Outliers & Robust Bayesian Regression

Readings: Hoff Chapter 9, West JRSSB 1984, Fúquene, Pérez & Pericchi 2015

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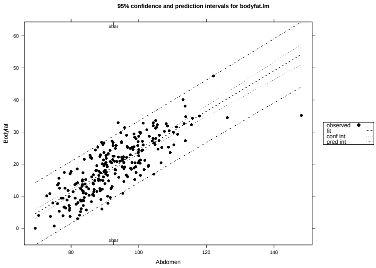
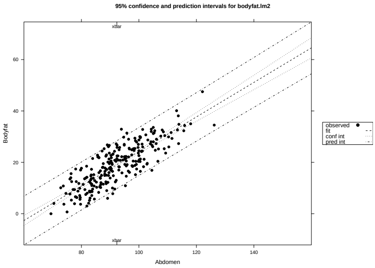
Body Fat Data: Intervals w/ All Data

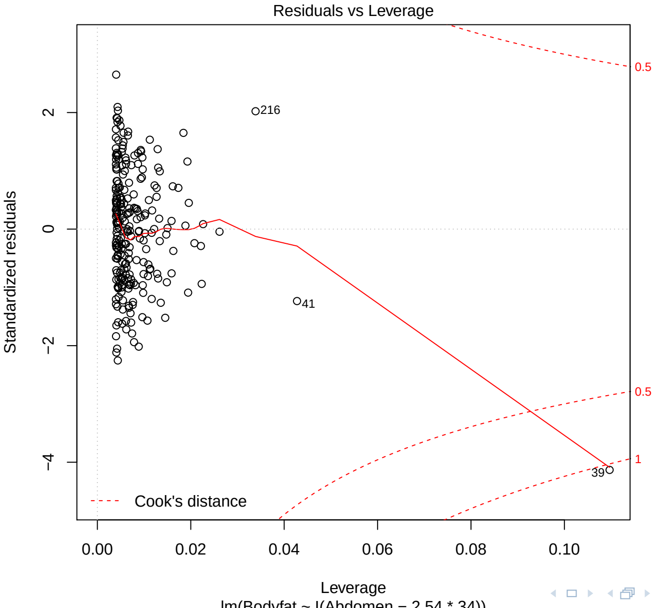
Response % Body Fat and Predictor Waist Circumference





Which analysis do we use? with Case 39 or not – or something different?

Cook's Distance



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- Model Averaging to Account for Model Uncertainty?
- Full model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{I}_n\delta + \epsilon$
- 2^n submodels $\gamma_i = 0 \Leftrightarrow \delta_i = 0$
- If $\gamma_i = 1$ then case i has a different mean "mean shift" outliers.

Mean Shift $=$ Variance Inflation

- Model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{I}_n\delta + \epsilon$
- Prior

$$\delta_i \mid \gamma_i \sim N(0, V\sigma^2\gamma_i)$$
$$\gamma_i \sim \text{Ber}(\pi)$$

Then ϵ_i given σ^2 is independent of δ_i and

$$\epsilon_i^* \equiv \epsilon_i + \delta_i \mid \sigma^2 \begin{cases} N(0, \sigma^2) & wp \quad (1-\pi) \\ N(0, \sigma^2(1+V)) & wp \qquad \pi \end{cases}$$

Model $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \epsilon^*$ “variance inflation”
$V + 1 = K = 7$ in the paper by Hoeting et al. package `BMA`

duke.eps

Simultaneous Outlier and Variable Selection

```
MC3.REG(all.y = bodyfat$Bodyfat, all.x = as.matrix(bodyfat$Abdom
        num.its = 10000, outliers = TRUE)

Model parameters: PI=0.02 K=7 nu=2.58 lambda=0.28 phi=2.85

  15  models were selected
 Best  5  models (cumulative posterior probability =  0.9939):

         prob    model 1  model 2  model 3  model 4  model 5
variables
  all.x  1          x        x        x        x        x
outliers
  39     0.94932  x        x        .        x        .
  204    0.04117  .        .        .        x        .
  207    0.10427  .        x        .        .        x

post prob         0.815    0.095    0.044    0.035    0.004
```

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Treat σ^2 as given, then *influence* of individual observations on the posterior distribution of $\boldsymbol{\beta}$ in the model where $\mathsf{E}[\mathsf{Y}_i] = \mathsf{x}_i^T \boldsymbol{\beta}$ is investigated through the score function:

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An outlying observation y_j is accommodated if the posterior distribution for $p(\boldsymbol{\beta} \mid \mathrm{Y}_{(i)})$ converges to $p(\boldsymbol{\beta} \mid \mathrm{Y})$ for all $\boldsymbol{\beta}$ as $|\mathrm{Y}_i| \to \infty$. Requires error models with influence functions that go to zero such as the Student t (O'Hagan, 1979)

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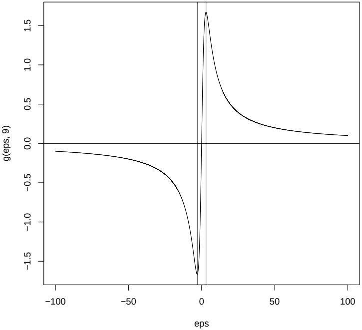

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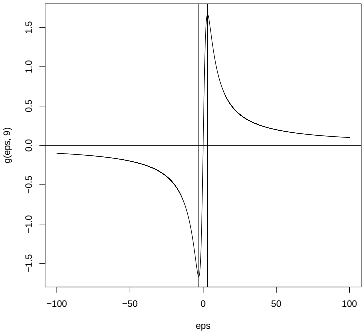

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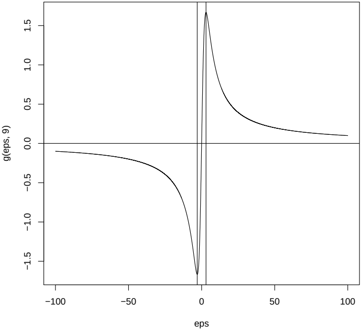

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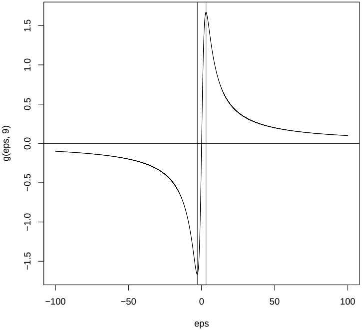

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Integrate out "latent" λ's to obtain marginal distribution.

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Model Specification via R2jags

```
rr.model = function() {
  for (i in 1:n) {
    mu[i] <- alpha0 + alpha1*(X[i] - Xbar)
    lambda[i] ~ dgamma(9/2, 9/2)
    prec[i] <- phi*lambda[i]
    Y[i] ~ dnorm(mu[i], prec[i])
  }
  phi ~ dgamma(1.0E-6, 1.0E-6)
  alpha0 ~ dnorm(0, 1.0E-6)
  alpha1 ~ dnorm(0,1.0E-6)
}
```

Specifying which Parameters to Save

The parameters to be monitored and returned to R are specified with the variable `parameters`

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Output

	mean	sd	2.5%	50%	97.5%
beta0	-41.70	2.75	-46.91	-41.67	-36.40
beta1	0.66	0.03	0.60	0.66	0.71
sigma	4.48	0.23	4.05	4.46	4.96
mu34	15.10	0.35	14.43	15.10	15.82
y34	14.94	5.15	4.37	15.21	24.65
lambda[39]	0.33	0.16	0.11	0.30	0.72

95% HPD interval for expected bodyfat $(14.5, 15.8)$

95% HPD interval for bodyfat $(5.1, 25.3)$

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Case 39 down weighted by λ_{39}

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\lambda_j \mid \text{rest}, Y \sim G\left(\frac{\nu+1}{2}, \frac{\phi(y_j - \alpha - \beta x_j)^2 + \nu}{2}\right)
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Weights

Under prior $E[\lambda_i] = 1$

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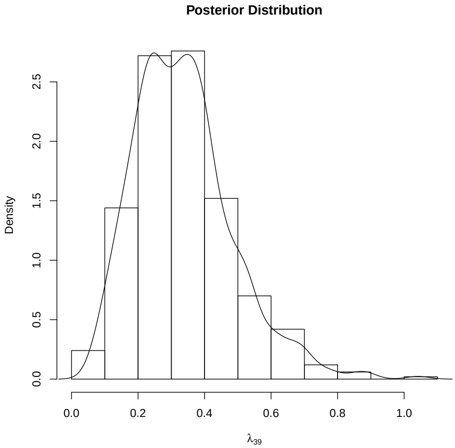
Under posterior, large residuals are down-weighted (approximately those bigger than $\sqrt{\nu}$)



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Sumary

- Classical diagnostics useful for EDA (checking data, potential outliers/influential points) or posterior predictive checks
- BMA/BVS and Bayesian robust regression avoid interactive decision making about outliers
- Robust Regression (Bayes) can still identify outliers through distribution on weights
- continuous versus mixture distribution on scale parameters
- Other mixtures (sub populations?) on scales and β?