Example

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2022

Contents

Alternative formats

It is recommended that you read this resource on the web but you can also download this resource as a PDF or download this resource as a Word document.

Arc length

We often need to know the length of a curve between two points, e.g. what is the length of the ropes holding Clifton suspension bridge (see Exercise Sheet 3).

Visualisation

Given a curve $y = y(x)$

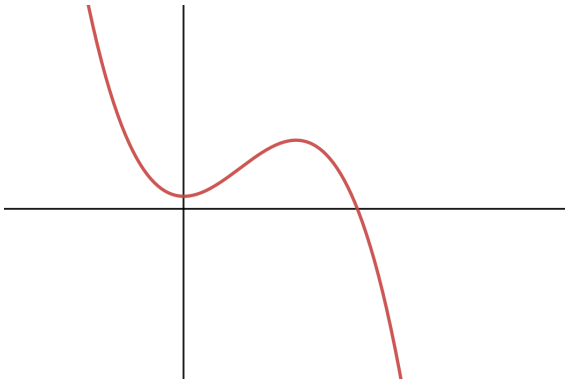

Figure 1: Accessible interactive graph at https://www.desmos.com/calculator/t8dz6vlmnz

Let S be the arc length and ΔS a short section of it.

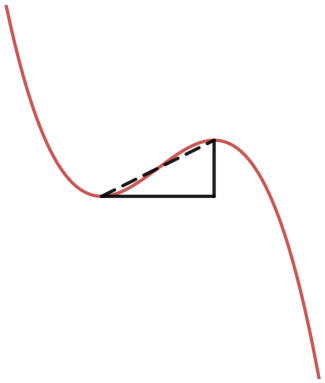

Figure 2: Accessible interactive graph at https://www.desmos.com/calculator/g5duc4kmfp

Derivation of Arc Length

By Pythagoras' Theorem,

$$\Delta S^2 \approx \Delta x^2 + \Delta y^2 \qquad \Rightarrow \qquad \left(\frac{\Delta S}{\Delta x}\right)^2 \approx 1 + \left(\frac{\Delta y}{\Delta x}\right)^2$$

As $\Delta x \to 0$ this becomes an identity

$$\left(\frac{dS}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2 \qquad \Rightarrow \qquad \frac{dS}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

The arclength between $x = a$ and $x = b$ is then

$$\begin{aligned} S(a,b) &= \int_a^b \frac{dS}{dx} dx \\ &= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx. \end{aligned}$$