STA 601: Lecture 4

Comparing Estimators & Prior/Posterior Checks

Merlise Clyde

9/7/2021

Normal Model Setup from Last Class

- independent observations $\mathbf{y} = (y_1, y_2, \ldots, y_n)^T$ where each $y_i \sim \mathsf{N}(\theta, 1/\tau)$ (iid)

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- The likelihood for θ is proportional to the sampling model

$$
\begin{aligned}
\mathcal{L}(\theta) &\propto \tau^{\frac{n}{2}} \ \exp\left\{ -\frac{1}{2}\tau \sum_{i=1}^{n} (y_i - \theta)^2 \right\} \\
&\propto \tau^{\frac{n}{2}} \ \exp\left\{ -\frac{1}{2}\tau \sum_{i=1}^{n} \left[(y_i - \bar{y}) - (\theta - \bar{y})\right]^2 \right\} \\
&\propto \tau^{\frac{n}{2}} \ \exp\left\{ -\frac{1}{2}\tau \left[\sum_{i=1}^{n} (y_i - \bar{y})^2 + \sum_{i=1}^{n} (\theta - \bar{y})^2 \right] \right\} \\
&\propto \tau^{\frac{n}{2}} \ \exp\left\{ -\frac{1}{2}\tau \left[\sum_{i=1}^{n} (y_i - \bar{y})^2 + n(\theta - \bar{y})^2 \right] \right\} \\
&\propto \exp\left\{ -\frac{1}{2}\tau n(\theta - \bar{y})^2 \right\}
\end{aligned}
$$

Exercises for Practice

Try this

1) Use $\mathcal{L}(\theta)$ based on n observations to find $\pi(\theta \mid y_1, \ldots, y_n)$ based on the sufficient statistics and prior $\theta \sim \mathsf{N}(\theta_0, 1/\tau_0)$

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2) Use $\pi(\theta \mid y_1, \ldots, y_n)$ to find the posterior predictive distribution for Y_{n+1}

After n observations

Posterior for θ

$$\theta \mid y_1, \ldots, y_n \sim \mathsf{N}\left(\frac{\tau_0\theta_0 + n\tau\bar{y}}{\tau_0 + n\tau}, \frac{1}{\tau_0 + n\tau}\right)$$

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- Shrinkage of the MLE to the prior mean

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Posterior Predictive $Y_{n+1} \mid y_1, \ldots, y_n \sim \textsf{N}\left(\bar{y}, \sigma^2(1 + \frac{1}{n})\right)$

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- For the MLE $\bar{Y}$ this is just the variance of $\bar{Y}$ or σ^2/n

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- Bias of Bayes Estimate

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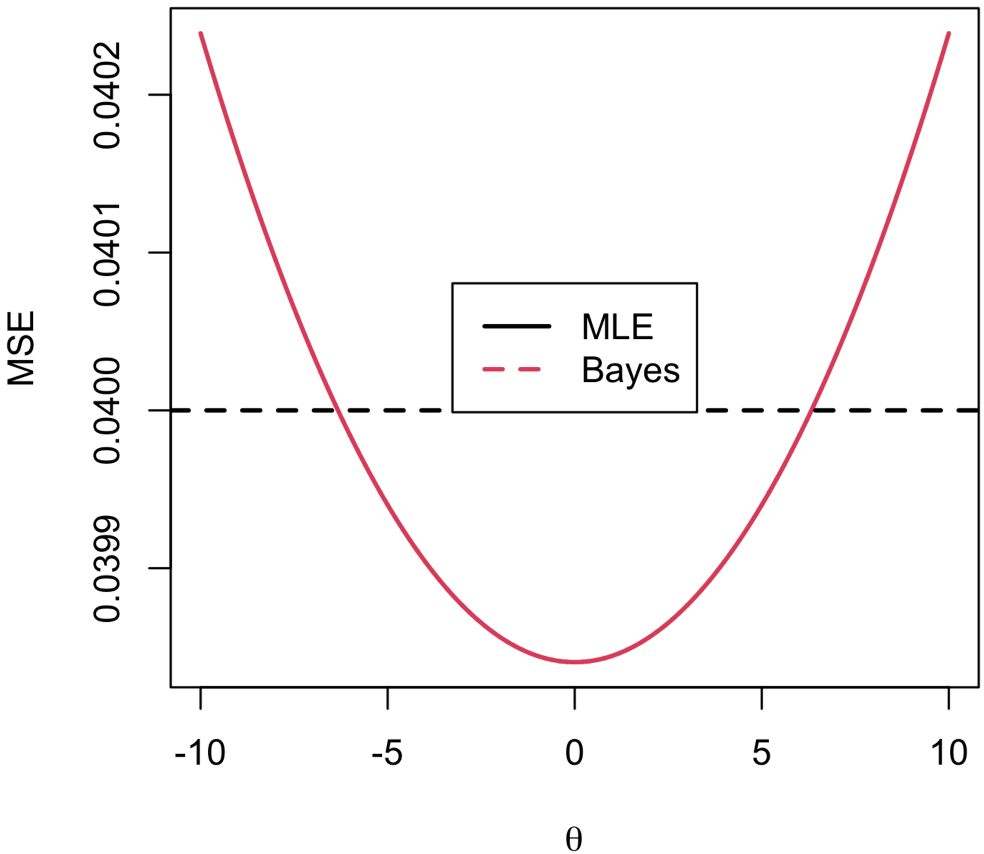
Behavior ?

Plot



Exercise

Repeat this for estimating a future Y under squared error loss using a proper prior and Jeffreys' prior

$$\mathsf{E}_{Y_{n+1}\mid\theta}\left[(Y_{n+1} - \mathsf{E}[Y_{n+1} \mid y_1, \ldots, n])^2]\right]$$

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- T samples from the posterior distribution
- Empirical Estimates & Quantiles from Monte Carlo Samples

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- "Useful" $\rightarrow$ model provides a good approximation; there aren't clear aspects of the data that are ignored or misspecified

Example

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- **zero-inflation** many more zero values than consistent with the poisson model
- Can we use the Posterior Predictive to diagnose whether these are issues with our observed data?

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- compare some feature of the observed data to the datasets simulated from the PP

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- How *extreme* is t_{obs} compared to the distribution of $t(\tilde{y}^{(n)})$

Example Over Dispersion

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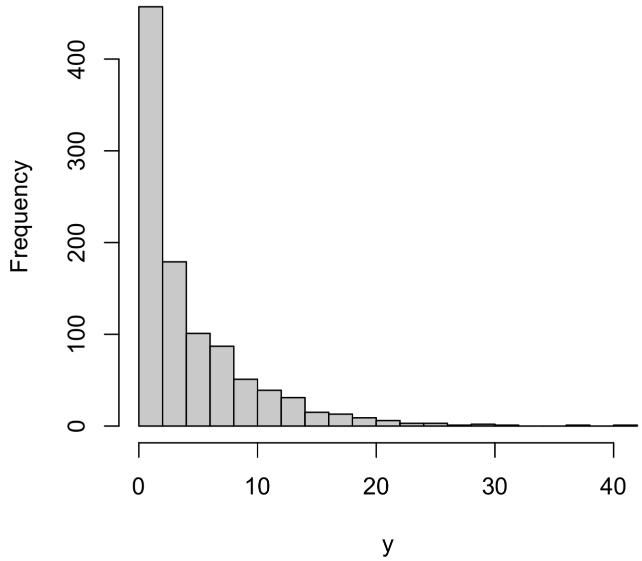
Better approach is to split the data use one piece to learn θ and the other to calculate t_{obs}

Zero Inflated Distribution



- Let the $t()$ be the proportion of zeros

$$t(y) = \frac{\sum_{i=1}^{n} 1(y_i = 0)}{n}$$

Posterior Predictive Distribution

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- Simple Two Stage Hierarchical Model

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- See Bayarri & Berger (2000) for more discussion about why PPP should not be used as a test