Bayesian Model Averaging

Hoff Chapter 9, Liang et al 2007, Hoeting et al (1999), Clyde & George (2004) Statistical Science

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where R^2 is the usual coefficient of determination for model $\mathcal{M}_\gamma$.

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Trade-off of model complexity versus goodness of fit

Lastly, assign distribution to space of models

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- algebra to simplify in from quadratic forms to R^2_γ

Priors on Model Space

$p(\mathcal{M}_\gamma) \Leftrightarrow p(\boldsymbol{\gamma})$

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  $p_{\boldsymbol{\gamma}} \sim \text{Bin}(p, .5)$
- $\gamma_j \mid \pi \overset{\text{iid}}{\sim} \text{Ber}(\pi)$ and $\pi \sim \text{Beta}(a, b)$ then $p_{\boldsymbol{\gamma}} \sim \text{BB}_p(a, b)$

$$p(p_{\boldsymbol{\gamma}} \mid p, a, b) = \frac{\Gamma(p+1)\Gamma(p_{\boldsymbol{\gamma}} + a)\Gamma(p - p_{\boldsymbol{\gamma}} + b)\Gamma(a+b)}{\Gamma(p_{\boldsymbol{\gamma}} + 1)\Gamma(p - p_{\boldsymbol{\gamma}} + 1)\Gamma(p + a + b)\Gamma(a)\Gamma(b)}$$

- $p_{\boldsymbol{\gamma}} \sim \text{BB}_p(1, 1) \sim \text{Unif}(0, p)$

USair Data

```r
library(BAS)
data(usair, package="HH")
poll.bma = bas.lm(log(SO2) ~ temp + log(mfgfirms) +
                             log(popn) + wind +
                             precip + raindays,
                  data=usair,
                  prior="g-prior",
                  alpha=nrow(usair), # g = n
                  n.models=2^6,
                  modelprior = uniform(),
                  method="deterministic")
```

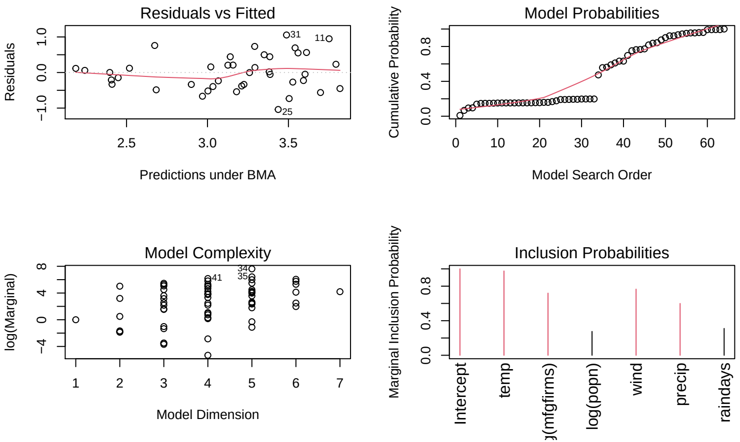
Summary

```
poll.bma

##
## Call:
## bas.lm(formula = log(SO2) ~ temp + log(mfgfirms) + log(popn)
##     wind + precip + raindays, data = usair, n.models = 2^6, p
##     alpha = nrow(usair), modelprior = uniform(), method = "de
##
##
##  Marginal Posterior Inclusion Probabilities:
##     Intercept          temp  log(mfgfirms)      log(popn)
##        1.0000        0.9755         0.7190         0.2757
##        precip      raindays
##        0.5994        0.3104
```

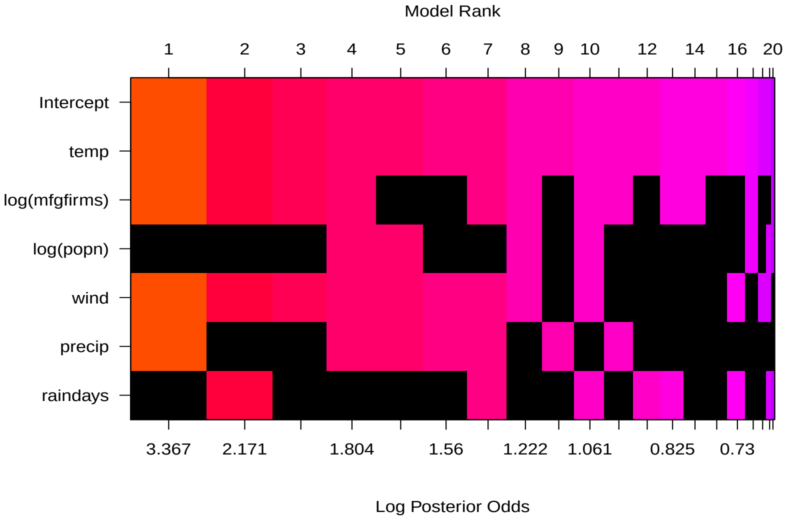
Plots

```r
par(mfrow=c(2,2))
plot(poll.bma, ask=F)
```



Posterior Distribution with Uniform Prior on Model Space

```r
image(poll.bma, rotate=FALSE)
```

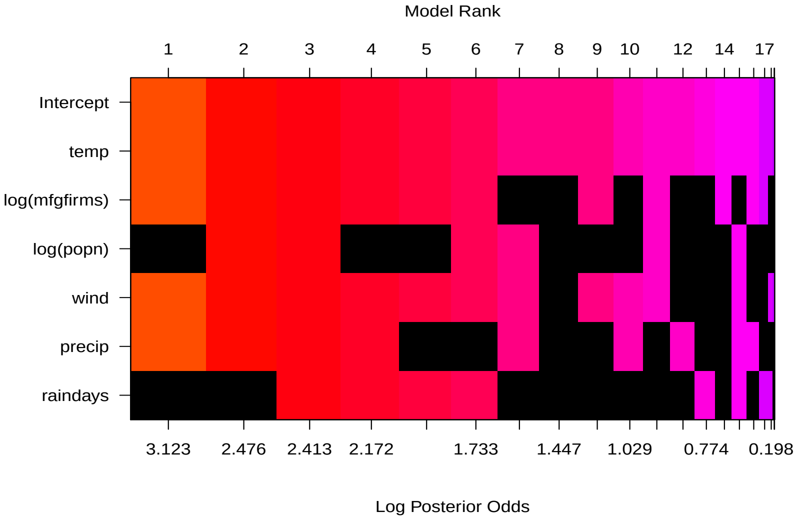


Posterior Distribution with BB(1,1) Prior on Model Space

```
poll.bb.bma = bas.lm(log(SO2) ~ temp + log(mfgfirms) +
                                log(popn) + wind +
                                precip + raindays,
                      data=usair,
                      prior="g-prior",
                      alpha=nrow(usair),
                      n.models=2^6,  #enumerate
                      modelprior=beta.binomial(1,1))
```

BB(1,1) Prior on Model Space

```
image(poll.bb.bma, rotate=FALSE)
```



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Bayes Factor	Interpretation
$B \geq 1$	H_0 supported
$1 > B \geq 10^{-\frac{1}{2}}$	minimal evidence against H_0
$10^{-\frac{1}{2}} > B \geq 10^{-1}$	substantial evidence against H_0
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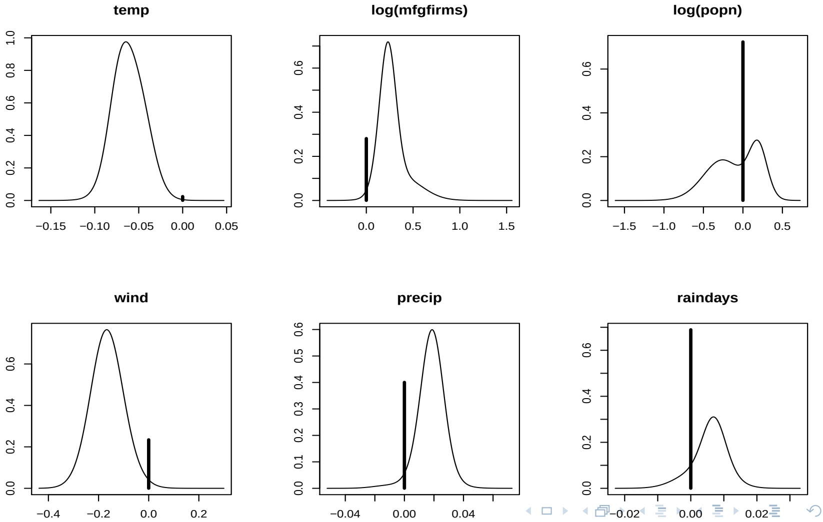
in context of testing one hypothesis with equal prior odds Kass & Raftery (JASA 1996)

Coefficients

```
beta = coef(poll.bma)
par(mfrow=c(2,3));  plot(beta, subset=2:7,ask=F)
```



Bartlett's Paradox

The Bayes factor for comparing $\mathcal{M}_\gamma$ to the null model:

$$BF(\mathcal{M}_\gamma : \mathcal{M}_0) = (1+g)^{(n-1-p_\gamma)/2}(1+g(1-R^2))^{-(n-1)/2}$$

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- What happens to BF as $g \to \infty$?

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- BF converges to a fixed constant $(1+g)^{-p_\gamma/2}$ (does not go to infinity

"Information Inconsistency" see Liang et al JASA 2008

Mixtures of g priors & Information consistency

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All have tails that behave like a Cauchy distribution

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- Is pollution associated with mortality after adjusting for other socio-economic and meteorological factors?
- 15 predictor variables implies $2^{15} = 32,768$ possible models

```r
data(ex1217, package="Sleuth3")
suppressWarnings(library(dplyr))
mortality = mutate(ex1217,
                   logHC = log(HC),
                   logNOX = log(NOX),
                   logSO2 = log(SO2)) %>%
            select(-CITY, -HC, -NOX, -SO2)
```

Jeffreys Zellner-Siow Cauchy Prior

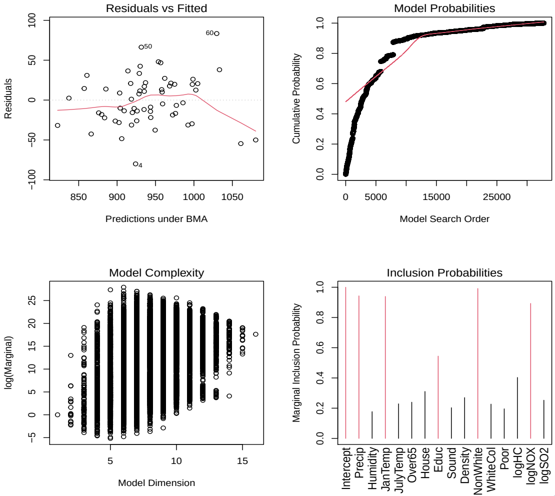
- Jeffreys "reference" prior on α and σ^2
- Zellner-Siow Cauchy prior

$$
\begin{aligned}
1/g &\sim G(1/2, n/2) \\
\boldsymbol{\beta}_\gamma \mid g, \sigma^2 &\sim N(0, g\sigma^2(\mathbf{X}_\gamma^T\mathbf{X}_\gamma)^{-1}) \\
\Rightarrow \boldsymbol{\beta}_\gamma \mid \sigma^2 &\sim C(0, \sigma^2(\mathbf{X}_\gamma^T\mathbf{X}_\gamma)^{-1})
\end{aligned}
$$

```
mort.bma = bas.lm(Mortality ~ ., data=mortality,
                  prior="JZS",
                  alpha=1,
                  n.models=2^15,
                  initprobs="eplogp",
                  method='BAS')
```

Posterior Plots

```
par(mfrow=c(2,2))
plot(mort.bma, ask=FALSE)
```



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poll.inclusion = (models[, 14:16] %*% rep(1, 3)) > 0
prob.poll = sum(poll.inclusion * mort.bma$postprobs)
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Model Space

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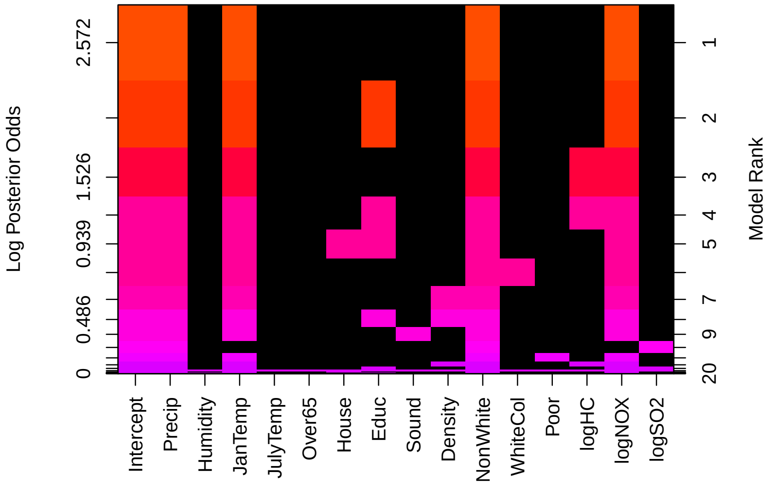
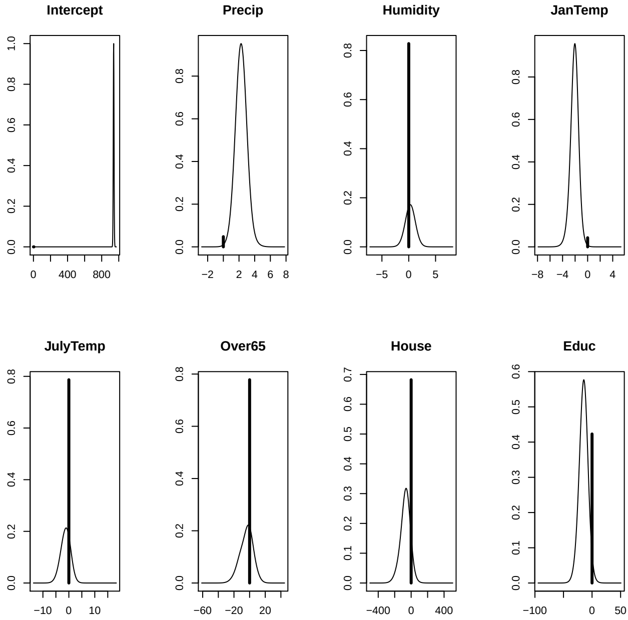
image(mort.bma)
```



Coefficients



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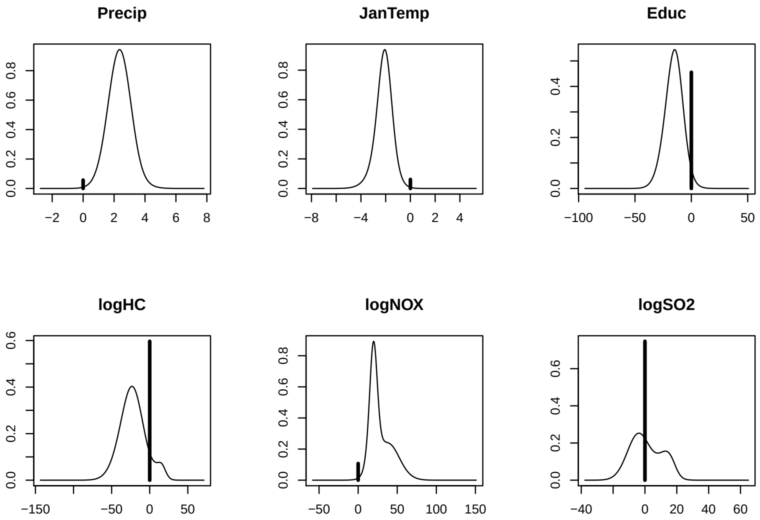

Effect Estimation

- Coefficients in each model are adjusted for other variables in the model
- OLS: leave out a predictor with a non-zero coefficient then estimates are biased!
- Model Selection in the presence of high correlation, may leave out "redundant" variables;
- improved MSE for prediction (Bias-variance tradeoff)
- in BMA all variables are included, but coefficients are shrunk to 0
- Care for "causal" questions and confounder adjustment!

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Model averaging versus Model Selection – what are objectives?