摘要

非线性特征值问题由于其重要的理论价值以及实际价值而广受自然科学和工程技术领域的专家学者关注。本文主要研究一类源自玻色-爱因斯坦凝聚态的非线性特征值问题，内容包括算法设计、收敛性分析、扰动性分析以及数值算例等。

Kolda和 Mayo在 Kofidis和 Regalia所提出的对称高阶幂法的基础上，通过添加平移项，提出了平移对称高阶幂法。受到 Kolda和 Mayo的工作启发，我们将平移对称高阶幂法由张量的特征值求解推广到一类源自玻色-爱因斯坦凝聚态的非线性特征值问题，并通过不动点分析区分了使用平移对称高阶幂法可以求得的特征对类型。

关于非线性特征值问题的平移对称高阶幂法的收敛性分析分成三个部分。首先，在非线性特征值问题的实特征向量个数有限的假设下，通过对平移项的相关分析，并利用凸函数相关性质，证明了采用平移对称高阶幂法求解非线性特征值问题将会得到收敛的结果。其次，通过非线性特征值问题相对应的约束优化问题转化为无约束优化问题，从而建立一个与平移对称高阶幂法产生的迭代序列相关联的新序列；通过 Kurdyka-Łojasiewicz不等式等工具证明了新序列的点列收敛性，进一步也就说明了平移对称高阶幂法的点列收敛性。最后，将多项式 Kurdyka-Łojasiewicz指数界的相关结果应用到非线性特征值问题相应优化问题的 Lagrange函数上，得到了平移对称高阶幂法的次线性收敛速度，从理论上解释了算法的效率。

关于扰动分析的情形，即考虑在非线性特征值问题的核心数据张量和矩阵发生扰动时，相应的非线性特征值会发生多大变化。具体地，即在张量和矩阵中，加入相应的正则扰动或是随机扰动；然后利用球面约束的条件，对扰动的非线性特征值问题对应的优化问题进行齐次化和张量的对称化；最后利用最大（或最小）特征值与对应特征向量的关系，推导出特征值的扰动量与张量、矩阵扰动之间的数量关系。

关于数值实验，计算了玻色-爱因斯坦凝聚态问题的基态解，即其最小特征值；而对于由其衍生出来的一般非线性特征值问题，则计算了相应的最大最小

特征值。此外，对于上面所述两种情形，从数值上刻画了其中发生张量或矩阵等数据扰动时，特征值会受到什么样的影响。总体而言，平移对称高阶幂法对于本文所考虑的非线性特征值问题来说，是一种稳定的数值方法。

关键词： 玻色-爱因斯坦凝聚态；非线性特征值；平移对称高阶幂法；收敛率；扰动分析.

Abstract

Nonlinear eigenvalue problems(NEPs) are widely concerned by the experts and scholars in the fields of natural science and engineering technology for its theoretical significance and application value. In this paper, we study a class of NEPs originated from Bose-Einstein condensate. The contents include algorithm design, convergence analysis, disturbance analysis and numerical examples.

Based on the work of Kofidis and Regalia, Kolda and Mayo put forward the shifted symmetric higher-order power method(SS-HOPM) by adding a shift term. Inspired by the work of Kolda and Mayo, we extend the SS-HOPM from the eigenvalue problems for tensors to the NEPs originated from Bose-Einstein condensate. Through the fixed point analysis, the types of eigenpairs that can be obtained by SS-HOPM are distinguished.

The convergence analysis of the SS-HOPM for NEPs we considered is divided into three parts. Firstly, under the assumption that the number of real eigenvectors of NEPs is limited, through analysis on the shift term, and by using the properties of convex function, we can prove the convergence of SS-HOPM for NEPs in this paper. Secondly, by transforming the constrained optimization problem corresponding to the NEP into unconstrained optimization problem, a new sequence associated with the iterative sequence of the corresponding SS-HOPM is established. Then, by means of Kurdyka-Ł ojasiewicz inequality and other tools, the convergence of the new sequence is proved, which further explains the convergence of the SS-HOPM. Finally, We apply the bound of the Kurdyka-Łojasiewicz exponent of polynomial to the Lagrange function of the optimization problem involved in this paper, then we obtain sublinear convergence rate of the SS-HOPM, which can explain the calculation efficiency of the algorithm theoretically.

We carry on the disturbance analysis on the eigenvalue of the NEPs when the core data is disturbed. Specifically, we add the regular or random disturbance to the tensor and matrix. After homogenizing and symmetrizing the disturbed NEPs with the spherical constraint conditions, we derive the quantitative relationship between the perturbations of eigenvalue and tensor or matrix with the relationship between the maximum (or minimum) eigenvalue and its corresponding eigenvector.

We calculate the ground state solution of Bose-Einstein condensate problem and the maximum and minimum eigenvalues of the general NEPs in the numerical experiment. For the two cases above, we describe the influence on the eigenvalue when the disturbance of tensor or matrix occurs. Generally speaking, the SS-HOPM is a stable numerical method for the NEPs considered in this paper.

Key Words: Bose-Einstein condensation; nonlinear eigenvalue; shifted symmetric higher-order power method; convergence rate; perturbation analysis.

目录