1. Assume 3 positive integers a, b, c, $a \neq b$ and $b \neq c$. Please prove for 3 numbers $a^3b - ab^3$, $b^3c - bc^3$, $c^3a - ca^3$, one of them is divisible by 10.
   Answer:
   $a^3b - ab^3 = ab(a^2 - b^2) = ab(a + b)(a - b)$
   $b^3c - bc^3 = bc(b^2 - c^2) = bc(b + c)(b - c)$
   $c^3a - ca^3 = ca(c^2 - a^2) = ca(c + a)(c - a)$

   If a, b, c are all odd numbers, then $a^3b - ab^3$, $b^3c - bc^3$, $c^3a - ca^3$ are all even number, so divisible by 2;
   If a, b, c are two odd, one even number, $a^3b - ab^3$, $b^3c - bc^3$, $c^3a - ca^3$ are all even number, so divisible by 2;
   If a, b, c are one odd, two even numbers, $a^3b - ab^3$, $b^3c - bc^3$, $c^3a - ca^3$ are all even number, so divisible by 2.

   If any one of a, b, c is divisible by 5, then one of 3 numbers $a^3b - ab^3$, $b^3c - bc^3$, $c^3a - ca^3$ is divisible by 5, then one of the 3 numbers $a^3b - ab^3$, $b^3c - bc^3$, $c^3a - ca^3$ is divisible by 10;
   If none of a, b, c is divisible by 5, then the unit digit of a, b, c must be 1, 2, 3, 4, 6, 7, 8,9, and likely unit digits of a^2, b^2, c^2 are 1, 4, 6, 9. Thus out of these 4 unit digits, there are always 3 of them allowing at least one of $(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - a^2)$ divisible by 5.

   Therefore, prove for 3 numbers $a^3b - ab^3$, $b^3c - bc^3$, $c^3a - ca^3$, one of them is divisible by 10.

2. Please prove: for whole number a, it is not divisible by 2 and 3, then $24|(a^2 -1)$.
   Answer:
   If a whole number a can't be divided by 2, then this number can be written as a = 2k + 1 where k is a whole number;
   If a whole number a can't be divided by 3, then this number can be written as a = 3m + 1 or a = 3n + 2 where m or n is a whole number.

   When a = 2k + 1, then $(a^2 - 1) = (2k + 1)^2 - 1 = 4k^2 + 4k + 1 - 1 = 4k^2 + 4k = 4k(k + 1) = 8p$ where p is a whole number and $p = k(k + 1)/2$. Thus $8|(a^2 - 1)$

   When a = 3m + 1, then $(a^2 - 1) = (3m + 1)^2 - 1 = 9m^2 + 6m + 1 - 1 = 3m(3m + 2)$. When a = 3m + 2, then $(a^2 - 1) = (3m + 2)^2 - 1 = 9m^2 + 12m + 4 - 1 = 3(3m^2 + 4m + 1)$. Thus $3|(a^2 - 1)$.

   Therefore, $24|(a^2 -1)$.
3. Numbers p and q are prime, and are bigger than 3, please prove $24 | (p^2 - q^2)$.
   Answer:
   Because p and q are prime and bigger than 3, therefore, we can assume p = 2m + 1 and q = 2n + 1 while both m and n bigger than 1, and m, n would allow p, q are primer numbers respectively, and also m, n are even numbers:
   Thus p2 – q2 = (p + q) (p – q) = (2m + 2n + 2)(2m – 2n) = 4(m + n + 1)(m – n) = 8(m + n + 1) p while m – n = 2p since both m and n are even numbers. Thus $8|(p^2 - q^2)$.

We can also assume $p = 3m + 1$, $q = 3n + 1$ or $p = 3m + 2$ and $q = 3n + 2$, then $(p^2 – q^2) = (p + q)(p – q) = (3m + 3n + 2 \text{ or } 4)(3m – 3n)$ which is divisible by 3;
Considering the scenario $p = 3m + 1$, $q = 3n + 2$ or $p = 3m + 2$ and $q = 3n + 1$, then $(p^2 – q^2) = (p + q)(p – q) = (3m + 3n + 3)(3m – 3n + \text{ or } - 1)$ which is divisible by 3. Thus $3|(p^2 – q^2)$.

Combine $8|(p^2 – q^2)$ and $3|(p^2 – q^2)$, we get $24|(p^2 – q^2)$.

4. $36x + 83y = 1$. If x and y are integers, please find x and y.
   Answer:
   Based on Euclidean algorithm, we get following:
   83 = 2 x 36 + 11 => 11 = 83 – 2 x 36
   36 = 3 x 11 + 3 => 3 = 36 – 3 x 11
   11 = 3 x 3 + 2 => 2 = 11 – 3 x 3
   3 = 2 + 1 => 1 = 3 – 2

   Thus 1 = 3 – 2 = 3 – (11 – 3 x 3) = 4 x 3 – 11 = 4 x (36 – 3 x 11) – 11 = 4 x 36 – 13 x 11
   = 4 x 36 – 13 x (83 – 2 x 36) = 36 x 30 – 83 x 13.

   Therefore, x = 30 and y = -13 is one set of the solution.