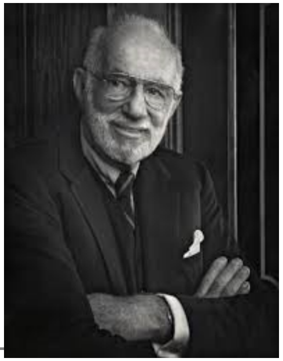
Comparisons Among Several Samples

MORE APPLICATIONS OF EXTRA-SUM-OF-SQUARES TEST

Spock Trial (Reminder)



- To test the claim, the Spock Judge's (which we will call S) recent venires are compared with 6 other Judge's recent venires (which we notate A to F)

- There are two key questions
  1. Is there evidence that women are unrepresented on S's venire relative to A to F's?
  2. **Is there evidence in a difference in women's representation on A to F's venires?**

- The question of interest is addressed by 1

- The strength of the result in 1. would be substantially diminished if 2 is true

New Notation for Hypothesis Tests

Reminder

(ESS)
EXTRA SUM OF SQUARES =

residual sum of squares(reduced) - residual sum of squares(full)

RSS(reduced) RSS(full)

Large extra sum of squares indicates the full model fits much better

- Did the data come from groups that **all** had the same mean?

(equivalent)

$H_0\!: \mu_1 = \mu_2 = \cdots = \mu_I = \mu$ (The **REDUCED OR EQUAL MEANS MODEL**)

$H_0\!: \underbrace{\mu, \mu, \dots, \mu}_{(I \text{ times})}$

- Did the data come from groups that **do not all** have the same mean

(equivalent)

$H_A\!: \mu_k \neq \mu_l$ for some k, l (The **FULL OR SEPARATE MEANS MODEL**)

$H_A\!: \mu_1, \mu_2, \dots, \mu_I$

Comparing Judges A-F: A First Take

In order to test if the A through F judges have the same mean, we can run:

H_0: $\mu_A = \mu_B = \cdots = \mu_F = \mu$

(equivalently, H_0: $\mu, \mu, \ldots, \mu$)

H_A: $\mu_k \neq \mu_l$ for some k, l

(equivalently, H_A: $\mu_A, \mu_B, \ldots, \mu_F$)

```
DATA spock_AtoF;
    SET spock;
    IF judge = "S" THEN delete;
RUN;

PROC GLM DATA = spock_AtoF ORDER=DATA;
     CLASS judge;
     MODEL percFemale = judge;
RUN;
```

This would be an ANOVA on the 6 other judges and would ignore the Spock judge

But, we saw earlier that it is better to use **all** the data if possible

(leading to a better estimate of the variance under equal variances assumption)

Comparing Judges A-F: A Better Approach

Let's add the Spock judge's data by including another parameter

$H_0\colon\ \mu_S, \mu, \mu, \dots, \mu$ (The **REDUCED MODEL**)

$H_A\colon\ \mu_S, \mu_A, \mu_B, \dots, \mu_F$ (The **FULL MODEL**)

```
DATA spock2;
    SET spock;
    IF judge ne "S" THEN OthersModel = "Others";
    ELSE OthersModel = "S";
RUN;
```

Comparing Judges A-F: A Better Approach

Let's add the Spock judge's data by including another parameter

```
DATA spock2;
    SET spock;
    IF judge ne "S" THEN OthersModel = "Others";
    ELSE OthersModel = "S";
RUN;
```

Obs	percFemale	judge	OthersModel
1	6.4	S	S
2	8.7	S	S
3	13.3	S	S
4	13.6	S	S
5	15.0	S	S
6	15.2	S	S
7	17.7	S	S
8	18.6	S	S
9	23.1	S	S
10	16.8	A	Others
11	30.8	A	Others
12	33.6	A	Others
13	40.5	A	Others
14	48.9	A	Others
15	27.0	B	Others
16	28.9	B	Others
17	32.0	B	Others
18	32.7	B	Others
19	35.5	B	Others
20	45.6	B	Others
21	21.0	C	Others
22	23.4	C	Others
23	27.5	C	Others
24	27.5	C	Others

H_0: $\mu_S, \mu, \mu, \dots, \mu$ (The **REDUCED MODEL**)

H_A: $\mu_S, \mu_A, \mu_B, \dots, \mu_F$ (The **FULL MODEL**)

Now, we need to test this hypothesis

→ Extra sum of squares test!

Comparing Judges A-F: A Better Approach

Let's add the Spock judge's data by including another parameter

```
DATA spock2;
    SET spock;
    IF judge ne "S" THEN OthersModel = "Others";
    ELSE OthersModel = "S";
RUN;
```

H_0: $\mu_S, \mu, \mu, \ldots, \mu$ (The **REDUCED MODEL**)

H_A: $\mu_S, \mu_A, \mu_B, \ldots, \mu_F$ (The **FULL MODEL**)

We need to compute:

- RSS(reduced)
- RSS(full)
- the degrees of freedom

Source	DF	SS	MS	F	Pr > F
Model (Between)	$I-1$	ESS	ESS/$(I-1)$	F-statistic	p-value
Error (Within)	$n-I$	RSS(full)	RSS(full)/$(n-I)$		
Corrected Total (Total)	$n-1$	RSS(reduced)			

Comparing Judges A-F: A Better Approach

A flexible way to do is via two calls to **PROC GLM**

$H_0: \mu, \mu, \ldots, \mu$

$H_A: \mu_S, \mu, \mu, \ldots, \mu$

```
PROC GLM DATA = spock2;
    CLASS OthersModel;
    MODEL percFemale = OthersModel;
RUN;
```

$H_0: \mu, \mu, \ldots, \mu$

$H_A: \mu_S, \mu_A, \mu_B, \ldots, \mu_F$

```
PROC GLM DATA = spock2;
    CLASS judge;
    MODEL percFemale = judge;
RUN;
```

Combine the output to test:

$H_0: \mu_S, \mu, \mu, \ldots, \mu$ (The **REDUCED MODEL**)

$H_A: \mu_S, \mu_A, \mu_B, \ldots, \mu_F$ (The **FULL MODEL**)

Obs	percFemale	judge	OthersModel
1	6.4	S	S
2	8.7	S	S
3	13.3	S	S
4	13.6	S	S
5	15.0	S	S
6	15.2	S	S
7	17.7	S	S
8	18.6	S	S
			S
			Others
			Others
			Others
13	40.5	A	Others
14	48.9	A	Others
	27.0	B	Others
	28.9	B	Others
	32.0	B	Others
	32.7	B	Others
19	35.5	B	Others
20	45.6	B	Others
21	21.0	C	Others
22	23.4	C	Others
23	27.5	C	Others
24	27.5	C	Others

Comparing Judges A-F: Extra Sums of Squares

A flexible way to do is via two calls to **PROC GLM**

(This "full" model is the new "reduced" model)

$H_0: \mu, \mu, \ldots, \mu$

$H_A: \mu_S, \mu, \mu, \ldots, \mu$

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	1600.622964	1600.622964	32.15	<.0001
Error	44	2190.903123	49.793253		
Corrected Total	45	3791.526087			

$H_0: \mu, \mu, \ldots, \mu$

$H_A: \mu_S, \mu_A, \mu_B, \ldots, \mu_F$

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	1927.080865	321.180144	6.72	<.0001
Error	39	1864.445222	47.806288		
Corrected Total	45	3791.526087			

Combine the output to test:

$H_0: \mu_S, \mu, \mu, \ldots, \mu$

$H_A: \mu_S, \mu_A, \mu_B, \ldots, \mu_F$

Source	DF	SS	MS	F	Pr > F
Model (ESS)	5	326.5	65.29	1.37	0.26
Error (Full)	39	1864.4	47.81		
Total (Reduced)	44	2190.9			

Summarizing

Reminder

(ESS)
EXTRA SUM OF SQUARES =

residual sum of squares(reduced) - residual sum of squares(full)

RSS(reduced) RSS(full)

Large extra sum of squares indicates the full model fits much better

We can test "nested" hypothesis; that is, where one is a special case of the other

Run two tests: the alternatives are the new null and alternative hypotheses

H_0: *something simple*
H_A: *something slightly complex*

H_0: *something simple*
H_A: *something even more complex*

H_0: *something slightly complex*
H_A: *something even more complex*