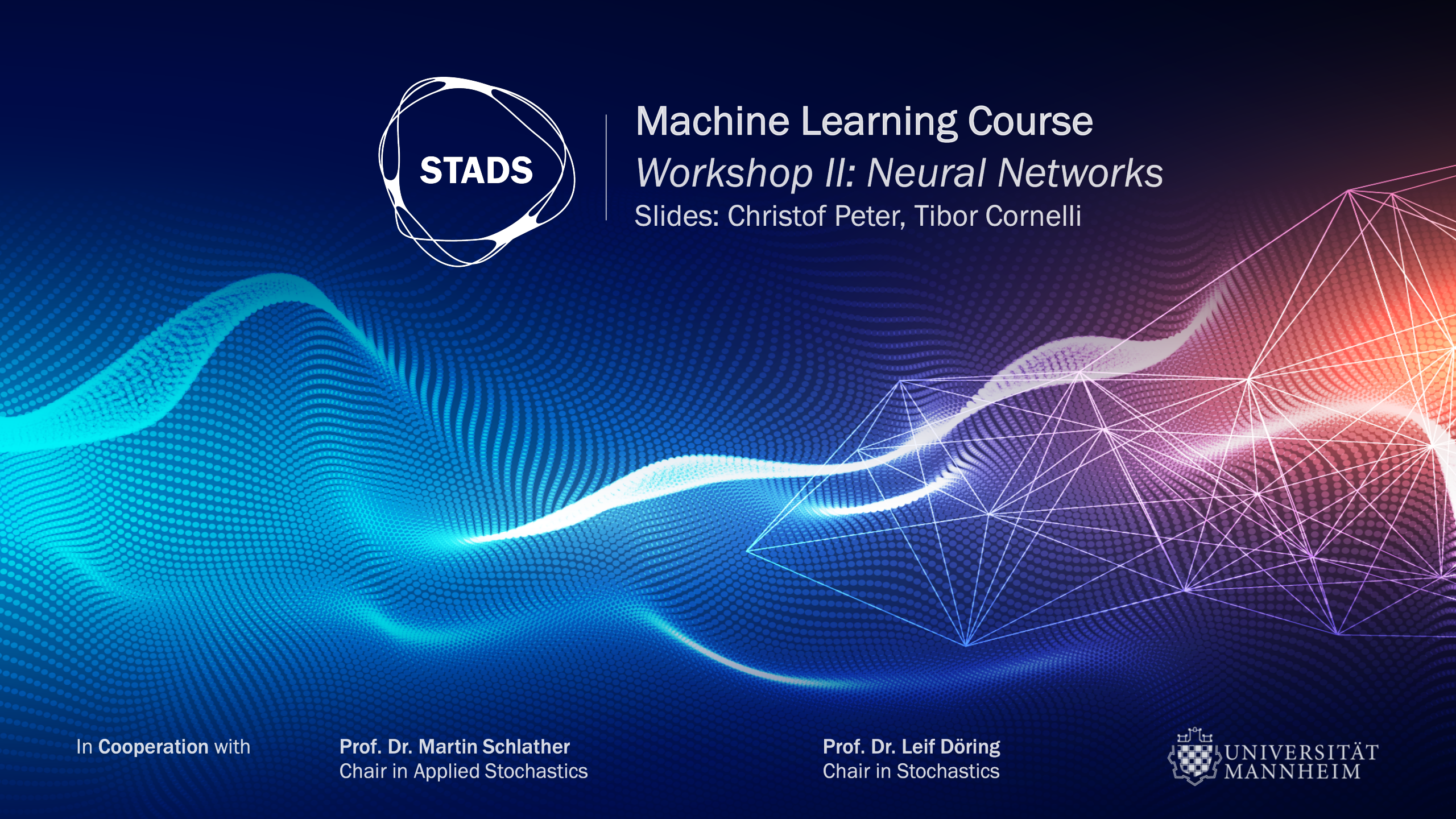

Machine Learning Course

Workshop II: Neural Networks

Slides: Christof Peter, Tibor Cornelli

In **Cooperation** with

Prof. Dr. Martin Schlather
Chair in Applied Stochastics

Prof. Dr. Leif Döring
Chair in Stochastics

UNIVERSITÄT MANNHEIM

Course structure

Part I: Basics of Machine Learning	04.10 – Introduction to machine learning **11.10 – Neural networks**
Part II: Computer Vision (CV)	18.10 – Convolutional Neural Networks 25.10 – Practical application of CNNs 01.11 – CV Challenge presentation
Part III: Natural Language Processing (NLP)	08.11 – Recurrent Neural Networks 15.11 – Practical application of RNNs 22.11 – NLP Challenge Presentation

Agenda

- Intro
- How do neural networks work?
- Practical implementation

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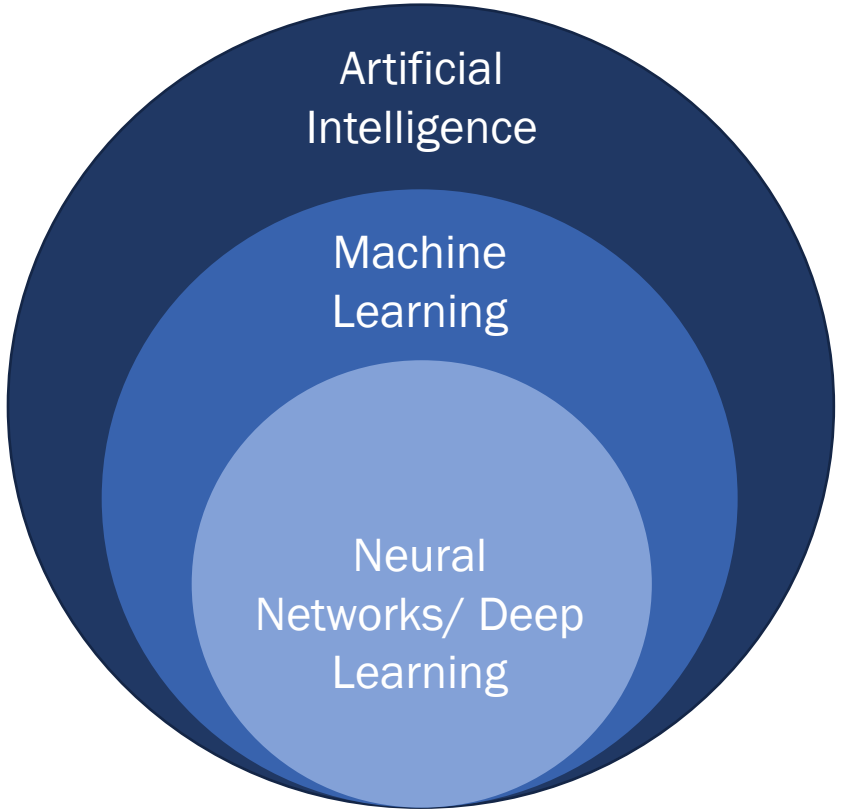
Classification & terminology

Our focus today lies on ANN‘s



AI	Artificial Intelligence
ML	Machine Learning
ANN	Artificial Neural Network
DNN	Deep Neural Network
CNN	Convolutional Neural Network

History

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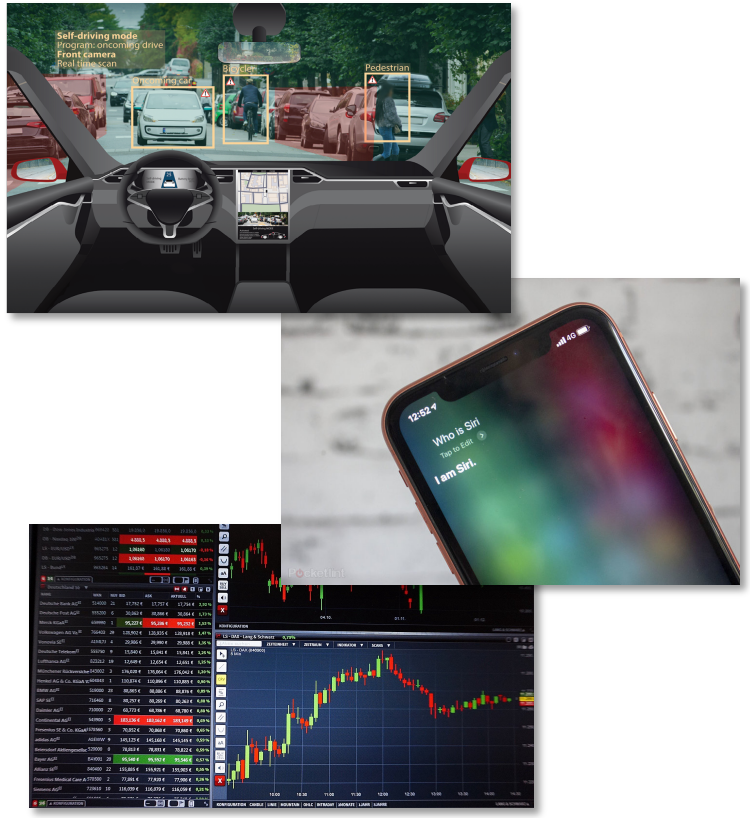
What are NNs used for?

Artificial neural networks have many applications. Currently, computer vision and natural language processing are the hottest topics.

- **Computer Vision**
  - Face Regocnition
  - Autonomous Driving
  - Charakter recognition
- **Natural Language Processing (NLP)**
  - Word suggestions
  - Translators
  - Chat-Bots
- **Other**
  - Logistics
  - Marketing
  - Stock Markets



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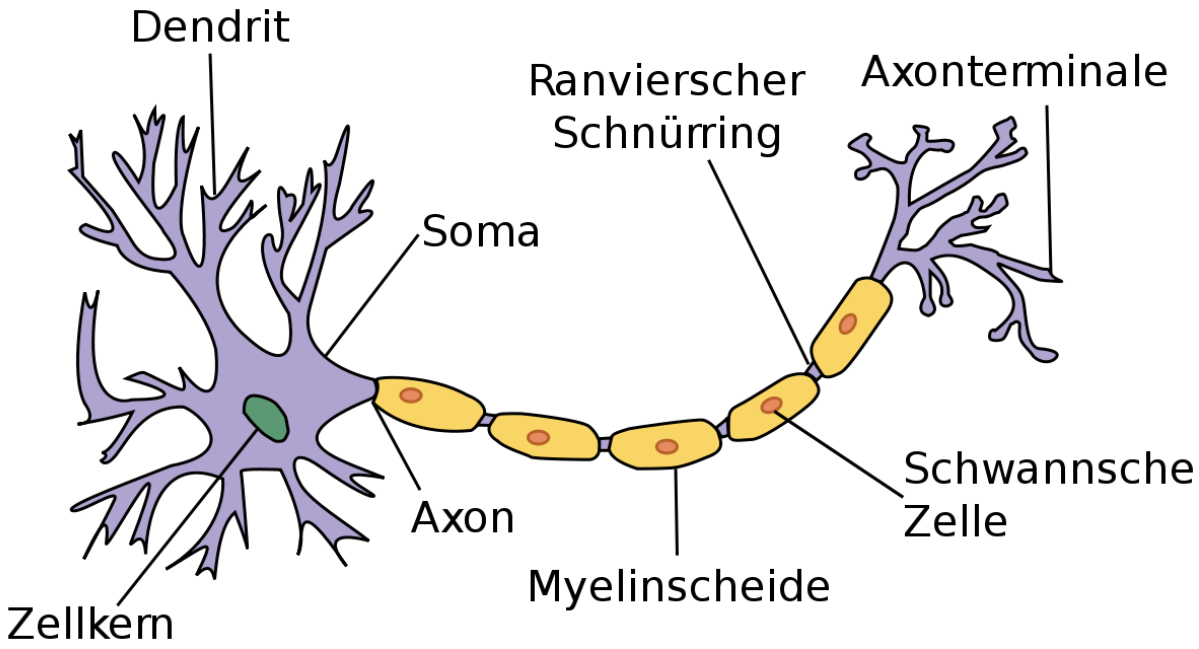
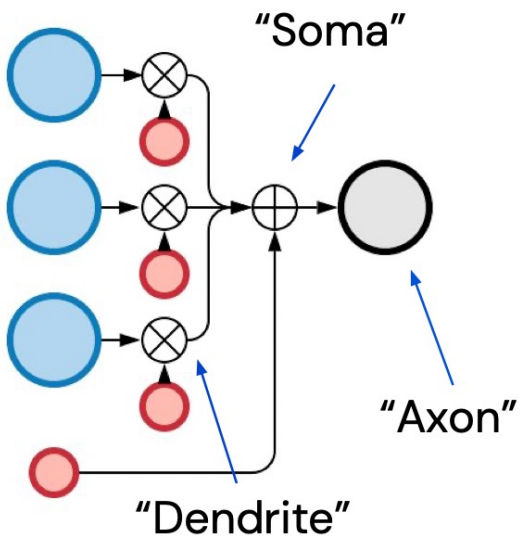
Why “neural”?



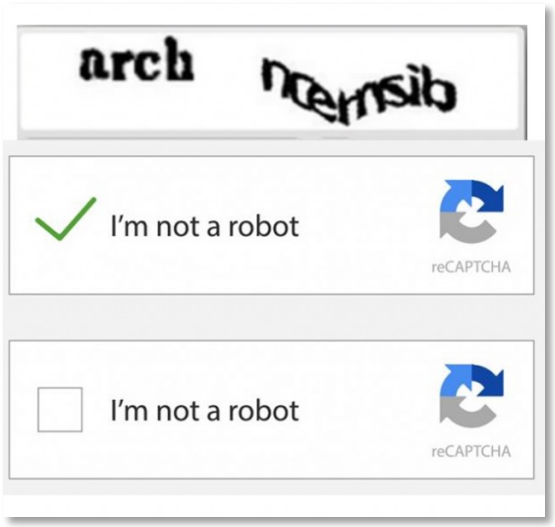


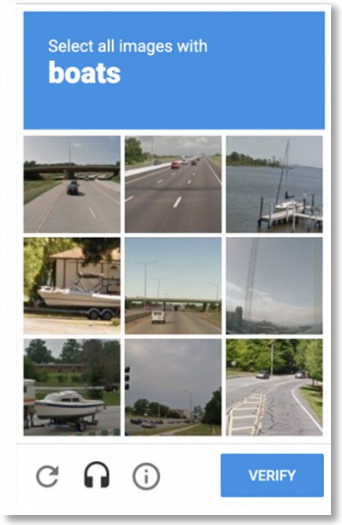
$$\sum_{i=1}^{d} \mathbf{w}_i \mathbf{x}_i + b$$

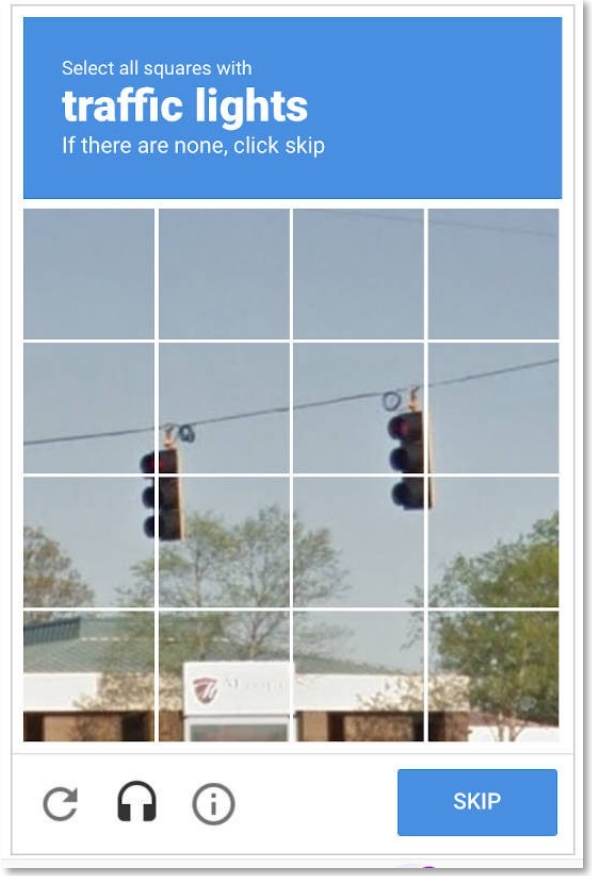
$$\sum_{i=0}^{d} \mathbf{w}_i \mathbf{x}_i \qquad \mathbf{x}_0 := 1$$

Excursion: Recaptcha

Quelle: https://www.techradar.com/news/captcha-if-you-can-how-youve-been-training-ai-for-years-without-realising-it







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Agenda

Intro

How do neural Networks work?

Practical implementation

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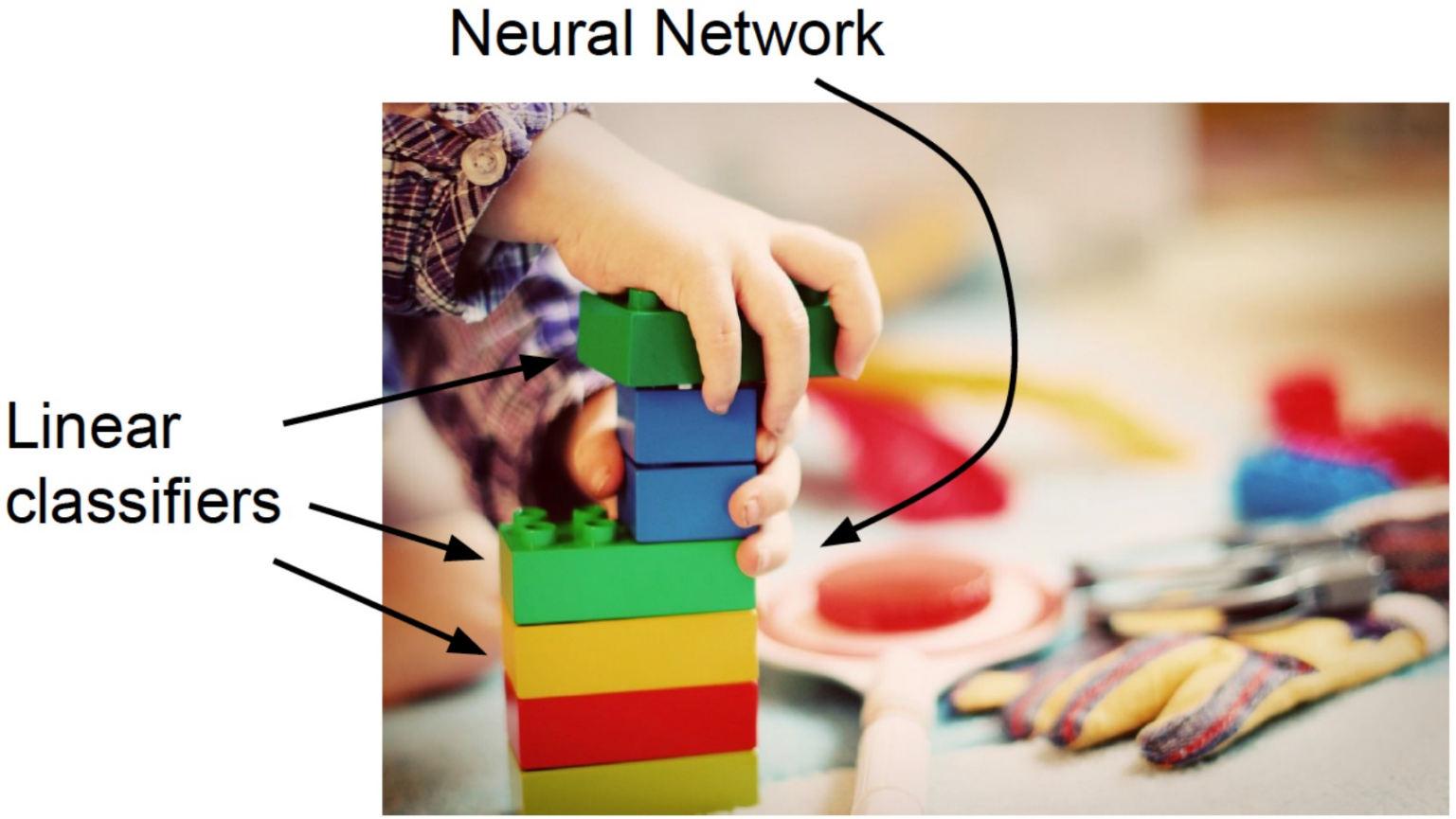
Idea behind Neural Networks

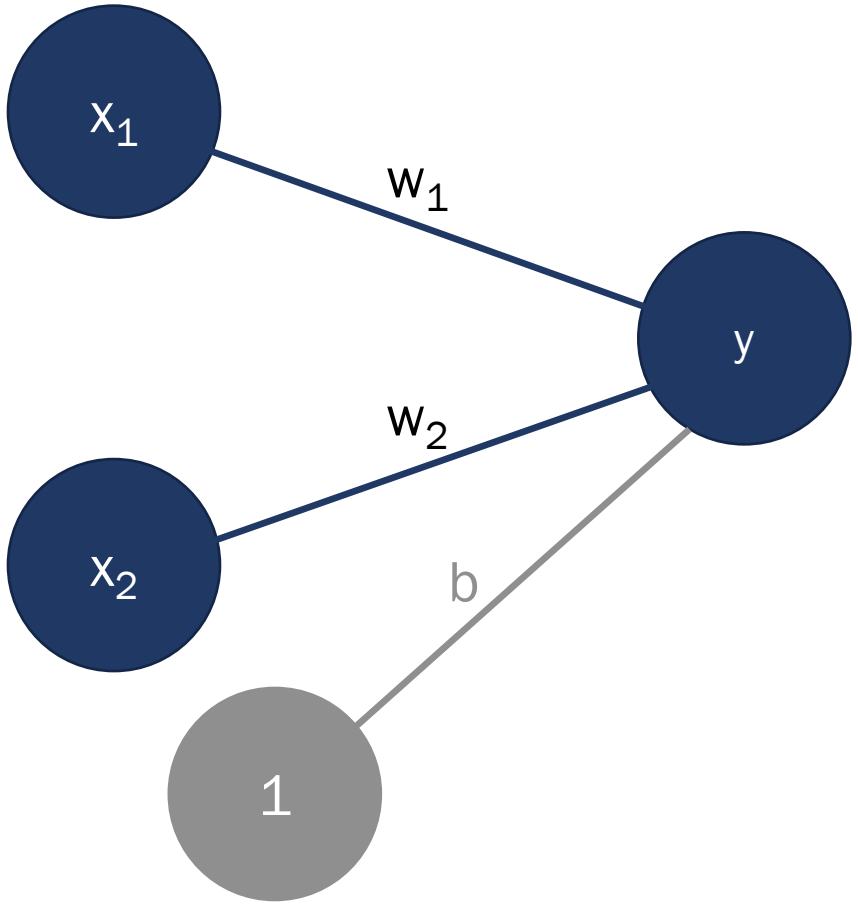


Linear Regression

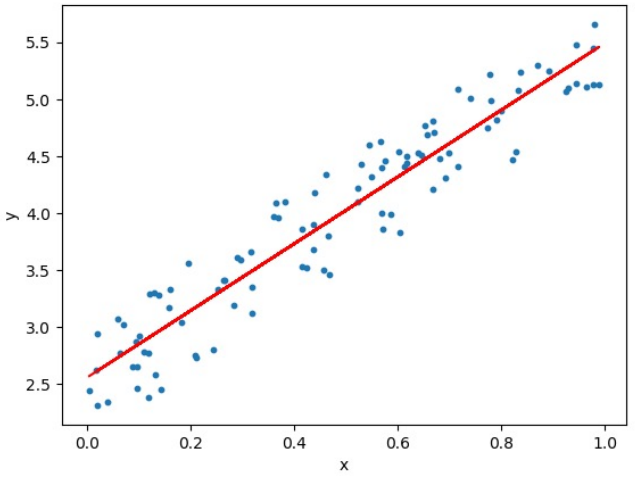
Predicitons general case

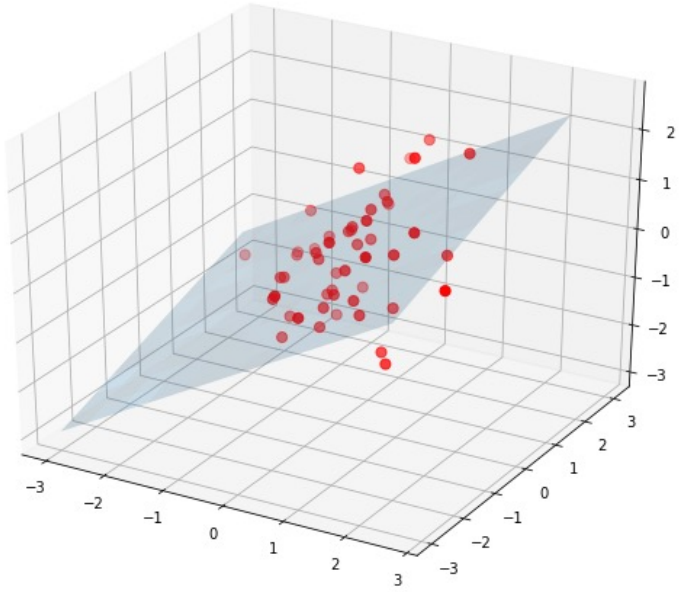
Input:

Output:



$= w_1x_1 + w_2x_2 + b$





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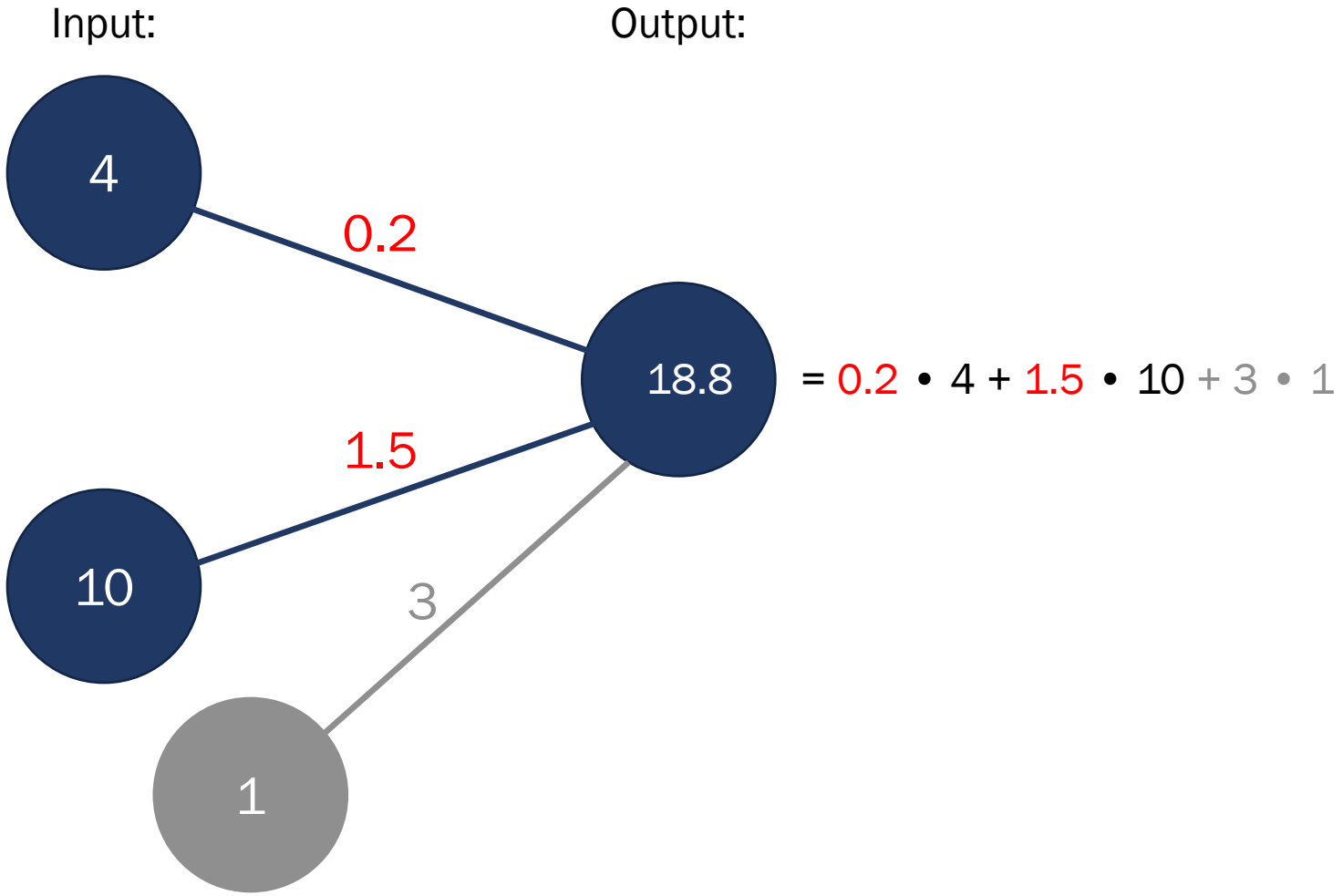
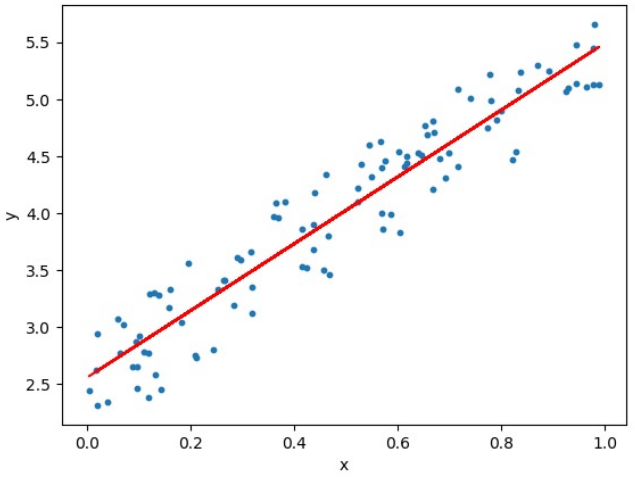
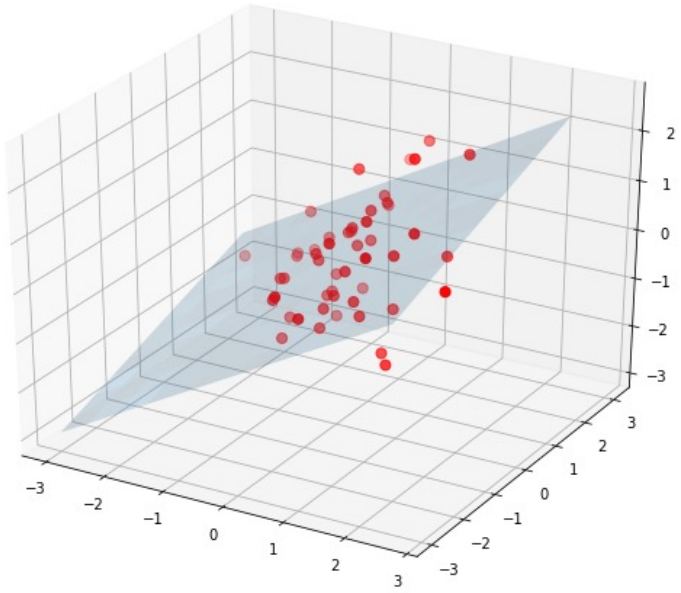
Linear Regression

Predicitons example



$= 0.2 \cdot 4 + 1.5 \cdot 10 + 3 \cdot 1$





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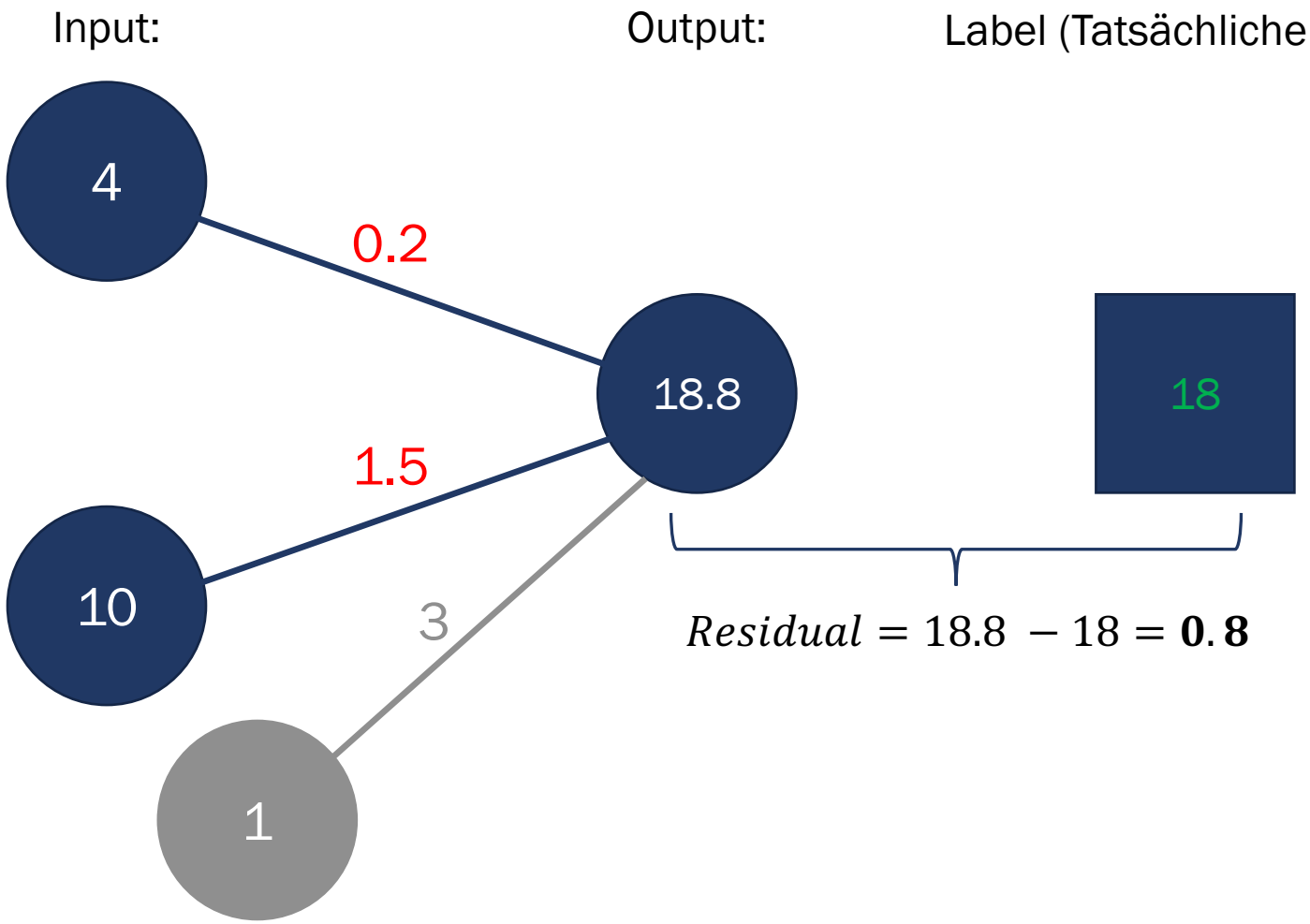
Linear Regression: Outcome



x_1	x_2	$\hat{y}$
4	10	18

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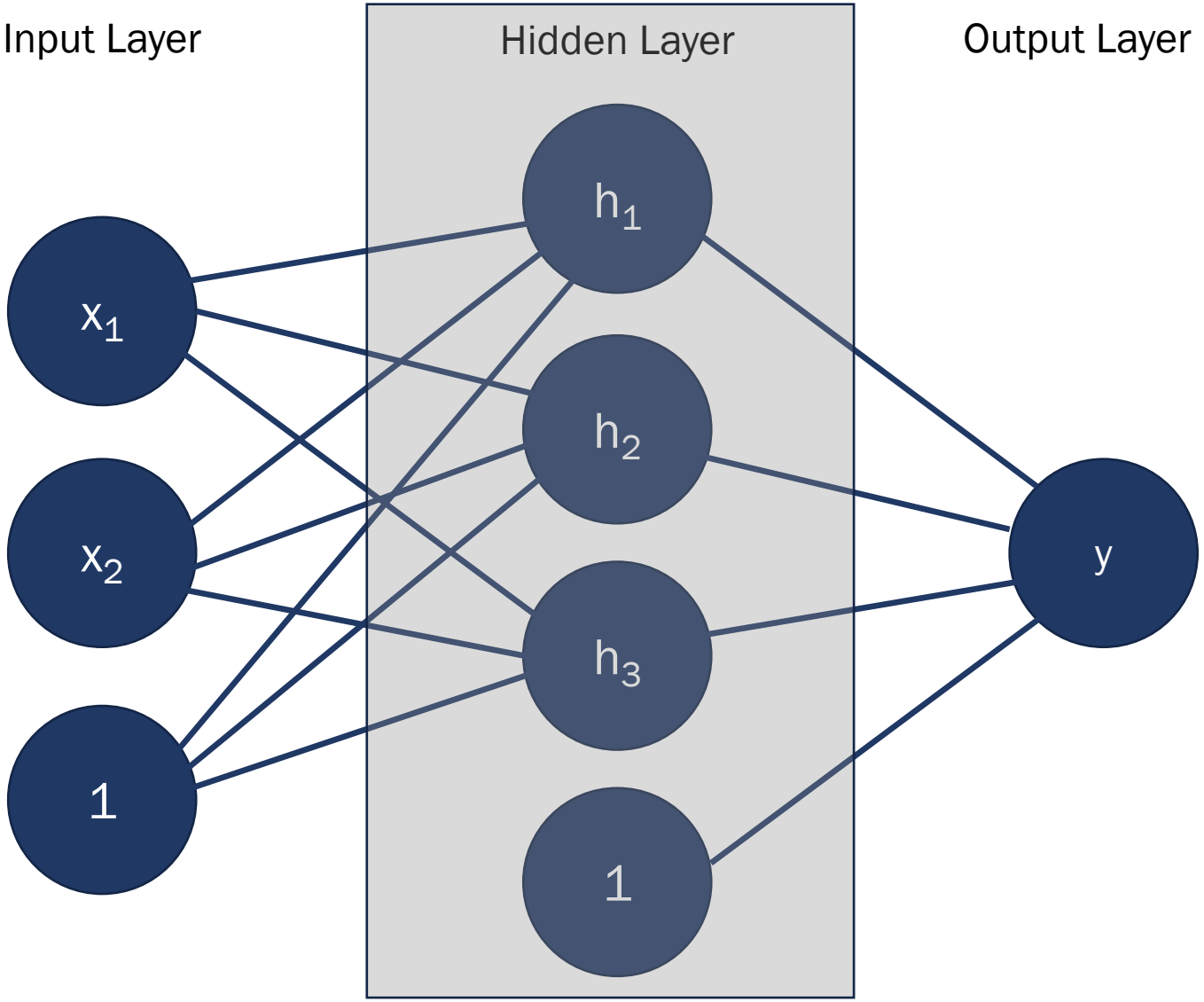
Structure



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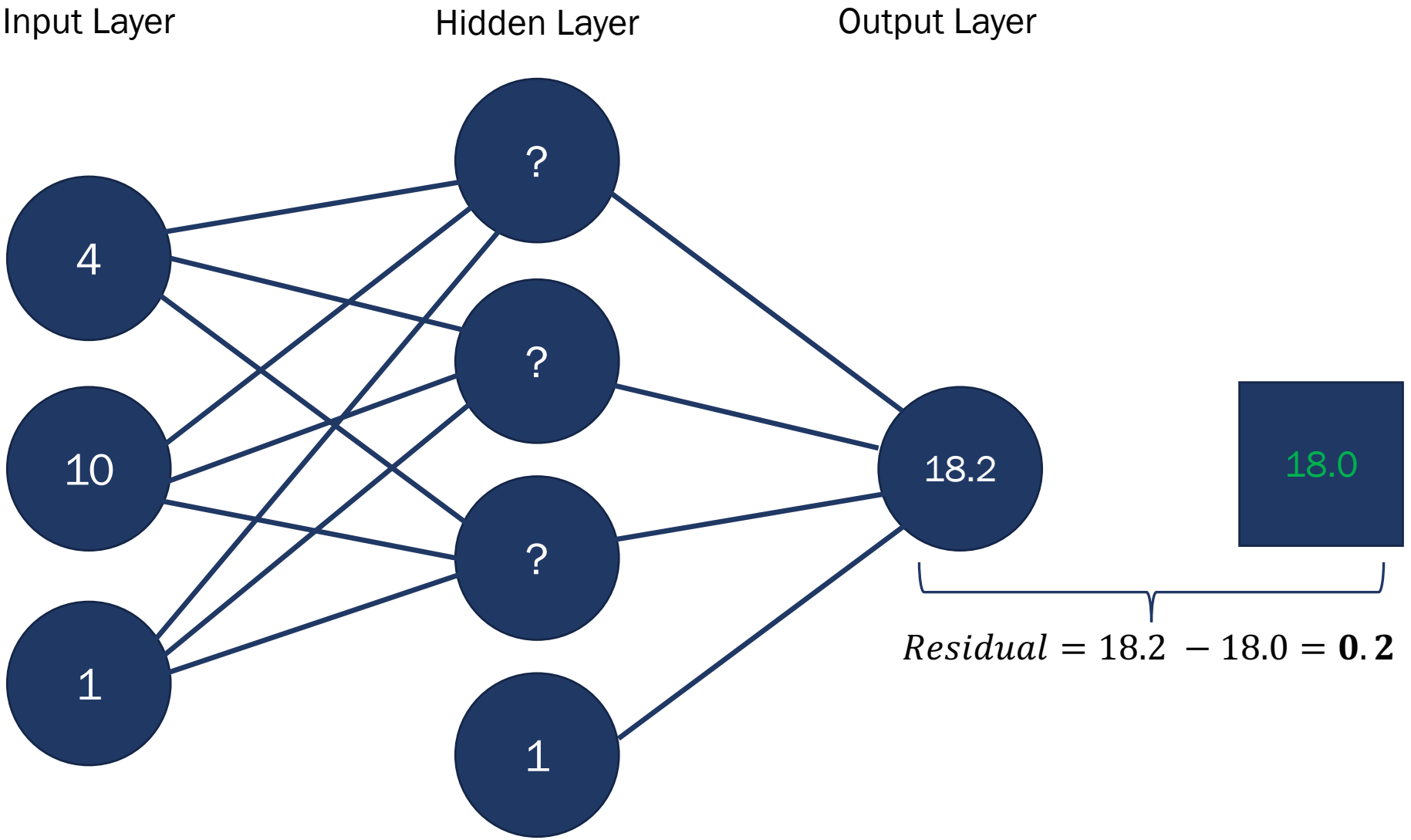
Example



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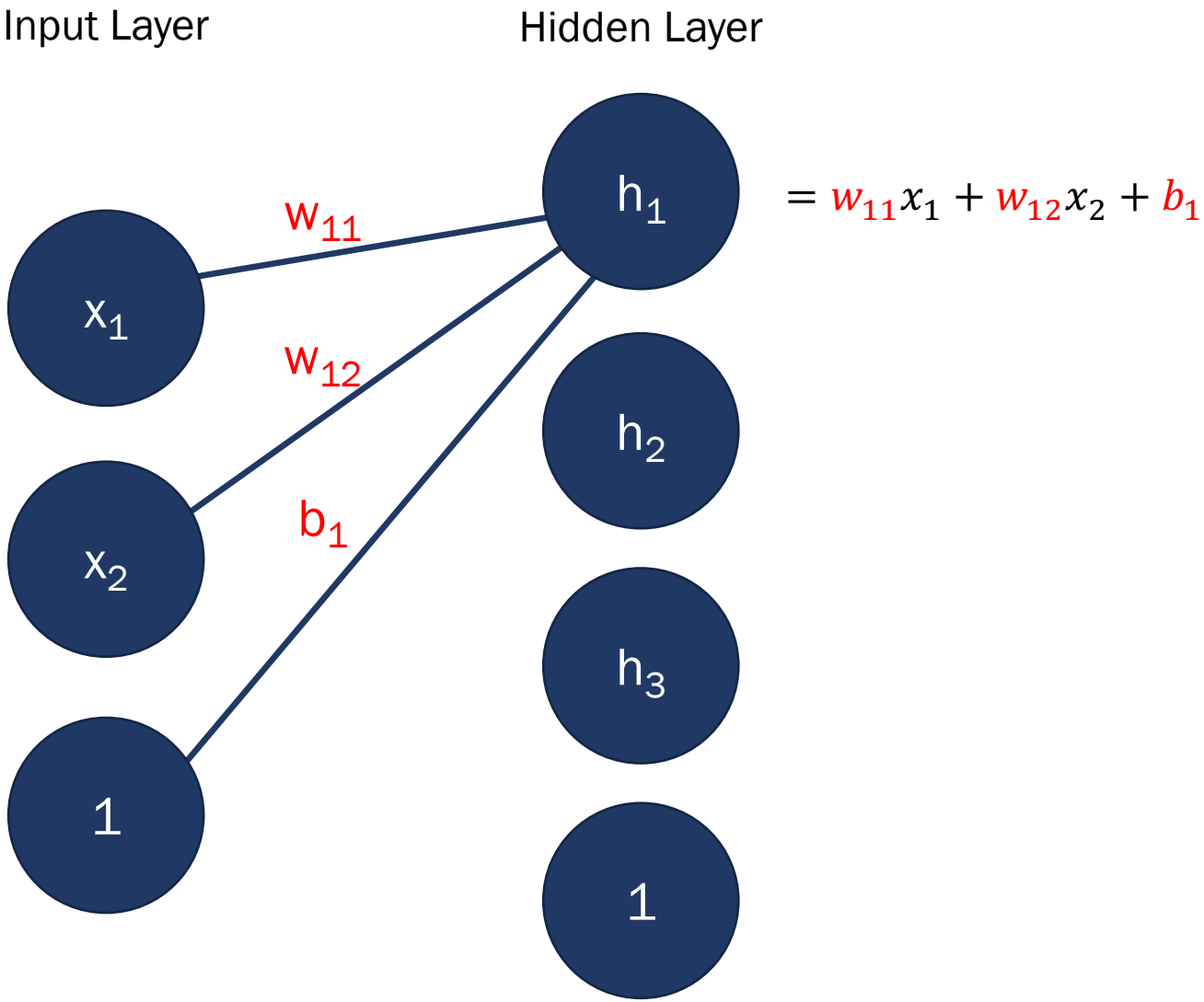
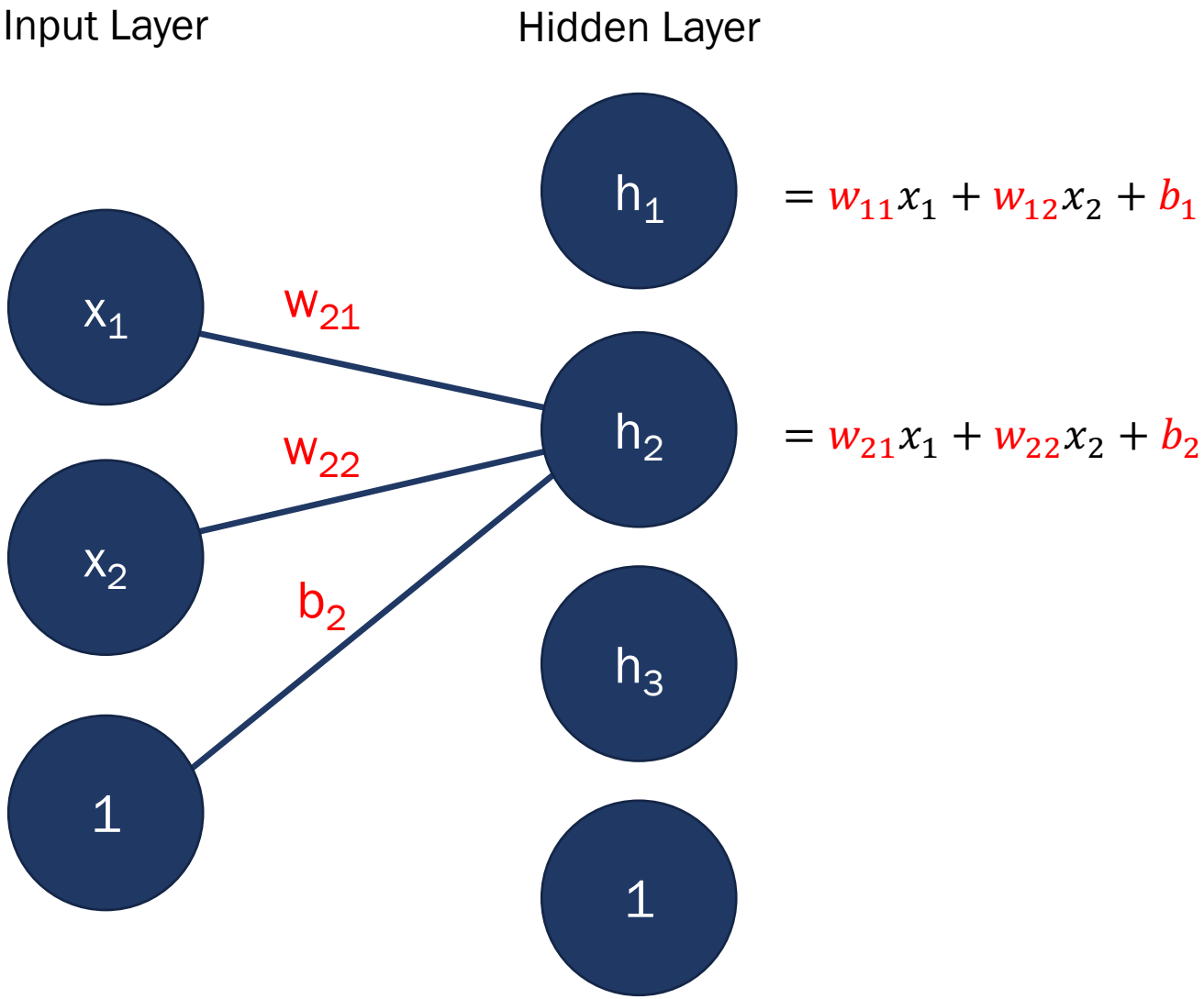
Weighted sum



Weighted sum



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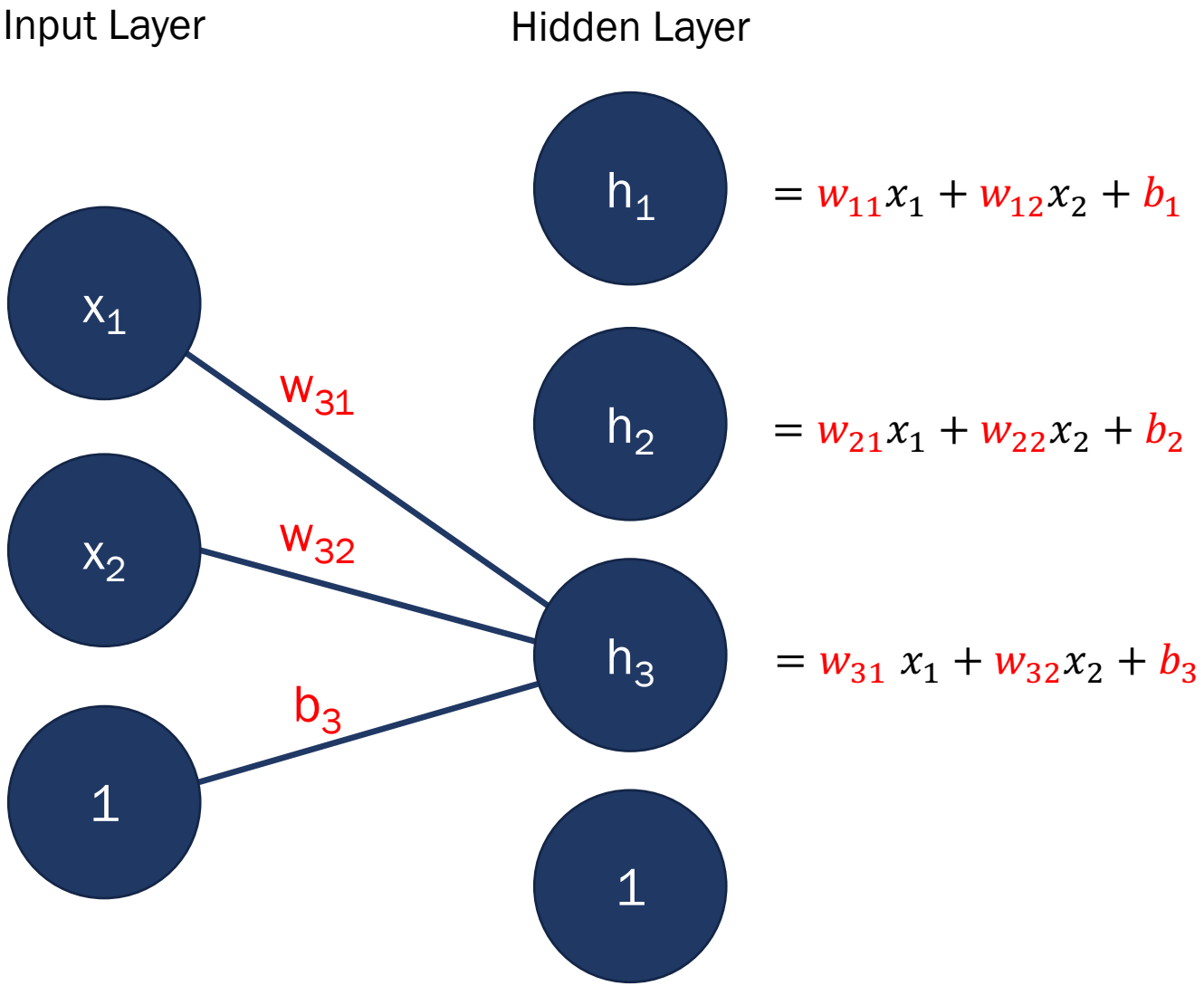
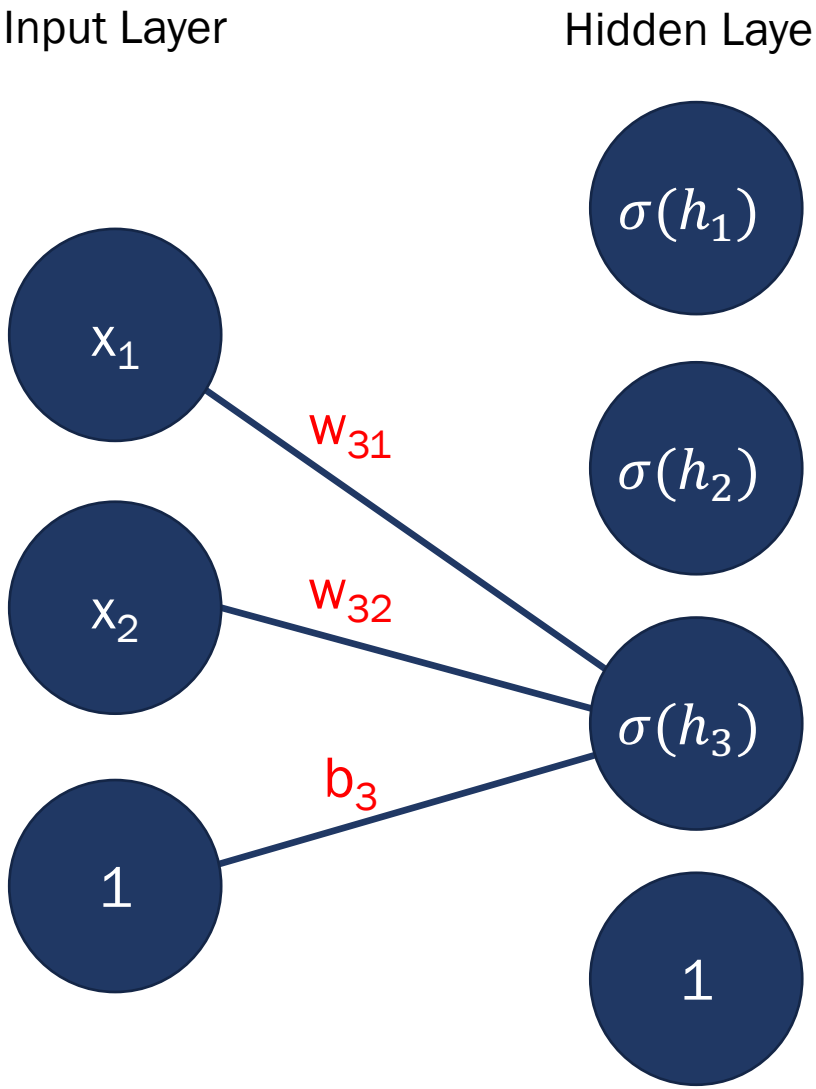
Weighted sum



Weighted sum with activation



$$= \sigma(w_{11}x_1 + w_{12}x_2 + b_1)$$

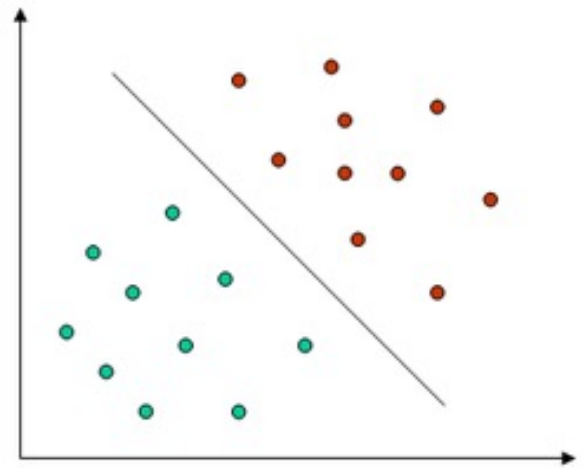
$$= \sigma(w_{21}x_1 + w_{22}x_2 + b_2)$$

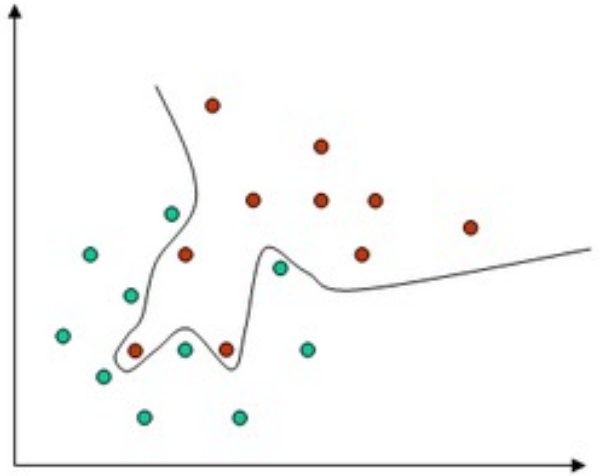
$$= \sigma(w_{31}x_1 + w_{32}x_2 + b_3)$$

Why Activationfunction?

Without nonlinear activation, neural networks are just linear regression!!!

- Without an activation function, the neural network would not capture nonlinear, complex relationships
- Via the activation function one can determine essential properties of the model



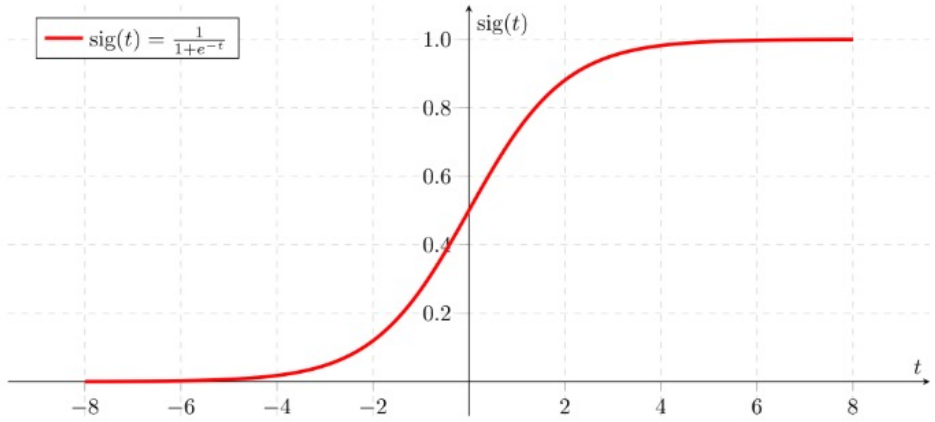


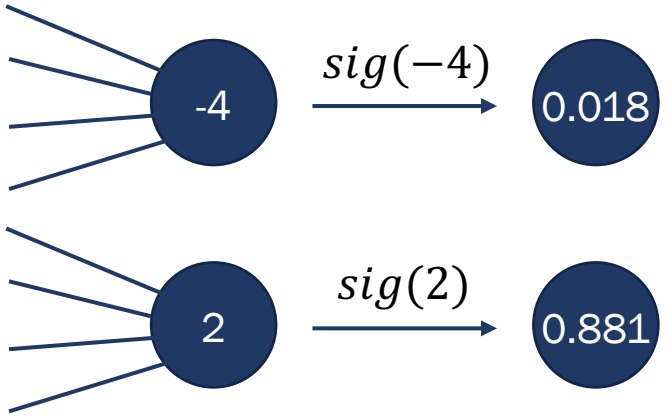
Activationfunction: Example

Different activation functions come with pros and cons. E.g. the sigmoid function is very computationally inefficient (exponential!) while ReLU is very efficient but loses probabilistic interpretation

$Sigmoid$

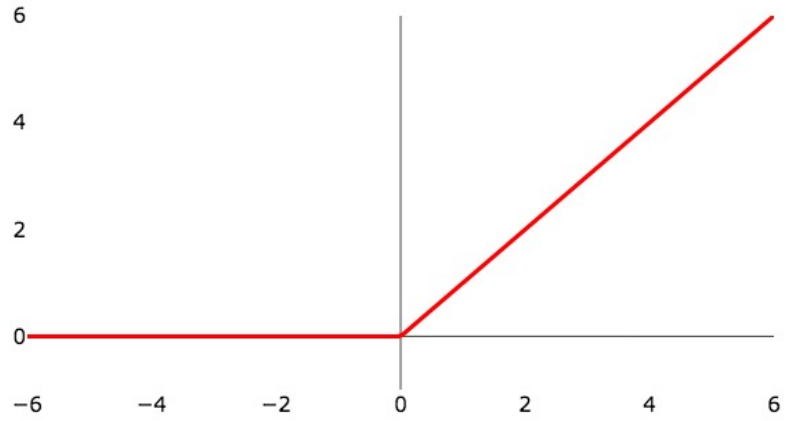
- Values between 0 and 1
- Mainly for classification
- Mostly used in **output layer**

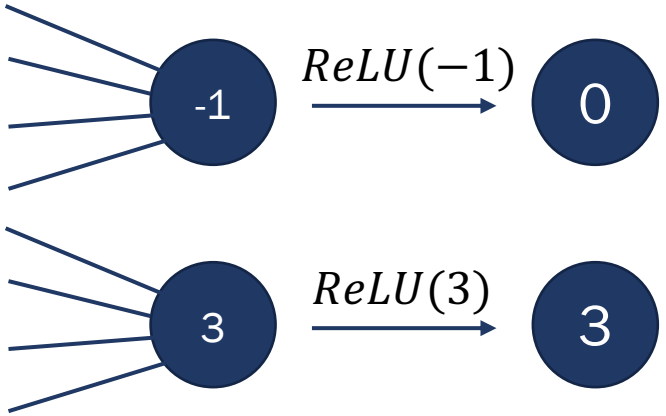




$ReLU(t) = max(0, t)$

- Negative values are set to zero
- Mostly used in **hidden layers**





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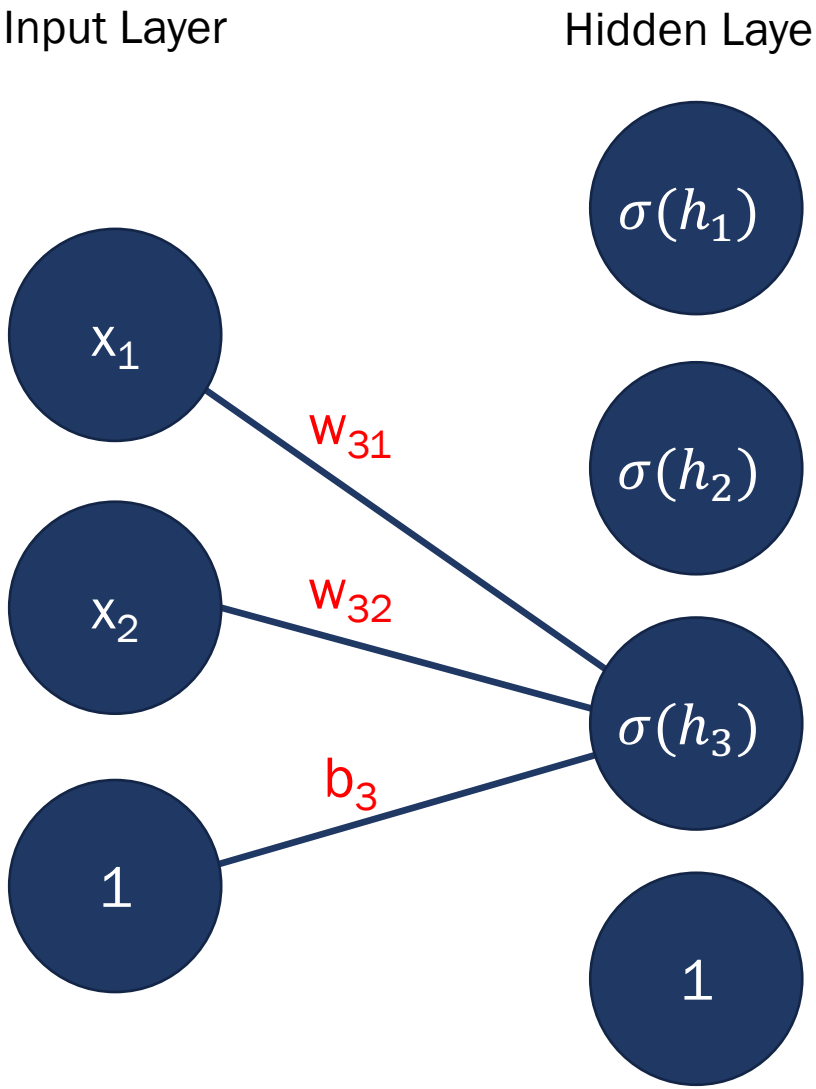
Weighted sum

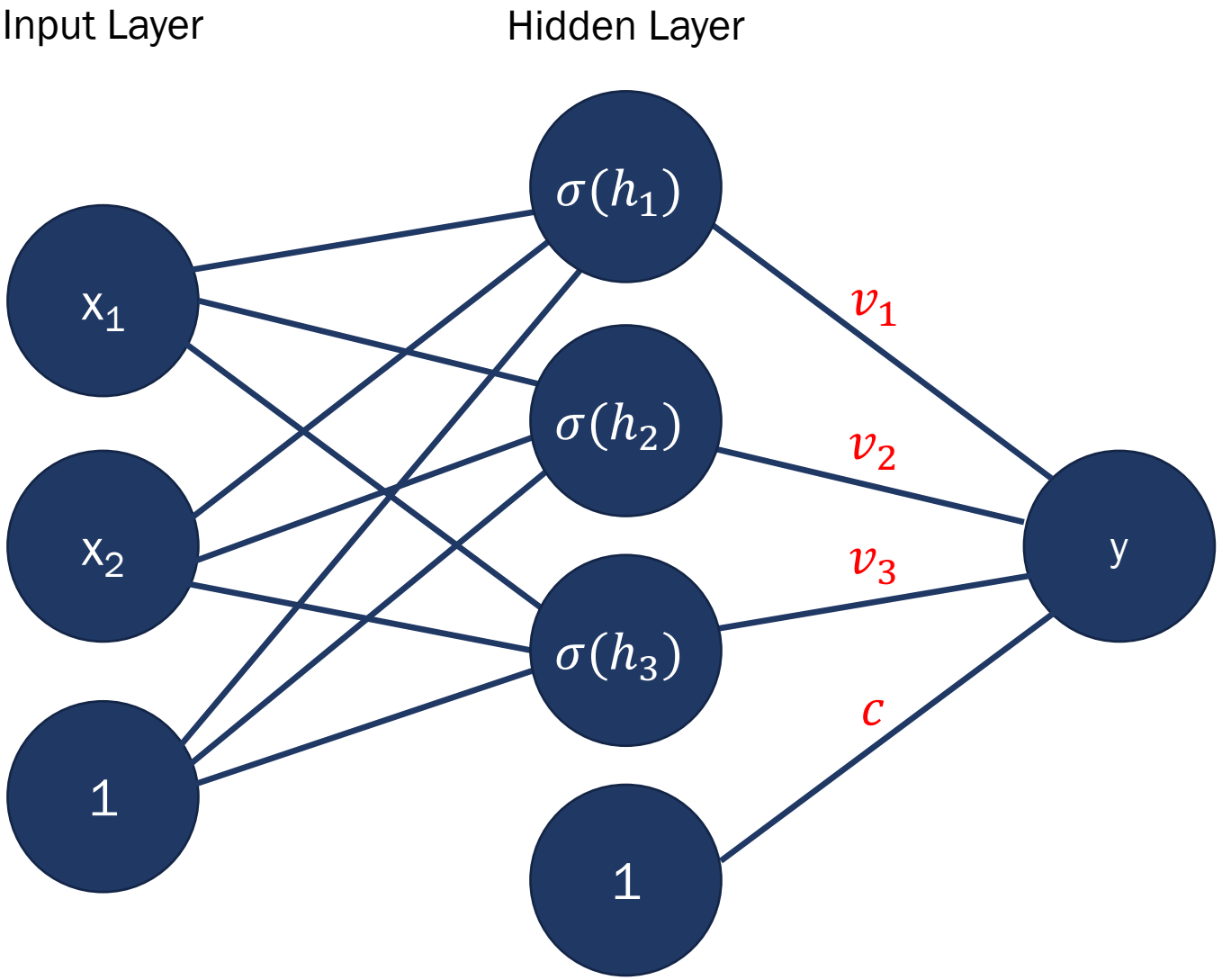


$$= ReLU(w_{11}x_1 + w_{12}x_2 + b_1)$$

$$= ReLU(w_{21}x_1 + w_{22}x_2 + b_2)$$

$$= ReLU(w_{31}x_1 + w_{32}x_2 + b_3)$$

Final Structure



$$= v_1\sigma(h_1) + v_2\sigma(h_2) + v_3\sigma(h_3) + c$$

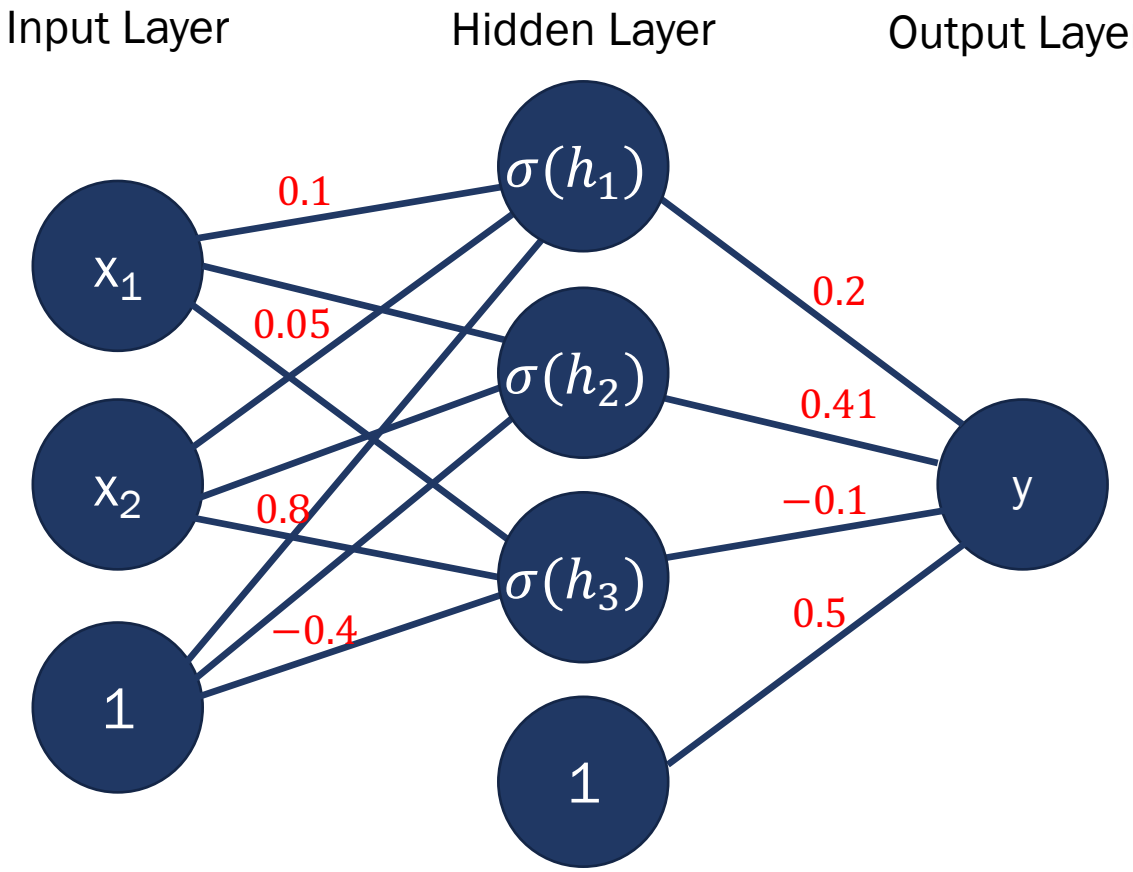
The most important step is still missing

Do you have an idea?

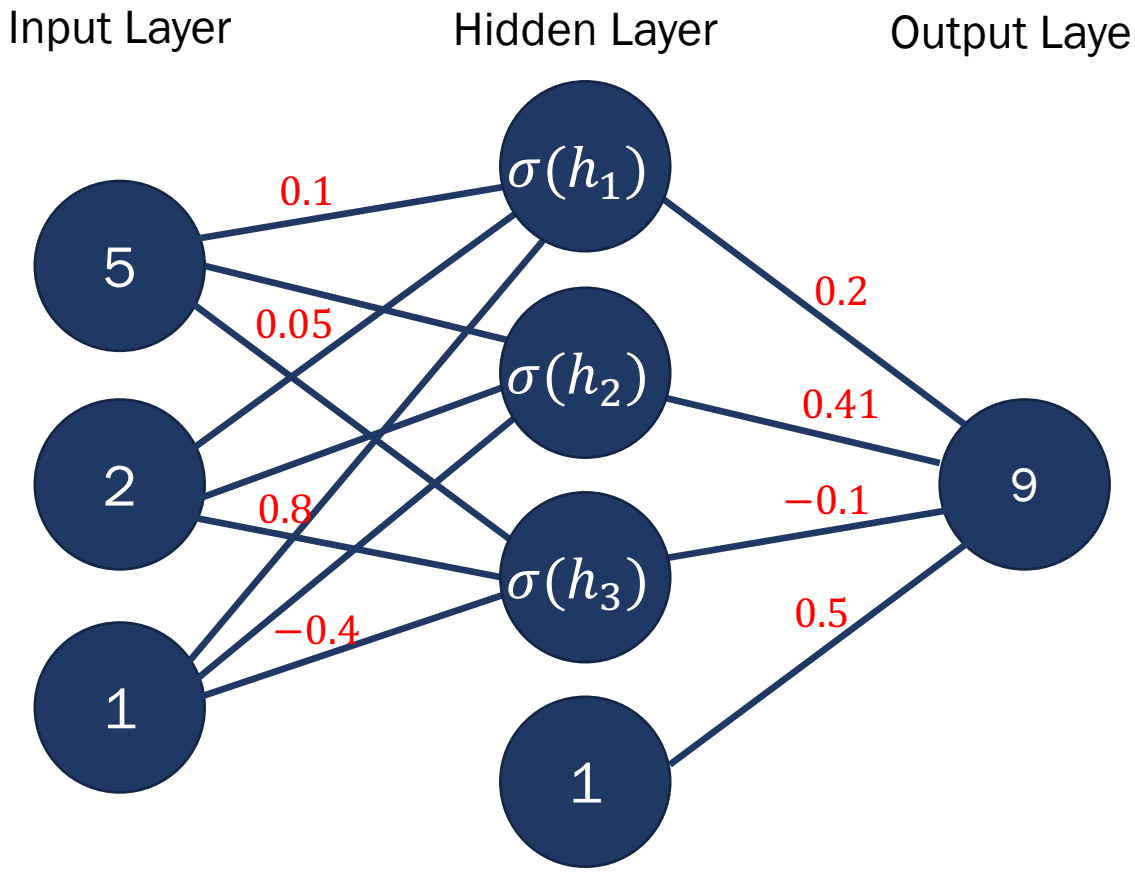
Goal: Fit the weights

Train a neural network = fit the weights to the data.

Initialize weights randomly



Insert data point



x_1	x_2	$\hat{y}$
5	2	22

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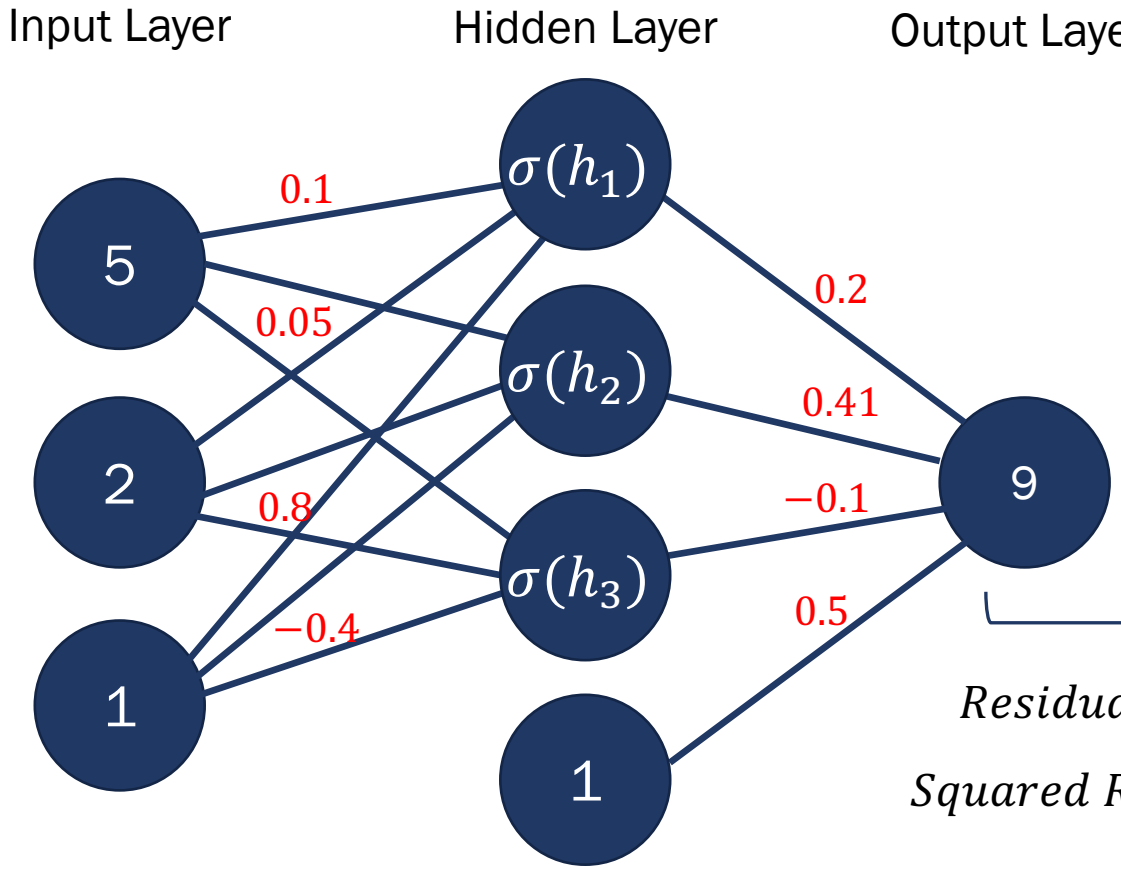
Compare prediction and label (residuals)

x_1	x_2	$\hat{y}$
5	2	22



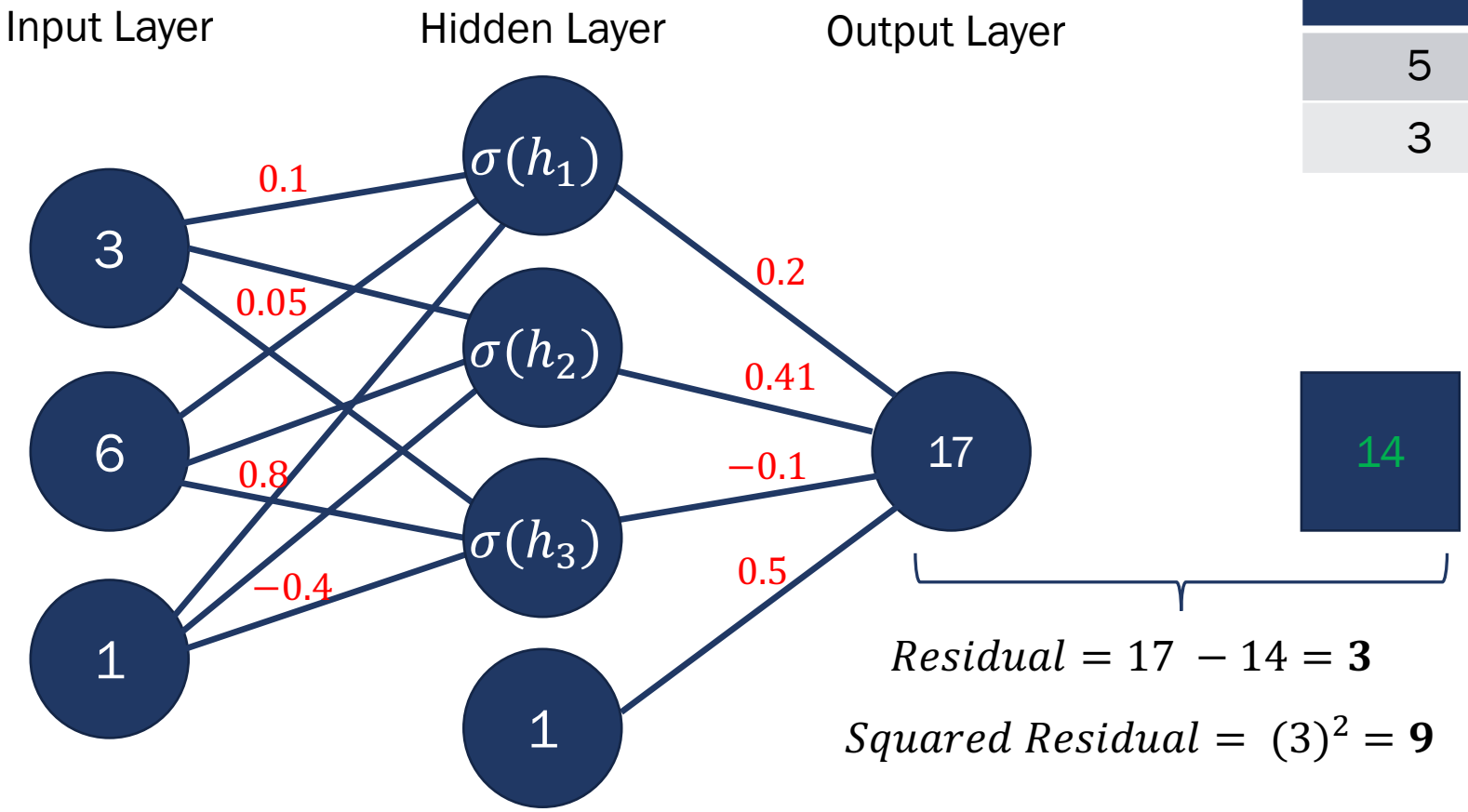
22

$$Residual = 9 - 22 = \mathbf{-13}$$

$$Squared\ Residual = (-13)^2 = \mathbf{169}$$

STADS

Compare prediction and label (residuals)



x_1	x_2	$\hat{y}$
5	2	22
3	6	14

14

$Residual = 17 - 14 = \mathbf{3}$

$Squared\ Residual = (3)^2 = \mathbf{9}$

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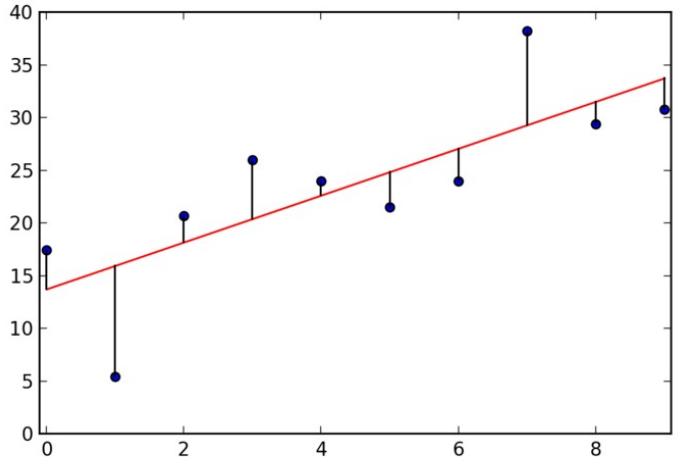
Loss-Function

In order to calculate the quality of forecasts, a so-called loss function is required. This serves as a measure for the deviation of the forecasts from the observed values. We use the residual sum of squares.

$Sum\ of\ squared\ residuals\ (SSR)$

- Sums the squares of the deviations/residuals
- Squaring makes values non-negative
- Easy to calculate and **differentiable**



$$SSR = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = (9 - 22)^2 + (17 - 14)^2 + (y_3 - \hat{y}_3)^2 + \cdots = 169 + 9 + (y_3 - \hat{y}_3)^2 + \ \ldots$$

We want to minimize this sum!

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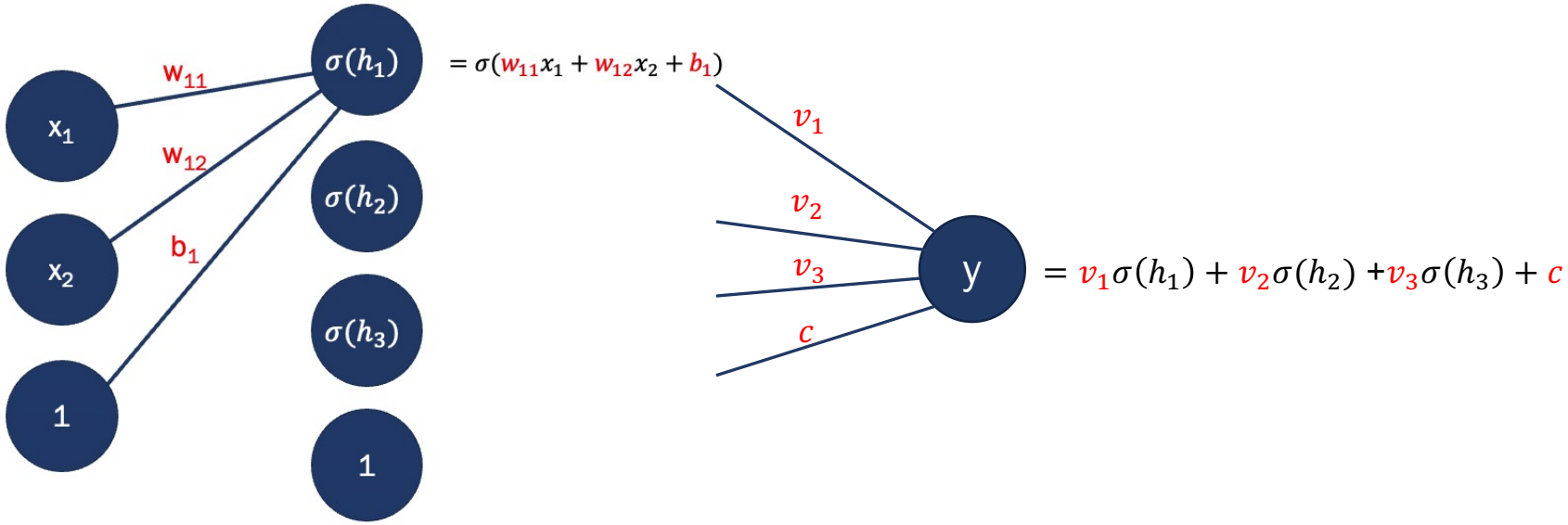
Minimize the sum of squared residuals

The goal is to change the weights & constants so that the error sum of squares is minimized.
Mathematically expressed: Approximate the minimum of the loss function

$$f(w_{11}, \dots, v_1, \dots, b_1, \dots, c) = (y - \hat{y})^2$$
$$= (v_1\sigma(h_1) + v_2\sigma(h_2) + v_3\sigma(h_3) + c\ - \hat{y})^2$$
$$= (v_1\sigma(w_{11}x_1 + w_{12}x_2 + b_1) + v_2\sigma(w_{21}x_1 + w_{22}x_2 + b_2) + v_3\sigma(w_{31}x_1 + w_{32}x_2 + b_3) + c\ - \hat{y})^2$$



Minimize the sum of squared residuals

The goal is to change the weights & constants so that the error sum of squares is minimized.
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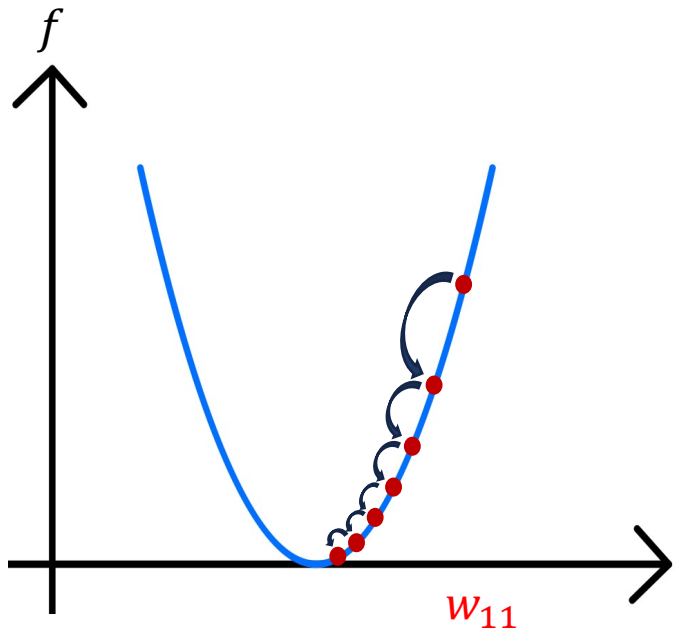
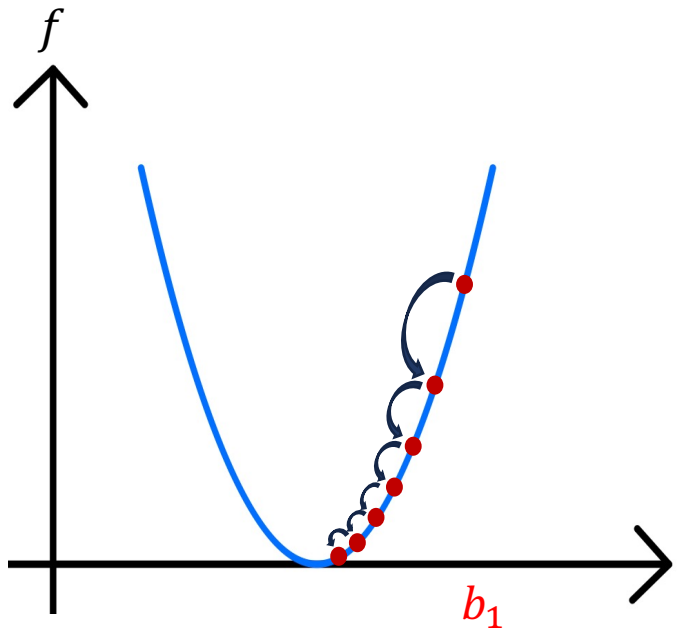
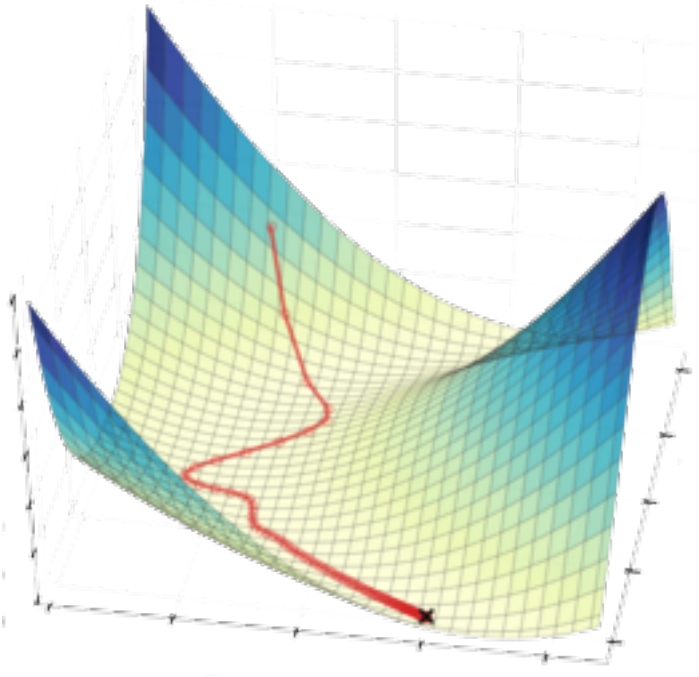
$$f(w_{11}, \dots, v_1, \dots, b_1, \dots, c) = (y - \hat{y})^2$$
$$= (v_1\sigma(h_1) + v_2\sigma(h_2) + v_3\sigma(h_3) + c\ - \hat{y})^2$$
$$= (v_1\sigma(w_{11}x_1 + w_{12}x_2 + b_1) + v_2\sigma(w_{21}x_1 + w_{22}x_2 + b_2) + v_3\sigma(w_{31}x_1 + w_{32}x_2 + b_3) + c\ - \hat{y})^2$$







Gradient descent

The gradient descent method is one of the best known optimization methods for neural networks. With the gradient method, a **local** minimum of the loss function can be easily approximated.

$$f(w_{11}, \dots, v_1, \dots, b_1, \dots, c) = (y - \hat{y})^2$$
$$= (v_1\sigma(h_1) + v_2\sigma(h_2) + v_3\sigma(h_3) + c\ - \hat{y})^2$$
$$= (v_1\sigma(w_{11}x_1 + w_{12}x_2 + b_1) + v_2\sigma(w_{21}x_1 + w_{22}x_2 + b_2) + v_3\sigma(w_{31}x_1 + w_{32}x_2 + b_3) + c\ - \hat{y})^2$$

Chain rule gives…

…for a data point: $\frac{\partial f(w_{11},\dots)}{\partial(w_{11})} = \frac{\partial(y-\hat{y})^2}{\partial(w_{11})} = 2\ (y - \hat{y}) \cdot \frac{\partial y}{\partial(w_{11})} = 2\ (y - \hat{y}) \cdot v_1 \cdot \frac{\partial\sigma(h_1)}{\partial(w_{11})} \cdot x_1$

…for the whole dataset: $\frac{\partial SSR}{\partial(w_{11})} = \frac{\partial}{\partial(w_{11})}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n}\frac{\partial(y_i-\hat{y}_i)^2}{\partial(w_{11})} = \sum_{i=1}^{n}\left[2\ (y_i - \hat{y}_i) \cdot\ v_1 \cdot \frac{\partial\sigma(h_1^i)}{\partial(w_{11})} \cdot x_1\right]$

Or in general: $\frac{\partial SSR}{\partial(\theta_j)} = \sum_{i=1}^{n}\left[2\ (y_i - \hat{y}_i) \cdot \frac{\partial y_i}{\partial(\theta_j)}\right]$

Learning Rate

Now the "learning" or the "training" begins. All parameters (=weights and constants) must be adjusted with the help of the derivation.

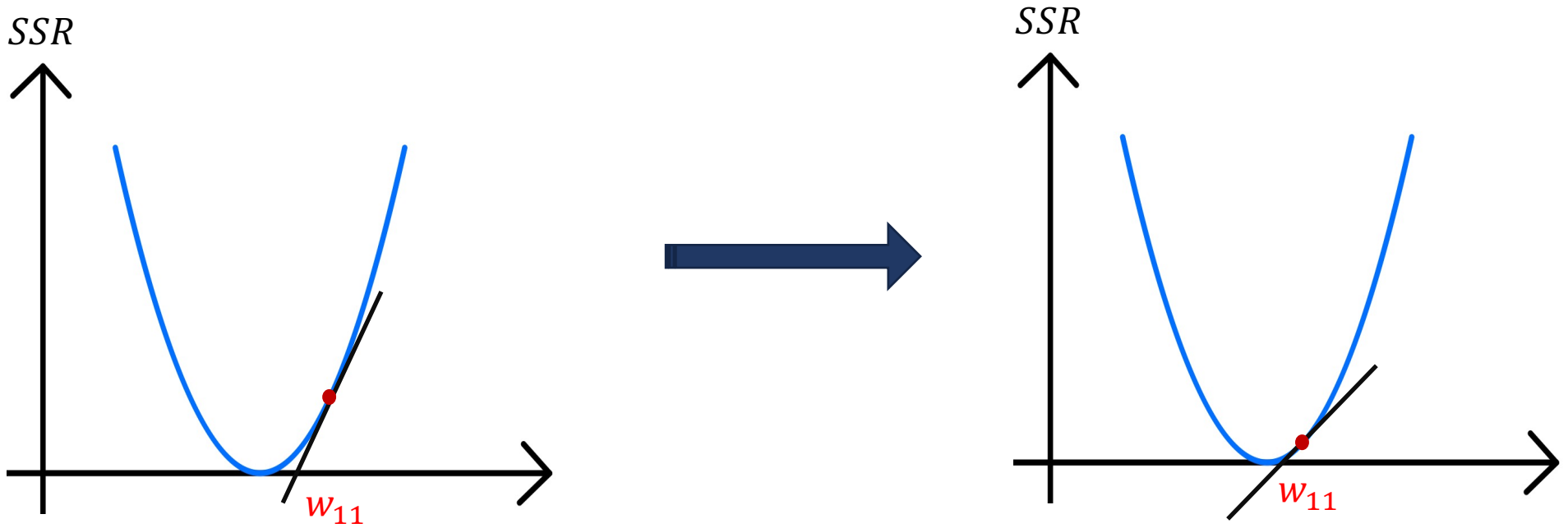
The most important step:

Set $w_{11} = w_{11} - \alpha \cdot SSR'(w_{11}, \dots)$

In general: $\theta_j = \theta_j - \alpha \cdot \frac{\partial SSR}{\partial(\theta_j)}$

Learning Rate α

- The learning rate indicates how much the parameters have to be corrected in each run
- E. g. $\alpha = 0.05$



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Process of Learning

Now the "learning" or the "training" begins. All parameters (=weights and constants) must be adjusted with the help of the derivation.

x_1	x_2	y	$\hat{y}$
5	2	9	22
3	6	17	14

x_1	x_2	y	$\hat{y}$
5	2	17.7	22
3	6	15.2	14

x_1	x_2	y	$\hat{y}$
5	2	21.3	22
3	6	14.5	14

Training & Testing

To train a model, one must use training data.
To evaluate the model, one must use the so-called test data. With them it is possible to check whether the model is reliable also on foreign data.

x_1	x_2	y	$\hat{y}$
5	2	21.3	22
3	6	14.5	14
7	-2	4.6	4
0.5	3	9.7	10

Training Data
Accuracy: 95%

x_1	x_2	y	$\hat{y}$
2	1	4.4	6
9	3	11.4	12.9

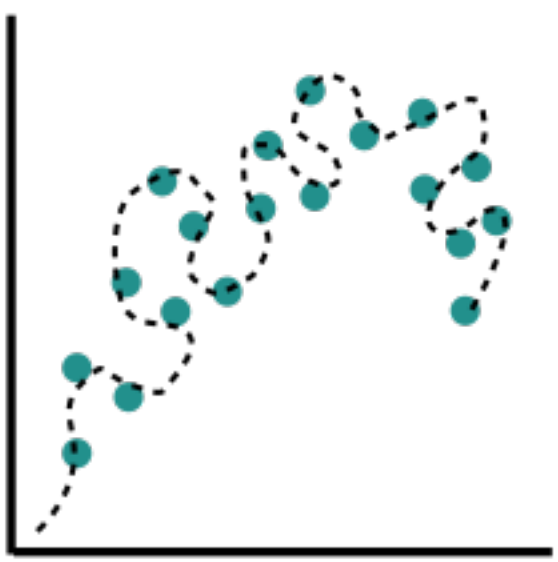
Test Data
Accuracy: 92%

- When you receive a data set, you must always split it into a training and a test data set
- Test data must not be used for training
- The model usually performs better on the training data than on the test data.

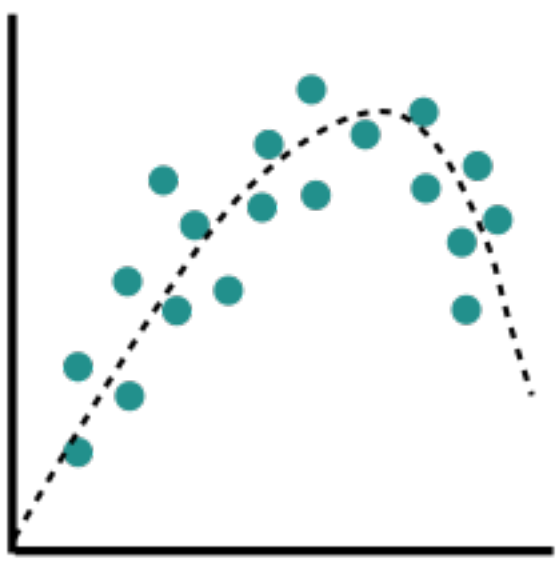
Overfitting vs. Underfitting

If you fit the model too much to the data, an overfit may take place. There are several methods to avoid this.

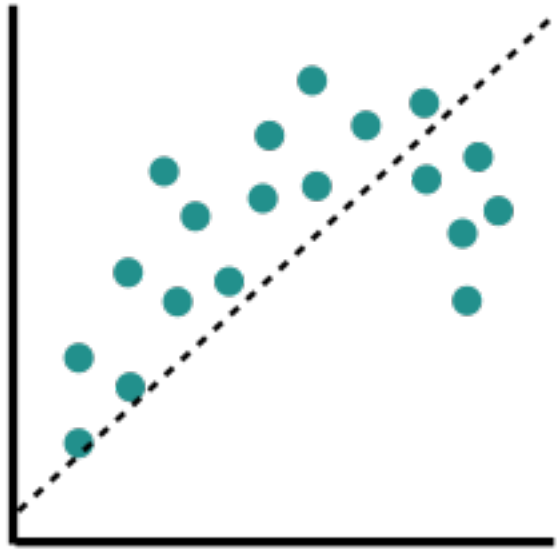
Overfit (Variance)



Robust



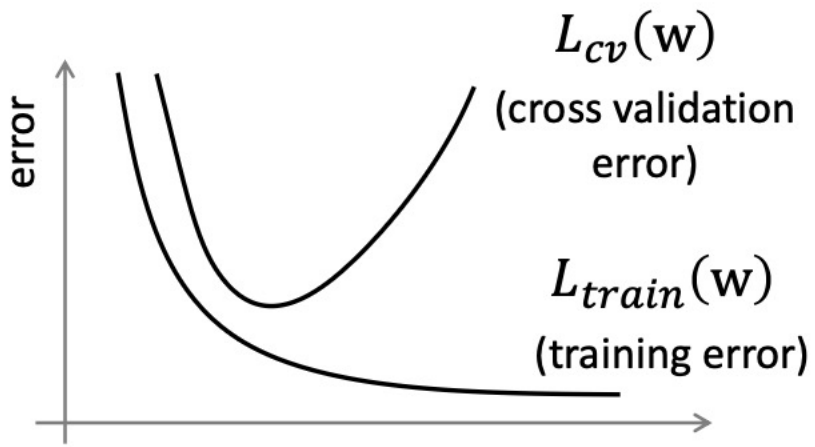
Underfit (Bias)



Overfitting vs. Underfitting: Data Set size

If you fit the model too much to the data, an overfit may take place. There are several methods to avoid this.

Problem



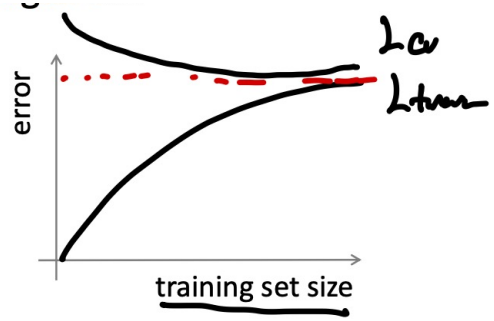
=Degree of overfitting

Bias (underfitting): Ltrain will be large and Lcv ≈ Ltrain

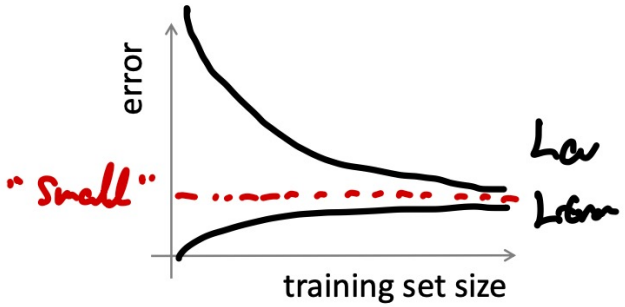
Variance (overfitting): Ltrain will be low and Lcv >> Ltrain

Solution

High Bias: getting more training data will **not** (by itself) **help much**



High Variance: getting more training data **may be likely to help**



Deep Neural Networks are very large models, but overfitting can be addressed by using large data sets

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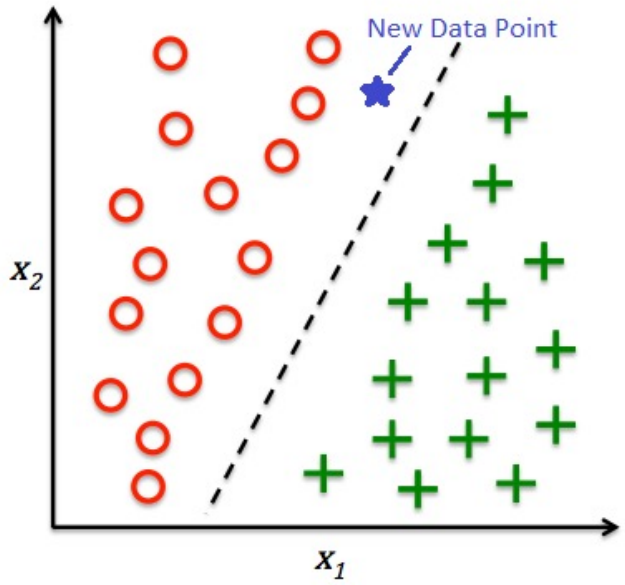
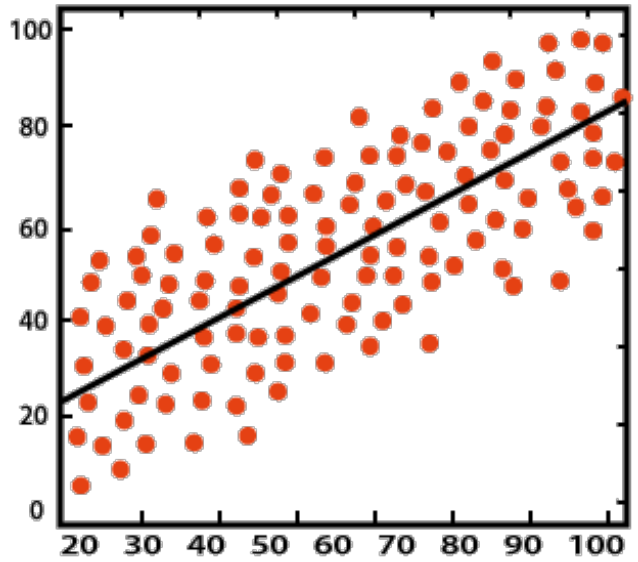
Classification vs. Reggression

Neural networks are divided into classification and regression models. Which of the two one chooses, depends on the facts.

Classification



Regression



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But...

What are possible disadvantages of neural networks?

Agenda

Intro

How do neural Networks work?

Practical implementation

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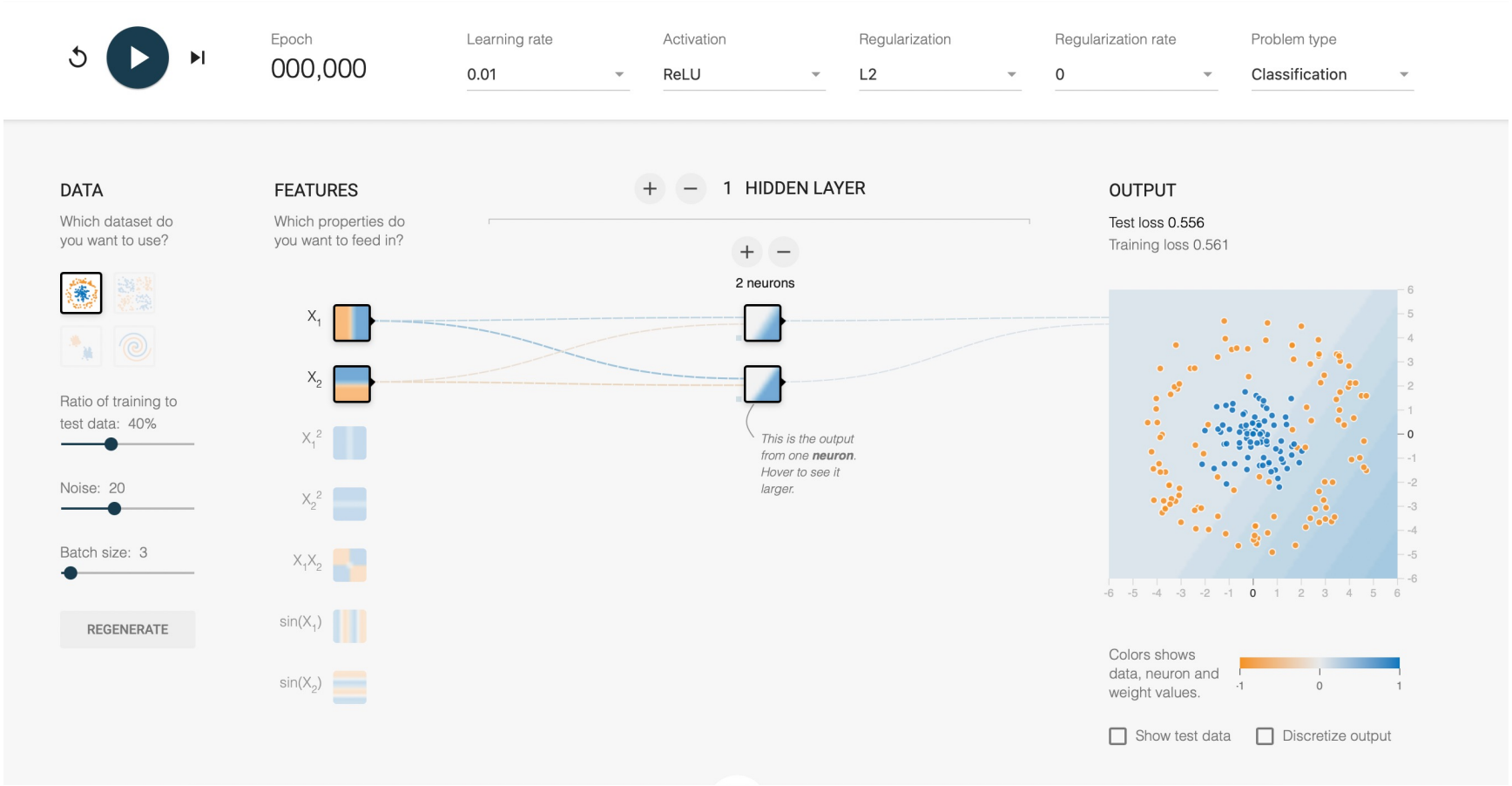
Playing around with neural networks...



playground.tensorflow.org

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