

1. Pick 10 hypotheses before seeing the data, each one represents some "discovery". Reject the hypotheses whose pvalues are smaller than 0.05.

Solution: Suppose the hypotheses are independent, meaning the pvalues are independent under the null distribution. Also suppose all of the null hypotheses are correct, then (the probability of having a false discovery/rejection) = (1 - the probability that there is no rejection) = $1 - 0.95^{10} = 0.4$. This means suppose all the null hypothesis are correct, then this procedure will very likely to declare a false discovery.

Remarks:

- Example [data](#)
- This is called multiple testing problem. It is easy(?) to avoid this problem in each study, but it is hard to avoid the whole community implicitly performing multiple testing. I think this is one of the reasons why people ask "Why are most scientific discoveries false?".
- Bonferroni correction: significance threshold divided by 10 will guarantee that the probability of making a false rejection is at most 0.05.
- More sophisticated correction: FDR assuming independence of the pvalues. In practice, p-value often tend to be correlated. So you can fix this problem precisely when you have absolute knowledge of the joint distribution of the pvalues.

2. Fix one hypothesis, pick 10 tests before seeing the data. No test is uniformly more powerful than any other one. Reject the hypothesis if any of the 10 pvalues is smaller than 0.05.

Solution: In this case, you become really strict about false rejections. The probability of the falsely rejecting the null will be lower than 0.05 and exact probability depends on the tests. As a tradeoff, you lose some power (the probability of rejecting under the alternative). Whether it is valid or not depends on the exact context and can be hard to judge, just like choosing a really small pvalue threshold. Think about normality test where you want to not reject null and the discovery setting when you want to reject the null.

3. Under the same setting as the previous one, reject the hypothesis if all of the 10 pvalues are smaller than 0.05.