Lecture 10: Gelman Hill Ch 7

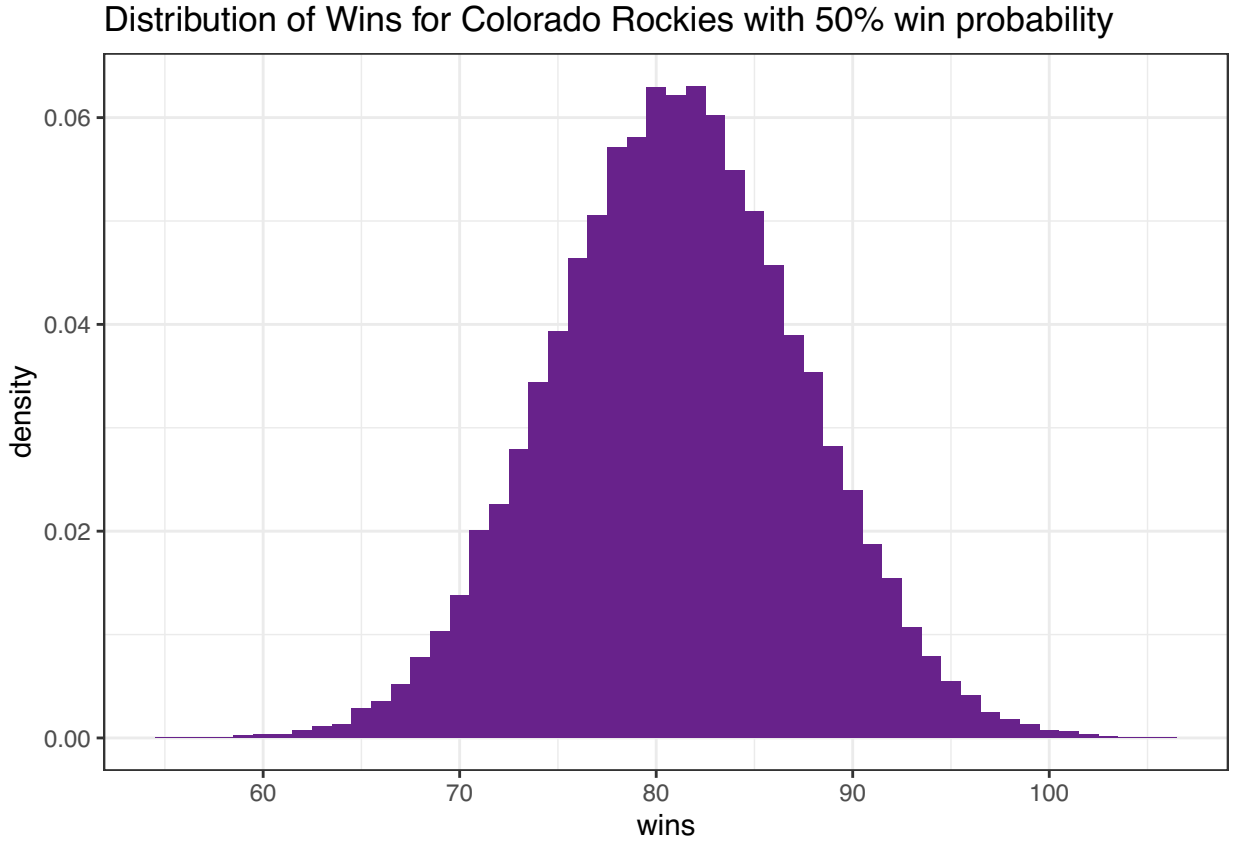
Simulation-Based Model Assessment

Simulation of Probabilistic Models

Suppose the Colorado Rockies have a talent level that corresponds to winning 50% of their games. What are the reasonable expectation for the number of games the Rockies will win this season (out of 162).

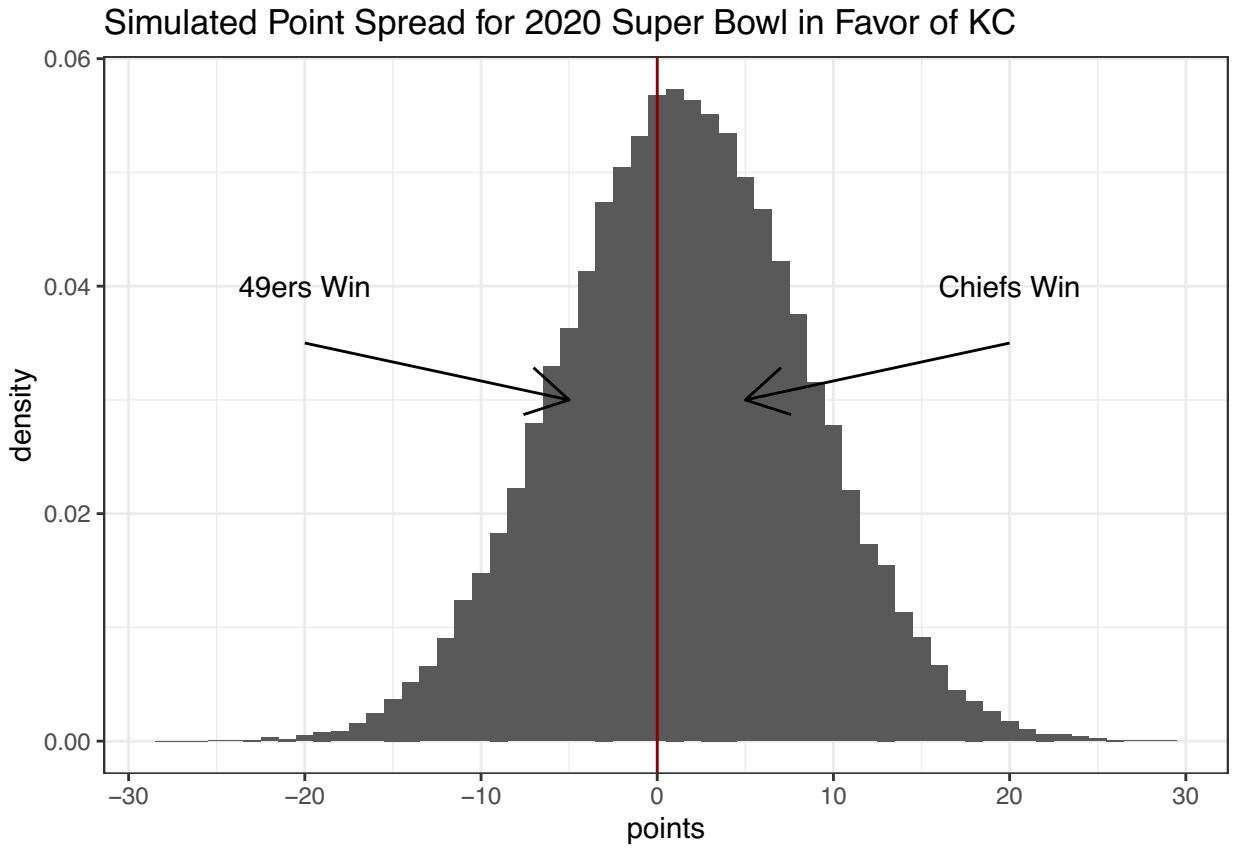
Specifically, we are interested in the distribution of outcomes that could be generated, given the Rockies are expected to win 50% of their games?

In one scenario, the Rockies could win 82. However, we are interested in the distribution of possible outcomes.



We can do the same thing for a continuous variable. Consider the 2020 Super Bowl where the Kansas City Chiefs were 1.5 point favorites. Assume that this point differential can be translated to a normal distribution with standard deviation of 7.

Then the range of possible outcomes can be simulated...



This can also be used to calculate the winning probability (under the assumed model), where the Chiefs would win with probability ‘

Summarizing linear regression with simulation

```
set.seed(02262020)
beer <- read_csv('http://math.montana.edu/ahoegh/Data/Brazil_cerveja.csv') %>%
  mutate(precip = scale(precip), max_tmp = scale(max_tmp))

lm_beer <- lm(consumed~ precip + max_tmp + weekend , data = beer)
display(lm_beer)
```

```
## lm(formula = consumed ~ precip + max_tmp + weekend, data = beer)
##             coef.est coef.se
## (Intercept) 23.92     0.14
## precip      -0.71     0.12
## max_tmp      2.89     0.12
## weekend      5.18     0.27
## ---
## n = 365, k = 4
## residual sd = 2.33, R-Squared = 0.72
```

Confidence intervals can be analytically computed... (but still require distributional assumptions or CLT asymptotics)

```
confint(lm_beer)
```

```
##                  2.5 %     97.5 %
## (Intercept) 23.6409989 24.2075184
## precip      -0.9539859 -0.4737965
## max_tmp      2.6460541  3.1266325
## weekend      4.6531170  5.7150484
```

Simulation can also be used construct intervals

sim function in the arm package

```
n.sims <- 1000
sim_vals <- arm::sim(lm_beer, n.sims )
head(coef(sim_vals))
```

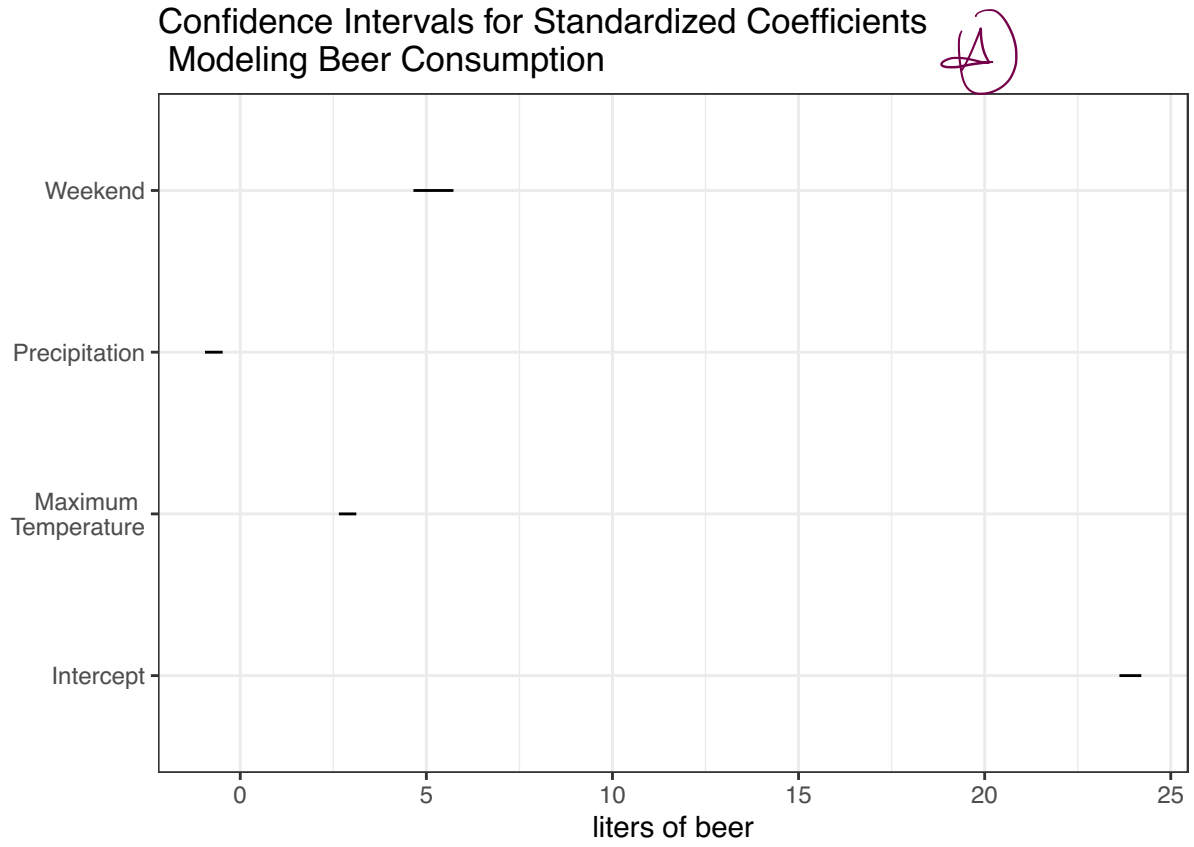
```
##      (Intercept)     precip  max_tmp  weekend
## [1,]    24.08906 -0.7182283 2.816890 5.146346
## [2,]    24.00334 -0.8814225 2.770705 4.979809
## [3,]    24.11486 -0.5414572 2.900834 5.396089
## [4,]    23.95317 -0.7030344 3.013430 4.869024
## [5,]    24.04089 -0.7942222 2.887384 4.957233
## [6,]    23.93099 -0.7383508 2.973810 5.349378
```

```
ci <- apply(coef(sim_vals),2, quantile, probs = c(.025, .975))  # simulation from arm package
round(ci,2)
```

```
##       (Intercept) precip max_tmp weekend
## 2.5%        23.62  -0.95    2.65    4.65
## 97.5%       24.21  -0.47    3.12    5.73
```

not a perfect match

```
tibble(var = colnames(coef(sim_vals)), lower = as.numeric(ci[1,]), upper = as.numeric(ci[2,])) %>%
  ggplot() + geom_segment(aes(x=lower, xend = upper, y = var, yend = var)) +
  ggtitle("Confidence Intervals for Standardized Coefficients \n Modeling Beer Consumption") +
  xlab('liters of beer') + theme_bw() + ylab('') +
  scale_y_discrete(labels = c('Intercept','Maximum \n Temperature','Precipitation','Weekend'))
```



Interpret this figure and explain the results to an owner of a liqour store.

As an aside, Megan Higgs has a blog about critical thinking in statistical inference (critical inference). You should read it https://critical-inference.com//. Here is an image from a discussion about power, see ([https://critical-inference.com/sample-size-without-power-yes-its-possible/]https://critical-inference.com/sample-size-without-power-yes-its-possible/

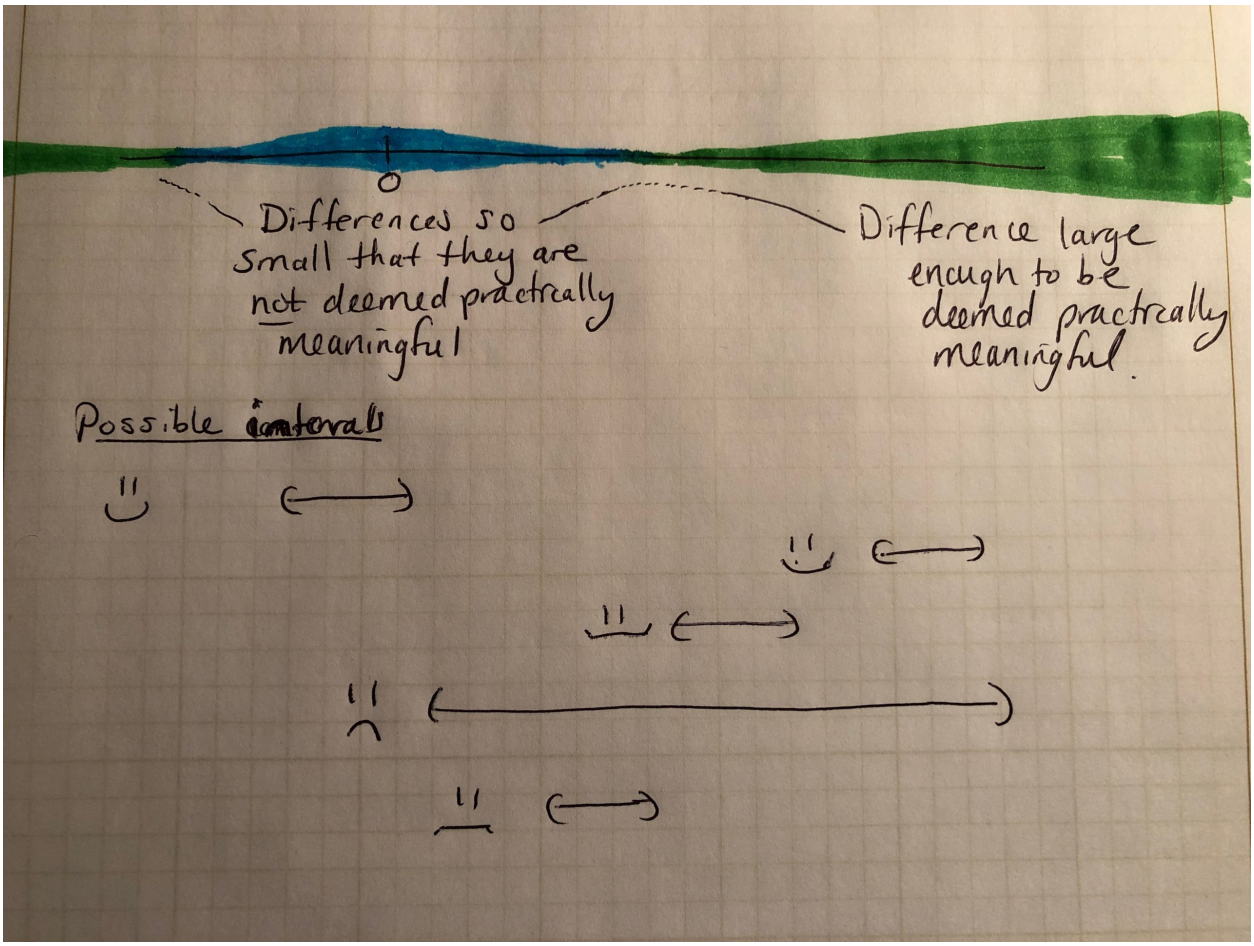


Figure 1: Critical Inference

Simulation for predictive inference

Suppose the liquour store owner is interested in the following results:

1. the mean predicted consumption on a weekend with average precipitation and temperature 1 sd greater than average.
2. the mean predicted consumption on a week day with average precipitation and temperature 1 sd less than average.
3. the mean predicted difference in consumption between the two days specified in point 1 and point 2.

how to answer these questions?

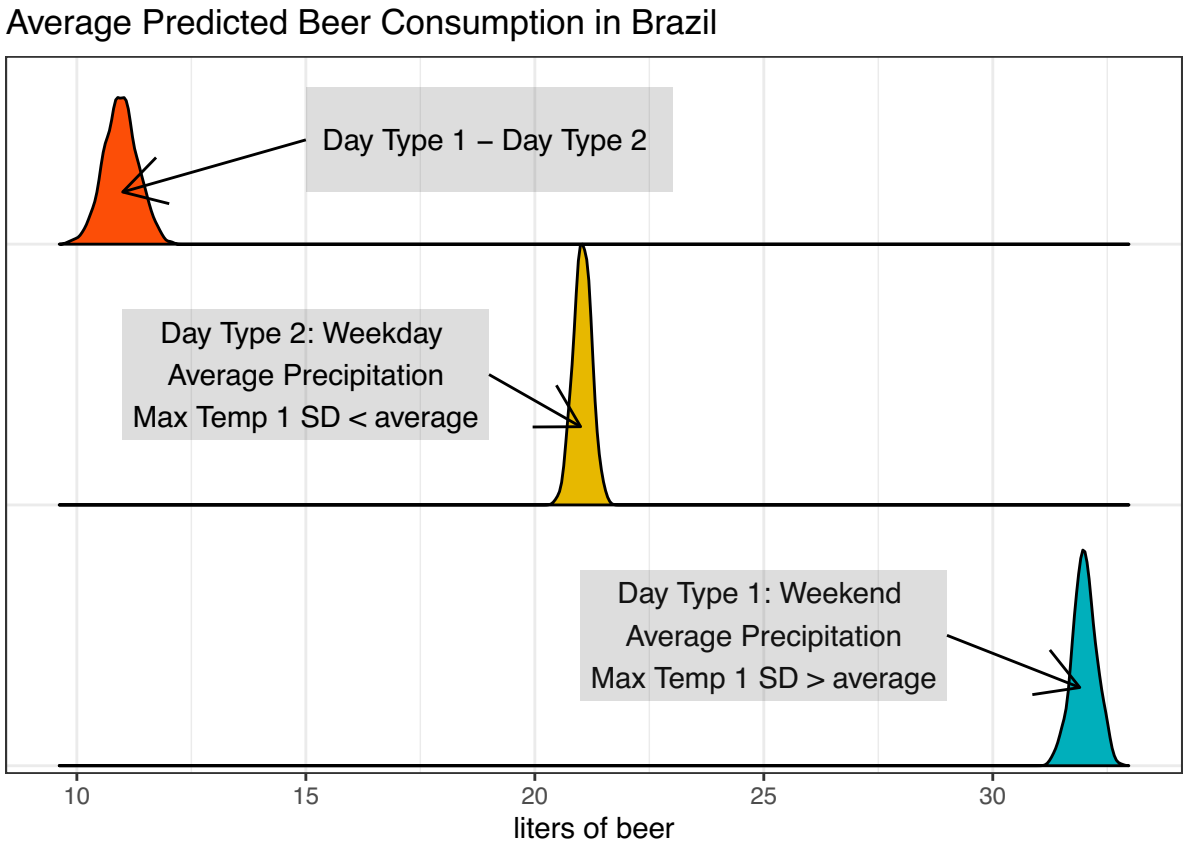
1000 x 4

pred1 ← coef(sim_vals) %*% matrix(c(1,0,1,1), nrow=4, ncol=1)
predict(lm_obj, dataframe)

Answering these questions is easy with simulation...

```
post_plot <- tibble( values = c(pred1, pred2, pred3), type = rep(c('pred1','pred2','pred3'),
                                                                 each = n.sims)) %>%
  ggplot(aes(x = values, y = type, fill = type)) + geom_density_ridges2(scale = 1) +
  scale_y_discrete(expand = c(0.01, .01,0.01,.7)) + ggtitle('Average Predicted Beer Consumption in Braz
  ylab('') + xlab('liters of beer') +
  annotate('text',label = 'Day Type 1: Weekend \n Average Precipitation \n Max Temp 1 SD > average ',
           x=25, y = 1.5) +
  annotate('rect',xmin = 21,xmax = 29, ymin = 1.25, ymax = 1.75, alpha = .2) +
  theme_bw() + theme(axis.text.y = element_blank(),axis.ticks.y = element_blank(),
                     legend.position = "none") +
  annotate('segment', x=29, xend = 31.9, y = 1.5, yend=1.3,arrow = arrow()) +
  annotate('rect', xmin = 11,xmax = 19, ymin = 2.25, ymax = 2.75, alpha = .2) +
  annotate('segment', x=19, xend = 21, y = 2.5, yend=2.3,arrow = arrow()) +
  annotate('text',label = 'Day Type 2: Weekday \n Average Precipitation \n Max Temp 1 SD < average ',
           x=15, y = 2.5) +
   annotate('rect', xmin = 15,xmax = 23, ymin = 3.2, ymax = 3.6, alpha = .2) +
  annotate('segment', x=15, xend = 11, y =3.4, yend=3.2,arrow = arrow()) +
  annotate('text',label = 'Day Type 1 - Day Type 2 ',x=19, y = 3.4) +
  scale_fill_manual(values = c("#00AFBB", "#E7B800", "#FC4E07"))
```

```
post_plot
```



So what exactly is the "mean predicted consumption, . . "? Does this correspond to predictions for a single day?

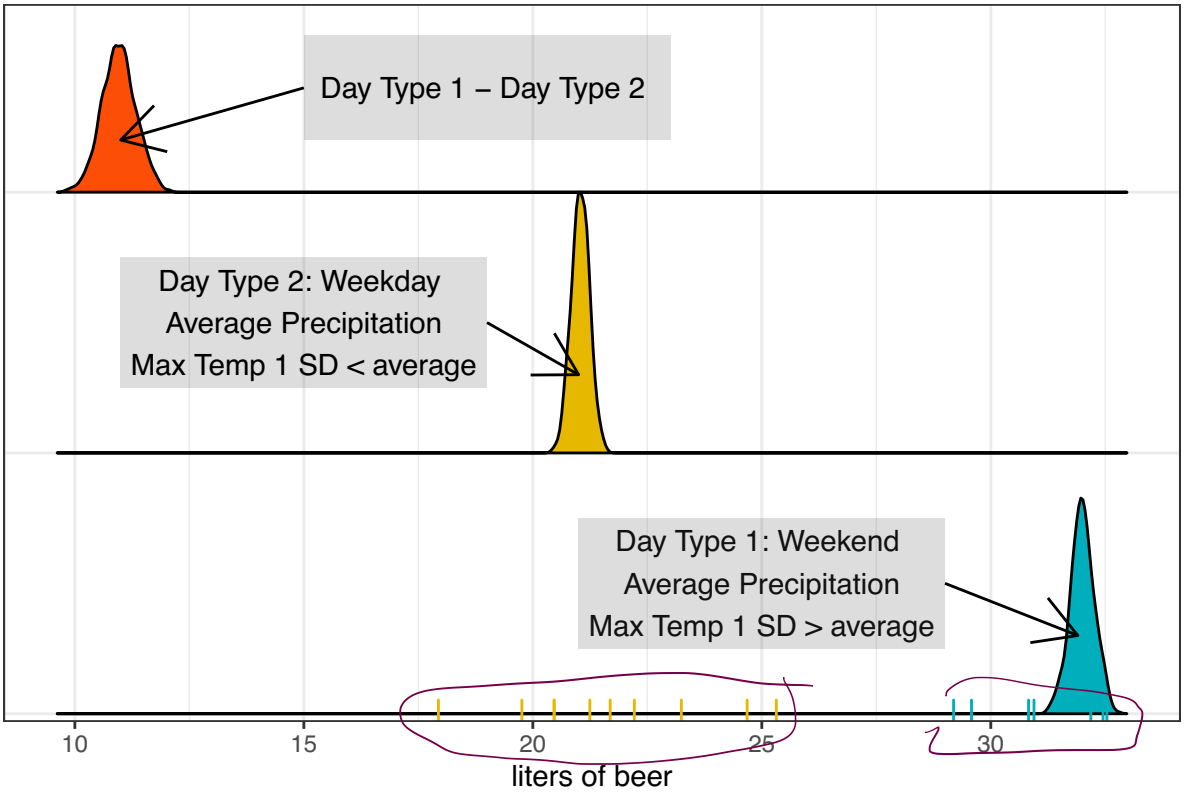
- This mean consumption for days of this type.

☆Put another way what does the uncertainty in these density plots correspond to?

belief about the mean consumption

```
post_plot +
  geom_rug(inherit.aes = F, data = beer %>% filter(weekend ==1 & max_tmp > .9 & max_tmp < 1.1),
           aes(x = consumed, y = NULL), sides = 'b', colour = "#00AFBB") +
  geom_rug(inherit.aes = F, data = beer %>% filter(weekend ==0  & max_tmp < -.9 & max_tmp > -1.1),
           aes(x = consumed, y = NULL), sides = 'b', colour = "#E7B800")
```

Average Predicted Beer Consumption in Brazil



Prediction for a single data point

Our model for a single data point is

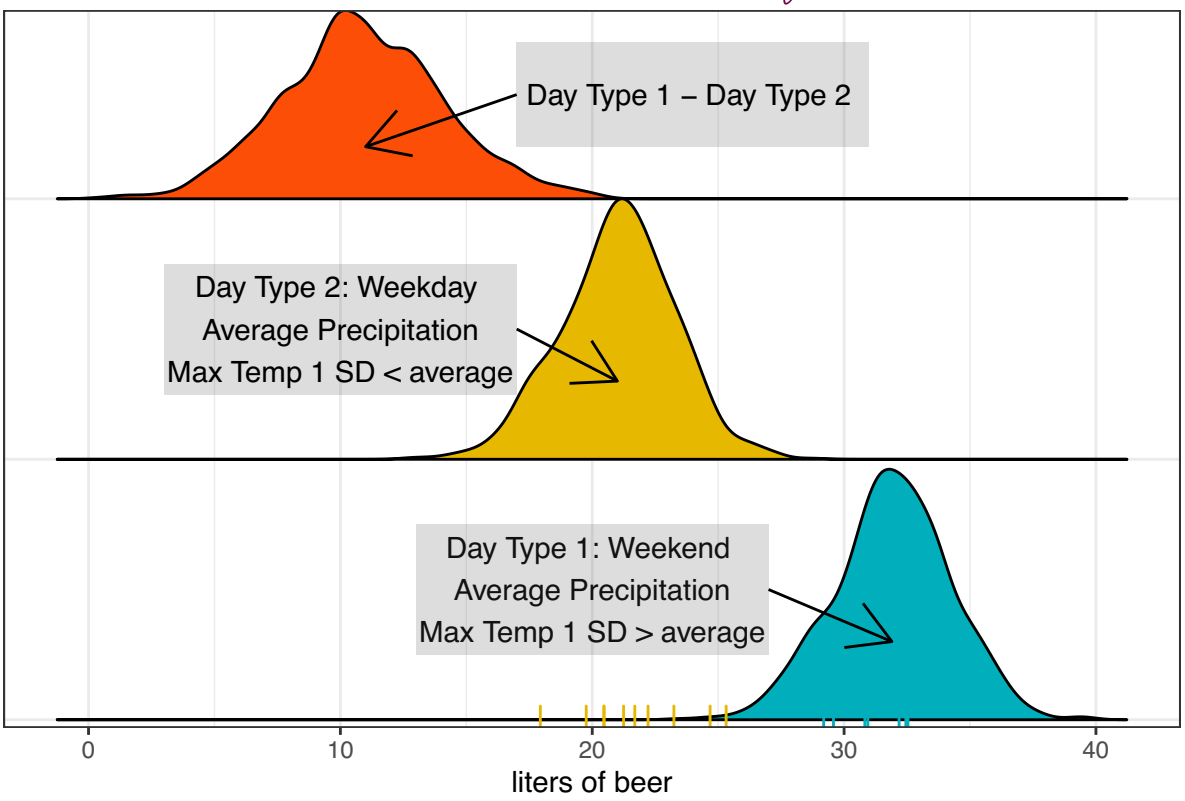
$$Y_i = X_i \underline{B} + \varepsilon_i \quad , \quad \text{where} \quad \varepsilon_i \sim N(0, \sigma^2)$$

$$Y_i \sim N(X_i \underline{B}, \sigma^2)$$

```
pred1_indiv <- rnorm(n = n.sims, mean = pred1, sd = sigma.hat(sim_vals) )
pred2_indiv <- rnorm(n = n.sims, mean = pred2, sd = sigma.hat(sim_vals) )
pred3_indiv <- pred1_indiv - pred2_indiv
```

Q1. Explain this figure ...

Q2. Is figure more useful than the figure (4) with confidence intervals for the coefficients?



Simulation Procedure

Simulation can be conducted using the `sim()` function in `arm`

1. Use classical regression of the n data points with k predictors to compute the estimate of regression coefficents

$$\hat{\beta} = (X^T X)^{-1} X^T y \qquad \sigma^2 = \hat{\sigma}^2$$

$$V_{\hat{\beta}} = \hat{\sigma}^2 (X^T X)^{-1}$$

2. For each of the `n.sims` iterations, create the coefficient vector $\underline{\hat{\beta}}^{iter}$ and residual standard deviation $\hat{\sigma}^{iter}$.

a. $\hat{\sigma}^{iter} = \hat{\sigma} \sqrt{\frac{(n-k)}{X}}$, where $X \sim \chi^2_{n-k}$

b. $\underline{\hat{\beta}}^{iter} \sim N(\hat{\beta}, V_{\beta})$ plugged in

Note: this is referred to as informal Bayesian analysis

can be used for the plot of means

SE $\sqrt{\frac{\sigma^2}{n}}$

rnorm(n.sims, XB, $\hat{\sigma}$)

9 however the individual days requires both

0 .0001