

ICTE in Transportation and Logistics 2018 (ICTE 2018)

The application of simulated annealing method for optimal route detection between objects

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Abstract

A shortcut search between different objects is one of the classic optimization goals. Simulated Annealing algorithms are often used for optimization purposes. The Simulated Annealing method is applied in combinatorial optimization tasks. Simulated Annealing is a stochastic optimization method that can be used to minimize the specified cost function given a combinatorial system with multiple degrees of freedom. The application of Simulated Annealing method and Travelling Salesman Problem is demonstrated in this paper and an experiment aimed to find the shortest route between Belarusian processing enterprises is performed. The paper also provides mapping of these objects using GPS location. The main goal of the paper is to show the possibilities of applying mathematical models in solving practical tasks - to determine the shortest route between the different objects.

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Peer review under responsibility of the scientific committee of the ICTE in Transportation and Logistics 2018 (ICTE2018).

Keywords: Logistic; Optimization; Thermal equilibrium; Travelling salesman problem; Simulated annealing

1. Introduction

Simulated Annealing (SA) is a stochastic optimization method used for the optimization of objective function (energy). It allows to find the global extreme for the function that has local minimums. SA principle was introduced in the classical work [1] and further developed in works [2] - [5]. SA is based on analogy with statistical mechanics,

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and in particular with solid state physics elements. A practical example can be drawn from metallurgy - what happens to the atomic structure of the body when it is rapidly cooled and its temperature is lowered. Sharp decrease in temperature may lead to the system's non-symmetric structure, in other words to a non-optimal state (with errors). Cooling eventually leads to a condition where the system gets stuck or freezes and thermal equilibrium sets up. SA differs from other gradient rollout optimization procedures by the fact that it does not “shrink” in the local minimum found. Even though SA is a relatively slow procedure, it guarantees the finding of a global optimal solution.

SA method is widely used in many combinatorial optimization tasks. It is utilized in graph theory (graph division, graph coloring), in integral circuit simulation [2], in neural network applications [6] and others. In the application of the SA algorithm, a well-known combinatorial task will be employed - Travelling Salesman Problem (TSP).

The task of the TSP is to find the shortest route between N cities - entering into each city only once and in the end returning to the original city. This is a combinatorial task that can be solved with a variety of combinatorics or graph theory techniques. In literature TSP solving methods with the SA algorithm are also viewed [6] - [8].

This paper demonstrates the application of the SA method in the combinatorial analysis TSP and an experiment aimed to find the shortest route between 8 Belarusian milk and meat processing companies is performed.

2. Simulated annealing process

The so-called Metropolis procedure [1] determines iterative steps controlling the best solution to be achieved. This algorithm is used in atomic equilibrium simulation at the given temperature. In each step of the algorithm, the atom is raised by a small probabilistic movement (shifting): $x_i + \varsigma$, and system energy change ΔE is calculated.

- If $E < 0$, then the movement is accepted and the configuration with the changed atomic states is used as the initial state in the next step
- If $E > 0$, then the probability when a new state is accepted, is:

$$P(\Delta E) = e^{\frac{-\Delta E}{kT}} \quad (1)$$

where k - Boltzmann's constant, T - temperature parameter

Using the energy system as a target function and defining the states of the system with $\{x_i\}$, it can be seen that the Metropolis procedure generates a series of states for the given optimization problem at a certain temperature.

To use the SA method practically, the following must be specified:

- The target function W (analogous to energy surface) with minimization as the purpose of this procedure
- A possible set of solutions according to the energy surface or the physical state of the system
- Configuration conditions, the variation generator
- The control parameter T , which characterizes an artificial system temperature, and the cooling mode (annealing schedule) that describes how the temperature will be lowered

The SA algorithm is based on the Boltzmann's probability distribution:

$$Pr(e) \sim e^{\frac{-E}{kT}} \quad (2)$$

This expression determines that if the system is in thermal equilibrium at a temperature T , its energy may be split between all different energy states E . Even at low temperatures, there is a possibility that the system may be in a high energy state. The system has an adequate probability of switching from the state with local energy minimum to a better, more global minimum.

To understand the essence of SA, the following analogy can be suggested. Assume a space of states is set as a basket of apples where each state is being represented as an apple. At each step one apple is drawn out and weighted. How to find the biggest apple? In the deterministic (full) searching an apple is drawn out of the basket, is

weighted and is not put back to the basket. The best apple is kept aside. In the SA method, an apple is weighted and put back to the basket. Hence, it can be drawn out and weighted several times.

The SA algorithm can be written as follows:

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 $T = T_0;$ 
While ( $T > T_{freeze}$ )
  Do until (Thermal Equilibrium is reached) {
    choice (Config.  $i \rightarrow$  Config.  $j$ );
    if ( $W_{ij} < 0$ ) then
      accept changes (Config.  $j$ );
    else
       $r = \text{random number } [0,1];$ 
      if ( $\exp(-W_{ij}/T) > r$ ) then
        accept changes (Config.  $j$ );
      else
        ignore changes (Config.  $i$ );
  }
   $T = T * T_f;$ 
where
   $T_0$  = initial temperature;
   $W_i$  = current configuration;
   $W_j$  = choice configuration;
   $T_f$  = temperature variation coefficient;
   $\exp(-\Delta W_{ij}/T)$  = Boltzmann's factor.

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3. The Travelling Salesman problem

The methods that solve combinatorial optimization problems can be divided into two groups: deterministic and stochastic. Deterministic methods not only require large computing resources, but also do not guarantee the sufficient accuracy in finding the global minimum. Stochastic methods use statistical data with stochastic parameters that influence the optimization process and solve the problem from a point of probability.

Stochastic methods are characterized by two phases:

- General phase - where the purpose of the functions is rated as a set of model points
- Local phase - where the set is restored in the direction of the local search

Moreover, in many methods these two phases are not strictly separable.

In general terms [2], the problem of combinatorial optimization can be formalized as (R, W) , where R -state space, W - the target function, $W: R \rightarrow R$. The problem is to find a state in which W accepts the minimum value, i.e. the state i_0 satisfies the following condition:

$$W_{opt} = W_{i_0} = \min W(i), \quad (3)$$

where W_{opt} indicates the optimum (minimum) value. Then, the Metropolis algorithm can be used to generate series of states $W(i)$ among which $W(i_0)$ could be.

TSP task is to find the minimum route between N cities - visiting each city only once and in the end returning to the departure city. This is a combinatorial task that can be solved with various combinatorial or graph theory methods. Literature also provides TSP solving methods using the SA algorithm [6] - [8].

In the n -city TSP, a distance matrix $D = (d_{ij})$, $i, j = 1, 2 \dots n$ is given. d_{ij} denotes the distance between cities i and j . A tour through the n cities is defined as a closed walk visiting each city only once. Each tour can be represented by an element π of the set of all cyclic permutations of the n cities $\{1, \dots, n\}$, if π is defined as $\pi(i)$, $i = 1, \dots, n$ is the successor of city i in the tour. Thus, the set of configurations consists of all cyclic permutations of $\{1, \dots, n\}$ (there are $\frac{1}{2}(n-1)!$ such permutations for a TSP with a symmetric distance matrix) and the cost of a permutation is defined as the length of the corresponding tour:

$$C(\pi) = \sum_{i=1}^n d_{i\pi(i)} \quad (4)$$

The task of TSP is to minimize the target function in all possible permutations. If n cities are created in the 2-dimensional Euclidean space, then d_{ij} is Euclidean distance between cities i and j , then C_{ij} is the shortest route for given distance matrix D .

The paper [2] provides some numerical expressions for the estimation of routes:

$$\lim_{n \rightarrow \infty} \frac{C_{opt}^{(D)}}{\sqrt{n}} = \theta \quad (5)$$

where $\theta \approx 0.7949$.

To use the SA algorithm for this type of tasks, some concepts need to be introduced. Each route can define a neighbor as a set of routes that can be reached from the current route during one transition. Such a neighboring structure mechanism for TSP is called k -opt transition. In the simplest case, the 2-opt transition is based on the fact that 2 cities are selected on the current route and the sequence of visiting these cities is reversed (see Fig. 1).

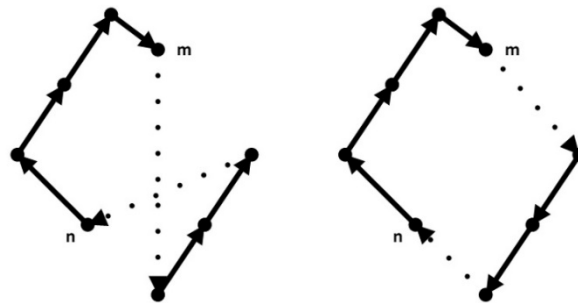


Fig. 1. Route: (a) the current route; (b) after reversing the sequence between m and n .

Route neighbors are now defined as a set of cities that can be reached from the current route through 2-opt transitions (are $\frac{1}{2}(n-1)!$ such neighbors).

For cooling mode, the temperature change coefficient T_f is 0.925 [2]. In this case, the execution time of each transition will be commensurate with $O(n^{p1})$, where $p1 < 1$, and the total execution time of the algorithm is commensurate with $O(n^p)$, where $1 < p < 2$.

4. The shortest route determination between Belarusian dairy companies

State Association Holding managing company “Concern Brestmiasomolprom” is one of the largest corporations in the meat and dairy industry in the Republic of Belarus. The enterprises of the concern have accumulated many years of experience in the production of meat and dairy products. Annual processing capacities - about 2 million tons of milk and up to 250 thousand tons of livestock. For today the corporation has around 13 thousand employees.

One of the main directions of the group's activities is the constant expansion of the products' range, the introduction of advanced technologies and new types of equipment that allow to mechanize and automate production processes (<http://brestmmp.by/en/about-us>).

The group consists of 8 dairy companies. Their brief characteristics are described below.

Joint public company “Liakhovich Milk Factory” was founded in August 1945. Every year the factory produces up to 1700 tons of butter, 1200 tons of industrial casein, more than 14000 tons of whole milk products. More than half of the total output is sold for export (http://www.molzavod.by/index_en.html).

The joint Belarusian-Russian open joint stock company “Belovezhskaya Cheese” brings joy and health to people creating high-quality and environmentally friendly products. The organization seeks to create the value of the products for the buyer. The modification of products, new forms of cheese, a new package design improve consumer product properties and increase its attractiveness in the eyes of the buyers (<http://belcheese.by/en/>).

Open joint stock company “Berioza Cheese-Producing Plant” is a company with a rich history. Today the plant is one of the leaders in dairy industry with a wide and varied range (<https://www.cheese.by/en/>).

Open joint stock company “Baranovitchi Dairy Integrated Plant” is a modern enterprise which offers only high-quality dairy products. Wide manufacturing capabilities, strict quality control and professional management help to earn confidence of customers and business partners (<http://www.ranitsamilk.by/en/>).

At joint stock company “Savushkin Product” the main law is - the company's products must be natural, useful and, at a great degree, qualitative. “Savushkin Product” is 99% plant – machine based. In the production process the milk has no contact with the external environment or people and it guarantees a maximum safety of dairy products and provides a high quality finished product (<http://www.savushkin.by/en/>).

The production in the open joint stock company “Pruzhan Milk Mill” meets the requirements of the market and the high quality criteria, with up to 65% of the output being sold for export (<http://www.prjmilk.by/en/>).

Joint stock company “Luninets dairy factory” has long traditions and wide experience in the production of milk and natural dairy products. Quality of products is closely monitored at all stages of the production process, from receipt of raw milk and tracking temperature of storage till test control of finished product samples, which are constantly tested in the laboratory of the factory. 70% of production is exported (<http://eng.lncmilk.by/>).

“Kobrin Butter and Cheese making Factory” complies with the requirements of the modern market and European standards of quality. 70 percent of the produced cheese is exported to the commercial enterprises in Moscow and to the various Russian regions (<http://www.kobrincheese.com/en/>).

The list of dairy enterprises (further in the text referred to as objects) and their GPS coordinates are shown in Table 1.

Table 1. Denotation and GPS coordinates of objects.

Name of object	Latitude	Longitude
“Liakhovich Milk Factory”	53,031017	26,257463
“Belovezhskaya Cheese”	52,372727	23,369195
“Berioza Cheese-Producing Plant”	52,542828	24,988239
“Baranovitchi Dairy Integrated Plant”	53,115339	25,94968
“Savushkin Product”	52,112929	23,765061
“Pruzhan Milk Mill”	52,550889	24,458614
“Luninets dairy factory”	52,245241	26,846835
“Kobrin Butter and Cheese making Factory”	52,211610	24,327513

For logistics purposes, a study was conducted: to find the shortest route between the given objects. It was a requirement to solve the TSP-8 task using the SA method, i.e. to determine the shortest route between 8 Belarusian dairy companies. The SA algorithm, in this case, was applied in 20 steps. As a result of the experiment, it can be concluded that the thermal equilibrium state is reached at the 20th step (see Fig. 2).

The shortest route computed by means of the SA algorithm was 648 km (see Fig. 3).

There is a possibility to remove individual objects from Table 1 and recalculate the route that allows efficiently planning transport logistics.

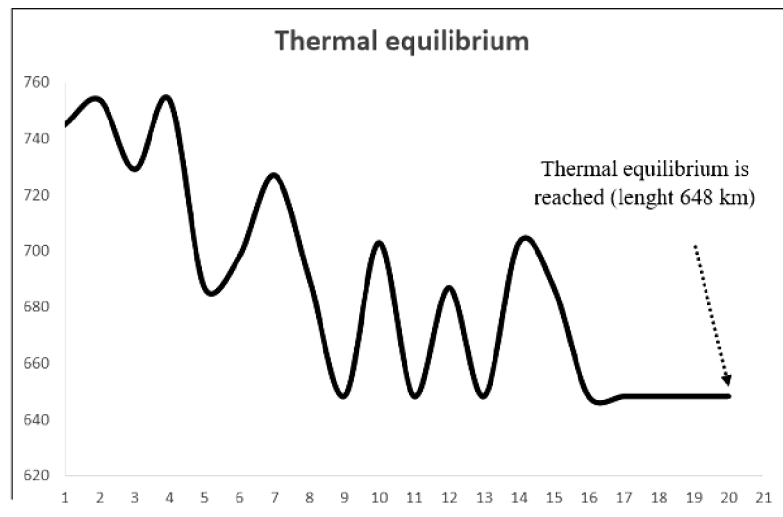


Fig. 2. Thermal equilibrium attainment.

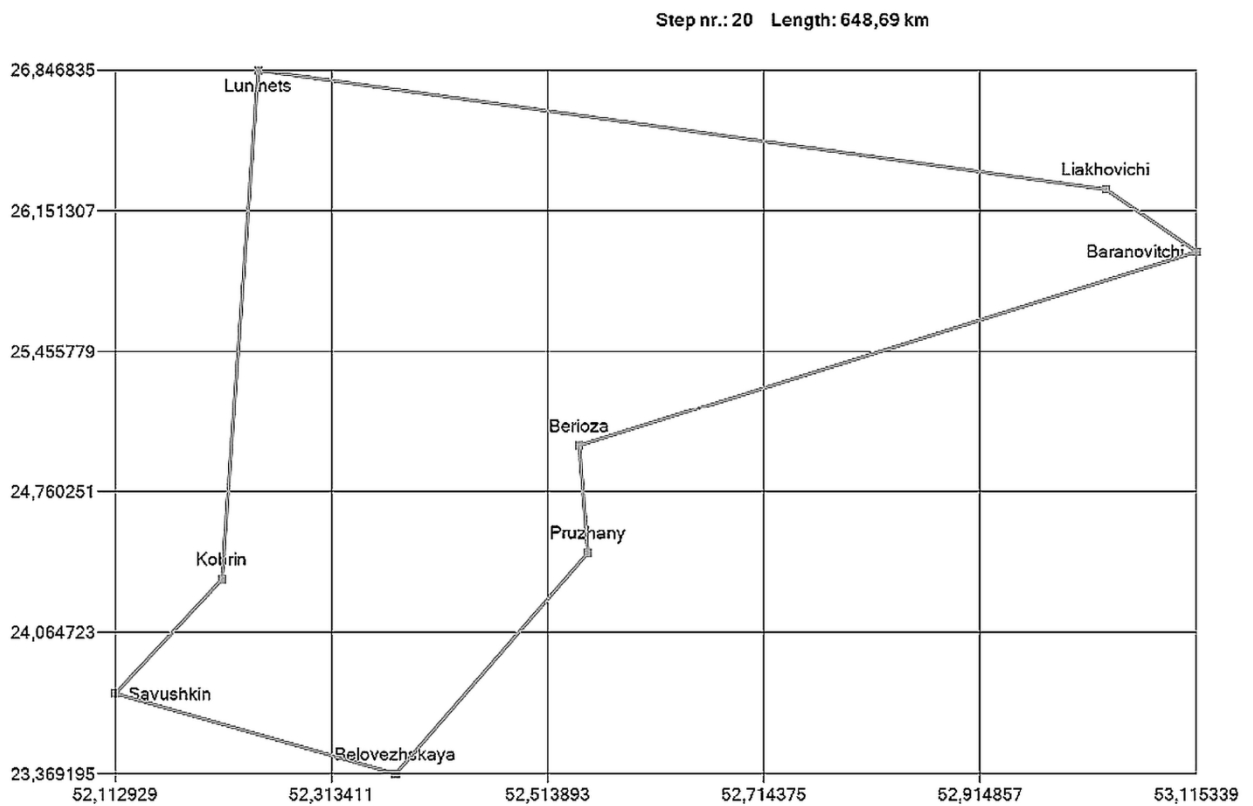


Fig. 3. The shortest route between the objects by means of the SA algorithm.

It can be concluded that the SA algorithm can be successfully applied in various optimization tasks, since SA is widely used in engineering design problems because due to its ability to find global optimum solutions [9] - [10].

5. Conclusion

This paper describes an optimization method called Simulated Annealing. The Simulated Annealing method is widely applied in various combinatorial optimization tasks. Simulated Annealing is a stochastic optimization method that can be used to minimize the specified cost function given a combinatorial system with multiple degrees of freedom. This method enables one to find a global extremum for a function that has local minimums.

In this paper the application of the Simulated Annealing method to a well - known task of combinatorial analysis, Travelling Salesman Problem, is demonstrated and an experiment aimed to find the shortest route between 8 Belarusian milk and meat processing companies is performed using the GPS coordinates.

In this research the software that allows to find the shortest route between different objects with a purpose to optimize and determine the shortest route has been developed.

We proposed that our simulation result is relatively simple, but in case it was needed to exclude an object from the existing network, it would allow to model overlapping of objects on the map and determine potentially shortest route to the chosen objects.

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