

**Math Competition (Grades 4-6)** October 26th, 2003

*Note.* The first fifteen problems for the three exams were identical, but in different orders.

1. A problem author for a math competition wrote 44 problems so far, but he needs a total of 50. How many more problems must he write?  
He must write  $50 - 44 = 6$  more problems.
2. Compute  $1.123 + 1.01 + 0.1111$ .  
Adding directly, we have  $1.123 + 1.01 + 0.1111 = 2.2441$ .
3. Express  $\frac{13}{4}$  in decimal form.  
*First Solution.* Note that  $\frac{13}{4} = \frac{12+1}{4} = \frac{12}{4} + \frac{1}{4} = 3 + \frac{1}{4} = 3 + 0.25 = 3.25$ .  
*Second Solution.* Dividing directly also yields  $13 \div 4 = 3.25$ .
4. If a bicycle costs \$35.55 and Tom has \$29.55, how many more dollars does he have to save in order to buy the bike?  
Tom needs to save  $\$35.55 - \$29.55 = \$6.00$  more.
5. Calculate  $1 + 2 \times 3$ .  
By order of operations,  $1 + 2 \times 3 = 1 + (2 \times 3) = 1 + 6 = 7$ .
6. Compute  $\frac{1}{2} - \frac{1}{3}$ .  
Using the common denominator 6, we have  $\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$ .
7. A rectangle has length 19 and width 14. What is its perimeter?  
The perimeter is  $19 + 14 + 19 + 14 = 66$ .  
*Note.* The perimeter of a rectangle with length  $a$  and width  $b$  is  $2 \times (a + b)$ .
8. My doctor told me that I am 5 feet and 10 inches tall. If there are 12 inches in a foot, how tall am I in inches?  
Since there are 12 inches in a foot, there are  $12 \times 5 = 60$  inches in 5 feet. Therefore, I am  $60 + 10 = 70$  inches tall.
9. What is the sum of the first six positive odd numbers?  
The first six odd numbers are 1, 3, 5, 7, 9, and 11. Adding them, we have  $1 + 3 + 5 + 7 + 9 + 11 = 36$ .  
*Note.* In general, the sum of the first  $n$  positive odd numbers is  $n^2$ .
10. In the word DIOPHANTINE, what fraction of the letters are vowels?  
There are 11 total letters and 5 vowels, so the answer is  $\frac{5}{11}$ .
11. My grandfather clock chimes on the hour, every hour. How many times will it chime between 3:30PM today and 3:30PM two days from now?  
There are 24 hours in a day, so 48 hours in 2 days. Therefore, the clock will chime 48 times.
12. A slice of pizza costs \$1. A pie, which is composed of 8 slices, costs \$6. Ann buys 5 pies. How much money did she save by doing this instead of buying all the slices individually?  
There are  $5 \times 8 = 40$  slices in 5 pies, so Ann would have paid  $40 \times 1 = 40$  dollars if she bought individual slices. However, she actually paid  $5 \times 6 = 30$  dollars, so she saved  $40 - 30 = 10$  dollars.
13. My favorite number is 1337. There is one special number that I can multiply by 7 to obtain 1337. What is this number?  
The number must be  $1337 \div 7 = 191$ .
14. Evaluate  $17 - 16 + 15 - 14 + \cdots + 3 - 2 + 1$ .  
We group the numbers into eight pairs and one leftover:  $(17 - 16) + (15 - 14) + (13 - 12) + \cdots + (5 - 4) + (3 - 2) + 1$ . The quantity inside each pair of parentheses is 1, and there are eight of them, so the expression is equal to  $8 + 1 = 9$ .

15. Mr. Holbrook bought \$5 worth of cheese balls, \$6 worth of donuts, \$2 worth of soda, and \$4 worth of candy. Since he is a regular customer at the store, he gets 50% off his purchase. If he pays with a \$20 bill, how much change will he receive?  
Mr. Holbrook purchased  $5 + 6 + 2 + 4 = 17$  dollars worth of goods without the discount. With the discount, he owes  $17 \times 0.50 = 8.50$  dollars. Therefore, he will receive  $20.00 - 8.50 = 11.50$  dollars change.
16. A member of a track team can run a mile in five minutes. If she can maintain this speed for as long as she wants, how many hours would it take her to run 36 miles?  
Since she needs 5 minutes to run one mile, she needs  $5 \times 36 = 180$  minutes to run 36 miles. Since 180 minutes is equivalent to  $180 \div 60 = 3$  hours, the answer is 3 hours.
17. Compute  $1.55 \times 21.4$ .  
Multiplying directly, we have  $1.55 \times 21.4 = 33.17$ .
18. In a baseball game, Chu hits three singles (1 base), one double (2 bases), one triple (3 bases), and one home run (4 bases). For this game, how many bases did Chu average per hit?  
Chu earned  $(3 \times 1) + (1 \times 2) + (1 \times 3) + (1 \times 4) = 3 + 2 + 3 + 4 = 12$  total bases, and he had  $3 + 1 + 1 + 1 = 6$  hits. Therefore, he averaged  $12 \div 6 = 2$  bases per hit.  
*Note.* In baseball, a batter's *slugging percentage* is calculated in a similar way, but instead of dividing the total bases by the number of hits, one divides the total bases by the number of at-bats. In the aforementioned game, Chu obviously did a fabulous job, for if we assume that he had those six hits in exactly six at-bats, he had a 2.000 *slugging percentage* for the game. As a side note, the highest *slugging percentage* obtained by a MLB hitter in a single season is 0.8635 (Barry Bonds, 2001).
19. Eve likes solving math problems as a hobby. It takes her 73 days to solve 6 of them. How many problems can she solve in three years if none of them are leap years?  
Since Eve can solve 6 problems in 73 days, she can solve  $6 \times 5 = 30$  problems in  $73 \times 5 = 365$  days. There are 365 days in a year, so she can solve  $30 \times 3 = 90$  problems in 3 years.
20. There are three whole numbers in a row, and their sum is 18. What is their product?  
From trial and error, one can easily deduce that the three numbers are 5, 6, and 7. Therefore, the answer is  $5 \times 6 \times 7 = 210$ .
21. What is 200% of 50% of 20% of 50?  
Calculating directly, we have  $50 \times 2.00 \times 0.50 \times 0.20 = 10$ .
22. How many positive multiples of 13 are less than 100?  
*First Solution.* We can simply count by 13's. The first couple multiples of 13 are 13, 26, 39, 52, 65, 78, 91, 104, and etc. Therefore, there are 7 multiples of 13 less than 100.  
*Second Solution.* We note that  $100 \div 13 = 7.69\dots$ , or  $100 = 13 \times 7.69\dots$ . Therefore,  $13 \times 1, 13 \times 2, 13 \times 3, \dots, 13 \times 7$  are all the multiples of 13 smaller than 100, and the answer is 7.  
*Note.* In general, the number of positive multiples of  $n$  less than or equal to a given number  $m$  is  $\lfloor \frac{m}{n} \rfloor$ , where  $\lfloor x \rfloor$  designates the greatest whole number less than or equal to  $x$ . One can prove this using the same idea invoked in the second solution above.
23. Jason is taking a math competition in which he must solve 50 problems in 1.5 hours. He made the mistake of staying up last night playing computer games, and falls asleep for the first thirty minutes of the competition. If he wishes to receive a perfect score, how many seconds, on average, should he spend per problem after his nap?  
Since 1.5 hours is equivalent to  $1.5 \times 60 = 90$  minutes, after his 30-minute nap, Jason has 60 minutes, or  $60 \times 60 = 3600$  seconds, to work. Therefore, he must spend, on average,  $3600 \div 50 = 72$  seconds per problem.
24. The Academy Math Team wants to purchase some tee shirts. The first tee shirt costs \$182, and each subsequent shirt costs \$2. If there are 90 people on the team, and the team equally distributes the cost amongst its members, how much does each member have to pay?  
The total cost is  $182 + (89 \times 2) = 360$  dollars, so each member needs to pay  $360 \div 90 = 4$  dollars.

25. A *prime number* is a positive whole number whose only positive divisors are 1 and itself. For instance, 2, 3, 5, and 7 are examples of *prime numbers*. What is the smallest *prime number* greater than 90? Since all even numbers greater than 2 are divisible by 2, none of them can be a *prime number*. Investigating only the odd numbers greater than 90, we see that  $91 = 7 \times 13$ ,  $93 = 3 \times 31$ , and  $95 = 5 \times 19$  are not *prime numbers*. However, 97 is not divisible by any number other than 1 and itself, so it must be the answer.
26. A gigantic rabbit weighs a thousand pounds. A *ripped man* can lift up to 160 pounds. How many *ripped men* are needed to lift this rabbit?  
We first note that  $1000 \div 160 = 6.25$ . Since 6 *ripped men* can only lift  $160 \times 6 = 960$  pounds, we need 7 *ripped men* to lift a thousand pounds.
27. Ralph Wiggum likes picking his nose. However, every time he does, the number of bacteria on his finger doubles. There are 3 bacteria on his finger right now. How many bacteria will be on his finger if he picks his nose seven times?  
He would accumulate  $3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 384$  bacteria after picking his nose seven times.
28. Harry is addicted to cheese balls. A tub of 48,000 cheese balls costs 5 dollars. He continuously eats cheese balls at a rate of 100 cheese balls per minute. However, he drops exactly one cheese ball per second, which he does not eat or notice. Wanting to enter the *Guinness Book of World Records*, Harry buys exactly enough cheese balls to last him five hours. How many dollars did he spend?  
Since Harry eats 100 and drops 60 cheese balls per minute, he needs 160 cheese balls to last one minute. There are  $5 \times 60 = 300$  minutes in 5 hours, so he needs  $160 \times 300 = 48,000$  cheese balls to last 5 hours. Therefore, he needs exactly \$5 for his momentous task.
29. I number the pages of a book from 1 to 55. How many times do I use the digit 2?  
The digit 2 occurs once in the following numbers: 2, 12, 20, 21, 23, 24, 25, 26, 27, 28, 29, 32, 42, 52. It occurs twice in 22. Therefore, I use it a total number of 16 times.
30. Calculate  $125 \times 125 \times 8 \times 8 \times 8$ .  
Simply note that  $125 \times 125 \times 8 \times 8 \times 8 = (125 \times 8) \times (125 \times 8) \times 8 = 1000 \times 1000 \times 8 = 8,000,000$ .
31. What two-digit number evenly divides both 323 and 391?  
*First Solution.* By trial and error, we find that 323 is divisible by 17, and also  $323 = 17 \times 19$ . It is now easy to show that 391 is divisible by 17, but not by 19, and therefore, the answer is 17.  
*Second Solution.* If two numbers are divisible by  $n$ , then their difference is also divisible by  $n$ . To prove this, we note that the two multiples of  $n$  can be expressed as  $nx$  and  $ny$  for some whole numbers  $x$  and  $y$ . Then, their difference  $nx - ny = n(x - y)$  is clearly a multiple of  $n$  as well. Applying this fact to the problem, we see that the number in question also divides  $391 - 323 = 68$ . Since  $68 = 4 \times 17$ , we see that 17 is the only possible candidate, and thus the answer.  
*Note.* The method used in the second solution is a fundamental method of solving *number theory* problems, which are problems that involve whole numbers and divisibility.
32. On a warm day, the temperature was  $77^\circ\text{F}$ . The conversion between Centigrade and Fahrenheit is:  $^\circ\text{C} = \frac{5}{9} \times (^\circ\text{F} - 32)$ . What was the temperature in degrees Centigrade?  
Substituting directly into the given expression, we compute that the temperature was  $\frac{5}{9} \times (77 - 32) = 25$  degrees Centigrade.  
*Note.* A question worth \$125,000 on the game show *Who Wants to Be a Millionaire?* asked what  $-40^\circ\text{F}$  is equivalent to in degrees Centigrade. (The conversion expression was not given) What is the answer? Also, a famous way of remembering the conversion between the two scales is as follows: Take the temperature in Fahrenheit, add 40, multiply five-ninths, and then subtract 40. Can you verify that this works, and find a similar way to convert from Celsius to Fahrenheit?
33. A painter mixes 4 gallons of white paint with 1 gallon of red paint to make 5 gallons of her signature pink paint. Each gallon of white paint costs \$2 and each gallon of red paint costs \$3. How much money does the painter need to make 400 gallons of pink paint?  
Since the painter mixes white paint and red paint in a 4:1 ratio, to produce 400 gallons of her pink

- paint, she needs  $400 \times \frac{4}{5} = 320$  gallons of white paint and  $400 \times \frac{1}{5} = 80$  gallons of red paint. The white paint will cost  $320 \times 2 = 640$  dollars and the red paint will cost  $80 \times 3 = 240$  dollars. Therefore, the painter needs  $640 + 240 = 880$  dollars.
34. My computer is capable of only one process: When I enter a two-digit number, it reverses its digits, subtracts 7 from the result, and gives this final number as its answer. For example, if I enter 34, the computer first reverses its digits to obtain 43, and then subtracts 7 to give its final answer 36. When I put in another number, the computer gave me 22 as its final answer. What number did I enter?  
We work backwards. Since the computer's final answer was 22, it must have had 29 before subtracting 7. Therefore, the number before that must have the digits of 29 reversed, so my original number was 92.
35. The Road Runner ran 200 miles in 10 minutes, walked 40 miles in 20 minutes, and then ran another 360 miles in 10 minutes. What was its average speed for the whole trip in miles per hour?  
For the entire trip, it ran  $200 + 40 + 360 = 600$  miles in  $10 + 20 + 10 = 40$  minutes. Since 40 minutes is equivalent to  $40 \div 60 = \frac{2}{3}$  hours, its average speed was  $600 \div \frac{2}{3} = 900$  miles per hour.  
*Note.* In problems involving units, make sure to be consistent and present the answers in the desired units.
36. There are 120 pieces of candy in a bowl. Adam takes a third of them. Then, Ben takes a fourth of what remains. Finally, Cory takes a fifth of what remains after Adam and Ben took their shares. After Cory leaves, David eats the rest of the candy in the bowl. How many pieces of candy did David eat?  
After Adam,  $\frac{2}{3}$  of the candy remain. After Ben,  $\frac{3}{4}$  of the remaining candy remain. After Cory,  $\frac{4}{5}$  of the remaining candy remain. Therefore, David must have eaten the leftover  $120 \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = 48$  pieces of candy.
37. The *blip*, a unit for measuring silliness, is equal to five *bloops*. A *hectablip* is equal to 100 *blips*, and a *kilobloop* is equal to 1000 *bloops*. How many *kilobloops* are equal to 200 *hectablips*?  
Since 1 *blip* = 5 *bloops*, we have 1 *hectablip* = 100 *blips* = 500 *bloops*. Therefore, 2 *hectablips* = 1000 *bloops* = 1 *kilobloop*, and 200 *hectablips* = 100 *kilobloops*.
38. What is the least positive number divisible by 4, 5, 6, and 9?  
*First Solution.* The number must be divisible by 2 at least 2 times, divisible by 3 at least 2 times, and divisible by 5 at least once to satisfy the given condition. Therefore, the minimal such number is  $2 \times 2 \times 3 \times 3 \times 5 = 180$ .  
*Second Solution.* The least common multiple of 4 and 5 is 20. The least common multiple of 20 and 6 is 60. The least common multiple of 60 and 9 is 180. Therefore, the answer is 180.
39. Find the number halfway between  $\frac{1}{21}$  and  $\frac{1}{23}$ .  
The number halfway between two given numbers is simply their average, which can be computed by dividing their sum by 2. Since  $\frac{1}{21} + \frac{1}{23} = \frac{44}{483}$ , the answer is  $\frac{44}{483} \div 2 = \frac{22}{483}$ .  
*Note.* The answer is *not*  $\frac{1}{22}$ , as seen by the above reasoning.
40. John chooses the numbers 10, 24, 18, and 8, and Joanne chooses 20, 9, 12, and 16. What does John obtain if he multiplies his numbers and divides the result by the product of Joanne's numbers?  
*First Solution.* The desired quantity is equal to  $\frac{10 \times 24 \times 18 \times 8}{20 \times 9 \times 12 \times 16} = \frac{10}{20} \times \frac{24}{9} \times \frac{18}{12} \times \frac{8}{16} = \frac{1}{2} \times \frac{8}{3} \times \frac{3}{2} \times \frac{1}{2} = \frac{24}{24} = 1$ .  
*Second Solution.* One can directly compute each product to obtain the answer as well, though this is far more difficult. The product of John's numbers is  $10 \times 24 \times 18 \times 8 = 34560$ , and the product of Joanne's numbers is  $20 \times 9 \times 12 \times 16 = 34560$ . Thus, the answer is  $34560 \div 34560 = 1$ .
41. We have 5 numbers whose average is 11. Suppose we include 29 as a sixth number. What is the new average of these 6 numbers?  
If 5 numbers have an average of 11, their sum must be  $11 \times 5 = 55$ . Then, the sum of all six numbers equals  $55 + 29 = 84$ , so their average is  $84 \div 6 = 14$ .
42. Define  $n!$  as the product of the first  $n$  counting numbers. For instance,  $4! = 1 \times 2 \times 3 \times 4 = 24$ . Compute  $11! \div 8!$ .  
We have  $\frac{11!}{8!} = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8} = 9 \times 10 \times 11 = 990$ .

43. Mark likes playing with averages. He chooses four numbers  $a$ ,  $b$ ,  $c$ , and  $d$ . He then calculates the average of  $a$  and  $b$  and calls it  $e$ . He also calculates the average of  $c$  and  $d$  and calls it  $f$ . He finds that the average of  $e$  and  $f$  is 14. What is  $a + b + c + d$ ?  
 Note that  $e = \frac{a+b}{2}$  and  $f = \frac{c+d}{2}$ , so  $e + f = \frac{a+b+c+d}{2}$ . Then, the average of  $e$  and  $f$  is  $\frac{e+f}{2} = \frac{a+b+c+d}{4}$ , and since it was given that this quantity equals 14,  $a + b + c + d$  must equal  $14 \times 4 = 56$ .
44. A number is said to *deny* another number if the remainder after dividing the first number by the second number is exactly 4. For instance, 39 *denies* 7, because when 39 is divided by 7, the quotient is 5 and the remainder is 4. How many whole numbers between 200 and 300 *deny* 6?  
 A number *denies* 6 if and only if it can be expressed in the form  $6a + 4$  where  $a$  is a whole number. We want  $6a + 4 > 200$  and  $6a + 4 < 300$ . Note that the first inequality is equivalent to  $6a > 196$ , or  $a > \frac{196}{6} \approx 32.67$ , and the second inequality is equivalent to  $6a < 296$ , or  $a < \frac{296}{6} \approx 49.33$ . Therefore,  $a$  can assume the values 33, 34, 35, ..., 49. There are 17 possible values for  $a$ , so there are 17 whole numbers between 200 and 300 that *deny* 6.
45. In the game of *Mafball*, points can only be scored in 3 points or 5 points. What is the largest unattainable score in *Mafball*?  
 The answer is 7, which is obviously unattainable. Note that 8 points can be achieved by scoring one 3-pointer and one 5-pointer, 9 points can be achieved by scoring three 3-pointers, and 10 points can be achieved by scoring two 5-pointers. Now, note that if  $n$  points can be attained, so can  $n + 3$ , since one only needs to score one additional 3-pointer. Since 8, 9, and 10 are attainable, so are 11, 12, and 13, and thus, 14, 15, and 16 are as well, and etc. Therefore, every score higher than 7 can be attained. *Note.* In general, if points can only be scored in  $a$  points or  $b$  points, where  $a$  and  $b$  are relatively prime, the largest unattainable score is  $a \times b - a - b$ .
46. Andy can paint a fence in 1 hour by himself, and Bobby can paint a fence in 2 hours by himself. How many minutes does it take Andy and Bobby to paint a fence together?  
*First Solution.* From the given, Andy can paint a fence per hour, whereas Bobby can paint half a fence per hour. Therefore, if they work together, they can paint  $1 + \frac{1}{2} = \frac{3}{2}$  of a fence per hour, so it would take them  $\frac{2}{3}$  hours to paint a fence. Since  $\frac{2}{3}$  hours is equal to  $\frac{2}{3} \times 60 = 40$  minutes, the answer is 40.  
*Second Solution.* Often, it is easier to understand the problem by assigning numbers. Let the total amount of fence to paint be  $2m^2$ . Then, Andy paints at a speed of  $2m^2/hr$ , and Bobby paints at a speed of  $1m^2/hr$ . Therefore, the two of them together can paint at a speed of  $3m^2/hr$ . Since one fence is  $2m^2$ , painting it would take them  $2m^2 \times \frac{1hr}{3m^2} = \frac{2}{3}hr$ , or  $\frac{2}{3} \times 60 = 40$  minutes.
47. If  $A$ ,  $B$ ,  $C$  are three distinct points such that all three do not lie on one line, how many parallelograms can be formed using  $A$ ,  $B$ ,  $C$ , and a fourth point?  
 In any quadrilateral with  $A$ ,  $B$ , and  $C$  as vertices, either  $\overline{AB}$ ,  $\overline{BC}$ , or  $\overline{CA}$  must be a diagonal. If  $\overline{AB}$  is a diagonal, reflect  $C$  across the midpoint of  $\overline{AB}$  to obtain the fourth point. We can construct two other points in a similar fashion. Therefore, the answer is 3.
48. A fruit company orders 4800 pounds of oranges at \$1.80 per pound. The shipping cost is \$3000. Suppose 10% of the oranges are spoiled during the shipping and the remaining oranges are all sold. What should the selling price per pound be, given that the fruit company wants to make a net 8% profit?  
 The total cost for the company is  $(4800 \times 1.80) + 3000 = 11640$  dollars, so to make a net 8% profit, the company wishes to earn  $11640 \times 1.08 = 12571.20$  dollars. Since only  $4800 \times 0.90 = 4320$  pounds of oranges remain, the company needs to sell them at  $12571.20 \div 4320 = 2.91$  dollars per pound.  
*Note.* Though the above solution is straightforward, there are two major obstacles: Multiplying 11640 and 1.08, and dividing 12571.20 by 4320. To avoid performing such long arithmetic, it is often necessary to simplify *less*. To illustrate this method, we note that the final answer may be written as  $\frac{11640 \times 1.08}{4800 \times 0.90}$ . We can separate all the powers of ten, leaving:  $\frac{1164 \times 10 \times 108 \times 0.01}{48 \times 100 \times 90 \times 0.01} = \frac{1164}{48} \times \frac{1}{100} \times \frac{10 \times 108}{90} = \frac{97}{4} \times \frac{1}{100} \times 12 = \frac{291}{100} = 2.91$ .
49. Let  $\lceil x \rceil$  denote the least whole number greater than or equal to  $x$ . For example,  $\lceil 3.6 \rceil = 4$ ,  $\lceil \frac{16}{7} \rceil = 3$ , and  $\lceil 5 \rceil = 5$ . Calculate  $\lceil \frac{1}{3} \rceil + \lceil \frac{2}{3} \rceil + \lceil \frac{3}{3} \rceil + \cdots + \lceil \frac{97}{3} \rceil + \lceil \frac{98}{3} \rceil + \lceil \frac{99}{3} \rceil$ .

Note that  $\lceil \frac{1}{3} \rceil = \lceil \frac{2}{3} \rceil = \lceil \frac{3}{3} \rceil = 1$ ,  $\lceil \frac{4}{3} \rceil = \lceil \frac{5}{3} \rceil = \lceil \frac{6}{3} \rceil = 2$ ,  $\lceil \frac{7}{3} \rceil = \lceil \frac{8}{3} \rceil = \lceil \frac{9}{3} \rceil = 3$ , ...,  $\lceil \frac{94}{3} \rceil = \lceil \frac{95}{3} \rceil = \lceil \frac{96}{3} \rceil = 32$ , and  $\lceil \frac{97}{3} \rceil = \lceil \frac{98}{3} \rceil = \lceil \frac{99}{3} \rceil = 33$ . Therefore, the given sum is equal to  $(3 \times 1) + (3 \times 2) + (3 \times 3) + \cdots + (3 \times 32) + (3 \times 33) = 3 \times (1 + 2 + 3 + \cdots + 32 + 33)$ . It is well known that the sum of the first  $n$  positive integers is  $\frac{1}{2} \times n \times (n + 1)$ , so  $1 + 2 + 3 + \cdots + 32 + 33 = \frac{1}{2} \times 33 \times 34 = 561$ . The answer is thus  $3 \times 561 = 1683$ .

*Note.* We will prove here that  $1 + 2 + 3 + \cdots + (n - 2) + (n - 1) + n = \frac{1}{2} \times n \times (n + 1)$ . Let  $S = 1 + 2 + 3 + \cdots + (n - 2) + (n - 1) + n$ , and then write  $S = n + (n - 1) + (n - 2) + \cdots + 3 + 2 + 1$ . If we add these two equations term by term, we have  $2S$  on the left side and  $(n + 1) + (n + 1) + (n + 1) + \cdots + (n + 1) + (n + 1) + (n + 1)$  on the right side. Since there are  $n$  of these  $(n + 1)$ 's, the right side is equal to  $n \times (n + 1)$ . Therefore, we have  $2S = n \times (n + 1)$ , so  $S = \frac{1}{2} \times n \times (n + 1)$ .

50. A *palindrome* is a number such that it is read the same regardless of whether the digits are read forwards or backwards. For example, 141, 7007, and 8888 are *palindromes*, whereas 345 and 5959 are not. How many even, four-digit numbers are *palindromes*?

First, note that every four-digit *palindrome* is defined by the number formed by its first two digits. For example, to obtain the *palindrome* 1331, one can take the number 13, reverse the digits to obtain 31, and attach this to the end of the original number. Every *palindrome* can clearly be formed this way. Since a number is even if and only if its last digit is even, a *palindrome* is even if and only if its first digit is even. Therefore, the number of even, four-digit *palindromes* is equal to the number of two-digit numbers whose tens digit is nonzero and even. There are 10 two-digit numbers that start with 2, 10 that start with 4, 10 that start with 6, and 10 that start with 8, so there are 40 total even, four-digit *palindromes*.