

## 摘要

随着互联网技术的飞速发展，图像信息在人们的生活中越来越重要。图像的传输与存储过程中涉及到图像的安全问题以及图像对空间的占用问题，因此需要对图像进行加密和压缩。传统的图像加密方法大多是基于光学的加密。后来有学者提出了矩阵奇异值分解（SVD）与光学加密相结合的加密方法，将矩阵知识应用到了图像加密之中。但由于光学加密计算较复杂，所以上述方法计算效率不够高。本文从张量的角度，提出了基于张量 TTr1SVD（Tensor-Train rank-1 SVD）的图像加密方法。其基本思想是将图像数据表示为张量，分解为有限项秩-1 正交因子的线性组合，并将分解后的因子作为密文进行传输或存储。该方法的密钥空间较大，密文对重组顺序敏感，计算效率高于矩阵 SVD 和光学加密相结合的方法。矩阵 SVD 在灰度图像压缩中也有所应用，但其在彩色图像压缩中的应用性有限。本文利用彩色图像自然的张量特性，提出了基于张量 TTr1SVD 的图像压缩方法。此方法得到的压缩图像与原图像的近似误差是可度量的，而且可以根据不同的图像峰值信噪比（PSNR）得到不同程度的压缩图像。通过数值实验对上述方法进行了实现并验证了基于张量 TTr1SVD 的图像加密和图像压缩方法的可行性。

**关键词：** 张量分解；TTr1SVD；图像加密；图像压缩

## Abstract

With the rapid development of Internet technology, image information is more and more important in people's life. In the process of image transmission and storage, the image security and image space occupation problems are involved, so it is necessary to encrypt and compress the image. Most of the traditional image encryption methods are based on optical encryption. Some scholars put forward the encryption method combining matrix singular value decomposition (SVD) and optical encryption methods, they applied the knowledge of matrix to image encryption. Since the calculation of optical encryption is relatively complicated, the calculation efficiency of the above method is not high enough. From a tensor perspective, this paper proposes an image encryption method based on TTr1SVD (Tensor-Train rank-1 SVD). The main idea is to represent the image data as a tensor, and then decompose the tensor into a linear combination of a finite number of rank-1 orthogonal factors, and transfer or store the decomposed factors as ciphertext. The key space of this method is large, the ciphertext is sensitive to the reorganization order, and the calculation efficiency is higher than the method of the combination of matrix SVD and optical encryption. Matrix SVD is also used in gray image compression, but its application in color image compression is limited. In this paper, a method of image compression based on TTr1SVD is proposed by using the natural tensor property of color image. The approximate error between the compressed image and the original image is measurable, and the compressed image can be obtained in different degrees according to different PSNR. The feasibility of image encryption and compression based on TTr1SVD is verified by numerical experiments.

**Key Words:** Tensor Decomposition; TTr1SVD; Image Encryption; Image Compression

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