Lecture 9: More MCMC: Adaptive Metropolis, Metropolis-Hastings, and Gibbs

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September 23

Example from Last Class

Priors with $\sigma^2 = 1$:

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- Use a Cauchy$(0, 1)$ prior on σ_μ independent of μ and
- Symmetric proposal for μ and σ_τ
- Independent normals centered at current values of μ and σ_μ with covariance $\frac{2.4^2}{d}$Cov(θ) where $d = 2$ (the dimension of θ)

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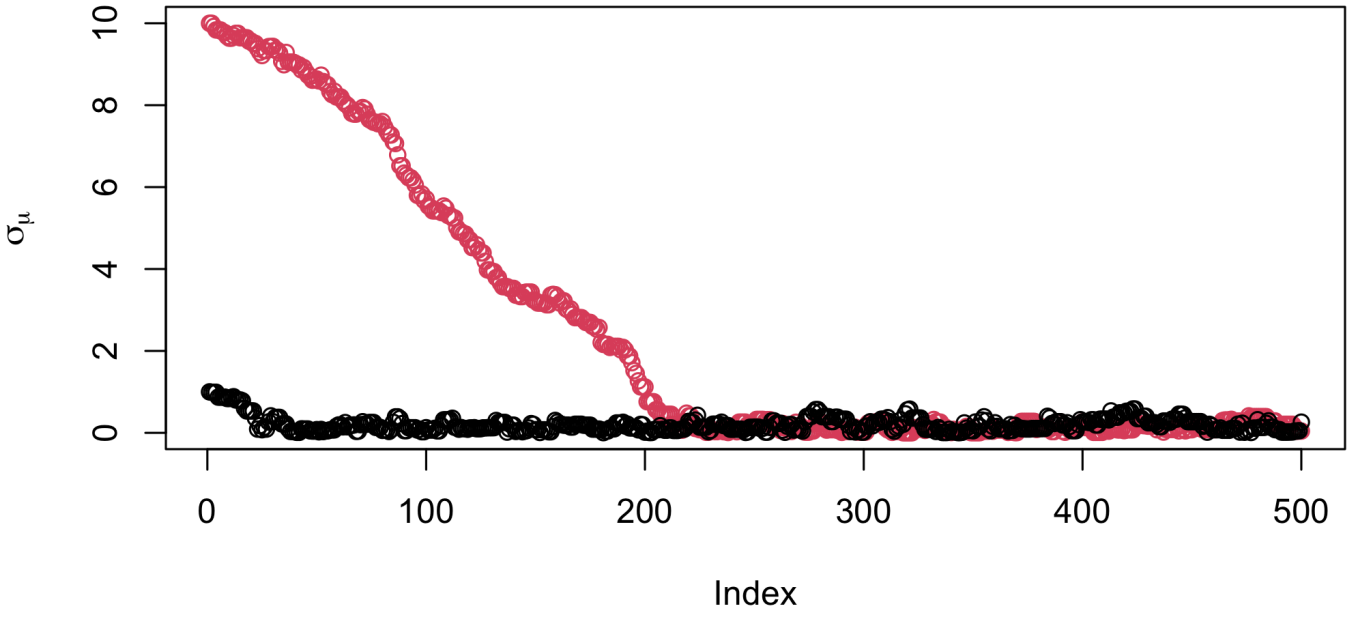

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- Note: version in R is slightly different

Gelman-Rubin Diagnostic

```
theta.mcmc = mcmc.list(mcmc(theta1, start=5000), mcmc(theta2, star
gelman.diag(theta.mcmc)
```

```
## Potential scale reduction factors:
##
##          Point est. Upper C.I.
## mu                1          1
## sigma_mu          1          1
##
## Multivariate psrf
##
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See also `gelman.plot`

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- The null hypothesis would be that the two parts of the chain are from the same distribution.
- The test statistic is a z-score with standard errors adjusted for autocorrelation, and if the p-value is significant for a variable, you need more draws.

Geweke Diagnostic

- The output is the z-score itself (not the p-value).

```
geweke.diag(theta.mcmc)
```

```
## [[1]]
##
## Fraction in 1st window = 0.1
## Fraction in 2nd window = 0.5
##
##       mu sigma_mu
##  -0.7779   0.7491
##
##
## [[2]]
##
## Fraction in 1st window = 0.1
## Fraction in 2nd window = 0.5
##
##       mu sigma_mu
##   0.4454   0.6377
```

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- The Gelman-Rubin approach is quite appealing in using multiple chains
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- Overly sensitive to minor departures from stationarity that do not impact inferences.
- Most common method of assessing convergence is visual examination of trace plots.

Improving

- more iterations and multiple chains

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- thinning to reduce correlations and increase ESS

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- change the proposal distribution q

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- ignore prior and use inverse of Fisher Information (covariance of MLE)

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  - need conditions for vanishing adaptation e.g. that the proposal depends less and less on recent states in the chain - Roberts & Rosenthal (2006) and other conditions

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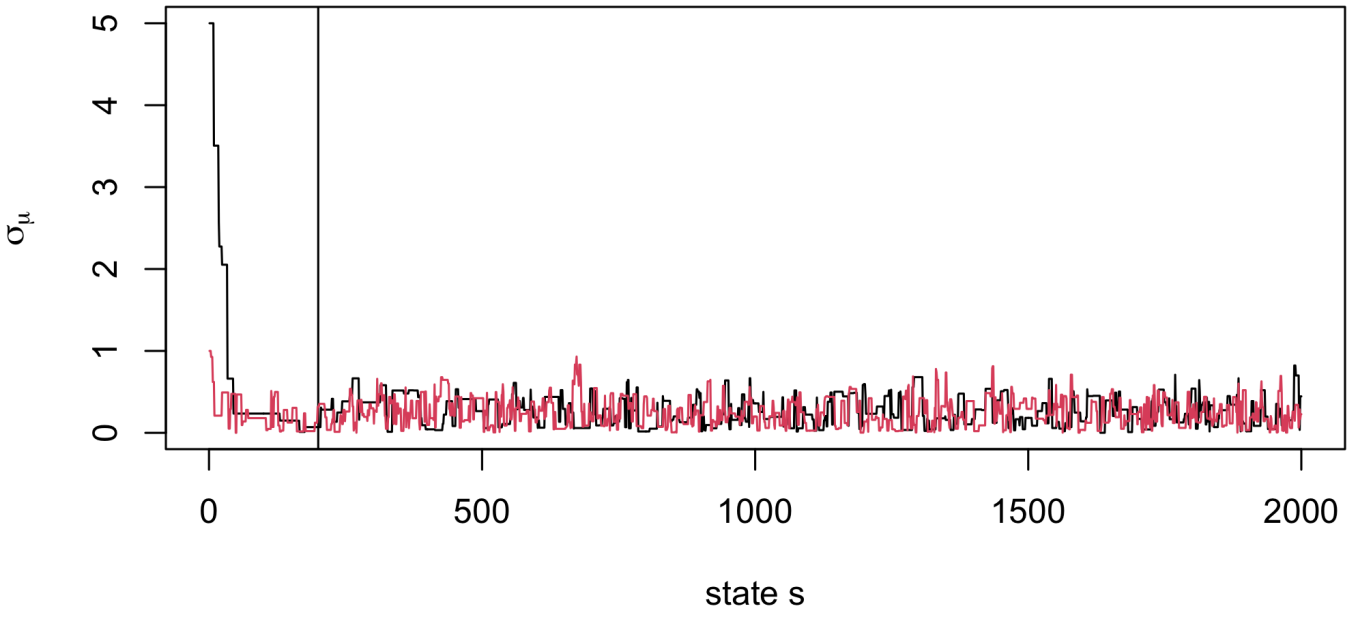
- if s_0 is too large will take longer for adaptation to be seen

Example again



Acceptance rate now around 30-35 % of 10,000 iterations!

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- adjustment for asymmetry in acceptance ratio is key to ensuring convergence to stationary distribution!

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- combinations of the above!

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- transformations of θ

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- convenient to break problems in to K blocks and update them separately

- $\theta = (\theta_{[1]}, \ldots, \theta_{[K]}) = (\theta_1, \ldots, \theta_p)$

At iteration s, for $k = 1, \ldots, K$ Cycle thru blocks: (fixed order or random order)

- propose $\theta^*_{[k]} \sim q_k(\theta_{[k]} \mid \theta^{(s)}_{[<k]}, \theta^{(s-1)}_{[>k]})$

- set $\theta^{(s)}_{[k]} = \theta^*_{[k]}$ with probability

$$\min\left\{1, \frac{\pi(\theta^{(s)}_{[<k]}, \theta^*_{[k]}, \theta^{(s-1)}_{[>k]})\mathcal{L}(\theta^{(s)}_{[<k]}, \theta^*_{[k]}, \theta^{(s-1)}_{[>k]})/q_k(\theta^*_{[k]} \mid \theta^{(s)}_{[<k]}, \theta^{(s-1)}_{[>k]})}{\pi(\theta^{(s)}_{[<k]}, \theta^{(s-1)}_{[k]}, \theta^{(s-1)}_{[>k]})\mathcal{L}(\theta^{(s)}_{[<k]}, \theta^{(s-1)}_{[k]}, \theta^{(s-1)}_{[>k]})/q_k(\theta^{(s-1)}_{[k]} \mid \theta^{(s)}_{[<k]}, \theta^{(s-1)}_{[>k]})}\right\}$$

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- even though joint distribution is messy, full conditionals may be (conditionally) conjugate and easy to sample from!

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- Adaptive Independence Metropolis Hastings (learn a mixture)