Class 8 Additional Problems   August 12, 2023   Name___________

1. There are 11 real numbers a1, a2, to a11, and 0 <= ai (i = 1, …, 11) <= 1. Please prove at least the absolute difference of two numbers would be less than or equal to 1/10.

   Answer: If we create ten buckets, first one for number(s) 0 <= ai < 0.1, second one for 0.1 <= ai < 0.2, …, 0.9 <= ai <= 1, then we put each one of the 11 real numbers into corresponding bucket matching to it's value. Since there are 10 buckets, and there are 11 numbers, there must be two numbers in one of the ten buckets. Because the differences of the numbers in any bucket are less than or equal to 1/10, then the proof.

2. There are 10 natural numbers from 1 to 10. Please prove: if selecting any 6 numbers from the 10 numbers, there must be 2 numbers of the 6 numbers, one number is the other number's multiple.

   Answer:

   If we group the 10 numbers to 5 sets:

   A1 = {1, 1 x 2, 1 x 2^2, 1 x 2^3}

   A2 = {3, 3 x 2}

   A3 = {5, 5 x 2}

   A4 = {7}

   A5 = {9}

   Thus if we pick 6 numbers, there must be two numbers from any one of the sets A1, A2, A3, and the smaller numbers in these 3 sets are the multiples of the bigger numbers.

3. In a class there are 40 students. Math teacher has 125 different math problems to give to these students. Is there any student will get 4 or more than 4 problems?

   Answer: if we consider 40 students as 40 sets/buckets, and 125 math problems as elements, then distribute these math problems evenly to each of the sets/buckets, then, each one student would receive 3 math problems. There would be still 1 math problem left to be distributed, thus at lease one of the students has to get 4.

4. In a basket there are balls in red, blue and yellow colors. If everyone takes turn to pull balls from the basket and can take two balls, and no one can pull twice, then how many people should be there to pull the balls to ensure at least two people would have balls in same color combinations.

   Answer:

   The balls pulled out could be the same colors i.e. all in red, all in blue, all in yellow 3 sets. For different colors, there are also 3 sets. Thus there would be totally 6 sets. Assuming for six people, each one of the six people as elements goes to each set, i.e. they pulled balls all in different color combinations, if we add one more person, then he would pull balls in one of 6 color sets. Thus we need at least seven people to ensure at least two people would have balls in the same color combinations.

5. There is a circle which has been divided to 12 sections from the center. If write 1, 2, to 12 numbers to each of the sections randomly, please prove there must be 3 connecting sections, the sum of the numbers in the 3 sections is 20 or bigger than 20.

   Answer:

Let us say the first section's has number a1, second section has number a2, …, a12, then the sum of the 3 connecting sections would be a1 + a2 + a 3, a2 + a3 + a4, …, a10 + a11 + a12. Add them all together we have 3(a1 + a2 + ,,, + a12) = 234, and 234/12 = 19.5. If we consider each one of the 12 sums as one bucket, and there would be 12 buckets, and average of the sums would be 19.5. Since ai (I is 1, 2, …, 12) are whole numbers, then the proof.