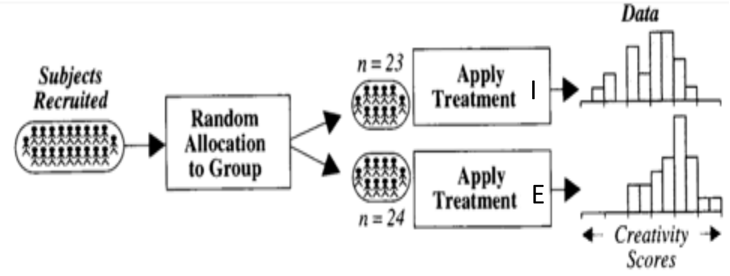
Inference Using t-Distributions

T-DISTRIBUTION FOR TWO SAMPLE INFERENCE

Two Sample Inference With the t-Distribution

Confidence Intervals

Creativity Study: Confidence Interval



→ Population mean: μ_I
→ Population sd: σ_I

→ Population mean: μ_E
→ Population sd: σ_E

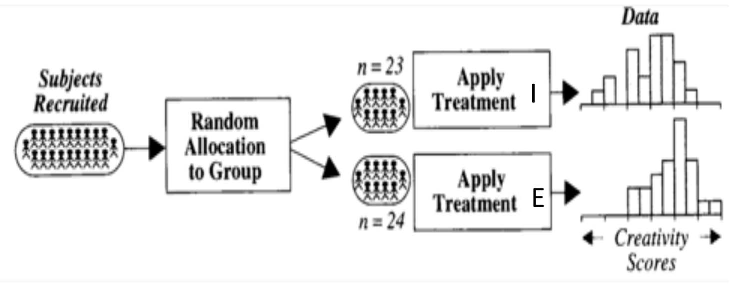
- We can form a 100(1-α)% confidence interval for μ_I - μ_E

$$\bar{Y}_I - \bar{Y}_E \pm t_{\alpha/2,(n-2)} SE(\bar{Y}_I - \bar{Y}_E)$$

(Here: the degrees of freedom is $n - 2 = n_I - 1 + n_E - 1$)

Reminder, we still need to assume that:
- The units are normally distributed or that the sample is large enough
- The intrinsic/extrinsic groups are independent and $\sigma_I = \sigma_E$

Creativity Study: Confidence Interval



→ Population mean: μ_I
→ Population sd: σ_I

→ Population mean: μ_E
→ Population sd: σ_E

- Important consideration: What confidence level do we desire?
- A $100(1\text{-}\alpha)\%$ confidence contains μ_I - μ_E $100(1\text{-}\alpha)\%$ of the time

(This means if we were to redo the experiment from the start 100 times, something like $100\alpha\%$ of the time, the interval would not contain μ_I - μ_E)

Creativity Study: Confidence Interval

- Let's demand a 99% confidence interval → $\alpha = 0.01$
- For this study, $n = 23 + 24 = 47$ → the degrees of freedom= $47\ - 2 = 45$

```
DATA quant;
     t   = QUANTILE('T',1-0.01/2,45);
RUN;

PROC PRINT DATA=quant;
RUN;
```

Obs	t
1	2.68959

→ $t_{\alpha/2,(n-2)}$ = 2.68959

Creativity Study: Confidence Interval

$\rightarrow \bar{y}_I - \bar{y}_E = 4.14$

$\bar{y}_I - \bar{y}_E = 19.88 - 15.74$

```
PROC SORT DATA = creativity;
    BY intrinsic;
RUN;
PROC UNIVARIATE data = creativity;
    BY intrinsic;
RUN;
```

$$s_p = \sqrt{\frac{(n_I - 1)\, s_I^2 + (n_E - 1)\, s_E^2}{n_I + n_E - 2}} = \sqrt{\frac{(23)4.44^2 + (22)5.25^2}{45}}$$

$\rightarrow s_p = 4.85$

intrinsic=1

Moments			
N	24	Sum Weights	24
Mean	19.8833333	Sum Observations	477.2
Std Deviation	4.43951296	Variance	19.7092754
Skewness	-0.074952	Kurtosis	0.08425798
Uncorrected SS	9941.64	Corrected SS	453.313333
Coeff Variation	22.3278104	Std Error Mean	0.90621179

intrinsic=0

Moments			
N	23	Sum Weights	23
Mean	15.7391304	Sum Observations	362
Std Deviation	5.25259582	Variance	27.5897628
Skewness	-0.76156	Kurtosis	-0.0935406
Uncorrected SS	6304.54	Corrected SS	606.974783
Coeff Variation	33.3728464	Std Error Mean	1.09524194

Creativity Study: Confidence Interval

To recap:

- $t_{\alpha/2,(n-2)}$ = 2.68959
- $\bar{y}_I - \bar{y}_E = 4.14$
- s_p = 4.85

What's left to be done?

$$SE(\bar{Y}_I - \bar{Y}_E) = s_p\sqrt{\frac{1}{n_I} + \frac{1}{n_E}} = 4.85\sqrt{\frac{1}{24} + \frac{1}{23}} = 1.42$$

Creativity Study: Confidence Interval

To recap:

- $t_{\alpha/2,(n-2)}$ = 2.68959
- $\bar{Y}_I - \bar{Y}_E = 4.14$
- s_p = 4.85
- $SE(\bar{Y}_I - \bar{Y}_E)$ = $s_p\sqrt{\frac{1}{n_I} + \frac{1}{n_E}} = 4.85\sqrt{\frac{1}{24} + \frac{1}{23}} = 1.42$

Putting it all together:

$$\bar{Y}_I - \bar{Y}_E \ \pm t_{\alpha/2,(n-2)} SE(\bar{Y}_I - \bar{Y}_E) \leftrightarrow [0.32, 7.96]$$

"A range of plausible values for μ_I - μ_E is $[0.32, 7.96]$ (units?) based on a 99% confidence interval"

Creativity Study: Confidence Interval with SAS

```
PROC TTEST DATA=creativity ALPHA = 0.01;
    CLASS intrinsic;
    VAR SCORE;
RUN;
```

The TTEST Procedure
Variable: score

intrinsic	N	Mean	Std Dev	Std Err	Minimum	Maximum
0	23	15.7391	5.2526	1.0952	5.0000	24.0000
1	24	19.8833	4.4395	0.9062	12.0000	29.7000
Diff (1-2)		-4.1442	4.8541	1.4164		

intrinsic	Method	Mean	99% CL Mean		Std Dev	99% CL Std Dev	
0		15.7391	12.6519	18.8264	5.2526	3.7660	8.3803
1		19.8833	17.3393	22.4274	4.4395	3.2032	6.9965
Diff (1-2)	Pooled	-4.1442	-7.9537	-0.3347	4.8541	3.8068	6.6041
Diff (1-2)	Satterthwaite	-4.1442	-7.9750	-0.3135			

Creativity Study: Confidence Interval with SAS

```
PROC TTEST DATA=creativity ALPHA = 0.01 ORDER=DATA;
    CLASS intrinsic;
    VAR SCORE;
RUN;
```

The TTEST Procedure
Variable: score

intrinsic	N	Mean	Std Dev	Std Err	Minimum	Maximum
1	24	19.8833	4.4395	0.9062	12.0000	29.7000
0	23	15.7391	5.2526	1.0952	5.0000	24.0000
Diff (1-2)		4.1442	4.8541	1.4164		

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Diff (1-2)	Satterthwaite	4.1442	0.3135	7.9750			

Hypothesis Test

Testing Hypothesis: Difference in Means

- Suppose that the observations are independent and normal (or n > 30)

- **Then:**

(ONE SAMPLE T-TEST STATISTIC)

T= $\frac{\bar{Y}-\mu}{s/\sqrt{n}}$ is "approx. distributed" t with *(n-1)* degrees of freedom

If additionally the groups are independent, then

$$\bar{Y}_I - \bar{Y}_E \pm t_{\alpha/2,(n-2)} SE(\bar{Y}_I - \bar{Y}_E)$$

(100(1-α)% CONFIDENCE INTERVAL FOR DIFFERENCE IN MEAN)

$H_0: \mu_I - \mu_E = \mu_0 = 0$
$H_A: \mu_I - \mu_E \neq 0$

t-statistic = $\frac{(\bar{Y}_I - \bar{Y}_E) - \mu_0}{SE(\bar{Y}_I - \bar{Y}_E)}$

"The P-VALUE is the probability of getting a draw from a t-distribution (w/ n-2 degrees of freedom) that is at least extreme as the t-statistic"

Testing Hypothesis: Difference in Means

```
PROC TTEST DATA=creativity ORDER=DATA;
    CLASS intrinsic;
    VAR SCORE;
RUN;
```

The TTEST Procedure
Variable: score

intrinsic	N	Mean	Std Dev	Std Err	Minimum	Maximum
1	24	19.8833	4.4395	0.9062	12.0000	29.7000
0	23	15.7391	5.2526	1.0952	5.0000	24.0000
Diff (1-2)		4.1442	4.8541	1.4164		

intrinsic	Method	Mean	95% CL Mean		Std Dev	95% CL Std Dev	
1		19.8833	18.0087	21.7580	4.4395	3.4504	6.2276
0		15.7391	13.4677	18.0105	5.2526	4.0623	7.4343
Diff (1-2)	Pooled	4.1442	1.2914	6.9970	4.8541	4.0261	6.1138
Diff (1-2)	Satterthwaite	4.1442	1.2776	7.0108			

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	45	2.93	0.0054
Satterthwaite	Unequal	43.108	2.92	0.0056

(STATISTICAL CONCLUSION)

"This experiment provides strong evidence that the intrinsic rather than extrinsic is associated with a higher scoring poem (p-value = 0.0054 from a two-sample t-test). The estimated treatment effect is 4.14 pts (99% confidence interval $[0.32, 7.96]$) on a 40 pt scale"

Testing Hypothesis: Difference in Means

(Randomization versus T-test in SAS)