOR COMPUTER SCIENCE: ASSIGNMENT 1

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Question 1

1.1.1) $8+6\div2\times(3+5)-4 = 8+6\div2\times(8)-4$
$= 8+6\div2\times8-4$
$= 8+3\times8-4$
$= 8+24-4$
$= 32-4$
$= 28$

1.1.2) $8+6\div2\times(3+5)-4 = 14\div2\times(3+5)-4$
$= 7\times(3+5)-4$
$= 7\times(8)-4$
$= 7\times8-4$
$= 56-4$
$= 52$

∴ The result in question 1.1.1) (28) is correct. I evaluated the expression using a different order as I solved the expression from left to right and did not use PEMDAS. I got 52 as the result of using a different order compared to the result of 28 I got from evaluating the expression using PEMDAS.

1.2) $\frac{2^3 . 4^{-1} . 8^2}{16^{-1} . 32^{\frac{1}{5}}} = \frac{2^3 . (2^2)^{-1} . (2^3)^2}{(2^4)^{-1} . (2^5)^{\frac{1}{5}}}$
$= \frac{2^3 . 2^{-2} . 2^6}{2^{-4} . 2^1}$
$= \frac{2^{3+(-2)+6}}{2^{-4+1}}$
$= \frac{2^7}{2^{-3}}$
$= 2^{7-(-3)}$
$= 2^{7+3}$
$= 2^{10}$

Question 2

2.1) $\dfrac{4x-3}{5} + \dfrac{5x+9}{4} = \dfrac{2x}{3} + \dfrac{15}{2} - \dfrac{7x}{6}$

$$\frac{12(4x-3)+15(5x+9)}{60} = \frac{20(2x)+30(15)-10(7x)}{60}$$

$$\frac{48x-36+75x+135}{60} = \frac{40x+450-70x}{60}$$

$$(60)\times\frac{123x+99}{60} = \frac{-30x+450}{60}\times(60)$$

$$123x+99 = -30x+450$$
$$153x = 351$$
$$x = \frac{351}{153} = \frac{39}{17}$$

2.2) $|5-4x| \leq 15$

if $5-4x > 0$ OR if $5-4x < 0$

$5-4x \leq 15$ | $-(5-4x) \leq 15$

$-10 \leq 4x$ | $-5+4x \leq 15$

$-\dfrac{5}{2} \leq x$ | $4x \leq 20$

$x \leq 5$

$\therefore \; -\dfrac{5}{2} \leq x \leq 5$

2.3) Let x = first number
Let y = second number

① $xy = 60$

② $(2)\,\dfrac{x+y}{2} = 8\,(2)$

$x+y = 16$

$y = 16-x$

$$② \rightarrow ①$$

$$xy = 60$$
$$x(16 - x) = 60$$

$$16x - x^2 = 60$$

$$x^2 - 16x + 60 = 0$$

$$(x - 10)(x - 6) = 0$$

$\therefore\ x - 10 = 0$ OR $x - 6 = 0$

③ $\rightarrow x = 10$ ④ $\rightarrow x = 6$

$\therefore$ if $x = 10$ OR if $x = 6$

③ $\rightarrow$ ① ④ $\rightarrow$ ①

$xy = 60$ $xy = 60$

$(10)y = 60$ $(6)y = 60$

$10y = 60$ $6y = 60$

$y = 6$ $y = 10$

$\therefore$ The two numbers are 6 and 10.

2.4) $\sqrt[3]{-8a^3b^6} \times \sqrt[4]{16x^8y^{12}}$

$$= \left(\sqrt[3]{-2^3}\ a^{\frac{3}{3}}\ b^{\frac{6}{3}}\right) \times \left(\sqrt[4]{4^2}\ x^{\frac{8}{4}}\ y^{\frac{12}{4}}\right)$$
$$= \left(\sqrt[3]{-8}\ ab^2\right) \times \left(\sqrt[4]{16}\ x^2y^3\right)$$
$$= (-2ab^2) \times (2x^2y^3)$$
$$= (-2)(2) \times ab^2 \times x^2y^3$$
$$= -4ab^2x^2y^3$$

Explanation:

- For the odd root, I simplified the cube root and got the coefficient of -2 as the cube root of -8 is -2 ($[-2]^3 = -8$). To simplify the variable part of the odd root, I used fractional exponents as they are equivalent to roots: $\rightarrow \sqrt[3]{a^3} = a^{\frac{3}{3}} = a$
$\rightarrow \sqrt[3]{b^6} = b^{\frac{6}{3}} = b^2$

$\therefore$ for the odd root, I got a simplified answer of $-2ab^2$

• For the even root, I simplified the fourth root and got the coefficient of 4 as the fourth root of $16 = 2$ $([2]^4 = 16)$. To simplify the variable part of the even root, I used fractional exponents as they are equivalent to roots : $\rightarrow \sqrt[4]{x^8} = x^{\frac{8}{4}} = x^2$

$\rightarrow \sqrt[4]{y^{12}} = y^{\frac{12}{4}} = y^3$

$\therefore$ For the even root, I got a simplified answer of $2x^2y^3$

• I multiplied the simplified even and odd roots together and have $-4ab^2x^2y^3$ as the final simplified expression.

2.5) $3 + \sqrt{4x-5} = 8 \longrightarrow ①$

$$\sqrt{4x-5} = 5$$
$$\left(\sqrt{4x-5}\right)^2 = (5)^2$$
$$4x - 5 = 25$$
$$4x = 30$$
$$x = \frac{15}{2} \longrightarrow ②$$

Verification of Solution :

② $\longrightarrow$ ①

$$3 + \sqrt{4\left(\tfrac{15}{2}\right) - 5} = 8$$
$$3 + \sqrt{30 - 5} = 8$$
$$3 + \sqrt{25} = 8$$
$$3 + 5 = 8$$
$$8 = 8$$

$\therefore$ $8 = 8$ is true, thus $x = \frac{15}{2}$ is the correct solution.

Question 3

3.1) Atomic Statement:

According to TutorialsPoint (n.d.), "An atomic statement is a simple, indivisible statement.

An example of an atomic statement in the context of programming would be:
isLoggedIn = true;

↳ This is an atomic statement because it only does one thing: it sets a variable called isLoggedIn to true. There is no combination of multiple statements happening here and no logical connectives are being used. It is just a single, straightforward statement that cannot be broken down any further and it does not include any additional conditions or actions. (stack Overflow, 2013).

Molecular statement:

According to TutorialsPoint (n.d.), molecular statements are " formed by combining two or more atomic statements using logical connectives. "

An example of a molecular statement in the context of programming would be:
while (balance > 0 && itemsInCart > 0)

↳ This is a molecular statement as it joins two different conditions together using a logical connective. One condition checks if the balance is still positive and the other one checks if there are items left in the cart. The && ($\wedge$: logical connective AND) operator joins these two conditions into one bigger statement, making it a molecular statement. (TutorialsPoint, n.d.).

3.2.1) $B = \{2,4,6\}$ $\quad C = \{1,2,3\}$

$\therefore B \cup C = \{1,2,3,4,6\}$

$U = \{1,2,\ldots,10\}$

$\therefore (B \cup C)^c = U \setminus (B \cup C) = \{5,7,8,9,10\}$

$\therefore (B \cup C)^c = \{5,7,8,9,10\}$

3.2.2) $A = \{1,2,3,4,5,6\}$ $\quad B = \{2,4,6\}$

$\therefore A \setminus B = \{1,3,5\}$

3.2.3) $D = \{7, 8, 9\}$ $\quad C = \{1, 2, 3\}$ $\quad U = \{1, 2, \dots, 10\}$

$\therefore C^c = U \setminus C = \{4, 5, 6, 7, 8, 9, 10\}$

$\therefore (D \cap C^c) = \{7, 8, 9\}$

$A = \{1, 2, 3, 4, 5, 6\}$ $\quad B = \{2, 4, 6\}$

$\therefore (A \cap B) = \{2, 4, 6)$

$U = \{1, 2 \dots, 10\}$

$\therefore (A \cap B)^c = U \setminus (A \cap B) = \{1, 3, 5, 7, 8, 9, 10\}$

$\therefore (D \cap C^c) \cup (A \cap B)^c = \{1, 3, 5, 7, 8, 9, 10\}$

3.3) $(c_n): 0, 1, 3, 7, 15, 31, \dots$

$+1 \quad +2 \quad +4 \quad +8 \quad +16$ } Powers of two

$2^0 \quad 2^1 \quad 2^2 \quad 2^3$

for first difference:

$a_n = 2^n$

$a(0) = 2^0 = 1$
$a(1) = 2^1 = 2$
$a(2) = 2^2 = 4$
$a(3) = 2^3 = 8$
$a(4) = 2^4 = 16$

} a_n matches the first difference and shows they are Powers of 2 and that this is a geometric sequence

$\therefore a_n$ can be used to find C_n

$C(0) = 0$
$C(1) = 1$
$C(2) = 3$
$C(3) = 7$
$C(4) = 15$

$C(0) = 0 \xrightarrow{+1} a(0) = 1$
$C(1) = 1 \xrightarrow{+1} a(1) = 2$
$C(2) = 3 \xrightarrow{+1} a(2) = 4$
$C(3) = 7 \xrightarrow{+1} a(3) = 8$
$C(4) = 15 \xrightarrow{+1} a(4) = 16$

$\therefore$ One is added to C_n to get to a_n

$\therefore C_n + 1 = a_n$

$C_n = a_n - 1$

$C_n = 2^n - 1$

$$T_n = \frac{n(n+1)}{2}$$

$$T(0) = \frac{0(0+1)}{2} = 0$$

$$T(1) = \frac{1(1+1)}{2} = 1$$

$$T(2) = \frac{2(2+1)}{2} = 3$$

$$T(3) = \frac{3(3+1)}{2} = 6$$

} $T_n: 0, 1, 3, 6$ does not match any of the differences or the sequence as the sequence is a geometric sequence $\therefore T_n$ cannot be used to find C_n.

$\therefore$ The closed formula for the sequence: $(c_n): 0, 1, 3, 7, 15, 31, \dots$ is :

$C_n = 2^n - 1$ for $n \geq 0$

Verification of the closed formula for the sequence:

$C_n = 2^n - 1$ for $n \geq 0$

$C(0) = 2^0 - 1 = 0$

$C(1) = 2^1 - 1 = 1$

$C(2) = 2^2 - 1 = 3$

$C(3) = 2^3 - 1 = 7$

$C(4) = 2^4 - 1 = 15$

$C(5) = 2^5 - 1 = 31$

$\therefore$ After substituting the n values into the closed formula, the answers match the values in the sequence : (c_n): 0, 1, 3, 7, 15, 31, ...

$\therefore$ The closed formula for the sequence is $C_n = 2^n - 1$ for $n \geq 0$

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