

7th Grade Competition

21 October 2007

1. A student has to compile 250 questions for a math competition. Since she is a procrastinator, she hasn't started yet. If she has 10 days left, how many questions must she write per day to complete the competition in time?
2. What is the area of a triangle with base 7 and height 4?
3. Let $v(x)$ be defined as $5x + 20$ for every real number x . What is the value of $(v(3) + v(5))$?
4. Find $|4 - 7| + |7 - 4|$
5. An extremely thirsty fish can drink one cubic yard of water in exactly 30 minutes. How many minutes would it take the fish to drink 36 cubic feet of water?
6. $\frac{3+6+9+12+\dots+291+294}{4+8+12+16+\dots+388+392} = ?$
7. Peter can mow a lawn that measures 600 square yards in 1.5 hours. At this rate, how many minutes would it take him to mow a lawn that measures 600 square feet?
8. Suppose that it takes Chirag on average 45 minutes to paint a fence. It takes Trent on average 30 minutes to paint the same fence. If Chirag and Trent worked together to paint the fence, how many minutes would it take them to finish?
9. Brian has 100 feet of fencing. He will use the fencing to enclose a rectangular play area for his puppy. What is the maximum number of square feet he can enclose?
10. Find the sum of the numbers from 1 to 15 inclusive.
11. Riding their bicycles, Alex and Brian leave from two different places at the same time and ride directly toward each other. Alex rides at 10 mi/h and Brian rides at 8 mi/h. If they meet after 40 minutes of riding, how far away were they at the beginning?

12. It takes 20 of Beowulf's men to beat a dragon in battle. If Beowulf has 170 men, how many dragons could they beat in battle?
13. If 55% of a number is 935, what is 70% of that number?
14. There are 9 teams in a school district competition. Each team plays each other team once. What is the total number of games played in the competition?
15. Blocks of molding clay are 9 inches by 6 inches by 3 inches. How many whole blocks are needed to mold a cylindrical sculpture 13 inches high and 6 inches in diameter? Use $\pi \approx 3.14$.
16. A club found that it could achieve a ratio of 2 adult members for every minor member either by inducting 24 adults or by expelling x minors. Find x .
17. After reading Shakespeare, Rachel decides to speak in iambic pentameter. This means that she speaks in lines with 5 sets of 2 syllables each. Ben rudely interrupts her after she has only spoken 13 lines and 70% of her 14th line. How many syllables has she said?
18. Two passenger trains traveling in opposite directions meet and pass each other. Each train is $\frac{1}{24}$ miles long and is traveling at 50 miles per hour. How many seconds after the front parts of the trains meet will their rear parts pass each other?
19. What is the 100th number in the arithmetic sequence 1, 4, 7, 10, 13, ...?
20. The sum of 4 consecutive odd numbers is 216. Find the smallest of these numbers.
21. Suppose that Michelle wishes to buy a laptop. The HP laptop costs \$1500 with an immediate 10% discount. Michelle can also later cash a \$100 rebate for the HP laptop. On the other hand, the Toshiba laptop costs \$1600 with a 25% discount, but no rebate. Assume that Michelle lives in a sales-tax-free nation. What is the price of the cheaper laptop?
22. What is the least number of people you could have in a group and still be guaranteed that at least 7 of them have birthdays in the same month?
23. Given that $5x + 7y = 9$ and $7x + 5y = 63$, what is the value of $x + y$?
24. The girls in physical education class sat around a large circle and spaced themselves evenly. To form teams, the instructor asked them to count off 1, 2, 3, 4, ... When they were finished counting, the girl who was 21st was sitting directly across from the girl who was 7th. How many girls were sitting around the circle?

25. If $7 \leq x \leq 12$, what is the value of $||x - 3| + |x - 24||$?
26. It takes Carrie 40 minutes to walk between her home and her school. One morning she walked half way to school and remembered that she had left her calculator at home. She ran home. It took 5 minutes to find her calculator when she got home. Then she ran all the way to school. She runs twice as fast as she walks. How many minutes more than usual did it take for her to get to school?
27. At age 8, Christine decided to start saving money from her allowance. She saved \$2 a month the first year, \$3 a month the second year, \$4 a month the third year, etc. She is turning 18 today. How much money has Christine saved so far?
28. On hypotenuse \overline{AB} of right triangle $\triangle ABC$, D is the point for which $\overline{CB} = \overline{BD}$. If $m\angle B = 40$ degrees, find $m\angle ACD$.
29. Marina selects 2 numbers at random from 1 to 8. If she can choose the same number twice, what is the probability that the sum of two numbers selected is 5?
30. The lengths of the sides of a triangle are 3, 4, and 6. What is the least possible perimeter of a similar triangle, one of whose sides has a length of 12?
31. A point P is chosen inside the square $ABCD$. What is the probability that the angle APB is obtuse? Express your answer as a decimal to the nearest hundredths. Use $\pi = 3.14$
32. Suppose the prime factorization of $6!$ is $2^a \times 3^b \times 5^c$. What is $a + b + c$?
33. Each person in Lauren's math class shakes hands with each of the others exactly once, and 120 handshakes are exchanged altogether. How many people are in her class?
34. Given that $81^m = 3$ and $m^n = 16$, evaluate mn .
35. Define a 3-digit number n to be a *multiplenumber* if the hundreds digit is the product of the tens and ones digit. The number 632 is a *multiplenumber* since $6 = 3 \cdot 2$. How many three-digit multiple numbers less than 500 exist?
36. Yoonjoo wants to sell all of her 60 pencils in combinations of 5 or 3 or both. In how many ways can the pencils be grouped?
37. Ian has a container holding 60 quarts of mixtures of 30% NaCl and 70% H₂O. Ian has a second container holding 40 quarts of mixtures of 50% NaCl and 50% H₂O. If he mixes them, what percent, to the nearest whole percent, the mixture will be NaCl?

38. On planet Holbrook, there are as many days in a week as there are weeks in a month. The number of months in a Holbrook year is twice the number of days in a month. If there are 1250 days in a Holbrook year, how many months are there?
39. The obtuse angle of an isosceles triangle is bisected and each resulting angle is 59 degrees larger than a base angle. How many degrees are in the measure of the obtuse angle?
40. Let ABCD be a square of side length 12 units. Let E, F, G and H be the midpoints of AB, BC, CD and DA respectively. What is the number of square units in the area of the triangle BHG?
41. An integer is represented by a two-digit base 10 numeral. If three times the sum of its digits is added to the integer, the result is the original integer with its digits reversed. For example, $12 + 3 * (1 + 2) = 21$. Including 12, how many such positive integers exist?

42. Find all real values x that satisfy

$$\frac{x^3 - x^2 - x + 1}{x^3 - x^2 + x - 1} = 0$$

43. Let $f(x) = x^2, g(x) = 2x, \phi(x) = f(g(x)) - g(f(x))$. What is $\phi(10)$?
44. Two standard dice are rolled. What is the probability that the product of the numbers on the top faces will be greater than 6? Express your answer as a common fraction in lowest terms.
45. What is the smallest integer that 4,320,000 must be multiplied by to get a number with exactly eight terminating zeroes?
46. What is the probability that two consecutive positive integers $x < y \leq 100$ satisfy $x \times y \leq 100$?
47. A chord of the larger circle of two concentric circles is tangent to the smaller (inner) circle and measures 14 inches. The number of square inches in the area outside the smaller circle and inside the larger circle can be expressed as $x\pi$. Find x .
48. How many ways can the letters of the word *NUMBER* be scrambled so that the first and the last letters are both vowels?
49. Ten tennis players participate in a tournament in which each player plays every other player exactly once. After each match, the two players shake hands. Then, both players shake hands with the umpire. After all of the matches, how many handshakes have been exchanged?
50. If $a > 0$ and if $(x + 1)(x + 2)(x + 3)(x + 4) + 1 = (ax^2 + bx + c)^2$, find the ordered triple (a, b, c) .