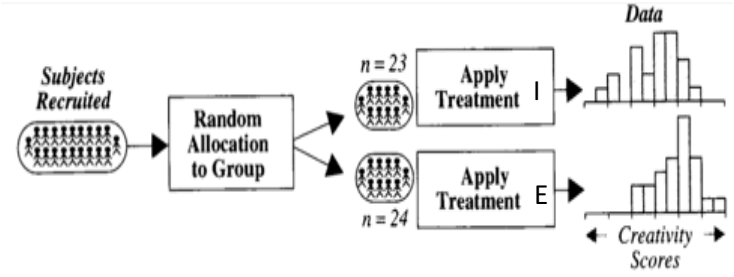
Inference Using t-Distributions

T-DISTRIBUTION FOR TWO SAMPLE INFERENCE

ONE-SIDED HYPOTHESIS TESTING AND CONFIDENCE INTERVALS

Creativity Study



→ Population mean: μ_I

→ Population mean: μ_E

- If the questionnaires had no effect, then we would expect:

$$H_0: \mu_I = \mu_E \leftrightarrow H_0: \mu_I - \mu_E = 0$$ **(NULL HYPOTHESIS)**

- What is the scientific question of interest? Is it:
  - Is there any difference in the intrinsic/extrinsic group means?

→ $H_A: \mu_I \neq \mu_E$

(ALTERNATIVE HYPOTHESES)

  - Is the intrinsic group mean larger than the extrinsic group mean?

→ $H_A: \mu_I > \mu_E$

One-Sided vs. Two-Sided

The core of p-values for t-tests is (heuristically):

$$\frac{\#\ of\ t\text{–}ratios\ (under\ H_0)\ as\ or\ more\ extreme\ \ than\ the\ observed\ t\text{–}statistic}{total\ \#\ of\ t\text{–}ratios}$$

Of course, there are an infinite number of t-ratios, so we really interpret this as a PROBABILITY

"The p-value is the probability of observing a t-ratio as or more extreme as the observed t-statistic if the null hypothesis is true"

- For this to be made rigorous, we need to define EXTREME
- This depends on the form of the alternative hypothesis

Two-Sided P-values

Is there any difference in the intrinsic/extrinsic group means?

$\rightarrow H_A: \mu_I \neq \mu_E$

- Evidence against H_0 comes from values of $\bar{Y}_I - \bar{Y}_E$ far from zero *in either direction*

- This is interpreted quantitatively as is
  - $\bar{Y}_I - \bar{Y}_E$ large?
  - $\bar{Y}_E - \bar{Y}_I$ large?

This can be summarized as:

Is $|\bar{Y}_I - \bar{Y}_E|$ large?

One-Sided P-values

- Is the intrinsic group mean larger than the extrinsic group mean?

  $\rightarrow$ H_A: $\mu_I > \mu_E$

- Evidence against H_0 comes from values of $\bar{Y}_I - \bar{Y}_E$ far from zero *in the positive direction only*

- This is interpreted quantitatively as is
  - Is $\bar{Y}_I - \bar{Y}_E$ large?

One-Sided P-values in SAS

```
PROC TTEST DATA=creativity ORDER=DATA SIDES=U;
    CLASS intrinsic;
    VAR SCORE;
RUN;
```

The TTEST Procedure
Variable: score

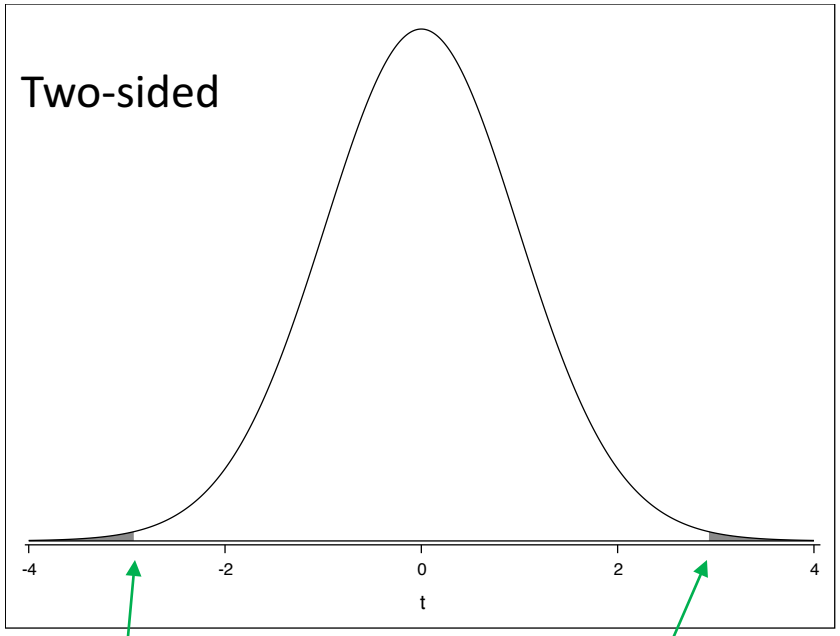
intrinsic	N	Mean	Std Dev	Std Err	Minimum	Maximum
1	24	19.8833	4.4395	0.9062	12.0000	29.7000
0	23	15.7391	5.2526	1.0952	5.0000	24.0000
Diff (1-2)		4.1442	4.8541	1.4164		

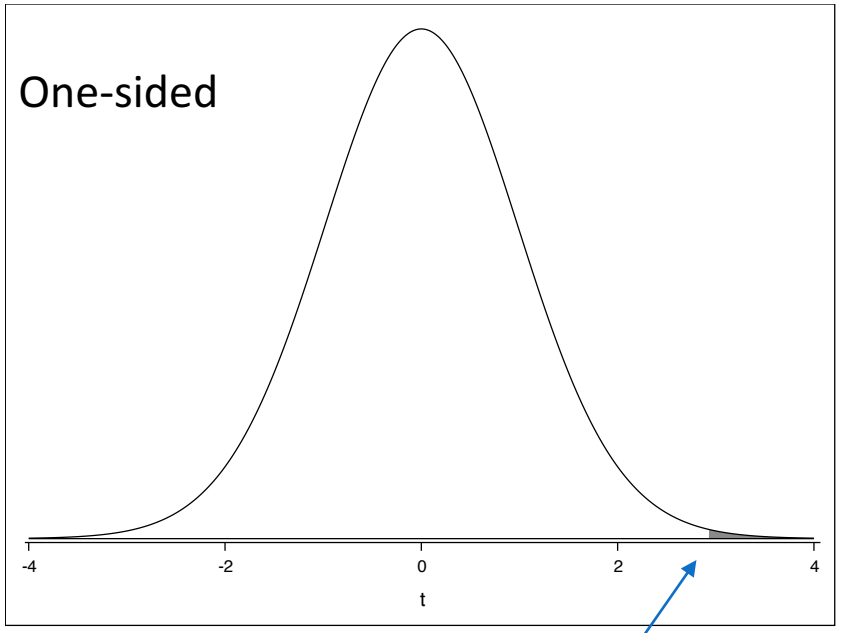
intrinsic	Method	Mean	95% CL Mean		Std Dev	95% CL Std Dev	
1		19.8833	18.0087	21.7580	4.4395	3.4504	6.2276
0		15.7391	13.4677	18.0105	5.2526	4.0623	7.4343
Diff (1-2)	Pooled	4.1442	1.7655	Infty	4.8541	4.0261	6.1138
Diff (1-2)	Satterthwaite	4.1442	1.7546	Infty			

Method	Variances	DF	t Value	Pr > t
Pooled	Equal	45	2.93	0.0027
Satterthwaite	Unequal	43.108	2.92	0.0028

One-sided CI

Comparing the Distributions





$$\frac{(\bar{y}_E - \bar{y}_I)}{s_p\sqrt{\frac{1}{n_I} + \frac{1}{n_E}}} = -4.14$$

$$\frac{(\bar{y}_I - \bar{y}_E)}{s_p\sqrt{\frac{1}{n_I} + \frac{1}{n_E}}} = 4.14$$

$$\frac{(\bar{y}_I - \bar{y}_E)}{s_p\sqrt{\frac{1}{n_I} + \frac{1}{n_E}}} = 4.14$$

One and Two Tailed Confidence Intervals

One-Sided P-values

- Is the intrinsic group mean larger than the extrinsic group mean?

  $\rightarrow$ H_A: $\mu_I > \mu_E$

- Evidence against H_0 comes from values of $\bar{Y}_I - \bar{Y}_E$ far from zero *in the positive direction only*

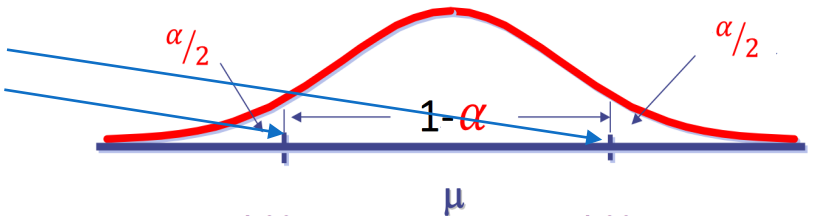
- This is interpreted quantitatively as is
  - Is $\bar{Y}_I - \bar{Y}_E$ large?

Related question:

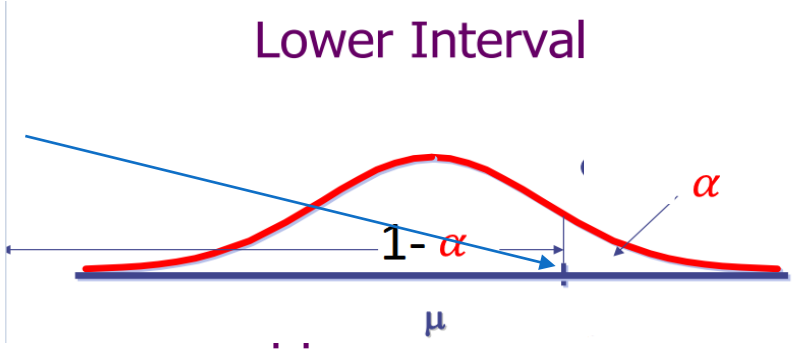
What is the smallest value of μ_I - μ_E that is plausible for this data?

One Sample Confidence Interval

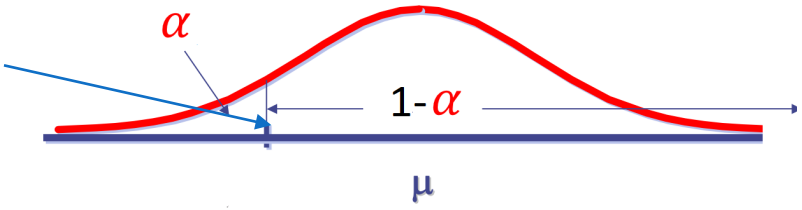
Two Tailed $\bar{Y} \pm t_{\alpha/2,(n-1)} \frac{s}{\sqrt{n}}$



Lower One Tailed $\bar{Y} + t_{\alpha,(n-1)} \frac{s}{\sqrt{n}}$



Upper One Tailed $\bar{Y} - t_{\alpha,(n-1)} \frac{s}{\sqrt{n}}$



Creativity Study: Confidence Interval

$\rightarrow \bar{y}_I - \bar{y}_E = 4.14$

$\bar{y}_I - \bar{y}_E = 19.88 - 15.74$

```
PROC SORT DATA = creativity;
    BY intrinsic;
RUN;
PROC UNIVARIATE data = creativity;
    BY intrinsic;
RUN;
```

intrinsic=1

Moments			
N	24	Sum Weights	24
Mean	19.8833333	Sum Observations	477.2
Std Deviation	4.43951296	Variance	19.7092754
Skewness	-0.074952	Kurtosis	0.08425798
Uncorrected SS	9941.64	Corrected SS	453.313333
Coeff Variation	22.3278104	Std Error Mean	0.90621179

intrinsic=0

Moments			
N	23	Sum Weights	23
Mean	15.7391304	Sum Observations	362
Std Deviation	5.25259582	Variance	27.5897628
Skewness	-0.76156	Kurtosis	-0.0935406
Uncorrected SS	6304.54	Corrected SS	606.974783
Coeff Variation	33.3728464	Std Error Mean	1.09524194

$$s_p = \sqrt{\frac{(n_I-1)\,s_I^2 + (n_E-1)\,s_E^2}{n_I+n_E-2}} = \sqrt{\frac{(23)4.44^2+(22)5.25^2}{45}}$$

$\rightarrow s_p = 4.85$

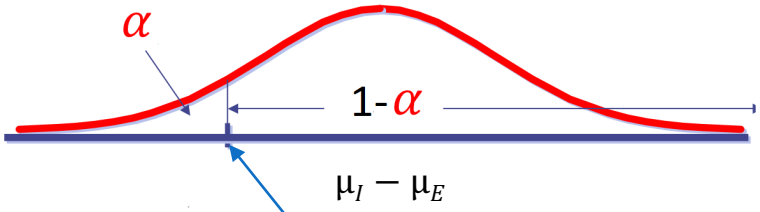
Constructing an Upper One-Tailed Confidence Interval

```
DATA quant;
    t = QUANTILE('T',.95,45);
RUN;

PROC PRINT DATA=quant;
RUN;
```

Obs	q
1	1.67943

Upper Interval



$$SE(\bar{Y}_I - \bar{Y}_E) = s_p\sqrt{\frac{1}{n_I} + \frac{1}{n_E}} = 4.85\sqrt{\frac{1}{24} + \frac{1}{23}} = 1.42$$

$$(\bar{y}_I - \bar{y}_E) - t_{\alpha,(n-2)}SE(\bar{Y}_I - \bar{Y}_E) = 4.14 - (1.68)(1.42) = 1.76$$

One-Sided P-values in SAS

The TTEST Procedure
Variable: score

intrinsic	N	Mean	Std Dev	Std Err	Minimum	Maximum
1	24	19.8833	4.4395	0.9062	12.0000	29.7000
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Method	Variances	DF	t Value	Pr > t
Pooled	Equal	45	2.93	0.0027
Satterthwaite	Unequal	43.108	2.92	0.0028

One-sided CI

"We are 95% confident that the difference in the mean creativity score for the intrinsic group vs. the extrinsic group is at least 1.7655 [units?]"

Five Steps to Hypothesis Testing

Five steps

(D = 0)

Step 1: Identify the null and alternative hypothesis.

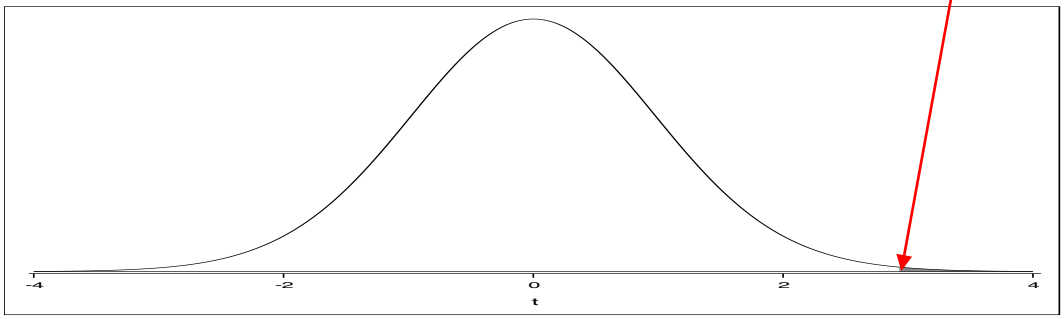
$H_0: \mu_I - \mu_E = D$

$H_A: \mu_I - \mu_E > D$

Step 2: Find the test statistic.

$$t = \frac{(\bar{y}_I - \bar{y}_E) - D}{s_p\sqrt{\frac{1}{n_I} + \frac{1}{n_E}}} = \frac{4.14}{4.85\sqrt{\frac{1}{24} + \frac{1}{23}}} = 2.93$$

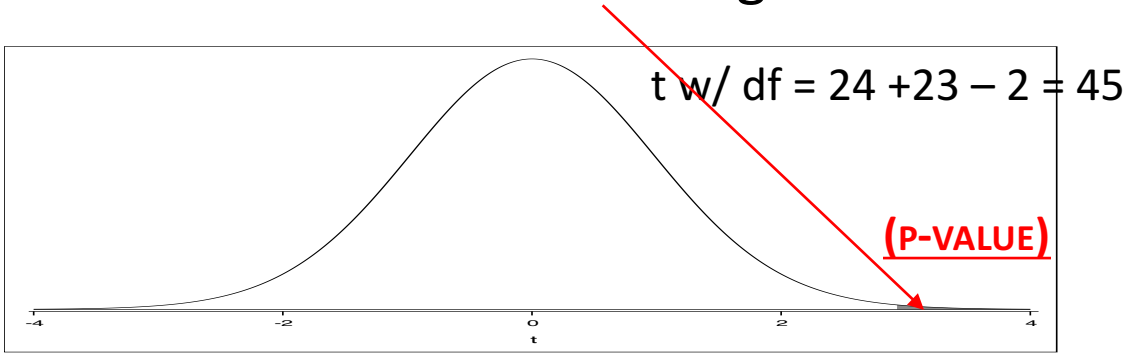
Step 3: Draw and shade the correct density according to H_A



t w/ df = 24 +23 – 2 = 45

Five steps

Step 4: Find the size of the shaded region



Step 5: Include the results in a statistical conclusion (hypothesis, p-value, and whether it is one-sided or two sided)

"There is convincing evidence that the intrinsic questionnaire mean is larger than the extrinsic (one-sided p-value =0.0027 from a pooled two-sample t-test for equal means)…"