Problem Set 09

Your Name

Last modified on August 24, 2024 11:03:35 Eastern Daylight Time

Background

First load the necessary packages:

R Code

```
library(tidyverse)
library(infer)
```

For this Problem Set, you will work with some grade-point-average (GPA) data for college freshman. The following will read in the data:

R Code

```
if(!dir.exists("./Data")){
  dir.create("./Data")
  }
url <- "https://rudeboybert.github.io/SDS220/static/PS/sat_gpa.csv"
if(!file.exists("./Data/sat_gpa.csv")){
  download.file(url, destfile = "./Data/sat_gpa.csv")
  }
sat_gpa <- read_csv("./Data/sat_gpa.csv")
dim(sat_gpa)
```

```
[1] 1000    7
```

```
# Show first 6 rows of sat_gpa
kable(head(sat_gpa))
```

...1	sex	sat_verbal	sat_math	sat_total	gpa_hs	gpa_fy
1	Male	65	62	127	high	3.18
2	Female	58	64	122	high	3.33
3	Female	56	60	116	high	3.25
4	Male	42	53	95	high	2.42
5	Male	55	52	107	high	2.63
6	Female	55	56	111	high	2.91

Be sure to take a look at the data in `sat_gpa`. Each row or case in this data frame is a student. The data includes the binary gender (`sex`) of each student; the math (`sat_math`), verbal (`sat_verbal`) and total SAT scores (`sat_total`) for each student; the GPA of each student in high school (`gpa_hs`) categorized as "low" or "high"; and the GPA of each student their first year of college on a numeric scale (`gpa_fy`).

Note

We will use hypothesis testing to answer the following questions:

- Is there a difference in male and female freshman GPAs?
- Is there a difference in total SAT score for students with a "low" and "high" high-school GPA?

Note, if you get stuck as you are working through this, it will be helpful to go back and read Chapter 9 in ModernDive.

Gender Differences in First-Year GPA?

Exploratory Data Analysis

Problem 1

Calculate the mean GPA score for each sex, using the `group_by` and `summarize` commands from the `dplyr` package. Store the result in `avg_gpa_sex`. What is the difference in sample mean GPA's? Make a guess: is this difference statistically significant?

Problem 1 Answers

```
# Type your code and comments inside the code chunk
```

- Delete this and put your text answer here.
- Delete this and put your text answer here.

Problem 2

Generate a data visualization that displays the GPAs of the two groups. Be sure to include a title and label your axes.

Problem 2 Answers

```
# Type your code and comments inside the code chunk
```

Stating a Null Hypothesis

We will now test the null hypothesis that there's no difference in population mean GPA between the sexes at the population level. We can write this out in mathematical notation

$$\begin{aligned} H_0 &: \mu_{\text{female}} = \mu_{\text{male}} \\ \text{vs } H_A &: \mu_{\text{female}} \neq \mu_{\text{male}} \end{aligned}$$

or expressed differently, that the difference is 0 or not:

$$\begin{aligned} H_0 &: \mu_{\text{female}} - \mu_{\text{male}} = 0 \\ \text{vs } H_A &: \mu_{\text{female}} - \mu_{\text{male}} \neq 0 \end{aligned}$$

Testing the Hypothesis

Here's how we use infer to run this hypothesis test:

Step 1: Calculate the Observed Difference

Note that the order we choose does not matter here (female then male)…but since we used `order = c("Female", "Male")` here, we should do the same in subsequent calculations!

R Code

```
obs_diff_gpa_sex <- sat_gpa %>%
  specify(gpa_fy ~ sex) %>%
  calculate(stat = "diff in means",
            order = c("Female", "Male")) %>%
  pull()

obs_diff_gpa_sex
```

```
[1] 0.1485209
```

```
# OR
obs_diff <- -diff(tapply(sat_gpa$gpa_fy, sat_gpa$sex, mean))
obs_diff
```

```
     Male
0.1485209
```

Note that this is the difference in the group means we calculated earlier!

R Code

```
obs_diff_gpa_sex
```

```
[1] 0.1485209
```

```
2.544587 - 2.396066
```

```
[1] 0.148521
```

```
avf_avm <- obs_diff_gpa_sex
avf_avm
```

```
[1] 0.1485209
```

Step 2. Generate the Null Distribution of δ

This step involves generating simulated values *as if* we lived in a world where there's no difference between the two groups. Going back to the idea of permutation, and tactile sampling, this is akin to shuffling the GPA scores between male and female labels (i.e. removing the structure to the data) just as we could have done with index cards.

R Code

```
gpas_in_null_world <- sat_gpa %>%
  specify(gpa_fy ~ sex) %>%
  hypothesize(null = "independence") %>%
  generate(reps = 2000, type = "permute")
kable(head(gpas_in_null_world))
```

gpa_fy	sex	replicate
1.50	Male	1
3.18	Female	1
2.38	Female	1
1.96	Male	1
3.07	Male	1
2.98	Female	1

Problem 3

What was the size of the "shuffled" (permuted) sample in each replicate?

Problem 3 Answers

- Delete this and put your text answer here.

Problem 4

How many times did we do a different "shuffle" (permute) here to the sample? How many rows are in the `gpas_in_null_world` data frame?

Problem 4 Answers

```
# Type your code and comments inside the code chunk
```

- Delete this and put your text answer here.
- Delete this and put your text answer here.

Step 3. Calculate the Differences Between Male and Females Under the Null Hypothesis

The following calculates the differences in mean GPA for males and females for "shuffled" (permuted) data.

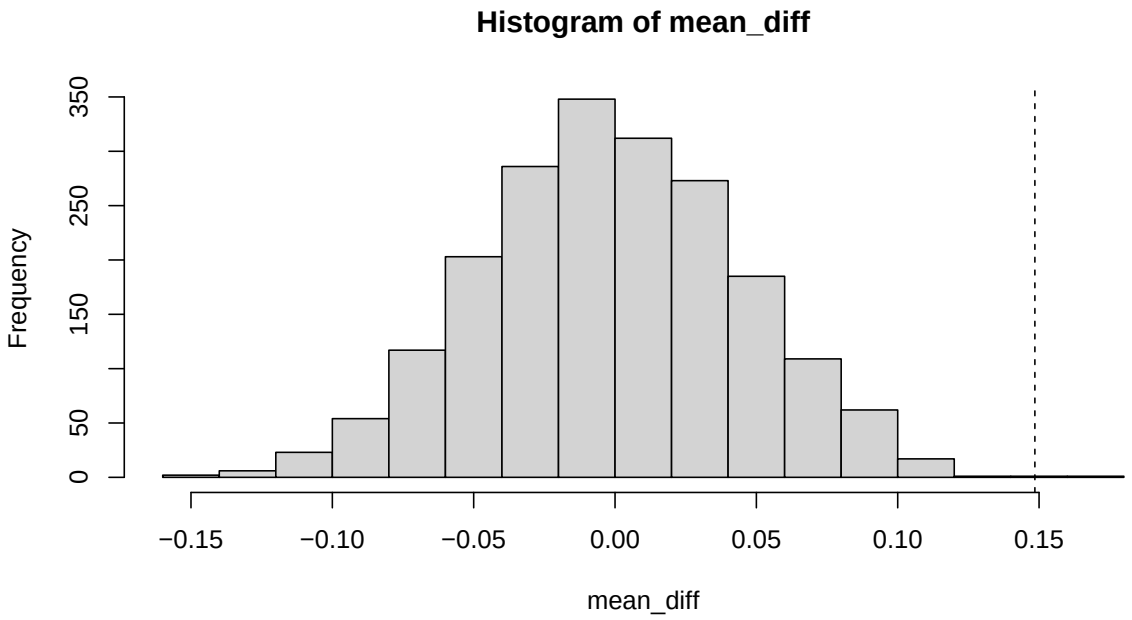
R Code

```
gpa_diff_under_null <- gpas_in_null_world %>%
  calculate(stat = "diff in means", order = c("Female", "Male"))

gpa_diff_under_null %>%
  slice(1:5) %>%  # show first five rows
  kable()
```

replicate	stat
1	-0.0225343
2	0.0044534
3	0.0204698
4	-0.0005518
5	-0.0045158

```r
# Done with a for loop
B <- 2000
mean_diff <- numeric(B)
for(i in 1:B){
  mean_diff[i] <- -diff(tapply(sat_gpa$gpa_fy, sample(sat_gpa$sex), mean))
}
hist(mean_diff)
abline(v = obs_diff, lty = "dashed")
```



Problem 5

How many rows are in the `gpa_diff_under_null` data frame? Why?

Problem 5 Answers

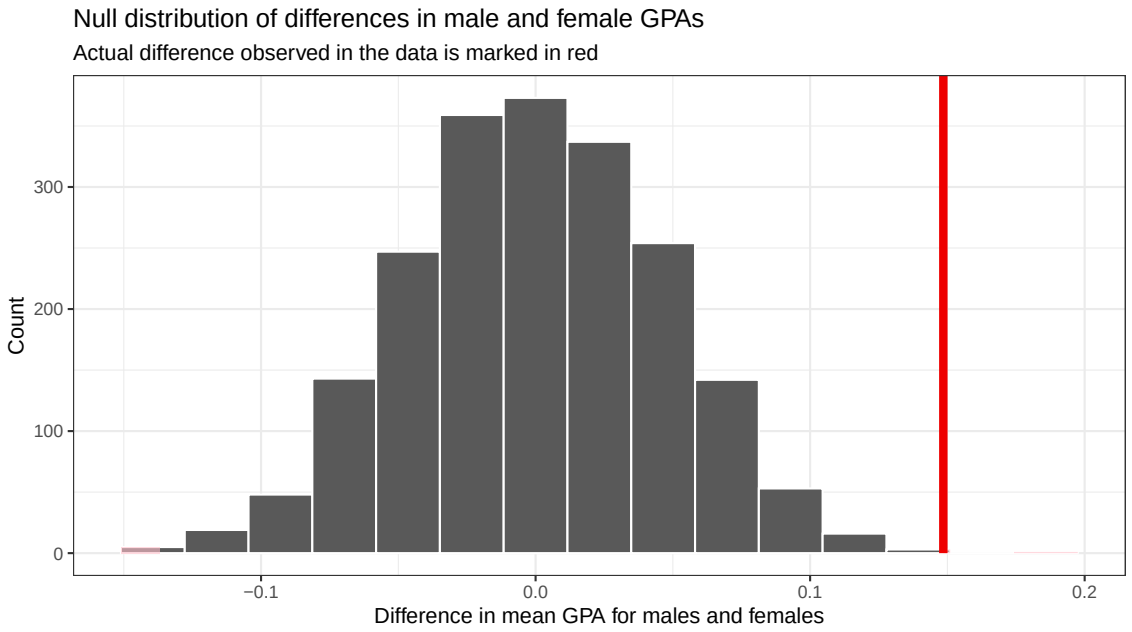
- Delete this and put your text answer here.

Step 4. Visualize how the Observed Difference Compares to the Null Distribution of δ

The following plots the δ values we calculated for each of the different "shuffled" replicates. This is the null distribution of δ. The red line shows the observed difference between male and female scores in the data (0.1485209) from Step 1.

R Code

```
gpa_diff_under_null %>%
  visualize() +
  labs(x = "Difference in mean GPA for males and females", y = "Count",
       title = "Null distribution of differences in male and female GPAs",
      subtitle = "Actual difference observed in the data is marked in red") +
  shade_p_value(obs_stat = obs_diff_gpa_sex, direction = "two-sided") +
  theme_bw()
```



Note

Note that zero is the center of this null distribution. The null hypothesis is that there is no difference between males and females in GPA score. In the permutations, zero was the most common value, because all structure was removed from the data. GPA values were sorted into male and female **at random**. Values as large as 0.1 and -0.1 occurred, but much less frequently, because they are just not as likely when structure is removed from the data.

Step 5: Calculate a p-value

R Code

```
gpa_diff_under_null %>%
  get_pvalue(obs_stat = obs_diff_gpa_sex, direction = "both") %>%
  pull() -> pvalue
pvalue
```

```
[1] 0.001
```

```
# OR from the for loop - slighlty more conservative approach
p_value <- (sum(mean_diff >= obs_diff)*2 + 1)/(B + 1)
p_value
```

```
[1] 0.002498751
```

The p-value indicates that there is a 0.001 or 0.0025 chance (very low even with the conservative p-value computation) that we would see a difference of 0.1485209 in GPA scores between males and females (or a bigger difference) if in fact there was truly no difference between the sexes in GPA scores at the population level.

Problem 6

Fill in the blanks and select the appropriate words below to write up the results & conclusions for this test:

Problem 6 Answers

The mean GPA scores for females in our sample ($\bar{x}_f =$ _____) was greater than that of males ($\bar{x}_m =$ _____). This difference **was/was not** statistically significant at $\alpha = 0.05$, (p = _____). Given this p-value, I **would/would not** reject the Null hypothesis and **find evidence/do not find evidence** that **females** have higher GPAs than **males** at the population level.

Step 6: Calculate a Confidence Interval for the Difference

The following will allow us to calculate a 95% bootstrap percentile confidence interval for the difference between mean GPA scores for females and males.

R Code

```
ci_diff_gpa_means <- sat_gpa %>%
  specify(gpa_fy ~ sex) %>%
  generate(reps = 2000, type = "bootstrap") %>%
  calculate(stat = "diff in means", order = c("Female", "Male")) %>%
  get_ci(level = 0.95, type = "percentile")
kable(ci_diff_gpa_means)
```

lower_ci	upper_ci
0.0544304	0.2400219

```
# Do the same thing with a for loop
sat_gpa %>%
  filter(sex == "Female") %>%
  select(gpa_fy) %>%
  pull() -> fem_gpa
sat_gpa %>%
  filter(sex == "Male") %>%
  select(gpa_fy) %>%
  pull() -> mal_gpa
mean_ds <- numeric(B)
for(i in 1:B){
  bss1 <- sample(fem_gpa, size = sum(!is.na(fem_gpa)), replace = TRUE)
  bss2 <- sample(mal_gpa, size = sum(!is.na(mal_gpa)), replace = TRUE)
  mean_ds[i] <- mean(bss1) - mean(bss2)
}
kable(quantile(mean_ds, probs =c(0.025, 0.975)))
```

	x
2.5%	0.0548812
97.5%	0.2355701

Complete all the Above Tasks with a t-test

Note that all the above steps can be done with one line of code **if a slew of assumptions** like normality and equal variance of the groups are met.

R Code

```
t.test(gpa_fy ~ sex, var.equal = TRUE, data = sat_gpa)
```

```
    Two Sample t-test

data:  gpa_fy by sex
t = 3.1828, df = 998, p-value = 0.001504
alternative hypothesis: true difference in means between group Female and group Male is not
95 percent confidence interval:
 0.05695029 0.24009148
sample estimates:
mean in group Female   mean in group Male
            2.544587             2.396066
```

Relationship Between High-School & First-Year GPA?

For this analysis `sat_total` is the outcome variable, and `gpa_hs` is the predictor variable, with two levels "low" and "high".

Exploratory Data Analysis

We can first calculate the mean total SAT score for each group (i.e students with a low and high GPA), using the `group_by` and `summarize` commands from the `dplyr` package.

R Code

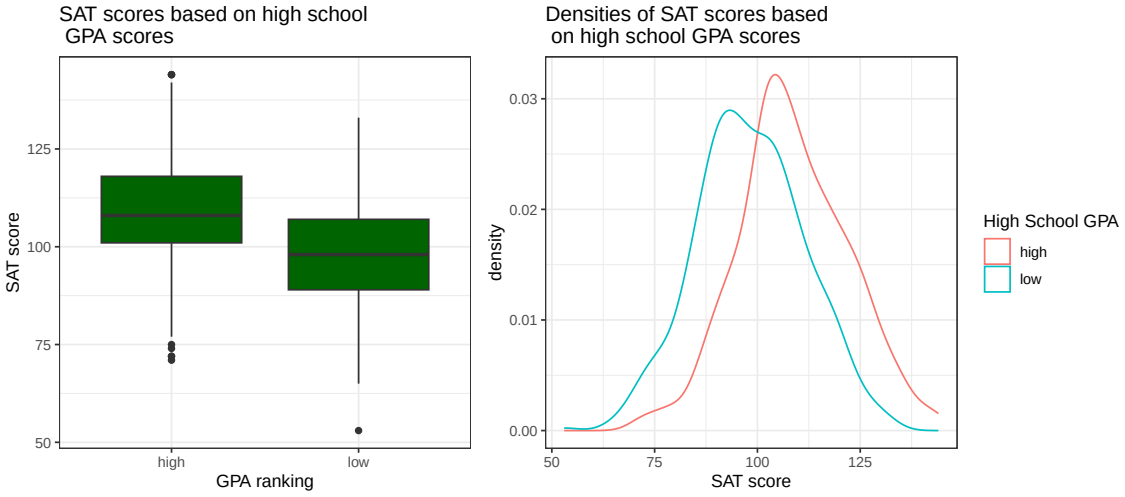
```
avg_sat_gpa <- sat_gpa %>%
  group_by(gpa_hs) %>%
  summarize(sat_total = mean(sat_total), n = n())
kable(avg_sat_gpa)
```

gpa_hs	sat_total	n
high	108.67828	488
low	98.23047	512

We will next generate a data visualization that displays the total SAT scores of the two groups. Be sure to include a title and label your axes.

R Code

```
library(patchwork)
p1 <- ggplot(sat_gpa, aes(x = gpa_hs, y = sat_total)) +
  geom_boxplot(fill = "darkgreen") +
  labs(title = "SAT scores based on high school \n GPA scores",
       x = "GPA ranking",
       y = "SAT score") +
  theme_bw()
# Or
p2 <- ggplot(data = sat_gpa,
             aes(x = sat_total, color = gpa_hs)) +
  geom_density() +
  theme_bw() +
  labs(color = "High School GPA",
       title = "Densities of SAT scores based \n on high school GPA scores",
       x = "SAT score")
p1 + p2
```



Stating a Null Hypothesis

Problem 7

State the null hypothesis that you are testing (using words and symbols).

Problem 7 Answers

- Delete this and put your text answer here.
- Delete this and put your text answer here.

Testing the Null Hypothesis

Problem 8

Calculate the observed difference between the mean total SAT scores of the low and high GPA high-school students. Store the result in an object named `obs_diff_sat_hs_gpa`

Problem 8 Answers

```
# Type your code and comments inside the code chunk
```

Problem 9

Generate the null distribution of δ. Here you need to generate simulated values *as if* we lived in a world where there's no difference in SAT scores between high school students with low and high GPAs. Use 2000 replications to generate the null distribution.

Problem 9 Answers

```
# Type your code and comments inside the code chunk
```

Problem 10

Calculate the differences in mean SAT scores between students with high and low GPA scores under the null hypothesis. Note you should use whatever order you chose in 7. Store your results in an object named `sat_diff_under_null`. Show the first six rows of `sat_diff_under_null`.

Problem 10 Answers

```
# Type your code and comments inside the code chunk
```

Problem 11

Visualize how the observed difference compares to the null distribution of δ. Generate a histogram of the null distribution, with a vertical red line showing the observed difference in SAT scores between high school students with a high and low GPA.

Problem 11 Answers

```
# Type your code and comments inside the code chunk
```

Problem 12

Calculate the p-value.

Problem 12 Answers

```
# Type your code and comments inside the code chunk
```

- Delete this and put your text answer here.

Problem 13

Write up the results & conclusions for this hypothesis test. Note, p-values less than 0.001 are often reported as $p < 0.001$.

Problem 13 Answers

- Delete this and put your text answer here.

Problem 14

Compute a 95% bootstrap percentile confidence interval for the difference in total SAT scores for students with high and low high-school GPA scores. Note that you should use whatever order you chose for your null hypothesis. That is either `order = c("low", "high")` or `order = c("high", "low")`. Provide a basic interpretation of your com-

puted interval.

Problem 14 Answers

```
# Type your code and comments inside the code chunk
```

- Delete this and put your text answer here.

Problem 15

Use a t-test to test the null hypothesis that average total SAT scores do not differ between students with high and low high school GPA scores at the population level.

Problem 15 Answers

```
# Type your code and comments inside the code chunk

# Use the bootstrap T distribution to test
```

Turning in Your Work

You will need to make sure you commit and push all of your changes to the github education repository where you obtained the lab.

Tip

- Make sure you **render a final copy with all your changes** and work.
- Look at your final html file to make sure it contains the work you expect and is formatted properly.

Logging out of the Server

There are many statistics classes and students using the Server. To keep the server running as fast as possible, it is best to sign out when you are done. To do so, follow all the same steps for closing Quarto document:

Tip

- Save all your work.
- Click on the orange button in the far right corner of the screen to quit `R`
- Choose **don't save** for the **Workspace image**
- When the browser refreshes, you can click on the sign out next to your name in the top right.
- You are signed out.

```
sessionInfo()
```

```
R version 4.4.1 (2024-06-14)
Platform: x86_64-redhat-linux-gnu
Running under: Red Hat Enterprise Linux 9.4 (Plow)

Matrix products: default
BLAS/LAPACK: FlexiBLAS OPENBLAS-OPENMP;  LAPACK version 3.9.0

locale:
 [1] LC_CTYPE=en_US.UTF-8       LC_NUMERIC=C
 [3] LC_TIME=en_US.UTF-8        LC_COLLATE=en_US.UTF-8
 [5] LC_MONETARY=en_US.UTF-8    LC_MESSAGES=en_US.UTF-8
 [7] LC_PAPER=en_US.UTF-8       LC_NAME=C
 [9] LC_ADDRESS=C               LC_TELEPHONE=C
[11] LC_MEASUREMENT=en_US.UTF-8 LC_IDENTIFICATION=C

time zone: America/New_York
tzcode source: system (glibc)

attached base packages:
[1] parallel  stats     graphics  grDevices utils     datasets  methods
[8] base

other attached packages:
 [1] patchwork_1.2.0  infer_1.0.7      lubridate_1.9.3  forcats_1.0.0
 [5] stringr_1.5.1    dplyr_1.1.4      purrr_1.0.2      readr_2.1.5
 [9] tidyr_1.3.1      tibble_3.2.1     ggplot2_3.5.1    tidyverse_2.0.0
[13] scales_1.3.0     doMC_1.3.8       iterators_1.0.14 foreach_1.5.2
[17] knitr_1.48

loaded via a namespace (and not attached):
 [1] utf8_1.2.4        generics_0.1.3    stringi_1.8.4     hms_1.1.3
```

```
 [5] digest_0.6.36     magrittr_2.0.3    evaluate_0.24.0   grid_4.4.1
 [9] timechange_0.3.0  fastmap_1.2.0     jsonlite_1.8.8    tinytex_0.52
[13] fansi_1.0.6       codetools_0.2-20  cli_3.6.3         crayon_1.5.3
[17] rlang_1.1.4       bit64_4.0.5       munsell_0.5.1     withr_3.0.1
[21] yaml_2.3.10       tools_4.4.1       tzdb_0.4.0        colorspace_2.1-1
[25] vctrs_0.6.5       R6_2.5.1          lifecycle_1.0.4   bit_4.0.5
[29] vroom_1.6.5       pkgconfig_2.0.3   pillar_1.9.0      gtable_0.3.5
[33] glue_1.7.0        xfun_0.47         tidyselect_1.2.1  rstudioapi_0.16.0
[37] farver_2.1.2      htmltools_0.5.8.1 labeling_0.4.3    rmarkdown_2.28
[41] compiler_4.4.1
```