Interest Word Problems
and
The Power of Compounding

Interest Problems are word problems that use formula for calculating Interests as well as future values:

- **Simple Interest and future value**
- **Compound Interest and future value**
- **Continuously Compounded Interest and future value**

Interest Formulas

Simple Interest Formula

$$I = Prt$$

I = Interest
P = Principal (Initial Value)
r = Interest Rate
t = time (years)

Continuously Compounded Interest

$$A = Pe^{rt}$$

A = Future Value
P = Principal (Initial Value)
r = Interest Rate
t = time

Compound Interest Formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = Future Value
P = Principal (Initial Value)
r = Interest Rate
n = number of times compounded in one "t"
t = time

- **Simple interest is applied only to the principal and not any accumulated interest.**
- **Compound interest is interest accruing on the principal and previously applied interest.**
- **The effect of compound interest depends on how frequently it is applied.**
- **For bonds, the bond equivalent yield is the expected annual return.**
- **Continuously compounding returns scale over multiple periods.**
- **Interest compounding at its highest frequency is said to be compounding continuously.**

Simple Interest and Future Value

Simple Interest Problem Example:

John wants to have an interest income of $3,000 a year. How much must he invest for one year at 8%?

Simple Interest Problem Example:

John wants to have an interest income of $3,000 a year. How much must he invest for one year at 8%?

Step 1: Write down the formula

$I = prt$

Step 2: Plug in the values

$3000 = p \times 0.08 \times 1$

$3000 = 0.08p$

$p = 37{,}500$

Answer: He must invest $37,500

Compound Interest and Future Value

Current Deposit Rates for 08/26/2023 - 09/01/2023

Savings Accounts:

Annual Percentage Yields (APYs) and Interest Rates shown are offered on accounts accepted by the Bank and effective for the dates shown above, unless otherwise noted. Interest Rates are subject to change without notice. Interest is compounded daily and paid monthly. Interest is calculated and accrued daily based on the daily collected balances in the account. Accrued interest is considered to be earned and will be paid only when the total interest accrued reaches $0.01 or more.

From https://www.wellsfargo.com/savings-cds/rates/

Compound Interest Problem Example:

Scott wants to invest $1000 for 1 year. At Bank A, his investment will collect 3% interest compounded daily while at Bank B, his investment will collect 3.50% interest compounded monthly. Which bank offers a better return? How much more will he receive by choosing that bank over the other?

Compound Interest Problem Example:

Scott wants to invest $1000 for 1 year. At Bank A, his investment will collect 3% interest compounded daily while at Bank B, his investment will collect 3.50% interest compounded monthly. Which bank offers a better return? How much more will he receive by choosing that bank over the other?

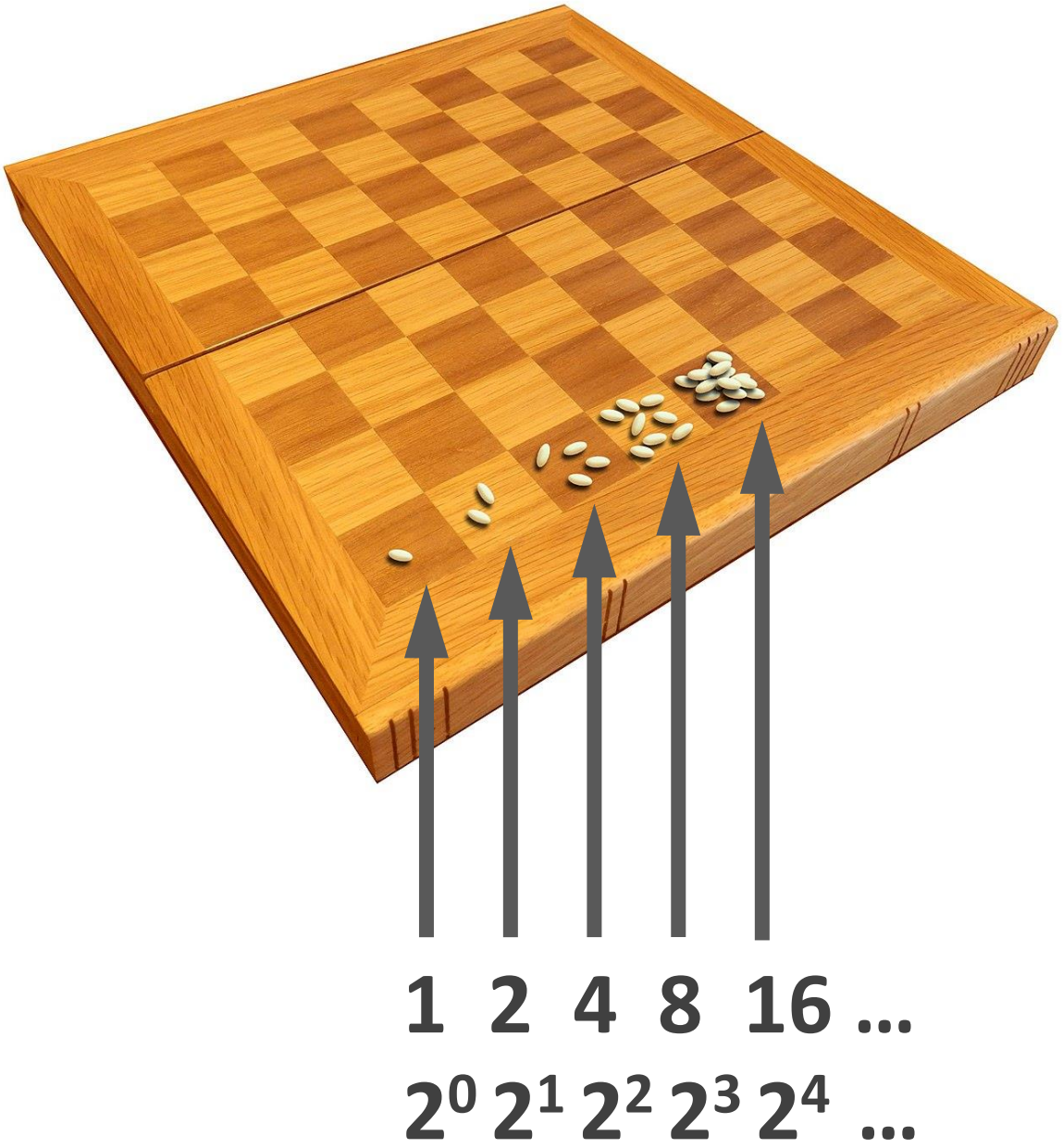
Bank B, $5.12

With compounding, how many years would it take for your money to double?

The inventor of chess (in some tellings Sessa, an ancient Indian Minister, year 1256) request his ruler give him wheat according to the wheat and chessboard problem. The ruler laughs it off as a meager prize for a brilliant invention, only to have court treasurers report the unexpectedly huge number of wheat grains would outstrip the ruler's resources.

The Wheat and Chess Board Story

The last grid alone holds 461 Billion Tons of wheat, which is over 590x world production



Would you rather have a million dollars or a penny on day 1, doubled every day until day 30?

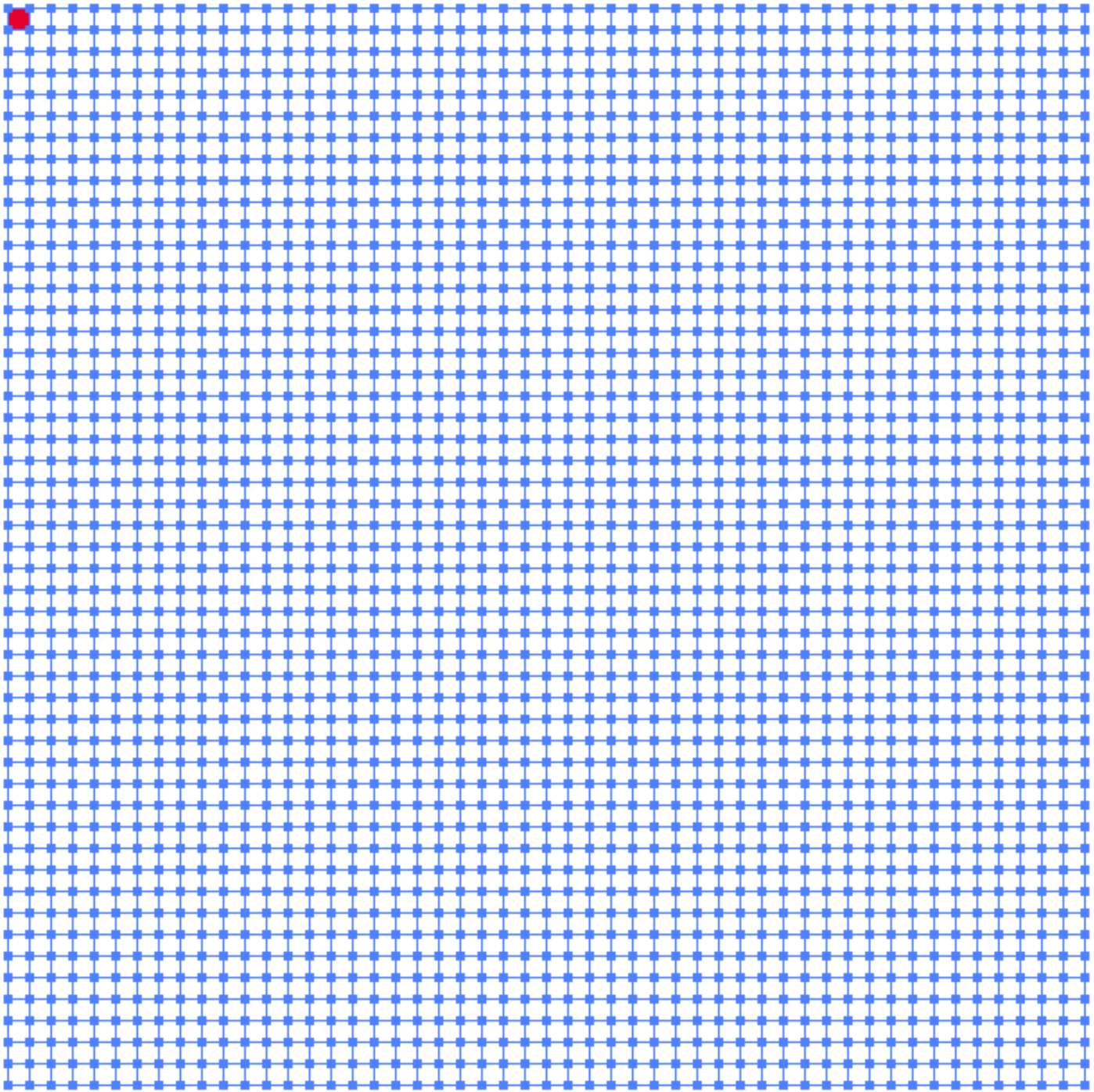
$2^{30}-1=1{,}073{,}741{,}823$ Cents

Or: over 10 million dollars

A blue pond 50x50, a red plant that doubles every day

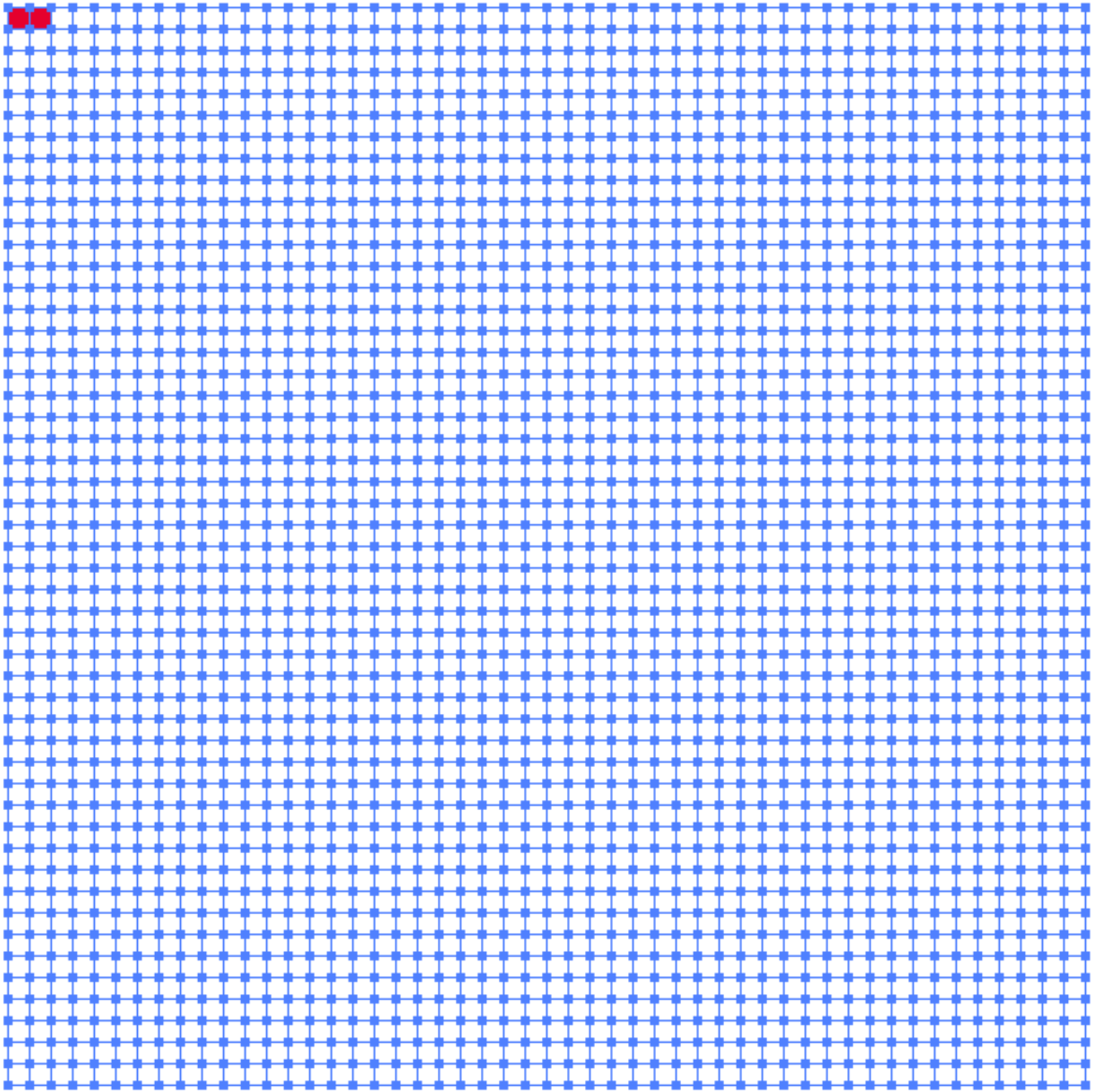
Day 0

2^0



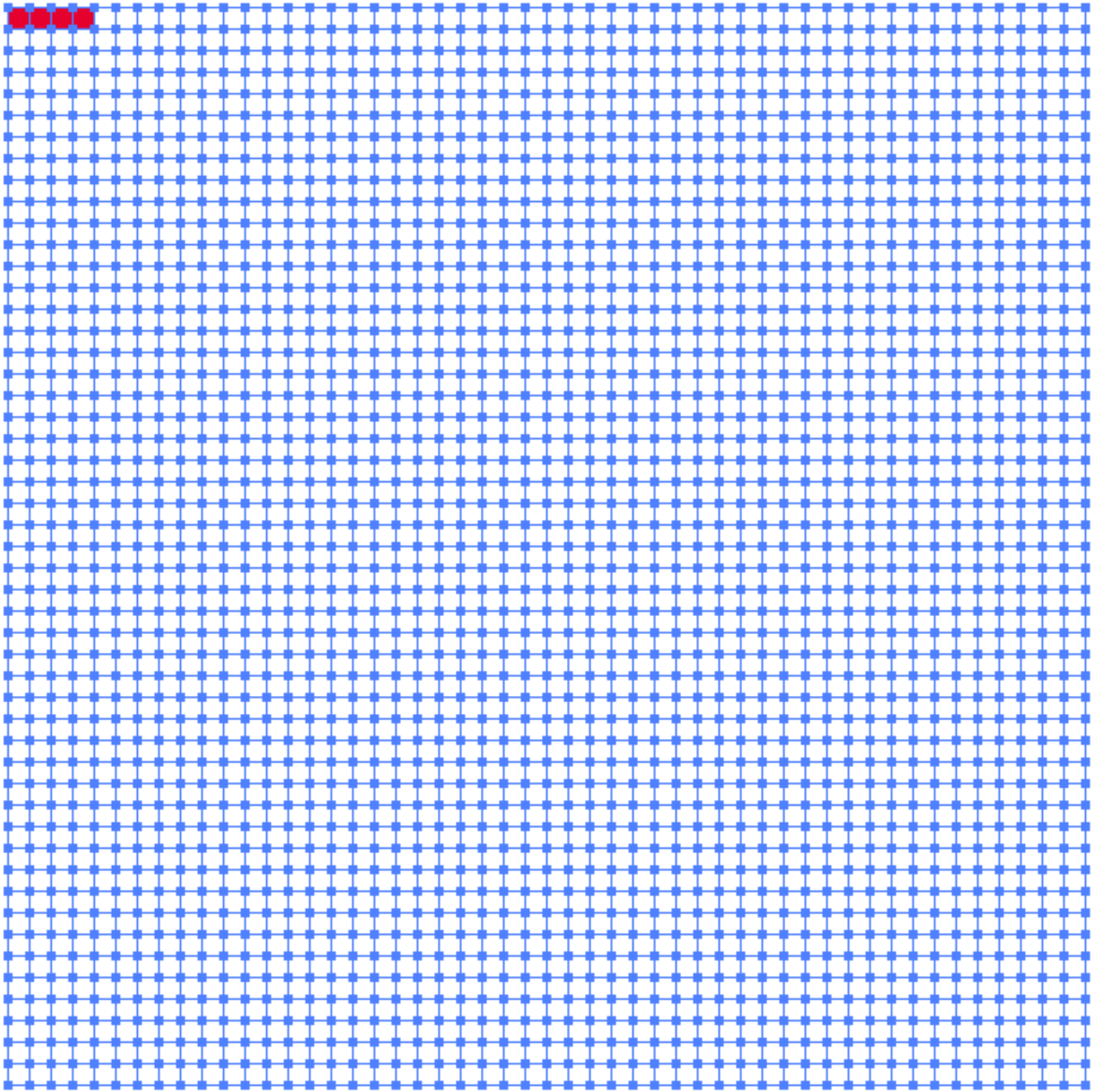
Day 1

2^1



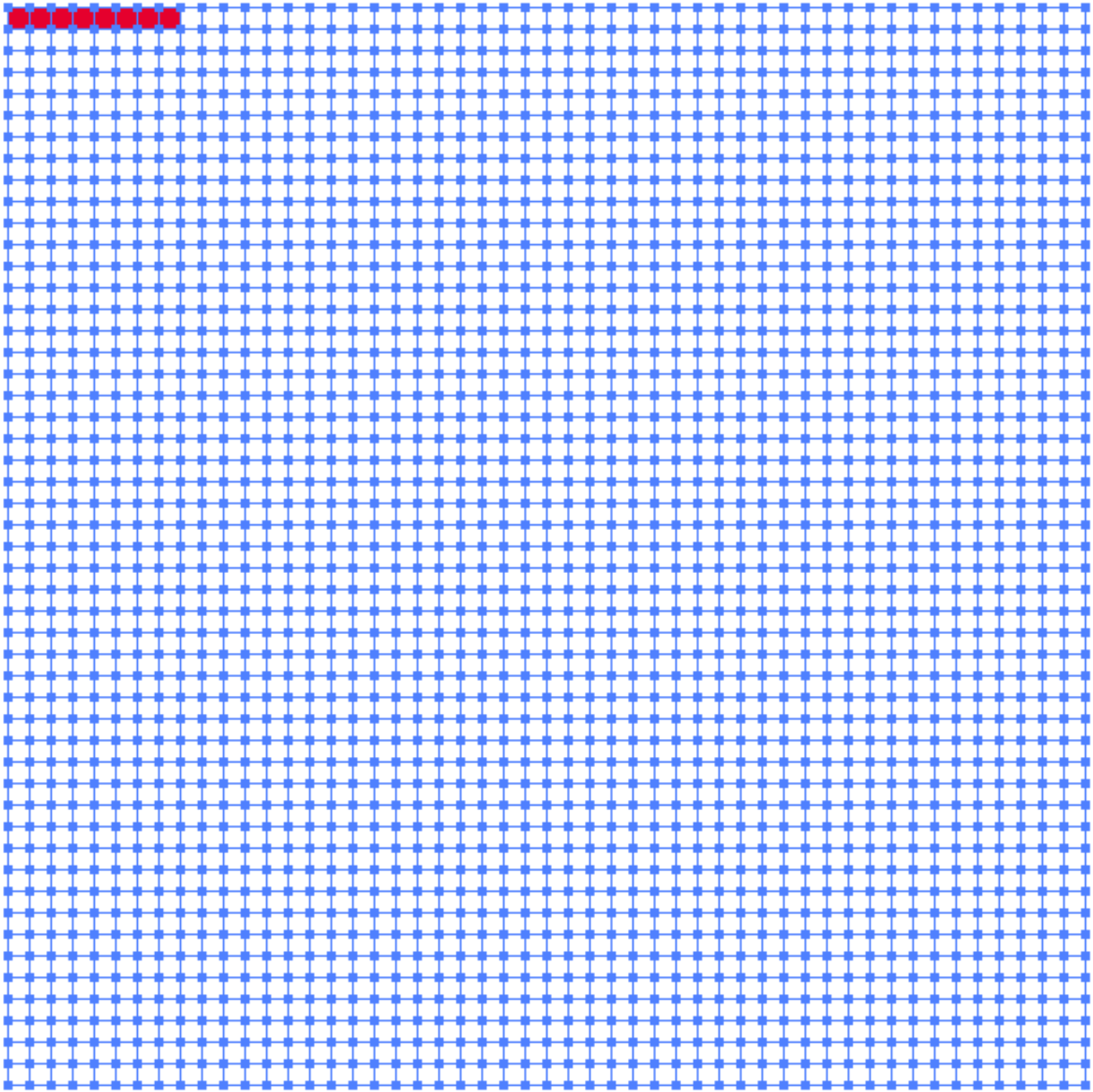
Day 2

2^2



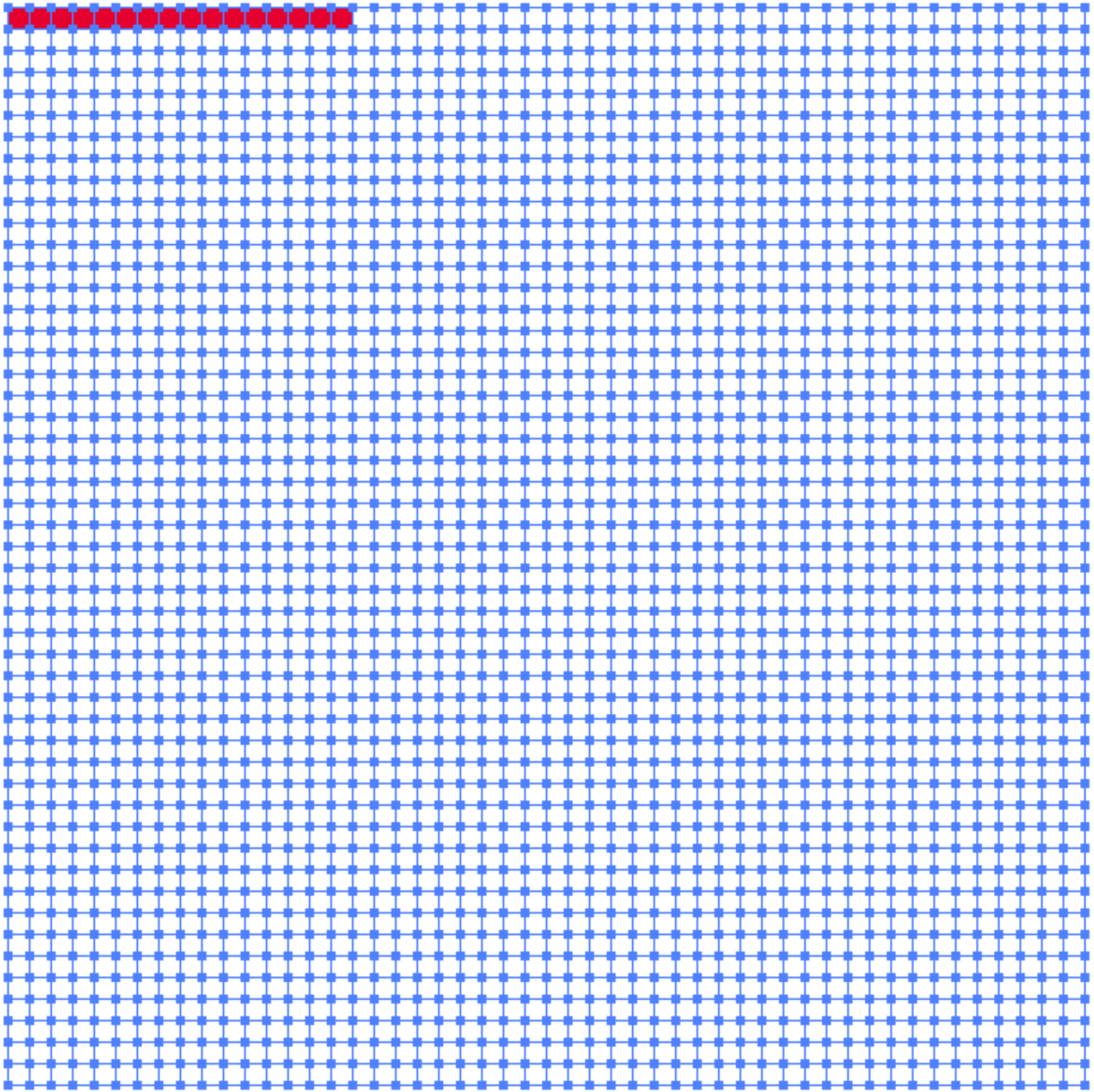
Day 3

2^3



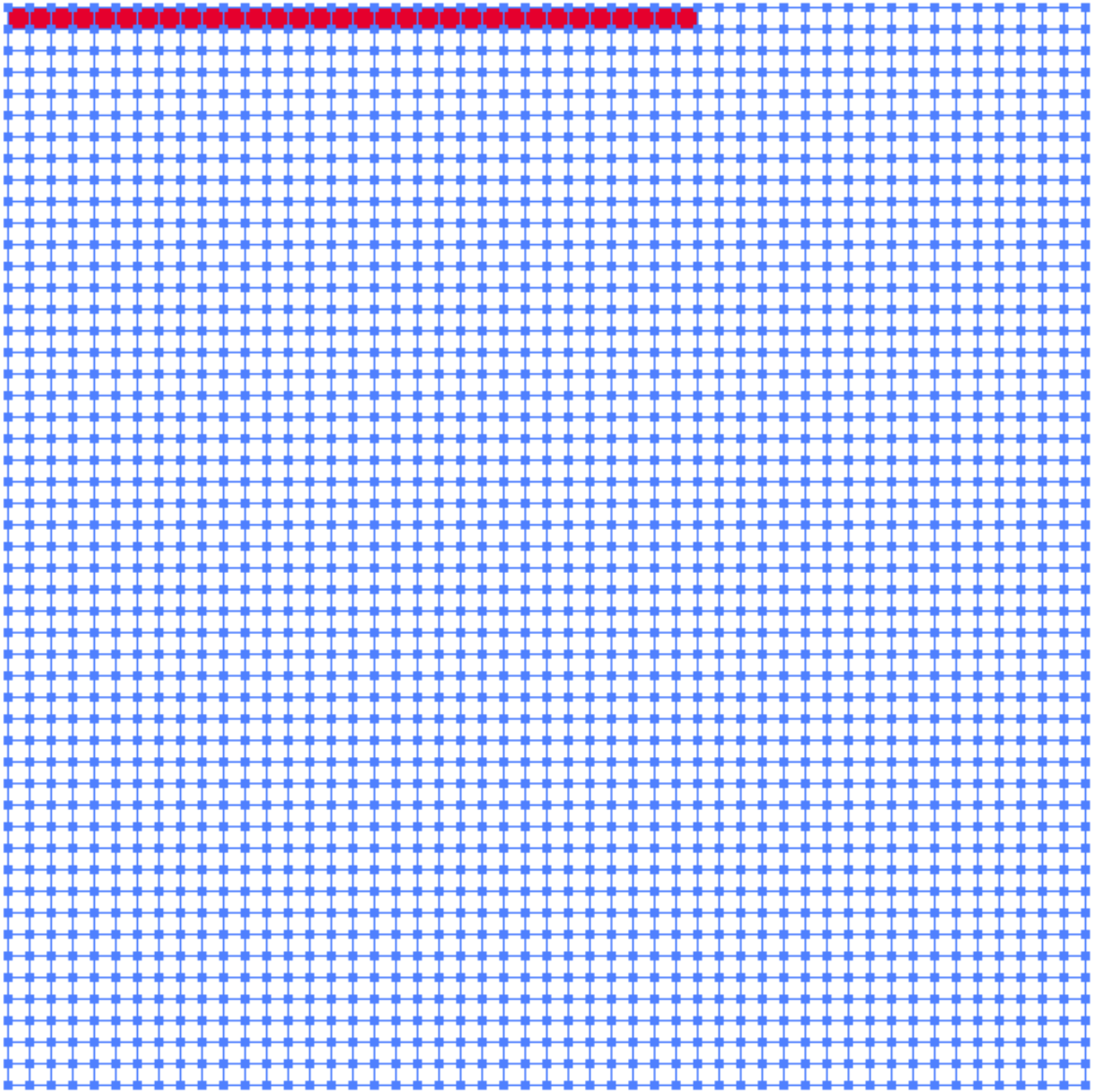
Day 4

2^4



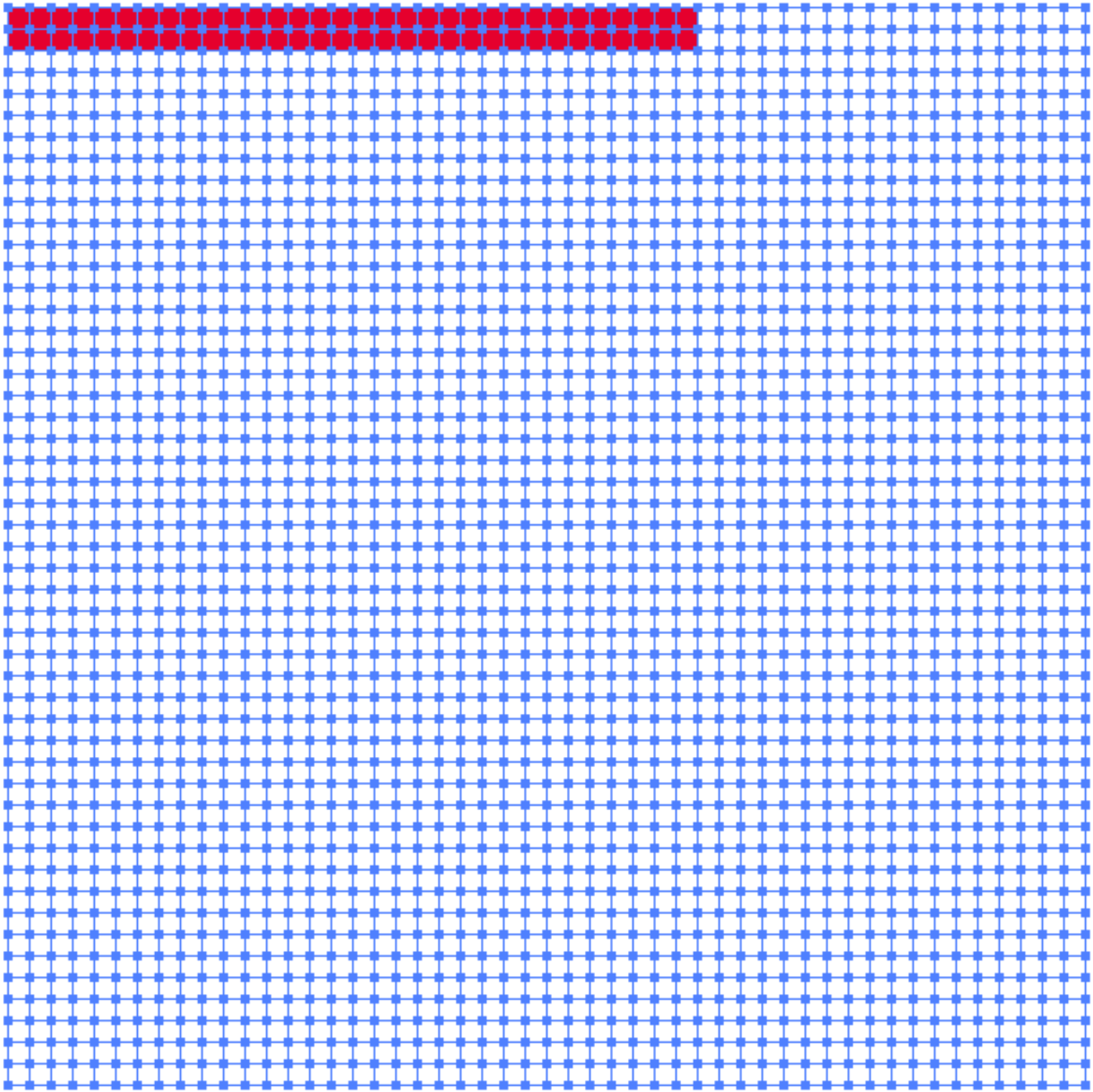
Day 5

2^5



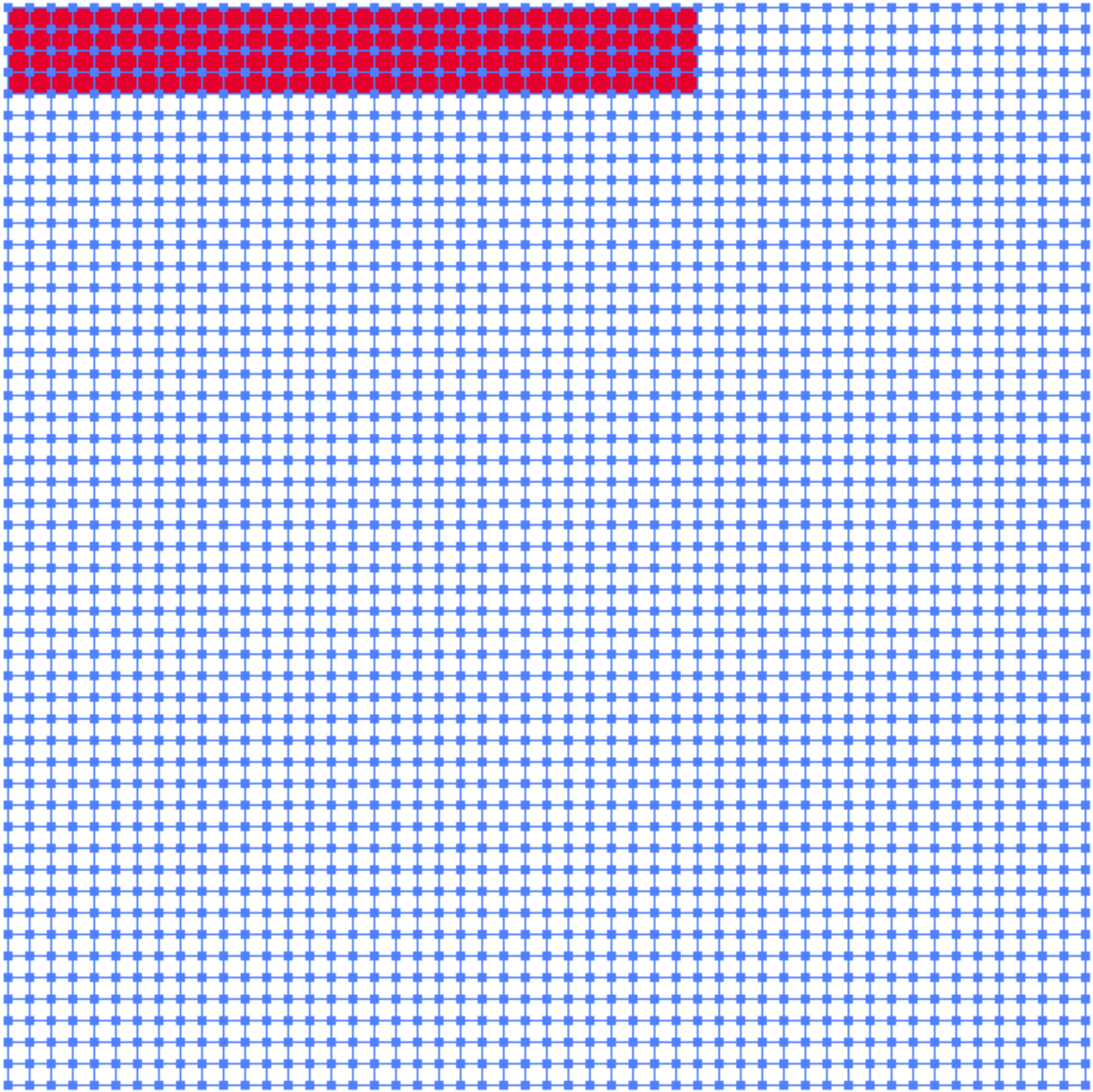
Day 6

2^6



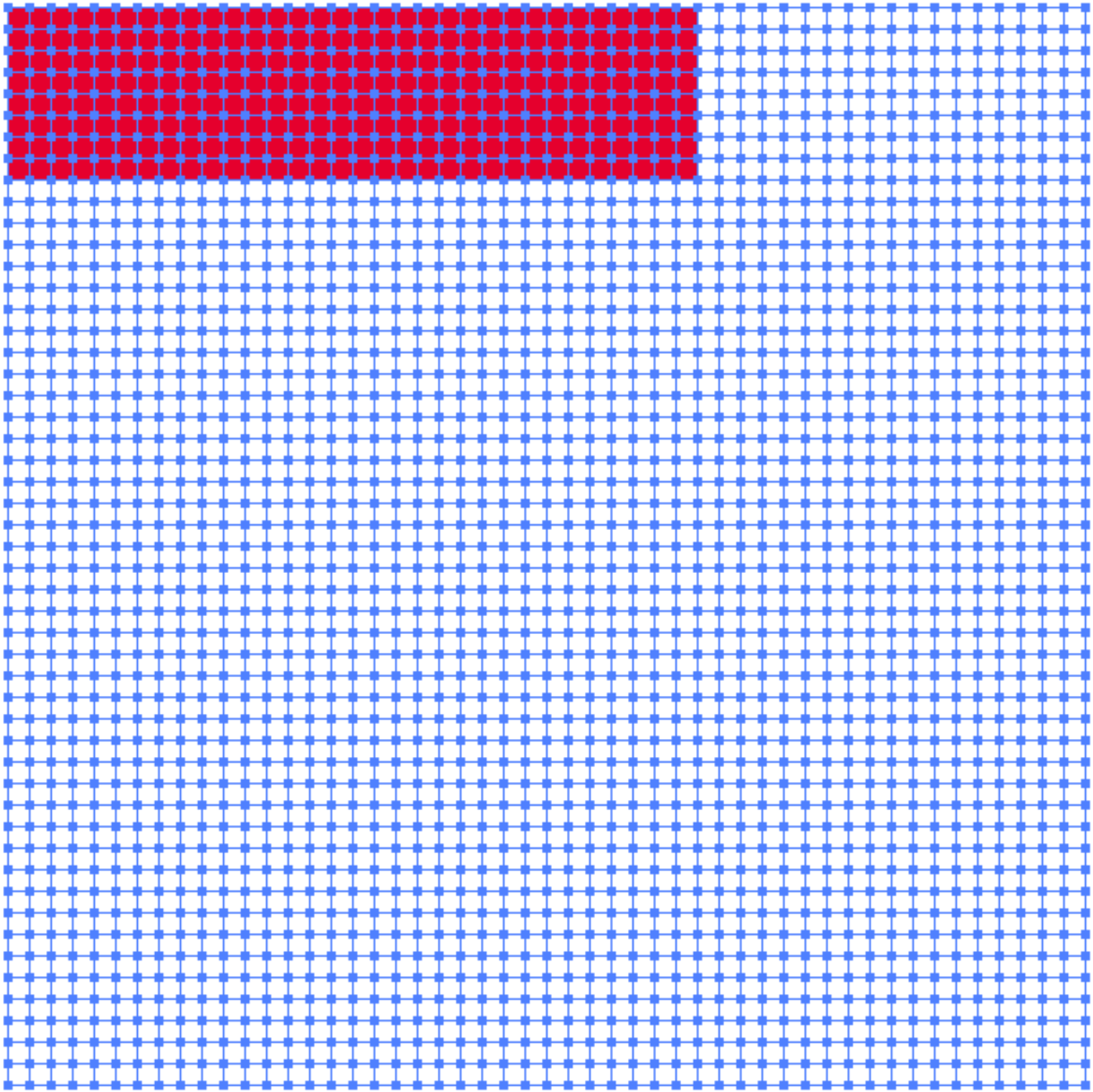
Day 7

2^7



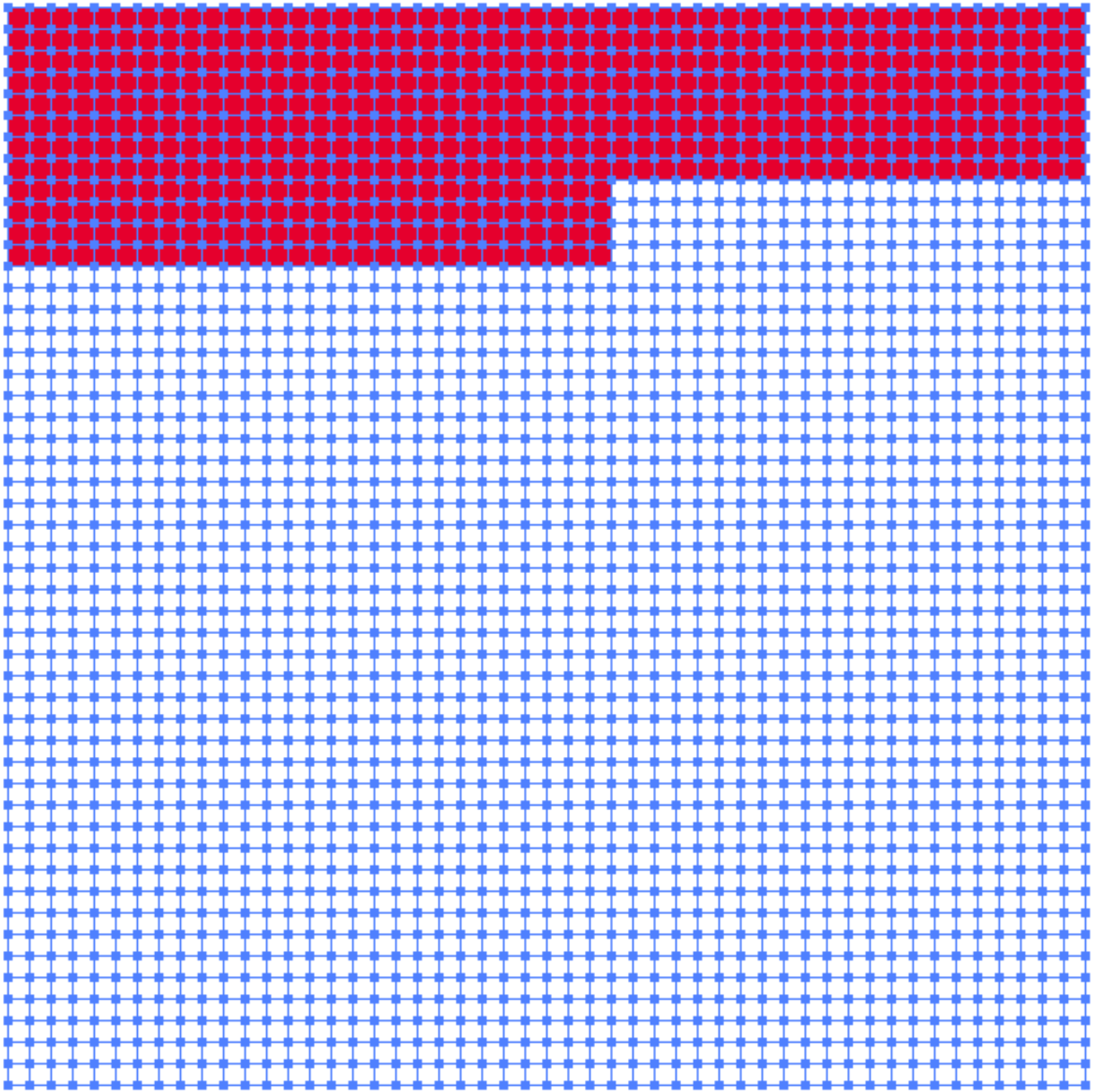
Day 8

2^8



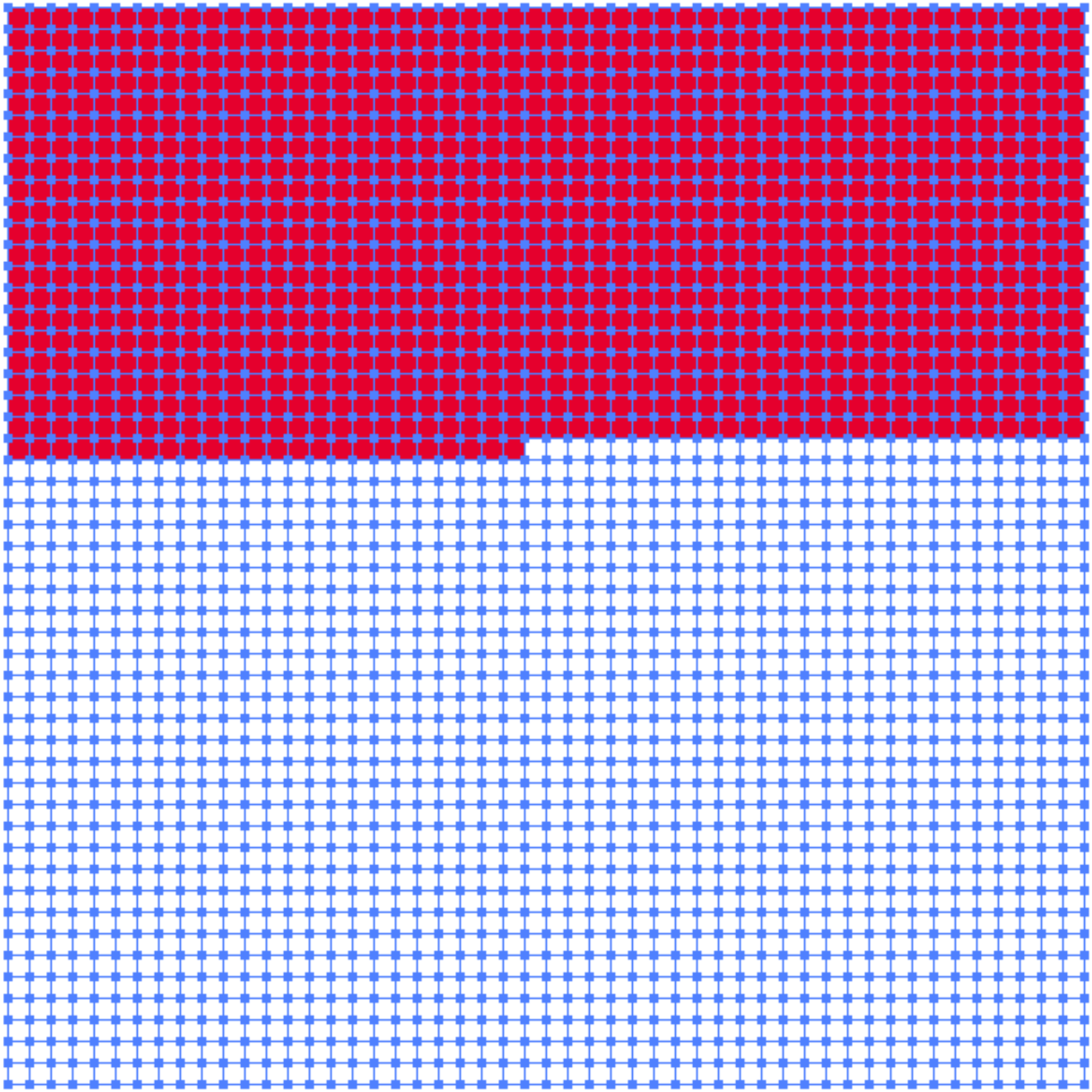
Day 9

2^9



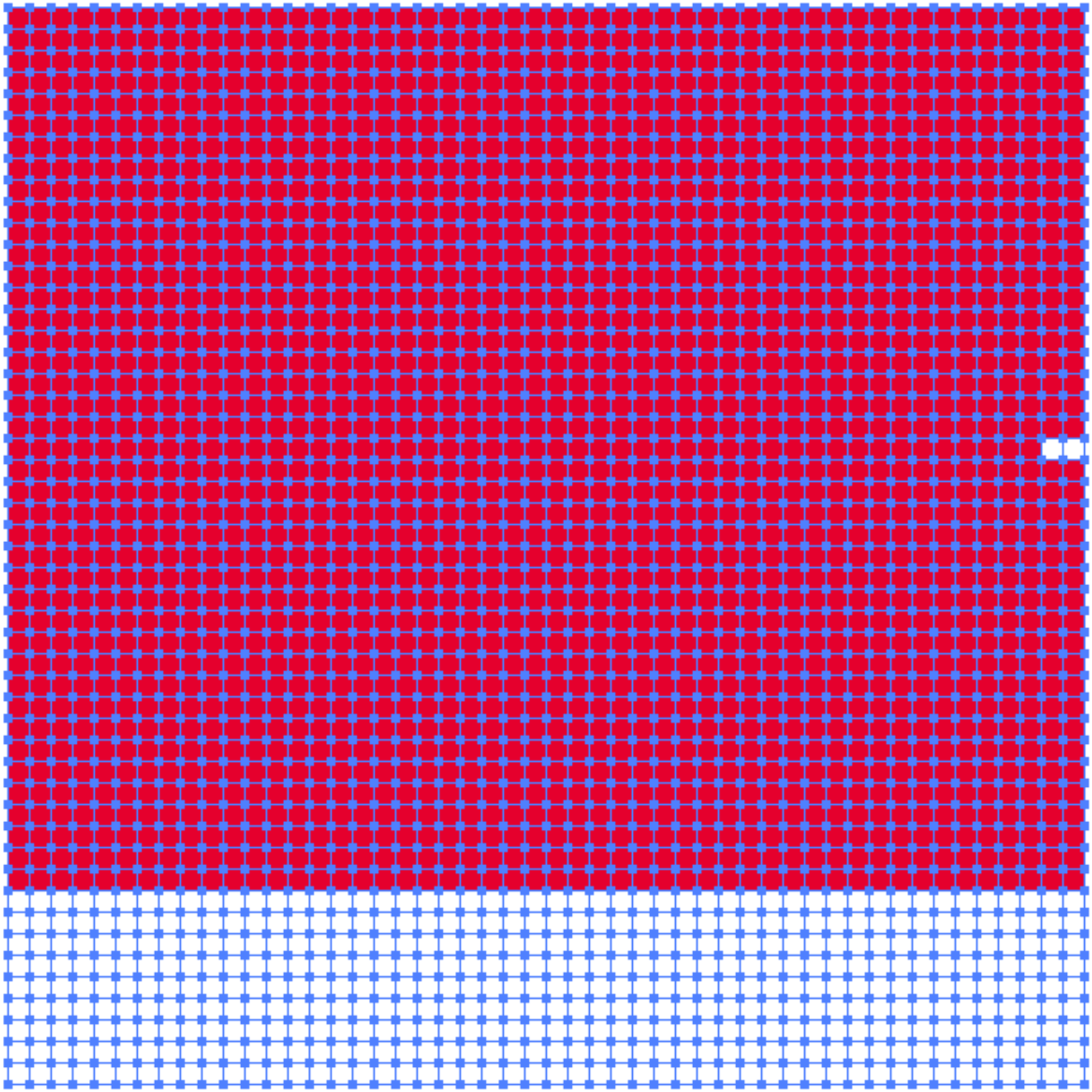
Day 10

2^{10}



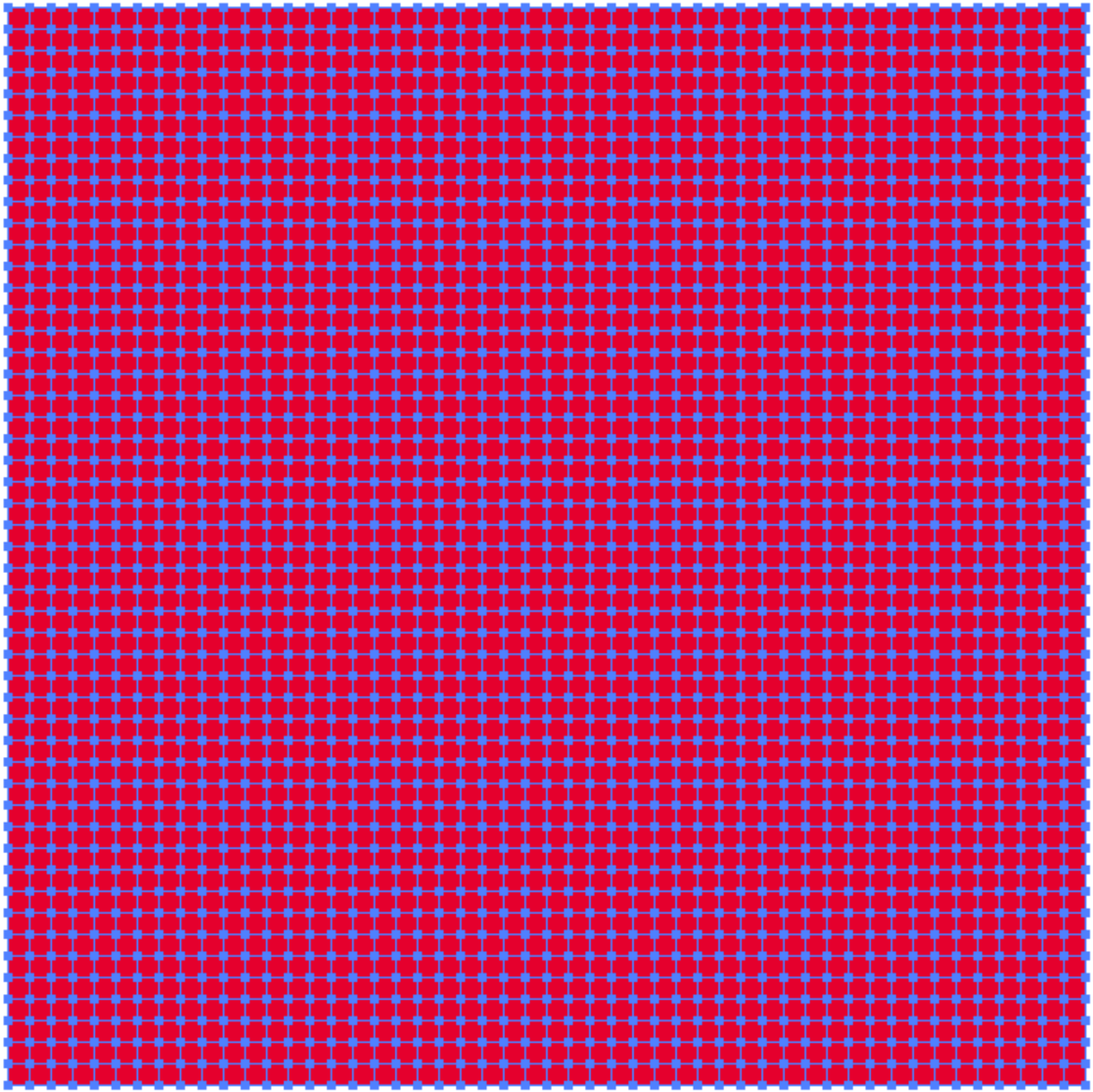
Day 11

2^{11}



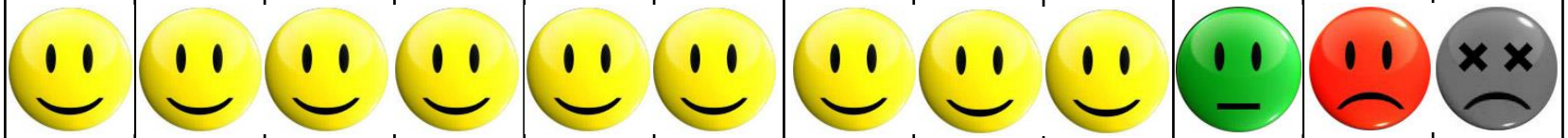
Day 11.3

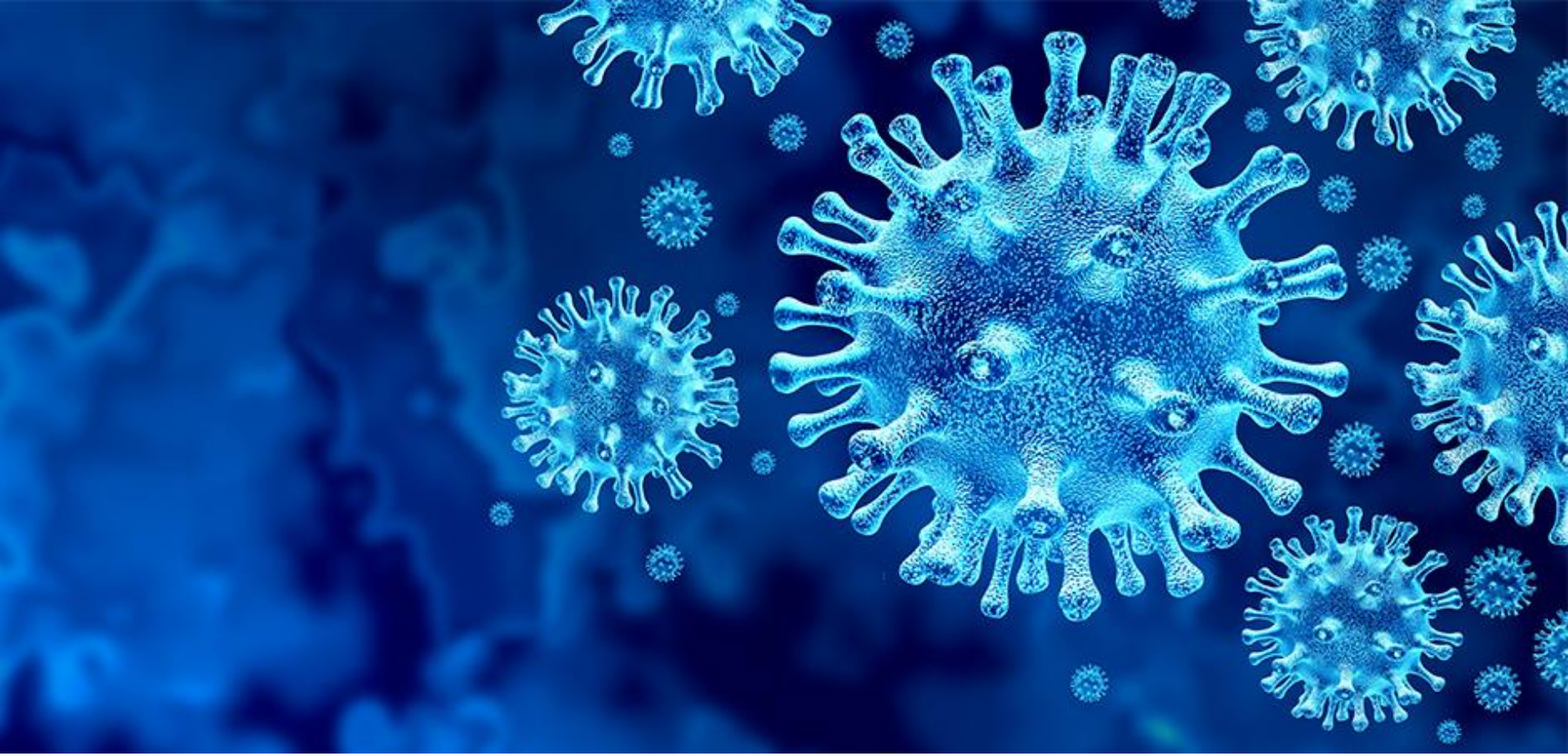
$2^{11.3}$



What does a fish feel in the pond?

- The fish is totally happy: < 25% covered
- Slightly concerned: > 25% covered
- Seriously concerned: > 50% covered
- Dead 100% covered

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10	Day 11	Day 11 7 AM
											



An example of compounding working against us: COVID-19

Exercise: Folding papers

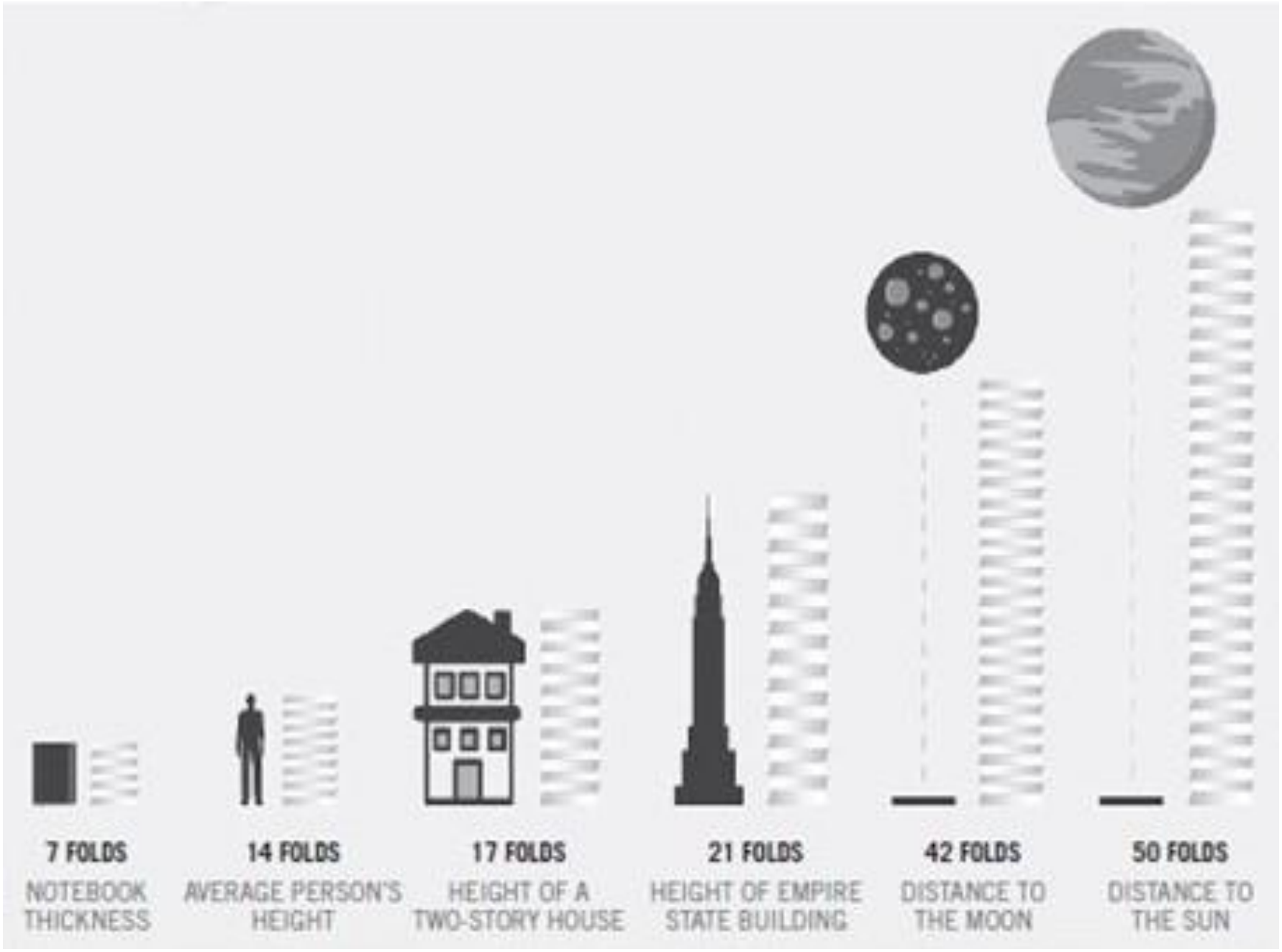
The thickness of a standard printing paper is about 0.1mm. Fold it once and it will be 0.2mm thick. If fold it again, it will be 0.4mm thick. Keeping folding it:

Times of Folding	The thickness of the Folded Paper
0	
1	
2	
7	
14	
21	
42	
50	

Exercise: Folding papers

The thickness of a standard printing paper is about 0.1mm. Fold it once and it will be 0.2mm thick. If fold it again, it will be 0.4mm thick. Keeping folding it:

Times of Folding	The thickness of the Folded Paper	2^x
0	0.1mm	0.1×2^0
1	0.2mm	0.1×2^1
2	0.4mm	0.1×2^2
7	128mm	0.1×2^7
14	1638.4mm = 1.63m	0.1×2^{14}
21	209715.2mm = 2,097m	0.1×2^{21}
42	4.4×10^{10}m	0.1×2^{42}
50	1.1×10^{13}m	0.1×2^{50}


7 FOLDS
NOTEBOOK THICKNESS
14 FOLDS
AVERAGE PERSON'S HEIGHT
17 FOLDS
HEIGHT OF A TWO-STORY HOUSE
21 FOLDS
HEIGHT OF EMPIRE STATE BUILDING
42 FOLDS
DISTANCE TO THE MOON
50 FOLDS
DISTANCE TO THE SUN

If your $1 doubles every year, how long does it take for you to become a Billionaire?

$1 to $1,000,000,000

- **In Finance, compounding is the process by which an asset's earnings, from either capital gains or interest, are reinvested to generate additional earnings over time.**

- **The formula for compound interest is:**

  $$A = P\,(1 + r/n)^{nt}$$

  Where:

  A = the future value of the investment/loan, including interest

  P = the principal investment amount (the initial deposit or loan amount)

  r = the annual interest rate or return

  n = the number of times that interest is compounded per unit t

  t = the time the money is invested or borrowed for

Compound Interest Calculator:

https://www.investor.gov/financial-tools-calculators/calculators/compound-interest-calculator

Exercise:

Calculate $1's worth at age 65 if investing the $1 at ages 20, 25, 30, 35, 40, 45, 50, 55

Starting Age of the $1 Investment	Ending Age of the $1 Investment	The Worth of $1 Investment At Ending Age
20	65	
25	65	
30	65	
35	65	
40	65	
45	65	
50	65	
55	65	

Step 1: Initial Investment

Initial Investment *

Amount of money that you have available to invest initially.

$1

Step 2: Contribute

Monthly Contribution

Amount that you plan to add to the principal every month, or a negative number for the amount that you plan to withdraw every month.

$0

Length of Time in Years *

Length of time, in years, that you plan to save.

45

Step 3: Interest Rate

Estimated Interest Rate *

Your estimated annual interest rate.

6

Interest rate variance range

Range of interest rates (above and below the rate set above) that you desire to see results for.

0

Step 4: Compound It

Compound Frequency

Times per year that interest will be compounded.

Annually

CALCULATE RESET

The Results Are In

In **45** years, you will have **$13.76**

Starting Age of the $1 Investment	Ending Age of the $1 Investment	The Worth of $1 Investment At Ending Age
20	65	$13.76
25	65	$10.29
30	65	$7.69
35	65	$5.74
40	65	$4.29
45	65	$3.21
50	65	$2.40
55	65	$1.79

Start early / Make your retirement savings work hard for you

For illustrative purposes only. Assumes a 6% return.

Have you started saving or investing?

Compounding Problem:

John invested $2,000 each year from year 2011 for first 8 years of a 40-year period with an annual compound rate of 10%. Kevin started to invest $2,000 a year from 2019 with the same rate and he has planned to continue to do that until 2050. Who will earn more by the end of year 2050? By how much?

Year	Early Funding		Late Funding		Continual Funding	
	Contribution	Year-End Value	Contribution	Year-End Value	Contribution	Year-End Value
1	$2,000	$2,200	$0	$0	$2,000	$2,200
2	$2,000	$4,620	$0	$0	$2,000	$4,620
3	$2,000	$7,282	$0	$0	$2,000	$7,282
4	$2,000	$10,210	$0	$0	$2,000	$10,210
5	$2,000	$13,431	$0	$0	$2,000	$13,431
6	$2,000	$16,974	$0	$0	$2,000	$16,974
7	$2,000	$20,871	$0	$0	$2,000	$20,871
8	$2,000	$25,158	$0	$0	$2,000	$25,158
9	$0	$27,674	$2,000	$2,200	$2,000	$29,874
10	$0	$30,441	$2,000	$4,620	$2,000	$35,061
11	$0	$33,485	$2,000	$7,282	$2,000	$40,767
12	$0	$36,834	$2,000	$10,210	$2,000	$47,044
13	$0	$40,517	$2,000	$13,431	$2,000	$53,948
14	$0	$44,569	$2,000	$16,974	$2,000	$61,643
15	$0	$49,026	$2,000	$20,871	$2,000	$69,897
16	$0	$53,929	$2,000	$25,158	$2,000	$79,087
17	$0	$59,322	$2,000	$29,874	$2,000	$89,196
18	$0	$65,254	$2,000	$35,061	$2,000	$100,316
19	$0	$71,779	$2,000	$40,767	$2,000	$112,548
20	$0	$78,957	$2,000	$47,044	$2,000	$126,003
21	$0	$86,853	$2,000	$53,948	$2,000	$140,803
22	$0	$95,583	$2,000	$61,643	$2,000	$157,083
23	$0	$105,092	$2,000	$69,897	$2,000	$174,991
24	$0	$115,601	$2,000	$79,087	$2,000	$194,690
25	$0	$127,161	$2,000	$89,196	$2,000	$216,359
26	$0	$139,877	$2,000	$100,316	$2,000	$240,195
27	$0	$153,865	$2,000	$112,548	$2,000	$266,415
28	$0	$169,252	$2,000	$126,003	$2,000	$295,257
29	$0	$186,177	$2,000	$140,803	$2,000	$326,983
30	$0	$204,795	$2,000	$157,083	$2,000	$361,881
31	$0	$225,275	$2,000	$174,991	$2,000	$400,269
32	$0	$247,803	$2,000	$194,690	$2,000	$442,496
33	$0	$272,583	$2,000	$216,359	$2,000	$488,741
34	$0	$299,841	$2,000	$240,195	$2,000	$539,615
35	$0	$329,825	$2,000	$266,415	$2,000	$595,576
36	$0	$362,808	$2,000	$295,257	$2,000	$657,134
37	$0	$399,089	$2,000	$326,983	$2,000	$724,847
38	$0	$438,998	$2,000	$361,881	$2,000	$799,332
39	$0	$482,898	$2,000	$400,269	$2,000	$881,265
40	$0	$531,188	$2,000	$442,496	$2,000	$971,339
Investment		**$16,000**		**$64,000**		**$80,000**
Earnings		**$515,188**		**$374,496**		**$891,339**

When to start investing matters!!!

Years	Principle	Deferred Growth
1	12000	13,200
2	12000	27,720
3	12000	43,692
4	12000	61,261
5	12000	80,587
6	12000	101,846
7	12000	125,231
8	12000	150,954
9	12000	179,249
10	12000	210,374
11		231,411
12		254,553
13		280,008
14		308,009
15		338,809
16		372,690
17		409,959
18		450,955
19		496,051
20		545,656
21		600,222
22		660,244
23		726,268
24		798,895
25		878,784
26		966,663
27		1,063,329
28		1,169,662
29		1,286,628
30		**1,415,291**

Years	Principle	Deferred Growth
1	12000	13,200
2	12000	27,720
3	12000	43,692
4	12000	61,261
5	12000	80,587
6	12000	101,846
7	12000	125,231
8	12000	150,954
9	12000	179,249
10	12000	210,374
11	12000	244,611
12	12000	282,273
13	12000	323,700
14	12000	369,270
15	12000	419,397
16	12000	474,536
17	12000	535,190
18	12000	601,909
19	12000	675,300
20	12000	756,030
21		831,633
22		914,796
23		1,006,276
24		1,106,904
25		1,217,594
26		1,339,353
27		1,473,289
28		1,620,617
29		1,782,679
30		**1,960,947**

Years	Principle	Deferred Growth
1	12000	13,200
2	12000	27,720
3	12000	43,692
4	12000	61,261
5	12000	80,587
6	12000	101,846
7	12000	125,231
8	12000	150,954
9	12000	179,249
10	12000	210,374
11	12000	244,611
12	12000	282,273
13	12000	323,700
14	12000	369,270
15	12000	419,397
16	12000	474,536
17	12000	535,190
18	12000	601,909
19	12000	675,300
20	12000	756,030
21	12000	844,833
22	12000	942,516
23	12000	1,049,968
24	12000	1,168,165
25	12000	1,298,181
26	12000	1,441,199
27	12000	1,598,519
28	12000	1,771,571
29	12000	1,961,928
30	12000	**2,171,321**

When to start investing matters!!!

纯利润

第一个十年：
$1,295,291

第二个十年：
$425,656

第三个十年：
$90,374

Assumptions: Same annual return of 10% (not guaranteed)

My wealth has come from a combination of living in America, some lucky genes, and compound interest.

Warren Buffett – The Giving Pledge, 2010.
www.givingpledge.org


The amount of initial investments matters!!!

Years	Principle	Deferred Growth
1	12000	13,200
2	12000	27,720
3	12000	43,692
4	12000	61,261
5	12000	80,587
6	12000	101,846
7	12000	125,231
8	12000	150,954
9	12000	179,249
10	12000	210,374
11		231,411
18		450,955
19		496,051
20		545,656
21		600,222
22		660,244
23		726,268
24		798,895
25		878,784
26		966,663
27		1,063,329
28		1,169,662
29		1,286,628
30		**1,415,291**

- Accounts allowing only low annual contributions
- Only saving low amount yearly

Years	Principle	Deferred Growth
1	40000	44,000
2	40000	92,400
3	40000	145,640
4		160,204
5		176,224
6		193,847
7		213,232
8		234,555
9		258,010
10		283,811
11		312,192
18		608,374
19		669,212
20		736,133
21		809,746
22		890,721
23		979,793
24		1,077,772
25		1,185,550
26		1,304,105
27		1,434,515
28		1,577,967
29		1,735,763
30		**1,909,340**

- Accounts allowing only high annual contributions
- Saving in higher amount yearly

The amount of initial investment matters!!!

Assumptions: Same total principle $120K, Same annual return of 10% (not guaranteed)

Interest rate or the return of your investment matters!!!

Rule of 72

- Divide 72 by the interest rate to estimate the number of years it takes for your money to double.

$4\overline{)72} = 18$

$$FV = PV \cdot (1+i)^n$$

$12\overline{)72} = 6$

Age	4%	Age	6%	Age	8%	Age	12%
29	$10,000	29	$10,000	29	$10,000	29	$10,000
47	$20,000	41	$20,000	38	$20,000	35	$20,000
65	$40,000	53	$40,000	47	$40,000	41	$40,000
		65	$80,000	56	$80,000	47	$80,000
				65	$160,000	53	$160,000
						59	$320,000
						65	$640,000

* All figures are for illustrative purposes only and do not reflect an actual investment in any product. They do not reflect the performance risks, expenses or charges associated with any actual investment. Past performance is not an indication of future performance. The Rule of 72 is a mathematical concept that approximates the number of years it would take to double the principal at a constant rate of return. The performance of investments fluctuates over time, and as a result, the actual time it will take an investment to double in value cannot be predicted with any certainty. Additionally, there are no guarantees that any investment or savings program can out-pace inflation.

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$4\overline{)72} = 18$

$$FV = PV \cdot (1+i)^n$$

$12\overline{)72} = 6$

Age	4%	Age	6%	Age	8%	Age	12%
29	$10,000	29	$10,000	29	$10,000	29	$10,000
47	$20,000	41	$20,000	38	$20,000	35	$20,000
65	$40,000	53	$40,000	47	$40,000	41	$40,000
		65	$80,000	56	$80,000	47	$80,000
				65	$160,000	53	$160,000
						59	$320,000
						65	$640,000

* All figures are for illustrative purposes only and do not reflect an actual investment in any product. They do not reflect the performance risks, expenses or charges associated with any actual investment. Past performance is not an indication of future performance. The Rule of 72 is a mathematical concept that approximates the number of years it would take to double the principal at a constant rate of return. The performance of investments fluctuates over time, and as a result, the actual time it will take an investment to double in value cannot be predicted with any certainty. Additionally, there are no guarantees that any investment or savings program can out-pace inflation.

Rule of 72

- Divide 72 by the interest rate to estimate the number of years it takes for your money to double.

$4\overline{)72} = 18$

$$FV = PV \cdot (1+i)^n$$

$12\overline{)72} = 6$

Age	4%	Age	6%	Age	8%	Age	12%
29	$10,000	29	$10,000	29	$10,000	29	$10,000
47	$20,000	41	$20,000	38	$20,000	35	$20,000
65	$40,000	53	$40,000	47	$40,000	41	$40,000
		65	$80,000	56	$80,000	47	$80,000
				65	$160,000	53	$160,000
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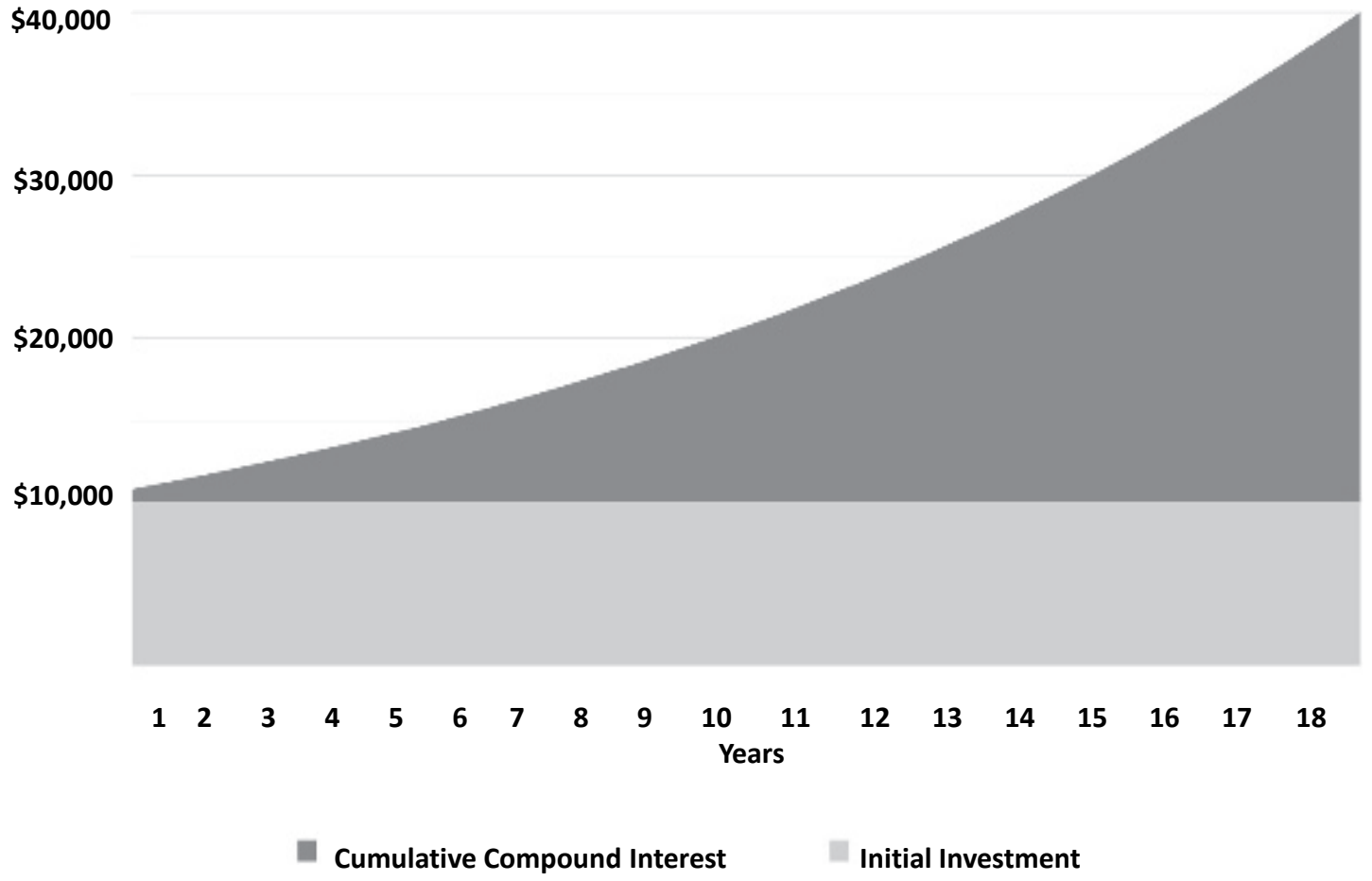
神奇的复利 – 自然常数e

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

“The Rule of 72”:

Simply assume a reasonable rate of return for planning purposes (between 7 percent and 9 percent over a multidecade time period) and divide 72 by that rate. This calculates the period of time it would take for your money to double.

Note: The Rule of 72 is an approximation. The equation is $2 = 1 \times (1 + \text{Rate of Return})^{Y}$, where Y is the time to double.



GROWTH OF $10,000 INVESTMENT WITH AN 8 PERCENT RETURN: IN I T I A L INVESTMENT VERSUS CUMULATI V E COMPOUND I NTEREST

Warren Buffett says, “Life is like a snowball. The important thing is finding wet snow and a long hill”:

- **The wet snow is the interest you reinvest to pick up even more interest as you roll along.**
- **The long hill is the multiple decades you give yourself if you start saving early.**

YouTube

课外作业

指数增长 妈咪说

常数e为什么代表了自然？一次看懂自然常数e的由来

317,987 views • Jan 7, 2019

4.9K 231 SHARE SAVE

《时间的朋友》2017

- 罗胖说:
- 以前认为挣钱最重要，后面发现增长(The growth)比挣钱重要；当你以为增长最重要的时候，后面发现增长的速度(The speed of growth)才是最重要的；当你在追求增长速度(The speed of growth)的时候，你又会发现超过市场预期的增长速度(The speed of growth acceleration)才重要。创业的本质是要增长，要预期中的增长，要超过预期的增长。
- 不仅要挣钱，还要明年要比今年挣得更多；不仅明年挣得多，还要明年多挣的部分大于今年多挣的；这还不行，还要多挣的增长速度也要大于去年的。 说白了，就是恨不得你任意阶导数大于零，这太残酷了！！

It is all about compounding!!!!

The Power of 1% Over a Single Year

- 1% Gain Every Day：
- $(1 + 1\%)^{365} = (1.01)^{365} = 37$

- 1% Loss Every Day：
- $(1 - 1\%)^{365} = (0.99)^{365} = 2.55\%$

The Power of 1% Over a Single Year

What have you learned today???

- **The formula for compound interest is:**

  $$A = P\,(1 + r/n)^{nt}$$

  Where:

  A = the future value of the investment/loan, including interest

  P = the principal investment amount (the initial deposit or loan amount)

  r = the annual interest rate or return

  n = the number of times that interest is compounded per unit t

  t = the time the money is invested or borrowed for

Compounding Summary

- T, when to invest matters.
- P, the amount of initial investment matters.
- R, the interest or return rate matters.

Compounding Summary

- It could be your BEST friend (wealth/business)
- It could be your WORST enemy (tax/COVID)

- Compounding Power comes at late stage
- How to manage life/study/investment?

- Understand its power and make use of it
- Success is not far away