Lecture 13: Bayesian Multiple Testing

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October 20

Normal Means Model

Recall normal model with

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- concern: is that # errors blows up with n (n = # tests = dimension of $\{\mu_i\}$)

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- e.g. $\gamma^{(n)} = (0, 1, 0, 0, \ldots, 1)^T$

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- Distribution of p_γ?

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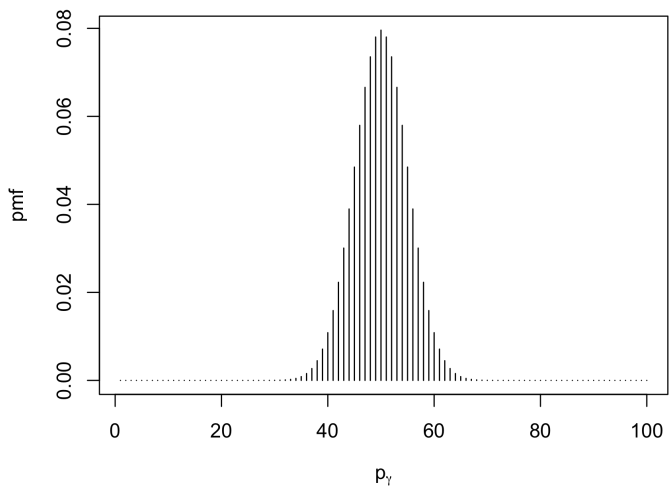

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- probability of at least one signal is $1 - 0.5^n \approx 1$

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- Conjugate or nice setups we can integrate out μ_i and then maximize marginal likelihood for π_0 and τ
- Numerical integration or EM algorithms to get $\hat{\pi}_0^{\mathsf{EB}}$ and $\hat{\tau}^{\mathsf{EB}}$
- Clyde & George (2000) Silverman & Johnstone (2004) for wavelet regression

Expectation-Maximization

- introduce latent variables so that "complete" data likelihood is nice! e.g. γ:

$$y_i \mid \gamma_i, \tau \overset{ind}{\sim} \mathsf{N}(0,1)^{1-\gamma_i}\mathsf{N}(0, 1+\tau)^{\gamma_i}$$

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  - E-step: find the expected values of the latent sufficient statistics given the data, $\hat{\pi}_0^{(t)}$, $\hat{\tau}^{(t)}$

$$\hat{\gamma}^{(t)} = \textsf{E}[\gamma_i \mid y, \hat{\pi}_0^{(t)}, \hat{\tau}^{(t)}]$$

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- This is good as it protects against Type I errors blowing up as n increases!
- However it becomes more and more difficult to find the few needles in a haystack!

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- $\gamma_i \sim \text{Bernoulli}(1 - \pi_0)$
- Update the prior for π_0 to include the info $\gamma_j = 0$ for $j = 2, \ldots, n$

$$\pi(\pi_0 \mid \gamma_2, \ldots, \gamma_n) \propto \pi_0^{a-1}(1 - \pi_0)^{b-1} \prod_{j=2}^{n} \pi_0^{1-\gamma_j}(1 - \pi_0)^{\gamma_j}$$

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Beta Posterior

$$\pi_0 \mid \gamma_2, \ldots, \gamma_n \sim \mathsf{Beta}(\mathsf{a} + \mathsf{n}\text{ - }\mathsf{1}, \mathsf{b})$$

with mean

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- Multiplicity adjustment as in the EB case
- Scott & Berger (2006 JSPI, 2010 AoS) show that above framework protects against increasing Type I errors with n; We also get FDR control automatically

Induced Prior on p_γ

Exercise: If $p_\gamma \mid \pi_0 \sim \textsf{Binomial}(n, 1 - \pi_0)$ and $\pi_0 \sim \textsf{Beta}(1,1)$

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Bottomline: We need to "learn" key parameters in our hierarchical prior or the magic doesn't work! Borrowing comes through using all the data to inform about "global" parameters in the prior, in this case π_0

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- Don't report selected hypotheses but report results under model averaging!

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- active area of research!