Regression Diagnostics

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Outline

- Leverage

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- Leverage
- Standardized Residuals

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- Leverage
- Standardized Residuals
- Outlier Test

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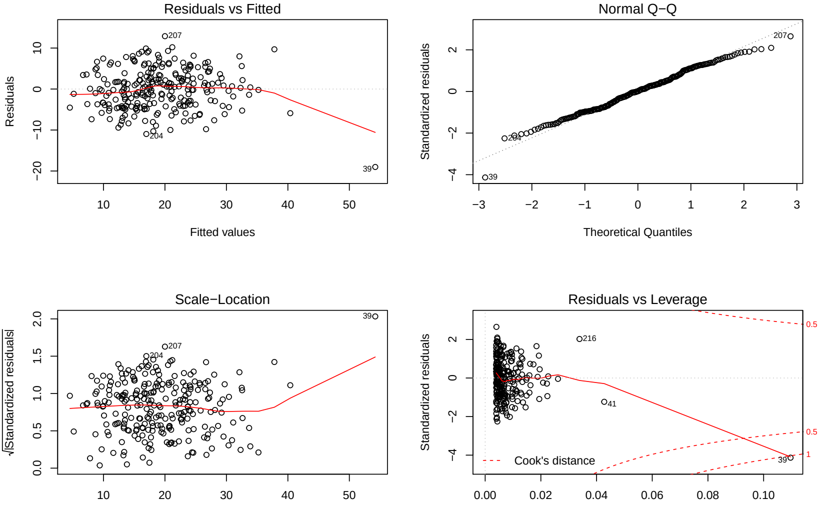
- Standardized Residuals
- Outlier Test
- Cook's Distance

Residual Plots

```r
bodyfat.lm = lm(Bodyfat ~ Abdomen, data=bodyfat)
par(mfrow=c(2,2))
plot(bodyfat.lm, ask=F)
```



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- standardized residuals versus leverage with contours of Cook's distance: shows influential points where points greater than 1 or $4/n$ are considered influential
- Case 39 appears to be influential and have a large standardized residual!

Hat Matrix

- predictions

$$\hat{\mathbf{Y}} = \mathbf{X}\hat{\beta} = \mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}$$

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- Leverage: measure of how far x_i is from center of data

$$h_{ii} = 1/n + (\mathbf{x}_i - \bar{\mathbf{x}})^T \left((\mathbf{X} - \mathbf{1}\bar{\mathbf{x}}^T)^T(\mathbf{X} - \mathbf{1}\bar{\mathbf{x}}^T)\right)^{-1}(\mathbf{x}_i - \bar{\mathbf{x}})$$

Residual Analysis

- residuals

$$\mathbf{e} = \mathbf{Y} - \hat{\mathbf{Y}} = (\mathbf{I} - \mathbf{H})\mathbf{Y}$$

$$\mathrm{var}(e_i) = \hat{\sigma}^2(1 - h_{ii})$$

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- if leverage is near 1 then residual is near 0 and variance is near 0 and r_i is approximately 0 (may not be helpful)

Predicted Residual

- Estimates without Case (i):

$$\hat{\beta}_{(i)} = (\mathbf{X}_{(i)}^T \mathbf{X}_{(i)})^{-1} \mathbf{X}_{(i)}^T \mathbf{Y}_{(i)}$$
$$= \hat{\beta} - \frac{(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{x}_i e_i}{1 - h_{ii}}$$

*Standardized predicted residual is

$$\frac{e_{(i)}}{\sqrt{\mathrm{var}(e_{(i)})}} = \frac{e_i/(1-h_{ii})}{\hat{\sigma}/\sqrt{1-h_{ii}}} = \frac{e_i}{\hat{\sigma}\sqrt{1-h_{ii}}}$$

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$$\mathrm{var}(e_{(i)}) = \frac{\sigma^2}{1 - h_{ii}}$$

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- Externally Standardized residuals

$$t_i = \frac{e_{(i)}}{\sqrt{\hat{\sigma}^2_{(i)}/(1 - h_{ii})}} = \frac{y_i - \mathbf{x}_i^T \hat{\beta}_{(i)}}{\sqrt{\hat{\sigma}^2_{(i)}/(1 - h_{ii})}} = r_i \left(\frac{n - p - 1}{n - p - r_i^2} \right)^{1/2}$$

Distribution of Externally Standardized Residuals

$$t_i = \frac{e_{(i)}}{\sqrt{\hat{\sigma}^2_{(i)}/(1-h_{ii})}} = \frac{y_i - \mathbf{x}_i^T\hat{\beta}_{(i)}}{\sqrt{\hat{\sigma}^2_{(i)}/(1-h_{ii})}} \sim \mathrm{St}(n-p-1)$$

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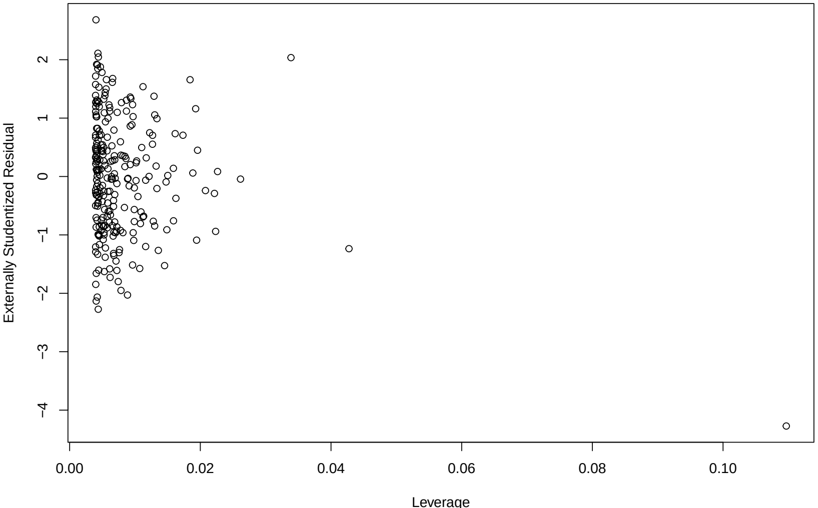
- Can extend to include multiple δ_i and δ_j to test that case i and j are both outliers
- Extreme case $\boldsymbol{\mu} = \mathbf{X}\beta + \mathbf{I}_n \boldsymbol{\alpha}$ all points have their own mean!

R Code

```r
plot(rstudent(bodyfat.lm) ~ hatvalues(bodyfat.lm),
     ylab="Externally Studentized Residual",
     xlab="Leverage")
```



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- P-value for test that observation with largest studentized residual is an outlier

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- Bonferroni compares p-values to α / n

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$$\begin{aligned}\text{FWER} &= P\left\{\bigcup_{i=1}^{n_0}\left(p_i \leq \frac{\alpha}{n}\right)\right\} \leq \sum_{i=1}^{n_0}\left\{P\left(p_i \leq \frac{\alpha}{n}\right)\right\} \leq n_0\frac{\alpha}{n} \leq n\frac{\alpha}{n} \\ &= \alpha\end{aligned}$$

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- Link https://en.wikipedia.org/wiki/Bonferroni_correction

Bonferroni Correction

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pval= 2*(1- pt(abs.ti, bodyfat.lm$df - 1))
min(pval) < .05/nrow(bodyfat)
```

```
## [1] TRUE
```

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sum(pval < .05/nrow(bodyfat))
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- Bonferroni multiplicity adjustment compare each p-value to α/n and reject null (point is not an outlier) if the p-value is less than α/n
- Start with max absolute value of t_i (or min p-value)
- Case 39 would be considered an outlier based on Bonferroni or other multiplicity adjustments. no other outliers

Cook's Distance

- Measure of influence of case i on predictions

$$D_i = \frac{\|\mathbf{Y} - \hat{\mathbf{Y}}_{(i)}\|^2}{\hat{\sigma}^2\, p}$$

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- Easier way to calculate

$$D_i = \frac{e_i^2}{\hat{\sigma}^2 p} \left[\frac{h_{ii}}{(1 - h_{ii})^2} \right],$$

$$D_i = \frac{r_{ii}}{p} \frac{h_{ii}}{1 - h_{ii}}$$

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- Impact on predictions?

Predictions with Case 39

```r
predict(bodyfat.lm, newdata=bodyfat[39,],
        se=T,  interval="prediction")
```

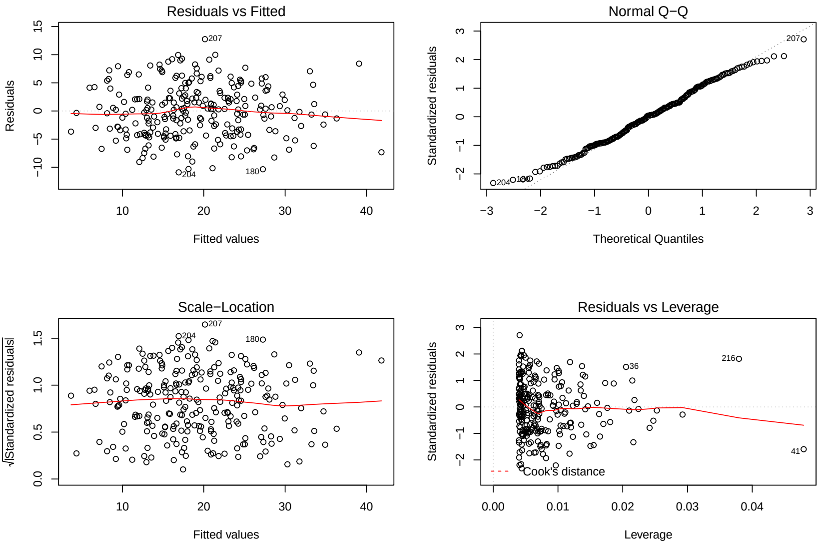
```
## $fit
##         fit     lwr      upr
## 39 54.21599 44.0967 64.33528
##
## $se.fit
## [1] 1.615311
##
## $df
## [1] 250
##
## $residual.scale
## [1] 4.877484
```

Predictions without Case 39

```
bodyfatsub.lm = lm(Bodyfat ~ Abdomen, data=bodyfat,
                   subset=c(-39))
predict(bodyfatsub.lm, newdata=bodyfat[39,],
        se=T, interval="prediction")
```

```
## $fit
##         fit      lwr      upr
## 39 56.55856 46.71172 66.40541
##
## $se.fit
## [1] 1.655744
##
## $df
## [1] 249
##
## $residual.scale
## [1] 4.717441
```

Residual Checks without Case 39



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- Next: Transformations