STA 702: Lecture 0

Course Overview

Merlise Clyde



Welcome to STA 702!



Learn the foundations and theory of Bayesian inference in the context of several models.



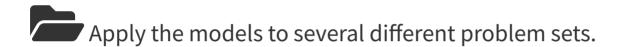
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A Bayesian version will usually make things better...







Instructor

Dr Merlise Clyde

- https://www2.stat.duke.edu/~clyde
- **1** 223 Old Chemistry
- https://www2.stat.duke.edu/courses/Fall22/sta702.001
- **See course website for OH**



TAs

Steven Winter

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- **See** course website for OH
- **1** See course website for location



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 - 1. Review Chapters 1 to 5 of the Casella and Berger book You can find the solution manual here
 - 2. Focus on the following topics:
 - basic probability theory, random variables, transformations of random variables and change of variables, expectations of random variables, common families of probability distribution functions including multivariate distributions
 - concepts of convergence, principles of statistical inference, likelihood based inference, sampling distributions and hypothesis testing.



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 - https://www2.stat.duke.edu/courses/Fall22/sta702.001/resources/.
 - Resources for the StaSci BootCamp 2021



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 - Resources for the StaSci BootCamp 2021
- Labs will introduce/review concepts



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- What is the difference between this course and STA360 or STA602?





■ See:

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- Check the dates for exams in the Course Calendar and let me know ASAP if there are issues



Topics

- Basics of Bayesian Models
- Loss Functions, Inference and Decision Making
- Predictive Distributions
- Predictive Distributions and Model Checking
- Bayesian Hypothesis Testing
- Multiple Testing
- MCMC (Gibbs & Metropolis Hastings Algorithms)
- Model Uncertainty
- Bayesian Generalized Linear Models
- Hiearchical Modeling and Random Effects
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- Grading
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 - 20% HW
 - 10% Lab
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- Confirm that you have access to Sakai, Gradescope, and GitHub.



Important Dates

- Tues, Aug 30 Classes begin
- Fri, Sept 9 Drop/Add ends
- Fri, Oct 7 Midterm I (*tentative*)
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Also refer to the schedule on the website for updated breakdown of the courses. Remember to refresh the page frequently. See here: Class Schedule.



Bayes Rules! Getting Started!



Basics of Bayesian inference

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- More to come later.



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- Typically uses large sample (asymptotic) theory to obtain confidence intervals and do hypothesis testing.



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 - a computational framework for model estimation, selection, decision making and validation.
 - builds on likelihood inference



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Pr(A) = marginal probability of event A, Pr(B|A) = conditional probability of event B given event A, and so on.



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- Now, how do we get from Step 1 to 3? Bayes' rule!

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We will use this over and over throughout the course!

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- -- Practical Bayes: Combines Subjective Bayes for aspects of a problem that one understands, and Objective Bayes elsewhere



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- Hence, it is very seldom the case that a weakly informative prior is not preferred over no prior. (Model selection is one case where one needs to be careful!)
- One (very important) role of the prior is to stabilize estimates (shrinkage) in the presence of limited data.



Next Steps

Work on Lab 0

Finally, here are some readings to entertain you. Make sure to glance through them within the next week. See here: Course Resources

- 1. Efron, B., 1986. Why isn't everyone a Bayesian?. The American Statistician, 40(1), pp. 1-5.
- 2. Gelman, A., 2008. Objections to Bayesian statistics. Bayesian Analysis, 3(3), pp. 445-449.
- 3. Diaconis, P., 1977. Finite forms of de Finetti's theorem on exchangeability. Synthese, 36(2), pp. 271-281.
- 4. Gelman, A., Meng, X. L. and Stern, H., 1996. Posterior predictive assessment of model fitness via realized discrepancies. Statistica sinica, pp. 733-760.
- 5. Dunson, D. B., 2018. Statistics in the big data era: Failures of the machine. Statistics & Probability Letters, 136, pp. 4-9.

