

1. Three whole numbers a, b, c . The sum of the squares of a, b, c is a perfect square. Prove at least two of the three numbers a, b, c are even numbers.

Answer:

Assume a, b, c , and u are whole numbers, then we need to prove $a^2 + b^2 + c^2 = u^2$ while at least two of the numbers a, b, c are even numbers or $a^2 + b^2 + c^2 \neq u^2$ while at least two of the numbers a, b, c are odd numbers.

Let us use the second approach to prove:

Assume c is the even number and a, b are odd number, then c can be written as $2k$, a can be written as $2m + 1$, b can be written as $2n + 1$.

Thus $a^2 + b^2 + c^2 = (2m + 1)^2 + (2n + 1)^2 + (2k)^2 = 4(m^2 + n^2 + k^2 + m + n) + 2$ which can not be a perfect number.

Similar approval can be applied to a, b, c if the 3 are all odd number:

$$a^2 + b^2 + c^2 = (2m + 1)^2 + (2n + 1)^2 + (2k + 1)^2 = 8q + 3.$$

Any odd number's square should be written as $8q + 1$ i.e. $(2p + 1)^2 = 4(p + 1)p + 1 = 8q + 1$, therefore, $8q + 3$ would not be a square number.

2. When dividing 1270 by a natural number (divisor), the quotient is 74. What are the divisor and remainder.

Answer:

If the remainder is r , and divisor is x , then $x > r = 1270 - 74x > 0$,

Therefore, $1270/75 < x < 1270/74$, we get $x = 17$ and $r = 12$

3. Four digit number \overline{abcd} can be divided by 11 with no remainder while a, b, c, d are numbers 0 to 9 and a is not equal to 0. $b + c = a$, and \overline{bc} is a perfect square. What is \overline{abcd} ?

Answer:

Since \overline{abcd} can be divided by 11, then $a + c - b - d = 0$, or $a + c - b - d = 11$.

Since \overline{bc} is a perfect square, then c can only be 0, 1, 4, 5, 6, 9.

First if $a + c - b - d = 0$. Since $b + c = a$, then $2c = d$, thus c could be 1, or 4.

If $c = 1$, then $d = 2$. To make \overline{bc} is a perfect square, b would be 0 or 8.

If $b = 0$, then $a = 1$; if $b = 8$, then $a = 9$. Then we found 9812, 1012 are the numbers.

If $c = 4$, use the same method, we will get 4048.

Second if $a + c - b - d = 11$, we will get 7161, 9361, 9097 respectively.

4. 1, 2, 3, 4, 5, 6 are 6 numbers forming \overline{abcdef} . $2|\overline{ab}$, $3|\overline{abc}$, $4|\overline{abcd}$, $5|\overline{abcde}$ and $6|\overline{abcdef}$.

Please find \overline{abcdef} .

Answer:

123654 or 361254

5. 17 classmates are having a meeting and shake hands. Please prove it is impossible that everyone shakes hands with 3 and only 3 people.