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Applying generalized tangent vectors to steady-state solutions to the Euler equations

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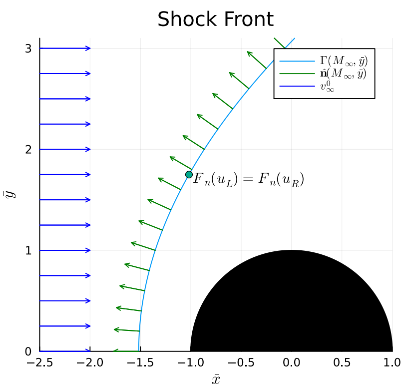
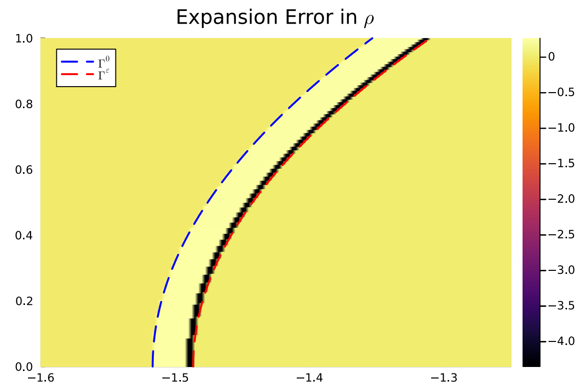
We are interested in steady-state supersonic flow over a blunt cylinder, and the relationship of various shock wave parameters (shock position, shape, etc) and flow parameters (post-shock state, wall temperature, etc) to perturbations in the state of the free-stream flow. To compute the effect of these perturbations on the shock wave, we apply the calculus developed by Bressan[1] to the steady-state Euler equations for perfect gases.

The main idea of Bressan's calculus is to separate the influence that the perturbation has on the solution data and its influence on the position of any discontinuities in the solution by using generalized tangent vectors. Explicitly treating the discontinuities allows the derivation of a Taylor-like expansion that is first order accurate to the true solution.





(a) A depiction of the problem scenario. Supersonic flow to the right ($M_\infty > 1$) impacts the cylinder of radius 1, and a shockwave develops. The shock curve can be parametrized in y, and the shock normals can be easily computed by hand or via algorithmic differentiation. In the steady-state case, the normal flux across the shock wave must be 0. We use the perfect gas relations to "close the system" and compute the conditions immediately behind the shock.

(b) The absolute error between the density component of the tangent vector expansion of the jump condition at a shock wave $(u_L^\varepsilon, u_R^\varepsilon)$ and the exactly computed jump condition (u_L, u_R). The dark-colored band near the perturbed shock wave Γ^ε is caused by *both* the error in the position of the perturbed shockwave as well as the computation error of the jump condition. The shock front and the error are symmetric about the x-axis.

The generalized tangent vectors are used to solve toy optimization problems in this scenario. This yields improvement in the convergence speed of the optimizer when compared against a "naïve" calculus that does not explicitly treat the discontinuities. To save computational effort, we use a parametrization of the shock wave[2]. However, the use of generalized tangent vectors improves the convergence rate of the optimizer independently of the choice of shock wave parametrization. We also use algorithmic differentiation (AD) to compute the individual components of the generalized tangent vectors.

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[1] Bressan et al., *Communications in Partial Differential Equations* **20** (1995)
[2] Billig. *Journal of Spacecraft and Rocketry* **6** 1967