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M161 6.1-6.6 Review

Solutions

1. $\int \frac{1}{x^2}\, dx =$

$$= \int x^{-2} dx$$

$$= -x^{-1} + C$$

$$= -\frac{1}{x} + C$$

2. $\int \sqrt[3]{x}\, dx =$

$$= \int x^{\frac{1}{3}} dx$$

$$= \frac{3}{4} x^{\frac{4}{3}} + C$$

3. $\int \frac{1}{x\sqrt{x}}\, dx =$

$$= \int \frac{1}{x\left(x^{\frac{1}{2}}\right)} dx$$

$$= \int \frac{1}{x^{\frac{3}{2}}} dx$$

$$= \int x^{-\frac{3}{2}} dx$$

$$= -2x^{-\frac{1}{2}} + C$$

$$= -\frac{2}{\sqrt{x}} + C$$

4. $\int \frac{1}{2x^3}\, dx =$

$$= \frac{1}{2} \int x^{-3} dx$$

$$= \frac{1}{2}\left[-\frac{1}{2} x^{-2} + C\right]$$

$$= -\frac{1}{4} x^{-2} + C$$

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$$= -\frac{1}{4x^2} + C$$

5. $\int (5 - x)\, dx =$

$$= 5x - \frac{1}{2}x^2 + C$$

6. $\int (4x^3 + 6x^2 - 1)\, dx =$

$$= \frac{4}{4}x^4 + \frac{6}{3}x^3 - x + C$$

$$= x^4 + 2x^3 - x + C$$

7. $\int (x^3 - 4x + 2)\, dx =$

$$= \frac{1}{4}x^4 - \frac{4}{2}x^2 + 2x + C$$

$$= \frac{1}{4}x^4 - 2x^2 + 2x + C$$

8. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)\, dx =$

$$= \int \left(x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right) dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}\left[2x^{\frac{1}{2}}\right] + C$$

$$= \frac{2}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} + C$$

$$= \frac{2}{3}\sqrt{x^3} + \sqrt{x} + C$$

9. $\int \frac{x^2+2x-3}{x^4}\, dx =$

$$= \int (x^{-2} + 2x^{-3} - 3x^{-4}) dx$$

$$= -x^{-1} + 2\left[-\frac{1}{2}x^{-2}\right] - 3\left[-\frac{1}{3}x^{-3}\right] + C$$

$$= -\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$$

10. $\int \sqrt{8x + 36}\, dx =$

$$= \int (8x + 36)^{\frac{1}{2}} dx$$

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$$u = 8x + 36;$$

$$du = 8dx; \frac{1}{8}du = dx$$

$$= \int u^{\frac{1}{2}} \left(\frac{1}{8}du\right)$$

$$= \frac{1}{8}\int u^{\frac{1}{2}} du$$

$$= \frac{1}{8}\left[\frac{2}{3}u^{\frac{3}{2}} + C\right]$$

$$= \frac{1}{12}u^{\frac{3}{2}} + C$$

$$= \frac{1}{12}(8x + 36)^{\frac{3}{2}} + C$$

11. $\int \frac{x}{x^2-4}\ dx =$

$$= \int \frac{1}{x^2 - 4}(xdx)$$

$$u = x^2 - 4$$

$$du = 2xdx; \frac{1}{2}du = dx$$

$$= \int \frac{1}{u}\left(\frac{1}{2}du\right)$$

$$= \frac{1}{2}\int \frac{1}{u}du$$

$$= \frac{1}{2}[\ln|u| + C]$$

$$= \frac{1}{2}\ln|x^2 - 4| + C$$

12. $\int \frac{3x}{2+6x^2}\ dx =$

$$= \int \frac{1}{2 + 6x^2}(3x)dx$$

$$u = 2 + 6x^2$$

$$du = 12xdx; \frac{1}{4}du = 3xdx$$

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$$= \int \frac{1}{u}\left(\frac{1}{4}du\right)$$

$$= \frac{1}{4}\int \frac{1}{u}du$$

$$= \frac{1}{4}[\ln|u| + C]$$

$$= \frac{1}{4}\ln|2 + 6x^2| + C$$

13. $\int \frac{1}{x(\ln x)^6}\ dx =$

$$= \int \frac{1}{(\ln x)^6}\left(\frac{1}{x}dx\right)$$

$$u = \ln x$$

$$du = \frac{1}{x}dx$$

$$= \int \frac{1}{u^6}du$$

$$= \int u^{-6}du$$

$$= -\frac{1}{5}u^{-5} + C$$

$$= -\frac{1}{5}(\ln x)^{-5} + C$$

14. $\int 8x^2e^{1-x^3}\ dx =$

$$= 8\int x^2e^{1-x^3}dx$$

$$u = 1 - x^3$$

$$du = -3x^2; -\frac{1}{3}du = x^2$$

$$= 8\int e^u\left(-\frac{1}{3}du\right)$$

$$= -\frac{8}{3}\int e^u du$$

$$= -\frac{8}{3}[e^u + C]$$

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$$= -\frac{8}{3}e^{1-x^3} + C$$

15. Find the general solution to $\frac{dy}{dx} = 5x - e^{3x}$.

$$dy = (5x - e^{3x})dx$$

$$\int 1dy = \int (5x - e^{3x})dx$$

$$y = 5\left[\frac{1}{2}x^2\right] - \frac{1}{3}[e^{3x}] + C$$

$$y = \frac{5}{2}x^2 - \frac{1}{3}e^{3x} + C$$

16. Find the specific solution to $\frac{dy}{dx} = 3x^2 + 2$ that passes through the point $(-1, 2)$.

$$dy = (3x^2 + 2)dx$$

$$\int 1dy = \int (3x^2 + 2)dx$$

$$y = x^3 + 2x + C$$

$$2 = (-1)^3 + 2(-1) + C$$

$$2 = -3 + C$$

$$C = 5$$

$$y = x^3 + 2x + 5$$

17. $\int_{-2}^{2} x^2\, dx =$

$$= \left[\frac{1}{3}x^3\right]_{-2}^{2}$$

$$= \frac{1}{3}[x^3]_{-2}^{2}$$

$$= \frac{1}{3}[(2^3) - (-2)^3]$$

$$= \frac{1}{3}[8 + 8]$$

$$= \frac{16}{3}$$

18. $\int_{0}^{1} (x^4 - 3x^3 + 1)\, dx =$

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$$= \left[\frac{1}{5}x^5 - 3\left[\frac{1}{4}x^4\right] + x\right]_0^1$$

$$= \left[\frac{1}{5}x^5 - \frac{3}{4}x^4 + x\right]_0^1$$

$$= \left[\left(\frac{1}{5} * 1^5 - \frac{3}{4} * 1^4 + 1\right) - \left(\frac{1}{5} * 0^5 - \frac{3}{4} * 0^4 + 0\right)\right]$$

$$= \frac{1}{5} - \frac{3}{4} + 1$$

$$= \frac{9}{20}$$

19. $\int_2^5 (2 + 2t + 3t^2)\, dt =$

$$= [2t + t^2 + t^3]_2^5$$

$$= [(2(5) + (5)^2 + (5)^3) - (2(2) + (2)^2 + (2)^3)]$$

$$= [(10 + 25 + 125) - (4 + 4 + 8)]$$

$$= 160 - 16$$

$$= 144$$

20. $\int_1^3 \left(1 + \frac{1}{x} + \frac{1}{x^2}\right)\, dx =$

$$= \int_1^3 \left(1 + \frac{1}{x} + x^{-2}\right) dx$$

$$= [x + \ln|x| - x^{-1}]_1^3$$

$$= \left[\left(3 + \ln 3 - \frac{1}{3}\right) - (1 + \ln 1 - 1)\right]$$

$$= \left(\frac{8}{3} + \ln 3\right) - (0)$$

$$= \frac{8}{3} + \ln 3$$

21. $\int_1^9 \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)\, dt =$

$$= \int_1^9 \left(t^{\frac{1}{2}} - t^{-\frac{1}{2}}\right) dt$$

$$= \left[\frac{2}{3}t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\right]_1^9$$

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$$= \left[\frac{2}{3}\left(\sqrt{t}\right)^3 - 2\sqrt{t}\right]_1^9$$

$$= \left[\left(\frac{2}{3}\left(\sqrt{9}\right)^3 - 2\sqrt{9}\right) - \left(\frac{2}{3}\left(\sqrt{1}\right)^3 - 2\sqrt{1}\right)\right]$$

$$= (18 - 6) - \left(\frac{2}{3} - 2\right)$$

$$= 12 - \left(-\frac{4}{3}\right)$$

$$= \frac{40}{3}$$

22. $\int_0^1 (t^3 + t)\sqrt{t^4 + 2t^2 + 1}\, dt =$

$$u = t^4 + 2t^2 + 1$$

$$du = (4t^3 + 4t)dt; du = 4(t^3 + t)dt; \frac{1}{4}du = (t^3 + t)dt$$

$$= \int_1^4 \sqrt{u}(4du)$$

$$= \frac{1}{4}\int_1^4 u^{\frac{1}{2}} du$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^4$$

$$= \frac{1}{6}\left[\left(\sqrt{u}\right)^3\right]_1^4$$

$$= \frac{1}{6}\left[\left(\sqrt{4}\right)^3 - \left(\sqrt{1}\right)^3\right]$$

$$= \frac{1}{6}[8 - 1]$$

$$= \frac{7}{6}$$

23. $\int_1^4 \frac{2}{x}\, dx =$

$$= 2\int_1^4 \frac{1}{x} dx$$

$$= 2[\ln|x|]_1^4$$

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$$= 2[(\ln 4) - (\ln 1)]$$

$$= 2\ln 4$$

24. $\int_{-1}^{0}(3x^5 - 3x^2 + 2x - 1)\, dx =$

$$= \left[\frac{1}{2}x^6 - x^3 + x^2 - x\right]_{-1}^{0}$$

$$= \left[\left(\frac{1}{2}(0)^6 - (0)^3 + (0)^2 - 0\right) - \left(\frac{1}{2}(-1)^6 - (-1)^3 + (-1)^2 - (-1)\right)\right]$$

$$= [(0) - \left(\frac{1}{2} + 1 + 1 + 1\right)$$

$$= -\frac{7}{2}$$

25. $\int_{0}^{4}(x-2)^2\, dx =$

$$u = x - 2$$

$$du = dx$$

$$= \int_{-2}^{2} u^2 du$$

$$= \left[\frac{1}{3}u^3\right]_{-2}^{2}$$

$$= \frac{1}{3}[(2)^3 - (-2)^3]$$

$$= \frac{1}{3}(8+8)$$

$$= \frac{16}{3}$$

26. $\int_{0}^{6} x^2(x-1)\, dx =$

$$= \int_{0}^{6}(x^3 - x^2)dx$$

$$= \left[\frac{1}{4}x^4 - \frac{1}{3}x^3\right]_{0}^{6}$$

$$= \left[\left(\frac{1}{4}(6)^4 - \frac{1}{3}(6)^3\right) - \left(\frac{1}{4}(0)^4 - \frac{1}{3}(0)^3\right)\right]$$

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$$= 6^3\left(\frac{1}{4}(6) - \frac{1}{3}\right)$$

$$= 6^3\left(\frac{7}{6}\right)$$

$$= 6^2(7)$$

$$= 252$$

27. $\int_{-3}^{0}(2x+6)^4\,dx =$

$$u = 2x + 6$$

$$du = 2dx; \frac{1}{2}du = dx$$

$$= \int_0^6 u^4\left(\frac{1}{2}du\right)$$

$$= \frac{1}{2}\int_0^6 u^4 du$$

$$= \frac{1}{2}\left[\frac{1}{5}u^5\right]_0^6$$

$$= \frac{1}{10}[6^5 - 6^0]$$

$$= \frac{1}{10}[7776 - 1]$$

$$= 777.5$$

28. $\int_{1}^{2}(2x-4)^5\,dx =$

$$u = 2x - 4$$

$$du = 2dx; \frac{1}{2}du = dx$$

$$= \int_{-2}^{0} u^5\left(\frac{1}{2}du\right)$$

$$= \frac{1}{2}\int_{-2}^{0} u^5 du$$

$$= \frac{1}{2}\left[\frac{1}{6}u^6\right]_{-2}^{0}$$

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$$= \frac{1}{12}[0^6 - (-2)^6]$$

$$= \frac{1}{12}[-64]$$

$$= -\frac{16}{3}$$

29. $\int_1^2 \frac{x^2}{(x^3+1)^2}\ dx =$

$$= \int_1^2 \frac{1}{(x^3+1)^2}(x^2 dx)$$

$$u = x^3 + 1$$

$$du = 3x^2 dx; \frac{1}{3} du = x^2 dx$$

$$= \int_2^9 \frac{1}{u^2}\left(\frac{1}{3} du\right)$$

$$= \frac{1}{3}\int_2^9 u^{-2} du$$

$$= \frac{1}{3}\left[-\frac{1}{u}\right]_2^9$$

$$= \frac{1}{3}\left[\left(-\frac{1}{9}\right) - \left(-\frac{1}{2}\right)\right]$$

$$= \frac{1}{3}\left(-\frac{1}{9} + \frac{1}{2}\right)$$

$$= \frac{1}{3}\left(\frac{7}{18}\right)$$

$$= \frac{7}{54}$$

30. $\int_0^4 \frac{1}{\sqrt{6t+1}}\ dt =$

$$u = 6t + 1$$

$$du = 6dt; \frac{1}{6} du = dt$$

$$= \int_1^{25} \frac{1}{\sqrt{u}}\left(\frac{1}{6} du\right)$$

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$$= \frac{1}{6}\int_{1}^{25} u^{-\frac{1}{2}} du$$

$$= \frac{1}{6}\left[2u^{\frac{1}{2}}\right]_{1}^{25}$$

$$= \frac{1}{3}\left[\sqrt{u}\right]_{1}^{25}$$

$$= \frac{1}{3}\left[\sqrt{25} - \sqrt{1}\right]$$

$$= \frac{1}{3}(4)$$

$$= \frac{4}{3}$$

31. $\int_{e}^{e^2} \frac{1}{x \ln x}\, dx =$

$$= \int_{e}^{e^2} \frac{1}{\ln x}\left(\frac{1}{x} dx\right)$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$= \int_{1}^{2} \frac{1}{u} du$$

$$= [\ln|u|]_{1}^{2}$$

$$= [\ln 2 - \ln 1]$$

$$= \ln 2$$

32. $\int_{\ln\frac{1}{2}}^{2} e^t\, dt =$

$$= [e^t]_{\ln\frac{1}{2}}^{2}$$

$$= \left[e^2 - e^{\ln\frac{1}{2}}\right]$$

$$= e^2 - \frac{1}{2}$$

33. $\int_{\ln 5}^{5} e^x\, dx =$

$$= [e^x]_{\ln 5}^{5}$$

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$$= \left[e^{5} - e^{\ln 5}\right]$$

$$= e^{5} - 5$$

34. $\int_0^2 \sqrt{2x+1}\, dx =$

$$u = 2x + 1$$

$$du = 2dx; \frac{1}{2} du = dx$$

$$= \int_1^5 \sqrt{u} \left(\frac{1}{2} du\right)$$

$$= \frac{1}{2} \int_1^5 u^{\frac{1}{2}} du$$

$$= \frac{1}{2} \left[\frac{2}{3} u^{\frac{3}{2}}\right]_1^5$$

$$= \frac{1}{3} \left[\left(\sqrt{u}\right)^3\right]_1^5$$

$$= \frac{1}{3} \left[\left(\sqrt{5}\right)^3 - \left(\sqrt{1}\right)^3\right]$$

$$= \frac{1}{3} \left[\sqrt{5}^3 - 1\right]$$

$$= \frac{1}{3} \left[5\sqrt{5} - 1\right]$$

35. $\int_0^1 e^{-4x}\, dx =$

$$u = -4x$$

$$du = -4dx; \; -\frac{1}{4} du = dx$$

$$= \int_0^{-4} e^u \left(-\frac{1}{4} du\right)$$

$$= -\int_{-4}^{0} e^u \left(-\frac{1}{4}\right) du$$

$$= \frac{1}{4} \int_{-4}^{0} e^u du$$

$$= \frac{1}{4} [e^u]_{-4}^{0}$$

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$$=\frac{1}{4}[e^0-e^{-4}]$$

$$=\frac{1}{4}\left[1-\frac{1}{e^4}\right]$$

36. The average value of $y=e^{3x}$ over the interval $[0,4]$ is…

$$\frac{1}{4-0}\int_0^4 e^{3x}\,dx$$

$$u=3x$$

$$du=3dx;\frac{1}{3}du=dx$$

$$=\frac{1}{4}\int_0^{12} e^u\left(\frac{1}{3}du\right)$$

$$=\frac{1}{12}[e^u]_0^{12}$$

$$=\frac{1}{12}[e^{12}-e^0]$$

$$=\frac{1}{12}[e^{12}-1]$$

37. The number of wolves in Yellowstone is given by $w(t)=t^2+3t+2$, where t is the number of years since 2000. What is the average number of wolves in Yellowstone between 2000 and 2005?

$$\frac{1}{5-0}\int_0^5 (t^2+3t+2)dt$$

$$=\frac{1}{5}\left[\frac{1}{3}t^3+\frac{3}{2}t^2+2t\right]_0^5$$

$$=\frac{1}{5}\left[\left(\frac{1}{3}(5)^3+\frac{3}{2}(5)^2+2(5)\right)-\left(\frac{1}{3}(0)^3+\frac{3}{2}(0)^2+2(0)\right)\right]$$

$$=\frac{1}{5}\left[\frac{125}{3}+\frac{75}{2}+10\right]$$

$$=\frac{107}{6}$$

38. If $\int_0^2(3x^3-kx^2+2k)\ dx=12$, then k must be…

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$$\left[\frac{3}{4}x^4 - \frac{k}{3}x^3 + 2kx\right]_0^2 = 12$$

$$\left[\left(\frac{3}{4}(2)^4 - \frac{k}{3}(2)^3 + 2k(2)\right) - \left(\frac{3}{4}(0)^4 - \frac{k}{3}(0)^3 + 2k(0)\right)\right] = 12$$

$$12 - \frac{8k}{3} + 4k = 12$$

$$-\frac{8k}{3} + 4k = 0$$

$$-8k + 12k = 0$$

$$k = 0$$

39. A study indicates that x months from now the population of a certain town will be increasing at the rate of $5 + 3x^{\frac{2}{3}}$ people per month. By how much will the population of the town increase over the next 8 months?

$$\int_0^8 \left(5 + 3x^{\frac{2}{3}}\right) dx$$

$$= \left[5x + 3\left(\frac{3}{5}x^{\frac{5}{3}}\right)\right]_0^8$$

$$= \left[5x + \frac{9}{5}\left(\sqrt[3]{x}\right)^5\right]_0^8$$

$$= \left[\left(5(8) + \frac{9}{5}\left(\sqrt[3]{8}\right)^5\right) - \left(5(0) + \frac{9}{5}\left(\sqrt[3]{0}\right)^5\right)\right]$$

$$= 40 + \frac{9}{5}(32)$$

$$= \frac{488}{5}$$

$$\approx 98 \text{ people}$$

40. An object is moving so that its speed after t minutes is $5 + 2t + 3t^2$ meters per minute. How far does the object travel during the first two minutes?

$$\int_0^2 (5 + 2t + 3t^2)dt$$

$$= [5t + t^2 + t^3]_0^2$$

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$$= [(5(2) + (2)^2 + (2)^3) - (5(0) + (0)^2 + (0)^3]$$

$$= 10 + 4 + 8$$

$$= 22 \text{ meters}$$