Poisson Regression

Gelman & Hill Chapter 6

February 5, 2017

Military Coups

Background: Sub-Sahara Africa has experienced a high proportion of regime changes due to military takeover of governments for a variety of reasons: ethnic fragmentation, arbitrary borders, economic problems, outside interventions, poorly developed government institutions, etc.

Data in Gill (page 551-552) is a subset from Bratton and Van de Valle (1994) to examine factors to try to explain military coups in 33 countries from each country's colonial independence to 1989.

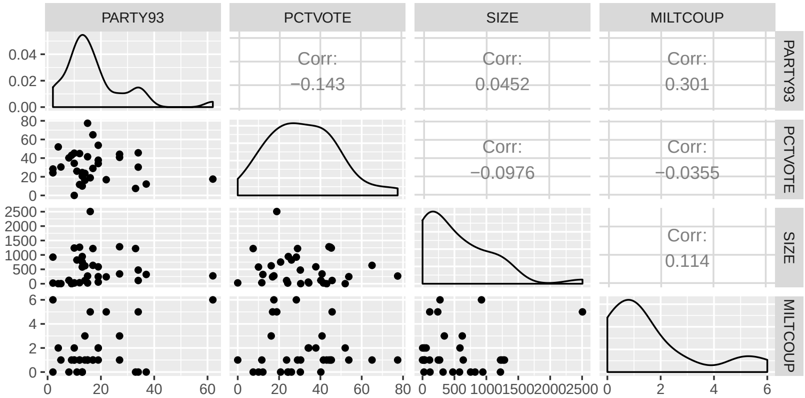
```
africa = read.table("africa.dat", header = T)
```

Variables

```
MILTCOUP     # of coups
MILTITARY    # of years of military oligarchy
POLLIB       (0=no civil rights, 1=limited, 2=extensive)
PARTY93      # of political parties
PCTVOTE      % legislative voting
PCTTURN      % registered voting
SIZE         of country (1000 km^2
POP          (in millions)
NUMREGIM     Number of regimes
NUMELEC      Number of elections
```

- Type of study?
- Are causal conclusions possible?

Distribution of Response



Response is non-negative

Poisson Distribution

$$Y_i \mid \lambda_i \sim P(\lambda_i)$$

$$p(y_i) = \frac{\lambda_i^{y_i} e^{-\lambda_i}}{y_i!} \qquad y_i = 0, 1, \ldots, \qquad \lambda_i > 0$$

- Used for counts with no upper limit
- $E(Y_i) = V(Y_i) = \lambda_i$

How to build in covariates into the mean?

- $\lambda > 0 \Leftrightarrow \log(\lambda) = \eta \in \mathbb{R}$
- log link

Generalized Linear Model

Canonical Link function for Poisson data is the log link

- $\log(\lambda_i) = \eta_i = \beta_0 + X_1\beta_1 + \ldots X_p\beta_p$ (linear predictor)
- $\lambda = \exp(\beta_0 + X_1\beta_1 + \ldots X_p\beta_p)$
- Holding all other X's fixed a 1 unit change in X_j

$$
\begin{aligned}
\lambda^* &= \exp(\beta_0 + X_1\beta_1 + \ldots (X_j + 1)\beta_j + \ldots X_p\beta_p) \\
\lambda^* &= \exp(\beta_j)\exp(\beta_0 + X_1\beta_1 + \ldots X_j\beta_j + \ldots X_p\beta_p) \\
\lambda^* &= \exp(\beta_j)\lambda \\
\lambda^*/\lambda &= \exp(\beta_j)
\end{aligned}
$$

- $\exp(\beta_j)$ is called a "relative risk" (risk relative to some baseline)

Output from glm

```
africa.glm = glm(MILTCOUP ~ MILITARY + POLLIB + PARTY93+
                   PCTVOTE + PCTTURN + SIZE*POP +
                   NUMREGIM*NUMELEC,
                   family=poisson, data=africa)
round(summary(africa.glm)$coef, 4)

##                   Estimate Std. Error z value Pr(>|z|)
## (Intercept)         2.9209     1.3368  2.1850   0.0289
## MILITARY            0.1709     0.0509  3.3575   0.0008
## POLLIB             -0.4654     0.3319 -1.4022   0.1609
## PARTY93             0.0247     0.0109  2.2792   0.0227
## PCTVOTE             0.0613     0.0217  2.8202   0.0048
## PCTTURN            -0.0361     0.0137 -2.6372   0.0084
## SIZE               -0.0018     0.0007 -2.5223   0.0117
## POP                -0.1188     0.0397 -2.9961   0.0027
## NUMREGIM           -0.8662     0.4571 -1.8949   0.0581
## NUMELEC            -0.4859     0.2118 -2.2948   0.0217
## SIZE:POP            0.0001     0.0000  3.0111   0.0026
## NUMREGIM:NUMELEC    0.1810     0.0689  2.6265   0.0086
```

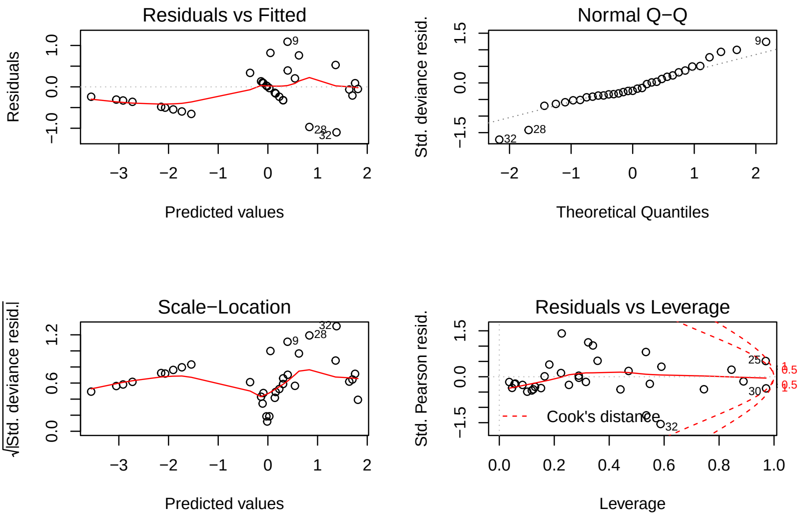
lack of fit?

- Under the hypothesis that the model is correct, residual deviance = -2 log likelihood has an asymptotic χ^2_{n-p} distribution
- Under the alternative that we have omitted important terms, expect to see a large residual deviance
- Compare observed deviance to χ^2 distribution

```
c(summary(africa.glm)$deviance, summary(africa.glm)$df.resi

## [1]  7.547369 21.000000

1 - pchisq(summary(africa.glm)$deviance, summary(africa.glm

## [1] 0.9967843
```

So no evidence of lack of fit (overdispersion).

Diagnostics



Residuals in GLMS

- residuals: $Y_i - \hat{\lambda}_i$ (observed - fitted)
- Pearson Goodness of Fit

$$X^2 = \sum_i \frac{(Y_i - \hat{\lambda}_i)^2}{\hat{\lambda}_i}$$

- Pearson Residuals:

$$r_i = \frac{Y_i - \hat{\lambda}_i}{\sqrt{\hat{\lambda}_i}}$$

```
residuals.glm(africa.glm, type="pearson")
```

- deviance: Change in likelihood for Model compared to Null model

$$\begin{aligned} D &= 2\left\{\sum_i y_i \log(y_i/\hat{\lambda}_i) - (y_i - \hat{\lambda}_i)\right\} \\ &= \sum d_i \end{aligned}$$

```
residuals.glm(africa.glm, type="deviance")
```

Coefficients

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	2.92	1.34	2.18	0.03
MILITARY	0.17	0.05	3.36	0.00
POLLIB	-0.47	0.33	-1.40	0.16
PARTY93	0.02	0.01	2.28	0.02
PCTVOTE	0.06	0.02	2.82	0.00
PCTTURN	-0.04	0.01	-2.64	0.01
SIZE	-0.00	0.00	-2.52	0.01
POP	-0.12	0.04	-3.00	0.00
NUMREGIM	-0.87	0.46	-1.89	0.06
NUMELEC	-0.49	0.21	-2.29	0.02
SIZE:POP	0.00	0.00	3.01	0.00
NUMREGIM:NUMELEC	0.18	0.07	2.63	0.01

Treat Political Liberties as a Factor?

```
africa.glm1 = glm(MILTCOUP ~ MILITARY + factor(POLLIB) +
                    PARTY93 + PCTVOTE+ PCTTURN +
                    SIZE*POP + NUMREGIM*NUMELEC,
                    family=poisson, data=africa)
anova(africa.glm, africa.glm1, test="Chi")

## Analysis of Deviance Table
##
## Model 1: MILTCOUP ~ MILITARY + POLLIB + PARTY93 + PCTVOT
##     SIZE * POP + NUMREGIM * NUMELEC
## Model 2: MILTCOUP ~ MILITARY + factor(POLLIB) + PARTY93
##     SIZE * POP + NUMREGIM * NUMELEC
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1        21     7.5474
## 2        20     7.1316  1  0.41581    0.519
```

Interpretation of Coefficients

- Asymptotic distribution (Frequentist)

$$(\beta_j - \hat{\beta}_j)/\mathrm{SE}(\beta_j) \sim N(0,1)$$

- 95% CI for coefficient of MILITARY:

$$0.171 \pm 1.96 \cdot 0.051 = (0.078, 0.282)$$

- relative risk is $\exp(0.171) = 1.186$
- 95% CI for relative risk e^{CI}

$$(\exp(0.078), \exp(0.282)) = (1.081, 1.325)$$

Keeping everything else constant, for every additional year of military oligarchy, the risk of a military coup increases by 8 to 32 percent