LaTeX to PDF and MathJax: Example 2

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Using this document

This is a second example of a document compiled from LaTeX into multiple formats.

- Standard print PDF
- Clearer print PDF
- Accessible web format

The outputs can be used to test setups and as a second example for students to try out.

1 The scalar product

Consider two vectors $\mathbf{a}$ and $\mathbf{b}$ drawn so their tails are at the same point.

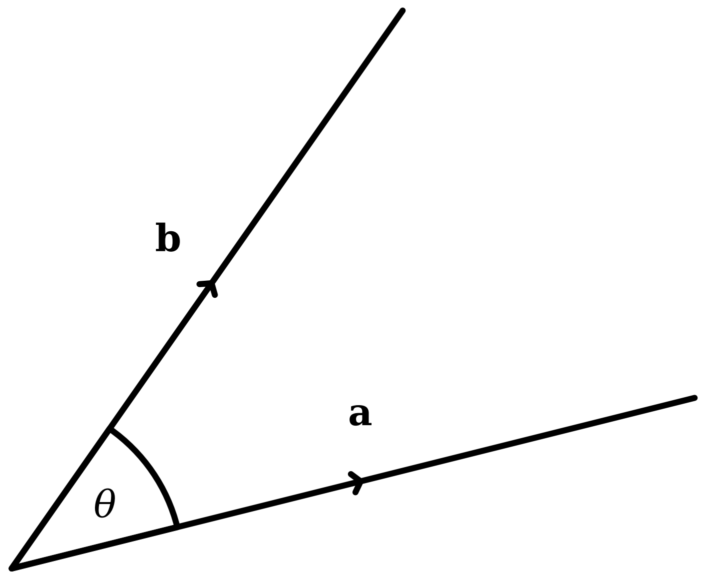

Figure 1: Two vectors with angle between them.

We define the scalar product of $\mathbf{a}$ and $\mathbf{b}$ as follows.

Definition 1.1 (Scalar product)**.**
The **scalar product** of $\mathbf{a}$ and $\mathbf{b}$ is

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$$

where

- $|\mathbf{a}|$ is the modulus of $\mathbf{a}$,
- $|\mathbf{b}|$ is the modulus of $\mathbf{b}$, and
- θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

Remark 1.2.
It is important to use the dot symbol for the scalar product (also called the dot product). You must not use a $\times$ symbol as this denotes the vector product which is defined differently.

Example 1.3.
Let

$$\mathbf{a} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}.$$

The angle between these vectors is $\theta = 45^\circ$. Then $|\mathbf{a}| = \sqrt{8}$ and $|\mathbf{b}| = 4$. So,

$$\begin{aligned}
\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \end{pmatrix} &= |\mathbf{a}||\mathbf{b}| \cos\theta \\
&= \sqrt{8} \times 4 \times \cos 45^\circ \\
&= 4\sqrt{8} \times \frac{1}{\sqrt{2}} = 4\frac{\sqrt{8}}{\sqrt{2}} = 4\sqrt{4} = 8.
\end{aligned}$$

Note that the result is a scalar, not a vector.

1.1 Vectors in cartesian form

When vectors are given in cartesian form there is an alternative formula for calculating the scalar product.

Proposition 1.4.
If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j}$ then

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2.$$

Proof. Consider the vector $\mathbf{b} - \mathbf{a} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \end{pmatrix}$. The modulus of this is

$$|\mathbf{b} - \mathbf{a}| = \sqrt{(b_1 - a_2)^2 + (b_2 - a_2)^2}.$$

Note from figure 2 that the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{b} - \mathbf{a}$ form a triangle:

Let θ denote the angle between $\mathbf{a}$ and $\mathbf{b}$. Then, the cosine rule yields:

$$|\mathbf{b} - \mathbf{a}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2|\mathbf{a}||\mathbf{b}| \cos\theta. \tag{1}$$

Substituting the definition of the scalar product of $\mathbf{a}$ and $\mathbf{b}$ into equation 1 gives:

$$|\mathbf{b} - \mathbf{a}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\,(\mathbf{a} \cdot \mathbf{b}).$$

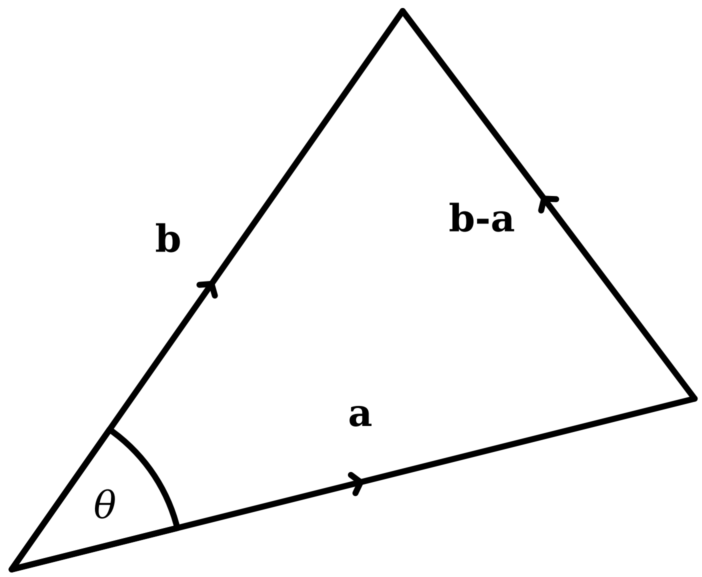

Figure 2: A triangle is formed by two vectors and their difference.

Rearranging:

$$(\mathbf{a}\cdot\mathbf{b}) = \frac{1}{2}\left(|\mathbf{a}|^2 + |\mathbf{b}|^2 - |\mathbf{b}-\mathbf{a}|^2\right).$$

Writing this in terms of components produces:

$$\begin{aligned}(\mathbf{a}\cdot\mathbf{b}) &= \frac{1}{2}\left(a_1^2 + a_2^2 + b_1^2 + b_2^2 - (b_1-a_1)^2 - (b_2-a_2)^2\right)\\ &= \frac{1}{2}\left(a_1^2 + a_2^2 + b_1^2 + b_2^2 - b_1^2 + 2b_1a_1 - a_1^2 - b_2^2 + 2b_2a_2 - a_2^2\right)\\ &= \frac{1}{2}\left(2b_1a_1 + 2b_2a_2\right)\\ &= a_1b_1 + a_2b_2\end{aligned}$$

as required. □

Example 1.5.
Consider again the vectors

$$\mathbf{a} = \begin{pmatrix}2\\2\end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix}4\\0\end{pmatrix}.$$

Calculating the scalar product using the components:

$$\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 = 2\times4 + 2\times0 = 8.$$

Note that if we are given vectors in this form, the scalar product may be used to calculate the angle between them. Since $\mathbf{a} \cdot \mathbf{b} = 8$ and we have:

$$|\mathbf{a}| = \sqrt{8}$$

$$|\mathbf{b}| = 4.$$

Hence,

$$8 = \mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

$$= 4\sqrt{8} \cos\theta.$$

Rearranging:

$$\theta = \cos^{-1}\left(\frac{8}{4\sqrt{8}}\right) = 45°.$$

2 Using Matlab

Two calculate the scalar product in Matlab the `dot` function is used.

Create two vectors:

```
> A = [4 -1 2];
> B = [2 -2 -1];
```

Calculate the scalar product:

```
> C = dot(A,B)

    C = 8
```