Problem 1

I.

Let us consider a stacked parallel-plate structure composed of two dielectrics and three metal electrodes, as shown in Fig. 1. Let us call the bottom and top dielectrics dielectric 1 and 2, respectively. Dielectrics 1 and 2 have dielectric permittivities ε_1 and ε_2 and thicknesses d_1 and d_2, respectively. A DC voltage source and a pulsed voltage source are connected to the top and bottom electrodes, as shown in Fig. 1. In the initial condition, the intermediate electrode sandwiched by dielectric 1 and 2 is electrically neutral. <u>Suppose that dielectric 1 has such a property that charges can flow through it when a voltage larger than a certain value is applied between the intermediate and the bottom electrodes</u>, but that such charge flow does not occur in dielectric 2. We can neglect the edge effect of the structure. Answer the following questions.

(1) When the pulsed voltage source is shorted and only a DC voltage V is applied between the top and the bottom electrodes, no charge flow is observed in dielectric 1. Obtain the sheet charge densities (charge per unit area) accumulated on the top electrode, q_{t0}, and on the bottom electrode, q_{b0}.

(2) Next, in addition to the DC voltage, we apply a single rectangular pulsed voltage of a duration Δt, whose polarity is the same as that of the DC voltage. When the pulsed voltage is applied, charge transfer takes place from the intermediate electrode to the bottom electrode through dielectric 1 and we observe a constant current density J during Δt. After the end of the pulsed voltage, the charge state of the intermediate electrode is preserved and the sheet charge density accumulated on the bottom electrode becomes q_{b}. Calculate q_{b}.

(3) As considered in Question (2), after the charge transfer takes place from the intermediate electrode to the bottom electrode, the sheet charge density accumulated on the bottom electrode changes from q_{b0} to q_{b}. To make q_{b} equal to q_{b0}, we need to adjust the DC voltage from V to $V + \Delta V$. You may assume that there is no current flow through dielectric 1 at $V + \Delta V$. Calculate ΔV.

(4) Discuss for what kind of device applications you can use the effect described here.

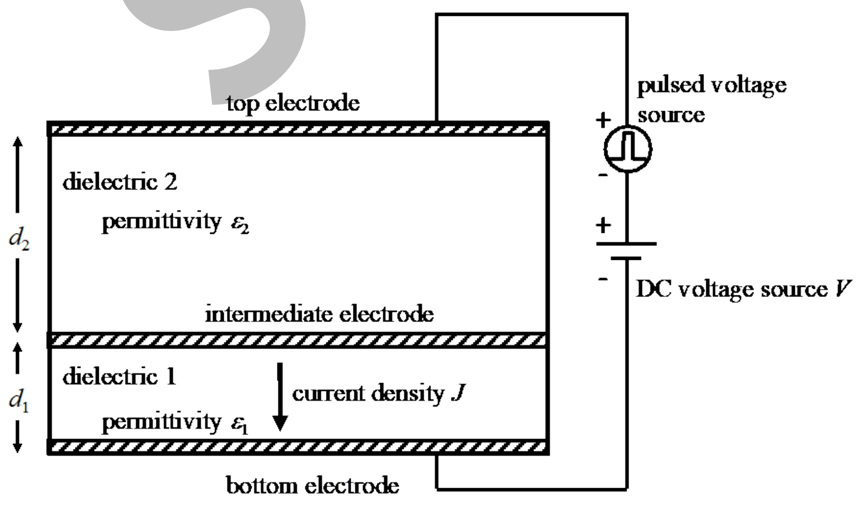


Fig. 1

II.

Let us take the xyz-coordinates as shown in Fig. 2 and consider a system in which two semi-infinitely large dielectrics 1 and 2 are touching with each other in the xy-plane. Dielectrics 1 and 2 have dielectric permittivities ε_1 and ε_2, respectively ($\varepsilon_2 > \varepsilon_1$). The two dielectrics have magnetic permeability μ_0. Consider a case in which an electromagnetic plane wave with an electric field component $\boldsymbol{E}_\mathbf{i} = (E_\mathrm{i}, 0, 0)e^{i(k_\mathrm{i}z-\omega t)}$ is propagating in dielectric 1 in the $+z$-direction and incident to the interface between the two dielectrics. Here, k_i is the wave number of the incident electromagnetic wave, ω is the angular frequency, z is the position coordinate on the z-axis, t is time, and i is the imaginary unit. <u>The electric and magnetic fields of the electromagnetic waves have only components which are parallel to the xy-plane and they are uniform in the in-plane direction</u>. Also, we can neglect losses and assume that there is no free current in the dielectrics or at the interface. Answer the following questions.

(1) By using Maxwell's equations, show that the magnetic field component of the incident electromagnetic wave, $\boldsymbol{H}_\mathbf{i}$, that propagates in dielectric 1 can be expressed as Eqs. (i) and (ii);

$$\boldsymbol{H}_\mathbf{i} = (0, H_\mathrm{i}, 0)e^{i(k_\mathrm{i}z-\omega t)} \tag{i}$$

$$H_\mathrm{i} = Y_1 E_\mathrm{i} \tag{ii}$$

Here, $Y_1 \equiv \sqrt{\frac{\varepsilon_1}{\mu_0}}$ and is called the admittance of dielectric 1.

(2) At the interface between dielectric 1 and dielectric 2, a part of the incident electromagnetic wave is reflected and the rest is transmitted. Let us write the electric field of the reflected wave as $\boldsymbol{E}_\mathbf{r} = (E_\mathrm{r}, 0, 0)e^{-i(k_\mathrm{r}z+\omega t)}$ and that of the transmitted wave as $\boldsymbol{E}_\mathbf{t} = (E_\mathrm{t}, 0, 0)e^{i(k_\mathrm{t}z-\omega t)}$. Here, k_r and k_t are the wave numbers of the reflected and the transmitted electromagnetic waves, respectively. Write down the boundary conditions for the electric and magnetic fields at the interface of dielectrics 1 and 2. Furthermore, express E_r and E_t by using E_i. If necessary, define the admittance of dielectric 2, Y_2, and use it in the calculation.

(3) Calculate the ratio of the reflected power to the incident power (reflection coefficient, R) and the ratio of the transmitted power to the incident power (transmission coefficient, T).

Next, we insert an infinitely thin conductive layer between the two dielectrics, as shown in Fig. 3. Due to this conductive layer, a current per unit width, $\boldsymbol{J} = \sigma\boldsymbol{E}$, flows at the interface and, therefore, the boundary conditions for the electric and magnetic fields at the interface need to be modified. Here, σ is a constant related to the conductivity of the conductive layer and $\boldsymbol{E}$ is the electric field component of the electromagnetic wave at the interface.

(4) Write down the boundary conditions for the electric and magnetic fields that need to be fulfilled at the interface of the two dielectrics when the conductive layer is inserted.

(5) Obtain the reflection coefficient R and the transmission coefficient T. Furthermore, calculate the ratio of the power absorbed in the conductive layer to the incident power (absorption coefficient, A).

(6) Explain how and why A changes when σ is increased from 0 to infinity. Furthermore, calculate the value of σ, when A becomes maximal, and also calculate the maximal A.

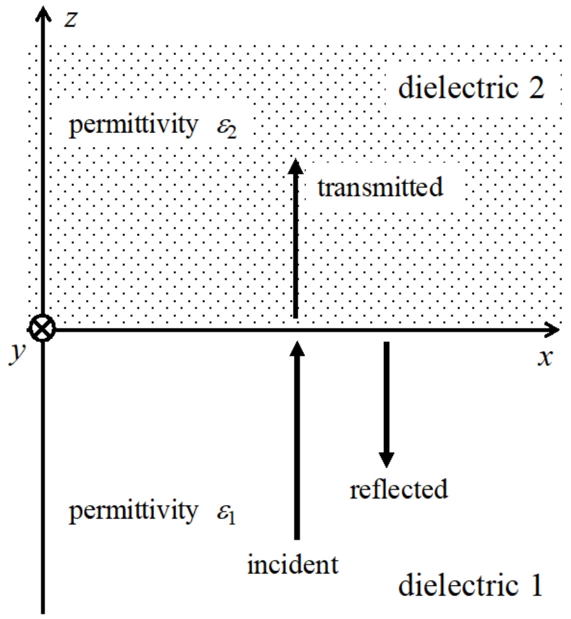

z
dielectric 2
permittivity ε_2
transmitted
y
x
reflected
permittivity ε_1
incident
dielectric 1

Fig. 2

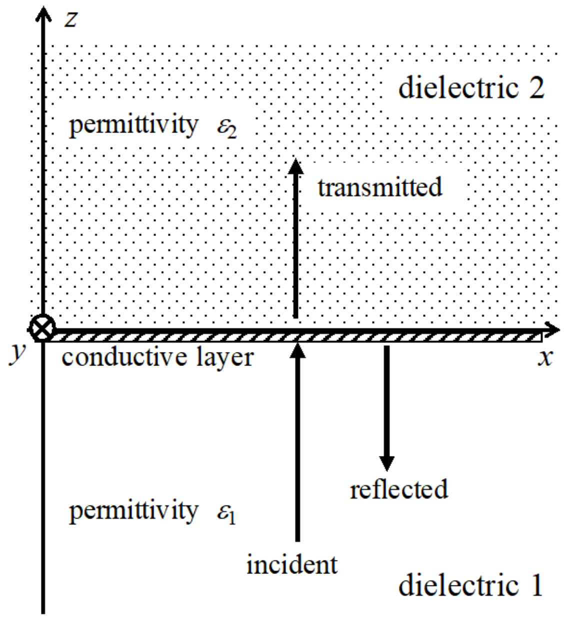

z
dielectric 2
permittivity ε_2
transmitted
y
conductive layer
x
reflected
permittivity ε_1
incident
dielectric 1

Fig. 3

Problem 2

I.

Answer the following questions related to linear circuits. Here, ω denotes the angular frequency. Use j as the imaginary unit in your answer.

(1) Find the admittance $\boldsymbol{Y}_0$ of circuit 1 in Fig. 1, using L and C.

(2) Find the angular frequency ω_0 ($\omega_0 > 0$) that gives the extreme value of $|\boldsymbol{Y}_0|^2$, and the value of $|\boldsymbol{Y}_0|^2$ at the angular frequency ω_0.

(3) Sketch a graph representing the dependency of $|\boldsymbol{Y}_0|$ on the angular frequency ω. In this question, let $L = 1$ H and $C = 1$ F. Draw the horizontal axis (angular frequency) using a logarithmic scale ranging from 0.01 to $100\ \mathrm{rad/s}$. Also draw the vertical axis using a logarithmic scale. In the graph, write the values of $|\boldsymbol{Y}_0|$ at the angular frequencies of 0.01, 0.1, 10, and $100\ \mathrm{rad/s}$, respectively.

(4) Find the impedance $\boldsymbol{Z}_1$ of circuit 2 in Fig. 2, using L, C, and R.

(5) Find the angular frequency ω_1 ($\omega_1 > 0$) that makes the imaginary part of $\boldsymbol{Z}_1$ zero. Give an expression for the impedance $\boldsymbol{Z}_1$ at the angular frequency ω_1. Find the condition on R in order that the angular frequency ω_1 exists.

(6) Choose the correct statement from the following: (a) $\omega_0 > \omega_1$, (b) $\omega_0 = \omega_1$, (c) $\omega_0 < \omega_1$. Note that <u>the values of L and C in Figs. 1 and 2 are respectively identical</u>, and the value of R ($R > 0$) satisfies the condition in Question (5).

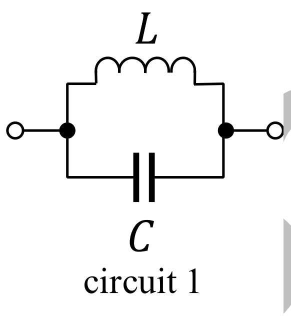


Fig. 1

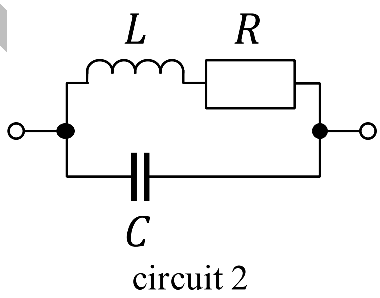
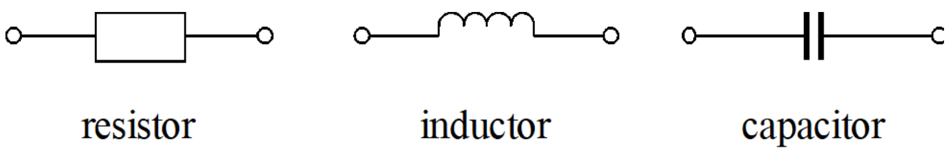


Fig. 2



Legend

II.

Answer the following questions related to the electronic circuits using the n-channel field effect transistor (n-FET) whose I-V characteristics are defined in Fig. 3, and the impedance $\boldsymbol{Z}_2(\omega)$ whose frequency response is defined in Fig. 4. Upon necessity, use the small signal equivalent circuit of the n-FET represented in Fig. 5. Here, g_{m} and r_{o} represent the transconductance and the drain resistance of the n-FET, respectively. In the following questions, <u>the small signal component is denoted in lower case, such as v_{GS} for gate-source voltage V_{GS}</u>. Let the small signal input v_{GS} be a sinusoidal signal with angular frequency ω. Regarding the capacitor C_D in Figs. 6, 7, and 8, the admittance value ωC_D is much larger than those of any other electrical components in the circuits. Use the following circuit parameters: Power supply voltage $V_{\mathrm{DD}} = 5.0$ V, resistance $R_{\mathrm{BIAS}} = 500\ \Omega$, and direct current (hereafter, DC) bias component of the gate-source voltage $V_{\mathrm{GS0}} = 0$ V.

(1) Regarding circuit 3 in Fig. 6, calculate the DC bias component of drain current, I_{D0}, and that of the drain-source voltage, V_{DS0}. Round each value to two significant figures.

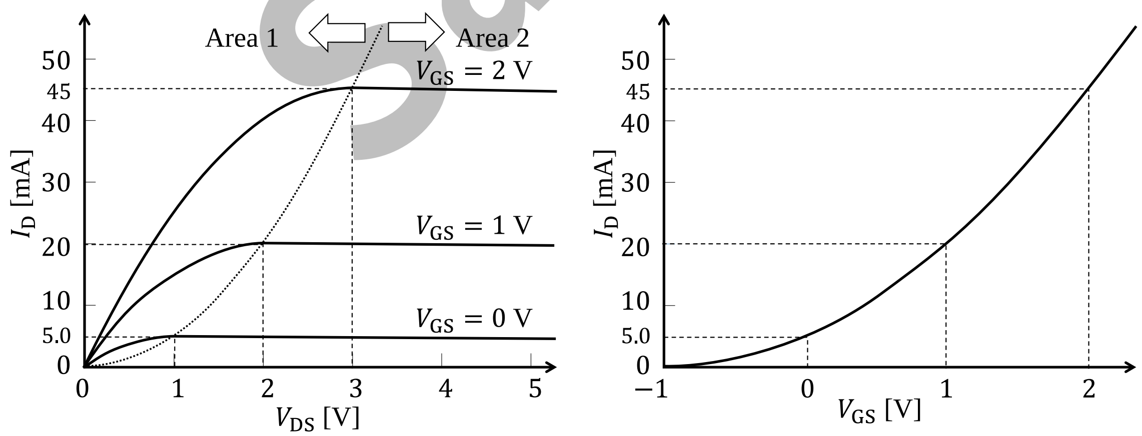
(2) A small signal v_{GS} is applied to the gate terminal of the circuit in Question (1). Draw the small signal equivalent circuit and find the equation that gives the absolute value of the voltage amplification factor

$|A_{\mathrm{v}}| = \left|\frac{v_{\mathrm{DS}}}{v_{\mathrm{GS}}}\right|$.

(3) Calculate the values of $|A_{\mathrm{v}}|$ from Question (2) at the angular frequencies $\omega = \omega_2$ and ω_3, respectively. Note that ω_2 and ω_3 are defined in Fig. 4.

(4) <u>Because of the poor soldering of the component R_{BIAS}</u>, the circuit topology was changed to circuit 4, as shown in Fig. 7. Compare the responses of circuits 3 and 4 at the angular frequency ω_3.

(5) Because of a mistake in component installation, the circuit topology was changed to circuit 5, as shown in Fig. 8. Discuss the behavior of circuit 5.



Area 1: $I_{\mathrm{D}} = 10(V_{\mathrm{GS}} + 1.0 - 0.50\, V_{\mathrm{DS}})\, V_{\mathrm{DS}}$ [mA]

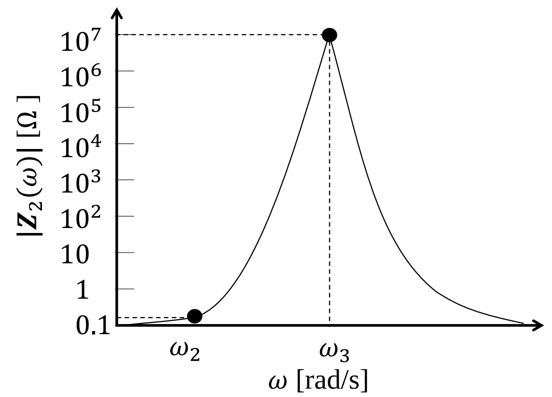
Area 2: $I_{\mathrm{D}} = 5.0(V_{\mathrm{GS}} + 1.0)^2$ [mA]

(a) $I_{\mathrm{D}} - V_{\mathrm{DS}}$ characteristics

$I_{\mathrm{D}} = 5.0(V_{\mathrm{GS}} + 1.0)^2$ [mA]
where $V_{\mathrm{DS}} \geq V_{\mathrm{GS}} + 1.0$

(b) $I_{\mathrm{D}} - V_{\mathrm{GS}}$ characteristics

Fig. 3



$\mathbf{Z}_2(0) = 0.10\ \Omega$

$\mathbf{Z}_2(\omega_2) = 0.10 + j0.10\ \Omega$

$\mathbf{Z}_2(\omega_3) = 1.0 \times 10^7\ \Omega$

Fig. 4

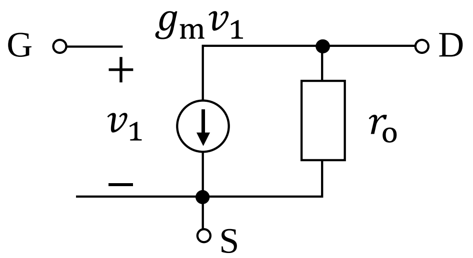
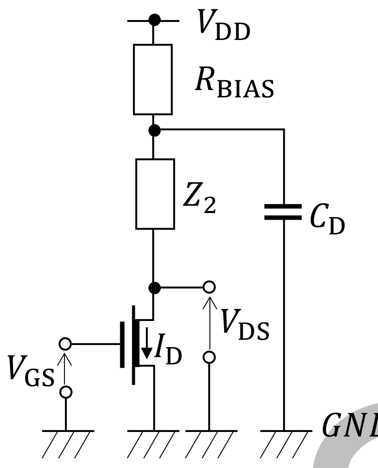
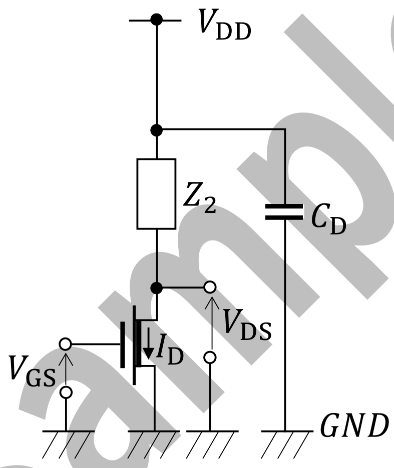


Fig. 5



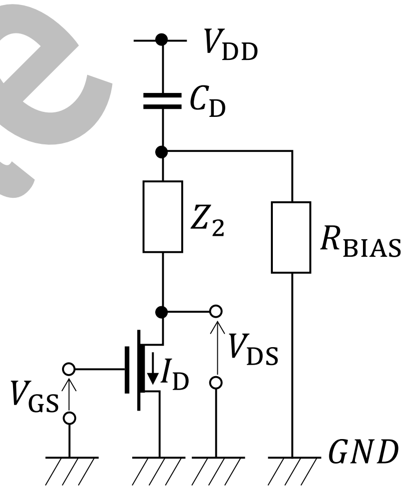
circuit 3

Fig. 6



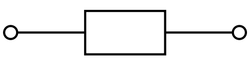
circuit 4

Fig. 7

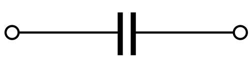


circuit 5

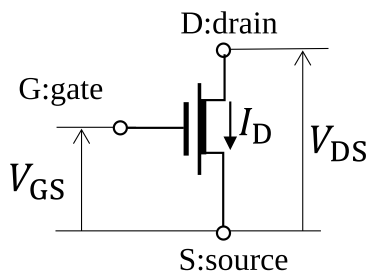
Fig. 8



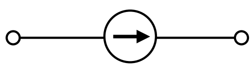
resistor



capacitor



n-FET



current source

Legend

Problem 3

I.

Answer the following questions on information theory. Suppose that we transmit information by using a time-discrete communication channel C, whose input and output are designated as $X \in \{-1,1\}$ and $Y \in \{-1,1\}$, respectively. The input and output relation of the i-th communication via C is represented as $Y_i = Z_i \times X_i$ ($i = 1, 2, \cdots$), where $\times$ means the multiplication of integers. $Z_i \in \{-1,1\}$ is an internal state of the channel at the i-th communication, and its value can change depending on the current or past states of the input and on the past states of the output. Both sender and receiver are unable to observe the value of Z_i directly although they can have knowledge about how Z_i changes depending on the input and output. Use the logarithm base 2 for your answers of the following questions. You may also use the following approximations upon necessity: $\log_2 3 = 1.585$, $\log_2 5 = 2.322$, and $\log_2 7 = 2.807$.

(1) Let X_i be an ideally independent random variable that takes $X_i = 1$ with probability μ and $X_i = -1$ with probability $1 - \mu$. Assume that Z_i becomes 1 with probability 1 when $X_i = 1$ and that it takes either 1 or -1 with equal probability when $X_i = -1$.

(1-i) Obtain the entropies $H[X]$ and $H[Y]$ and the conditional entropy $H[Y|X]$ of C.

(1-ii) Obtain the channel capacity of C.

(2) Assume that Z_1 takes either 1 or -1 with equal probability and that, for $i \geq 2$, the value of Z_i becomes the same as the previous output value Y_{i-1} with probability 1 as $Z_i = Y_{i-1}$. Obtain the maximum bits that can be transmitted by using this channel n times.

(3) Assume that Z_i takes either 1 or -1 with equal probability when i is odd and that Z_i keeps its previous value with probability 1 as $Z_i = Z_{i-1}$ when i is even. Obtain the channel capacity of C and show a code that can achieve the capacity.

(4) Assume that $Z_1 = 1$ with probability 1 and that, for $i \geq 2$, the value of Z_i becomes the same as the previous input value X_{i-1} with probability 1 as $Z_i = X_{i-1}$. Let X_i be an ideally independent random variable that takes $X_i = 1$ with probability μ and $X_i = -1$ with probability $1 - \mu$. Calculate the probability q that $Y_i = 1$ at the stationary state for sufficiently large i.

II.

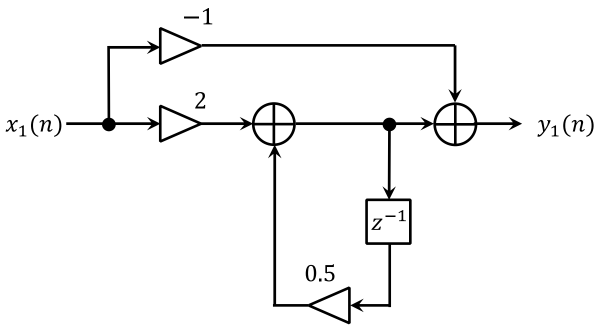
Answer the following questions on signal processing. Consider the two infinite impulse response systems shown in Figs. 1 and 2. $x_1(n)$ and $y_1(n)$ are the input and output signal sequences of system 1 in Fig. 1, respectively, and represent the signal values at time $nT(T > 0)$ for $n = 0,1,\cdots$. Similarly, $x_2(n)$ and $y_2(n)$ are the input and output sequences of system 2 in Fig. 2. The circuits consist of adders, coefficient multipliers, and delays, whose respective functions are described in Fig. 3.

(1) Obtain the impulse response of system 1, $h_1(n)$, and its *z*-transform $H_1(z)$.

(2) Calculate the frequency response of system 1 and explain the filtering function of this system on the input signal.

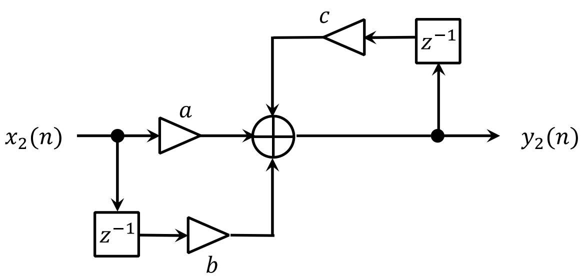
(3) Obtain the parameter values of a, b, and c that makes system 2 equivalent to system 1.

(4) Draw an equivalent circuit of system 2 that has a smaller number of delays than the original system 2 shown in Fig. 2.



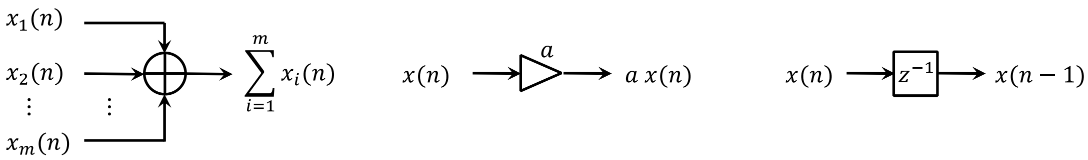
System 1

Fig. 1



System 2

Fig. 2



Adder | Coefficient multiplier | Delay

Fig. 3

Problem 4

I.

Answer the following questions on sequential circuits. We consider a circuit that has four states S={s0, s1, s2, s3}. The circuit changes its state and output Y according to the value of input X. Table 1 shows the state transition table of the circuit.

(1) Draw the state transition diagram that is equivalent to the state transition table in Table 1.

(2) Show the shortest input sequence that outputs Y=1 when the initial state is s2.

Next, assume that we represent the state S with two bits as s0=00, s1=01, s2=10, s3=11. We consider designing the circuit with two JK-Flip Flop (JK-FF) F_A and F_B. We represent the higher bit of the states with Q_A, and the lower bit with Q_B such as s=Q_AQ_B. F_A and F_B hold the states Q_A and Q_B in the circuits, respectively. A JK-FF has one bit state Q, and the state transits as shown in Fig. 1 according to inputs J and K.

(3) Copy Table 1 to your answer sheet and complete the table by filling the blanks. Here, (J_A, K_A) and (J_B, K_B) are the inputs to F_A and F_B, respectively. Use *d* for "don't care" conditions.

(4) Simplify the input functions of J_A, K_A, J_B, K_B and output function of Y by using Karnaugh maps.

(5) Draw the circuit using the symbols shown in Fig. 2.

Table 1

Current state S(t)	Input X	Next state S(t+1)	Output Y	$Q_A(t)$	$Q_B(t)$	$Q_A(t+1)$	$Q_B(t+1)$	J_A	K_A	J_B	K_B
s0	0	s1	0								
s0	1	s2	0								
s1	0	s3	0								
s1	1	s2	0								
s2	0	s0	0								
s2	1	s2	0								
s3	0	s0	0								
s3	1	s2	1								

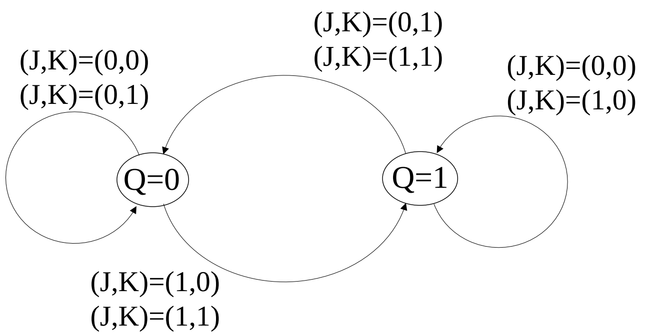


Fig. 1

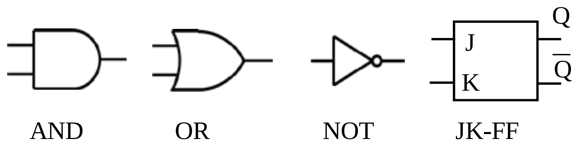


Fig. 2

II.

Answer the following questions on heapsort. Assume that max heap is an array representation of a binary tree where the value in a parent node is greater than the values in its two children nodes. The structure of the array is as follows: assuming that the index i is an integer greater than or equal to 1, the value in the left child node of node i is stored in the $(i \times 2)$-th element of the array, and the value in the right child node of node i is stored in the $(i \times 2 + 1)$-th element of the array.

The function `make_heap` in Program 1 is a function that generates the max heap from an `array` with `n` items. The first element of the array is stored in `array[1]`. The function `swap(array, i, j)` in Program 1 is a function that switches the values of `array[i]` and `array[j]`. Here, the values in the array are integers, and there is no duplication among them.

(1) Assume that the number of items is 7 (`n`=7), and the input `array` stores the values {1, 7, 8, 9, 6, 4, 5} in this order. Find the max heap that is generated by `make_heap` in Program 1.

(2) Draw the process of generating the max heap in Question (1) with binary trees. The arrays follow the same structure as the max heap even during the process of making max heap. You do not need to draw the trees that remain unmodified in the process.

(3) Give the order of the time complexity of `make_heap` according to the number of comparisons of the values.

Assume that we sort a max heap by iteratively taking the maximum value and rebuilding a max heap with the remaining items.

(4) Describe a function by using the C programming language in as few lines as possible to sort a max heap `array` with `n` items in descending order. Use the function `make_heap` in Program 1 for rebuilding the max heap in your function.

(5) Describe an overview of an algorithm to sort an array using max heap, which is more computationally efficient than that obtained in Question (4). The sort order can be either ascending order or descending order.

```
/*  Program 1 */
void push_down(int array[], int n, int node) {
    if(node * 2 > n)    return;
    int    parent = node, child;
    do {
        child = parent * 2;
        if ((child < n) && (array[child] < array[child + 1]))
            child = child + 1;
        if (array[child] < array[parent])
            break;
        swap(array, child, parent);
        parent = child;
    }while (parent * 2 <= n);
}

void make_heap(int array[], int n){
    int    node;
    for (node = n / 2; node >= 1; --node)
        push_down(array, n, node);
}
```

Problem 5

I.

Suppose that the potential in an one-dimensional space along the x-axis is given as $V(x) = 0$ for Region A ($x < 0$) and $V(x) = V_0$ ($V_0 > 0$) for Region B ($x \geq 0$), as shown in Fig. 1. Also suppose a wave function $\Psi(x,t) = \phi(x)e^{-i\omega t}$ for a particle with mass m in this space. Here, i is the imaginary unit, ω is the angular frequency and t is time. $\phi(x)$ is given as a solution of the time-independent Schrödinger equation:

$$\left\{-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right\}\phi(x) = E\phi(x), \qquad \text{(i)}$$

where E ($E > 0$) is the energy of this particle and $\hbar$ is the reduced Planck constant (given by the Planck constant divided by 2π). Answer the following questions.

(1) Prove that a general solution of Eq. (i) in Region A is given by the following equation (ii) with arbitrary constants C_1 and C_2. In addition, give an expression for α, where α is a positive real number.

$$\phi(x) = C_1 e^{-i\alpha x} + C_2 e^{i\alpha x}. \qquad \text{(ii)}$$

(2) Give a general solution of $\phi(x)$ in Region B satisfying Eq. (i) when $E < V_0$.

Next, suppose that the particle in Region A moves towards the positive direction in the x-axis. Consider the reflection of this particle by the potential interface existing at $x = 0$, and the penetration of the particle into Region B.

(3) Suppose that the wave function of the injected particle is given by $\Psi_{\text{in}}(x,t) = \phi_{\text{in}}(x)e^{-i\omega t}$, choose an appropriate term for $\phi_{\text{in}}(x)$ from the two terms in the right side of Eq. (ii), and briefly describe the reason why.

(4) Explain the boundary conditions for $\phi(x)$ and $\frac{d}{dx}\phi(x)$ which must be satisfied at the interface at $x = 0$.

(5) When the energy E of the injected particle is equal to $\frac{V_0}{2}$, give an expression for C_1 using C_2. Then, choose the correct statement from the following: (a) $|C_1| > |C_2|$, (b) $|C_1| = |C_2|$, or (c) $|C_1| < |C_2|$.

(6) Under the same conditions as Question (5), and when the wave function of the particle in Region B is given by $\Psi_{\text{B}}(x,t) = \phi_{\text{B}}(x)e^{-i\omega t}$, show that $|\phi_{\text{B}}(0)|$ is larger than $|\phi_{\text{in}}(x)|$. In addition, explain the reason why this relationship does not conflict with the energy conservation law for the energy transport by this particle, in about 3 lines.

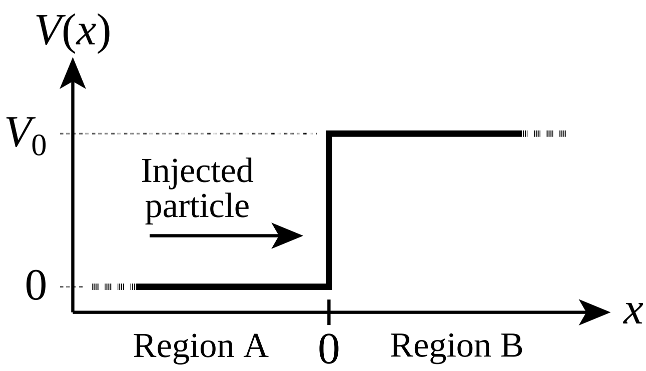


Fig. 1

II.

Consider a semiconductor, in which the bandgap E_g is 1.0 eV, the effective masses m_e and m_h of an electron in the conduction band and a hole in the valence band are $0.10m_0$ and $0.30m_0$ (m_0 is the mass of a free electron in a vacuum), respectively, and the relative dielectric constant ε_r is 10. Also suppose that impurities which become monovalent ions are doped in this semiconductor to make it a n-type semiconductor, and that such impurities create an impurity level at E_{imp} near the bottom of the conduction band, as shown in Fig. 2. Answer the following questions.

(1) Give the sign of charge of the impurity ionized in the semiconductor.

(2) For an absolute temperature T, the free carrier density ρ varies with respect to $1/T$, as shown in Fig. 3. For the points labeled (a) and (b) in Fig. 3, choose the most appropriate Fermi level positions from V to Z in Fig. 2, and describe the reason why, in about 3 lines each. You may omit the temperature dependence of E_g.

(3) Around the point labeled (c) in Fig. 3, the free carrier density ρ is almost independent of temperature. Describe the reason for this, in about 3 lines.

(4) In a hydrogen atom, an electron with a negative single charge is bound by an atomic nucleus with a positive single charge. In the hydrogen atomic model, the ground state energy E_1 and the Bohr radius a_B are given by

$$E_1 = -\frac{e^2}{2(4\pi\varepsilon_0)a_B} \cong -14\ \text{[eV]} \quad \text{and}$$
$$a_B = \frac{\varepsilon_0 h^2}{\pi m_0 e^2} \cong 0.053\ \text{[nm]},$$

where e, ε_0 and h are the elementary charge, the dielectric constant of vacuum and the Planck constant, respectively. Based on this hydrogen atomic model, obtain approximate values of the ionization energy ΔE_{imp} and the effective Bohr radius a_B^* for the present impurity.

(5) Choose the most appropriate point from (a) to (c) in Fig. 3 which corresponds to the situation at room temperature (300 K), and describe the reason why, using values of the Boltzman constant k_B ($\cong 1.4 \times 10^{-23}$ [J/K]) and the elementary charge e ($\cong 1.6 \times 10^{-19}$ [C]).

(6) For an impurity which becomes a divalent ion, the energy level of the impurity is not normally given based on the hydrogen atomic model in Question (4). Describe the reason for this, in about 5 lines. In addition, briefly describe which is larger, the ionization energy for the divalent ion or ΔE_{imp} for the monovalent ion.

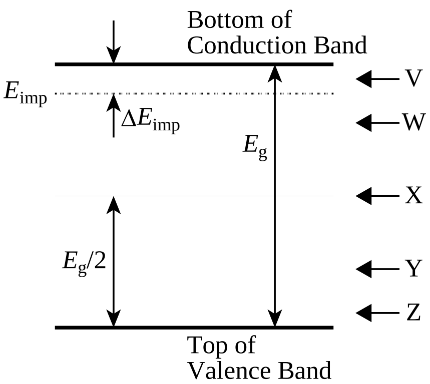


Fig. 2

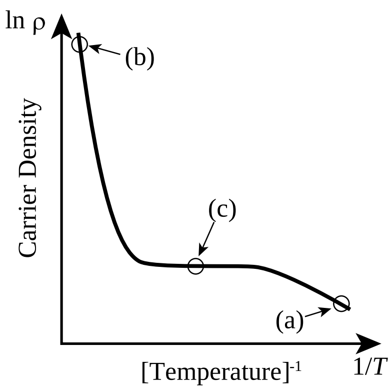


Fig. 3

Problem 6

I.

Consider a speed control system of the DC servomotor represented by Eq. (i). Here, t is time, $\omega(t)$ is the rotational angular velocity, and $u(t)$ is the control input. In addition, s is the Laplace operator. $W(s)$ and $U(s)$ are the Laplace transformations of $\omega(t)$ and $u(t)$, respectively. Answer the following questions.

$$\frac{d\omega(\mathrm{t})}{dt} = 2u(t) - 4\omega(t) \qquad \text{(i)}$$

(1) Derive the transfer function $P_0(s) = W(s) \,/\, U(s)$ of this plant.

(2) For this plant, a feedback controller $C(s)$ is designed as the proportional-integral controller expressed by Eq. (ii).

$$U(s) = C(s)\big(R(s) - W(s)\big), \qquad C(s) = K_\mathrm{p}\left(1 + \frac{1}{\tau_\mathrm{i} s}\right) \qquad \text{(ii)}$$

Here, $R(s)$ is the Laplace transformation of the speed command $r(t)$, K_p is the proportional gain, and τ_i is the integration time.

(2-i) Derive the transfer function $G(s) = W(s) \,/\, R(s)$ of the closed loop system.

(2-ii) Find the controller parameters K_p and τ_i for placing the poles of the closed loop system at -40 and -50.

(2-iii) Calculate the time response of $\omega(t)$ when a unit step function is given to the speed command $r(t)$ for the closed-loop system obtained in Question (2-ii).

(3) Draw the root locus of the closed-loop system when τ_i is fixed to 0.05 and K_p is changed from 0 to ∞ in the controller of Eq. (ii).

(4) Find K_p that gives multiple roots of the closed-loop system in Question (3).

(5) Consider the stability when the plant has a modeling error of the mechanical resonance mode that is expressed by Eq. (iii) with $0 < \zeta < 1$, for the controller $C(s)$ obtained in Question (2-ii).

$$P_1(s) = \frac{\omega_\mathrm{p}^2}{s^2 + 2\zeta\omega_\mathrm{p} s + \omega_\mathrm{p}^2} \qquad \text{(iii)}$$

(5-i) When $\omega_\mathrm{p} = 1000$ and $\zeta = 0.1$, sketch the Bode diagram of the open-loop transfer function $P_0(s)P_1(s)C(s)$ that has this modeling error using asymptotic approximations. Indicate numerical values such as the angular-frequency break-points, the slope of the gain diagram, and the angle of the phase diagram.

(5-ii) For $\omega_\mathrm{p} = 1000$, find the range of ζ that can guarantee the stability of the closed loop system.

II.

Consider a separately excited DC motor. Figure 1 shows the armature circuit. Let s be the Laplace operator. Answer the following questions.

(1) Consider deriving a mathematical model from the circuit equation and the equation of motion.

(1-i) Let $V_{\mathrm{a}}(s)$, $I(s)$, and $W(s)$ be the Laplace transformations of the terminal voltage, armature current, and rotational angular velocity, respectively. Let R and L be the armature resistance and the armature inductance, respectively. Describe the armature circuit equation in the s domain. Here the back electromotive force $V_{\mathrm{e}}(s)$ can be expressed as $K_{\mathrm{e}}W(s)$, where K_{e} is the back electromotive force coefficient.

(1-ii) Describe the equation of motion of the rotor in the s domain where J is the moment of inertia. The torque generated by the motor and the viscous friction torque are represented as $T_{\mathrm{m}}(s) = K_{\mathrm{t}}I(s)$ and $T_{\mathrm{v}}(s) = D_{\mathrm{v}}W(s)$, respectively, where K_{t} is the torque coefficient and D_{v} is the viscous friction coefficient.

(2) <u>To what kinds of work is the input power from voltage source V_{a} converted</u>? Explain it using equations in time domain based on the circuit model obtained in Question (1-i). You may define the necessary variables by yourself.

(3) Consider applying a current feedback control system to the motor in Question (1). Draw the block diagram. Also, show that the transfer function from the current command $I^{*}(s)$ to the rotational angular velocity $W(s)$ can be approximately expressed as a first-order system when the controller has a sufficiently high gain.

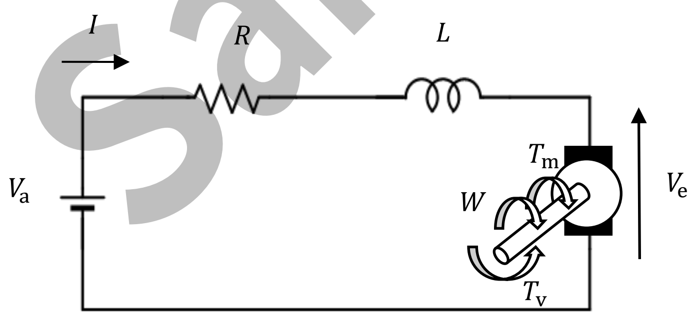


Fig. 1