

# **§0.1: Draft Answers to Exercises**

**Answers 1** (A simple t-test? from page 66.)

- A. *Use with the Central Limit Theorem (CLT) and its assumptions. Re-arrange the convergence result until only  $N(0, 1)$  is on the right-hand-side. Use “plug-in” estimates, based on data, for unknown parameters (the variance in this case). Then recall that the  $t$ -distribution converges to  $N(0, 1)$  as the sample size (degrees of freedom) increases. Also, remember that the standard  $t$ -test is a test about a mean. So if the data are non-normal you can rely on the CLT if the sample size is “big enough”.*
- B. *Usually the data are assumed to be IID for a  $t$ -test. But the CLT has versions that apply in cases where this is not true, as well. In practice you need good estimates of the applicable covariance terms. This is feasible, and indeed common, in a time series context.*

**Answers 2** (T-test interpretation from page 69.)

A. The tables display the sample size and pieces for each  $t$ -statistic. The key is check the  $t$ -statistic in the  $T$ -distribution cdf. This value is the  $p_0$ -value. When this value is close to zero or one there is evidence against the null hypothesis (of zero mean). The confidence intervals could also be used directly to test the null hypothesis: if the interval excludes zero then this entails extreme  $p_0$ -values.

The actual results suggest there is little empirical evidence against the null hypotheses of zero means. But we have not accounted for the fact that the test may lack power (so we shouldn't conclude that the true means of these assets are really zero). Notice that the paired tests also suggest that there is very little evidence against the null hypotheses that the individual assets have different mean to the market index (only Apple's confidence interval excludes zero, and even then only just).

B. We will get many extreme looking  $p_0$ -values even by chance under the null hypotheses of zero mean. This may cause confusion and is known as the problem of multiplicity! We return to this problem in section 4, page 109.

*C. Quite possibly there exist “regimes” in which the series have a different distribution. For example, Apple may have a return similar to the market when market volatility is high but very different when market volatility is low. A key problem with this is the problem of multiplicity.*

**Answers 3** (Bootstrap cut-offs from page 76.)

*A. Because it is a subjective matter about what counts as “extreme”. This is the same in the typical testing context.*

**Answers 4** (Bootstrapping from page 82.)

- A. *The results are very similar in terms of confidence intervals. This would be expected, for example, when the sample size is large enough (and the assumptions approximately hold with respect to the data being from the same distribution), ensuring the key implication of the Central Limit Theorem is practical.*
- B. *Bootstrap the averages of data series based on period-by-period differences between Apple and other assets. Then check to see whether the confidence intervals embrace zero. If a particular one excludes then there is evidence against that particular null hypothesis that the true mean difference is zero.*

**Answers 5** (Over-fitting and Multiplicity from page 117.)

- A. *Assuming the null hypothesis (and the DGP and other assumptions underlying it) the  $p$ -value is the probability of obtaining a “more extreme” test statistic (or perhaps “more unusual data” under the null, if you like.) The  $p_0$ -value is a similar measure which relates to the exactly where within the null cdf the test statistic falls (which in turn enables a judgement about how often you would expect such an extreme cdf value if the null were true.)*
- B. *Nothing — the  $p$ -value is simply a measure of how “consistent” or “unusual” the data are relative to a hypothesis about the data.*
- C. *The first,  $\text{over-fitting}_1$ , is tractable to the extent it is a mathematical measure of the flexibility of the model itself. The second,  $\text{over-fitting}_2$ , relates to more subtle flexibility about modelling procedures and interactions between research and models. In other words,  $\text{over-fitting}_1$  is more mathematical!*