STA 702: Lecture 0

Course Overview

Merlise Clyde

Welcome to STA 702!

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> A Bayesian version will usually make things better...
>
> -- Andrew Gelman.

Instructor

Dr Merlise Clyde

- clyde@duke.edu
- https://www2.stat.duke.edu/~clyde
- 223 Old Chemistry
- https://www2.stat.duke.edu/courses/Fall22/sta702.001
- See course website for OH

TAs

Rick Presman

- rick.presman@duke.edu
- See course website for OH
- See course website for location

FAQs

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  1. Review Chapters 1 to 5 of the Casella and Berger book You can find the solution manual here
  2. Focus on the following topics:
     - basic probability theory, random variables, transformations of random variables and change of variables, expectations of random variables, common families of probability distribution functions including multivariate distributions
     - concepts of convergence, principles of statistical inference, likelihood based inference, sampling distributions and hypothesis testing.

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  - Resources for the StaSci BootCamp 2021

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- Labs will introduce/review concepts

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- What is the difference between this course and STA360 or STA602 ?

Course structure and policies

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- Check the dates for exams in the Course Calendar and let me know ASAP if there are issues

Topics

- Basics of Bayesian Models
- Loss Functions, Inference and Decision Making
- Predictive Distributions
- Predictive Distributions and Model Checking
- Bayesian Hypothesis Testing
- Multiple Testing
- MCMC (Gibbs & Metropolis Hastings Algorithms)
- Model Uncertainty
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Also refer to the Class Schedule.

Other details

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- Confirm that you have access to Sakai, Gradescope, and GitHub.

Important Dates

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- Fri, Sept 9   Drop/Add ends
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Also refer to the schedule on the website for updated breakdown of the courses. Remember to refresh the page frequently. See here: Class Schedule.

Bayes Rules! Getting Started!

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- More to come later.

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- Frequentist ML estimation finds the one value of θ that maximizes the likelihood.
- Typically uses large sample (asymptotic) theory to obtain confidence intervals and do hypothesis testing.

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$\Pr(A)$ = marginal probability of event A, $\Pr(B|A)$ = conditional probability of event B given event A, and so on.

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- Now, how do we get from Step 1 to 3? Bayes' rule!

$$p(\theta|y) = \frac{p(\theta)p(y|\theta)}{\int_\Theta p(\tilde{\theta})p(y|\tilde{\theta})\mathrm{d}\tilde{\theta}} = \frac{p(\theta)p(y|\theta)}{p(y)}$$

We will use this over and over throughout the course!

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-- Practical Bayes: Combines Subjective Bayes for aspects of a problem that one understands, and Objective Bayes elsewhere

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- Hence, it is very seldom the case that a weakly informative prior is not preferred over no prior. (Model selection is one case where one needs to be careful!)
- One (very important) role of the prior is to stabilize estimates (shrinkage) in the presence of limited data.

Next Steps

Finally, here are some readings to entertain you. Make sure to glance through them within the next week. See here: Course Resources

1. Efron, B., 1986. Why isn't everyone a Bayesian?. The American Statistician, 40(1), pp. 1-5.
2. Gelman, A., 2008. Objections to Bayesian statistics. Bayesian Analysis, 3(3), pp. 445-449.
3. Diaconis, P., 1977. Finite forms of de Finetti's theorem on exchangeability. Synthese, 36(2), pp. 271-281.
4. Gelman, A., Meng, X. L. and Stern, H., 1996. Posterior predictive assessment of model fitness via realized discrepancies. Statistica sinica, pp. 733-760.
5. Dunson, D. B., 2018. Statistics in the big data era: Failures of the machine. Statistics & Probability Letters, 136, pp. 4-9.