Lecture 6: Bayesian Hypothesis Testing Continued

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4. Obtain posterior probabilities of $\mathcal{H}_0$ and $\mathcal{H}_1$ via Bayes Theorem or Bayes Factors
5. Report based on loss (optional)

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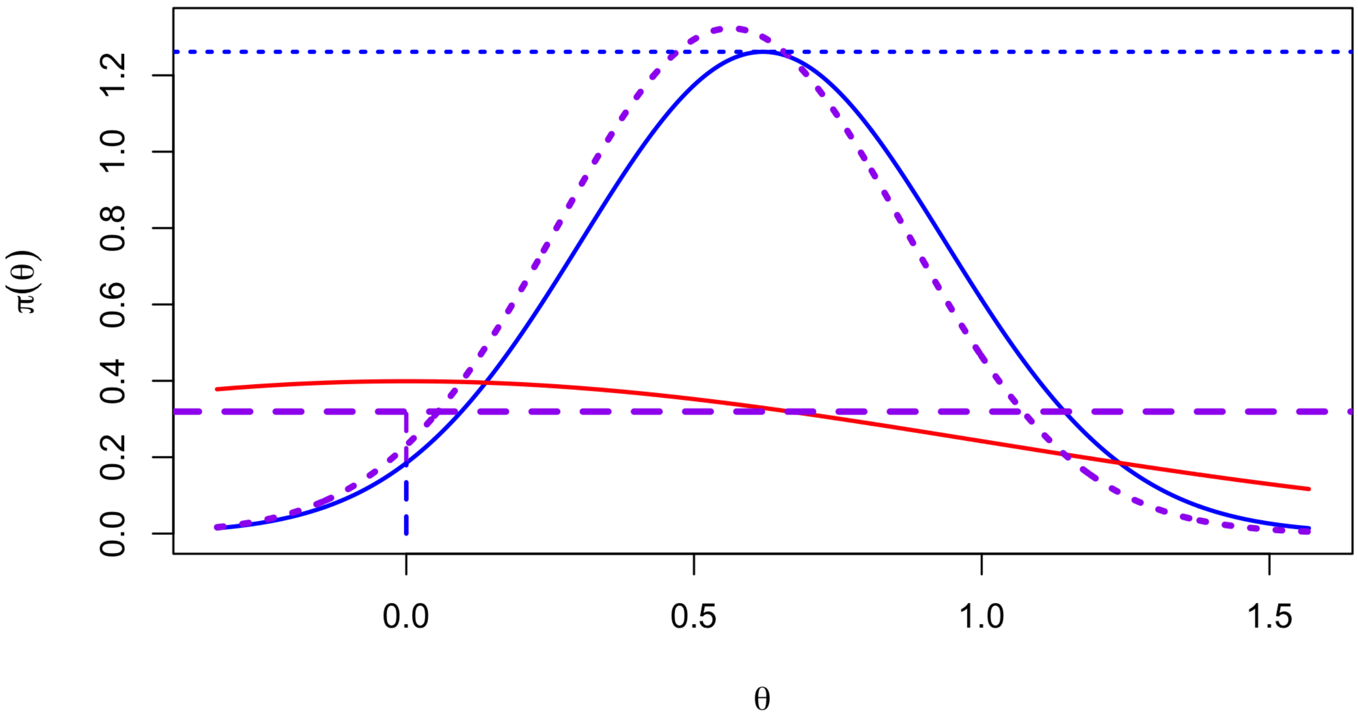
Alternative expression for Bayes Factor

$$\mathcal{BF}_{10} = \frac{p(y^{(n)} \mid \mathcal{H}_1)}{p(y^{(n)} \mid \mathcal{H}_0)} = \frac{\pi_\theta(0 \mid \mathcal{H}_1)}{\pi_\theta(0 \mid y^{(n)}, \mathcal{H}_1)}$$

Marginal Likelihoods & Evidence



$\mathcal{BF}_{10}$ = 1.73 Posterior Probability of $\mathcal{H}_0$ = 0.3665 versus p-value of 0.05

Decisions

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  - if $\pi(\mathcal{H}_1 \mid y^{(n)}) > .5$ choose $\mathcal{H}_1$,
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- report $\hat{\theta}$ that minimizes Bayes expected loss

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- marginal posterior distribution of θ

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Prior on θ is a mixture model:

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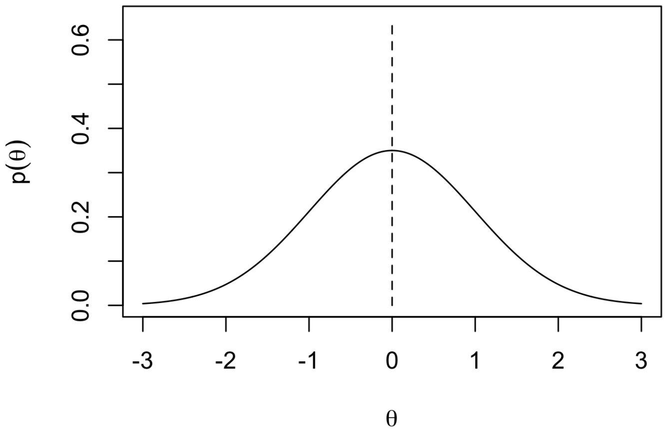

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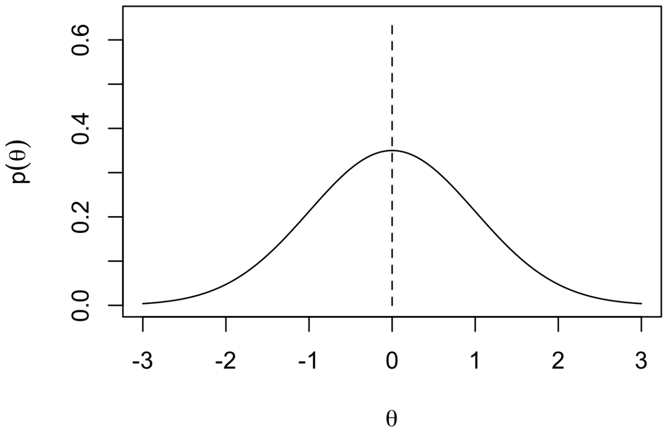

- how to sample from prior?

Posterior under Spike & Slab Prior

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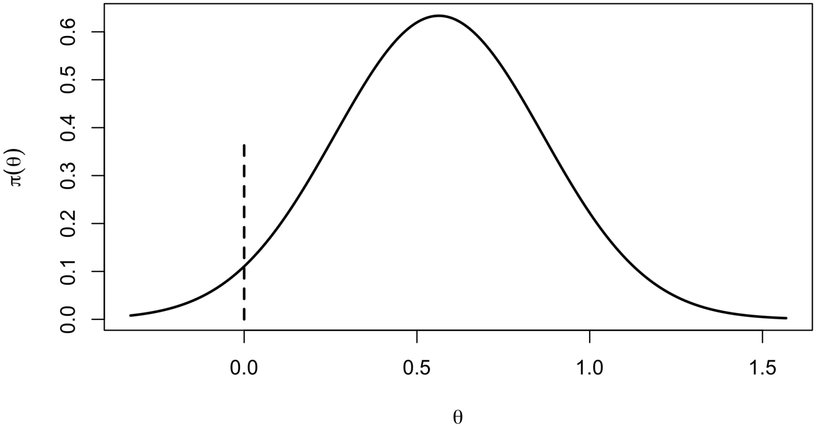
- updated "slab" hyperparameters



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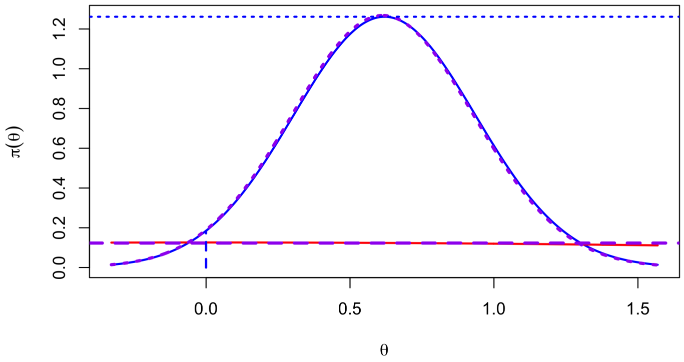
- Bayes Factor and posterior probabilities of $\mathcal{H}_i$ depend on τ_0 through $p(y^{(n)} \mid \mathcal{H}_1)$

1. What is impact of τ_0 on $\mathcal{BF}_{01}$?
2. How do we choose τ_0?

Question 1.

$$\mathcal{BF}_{01} = \frac{\pi(0 \mid \mathcal{H}_1, y^{(n)})}{\pi(0 \mid \mathcal{H}_1)}$$

Precision



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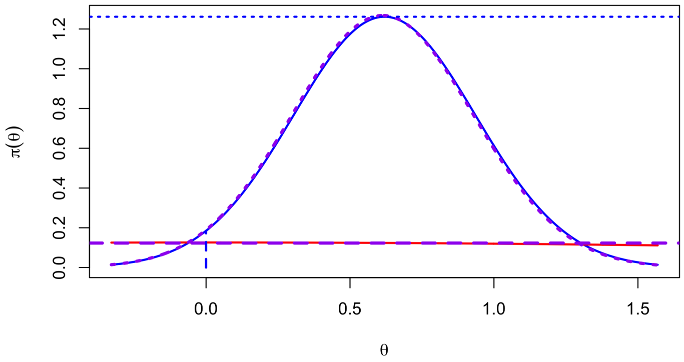

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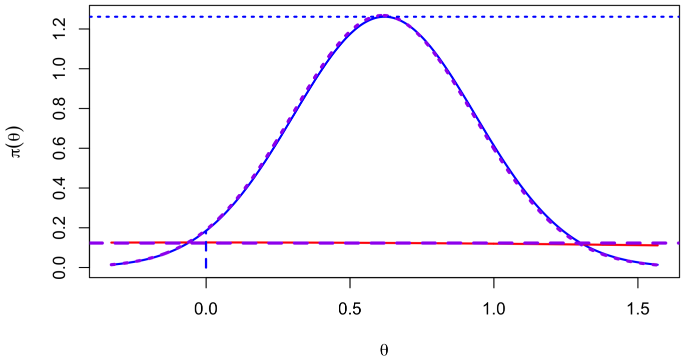

- $\tau_0 = 1/10$
- Bayes Factor for $\mathcal{H}_0$ to $\mathcal{H}_1$ is 1.5
- Posterior Probability of $\mathcal{H}_0$ = 0.6001

What about even more vague priors?

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Default UIP

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- Is a fixed τ_0 consistent as $n \to \infty$?

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- no closed form expressions for marginal likelihood!

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- **intrinsic prior** on θ that leads to the IBF