STA 610L: Review

Random vectors and matrices

Dr. Olanrewaju Michael Akande

Random vector

Suppose

$$\mathbf{Y} = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix}$$

is a vector of random variables with $E(Y_i) = \mu_i$, $\mathrm{Var}(Y_i) = \sigma_{ii}$, and $\mathrm{Cov}(Y_i, Y_j) = \sigma_{ij}$.

STA 610L

Expectation of a vector

The expectation of the random vector $\mathbf{Y}$ is defined

$$E(\mathbf{Y}) = \begin{pmatrix} E(Y_1) \\ E(Y_2) \\ \vdots \\ E(Y_n) \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix} = \mu.$$

Expectation of a matrix

Suppose $\mathbf{Z}$ is an $(n \times p)$ matrix of random variables. Then

$$E(\mathbf{Z}) = \begin{pmatrix} E(Z_{11}) & \cdots & E(Z_{1p}) \\ \vdots & \ldots & \vdots \\ E(Z_{n1}) & \cdots & E(Z_{np}) \end{pmatrix}.$$

Thus the expectation of a random matrix is the matrix of the expectations.

STA 610L

Covariance

For $\mathbf{Y}$ an $(n \times 1)$ random vector, the *covariance matrix* of $\mathbf{Y}$ is defined as

$$\begin{aligned}
\mathrm{Cov}(\mathbf{Y}) &= E\left[(\mathbf{Y}-\mu)(\mathbf{Y}-\mu)'\right] \\
&= \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{n1} & \cdots & \cdots & \sigma_{nn} \end{pmatrix},
\end{aligned}$$

where $\sigma_{ij} = E[(Y_i - \mu_i)(Y_j - \mu_j)]$, $i, j = 1, \ldots, n$.

STA 610L

Linear combinations

Suppose $\mathbf{Y}_{n\times 1}$ is a random vector with mean $\mu = E(\mathbf{Y})$ and covariance matrix $\Sigma = \mathrm{Cov}(\mathbf{Y})$.

In addition, suppose $\mathbf{A}_{r\times n}$ is a matrix of constants and $\mathbf{b}_{r\times 1}$ is a vector of constants.

Then

$$E(\mathbf{AY} + \mathbf{b}) = \mathbf{A}E(\mathbf{Y}) + \mathbf{b} = \mathbf{A}\mu + \mathbf{b}$$

and

$$\mathrm{Cov}(\mathbf{AY} + \mathbf{b}) = \mathbf{A}\mathrm{Cov}(\mathbf{Y})\mathbf{A}' = \mathbf{A}\Sigma\mathbf{A}'.$$

Linear combinations

Let $\mathbf{W}_{r\times 1}$ be a random vector with $E(\mathbf{W}) = \gamma$. Then

$$\mathrm{Cov}(\mathbf{W}, \mathbf{Y}) = E\left[(\mathbf{W} - \gamma)(\mathbf{Y} - \mu)'\right],$$

where $\mathrm{Cov}(\mathbf{W}, \mathbf{Y})$ is an $(r \times n)$ matrix of covariances with ij^{th} element equal to $\mathrm{Cov}(W_i, Y_j)$.

STA 610L

What's next?

Move on to the readings for the next module!

STA 610L