

HW8

1. Separation (12 points)

a. (4 points)

Assume $y \sim \text{Binomial}(n, p)$, what is the MLE of p ? Using your MLE what is the point estimate and confidence interval for p when $y = n$?

b. (4 points)

Now consider a logistic regression model

$$\begin{aligned} y_i &\sim \text{Bernoulli}(p_i) \\ \text{logit}(p_i) &= \beta_0 + \beta_1 I(\text{zipcode} = \text{group2}) \end{aligned}$$

If all of the y_i values for group 1 are equal to 0 and all the y_i values for group 2 are equal to 1, what should be the MLE for β_0 ? Hint: you can use the `invlogit` function in the `arm` package to explore this.

c. (4 points)

Diagnose the model

```
seattle %>% group_by(zipcode) %>% summarise(mean(waterfront))

## # A tibble: 2 x 2
##   zipcode `mean(waterfront)`
##   <fct>      <dbl>
## 1 98070      1
## 2 98102      0

glm_separation <- glm(waterfront ~ zipcode, data = seattle, family = binomial)

## Warning: glm.fit: algorithm did not converge

display(glm_separation)

## glm(formula = waterfront ~ zipcode, family = binomial, data = seattle)
##               coef.est coef.se
## (Intercept)    26.57 72693.36
## zipcode98102  -53.13 80574.03
## ---
##   n = 129, k = 2
##   residual deviance = 0.0, null deviance = 124.0 (difference = 124.0)
```

2. Diagnostics / Simulation (8 points)

a. (4 points)

Simulate data and fit a logistic regression model to this data. Comment on your results.

b. (4 points)

Repeat this simulation many times and create diagnostics plots to look at the residuals. Present one plot and comment on the results

3. Titanic Prediction (10 points)

The `titanic` package in R contains survival data on passengers from the Titanic. Use the `titanic_train` dataset to fit a logistic regression model for passenger survival. Then report your classification error on the `titanic_test` dataset. Look at a few different model specifications and summarize what you found to work best.

```
library(titanic)
tibble(titanic_train)

## # A tibble: 891 x 1
##   titanic_train$P... $Survived $Pclass $Name $Sex   $Age $SibSp $Parch $Ticket
##           <int>      <int>   <int> <chr> <chr> <dbl> <int> <int> <chr>
## 1             1          0       3 Brau... male    22     1     0 A/5 21...
## 2             2          1       1 Cumi... fema... 38     1     0 PC 175...
## 3             3          1       3 Heik... fema... 26     0     0 STON/0...
## 4             4          1       1 Futr... fema... 35     1     0 113803
## 5             5          0       3 Alle... male   35     0     0 373450
## 6             6          0       3 Mora... male   NA     0     0 330877
## 7             7          0       1 McCa... male   54     0     0 17463
## 8             8          0       3 Pals... male    2     3     1 349909
## 9             9          1       3 John... fema... 27     0     2 347742
## 10           10          1       2 Nass... fema... 14     1     0 237736
## # ... with 881 more rows, and 3 more variables: $Fare <dbl>, $Cabin <chr>,
## #   $Embarked <chr>
tibble(titanic_test)

## # A tibble: 418 x 1
##   titanic_test$Pa... $Pclass $Name $Sex   $Age $SibSp $Parch $Ticket $Fare $Cabin
##           <int>      <int> <chr> <chr> <dbl> <int> <int> <chr>   <dbl> <chr>
## 1             892          3 Kell... male   34.5     0     0 330911   7.83 ""
## 2             893          3 Wilk... fema... 47     1     0 363272    7 ""
## 3             894          2 Myle... male    62     0     0 240276   9.69 ""
## 4             895          3 Wirz... male    27     0     0 315154   8.66 ""
## 5             896          3 Hirv... fema... 22     1     1 3101298 12.3 ""
## 6             897          3 Sven... male    14     0     0 7538     9.22 ""
## 7             898          3 Conn... fema... 30     0     0 330972   7.63 ""
## 8             899          2 Cald... male    26     1     1 248738   29 ""
## 9             900          3 Abra... fema... 18     0     0 2657     7.23 ""
## 10           901          3 Davi... male    21     2     0 A/4 48... 24.2 ""
## # ... with 408 more rows, and 1 more variable: $Embarked <chr>
```