STA 601: Lecture 1

Basics of Bayesian Statistics

Merlise Clyde

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(4) **Loss Function** Depends on what you want to report; estimate of θ, predict future Y_{n+1}, etc

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  Note: the marginal likelihood and maximized likelihood are *very* different!

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Recall Derivation:

Marginal Likelihood

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"Averaging" likelihood over prior

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Beta Prior Distributions

Beta(a,b) is a probability density function (pdf) on (0,1),

$$\pi(\theta) = \frac{1}{B(a,b)}\theta^{a-1}(1-\theta)^{b-1}$$

Use the "**kernel**" trick

$$\pi(\theta \mid y) \propto \mathcal{L}(\theta \mid y)\pi(\theta)$$

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  - $a + b$ "prior sample size" (number of trials in a hypothetical experiment)
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- Should be easy to do "prior elicitation " (process of choosing the prior hyperparamters)

Summaries & Properties

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Weighted average of prior mean and MLE where weight for $\theta_0 \propto n_0$ and weight for $\hat{\theta} \propto n$

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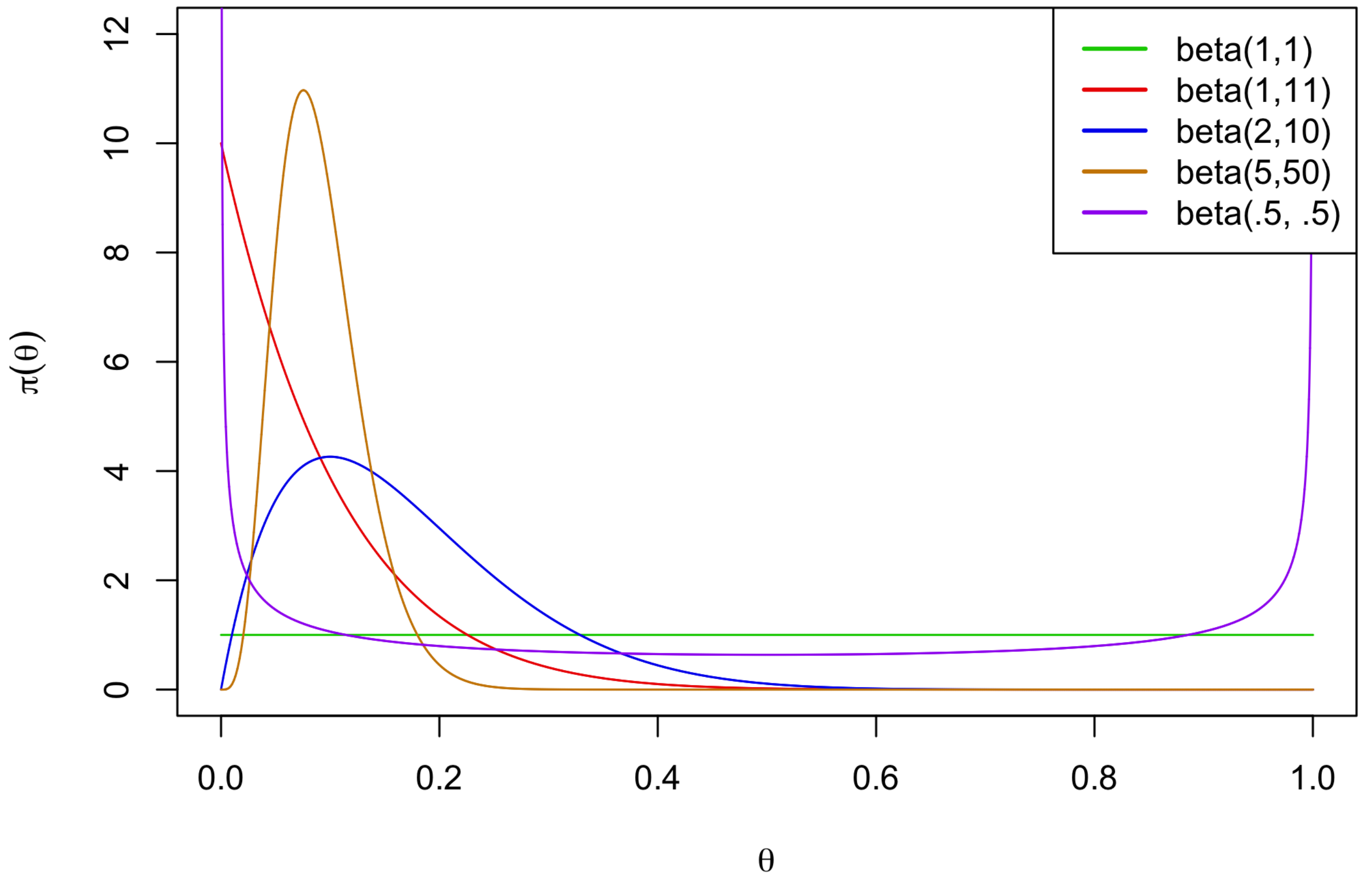
- as we get more information from the data $n \to \infty$ we have $\tilde{\theta} \to \hat{\theta}$ and **consistency** ! As $n \to \infty$, $\mathsf{E}[\tilde{\theta}] \to \theta_{\text{true}}$

Some possible prior densities



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- limiting case of a Beta $a \to 0$ and $b \to 0$ (Haldane's prior)

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- no shrinkage $\textsf{E}[\theta \mid y] = \frac{y}{n} = \tilde{\theta} = \hat{\theta}$

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Thus Jefferys' prior is a $\textsf{Beta}(1/2, 1/2)$

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Chain Rule!

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Find information matrix for ρ from $I(\theta)$

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Show that the prior satisfies the invariance property that