

HW5 - Individual Theory

Theory (work individually) - you may use LaTeX or write out neatly by hand.

1. (6 points) For the linear model, assume that the X have been centered so that they all have mean 0. For the linear model

$$Y \sim N(1_n\beta_0 + X\beta, I_n/\phi)$$

using Zellner's g -prior for β with

$$\beta \mid \beta_0, \phi \sim N(0, g(X^T X)^{-1}/\phi)$$

and the improper independent Jeffrey's prior

$$p(\beta_0, \phi) \propto 1/\phi$$

find the a) posterior distribution of $\beta \mid Y, g, \phi$, b) posterior distribution of $\mu_i = x_i^T \beta \mid Y, g, \phi$ and c) the posterior predictive distribution of $Y^{test} \mid Y, g, \phi$ as functions of the OLS/MLE summaries. (*you may use results in notes - just quote - or derive*)

2. (6 points) What are the corresponding distributions from above unconditional on ϕ ? (hint recall theorem from class) Are β_0 and β still independent? Explain.
3. (3 points) Let $\tau = 1/g$ and substitute that in the prior for β

$$\beta \mid \beta_0, \phi \sim N(0, (X^T X)^{-1}/(\tau\phi))$$

If $\tau \sim G(1/2, n/2)$, show that the prior on β is a Cauchy Distribution

$$\beta \mid \phi, \beta_0 \sim C(0, (X^T X/n)^{-1}/\phi)$$

(*a Cauchy distribution is a Student t with 1 df - see notes for density*)