Kolmogorov Smirnov Test for Testing Population Means

Final Project ST541

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Project Objectives

- Is the Kolmogorov Smirnov (KS) test asymptotically exact for testing the means of two distributions?

- Is the Kolmogorov Smirnov (KS) test asymptotically consistent for testing the means of two distributions?

Kolmogorov Smirnov Test

o Testing two sample distributions

    o Null Hypothesis: $H_0: F_Y = F_X$

o Test Statistics : $D = \sup_y |\hat{F}_Y(y) - \hat{F}_X(x)|$

o $\sqrt{\dfrac{mn}{m+n}} D \rightarrow^d K$
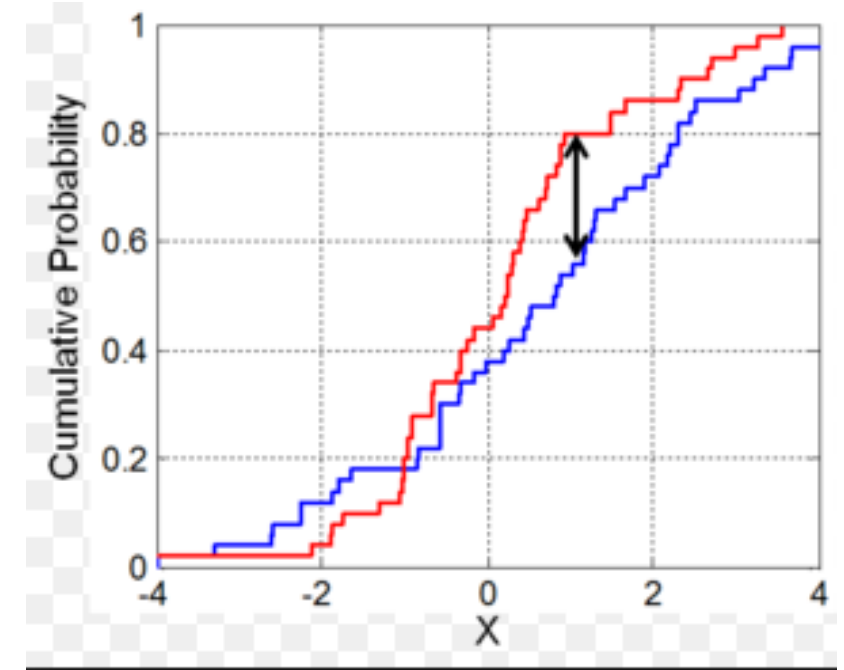
First Question (Testing the Exactness)

o Null hypothesis: $\mu_Y = \mu_X$

o Distributions with the population mean of 0.5:

1. $Normal(\mu = \frac{1}{2}, \sigma^2 = 1)$
2. $exponential(\lambda = 2)$
3. $beta(\alpha = 2, \beta = 2)$
4. $gamma\left(\alpha = 2, \beta = \frac{1}{4}\right)$

rejection_rate_10	rejection_rate_50	rejection_rate_100	dist1	dist2
0.042	0.885	1	normal	exponential
0.089	0.999	1	normal	beta
0.063	0.951	1	normal	gamma
0.049	0.658	0.951	exponential	beta
0.015	0.218	0.425	exponential	gamma
0.021	0.199	0.366	beta	gamma

Second Question (Testing Consistency)

o Null hypothesis: $\mu_Y = \mu_X$

o Distributions:

1. $Normal\left(\mu = \frac{1}{2}, \sigma^2 = 1\right)$, $Population\ mean = \frac{1}{2}$
2. $exponential(\lambda = 1)$, $Population\ mean = 1$
3. $beta(\alpha = 2, \beta = 1)$, $Population\ mean = \frac{2}{3}$
4. $gamma(\alpha = 2, \beta = 1)$, $Population\ mean = 2$

rejection_rate_10	rejection_rate_50	rejection_rate_100	dist1	dist2
0.037	0.777	0.997	normal	exponential
0.095	0.998	1	normal	beta
0.353	1	1	normal	gamma
0.066	0.98	1	exponential	beta
0.215	0.984	1	exponential	gamma
0.816	1	1	beta	gamma

Conclusion

o KS test is not asymptotically exact for testing the means

o KS test is asymptotically consistent for testing the means

Thank You