LaTeX to PDF and MathJax: Example 2

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Using this document

This is a second example of a document compiled from LaTeX into multiple formats.

- Standard print PDF
- Clearer print PDF
- Accessible web format

The outputs can be used to test setups and as a second example for students to try out.

1 The scalar product

Consider two vectors **a** and **b** drawn so their tails are at the same point.

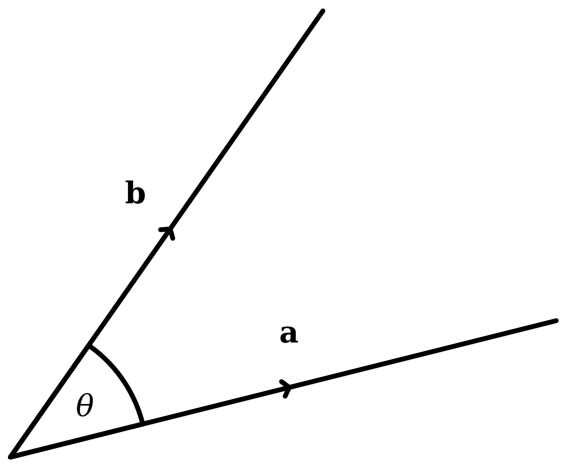

Figure 1: Two vectors with angle between them.

We define the scalar product of **a** and **b** as follows.

Definition 1.1 (Scalar product)**.** The *scalar product* of **a** and **b** is

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

where

- $|\mathbf{a}|$ is the modulus of **a**,
- $|\mathbf{b}|$ is the modulus of **b**, and
- θ is the angle between **a** and **b**.

Remark 1.2. It is important to use the dot symbol for the scalar product (also called the dot product). You must not use a $\times$ symbol as this denotes the vector product which is defined differently.

Example 1.3. Let

$$\mathbf{a} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}.$$

The angle between these vectors is $\theta = 45°$. Then $|\mathbf{a}| = \sqrt{8}$ and $|\mathbf{b}| = 4$. So,

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \end{pmatrix} &= |\mathbf{a}||\mathbf{b}| \cos\theta \\ &= \sqrt{8} \times 4 \times \cos 45° \\ &= 4\sqrt{8} \times \frac{1}{\sqrt{2}} = 4\frac{\sqrt{8}}{\sqrt{2}} = 4\sqrt{4} = 8. \end{aligned}$$

Note that the result is a scalar, not a vector.

1.1 Vectors in cartesian form

When vectors are given in cartesian form there is an alternative formula for calculating the scalar product.

Proposition 1.4. *If* $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$ *and* $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j}$ *then*

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2.$$

Proof. Consider the vector $\mathbf{b} - \mathbf{a} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \end{pmatrix}$. The modulus of this is

$$|\mathbf{b} - \mathbf{a}| = \sqrt{(b_1 - a_2)^2 + (b_2 - a_2)^2}.$$

Note from figure 2 that the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{b} - \mathbf{a}$ form a triangle:

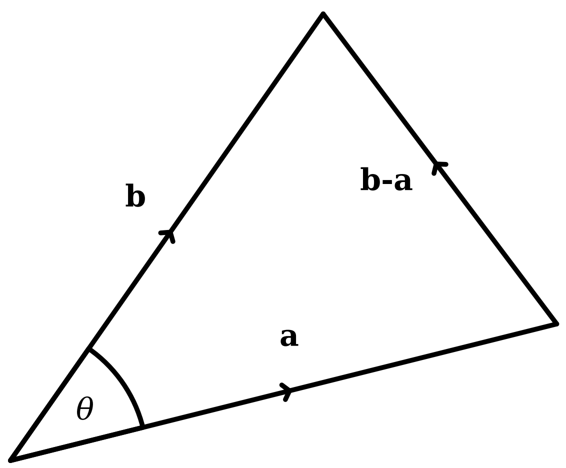

Figure 2: A triangle is formed by two vectors and their difference.

Let θ denote the angle between $\mathbf{a}$ and $\mathbf{b}$. Then, the cosine rule yields:

$$|\mathbf{b} - \mathbf{a}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2|\mathbf{a}||\mathbf{b}|\cos\theta. \tag{1}$$

Substituting the definition of the scalar product of $\mathbf{a}$ and $\mathbf{b}$ into equation 1 gives:

$$|\mathbf{b} - \mathbf{a}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\left(\mathbf{a} \cdot \mathbf{b}\right).$$

Rearranging:

$$\left(\mathbf{a} \cdot \mathbf{b}\right) = \frac{1}{2}\left(|\mathbf{a}|^2 + |\mathbf{b}|^2 - |\mathbf{b} - \mathbf{a}|^2\right).$$

Writing this in terms of components produces:

$$\begin{aligned}
\left(\mathbf{a} \cdot \mathbf{b}\right) &= \frac{1}{2}\left(a_1^2 + a_2^2 + b_1^2 + b_2^2 - (b_1 - a_1)^2 - (b_2 - a_2)^2\right) \\
&= \frac{1}{2}\left(a_1^2 + a_2^2 + b_1^2 + b_2^2 - b_1^2 + 2b_1a_1 - a_1^2 - b_2^2 + 2b_2a_2 - a_2^2\right) \\
&= \frac{1}{2}\left(2b_1a_1 + 2b_2a_2\right) \\
&= a_1b_1 + a_2b_2
\end{aligned}$$

as required. $\square$

Example 1.5. Consider again the vectors

$$\mathbf{a} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}.$$

Calculating the scalar product using the components:

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 = 2 \times 4 + 2 \times 0 = 8.$$

Note that if we are given vectors in this form, the scalar product may be used to calculate the angle between them. Since $\mathbf{a} \cdot \mathbf{b} = 8$ and we have:

$$\begin{aligned} |\mathbf{a}| &= \sqrt{8} \\ |\mathbf{b}| &= 4. \end{aligned}$$

Hence,

$$\begin{aligned} 8 = \mathbf{a} \cdot \mathbf{b} &= |\mathbf{a}||\mathbf{b}| \cos\theta \\ &= 4\sqrt{8} \cos\theta. \end{aligned}$$

Rearranging:

$$\theta = \cos^{-1}\left(\frac{8}{4\sqrt{8}}\right) = 45^\circ.$$

2 Using Matlab

Two calculate the scalar product in Matlab the `dot` function is used.

Create two vectors:

```
> A = [4 -1 2];
> B = [2 -2 -1];
```

Calculate the scalar product:

```
> C = dot(A,B)

    C = 8
```