Class 8 Homework August 12, 2023 Name_____

1. There are 11 real numbers a1, a2, to a 11, and 0 <= ai (I = 1, …, 11) <= 1. Please prove at least the absolute difference of two numbers would be less than 1/10.

   Answer: If we create ten buckets, first one for number(s) 0 <= ai < 0.1, second one for 0.1 <= ai < 0.2, …, 0.9 <= ai <= 1, then we put each one of the 11 real numbers into corresponding bucket matching to it's value. Since there are 10 buckets, and there are 11 numbers, there must be two numbers in one of the ten buckets. Because the differences of the numbers in any bucket are less than or equal to 1/10, then the proof.

2. There are 10 natural numbers from 1 to 10. Please prove: if selecting any 6 numbers from the 10 numbers, there must be 2 numbers of the 6 numbers, one number is the other number's multiple.

   If we group the 10 numbers to 5 sets:

   A1 = {1, 1 x 2, 1 x 2^2, 1 x 2^3}

   A2 = {3, 3 x 2}

   A3 = {5, 5 x 2}

   A4 = {7}

   A5 = {9}

   Thus if we pick 6 numbers, there must be two numbers from any one of the sets A1, A2, A3, and the smaller numbers in these 3 sets are the multiples of the bigger numbers.

3. How many lines can you draw using 5 non collinear (not in a single line) points A, B，C, D and E on a plane?

4. How many 4 digit numbers can you make using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 without repeating the digits?

5. We need to form a team of 5 students in a class of 20 students. How many different teams can be formed?

6. In how many ways can you arrange 7 different books on a shelf?

7. How many triangles can you make using 6 non collinear points on a plane?

8. In a certain country, the car number plate is formed by 3 digits from the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 followed by 3 letters from the alphabet. How many number plates can be formed if neither the digits nor the letters are repeated?

9. A committee including 5 boys and 4 girls is to be formed from a group of 10 boys and 12 girls. How many different committees can be formed from the group?

10. Out of 8 consonants and 5 vowels, how many words of 3 consonants and 2 vowels can be formed?

11. A coin is tossed 5 times. Find out the number of possible outcomes.

When a coin is tossed once, there are two possible outcomes: Head (H) and Tail (T). Hence, when a coin is tossed 5 times, the number of possible outcomes $=2 \times 2 \times 2 \times 2 \times 2=32$. (The possible outcomes are HHHHH, HHHHT, HHHTT, HHTTT, HTTTT, HHHTH, …… TTTTT)