Regression and Other Stories: Ch 3

Chapter 3 focuses on important statistical and mathematical understanding for this class. As the semester progresses, and depending on how 506 will be taught, the treatment of these things might differ.

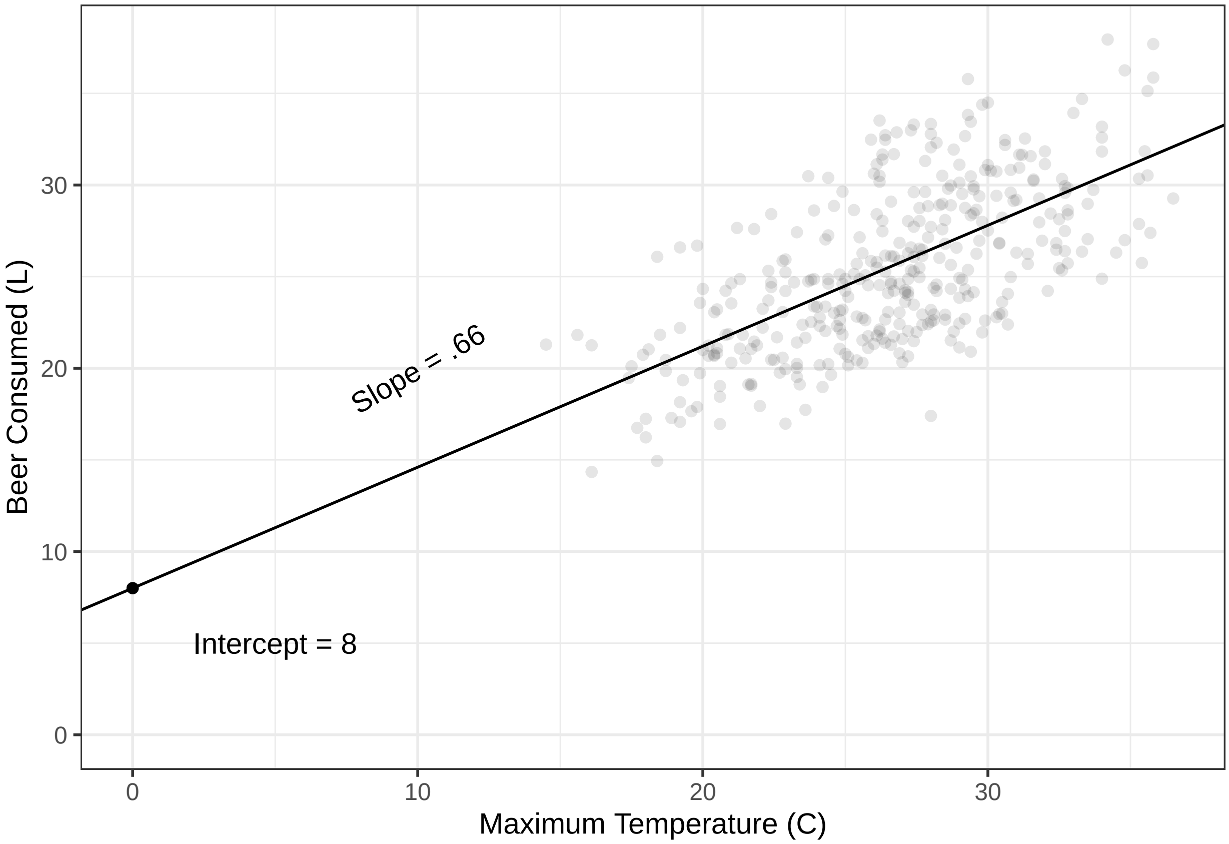
Recall the Brazilian Beer dataset. Let y_i be the response variable, beer consumption, and x_i be the maximum temperature, both on day i.

Then using scalar notation, we can write out the model as

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i \tag{1}$$

where β_0 is the intercept (predicted response with x is zero), β_1 is the slope term (predicted increase/decrease with a one unit change in x) and $epsilon_i$ is an error term (more soon)

This model results in a linear relationship between y and x, for the deterministic portion. *($\beta_0 + \beta_1 x_i$) Sketch this model*



Linear Algebra

Using matrix algebra, this model can also be formulated as

$$\mathbf{y} = X\beta + \epsilon, \tag{2}$$

where $\mathbf{y}$ is a $n \times 1$ vector, X is a $n \times 2$ matrix, β is a 2×1 vector, and ϵ is a $n \times 1$ vector.

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix}$$

$$\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}$$

Non-linear relationships

While linear models assume a linear relationship between y and x, we can also transform covariates.

```r
## Simulate logarithmic relationship
dat <- tibble(x = runif(1000, min = 1, max = 10)) %>%
  mutate(log_x = log(x), y = log_x + rnorm(1000, mean = 0, sd =.1))
```

```r
fig1 <- dat %>% ggplot(aes(y=y,x=x)) + geom_point(alpha = .2) +
  geom_smooth(method = 'lm', formula = 'y~x', color = 'red', linetype = 2) +
  geom_smooth(method = 'loess', formula = 'y~x') + theme_bw()
fig2 <- dat %>% ggplot(aes(y=y,x=log_x)) + geom_point(alpha = .2) + theme_bw() +
  geom_smooth(method = 'lm', formula = 'y~x')
grid.arrange(fig1, fig2, nrow = 1, ncol = 2)
```

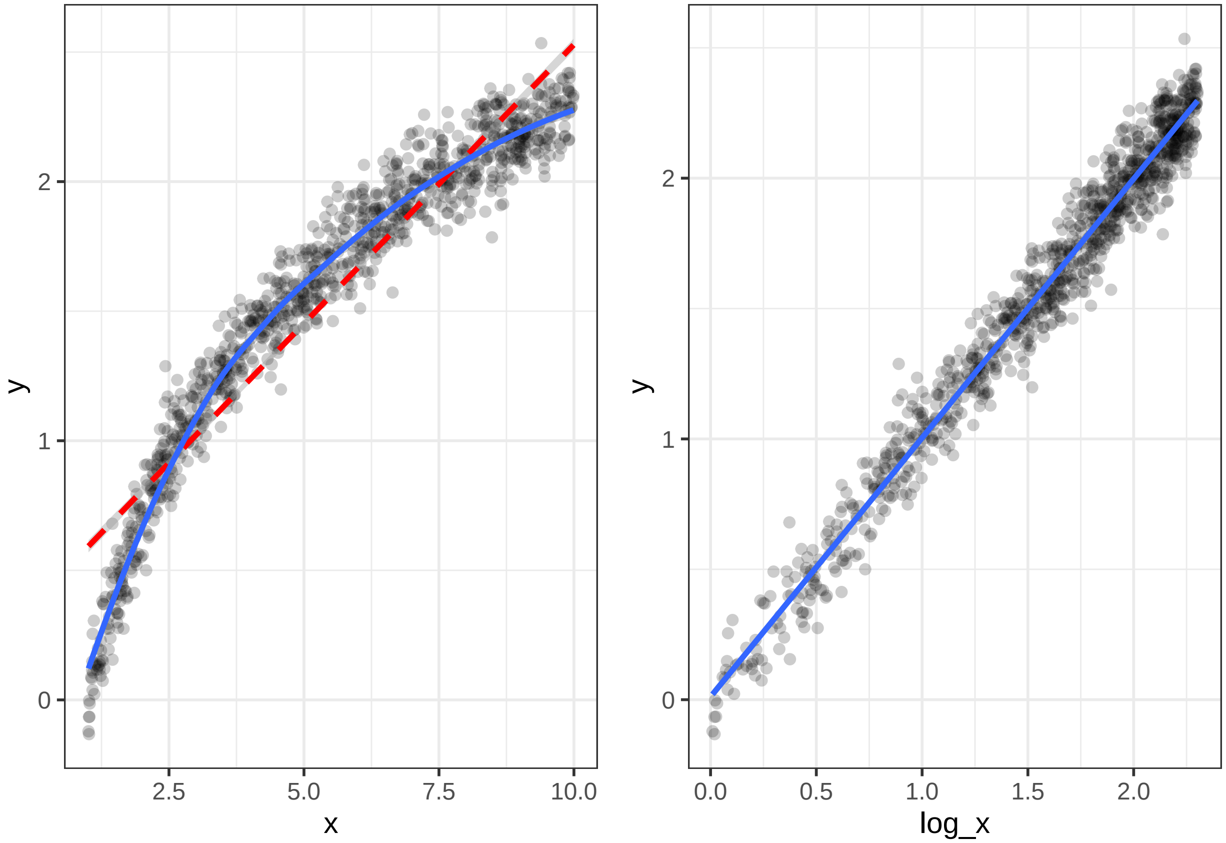


Figure 1: Plots of y vs. x (left panel) and log(x) (right panel). The figure in the left shows a non-linear relationship between y and x. The blue line is a LOESS curve and the red line is a linear relationship. The panal on the right shows a linear relationship between y and log(x).

Probability Distributions

While regression models allow us to fit linear (and non-linear) functional relationships, *assessment and interpretation of uncertainty is an important part of the statistical process.*

The error term (ϵ) controls the randomness fundamental to data and that provides a mechanism to *capture and report uncertainty.*

Understanding uncertainty begins with a discussion about fundamental statistical terms of a probability distribution.

- mean: *or expectation $E[y]$, is the value obtained, on average, from a random sample from the distribution*

- variance: $E[(y - \mu_y)^2]$ *mean of squared differences from the mean.*

- standard deviation: *square root of the variance.*

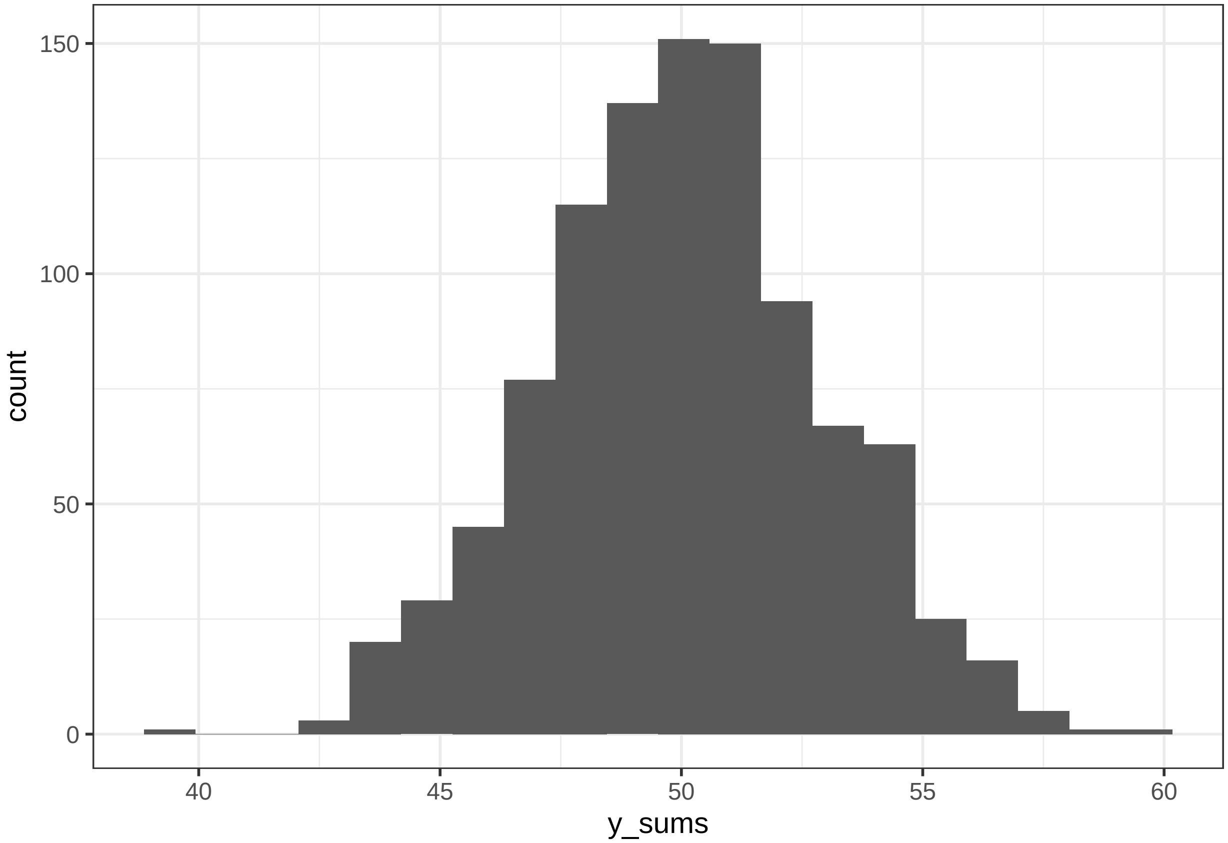
Normal Distribution The Central Limit Theorem (CLT) states

> that the sum of many small, independent random variables will be a random variable that approximates a normal distribution.

The CLT is the reason that we can say the confidence interval is ± 2 standard errors from the mean.

The following code demonstrates the CLT. **Q** what does each line of code do? Then complete the caption and/or title on the figure.

```
num_sims <- 1000
num_obs <- 100
y <- matrix(runif(num_obs* num_sims), nrow = num_sims, ncol = num_obs)
y_sums <- tibble(y_sums = rowSums(y))
y_sums %>% ggplot(aes(x=y_sums)) +
  geom_histogram(bins = 20) +
  theme_bw()
```



In R we can:

1. Simulate from distributions

```
rnorm(n = 1, mean = 0 , sd = 1)
```

```
## [1] -0.8838417
```

2. Find quantiles from distributions

```
qnorm(.975, mean = 0, sd = 1)
```

```
## [1] 1.959964
```

3. We can also evaluate probability densities and cumulative distribution functions with `d` and `p`.

If a random variable, x is normally distributed, then a linear transformation of x is also normally distributed.

Assume $x \sim N(0, 1)$, then

- $ax \sim N(0, a^2)$
- $x + b \sim N(b, 1)$
- $ax + b \sim N(b, a^2)$

Other probability Distributions

More on these later:

- *Lognormal distribution: continuous values greater than 0*
- *binomial distribution: binary trials*
- *Poisson distribution: (one way to model) count data*