1. $f(x) = ax^2 + bx + c$, where a, b, c are known integers. For odd number m, $am^2 + bm + c$ is odd number and c is also odd number. Please prove for $ax^2 + bx + c = 0$, x would not be an odd number.

   Answer:

   Since c is odd number, when m is odd number, $m(am + b) + c$ is odd, then $am + b$ is even number. Thus am and b must be odd at the same time, or even at the same time.

   If am and b are odd the same time, since m is odd, then a is also odd number;
   If am and b are both even number, since m is odd, then a is also even number. Therefore, a and b must even or odd at the same time.

   Assume x is odd number,
   if both a and b are odd numbers, then $x(ax + b)$ is an even number, thus $ax^2 + bx + c$ can not equal to 0;
   if both a and b are even numbers, then $x(ax + b)$ is also an even number, thus $ax^2 + bx + c$ can not equal to 0.

   Therefore, x would not be an odd number

2. a, b, c, d are 4 natural numbers, and $a + b + c + d = 1989$, please prove $a^3 + b^3 + c^3 + d^3$ is not an even number.

   Answer:

   Since $a + b + c + d$ is an odd number, then either one of them is an odd number while other three are even numbers or one of them is an even number while other three are odd numbers. It can't be that four of them are all even number, or all odd number, or two of them are odd and two of them are even numbers.

   We know any odd or even number's 3 power would still be odd or even, therefore, $a^3 + b^3 + c^3 + d^3$ is not an even number.

3. a, b are two integers and b would not equal 0. If integer q, then $a = b * q$, then we can say a is divisible by b, written as $b|a$. Now we know $7|(13x + 8y)$, please prove $7|(9x + 5y)$.

   Answer:

   Since $7|(13x + 8y)$ then it must be $7|13x$ and $7|8y$, then $7|x$ and $7|y$. Therefore, $7|(9x + 5y)$.
4. We know m and n are odd numbers. When $4|k$, please prove $8|(m^k - n^k)$.

   Answer:

   Since $4|k$, then $m^k - n^k = (m + k)(m - k)(m^2 + n^2)$……..

   Since m and n are odd numbers, then $m + k$, $m - k$, and $m^2 + n^2$ are even numbers, therefore, $8|(m^k - n^k)$.