8th Grade

19 October 2008

1. Before taking the USAMO, a student notices that he has two bags of Doritos, two bags of Fritos, three cans of soda, a bottle of water, four bags of skittles, half a box of Cheez-its, two bags of Cheetos, and five mini Hersheys bars on his desk. Each bag of Doritos has 130 calories, each bag of Fritos has 120 calories, each can of soda has 140 calories, each bag of Skittles has 60 calories, each box of Cheez-its has 1200 calories, each mini Hersheys bar has 60 calories, each bag of Cheetos has 110 calories, and each bottle of water has no calories. If, during the test, the student consumes everything on his desk, how many calories will he have consumed?

2. $8+88+888+8888+88888=?$

3. On Monday, Matt hits a golf ball 100 yards. Disappointed, he goes to the gym and lifts weights for two hours. The next day, he can hit the golf ball 25 yards further than the day before. Still disappointed, he goes back to the gym that night and lifts weights again. Wednesday, he could hit the ball 25 yards further than on Tuesday. If this process continues through Thursday night, then how many yards will Matt be able to hit a golf ball on Friday of that week?

4. Josh and Kun-Soo have 2008 Frosted Flakes each. Josh gives Kun-Soo half of his flakes. After this, Kun-Soo gives a quarter of his new number of flakes back to Josh. How many flakes does Josh have now?

5. If someone buys 3 cokes at \$1.77 each, and tax is 35 cents total, how much change would that person get from a 10 dollar bill?

6. Compute $123456789+876543211$. Write your answer in the form 10^a, where a is a whole number.

7. David Rush is in a hurry to get to class. His dorm at Philips Exeter is 840 meters away from his first class. He runs the first quarter of the distance at a brisk 7 meters per second. After this, he (instantly) slows down to 3 meters per second and finishes running at this rate. How many seconds did it take him to get to class?

8. James wants to paint his 12×9 ft^2 bedroom wall. However, he does not need to paint his bedroom door, which has dimensions of 3×6 ft^2. If each square foot of wall requires 2 milliliters of paint, then how many milliliters of paint does James need to complete his job?

9. A square has perimeter 104. What is the area of the square?

10. Angel sees the time 52:11 on his digital clock. He then realizes this does not make sense because he is looking into a mirror. What time is it, assuming his clock is correct?

11. n minutes before the time in problem 10, the time in the mirror matched the time in real life. What is the smallest possible value of n?

12. Find $76648\div 13$.

13. If $a@b=\dfrac{a^2-2b^2}{a-b}$, compute $3@4-4@7$.

14. If $T(x,y)=\dfrac{x^2}{y^x}$, what is $T(T(1,2),T(2,1))$?

15. Simplify $\dfrac{7+\imath}{1-2\imath}$ given that $\imath=\sqrt{-1}$.

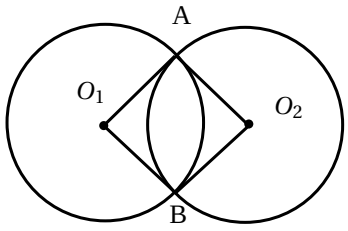
16. What is the probability of rolling an 11 with a pair of fair 6-sided dice?

17. If $\gamma = \frac{1}{2}$, then evaluate $\gamma + \gamma^2 + \gamma^3 + \gamma^4$.

18. Jenny and Hyungie are playing a variant of baseball. Jenny has a $\frac{4}{7}$ chance of winning an inning while Hyungie has a $\frac{3}{7}$ of winning the inning. (There are no ties.) What is the probability that Hyungie will be winning after two innings?

19. If 96346791 beads are split among 12 people so that each person has the same whole number of beads, how many are left over?

20. The circles centered at O_1 and O_2 both have radii of length 4. They intersect at A and B, and $\angle AO_1B = \angle AO_2B = 90^o$. The area of the intersection of both circles can be written in the form $p\pi + q$. Find $p + q$.



21. How many 1s does Nikhil need to write if he writes all the page numbers for a book that has 416 pages and starts on page 1?

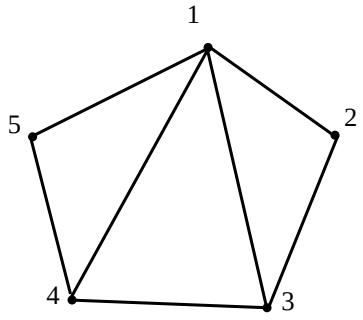
22. Find the largest integer value of a for which the statement "6144 is divisible by 2^a" is true.

23. Compute all real x such that $x^6 - x^3 = 0$.

24. Express $\dfrac{b^3-1}{b-1}$ in base b. Write your answer without the subscript b.

25. Nikhil is writing the page numbers in a book that has 186 pages. After a certain page number, Nikhil notices that he has written the same number of digits as he still has to write in order to finish numbering all 186 pages. What page number is this?

26. A construction company must pave every road in Aganon, a region shown below. There are five cities in Aganon. If every road must be traversed exactly once, then the construction company may start in only two cities, cities a and b. Find $10a + b$, given that $a < b$.



27. Austin runs an animal hospital that takes care of cats, dogs, and birds. The cats here only have three legs and one tail each, the birds have one leg and two tails each, and the dogs have four legs each but no tail. Victoria, the

hospital inspector, walks in one day and counts 15 heads, 42 legs, and 13 tails. How many cats were there?

28. Scoring in a frisbee game requires a long pass, two medium passes, or four short passes. A team cannot combine different types of passes to score. If the probabilities of successfully completing these passes are 0.3, 0.6, and 0.8 respectively, which is the best strategy to score? Your answer should be exactly one of the words 'long,' 'short,' and 'medium.'

29. What are the last two digits of 7^{2008}?

30. Two positive integers are relatively prime if they share no factors other than 1. $\phi(n)$ is the defined as the number of numbers less than or equal to n that are relatively prime to n. For example, $\phi(6) = 1$. Find $\phi(120)$.

31. If you place a nickel on the table and surround the nickel with other nickels so that the inner nickel is touching every other nickel and the outer nickels are each touching the inner nickel and two other nickels, how many nickels are on the table?

32. Let A be the set of 3-digit positive palindromes and let B be the set of 4-digit numbers that are divisible by 3. If $|X|$ is the number of elements in set X, then compute $|B| - |A|$.

33. How many distinct rearrangements of the letters in 'ippisssissim' are there?

34. Ben is making a five hour lecture to a class of 20 students, $\frac{9}{10}$ of which are always paying attention. If the probability of a student who is not always paying attention being asleep at any time is $\frac{1}{5}$, then what is the expected number of minutes that the entire class will be awake?

35. Evaluate $2 + \dfrac{1}{2 + \frac{1}{2 + \frac{1}{2+\ldots}}}$.

36. By definition, $|a + b\imath| = \sqrt{a^2 + b^2}$, and $\imath = \sqrt{-1}$. Find $\left|\dfrac{5 + 12\imath}{3 - 4\imath}\right|$.

37. The area of an isosceles triangle with base length 4 is 30. Find the perimeter of the triangle.

38. A sphere inside a cylinder is tangent to both of the cylinder's bases and touches the rounded part. If the area of the cylinder is 96, compute the area of the sphere.

39. Evaluate $\log_2(\log_4 16)$.

40. Your mom is mixing batter for cookies one day. She puts in 100 milliliters of flour and .9 liters of water and starts stirring. However, .2 liters of the evenly-mixed mixture spills out. She accidentally replaces the lost batter with .1 liters of hydrochloric acid and keeps stirring. After the hydrochloric acid is evenly distributed, she pours 300 milliliters of the final mixture into a cup. How many milliliters of water are in the cup?

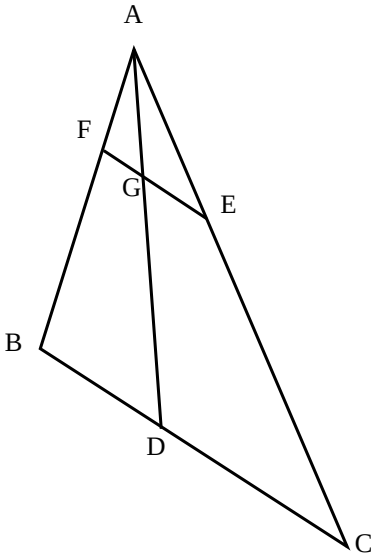
41. Find $\cos 30^o \tan 30^o$.

42. Compute $\dfrac{1}{\sqrt{2}+\sqrt{1}} + \dfrac{1}{\sqrt{3}+\sqrt{2}} + \dfrac{1}{\sqrt{4}+\sqrt{3}} + \ldots + \dfrac{1}{\sqrt{99}+\sqrt{98}} + \dfrac{1}{\sqrt{100}+\sqrt{99}}$.

43. A and B are digits. Find the sum of all two-digit numbers AB (not the product AB but rather the two-digit number 10A + B) such that 836A7B is divisible by 44.

44. Express 31 in base 2.

45. In the diagram, $\angle BAD = \angle DAC$ and $\angle AFE = \angle ABC$. Given that $\overline{AF} = 5$, $\overline{AE} = 10$, $\overline{FG} = 3$, and $\overline{EC} = 20$, compute $\overline{BC}$.



46. Compute $0\binom{7}{0} + 1\binom{7}{1} + 2\binom{7}{2} + 3\binom{7}{3} + \ldots + 6\binom{7}{6} + 7\binom{7}{7}$.

47. Find all ordered pairs (p, q) of prime numbers with $p > q$ such that $p^2 - q^2$ is a prime number.

48. Tolga is playing Young Duck in a game involving coin flips. Tolga wins if the coin comes up heads two times in a row while Young Duck wins after the coin comes up tails just once. If $P(\text{Y})$ is the probability that Young Duck wins, and $P(\text{T})$ is the probability of Tolga winning, then find $P(\text{Y})$ - $P(\text{T})$.

49. Express $BCA_{13} + 15_6 + 1337_8$ in base 10.

50. In how many ways can a 2×9 board be tiled with nine 2×1 tiles? Overlap of tiles is not allowed.