

# Linear Algebra Primer

**Matrices / Vectors** A matrix is an  $n \times p$  object. Matrices are often denoted by a capital letter (or Greek symbol). A few common matrices will be

Vectors are essentially one-dimension vectors and will be denoted with an underline. We will assume vectors are  $q \times 1$  dimension unless noted with a transpose.

The transpose operator will be denoted by  $\underline{y}^T = (y_1 \ y_2 \ \cdots \ y_n)$  or  $\underline{y}'$ , both of which would result in a  $1 \times n$  vector.

**Matrix Multiplication** The most important component in matrix multiplication is tracking dimensions.

Consider a simple case with

$$\hat{\underline{y}} = X \times \hat{\underline{\beta}},$$

where  $X$  is a  $2 \times 2$  matrix,  $\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$  and  $\hat{\underline{\beta}} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ .

In R, we use `%*` for matrix multiplication.

```
X <- matrix(c(1,2, 1 ,-1), nrow = 2, ncol = 2, byrow = T); X
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]    1   -1
```

```
#X <- matrix(c(1, 1, 2 ,-1), nrow = 2, ncol = 2); X
beta_hat <- matrix(c(3,2),nrow =2, ncol = 1); beta_hat
```

```
##      [,1]
## [1,]    3
## [2,]    2
```

```
y_hat <- X %*% beta_hat; y_hat
```

```
##      [,1]
## [1,]    7
## [2,]    1
```

**Kronecker Product** Kronecker product  $\otimes$ , enables a different type of matrix multiplication.

If

$$A = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

and  $B = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ , then

## More about matrices

**Nonsingular matrix and Matrix Inverse** If  $X$  is a square matrix, then  $X$  is nonsingular if there exists another matrix s.t.

$$XX^{-1} = I \text{ and } X^{-1}X = I$$

$X^{-1}$  is the inverse of a matrix. We can calculate the inverse of a matrix for a  $1 \times 1$  matrix, perhaps as  $2 \times 2$ , matrix and maybe even a  $3 \times 3$  matrix. However, beyond that it is quite challenging and time consuming. Furthermore, it is also (relatively) time intensive for your computer.

**Orthogonal matrices** If a matrix  $X$  has an inverse that is also the transpose,  $XX^T = I$ , then  $X$  is an orthogonal matrix.

**Motivating Dataset: Washington (DC) housing dataset** Hopefully the connections to statistics are clear, using  $X$  and  $\beta$ , but let's consider a motivating dataset.

This dataset contains housing information from Washington, D.C. It was used for a STAT532 exam, so apologize in advance for any scar tissue.

```
DC <- read_csv('https://math.montana.edu/ahoegh/teaching/stat532/data/DC.csv')

## Parsed with column specification:
## cols(
##   BATHRM = col_double(),
##   HF_BATHRM = col_double(),
##   AC = col_character(),
##   BEDRM = col_double(),
##   STORIES = col_double(),
##   PRICE = col_double(),
##   CNDTN = col_character(),
##   LANDAREA = col_double(),
##   FULLADDRESS = col_character(),
##   ASSESSMENT_NBHD = col_character(),
##   WARD = col_character(),
##   QUADRANT = col_character()
## )

DC %>% group_by(WARD) %>%
  summarize(`Average Price (millions of dollars)` = mean(PRICE)/1000000, .groups = 'drop') %>%
  kable(digits = 3)
```

WARD	Average Price (millions of dollars)
Ward 1	0.879
Ward 2	1.919
Ward 3	1.294
Ward 4	0.693
Ward 5	0.592
Ward 6	0.856
Ward 7	0.321
Ward 8	0.306

```
DC %>% group_by(BEDRM) %>%
  summarize(`Average Price (millions of dollars)` = mean(PRICE)/1000000, .groups = 'drop') %>%
  kable(digits = 3)
```

BEDRM	Average Price (millions of dollars)
0	0.195
1	0.442
2	0.479
3	0.620
4	0.832
5	1.355
6	1.849
7	1.666
9	7.365

## Regression Model

There are many factors in this dataset that can be useful to predict housing prices.

$$y_i = \beta_0 + \beta_1 * x_{SQFT,i} + \beta_2 x_{BEDRM,i} + \epsilon_i, \quad (1)$$

where  $y_i$  is the sales price of the  $i^{th}$  house,  $x_{SQFT,i}$  is the living square footage of the  $i^{th}$  house, and  $x_{BEDRM,i}$  is the number of bedrooms for the  $i^{th}$  house. Note this implies that we are treating bedrooms as continuous variables as opposed to categorical.

we usually write  $\epsilon_i \sim N(0, \sigma^2)$ . More on that soon.

In R we often write something like: `price ~ LANDUSE + BEDRM`.

Now let's write this model in matrix notation:

$$\underline{y} = X\underline{\beta} + \underline{\epsilon}, \quad (2)$$

$$\text{where } \underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, X = \begin{bmatrix} 1 & x_{SQFT,1} & x_{BEDRM,1} \\ 1 & x_{SQFT,2} & x_{BEDRM,2} \\ \vdots & \vdots & \vdots \\ 1 & x_{SQFT,n} & x_{BEDRM,n} \end{bmatrix}, \underline{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}, \text{ and } \underline{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}$$