中文摘要

本文研究了几类结构张量的理论性质及其张量互补问题，主要讨论了非负Q-张量的张量互补问题解集的具体上下界；Cauchy-Hankel张量特征值的上界；矩形Z-张量、矩形P-张量的性质及相应互补问题解的存在情况。并运用相关算法来计算张量的特征值。论文结构如下：

首先，对由张量互补问题可解性定义的Q-张量给出一些新的结果。对于这类结构张量，我们给出了充分条件来保证其相应的张量互补性问题的非零解至少包含两个非零分量，并讨论了它与其它结构张量之间的关系。此外，关于非负Q-张量的张量互补问题，我们得到了其解集的上下界，并且证明了该张量的特征值与此解集密切相关。

其次，我们给出了有限维和无限维Cauchy-Hankel张量特征值的上下界，并证明了由 m 阶无限维Cauchy-Hankel张量定义的算子是一个从 l^1 到 $l^p(1 < p < \infty)$ 的有界、正 $(m-1)$-齐次算子。同时给出了相对应的两个正齐次算子范数的上界。此外，对于四阶实部分对称Cauchy-Hankel张量，我们得到了M-正定性的充分必要条件，并且给出了M-特征值的上界，通过数值实验可以看出，该上界与由幂法计算出的上界很接近。

最后，我们讨论了矩形Z-张量的性质，证明了一个矩形Z-张量是一个矩形M-张量当且仅当它的 V^+-奇异值都是非负的，并证明了矩形M-张量的最大对角元素是非负的。此外，关于矩形强M-张量的结果也相应地得到。同时，我们证明了一个偶数阶严格对角占优矩形张量是一个矩形P-张量，并且证明了矩形P-张量的矩形张量互补问题对于任何正向量都只有零解。此外，给出了非负矩形张量的矩形张量互补问题无解的充分条件。最后，通过数值实验来说明提出算法的有效性。

关键词： 结构张量；张量互补问题；特征值；上下界；范数；M-正定

Abstract

This thesis studies the theoretical properties of several structured tensors and complementarity problem, mainly discusses the specific upper and lower bounds of solution set of tensor complementarity problem for nonnegative Q-tensors; the upper bounds of the eigenvalues of Cauchy-Hankel tensors; the properties of rectangular Z-tensors and rectangular P-tensors, and the existence of solutions for the corresponding complementarity problems. And calculate the eigenvalues of tensors by related algorithms. The thesis is organized as follows.

First, we present some new results on Q-tensors, which are defined by the solvability of the corresponding tensor complementarity problem. For such structured tensors, we give a sufficient condition to guarantee the nonzero solution of the corresponding tensor complementarity problem with a vector containing at least two nonzero components, and discuss their relationships with some other structured tensors. Furthermore, with respect to the tensor complementarity problem with a non-negative Q-tensor, we obtain the upper and lower bounds of its solution set, and by the way, we show that the eigenvalues of such a tensor are closely related to this solution set.

Next, we present upper bounds of eigenvalues for finite and infinite dimensional Cauchy-Hankel tensors. It is proved that an m-order infinite dimensional Cauchy-Hankel tensor defines a bounded and positively $(m-1)$-homogeneous operator from l^1 into $l^p (1 < p < \infty)$, and two upper bounds of corresponding positively homogeneous operator norms are given. Moreover, for a fourth-order real partially symmetric Cauchy-Hankel tensor, sufficient and necessary conditions of M-positive definiteness are obtained, and an upper bound of M-eigenvalue is also given. And numerical experiments show that the upper bound is very close to that calculated by power method.

Last, we discuss some properties of rectangular Z-tensors. It is proved that a rectangular Z-tensor is a rectangular M-tensor if and only if all of its V^+-singular value are nonnegative, we present that the maximal diagonal element of rectangular M-tensors is nonnegative. Moreover, some corresponding results of strong rectangular M-tensors are also obtained. In addition, we prove that an even order strictly diagonally dominated rectangular tensor is a rectangular P-tensor, and also establish that rectangular tensor complementarity problem, corresponding to a rectangular P-tensor, has only zero vector solution for any positive vectors. Besides, some sufficient conditions are given for the rectangular tensor complementarity problem with nonnegative rectangular tensors to have no solution. At last, numerical experiments are showed to test the efficiency of the proposed algorithm.

Key Words: Structured tensor; Tensor complementarity problems; Eigenvalues; Upper and lower bounds; Norm; M-positive definite

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