6th Grade Competition

Math Competition 19 October 2008

1. Before taking the AIME, a student notices that he has two can of soda, three bags of chips, and six bars of chocolate on his desk. If each can of soda has 140 calories, each bag of chips has 120 calories, and each bar of chocolate has 60 calories, and the student devours everything on his desk during the test, then how many calories did the student consume?

2. In order to walk completely around his square backyard, Mike must walk 84 yards. How long is one side of his field in yards?

3. If $a\#b = 2a - b + 117$, then what is $(3\#4) - (5\#2)$?

4. An airplane ascends from sea level to 30,000 feet above sea level at 500 feet per minute. How many minutes does it take the airplane to reach its target height?

5. The sum of five consecutive terms of an arithmetic sequence is 30. If the common difference is 6, then find the difference between the largest and smallest of the five terms.

6. One Friday night, Mr. Holbrook had 219 Tootsie Rolls in his candy cabinet. (Not many people at math team like to eat Tootsie Rolls.) The next morning, he gets a shipment of 87 Tootsie Rolls. How many Tootsie Rolls does Mr. Holbrook have now?

7. Find the sum of the first 8 terms in the pattern 26, 23, 20, 17, ...

8. $6 + 66 + 666 + 6666 + 66666 = ?$

9. The formula for converting Fahrenheit temperatures to Celsius temperatures is $C = \frac{5}{9}(F - 32)$. Convert 50^o Fahrenheit to degrees Celsius.

10. Find the largest prime factor of 111.

11. $9 \cdot (47 + 53) = 3 \cdot (7 + ?)$

12. $9 \cdot (1001 + 999) = .9 \cdot (10001 + \ ?)$.

13. 15% of a number is equal to 9. Find the number.

14. Alex Anderson is typing up notes for his geometry class. If it takes him a half hour to make a diagram on the computer and fifteen minutes to type each solution, then how many hours will it take him to type up four solutions and two diagrams?

15. Find the area of a circle with circumference 24π.

16. Ben drives at 40 miles per hour towards North Dakota, 2000 miles away. Halfway there, he decides that he doesn't want to go to North Dakota, and also that he should travel faster. Thus, he turns back home, and drives back at an average speed of 50 miles per hour. How long, in hours, did Ben's trip take?

17. What is the prime factorization of 6534?

18. If n is a positive integer, then how many values of $\imath^n$ are distinct? $\imath = \sqrt{-1}$.

19. What is the smallest positive number that must be added to 83 to get a number that ends in two 0's?

20. During the first quarter of a two-hour flight, Jeff can see the ground the entire time. However, exactly one-quarter of the way into the flight, clouds begin to block Jeff's view of the ground. If the cloud cover persists until the end of the flight, how many seconds of the flight could Jeff not see the ground?

21. Find the last digit of the number $2008^2 + 2^{2008}$.

22. Compute 8635×2121.

23. Each day David does not shower, the number of bacteria on his body doubles. Every day he does shower, $\frac{15}{16}$ of the bacteria on his body are killed. David comes to math camp with 6 bacteria on his body and doesn't shower for a week. The next day, he showers and then leaves math camp and arrives home. How may bacteria are on his body when he gets home?

24. Students at math team consume bags of chips at a rate that is directly proportional to the time that they have been at math team that day. If a math team member consumes three bags of chips per hour after being at math team for one hour, then what is the rate of consumption of chips, in bags per hour, of a member who has been at math team for three hours?

25. During his first week in America, Tolga goes to Six Flags and spends \$40 on games. If all he wins is a basketball, and he sells the basketball to his friend John for \$5 at Six Flags, then how many more dollars did Tolga enter Six Flags with than he left it?

26. What is the volume of a sphere with radius $\dfrac{12}{\sqrt[3]{\pi}}$?

27. John draws a card from a standard deck of 52 cards, replaces it, and then chooses another card. What are the chances that he chooses a spade the first time and a four the second time?

28. A palindrome is a number that is read the same way forwards as it is backwards. For example, 70107 is a palindrome, but 502 is not. Find the sum of all two-digit palindromes.

29. Fully simplify $3\sqrt{27\sqrt{9}}$.

30. Compute the length of a diagonal of rectangle $ABCD$ given $\overline{AB} = 9$ and that the area of $ABCD$ is 81.

31. One day, Mark was mad, and dug a 7-foot deep hole in his backyard. However, since he didn't toss the dirt far enough away from the hole, 8 inches of dirt fell back into the hole. How deep in inches is the hole now?

32. At what point do the lines $2x + 3y = 5$ and $x + y = 2$ intersect? Give your answer in the form (x, y), including the parentheses and the comma.

33. Given points $A(3,4)$ and $B(5,12)$, find the distance $\overline{AB}$.

34. Find the number of two-digit numbers n such that the sum of n and the number obtained by switching the digits of n is divisible by 11.

35. Tom has an 89 average on his first two tests of the year. If the maximum possible grade on any test is a 100, find the lowest possible grade he can get on his next test in order to be able to get a 93 average at the end of the year (there are 4 tests in Tom's class this year).

36. Jenny buys a twelve pack of soda cans at the gas station for \$2.40. How many cents did she pay for each can?

37. The next day (refer to problem 36), Jenny goes back to the gas station and buys another twelve pack of soda, but this time, it costs \$3.60. How many more cents did she pay for a can of soda this time than she did in problem 36?

38. Ben is trying to throw darts at a dartboard. Each throw he makes, the dart ends up at most one foot away from some part of the dartboard (sometimes, he hits the dartboard). Also, any point in this area has an equal chance of being hit. If the dartboard is a circle of radius two feet, then what is the probability that Ben hits the dartboard?

39. Find the area of a rhombus with diagonals of lengths 2 and 18.

40. How many 2×3 tiles does it take to cover a 6×6 square?

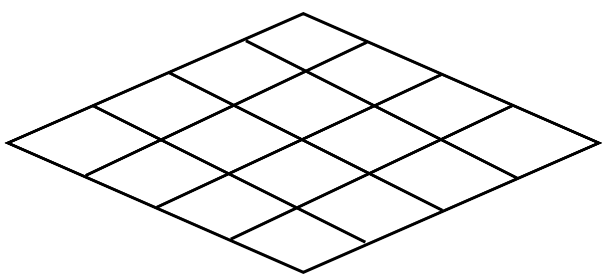
41. If the three solutions to $x^3 - 12x^2 + 27x = 0$ are a, b, and c and $a \le b \le c$, compute $a + c^b$.

42. A coin is flipped five times. What is the probability of heads coming up exactly twice?

43. Two positive numbers are relatively prime if they share no factors other than 1. How many numbers less than or equal to 21 are relatively prime to 21?

44. A sphere fits exactly inside of a cube – it just touches all six faces of the cube. If the volume of the cube is 27, then what is the radius of the sphere?

45. What is the least number of colors needed to color each of the parallelograms below such that no two parallelograms that share an edge are colored the same color?

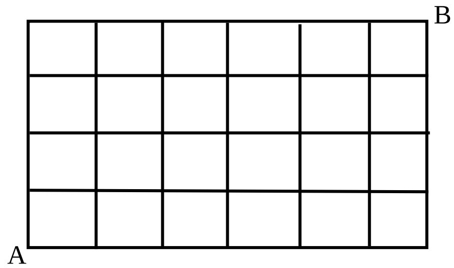


46. Tolga is throwing a basketball at the wall. Each time he throws it, the strength of the wall is reduced to $\frac{9}{10}$ of its strength before that throw. If the wall was at full strength before Tolga started throwing the ball at the wall and if the wall breaks when its strength is reduced to below 50% of its original strength, then the wall will break after the n^{th} throw. Find n.

47. A golf coach distributes 54 tees and 30 golf balls to a team of n people so that each of the n players gets the same number of tees and each player gets the same number of balls (although the number of tees each player receives may be different from the number of balls they receive). If there are no tees or balls left over, then what is the largest possible value of n?

48. How many composite numbers are less than 30?

49. In how many ways can Lena get from point A to point B if Lena can only move up and right?



50. What is the distance from the origin to the line $8x+6y=72$?