HW2 STA521 Fall18

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Due September 23, 2018 5pm

Backgound Reading

Readings: Chapters 3-4 in Weisberg Applied Linear Regression

This exercise involves the UN data set from `alr3` package. Install `alr3` and the `car` packages and load the data to answer the following questions adding your code in the code chunks. Please add appropriate code to the chunks to suppress messages and warnings as needed once you are sure the code is working properly and remove instructions if no longer needed. Figures should have informative captions. Please switch the output to pdf for your final version to upload to Sakai. **Remove these instructions for final submission**

Exploratory Data Analysis

0. Preliminary read in the data. After testing, modify the code chunk so that output, messages and warnings are suppressed. *Exclude text from final*

```
library(alr3)
```

```
## Loading required package: car
```

```
## Loading required package: carData
```

```
##
## Attaching package: 'car'
```

```
## The following object is masked from 'package:dplyr':
##
##     recode
```

```
data(UN3, package="alr3")
help(UN3)
library(car)
```

1. Create a summary of the data. How many variables have missing data? Which are quantitative and which are qualtitative?

```
#Summary of Data
summary(UN3)
```

```
##     ModernC          Change           PPgdp           Frate
##  Min.   : 1.00   Min.   :-1.100   Min.   :   90   Min.   : 2.00
##  1st Qu.:19.00   1st Qu.: 0.580   1st Qu.:  479   1st Qu.:39.50
##  Median :40.50   Median : 1.400   Median : 2046   Median :49.00
##  Mean   :38.72   Mean   : 1.418   Mean   : 6527   Mean   :48.31
##  3rd Qu.:55.00   3rd Qu.: 2.270   3rd Qu.: 8461   3rd Qu.:58.00
##  Max.   :83.00   Max.   : 4.170   Max.   :44579   Max.   :91.00
##  NA's   :58      NA's   :1        NA's   :9       NA's   :43
##       Pop             Fertility         Purban
##  Min.   :      2.3   Min.   :1.000   Min.   :  6.00
##  1st Qu.:    767.2   1st Qu.:1.897   1st Qu.: 36.25
##  Median :   5469.5   Median :2.700   Median : 57.00
```

```
##  Mean   :  30281.9   Mean   :3.214   Mean   : 56.20
##  3rd Qu.:  18913.5   3rd Qu.:4.395   3rd Qu.: 75.00
##  Max.   :1304196.0   Max.   :8.000   Max.   :100.00
##  NA's   :2           NA's   :10
```

```
#Additional Missing Data
help(UN3)

#Data Type of Predictors
sapply(UN3, class)
```

```
##    ModernC     Change      PPgdp      Frate        Pop Fertility     Purban
## "integer" "numeric" "integer" "integer" "numeric" "numeric" "integer"
```

According to the summary function, there are 58 data missing from ModernC, 1 missing from Change, 9 missing from PPgdp, 43 from Frate, 2 missing from Pop, 10 missing from Fertility, and 0 missing from Purban.

All the predictors are quantitative, according to the description of each predictor from R.

2. What is the mean and standard deviation of each quantitative predictor? Provide in a nicely formatted table.

```
#Calculating the Mean and SD, w/ NA's Removed from Predictors
kable(sapply(UN3, mean, na.rm = TRUE), format = "markdown", col.names = "Mean")
```

	Mean
ModernC	38.717105
Change	1.418373
PPgdp	6527.388060
Frate	48.305389
Pop	30281.871428
Fertility	3.214000
Purban	56.200000

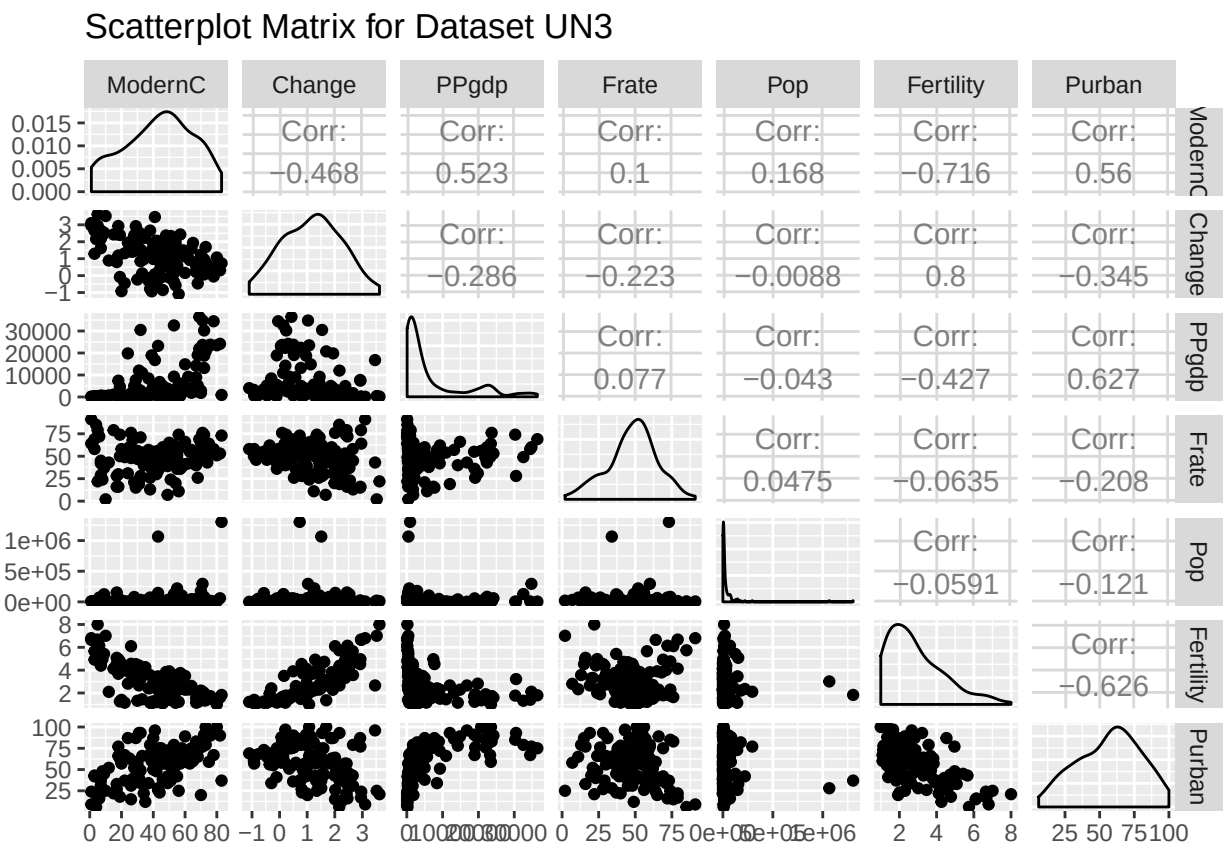
```
kable(sapply(UN3, sd, na.rm = TRUE), format = "markdown", col.names = "Standard Deviation")
```

	Standard Deviation
ModernC	2.263661e+01
Change	1.133133e+00
PPgdp	9.325189e+03
Frate	1.653245e+01
Pop	1.206767e+05
Fertility	1.706918e+00
Purban	2.410976e+01

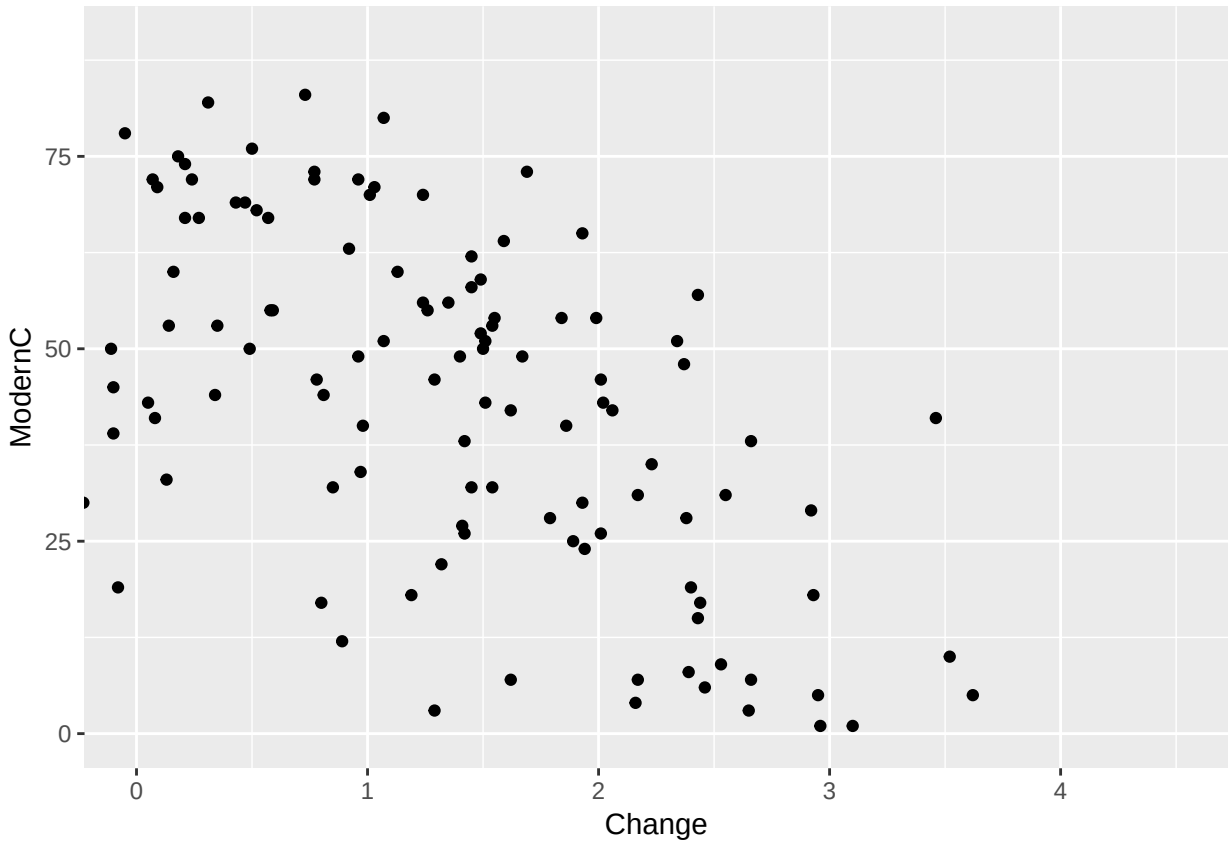
3. Investigate the predictors graphically, using scatterplots or other tools of your choice. Create some plots highlighting the relationships among the predictors. Comment on your findings regarding trying to predict `ModernC` from the other variables. Are there potential outliers, nonlinear relationships or transformations that appear to be needed based on your graphical EDA?

```
#Removing NA's from Data
UN3_No_NA = na.omit(UN3)

#Predictors
ModernC = UN3_No_NA$ModernC
```

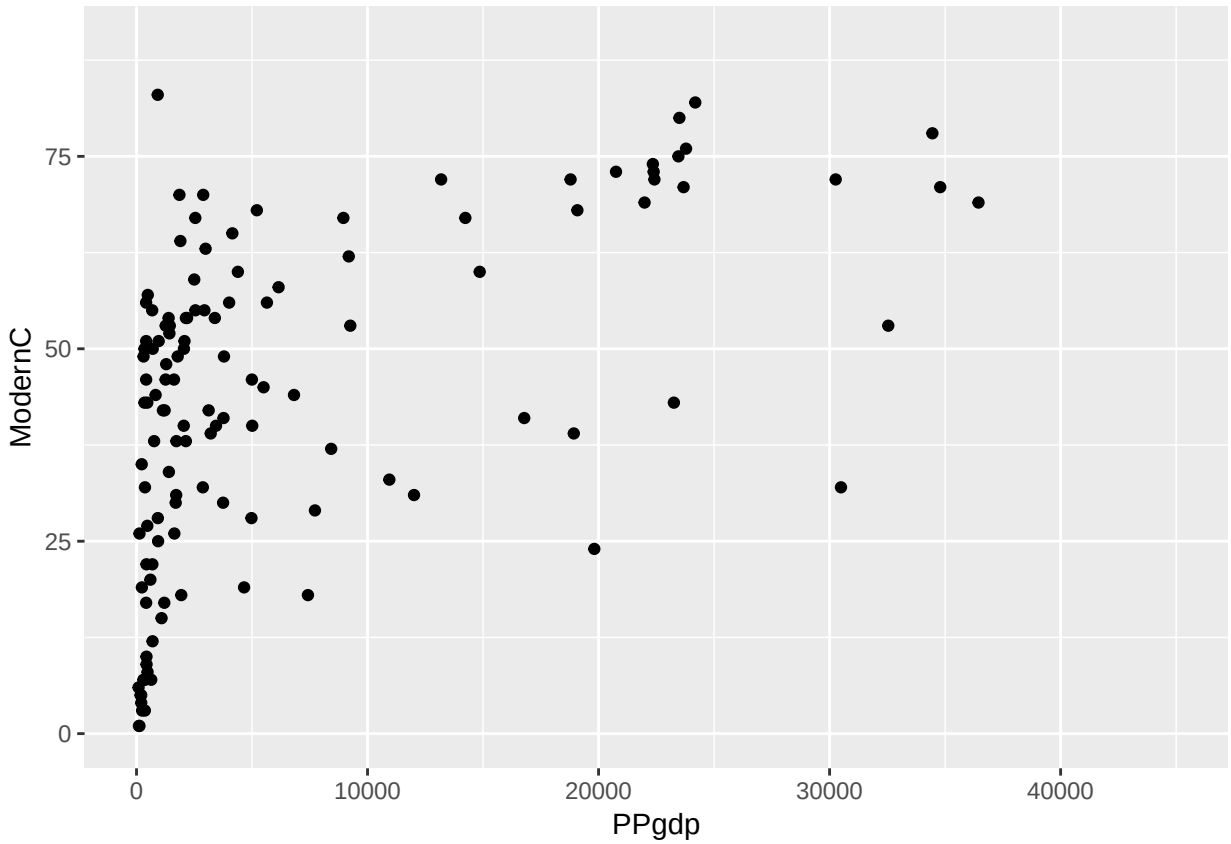
```
Change = UN3_No_NA$Change
PPgdp = UN3_No_NA$PPgdp
Frate = UN3_No_NA$Frate
Pop = UN3_No_NA$Pop
Fertility = UN3_No_NA$Fertility
Purban = UN3_No_NA$Purban

#Plots
ggpairs(UN3_No_NA, progress = FALSE, title = "Scatterplot Matrix for Dataset UN3")
```



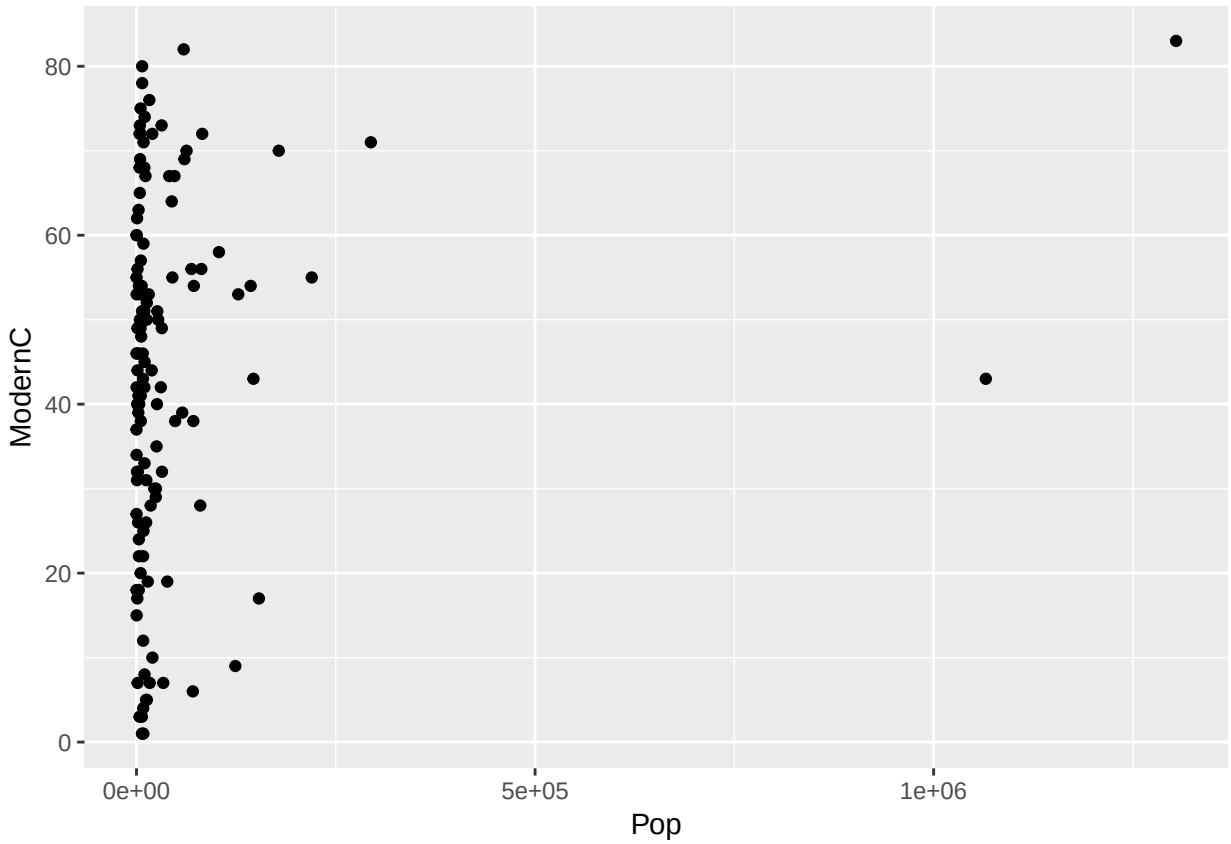

```
ggplot(UN3_No_NA, aes(Change, ModernC)) + geom_point() + coord_cartesian(xlim = c(0,4.5), ylim = c(0,90)
```



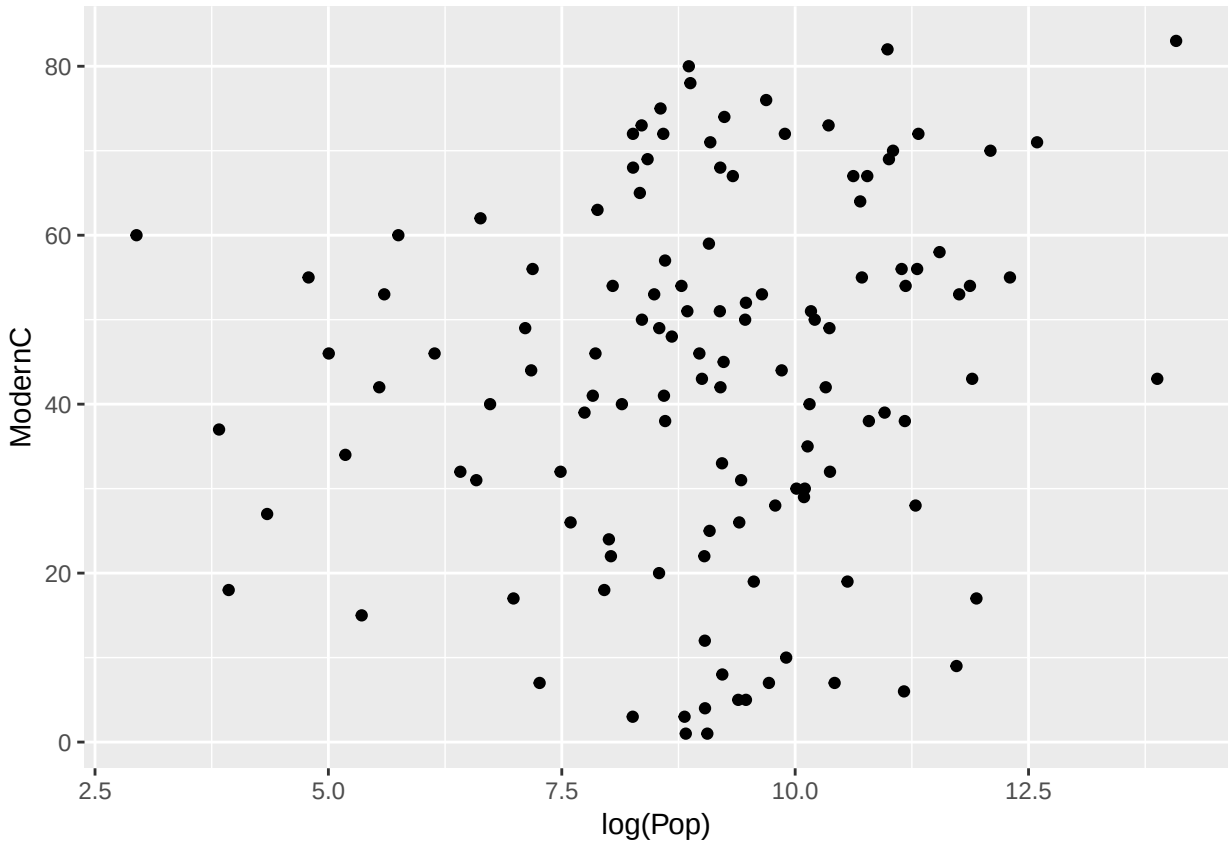

```
ggplot(UN3_No_NA, aes(PPgdp, ModernC)) + geom_point() + coord_cartesian(xlim = c(0,45000), ylim = c(0,9
```




```
ggplot(UN3_No_NA, aes(Pop, ModernC)) + geom_point()
```




```
ggplot(UN3_No_NA, aes(log(Pop), ModernC)) + geom_point()
```



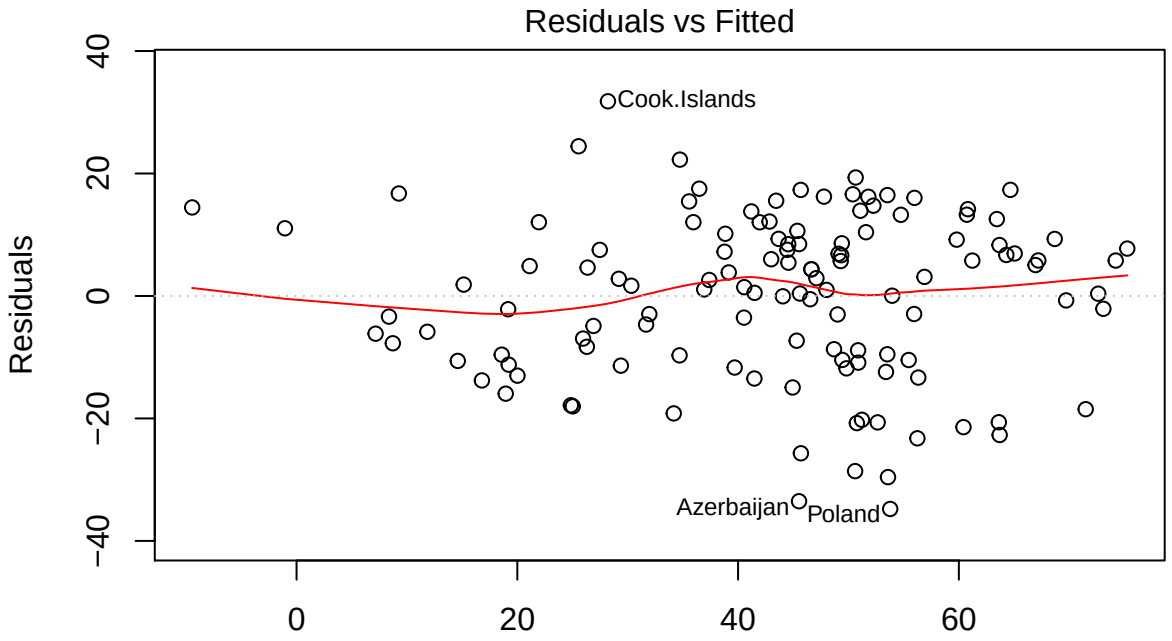
By visual inspection, I was not able to identify potential outliers. However, I was able to observe that ModernC and PPgdp, PPgdp and Fertility, and PPgdp and Purban all have noticeable non-linear relationships, which may demand some transformations.

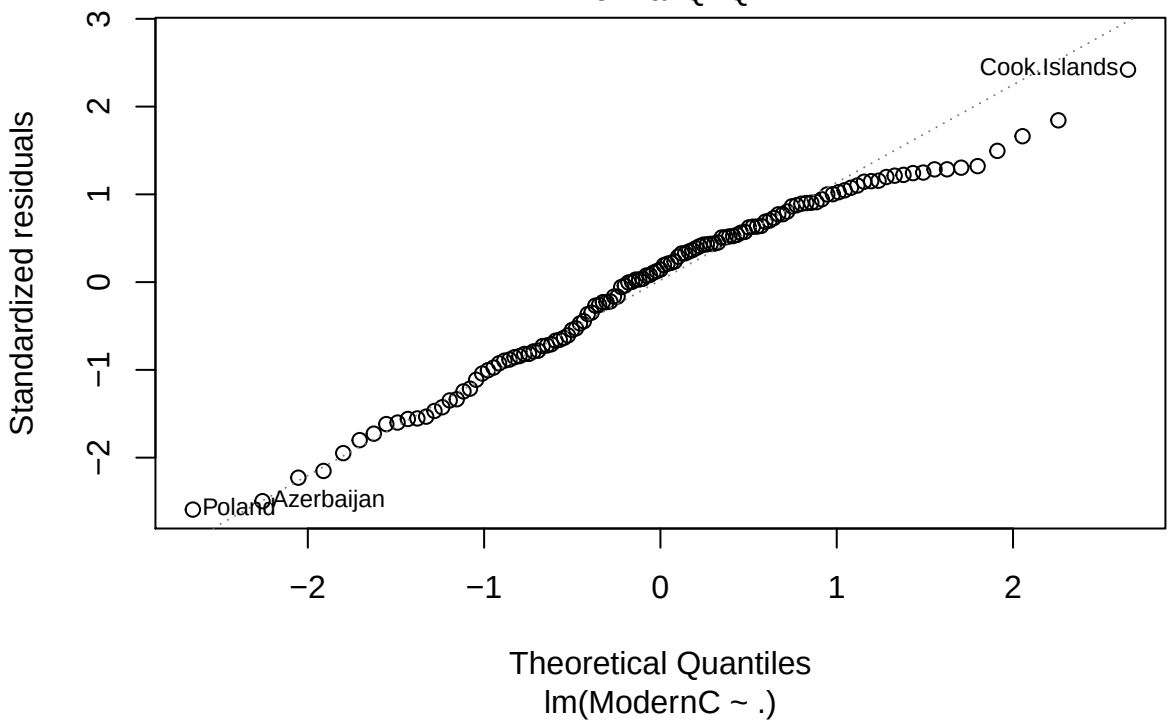
As for the findings regarding predicting "ModernC" using other variables, I concluded that the predictor "Fertility" would be the best pre-transformed variable to predict the response, since "Fertility" has the most linear relationship, among the predictors, with "ModernC." I also concluded that "PPgdp" would be the worst pre-transformed variable to predict the response, since "PPgdp" has the most noticeable non-linear relationship, among the predictors, with "ModernC."

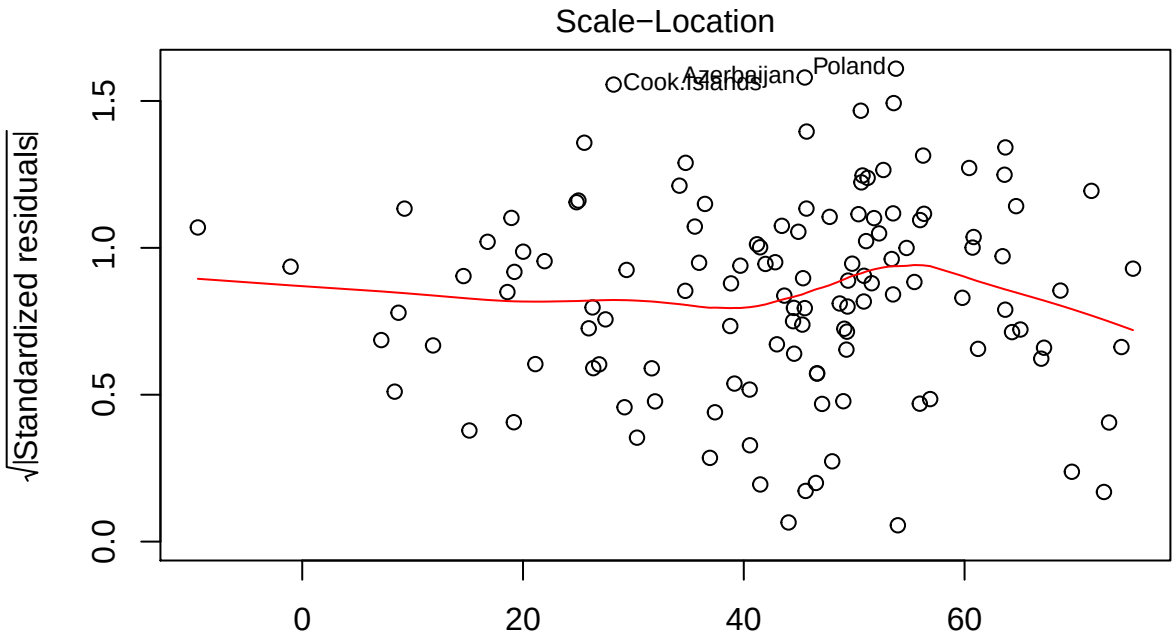
Model Fitting

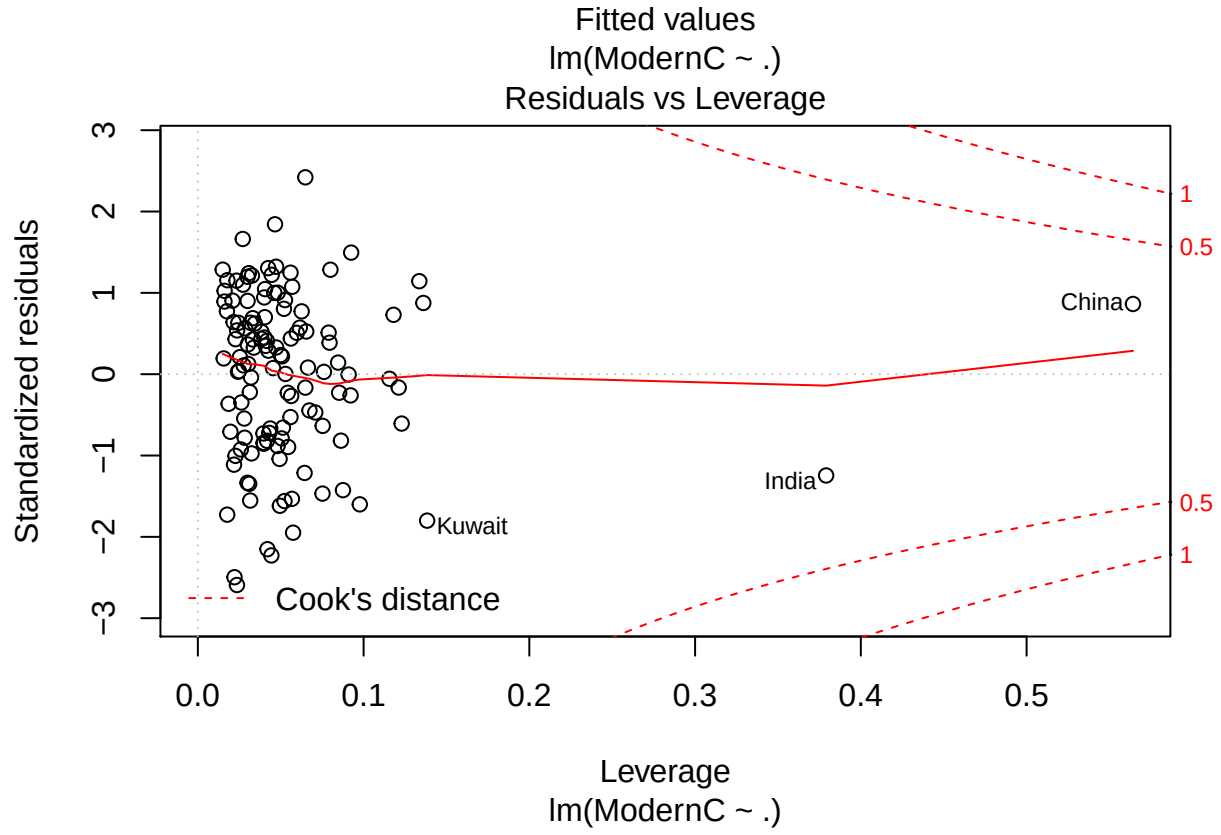
4. Use the `lm()` function to perform a multiple linear regression with `ModernC` as the response and all other variables as the predictors, using the formula `ModernC ~ .`, where the `.` includes all remaining variables in the dataframe. Create diagnostic residual plot from the linear model object and comment on results regarding assumptions. How many observations are used in your model fitting?

```
#Multiplie Regression
LM_ModernC = lm(ModernC ~ ., data = UN3_No_NA)

#Multiple Regression Plot
plot(LM_ModernC)
```


Residuals vs Fitted
Residuals
Fitted values
lm(ModernC ~ .)
Cook.Islands
Azerbaijan
Poland
40
20
0
−20
−40
0
20
40
60


Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(ModernC ~ .)
Cook.Islands
Azerbaijan
Poland
3
2
1
0
−1
−2
−2
−1
0
1
2





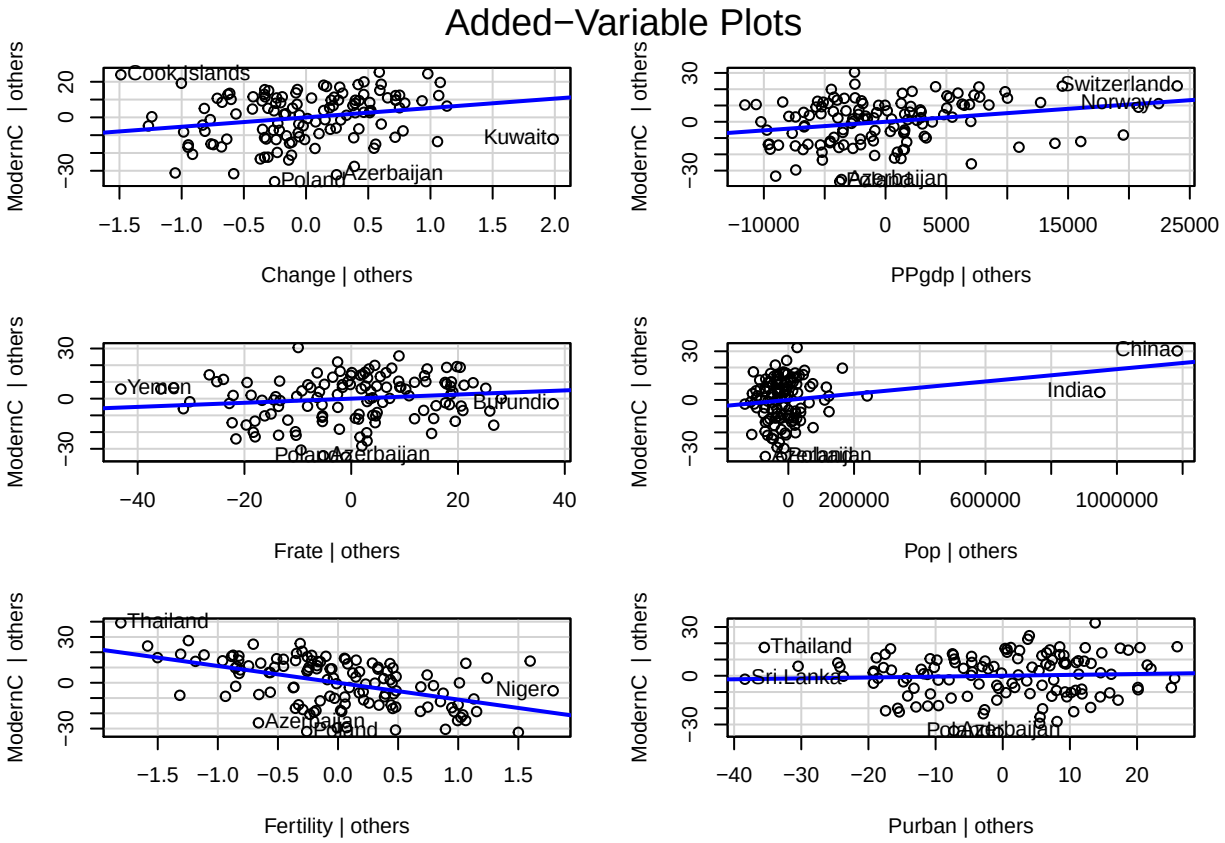

```
#Observations Used in Multiple Regression
nobs(LM_ModernC)
```

```
## [1] 125
```

5. Examine added variable plots `car::avPlot` or `car::avPlots` for your model above. Are there any plots that suggest that transformations are needed for any of the terms in the model? Describe. Is it

likely that any of the localities are influential for any of the terms? Which localities? Which terms?

```
#Added Variable Plot
avPlots(LM_ModernC)
```



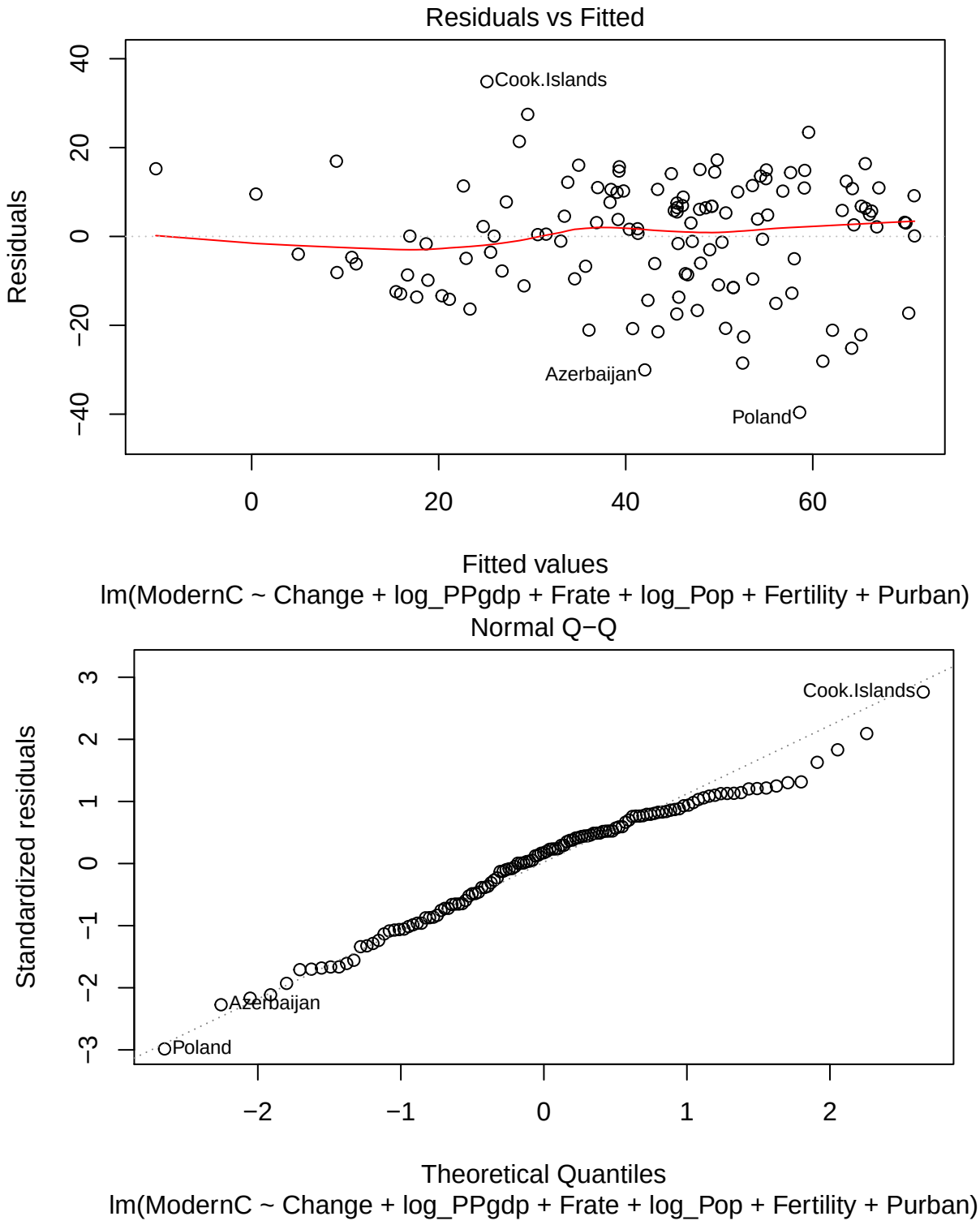
Comment: After visual inspection, I would like to suggest transforming the predictors Pop and PPgdp. The predictor "Pop" is too clustered around one section of the line, which I believe would become more spread out after a transformation. The predictor "PPgdp," on the other hand, has a large x-scale, which hints that transformation would significantly change the plot.

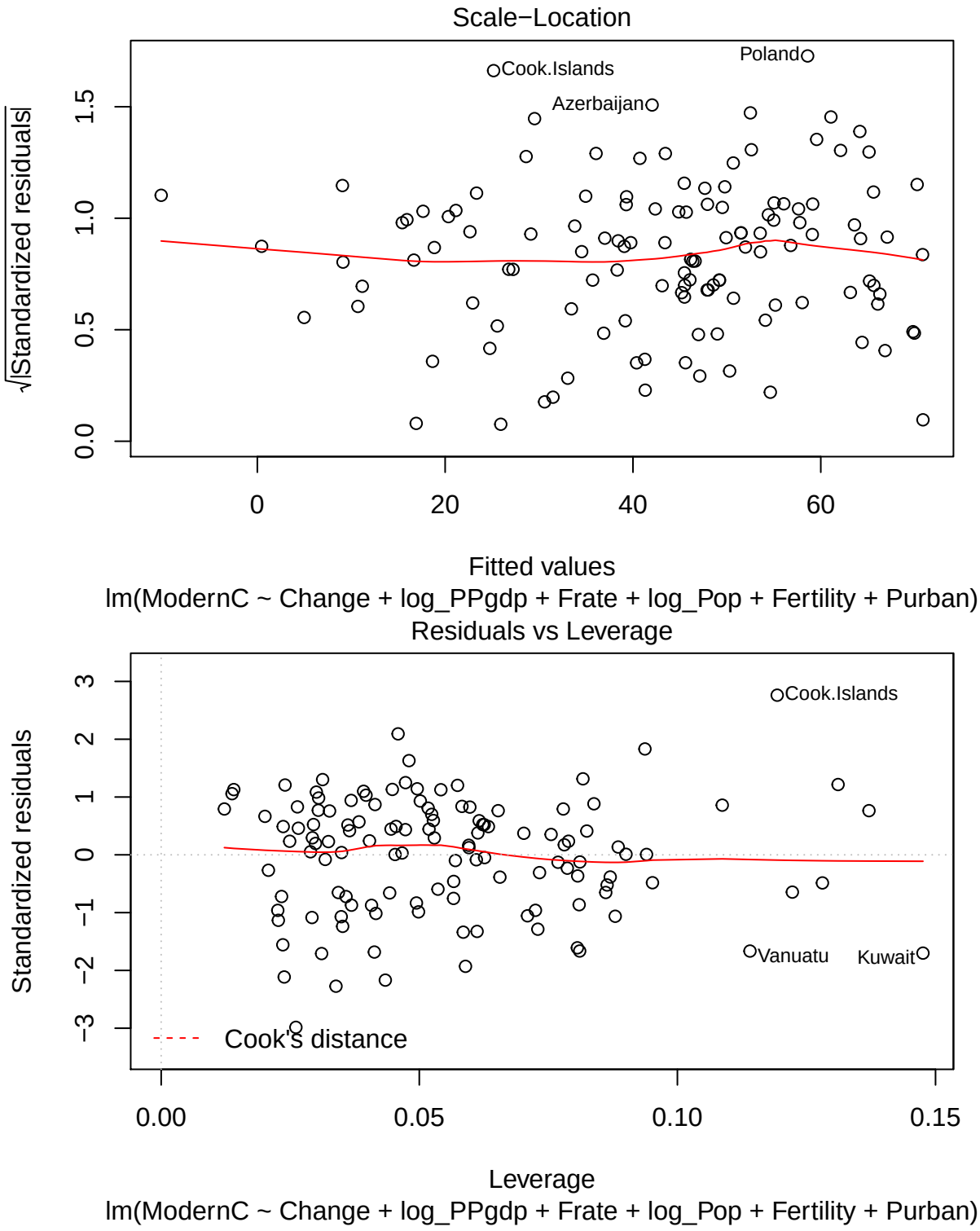
As for the question about any of localities being influential for any of the terms, I would say China and India look like influential localities in ModernC|Others ~ Pop|Others graph. The presence of the two points is judged - subjectively - to be creating a positive relationship between the response and the predictor.

6. Using the Box-Tidwell `car::boxTidwell` or graphical methods find appropriate transformations of the predictor variables to be used as predictors in the linear model. If any predictors are negative, you may need to transform so that they are non-negative. Describe your method and the resulting transformations.

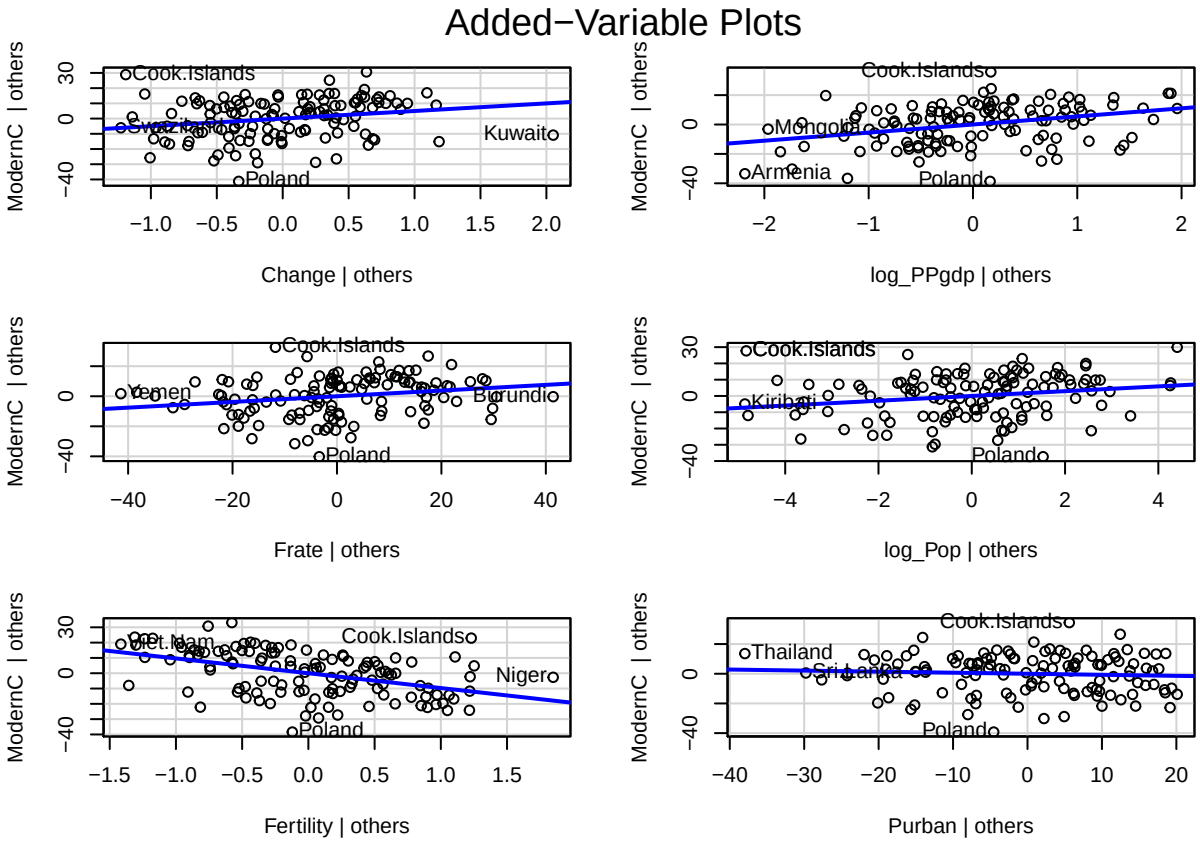
```
#Transformed Variables
log_PPgdp = log(PPgdp)
log_Pop = log(Pop)

#New Data Frame
UN3_No_NA_Log = UN3_No_NA %>%
  rownames_to_column("Country") %>%
  mutate("log_PPgdp" = log(PPgdp), "log_Pop" = log(Pop)) %>%
  column_to_rownames("Country")
```

```r
#Transformed Multipled Regression
LM_ModernC_Transformed = lm(ModernC ~ Change + log_PPgdp + Frate + log_Pop + Fertility + Purban, data =

#Plot for Regression
plot(LM_ModernC_Transformed)
```






```
#AVPlot for Regression
avPlots(LM_ModernC_Transformed)
```



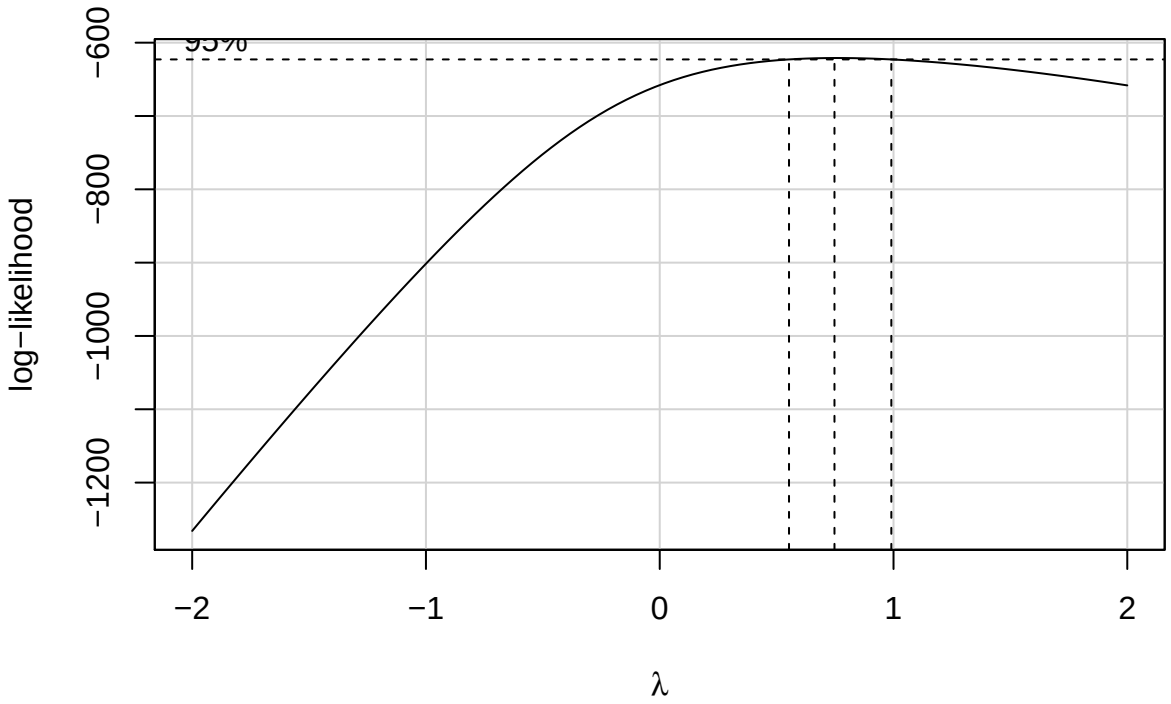

```
#BoxTidwell
boxTidwell(ModernC ~ Pop + PPgdp, other.x = ~ Change + Frate + Fertility + Purban, data = UN3_No_NA)
```

```
##       MLE of lambda Score Statistic (z) Pr(>|z|)
## Pop         0.40749             -0.7874   0.4310
## PPgdp      -0.12921             -1.1410   0.2539
##
## iterations =  4
```

Comment: After running boxTidwell for predictors Pop and PPgdp, I obtained outputs that suggested statistical insignificance of doing predictors' transformations. However, to not to overlook suggestions about transformations I had made in #3 and #5, I log-transform the predictors, to see if I really end up with the outputs provided by boxTidwell. The results were different from what boxTidwell provided. Log transforming the predictors PPgdp and Pop actually altered the plots, providing avPlots with more linear trend, less pronounced residuals vs. fitted plot, and less pronounced residuals vs. leverage plot. Since log-transformations ended up improving many aspects of the linear model, despite the insignificance message from boxTidwell, I concluded to log-transform PPgdp and Pop.

7. Given the selected transformations of the predictors, select a transformation of the response using `MASS::boxcox` or `car::boxCox` and justify.

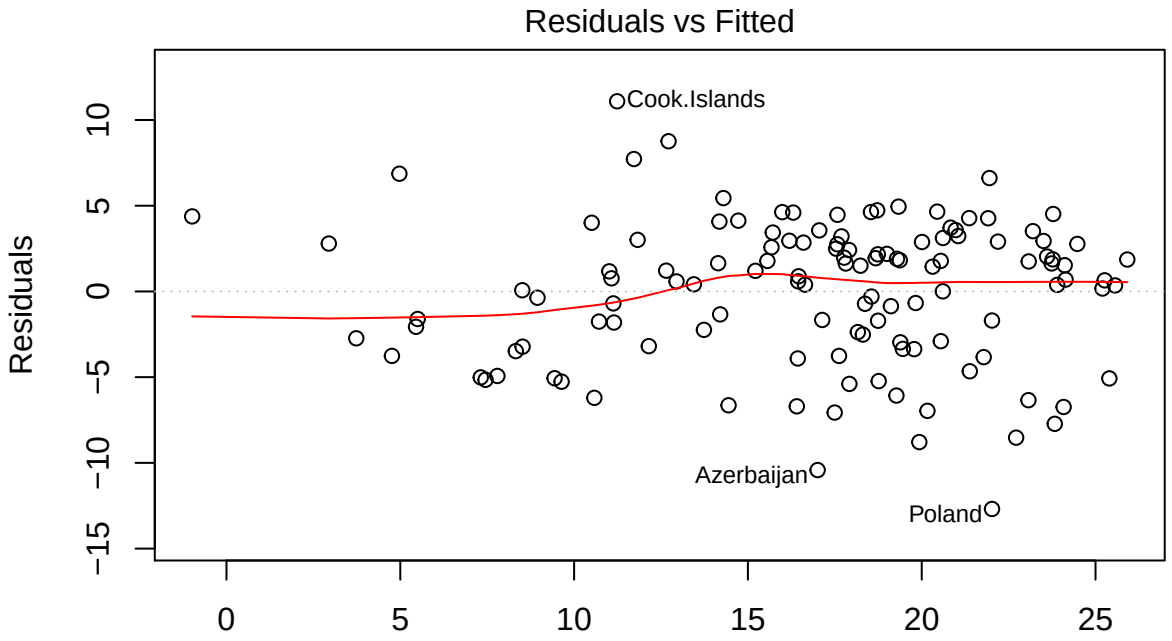
```
#Plot for BoxCox
boxCox(LM_ModernC_Transformed)
```

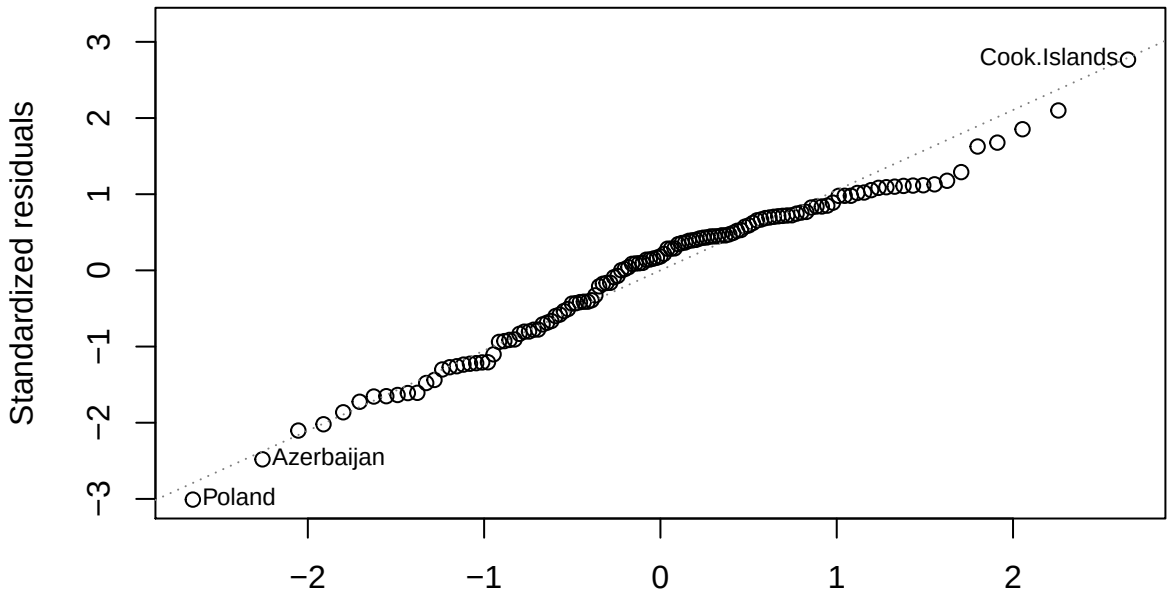



```
#Lambda for Optimal Transformation
powerTransform(LM_ModernC_Transformed)
```

```
## Estimated transformation parameter
##        Y1
## 0.7585897
```

```
#Response Transformed Multiple Regression
boxcox.transformed.lm = lm((ModernC)^(.7585897) ~ log_PPgdp + log_Pop + Change + Frate + Fertility + Pu

#Comparison of Response Transformed Multiple Regression to Predictors Transformed Regression
plot(boxcox.transformed.lm)
```


Residuals vs Fitted
Cook.Islands
Azerbaijan
Poland
Residuals
10
5
0
−5
−10
−15
0
5
10
15
20
25
Fitted values
lm((ModernC)^(0.7585897) ~ log_PPgdp + log_Pop + Change + Frate + Fertility ...


Normal Q−Q
Cook.Islands
Azerbaijan
Poland
Standardized residuals
3
2
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−3
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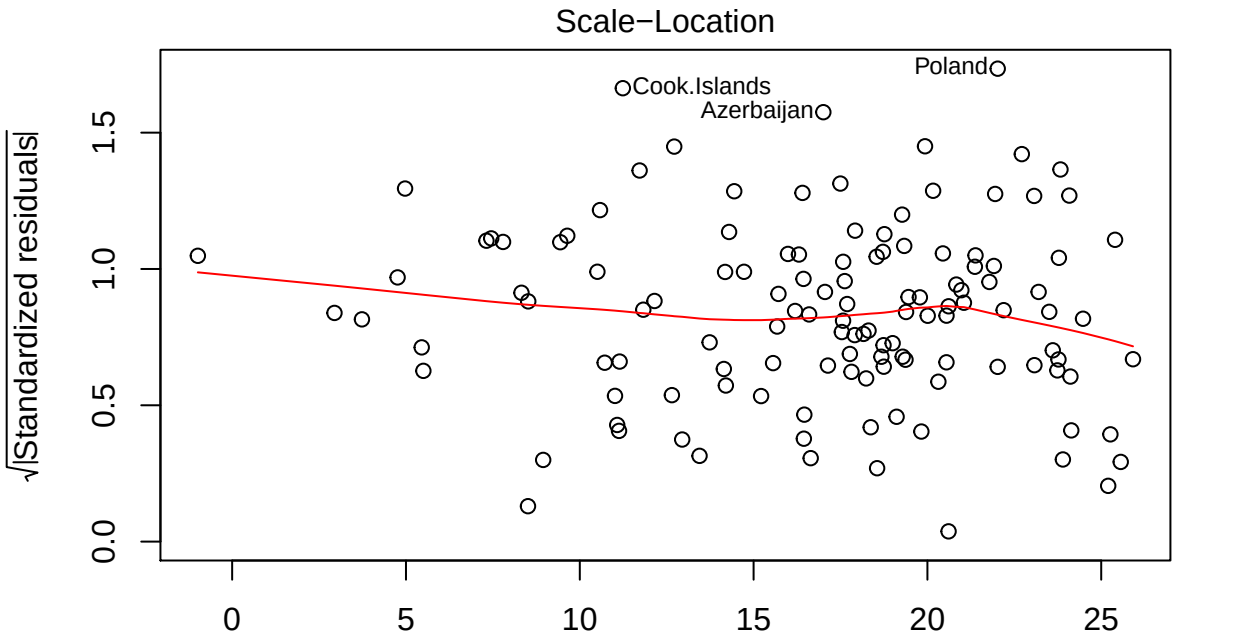
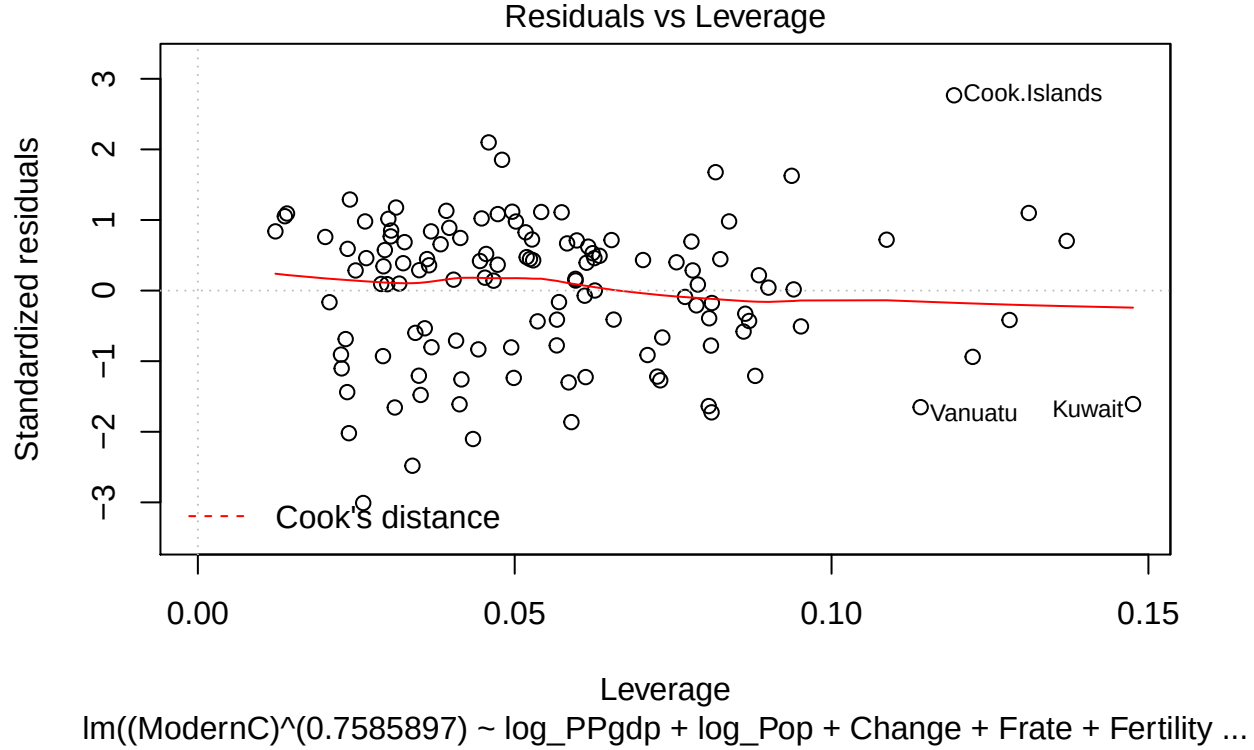
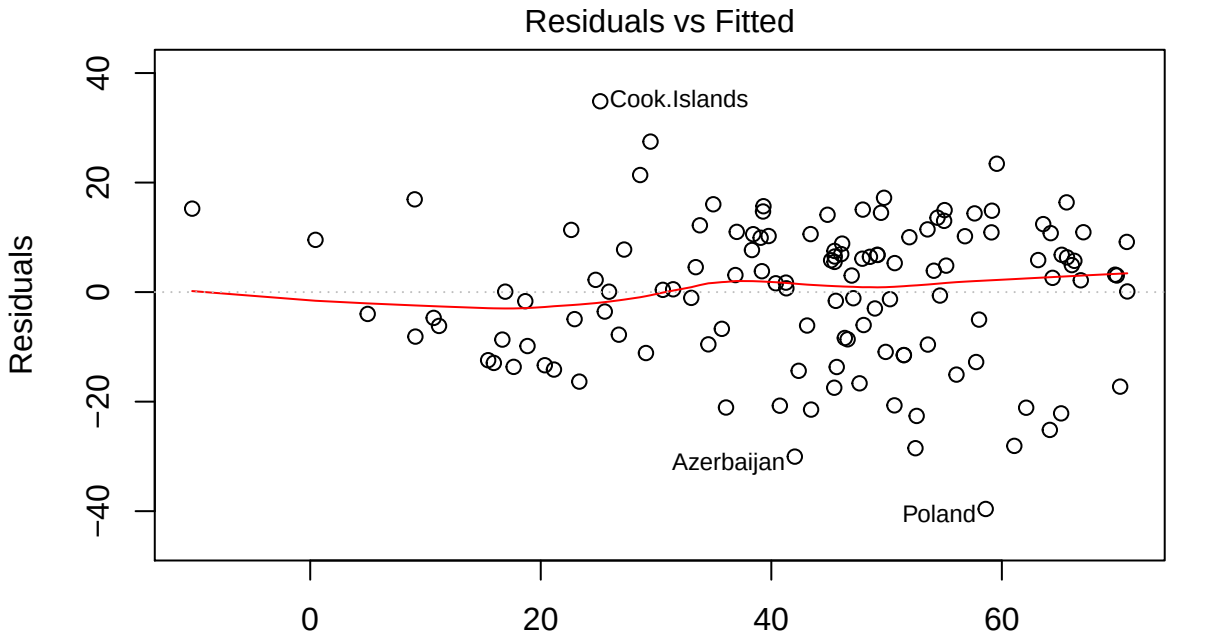
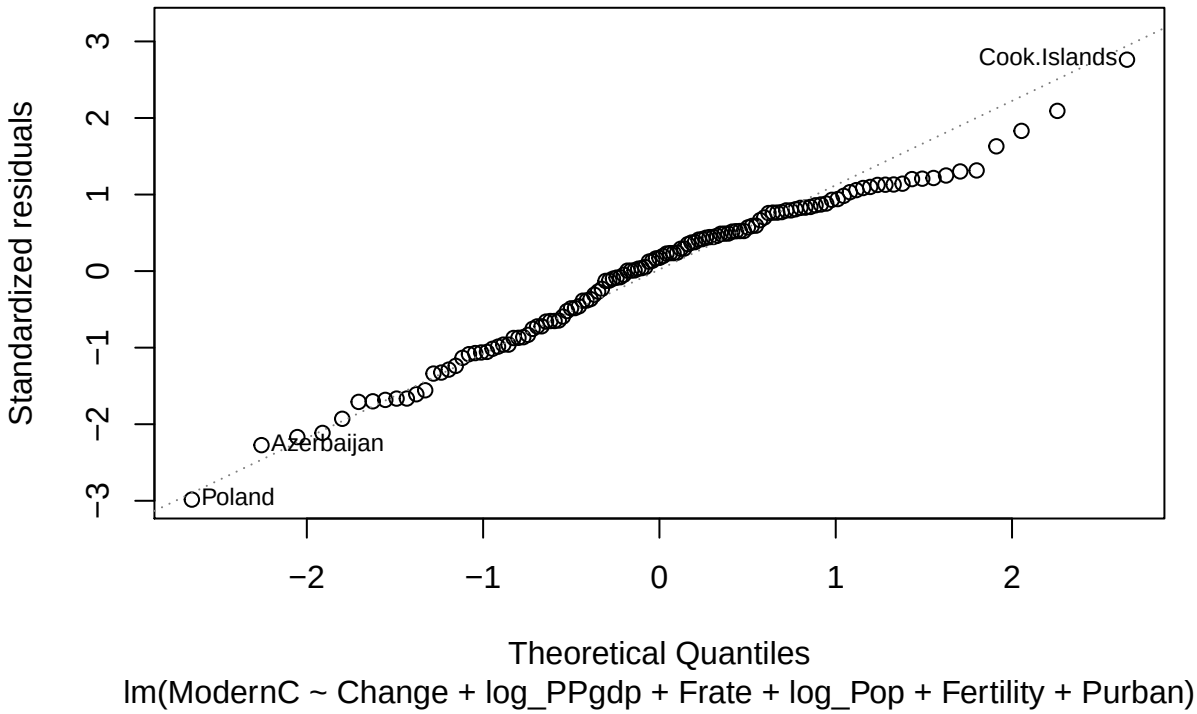
Theoretical Quantiles
lm((ModernC)^(0.7585897) ~ log_PPgdp + log_Pop + Change + Frate + Fertility ...






```
plot(LM_ModernC_Transformed)
```


Residuals vs Fitted
Residuals
Fitted values
lm(ModernC ~ Change + log_PPgdp + Frate + log_Pop + Fertility + Purban)
Cook.Islands
Azerbaijan
Poland
40
20
0
−20
−40
0
20
40
60


Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(ModernC ~ Change + log_PPgdp + Frate + log_Pop + Fertility + Purban)
Cook.Islands
Azerbaijan
Poland
3
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−2
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−2
−1
0
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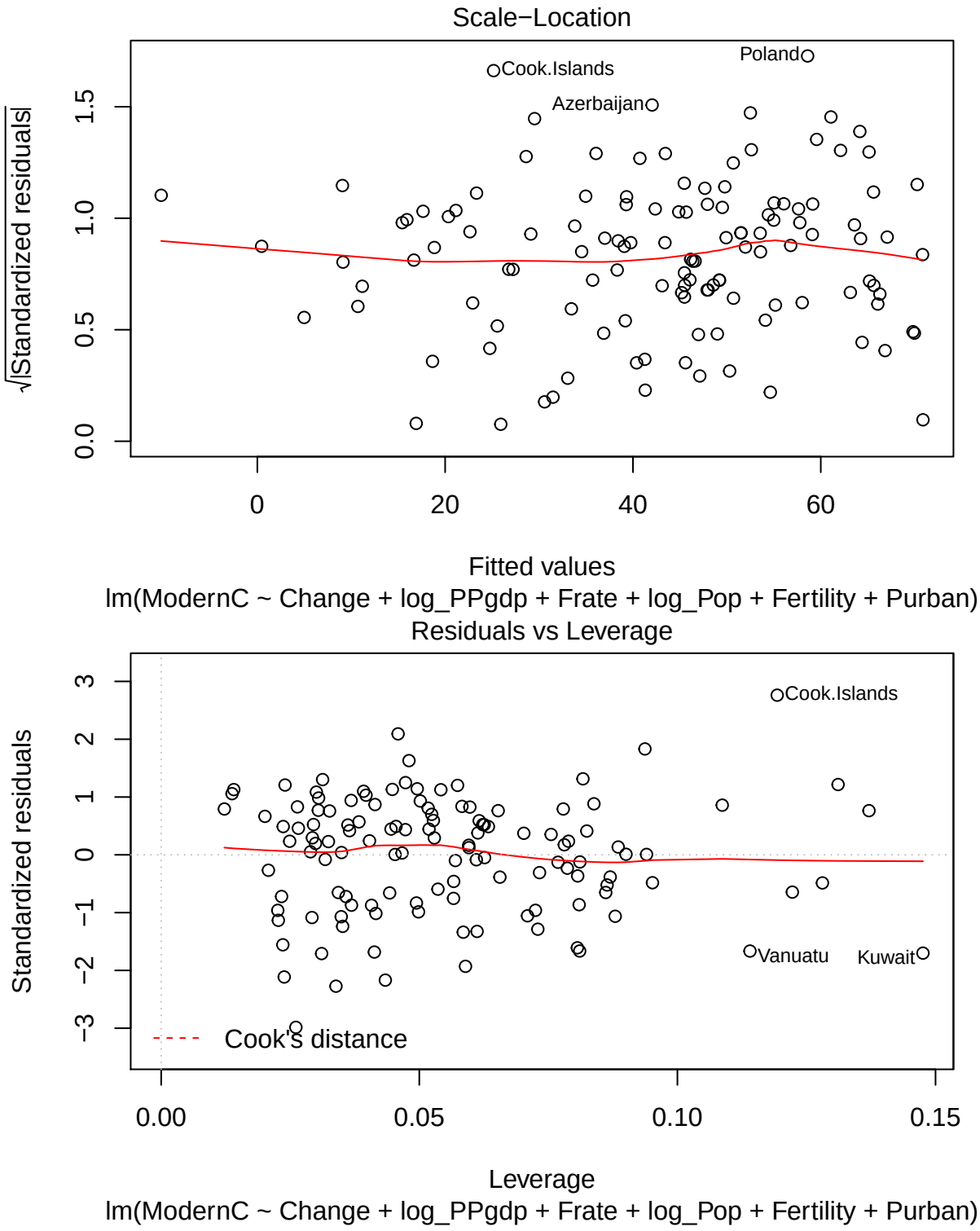


Comment: BoxCox transformation suggested applying a power of .7585897 to the response variable, which I have implemented by constructing a new linear regression model. However, I noticed that the residuals plots generated from the response-transformed model were not significantly different from those generated from predictors-only transformed model, compelling me to conclude that the response transformation is not being helpful in improving the model. Thus, I elected not to transform the response.

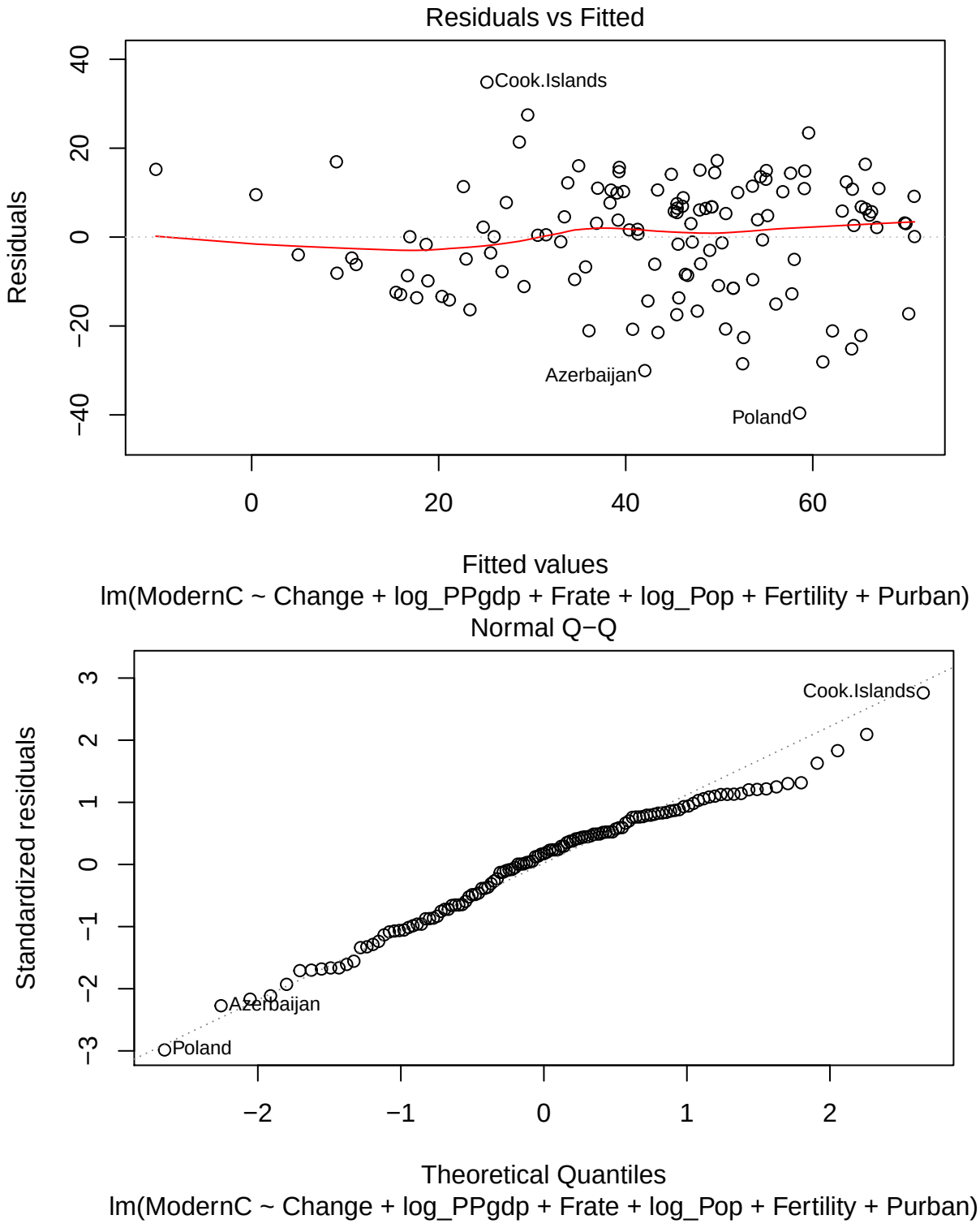
8. Fit the regression using the transformed variables. Provide residual plots and added variables plots

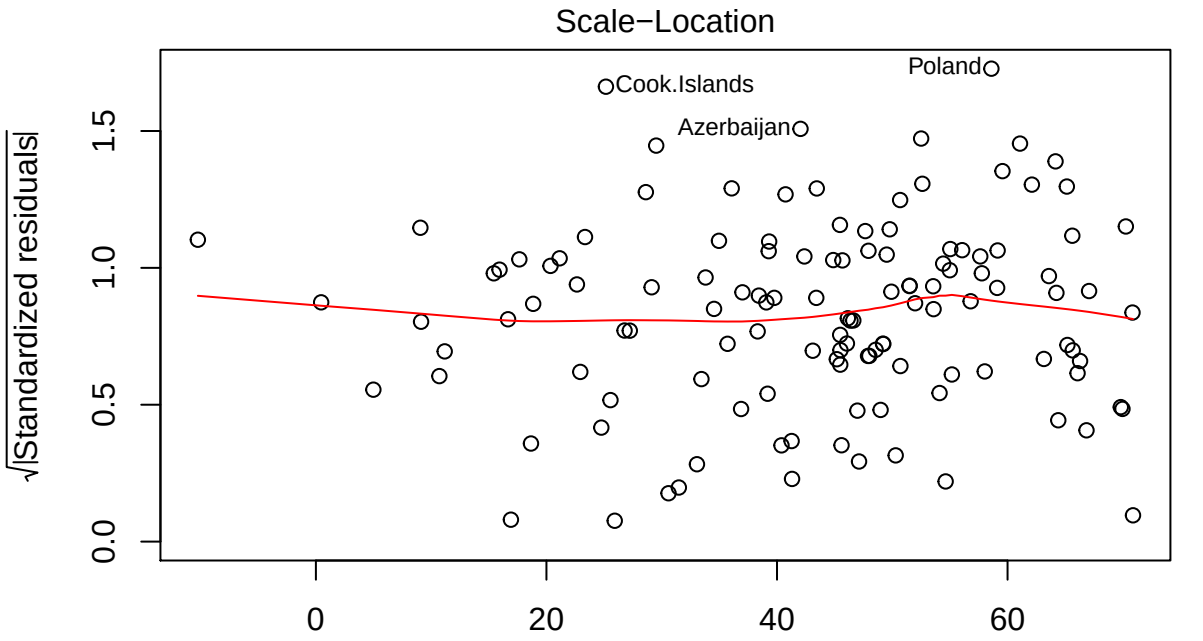
and comment. If you feel that you need additional transformations of either the response or predictors, repeat any steps until you feel satisfied.

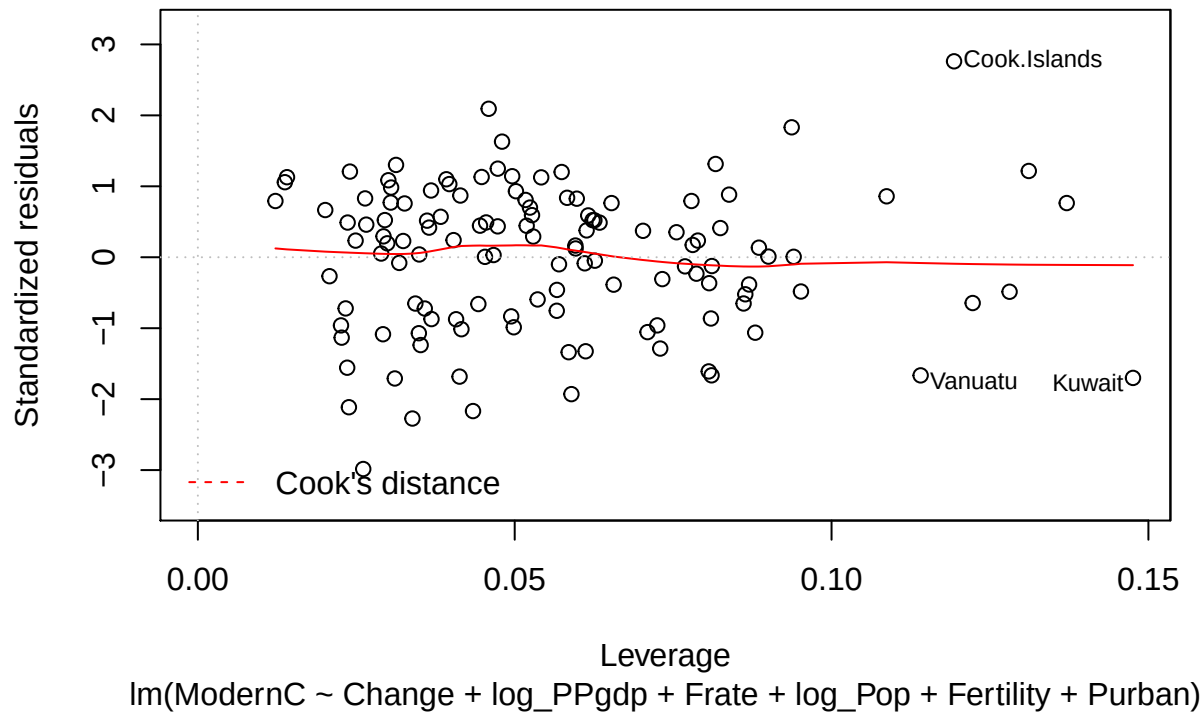
```
#Regression Using the Transformed Variables
summary(LM_ModernC_Transformed)
```

```
##
## Call:
## lm(formula = ModernC ~ Change + log_PPgdp + Frate + log_Pop +
##     Fertility + Purban, data = UN3_No_NA_Log)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -39.597  -9.540   2.238  10.024  34.840
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.11547   14.50854   0.284 0.777169
## Change       4.99296    2.07709   2.404 0.017781 *
## log_PPgdp    5.50728    1.40505   3.920 0.000149 ***
## Frate        0.18939    0.07711   2.456 0.015500 *
## log_Pop      1.47207    0.62875   2.341 0.020897 *
## Fertility   -9.67594    1.76561  -5.480 2.44e-07 ***
## Purban      -0.07077    0.09760  -0.725 0.469829
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 13.44 on 118 degrees of freedom
## Multiple R-squared:  0.626,  Adjusted R-squared:  0.6069
## F-statistic: 32.91 on 6 and 118 DF,  p-value: < 2.2e-16
```

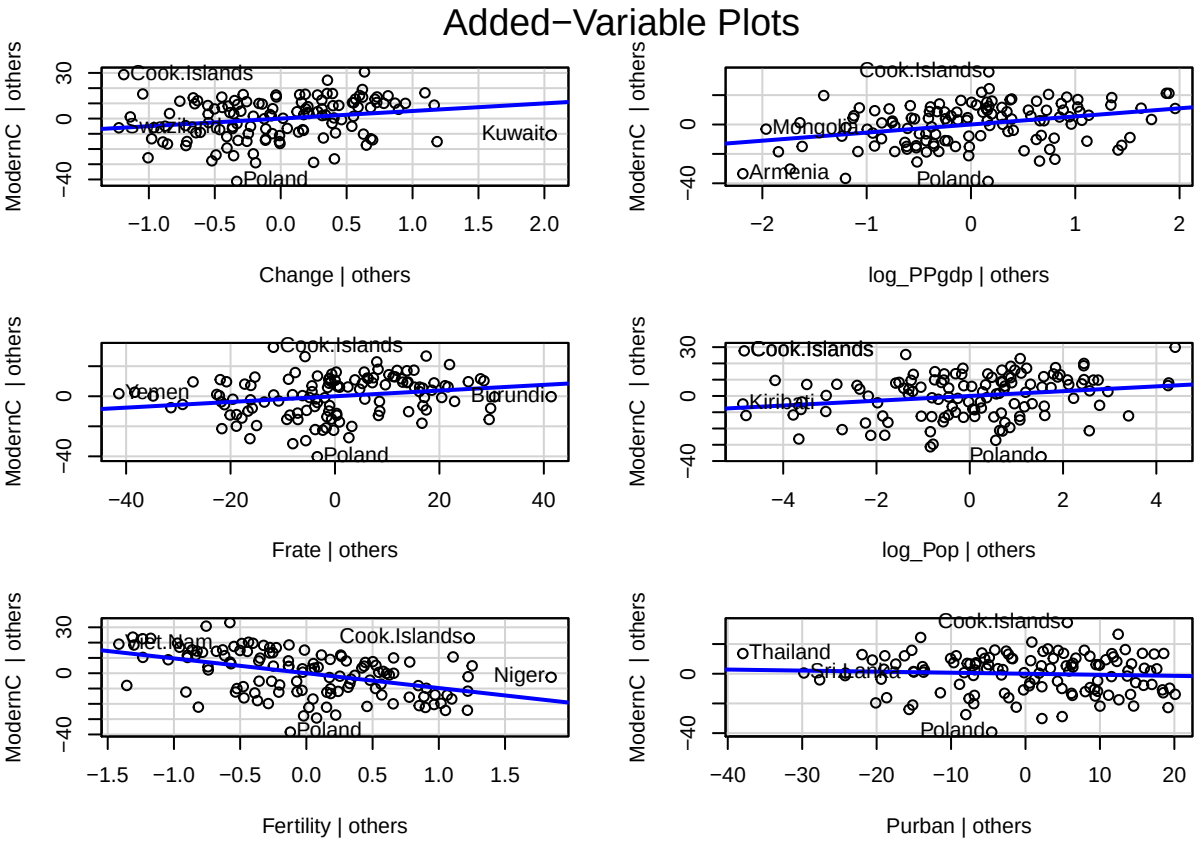
```
#Residual Plot
plot(LM_ModernC_Transformed)
```


Residuals vs Fitted
Residuals
Fitted values
lm(ModernC ~ Change + log_PPgdp + Frate + log_Pop + Fertility + Purban)
Cook.Islands
Azerbaijan
Poland
Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(ModernC ~ Change + log_PPgdp + Frate + log_Pop + Fertility + Purban)
Cook.Islands
Azerbaijan
Poland






```
#Added Variables Plot
avPlots(LM_ModernC_Transformed)
```

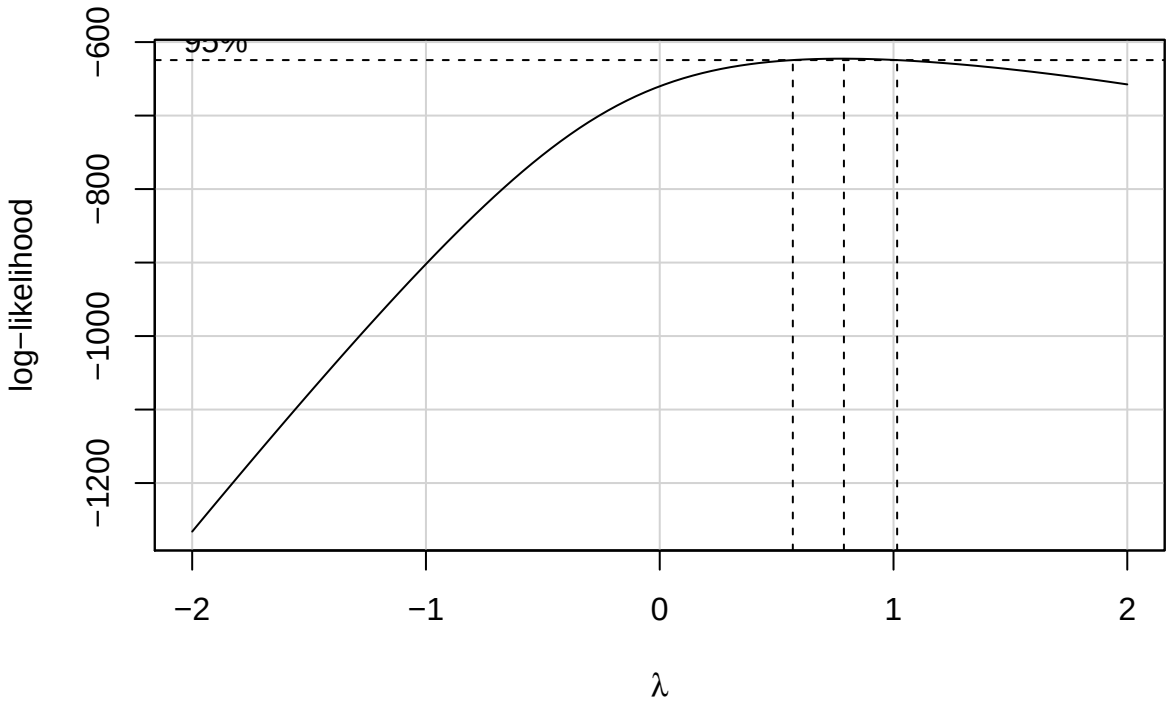


Note: Please note that since I elected only to transform the predictors, my regression of the transformed variables is identical to the regression model from #6.

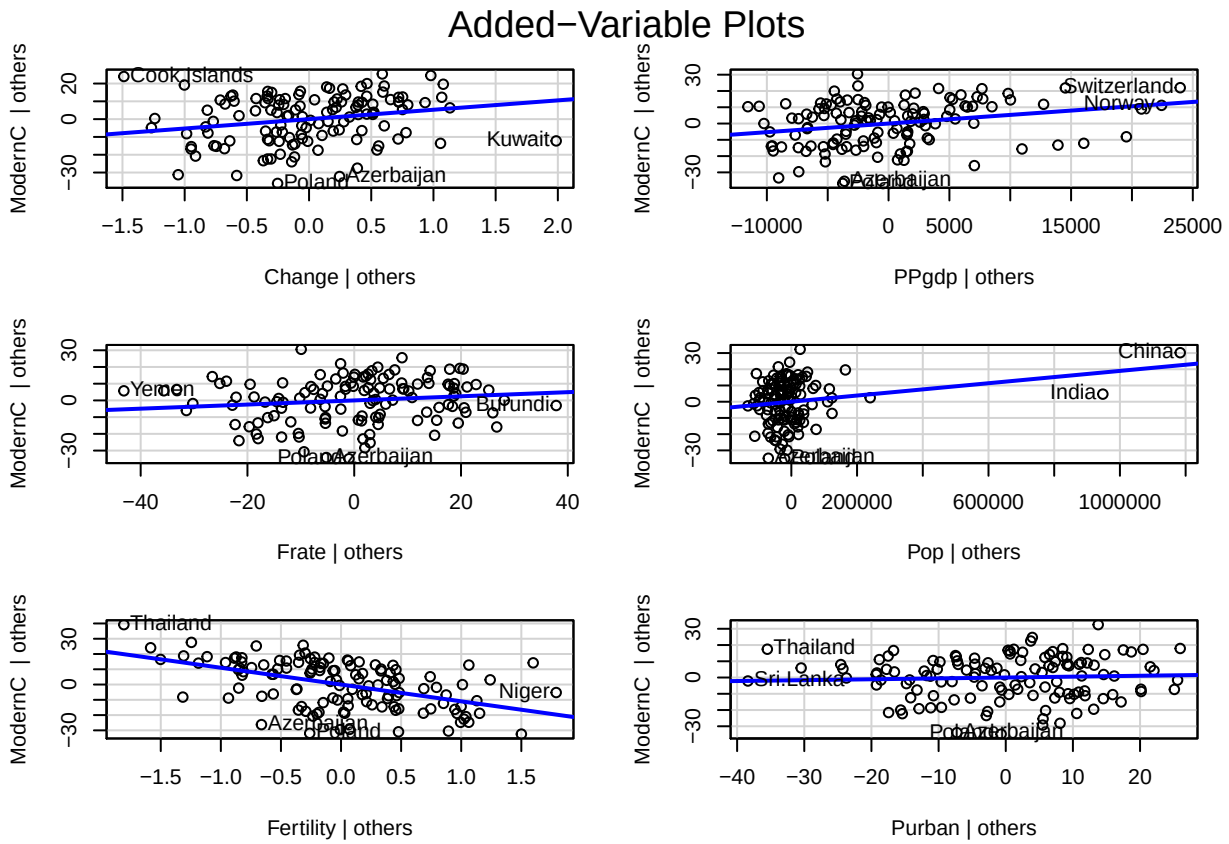
Comment: After the transformations were done on the predictors, I observed improvements in residuals plots. The red curve on "Residual vs. Fitted plot" became less pronounced; the right-tail of Normal Q-Q plot bent toward the perfect normal Q-Q line; the red curve on "Scale-Location" became less pronounced; and the red curve "Residuals vs. Leverage" is flatter. These evidences support that transformations done on the predictors improved the model.

9. Start by finding the best transformation of the response and then find transformations of the predictors. Do you end up with a different model than in 8?

```
#BoxCox
boxCox(LM_ModernC)
```




```
#AVPlot to Detect Predictors that Need Transformations
avPlots(LM_ModernC)
```




```
#BoxTidwell
boxTidwell(ModernC ~ Pop + PPgdp, other.x = ~ Change + Frate + Purban + Fertility, data = UN3_No_NA)
```

```
##       MLE of lambda Score Statistic (z) Pr(>|z|)
## Pop         0.40749             -0.7874   0.4310
## PPgdp      -0.12921             -1.1410   0.2539
##
## iterations =  4
```

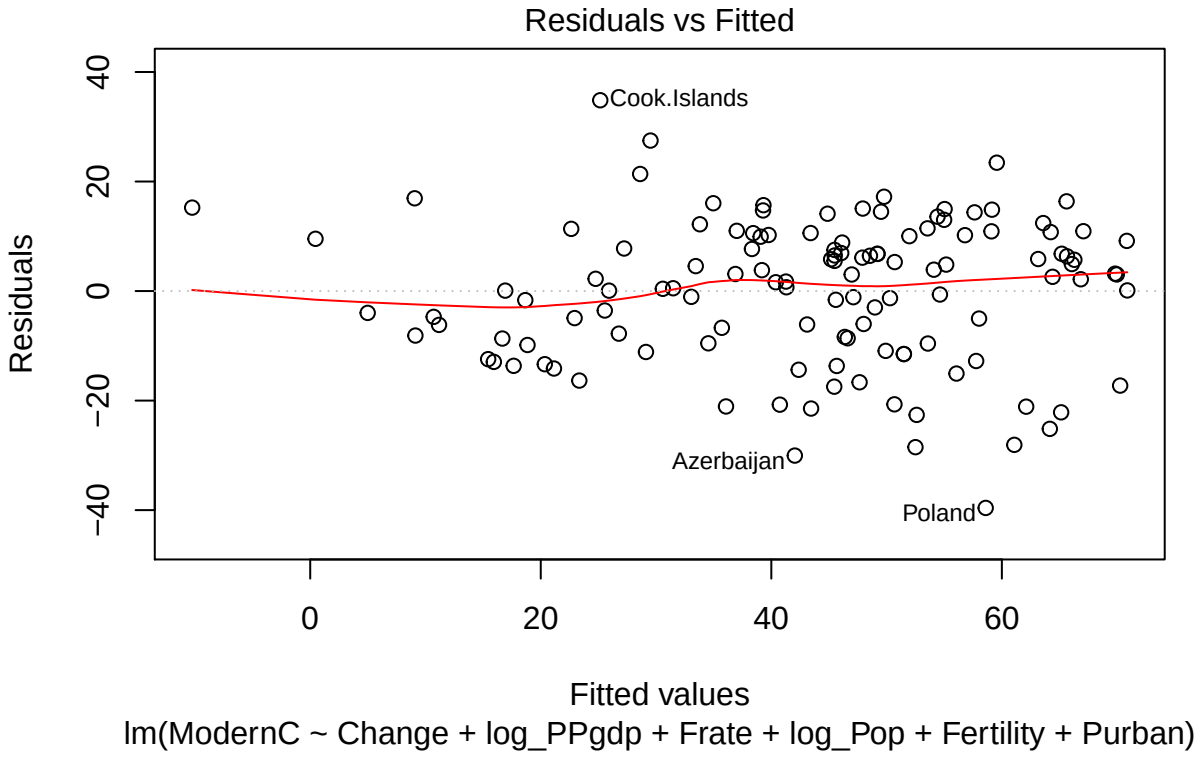
Comment: Unlike the previous problem, boxCox function for this question is not suggesting to transform the response, by including 1 in its 95% confidence interval - inclusion of 1 suggests that the power of the response is not statistically different from 1. As for BoxTidwell, it is once again informing me to not transform predictors, by providing me with relatively high probabilities - I selected to transform PPgdp and Pop after looking at the avPlot of linear model. *INCOMPLETE*

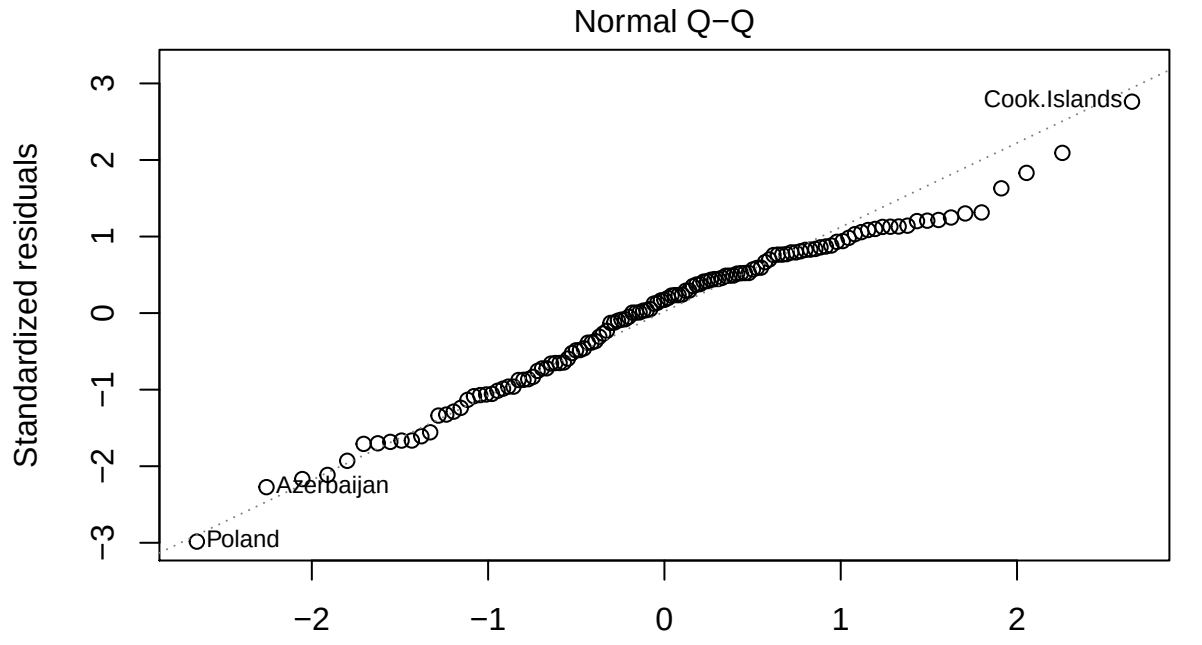
10. Are there any outliers or influential points in the data? Explain. If so, refit the model after removing any outliers and comment on residual plots.

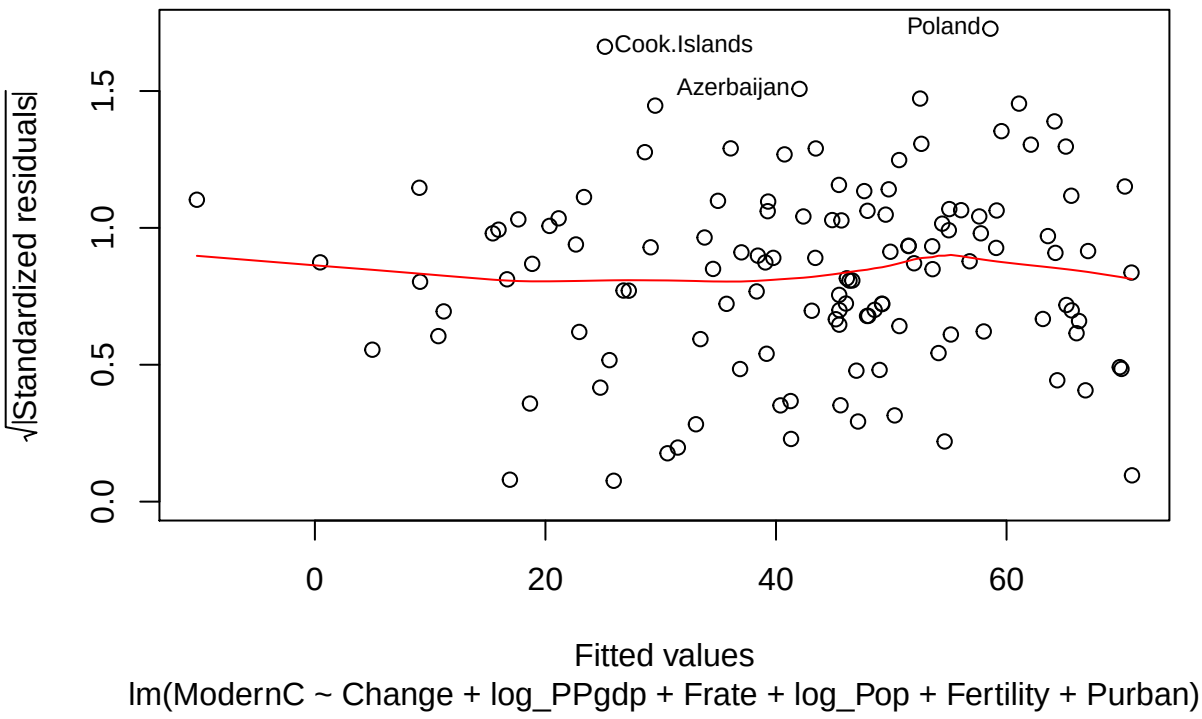
```
#Bonferroni Correction for Outliers
pval = 2*(1 - pt(abs(rstudent(LM_ModernC_Transformed)), LM_ModernC_Transformed$df - 1))
rownames(UN3_No_NA_Log)[pval < .05/nrow(UN3_No_NA_Log)]
```

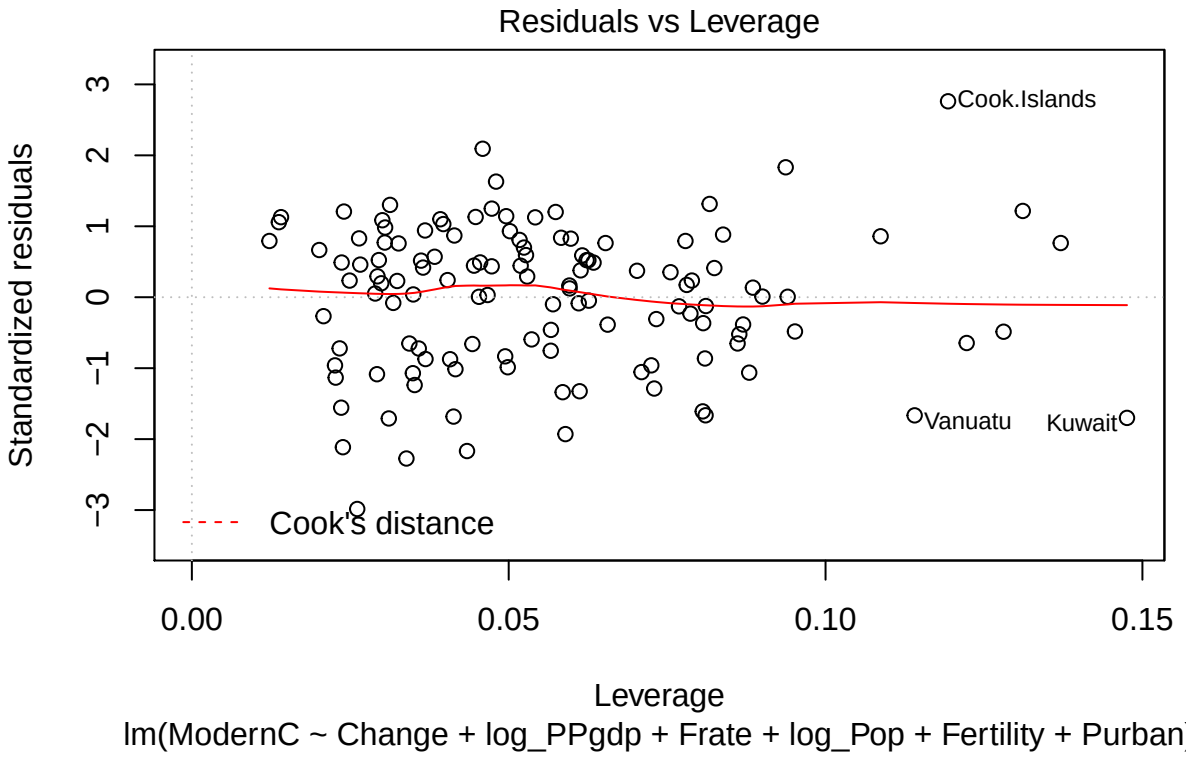
```
## character(0)
```

```
#Cook's Distance for Influential Points
plot(LM_ModernC_Transformed)
```




Normal Q−Q
Standardized residuals
Cook.Islands
Azerbaijan
Poland
−3
−2
−1
0
1
2
3
Theoretical Quantiles
lm(ModernC ~ Change + log_PPgdp + Frate + log_Pop + Fertility + Purban)


Scale−Location
√|Standardized residuals|
Cook.Islands
Poland
Azerbaijan
0.0
0.5
1.0
1.5
0
20
40
60
Fitted values
lm(ModernC ~ Change + log_PPgdp + Frate + log_Pop + Fertility + Purban)



According to Bonferroni correction, there are no outliers in the data and therefore no data need to be removed. As for influential points, cook's distance was absent on the residuals plot, which suggests that no data are influential points.

Summary of Results

11. For your final model, provide summaries of coefficients with 95% confidence intervals in a nice table with interpretations of each coefficient. These should be in terms of the original units!

```
#Multiple Regression with Purban Removed
lm3 = lm(ModernC ~ Change + log_Pop + log_PPgdp + Frate + Fertility, data = UN3_No_NA_Log)

#Justification of Removing Purban
anova(lm3, LM_ModernC_Transformed)
```

```
## Analysis of Variance Table
##
## Model 1: ModernC ~ Change + log_Pop + log_PPgdp + Frate + Fertility
## Model 2: ModernC ~ Change + log_PPgdp + Frate + log_Pop + Fertility +
##     Purban
##   Res.Df   RSS Df Sum of Sq      F Pr(>F)
## 1    119 21420
## 2    118 21325  1    95.016 0.5258 0.4698
```

```
confint_predictors = as.data.frame(confint(lm3))
confint_predictors["log_PPgdp",] = exp(confint_predictors["log_PPgdp",])
confint_predictors["log_Pop" ,] = exp(confint_predictors["log_Pop",])
rownames(confint_predictors) = c("(Intercept)", "Change", "PPgdp", "Frate", "Pop", "Fertility")
kable(confint_predictors)
```

	2.5 %	97.5 %
(Intercept)	-24.5689501	32.7731131
Change	0.6727287	8.7227854
PPgdp	1.2233091	14.5980723
Frate	15.1291534	1098.9398923
Pop	0.0496977	0.3493943
Fertility	-12.5950756	-5.9617672

Comment: For the final model, I elected to remove Purban. I ran anova and observed that adding Purban to the linear model is not statistically significant.

12. Provide a paragraph summarizing your final model and findings suitable for the US envoy to the UN after adjusting for outliers or influential points. You should provide a justification for any case deletions in your final model

Methodology

13. Prove that the intercept in the added variable scatter plot will always be zero. _Hint: use the fact that if H is the project matrix which contains a column of ones, then $1_n^T(I-H)=0$. Use this to show that the sample mean of residuals will always be zero if there is an intercept.

14. For multiple regression with more than 2 predictors, say a full model given by `Y ~ X1 + X2 + ... Xp` we create the added variable plot for variable `j` by regressing `Y` on all of the `X`'s except `Xj` to form `e_Y` and then regressing `Xj` on all of the other X's to form `e_X`. Confirm that the slope in a manually constructed added variable plot for one of the predictors in Ex. 10 is the same as the estimate from your model.

```
#e_Y on e_X
e_Y = residuals(lm3)
e_X = residuals(lm(Purban ~ Change + log_Pop + log_PPgdp + Frate + Fertility, data = UN3_No_NA_Log))

#Slope Comparison
lm_e_Y_on_e_X = lm(e_Y ~ e_X)
LM_ModernC_Transformed$coefficients["Purban"]
```

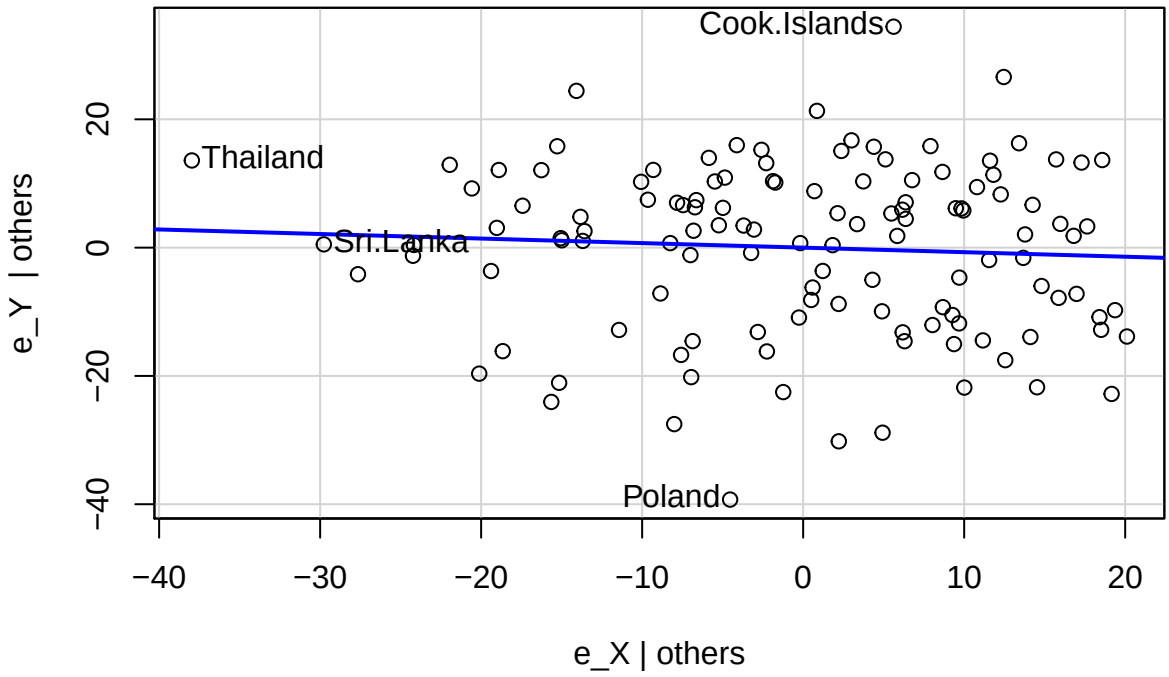
```
##      Purban
## -0.07076799
```

```
x = data.frame(e_X = lm(e_Y ~ e_X)$coefficients["e_X"], LM_ModernC_Transformed$coefficients["Purban"], 

kable(x, col.names = c("e_X", "Model"), format = "markdown")
```

	e_X	Model
Coefficients	-0.070768	-0.070768

```
#AVPlots
avPlots(lm_e_Y_on_e_X)
```


Cook.Islands
Thailand
Sri.Lanka
Poland
e_Y | others
e_X | others
20
0
−20
−40
−40
−30
−20
−10
0
10
20