北京邮电大学 2020——2021 学年第 二 学期

《 离散数学(上) 》期中考试试题

考试注意事项	一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。 二、书本、参考资料、书包等物品一律放到考场指定位置。 三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。 四、学生必须将答题内容做在试题答卷上，做在试题及草稿纸上一律无效。

考试课程	离散数学(上)	考试时间	2021 年 4 月 25 日

题号	一	二	三	四	五	六	七	八	九	十	十一	十二	十三	十四	总分
满分	10	5	10	5	6	5	6	8	10	8	4	8	8	7	
得分															
阅卷教师															

姓名：　班内序号：　学号：　班级：　专业：

装　订　线　封　密

1. [10 points] Which of these are propositions? What are the truth values of those that are proposition?

   1) Discrete mathematics is a required course in the Department of computer science.

   2) There is no maximum prime.

   3) This year marks the 100th anniversary of the founding of the Communist Party of China (CPC).

4) $9+5<13$

5) No littering!

2. [5 points] Let P and Q be the propositions

P : It is below freezing.

Q : It is snowing.

Write these propositions using P and Q and logical connectives (including negations).

1) It is below freezing and snowing.

2) It is not below freezing and it is not snowing.

3) It is either snowing or below freezing (or both).

4) If it is below freezing, it is also snowing.

5) That it is below freezing is necessary and sufficient for it to be snowing.

3. [10 points] Construct a truth table for each of these compound propositions.

a) $(p\wedge\neg q)\vee(r\wedge q)\rightarrow r$

b) $((\neg p \to (p \land \neg q)) \to r) \lor (q \land \neg r)$

4. [5 points] Put the statement in major disjunctive normal form:

$$(P \land Q) \lor (\neg P \land R) \lor (Q \land R)$$

5. [6 points] In the questions below suppose the variable x represents students and y represents courses, and:

$U(y)$: y is an upper-level course $M(y)$: y is a math course $F(x)$: x is a freshman

$A(x)$: x is a part-time student $T(x,y)$: student x is taking course y.

Write the statement using these predicates and any needed quantifiers.

a) Every student is taking at least one course.

b) There is a part-time student who is not taking any math course.

c) Every part-time freshman is taking some upper-level course.

6. [5 points] Put the statement in prenex normal form
$((\forall x)P(x) \vee (\exists y)Q(y)) \rightarrow (\forall x)R(x)$

7. [6 points] Determine whether the following two propositions are logically equivalent:

(1) $\exists x \forall y (A(x) \rightarrow B(y))$ and $\forall x A(x) \rightarrow \forall y B(y)$ [ ]

(2) $\exists x (P(x) \wedge Q(x))$ and $\exists x P(x) \wedge \exists x Q(x)$ [ ]

(3) $\neg \forall x (P(x) \rightarrow Q(x))$ and $\exists x (P(x) \wedge \neg Q(x))$ [ ]

8. [8 points] In the questions below suppose the variable x represents students, $F(x)$ means "x is a freshman", and $M(x)$ means "x is a math major". Match the statement in symbols with one of the English statements in this list:

(1) Some freshmen are math majors.

(2) Every math major is a freshman.

(3) No math major is a freshman.

a) $\forall x(M(x) \rightarrow \neg F(x))$. [ ]

b) $\neg \exists x(M(x) \wedge \neg F(x))$. [ ]

c) $\forall x(F(x) \rightarrow \neg M(x))$. [ ]

d) $\exists x(F(x) \wedge M(x))$. [ ]

e) $\forall x(M(x) \rightarrow F(x))$. [ ]

f) $\neg \forall x(\neg F(x) \vee \neg M(x))$. [ ]

g) $\forall x(\neg(M(x) \wedge \neg F(x)))$. [ ]

h) $\forall x(\neg M(x) \vee \neg F(x))$. [ ]

9. [10 points] In the questions below suppose $P(x,y)$ is a predicate and the universe for the variables x and y is {1,2,3}. Suppose $P(1,3)$, $P(2,1)$, $P(2,2)$, $P(2,3)$, $P(3,1)$, $P(3,2)$ are true, and $P(x,y)$ is false otherwise. Determine whether the following statements are true.

a) $\forall x \exists y P(x,y)$. [ ]

b) $\exists x \forall y P(x,y)$. [ ]

c) $\neg \exists x \exists y (P(x,y) \wedge \neg P(y,x))$. [ ]

d) $\forall y \exists x (P(x,y) \rightarrow P(y,x))$. [ ]

e) $\forall x \forall y (x \neq y \rightarrow (P(x,y) \vee P(y,x))$. [ ]

10. [8 points] Show that the arguments. If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.

11. [4 points] Are these propositions consistent?

Rainy days make gardens grow.
Gardens don't grow if it is not hot.
It always rains on a day if it is not hot.

12. [8 points] Hypotheses: Everyone in the class has a graphing calculator. Everyone who has a graphing calculator understands the trigonometric functions. Conclusion: Ralphie, who is in the class, understands the trigonometric functions. Give an argument using the rules of inference to show that conclusion from the hypotheses.

13. [8 points] Prove or disprove that

1) If $a+b$ is irrational, then a or b is irrational.

2) If $a+b$ is irrational, then a and b are irrational.

14. [7 points] Let $a \$ b = \min\{a, b\} = a$ if $a \leqslant b$,

otherwise $a \$ b = \min\{a, b\} = b$.

Show that for all real numbers a, b, c, $(a \$ b) \$ c = a \$ (b \$ c)$.