Regression and Other Stories: Continuous Predictor

Regression Example

We will once again use the Brazilian beer dataset to illustrate the regression process.

```
beer <- read_csv('http://math.montana.edu/ahoegh/Data/Brazil_cerveja.csv')
```

- Fit Model

```
stan_fit <- stan_glm(consumed ~ max_tmp, data = beer, refresh = 0)
print(stan_fit)
```

```
## stan_glm
##  family:       gaussian [identity]
##  formula:      consumed ~ max_tmp
##  observations: 365
##  predictors:   2
## ------
##             Median MAD_SD
## (Intercept) 8.0    1.1
## max_tmp     0.7    0.0
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 3.4    0.1
##
## ------
## * For help interpreting the printed output see ?print.stanreg
## * For info on the priors used see ?prior_summary.stanreg
```

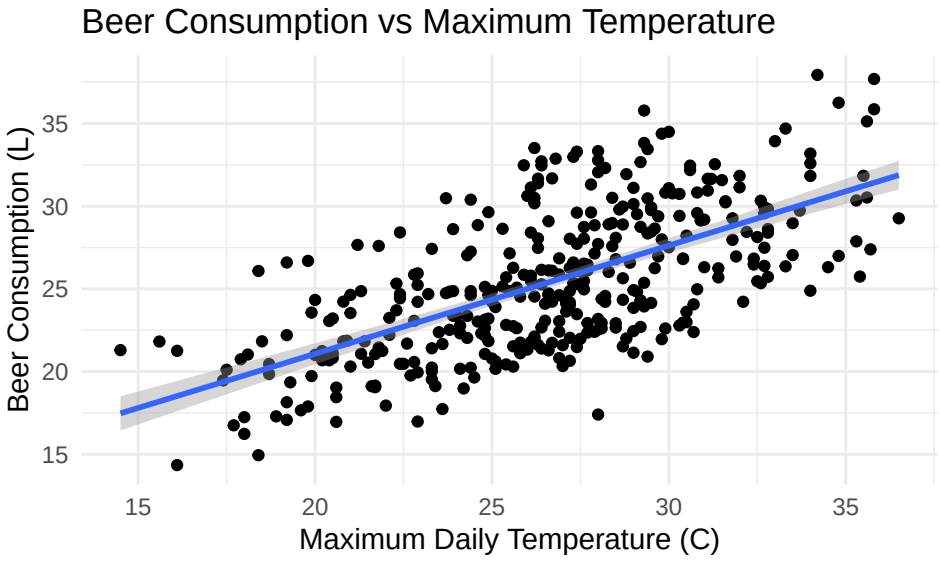
- the fitted regression line is $\hat{y} = 8 + .7x$, *where* $\hat{y}$ *is the beer consumption in liters and* x *is the daily maximum temperature.*

- At $x = 0$ *(maximum temperature of O) the daily consumption is predicted to be 8 liters*

- Each additional degree (maximum temperature) corresponds *to an expected daily consumption that is 0.7 liter greater than 8 liters.*

- The standard errors around the coefficients are quite small.

- The estimated *residual standard deviation* is 3.4. *To interpret this value, roughly 68% of the daily consumption values will be within* $\pm$ *3.4 liters of the fitted regression line and 95% will fall within* $\approx \pm 2 \times 3.4$ *liters of the regression line.*

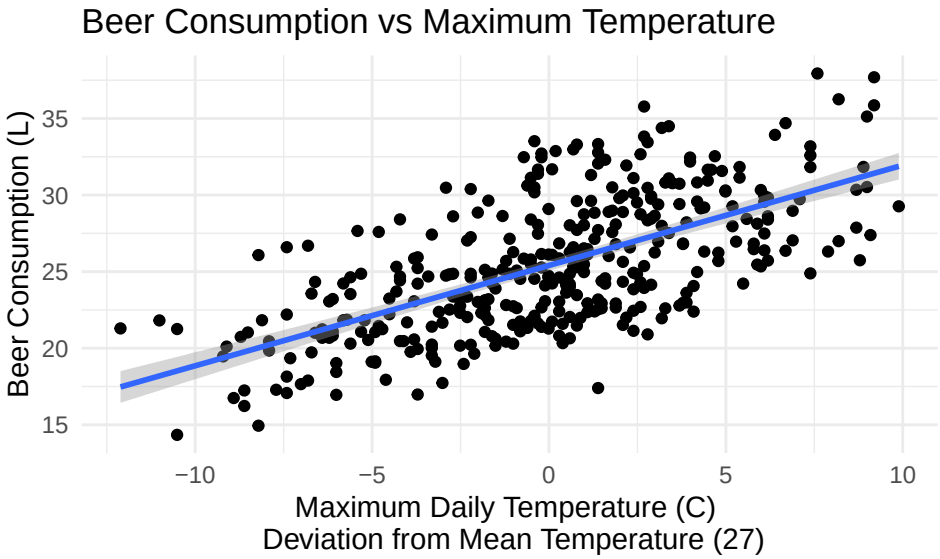



```
ave_tmp <- beer %>% summarize(ave_tmp = mean(max_tmp)) %>% pull()
```

Centering Data *The average daily maximum temperature is 27 Celsius, which corresponds to 80 F.*

We can center the temperature by subtracting the average temperature. Thus the new variable corresponds to deviation from the average temperature.

```
beer <- beer %>% mutate(tmp_centered = max_tmp - ave_tmp)
```




```
stan_fit <- stan_glm(consumed ~ tmp_centered, data = beer, refresh = 0)
print(stan_fit)
```

```
## stan_glm
##  family:       gaussian [identity]
##  formula:      consumed ~ tmp_centered
##  observations: 365
##  predictors:   2
## ------
##              Median MAD_SD
## (Intercept)  25.4    0.2
## tmp_centered  0.7    0.0
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 3.4    0.1
##
## ------
## * For help interpreting the printed output see ?print.stanreg
## * For info on the priors used see ?prior_summary.stanreg
```

- the fitted regression line is $\hat{y} = 25.4 + .7x^{'}$, *where* $\hat{y}$ *is the beer consumption in liters and* $x^{'}$ *is the deviation from the average daily maximum temperature (27C).*

- At $x^{'} = 0$ *(average daily maximum temperature) the daily consumption is predicted to be 25.4 liters*

- Each degree different from the daily average maximum temperature corresponds *to an expected daily consumption that is 0.7 liter greater/less than 25.4 liters.*

- The standard errors around the coefficients are quite small.

- The estimated *residual standard deviation* is 3.4. *To interpret this value, roughly 68% of the daily consumption values will be within* $\pm$ *3.4 liters of the fitted regression line and 95% will fall within* $\approx \pm 2 \times 3.4$ *liters of the regression line.*

Comparisons of a mean and linear models

Consider comparing the mean beer consumption between weekend and weekdays.

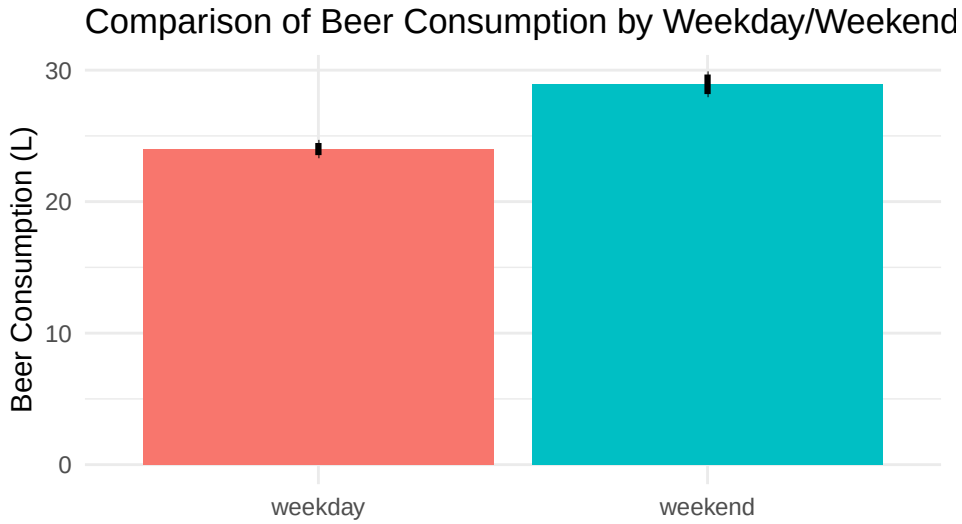


Figure 1: Comparison of beer consumption by day of week.

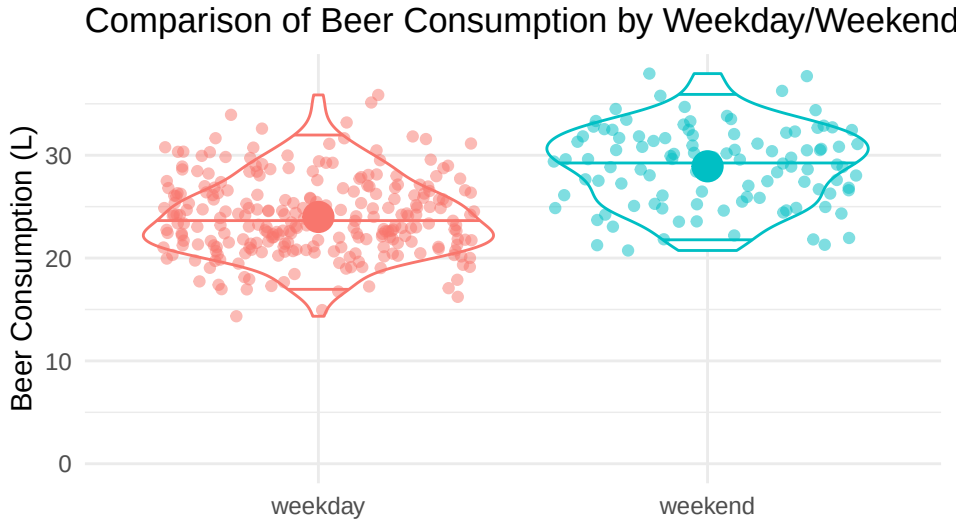


Figure 2: Comparison of beer consumption by day of week. The lines on the violin plots correspond to the median, and the .025 and .975 quantiles. The large circles represent the mean comsumption for each group.

A t-test is a common procedure for comparing whether the mean differs between two populations.

```
weekend <- beer %>% filter(weekend == 1) %>% dplyr::select(consumed) %>% pull()
weekday <- beer %>% filter(weekend == 0) %>% dplyr::select(consumed) %>% pull()
t.test(weekend, weekday)
```

```
##
##  Welch Two Sample t-test
##
## data:  weekend and weekday
## t = 11.123, df = 187.62, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  4.051098 5.797899
## sample estimates:
## mean of x mean of y
##  28.92272  23.99822
```

Formally this can be expressed as a linear model.

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

where y_i is the consumption on day i, x_i is a indicator variable for whether day i is a weekend. The parameter β_0 is the mean value for the reference case (weekday in this setting) and β_1 is the mean difference between the two categories.

```
beer %>% mutate(weekend = as.factor(weekend)) %>%
stan_glm(consumed ~ weekend, data = ., refresh = 0)
```

```
## stan_glm
##  family:       gaussian [identity]
##  formula:      consumed ~ weekend
##  observations: 365
##  predictors:   2
## ------
##             Median MAD_SD
## (Intercept) 24.0    0.2
## weekend1     4.9    0.4
##
## Auxiliary parameter(s):
##       Median MAD_SD
## sigma 3.8    0.1
##
## ------
## * For help interpreting the printed output see ?print.stanreg
## * For info on the priors used see ?prior_summary.stanreg
```

With the t test, the goal is to determine whether the mean difference between the two groups is different from zero. This test statistics corresponds to β_1 in the regression model specified above.

Similarly other "named models" such as ANOVA (ANalysis Of VAriance), ANCOVA (ANalysis of COVAriance) are just special cases of a linear model.