HW5 - Theory

Theory (work individually) - you may use LaTeX or write out neatly by hand.

1. (3 points) For the linear model, assume that the X have been centered so that they all have mean 0. For the linear model
$$Y \sim N(1_n\beta_0 + X\beta, I_n/\phi)$$
using Zellner's g-prior for β with
$$\beta \mid \beta_0, \phi \sim N(0, g(X^TX)^{-1}/\phi)$$
and the improper independent Jeffrey's prior
$$p(\beta_0, \phi) \propto 1/\phi$$
find the a) posterior distriubtion of $\beta \mid Y, g, \phi$, b) posterior distribution of $\mu_i = x_i^T\beta \mid Y, g, \phi$ and c) the posterior predictive distribution of $Y^{test} \mid Y, g, \phi$ as functions of the OLS/MLE summaries. *(you may use results in notes - just quote - or derive)*

2. (1 point) What are the corresponding distributions from above unconditional on ϕ? (hint recall theorem from class) Are β_0 and β still independent? Explain.

3. (1 point) Let $\tau = 1/g$ and substitute that in the prior for β
$$\beta \mid \beta_0, \phi \sim N(0, (X^TX)^{-1}/(\tau\phi))$$
If $\tau \sim G(1/2, n/2)$, show that the prior on β is a Cauchy Distribution
$$\beta \mid \phi, \beta_0 \sim C(0, (X^TX/n)^{-1}/\phi)$$
(a Cauchy distribution is a Student t with 1 df - see notes for density)